

Computer algebra independent integration tests

6-Hyperbolic-functions/6.1-Hyperbolic-sine/6.1.7-hyper^m-a+b-sinhⁿ-^p

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Contents

1	Introduction	23
1.1	Listing of CAS systems tested	23
1.2	Results	24
1.3	Performance	27
1.4	list of integrals that has no closed form antiderivative	28
1.5	list of integrals solved by CAS but has no known antiderivative	28
1.6	list of integrals solved by CAS but failed verification	28
1.7	Timing	29
1.8	Verification	29
1.9	Important notes about some of the results	29
1.10	Design of the test system	31
2	detailed summary tables of results	33
2.1	List of integrals sorted by grade for each CAS	33
2.2	Detailed conclusion table per each integral for all CAS systems	38
2.3	Detailed conclusion table specific for Rubi results	143
3	Listing of integrals	161
3.1	$\int \sinh^4(c + dx) (a + b \sinh^2(c + dx)) dx$	161
3.2	$\int \sinh^3(c + dx) (a + b \sinh^2(c + dx)) dx$	165
3.3	$\int \sinh^2(c + dx) (a + b \sinh^2(c + dx)) dx$	169
3.4	$\int \sinh(c + dx) (a + b \sinh^2(c + dx)) dx$	173

3.5	$\int (a + b \sinh^2(c + dx)) dx$	176
3.6	$\int \operatorname{csch}(c + dx) (a + b \sinh^2(c + dx)) dx$	179
3.7	$\int \operatorname{csch}^2(c + dx) (a + b \sinh^2(c + dx)) dx$	182
3.8	$\int \operatorname{csch}^3(c + dx) (a + b \sinh^2(c + dx)) dx$	185
3.9	$\int \operatorname{csch}^4(c + dx) (a + b \sinh^2(c + dx)) dx$	189
3.10	$\int \sinh^4(c + dx) (a + b \sinh^2(c + dx))^2 dx$	193
3.11	$\int \sinh^3(c + dx) (a + b \sinh^2(c + dx))^2 dx$	199
3.12	$\int \sinh^2(c + dx) (a + b \sinh^2(c + dx))^2 dx$	203
3.13	$\int \sinh(c + dx) (a + b \sinh^2(c + dx))^2 dx$	207
3.14	$\int (a + b \sinh^2(c + dx))^2 dx$	211
3.15	$\int \operatorname{csch}(c + dx) (a + b \sinh^2(c + dx))^2 dx$	214
3.16	$\int \operatorname{csch}^2(c + dx) (a + b \sinh^2(c + dx))^2 dx$	218
3.17	$\int \operatorname{csch}^3(c + dx) (a + b \sinh^2(c + dx))^2 dx$	222
3.18	$\int \operatorname{csch}^4(c + dx) (a + b \sinh^2(c + dx))^2 dx$	227
3.19	$\int \sinh^4(c + dx) (a + b \sinh^2(c + dx))^3 dx$	231
3.20	$\int \sinh^3(c + dx) (a + b \sinh^2(c + dx))^3 dx$	237
3.21	$\int \sinh^2(c + dx) (a + b \sinh^2(c + dx))^3 dx$	241
3.22	$\int \sinh(c + dx) (a + b \sinh^2(c + dx))^3 dx$	246
3.23	$\int (a + b \sinh^2(c + dx))^3 dx$	250
3.24	$\int \operatorname{csch}(c + dx) (a + b \sinh^2(c + dx))^3 dx$	254
3.25	$\int \operatorname{csch}^2(c + dx) (a + b \sinh^2(c + dx))^3 dx$	259
3.26	$\int \operatorname{csch}^3(c + dx) (a + b \sinh^2(c + dx))^3 dx$	264
3.27	$\int \operatorname{csch}^4(c + dx) (a + b \sinh^2(c + dx))^3 dx$	269
3.28	$\int \frac{\sinh^7(c+dx)}{a+b \sinh^2(c+dx)} dx$	273
3.29	$\int \frac{\sinh^6(c+dx)}{a+b \sinh^2(c+dx)} dx$	279
3.30	$\int \frac{\sinh^5(c+dx)}{a+b \sinh^2(c+dx)} dx$	285
3.31	$\int \frac{\sinh^4(c+dx)}{a+b \sinh^2(c+dx)} dx$	290
3.32	$\int \frac{\sinh^3(c+dx)}{a+b \sinh^2(c+dx)} dx$	295
3.33	$\int \frac{\sinh^2(c+dx)}{a+b \sinh^2(c+dx)} dx$	299
3.34	$\int \frac{\sinh(c+dx)}{a+b \sinh^2(c+dx)} dx$	303

3.35	$\int \frac{1}{a+b \sinh^2(c+dx)} dx$	307
3.36	$\int \frac{\operatorname{csch}(c+dx)}{a+b \sinh^2(c+dx)} dx$	313
3.37	$\int \frac{\operatorname{csch}^2(c+dx)}{a+b \sinh^2(c+dx)} dx$	317
3.38	$\int \frac{\operatorname{csch}^3(c+dx)}{a+b \sinh^2(c+dx)} dx$	321
3.39	$\int \frac{\operatorname{csch}^4(c+dx)}{a+b \sinh^2(c+dx)} dx$	326
3.40	$\int \frac{\operatorname{csch}^5(c+dx)}{a+b \sinh^2(c+dx)} dx$	331
3.41	$\int \frac{\operatorname{csch}^6(c+dx)}{a+b \sinh^2(c+dx)} dx$	339
3.42	$\int \frac{\sinh^4(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$	345
3.43	$\int \frac{\sinh^3(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$	351
3.44	$\int \frac{\sinh^2(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$	356
3.45	$\int \frac{\sinh(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$	361
3.46	$\int \frac{1}{(a+b \sinh^2(c+dx))^2} dx$	366
3.47	$\int \frac{\operatorname{csch}(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$	371
3.48	$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$	377
3.49	$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$	383
3.50	$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$	393
3.51	$\int \frac{\sinh^4(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$	401
3.52	$\int \frac{\sinh^3(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$	408
3.53	$\int \frac{\sinh^2(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$	416
3.54	$\int \frac{\sinh(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$	424
3.55	$\int \frac{1}{(a+b \sinh^2(c+dx))^3} dx$	431
3.56	$\int \frac{\operatorname{csch}(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$	439
3.57	$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$	450

3.58	$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$	461
3.59	$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$	467
3.60	$\int \frac{1}{1+\sinh^2(x)} dx$	473
3.61	$\int \frac{1}{(1+\sinh^2(x))^2} dx$	476
3.62	$\int \frac{1}{(1+\sinh^2(x))^3} dx$	480
3.63	$\int \frac{1}{1-\sinh^2(x)} dx$	484
3.64	$\int \frac{1}{(1-\sinh^2(x))^2} dx$	488
3.65	$\int \frac{1}{(1-\sinh^2(x))^3} dx$	492
3.66	$\int \sinh^3(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	497
3.67	$\int \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	503
3.68	$\int \operatorname{csch}(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	508
3.69	$\int \operatorname{csch}^3(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	515
3.70	$\int \operatorname{csch}^5(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	520
3.71	$\int \sinh^4(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	528
3.72	$\int \sinh^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	533
3.73	$\int \sqrt{a+b \sinh^2(e+fx)} dx$	538
3.74	$\int \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	542
3.75	$\int \operatorname{csch}^4(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	547
3.76	$\int \sinh^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	552
3.77	$\int \sinh(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	559
3.78	$\int \operatorname{csch}(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	565
3.79	$\int \operatorname{csch}^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	573
3.80	$\int \operatorname{csch}^5(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	582
3.81	$\int \operatorname{csch}^7(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	588
3.82	$\int \sinh^4(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	597
3.83	$\int \sinh^2(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	603
3.84	$\int (a+b \sinh^2(e+fx))^{3/2} dx$	608
3.85	$\int \operatorname{csch}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	613

3.86	$\int \operatorname{csch}^4(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	618
3.87	$\int (a+b \sinh^2(c+dx))^{5/2} dx$	623
3.88	$\int \sqrt{1+\sinh^2(x)} dx$	628
3.89	$\int \sqrt{-1-\sinh^2(x)} dx$	631
3.90	$\int \sqrt{1-\sinh^2(x)} dx$	634
3.91	$\int \sqrt{-1+\sinh^2(x)} dx$	637
3.92	$\int \sqrt{a+b \sinh^2(x)} dx$	640
3.93	$\int (1+\sinh^2(x))^{3/2} dx$	644
3.94	$\int (-1-\sinh^2(x))^{3/2} dx$	648
3.95	$\int (1-\sinh^2(x))^{3/2} dx$	652
3.96	$\int (-1+\sinh^2(x))^{3/2} dx$	656
3.97	$\int (a+b \sinh^2(x))^{3/2} dx$	660
3.98	$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	665
3.99	$\int \frac{\sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	670
3.100	$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	675
3.101	$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	679
3.102	$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	684
3.103	$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	689
3.104	$\int \frac{1}{\sqrt{a+b \sinh^2(e+fx)}} dx$	693
3.105	$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	697
3.106	$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	702
3.107	$\int \frac{\sinh^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$	707
3.108	$\int \frac{\sinh(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$	713
3.109	$\int \frac{\operatorname{csch}(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$	717
3.110	$\int \frac{\operatorname{csch}^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$	722

3.111	$\int \frac{\sinh^6(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx \dots\dots\dots$	729
3.112	$\int \frac{\sinh^4(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx \dots\dots\dots$	735
3.113	$\int \frac{\sinh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx \dots\dots\dots$	740
3.114	$\int \frac{1}{(a+b \sinh^2(e+fx))^{3/2}} dx \dots\dots\dots$	745
3.115	$\int \frac{\operatorname{csch}^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx \dots\dots\dots$	749
3.116	$\int \frac{\sinh^5(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx \dots\dots\dots$	755
3.117	$\int \frac{\sinh^3(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx \dots\dots\dots$	764
3.118	$\int \frac{\sinh(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx \dots\dots\dots$	769
3.119	$\int \frac{\operatorname{csch}(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx \dots\dots\dots$	774
3.120	$\int \frac{\sinh^6(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx \dots\dots\dots$	782
3.121	$\int \frac{\sinh^4(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx \dots\dots\dots$	788
3.122	$\int \frac{\sinh^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx \dots\dots\dots$	793
3.123	$\int \frac{1}{(a+b \sinh^2(e+fx))^{5/2}} dx \dots\dots\dots$	798
3.124	$\int \frac{\operatorname{csch}^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx \dots\dots\dots$	803
3.125	$\int \frac{1}{\sqrt{1+\sinh^2(x)}} dx \dots\dots\dots$	810
3.126	$\int \frac{1}{\sqrt{1-\sinh^2(x)}} dx \dots\dots\dots$	814
3.127	$\int \frac{1}{\sqrt{-1+\sinh^2(x)}} dx \dots\dots\dots$	817
3.128	$\int \frac{1}{\sqrt{-1-\sinh^2(x)}} dx \dots\dots\dots$	821
3.129	$\int \frac{1}{\sqrt{a+b \sinh^2(x)}} dx \dots\dots\dots$	825
3.130	$\int (d \sinh(e+fx))^m (a+b \sinh^2(e+fx))^p dx \dots\dots\dots$	829
3.131	$\int \sinh^5(e+fx) (a+b \sinh^2(e+fx))^p dx \dots\dots\dots$	833
3.132	$\int \sinh^3(e+fx) (a+b \sinh^2(e+fx))^p dx \dots\dots\dots$	838
3.133	$\int \sinh(e+fx) (a+b \sinh^2(e+fx))^p dx \dots\dots\dots$	842
3.134	$\int \operatorname{csch}(e+fx) (a+b \sinh^2(e+fx))^p dx \dots\dots\dots$	846

3.135	$\int \operatorname{csch}^3(e+fx)(a+b\sinh^2(e+fx))^p dx$	850
3.136	$\int \operatorname{csch}^5(e+fx)(a+b\sinh^2(e+fx))^p dx$	854
3.137	$\int \sinh^4(e+fx)(a+b\sinh^2(e+fx))^p dx$	858
3.138	$\int \sinh^2(e+fx)(a+b\sinh^2(e+fx))^p dx$	862
3.139	$\int \operatorname{csch}^2(e+fx)(a+b\sinh^2(e+fx))^p dx$	866
3.140	$\int \operatorname{csch}^4(e+fx)(a+b\sinh^2(e+fx))^p dx$	870
3.141	$\int \sinh^4(c+dx)(a+b\sinh^3(c+dx)) dx$	874
3.142	$\int \sinh^3(c+dx)(a+b\sinh^3(c+dx)) dx$	878
3.143	$\int \sinh^2(c+dx)(a+b\sinh^3(c+dx)) dx$	882
3.144	$\int \sinh(c+dx)(a+b\sinh^3(c+dx)) dx$	886
3.145	$\int (a+b\sinh^3(c+dx)) dx$	890
3.146	$\int \operatorname{csch}(c+dx)(a+b\sinh^3(c+dx)) dx$	893
3.147	$\int \operatorname{csch}^2(c+dx)(a+b\sinh^3(c+dx)) dx$	897
3.148	$\int \operatorname{csch}^3(c+dx)(a+b\sinh^3(c+dx)) dx$	901
3.149	$\int \operatorname{csch}^4(c+dx)(a+b\sinh^3(c+dx)) dx$	905
3.150	$\int \sinh^3(c+dx)(a+b\sinh^3(c+dx))^2 dx$	909
3.151	$\int \sinh^2(c+dx)(a+b\sinh^3(c+dx))^2 dx$	914
3.152	$\int \sinh(c+dx)(a+b\sinh^3(c+dx))^2 dx$	919
3.153	$\int (a+b\sinh^3(c+dx))^2 dx$	923
3.154	$\int \operatorname{csch}(c+dx)(a+b\sinh^3(c+dx))^2 dx$	927
3.155	$\int \operatorname{csch}^2(c+dx)(a+b\sinh^3(c+dx))^2 dx$	932
3.156	$\int \operatorname{csch}^3(c+dx)(a+b\sinh^3(c+dx))^2 dx$	936
3.157	$\int \operatorname{csch}^4(c+dx)(a+b\sinh^3(c+dx))^2 dx$	941
3.158	$\int \operatorname{csch}^5(c+dx)(a+b\sinh^3(c+dx))^2 dx$	946
3.159	$\int \operatorname{csch}^6(c+dx)(a+b\sinh^3(c+dx))^2 dx$	951
3.160	$\int \operatorname{csch}^7(c+dx)(a+b\sinh^3(c+dx))^2 dx$	956
3.161	$\int \sinh^2(c+dx)(a+b\sinh^3(c+dx))^3 dx$	962
3.162	$\int \sinh(c+dx)(a+b\sinh^3(c+dx))^3 dx$	967
3.163	$\int (a+b\sinh^3(c+dx))^3 dx$	972
3.164	$\int \operatorname{csch}(c+dx)(a+b\sinh^3(c+dx))^3 dx$	977
3.165	$\int \operatorname{csch}^2(c+dx)(a+b\sinh^3(c+dx))^3 dx$	983
3.166	$\int \operatorname{csch}^3(c+dx)(a+b\sinh^3(c+dx))^3 dx$	988
3.167	$\int \operatorname{csch}^4(c+dx)(a+b\sinh^3(c+dx))^3 dx$	995

3.168	$\int \operatorname{csch}^5(c+dx) (a+b \sinh^3(c+dx))^3 dx$.1002
3.169	$\int \operatorname{csch}^6(c+dx) (a+b \sinh^3(c+dx))^3 dx$.1009
3.170	$\int \operatorname{csch}^7(c+dx) (a+b \sinh^3(c+dx))^3 dx$.1016
3.171	$\int \frac{\sinh^6(c+dx)}{a+b \sinh^3(c+dx)} dx$.1024
3.172	$\int \frac{\sinh^5(c+dx)}{a+b \sinh^3(c+dx)} dx$.1029
3.173	$\int \frac{\sinh^4(c+dx)}{a+b \sinh^3(c+dx)} dx$.1034
3.174	$\int \frac{\sinh^3(c+dx)}{a+b \sinh^3(c+dx)} dx$.1039
3.175	$\int \frac{\sinh^2(c+dx)}{a+b \sinh^3(c+dx)} dx$.1044
3.176	$\int \frac{\sinh(c+dx)}{a+b \sinh^3(c+dx)} dx$.1049
3.177	$\int \frac{1}{a+b \sinh^3(c+dx)} dx$.1054
3.178	$\int \frac{\operatorname{csch}(c+dx)}{a+b \sinh^3(c+dx)} dx$.1058
3.179	$\int \frac{\operatorname{csch}^2(c+dx)}{a+b \sinh^3(c+dx)} dx$.1063
3.180	$\int \frac{\operatorname{csch}^3(c+dx)}{a+b \sinh^3(c+dx)} dx$.1068
3.181	$\int \frac{\operatorname{csch}^4(c+dx)}{a+b \sinh^3(c+dx)} dx$.1073
3.182	$\int \frac{1}{1+\sinh^3(x)} dx$.1078
3.183	$\int \frac{1}{1-\sinh^3(x)} dx$.1084
3.184	$\int \sinh^4(c+dx) (a+b \sinh^4(c+dx)) dx$.1093
3.185	$\int \sinh^3(c+dx) (a+b \sinh^4(c+dx)) dx$.1098
3.186	$\int \sinh^2(c+dx) (a+b \sinh^4(c+dx)) dx$.1102
3.187	$\int \sinh(c+dx) (a+b \sinh^4(c+dx)) dx$.1107
3.188	$\int (a+b \sinh^4(c+dx)) dx$.1110
3.189	$\int \operatorname{csch}(c+dx) (a+b \sinh^4(c+dx)) dx$.1114
3.190	$\int \operatorname{csch}^2(c+dx) (a+b \sinh^4(c+dx)) dx$.1118
3.191	$\int \operatorname{csch}^3(c+dx) (a+b \sinh^4(c+dx)) dx$.1122
3.192	$\int \operatorname{csch}^4(c+dx) (a+b \sinh^4(c+dx)) dx$.1126
3.193	$\int \operatorname{csch}^5(c+dx) (a+b \sinh^4(c+dx)) dx$.1130
3.194	$\int \operatorname{csch}^6(c+dx) (a+b \sinh^4(c+dx)) dx$.1135
3.195	$\int \operatorname{csch}^7(c+dx) (a+b \sinh^4(c+dx)) dx$.1139
3.196	$\int \sinh^3(c+dx) (a+b \sinh^4(c+dx))^2 dx$.1145
3.197	$\int \sinh^2(c+dx) (a+b \sinh^4(c+dx))^2 dx$.1149
3.198	$\int \sinh(c+dx) (a+b \sinh^4(c+dx))^2 dx$.1155

3.199	$\int (a + b \sinh^4(c + dx))^2 dx$	1159
3.200	$\int \operatorname{csch}(c + dx) (a + b \sinh^4(c + dx))^2 dx$	1164
3.201	$\int \operatorname{csch}^2(c + dx) (a + b \sinh^4(c + dx))^2 dx$	1169
3.202	$\int \operatorname{csch}^3(c + dx) (a + b \sinh^4(c + dx))^2 dx$	1174
3.203	$\int \operatorname{csch}^4(c + dx) (a + b \sinh^4(c + dx))^2 dx$	1180
3.204	$\int \operatorname{csch}^5(c + dx) (a + b \sinh^4(c + dx))^2 dx$	1185
3.205	$\int \operatorname{csch}^6(c + dx) (a + b \sinh^4(c + dx))^2 dx$	1192
3.206	$\int \operatorname{csch}^7(c + dx) (a + b \sinh^4(c + dx))^2 dx$	1197
3.207	$\int \sinh^5(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1205
3.208	$\int \sinh^3(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1210
3.209	$\int \sinh(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1215
3.210	$\int \operatorname{csch}(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1220
3.211	$\int \operatorname{csch}^3(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1227
3.212	$\int \operatorname{csch}^5(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1234
3.213	$\int \operatorname{csch}^7(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1243
3.214	$\int \operatorname{csch}^9(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1253
3.215	$\int \operatorname{csch}^{11}(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1265
3.216	$\int \operatorname{csch}^{13}(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1271
3.217	$\int \sinh^2(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1277
3.218	$\int (a + b \sinh^4(c + dx))^3 dx$	1283
3.219	$\int \operatorname{csch}^2(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1289
3.220	$\int \operatorname{csch}^4(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1295
3.221	$\int \operatorname{csch}^6(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1301
3.222	$\int \operatorname{csch}^8(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1307
3.223	$\int \operatorname{csch}^{10}(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1313
3.224	$\int \operatorname{csch}^{12}(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1319
3.225	$\int \operatorname{csch}^{14}(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1325
3.226	$\int \operatorname{csch}^{16}(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1332
3.227	$\int \operatorname{csch}^{18}(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1340
3.228	$\int \operatorname{csch}^{20}(c + dx) (a + b \sinh^4(c + dx))^3 dx$	1349
3.229	$\int \frac{\sinh^7(c+dx)}{a-b \sinh^4(c+dx)} dx$	1360

3.230	$\int \frac{\sinh^5(c+dx)}{a-b \sinh^4(c+dx)} dx$1366
3.231	$\int \frac{\sinh^3(c+dx)}{a-b \sinh^4(c+dx)} dx$1371
3.232	$\int \frac{\sinh(c+dx)}{a-b \sinh^4(c+dx)} dx$1376
3.233	$\int \frac{\operatorname{csch}(c+dx)}{a-b \sinh^4(c+dx)} dx$1381
3.234	$\int \frac{\operatorname{csch}^3(c+dx)}{a-b \sinh^4(c+dx)} dx$1387
3.235	$\int \frac{\sinh^6(c+dx)}{a-b \sinh^4(c+dx)} dx$1393
3.236	$\int \frac{\sinh^4(c+dx)}{a-b \sinh^4(c+dx)} dx$1399
3.237	$\int \frac{\sinh^2(c+dx)}{a-b \sinh^4(c+dx)} dx$1404
3.238	$\int \frac{1}{a-b \sinh^4(c+dx)} dx$1408
3.239	$\int \frac{\operatorname{csch}^2(c+dx)}{a-b \sinh^4(c+dx)} dx$1412
3.240	$\int \frac{\operatorname{csch}^4(c+dx)}{a-b \sinh^4(c+dx)} dx$1417
3.241	$\int \frac{\sinh^9(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1423
3.242	$\int \frac{\sinh^7(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1433
3.243	$\int \frac{\sinh^5(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1442
3.244	$\int \frac{\sinh^3(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1451
3.245	$\int \frac{\sinh(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1459
3.246	$\int \frac{\operatorname{csch}(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1468
3.247	$\int \frac{\sinh^8(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1478
3.248	$\int \frac{\sinh^6(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1488
3.249	$\int \frac{\sinh^4(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1496
3.250	$\int \frac{\sinh^2(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1504
3.251	$\int \frac{1}{(a-b \sinh^4(c+dx))^2} dx$1513
3.252	$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$1521

3.253	$\int \frac{\sinh^9(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1531
3.254	$\int \frac{\sinh^7(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1538
3.255	$\int \frac{\sinh^5(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1545
3.256	$\int \frac{\sinh^3(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1553
3.257	$\int \frac{\sinh(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1560
3.258	$\int \frac{\operatorname{csch}(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1568
3.259	$\int \frac{\sinh^8(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1577
3.260	$\int \frac{\sinh^6(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1585
3.261	$\int \frac{\sinh^4(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1592
3.262	$\int \frac{\sinh^2(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1599
3.263	$\int \frac{1}{(a-b \sinh^4(c+dx))^3} dx$1606
3.264	$\int \frac{\operatorname{csch}^2(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$1613
3.265	$\int \frac{1}{1-\sinh^4(x)} dx$1621
3.266	$\int \frac{1}{1+\sinh^4(x)} dx$1625
3.267	$\int \frac{1}{a+b \sinh^5(x)} dx$1630
3.268	$\int \frac{1}{a+b \sinh^6(x)} dx$1635
3.269	$\int \frac{1}{a+b \sinh^8(x)} dx$1639
3.270	$\int \frac{1}{1+\sinh^5(x)} dx$1643
3.271	$\int \frac{1}{1+\sinh^6(x)} dx$1650
3.272	$\int \frac{1}{1+\sinh^8(x)} dx$1655
3.273	$\int \frac{1}{1-\sinh^5(x)} dx$1661
3.274	$\int \frac{1}{1-\sinh^6(x)} dx$1668
3.275	$\int \frac{1}{1-\sinh^8(x)} dx$1672
3.276	$\int \frac{\cosh^5(x)}{a+a \sinh^2(x)} dx$1677

3.277	$\int \frac{\cosh^4(x)}{a+a \sinh^2(x)} dx$1681
3.278	$\int \frac{\cosh^3(x)}{a+a \sinh^2(x)} dx$1685
3.279	$\int \frac{\cosh^2(x)}{a+a \sinh^2(x)} dx$1688
3.280	$\int \frac{\cosh(x)}{a+a \sinh^2(x)} dx$1691
3.281	$\int \frac{\operatorname{sech}(x)}{a+a \sinh^2(x)} dx$1694
3.282	$\int \frac{\operatorname{sech}^3(x)}{a+a \sinh^2(x)} dx$1698
3.283	$\int \cosh^4(c+dx) (a+b \sinh^2(c+dx)) dx$1702
3.284	$\int \cosh^3(c+dx) (a+b \sinh^2(c+dx)) dx$1706
3.285	$\int \cosh^2(c+dx) (a+b \sinh^2(c+dx)) dx$1710
3.286	$\int \cosh(c+dx) (a+b \sinh^2(c+dx)) dx$1714
3.287	$\int \operatorname{sech}(c+dx) (a+b \sinh^2(c+dx)) dx$1717
3.288	$\int \operatorname{sech}^2(c+dx) (a+b \sinh^2(c+dx)) dx$1721
3.289	$\int \operatorname{sech}^3(c+dx) (a+b \sinh^2(c+dx)) dx$1724
3.290	$\int \operatorname{sech}^4(c+dx) (a+b \sinh^2(c+dx)) dx$1728
3.291	$\int \operatorname{sech}^5(c+dx) (a+b \sinh^2(c+dx)) dx$1731
3.292	$\int \operatorname{sech}^6(c+dx) (a+b \sinh^2(c+dx)) dx$1736
3.293	$\int \cosh^4(c+dx) (a+b \sinh^2(c+dx))^2 dx$1740
3.294	$\int \cosh^3(c+dx) (a+b \sinh^2(c+dx))^2 dx$1746
3.295	$\int \cosh^2(c+dx) (a+b \sinh^2(c+dx))^2 dx$1750
3.296	$\int \cosh(c+dx) (a+b \sinh^2(c+dx))^2 dx$1755
3.297	$\int \operatorname{sech}(c+dx) (a+b \sinh^2(c+dx))^2 dx$1759
3.298	$\int \operatorname{sech}^2(c+dx) (a+b \sinh^2(c+dx))^2 dx$1763
3.299	$\int \operatorname{sech}^3(c+dx) (a+b \sinh^2(c+dx))^2 dx$1767
3.300	$\int \operatorname{sech}^4(c+dx) (a+b \sinh^2(c+dx))^2 dx$1772
3.301	$\int \operatorname{sech}^5(c+dx) (a+b \sinh^2(c+dx))^2 dx$1776
3.302	$\int \operatorname{sech}^6(c+dx) (a+b \sinh^2(c+dx))^2 dx$1781
3.303	$\int \operatorname{sech}^7(c+dx) (a+b \sinh^2(c+dx))^2 dx$1785
3.304	$\int \cosh^4(c+dx) (a+b \sinh^2(c+dx))^3 dx$1792
3.305	$\int \cosh^3(c+dx) (a+b \sinh^2(c+dx))^3 dx$1798
3.306	$\int \cosh^2(c+dx) (a+b \sinh^2(c+dx))^3 dx$1802
3.307	$\int \cosh(c+dx) (a+b \sinh^2(c+dx))^3 dx$1808
3.308	$\int \operatorname{sech}(c+dx) (a+b \sinh^2(c+dx))^3 dx$1812

3.309	$\int \operatorname{sech}^2(c+dx) (a+b \sinh^2(c+dx))^3 dx$.1817
3.310	$\int \operatorname{sech}^3(c+dx) (a+b \sinh^2(c+dx))^3 dx$.1822
3.311	$\int \operatorname{sech}^4(c+dx) (a+b \sinh^2(c+dx))^3 dx$.1828
3.312	$\int \operatorname{sech}^5(c+dx) (a+b \sinh^2(c+dx))^3 dx$.1832
3.313	$\int \operatorname{sech}^6(c+dx) (a+b \sinh^2(c+dx))^3 dx$.1838
3.314	$\int \operatorname{sech}^7(c+dx) (a+b \sinh^2(c+dx))^3 dx$.1843
3.315	$\int \operatorname{sech}^8(c+dx) (a+b \sinh^2(c+dx))^3 dx$.1851
3.316	$\int \frac{\cosh^7(c+dx)}{a+b \sinh^2(c+dx)} dx$.1856
3.317	$\int \frac{\cosh^6(c+dx)}{a+b \sinh^2(c+dx)} dx$.1862
3.318	$\int \frac{\cosh^5(c+dx)}{a+b \sinh^2(c+dx)} dx$.1869
3.319	$\int \frac{\cosh^4(c+dx)}{a+b \sinh^2(c+dx)} dx$.1874
3.320	$\int \frac{\cosh^3(c+dx)}{a+b \sinh^2(c+dx)} dx$.1879
3.321	$\int \frac{\cosh^2(c+dx)}{a+b \sinh^2(c+dx)} dx$.1883
3.322	$\int \frac{\cosh(c+dx)}{a+b \sinh^2(c+dx)} dx$.1887
3.323	$\int \frac{\operatorname{sech}(c+dx)}{a+b \sinh^2(c+dx)} dx$.1891
3.324	$\int \frac{\operatorname{sech}^2(c+dx)}{a+b \sinh^2(c+dx)} dx$.1896
3.325	$\int \frac{\operatorname{sech}^3(c+dx)}{a+b \sinh^2(c+dx)} dx$.1900
3.326	$\int \frac{\operatorname{sech}^4(c+dx)}{a+b \sinh^2(c+dx)} dx$.1905
3.327	$\int \frac{\operatorname{sech}^5(c+dx)}{a+b \sinh^2(c+dx)} dx$.1910
3.328	$\int \frac{\operatorname{sech}^6(c+dx)}{a+b \sinh^2(c+dx)} dx$.1918
3.329	$\int \frac{\cosh^6(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$.1925
3.330	$\int \frac{\cosh^5(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$.1933
3.331	$\int \frac{\cosh^4(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$.1939
3.332	$\int \frac{\cosh^3(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$.1945
3.333	$\int \frac{\cosh^2(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$.1950
3.334	$\int \frac{\cosh(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$.1955

3.335	$\int \frac{\operatorname{sech}(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$1960
3.336	$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$1966
3.337	$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$1972
3.338	$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$1981
3.339	$\int \frac{\cosh^6(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$1990
3.340	$\int \frac{\cosh^5(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$1999
3.341	$\int \frac{\cosh^4(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$2007
3.342	$\int \frac{\cosh^3(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$2014
3.343	$\int \frac{\cosh^2(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$2021
3.344	$\int \frac{\cosh(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$2028
3.345	$\int \frac{\operatorname{sech}(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$2035
3.346	$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$2045
3.347	$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$2055
3.348	$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$2062
3.349	$\int \frac{\cosh^2(x)}{1-\sinh^2(x)} dx$2068
3.350	$\int \frac{\cosh^3(x)}{1-\sinh^2(x)} dx$2072
3.351	$\int \frac{\cosh^4(x)}{1-\sinh^2(x)} dx$2076
3.352	$\int \cosh^3(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$2080
3.353	$\int \cosh(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$2086
3.354	$\int \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$2091
3.355	$\int \operatorname{sech}^3(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$2098
3.356	$\int \operatorname{sech}^5(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$2103
3.357	$\int \cosh^4(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$2111

3.358	$\int \cosh^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	2116
3.359	$\int \sqrt{a+b \sinh^2(e+fx)} dx$	2121
3.360	$\int \operatorname{sech}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	2125
3.361	$\int \operatorname{sech}^4(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$	2129
3.362	$\int \cosh^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2134
3.363	$\int \cosh(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2141
3.364	$\int \operatorname{sech}(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2147
3.365	$\int \operatorname{sech}^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2155
3.366	$\int \operatorname{sech}^5(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2164
3.367	$\int \operatorname{sech}^7(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2170
3.368	$\int \cosh^4(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2180
3.369	$\int \cosh^2(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2186
3.370	$\int (a+b \sinh^2(e+fx))^{3/2} dx$	2191
3.371	$\int \operatorname{sech}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2196
3.372	$\int \operatorname{sech}^4(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$	2201
3.373	$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	2206
3.374	$\int \frac{\cosh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	2211
3.375	$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	2216
3.376	$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	2220
3.377	$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	2226
3.378	$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	2231
3.379	$\int \frac{1}{\sqrt{a+b \sinh^2(e+fx)}} dx$	2236
3.380	$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	2240
3.381	$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$	2245
3.382	$\int \frac{\cosh^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$	2250
3.383	$\int \frac{\cosh(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$	2256

3.384	$\int \frac{\operatorname{sech}(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2260
3.385	$\int \frac{\operatorname{sech}^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2266
3.386	$\int \frac{\cosh^6(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2274
3.387	$\int \frac{\cosh^4(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2280
3.388	$\int \frac{\cosh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2285
3.389	$\int \frac{1}{(a+b \sinh^2(e+fx))^{3/2}} dx$2289
3.390	$\int \frac{\operatorname{sech}^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2293
3.391	$\int \frac{\cosh^5(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2298
3.392	$\int \frac{\cosh^3(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2306
3.393	$\int \frac{\cosh(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2311
3.394	$\int \frac{\operatorname{sech}(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2316
3.395	$\int \frac{\cosh^6(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2325
3.396	$\int \frac{\cosh^4(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2331
3.397	$\int \frac{\cosh^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2336
3.398	$\int \frac{1}{(a+b \sinh^2(e+fx))^{5/2}} dx$2341
3.399	$\int \frac{\operatorname{sech}^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2346
3.400	$\int (d \cosh(e+fx))^m (a+b \sinh^2(e+fx))^p dx$2352
3.401	$\int \cosh^5(e+fx) (a+b \sinh^2(e+fx))^p dx$2356
3.402	$\int \cosh^3(e+fx) (a+b \sinh^2(e+fx))^p dx$2361
3.403	$\int \cosh(e+fx) (a+b \sinh^2(e+fx))^p dx$2365
3.404	$\int \operatorname{sech}(e+fx) (a+b \sinh^2(e+fx))^p dx$2369
3.405	$\int \operatorname{sech}^3(e+fx) (a+b \sinh^2(e+fx))^p dx$2373
3.406	$\int \cosh^4(e+fx) (a+b \sinh^2(e+fx))^p dx$2377
3.407	$\int \cosh^2(e+fx) (a+b \sinh^2(e+fx))^p dx$2381
3.408	$\int (a+b \sinh^2(e+fx))^p dx$2385

3.409	$\int \operatorname{sech}^2(e + fx) (a + b \sinh^2(e + fx))^p dx$.2389
3.410	$\int \operatorname{sech}^4(e + fx) (a + b \sinh^2(e + fx))^p dx$.2393
3.411	$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}} dx$.2397
3.412	$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}} dx$.2403
3.413	$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}} dx$.2407
3.414	$\int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}} dx$.2411
3.415	$\int \frac{\cosh^5(c+dx)}{(a+b\sqrt{\sinh(c+dx)})^2} dx$.2417
3.416	$\int \frac{\cosh^3(c+dx)}{(a+b\sqrt{\sinh(c+dx)})^2} dx$.2424
3.417	$\int \frac{\cosh(c+dx)}{(a+b\sqrt{\sinh(c+dx)})^2} dx$.2429
3.418	$\int \frac{\operatorname{sech}(c+dx)}{(a+b\sqrt{\sinh(c+dx)})^2} dx$.2433
3.419	$\int \frac{\cosh^5(c+dx)}{a+b \sinh^n(c+dx)} dx$.2439
3.420	$\int \frac{\cosh^3(c+dx)}{a+b \sinh^n(c+dx)} dx$.2443
3.421	$\int \frac{\cosh(c+dx)}{a+b \sinh^n(c+dx)} dx$.2447
3.422	$\int \frac{\cosh^5(c+dx)}{(a+b \sinh^n(c+dx))^2} dx$.2450
3.423	$\int \frac{\cosh^3(c+dx)}{(a+b \sinh^n(c+dx))^2} dx$.2454
3.424	$\int \frac{\cosh(c+dx)}{(a+b \sinh^n(c+dx))^2} dx$.2458
3.425	$\int \frac{\coth(x)}{1-\sinh^2(x)} dx$.2462
3.426	$\int \sqrt{a + a \sinh^2(e + fx)} \tanh^5(e + fx) dx$.2466
3.427	$\int \sqrt{a + a \sinh^2(e + fx)} \tanh^3(e + fx) dx$.2471
3.428	$\int \sqrt{a + a \sinh^2(e + fx)} \tanh(e + fx) dx$.2475
3.429	$\int \coth(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx$.2479
3.430	$\int \coth^3(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx$.2484
3.431	$\int \sqrt{a + a \sinh^2(e + fx)} \tanh^6(e + fx) dx$.2489
3.432	$\int \sqrt{a + a \sinh^2(e + fx)} \tanh^4(e + fx) dx$.2495
3.433	$\int \sqrt{a + a \sinh^2(e + fx)} \tanh^2(e + fx) dx$.2501
3.434	$\int \coth^2(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx$.2505

3.435	$\int \coth^4(e+fx) \sqrt{a+a \sinh^2(e+fx)} dx$.2509
3.436	$\int \coth^6(e+fx) \sqrt{a+a \sinh^2(e+fx)} dx$.2514
3.437	$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2520
3.438	$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2525
3.439	$\int \frac{\tanh(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2529
3.440	$\int \frac{\coth(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2533
3.441	$\int \frac{\coth^3(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2537
3.442	$\int \frac{\tanh^4(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2542
3.443	$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2547
3.444	$\int \frac{\coth^2(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2552
3.445	$\int \frac{\coth^4(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2556
3.446	$\int \frac{\coth^6(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$.2560
3.447	$\int \frac{\tanh^5(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$.2566
3.448	$\int \frac{\tanh^3(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$.2572
3.449	$\int \frac{\tanh(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$.2577
3.450	$\int \frac{\coth(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$.2581
3.451	$\int \frac{\coth^3(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$.2586
3.452	$\int \frac{\tanh^2(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$.2591
3.453	$\int \frac{\coth^2(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$.2596
3.454	$\int \frac{\coth^4(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$.2601
3.455	$\int \frac{\coth^6(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$.2606

3.456	$\int \frac{\coth^8(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$.2612
3.457	$\int \sqrt{a+b \sinh^2(e+fx)} \tanh^5(e+fx) dx$.2619
3.458	$\int \sqrt{a+b \sinh^2(e+fx)} \tanh^3(e+fx) dx$.2627
3.459	$\int \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx) dx$.2633
3.460	$\int \coth(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$.2637
3.461	$\int \coth^3(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$.2641
3.462	$\int \coth^5(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$.2647
3.463	$\int \sqrt{a+b \sinh^2(e+fx)} \tanh^4(e+fx) dx$.2654
3.464	$\int \sqrt{a+b \sinh^2(e+fx)} \tanh^2(e+fx) dx$.2659
3.465	$\int \sqrt{a+b \sinh^2(e+fx)} dx$.2664
3.466	$\int \coth^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$.2668
3.467	$\int \coth^4(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx$.2673
3.468	$\int (a+b \sinh^2(e+fx))^{3/2} \tanh^5(e+fx) dx$.2678
3.469	$\int (a+b \sinh^2(e+fx))^{3/2} \tanh^3(e+fx) dx$.2687
3.470	$\int (a+b \sinh^2(e+fx))^{3/2} \tanh(e+fx) dx$.2693
3.471	$\int \coth(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$.2698
3.472	$\int \coth^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$.2703
3.473	$\int \coth^5(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$.2709
3.474	$\int (a+b \sinh^2(e+fx))^{3/2} \tanh^4(e+fx) dx$.2717
3.475	$\int (a+b \sinh^2(e+fx))^{3/2} \tanh^2(e+fx) dx$.2723
3.476	$\int (a+b \sinh^2(e+fx))^{3/2} dx$.2728
3.477	$\int \coth^2(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$.2733
3.478	$\int \coth^4(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx$.2738
3.479	$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$.2744
3.480	$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$.2751
3.481	$\int \frac{\tanh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$.2756
3.482	$\int \frac{\coth(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$.2760

3.483	$\int \frac{\coth^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$2764
3.484	$\int \frac{\coth^5(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$2769
3.485	$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$2775
3.486	$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$2780
3.487	$\int \frac{1}{\sqrt{a+b \sinh^2(e+fx)}} dx$2785
3.488	$\int \frac{\coth^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$2789
3.489	$\int \frac{\coth^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$2794
3.490	$\int \frac{\tanh^5(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2799
3.491	$\int \frac{\tanh^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2809
3.492	$\int \frac{\tanh(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2816
3.493	$\int \frac{\coth(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2821
3.494	$\int \frac{\coth^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2826
3.495	$\int \frac{\coth^5(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2833
3.496	$\int \frac{\tanh^4(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2842
3.497	$\int \frac{\tanh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2847
3.498	$\int \frac{1}{(a+b \sinh^2(e+fx))^{3/2}} dx$2852
3.499	$\int \frac{\coth^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2856
3.500	$\int \frac{\coth^4(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$2861
3.501	$\int \frac{\tanh^5(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2867
3.502	$\int \frac{\tanh^3(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2872
3.503	$\int \frac{\tanh(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$2882

3.504	$\int \frac{\coth(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$.2889
3.505	$\int \frac{\coth^3(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$.2895
3.506	$\int \frac{\coth^5(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$.2904
3.507	$\int \frac{\tanh^4(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$.2909
3.508	$\int \frac{\tanh^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$.2915
3.509	$\int \frac{1}{(a+b \sinh^2(e+fx))^{5/2}} dx$.2920
3.510	$\int \frac{\coth^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$.2925
3.511	$\int \frac{\coth^4(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$.2931
3.512	$\int (a+b \sinh^2(e+fx))^p (d \tanh(e+fx))^m dx$.2938
3.513	$\int (a+b \sinh^2(c+dx))^p \tanh^3(c+dx) dx$.2942
3.514	$\int (a+b \sinh^2(c+dx))^p \tanh(c+dx) dx$.2946
3.515	$\int \coth(c+dx) (a+b \sinh^2(c+dx))^p dx$.2949
3.516	$\int \coth^3(c+dx) (a+b \sinh^2(c+dx))^p dx$.2952
3.517	$\int (a+b \sinh^2(c+dx))^p \tanh^4(c+dx) dx$.2956
3.518	$\int (a+b \sinh^2(c+dx))^p \tanh^2(c+dx) dx$.2960
3.519	$\int \coth^2(c+dx) (a+b \sinh^2(c+dx))^p dx$.2964
3.520	$\int \coth^4(c+dx) (a+b \sinh^2(c+dx))^p dx$.2968
3.521	$\int \frac{\coth^3(x)}{a+b \sinh^3(x)} dx$.2972
3.522	$\int \frac{\coth(x)}{\sqrt{a+b \sinh^3(x)}} dx$.2979
3.523	$\int \coth(x) \sqrt{a+b \sinh^3(x)} dx$.2983
3.524	$\int \frac{\coth(x)}{\sqrt{a+b \sinh^n(x)}} dx$.2988
3.525	$\int \coth(x) \sqrt{a+b \sinh^n(x)} dx$.2992

4 Listing of Grading functions

2997

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [525]. This is test number [164].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (525)	% 0. (0)
Mathematica	% 95.43 (501)	% 4.57 (24)
Maple	% 92.95 (488)	% 7.05 (37)
Maxima	% 37.33 (196)	% 62.67 (329)
Fricas	% 69.71 (366)	% 30.29 (159)
Sympy	% 12.95 (68)	% 87.05 (457)
Giac	% 51.24 (269)	% 48.76 (256)

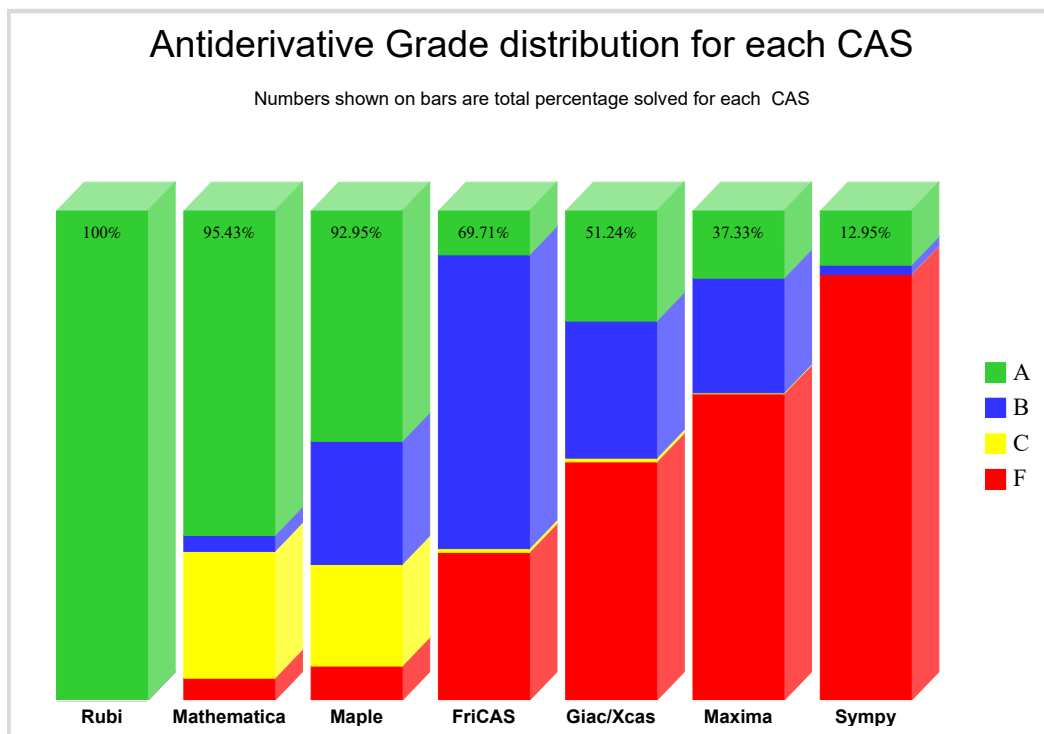
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

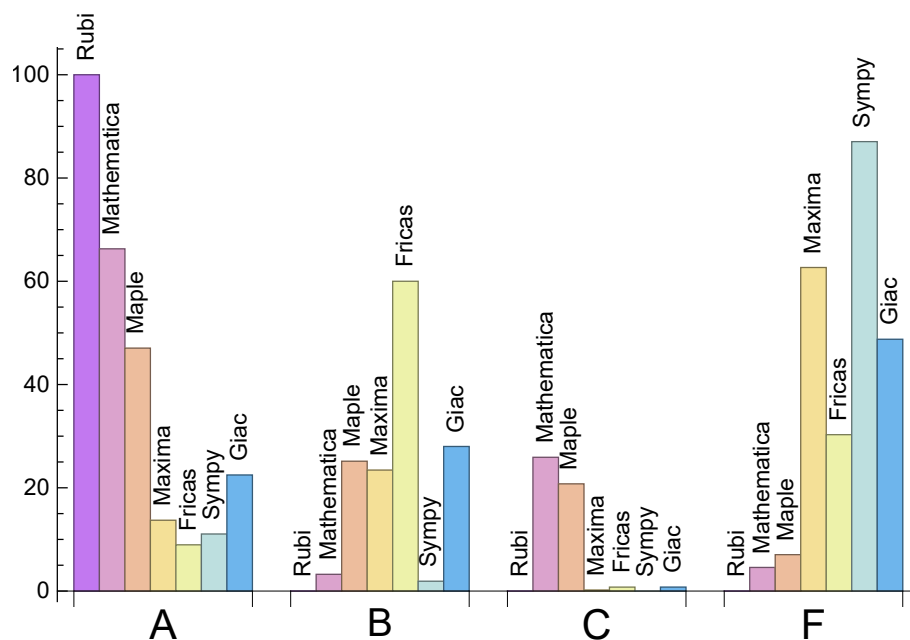
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	66.29	3.24	25.9	4.57
Maple	47.05	25.14	20.76	7.05
Maxima	13.71	23.43	0.19	62.67
Fricas	8.95	60.	0.76	30.29
Sympy	11.05	1.9	0.	87.05
Giac	22.48	28.	0.76	48.76

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	133.48	1.	114.	1.
Mathematica	0.97	155.03	1.19	117.	0.97
Maple	0.08	344.59	2.36	130.	1.23
Maxima	1.32	448.83	4.89	254.5	3.16
Fricas	2.58	5148.96	43.06	2792.	34.94
Sympy	24.51	371.74	5.11	205.	2.54
Giac	1.8	295.92	2.9	209.	2.57

1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {76, 116, 138, 247, 299, 301, 303, 310, 312, 314, 356, 367, 376, 384, 385, 394}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via `sagemath`) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

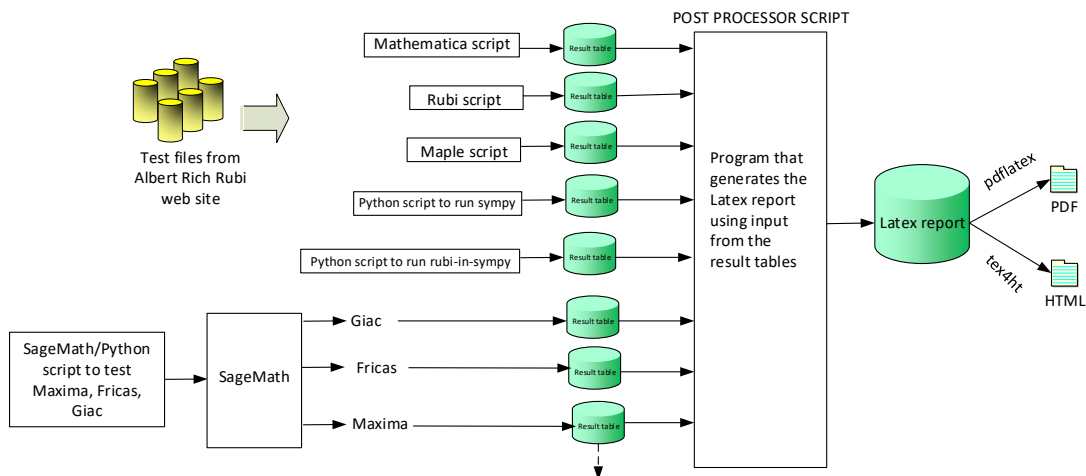
```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

    except Exception as ee:
        leafCount =1
```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507,

508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 37, 39, 41, 42, 44, 46, 48, 50, 51, 53, 55, 57, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 72, 73, 76, 77, 78, 79, 80, 81, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 107, 108, 109, 110, 113, 114, 116, 117, 118, 119, 122, 123, 125, 126, 127, 128, 129, 133, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 265, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 302, 304, 305, 306, 307, 308, 309, 311, 313, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 359, 362, 363, 364, 365, 370, 373, 374, 375, 379, 382, 383, 389, 391, 392, 393, 398, 402, 403, 411, 412, 413, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 460, 461, 462, 465, 468, 469, 470, 471, 472, 473, 476, 479, 480, 481, 482, 483, 484, 487, 498, 509, 513, 514, 515, 516, 521, 522, 523, 524, 525 }

B grade: { 6, 8, 17, 18, 26, 138, 159, 193, 195, 206, 225, 226, 227, 228, 315, 345, 355 }

C grade: { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 71, 74, 75, 82, 85, 86, 102, 105, 106, 111, 112, 115, 120, 121, 124, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 229, 230, 231, 232, 233, 234, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 266, 267, 268, 269, 270, 271, 272, 273, 274, 299, 301, 303, 310, 312, 314, 356, 357, 358, 360, 361, 366, 367, 368, 369, 371, 372, 376, 377, 378, 380, 381, 384, 385, 386, 387, 388, 390, 394, 395, 396, 397, 399, 414, 418, 453, 463, 464, 466, 467, 474, 475, 477, 478, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 510, 511 }

F grade: { 63, 130, 131, 132, 134, 135, 136, 137, 139, 140, 400, 401, 404, 405, 406, 407, 408, 409, 410, 512, 517, 518, 519, 520 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 34, 36, 38, 40, 71, 72, 73, 74, 75, 82, 83, 84, 85, 86, 88, 89, 91, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 129, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165,

166, 167, 168, 169, 170, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 278, 280, 283, 284, 285, 286, 287, 288, 291, 292, 293, 294, 295, 296, 297, 298, 304, 306, 307, 308, 309, 311, 322, 334, 344, 353, 357, 358, 359, 360, 361, 363, 368, 369, 370, 371, 372, 374, 377, 378, 379, 380, 381, 383, 386, 387, 388, 389, 390, 393, 398, 428, 431, 432, 433, 434, 435, 436, 439, 442, 443, 444, 445, 446, 449, 452, 453, 454, 455, 456, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 485, 486, 487, 488, 489, 496, 497, 498, 499, 500, 507, 509, 510, 522, 523, 524, 525 }

B grade: { 28, 29, 30, 31, 32, 33, 35, 37, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 76, 77, 78, 79, 80, 81, 87, 90, 92, 97, 98, 99, 100, 101, 109, 110, 120, 121, 122, 128, 229, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 265, 276, 277, 281, 282, 289, 290, 299, 300, 301, 302, 303, 305, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 395, 396, 397, 399, 413, 417, 425, 508, 511 }

C grade: { 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 279, 352, 354, 355, 356, 362, 364, 365, 366, 367, 373, 375, 376, 382, 384, 385, 391, 392, 394, 411, 412, 414, 415, 416, 418, 426, 427, 429, 430, 437, 438, 440, 441, 447, 448, 450, 451, 457, 458, 459, 460, 461, 462, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 495, 501, 502, 503, 504, 505, 506, 521 }

F grade: { 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 419, 420, 421, 422, 423, 424, 512, 513, 514, 515, 516, 517, 518, 519, 520 }

2.1.4 Maxima

A grade: { 1, 3, 5, 6, 7, 10, 12, 14, 16, 19, 21, 23, 25, 27, 88, 93, 125, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 168, 184, 186, 188, 189, 190, 197, 199, 201, 203, 217, 218, 219, 220, 277, 279, 280, 283, 285, 286, 287, 293, 295, 296, 304, 306, 307, 428, 429, 430, 433, 439, 440, 441, 450, 451 }

B grade: { 2, 4, 8, 9, 11, 13, 15, 17, 18, 20, 22, 24, 26, 60, 61, 62, 63, 64, 65, 89, 94, 108, 117, 118, 148, 149, 156, 157, 158, 159, 160, 167, 169, 170, 185, 187, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 224, 225, 226, 227, 228, 265, 276, 278, 281, 282, 284, 288, 289, 290, 291, 292, 294, 297, 298, 299, 300, 301, 302, 303, 305, 308, 309, 310, 311, 312, 313, 314, 315, 349, 350, 351, 383, 392, 393, 425, 426, 427, 431, 432, 434, 435, 436, 437, 438, 442, 443, 444, 445, 446, 447, 448, 449, 452, 453, 454, 455, 456 }

C grade: { 128 }

F grade: { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 229, 230, 231, 232, 233, 234, }

235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 5, 10, 12, 14, 16, 19, 21, 23, 25, 88, 93, 125, 141, 142, 143, 144, 145, 147, 151, 152, 153, 155, 162, 184, 186, 188, 190, 199, 276, 277, 278, 279, 280, 283, 284, 285, 286, 293, 295, 298, 304, 306, 524, 525 }

B grade: { 2, 6, 7, 8, 9, 11, 13, 15, 17, 18, 20, 22, 24, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 76, 77, 78, 79, 80, 81, 98, 99, 100, 101, 107, 108, 109, 110, 116, 117, 118, 119, 146, 148, 149, 150, 154, 156, 157, 158, 159, 160, 161, 163, 164, 165, 166, 167, 168, 169, 170, 182, 183, 185, 187, 189, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 265, 266, 270, 271, 272, 273, 274, 275, 281, 282, 287, 288, 289, 290, 291, 292, 294, 296, 297, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 349, 350, 351, 352, 353, 354, 355, 356, 362, 363, 364, 365, 366, 367, 373, 374, 375, 376, 382, 383, 384, 385, 391, 392, 393, 394, 411, 412, 413, 415, 416, 417, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 495, 502, 503, 504, 505, 523 }

C grade: { 89, 94, 128, 521 }

F grade: { 58, 59, 71, 72, 73, 74, 75, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 102, 103, 104, 105, 106, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 215, 216, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 269, 347, 348, 357, 358, 359, 360, 361, 368, 369, 370, 371, 372, 377, 378, 379, 380, 381, 386, 387, 388, 389, 390, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 414, 418, 419, 420, 421, 422, 423, 424, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 485, 486, 487, 488, 489, 496, 497, 498, 499, 500, 501, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 35, 141, 142, 143, 144, 145, 150, 151, 152, 153, 161, 162, 163, 184, 185, 186, 187, 188, 196, 197, 198, 199, 209, 218, 279, 280, 283, 284, 285, 286, 293, 294, 295, 296, 304, 305, 306, 307, 322, 334, 344, 413, 417 }

B grade: { 60, 61, 62, 63, 182, 183, 276, 277, 278, 350 }

C grade: { }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 146, 147, 148, 149, 154, 155, 156, 157, 158, 159, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 189, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 281, 282, 287, 288, 289, 290, 291, 292, 297, 298, 299, 300, 301, 302, 303, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

2.1.7 Giac

A grade: { 7, 9, 25, 27, 33, 35, 37, 39, 42, 44, 46, 48, 50, 57, 59, 61, 62, 65, 88, 93, 125, 141, 142, 143, 144, 145, 146, 149, 150, 151, 152, 153, 159, 160, 161, 162, 163, 164, 182, 183, 184, 186, 188, 192, 203, 235, 236, 237, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 268, 269, 271, 272, 275, 276, 277, 279, 280, 285, 287, 288, 290, 292, 293, 295, 304, 306, 321, 324, 326, 333, 336, 413, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 521 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 29, 31, 41, 51, 53, 55, 60, 63, 64, 69, 70, 100, 108, 109, 117, 118, 119, 147, 148, 154, 155, 156, 157, 158, 165, 166, 167, 168, 169, 170, 185, 187, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 265, 274, 278, 281, 282, 283, 284, 286, 289, 291, 294, 296, 297, 298, 299, 300, 301, 302, 303, }

305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 319, 323, 328, 329, 331, 337, 338, 339, 341, 343, 346, 348, 349, 350, 351, 355, 356, 375, 376, 383, 384, 392, 393, 394 }

C grade: { 89, 94, 128, 266 }

F grade: { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 229, 230, 231, 232, 233, 234, 238, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 267, 270, 273, 316, 318, 320, 322, 325, 327, 330, 332, 334, 335, 340, 342, 344, 345, 347, 352, 353, 354, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 377, 378, 379, 380, 381, 382, 385, 386, 387, 388, 389, 390, 391, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 451, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	68	88	203	319	258	221
normalized size	1	1.	0.76	0.99	2.28	3.58	2.9	2.48
time (sec)	N/A	0.055	0.108	0.014	1.034	1.952	4.778	1.323

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	77	56	190	273	105	166
normalized size	1	1.	1.45	1.06	3.58	5.15	1.98	3.13
time (sec)	N/A	0.056	0.026	0.013	1.041	1.823	2.48	1.255

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	66	131	163	158	161
normalized size	1	1.	0.77	1.08	2.15	2.67	2.59	2.64
time (sec)	N/A	0.043	0.085	0.013	1.059	1.909	1.428	1.369

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	53	34	90	127	56	96
normalized size	1	1.	1.66	1.06	2.81	3.97	1.75	3.
time (sec)	N/A	0.029	0.024	0.013	1.015	1.875	0.769	1.267

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	36	32	51	74	51	72
normalized size	1	1.	1.2	1.07	1.7	2.47	1.7	2.4
time (sec)	N/A	0.016	0.032	0.007	1.044	1.841	0.409	1.308

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	62	24	58	374	0	78
normalized size	1	1.	2.48	0.96	2.32	14.96	0.	3.12
time (sec)	N/A	0.035	0.032	0.029	1.041	1.974	0.	1.214

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	31	89	0	41
normalized size	1	1.	1.	1.38	1.94	5.56	0.	2.56
time (sec)	N/A	0.027	0.02	0.027	1.024	1.85	0.	1.293

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	99	40	169	1323	0	136
normalized size	1	1.	2.48	1.	4.22	33.08	0.	3.4
time (sec)	N/A	0.041	0.032	0.034	1.055	1.936	0.	1.253

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	49	35	153	425	0	82
normalized size	1	1.	1.14	0.81	3.56	9.88	0.	1.91
time (sec)	N/A	0.039	0.031	0.034	1.025	1.807	0.	1.173

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	133	150	360	589	490	429
normalized size	1	1.	0.91	1.03	2.47	4.03	3.36	2.94
time (sec)	N/A	0.18	0.204	0.048	1.044	1.973	15.386	1.363

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	154	102	333	554	204	332
normalized size	1	1.	1.81	1.2	3.92	6.52	2.4	3.91
time (sec)	N/A	0.097	0.038	0.017	1.06	2.249	8.167	1.325

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	117	99	118	255	373	332	316
normalized size	1	1.06	0.9	1.07	2.32	3.39	3.02	2.87
time (sec)	N/A	0.11	0.192	0.017	1.043	2.145	4.88	1.356

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	111	70	212	309	128	220
normalized size	1	1.	1.95	1.23	3.72	5.42	2.25	3.86
time (sec)	N/A	0.06	0.036	0.015	1.046	2.018	2.35	1.25

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	79	142	193	168	204
normalized size	1	1.	0.83	1.1	1.97	2.68	2.33	2.83
time (sec)	N/A	0.022	0.127	0.013	1.008	1.878	1.277	1.226

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	104	50	138	1280	0	174
normalized size	1	1.	2.	0.96	2.65	24.62	0.	3.35
time (sec)	N/A	0.066	0.034	0.032	1.049	1.851	0.	1.28

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	64	56	52	85	215	0	189
normalized size	1	1.28	1.12	1.04	1.7	4.3	0.	3.78
time (sec)	N/A	0.081	0.163	0.033	1.014	1.87	0.	1.228

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	134	53	212	2310	0	180
normalized size	1	1.	2.39	0.95	3.79	41.25	0.	3.21
time (sec)	N/A	0.088	0.064	0.041	1.075	1.952	0.	1.359

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	85	47	163	428	0	109
normalized size	1	1.	2.12	1.18	4.08	10.7	0.	2.72
time (sec)	N/A	0.074	0.712	0.04	1.063	1.893	0.	1.345

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	162	222	547	1013	777	679
normalized size	1	1.	0.62	0.85	2.1	3.88	2.98	2.6
time (sec)	N/A	0.43	0.422	0.052	1.077	1.81	37.062	1.399

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	127	158	508	940	330	544
normalized size	1	1.	1.1	1.37	4.42	8.17	2.87	4.73
time (sec)	N/A	0.129	0.797	0.049	1.083	1.82	21.724	1.414

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	130	180	413	667	561	521
normalized size	1	1.	0.72	0.99	2.28	3.69	3.1	2.88
time (sec)	N/A	0.193	0.291	0.014	1.105	1.817	14.363	1.408

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	94	116	355	589	221	386
normalized size	1	1.	1.19	1.47	4.49	7.46	2.8	4.89
time (sec)	N/A	0.087	0.304	0.016	1.053	1.774	7.788	1.32

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	95	131	266	398	350	362
normalized size	1	1.	0.74	1.02	2.08	3.11	2.73	2.83
time (sec)	N/A	0.1	0.259	0.013	1.045	1.823	5.044	1.322

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	86	261	2889	0	311
normalized size	1	1.	1.	1.04	3.14	34.81	0.	3.75
time (sec)	N/A	0.087	0.215	0.03	1.065	2.065	0.	1.322

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	113	94	176	409	0	257
normalized size	1	1.	0.82	0.69	1.28	2.99	0.	1.88
time (sec)	N/A	0.192	1.917	0.03	1.065	1.841	0.	1.363

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	210	79	293	4575	0	261
normalized size	1	1.	2.53	0.95	3.53	55.12	0.	3.14
time (sec)	N/A	0.107	4.7	0.042	1.075	2.084	0.	1.363

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	107	77	217	668	0	217
normalized size	1	1.	0.95	0.68	1.92	5.91	0.	1.92
time (sec)	N/A	0.141	2.504	0.039	1.03	1.764	0.	1.368

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	165	448	0	7640	0	0
normalized size	1	1.	1.51	4.11	0.	70.09	0.	0.
time (sec)	N/A	0.149	0.868	0.044	0.	2.284	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	97	670	0	4313	0	296
normalized size	1	1.	0.8	5.54	0.	35.64	0.	2.45
time (sec)	N/A	0.235	0.475	0.08	0.	2.054	0.	1.209

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	134	227	0	3993	0	0
normalized size	1	1.	1.7	2.87	0.	50.54	0.	0.
time (sec)	N/A	0.108	0.434	0.033	0.	1.97	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	454	0	2191	0	181
normalized size	1	1.	0.9	5.75	0.	27.73	0.	2.29
time (sec)	N/A	0.121	0.25	0.043	0.	2.129	0.	1.238

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	107	98	0	1894	0	0
normalized size	1	1.	1.91	1.75	0.	33.82	0.	0.
time (sec)	N/A	0.085	0.234	0.03	0.	1.916	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	312	0	1122	0	88
normalized size	1	1.	1.	6.24	0.	22.44	0.	1.76
time (sec)	N/A	0.087	0.131	0.033	0.	2.02	0.	1.265

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	91	51	0	1293	0	0
normalized size	1	1.	2.28	1.27	0.	32.32	0.	0.
time (sec)	N/A	0.047	0.123	0.016	0.	2.016	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	267	0	1068	3859	63
normalized size	1	1.	1.	6.68	0.	26.7	96.48	1.58
time (sec)	N/A	0.027	0.071	0.042	0.	1.928	118.484	1.258

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	124	74	0	1544	0	0
normalized size	1	1.	2.07	1.23	0.	25.73	0.	0.
time (sec)	N/A	0.074	0.21	0.049	0.	2.014	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	319	0	1661	0	100
normalized size	1	1.	1.	5.6	0.	29.14	0.	1.75
time (sec)	N/A	0.081	0.292	0.061	0.	2.214	0.	1.308

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	201	133	0	4867	0	0
normalized size	1	1.	2.28	1.51	0.	55.31	0.	0.
time (sec)	N/A	0.123	0.694	0.062	0.	2.819	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	126	401	0	4710	0	159
normalized size	1	1.	1.62	5.14	0.	60.38	0.	2.04
time (sec)	N/A	0.12	0.693	0.071	0.	2.378	0.	1.425

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	574	232	0	14357	0	0
normalized size	1	1.	4.42	1.78	0.	110.44	0.	0.
time (sec)	N/A	0.204	6.283	0.066	0.	3.037	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	155	519	0	10761	0	289
normalized size	1	1.	1.41	4.72	0.	97.83	0.	2.63
time (sec)	N/A	0.136	1.458	0.074	0.	2.781	0.	1.493

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	99	798	0	4199	0	228
normalized size	1	1.	0.97	7.82	0.	41.17	0.	2.24
time (sec)	N/A	0.169	0.833	0.051	0.	2.56	0.	1.283

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	141	341	0	4536	0	0
normalized size	1	1.	1.57	3.79	0.	50.4	0.	0.
time (sec)	N/A	0.113	0.597	0.032	0.	2.432	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	81	428	0	3564	0	184
normalized size	1	1.	0.96	5.1	0.	42.43	0.	2.19
time (sec)	N/A	0.086	0.444	0.041	0.	2.392	0.	1.402

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	130	256	0	3999	0	0
normalized size	1	1.	1.6	3.16	0.	49.37	0.	0.
time (sec)	N/A	0.065	0.348	0.027	0.	2.395	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	749	0	3767	0	196
normalized size	1	1.	1.01	7.88	0.	39.65	0.	2.06
time (sec)	N/A	0.069	0.303	0.046	0.	2.437	0.	1.213

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	176	350	0	6178	0	0
normalized size	1	1.	1.6	3.18	0.	56.16	0.	0.
time (sec)	N/A	0.149	0.644	0.062	0.	2.911	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	170	810	0	6962	0	312
normalized size	1	1.	1.2	5.7	0.	49.03	0.	2.2
time (sec)	N/A	0.153	0.811	0.076	0.	2.621	0.	1.485

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	350	415	0	19038	0	0
normalized size	1	1.	2.17	2.58	0.	118.25	0.	0.
time (sec)	N/A	0.273	1.406	0.079	0.	3.669	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	210	890	0	16662	0	301
normalized size	1	1.	1.21	5.11	0.	95.76	0.	1.73
time (sec)	N/A	0.206	1.259	0.089	0.	3.163	0.	1.508

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	104	768	0	11786	0	386
normalized size	1	1.	0.84	6.19	0.	95.05	0.	3.11
time (sec)	N/A	0.123	1.369	0.052	0.	2.942	0.	1.565

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	170	961	0	14074	0	0
normalized size	1	1.	1.26	7.12	0.	104.25	0.	0.
time (sec)	N/A	0.141	1.222	0.042	0.	2.963	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	121	1408	0	12358	0	378
normalized size	1	1.	0.87	10.13	0.	88.91	0.	2.72
time (sec)	N/A	0.157	1.346	0.049	0.	3.003	0.	1.526

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	149	964	0	12081	0	0
normalized size	1	1.	1.26	8.17	0.	102.38	0.	0.
time (sec)	N/A	0.083	0.747	0.034	0.	2.862	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	132	1768	0	13509	0	413
normalized size	1	1.	0.86	11.48	0.	87.72	0.	2.68
time (sec)	N/A	0.155	1.219	0.053	0.	2.969	0.	1.462

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	237	1145	0	22814	0	0
normalized size	1	1.	1.43	6.9	0.	137.43	0.	0.
time (sec)	N/A	0.267	3.426	0.069	0.	4.056	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	225	1850	0	21173	0	455
normalized size	1	1.	1.05	8.6	0.	98.48	0.	2.12
time (sec)	N/A	0.287	1.813	0.089	0.	3.586	0.	1.6

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	419	1225	0	0	0	0
normalized size	1	1.	1.87	5.47	0.	0.	0.	0.
time (sec)	N/A	0.439	2.943	0.091	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	167	1930	0	0	0	518
normalized size	1	1.	0.64	7.45	0.	0.	0.	2.
time (sec)	N/A	0.338	2.762	0.111	0.	0.	0.	1.609

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	17	14	70	14	14
normalized size	1	1.	1.	8.5	7.	35.	7.	7.
time (sec)	N/A	0.016	0.004	0.016	1.026	1.758	0.895	1.298

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	36	66	286	104	24
normalized size	1	1.	1.55	3.27	6.	26.	9.45	2.18
time (sec)	N/A	0.019	0.003	0.015	1.039	1.793	4.906	1.275

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	27	52	150	628	260	32
normalized size	1	1.	1.42	2.74	7.89	33.05	13.68	1.68
time (sec)	N/A	0.02	0.003	0.019	1.045	1.78	18.192	1.275

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	B	B	B	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	0	40	82	215	209	50
normalized size	1	1.	0.	2.67	5.47	14.33	13.93	3.33
time (sec)	N/A	0.013	0.022	0.016	1.53	1.843	3.567	1.266

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	92	117	729	0	84
normalized size	1	1.	0.95	2.49	3.16	19.7	0.	2.27
time (sec)	N/A	0.027	0.128	0.02	1.549	1.854	0.	1.277

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	124	150	1936	0	100
normalized size	1	1.	0.93	2.25	2.73	35.2	0.	1.82
time (sec)	N/A	0.058	0.178	0.023	1.566	2.069	0.	1.277

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	339	0	7794	0	0
normalized size	1	1.	0.88	2.61	0.	59.95	0.	0.
time (sec)	N/A	0.148	0.651	0.114	0.	3.896	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	200	0	5562	0	0
normalized size	1	1.	1.18	2.44	0.	67.83	0.	0.
time (sec)	N/A	0.07	0.162	0.066	0.	3.464	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	97	174	0	11636	0	0
normalized size	1	1.	1.15	2.07	0.	138.52	0.	0.
time (sec)	N/A	0.105	0.124	0.098	0.	3.265	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	104	230	0	3421	0	938
normalized size	1	1.	1.18	2.61	0.	38.88	0.	10.66
time (sec)	N/A	0.118	0.293	0.09	0.	2.279	0.	1.373

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	129	381	0	8303	0	3086
normalized size	1	1.	0.9	2.65	0.	57.66	0.	21.43
time (sec)	N/A	0.15	0.538	0.099	0.	4.097	0.	1.574

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	210	512	0	0	0	0
normalized size	1	1.	0.7	1.71	0.	0.	0.	0.
time (sec)	N/A	0.327	1.369	0.086	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	170	343	0	0	0	0
normalized size	1	1.	0.96	1.94	0.	0.	0.	0.
time (sec)	N/A	0.213	0.841	0.065	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	0	0	0
normalized size	1	1.	1.15	2.33	0.	0.	0.	0.
time (sec)	N/A	0.036	0.096	0.065	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	151	160	0	0	0	0
normalized size	1	1.	0.76	0.8	0.	0.	0.	0.
time (sec)	N/A	0.182	0.594	0.072	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	208	436	0	0	0	0
normalized size	1	1.	0.75	1.58	0.	0.	0.	0.
time (sec)	N/A	0.282	3.028	0.121	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	151	483	0	11580	0	0
normalized size	1	1.	0.85	2.73	0.	65.42	0.	0.
time (sec)	N/A	0.181	0.582	0.091	0.	3.807	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	111	336	0	7737	0	0
normalized size	1	1.	0.92	2.78	0.	63.94	0.	0.
time (sec)	N/A	0.097	0.312	0.071	0.	2.926	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	136	268	0	14642	0	0
normalized size	1	1.	1.07	2.11	0.	115.29	0.	0.
time (sec)	N/A	0.165	0.554	0.086	0.	3.685	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	143	297	0	17554	0	0
normalized size	1	1.	1.1	2.28	0.	135.03	0.	0.
time (sec)	N/A	0.168	0.711	0.097	0.	4.753	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	123	379	0	8128	0	0
normalized size	1	1.	0.91	2.81	0.	60.21	0.	0.
time (sec)	N/A	0.141	0.63	0.095	0.	4.112	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	174	569	0	18294	0	0
normalized size	1	1.	0.87	2.86	0.	91.93	0.	0.
time (sec)	N/A	0.199	1.012	0.122	0.	10.174	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	262	743	0	0	0	0
normalized size	1	1.	0.71	2.02	0.	0.	0.	0.
time (sec)	N/A	0.471	2.873	0.074	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	213	535	0	0	0	0
normalized size	1	1.	0.9	2.27	0.	0.	0.	0.
time (sec)	N/A	0.325	1.363	0.071	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	169	416	0	0	0	0
normalized size	1	1.	0.97	2.39	0.	0.	0.	0.
time (sec)	N/A	0.194	0.679	0.069	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	155	243	0	0	0	0
normalized size	1	1.	0.76	1.19	0.	0.	0.	0.
time (sec)	N/A	0.211	1.047	0.079	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	213	454	0	0	0	0
normalized size	1	1.	0.8	1.7	0.	0.	0.	0.
time (sec)	N/A	0.306	4.061	0.087	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	208	609	0	0	0	0
normalized size	1	1.	0.9	2.62	0.	0.	0.	0.
time (sec)	N/A	0.304	1.394	0.085	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	15	12	0	15
normalized size	1	1.	1.	1.27	1.36	1.09	0.	1.36
time (sec)	N/A	0.024	0.007	0.049	1.558	1.874	0.	1.257

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	34	38	0	15
normalized size	1	1.	1.	1.15	2.62	2.92	0.	1.15
time (sec)	N/A	0.026	0.005	0.033	1.553	1.768	0.	1.301

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	51	0	0	0	0
normalized size	1	1.	1.	4.64	0.	0.	0.	0.
time (sec)	N/A	0.01	0.024	0.07	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	0	0	0
normalized size	1	1.	1.	1.85	0.	0.	0.	0.
time (sec)	N/A	0.02	0.033	0.064	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	54	109	0	0	0	0
normalized size	1	1.	1.29	2.6	0.	0.	0.	0.
time (sec)	N/A	0.032	0.044	0.067	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	21	31	62	0	34
normalized size	1	1.	0.79	0.72	1.07	2.14	0.	1.17
time (sec)	N/A	0.026	0.02	0.041	1.558	1.788	0.	1.227

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	25	21	72	81	0	34
normalized size	1	1.	0.76	0.64	2.18	2.45	0.	1.03
time (sec)	N/A	0.029	0.006	0.043	1.55	1.806	0.	1.199

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	103	0	0	0	0
normalized size	1	1.	1.	2.29	0.	0.	0.	0.
time (sec)	N/A	0.064	0.069	0.077	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	78	106	0	0	0	0
normalized size	1	1.	0.9	1.22	0.	0.	0.	0.
time (sec)	N/A	0.086	0.122	0.079	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	132	329	0	0	0	0
normalized size	1	1.	1.07	2.67	0.	0.	0.	0.
time (sec)	N/A	0.163	0.379	0.07	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	98	204	0	5576	0	0
normalized size	1	1.	1.18	2.46	0.	67.18	0.	0.
time (sec)	N/A	0.099	0.269	0.109	0.	2.562	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	49	108	0	4286	0	0
normalized size	1	1.	1.2	2.63	0.	104.54	0.	0.
time (sec)	N/A	0.051	0.109	0.054	0.	2.362	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	49	113	0	1561	0	143
normalized size	1	1.	1.17	2.69	0.	37.17	0.	3.4
time (sec)	N/A	0.08	0.178	0.069	0.	2.077	0.	1.401

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	102	232	0	3468	0	0
normalized size	1	1.	1.15	2.61	0.	38.97	0.	0.
time (sec)	N/A	0.115	0.319	0.091	0.	2.567	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	168	344	0	0	0	0
normalized size	1	1.	0.73	1.5	0.	0.	0.	0.
time (sec)	N/A	0.218	0.952	0.068	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	89	113	0	0	0	0
normalized size	1	1.	0.7	0.88	0.	0.	0.	0.
time (sec)	N/A	0.136	0.257	0.066	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	0	0	0
normalized size	1	1.	1.13	1.43	0.	0.	0.	0.
time (sec)	N/A	0.036	0.081	0.047	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	150	189	0	0	0	0
normalized size	1	1.	1.12	1.41	0.	0.	0.	0.
time (sec)	N/A	0.149	0.515	0.079	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	201	456	0	0	0	0
normalized size	1	1.	0.75	1.71	0.	0.	0.	0.
time (sec)	N/A	0.284	3.795	0.098	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	98	146	0	7602	0	0
normalized size	1	1.	1.18	1.76	0.	91.59	0.	0.
time (sec)	N/A	0.112	0.422	0.109	0.	3.168	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	43	32	319	738	0	207
normalized size	1	1.	1.19	0.89	8.86	20.5	0.	5.75
time (sec)	N/A	0.053	0.155	0.051	1.606	2.172	0.	1.229

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	98	154	0	4082	0	331
normalized size	1	1.	1.17	1.83	0.	48.6	0.	3.94
time (sec)	N/A	0.108	0.401	0.123	0.	2.585	0.	1.424

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	134	251	0	10539	0	0
normalized size	1	1.	0.96	1.81	0.	75.82	0.	0.
time (sec)	N/A	0.182	0.712	0.132	0.	4.792	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	211	500	0	0	0	0
normalized size	1	1.	0.62	1.47	0.	0.	0.	0.
time (sec)	N/A	0.341	1.252	0.102	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	156	313	0	0	0	0
normalized size	1	1.	0.61	1.22	0.	0.	0.	0.
time (sec)	N/A	0.235	1.05	0.093	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	151	127	0	0	0	0
normalized size	1	1.	0.87	0.73	0.	0.	0.	0.
time (sec)	N/A	0.212	0.481	0.084	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	100	252	0	0	0	0
normalized size	1	1.	0.87	2.19	0.	0.	0.	0.
time (sec)	N/A	0.062	0.159	0.078	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	185	284	0	0	0	0
normalized size	1	1.	0.64	0.98	0.	0.	0.	0.
time (sec)	N/A	0.31	1.262	0.104	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	130	230	0	18590	0	0
normalized size	1	1.	0.91	1.61	0.	130.	0.	0.
time (sec)	N/A	0.166	0.871	0.16	0.	6.888	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	67	58	1251	2873	0	699
normalized size	1	1.	0.77	0.67	14.38	33.02	0.	8.03
time (sec)	N/A	0.107	0.329	0.077	1.774	3.411	0.	1.339

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	63	57	655	2789	0	622
normalized size	1	1.	0.8	0.72	8.29	35.3	0.	7.87
time (sec)	N/A	0.068	0.18	0.066	1.675	3.325	0.	1.378

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	130	236	0	12269	0	857
normalized size	1	1.	0.96	1.74	0.	90.21	0.	6.3
time (sec)	N/A	0.171	0.798	0.176	0.	5.343	0.	1.518

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	207	868	0	0	0	0
normalized size	1	1.	0.6	2.52	0.	0.	0.	0.
time (sec)	N/A	0.366	2.035	0.13	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	198	659	0	0	0	0
normalized size	1	1.	0.81	2.7	0.	0.	0.	0.
time (sec)	N/A	0.249	1.609	0.107	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	187	598	0	0	0	0
normalized size	1	1.	0.78	2.48	0.	0.	0.	0.
time (sec)	N/A	0.326	1.403	0.114	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	0	0	0
normalized size	1	1.	0.76	1.62	0.	0.	0.	0.
time (sec)	N/A	0.28	1.32	0.198	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	234	747	0	0	0	0
normalized size	1	1.	0.61	1.94	0.	0.	0.	0.
time (sec)	N/A	0.456	2.311	0.128	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	19	15	7	39	0	7
normalized size	1	1.	1.36	1.07	0.5	2.79	0.	0.5
time (sec)	N/A	0.024	0.011	0.05	1.657	2.036	0.	1.146

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	0	0	0	0
normalized size	1	1.	1.	3.73	0.	0.	0.	0.
time (sec)	N/A	0.011	0.04	0.147	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	0	0	0
normalized size	1	1.	1.	1.85	0.	0.	0.	0.
time (sec)	N/A	0.022	0.041	0.068	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	34	7	39	0	7
normalized size	1	1.	1.31	2.12	0.44	2.44	0.	0.44
time (sec)	N/A	0.024	0.008	0.04	1.925	1.775	0.	1.15

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	63	0	0	0	0
normalized size	1	1.	1.26	1.5	0.	0.	0.	0.
time (sec)	N/A	0.033	0.051	0.059	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	8.657	0.641	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.259	11.434	0.629	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	15.097	0.436	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.207	0.331	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	4.189	0.264	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	7.895	0.294	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	15.005	0.273	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	10.263	0.497	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	250	0	0	0	0	0
normalized size	1	1.	2.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.745	0.356	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	5.551	0.26	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	10.55	0.249	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	81	82	221	539	192	213
normalized size	1	1.	0.76	0.77	2.08	5.08	1.81	2.01
time (sec)	N/A	0.094	0.103	0.016	1.197	1.931	8.107	1.212

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	66	72	193	375	194	177
normalized size	1	1.	0.67	0.73	1.95	3.79	1.96	1.79
time (sec)	N/A	0.108	0.129	0.015	1.116	1.884	4.725	1.153

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	79	60	162	302	117	149
normalized size	1	1.	1.13	0.86	2.31	4.31	1.67	2.13
time (sec)	N/A	0.071	0.095	0.013	1.118	2.054	2.307	1.165

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	45	50	100	171	121	113
normalized size	1	1.	0.75	0.83	1.67	2.85	2.02	1.88
time (sec)	N/A	0.068	0.126	0.014	1.151	1.802	1.248	1.172

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	80	128	41	72
normalized size	1	1.	1.06	0.88	2.5	4.	1.28	2.25
time (sec)	N/A	0.021	0.011	0.005	1.155	1.905	0.55	1.124

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	72	40	68	721	0	97
normalized size	1	1.	1.8	1.	1.7	18.02	0.	2.42
time (sec)	N/A	0.056	0.054	0.036	1.145	1.998	0.	1.139

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	23	63	103	0	84
normalized size	1	1.	1.46	0.96	2.62	4.29	0.	3.5
time (sec)	N/A	0.05	0.029	0.036	1.135	1.951	0.	1.204

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	63	37	123	1413	0	108
normalized size	1	1.	1.62	0.95	3.15	36.23	0.	2.77
time (sec)	N/A	0.063	0.013	0.04	1.099	2.126	0.	1.161

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	76	36	177	1773	0	88
normalized size	1	1.	1.85	0.88	4.32	43.24	0.	2.15
time (sec)	N/A	0.057	0.03	0.043	1.162	2.029	0.	1.194

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	125	128	367	973	325	369
normalized size	1	1.	0.65	0.67	1.91	5.07	1.69	1.92
time (sec)	N/A	0.179	0.683	0.02	1.189	1.918	31.015	1.269

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	133	122	320	734	340	317
normalized size	1	1.	0.74	0.68	1.78	4.08	1.89	1.76
time (sec)	N/A	0.154	0.13	0.021	1.152	1.909	15.338	1.251

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	92	96	243	594	219	266
normalized size	1	1.	0.71	0.74	1.87	4.57	1.68	2.05
time (sec)	N/A	0.119	0.41	0.019	1.073	1.903	8.429	1.203

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	94	85	204	421	212	212
normalized size	1	1.	0.82	0.75	1.79	3.69	1.86	1.86
time (sec)	N/A	0.083	0.146	0.017	1.142	1.987	5.482	1.128

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	96	76	189	2851	0	240
normalized size	1	1.	1.09	0.86	2.15	32.4	0.	2.73
time (sec)	N/A	0.102	0.302	0.056	1.176	2.16	0.	1.254

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	92	65	153	360	0	224
normalized size	1	1.	1.12	0.79	1.87	4.39	0.	2.73
time (sec)	N/A	0.098	0.285	0.046	1.076	1.912	0.	1.229

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	105	63	205	4246	0	244
normalized size	1	1.	1.36	0.82	2.66	55.14	0.	3.17
time (sec)	N/A	0.105	0.031	0.059	1.22	2.207	0.	1.229

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	81	65	230	4551	0	219
normalized size	1	1.	1.07	0.86	3.03	59.88	0.	2.88
time (sec)	N/A	0.091	0.399	0.069	1.26	2.081	0.	1.253

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	149	66	254	5495	0	246
normalized size	1	1.	1.66	0.73	2.82	61.06	0.	2.73
time (sec)	N/A	0.132	0.038	0.109	1.122	2.27	0.	1.267

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	197	73	409	6268	0	197
normalized size	1	1.	2.24	0.83	4.65	71.23	0.	2.24
time (sec)	N/A	0.103	0.941	0.108	1.153	2.312	0.	1.248

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	235	90	427	9393	0	281
normalized size	1	1.	1.77	0.68	3.21	70.62	0.	2.11
time (sec)	N/A	0.175	0.053	0.099	1.197	2.284	0.	1.215

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	194	188	522	1546	498	545
normalized size	1	1.	0.67	0.65	1.79	5.31	1.71	1.87
time (sec)	N/A	0.209	0.477	0.06	1.09	1.931	83.459	1.442

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	184	168	429	1193	496	474
normalized size	1	1.	0.69	0.63	1.61	4.47	1.86	1.78
time (sec)	N/A	0.222	0.459	0.023	1.147	2.006	97.203	1.476

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	159	141	378	1017	340	405
normalized size	1	1.	0.78	0.69	1.85	4.99	1.67	1.99
time (sec)	N/A	0.128	0.251	0.02	1.177	1.813	33.478	1.163

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	158	138	347	7191	0	427
normalized size	1	1.	0.79	0.69	1.73	35.78	0.	2.12
time (sec)	N/A	0.189	0.219	0.072	1.184	2.071	0.	1.407

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	140	111	297	809	0	410
normalized size	1	1.	0.92	0.73	1.95	5.32	0.	2.7
time (sec)	N/A	0.138	1.144	0.06	1.201	1.871	0.	1.448

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	150	115	329	9546	0	428
normalized size	1	1.	0.96	0.74	2.11	61.19	0.	2.74
time (sec)	N/A	0.169	3.702	0.066	1.173	2.618	0.	1.452

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	169	101	351	10166	0	419
normalized size	1	1.	1.31	0.78	2.72	78.81	0.	3.25
time (sec)	N/A	0.12	0.445	0.077	1.208	2.437	0.	1.437

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	218	108	344	11867	0	475
normalized size	1	1.	1.47	0.73	2.32	80.18	0.	3.21
time (sec)	N/A	0.173	6.132	0.078	1.161	2.602	0.	1.409

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	225	99	493	12388	0	389
normalized size	1	1.	1.72	0.76	3.76	94.56	0.	2.97
time (sec)	N/A	0.125	1.846	0.085	1.087	2.366	0.	1.442

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	236	119	479	16188	0	455
normalized size	1	1.	1.42	0.72	2.89	97.52	0.	2.74
time (sec)	N/A	0.197	1.552	0.083	1.084	2.782	0.	1.461

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	168	259	0	0	0	0
normalized size	1	1.	0.51	0.79	0.	0.	0.	0.
time (sec)	N/A	0.739	0.349	0.066	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	299	207	0	0	0	0
normalized size	1	1.	1.01	0.7	0.	0.	0.	0.
time (sec)	N/A	0.559	0.316	0.06	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	214	128	0	0	0	0
normalized size	1	1.	0.71	0.42	0.	0.	0.	0.
time (sec)	N/A	0.545	0.345	0.052	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	145	129	0	0	0	0
normalized size	1	1.	0.49	0.44	0.	0.	0.	0.
time (sec)	N/A	0.44	0.231	0.043	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	275	78	0	0	0	0
normalized size	1	1.	1.05	0.3	0.	0.	0.	0.
time (sec)	N/A	0.271	0.178	0.042	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	199	82	0	0	0	0
normalized size	1	1.	0.69	0.28	0.	0.	0.	0.
time (sec)	N/A	0.336	0.236	0.038	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	131	87	0	0	0	0
normalized size	1	1.	0.47	0.31	0.	0.	0.	0.
time (sec)	N/A	0.251	0.174	0.04	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	295	100	0	0	0	0
normalized size	1	1.	1.03	0.35	0.	0.	0.	0.
time (sec)	N/A	0.432	0.254	0.072	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	230	123	0	0	0	0
normalized size	1	1.	0.76	0.4	0.	0.	0.	0.
time (sec)	N/A	0.478	0.39	0.075	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	178	146	0	0	0	0
normalized size	1	1.	0.55	0.45	0.	0.	0.	0.
time (sec)	N/A	0.457	0.497	0.089	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	370	178	0	0	0	0
normalized size	1	1.	1.17	0.56	0.	0.	0.	0.
time (sec)	N/A	0.433	6.062	0.087	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	156	82	0	591	1423	140
normalized size	1	1.	1.12	0.59	0.	4.25	10.24	1.01
time (sec)	N/A	0.188	1.399	0.029	0.	1.935	174.633	1.163

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	156	80	0	590	3742	146
normalized size	1	1.	1.17	0.6	0.	4.44	28.14	1.1
time (sec)	N/A	0.189	1.219	0.029	0.	1.851	72.418	1.18

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	82	98	236	467	306	258
normalized size	1	1.	0.74	0.88	2.13	4.21	2.76	2.32
time (sec)	N/A	0.143	0.142	0.014	1.141	1.63	14.166	1.203

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	93	66	212	439	128	198
normalized size	1	1.	1.39	0.99	3.16	6.55	1.91	2.96
time (sec)	N/A	0.066	0.029	0.014	1.129	1.635	8.487	1.162

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	63	76	165	292	206	193
normalized size	1	1.	0.76	0.92	1.99	3.52	2.48	2.33
time (sec)	N/A	0.098	0.077	0.014	1.114	1.604	4.94	1.166

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	69	44	131	250	80	132
normalized size	1	1.	1.5	0.96	2.85	5.43	1.74	2.87
time (sec)	N/A	0.034	0.022	0.013	1.056	1.67	2.538	1.186

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	44	89	155	100	99
normalized size	1	1.	0.94	0.85	1.71	2.98	1.92	1.9
time (sec)	N/A	0.034	0.054	0.005	1.164	1.619	1.248	1.129

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	70	36	96	1100	0	126
normalized size	1	1.	1.67	0.86	2.29	26.19	0.	3.
time (sec)	N/A	0.043	0.024	0.029	1.161	1.69	0.	1.181

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	45	39	73	185	0	124
normalized size	1	1.	1.15	1.	1.87	4.74	0.	3.18
time (sec)	N/A	0.052	0.119	0.03	1.16	1.672	0.	1.154

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	82	38	155	1867	0	155
normalized size	1	1.	1.74	0.81	3.3	39.72	0.	3.3
time (sec)	N/A	0.054	0.026	0.04	1.237	1.742	0.	1.199

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	40	33	131	333	0	61
normalized size	1	1.	1.29	1.06	4.23	10.74	0.	1.97
time (sec)	N/A	0.05	0.015	0.039	1.165	1.673	0.	1.162

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	139	54	235	3906	0	174
normalized size	1	1.	2.17	0.84	3.67	61.03	0.	2.72
time (sec)	N/A	0.063	0.029	0.039	1.04	1.86	0.	1.165

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	71	45	308	903	0	131
normalized size	1	1.	1.51	0.96	6.55	19.21	0.	2.79
time (sec)	N/A	0.043	0.034	0.036	1.043	1.661	0.	1.149

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	199	78	362	8357	0	285
normalized size	1	1.	2.16	0.85	3.93	90.84	0.	3.1
time (sec)	N/A	0.084	0.035	0.041	1.056	1.881	0.	1.174

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	207	132	414	1111	280	443
normalized size	1	1.	1.72	1.1	3.45	9.26	2.33	3.69
time (sec)	N/A	0.129	0.064	0.023	1.046	1.632	74.891	1.32

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	139	148	351	802	484	428
normalized size	1	1.	0.86	0.92	2.18	4.98	3.01	2.66
time (sec)	N/A	0.281	0.352	0.022	1.102	1.651	60.839	1.348

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	164	100	305	740	204	331
normalized size	1	1.	1.78	1.09	3.32	8.04	2.22	3.6
time (sec)	N/A	0.086	0.044	0.021	1.047	1.632	43.727	1.277

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	92	111	247	522	332	319
normalized size	1	1.	0.74	0.89	1.98	4.18	2.66	2.55
time (sec)	N/A	0.162	0.166	0.017	1.117	1.629	19.201	1.19

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	146	82	239	4263	0	301
normalized size	1	1.	1.59	0.89	2.6	46.34	0.	3.27
time (sec)	N/A	0.093	0.041	0.039	1.051	1.921	0.	1.303

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	77	91	197	562	0	273
normalized size	1	1.	0.75	0.88	1.91	5.46	0.	2.65
time (sec)	N/A	0.194	0.306	0.038	1.088	1.708	0.	1.293

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	144	74	275	6047	0	275
normalized size	1	1.	1.57	0.8	2.99	65.73	0.	2.99
time (sec)	N/A	0.135	0.046	0.048	1.118	1.982	0.	1.284

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	68	75	223	761	0	207
normalized size	1	1.	0.75	0.82	2.45	8.36	0.	2.27
time (sec)	N/A	0.165	0.325	0.044	1.048	1.748	0.	1.317

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	186	79	316	8741	0	262
normalized size	1	1.	1.84	0.78	3.13	86.54	0.	2.59
time (sec)	N/A	0.151	0.043	0.057	1.126	2.072	0.	1.318

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	67	74	360	1185	0	234
normalized size	1	1.	0.8	0.88	4.29	14.11	0.	2.79
time (sec)	N/A	0.146	0.635	0.049	1.065	1.714	0.	1.279

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	240	92	404	11957	0	338
normalized size	1	1.	2.16	0.83	3.64	107.72	0.	3.05
time (sec)	N/A	0.174	0.036	0.048	1.041	2.119	0.	1.278

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	288	258	810	2898	0	934
normalized size	1	1.	1.31	1.17	3.68	13.17	0.	4.25
time (sec)	N/A	0.218	2.266	0.096	1.076	1.787	0.	1.794

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	185	218	676	2207	0	779
normalized size	1	1.	1.01	1.19	3.69	12.06	0.	4.26
time (sec)	N/A	0.175	2.47	0.059	1.08	1.731	0.	1.721

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	157	176	539	1620	377	621
normalized size	1	1.	1.1	1.23	3.77	11.33	2.64	4.34
time (sec)	N/A	0.157	0.901	0.02	1.045	1.736	169.693	1.678

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	139	148	441	10637	0	570
normalized size	1	1.	0.88	0.94	2.79	67.32	0.	3.61
time (sec)	N/A	0.135	0.381	0.039	1.066	2.04	0.	1.65

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	155	130	451	13697	0	455
normalized size	1	1.	1.05	0.88	3.05	92.55	0.	3.07
time (sec)	N/A	0.209	0.34	0.049	1.079	2.122	0.	1.669

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	173	125	459	17403	0	404
normalized size	1	1.	1.22	0.88	3.23	122.56	0.	2.85
time (sec)	N/A	0.273	0.383	0.046	1.058	2.428	0.	1.743

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	223	128	527	22631	0	463
normalized size	1	1.	1.43	0.82	3.38	145.07	0.	2.97
time (sec)	N/A	0.304	0.703	0.047	1.114	2.397	0.	1.684

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	219	143	625	30027	0	473
normalized size	1	1.	1.28	0.84	3.65	175.6	0.	2.77
time (sec)	N/A	0.332	1.694	0.083	1.132	2.77	0.	1.703

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	265	166	774	0	0	653
normalized size	1	1.	1.4	0.88	4.1	0.	0.	3.46
time (sec)	N/A	0.369	2.473	0.081	1.148	0.	0.	1.704

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	246	202	972	0	0	730
normalized size	1	1.	1.12	0.92	4.42	0.	0.	3.32
time (sec)	N/A	0.398	2.069	0.085	1.152	0.	0.	1.651

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	189	240	597	1688	0	756
normalized size	1	1.	0.74	0.94	2.34	6.62	0.	2.96
time (sec)	N/A	0.557	0.68	0.054	1.218	1.405	0.	1.703

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	156	193	464	1200	666	601
normalized size	1	1.	0.74	0.91	2.2	5.69	3.16	2.85
time (sec)	N/A	0.387	0.39	0.056	1.112	1.484	103.123	1.141

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	134	163	383	1247	0	531
normalized size	1	1.	0.74	0.9	2.12	6.89	0.	2.93
time (sec)	N/A	0.438	0.771	0.039	1.092	1.445	0.	1.704

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	131	137	381	1476	0	423
normalized size	1	1.	0.81	0.85	2.37	9.17	0.	2.63
time (sec)	N/A	0.384	0.782	0.048	1.108	1.368	0.	1.704

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	110	126	485	2006	0	414
normalized size	1	1.	0.74	0.85	3.28	13.55	0.	2.8
time (sec)	N/A	0.355	1.16	0.046	1.066	1.523	0.	1.707

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	106	121	725	2464	0	356
normalized size	1	1.	0.8	0.91	5.45	18.53	0.	2.68
time (sec)	N/A	0.289	0.692	0.089	1.148	1.493	0.	1.707

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	115	130	1137	3646	0	495
normalized size	1	1.	0.82	0.93	8.12	26.04	0.	3.54
time (sec)	N/A	0.223	0.579	0.084	1.217	1.567	0.	1.715

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	239	145	1743	4510	0	485
normalized size	1	1.	1.63	0.99	11.86	30.68	0.	3.3
time (sec)	N/A	0.149	6.105	0.086	1.24	1.721	0.	1.699

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	350	177	2587	6778	0	760
normalized size	1	1.	2.43	1.23	17.97	47.07	0.	5.28
time (sec)	N/A	0.131	3.207	0.098	1.304	1.803	0.	1.667

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	404	218	3687	8951	0	838
normalized size	1	1.	2.22	1.2	20.26	49.18	0.	4.6
time (sec)	N/A	0.159	4.51	0.085	1.381	1.89	0.	1.707

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	494	258	5021	11491	0	917
normalized size	1	1.	2.24	1.17	22.72	52.	0.	4.15
time (sec)	N/A	0.192	6.181	0.085	1.297	1.909	0.	1.654

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	548	298	6592	13925	0	995
normalized size	1	1.	2.21	1.2	26.58	56.15	0.	4.01
time (sec)	N/A	0.223	6.168	0.09	1.29	2.261	0.	1.664

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	390	270	0	3729	0	0
normalized size	1	1.	2.64	1.82	0.	25.2	0.	0.
time (sec)	N/A	0.254	0.336	0.082	0.	2.639	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	235	175	0	2755	0	0
normalized size	1	1.	1.69	1.26	0.	19.82	0.	0.
time (sec)	N/A	0.198	0.26	0.048	0.	2.111	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	365	142	0	2202	0	0
normalized size	1	1.	3.17	1.23	0.	19.15	0.	0.
time (sec)	N/A	0.115	0.177	0.029	0.	1.902	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	221	126	0	2256	0	0
normalized size	1	1.	1.77	1.01	0.	18.05	0.	0.
time (sec)	N/A	0.104	0.168	0.04	0.	1.981	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	385	159	0	2431	0	0
normalized size	1	1.	2.83	1.17	0.	17.88	0.	0.
time (sec)	N/A	0.163	0.249	0.053	0.	2.261	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	265	190	0	4716	0	0
normalized size	1	1.	1.44	1.03	0.	25.63	0.	0.
time (sec)	N/A	0.201	0.379	0.066	0.	2.586	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	158	223	0	3123	0	82
normalized size	1	1.	0.9	1.27	0.	17.85	0.	0.47
time (sec)	N/A	0.258	0.867	0.051	0.	2.398	0.	1.308

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	143	144	0	2128	0	18
normalized size	1	1.	1.13	1.13	0.	16.76	0.	0.14
time (sec)	N/A	0.206	0.45	0.033	0.	2.164	0.	1.323

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	127	94	0	2130	0	1
normalized size	1	1.	1.02	0.75	0.	17.04	0.	0.01
time (sec)	N/A	0.121	0.349	0.031	0.	2.213	0.	1.616

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	128	102	0	2130	0	0
normalized size	1	1.	1.11	0.89	0.	18.52	0.	0.
time (sec)	N/A	0.098	0.239	0.033	0.	2.163	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	143	135	0	2840	0	28
normalized size	1	1.	1.03	0.97	0.	20.43	0.	0.2
time (sec)	N/A	0.181	0.814	0.059	0.	2.286	0.	2.003

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	165	179	0	5195	0	46
normalized size	1	1.	1.11	1.21	0.	35.1	0.	0.31
time (sec)	N/A	0.211	2.193	0.075	0.	2.219	0.	2.853

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	615	1191	0	17667	0	0
normalized size	1	1.	2.62	5.07	0.	75.18	0.	0.
time (sec)	N/A	0.486	0.949	0.082	0.	3.045	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	737	1200	0	14307	0	0
normalized size	1	1.	3.51	5.71	0.	68.13	0.	0.
time (sec)	N/A	0.37	0.663	0.184	0.	2.538	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	597	1116	0	13905	0	0
normalized size	1	1.	2.75	5.14	0.	64.08	0.	0.
time (sec)	N/A	0.294	0.551	0.077	0.	2.66	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	422	767	0	11888	0	0
normalized size	1	1.	2.27	4.12	0.	63.91	0.	0.
time (sec)	N/A	0.205	0.485	0.058	0.	2.364	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	597	1112	0	13570	0	0
normalized size	1	1.	2.7	5.03	0.	61.4	0.	0.
time (sec)	N/A	0.299	0.354	0.072	0.	2.731	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	761	966	0	17785	0	0
normalized size	1	1.	2.34	2.97	0.	54.72	0.	0.
time (sec)	N/A	0.361	0.838	0.096	0.	4.222	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	262	574	0	15757	0	201
normalized size	1	1.	0.82	1.79	0.	49.24	0.	0.63
time (sec)	N/A	0.461	4.637	0.063	0.	3.88	0.	1.193

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	238	716	0	13423	0	205
normalized size	1	1.	1.02	3.07	0.	57.61	0.	0.88
time (sec)	N/A	0.338	2.59	0.054	0.	3.555	0.	4.439

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	225	490	0	12467	0	171
normalized size	1	1.	1.15	2.51	0.	63.93	0.	0.88
time (sec)	N/A	0.249	4.137	0.046	0.	2.729	0.	4.337

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	253	708	0	14390	0	205
normalized size	1	1.	1.15	3.22	0.	65.41	0.	0.93
time (sec)	N/A	0.298	2.064	0.069	0.	3.52	0.	4.23

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	230	534	0	14862	0	171
normalized size	1	1.	1.1	2.54	0.	70.77	0.	0.81
time (sec)	N/A	0.246	2.932	0.062	0.	3.535	0.	4.289

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	272	765	0	20254	0	321
normalized size	1	1.	1.15	3.23	0.	85.46	0.	1.35
time (sec)	N/A	0.534	1.873	0.09	0.	4.285	0.	5.799

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	1021	3542	0	0	0	0
normalized size	1	1.	3.24	11.24	0.	0.	0.	0.
time (sec)	N/A	0.577	1.689	0.099	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	802	2563	0	0	0	0
normalized size	1	1.	2.77	8.84	0.	0.	0.	0.
time (sec)	N/A	0.467	1.295	0.088	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	1019	3511	0	0	0	0
normalized size	1	1.	3.26	11.22	0.	0.	0.	0.
time (sec)	N/A	0.498	1.91	0.096	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	802	2916	0	0	0	0
normalized size	1	1.	2.78	10.12	0.	0.	0.	0.
time (sec)	N/A	0.519	1.3	0.082	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	1018	3512	0	0	0	0
normalized size	1	1.	3.25	11.22	0.	0.	0.	0.
time (sec)	N/A	0.463	1.367	0.106	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	617	617	1189	3159	0	0	0	0
normalized size	1	1.	1.93	5.12	0.	0.	0.	0.
time (sec)	N/A	0.819	5.757	0.138	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	331	2236	0	0	0	527
normalized size	1	1.	1.04	7.01	0.	0.	0.	1.65
time (sec)	N/A	0.485	3.953	0.092	0.	0.	0.	17.417

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	351	2681	0	0	0	609
normalized size	1	1.	1.02	7.77	0.	0.	0.	1.77
time (sec)	N/A	0.733	3.307	0.088	0.	0.	0.	18.204

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	316	2214	0	0	0	489
normalized size	1	1.	1.01	7.05	0.	0.	0.	1.56
time (sec)	N/A	0.648	4.751	0.083	0.	0.	0.	17.54

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	343	2670	0	0	0	608
normalized size	1	1.	0.99	7.67	0.	0.	0.	1.75
time (sec)	N/A	0.664	4.873	0.095	0.	0.	0.	17.205

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	333	2290	0	0	0	529
normalized size	1	1.	1.04	7.16	0.	0.	0.	1.65
time (sec)	N/A	0.609	2.988	0.103	0.	0.	0.	17.829

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	357	2747	0	0	0	660
normalized size	1	1.	0.99	7.65	0.	0.	0.	1.84
time (sec)	N/A	1.158	3.456	0.158	0.	0.	0.	23.584

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	55	93	382	0	65
normalized size	1	1.	0.96	2.2	3.72	15.28	0.	2.6
time (sec)	N/A	0.018	0.1	0.02	1.541	2.041	0.	1.149

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	45	44	0	1874	0	293
normalized size	1	1.	0.26	0.25	0.	10.65	0.	1.66
time (sec)	N/A	0.159	0.072	0.024	0.	2.257	0.	1.242

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	141	113	0	0	0	0
normalized size	1	1.	0.32	0.26	0.	0.	0.	0.
time (sec)	N/A	0.975	0.326	0.032	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	134	128	0	0	0	1
normalized size	1	1.	0.77	0.73	0.	0.	0.	0.01
time (sec)	N/A	0.279	0.169	0.03	0.	0.	0.	1.379

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	160	162	0	0	0	1
normalized size	1	1.	0.65	0.66	0.	0.	0.	0.
time (sec)	N/A	0.517	0.246	0.033	0.	0.	0.	1.904

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	439	124	0	11367	0	0
normalized size	1	1.	1.81	0.51	0.	46.97	0.	0.
time (sec)	N/A	0.518	0.916	0.033	0.	3.486	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	87	61	0	2303	0	14
normalized size	1	1.	1.23	0.86	0.	32.44	0.	0.2
time (sec)	N/A	0.138	0.201	0.033	0.	1.847	0.	1.274

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	127	64	0	11979	0	1
normalized size	1	1.	0.98	0.5	0.	92.86	0.	0.01
time (sec)	N/A	0.198	0.123	0.036	0.	2.875	0.	1.279

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	437	124	0	11359	0	0
normalized size	1	1.	1.92	0.54	0.	49.82	0.	0.
time (sec)	N/A	0.417	0.868	0.036	0.	4.086	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	70	160	0	505	0	193
normalized size	1	1.	0.84	1.93	0.	6.08	0.	2.33
time (sec)	N/A	0.1	0.431	0.032	0.	1.744	0.	1.175

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	99	0	2198	0	65
normalized size	1	1.	0.93	1.43	0.	31.86	0.	0.94
time (sec)	N/A	0.077	0.483	0.033	0.	1.858	0.	1.389

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	67	46	65	124	39
normalized size	1	1.	1.06	3.72	2.56	3.61	6.89	2.17
time (sec)	N/A	0.052	0.003	0.025	1.031	1.476	10.048	1.141

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	78	34	39	153	38
normalized size	1	1.	0.9	3.9	1.7	1.95	7.65	1.9
time (sec)	N/A	0.05	0.002	0.022	1.053	1.462	6.117	1.131

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	23	15	17	19
normalized size	1	1.	1.	1.17	3.83	2.5	2.83	3.17
time (sec)	N/A	0.044	0.002	0.012	1.171	1.441	3.295	1.11

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	11	7	7	2	7
normalized size	1	1.	1.	2.2	1.4	1.4	0.4	1.4
time (sec)	N/A	0.042	0.	0.012	1.114	1.423	1.772	1.155

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	12	8	14	42	5	11
normalized size	1	1.	1.71	1.14	2.	6.	0.71	1.57
time (sec)	N/A	0.028	0.003	0.008	1.576	1.446	0.251	1.108

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	50	54	522	0	70
normalized size	1	1.	0.91	2.27	2.45	23.73	0.	3.18
time (sec)	N/A	0.045	0.004	0.021	1.523	1.487	0.	1.137

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	94	93	1607	0	90
normalized size	1	1.	0.97	2.69	2.66	45.91	0.	2.57
time (sec)	N/A	0.06	0.004	0.026	1.517	1.491	0.	1.158

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	63	95	205	306	250	221
normalized size	1	1.	0.71	1.07	2.3	3.44	2.81	2.48
time (sec)	N/A	0.057	0.168	0.059	1.013	1.488	4.286	1.2

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	48	65	184	220	85	166
normalized size	1	1.	1.04	1.41	4.	4.78	1.85	3.61
time (sec)	N/A	0.042	0.139	0.053	1.013	1.482	2.047	1.174

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	43	70	103	153	150	124
normalized size	1	1.	0.7	1.15	1.69	2.51	2.46	2.03
time (sec)	N/A	0.049	0.07	0.026	1.007	1.459	1.14	1.126

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	39	25	35	103	36	97
normalized size	1	1.	1.39	0.89	1.25	3.68	1.29	3.46
time (sec)	N/A	0.022	0.012	0.01	1.033	1.449	0.487	1.133

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	37	39	76	282	0	61
normalized size	1	1.	1.32	1.39	2.71	10.07	0.	2.18
time (sec)	N/A	0.032	0.014	0.028	1.519	1.515	0.	1.128

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	36	29	63	101	0	46
normalized size	1	1.	1.89	1.53	3.32	5.32	0.	2.42
time (sec)	N/A	0.034	0.014	0.056	1.05	1.51	0.	1.151

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	71	82	184	903	0	143
normalized size	1	1.	1.69	1.95	4.38	21.5	0.	3.4
time (sec)	N/A	0.04	0.023	0.069	1.525	1.523	0.	1.184

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	44	65	250	427	0	63
normalized size	1	1.	1.38	2.03	7.81	13.34	0.	1.97
time (sec)	N/A	0.034	0.007	0.062	1.045	1.387	0.	1.186

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	60	124	308	2838	0	209
normalized size	1	1.	0.86	1.77	4.4	40.54	0.	2.99
time (sec)	N/A	0.047	0.149	0.072	1.537	1.6	0.	1.152

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	102	85	656	930	0	112
normalized size	1	1.	1.89	1.57	12.15	17.22	0.	2.07
time (sec)	N/A	0.047	0.05	0.066	1.07	1.458	0.	1.155

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	98	172	304	532	481	354
normalized size	1	1.	0.62	1.08	1.91	3.35	3.03	2.23
time (sec)	N/A	0.174	0.309	0.031	1.11	1.49	12.886	1.234

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	128	327	463	136	332
normalized size	1	1.	0.86	1.73	4.42	6.26	1.84	4.49
time (sec)	N/A	0.072	0.115	0.031	1.082	1.464	6.897	1.221

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	79	134	231	347	314	277
normalized size	1	1.	0.66	1.13	1.94	2.92	2.64	2.33
time (sec)	N/A	0.143	0.295	0.032	1.152	1.447	4.585	1.186

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	41	61	261	58	221
normalized size	1	1.	0.9	0.84	1.24	5.33	1.18	4.51
time (sec)	N/A	0.037	0.056	0.011	1.165	1.478	2.017	1.173

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	70	89	180	1143	0	159
normalized size	1	1.	1.27	1.62	3.27	20.78	0.	2.89
time (sec)	N/A	0.058	0.228	0.038	1.724	1.604	0.	1.182

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	71	161	235	0	184
normalized size	1	1.	0.94	1.34	3.04	4.43	0.	3.47
time (sec)	N/A	0.095	0.292	0.037	1.216	1.535	0.	1.189

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	253	169	316	1916	0	225
normalized size	1	1.	3.95	2.64	4.94	29.94	0.	3.52
time (sec)	N/A	0.08	7.367	0.047	1.893	1.626	0.	1.204

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	57	96	360	495	0	132
normalized size	1	1.	1.21	2.04	7.66	10.53	0.	2.81
time (sec)	N/A	0.063	0.332	0.046	1.116	1.482	0.	1.166

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	303	237	468	3672	0	297
normalized size	1	1.	3.16	2.47	4.88	38.25	0.	3.09
time (sec)	N/A	0.093	5.946	0.051	1.725	1.592	0.	1.16

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	158	942	1040	0	173
normalized size	1	1.	1.21	2.77	16.53	18.25	0.	3.04
time (sec)	N/A	0.06	0.373	0.048	1.127	1.46	0.	1.222

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	715	302	652	7356	0	394
normalized size	1	1.	5.46	2.31	4.98	56.15	0.	3.01
time (sec)	N/A	0.133	10.168	0.082	1.791	1.714	0.	1.179

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	144	267	490	906	774	598
normalized size	1	1.	0.61	1.12	2.06	3.81	3.25	2.51
time (sec)	N/A	0.332	0.521	0.032	1.104	1.497	34.767	1.262

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	83	209	471	787	182	506
normalized size	1	1.	0.85	2.13	4.81	8.03	1.86	5.16
time (sec)	N/A	0.09	0.234	0.035	1.099	1.52	19.494	1.26

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	120	216	387	626	559	482
normalized size	1	1.	0.59	1.06	1.91	3.08	2.75	2.37
time (sec)	N/A	0.268	0.302	0.032	1.12	1.481	13.704	1.279

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	59	56	85	512	75	387
normalized size	1	1.	0.88	0.84	1.27	7.64	1.12	5.78
time (sec)	N/A	0.045	0.116	0.011	1.054	1.548	6.669	1.284

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	100	155	315	2795	0	309
normalized size	1	1.	1.16	1.8	3.66	32.5	0.	3.59
time (sec)	N/A	0.076	0.531	0.056	1.695	1.622	0.	1.279

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	78	131	290	435	0	284
normalized size	1	1.	0.85	1.42	3.15	4.73	0.	3.09
time (sec)	N/A	0.129	0.511	0.039	1.091	1.551	0.	1.308

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	347	286	482	4188	0	354
normalized size	1	1.	3.81	3.14	5.3	46.02	0.	3.89
time (sec)	N/A	0.096	7.326	0.055	1.592	1.812	0.	1.43

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	84	148	516	788	0	290
normalized size	1	1.	1.02	1.8	6.29	9.61	0.	3.54
time (sec)	N/A	0.108	0.738	0.054	1.144	1.598	0.	1.372

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	472	376	660	5520	0	412
normalized size	1	1.	4.58	3.65	6.41	53.59	0.	4.
time (sec)	N/A	0.132	9.938	0.062	1.71	1.765	0.	1.358

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	86	199	1112	1287	0	288
normalized size	1	1.	1.16	2.69	15.03	17.39	0.	3.89
time (sec)	N/A	0.08	0.732	0.058	1.242	1.568	0.	1.38

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	1192	467	872	9044	0	518
normalized size	1	1.	7.74	3.03	5.66	58.73	0.	3.36
time (sec)	N/A	0.153	14.262	0.062	1.832	1.814	0.	1.296

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	163	289	2368	2063	0	351
normalized size	1	1.	2.04	3.61	29.6	25.79	0.	4.39
time (sec)	N/A	0.074	0.569	0.089	1.281	1.54	0.	1.267

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	117	1656	0	7526	0	0
normalized size	1	1.	1.08	15.33	0.	69.69	0.	0.
time (sec)	N/A	0.113	0.477	0.081	0.	1.859	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	106	1497	0	4566	0	320
normalized size	1	1.	0.88	12.37	0.	37.74	0.	2.64
time (sec)	N/A	0.221	0.317	0.063	0.	1.759	0.	1.301

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	79	1148	0	3804	0	0
normalized size	1	1.	1.03	14.91	0.	49.4	0.	0.
time (sec)	N/A	0.093	0.245	0.059	0.	1.72	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	993	0	2222	0	194
normalized size	1	1.	0.99	12.26	0.	27.43	0.	2.4
time (sec)	N/A	0.133	0.15	0.054	0.	1.709	0.	1.33

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	732	0	1754	0	0
normalized size	1	1.	0.96	14.08	0.	33.73	0.	0.
time (sec)	N/A	0.07	0.043	0.049	0.	1.613	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	577	0	1088	0	93
normalized size	1	1.	1.	11.54	0.	21.76	0.	1.86
time (sec)	N/A	0.082	0.082	0.04	0.	1.625	0.	1.281

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	1207	128	0
normalized size	1	1.	1.	0.75	0.	37.72	4.	0.
time (sec)	N/A	0.037	0.01	0.01	0.	1.573	5.011	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	481	0	1341	0	289
normalized size	1	1.	0.92	8.15	0.	22.73	0.	4.9
time (sec)	N/A	0.067	0.126	0.058	0.	1.661	0.	1.431

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	335	0	1770	0	111
normalized size	1	1.	1.	5.58	0.	29.5	0.	1.85
time (sec)	N/A	0.079	0.154	0.06	0.	1.631	0.	1.225

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	91	662	0	4370	0	0
normalized size	1	1.	0.99	7.2	0.	47.5	0.	0.
time (sec)	N/A	0.105	0.222	0.068	0.	1.923	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	84	535	0	5762	0	192
normalized size	1	1.	0.95	6.08	0.	65.48	0.	2.18
time (sec)	N/A	0.105	0.491	0.07	0.	1.82	0.	1.411

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	139	1023	0	13526	0	0
normalized size	1	1.	1.01	7.41	0.	98.01	0.	0.
time (sec)	N/A	0.162	0.507	0.073	0.	2.41	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	119	907	0	14236	0	348
normalized size	1	1.	0.94	7.2	0.	112.98	0.	2.76
time (sec)	N/A	0.147	0.894	0.075	0.	2.133	0.	1.42

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	118	1659	0	8500	0	421
normalized size	1	1.	0.75	10.5	0.	53.8	0.	2.66
time (sec)	N/A	0.258	0.577	0.073	0.	1.906	0.	1.17

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	106	1403	0	6456	0	0
normalized size	1	1.	1.02	13.49	0.	62.08	0.	0.
time (sec)	N/A	0.138	0.283	0.065	0.	1.822	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	108	1116	0	3749	0	243
normalized size	1	1.	1.08	11.16	0.	37.49	0.	2.43
time (sec)	N/A	0.138	0.591	0.062	0.	1.752	0.	1.193

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	808	0	3937	0	0
normalized size	1	1.	0.97	10.49	0.	51.13	0.	0.
time (sec)	N/A	0.078	0.327	0.057	0.	1.656	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	404	0	3321	0	173
normalized size	1	1.	0.99	5.11	0.	42.04	0.	2.19
time (sec)	N/A	0.077	0.206	0.053	0.	1.68	0.	1.299

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	57	0	3372	428	0
normalized size	1	1.	0.97	0.86	0.	51.09	6.48	0.
time (sec)	N/A	0.047	0.046	0.013	0.	1.632	31.67	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	174	1080	0	5192	0	0
normalized size	1	1.	1.64	10.19	0.	48.98	0.	0.
time (sec)	N/A	0.108	0.377	0.082	0.	2.003	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	105	798	0	7185	0	200
normalized size	1	1.	0.92	7.	0.	63.03	0.	1.75
time (sec)	N/A	0.179	0.855	0.075	0.	2.089	0.	2.441

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	230	1265	0	15107	0	1322
normalized size	1	1.	1.46	8.06	0.	96.22	0.	8.42
time (sec)	N/A	0.193	0.875	0.089	0.	3.007	0.	1.613

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	130	998	0	17920	0	377
normalized size	1	1.	0.91	6.98	0.	125.31	0.	2.64
time (sec)	N/A	0.207	1.894	0.092	0.	2.667	0.	1.38

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	164	2048	0	12741	0	481
normalized size	1	1.	1.02	12.8	0.	79.63	0.	3.01
time (sec)	N/A	0.236	1.34	0.076	0.	2.557	0.	1.194

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	149	1884	0	13445	0	0
normalized size	1	1.	1.12	14.17	0.	101.09	0.	0.
time (sec)	N/A	0.125	0.352	0.062	0.	2.451	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	102	664	0	10175	0	331
normalized size	1	1.	0.89	5.82	0.	89.25	0.	2.9
time (sec)	N/A	0.095	0.654	0.064	0.	2.288	0.	1.361

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	114	1348	0	11478	0	0
normalized size	1	1.	0.97	11.52	0.	98.1	0.	0.
time (sec)	N/A	0.093	0.666	0.068	0.	2.331	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	124	1322	0	11760	0	365
normalized size	1	1.	0.87	9.24	0.	82.24	0.	2.55
time (sec)	N/A	0.126	0.888	0.06	0.	2.353	0.	1.398

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	79	85	0	9451	915	0
normalized size	1	1.	0.82	0.89	0.	98.45	9.53	0.
time (sec)	N/A	0.057	0.171	0.013	0.	2.188	175.048	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	321	2118	0	18737	0	0
normalized size	1	1.	2.02	13.32	0.	117.84	0.	0.
time (sec)	N/A	0.18	0.731	0.092	0.	3.158	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	165	1694	0	21442	0	509
normalized size	1	1.	0.96	9.85	0.	124.66	0.	2.96
time (sec)	N/A	0.265	1.079	0.084	0.	2.843	0.	1.398

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	222	2307	0	0	0	0
normalized size	1	1.	1.02	10.63	0.	0.	0.	0.
time (sec)	N/A	0.303	1.791	0.104	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	169	1892	0	0	0	608
normalized size	1	1.	0.83	9.32	0.	0.	0.	3.
time (sec)	N/A	0.341	2.2	0.108	0.	0.	0.	1.498

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	54	86	220	0	55
normalized size	1	1.	1.26	2.84	4.53	11.58	0.	2.89
time (sec)	N/A	0.045	0.076	0.023	1.703	1.569	0.	1.175

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	14	50	53	275	129	50
normalized size	1	1.	1.4	5.	5.3	27.5	12.9	5.
time (sec)	N/A	0.039	0.01	0.028	1.106	1.495	2.309	1.171

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	32	98	101	552	0	82
normalized size	1	1.	1.07	3.27	3.37	18.4	0.	2.73
time (sec)	N/A	0.059	0.069	0.032	1.532	1.513	0.	1.154

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	124	52	0	8343	0	0
normalized size	1	1.	1.06	0.44	0.	71.31	0.	0.
time (sec)	N/A	0.114	0.663	0.09	0.	2.459	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	96	62	0	6229	0	0
normalized size	1	1.	1.33	0.86	0.	86.51	0.	0.
time (sec)	N/A	0.056	0.245	0.013	0.	2.091	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	130	51	0	13256	0	0
normalized size	1	1.	1.53	0.6	0.	155.95	0.	0.
time (sec)	N/A	0.092	0.71	0.091	0.	2.644	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	175	35	0	3534	0	938
normalized size	1	1.	2.03	0.41	0.	41.09	0.	10.91
time (sec)	N/A	0.096	1.514	0.085	0.	1.971	0.	1.298

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	684	35	0	9495	0	3009
normalized size	1	1.	4.53	0.23	0.	62.88	0.	19.93
time (sec)	N/A	0.143	12.423	0.104	0.	3.429	0.	1.417

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	211	521	0	0	0	0
normalized size	1	1.	0.7	1.73	0.	0.	0.	0.
time (sec)	N/A	0.298	1.376	0.106	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	168	339	0	0	0	0
normalized size	1	1.	0.75	1.52	0.	0.	0.	0.
time (sec)	N/A	0.203	0.698	0.085	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	0	0	0
normalized size	1	1.	1.15	2.33	0.	0.	0.	0.
time (sec)	N/A	0.036	0.079	0.	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	148	177	0	0	0	0
normalized size	1	1.	2.11	2.53	0.	0.	0.	0.
time (sec)	N/A	0.084	0.484	0.101	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	204	318	0	0	0	0
normalized size	1	1.	0.99	1.54	0.	0.	0.	0.
time (sec)	N/A	0.167	2.837	0.132	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	149	77	0	11570	0	0
normalized size	1	1.	0.95	0.49	0.	73.69	0.	0.
time (sec)	N/A	0.129	1.2	0.085	0.	3.632	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	90	0	8070	0	0
normalized size	1	1.	0.89	0.87	0.	77.6	0.	0.
time (sec)	N/A	0.069	0.485	0.011	0.	2.758	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	142	63	0	16424	0	0
normalized size	1	1.	1.14	0.5	0.	131.39	0.	0.
time (sec)	N/A	0.139	0.453	0.085	0.	4.022	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	150	63	0	19290	0	0
normalized size	1	1.	1.13	0.47	0.	145.04	0.	0.
time (sec)	N/A	0.146	0.728	0.091	0.	4.642	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	66	63	0	7839	0	0
normalized size	1	1.	0.52	0.5	0.	62.21	0.	0.
time (sec)	N/A	0.121	0.109	0.107	0.	3.801	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	959	63	0	19302	0	0
normalized size	1	1.	4.68	0.31	0.	94.16	0.	0.
time (sec)	N/A	0.172	15.325	0.154	0.	10.114	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	256	730	0	0	0	0
normalized size	1	1.	0.72	2.04	0.	0.	0.	0.
time (sec)	N/A	0.393	2.531	0.167	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	213	535	0	0	0	0
normalized size	1	1.	0.71	1.79	0.	0.	0.	0.
time (sec)	N/A	0.291	1.327	0.12	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	169	416	0	0	0	0
normalized size	1	1.	0.97	2.39	0.	0.	0.	0.
time (sec)	N/A	0.19	0.633	0.	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	160	334	0	0	0	0
normalized size	1	1.	0.76	1.59	0.	0.	0.	0.
time (sec)	N/A	0.201	0.939	0.115	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	197	324	0	0	0	0
normalized size	1	1.	1.02	1.68	0.	0.	0.	0.
time (sec)	N/A	0.188	1.965	0.116	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	77	35	0	6375	0	0
normalized size	1	1.	0.97	0.44	0.	80.7	0.	0.
time (sec)	N/A	0.09	0.103	0.073	0.	2.482	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	0	5052	0	0
normalized size	1	1.	1.	0.89	0.	132.95	0.	0.
time (sec)	N/A	0.047	0.016	0.013	0.	2.105	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	0	1615	0	113
normalized size	1	1.	1.	0.76	0.	35.11	0.	2.46
time (sec)	N/A	0.069	0.031	0.079	0.	1.874	0.	1.439

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	443	35	0	4010	0	976
normalized size	1	1.	4.57	0.36	0.	41.34	0.	10.06
time (sec)	N/A	0.107	9.479	0.096	0.	2.363	0.	2.01

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	179	344	0	0	0	0
normalized size	1	1.	0.74	1.43	0.	0.	0.	0.
time (sec)	N/A	0.219	0.822	0.109	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	95	86	0	0	0	0
normalized size	1	1.	0.54	0.49	0.	0.	0.	0.
time (sec)	N/A	0.148	0.241	0.082	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	0	0	0
normalized size	1	1.	1.13	1.43	0.	0.	0.	0.
time (sec)	N/A	0.038	0.069	0.	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	159	133	0	0	0	0
normalized size	1	1.	0.99	0.83	0.	0.	0.	0.
time (sec)	N/A	0.181	0.622	0.138	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	219	343	0	0	0	0
normalized size	1	1.	1.	1.57	0.	0.	0.	0.
time (sec)	N/A	0.195	2.152	0.22	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	89	35	0	7876	0	0
normalized size	1	1.	1.16	0.45	0.	102.29	0.	0.
time (sec)	N/A	0.099	0.17	0.065	0.	2.726	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	319	641	0	211
normalized size	1	1.	1.	0.97	11.	22.1	0.	7.28
time (sec)	N/A	0.044	0.028	0.012	1.656	1.829	0.	1.245

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	315	101	0	4262	0	385
normalized size	1	1.	3.71	1.19	0.	50.14	0.	4.53
time (sec)	N/A	0.097	7.552	0.115	0.	2.482	0.	1.474

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	231	95	0	11344	0	0
normalized size	1	1.	1.63	0.67	0.	79.89	0.	0.
time (sec)	N/A	0.165	5.364	0.149	0.	4.657	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	196	498	0	0	0	0
normalized size	1	1.	0.6	1.53	0.	0.	0.	0.
time (sec)	N/A	0.301	1.07	0.118	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	155	322	0	0	0	0
normalized size	1	1.	0.64	1.32	0.	0.	0.	0.
time (sec)	N/A	0.218	0.602	0.116	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	143	181	0	0	0	0
normalized size	1	1.	1.57	1.99	0.	0.	0.	0.
time (sec)	N/A	0.101	0.307	0.097	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	100	252	0	0	0	0
normalized size	1	1.	0.87	2.19	0.	0.	0.	0.
time (sec)	N/A	0.061	0.143	0.001	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	178	345	0	0	0	0
normalized size	1	1.	0.82	1.59	0.	0.	0.	0.
time (sec)	N/A	0.203	1.177	0.148	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	126	65	0	15815	0	0
normalized size	1	1.	0.94	0.49	0.	118.02	0.	0.
time (sec)	N/A	0.141	0.864	0.082	0.	5.854	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	50	65	1251	2267	0	801
normalized size	1	1.	0.68	0.89	17.14	31.05	0.	10.97
time (sec)	N/A	0.093	0.105	0.082	1.757	3.102	0.	1.638

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	47	56	655	2161	0	702
normalized size	1	1.	0.72	0.86	10.08	33.25	0.	10.8
time (sec)	N/A	0.058	0.045	0.011	1.655	3.099	0.	1.695

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	169	0	12320	0	1831
normalized size	1	1.	9.93	1.26	0.	91.94	0.	13.66
time (sec)	N/A	0.149	9.303	0.133	0.	5.135	0.	3.258

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	206	812	0	0	0	0
normalized size	1	1.	0.62	2.46	0.	0.	0.	0.
time (sec)	N/A	0.331	2.133	0.133	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	178	597	0	0	0	0
normalized size	1	1.	0.8	2.68	0.	0.	0.	0.
time (sec)	N/A	0.211	1.378	0.253	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	193	662	0	0	0	0
normalized size	1	1.	0.85	2.9	0.	0.	0.	0.
time (sec)	N/A	0.204	1.402	0.151	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	0	0	0
normalized size	1	1.	0.76	1.62	0.	0.	0.	0.
time (sec)	N/A	0.281	1.264	0.	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	260	1002	0	0	0	0
normalized size	1	1.	0.89	3.43	0.	0.	0.	0.
time (sec)	N/A	0.314	3.46	0.166	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	8.19	0.51	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	10.606	0.645	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	119	120	0	0	0	0	0
normalized size	1	0.95	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.202	0.453	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.026	0.341	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	3.354	0.255	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	5.454	0.295	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	9.884	0.482	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	10.074	0.365	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	2.137	0.145	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	4.189	0.257	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	7.957	0.256	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	220	780	0	6338	0	0
normalized size	1	1.	0.85	3.01	0.	24.47	0.	0.
time (sec)	N/A	0.296	0.444	0.105	0.	8.191	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	117	359	0	2182	0	0
normalized size	1	1.	0.86	2.64	0.	16.04	0.	0.
time (sec)	N/A	0.157	0.151	0.1	0.	7.326	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	89	0	563	68	92
normalized size	1	1.	0.95	2.07	0.	13.09	1.58	2.14
time (sec)	N/A	0.05	0.027	0.013	0.	6.355	1.9	1.237

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	229	206	0	0	0	0
normalized size	1	1.	0.8	0.72	0.	0.	0.	0.
time (sec)	N/A	0.482	0.248	0.108	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	288	955	0	12573	0	0
normalized size	1	1.	1.07	3.54	0.	46.57	0.	0.
time (sec)	N/A	0.323	0.619	0.218	0.	8.894	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	123	481	0	5027	0	0
normalized size	1	1.	0.87	3.39	0.	35.4	0.	0.
time (sec)	N/A	0.164	0.464	0.207	0.	7.246	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	144	0	1370	151	0
normalized size	1	1.	0.86	2.94	0.	27.96	3.08	0.
time (sec)	N/A	0.055	0.061	0.021	0.	1.776	4.926	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	280	567	0	0	0	0
normalized size	1	1.	0.73	1.48	0.	0.	0.	0.
time (sec)	N/A	0.631	0.733	0.234	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.142	0.254	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.055	0.22	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.009	0.224	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.111	5.743	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.052	5.335	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.008	5.571	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	23	41	61	165	0	34
normalized size	1	1.	1.35	2.41	3.59	9.71	0.	2.
time (sec)	N/A	0.032	0.014	0.029	1.109	1.782	0.	1.17

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	51	42	394	2361	0	108
normalized size	1	1.	0.81	0.67	6.25	37.48	0.	1.71
time (sec)	N/A	0.128	0.1	0.145	1.799	1.859	0.	1.301

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	42	143	829	0	72
normalized size	1	1.	0.76	1.11	3.76	21.82	0.	1.89
time (sec)	N/A	0.113	0.087	0.105	1.791	1.893	0.	1.255

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	43	365	0	35
normalized size	1	1.	1.	1.06	2.39	20.28	0.	1.94
time (sec)	N/A	0.069	0.04	0.026	1.758	1.806	0.	1.159

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	42	92	549	0	69
normalized size	1	1.	0.84	0.84	1.84	10.98	0.	1.38
time (sec)	N/A	0.098	0.059	0.089	1.781	1.853	0.	1.206

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	54	170	2056	0	120
normalized size	1	1.	0.89	0.62	1.95	23.63	0.	1.38
time (sec)	N/A	0.142	0.261	0.092	1.947	1.976	0.	1.259

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	75	85	1203	4517	0	134
normalized size	1	1.	0.62	0.71	10.02	37.64	0.	1.12
time (sec)	N/A	0.13	0.343	0.109	1.904	2.045	0.	1.403

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	55	69	522	2003	0	100
normalized size	1	1.	0.6	0.76	5.74	22.01	0.	1.1
time (sec)	N/A	0.125	0.194	0.098	1.675	1.962	0.	1.285

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	40	41	68	497	0	51
normalized size	1	1.	0.7	0.72	1.19	8.72	0.	0.89
time (sec)	N/A	0.109	0.053	0.088	1.761	1.971	0.	1.267

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	35	42	169	829	0	77
normalized size	1	1.	0.62	0.75	3.02	14.8	0.	1.38
time (sec)	N/A	0.114	0.075	0.085	1.734	1.788	0.	1.292

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	47	55	657	2361	0	108
normalized size	1	1.	0.52	0.6	7.22	25.95	0.	1.19
time (sec)	N/A	0.122	0.073	0.092	1.783	1.97	0.	1.272

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	67	65	1418	4725	0	140
normalized size	1	1.	0.54	0.52	11.44	38.1	0.	1.13
time (sec)	N/A	0.125	0.193	0.092	1.784	2.114	0.	1.299

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	43	41	602	3776	0	128
normalized size	1	1.	0.65	0.62	9.12	57.21	0.	1.94
time (sec)	N/A	0.129	0.09	0.102	2.084	1.895	0.	1.399

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	41	248	1692	0	88
normalized size	1	1.	0.74	0.98	5.9	40.29	0.	2.1
time (sec)	N/A	0.118	0.07	0.098	1.899	1.761	0.	1.344

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	45	428	0	39
normalized size	1	1.	1.	1.05	2.37	22.53	0.	2.05
time (sec)	N/A	0.07	0.032	0.024	1.774	1.754	0.	1.26

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	49	33	54	455	0	49
normalized size	1	1.	1.58	1.06	1.74	14.68	0.	1.58
time (sec)	N/A	0.086	0.053	0.07	1.866	1.842	0.	1.322

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	42	135	1409	0	109
normalized size	1	1.	0.98	0.64	2.05	21.35	0.	1.65
time (sec)	N/A	0.129	0.122	0.086	1.904	1.949	0.	1.367

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	66	68	848	3584	0	130
normalized size	1	1.	0.73	0.75	9.32	39.38	0.	1.43
time (sec)	N/A	0.144	0.133	0.108	1.955	2.048	0.	1.37

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	44	51	293	1343	0	85
normalized size	1	1.	0.71	0.82	4.73	21.66	0.	1.37
time (sec)	N/A	0.121	0.057	0.224	1.808	1.841	0.	1.314

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	136	428	0	39
normalized size	1	1.	1.	1.28	5.44	17.12	0.	1.56
time (sec)	N/A	0.104	0.037	0.07	1.849	1.874	0.	1.379

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	37	44	751	1692	0	88
normalized size	1	1.	0.61	0.72	12.31	27.74	0.	1.44
time (sec)	N/A	0.115	0.061	0.098	2.	1.75	0.	1.497

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	49	54	1662	3776	0	128
normalized size	1	1.	0.51	0.56	17.31	39.33	0.	1.33
time (sec)	N/A	0.123	0.082	0.113	2.242	1.895	0.	1.561

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	44	791	6649	0	132
normalized size	1	1.	0.75	0.65	11.63	97.78	0.	1.94
time (sec)	N/A	0.141	0.112	0.107	2.218	2.141	0.	1.733

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	34	44	362	3571	0	92
normalized size	1	1.	0.77	1.	8.23	81.16	0.	2.09
time (sec)	N/A	0.126	0.123	0.098	1.88	1.892	0.	1.536

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	82	1511	0	43
normalized size	1	1.	1.	0.95	3.9	71.95	0.	2.05
time (sec)	N/A	0.077	0.039	0.018	1.898	1.847	0.	1.316

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	41	44	103	706	0	84
normalized size	1	1.	0.77	0.83	1.94	13.32	0.	1.58
time (sec)	N/A	0.108	0.062	0.066	1.793	1.853	0.	1.351

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	36	135	1458	0	0
normalized size	1	1.	1.02	0.55	2.05	22.09	0.	0.
time (sec)	N/A	0.139	0.132	0.069	2.112	1.875	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	58	69	498	3698	0	126
normalized size	1	1.	0.55	0.65	4.7	34.89	0.	1.19
time (sec)	N/A	0.16	0.094	0.142	1.899	2.001	0.	1.422

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	46	51	433	652	0	58
normalized size	1	1.	0.72	0.8	6.77	10.19	0.	0.91
time (sec)	N/A	0.132	0.079	0.093	1.718	1.806	0.	1.43

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	1111	1511	0	43
normalized size	1	1.	0.76	0.92	29.24	39.76	0.	1.13
time (sec)	N/A	0.126	0.055	0.078	1.986	1.781	0.	1.652

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	41	67	2067	3571	0	92
normalized size	1	1.	0.53	0.87	26.84	46.38	0.	1.19
time (sec)	N/A	0.144	0.109	0.132	2.326	1.944	0.	1.803

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	51	57	2992	6649	0	132
normalized size	1	1.	0.44	0.5	26.02	57.82	0.	1.15
time (sec)	N/A	0.15	0.136	0.105	2.601	2.143	0.	1.943

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	151	43	0	12141	0	0
normalized size	1	1.	0.81	0.23	0.	64.93	0.	0.
time (sec)	N/A	0.225	0.572	0.341	0.	9.015	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	88	43	0	4352	0	0
normalized size	1	1.	0.7	0.34	0.	34.54	0.	0.
time (sec)	N/A	0.125	0.406	0.102	0.	7.675	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	65	41	0	1705	0	0
normalized size	1	1.	1.05	0.66	0.	27.5	0.	0.
time (sec)	N/A	0.064	0.052	0.079	0.	7.164	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	46	0	1656	0	0
normalized size	1	1.	0.98	0.85	0.	30.67	0.	0.
time (sec)	N/A	0.074	0.049	0.085	0.	3.907	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	69	58	0	3903	0	0
normalized size	1	1.	0.65	0.55	0.	36.82	0.	0.
time (sec)	N/A	0.122	0.363	0.214	0.	4.977	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	102	80	0	9559	0	0
normalized size	1	1.	0.61	0.48	0.	57.24	0.	0.
time (sec)	N/A	0.18	0.564	0.131	0.	5.962	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	214	369	0	0	0	0
normalized size	1	1.	0.73	1.26	0.	0.	0.	0.
time (sec)	N/A	0.306	1.947	0.177	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	150	233	0	0	0	0
normalized size	1	1.	0.89	1.39	0.	0.	0.	0.
time (sec)	N/A	0.177	0.494	0.135	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	0	0	0
normalized size	1	1.	1.15	2.33	0.	0.	0.	0.
time (sec)	N/A	0.038	0.084	0.	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	154	215	0	0	0	0
normalized size	1	1.	0.76	1.06	0.	0.	0.	0.
time (sec)	N/A	0.199	0.57	0.142	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	210	519	0	0	0	0
normalized size	1	1.	0.78	1.92	0.	0.	0.	0.
time (sec)	N/A	0.296	3.067	0.17	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	169	71	0	16471	0	0
normalized size	1	1.	0.73	0.31	0.	71.	0.	0.
time (sec)	N/A	0.29	1.636	0.122	0.	9.431	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	122	71	0	6527	0	0
normalized size	1	1.	0.78	0.46	0.	41.84	0.	0.
time (sec)	N/A	0.159	0.542	0.125	0.	7.387	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	86	69	0	2827	0	0
normalized size	1	1.	0.96	0.77	0.	31.41	0.	0.
time (sec)	N/A	0.09	0.146	0.296	0.	6.381	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	62	0	2696	0	0
normalized size	1	1.	0.88	0.79	0.	34.56	0.	0.
time (sec)	N/A	0.092	0.122	0.078	0.	3.797	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	90	84	0	6418	0	0
normalized size	1	1.	0.64	0.6	0.	45.84	0.	0.
time (sec)	N/A	0.148	0.446	0.124	0.	5.472	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	199	123	113	0	14438	0	0
normalized size	1	0.98	0.61	0.56	0.	71.12	0.	0.
time (sec)	N/A	0.232	0.79	0.254	0.	6.137	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	224	389	0	0	0	0
normalized size	1	1.	0.73	1.28	0.	0.	0.	0.
time (sec)	N/A	0.38	2.796	0.167	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	188	413	0	0	0	0
normalized size	1	1.	0.72	1.59	0.	0.	0.	0.
time (sec)	N/A	0.245	2.887	0.148	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	169	416	0	0	0	0
normalized size	1	1.	0.97	2.39	0.	0.	0.	0.
time (sec)	N/A	0.182	0.641	0.	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	184	327	0	0	0	0
normalized size	1	1.	0.72	1.28	0.	0.	0.	0.
time (sec)	N/A	0.272	2.126	0.156	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	229	540	0	0	0	0
normalized size	1	1.	0.75	1.76	0.	0.	0.	0.
time (sec)	N/A	0.359	4.701	0.288	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	116	43	0	9879	0	0
normalized size	1	1.	0.82	0.3	0.	69.57	0.	0.
time (sec)	N/A	0.193	0.463	0.125	0.	3.74	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	85	43	0	3425	0	0
normalized size	1	1.	0.96	0.48	0.	38.48	0.	0.
time (sec)	N/A	0.112	0.117	0.129	0.	2.747	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	41	0	1168	0	0
normalized size	1	1.	1.07	1.	0.	28.49	0.	0.
time (sec)	N/A	0.058	0.042	0.093	0.	2.354	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	0	1111	0	0
normalized size	1	1.	1.	1.06	0.	33.67	0.	0.
time (sec)	N/A	0.071	0.035	0.073	0.	2.228	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	72	44	0	2957	0	0
normalized size	1	1.	0.94	0.57	0.	38.4	0.	0.
time (sec)	N/A	0.102	0.202	0.107	0.	2.492	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	100	54	0	7682	0	0
normalized size	1	1.	0.79	0.43	0.	60.97	0.	0.
time (sec)	N/A	0.147	0.362	0.102	0.	2.846	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	206	366	0	0	0	0
normalized size	1	1.	0.94	1.67	0.	0.	0.	0.
time (sec)	N/A	0.196	2.105	0.289	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	109	239	0	0	0	0
normalized size	1	1.	0.7	1.53	0.	0.	0.	0.
time (sec)	N/A	0.181	0.382	0.161	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	0	0	0
normalized size	1	1.	1.13	1.43	0.	0.	0.	0.
time (sec)	N/A	0.037	0.071	0.001	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	105	217	0	0	0	0
normalized size	1	1.	0.51	1.05	0.	0.	0.	0.
time (sec)	N/A	0.197	0.389	0.179	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	208	522	0	0	0	0
normalized size	1	1.	0.73	1.83	0.	0.	0.	0.
time (sec)	N/A	0.305	3.549	0.187	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	113	103	0	24184	0	0
normalized size	1	1.	0.6	0.55	0.	129.33	0.	0.
time (sec)	N/A	0.251	0.468	0.225	0.	6.766	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	79	103	0	9603	0	0
normalized size	1	1.	0.65	0.84	0.	78.71	0.	0.
time (sec)	N/A	0.14	0.118	0.195	0.	3.543	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	93	0	3475	0	0
normalized size	1	1.	0.84	1.35	0.	50.36	0.	0.
time (sec)	N/A	0.074	0.078	0.123	0.	2.672	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	35	0	2955	0	0
normalized size	1	1.	0.81	0.61	0.	51.84	0.	0.
time (sec)	N/A	0.084	0.06	0.063	0.	2.386	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	69	43	0	7960	0	0
normalized size	1	1.	0.63	0.39	0.	72.36	0.	0.
time (sec)	N/A	0.136	0.103	0.104	0.	3.056	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	94	43	0	19003	0	0
normalized size	1	1.	0.56	0.26	0.	113.79	0.	0.
time (sec)	N/A	0.201	0.327	0.09	0.	4.582	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	212	354	0	0	0	0
normalized size	1	1.	0.77	1.29	0.	0.	0.	0.
time (sec)	N/A	0.276	2.184	0.204	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	158	256	0	0	0	0
normalized size	1	1.	0.73	1.18	0.	0.	0.	0.
time (sec)	N/A	0.213	1.282	0.185	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	100	252	0	0	0	0
normalized size	1	1.	0.87	2.19	0.	0.	0.	0.
time (sec)	N/A	0.068	0.15	0.	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	153	219	0	0	0	0
normalized size	1	1.	0.65	0.92	0.	0.	0.	0.
time (sec)	N/A	0.27	0.836	0.282	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	214	522	0	0	0	0
normalized size	1	1.	0.63	1.53	0.	0.	0.	0.
time (sec)	N/A	0.408	3.22	0.195	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	114	213	0	0	0	0
normalized size	1	1.	0.49	0.92	0.	0.	0.	0.
time (sec)	N/A	0.316	0.589	0.298	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	82	213	0	24677	0	0
normalized size	1	1.	0.5	1.31	0.	151.39	0.	0.
time (sec)	N/A	0.166	0.122	0.266	0.	6.75	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	60	173	0	9900	0	0
normalized size	1	1.	0.61	1.75	0.	100.	0.	0.
time (sec)	N/A	0.093	0.087	0.218	0.	3.48	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	49	65	0	7507	0	0
normalized size	1	1.	0.59	0.78	0.	90.45	0.	0.
time (sec)	N/A	0.1	0.065	0.086	0.	2.807	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	69	73	0	18669	0	0
normalized size	1	1.	0.48	0.51	0.	130.55	0.	0.
time (sec)	N/A	0.152	0.3	0.118	0.	4.733	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	117	73	0	0	0	0
normalized size	1	1.	0.56	0.35	0.	0.	0.	0.
time (sec)	N/A	0.225	0.424	0.122	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	252	661	0	0	0	0
normalized size	1	1.	0.76	1.98	0.	0.	0.	0.
time (sec)	N/A	0.399	3.37	0.217	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	215	799	0	0	0	0
normalized size	1	1.	0.78	2.92	0.	0.	0.	0.
time (sec)	N/A	0.28	2.601	0.192	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	0	0	0
normalized size	1	1.	0.76	1.62	0.	0.	0.	0.
time (sec)	N/A	0.288	1.256	0.	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	226	640	0	0	0	0
normalized size	1	1.	0.64	1.82	0.	0.	0.	0.
time (sec)	N/A	0.412	2.677	0.24	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	247	924	0	0	0	0
normalized size	1	1.	0.64	2.4	0.	0.	0.	0.
time (sec)	N/A	0.562	2.788	0.226	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	9.498	0.227	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	90	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.239	0.36	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.071	0.255	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.06	0.279	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	71	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.379	0.265	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	7.105	0.227	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	5.423	0.222	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	6.369	0.218	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	6.635	0.216	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	136	132	0	2762	0	282
normalized size	1	1.	0.89	0.87	0.	18.17	0.	1.86
time (sec)	N/A	0.252	0.338	0.06	0.	11.757	0.	1.238

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	0	0	0
normalized size	1	1.	1.	0.75	0.	0.	0.	0.
time (sec)	N/A	0.076	0.024	0.214	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	0	5184	0	0
normalized size	1	1.	1.	0.76	0.	115.2	0.	0.
time (sec)	N/A	0.082	0.023	0.042	0.	15.551	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	390	0	0
normalized size	1	1.	1.	0.83	0.	13.45	0.	0.
time (sec)	N/A	0.087	0.024	0.033	0.	1.835	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	0	552	0	0
normalized size	1	1.	0.96	0.81	0.	11.74	0.	0.
time (sec)	N/A	0.092	0.021	0.016	0.	1.818	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. The column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [182] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	21	0.143
2	A	3	2	1.	21	0.095
3	A	3	3	1.	21	0.143
4	A	2	1	1.	19	0.053
5	A	3	2	1.	12	0.167
6	A	2	2	1.	19	0.105
7	A	2	2	1.	21	0.095
8	A	2	2	1.	21	0.095
9	A	3	3	1.	21	0.143
10	A	6	6	1.	23	0.261
11	A	3	2	1.	23	0.087
12	A	2	2	1.06	23	0.087
13	A	3	2	1.	21	0.095
14	A	1	1	1.	14	0.071
15	A	4	3	1.	21	0.143
16	A	4	4	1.28	23	0.174
17	A	5	4	1.	23	0.174
18	A	4	3	1.	23	0.13
19	A	7	6	1.	23	0.261
20	A	3	2	1.	23	0.087
21	A	3	2	1.	23	0.087
22	A	3	2	1.	21	0.095
23	A	2	2	1.	14	0.143
24	A	4	3	1.	21	0.143
25	A	5	5	1.	23	0.217
26	A	5	4	1.	23	0.174
27	A	5	4	1.	23	0.174
28	A	4	3	1.	23	0.13
29	A	6	6	1.	23	0.261
30	A	4	3	1.	23	0.13
31	A	5	5	1.	23	0.217

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	3	3	1.	23	0.13
33	A	3	3	1.	23	0.13
34	A	2	2	1.	21	0.095
35	A	2	2	1.	14	0.143
36	A	4	4	1.	21	0.19
37	A	3	3	1.	23	0.13
38	A	5	5	1.	23	0.217
39	A	4	3	1.	23	0.13
40	A	6	6	1.	23	0.261
41	A	4	3	1.	23	0.13
42	A	5	5	1.	23	0.217
43	A	3	3	1.	23	0.13
44	A	4	4	1.	23	0.174
45	A	3	3	1.	21	0.143
46	A	4	4	1.	14	0.286
47	A	5	5	1.	21	0.238
48	A	4	4	1.	23	0.174
49	A	6	6	1.	23	0.261
50	A	5	4	1.	23	0.174
51	A	4	3	1.	23	0.13
52	A	4	4	1.	23	0.174
53	A	5	4	1.	23	0.174
54	A	4	3	1.	21	0.143
55	A	5	5	1.	14	0.357
56	A	6	6	1.	21	0.286
57	A	5	5	1.	23	0.217
58	A	7	6	1.	23	0.261
59	A	6	5	1.	23	0.217
60	A	3	3	1.	8	0.375
61	A	3	2	1.	8	0.25
62	A	3	2	1.	8	0.25
63	A	2	2	1.	10	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	4	4	1.	10	0.4
65	A	5	5	1.	10	0.5
66	A	5	5	1.	25	0.2
67	A	4	4	1.	23	0.174
68	A	6	5	1.	23	0.217
69	A	4	4	1.	25	0.16
70	A	5	5	1.	25	0.2
71	A	7	7	1.	25	0.28
72	A	6	6	1.	25	0.24
73	A	2	2	1.	16	0.125
74	A	7	7	1.	25	0.28
75	A	7	7	1.	25	0.28
76	A	6	5	1.	25	0.2
77	A	5	4	1.	23	0.174
78	A	7	6	1.	23	0.261
79	A	7	6	1.	25	0.24
80	A	5	4	1.	25	0.16
81	A	6	5	1.	25	0.2
82	A	8	7	1.	25	0.28
83	A	7	6	1.	25	0.24
84	A	6	6	1.	16	0.375
85	A	6	6	1.	25	0.24
86	A	7	7	1.	25	0.28
87	A	7	7	1.	16	0.438
88	A	3	3	1.	10	0.3
89	A	3	3	1.	12	0.25
90	A	1	1	1.	12	0.083
91	A	2	2	1.	10	0.2
92	A	2	2	1.	12	0.167
93	A	4	4	1.	10	0.4
94	A	4	4	1.	12	0.333
95	A	4	4	1.	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	6	6	1.	10	0.6
97	A	6	6	1.	12	0.5
98	A	4	4	1.	25	0.16
99	A	3	3	1.	23	0.13
100	A	3	3	1.	23	0.13
101	A	4	4	1.	25	0.16
102	A	6	6	1.	25	0.24
103	A	5	5	1.	25	0.2
104	A	2	2	1.	16	0.125
105	A	5	5	1.	25	0.2
106	A	7	7	1.	25	0.28
107	A	4	4	1.	25	0.16
108	A	2	2	1.	23	0.087
109	A	4	4	1.	23	0.174
110	A	6	6	1.	25	0.24
111	A	7	7	1.	25	0.28
112	A	6	6	1.	25	0.24
113	A	6	6	1.	25	0.24
114	A	4	4	1.	16	0.25
115	A	7	7	1.	25	0.28
116	A	5	5	1.	25	0.2
117	A	3	3	1.	25	0.12
118	A	3	3	1.	23	0.13
119	A	6	6	1.	23	0.261
120	A	7	7	1.	25	0.28
121	A	5	5	1.	25	0.2
122	A	7	6	1.	25	0.24
123	A	7	7	1.	16	0.438
124	A	8	8	1.	25	0.32
125	A	3	3	1.	10	0.3
126	A	1	1	1.	12	0.083
127	A	2	2	1.	10	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	3	3	1.	12	0.25
129	A	2	2	1.	12	0.167
130	A	3	3	1.	25	0.12
131	A	5	5	1.	23	0.217
132	A	4	4	1.	23	0.174
133	A	3	3	1.	21	0.143
134	A	3	3	1.	21	0.143
135	A	3	3	1.	23	0.13
136	A	3	3	1.	23	0.13
137	A	3	3	1.	23	0.13
138	A	3	3	1.	23	0.13
139	A	3	3	1.	23	0.13
140	A	3	3	1.	23	0.13
141	A	7	4	1.	21	0.19
142	A	8	4	1.	21	0.19
143	A	6	4	1.	21	0.19
144	A	6	4	1.	19	0.21
145	A	3	1	1.	12	0.083
146	A	5	4	1.	19	0.21
147	A	5	4	1.	21	0.19
148	A	4	3	1.	21	0.143
149	A	5	3	1.	21	0.143
150	A	10	4	1.	23	0.174
151	A	11	4	1.	23	0.174
152	A	8	5	1.	21	0.238
153	A	8	4	1.	14	0.286
154	A	7	5	1.	21	0.238
155	A	8	5	1.	23	0.217
156	A	6	4	1.	23	0.174
157	A	7	5	1.	23	0.217
158	A	8	6	1.	23	0.261
159	A	6	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	9	4	1.	23	0.174
161	A	13	4	1.	23	0.174
162	A	14	5	1.	21	0.238
163	A	10	4	1.	14	0.286
164	A	12	5	1.	21	0.238
165	A	10	6	1.	23	0.261
166	A	10	6	1.	23	0.261
167	A	9	6	1.	23	0.261
168	A	11	7	1.	23	0.304
169	A	8	5	1.	23	0.217
170	A	11	6	1.	23	0.261
171	A	15	6	1.	23	0.261
172	A	15	7	1.	23	0.304
173	A	14	5	1.	23	0.217
174	A	13	5	1.	23	0.217
175	A	11	5	1.	23	0.217
176	A	11	4	1.	21	0.19
177	A	11	4	1.	14	0.286
178	A	14	6	1.	21	0.286
179	A	15	6	1.	23	0.261
180	A	15	7	1.	23	0.304
181	A	16	7	1.	23	0.304
182	A	12	8	1.	8	1.
183	A	12	8	1.	10	0.8
184	A	6	6	1.	21	0.286
185	A	3	2	1.	21	0.095
186	A	5	5	1.	21	0.238
187	A	2	1	1.	19	0.053
188	A	4	2	1.	12	0.167
189	A	4	3	1.	19	0.158
190	A	4	4	1.	21	0.19
191	A	4	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	4	3	1.	21	0.143
193	A	4	4	1.	21	0.19
194	A	3	2	1.	21	0.095
195	A	5	5	1.	21	0.238
196	A	3	2	1.	23	0.087
197	A	7	6	1.	23	0.261
198	A	3	2	1.	21	0.095
199	A	6	5	1.	14	0.357
200	A	4	3	1.	21	0.143
201	A	6	5	1.	23	0.217
202	A	5	4	1.	23	0.174
203	A	6	5	1.	23	0.217
204	A	6	5	1.	23	0.217
205	A	5	4	1.	23	0.174
206	A	6	5	1.	23	0.217
207	A	3	2	1.	23	0.087
208	A	3	2	1.	23	0.087
209	A	3	2	1.	21	0.095
210	A	4	3	1.	21	0.143
211	A	5	4	1.	23	0.174
212	A	6	5	1.	23	0.217
213	A	7	5	1.	23	0.217
214	A	8	5	1.	23	0.217
215	A	8	5	1.	23	0.217
216	A	8	5	1.	23	0.217
217	A	9	6	1.	23	0.261
218	A	8	5	1.	14	0.357
219	A	8	5	1.	23	0.217
220	A	8	5	1.	23	0.217
221	A	7	5	1.	23	0.217
222	A	6	5	1.	23	0.217
223	A	5	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	4	3	1.	23	0.13
225	A	3	2	1.	23	0.087
226	A	3	2	1.	23	0.087
227	A	3	2	1.	23	0.087
228	A	3	2	1.	23	0.087
229	A	6	5	1.	24	0.208
230	A	6	5	1.	24	0.208
231	A	4	4	1.	24	0.167
232	A	4	4	1.	22	0.182
233	A	7	6	1.	22	0.273
234	A	7	6	1.	24	0.25
235	A	7	5	1.	24	0.208
236	A	7	5	1.	24	0.208
237	A	4	3	1.	24	0.125
238	A	4	3	1.	15	0.2
239	A	6	4	1.	24	0.167
240	A	6	4	1.	24	0.167
241	A	7	6	1.	24	0.25
242	A	5	5	1.	24	0.208
243	A	5	5	1.	24	0.208
244	A	5	5	1.	24	0.208
245	A	5	5	1.	22	0.227
246	A	11	7	1.	22	0.318
247	A	14	9	1.	24	0.375
248	A	6	5	1.	24	0.208
249	A	7	6	1.	24	0.25
250	A	5	4	1.	24	0.167
251	A	5	4	1.	15	0.267
252	A	7	5	1.	24	0.208
253	A	6	6	1.	24	0.25
254	A	6	6	1.	24	0.25
255	A	6	6	1.	24	0.25

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	6	5	1.	24	0.208
257	A	6	6	1.	22	0.273
258	A	16	7	1.	22	0.318
259	A	9	7	1.	24	0.292
260	A	6	5	1.	24	0.208
261	A	6	5	1.	24	0.208
262	A	6	5	1.	24	0.208
263	A	6	5	1.	15	0.333
264	A	8	6	1.	24	0.25
265	A	3	3	1.	10	0.3
266	A	10	6	1.	8	0.75
267	A	17	5	1.	10	0.5
268	A	7	3	1.	10	0.3
269	A	9	3	1.	10	0.3
270	A	17	6	1.	8	0.75
271	A	8	6	1.	8	0.75
272	A	9	3	1.	8	0.375
273	A	17	6	1.	10	0.6
274	A	7	3	1.	10	0.3
275	A	10	6	1.	10	0.6
276	A	3	2	1.	15	0.133
277	A	3	3	1.	15	0.2
278	A	2	2	1.	15	0.133
279	A	2	2	1.	15	0.133
280	A	2	2	1.	13	0.154
281	A	3	3	1.	13	0.231
282	A	4	3	1.	15	0.2
283	A	5	4	1.	21	0.19
284	A	3	2	1.	21	0.095
285	A	4	4	1.	21	0.19
286	A	2	1	1.	19	0.053
287	A	3	3	1.	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	3	3	1.	21	0.143
289	A	3	3	1.	21	0.143
290	A	2	1	1.	21	0.048
291	A	4	4	1.	21	0.19
292	A	3	2	1.	21	0.095
293	A	6	5	1.	23	0.217
294	A	3	2	1.	23	0.087
295	A	5	5	1.	23	0.217
296	A	3	2	1.	21	0.095
297	A	4	3	1.	21	0.143
298	A	5	4	1.	23	0.174
299	A	5	4	1.	23	0.174
300	A	4	3	1.	23	0.13
301	A	4	4	1.	23	0.174
302	A	3	2	1.	23	0.087
303	A	5	5	1.	23	0.217
304	A	7	6	1.	23	0.261
305	A	3	2	1.	23	0.087
306	A	6	6	1.	23	0.261
307	A	3	2	1.	21	0.095
308	A	4	3	1.	21	0.143
309	A	6	5	1.	23	0.217
310	A	5	4	1.	23	0.174
311	A	5	4	1.	23	0.174
312	A	6	5	1.	23	0.217
313	A	4	3	1.	23	0.13
314	A	5	5	1.	23	0.217
315	A	3	2	1.	23	0.087
316	A	4	3	1.	23	0.13
317	A	6	6	1.	23	0.261
318	A	4	3	1.	23	0.13
319	A	5	5	1.	23	0.217

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
320	A	3	3	1.	23	0.13
321	A	4	4	1.	23	0.174
322	A	2	2	1.	21	0.095
323	A	4	4	1.	21	0.19
324	A	3	3	1.	23	0.13
325	A	5	5	1.	23	0.217
326	A	4	3	1.	23	0.13
327	A	6	6	1.	23	0.261
328	A	4	3	1.	23	0.13
329	A	6	6	1.	23	0.261
330	A	5	4	1.	23	0.174
331	A	5	5	1.	23	0.217
332	A	3	3	1.	23	0.13
333	A	3	3	1.	23	0.13
334	A	3	3	1.	21	0.143
335	A	5	5	1.	21	0.238
336	A	5	4	1.	23	0.174
337	A	6	6	1.	23	0.261
338	A	5	4	1.	23	0.174
339	A	6	6	1.	23	0.261
340	A	4	4	1.	23	0.174
341	A	4	3	1.	23	0.13
342	A	4	4	1.	23	0.174
343	A	4	4	1.	23	0.174
344	A	4	3	1.	21	0.143
345	A	6	6	1.	21	0.286
346	A	6	5	1.	23	0.217
347	A	7	6	1.	23	0.261
348	A	6	5	1.	23	0.217
349	A	4	3	1.	15	0.2
350	A	3	3	1.	15	0.2
351	A	5	4	1.	15	0.267

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	5	5	1.	25	0.2
353	A	4	4	1.	23	0.174
354	A	6	6	1.	23	0.261
355	A	4	4	1.	25	0.16
356	A	5	5	1.	25	0.2
357	A	7	7	1.	25	0.28
358	A	6	6	1.	25	0.24
359	A	2	2	1.	16	0.125
360	A	2	2	1.	25	0.08
361	A	5	5	1.	25	0.2
362	A	6	5	1.	25	0.2
363	A	5	4	1.	23	0.174
364	A	7	7	1.	23	0.304
365	A	7	7	1.	25	0.28
366	A	5	4	1.	25	0.16
367	A	6	5	1.	25	0.2
368	A	8	7	1.	25	0.28
369	A	7	7	1.	25	0.28
370	A	6	6	1.	16	0.375
371	A	6	6	1.	25	0.24
372	A	5	5	1.	25	0.2
373	A	4	4	1.	25	0.16
374	A	3	3	1.	23	0.13
375	A	3	3	1.	23	0.13
376	A	4	4	1.	25	0.16
377	A	6	6	1.	25	0.24
378	A	5	5	1.	25	0.2
379	A	2	2	1.	16	0.125
380	A	7	7	1.	25	0.28
381	A	5	5	1.	25	0.2
382	A	4	4	1.	25	0.16
383	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
384	A	4	4	1.	23	0.174
385	A	6	6	1.	25	0.24
386	A	7	7	1.	25	0.28
387	A	6	6	1.	25	0.24
388	A	2	2	1.	25	0.08
389	A	4	4	1.	16	0.25
390	A	5	5	1.	25	0.2
391	A	5	5	1.	25	0.2
392	A	3	3	1.	25	0.12
393	A	3	3	1.	23	0.13
394	A	6	6	1.	23	0.261
395	A	7	7	1.	25	0.28
396	A	5	5	1.	25	0.2
397	A	5	5	1.	25	0.2
398	A	7	7	1.	16	0.438
399	A	6	6	1.	25	0.24
400	A	3	3	1.	25	0.12
401	A	5	5	1.	23	0.217
402	A	4	4	0.95	23	0.174
403	A	3	3	1.	21	0.143
404	A	3	3	1.	21	0.143
405	A	3	3	1.	23	0.13
406	A	3	3	1.	23	0.13
407	A	3	3	1.	23	0.13
408	A	3	3	1.	14	0.214
409	A	3	3	1.	23	0.13
410	A	3	3	1.	23	0.13
411	A	4	3	1.	25	0.12
412	A	4	3	1.	25	0.12
413	A	4	3	1.	23	0.13
414	A	19	13	1.	23	0.565
415	A	4	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
416	A	4	3	1.	25	0.12
417	A	4	3	1.	23	0.13
418	A	19	13	1.	23	0.565
419	A	6	4	1.	23	0.174
420	A	5	4	1.	23	0.174
421	A	2	2	1.	21	0.095
422	A	6	4	1.	23	0.174
423	A	5	4	1.	23	0.174
424	A	2	2	1.	21	0.095
425	A	4	4	1.	13	0.308
426	A	5	4	1.	25	0.16
427	A	5	4	1.	25	0.16
428	A	4	4	1.	23	0.174
429	A	5	5	1.	23	0.217
430	A	7	7	1.	25	0.28
431	A	7	6	1.	25	0.24
432	A	6	6	1.	25	0.24
433	A	5	5	1.	25	0.2
434	A	5	4	1.	25	0.16
435	A	5	4	1.	25	0.16
436	A	5	4	1.	25	0.16
437	A	5	4	1.	25	0.16
438	A	5	4	1.	25	0.16
439	A	4	4	1.	23	0.174
440	A	4	4	1.	23	0.174
441	A	6	6	1.	25	0.24
442	A	5	4	1.	25	0.16
443	A	4	4	1.	25	0.16
444	A	4	4	1.	25	0.16
445	A	4	3	1.	25	0.12
446	A	5	4	1.	25	0.16
447	A	5	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
448	A	5	4	1.	25	0.16
449	A	4	4	1.	23	0.174
450	A	5	5	1.	23	0.217
451	A	6	6	1.	25	0.24
452	A	5	5	1.	25	0.2
453	A	5	5	1.	25	0.2
454	A	4	4	1.	25	0.16
455	A	5	4	1.	25	0.16
456	A	5	4	1.	25	0.16
457	A	6	6	1.	25	0.24
458	A	5	5	1.	25	0.2
459	A	4	4	1.	23	0.174
460	A	4	4	1.	23	0.174
461	A	5	5	1.	25	0.2
462	A	6	6	1.	25	0.24
463	A	7	7	1.	25	0.28
464	A	6	6	1.	25	0.24
465	A	2	2	1.	16	0.125
466	A	6	6	1.	25	0.24
467	A	7	7	1.	25	0.28
468	A	7	6	1.	25	0.24
469	A	6	5	1.	25	0.2
470	A	5	4	1.	23	0.174
471	A	5	4	1.	23	0.174
472	A	6	5	1.	25	0.2
473	A	7	6	0.98	25	0.24
474	A	8	8	1.	25	0.32
475	A	7	7	1.	25	0.28
476	A	6	6	1.	16	0.375
477	A	7	7	1.	25	0.28
478	A	8	8	1.	25	0.32
479	A	5	5	1.	25	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
480	A	4	4	1.	25	0.16
481	A	3	3	1.	23	0.13
482	A	3	3	1.	23	0.13
483	A	4	4	1.	25	0.16
484	A	5	5	1.	25	0.2
485	A	5	5	1.	25	0.2
486	A	6	6	1.	25	0.24
487	A	2	2	1.	16	0.125
488	A	6	6	1.	25	0.24
489	A	7	7	1.	25	0.28
490	A	6	6	1.	25	0.24
491	A	5	5	1.	25	0.2
492	A	4	4	1.	23	0.174
493	A	4	4	1.	23	0.174
494	A	5	5	1.	25	0.2
495	A	6	6	1.	25	0.24
496	A	6	6	1.	25	0.24
497	A	5	5	1.	25	0.2
498	A	4	4	1.	16	0.25
499	A	7	7	1.	25	0.28
500	A	8	7	1.	25	0.28
501	A	7	6	1.	25	0.24
502	A	6	5	1.	25	0.2
503	A	5	4	1.	23	0.174
504	A	5	4	1.	23	0.174
505	A	6	5	1.	25	0.2
506	A	7	6	1.	25	0.24
507	A	7	6	1.	25	0.24
508	A	6	6	1.	25	0.24
509	A	7	7	1.	16	0.438
510	A	8	8	1.	25	0.32
511	A	9	8	1.	25	0.32

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
512	A	3	3	1.	25	0.12
513	A	3	3	1.	23	0.13
514	A	2	2	1.	21	0.095
515	A	2	2	1.	21	0.095
516	A	3	3	1.	23	0.13
517	A	3	3	1.	23	0.13
518	A	3	3	1.	23	0.13
519	A	3	3	1.	23	0.13
520	A	3	3	1.	23	0.13
521	A	12	11	1.	15	0.733
522	A	4	4	1.	15	0.267
523	A	5	5	1.	15	0.333
524	A	4	4	1.	15	0.267
525	A	5	5	1.	15	0.333

Chapter 3

Listing of integrals

3.1 $\int \sinh^4(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=89

$$\frac{(6a - 5b) \sinh^3(c + dx) \cosh(c + dx)}{24d} - \frac{(6a - 5b) \sinh(c + dx) \cosh(c + dx)}{16d} + \frac{1}{16}x(6a - 5b) + \frac{b \sinh^5(c + dx) \cosh(c + dx)}{6d}$$

[Out] $((6*a - 5*b)*x)/16 - ((6*a - 5*b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(16*d) + ((6*a - 5*b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x]^3)/(24*d) + (b*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x]^5)/(6*d)$

Rubi [A] time = 0.0548921, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3014, 2635, 8}

$$\frac{(6a - 5b) \sinh^3(c + dx) \cosh(c + dx)}{24d} - \frac{(6a - 5b) \sinh(c + dx) \cosh(c + dx)}{16d} + \frac{1}{16}x(6a - 5b) + \frac{b \sinh^5(c + dx) \cosh(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^4*(a + b*\text{Sinh}[c + d*x]^2), x]$

[Out] $((6*a - 5*b)*x)/16 - ((6*a - 5*b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(16*d) + ((6*a - 5*b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x]^3)/(24*d) + (b*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x]^5)/(6*d)$

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \sinh^4(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{b \cosh(c + dx) \sinh^5(c + dx)}{6d} + \frac{1}{6}(6a - 5b) \int \sinh^4(c + dx) dx \\ &= \frac{(6a - 5b) \cosh(c + dx) \sinh^3(c + dx)}{24d} + \frac{b \cosh(c + dx) \sinh^5(c + dx)}{6d} + \frac{1}{8}(-6a + 5b) \int \sinh^2(c + dx) dx \\ &= -\frac{(6a - 5b) \cosh(c + dx) \sinh(c + dx)}{16d} + \frac{(6a - 5b) \cosh(c + dx) \sinh^3(c + dx)}{24d} \\ &= \frac{1}{16}(6a - 5b)x - \frac{(6a - 5b) \cosh(c + dx) \sinh(c + dx)}{16d} + \frac{(6a - 5b) \cosh(c + dx) \sinh^3(c + dx)}{24d} \end{aligned}$$

Mathematica [A] time = 0.108146, size = 68, normalized size = 0.76

$$\frac{(45b - 48a) \sinh(2(c + dx)) + (6a - 9b) \sinh(4(c + dx)) + 72ac + 72adx + b \sinh(6(c + dx)) - 60bc - 60bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2), x]
```

```
[Out] (72*a*c - 60*b*c + 72*a*d*x - 60*b*d*x + (-48*a + 45*b)*Sinh[2*(c + d*x)] + (6*a - 9*b)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.014, size = 88, normalized size = 1.

$$\frac{1}{d} \left(b \left(\left(\frac{(\sinh(dx+c))^5}{6} - \frac{5(\sinh(dx+c))^3}{24} + \frac{5\sinh(dx+c)}{16} \right) \cosh(dx+c) - \frac{5dx}{16} - \frac{5c}{16} \right) + a \left(\left(\frac{(\sinh(dx+c))^3}{4} - \frac{3\sinh(dx+c)}{8} \right) \cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)

[Out] 1/d*(b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))

Maxima [A] time = 1.034, size = 203, normalized size = 2.28

$$\frac{1}{64} a \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{384} b \left(\frac{(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/384*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)

Fricas [A] time = 1.95221, size = 319, normalized size = 3.58

$$\frac{3b \cosh(dx+c) \sinh(dx+c)^5 + 2(5b \cosh(dx+c)^3 + 3(2a-3b) \cosh(dx+c)) \sinh(dx+c)^3 + 6(6a-5b)dx + 3c}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - 5*b)*d*x + 3*(b*cosh(d*x + c)

$$\frac{(c+x)^5 + 2(2a - 3b)\cosh(dx + c)^3 - (16a - 15b)\cosh(dx + c)\sinh(dx + c)}{d}$$

Sympy [A] time = 4.7777, size = 258, normalized size = 2.9

$$\frac{\left\{ \begin{array}{l} \frac{3ax \sinh^4(c+dx)}{8} - \frac{3ax \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3ax \cosh^4(c+dx)}{8} + \frac{5a \sinh^3(c+dx) \cosh(c+dx)}{8d} - \frac{3a \sinh(c+dx) \cosh^3(c+dx)}{8d} + \frac{5bx \sinh^6(c+dx)}{16} \\ x(a + b \sinh^2(c)) \sinh^4(c) \end{array} \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)

[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b*x*sinh(c + d*x)**6/16 - 15
*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c
+ d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d
*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x
) *cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c)**4, True))

Giac [B] time = 1.32322, size = 221, normalized size = 2.48

$$\frac{24(dx + c)(6a - 5b) + be^{6dx+6c} + 6ae^{4dx+4c} - 9be^{4dx+4c} - 48ae^{2dx+2c} + 45be^{2dx+2c} - (132ae^{6dx+6c} - 110be^{6dx+6c})}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] 1/384*(24*(d*x + c)*(6*a - 5*b) + b*e^(6*d*x + 6*c) + 6*a*e^(4*d*x + 4*c) -
9*b*e^(4*d*x + 4*c) - 48*a*e^(2*d*x + 2*c) + 45*b*e^(2*d*x + 2*c) - (132*a
*e^(6*d*x + 6*c) - 110*b*e^(6*d*x + 6*c) - 48*a*e^(4*d*x + 4*c) + 45*b*e^(4
*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 9*b*e^(2*d*x + 2*c) + b)*e^(-6*d*x - 6*
c))/d

3.2 $\int \sinh^3(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=53

$$\frac{(a - 2b) \cosh^3(c + dx)}{3d} - \frac{(a - b) \cosh(c + dx)}{d} + \frac{b \cosh^5(c + dx)}{5d}$$

[Out] -(((a - b)*Cosh[c + d*x])/d) + ((a - 2*b)*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c + d*x]^5)/(5*d)

Rubi [A] time = 0.0557225, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3013, 373}

$$\frac{(a - 2b) \cosh^3(c + dx)}{3d} - \frac{(a - b) \cosh(c + dx)}{d} + \frac{b \cosh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2), x]

[Out] -(((a - b)*Cosh[c + d*x])/d) + ((a - 2*b)*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c + d*x]^5)/(5*d)

Rule 3013

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)], x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \sinh^3(c+dx)(a+b\sinh^2(c+dx))dx &= -\frac{\text{Subst}\left(\int(1-x^2)(a-b+bx^2)dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\text{Subst}\left(\int\left(a\left(1-\frac{b}{a}\right)-(a-2b)x^2-bx^4\right)dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{(a-b)\cosh(c+dx)}{d} + \frac{(a-2b)\cosh^3(c+dx)}{3d} + \frac{b\cosh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.0255729, size = 77, normalized size = 1.45

$$-\frac{3a\cosh(c+dx)}{4d} + \frac{a\cosh(3(c+dx))}{12d} + \frac{5b\cosh(c+dx)}{8d} - \frac{5b\cosh(3(c+dx))}{48d} + \frac{b\cosh(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2), x]

[Out] (-3*a*Cosh[c + d*x])/(4*d) + (5*b*Cosh[c + d*x])/(8*d) + (a*Cosh[3*(c + d*x)])/(12*d) - (5*b*Cosh[3*(c + d*x)])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d)

Maple [A] time = 0.013, size = 56, normalized size = 1.1

$$\frac{1}{d} \left(b \left(\frac{8}{15} + \frac{(\sinh(dx+c))^4}{5} - \frac{4(\sinh(dx+c))^2}{15} \right) \cosh(dx+c) + a \left(-\frac{2}{3} + \frac{(\sinh(dx+c))^2}{3} \right) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))

Maxima [B] time = 1.04109, size = 190, normalized size = 3.58

$$\frac{1}{480} b \left(\frac{3e^{5dx+5c}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d} \right) + \frac{1}{24} a \left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{1}{480}b(3e^{(5dx+5c)}/d - 25e^{(3dx+3c)}/d + 150e^{(dx+c)}/d + 150e^{(-dx-c)}/d - 25e^{(-3dx-3c)}/d + 3e^{(-5dx-5c)}/d) + \frac{1}{24}a(e^{(3dx+3c)}/d - 9e^{(dx+c)}/d - 9e^{(-dx-c)}/d + e^{(-3dx-3c)}/d)$

Fricas [B] time = 1.82341, size = 273, normalized size = 5.15

$$\frac{3b \cosh(dx+c)^5 + 15b \cosh(dx+c) \sinh(dx+c)^4 + 5(4a-5b) \cosh(dx+c)^3 + 15(2b \cosh(dx+c)^3 + (4a-5b))}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $\frac{1}{240}(3b \cosh(dx+c)^5 + 15b \cosh(dx+c) \sinh(dx+c)^4 + 5(4a-5b) \cosh(dx+c)^3 + 15(2b \cosh(dx+c)^3 + (4a-5b) \sinh(dx+c)^2 - 30(6a-5b) \cosh(dx+c)))/d$

Sympy [A] time = 2.47998, size = 105, normalized size = 1.98

$$\begin{cases} \frac{a \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2a \cosh^3(c+dx)}{3d} + \frac{b \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{4b \sinh^2(c+dx) \cosh^3(c+dx)}{3d} + \frac{8b \cosh^5(c+dx)}{15d} & \text{for } d \neq 0 \\ x(a + b \sinh^2(c)) \sinh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)

[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d) + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c)**3, True))

Giac [B] time = 1.25488, size = 166, normalized size = 3.13

$$\frac{3be^{(5dx+5c)} + 20ae^{(3dx+3c)} - 25be^{(3dx+3c)} - 180ae^{(dx+c)} + 150be^{(dx+c)} - (180ae^{(4dx+4c)} - 150be^{(4dx+4c)} - 20ae^{(2dx+2c)} + 25be^{(2dx+2c)} - 3b)e^{(-5dx-5c)}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] 1/480*(3*b*e^(5*d*x + 5*c) + 20*a*e^(3*d*x + 3*c) - 25*b*e^(3*d*x + 3*c) - 180*a*e^(d*x + c) + 150*b*e^(d*x + c) - (180*a*e^(4*d*x + 4*c) - 150*b*e^(4*d*x + 4*c) - 20*a*e^(2*d*x + 2*c) + 25*b*e^(2*d*x + 2*c) - 3*b)*e^(-5*d*x - 5*c))/d

3.3 $\int \sinh^2(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=61

$$\frac{(4a - 3b) \sinh(c + dx) \cosh(c + dx)}{8d} - \frac{1}{8}x(4a - 3b) + \frac{b \sinh^3(c + dx) \cosh(c + dx)}{4d}$$

[Out] $-\frac{((4*a - 3*b)*x)}{8} + \frac{((4*a - 3*b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])}{(8*d)} + (b*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x]^3)/(4*d)$

Rubi [A] time = 0.0428877, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3014, 2635, 8}

$$\frac{(4a - 3b) \sinh(c + dx) \cosh(c + dx)}{8d} - \frac{1}{8}x(4a - 3b) + \frac{b \sinh^3(c + dx) \cosh(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^2*(a + b*\text{Sinh}[c + d*x]^2), x]$

[Out] $-\frac{((4*a - 3*b)*x)}{8} + \frac{((4*a - 3*b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])}{(8*d)} + (b*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x]^3)/(4*d)$

Rule 3014

$\text{Int}[\frac{(b_*)*\sin[(e_*) + (f_*)(x_*)]}{(c_*) + (d_*)(x_*)}]^{(m_*)} * ((A_*) + (C_*)*\sin[(e_*) + (f_*)(x_*)])^{(n_*)}, x_Symbol] :> -\text{Simp}[\frac{(C*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})}{(b*f*(m+2))}, x] + \text{Dist}[\frac{(A*(m+2) + C*(m+1))}{(m+2)}, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{b, e, f, A, C, m\}, x] \&\& !\text{LtQ}[m, -1]$

Rule 2635

$\text{Int}[\frac{(b_*)*\sin[(c_*) + (d_*)(x_*)]}{(e_*) + (f_*)(x_*)}]^{(n_*)}, x_Symbol] :> -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a_*, x_Symbol] :> \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned}
\int \sinh^2(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{b \cosh(c + dx) \sinh^3(c + dx)}{4d} - \frac{1}{4}(-4a + 3b) \int \sinh^2(c + dx) dx \\
&= \frac{(4a - 3b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b \cosh(c + dx) \sinh^3(c + dx)}{4d} - \frac{1}{8}(4a - 3b)x \\
&= -\frac{1}{8}(4a - 3b)x + \frac{(4a - 3b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b \cosh(c + dx) \sinh^3(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.0854699, size = 47, normalized size = 0.77

$$\frac{-4(4a - 3b)(c + dx) + 8(a - b) \sinh(2(c + dx)) + b \sinh(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2), x]

[Out] (-4*(4*a - 3*b)*(c + d*x) + 8*(a - b)*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)])/(32*d)

Maple [A] time = 0.013, size = 66, normalized size = 1.1

$$\frac{1}{d} \left(b \left(\left(\frac{(\sinh(dx + c))^3}{4} - \frac{3 \sinh(dx + c)}{8} \right) \cosh(dx + c) + \frac{3dx}{8} + \frac{3c}{8} \right) + a \left(\frac{\cosh(dx + c) \sinh(dx + c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))

Maxima [A] time = 1.05928, size = 131, normalized size = 2.15

$$\frac{1}{64} b \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) - \frac{1}{8} a \left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{1}{64}b(24x + e^{(4dx + 4c)})/d - 8e^{(2dx + 2c)}/d + 8e^{(-2dx - 2c)}/d - e^{(-4dx - 4c)}/d - \frac{1}{8}a(4x - e^{(2dx + 2c)})/d + e^{(-2dx - 2c)}/d$

Fricas [A] time = 1.90866, size = 163, normalized size = 2.67

$$\frac{b \cosh(dx + c) \sinh(dx + c)^3 - (4a - 3b)dx + (b \cosh(dx + c)^3 + 4(a - b) \cosh(dx + c)) \sinh(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $\frac{1}{8}(b \cosh(dx + c) \sinh(dx + c)^3 - (4a - 3b)dx + (b \cosh(dx + c)^3 + 4(a - b) \cosh(dx + c)) \sinh(dx + c))/d$

Sympy [A] time = 1.42794, size = 158, normalized size = 2.59

$$\left\{ \frac{ax \sinh^2(c+dx)}{2} - \frac{ax \cosh^2(c+dx)}{2} + \frac{a \sinh(c+dx) \cosh(c+dx)}{2d} + \frac{3bx \sinh^4(c+dx)}{8} - \frac{3bx \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3bx \cosh^4(c+dx)}{8} + \frac{5b \sinh^3(c)}{8} \right\} x (a + b \sinh^2(c)) \sinh^2(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)

[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c)**2, True))

Giac [B] time = 1.36898, size = 161, normalized size = 2.64

$$\frac{8(dx+c)(4a-3b) - be^{(4dx+4c)} - 8ae^{(2dx+2c)} + 8be^{(2dx+2c)} - (24ae^{(4dx+4c)} - 18be^{(4dx+4c)} - 8ae^{(2dx+2c)} + 8be^{(2dx+2c)})}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] $-1/64*(8*(d*x + c)*(4*a - 3*b) - b*e^{(4*d*x + 4*c)} - 8*a*e^{(2*d*x + 2*c)} + 8*b*e^{(2*d*x + 2*c)} - (24*a*e^{(4*d*x + 4*c)} - 18*b*e^{(4*d*x + 4*c)} - 8*a*e^{(2*d*x + 2*c)} + 8*b*e^{(2*d*x + 2*c)} - b)*e^{(-4*d*x - 4*c)})/d$

3.4 $\int \sinh(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=32

$$\frac{(a - b) \cosh(c + dx)}{d} + \frac{b \cosh^3(c + dx)}{3d}$$

[Out] $((a - b) * \text{Cosh}[c + d * x]) / d + (b * \text{Cosh}[c + d * x]^3) / (3 * d)$

Rubi [A] time = 0.0289156, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3013}

$$\frac{(a - b) \cosh(c + dx)}{d} + \frac{b \cosh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d * x] * (a + b * \text{Sinh}[c + d * x]^2), x]$

[Out] $((a - b) * \text{Cosh}[c + d * x]) / d + (b * \text{Cosh}[c + d * x]^3) / (3 * d)$

Rule 3013

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)} * ((A_.) + (C_.) * \sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -\text{Dist}[f^{(-1)}, \text{Subst}[\text{Int}[(1 - x^2)^{((m - 1)/2)} * (A + C - C * x^2)], x], x, \text{Cos}[e + f * x]], x] /; \text{FreeQ}\{e, f, A, C\}, x \ \&\& \ \text{IGtQ}[(m + 1)/2, 0]$

Rubi steps

$$\begin{aligned} \int \sinh(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int (a - b + bx^2) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{(a - b) \cosh(c + dx)}{d} + \frac{b \cosh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0240816, size = 53, normalized size = 1.66

$$\frac{a \sinh(c) \sinh(dx)}{d} + \frac{a \cosh(c) \cosh(dx)}{d} - \frac{3b \cosh(c + dx)}{4d} + \frac{b \cosh(3(c + dx))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]

[Out] (a*Cosh[c]*Cosh[d*x])/d - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d) + (a*Sinh[c]*Sinh[d*x])/d

Maple [A] time = 0.013, size = 34, normalized size = 1.1

$$\frac{1}{d} \left(b \left(-\frac{2}{3} + \frac{(\sinh(dx+c))^2}{3} \right) \cosh(dx+c) + a \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x)

[Out] 1/d*(b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a*cosh(d*x+c))

Maxima [B] time = 1.0147, size = 90, normalized size = 2.81

$$\frac{1}{24} b \left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d} \right) + \frac{a \cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a*cosh(d*x + c)/d

Fricas [A] time = 1.87512, size = 127, normalized size = 3.97

$$\frac{b \cosh(dx+c)^3 + 3b \cosh(dx+c) \sinh(dx+c)^2 + 3(4a-3b) \cosh(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $\frac{1}{12}*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + 3*(4*a - 3*b)*\cosh(d*x + c))/d$

Sympy [A] time = 0.768842, size = 56, normalized size = 1.75

$$\begin{cases} \frac{a \cosh(c+dx)}{d} + \frac{b \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2b \cosh^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a + b \sinh^2(c)) \sinh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2),x)

[Out] Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b*cosh(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c), True))

Giac [B] time = 1.26747, size = 96, normalized size = 3.

$$\frac{be^{(3dx+3c)} + 12ae^{(dx+c)} - 9be^{(dx+c)} + (12ae^{(2dx+2c)} - 9be^{(2dx+2c)} + b)e^{(-3dx-3c)}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] $\frac{1}{24}*(b*e^{(3*d*x + 3*c)} + 12*a*e^{(d*x + c)} - 9*b*e^{(d*x + c)} + (12*a*e^{(2*d*x + 2*c)} - 9*b*e^{(2*d*x + 2*c)} + b)*e^{(-3*d*x - 3*c)})/d$

3.5 $\int (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=30

$$ax + \frac{b \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{bx}{2}$$

[Out] a*x - (b*x)/2 + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)

Rubi [A] time = 0.0160623, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2635, 8}

$$ax + \frac{b \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{bx}{2}$$

Antiderivative was successfully verified.

[In] Int[a + b*Sinh[c + d*x]^2, x]

[Out] a*x - (b*x)/2 + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int (a + b \sinh^2(c + dx)) dx &= ax + b \int \sinh^2(c + dx) dx \\ &= ax + \frac{b \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{1}{2}b \int 1 dx \\ &= ax - \frac{bx}{2} + \frac{b \cosh(c + dx) \sinh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.0316776, size = 36, normalized size = 1.2

$$ax + \frac{b(-c - dx)}{2d} + \frac{b \sinh(2(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Sinh[c + d*x]^2,x]

[Out] a*x + (b*(-c - d*x))/(2*d) + (b*Sinh[2*(c + d*x)])/(4*d)

Maple [A] time = 0.007, size = 32, normalized size = 1.1

$$ax + \frac{b}{d} \left(\frac{\cosh(dx + c) \sinh(dx + c)}{2} - \frac{dx}{2} - \frac{c}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b*sinh(d*x+c)^2,x)

[Out] a*x+b/d*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)

Maxima [A] time = 1.04378, size = 51, normalized size = 1.7

$$-\frac{1}{8} b \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="maxima")

[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*x

Fricas [A] time = 1.84118, size = 74, normalized size = 2.47

$$\frac{(2a - b)dx + b \cosh(dx + c) \sinh(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="fricas")

[Out] 1/2*((2*a - b)*d*x + b*cosh(d*x + c)*sinh(d*x + c))/d

Sympy [A] time = 0.408983, size = 51, normalized size = 1.7

$$ax + b \left(\begin{cases} \frac{x \sinh^2(c+dx)}{2} - \frac{x \cosh^2(c+dx)}{2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2d} & \text{for } d \neq 0 \\ x \sinh^2(c) & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sinh(d*x+c)**2,x)

[Out] a*x + b*Piecewise((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)), (x*sinh(c)**2, True))

Giac [B] time = 1.30807, size = 72, normalized size = 2.4

$$ax - \frac{(4dx - (2e^{(2dx+2c)} - 1)e^{(-2dx-2c)} + 4c - e^{(2dx+2c)})b}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="giac")

[Out] a*x - 1/8*(4*d*x - (2*e^(2*d*x + 2*c) - 1)*e^(-2*d*x - 2*c) + 4*c - e^(2*d*x + 2*c))*b/d

3.6 $\int \operatorname{csch}(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=25

$$\frac{b \cosh(c + dx)}{d} - \frac{a \tanh^{-1}(\cosh(c + dx))}{d}$$

[Out] $-\left(\frac{a \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) + \frac{b \operatorname{Cosh}[c + d*x]}{d}$

Rubi [A] time = 0.0350184, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3014, 3770}

$$\frac{b \cosh(c + dx)}{d} - \frac{a \tanh^{-1}(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Sinh}[c + d*x]^2), x]$

[Out] $-\left(\frac{a \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) + \frac{b \operatorname{Cosh}[c + d*x]}{d}$

Rule 3014

$\operatorname{Int}[(b \sin(e \cdot x) + f \cdot x)^m (A + C \sin(e \cdot x) + f \cdot x)^2], x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(C \cos[e + f \cdot x] (b \sin[e + f \cdot x])^{m+1}) / (b f (m+2)), x] + \operatorname{Dist}[(A(m+2) + C(m+1)) / (m+2), \operatorname{Int}[(b \sin[e + f \cdot x])^m, x], x] /;$ $\operatorname{FreeQ}\{b, e, f, A, C, m\}, x] \ \&\& \ \operatorname{!LtQ}[m, -1]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c \cdot x) + (d \cdot x)], x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d \cdot x]] / d, x] /;$ $\operatorname{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{b \cosh(c + dx)}{d} + a \int \operatorname{csch}(c + dx) dx \\ &= -\frac{a \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b \cosh(c + dx)}{d} \end{aligned}$$

Mathematica [B] time = 0.0315701, size = 62, normalized size = 2.48

$$\frac{a \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{b \sinh(c) \sinh(dx)}{d} + \frac{b \cosh(c) \cosh(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2), x]

[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (d*x)/2]])/d + (b*Sinh[c]*Sinh[d*x])/d

Maple [A] time = 0.029, size = 24, normalized size = 1.

$$\frac{-2a \operatorname{Artanh}\left(e^{dx+c}\right) + b \cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*cosh(d*x+c))

Maxima [A] time = 1.04059, size = 58, normalized size = 2.32

$$\frac{1}{2} b \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) + \frac{a \log\left(\tanh\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2), x, algorithm="maxima")

[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a*log(tanh(1/2*d*x + 1/2*c))/d

Fricas [B] time = 1.9737, size = 374, normalized size = 14.96

$$\frac{b \cosh(dx+c)^2 + 2b \cosh(dx+c) \sinh(dx+c) + b \sinh(dx+c)^2 - 2(a \cosh(dx+c) + a \sinh(dx+c)) \log(\cosh(dx+c) + \sinh(dx+c))}{2(d \cosh(dx+c) + d \sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2144, size = 78, normalized size = 3.12

$$\frac{be^{(dx+c)}}{2d} + \frac{be^{(-dx-c)}}{2d} - \frac{a \log(e^{(dx+c)} + 1)}{d} + \frac{a \log(|e^{(dx+c)} - 1|)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*b*e^(d*x + c)/d + 1/2*b*e^(-d*x - c)/d - a*log(e^(d*x + c) + 1)/d + a*log(abs(e^(d*x + c) - 1))/d
```

3.7 $\int \operatorname{csch}^2(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=16

$$bx - \frac{a \operatorname{coth}(c + dx)}{d}$$

[Out] b*x - (a*Coth[c + d*x])/d

Rubi [A] time = 0.0266243, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3012, 8}

$$bx - \frac{a \operatorname{coth}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2), x]

[Out] b*x - (a*Coth[c + d*x])/d

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) (a + b \sinh^2(c + dx)) dx &= -\frac{a \operatorname{coth}(c + dx)}{d} + b \int 1 dx \\ &= bx - \frac{a \operatorname{coth}(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.0195822, size = 16, normalized size = 1.

$$bx - \frac{a \coth(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2), x]

[Out] b*x - (a*Coth[c + d*x])/d

Maple [A] time = 0.027, size = 22, normalized size = 1.4

$$\frac{-\coth(dx + c)a + (dx + c)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(-coth(d*x+c)*a+(d*x+c)*b)

Maxima [A] time = 1.02413, size = 31, normalized size = 1.94

$$bx + \frac{2a}{d(e^{-2dx-2c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2), x, algorithm="maxima")

[Out] b*x + 2*a/(d*(e^(-2*d*x - 2*c) - 1))

Fricas [B] time = 1.8503, size = 89, normalized size = 5.56

$$\frac{a \cosh(dx + c) - (bdx + a) \sinh(dx + c)}{d \sinh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $-(a*\cosh(d*x + c) - (b*d*x + a)*\sinh(d*x + c))/(d*\sinh(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [A] time = 1.29289, size = 41, normalized size = 2.56

$$\frac{(dx + c)b}{d} - \frac{2a}{d(e^{2dx+2c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] $(d*x + c)*b/d - 2*a/(d*(e^{2*d*x + 2*c} - 1))$

3.8 $\int \operatorname{csch}^3(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=40

$$\frac{(a - 2b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d}$$

[Out] $((a - 2*b)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(2*d) - (a*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(2*d)$

Rubi [A] time = 0.04142, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3012, 3770}

$$\frac{(a - 2b) \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^3*(a + b*\operatorname{Sinh}[c + d*x]^2), x]$

[Out] $((a - 2*b)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(2*d) - (a*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(2*d)$

Rule 3012

$\operatorname{Int}[(b_* \sin[e_*] + (f_*)*(x_*))^{(m_*)}*((A_*) + (C_*)*\sin[e_*] + (f_*)*(x_*))^{(2)}, x_Symbol] \rightarrow \operatorname{Simp}[(A*\operatorname{Cos}[e + f*x]*(b*\operatorname{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 1)), x] + \operatorname{Dist}[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), \operatorname{Int}[(b*\operatorname{Sin}[e + f*x])^{(m + 2)}, x], x] /; \operatorname{FreeQ}\{b, e, f, A, C\}, x] \&\& \operatorname{LtQ}[m, -1]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_*) + (d_*)*(x_*)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rubi steps

$$\int \operatorname{csch}^3(c+dx) (a+b \sinh^2(c+dx)) dx = -\frac{a \coth(c+dx) \operatorname{csch}(c+dx)}{2d} - \frac{1}{2}(a-2b) \int \operatorname{csch}(c+dx) dx$$

$$= \frac{(a-2b) \tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a \coth(c+dx) \operatorname{csch}(c+dx)}{2d}$$

Mathematica [B] time = 0.0320482, size = 99, normalized size = 2.48

$$-\frac{a \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a \operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{b \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2), x]

[Out] -(a*Csch[(c + d*x)/2]^2)/(8*d) - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d)

Maple [A] time = 0.034, size = 40, normalized size = 1.

$$\frac{1}{d} \left(a \left(-\frac{\operatorname{csch}(dx+c) \coth(dx+c)}{2} + \operatorname{Artanh}(e^{dx+c}) \right) - 2b \operatorname{Artanh}(e^{dx+c}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-2*b*arctanh(exp(d*x+c)))

Maxima [B] time = 1.05464, size = 169, normalized size = 4.22

$$\frac{1}{2} a \left(\frac{\log(e^{-dx-c} + 1)}{d} - \frac{\log(e^{-dx-c} - 1)}{d} + \frac{2(e^{-dx-c} + e^{-3dx-3c})}{d(2e^{-2dx-2c} - e^{-4dx-4c} - 1)} \right) - b \left(\frac{\log(e^{-dx-c} + 1)}{d} - \frac{\log(e^{-dx-c} - 1)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{1}{2}a \left(\frac{\log(e^{-d*x - c} + 1)}{d} - \frac{\log(e^{-d*x - c} - 1)}{d} + 2 \frac{(e^{-d*x - c} + e^{-3*d*x - 3*c})}{(d*(2*e^{-2*d*x - 2*c} - e^{-4*d*x - 4*c} - 1))} \right) - b \left(\frac{\log(e^{-d*x - c} + 1)}{d} - \frac{\log(e^{-d*x - c} - 1)}{d} \right)$

Fricas [B] time = 1.9364, size = 1323, normalized size = 33.08

$$\frac{2 a \cosh (d x + c)^3 + 6 a \cosh (d x + c) \sinh (d x + c)^2 + 2 a \sinh (d x + c)^3 + 2 a \cosh (d x + c) - ((a - 2 b) \cosh (d x + c)^4}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\frac{-1/2*(2*a*\cosh(d*x + c)^3 + 6*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*a*\sinh(d*x + c)^3 + 2*a*\cosh(d*x + c) - ((a - 2*b)*\cosh(d*x + c)^4 + 4*(a - 2*b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a - 2*b)*\sinh(d*x + c)^4 - 2*(a - 2*b)*\cosh(d*x + c)^2 + 2*(3*(a - 2*b)*\cosh(d*x + c)^2 - a + 2*b)*\sinh(d*x + c)^2 + 4*((a - 2*b)*\cosh(d*x + c)^3 - (a - 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a - 2*b)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + ((a - 2*b)*\cosh(d*x + c)^4 + 4*(a - 2*b)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a - 2*b)*\sinh(d*x + c)^4 - 2*(a - 2*b)*\cosh(d*x + c)^2 + 2*(3*(a - 2*b)*\cosh(d*x + c)^2 - a + 2*b)*\sinh(d*x + c)^2 + 4*((a - 2*b)*\cosh(d*x + c)^3 - (a - 2*b)*\cosh(d*x + c))*\sinh(d*x + c) + a - 2*b)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(3*a*\cosh(d*x + c)^2 + a)*\sinh(d*x + c))/(d*\cosh(d*x + c)^4 + 4*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + d*\sinh(d*x + c)^4 - 2*d*\cosh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^2 + 4*(d*\cosh(d*x + c)^3 - d*\cosh(d*x + c))*\sinh(d*x + c) + d)}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [B] time = 1.25302, size = 136, normalized size = 3.4

$$\frac{(a - 2b) \log(e^{(dx+c)} + e^{(-dx-c)} + 2)}{4d} - \frac{(a - 2b) \log(e^{(dx+c)} + e^{(-dx-c)} - 2)}{4d} - \frac{a(e^{(dx+c)} + e^{(-dx-c)})}{\left((e^{(dx+c)} + e^{(-dx-c)})^2 - 4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] 1/4*(a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/4*(a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d - a*(e^(d*x + c) + e^(-d*x - c))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)*d)

3.9 $\int \operatorname{csch}^4(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=43

$$\frac{(2a - 3b) \operatorname{coth}(c + dx)}{3d} - \frac{a \operatorname{coth}(c + dx) \operatorname{csch}^2(c + dx)}{3d}$$

[Out] $((2*a - 3*b)*\operatorname{Coth}[c + d*x])/(3*d) - (a*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x]^2)/(3*d)$

Rubi [A] time = 0.0394368, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3012, 3767, 8}

$$\frac{(2a - 3b) \operatorname{coth}(c + dx)}{3d} - \frac{a \operatorname{coth}(c + dx) \operatorname{csch}^2(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^4*(a + b*\operatorname{Sinh}[c + d*x]^2), x]$

[Out] $((2*a - 3*b)*\operatorname{Coth}[c + d*x])/(3*d) - (a*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x]^2)/(3*d)$

Rule 3012

$\operatorname{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((A_.) + (C_.)*\sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] \rightarrow \operatorname{Simp}[(A*\operatorname{Cos}[e + f*x]*(b*\operatorname{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 1)), x] + \operatorname{Dist}[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), \operatorname{Int}[(b*\operatorname{Sin}[e + f*x])^{(m + 2)}, x], x] /; \operatorname{FreeQ}\{b, e, f, A, C\}, x] \&\& \operatorname{LtQ}[m, -1]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}\{c, d\}, x] \&\& \operatorname{IGtQ}[n/2, 0]$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c+dx) (a+b \sinh^2(c+dx)) dx &= -\frac{a \operatorname{coth}(c+dx) \operatorname{csch}^2(c+dx)}{3d} + \frac{1}{3}(-2a+3b) \int \operatorname{csch}^2(c+dx) dx \\ &= -\frac{a \operatorname{coth}(c+dx) \operatorname{csch}^2(c+dx)}{3d} + \frac{(i(2a-3b)) \operatorname{Subst}(\int 1 dx, x, -i \operatorname{coth}(c+dx))}{3d} \\ &= \frac{(2a-3b) \operatorname{coth}(c+dx)}{3d} - \frac{a \operatorname{coth}(c+dx) \operatorname{csch}^2(c+dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0314179, size = 49, normalized size = 1.14

$$\frac{2a \operatorname{coth}(c+dx)}{3d} - \frac{a \operatorname{coth}(c+dx) \operatorname{csch}^2(c+dx)}{3d} - \frac{b \operatorname{coth}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2), x]

[Out] (2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)

Maple [A] time = 0.034, size = 35, normalized size = 0.8

$$\frac{1}{d} \left(a \left(\frac{2}{3} - \frac{(\operatorname{csch}(dx+c))^2}{3} \right) \operatorname{coth}(dx+c) - b \operatorname{coth}(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-b*coth(d*x+c))

Maxima [B] time = 1.0247, size = 153, normalized size = 3.56

$$\frac{4}{3} a \left(\frac{3 e^{(-2dx-2c)}}{d(3 e^{(-2dx-2c)} - 3 e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3 e^{(-2dx-2c)} - 3 e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right) + \frac{2b}{d(e^{(-2dx-2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{4}{3}a \frac{(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)}{(d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1))} - \frac{1}{(d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1))} + \frac{2b}{(d(e^{(-2dx-2c)} - 1))}$

Fricas [B] time = 1.80659, size = 425, normalized size = 9.88

$$\frac{4((a-3b)\cosh(dx+c)^2 - 2a\cosh(dx+c)\sinh(dx+c) + (a-3b)\sinh(dx+c)^2 - 3a + 3b)}{3(d\cosh(dx+c)^4 + 4d\cosh(dx+c)\sinh(dx+c)^3 + d\sinh(dx+c)^4 - 4d\cosh(dx+c)^2 + 2(3d\cosh(dx+c)^2 - 2d)\sinh(dx+c)^2 + 4(d\cosh(dx+c)^3 - d\cosh(dx+c))\sinh(dx+c) + 3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $\frac{4}{3} \frac{((a-3b)\cosh(dx+c)^2 - 2a\cosh(dx+c)\sinh(dx+c) + (a-3b)\sinh(dx+c)^2 - 3a + 3b)}{(d\cosh(dx+c)^4 + 4d\cosh(dx+c)\sinh(dx+c)^3 + d\sinh(dx+c)^4 - 4d\cosh(dx+c)^2 + 2(3d\cosh(dx+c)^2 - 2d)\sinh(dx+c)^2 + 4(d\cosh(dx+c)^3 - d\cosh(dx+c))\sinh(dx+c) + 3d)}$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [A] time = 1.17306, size = 82, normalized size = 1.91

$$\frac{2(3be^{(4dx+4c)} + 6ae^{(2dx+2c)} - 6be^{(2dx+2c)} - 2a + 3b)}{3d(e^{(2dx+2c)} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) - 2*a + 3*b)/(d*(e^(2*d*x + 2*c) - 1)^3)
```


3.10 $\int \sinh^4(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=146

$$\frac{(48a^2 - 208ab + 139b^2) \sinh(c + dx) \cosh^3(c + dx)}{192d} - \frac{(80a^2 - 176ab + 93b^2) \sinh(c + dx) \cosh(c + dx)}{128d} + \frac{1}{128}x(48a^2 - 208ab + 139b^2)$$

```
[Out] ((48*a^2 - 80*a*b + 35*b^2)*x)/128 - ((80*a^2 - 176*a*b + 93*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 208*a*b + 139*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + ((16*a - 13*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x]^5)/(8*d)
```

Rubi [A] time = 0.1804, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3187, 463, 455, 1157, 385, 206}

$$\frac{(48a^2 - 208ab + 139b^2) \sinh(c + dx) \cosh^3(c + dx)}{192d} - \frac{(80a^2 - 176ab + 93b^2) \sinh(c + dx) \cosh(c + dx)}{128d} + \frac{1}{128}x(48a^2 - 208ab + 139b^2)$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((48*a^2 - 80*a*b + 35*b^2)*x)/128 - ((80*a^2 - 176*a*b + 93*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 208*a*b + 139*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + ((16*a - 13*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x]^5)/(8*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 463

```
Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x]
```

```
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
+ 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
(IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sinh^4(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{x^4(a-(a-b)x^2)^2}{(1-x^2)^5} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b^2 \cosh^3(c + dx) \sinh^5(c + dx)}{8d} - \frac{\text{Subst}\left(\int \frac{x^4(-8a^2+5b^2+8(a-b)^2x^2)}{(1-x^2)^4} dx, x, \tanh(c + dx)\right)}{8d} \\
&= \frac{(16a - 13b)b \cosh^5(c + dx) \sinh(c + dx)}{48d} + \frac{b^2 \cosh^3(c + dx) \sinh^5(c + dx)}{8d} \\
&= \frac{(48a^2 - 208ab + 139b^2) \cosh^3(c + dx) \sinh(c + dx)}{192d} + \frac{(16a - 13b)b \cosh^5(c + dx) \sinh(c + dx)}{48d} \\
&= -\frac{(80a^2 - 176ab + 93b^2) \cosh(c + dx) \sinh(c + dx)}{128d} + \frac{(48a^2 - 208ab + 139b^2) \cosh^3(c + dx) \sinh(c + dx)}{192d} \\
&= \frac{1}{128} (48a^2 - 80ab + 35b^2) x - \frac{(80a^2 - 176ab + 93b^2) \cosh(c + dx) \sinh(c + dx)}{128d}
\end{aligned}$$

Mathematica [A] time = 0.203947, size = 133, normalized size = 0.91

$$\frac{-96(8a^2 - 15ab + 7b^2) \sinh(2(c + dx)) + 24(4a^2 - 12ab + 7b^2) \sinh(4(c + dx)) + 1152a^2c + 1152a^2dx + 32ab \sinh(6(c + dx)) + 3b^2 \sinh(8(c + dx))}{3072d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (1152*a^2*c - 1920*a*b*c + 840*b^2*c + 1152*a^2*d*x - 1920*a*b*d*x + 840*b^2*d*x - 96*(8*a^2 - 15*a*b + 7*b^2)*Sinh[2*(c + d*x)] + 24*(4*a^2 - 12*a*b + 7*b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] - 32*b^2*Sinh[6*(c + d*x)] + 3*b^2*Sinh[8*(c + d*x)])/(3072*d)

Maple [A] time = 0.048, size = 150, normalized size = 1.

$$\frac{1}{d} \left(b^2 \left(\frac{(\sinh(dx + c))^7}{8} - \frac{7(\sinh(dx + c))^5}{48} + \frac{35(\sinh(dx + c))^3}{192} - \frac{35 \sinh(dx + c)}{128} \right) \cosh(dx + c) + \frac{35 dx}{128} + \frac{35 c}{128} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)`

[Out] $\frac{1}{d} * (b^2 * ((\frac{1}{8} * \sinh(d*x+c)^7 - \frac{7}{48} * \sinh(d*x+c)^5 + \frac{35}{192} * \sinh(d*x+c)^3 - \frac{35}{128} * \sinh(d*x+c)) * \cosh(d*x+c) + \frac{35}{128} * d*x + \frac{35}{128} * c) + 2 * a * b * ((\frac{1}{6} * \sinh(d*x+c)^5 - \frac{5}{24} * \sinh(d*x+c)^3 + \frac{5}{16} * \sinh(d*x+c)) * \cosh(d*x+c) - \frac{5}{16} * d*x - \frac{5}{16} * c) + a^2 * ((\frac{1}{4} * \sinh(d*x+c)^3 - \frac{3}{8} * \sinh(d*x+c)) * \cosh(d*x+c) + \frac{3}{8} * d*x + \frac{3}{8} * c))$

Maxima [A] time = 1.04378, size = 360, normalized size = 2.47

$$\frac{1}{64} a^2 \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) - \frac{1}{6144} b^2 \left(\frac{(32e^{-2dx-2c} - 168e^{-4dx-4c} + 672e^{-6dx-6c})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $\frac{1}{64} * a^2 * (24 * x + \frac{e^{(4 * d * x + 4 * c)}}{d} - \frac{8 * e^{(2 * d * x + 2 * c)}}{d} + \frac{8 * e^{(-2 * d * x - 2 * c)}}{d} - \frac{e^{(-4 * d * x - 4 * c)}}{d}) - \frac{1}{6144} * b^2 * ((32 * e^{(-2 * d * x - 2 * c)} - 168 * e^{(-4 * d * x - 4 * c)} + 672 * e^{(-6 * d * x - 6 * c)} - 3) * e^{(8 * d * x + 8 * c)} / d - 1680 * (d * x + c) / d - (672 * e^{(-2 * d * x - 2 * c)} - 168 * e^{(-4 * d * x - 4 * c)} + 32 * e^{(-6 * d * x - 6 * c)} - 3 * e^{(-8 * d * x - 8 * c)}) / d - \frac{1}{192} * a * b * ((9 * e^{(-2 * d * x - 2 * c)} - 45 * e^{(-4 * d * x - 4 * c)} - 1) * e^{(6 * d * x + 6 * c)} / d + 120 * (d * x + c) / d + (45 * e^{(-2 * d * x - 2 * c)} - 9 * e^{(-4 * d * x - 4 * c)} + e^{(-6 * d * x - 6 * c)}) / d)$

Fricas [A] time = 1.97267, size = 589, normalized size = 4.03

$$\frac{3b^2 \cosh(dx+c) \sinh(dx+c)^7 + 3(7b^2 \cosh(dx+c)^3 + 8(ab-b^2) \cosh(dx+c)) \sinh(dx+c)^5 + (21b^2 \cosh(dx+c) + 8(a*b - b^2) \cosh(dx+c)) \sinh(dx+c)^3 + 3(48a^2 - 80a*b + 35b^2) * d*x + 3(b^2 \cosh(dx+c)^7 + 8(a*b - b^2) \cosh(dx+c)^5 + 4(4a^2 - 12a*b + 7b^2) \cosh(dx+c)^3 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $\frac{1}{384} * (3 * b^2 * \cosh(d*x + c) * \sinh(d*x + c)^7 + 3 * (7 * b^2 * \cosh(d*x + c)^3 + 8 * (a * b - b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^5 + (21 * b^2 * \cosh(d*x + c)^5 + 80 * (a * b - b^2) * \cosh(d*x + c)^3 + 12 * (4 * a^2 - 12 * a * b + 7 * b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 3 * (48 * a^2 - 80 * a * b + 35 * b^2) * d * x + 3 * (b^2 * \cosh(d*x + c)^7 + 8 * (a * b - b^2) * \cosh(d*x + c)^5 + 4 * (4 * a^2 - 12 * a * b + 7 * b^2) * \cosh(d*x + c)^3 -$

$$8*(8*a^2 - 15*a*b + 7*b^2)*\cosh(d*x + c)*\sinh(d*x + c))/d$$

Sympy [A] time = 15.3862, size = 490, normalized size = 3.36

$$\left\{ \begin{array}{l} \frac{3a^2x \sinh^4(c+dx)}{8} - \frac{3a^2x \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3a^2x \cosh^4(c+dx)}{8} + \frac{5a^2 \sinh^3(c+dx) \cosh(c+dx)}{8d} - \frac{3a^2 \sinh(c+dx) \cosh^3(c+dx)}{8d} + \frac{5abx \sinh^4(c)}{8} \\ x \left(a + b \sinh^2(c) \right)^2 \sinh^4(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 + 5*a**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 3*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + 35*b**2*x*sinh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**4, True))

Giac [B] time = 1.3632, size = 429, normalized size = 2.94

$$3b^2e^{(8dx+8c)} + 32abe^{(6dx+6c)} - 32b^2e^{(6dx+6c)} + 96a^2e^{(4dx+4c)} - 288abe^{(4dx+4c)} + 168b^2e^{(4dx+4c)} - 768a^2e^{(2dx+2c)} + 1440a^2e^{(2dx+2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/6144*(3*b^2*e^(8*d*x + 8*c) + 32*a*b*e^(6*d*x + 6*c) - 32*b^2*e^(6*d*x + 6*c) + 96*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 168*b^2*e^(4*d*x + 4*c) - 768*a^2*e^(2*d*x + 2*c) + 1440*a*b*e^(2*d*x + 2*c) - 672*b^2*e^(2*d*x + 2*c) + 48*(48*a^2 - 80*a*b + 35*b^2)*(d*x + c) - (2400*a^2*e^(8*d*x + 8*c) - 4000*a*b*e^(8*d*x + 8*c) + 1750*b^2*e^(8*d*x + 8*c) - 768*a^2*e^(6*d*x + 6*c) + 32*a*b*e^(6*d*x + 6*c) - 32*b^2*e^(6*d*x + 6*c) + 96*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 168*b^2*e^(4*d*x + 4*c) - 768*a^2*e^(2*d*x + 2*c) + 1440*a*b*e^(2*d*x + 2*c) - 672*b^2*e^(2*d*x + 2*c) + 48*(48*a^2 - 80*a*b + 35*b^2)*(d*x + c) - (2400*a^2*e^(8*d*x + 8*c) - 4000*a*b*e^(8*d*x + 8*c) + 1750*b^2*e^(8*d*x + 8*c) - 768*a^2*e^(6*d*x + 6*c) + 32*a*b*e^(6*d*x + 6*c) - 32*b^2*e^(6*d*x + 6*c) + 96*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 168*b^2*e^(4*d*x + 4*c) - 768*a^2*e^(2*d*x + 2*c) + 1440*a*b*e^(2*d*x + 2*c) - 672*b^2*e^(2*d*x + 2*c) + 48*(48*a^2 - 80*a*b + 35*b^2)*(d*x + c))

$$\begin{aligned} & d*x + 6*c) + 1440*a*b*e^{(6*d*x + 6*c)} - 672*b^2*e^{(6*d*x + 6*c)} + 96*a^2*e^{(4*d*x + 4*c)} \\ & - 288*a*b*e^{(4*d*x + 4*c)} + 168*b^2*e^{(4*d*x + 4*c)} + 32*a*b* \\ & e^{(2*d*x + 2*c)} - 32*b^2*e^{(2*d*x + 2*c)} + 3*b^2)*e^{(-8*d*x - 8*c)})/d \end{aligned}$$

3.11 $\int \sinh^3(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=85

$$\frac{b(2a - 3b) \cosh^5(c + dx)}{5d} + \frac{(a - 3b)(a - b) \cosh^3(c + dx)}{3d} - \frac{(a - b)^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh^7(c + dx)}{7d}$$

[Out] -(((a - b)^2*Cosh[c + d*x])/d) + ((a - 3*b)*(a - b)*Cosh[c + d*x]^3)/(3*d) + ((2*a - 3*b)*b*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d)

Rubi [A] time = 0.0974927, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3186, 373}

$$\frac{b(2a - 3b) \cosh^5(c + dx)}{5d} + \frac{(a - 3b)(a - b) \cosh^3(c + dx)}{3d} - \frac{(a - b)^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] -(((a - b)^2*Cosh[c + d*x])/d) + ((a - 3*b)*(a - b)*Cosh[c + d*x]^3)/(3*d) + ((2*a - 3*b)*b*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 373

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\int \sinh^3(c + dx) (a + b \sinh^2(c + dx))^2 dx = -\frac{\text{Subst}\left(\int (1-x^2)(a-b+bx^2)^2 dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int ((a-b)^2 + (a-3b)(-a+b)x^2 - (2a-3b)bx^4 - b^2x^6) dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)^2 \cosh(c+dx)}{d} + \frac{(a-3b)(a-b) \cosh^3(c+dx)}{3d} + \frac{(2a-3b)b \cosh^5(c+dx)}{5d}$$

Mathematica [A] time = 0.0384168, size = 154, normalized size = 1.81

$$-\frac{3a^2 \cosh(c+dx)}{4d} + \frac{a^2 \cosh(3(c+dx))}{12d} + \frac{5ab \cosh(c+dx)}{4d} - \frac{5ab \cosh(3(c+dx))}{24d} + \frac{ab \cosh(5(c+dx))}{40d} - \frac{35b^2 \cosh(c+dx)}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] $(-3*a^2*\text{Cosh}[c + d*x])/(4*d) + (5*a*b*\text{Cosh}[c + d*x])/(4*d) - (35*b^2*\text{Cosh}[c + d*x])/(64*d) + (a^2*\text{Cosh}[3*(c + d*x)])/(12*d) - (5*a*b*\text{Cosh}[3*(c + d*x)])/(24*d) + (7*b^2*\text{Cosh}[3*(c + d*x)])/(64*d) + (a*b*\text{Cosh}[5*(c + d*x)])/(40*d) - (7*b^2*\text{Cosh}[5*(c + d*x)])/(320*d) + (b^2*\text{Cosh}[7*(c + d*x)])/(448*d)$

Maple [A] time = 0.017, size = 102, normalized size = 1.2

$$\frac{1}{d} \left(b^2 \left(-\frac{16}{35} + \frac{(\sinh(dx+c))^6}{7} - \frac{6(\sinh(dx+c))^4}{35} + \frac{8(\sinh(dx+c))^2}{35} \right) \cosh(dx+c) + 2ab \left(\frac{8}{15} + \frac{1}{5} (\sinh(dx+c))^4 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)

[Out] $1/d*(b^2*(-16/35+1/7*\sinh(d*x+c)^6-6/35*\sinh(d*x+c)^4+8/35*\sinh(d*x+c)^2)*\cosh(d*x+c)+2*a*b*(8/15+1/5*\sinh(d*x+c)^4-4/15*\sinh(d*x+c)^2)*\cosh(d*x+c)+a^2*(-2/3+1/3*\sinh(d*x+c)^2)*\cosh(d*x+c)$

Maxima [B] time = 1.05969, size = 333, normalized size = 3.92

$$-\frac{1}{4480} b^2 \left(\frac{(49 e^{(-2dx-2c)} - 245 e^{(-4dx-4c)} + 1225 e^{(-6dx-6c)} - 5) e^{(7dx+7c)}}{d} + \frac{1225 e^{(-dx-c)} - 245 e^{(-3dx-3c)} + 49 e^{(-5dx-5c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$-1/4480*b^2*((49*e^{(-2*d*x - 2*c)} - 245*e^{(-4*d*x - 4*c)} + 1225*e^{(-6*d*x - 6*c)} - 5)*e^{(7*d*x + 7*c)}/d + (1225*e^{(-d*x - c)} - 245*e^{(-3*d*x - 3*c)} + 49*e^{(-5*d*x - 5*c)} - 5*e^{(-7*d*x - 7*c)})/d) + 1/240*a*b*(3*e^{(5*d*x + 5*c)}/d - 25*e^{(3*d*x + 3*c)}/d + 150*e^{(d*x + c)}/d + 150*e^{(-d*x - c)}/d - 25*e^{(-3*d*x - 3*c)}/d + 3*e^{(-5*d*x - 5*c)}/d) + 1/24*a^2*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d)$$

Fricas [B] time = 2.24913, size = 554, normalized size = 6.52

$$15 b^2 \cosh(dx + c)^7 + 105 b^2 \cosh(dx + c) \sinh(dx + c)^6 + 21 (8 ab - 7 b^2) \cosh(dx + c)^5 + 105 (5 b^2 \cosh(dx + c)^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$1/6720*(15*b^2*\cosh(d*x + c)^7 + 105*b^2*\cosh(d*x + c)*\sinh(d*x + c)^6 + 21*(8*a*b - 7*b^2)*\cosh(d*x + c)^5 + 105*(5*b^2*\cosh(d*x + c)^3 + (8*a*b - 7*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^4 + 35*(16*a^2 - 40*a*b + 21*b^2)*\cosh(d*x + c)^3 + 105*(3*b^2*\cosh(d*x + c)^5 + 2*(8*a*b - 7*b^2)*\cosh(d*x + c)^3 + (16*a^2 - 40*a*b + 21*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 105*(48*a^2 - 80*a*b + 35*b^2)*\cosh(d*x + c))/d$$

Sympy [A] time = 8.16706, size = 204, normalized size = 2.4

$$\left\{ \frac{a^2 \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2a^2 \cosh^3(c+dx)}{3d} + \frac{2ab \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{8ab \sinh^2(c+dx) \cosh^3(c+dx)}{3d} + \frac{16ab \cosh^5(c+dx)}{15d} + \frac{b^2 \sinh^6(c+dx)}{d} \right\} x (a + b \sinh^2(c))^2 \sinh^3(c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/(3*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*d) + b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**3, True))
```

Giac [B] time = 1.32485, size = 332, normalized size = 3.91

$$15b^2e^{(7dx+7c)} + 168abe^{(5dx+5c)} - 147b^2e^{(5dx+5c)} + 560a^2e^{(3dx+3c)} - 1400abe^{(3dx+3c)} + 735b^2e^{(3dx+3c)} - 5040a^2e^{(dx+c)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*b^2*e^(7*d*x + 7*c) + 168*a*b*e^(5*d*x + 5*c) - 147*b^2*e^(5*d*x + 5*c) + 560*a^2*e^(3*d*x + 3*c) - 1400*a*b*e^(3*d*x + 3*c) + 735*b^2*e^(3*d*x + 3*c) - 5040*a^2*e^(d*x + c) + 8400*a*b*e^(d*x + c) - 3675*b^2*e^(d*x + c) - (5040*a^2*e^(6*d*x + 6*c) - 8400*a*b*e^(6*d*x + 6*c) + 3675*b^2*e^(6*d*x + 6*c) - 560*a^2*e^(4*d*x + 4*c) + 1400*a*b*e^(4*d*x + 4*c) - 735*b^2*e^(4*d*x + 4*c) - 168*a*b*e^(2*d*x + 2*c) + 147*b^2*e^(2*d*x + 2*c) - 15*b^2)*e^(-7*d*x - 7*c))/d
```

3.12 $\int \sinh^2(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=110

$$\frac{(8a^2 - 20ab + 11b^2) \sinh(c + dx) \cosh(c + dx)}{16d} - \frac{1}{16}x(8a^2 - 12ab + 5b^2) + \frac{b(4a - 3b) \sinh(c + dx) \cosh^3(c + dx)}{8d} + \frac{b^2}{8d}$$

```
[Out] -((8*a^2 - 12*a*b + 5*b^2)*x)/16 + ((8*a^2 - 20*a*b + 11*b^2)*Cosh[c + d*x]
*Sinh[c + d*x])/(16*d) + ((4*a - 3*b)*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*d
) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.110067, antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3170, 3169}

$$\frac{(16a^2 - 36ab + 15b^2) \sinh(c + dx) \cosh(c + dx)}{48d} - \frac{1}{16}x(8a^2 - 12ab + 5b^2) + \frac{b(4a - 5b) \sinh^3(c + dx) \cosh(c + dx)}{24d} + \frac{b^2}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((8*a^2 - 12*a*b + 5*b^2)*x)/16 + ((16*a^2 - 36*a*b + 15*b^2)*Cosh[c + d*x]
]*Sinh[c + d*x])/(48*d) + ((4*a - 5*b)*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24
*d) + (Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*d)
```

Rule 3170

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)]^2), x_Symbol] := -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3169

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)*((A_) + (B_)*sin[(e_) + (f_
)*(x_)]^2), x_Symbol] := Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
```

x]

Rubi steps

$$\int \sinh^2(c + dx) (a + b \sinh^2(c + dx))^2 dx = \frac{\cosh(c + dx) \sinh(c + dx) (a + b \sinh^2(c + dx))^2}{6d} - \frac{1}{6} \int (a - (4a - 5b) \sinh^2(c + dx)) \cosh(c + dx) \sinh(c + dx) dx$$

$$= -\frac{1}{16} (8a^2 - 12ab + 5b^2) x + \frac{(16a^2 - 36ab + 15b^2) \cosh(c + dx) \sinh(c + dx)}{48d}$$

Mathematica [A] time = 0.191873, size = 99, normalized size = 0.9

$$\frac{(48a^2 - 96ab + 45b^2) \sinh(2(c + dx)) - 96a^2c - 96a^2dx + 3b(4a - 3b) \sinh(4(c + dx)) + 144abc + 144abdx + b^2 \sinh(6(c + dx))}{192d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] $(-96*a^2*c + 144*a*b*c - 60*b^2*c - 96*a^2*d*x + 144*a*b*d*x - 60*b^2*d*x + (48*a^2 - 96*a*b + 45*b^2)*\text{Sinh}[2*(c + d*x)] + 3*(4*a - 3*b)*b*\text{Sinh}[4*(c + d*x)] + b^2*\text{Sinh}[6*(c + d*x)])/(192*d)$

Maple [A] time = 0.017, size = 118, normalized size = 1.1

$$\frac{1}{d} \left(b^2 \left(\frac{(\sinh(dx + c))^5}{6} - \frac{5(\sinh(dx + c))^3}{24} + \frac{5 \sinh(dx + c)}{16} \right) \cosh(dx + c) - \frac{5dx}{16} - \frac{5c}{16} \right) + 2ab \left(\frac{1}{4} (\sinh(dx + c))^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)

[Out] $1/d*(b^2*((1/6*\sinh(d*x+c)^5-5/24*\sinh(d*x+c)^3+5/16*\sinh(d*x+c))*\cosh(d*x+c)-5/16*d*x-5/16*c)+2*a*b*((1/4*\sinh(d*x+c)^3-3/8*\sinh(d*x+c))*\cosh(d*x+c)+3/8*d*x+3/8*c)+a^2*(1/2*\cosh(d*x+c)*\sinh(d*x+c)-1/2*d*x-1/2*c))$

Maxima [A] time = 1.04317, size = 255, normalized size = 2.32

$$\frac{1}{32} ab \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{8} a^2 \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) - \frac{1}{384} b^2 \left(\frac{9e^{(-2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{32} a b (24 x + \frac{e^{(4 d x + 4 c)}}{d} - \frac{8 e^{(2 d x + 2 c)}}{d} + \frac{8 e^{(-2 d x - 2 c)}}{d} - \frac{e^{(-4 d x - 4 c)}}{d}) - \frac{1}{8} a^2 (4 x - \frac{e^{(2 d x + 2 c)}}{d} + \frac{e^{(-2 d x - 2 c)}}{d}) - \frac{1}{384} b^2 ((9 e^{(-2 d x - 2 c)} - 45 e^{(-4 d x - 4 c)} - 1) e^{(6 d x + 6 c)} / d + 120 (d x + c) / d + (45 e^{(-2 d x - 2 c)} - 9 e^{(-4 d x - 4 c)} + e^{(-6 d x - 6 c)}) / d)$

Fricas [A] time = 2.14473, size = 373, normalized size = 3.39

$$\frac{3 b^2 \cosh (d x + c) \sinh (d x + c)^5 + 2 \left(5 b^2 \cosh (d x + c)^3 + 3 \left(4 a b - 3 b^2 \right) \cosh (d x + c) \right) \sinh (d x + c)^3 - 6 \left(8 a^2 - 12 a b \right) \sinh (d x + c)}{96}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $\frac{1}{96} (3 b^2 \cosh (d x + c) \sinh (d x + c)^5 + 2 (5 b^2 \cosh (d x + c)^3 + 3 (4 a b - 3 b^2) \cosh (d x + c)) \sinh (d x + c)^3 - 6 (8 a^2 - 12 a b + 5 b^2) d x + 3 (b^2 \cosh (d x + c)^5 + 2 (4 a b - 3 b^2) \cosh (d x + c)^3 + (16 a^2 - 32 a b + 15 b^2) \cosh (d x + c)) \sinh (d x + c)) / d$

Sympy [A] time = 4.87993, size = 332, normalized size = 3.02

$$\left\{ \frac{a^2 x \sinh^2 (c+d x)}{2} - \frac{a^2 x \cosh^2 (c+d x)}{2} + \frac{a^2 \sinh (c+d x) \cosh (c+d x)}{2 d} + \frac{3 a b x \sinh^4 (c+d x)}{4} - \frac{3 a b x \sinh^2 (c+d x) \cosh^2 (c+d x)}{2} + \frac{3 a b x \cosh^4 (c+d x)}{4} + \frac{5 a^2 x \sinh^2 (c)}{2} \right\} \sinh^2 (c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) + 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b**2*x*cosh(c + d*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**2, True))
```

Giac [B] time = 1.35627, size = 316, normalized size = 2.87

$$b^2 e^{(6dx+6c)} + 12abe^{(4dx+4c)} - 9b^2 e^{(4dx+4c)} + 48a^2 e^{(2dx+2c)} - 96abe^{(2dx+2c)} + 45b^2 e^{(2dx+2c)} - 24(8a^2 - 12ab + 5b^2)(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/384*(b^2*e^(6*d*x + 6*c) + 12*a*b*e^(4*d*x + 4*c) - 9*b^2*e^(4*d*x + 4*c) + 48*a^2*e^(2*d*x + 2*c) - 96*a*b*e^(2*d*x + 2*c) + 45*b^2*e^(2*d*x + 2*c) - 24*(8*a^2 - 12*a*b + 5*b^2)*(d*x + c) + (176*a^2*e^(6*d*x + 6*c) - 264*a*b*e^(6*d*x + 6*c) + 110*b^2*e^(6*d*x + 6*c) - 48*a^2*e^(4*d*x + 4*c) + 96*a*b*e^(4*d*x + 4*c) - 45*b^2*e^(4*d*x + 4*c) - 12*a*b*e^(2*d*x + 2*c) + 9*b^2*e^(2*d*x + 2*c) - b^2)*e^(-6*d*x - 6*c))/d
```

3.13 $\int \sinh(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=57

$$\frac{2b(a-b) \cosh^3(c+dx)}{3d} + \frac{(a-b)^2 \cosh(c+dx)}{d} + \frac{b^2 \cosh^5(c+dx)}{5d}$$

[Out] $((a - b)^2 \text{Cosh}[c + d*x])/d + (2*(a - b)*b*\text{Cosh}[c + d*x]^3)/(3*d) + (b^2*\text{Cosh}[c + d*x]^5)/(5*d)$

Rubi [A] time = 0.0600295, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3186, 194}

$$\frac{2b(a-b) \cosh^3(c+dx)}{3d} + \frac{(a-b)^2 \cosh(c+dx)}{d} + \frac{b^2 \cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]*(a + b*\text{Sinh}[c + d*x]^2)^2, x]$

[Out] $((a - b)^2 \text{Cosh}[c + d*x])/d + (2*(a - b)*b*\text{Cosh}[c + d*x]^3)/(3*d) + (b^2*\text{Cosh}[c + d*x]^5)/(5*d)$

Rule 3186

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - b*ff^2*x^2)^p, x], x, \text{Cos}[e + f*x]/ff], x]] /; \text{FreeQ}\{a, b, e, f, p, x\} \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

Rule 194

$\text{Int}[(a_. + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{IGtQ}[p, 0]$

Rubi steps

$$\begin{aligned} \int \sinh(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int (a - b + bx^2)^2 dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(a^2 \left(1 + \frac{b(-2a+b)}{a^2}\right) + 2ab \left(1 - \frac{b}{a}\right) x^2 + b^2 x^4\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{(a - b)^2 \cosh(c + dx)}{d} + \frac{2(a - b)b \cosh^3(c + dx)}{3d} + \frac{b^2 \cosh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.0362276, size = 111, normalized size = 1.95

$$\frac{a^2 \sinh(c) \sinh(dx)}{d} + \frac{a^2 \cosh(c) \cosh(dx)}{d} - \frac{3ab \cosh(c + dx)}{2d} + \frac{ab \cosh(3(c + dx))}{6d} + \frac{5b^2 \cosh(c + dx)}{8d} - \frac{5b^2 \cosh(3(c + dx))}{48d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (a^2*Cosh[c]*Cosh[d*x])/d - (3*a*b*Cosh[c + d*x])/(2*d) + (5*b^2*Cosh[c + d*x])/d - (5*b^2*Cosh[3*(c + d*x)])/(48*d) + (a*b*Cosh[3*(c + d*x)])/(6*d) - (5*b^2*Cosh[5*(c + d*x)])/(80*d) + (a^2*Sinh[c]*Sinh[d*x])/d

Maple [A] time = 0.015, size = 70, normalized size = 1.2

$$\frac{1}{d} \left(b^2 \left(\frac{8}{15} + \frac{(\sinh(dx+c))^4}{5} - \frac{4(\sinh(dx+c))^2}{15} \right) \cosh(dx+c) + 2ab \left(-\frac{2}{3} + \frac{1}{3} (\sinh(dx+c))^2 \right) \cosh(dx+c) + a^2 \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*(b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*cosh(d*x+c))

Maxima [B] time = 1.04599, size = 212, normalized size = 3.72

$$\frac{1}{480} b^2 \left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d} \right) + \frac{1}{12} ab \left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{480}b^2\left(\frac{3e^{(5d*x+5c)}}{d} - \frac{25e^{(3d*x+3c)}}{d} + \frac{150e^{(d*x+c)}}{d} + \frac{150e^{(-d*x-c)}}{d} - \frac{25e^{(-3d*x-3c)}}{d} + \frac{3e^{(-5d*x-5c)}}{d}\right) + \frac{1}{12}a*b\left(\frac{e^{(3d*x+3c)}}{d} - \frac{9e^{(d*x+c)}}{d} - \frac{9e^{(-d*x-c)}}{d} + \frac{e^{(-3d*x-3c)}}{d}\right) + a^2\frac{\cosh(d*x+c)}{d}$

Fricas [B] time = 2.01757, size = 309, normalized size = 5.42

$$\frac{3b^2 \cosh(dx+c)^5 + 15b^2 \cosh(dx+c) \sinh(dx+c)^4 + 5(8ab - 5b^2) \cosh(dx+c)^3 + 15(2b^2 \cosh(dx+c)^3 + (8ab - 5b^2) \cosh(dx+c) \sinh(dx+c)^2 + 30(8a^2 - 12ab + 5b^2) \cosh(dx+c))}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $\frac{1}{240}\left(3b^2\cosh(d*x+c)^5 + 15b^2\cosh(d*x+c)\sinh(d*x+c)^4 + 5(8ab - 5b^2)\cosh(d*x+c)^3 + 15(2b^2\cosh(d*x+c)^3 + (8ab - 5b^2)\cosh(d*x+c)\sinh(d*x+c)^2 + 30(8a^2 - 12ab + 5b^2)\cosh(d*x+c)\right)/d$

Sympy [A] time = 2.35011, size = 128, normalized size = 2.25

$$\left\{ \begin{array}{l} \frac{a^2 \cosh(c+dx)}{d} + \frac{2ab \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{4ab \cosh^3(c+dx)}{3d} + \frac{b^2 \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{4b^2 \sinh^2(c+dx) \cosh^3(c+dx)}{3d} + \frac{8b^2 \cosh^5(c+dx)}{15d} \\ x \left(a + b \sinh^2(c) \right)^2 \sinh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 4*a*b*cosh(c + d*x)**3/(3*d) + b**2*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b**2*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b**2*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c), True))

Giac [B] time = 1.25033, size = 220, normalized size = 3.86

$$\frac{3b^2e^{(5dx+5c)} + 40abe^{(3dx+3c)} - 25b^2e^{(3dx+3c)} + 240a^2e^{(dx+c)} - 360abe^{(dx+c)} + 150b^2e^{(dx+c)} + (240a^2e^{(4dx+4c)} - 360abe^{(4dx+4c)})}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c))^2,x, algorithm="giac")

[Out] 1/480*(3*b^2*e^(5*d*x + 5*c) + 40*a*b*e^(3*d*x + 3*c) - 25*b^2*e^(3*d*x + 3*c) + 240*a^2*e^(d*x + c) - 360*a*b*e^(d*x + c) + 150*b^2*e^(d*x + c) + (240*a^2*e^(4*d*x + 4*c) - 360*a*b*e^(4*d*x + 4*c) + 150*b^2*e^(4*d*x + 4*c) + 40*a*b*e^(2*d*x + 2*c) - 25*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-5*d*x - 5*c)) /d

3.14 $\int (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=72

$$\frac{1}{8}x(8a^2 - 8ab + 3b^2) + \frac{b(8a - 3b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{b^2 \sinh^3(c + dx) \cosh(c + dx)}{4d}$$

[Out] $((8*a^2 - 8*a*b + 3*b^2)*x)/8 + ((8*a - 3*b)*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)$

Rubi [A] time = 0.0217836, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3179}

$$\frac{1}{8}x(8a^2 - 8ab + 3b^2) + \frac{b(8a - 3b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{b^2 \sinh^3(c + dx) \cosh(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^2,x]

[Out] $((8*a^2 - 8*a*b + 3*b^2)*x)/8 + ((8*a - 3*b)*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)$

Rule 3179

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> Simp[((8*a^2 + 8*a*b + 3*b^2)*x)/8, x] + (-Simp[(b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[(b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f}, x]

Rubi steps

$$\int (a + b \sinh^2(c + dx))^2 dx = \frac{1}{8} (8a^2 - 8ab + 3b^2) x + \frac{(8a - 3b)b \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b^2 \cosh(c + dx) \sinh^3(c + dx)}{4d}$$

Mathematica [A] time = 0.126727, size = 60, normalized size = 0.83

$$\frac{4(8a^2 - 8ab + 3b^2)(c + dx) + 8b(2a - b) \sinh(2(c + dx)) + b^2 \sinh(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (4*(8*a^2 - 8*a*b + 3*b^2)*(c + d*x) + 8*(2*a - b)*b*Sinh[2*(c + d*x)] + b^2*Sinh[4*(c + d*x)])/(32*d)

Maple [A] time = 0.013, size = 79, normalized size = 1.1

$$\frac{1}{d} \left(b^2 \left(\left(\frac{(\sinh(dx+c))^3}{4} - \frac{3 \sinh(dx+c)}{8} \right) \cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8} \right) + 2ab \left(\frac{1}{2} \cosh(dx+c) \sinh(dx+c) - \frac{1}{2} dx - \frac{1}{2} c \right) + a^2 (dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*(b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a^2*(d*x+c))

Maxima [A] time = 1.00849, size = 142, normalized size = 1.97

$$\frac{1}{64} b^2 \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) - \frac{1}{4} ab \left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d} \right) + a^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^2*x

Fricas [A] time = 1.87806, size = 193, normalized size = 2.68

$$\frac{b^2 \cosh(dx+c) \sinh(dx+c)^3 + (8a^2 - 8ab + 3b^2)dx + (b^2 \cosh(dx+c)^3 + 4(2ab - b^2) \cosh(dx+c)) \sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $\frac{1}{8}(b^2 \cosh(dx+c) \sinh(dx+c)^3 + (8a^2 - 8ab + 3b^2)dx + (b^2 \cosh(dx+c)^3 + 4(2ab - b^2) \cosh(dx+c) \sinh(dx+c)))/d$

Sympy [A] time = 1.27699, size = 168, normalized size = 2.33

$$\left\{ \begin{array}{l} a^2x + abx \sinh^2(c+dx) - abx \cosh^2(c+dx) + \frac{ab \sinh(c+dx) \cosh(c+dx)}{d} + \frac{3b^2x \sinh^4(c+dx)}{8} - \frac{3b^2x \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3b^2}{8} \\ x(a + b \sinh^2(c))^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)**2)**2,x)

[Out] Piecewise((a**2*x + a*b*x*sinh(c + d*x)**2 - a*b*x*cosh(c + d*x)**2 + a*b*sinh(c + d*x)*cosh(c + d*x)/d + 3*b**2*x*sinh(c + d*x)**4/8 - 3*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*b**2*x*cosh(c + d*x)**4/8 + 5*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2, True))

Giac [B] time = 1.22594, size = 204, normalized size = 2.83

$$\frac{b^2 e^{(4dx+4c)} + 16 abe^{(2dx+2c)} - 8 b^2 e^{(2dx+2c)} + 8(8a^2 - 8ab + 3b^2)(dx+c) - (48a^2 e^{(4dx+4c)} - 48abe^{(4dx+4c)} + 18b^2 e^{(4dx+4c)})}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{64}(b^2 e^{(4dx+4c)} + 16ab e^{(2dx+2c)} - 8b^2 e^{(2dx+2c)} + 8(8a^2 - 8ab + 3b^2)(dx+c) - (48a^2 e^{(4dx+4c)} - 48ab e^{(4dx+4c)} + 18b^2 e^{(4dx+4c)}) + 16ab e^{(2dx+2c)} - 8b^2 e^{(2dx+2c)} + b^2) e^{(-4dx-4c)})/d$

3.15 $\int \operatorname{csch}(c + dx) \left(a + b \sinh^2(c + dx) \right)^2 dx$

Optimal. Leaf size=52

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b(2a - b) \cosh(c + dx)}{d} + \frac{b^2 \cosh^3(c + dx)}{3d}$$

[Out] $-\left(\frac{a^2 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) + \left(\frac{(2*a - b)*b*\operatorname{Cosh}[c + d*x]}{d}\right) + \left(\frac{b^2*\operatorname{Cosh}[c + d*x]^3}{3*d}\right)$

Rubi [A] time = 0.0657536, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3186, 390, 206}

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b(2a - b) \cosh(c + dx)}{d} + \frac{b^2 \cosh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Sinh}[c + d*x]^2)^2, x]$

[Out] $-\left(\frac{a^2 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) + \left(\frac{(2*a - b)*b*\operatorname{Cosh}[c + d*x]}{d}\right) + \left(\frac{b^2*\operatorname{Cosh}[c + d*x]^3}{3*d}\right)$

Rule 3186

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - b*ff^2*x^2)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x]] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 390

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(n_.)]^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(a + b*x^n)^p, (c + d*x^n)^{-q}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{ILtQ}[q, 0] \&\& \operatorname{GeQ}[p, -q]$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& \operatorname{Gt}$

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c+dx) (a+b \sinh^2(c+dx))^2 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^2}{1-x^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(-2a-b\right)b - b^2x^2 + \frac{a^2}{1-x^2}\right) dx, x, \cosh(c+dx)}{d} \\ &= \frac{(2a-b)b \cosh(c+dx)}{d} + \frac{b^2 \cosh^3(c+dx)}{3d} - \frac{a^2 \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{a^2 \tanh^{-1}(\cosh(c+dx))}{d} + \frac{(2a-b)b \cosh(c+dx)}{d} + \frac{b^2 \cosh^3(c+dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0339139, size = 104, normalized size = 2.

$$\frac{a^2 \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a^2 \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{2ab \sinh(c) \sinh(dx)}{d} + \frac{2ab \cosh(c) \cosh(dx)}{d} - \frac{3b^2 \cosh(c+dx)}{4d} +$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (2*a*b*Cosh[c]*Cosh[d*x])/d - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)]/(12*d) - (a^2*Log[Cosh[c/2 + (d*x)/2]])/d + (a^2*Log[Sinh[c/2 + (d*x)/2]])/d + (2*a*b*Sinh[c]*Sinh[d*x])/d

Maple [A] time = 0.032, size = 50, normalized size = 1.

$$\frac{1}{d} \left(-2a^2 \operatorname{Artanh}(e^{dx+c}) + 2ab \cosh(dx+c) + b^2 \left(-\frac{2}{3} + \frac{(\sinh(dx+c))^2}{3} \right) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)

[Out] $1/d*(-2*a^2*\operatorname{arctanh}(\exp(d*x+c))+2*a*b*\cosh(d*x+c)+b^2*(-2/3+1/3*\sinh(d*x+c))^2*\cosh(d*x+c))$

Maxima [B] time = 1.04856, size = 138, normalized size = 2.65

$$\frac{1}{24} b^2 \left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d} \right) + ab \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) + \frac{a^2 \log \left(\tanh \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $1/24*b^2*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d) + a*b*(e^{(d*x + c)}/d + e^{(-d*x - c)}/d) + a^2*\log(\tanh(1/2*d*x + 1/2*c))/d$

Fricas [B] time = 1.85076, size = 1280, normalized size = 24.62

$$b^2 \cosh(dx + c)^6 + 6b^2 \cosh(dx + c) \sinh(dx + c)^5 + b^2 \sinh(dx + c)^6 + 3(8ab - 3b^2) \cosh(dx + c)^4 + 3(5b^2 \cosh(dx + c)^2 + 8ab - 3b^2) \sinh(dx + c)^4 + 4(5b^2 \cosh(dx + c)^3 + 3(8ab - 3b^2) \cosh(dx + c)) \sinh(dx + c)^3 + 3(8ab - 3b^2) \cosh(dx + c)^2 + 3(5b^2 \cosh(dx + c)^4 + 6(8ab - 3b^2) \cosh(dx + c)^2 + 8ab - 3b^2) \sinh(dx + c)^2 + b^2 - 24(a^2 \cosh(dx + c)^3 + 3a^2 \cosh(dx + c)^2 \sinh(dx + c) + 3a^2 \cosh(dx + c) \sinh(dx + c)^2 + a^2 \sinh(dx + c)^3) \log(\cosh(dx + c) + \sinh(dx + c) + 1) + 24(a^2 \cosh(dx + c)^3 + 3a^2 \cosh(dx + c)^2 \sinh(dx + c) + 3a^2 \cosh(dx + c) \sinh(dx + c)^2 + a^2 \sinh(dx + c)^3) \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 6(b^2 \cosh(dx + c)^5 + 2(8ab - 3b^2) \cosh(dx + c)^3 + (8ab - 3b^2) \cosh(dx + c)) \sinh(dx + c) / (d \cosh(dx + c)^3 + 3d \cosh(dx + c)^2 \sinh(dx + c) + 3d \cosh(dx + c) \sinh(dx + c)^2 + d \sinh(dx + c)^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $1/24*(b^2*\cosh(d*x + c)^6 + 6*b^2*\cosh(d*x + c)*\sinh(d*x + c)^5 + b^2*\sinh(d*x + c)^6 + 3*(8*a*b - 3*b^2)*\cosh(d*x + c)^4 + 3*(5*b^2*\cosh(d*x + c)^2 + 8*a*b - 3*b^2)*\sinh(d*x + c)^4 + 4*(5*b^2*\cosh(d*x + c)^3 + 3*(8*a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(8*a*b - 3*b^2)*\cosh(d*x + c)^2 + 3*(5*b^2*\cosh(d*x + c)^4 + 6*(8*a*b - 3*b^2)*\cosh(d*x + c)^2 + 8*a*b - 3*b^2)*\sinh(d*x + c)^2 + b^2 - 24*(a^2*\cosh(d*x + c)^3 + 3*a^2*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a^2*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^2*\sinh(d*x + c)^3)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 24*(a^2*\cosh(d*x + c)^3 + 3*a^2*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a^2*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^2*\sinh(d*x + c)^3)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 6*(b^2*\cosh(d*x + c)^5 + 2*(8*a*b - 3*b^2)*\cosh(d*x + c)^3 + (8*a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c) / (d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + d*\sinh(d*x + c)^3)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.28008, size = 174, normalized size = 3.35

$$-\frac{a^2 \log(e^{(dx+c)} + 1)}{d} + \frac{a^2 \log(|e^{(dx+c)} - 1|)}{d} + \frac{(24 abe^{(2dx+2c)} - 9 b^2 e^{(2dx+2c)} + b^2) e^{(-3dx-3c)}}{24d} + \frac{b^2 d^2 e^{(3dx+3c)} + 24 abd^2 e^{(3dx+3c)}}{24d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] -a^2*log(e^(d*x + c) + 1)/d + a^2*log(abs(e^(d*x + c) - 1))/d + 1/24*(24*a*b*e^(2*d*x + 2*c) - 9*b^2*e^(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c)/d + 1/24*(b^2*d^2*e^(3*d*x + 3*c) + 24*a*b*d^2*e^(d*x + c) - 9*b^2*d^2*e^(d*x + c))/d^3

3.16 $\int \operatorname{csch}^2(c + dx) \left(a + b \sinh^2(c + dx)\right)^2 dx$

Optimal. Leaf size=50

$$-\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{1}{2}bx(4a - b) + \frac{b^2 \sinh(c + dx) \cosh(c + dx)}{2d}$$

[Out] $((4*a - b)*b*x)/2 - (a^2*\operatorname{Coth}[c + d*x])/d + (b^2*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/ (2*d)$

Rubi [A] time = 0.0813633, antiderivative size = 64, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3187, 462, 385, 206}

$$\frac{(2a^2 + b^2) \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{a^2 \cosh^2(c + dx) \operatorname{coth}(c + dx)}{d} + \frac{1}{2}bx(4a - b)$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^2*(a + b*\operatorname{Sinh}[c + d*x]^2)^2, x]$

[Out] $((4*a - b)*b*x)/2 - (a^2*\operatorname{Cosh}[c + d*x]^2*\operatorname{Coth}[c + d*x])/d + ((2*a^2 + b^2)*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/ (2*d)$

Rule 3187

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^{(m/2 + p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x] \}; \operatorname{FreeQ}\{a, b, e, f\}, x\} \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 462

$\operatorname{Int}[(e_.)*(x_.)^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^2, x_Symbol] \rightarrow \operatorname{Simp}[(c^2*(e*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)})/(a*e^{(m + 1)}), x] - \operatorname{Dist}[1/(a*e^n*(m + 1)), \operatorname{Int}[(e*x)^{(m + n)}*(a + b*x^n)^p*\operatorname{Simp}[b*c^2*n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x] \}; \operatorname{FreeQ}\{a, b, c, d, e, p\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{GtQ}[n, 0]$

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[
((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a - (a-b)x^2)^2}{x^2(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^2 \cosh^2(c + dx) \coth(c + dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{a(a+2b) + (a-b)^2 x^2}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a^2 \cosh^2(c + dx) \coth(c + dx)}{d} + \frac{(2a^2 + b^2) \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{2abx}{d} \\ &= \frac{1}{2}(4a - b)bx - \frac{a^2 \cosh^2(c + dx) \coth(c + dx)}{d} + \frac{(2a^2 + b^2) \cosh(c + dx) \sinh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.162947, size = 56, normalized size = 1.12

$$-\frac{a^2 \coth(c + dx)}{d} + 2abx + \frac{b^2(-c - dx)}{2d} + \frac{b^2 \sinh(2(c + dx))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] 2*a*b*x + (b^2*(-c - d*x))/(2*d) - (a^2*Coth[c + d*x])/d + (b^2*Sinh[2*(c + d*x)])/(4*d)

Maple [A] time = 0.033, size = 52, normalized size = 1.

$$\frac{1}{d} \left(-a^2 \coth(dx+c) + 2ab(dx+c) + b^2 \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*(d*x+c)+b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))

Maxima [A] time = 1.01429, size = 85, normalized size = 1.7

$$-\frac{1}{8} b^2 \left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d} \right) + 2abx + \frac{2a^2}{d(e^{-2dx-2c} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1))

Fricas [A] time = 1.87027, size = 215, normalized size = 4.3

$$\frac{b^2 \cosh(dx+c)^3 + 3b^2 \cosh(dx+c) \sinh(dx+c)^2 - (8a^2 + b^2) \cosh(dx+c) + 4((4ab - b^2)dx + 2a^2) \sinh(dx+c)}{8d \sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/8*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a^2 + b^2)*cosh(d*x + c) + 4*((4*a*b - b^2)*d*x + 2*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.22766, size = 189, normalized size = 3.78

$$\frac{b^2 e^{(2dx+2c)}}{8d} + \frac{(4ab - b^2)(dx + c)}{2d} - \frac{4abe^{(4dx+4c)} - b^2e^{(4dx+4c)} + 16a^2e^{(2dx+2c)} - 4abe^{(2dx+2c)} + 2b^2e^{(2dx+2c)} - b^2}{8d(e^{(4dx+4c)} - e^{(2dx+2c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/8*b^2*e^(2*d*x + 2*c)/d + 1/2*(4*a*b - b^2)*(d*x + c)/d - 1/8*(4*a*b*e^(4*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 4*a*b*e^(2*d*x + 2*c) + 2*b^2*e^(2*d*x + 2*c) - b^2)/(d*(e^(4*d*x + 4*c) - e^(2*d*x + 2*c)))

3.17 $\int \operatorname{csch}^3(c + dx) \left(a + b \sinh^2(c + dx)\right)^2 dx$

Optimal. Leaf size=56

$$-\frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{a(a - 4b) \tanh^{-1}(\cosh(c + dx))}{2d} + \frac{b^2 \cosh(c + dx)}{d}$$

[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b^2*Cosh[c + d*x])/d - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rubi [A] time = 0.0880729, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3186, 390, 385, 206}

$$-\frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{a(a - 4b) \tanh^{-1}(\cosh(c + dx))}{2d} + \frac{b^2 \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b^2*Cosh[c + d*x])/d - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 385

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b

c(n*(p + 1) + 1)/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^3(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^2}{(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left(b^2 + \frac{a(a-2b)+2abx^2}{(1-x^2)^2}\right) dx, x, \cosh(c + dx)\right)}{d} \\
 &= \frac{b^2 \cosh(c + dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{a(a-2b)+2abx^2}{(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\
 &= \frac{b^2 \cosh(c + dx)}{d} - \frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{(a(a-4b)) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c + dx)\right)}{2d} \\
 &= \frac{a(a-4b) \tanh^{-1}(\cosh(c + dx))}{2d} + \frac{b^2 \cosh(c + dx)}{d} - \frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d}
 \end{aligned}$$

Mathematica [B] time = 0.0639887, size = 134, normalized size = 2.39

$$-\frac{a^2 \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a^2 \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a^2 \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{2d} + \frac{2ab \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{2ab \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) - (2*a*b*Log[Cosh[c/2 + (d*x)/2]])/d + (2*a*b*Log[Sinh[c/2 + (d*x)/2]])/d - (a^2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d) + (b^2*Sinh[c]*Sinh[d*x])/d

Maple [A] time = 0.041, size = 53, normalized size = 1.

$$\frac{1}{d} \left(a^2 \left(-\frac{\operatorname{csch}(dx+c) \operatorname{coth}(dx+c)}{2} + \operatorname{Artanh}(e^{dx+c}) \right) - 4ab \operatorname{Artanh}(e^{dx+c}) + b^2 \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)`

[Out] `1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-4*a*b*arctanh(exp(d*x+c))+b^2*cosh(d*x+c))`

Maxima [B] time = 1.07468, size = 212, normalized size = 3.79

$$\frac{1}{2} b^2 \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) + \frac{1}{2} a^2 \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2(e^{(-dx-c)} + e^{(-3dx-3c)})}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right) - 2ab \left(\frac{\log(e^{(-dx-c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] `1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)`

Fricas [B] time = 1.95247, size = 2310, normalized size = 41.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] `1/2*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - (2*a^2 + b^2)*cosh(d*x + c)^4 + (15*b^2*cosh(d*x + c)^2 - 2*a^2 - b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x`

$$\begin{aligned}
& + c))\sinh(dx + c)^3 - (2a^2 + b^2)\cosh(dx + c)^2 + (15b^2\cosh(dx + c)^4 - 6(2a^2 + b^2)\cosh(dx + c)^2 - 2a^2 - b^2)\sinh(dx + c)^2 + b^2 \\
& + ((a^2 - 4ab)\cosh(dx + c)^5 + 5(a^2 - 4ab)\cosh(dx + c)\sinh(dx + c)^4 + (a^2 - 4ab)\sinh(dx + c)^5 - 2(a^2 - 4ab)\cosh(dx + c)^3 + \\
& + 2(5(a^2 - 4ab)\cosh(dx + c)^2 - a^2 + 4ab)\sinh(dx + c)^3 + 2(5(a^2 - 4ab)\cosh(dx + c)^3 - 3(a^2 - 4ab)\cosh(dx + c))\sinh(dx + c)^2 \\
& + (a^2 - 4ab)\cosh(dx + c) + (5(a^2 - 4ab)\cosh(dx + c)^4 - 6(a^2 - 4ab)\cosh(dx + c)^2 + a^2 - 4ab)\sinh(dx + c))\log(\cosh(dx + c) \\
& + \sinh(dx + c) + 1) - ((a^2 - 4ab)\cosh(dx + c)^5 + 5(a^2 - 4ab)\cosh(dx + c)\sinh(dx + c)^4 + (a^2 - 4ab)\sinh(dx + c)^5 - 2(a^2 - 4ab) \\
&)\cosh(dx + c)^3 + 2(5(a^2 - 4ab)\cosh(dx + c)^2 - a^2 + 4ab)\sinh(dx + c)^3 + 2(5(a^2 - 4ab)\cosh(dx + c)^3 - 3(a^2 - 4ab)\cosh(dx + c))\sinh(dx + c)^2 \\
& + (a^2 - 4ab)\cosh(dx + c) + (5(a^2 - 4ab)\cosh(dx + c)^4 - 6(a^2 - 4ab)\cosh(dx + c)^2 + a^2 - 4ab)\sinh(dx + c)) \\
& * \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 2(3b^2\cosh(dx + c)^5 - 2(2a^2 + b^2)\cosh(dx + c)^3 - (2a^2 + b^2)\cosh(dx + c))\sinh(dx + c) / (d \\
& \cosh(dx + c)^5 + 5d\cosh(dx + c)\sinh(dx + c)^4 + d\sinh(dx + c)^5 - 2d\cosh(dx + c)^3 + 2(5d\cosh(dx + c)^2 - d)\sinh(dx + c)^3 + 2(5d\cosh(dx + c)^3 - 3d\cosh(dx + c))\sinh(dx + c)^2 \\
& + d\cosh(dx + c) + (5d\cosh(dx + c)^4 - 6d\cosh(dx + c)^2 + d)\sinh(dx + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)**3*(a+b*sinh(dx+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.35923, size = 180, normalized size = 3.21

$$\frac{b^2(e^{dx+c} + e^{-dx-c})}{2d} - \frac{a^2(e^{dx+c} + e^{-dx-c})}{\left((e^{dx+c} + e^{-dx-c})^2 - 4\right)d} + \frac{(a^2 - 4ab)\log(e^{dx+c} + e^{-dx-c} + 2)}{4d} - \frac{(a^2 - 4ab)\log(e^{dx+c} + e^{-dx-c})}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^3*(a+b*sinh(dx+c)^2)^2,x, algorithm="giac")

```
[Out] 1/2*b^2*(e^(d*x + c) + e^(-d*x - c))/d - a^2*(e^(d*x + c) + e^(-d*x - c))/(
((e^(d*x + c) + e^(-d*x - c))^2 - 4)*d) + 1/4*(a^2 - 4*a*b)*log(e^(d*x + c)
+ e^(-d*x - c) + 2)/d - 1/4*(a^2 - 4*a*b)*log(e^(d*x + c) + e^(-d*x - c) -
2)/d
```

3.18 $\int \operatorname{csch}^4(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=40

$$-\frac{a^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{a(a - 2b) \operatorname{coth}(c + dx)}{d} + b^2 x$$

[Out] $b^2 x + (a(a - 2b) \operatorname{Coth}[c + d*x])/d - (a^2 \operatorname{Coth}[c + d*x]^3)/(3*d)$

Rubi [A] time = 0.0741049, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3187, 461, 207}

$$-\frac{a^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{a(a - 2b) \operatorname{coth}(c + dx)}{d} + b^2 x$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^4*(a + b*\operatorname{Sinh}[c + d*x]^2)^2, x]$

[Out] $b^2 x + (a(a - 2b) \operatorname{Coth}[c + d*x])/d - (a^2 \operatorname{Coth}[c + d*x]^3)/(3*d)$

Rule 3187

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^{(m/2 + p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]\} /; \operatorname{FreeQ}\{a, b, e, f\}, x\} \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 461

$\operatorname{Int}[(((e_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)})/((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(e*x)^m*(a + b*x^n)^p/(c + d*x^n), x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, m\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{IGtQ}[p, 0] \&\& (\operatorname{IntegerQ}[m] \mid \mid \operatorname{IGtQ}[2*(m + 1), 0] \mid \mid \operatorname{!RationalQ}[m])$

Rule 207

$\operatorname{Int}[((a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{LtQ}[a$

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^4(c+dx) (a+b \sinh^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-(a-b)x^2)^2}{x^4(1-x^2)} dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2}{x^4} - \frac{a(a-2b)}{x^2} - \frac{b^2}{-1+x^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{a(a-2b) \operatorname{coth}(c+dx)}{d} - \frac{a^2 \operatorname{coth}^3(c+dx)}{3d} - \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \tanh(c+dx)\right)}{d} \\
 &= b^2 x + \frac{a(a-2b) \operatorname{coth}(c+dx)}{d} - \frac{a^2 \operatorname{coth}^3(c+dx)}{3d}
 \end{aligned}$$

Mathematica [B] time = 0.712014, size = 85, normalized size = 2.12

$$\frac{4 \sinh^4(c+dx) \left(\operatorname{acsch}^2(c+dx) + b\right)^2 \left(3b^2(c+dx) - a \operatorname{coth}(c+dx) \left(\operatorname{acsch}^2(c+dx) - 2a + 6b\right)\right)}{3d(2a + b \cosh(2(c+dx)) - b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (4*(b + a*Csch[c + d*x]^2)^2*(3*b^2*(c + d*x) - a*Coth[c + d*x]*(-2*a + 6*b + a*Csch[c + d*x]^2))*Sinh[c + d*x]^4)/(3*d*(2*a - b + b*Cosh[2*(c + d*x)]^2)

Maple [A] time = 0.04, size = 47, normalized size = 1.2

$$\frac{1}{d} \left(a^2 \left(\frac{2}{3} - \frac{(\operatorname{csch}(dx+c))^2}{3} \right) \operatorname{coth}(dx+c) - 2ab \operatorname{coth}(dx+c) + b^2(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)

[Out] $1/d*(a^2*(2/3-1/3*\operatorname{csch}(d*x+c)^2)*\operatorname{coth}(d*x+c)-2*a*b*\operatorname{coth}(d*x+c)+b^2*(d*x+c))$

Maxima [B] time = 1.06279, size = 163, normalized size = 4.08

$$b^2x + \frac{4}{3}a^2 \left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right) + \frac{4ab}{d(e^{(-2dx-2c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $b^2x + 4/3*a^2*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1))) + 4*a*b/(d*(e^{(-2*d*x - 2*c)} - 1))$

Fricas [B] time = 1.89286, size = 428, normalized size = 10.7

$$\frac{2(a^2 - 3ab) \cosh(dx + c)^3 + 6(a^2 - 3ab) \cosh(dx + c) \sinh(dx + c)^2 + (3b^2dx - 2a^2 + 6ab) \sinh(dx + c)^3 - 6(a^2 - 3ab) \cosh(dx + c) \sinh(dx + c)}{3(d \sinh(dx + c))^3 + 3(d \cosh(dx + c))^2 - d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $1/3*(2*(a^2 - 3*a*b)*\cosh(d*x + c)^3 + 6*(a^2 - 3*a*b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (3*b^2*d*x - 2*a^2 + 6*a*b)*\sinh(d*x + c)^3 - 6*(a^2 - 3*a*b)*\cosh(d*x + c) - 3*(3*b^2*d*x - (3*b^2*d*x - 2*a^2 + 6*a*b)*\cosh(d*x + c)^2 - 2*a^2 + 6*a*b)*\sinh(d*x + c))/(d*\sinh(d*x + c)^3 + 3*(d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.34544, size = 109, normalized size = 2.72

$$\frac{(dx+c)b^2}{d} - \frac{4(3abe^{4dx+4c} + 3a^2e^{2dx+2c} - 6abe^{2dx+2c} - a^2 + 3ab)}{3d(e^{2dx+2c} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] (d*x + c)*b^2/d - 4/3*(3*a*b*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) - a^2 + 3*a*b)/(d*(e^(2*d*x + 2*c) - 1)^3)

3.19 $\int \sinh^4(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=261

$$\frac{(-272a^2b + 48a^3 + 314ab^2 - 105b^3) \sinh(c + dx) \cosh^3(c + dx)}{640d} - \frac{(-1744a^2b + 576a^3 + 1678ab^2 - 525b^3) \sinh(c + dx)}{1280d}$$

```
[Out] (3*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*x)/256 - ((576*a^3 - 1744*a^2*b + 1
678*a*b^2 - 525*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(1280*d) + ((48*a^3 - 272
*a^2*b + 314*a*b^2 - 105*b^3)*Cosh[c + d*x]^3*Sinh[c + d*x])/(640*d) + (3*(
2*a - 3*b)*Cosh[c + d*x]^5*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^2)
/(80*d) + (Cosh[c + d*x]^7*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^3)
/(10*d) - (b*Cosh[c + d*x]^3*Sinh[c + d*x]^3*(a*(14*a - 9*b) - (22*a - 21*b
)*(a - b)*Tanh[c + d*x]^2))/(160*d)
```

Rubi [A] time = 0.429778, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3187, 467, 577, 455, 385, 206}

$$\frac{(-272a^2b + 48a^3 + 314ab^2 - 105b^3) \sinh(c + dx) \cosh^3(c + dx)}{640d} - \frac{(-1744a^2b + 576a^3 + 1678ab^2 - 525b^3) \sinh(c + dx)}{1280d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*x)/256 - ((576*a^3 - 1744*a^2*b + 1
678*a*b^2 - 525*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(1280*d) + ((48*a^3 - 272
*a^2*b + 314*a*b^2 - 105*b^3)*Cosh[c + d*x]^3*Sinh[c + d*x])/(640*d) + (3*(
2*a - 3*b)*Cosh[c + d*x]^5*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^2)
/(80*d) + (Cosh[c + d*x]^7*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^3)
/(10*d) - (b*Cosh[c + d*x]^3*Sinh[c + d*x]^3*(a*(14*a - 9*b) - (22*a - 21*b
)*(a - b)*Tanh[c + d*x]^2))/(160*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1),
x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
```

IntegerQ[p]

Rule 467

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 577

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n*q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && !(EqQ[q, 1] && SimplifierQ[b*c - a*d, b*e - a*f])

Rule 455

Int[(x_)^(m_)*((a_) + (b_)*(x_)^2)^(p_)*((c_) + (d_)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 385

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \sinh^4(c + dx) (a + b \sinh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x^4(a-(a-b)x^2)^3}{(1-x^2)^6} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{\cosh^7(c + dx) \sinh^3(c + dx) (a - (a - b) \tanh^2(c + dx))^3}{10d} - \frac{\text{Subst}\left(\int \frac{x^2(3a - 3b + 3bx^2)}{(1-x^2)^6} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{3(2a - 3b) \cosh^5(c + dx) \sinh^3(c + dx) (a - (a - b) \tanh^2(c + dx))^2}{80d} + \frac{\cosh^3(c + dx) \sinh^3(c + dx) (a - (a - b) \tanh^2(c + dx))^2}{80d} \\
 &= \frac{3(2a - 3b) \cosh^5(c + dx) \sinh^3(c + dx) (a - (a - b) \tanh^2(c + dx))^2}{80d} + \frac{\cosh^3(c + dx) \sinh^3(c + dx) (a - (a - b) \tanh^2(c + dx))^2}{80d} \\
 &= \frac{(48a^3 - 272a^2b + 314ab^2 - 105b^3) \cosh^3(c + dx) \sinh(c + dx)}{640d} + \frac{3(2a - 3b) \cosh^3(c + dx) \sinh^3(c + dx)}{640d} \\
 &= -\frac{(576a^3 - 1744a^2b + 1678ab^2 - 525b^3) \cosh(c + dx) \sinh(c + dx)}{1280d} + \frac{(48a^3 - 272a^2b + 314ab^2 - 105b^3) \cosh^3(c + dx) \sinh(c + dx)}{1280d} \\
 &= \frac{3}{256} (4a - 3b) (8a^2 - 14ab + 7b^2) x - \frac{(576a^3 - 1744a^2b + 1678ab^2 - 525b^3) \cosh(c + dx) \sinh(c + dx)}{1280d}
 \end{aligned}$$

Mathematica [A] time = 0.42218, size = 162, normalized size = 0.62

$$\frac{120(4a - 3b)(8a^2 - 14ab + 7b^2)(c + dx) + 10b(16a^2 - 32ab + 15b^2) \sinh(6(c + dx)) - 20(-360a^2b + 128a^3 + 336ab^2 - 105b^3) \cosh(2(c + dx)) + 40(8a^3 - 36a^2b + 42ab^2 - 15b^3) \cosh(4(c + dx)) + 10b(16a^2 - 32ab + 15b^2) \cosh(6(c + dx)) + 5(6a - 5b)b^2 \cosh(8(c + dx)) + 2b^3 \cosh(10(c + dx))}{10240d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (120*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*(c + d*x) - 20*(128*a^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*Sinh[4*(c + d*x)] + 10*b*(16*a^2 - 32*a*b + 15*b^2)*Sinh[6*(c + d*x)] + 5*(6*a - 5*b)*b^2*Sinh[8*(c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)

Maple [A] time = 0.052, size = 222, normalized size = 0.9

$$\frac{1}{d} \left(b^3 \left(\frac{(\sinh(dx+c))^9}{10} - \frac{9(\sinh(dx+c))^7}{80} + \frac{21(\sinh(dx+c))^5}{160} - \frac{21(\sinh(dx+c))^3}{128} + \frac{63\sinh(dx+c)}{256} \right) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)`

[Out] `1/d*(b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/128*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+3*a*b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+3*a^2*b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))`

Maxima [A] time = 1.0771, size = 547, normalized size = 2.1

$$\frac{1}{64} a^3 \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) - \frac{1}{20480} b^3 \left(\frac{25e^{-2dx-2c} - 150e^{-4dx-4c} + 600e^{-6dx-6c}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] `1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 1/2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) - 1/128*a^2*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)`

Fricas [A] time = 1.80972, size = 1013, normalized size = 3.88

$$5b^3 \cosh(dx + c) \sinh(dx + c)^9 + 10(6b^3 \cosh(dx + c)^3 + (6ab^2 - 5b^3) \cosh(dx + c)) \sinh(dx + c)^7 + (126b^3 \cosh(dx + c) \sinh(dx + c)^5 + 70(6a^2b - 5b^3) \cosh(dx + c)^3 + 15(16a^2b - 32ab^2 + 15b^3) \cosh(dx + c) \sinh(dx + c)^5 + 10(6b^3 \cosh(dx + c)^7 + 7(6a^2b - 5b^3) \cosh(dx + c)^5 + 5(16a^2b - 32ab^2 + 15b^3) \cosh(dx + c)^3 + 4(8a^3 - 36a^2b + 42ab^2 - 15b^3) \cosh(dx + c) \sinh(dx + c)^3 + 30(32a^3 - 80a^2b + 70ab^2 - 21b^3) dx + 5(b^3 \cosh(dx + c)^9 + 2(6a^2b - 5b^3) \cosh(dx + c)^7 + 3(16a^2b - 32ab^2 + 15b^3) \cosh(dx + c)^5 + 8(8a^3 - 36a^2b + 42ab^2 - 15b^3) \cosh(dx + c)^3 - 2(128a^3 - 360a^2b + 336ab^2 - 105b^3) \cosh(dx + c) \sinh(dx + c)) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 + (6*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^5 + 70*(6*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 5*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d*x + c)^3 + 4*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*d*x + 5*(b^3*cosh(d*x + c)^9 + 2*(6*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 3*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d*x + c)^5 + 8*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c)^3 - 2*(128*a^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*cosh(d*x + c))*sinh(d*x + c))/d

Sympy [A] time = 37.0623, size = 777, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Piecewise(((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 + 5*a**3*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 3*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 15*a**2*b*x*sinh(c + d*x)**6/16 - 45*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 45*a**2*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 15*a**2*b*x*cosh(c + d*x)**6/16 + 33*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*a**2*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 105*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x)**8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 63*b**3*x*sinh(c + d*x)**10/256 - 315*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**

```

2/256 + 315*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 315*b**3*x*sinh(
c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**3*x*sinh(c + d*x)**2*cosh(c + d*x
)**8/256 - 63*b**3*x*cosh(c + d*x)**10/256 + 193*b**3*sinh(c + d*x)**9*cosh
(c + d*x)/(256*d) - 237*b**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21
*b**3*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 147*b**3*sinh(c + d*x)**3*
cosh(c + d*x)**7/(128*d) + 63*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d),
Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**4, True))

```

Giac [B] time = 1.39929, size = 679, normalized size = 2.6

$$2b^3e^{10dx+10c} + 30ab^2e^{8dx+8c} - 25b^3e^{8dx+8c} + 160a^2be^{6dx+6c} - 320ab^2e^{6dx+6c} + 150b^3e^{6dx+6c} + 320a^3e^{4dx+4c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/20480*(2*b^3*e^(10*d*x + 10*c) + 30*a*b^2*e^(8*d*x + 8*c) - 25*b^3*e^(8*d
*x + 8*c) + 160*a^2*b*e^(6*d*x + 6*c) - 320*a*b^2*e^(6*d*x + 6*c) + 150*b^3
*e^(6*d*x + 6*c) + 320*a^3*e^(4*d*x + 4*c) - 1440*a^2*b*e^(4*d*x + 4*c) + 1
680*a*b^2*e^(4*d*x + 4*c) - 600*b^3*e^(4*d*x + 4*c) - 2560*a^3*e^(2*d*x + 2
*c) + 7200*a^2*b*e^(2*d*x + 2*c) - 6720*a*b^2*e^(2*d*x + 2*c) + 2100*b^3*e^
(2*d*x + 2*c) + 240*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*(d*x + c) - (87
68*a^3*e^(10*d*x + 10*c) - 21920*a^2*b*e^(10*d*x + 10*c) + 19180*a*b^2*e^(1
0*d*x + 10*c) - 5754*b^3*e^(10*d*x + 10*c) - 2560*a^3*e^(8*d*x + 8*c) + 720
0*a^2*b*e^(8*d*x + 8*c) - 6720*a*b^2*e^(8*d*x + 8*c) + 2100*b^3*e^(8*d*x +
8*c) + 320*a^3*e^(6*d*x + 6*c) - 1440*a^2*b*e^(6*d*x + 6*c) + 1680*a*b^2*e^
(6*d*x + 6*c) - 600*b^3*e^(6*d*x + 6*c) + 160*a^2*b*e^(4*d*x + 4*c) - 320*a
*b^2*e^(4*d*x + 4*c) + 150*b^3*e^(4*d*x + 4*c) + 30*a*b^2*e^(2*d*x + 2*c) -
25*b^3*e^(2*d*x + 2*c) + 2*b^3)*e^(-10*d*x - 10*c))/d
```

3.20 $\int \sinh^3(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=115

$$\frac{b^2(3a - 4b) \cosh^7(c + dx)}{7d} + \frac{3b(a - 2b)(a - b) \cosh^5(c + dx)}{5d} + \frac{(a - 4b)(a - b)^2 \cosh^3(c + dx)}{3d} - \frac{(a - b)^3 \cosh(c + dx)}{d}$$

[Out] $-\frac{((a - b)^3 \text{Cosh}[c + d*x])}{d} + \frac{((a - 4*b)*(a - b)^2 \text{Cosh}[c + d*x]^3)}{(3*d)} + \frac{(3*(a - 2*b)*(a - b)*b \text{Cosh}[c + d*x]^5)}{(5*d)} + \frac{((3*a - 4*b)*b^2 \text{Cosh}[c + d*x]^7)}{(7*d)} + \frac{(b^3 \text{Cosh}[c + d*x]^9)}{(9*d)}$

Rubi [A] time = 0.129173, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3186, 373}

$$\frac{b^2(3a - 4b) \cosh^7(c + dx)}{7d} + \frac{3b(a - 2b)(a - b) \cosh^5(c + dx)}{5d} + \frac{(a - 4b)(a - b)^2 \cosh^3(c + dx)}{3d} - \frac{(a - b)^3 \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^3*(a + b*\text{Sinh}[c + d*x]^2)^3, x]$

[Out] $-\frac{((a - b)^3 \text{Cosh}[c + d*x])}{d} + \frac{((a - 4*b)*(a - b)^2 \text{Cosh}[c + d*x]^3)}{(3*d)} + \frac{(3*(a - 2*b)*(a - b)*b \text{Cosh}[c + d*x]^5)}{(5*d)} + \frac{((3*a - 4*b)*b^2 \text{Cosh}[c + d*x]^7)}{(7*d)} + \frac{(b^3 \text{Cosh}[c + d*x]^9)}{(9*d)}$

Rule 3186

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - b*ff^2*x^2)^p, x], x, \text{Cos}[e + f*x]/ff], x]] \text{ ; FreeQ}\{a, b, e, f, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

Rule 373

$\text{Int}[(a_. + (b_.)*(x_.)^{(n_.)})^{(p_.)*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x^n)^p*(c + d*x^n)^q, x], x] \text{ ; FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IGtQ}[q, 0]$

Rubi steps

$$\int \sinh^3(c + dx) (a + b \sinh^2(c + dx))^3 dx = -\frac{\text{Subst}\left(\int (1 - x^2)(a - b + bx^2)^3 dx, x, \cosh(c + dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int ((a - b)^3 - (a - 4b)(a - b)^2 x^2 + 3(a - 2b)b(-a + b)x^4 - (3a - 4b)b^2 x^6) dx, x, \cosh(c + dx)\right)}{d}$$

$$= -\frac{(a - b)^3 \cosh(c + dx)}{d} + \frac{(a - 4b)(a - b)^2 \cosh^3(c + dx)}{3d} + \frac{3(a - 2b)(a - b)b^2 \cosh^5(c + dx)}{5d}$$

Mathematica [A] time = 0.796978, size = 127, normalized size = 1.1

$$\frac{-1890(4a - 3b)(8a^2 - 14ab + 7b^2) \cosh(c + dx) + 420(-60a^2b + 16a^3 + 63ab^2 - 21b^3) \cosh(3(c + dx)) + 135b^2(4a - 3b) \cosh(5(c + dx)) + 35b^3 \cosh(7(c + dx)) + 35b^3 \cosh(9(c + dx))}{80640d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (-1890*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*Cosh[c + d*x] + 420*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*Cosh[3*(c + d*x)] + 756*(4*a - 3*b)*(a - b)*b*Cosh[5*(c + d*x)] + 135*(4*a - 3*b)*b^2*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(c + d*x)])/(80640*d)

Maple [A] time = 0.049, size = 158, normalized size = 1.4

$$\frac{1}{d} \left(b^3 \left(\frac{128}{315} + \frac{(\sinh(dx + c))^8}{9} - \frac{8(\sinh(dx + c))^6}{63} + \frac{16(\sinh(dx + c))^4}{105} - \frac{64(\sinh(dx + c))^2}{315} \right) \cosh(dx + c) + 3ab^2 \left(-\frac{16}{35} + \frac{(\sinh(dx + c))^6}{7} - \frac{6(\sinh(dx + c))^4}{35} + \frac{8(\sinh(dx + c))^2}{35} - \frac{4}{15} \right) \cosh(dx + c) + a^3 \left(-\frac{2}{3} + \frac{(\sinh(dx + c))^2}{3} \right) \cosh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/d*(b^3*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)

Maxima [B] time = 1.08331, size = 508, normalized size = 4.42

$$-\frac{1}{161280} b^3 \left(\frac{(405 e^{(-2dx-2c)} - 2268 e^{(-4dx-4c)} + 8820 e^{(-6dx-6c)} - 39690 e^{(-8dx-8c)} - 35) e^{(9dx+9c)}}{d} - \frac{39690 e^{(-dx-c)} - 8820 e^{(-3dx-3c)} + 2268 e^{(-5dx-5c)} - 405 e^{(-7dx-7c)}}{d} + \frac{35 e^{(-9dx-9c)}}{d} \right) - \frac{3}{4480} a b^2 \left(\frac{(49 e^{(-2dx-2c)} - 245 e^{(-4dx-4c)} + 1225 e^{(-6dx-6c)} - 5) e^{(7dx+7c)}}{d} + \frac{(1225 e^{(-dx-c)} - 245 e^{(-3dx-3c)} + 49 e^{(-5dx-5c)} - 5 e^{(-7dx-7c)})}{d} \right) + \frac{1}{160} a^2 b \left(\frac{3 e^{(5dx+5c)}}{d} - \frac{25 e^{(3dx+3c)}}{d} + \frac{150 e^{(dx+c)}}{d} + \frac{150 e^{(-dx-c)}}{d} - \frac{25 e^{(-3dx-3c)}}{d} + \frac{3 e^{(-5dx-5c)}}{d} \right) + \frac{1}{24} a^3 \left(\frac{e^{(3dx+3c)}}{d} - \frac{9 e^{(dx+c)}}{d} - \frac{9 e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/160*a^2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)

Fricas [B] time = 1.82041, size = 940, normalized size = 8.17

$$35 b^3 \cosh(dx + c)^9 + 315 b^3 \cosh(dx + c) \sinh(dx + c)^8 + 135 (4 a b^2 - 3 b^3) \cosh(dx + c)^7 + 105 (28 b^3 \cosh(dx + c)^5 + 35 (4 a b^2 - 3 b^3) \cosh(dx + c) \sinh(dx + c)^4 + 756 (4 a^2 b - 7 a b^2 + 3 b^3) \cosh(dx + c)^3 + 315 (14 b^3 \cosh(dx + c)^5 + 15 (4 a b^2 - 3 b^3) \cosh(dx + c) \sinh(dx + c)^3 + 12 (4 a^2 b - 7 a b^2 + 3 b^3) \cosh(dx + c) \sinh(dx + c)^4 + 420 (16 a^3 - 60 a^2 b + 63 a b^2 - 21 b^3) \cosh(dx + c)^3 + 315 (4 b^3 \cosh(dx + c)^7 + 9 (4 a b^2 - 3 b^3) \cosh(dx + c)^5 + 24 (4 a^2 b - 7 a b^2 + 3 b^3) \cosh(dx + c)^3 + 4 (16 a^3 - 60 a^2 b + 63 a b^2 - 21 b^3) \cosh(dx + c) \sinh(dx + c)^2 - 1890 (32 a^3 - 80 a^2 b + 70 a b^2 - 21 b^3) \cosh(dx + c))/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/80640*(35*b^3*cosh(d*x + c)^9 + 315*b^3*cosh(d*x + c)*sinh(d*x + c)^8 + 135*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^7 + 105*(28*b^3*cosh(d*x + c)^5 + 35*(4*a*b^2 - 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + 756*(4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 315*(14*b^3*cosh(d*x + c)^5 + 15*(4*a*b^2 - 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 12*(4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + 420*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*cosh(d*x + c)^3 + 315*(4*b^3*cosh(d*x + c)^7 + 9*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + 24*(4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*cosh(d*x + c)*sinh(d*x + c)^2 - 1890*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*cosh(d*x + c))/d

Sympy [A] time = 21.724, size = 330, normalized size = 2.87

$$\left\{ \frac{a^3 \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2a^3 \cosh^3(c+dx)}{3d} + \frac{3a^2 b \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{4a^2 b \sinh^2(c+dx) \cosh^3(c+dx)}{d} + \frac{8a^2 b \cosh^5(c+dx)}{5d} + \frac{3ab^2 \sinh^6(c+dx)}{6d} \right\} x (a + b \sinh^2(c))^3 \sinh^3(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Piecewise((a**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**3*cosh(c + d*x)**3/(3*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/(5*d) + 3*a*b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 6*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 2*4*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 48*a*b**2*cosh(c + d*x)**7/(35*d) + b**3*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**3*sinh(c + d*x)**6*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**3*cosh(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**3, True))

Giac [B] time = 1.41424, size = 544, normalized size = 4.73

$$35 b^3 e^{(9 dx+9 c)} + 540 a b^2 e^{(7 dx+7 c)} - 405 b^3 e^{(7 dx+7 c)} + 3024 a^2 b e^{(5 dx+5 c)} - 5292 a b^2 e^{(5 dx+5 c)} + 2268 b^3 e^{(5 dx+5 c)} + 6720 a^3 e^{(3 dx+3 c)} - 25200 a^2 b e^{(3 dx+3 c)} + 26460 a b^2 e^{(3 dx+3 c)} - 8820 b^3 e^{(3 dx+3 c)} - 60480 a^3 e^{(d x+c)} + 151200 a^2 b e^{(d x+c)} - 132300 a b^2 e^{(d x+c)} + 39690 b^3 e^{(d x+c)} - (60480 a^3 e^{(8 d x+8 c)} - 151200 a^2 b e^{(8 d x+8 c)} + 132300 a b^2 e^{(8 d x+8 c)} - 39690 b^3 e^{(8 d x+8 c)} - 6720 a^3 e^{(6 d x+6 c)} + 25200 a^2 b e^{(6 d x+6 c)} - 26460 a b^2 e^{(6 d x+6 c)} + 8820 b^3 e^{(6 d x+6 c)} - 3024 a^2 b e^{(4 d x+4 c)} + 5292 a b^2 e^{(4 d x+4 c)} - 2268 b^3 e^{(4 d x+4 c)} - 540 a b^2 e^{(2 d x+2 c)} + 405 b^3 e^{(2 d x+2 c)} - 35 b^3) e^{(-9 d x-9 c)}/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/161280*(35*b^3*e^(9*d*x + 9*c) + 540*a*b^2*e^(7*d*x + 7*c) - 405*b^3*e^(7*d*x + 7*c) + 3024*a^2*b*e^(5*d*x + 5*c) - 5292*a*b^2*e^(5*d*x + 5*c) + 2268*b^3*e^(5*d*x + 5*c) + 6720*a^3*e^(3*d*x + 3*c) - 25200*a^2*b*e^(3*d*x + 3*c) + 26460*a*b^2*e^(3*d*x + 3*c) - 8820*b^3*e^(3*d*x + 3*c) - 60480*a^3*e^(d*x + c) + 151200*a^2*b*e^(d*x + c) - 132300*a*b^2*e^(d*x + c) + 39690*b^3*e^(d*x + c) - (60480*a^3*e^(8*d*x + 8*c) - 151200*a^2*b*e^(8*d*x + 8*c) + 132300*a*b^2*e^(8*d*x + 8*c) - 39690*b^3*e^(8*d*x + 8*c) - 6720*a^3*e^(6*d*x + 6*c) + 25200*a^2*b*e^(6*d*x + 6*c) - 26460*a*b^2*e^(6*d*x + 6*c) + 8820*b^3*e^(6*d*x + 6*c) - 3024*a^2*b*e^(4*d*x + 4*c) + 5292*a*b^2*e^(4*d*x + 4*c) - 2268*b^3*e^(4*d*x + 4*c) - 540*a*b^2*e^(2*d*x + 2*c) + 405*b^3*e^(2*d*x + 2*c) - 35*b^3)*e^(-9*d*x - 9*c))/d

3.21 $\int \sinh^2(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=181

$$\frac{b(24a^2 - 64ab + 35b^2) \sinh^3(c + dx) \cosh(c + dx)}{192d} + \frac{(-376a^2b + 96a^3 + 360ab^2 - 105b^3) \sinh(c + dx) \cosh(c + dx)}{384d}$$

[Out] $-\frac{(64a^3 - 144a^2b + 120ab^2 - 35b^3)x}{128} + \frac{(96a^3 - 376a^2b + 360ab^2 - 105b^3) \cosh[c + dx] \sinh[c + dx]}{384d} + \frac{b(24a^2 - 64ab + 35b^2) \cosh[c + dx] \sinh^3[c + dx]}{192d} + \frac{(6a - 7b) \cosh[c + dx] \sinh[c + dx] (a + b \sinh^2[c + dx])^2}{48d} + \frac{\cosh[c + dx] \sinh[c + dx] (a + b \sinh^2[c + dx])^3}{8d}$

Rubi [A] time = 0.192777, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3170, 3169}

$$\frac{b(24a^2 - 64ab + 35b^2) \sinh^3(c + dx) \cosh(c + dx)}{192d} + \frac{(-376a^2b + 96a^3 + 360ab^2 - 105b^3) \sinh(c + dx) \cosh(c + dx)}{384d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\sinh^2[c + dx] (a + b \sinh^2[c + dx])^3, x]$

[Out] $-\frac{(64a^3 - 144a^2b + 120ab^2 - 35b^3)x}{128} + \frac{(96a^3 - 376a^2b + 360ab^2 - 105b^3) \cosh[c + dx] \sinh[c + dx]}{384d} + \frac{b(24a^2 - 64ab + 35b^2) \cosh[c + dx] \sinh^3[c + dx]}{192d} + \frac{(6a - 7b) \cosh[c + dx] \sinh[c + dx] (a + b \sinh^2[c + dx])^2}{48d} + \frac{\cosh[c + dx] \sinh[c + dx] (a + b \sinh^2[c + dx])^3}{8d}$

Rule 3170

$\text{Int}[(a_ + (b_.) \sin[e_.] + (f_.) (x_.)^2)^{(p_)} ((A_.) + (B_.) \sin[e_.] + (f_.) (x_.)^2), x_Symbol] := -\text{Simp}[(B \cos[e + f x] \sin[e + f x] (a + b \sin[e + f x]^2)^p) / (2 f (p + 1)), x] + \text{Dist}[1 / (2 (p + 1)), \text{Int}[(a + b \sin[e + f x]^2)^{(p - 1)} \text{Simp}[a B + 2 a A (p + 1) + (2 A b (p + 1) + B (b + 2 a p + 2 b p)) \sin[e + f x]^2, x], x], x] /;$ FreeQ[{a, b, e, f, A, B}, x] && GtQ[p, 0]

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] + (-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a + 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B}, x]
```

Rubi steps

$$\begin{aligned} \int \sinh^2(c + dx) (a + b \sinh^2(c + dx))^3 dx &= \frac{\cosh(c + dx) \sinh(c + dx) (a + b \sinh^2(c + dx))^3}{8d} - \frac{1}{8} \int (a - (6a - 7b) \sinh^2(c + dx)) \cosh(c + dx) \sinh^2(c + dx) dx \\ &= \frac{(6a - 7b) \cosh(c + dx) \sinh(c + dx) (a + b \sinh^2(c + dx))^2}{48d} + \frac{\cosh(c + dx) \sinh^3(c + dx)}{24d} \\ &= -\frac{1}{128} (64a^3 - 144a^2b + 120ab^2 - 35b^3) x + \frac{(96a^3 - 376a^2b + 360ab^2 - 105b^3) \cosh(c + dx) \sinh^3(c + dx)}{384d} \end{aligned}$$

Mathematica [A] time = 0.29103, size = 130, normalized size = 0.72

$$\frac{-24(-144a^2b + 64a^3 + 120ab^2 - 35b^3)(c + dx) + 24b(12a^2 - 18ab + 7b^2) \sinh(4(c + dx)) + 48(-48a^2b + 16a^3 + 45ab^2) \cosh(4(c + dx))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-24*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*(c + d*x) + 48*(16*a^3 - 48*a^2*b + 45*a*b^2 - 14*b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 18*a*b + 7*b^2)*Sinh[4*(c + d*x)] + 16*(3*a - 2*b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh[8*(c + d*x)])/(3072*d)
```

Maple [A] time = 0.014, size = 180, normalized size = 1.

$$\frac{1}{d} \left(b^3 \left(\frac{(\sinh(dx + c))^7}{8} - \frac{7(\sinh(dx + c))^5}{48} + \frac{35(\sinh(dx + c))^3}{192} - \frac{35 \sinh(dx + c)}{128} \right) \cosh(dx + c) + \frac{35 dx}{128} + \frac{35 c}{128} \right) + \frac{b^3 \sinh^3(dx + c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

[Out] $\frac{1}{d} \cdot (b^3 \cdot ((\frac{1}{8} \sinh(dx+c))^7 - \frac{7}{48} \sinh(dx+c)^5 + \frac{35}{192} \sinh(dx+c)^3 - \frac{35}{128} \sinh(dx+c)) \cdot \cosh(dx+c) + \frac{35}{128} dx + \frac{35}{128} c) + 3a \cdot b^2 \cdot ((\frac{1}{6} \sinh(dx+c))^5 - \frac{5}{24} \sinh(dx+c)^3 + \frac{5}{16} \sinh(dx+c)) \cdot \cosh(dx+c) - \frac{5}{16} dx - \frac{5}{16} c) + 3a^2 \cdot b \cdot ((\frac{1}{4} \sinh(dx+c))^3 - \frac{3}{8} \sinh(dx+c)) \cdot \cosh(dx+c) + \frac{3}{8} dx + \frac{3}{8} c) + a^3 \cdot (\frac{1}{2} \cosh(dx+c) \cdot \sinh(dx+c) - \frac{1}{2} dx - \frac{1}{2} c)$

Maxima [A] time = 1.10491, size = 413, normalized size = 2.28

$$\frac{3}{64} a^2 b \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) - \frac{1}{8} a^3 \left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d} \right) - \frac{1}{6144} b^3 \left(\frac{32e^{4dx+4c}}{d} - \frac{16e^{2dx+2c}}{d} + \frac{16e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(dx+c)^2*(a+b*sinh(dx+c)^2)^3,x, algorithm="maxima")`

[Out] $\frac{3}{64} a^2 b \cdot (24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d}) - \frac{1}{8} a^3 \cdot (4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d}) - \frac{1}{6144} b^3 \cdot (\frac{32e^{4dx+4c}}{d} - \frac{16e^{2dx+2c}}{d} + \frac{16e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d})$

Fricas [A] time = 1.81714, size = 667, normalized size = 3.69

$$3b^3 \cosh(dx+c) \sinh(dx+c)^7 + 3(7b^3 \cosh(dx+c)^3 + 4(3ab^2 - 2b^3) \cosh(dx+c)) \sinh(dx+c)^5 + (21b^3 \cosh(dx+c)^5 + 40(3ab^2 - 2b^3) \cosh(dx+c)^3 + 12(12a^2b - 18ab^2 + 7b^3) \cosh(dx+c) \sinh(dx+c)^3 - 3(64a^3 - 144a^2b + 120ab^2 - 35b^3) dx + 3(b^3 \cosh(dx+c)^7 + 4(3ab^2 - 2b^3) \cosh(dx+c)^5 + 4(12a^2b - 18ab^2 + 7b^3) \cosh(dx+c)^3 + 4(16a^3 - 48a^2b + 45ab^2 - 14b^3) \sinh(dx+c)^3 - 4(12a^2b - 18ab^2 + 7b^3) \sinh(dx+c)^5 - 4(16a^3 - 48a^2b + 45ab^2 - 14b^3) dx - 4(16a^3 - 48a^2b + 45ab^2 - 14b^3) c) \sinh(dx+c)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(dx+c)^2*(a+b*sinh(dx+c)^2)^3,x, algorithm="fricas")`

[Out] $\frac{1}{384} \cdot (3b^3 \cosh(dx+c) \sinh(dx+c)^7 + 3(7b^3 \cosh(dx+c)^3 + 4(3ab^2 - 2b^3) \cosh(dx+c)) \sinh(dx+c)^5 + (21b^3 \cosh(dx+c)^5 + 40(3ab^2 - 2b^3) \cosh(dx+c)^3 + 12(12a^2b - 18ab^2 + 7b^3) \cosh(dx+c) \sinh(dx+c)^3 - 3(64a^3 - 144a^2b + 120ab^2 - 35b^3) dx + 3(b^3 \cosh(dx+c)^7 + 4(3ab^2 - 2b^3) \cosh(dx+c)^5 + 4(12a^2b - 18ab^2 + 7b^3) \cosh(dx+c)^3 + 4(16a^3 - 48a^2b + 45ab^2 - 14b^3) \sinh(dx+c)^3 - 4(12a^2b - 18ab^2 + 7b^3) \sinh(dx+c)^5 - 4(16a^3 - 48a^2b + 45ab^2 - 14b^3) dx - 4(16a^3 - 48a^2b + 45ab^2 - 14b^3) c) \sinh(dx+c)^3$

$$- 14*b^3*\cosh(d*x + c))*\sinh(d*x + c))/d$$

Sympy [A] time = 14.3627, size = 561, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*a**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a*b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 35*b**3*x*sinh(c + d*x)**8/128 - 35*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b**3*x*cosh(c + d*x)**8/128 + 93*b**3*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b**3*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**3*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**3*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**2, True))

Giac [B] time = 1.40784, size = 521, normalized size = 2.88

$$3b^3e^{(8dx+8c)} + 48ab^2e^{(6dx+6c)} - 32b^3e^{(6dx+6c)} + 288a^2be^{(4dx+4c)} - 432ab^2e^{(4dx+4c)} + 168b^3e^{(4dx+4c)} + 768a^3e^{(2dx+2c)} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/6144*(3*b^3*e^(8*d*x + 8*c) + 48*a*b^2*e^(6*d*x + 6*c) - 32*b^3*e^(6*d*x + 6*c) + 288*a^2*b*e^(4*d*x + 4*c) - 432*a*b^2*e^(4*d*x + 4*c) + 168*b^3*e^(4*d*x + 4*c) + 768*a^3*e^(2*d*x + 2*c) - 2304*a^2*b*e^(2*d*x + 2*c) + 2160*a*b^2*e^(2*d*x + 2*c) - 672*b^3*e^(2*d*x + 2*c) - 48*(64*a^3 - 144*a^2*b +

$$\begin{aligned} & 120*a*b^2 - 35*b^3)*(d*x + c) + (3200*a^3*e^(8*d*x + 8*c) - 7200*a^2*b*e^(8*d*x + 8*c) + 6000*a*b^2*e^(8*d*x + 8*c) - 1750*b^3*e^(8*d*x + 8*c) - 768*a^3*e^(6*d*x + 6*c) + 2304*a^2*b*e^(6*d*x + 6*c) - 2160*a*b^2*e^(6*d*x + 6*c) + 672*b^3*e^(6*d*x + 6*c) - 288*a^2*b*e^(4*d*x + 4*c) + 432*a*b^2*e^(4*d*x + 4*c) - 168*b^3*e^(4*d*x + 4*c) - 48*a*b^2*e^(2*d*x + 2*c) + 32*b^3*e^(2*d*x + 2*c) - 3*b^3)*e^(-8*d*x - 8*c))/d \end{aligned}$$

3.22 $\int \sinh(c + dx) \left(a + b \sinh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=79

$$\frac{3b^2(a-b)\cosh^5(c+dx)}{5d} + \frac{b(a-b)^2\cosh^3(c+dx)}{d} + \frac{(a-b)^3\cosh(c+dx)}{d} + \frac{b^3\cosh^7(c+dx)}{7d}$$

[Out] $((a - b)^3 \text{Cosh}[c + d*x])/d + ((a - b)^2 * b * \text{Cosh}[c + d*x]^3)/d + (3*(a - b) * b^2 * \text{Cosh}[c + d*x]^5)/(5*d) + (b^3 * \text{Cosh}[c + d*x]^7)/(7*d)$

Rubi [A] time = 0.0873647, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3186, 194}

$$\frac{3b^2(a-b)\cosh^5(c+dx)}{5d} + \frac{b(a-b)^2\cosh^3(c+dx)}{d} + \frac{(a-b)^3\cosh(c+dx)}{d} + \frac{b^3\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]*(a + b*\text{Sinh}[c + d*x]^2)^3, x]$

[Out] $((a - b)^3 \text{Cosh}[c + d*x])/d + ((a - b)^2 * b * \text{Cosh}[c + d*x]^3)/d + (3*(a - b) * b^2 * \text{Cosh}[c + d*x]^5)/(5*d) + (b^3 * \text{Cosh}[c + d*x]^7)/(7*d)$

Rule 3186

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - b*ff^2*x^2)^p, x], x, \text{Cos}[e + f*x]/ff], x]] /; \text{FreeQ}\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[(m-1)/2]$

Rule 194

$\text{Int}[(a_.) + (b_.)*(x_.)^{(n_.)]^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{IGtQ}[p, 0]$

Rubi steps

$$\int \sinh(c + dx) (a + b \sinh^2(c + dx))^3 dx = \frac{\text{Subst}\left(\int (a - b + bx^2)^3 dx, x, \cosh(c + dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^3 \left(1 - \frac{b(3a^2 - 3ab + b^2)}{a^3}\right) + 3a^2b \left(1 + \frac{b(-2a+b)}{a^2}\right)x^2 + 3ab^2 \left(1 - \frac{b}{a}\right)x^4\right) dx, x, \cosh(c + dx)\right)}{d}$$

$$= \frac{(a - b)^3 \cosh(c + dx)}{d} + \frac{(a - b)^2 b \cosh^3(c + dx)}{d} + \frac{3(a - b)b^2 \cosh^5(c + dx)}{5d}$$

Mathematica [A] time = 0.303804, size = 94, normalized size = 1.19

$$\frac{\cosh(c + dx) \left(b(560a^2 - 784ab + 299b^2) \cosh(2(c + dx)) - 2800a^2b + 1120a^3 + 6b^2(14a - 9b) \cosh(4(c + dx)) + 2492ab^2 \right)}{1120d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (Cosh[c + d*x]*(1120*a^3 - 2800*a^2*b + 2492*a*b^2 - 762*b^3 + b*(560*a^2 - 784*a*b + 299*b^2)*Cosh[2*(c + d*x)] + 6*(14*a - 9*b)*b^2*Cosh[4*(c + d*x)] + 5*b^3*Cosh[6*(c + d*x)])/(1120*d)

Maple [A] time = 0.016, size = 116, normalized size = 1.5

$$\frac{1}{d} \left(b^3 \left(-\frac{16}{35} + \frac{(\sinh(dx + c))^6}{7} - \frac{6(\sinh(dx + c))^4}{35} + \frac{8(\sinh(dx + c))^2}{35} \right) \cosh(dx + c) + 3ab^2 \left(\frac{8}{15} + \frac{1}{5} (\sinh(dx + c))^2 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/d*(b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(8/15+1/5*sinh(d*x+c)^2-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*cosh(d*x+c))

Maxima [B] time = 1.05319, size = 355, normalized size = 4.49

$$-\frac{1}{4480} b^3 \left(\frac{(49 e^{(-2dx-2c)} - 245 e^{(-4dx-4c)} + 1225 e^{(-6dx-6c)} - 5) e^{(7dx+7c)}}{d} + \frac{1225 e^{(-dx-c)} - 245 e^{(-3dx-3c)} + 49 e^{(-5dx-5c)} - 5 e^{(-7dx-7c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/160*a*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/8*a^2*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^3*cosh(d*x + c)/d

Fricas [B] time = 1.77414, size = 589, normalized size = 7.46

$$5b^3 \cosh(dx+c)^7 + 35b^3 \cosh(dx+c) \sinh(dx+c)^6 + 7(12ab^2 - 7b^3) \cosh(dx+c)^5 + 35(5b^3 \cosh(dx+c)^3 + (12a^2b - 7b^3) \cosh(dx+c) \sinh(dx+c)^2) \cosh(dx+c) + 35(12a^2b - 7b^3) \cosh(dx+c) \sinh(dx+c)^4 + 35(16a^2b - 20ab^2 + 7b^3) \cosh(dx+c) \sinh(dx+c)^3 + 35(3b^3 \cosh(dx+c)^5 + 2(12a^2b - 7b^3) \cosh(dx+c) \sinh(dx+c)^2 + 3(16a^2b - 20ab^2 + 7b^3) \cosh(dx+c) \sinh(dx+c)) \sinh(dx+c) + 35(64a^3 - 144a^2b + 120ab^2 - 35b^3) \cosh(dx+c) \sinh(dx+c)^2) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/2240*(5*b^3*cosh(d*x + c)^7 + 35*b^3*cosh(d*x + c)*sinh(d*x + c)^6 + 7*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^3 + (12*a*b^2 - 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^2) *cosh(d*x + c) + 35*(16*a^2*b - 20*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 35*(3*b^3*cosh(d*x + c)^5 + 2*(12*a*b^2 - 7*b^3)*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(16*a^2*b - 20*a*b^2 + 7*b^3)*cosh(d*x + c)*sinh(d*x + c)) *sinh(d*x + c) + 35*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*cosh(d*x + c)*sinh(d*x + c)^2)/d

Sympy [A] time = 7.78788, size = 221, normalized size = 2.8

$$\left\{ \frac{a^3 \cosh(c+dx)}{d} + \frac{3a^2b \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2a^2b \cosh^3(c+dx)}{d} + \frac{3ab^2 \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{4ab^2 \sinh^2(c+dx) \cosh^3(c+dx)}{d} + \frac{8ab^2 \cosh^5(c+dx)}{5d} \right\} x (a + b \sinh^2(c)) \sinh(c)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/d
- 2*a**2*b*cosh(c + d*x)**3/d + 3*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)/d
- 4*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a*b**2*cosh(c + d*x)**5/
(5*d) + b**3*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**3*sinh(c + d*x)**4*cos
h(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**3*
cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c), True)
)
```

Giac [B] time = 1.31953, size = 386, normalized size = 4.89

$$5b^3e^{(7dx+7c)} + 84ab^2e^{(5dx+5c)} - 49b^3e^{(5dx+5c)} + 560a^2be^{(3dx+3c)} - 700ab^2e^{(3dx+3c)} + 245b^3e^{(3dx+3c)} + 2240a^3e^{(dx+c)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*b^3*e^(7*d*x + 7*c) + 84*a*b^2*e^(5*d*x + 5*c) - 49*b^3*e^(5*d*x
+ 5*c) + 560*a^2*b*e^(3*d*x + 3*c) - 700*a*b^2*e^(3*d*x + 3*c) + 245*b^3*e^
(3*d*x + 3*c) + 2240*a^3*e^(d*x + c) - 5040*a^2*b*e^(d*x + c) + 4200*a*b^2*
e^(d*x + c) - 1225*b^3*e^(d*x + c) + (2240*a^3*e^(6*d*x + 6*c) - 5040*a^2*b
*e^(6*d*x + 6*c) + 4200*a*b^2*e^(6*d*x + 6*c) - 1225*b^3*e^(6*d*x + 6*c) +
560*a^2*b*e^(4*d*x + 4*c) - 700*a*b^2*e^(4*d*x + 4*c) + 245*b^3*e^(4*d*x +
4*c) + 84*a*b^2*e^(2*d*x + 2*c) - 49*b^3*e^(2*d*x + 2*c) + 5*b^3)*e^(-7*d*x
- 7*c))/d
```

3.23 $\int (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=128

$$\frac{b(64a^2 - 54ab + 15b^2) \sinh(c + dx) \cosh(c + dx)}{48d} + \frac{1}{16}x(2a - b)(8a^2 - 8ab + 5b^2) + \frac{5b^2(2a - b) \sinh^3(c + dx) \cosh(c + dx)}{24d}$$

[Out] ((2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*x)/16 + (b*(64*a^2 - 54*a*b + 15*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(48*d) + (5*(2*a - b)*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*d)

Rubi [A] time = 0.100085, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3180, 3169}

$$\frac{b(64a^2 - 54ab + 15b^2) \sinh(c + dx) \cosh(c + dx)}{48d} + \frac{1}{16}x(2a - b)(8a^2 - 8ab + 5b^2) + \frac{5b^2(2a - b) \sinh^3(c + dx) \cosh(c + dx)}{24d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*x)/16 + (b*(64*a^2 - 54*a*b + 15*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(48*d) + (5*(2*a - b)*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*d)

Rule 3180

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[1/(2*p), Int[(a + b*Sinh[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && GtQ[p, 1]

Rule 3169

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] + (-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a

+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B}, x]

Rubi steps

$$\int (a + b \sinh^2(c + dx))^3 dx = \frac{b \cosh(c + dx) \sinh(c + dx) (a + b \sinh^2(c + dx))^2}{6d} + \frac{1}{6} \int (a + b \sinh^2(c + dx)) (a(6a - 5b) + b \sinh^2(c + dx)) dx$$

$$= \frac{1}{16} (2a - b) (8a^2 - 8ab + 5b^2) x + \frac{b (64a^2 - 54ab + 15b^2) \cosh(c + dx) \sinh(c + dx)}{48d} + \dots$$

Mathematica [A] time = 0.259497, size = 95, normalized size = 0.74

$$\frac{12(2a - b) (8a^2 - 8ab + 5b^2) (c + dx) + 9b (16a^2 - 16ab + 5b^2) \sinh(2(c + dx)) + 9b^2(2a - b) \sinh(4(c + dx)) + b^3 \sinh(6(c + dx))}{192d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^3, x]

[Out] (12*(2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*(c + d*x) + 9*b*(16*a^2 - 16*a*b + 5*b^2)*Sinh[2*(c + d*x)] + 9*(2*a - b)*b^2*Sinh[4*(c + d*x)] + b^3*Sinh[6*(c + d*x)])/(192*d)

Maple [A] time = 0.013, size = 131, normalized size = 1.

$$\frac{1}{d} \left(b^3 \left(\left(\frac{(\sinh(dx + c))^5}{6} - \frac{5(\sinh(dx + c))^3}{24} + \frac{5 \sinh(dx + c)}{16} \right) \cosh(dx + c) - \frac{5dx}{16} - \frac{5c}{16} \right) + 3ab^2 \left(\frac{1}{4} (\sinh(dx + c))^3 - \frac{3}{8} \sinh(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(d*x+c)^2)^3, x)

[Out] 1/d*(b^3*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a^3*(d*x+c))

Maxima [A] time = 1.04539, size = 266, normalized size = 2.08

$$\frac{3}{64} ab^2 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{3}{8} a^2 b \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) + a^3 x - \frac{1}{384} b^3 \left(\dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a^3*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)

Fricas [A] time = 1.82293, size = 398, normalized size = 3.11

$$3b^3 \cosh(dx+c) \sinh(dx+c)^5 + 2(5b^3 \cosh(dx+c)^3 + 9(2ab^2 - b^3) \cosh(dx+c) \sinh(dx+c)^3 + 6(16a^3 - 24a^2b + 18ab^2 - b^3) \cosh(dx+c) \sinh(dx+c)) \sinh(dx+c)^3 + 6(16a^3 - 24a^2b + 18ab^2 - b^3) \cosh(dx+c) \sinh(dx+c)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/96*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^3*cosh(d*x + c)^3 + 9*(2*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(16*a^3 - 24*a^2*b + 18*a*b^2 - 5*b^3)*d*x + 3*(b^3*cosh(d*x + c)^5 + 6*(2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(16*a^2*b - 16*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/d

Sympy [A] time = 5.04416, size = 350, normalized size = 2.73

$$\left\{ \begin{array}{l} a^3 x + \frac{3a^2 b x \sinh^2(c+dx)}{2} - \frac{3a^2 b x \cosh^2(c+dx)}{2} + \frac{3a^2 b \sinh(c+dx) \cosh(c+dx)}{2d} + \frac{9ab^2 x \sinh^4(c+dx)}{8} - \frac{9ab^2 x \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{9ab^2 x \cosh^4(c+dx)}{8} \\ x(a + b \sinh^2(c))^3 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)**2)**3,x)

```
[Out] Piecewise((a**3*x + 3*a**2*b*x*sinh(c + d*x)**2/2 - 3*a**2*b*x*cosh(c + d*x)
)**2/2 + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 9*a*b**2*x*sinh(c + d
*x)**4/8 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a*b**2*x*cosh
(c + d*x)**4/8 + 15*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a*b**2*
sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b**3*x*sinh(c + d*x)**6/16 - 15*b*
**3*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b**3*x*sinh(c + d*x)**2*cosh
(c + d*x)**4/16 - 5*b**3*x*cosh(c + d*x)**6/16 + 11*b**3*sinh(c + d*x)**5*c
osh(c + d*x)/(16*d) - 5*b**3*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*
**3*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*
**3, True))
```

Giac [B] time = 1.3218, size = 362, normalized size = 2.83

$$b^3 e^{(6dx+6c)} + 18 ab^2 e^{(4dx+4c)} - 9 b^3 e^{(4dx+4c)} + 144 a^2 b e^{(2dx+2c)} - 144 ab^2 e^{(2dx+2c)} + 45 b^3 e^{(2dx+2c)} + 24 (16 a^3 - 24 a^2 b +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/384*(b^3*e^(6*d*x + 6*c) + 18*a*b^2*e^(4*d*x + 4*c) - 9*b^3*e^(4*d*x + 4*
c) + 144*a^2*b*e^(2*d*x + 2*c) - 144*a*b^2*e^(2*d*x + 2*c) + 45*b^3*e^(2*d*
x + 2*c) + 24*(16*a^3 - 24*a^2*b + 18*a*b^2 - 5*b^3)*(d*x + c) - (352*a^3*e
^(6*d*x + 6*c) - 528*a^2*b*e^(6*d*x + 6*c) + 396*a*b^2*e^(6*d*x + 6*c) - 11
0*b^3*e^(6*d*x + 6*c) + 144*a^2*b*e^(4*d*x + 4*c) - 144*a*b^2*e^(4*d*x + 4*
c) + 45*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*c) - 9*b^3*e^(2*d*x + 2
*c) + b^3)*e^(-6*d*x - 6*c))/d
```

3.24 $\int \operatorname{csch}(c + dx) \left(a + b \sinh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=83

$$\frac{b(3a^2 - 3ab + b^2) \cosh(c + dx)}{d} - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2(3a - 2b) \cosh^3(c + dx)}{3d} + \frac{b^3 \cosh^5(c + dx)}{5d}$$

[Out] $-\left(\frac{a^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) + \frac{b*(3*a^2 - 3*a*b + b^2)*\operatorname{Cosh}[c + d*x]}{d} + \frac{((3*a - 2*b)*b^2*\operatorname{Cosh}[c + d*x]^3)}{(3*d)} + \frac{b^3*\operatorname{Cosh}[c + d*x]^5}{(5*d)}$

Rubi [A] time = 0.0869544, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3186, 390, 206}

$$\frac{b(3a^2 - 3ab + b^2) \cosh(c + dx)}{d} - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2(3a - 2b) \cosh^3(c + dx)}{3d} + \frac{b^3 \cosh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Sinh}[c + d*x]^2)^3, x]$

[Out] $-\left(\frac{a^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) + \frac{b*(3*a^2 - 3*a*b + b^2)*\operatorname{Cosh}[c + d*x]}{d} + \frac{((3*a - 2*b)*b^2*\operatorname{Cosh}[c + d*x]^3)}{(3*d)} + \frac{b^3*\operatorname{Cosh}[c + d*x]^5}{(5*d)}$

Rule 3186

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - b*ff^2*x^2)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 390

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(a + b*x^n)^p, (c + d*x^n)^{-q}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{IGtQ}[n, 0] \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{ILtQ}[q, 0] \&\& \operatorname{GeQ}[p, -q]$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2]^{(-1)}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{Gt}$

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c+dx) (a+b \sinh^2(c+dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a-bx^2)^3}{1-x^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(-b(3a^2-3ab+b^2) - (3a-2b)b^2x^2 - b^3x^4 + \frac{a^3}{1-x^2}\right) dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{b(3a^2-3ab+b^2) \cosh(c+dx)}{d} + \frac{(3a-2b)b^2 \cosh^3(c+dx)}{3d} + \frac{b^3 \cosh^5(c+dx)}{5d} \\ &= -\frac{a^3 \tanh^{-1}(\cosh(c+dx))}{d} + \frac{b(3a^2-3ab+b^2) \cosh(c+dx)}{d} + \frac{(3a-2b)b^2 \cosh^3(c+dx)}{3d} + \frac{b^3 \cosh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.214731, size = 83, normalized size = 1.

$$\frac{30b(24a^2 - 18ab + 5b^2) \cosh(c+dx) + 3\left(80a^3 \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + b^3 \cosh(5(c+dx))\right) + 5b^2(12a - 5b) \cosh(3(c+dx))}{240d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (30*b*(24*a^2 - 18*a*b + 5*b^2)*Cosh[c + d*x] + 5*(12*a - 5*b)*b^2*Cosh[3*(c + d*x)] + 3*(b^3*Cosh[5*(c + d*x)] + 80*a^3*Log[Tanh[(c + d*x)/2]])/(240*d)

Maple [A] time = 0.03, size = 86, normalized size = 1.

$$\frac{1}{d} \left(-2a^3 \operatorname{Artanh}(e^{dx+c}) + 3a^2b \cosh(dx+c) + 3ab^2 \left(-\frac{2}{3} + \frac{1}{3} (\sinh(dx+c))^2 \right) \cosh(dx+c) + b^3 \left(\frac{8}{15} + \frac{(\sinh(dx+c))^5}{5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)

[Out] $1/d*(-2*a^3*\operatorname{arctanh}(\exp(d*x+c))+3*a^2*b*\cosh(d*x+c)+3*a*b^2*(-2/3+1/3*\sinh(d*x+c)^2)*\cosh(d*x+c)+b^3*(8/15+1/5*\sinh(d*x+c)^4-4/15*\sinh(d*x+c)^2)*\cosh(d*x+c))$

Maxima [B] time = 1.06502, size = 261, normalized size = 3.14

$$\frac{1}{480} b^3 \left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d} \right) + \frac{1}{8} ab^2 \left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $1/480*b^3*(3*e^{(5*d*x + 5*c)}/d - 25*e^{(3*d*x + 3*c)}/d + 150*e^{(d*x + c)}/d + 150*e^{(-d*x - c)}/d - 25*e^{(-3*d*x - 3*c)}/d + 3*e^{(-5*d*x - 5*c)}/d) + 1/8*a*b^2*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d) + 3/2*a^2*b*(e^{(d*x + c)}/d + e^{(-d*x - c)}/d) + a^3*\log(\tanh(1/2*d*x + 1/2*c))/d$

Fricas [B] time = 2.06531, size = 2889, normalized size = 34.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")`

[Out] $1/480*(3*b^3*\cosh(d*x + c)^{10} + 30*b^3*\cosh(d*x + c)*\sinh(d*x + c)^9 + 3*b^3*\sinh(d*x + c)^{10} + 5*(12*a*b^2 - 5*b^3)*\cosh(d*x + c)^8 + 5*(27*b^3*\cosh(d*x + c)^2 + 12*a*b^2 - 5*b^3)*\sinh(d*x + c)^8 + 40*(9*b^3*\cosh(d*x + c)^3 + (12*a*b^2 - 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 30*(24*a^2*b - 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 10*(63*b^3*\cosh(d*x + c)^4 + 72*a^2*b - 54*a*b^2 + 15*b^3 + 14*(12*a*b^2 - 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(189*b^3*\cosh(d*x + c)^5 + 70*(12*a*b^2 - 5*b^3)*\cosh(d*x + c)^3 + 45*(24*a^2*b - 18*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 30*(24*a^2*b - 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 10*(63*b^3*\cosh(d*x + c)^6 + 35*(12*a*b^2 - 5*b^3)*\cosh(d*x + c)^4 + 72*a^2*b - 54*a*b^2 + 15*b^3 + 45*(24*a^2*b - 18*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 40*(9*b^3*\cosh(d*x + c)^7 + 7*(12*a*b^2 - 5*b^3)*\cosh(d*x + c)^5 + 15*(24*a^2*b - 18*a*b^2 + 5*b^3)$


```
*cosh(d*x + c)^3 + 3*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*b^3 + 5*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2 + 5*(27*b^3*cosh(d*x
+ c)^8 + 28*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 90*(24*a^2*b - 18*a*b^2 +
5*b^3)*cosh(d*x + c)^4 + 12*a*b^2 - 5*b^3 + 36*(24*a^2*b - 18*a*b^2 + 5*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(
d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*
cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3
*sinh(d*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 480*(a^3*cosh(d*
x + c)^5 + 5*a^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*si
nh(d*x + c)^2 + 10*a^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)
*sinh(d*x + c)^4 + a^3*sinh(d*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) -
1) + 10*(3*b^3*cosh(d*x + c)^9 + 4*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 18
*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 12*(24*a^2*b - 18*a*b^2 +
5*b^3)*cosh(d*x + c)^3 + (12*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(
d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)
)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.32194, size = 311, normalized size = 3.75

$$-\frac{a^3 \log(e^{(dx+c)} + 1)}{d} + \frac{a^3 \log(|e^{(dx+c)} - 1|)}{d} + \frac{(720 a^2 b e^{(4 dx+4 c)} - 540 a b^2 e^{(4 dx+4 c)} + 150 b^3 e^{(4 dx+4 c)} + 60 a b^2 e^{(2 dx+2 c)} - 60 a^2 b e^{(4 dx+4 c)} + 540 a b^2 e^{(4 dx+4 c)} - 150 b^3 e^{(4 dx+4 c)} - 60 a b^2 e^{(2 dx+2 c)})}{480 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $-a^3 \log(e^{(d*x + c)} + 1)/d + a^3 \log(\text{abs}(e^{(d*x + c)} - 1))/d + 1/480*(720*a^2*b*e^{(4*d*x + 4*c)} - 540*a*b^2*e^{(4*d*x + 4*c)} + 150*b^3*e^{(4*d*x + 4*c)} - 60*a*b^2*e^{(2*d*x + 2*c)} - 60*a^2*b*e^{(4*d*x + 4*c)} + 540*a*b^2*e^{(4*d*x + 4*c)} - 150*b^3*e^{(4*d*x + 4*c)} - 60*a*b^2*e^{(2*d*x + 2*c)})$

$$\begin{aligned} &+ 60*a*b^2*e^{(2*d*x + 2*c)} - 25*b^3*e^{(2*d*x + 2*c)} + 3*b^3)*e^{(-5*d*x - 5 \\ &*c)}/d + 1/480*(3*b^3*d^4*e^{(5*d*x + 5*c)} + 60*a*b^2*d^4*e^{(3*d*x + 3*c)} - 2 \\ &5*b^3*d^4*e^{(3*d*x + 3*c)} + 720*a^2*b*d^4*e^{(d*x + c)} - 540*a*b^2*d^4*e^{(d* \\ &x + c)} + 150*b^3*d^4*e^{(d*x + c)})/d^5 \end{aligned}$$

3.25 $\int \operatorname{csch}^2(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=137

$$\frac{3}{8}bx(8a^2 - 4ab + b^2) - \frac{a(2a + b)(4a + b) \operatorname{coth}(c + dx)}{8d} + \frac{b \cosh^4(c + dx) \operatorname{coth}(c + dx) (a - (a - b) \tanh^2(c + dx))^2}{4d} + \dots$$

```
[Out] (3*b*(8*a^2 - 4*a*b + b^2)*x)/8 - (a*(2*a + b)*(4*a + b)*Coth[c + d*x])/(8*d) + (b*Cosh[c + d*x]^4*Coth[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(4*d) + (b*Cosh[c + d*x]^2*Coth[c + d*x]*(a*(4*a + b) - (4*a - 3*b)*(a - b)*Tanh[c + d*x]^2))/(8*d)
```

Rubi [A] time = 0.191937, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3187, 468, 577, 453, 206}

$$\frac{3}{8}bx(8a^2 - 4ab + b^2) - \frac{a(2a + b)(4a + b) \operatorname{coth}(c + dx)}{8d} + \frac{b \cosh^4(c + dx) \operatorname{coth}(c + dx) (a - (a - b) \tanh^2(c + dx))^2}{4d} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*b*(8*a^2 - 4*a*b + b^2)*x)/8 - (a*(2*a + b)*(4*a + b)*Coth[c + d*x])/(8*d) + (b*Cosh[c + d*x]^4*Coth[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(4*d) + (b*Cosh[c + d*x]^2*Coth[c + d*x]*(a*(4*a + b) - (4*a - 3*b)*(a - b)*Tanh[c + d*x]^2))/(8*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)]/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
```

```
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_
))^ (q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] && !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 453

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n
_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^2(c+dx) (a+b \sinh^2(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-(a-b)x^2)^3}{x^2(1-x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b \cosh^4(c+dx) \coth(c+dx) (a-(a-b) \tanh^2(c+dx))^2}{4d} + \frac{\operatorname{Subst}\left(\int \frac{(a-(a-b)x^2)^3}{x^2(1-x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b \cosh^4(c+dx) \coth(c+dx) (a-(a-b) \tanh^2(c+dx))^2}{4d} + \frac{b \cosh^2(c+dx) (a-(a-b) \tanh^2(c+dx))}{4d} \\
&= -\frac{a(2a+b)(4a+b) \coth(c+dx)}{8d} + \frac{b \cosh^4(c+dx) \coth(c+dx) (a-(a-b) \tanh^2(c+dx))^2}{4d} \\
&= \frac{3}{8} b (8a^2 - 4ab + b^2) x - \frac{a(2a+b)(4a+b) \coth(c+dx)}{8d} + \frac{b \cosh^4(c+dx) \coth(c+dx) (a-(a-b) \tanh^2(c+dx))^2}{4d}
\end{aligned}$$

Mathematica [A] time = 1.91726, size = 113, normalized size = 0.82

$$\frac{\sinh^6(c+dx) (a \operatorname{csch}^2(c+dx) + b)^3 (12b(8a^2 - 4ab + b^2)(c+dx) - 32a^3 \coth(c+dx) + 8b^2(3a-b) \sinh(2(c+dx))) + 4d(2a+b \cosh(2(c+dx)) - b)^3}{4d(2a+b \cosh(2(c+dx)) - b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((b + a*Csch[c + d*x]^2)^3*Sinh[c + d*x]^6*(12*b*(8*a^2 - 4*a*b + b^2)*(c + d*x) - 32*a^3*Coth[c + d*x] + 8*(3*a - b)*b^2*Sinh[2*(c + d*x)] + b^3*Sinh[4*(c + d*x)])/(4*d*(2*a - b + b*Cosh[2*(c + d*x)])^3)

Maple [A] time = 0.03, size = 94, normalized size = 0.7

$$\frac{1}{d} \left(-a^3 \coth(dx+c) + 3a^2b(dx+c) + 3ab^2 \left(\frac{1}{2} \cosh(dx+c) \sinh(dx+c) - \frac{1}{2} dx - c/2 \right) + b^3 \left(\frac{(\sinh(dx+c))^3}{4} - \frac{3}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)

[Out] $1/d*(-a^3*\coth(d*x+c)+3*a^2*b*(d*x+c)+3*a*b^2*(1/2*\cosh(d*x+c)*\sinh(d*x+c)-1/2*d*x-1/2*c)+b^3*((1/4*\sinh(d*x+c)^3-3/8*\sinh(d*x+c))*\cosh(d*x+c)+3/8*d*x+3/8*c))$

Maxima [A] time = 1.06488, size = 176, normalized size = 1.28

$$\frac{1}{64} b^3 \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) - \frac{3}{8} ab^2 \left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d} \right) + 3a^2bx + \frac{2}{d(e^{-2d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $1/64*b^3*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) - 3/8*a*b^2*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) + 3*a^2*b*x + 2*a^3/(d*(e^{(-2*d*x - 2*c)} - 1))$

Fricas [A] time = 1.84056, size = 409, normalized size = 2.99

$$\frac{b^3 \cosh(dx + c)^5 + 5b^3 \cosh(dx + c) \sinh(dx + c)^4 + 3(8ab^2 - 3b^3) \cosh(dx + c)^3 + (10b^3 \cosh(dx + c)^3 + 9(8ab^2 - 3b^3) \cosh(dx + c) \sinh(dx + c)^2 - 8(8a^3 + 3ab^2 - b^3) \cosh(dx + c) + 8(8a^3 + 3(8a^2b - 4ab^2 + b^3)d*x) \sinh(dx + c))}{64 d \sinh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")`

[Out] $1/64*(b^3*\cosh(d*x + c)^5 + 5*b^3*\cosh(d*x + c)*\sinh(d*x + c)^4 + 3*(8*a*b^2 - 3*b^3)*\cosh(d*x + c)^3 + (10*b^3*\cosh(d*x + c)^3 + 9*(8*a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 8*(8*a^3 + 3*a*b^2 - b^3)*\cosh(d*x + c) + 8*(8*a^3 + 3*(8*a^2*b - 4*a*b^2 + b^3)*d*x)*\sinh(d*x + c))/(d*\sinh(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [A] time = 1.36316, size = 257, normalized size = 1.88

$$\frac{3(8a^2b - 4ab^2 + b^3)(dx + c)}{8d} - \frac{2a^3}{d(e^{2dx+2c} - 1)} - \frac{(144a^2be^{4dx+4c} - 72ab^2e^{4dx+4c} + 18b^3e^{4dx+4c} + 24ab^2e^{2dx+2c})}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 3/8*(8*a^2*b - 4*a*b^2 + b^3)*(d*x + c)/d - 2*a^3/(d*(e^(2*d*x + 2*c) - 1)) - 1/64*(144*a^2*b*e^(4*d*x + 4*c) - 72*a*b^2*e^(4*d*x + 4*c) + 18*b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c)/d + 1/64*(b^3*d*e^(4*d*x + 4*c) + 24*a*b^2*d*e^(2*d*x + 2*c) - 8*b^3*d*e^(2*d*x + 2*c))/d^2

3.26 $\int \operatorname{csch}^3(c + dx) \left(a + b \sinh^2(c + dx)\right)^3 dx$

Optimal. Leaf size=83

$$\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b^2(3a-b)\cosh(c+dx)}{d} + \frac{b^3\cosh^3(c+dx)}{3d}$$

[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + ((3*a - b)*b^2*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rubi [A] time = 0.106845, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3186, 390, 385, 206}

$$\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b^2(3a-b)\cosh(c+dx)}{d} + \frac{b^3\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + ((3*a - b)*b^2*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 385

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b

c(n*(p + 1) + 1)/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c + dx) (a + b \sinh^2(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^3}{(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left((3a-b)b^2 + b^3x^2 + \frac{a^2(a-3b)+3a^2bx^2}{(1-x^2)^2}\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{(3a-b)b^2 \cosh(c + dx)}{d} + \frac{b^3 \cosh^3(c + dx)}{3d} + \frac{\operatorname{Subst}\left(\int \frac{a^2(a-3b)+3a^2bx^2}{(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{(3a-b)b^2 \cosh(c + dx)}{d} + \frac{b^3 \cosh^3(c + dx)}{3d} - \frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} \\ &= \frac{a^2(a-6b) \tanh^{-1}(\cosh(c + dx))}{2d} + \frac{(3a-b)b^2 \cosh(c + dx)}{d} + \frac{b^3 \cosh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [B] time = 4.70042, size = 210, normalized size = 2.53

$$\frac{(a + b \sinh^2(c + dx))^3 \left(-72a^2b \log\left(\sinh\left(\frac{1}{2}(c + dx)\right)\right) + 72a^2b \log\left(\cosh\left(\frac{1}{2}(c + dx)\right)\right) + 3a^3 \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right) + 3a^3 \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] -((-18*(4*a - b)*b^2*Cosh[c]*Cosh[d*x] - 2*b^3*Cosh[3*c]*Cosh[3*d*x] + 3*a^3*Csch[(c + d*x)/2]^2 - 12*a^3*Log[Cosh[(c + d*x)/2]] + 72*a^2*b*Log[Cosh[(c + d*x)/2]] + 12*a^3*Log[Sinh[(c + d*x)/2]] - 72*a^2*b*Log[Sinh[(c + d*x)/2]] + 3*a^3*Sech[(c + d*x)/2]^2 - 72*a*b^2*Sinh[c]*Sinh[d*x] + 18*b^3*Sinh[

$c] * \text{Sinh}[d*x] - 2*b^3 * \text{Sinh}[3*c] * \text{Sinh}[3*d*x]) * (a + b * \text{Sinh}[c + d*x]^2)^3 / (3*d * (2*a - b + b * \text{Cosh}[2*(c + d*x)])^3)$

Maple [A] time = 0.042, size = 79, normalized size = 1.

$$\frac{1}{d} \left(a^3 \left(-\frac{\text{csch}(dx+c) \coth(dx+c)}{2} + \text{Artanh}(e^{dx+c}) \right) - 6a^2b \text{Artanh}(e^{dx+c}) + 3ab^2 \cosh(dx+c) + b^3 \left(-\frac{2}{3} + \frac{\sinh(dx+c)}{3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)`

[Out] $1/d * (a^3 * (-1/2 * \text{csch}(d*x+c) * \coth(d*x+c) + \text{arctanh}(\exp(d*x+c))) - 6*a^2*b * \text{arctanh}(\exp(d*x+c)) + 3*a*b^2 * \cosh(d*x+c) + b^3 * (-2/3 + 1/3 * \sinh(d*x+c)^2) * \cosh(d*x+c))$

Maxima [B] time = 1.07542, size = 293, normalized size = 3.53

$$\frac{1}{24} b^3 \left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d} \right) + \frac{3}{2} ab^2 \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) + \frac{1}{2} a^3 \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $1/24*b^3*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d) + 3/2*a*b^2*(e^{(d*x + c)}/d + e^{(-d*x - c)}/d) + 1/2*a^3*(\log(e^{(-d*x - c)} + 1)/d - \log(e^{(-d*x - c)} - 1)/d + 2*(e^{(-d*x - c)} + e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1))) - 3*a^2*b*(\log(e^{(-d*x - c)} + 1)/d - \log(e^{(-d*x - c)} - 1)/d)$

Fricas [B] time = 2.0839, size = 4575, normalized size = 55.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")`

```
[Out] 1/24*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sin
h(d*x + c)^10 + (36*a*b^2 - 11*b^3)*cosh(d*x + c)^8 + (45*b^3*cosh(d*x + c)
^2 + 36*a*b^2 - 11*b^3)*sinh(d*x + c)^8 + 8*(15*b^3*cosh(d*x + c)^3 + (36*a
*b^2 - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(12*a^3 + 18*a*b^2 - 5*b^
3)*cosh(d*x + c)^6 + 2*(105*b^3*cosh(d*x + c)^4 - 12*a^3 - 18*a*b^2 + 5*b^3
+ 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*b^3*cosh
(d*x + c)^5 + 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^3 - 3*(12*a^3 + 18*a*b^2
- 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(12*a^3 + 18*a*b^2 - 5*b^3)*co
sh(d*x + c)^4 + 2*(105*b^3*cosh(d*x + c)^6 + 35*(36*a*b^2 - 11*b^3)*cosh(d*
x + c)^4 - 12*a^3 - 18*a*b^2 + 5*b^3 - 15*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*b^3*cosh(d*x + c)^7 + 7*(36*a*b^2 - 11*
b^3)*cosh(d*x + c)^5 - 5*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - (12*
a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + (36*a*b^2 -
11*b^3)*cosh(d*x + c)^2 + (45*b^3*cosh(d*x + c)^8 + 28*(36*a*b^2 - 11*b^3)*
cosh(d*x + c)^6 - 30*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 36*a*b^2
- 11*b^3 - 12*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 12*((a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 - 6*a^2*b)*cosh(d*x + c)*si
nh(d*x + c)^6 + (a^3 - 6*a^2*b)*sinh(d*x + c)^7 - 2*(a^3 - 6*a^2*b)*cosh(d*
x + c)^5 - (2*a^3 - 12*a^2*b - 21*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x
+ c)^5 + 5*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 2*(a^3 - 6*a^2*b)*cosh(d*x
+ c))*sinh(d*x + c)^4 + (a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (35*(a^3 - 6*a^2
*b)*cosh(d*x + c)^4 + a^3 - 6*a^2*b - 20*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + (21*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 20*(a^3 - 6*a^2*b)*c
osh(d*x + c)^3 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(a^3
- 6*a^2*b)*cosh(d*x + c)^6 - 10*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 3*(a^3 -
6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
+ 1) - 12*((a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 - 6*a^2*b)*cosh(d*x +
c)*sinh(d*x + c)^6 + (a^3 - 6*a^2*b)*sinh(d*x + c)^7 - 2*(a^3 - 6*a^2*b)*co
sh(d*x + c)^5 - (2*a^3 - 12*a^2*b - 21*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sin
h(d*x + c)^5 + 5*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 2*(a^3 - 6*a^2*b)*cos
h(d*x + c))*sinh(d*x + c)^4 + (a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (35*(a^3 -
6*a^2*b)*cosh(d*x + c)^4 + a^3 - 6*a^2*b - 20*(a^3 - 6*a^2*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^3 + (21*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 20*(a^3 - 6*a^2
*b)*cosh(d*x + c)^3 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7
*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 - 10*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 3*(
a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
+ c) - 1) + 2*(5*b^3*cosh(d*x + c)^9 + 4*(36*a*b^2 - 11*b^3)*cosh(d*x + c)
^7 - 6*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^5 - 4*(12*a^3 + 18*a*b^2 -
5*b^3)*cosh(d*x + c)^3 + (36*a*b^2 - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)
)/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7
- 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 5*(
7*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^
3 + (35*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21
*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c
)^2 + (7*d*cosh(d*x + c)^6 - 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*si
```

nh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.36298, size = 261, normalized size = 3.14

$$-\frac{a^3(e^{dx+c} + e^{-dx-c})}{\left((e^{dx+c} + e^{-dx-c})^2 - 4\right)d} + \frac{(a^3 - 6a^2b)\log(e^{dx+c} + e^{-dx-c} + 2)}{4d} - \frac{(a^3 - 6a^2b)\log(e^{dx+c} + e^{-dx-c} - 2)}{4d} + \frac{b^3d^2(e^{dx+c} + e^{-dx-c})^3}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $-a^3(e^{d*x + c} + e^{-d*x - c})/(((e^{d*x + c} + e^{-d*x - c})^2 - 4)*d) + 1/4*(a^3 - 6*a^2*b)*\log(e^{d*x + c} + e^{-d*x - c} + 2)/d - 1/4*(a^3 - 6*a^2*b)*\log(e^{d*x + c} + e^{-d*x - c} - 2)/d + 1/24*(b^3*d^2*(e^{d*x + c} + e^{-d*x - c})^3 + 36*a*b^2*d^2*(e^{d*x + c} + e^{-d*x - c}) - 12*b^3*d^2*(e^{d*x + c} + e^{-d*x - c}))/d^3$

3.27 $\int \operatorname{csch}^4(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=113

$$\frac{a(2a^2 - 5ab - 2b^2) \operatorname{coth}(c + dx)}{2d} - \frac{a^2(2a + 3b) \operatorname{coth}^3(c + dx)}{6d} + \frac{1}{2} b^2 x(6a - b) + \frac{b \cosh^2(c + dx) \operatorname{coth}^3(c + dx) (a - (a - b) \operatorname{Tanh}^2(c + dx))}{2d}$$

[Out] $((6*a - b)*b^2*x)/2 + (a*(2*a^2 - 5*a*b - 2*b^2)*\operatorname{Coth}[c + d*x])/(2*d) - (a^2*(2*a + 3*b)*\operatorname{Coth}[c + d*x]^3)/(6*d) + (b*\operatorname{Cosh}[c + d*x]^2*\operatorname{Coth}[c + d*x]^3*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2)^2)/(2*d)$

Rubi [A] time = 0.141387, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3187, 468, 570, 207}

$$\frac{a(2a^2 - 5ab - 2b^2) \operatorname{coth}(c + dx)}{2d} - \frac{a^2(2a + 3b) \operatorname{coth}^3(c + dx)}{6d} + \frac{1}{2} b^2 x(6a - b) + \frac{b \cosh^2(c + dx) \operatorname{coth}^3(c + dx) (a - (a - b) \operatorname{Tanh}^2(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^4*(a + b*\operatorname{Sinh}[c + d*x]^2)^3, x]$

[Out] $((6*a - b)*b^2*x)/2 + (a*(2*a^2 - 5*a*b - 2*b^2)*\operatorname{Coth}[c + d*x])/(2*d) - (a^2*(2*a + 3*b)*\operatorname{Coth}[c + d*x]^3)/(6*d) + (b*\operatorname{Cosh}[c + d*x]^2*\operatorname{Coth}[c + d*x]^3*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2)^2)/(2*d)$

Rule 3187

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^{(m/2 + p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]] /;$ $\operatorname{FreeQ}[\{a, b, e, f\}, x] \ \&\& \ \operatorname{IntegerQ}[m/2] \ \&\& \ \operatorname{IntegerQ}[p]$

Rule 468

$\operatorname{Int}[(e_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(c*b - a*d)*(e*x)^{(m + 1)}*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q - 1)}/(a*b*e*n*(p + 1)), x] + \operatorname{Dist}[1/(a*b*n*(p + 1)), \operatorname{Int}[(e*x)^m*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q - 2)}*\operatorname{Simp}[c*(c*b*n*(p + 1) + (c*b - a*d)*(m + 1)] + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,$

`x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]`

Rule 570

`Int[((g_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((e_) + (f_.)*(x_)^(n_.))^(r_.), x_Symbol] := Int[ExpandIntegrand[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]`

Rule 207

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])`

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^4(c+dx) (a+b \sinh^2(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-(a-b)x^2)^3}{x^4(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{b \cosh^2(c+dx) \coth^3(c+dx) (a-(a-b) \tanh^2(c+dx))^2}{2d} + \operatorname{Subst}\left(\int \frac{(a/2)}{x^4} dx, x, \tanh(c+dx)\right) \\
 &= \frac{b \cosh^2(c+dx) \coth^3(c+dx) (a-(a-b) \tanh^2(c+dx))^2}{2d} + \operatorname{Subst}\left(\int \left(\frac{a^2}{x^4} - \frac{a}{x^2}\right) dx, x, \tanh(c+dx)\right) \\
 &= \frac{a(2a^2-5ab-2b^2) \coth(c+dx)}{2d} - \frac{a^2(2a+3b) \coth^3(c+dx)}{6d} + \frac{b \cosh^2(c+dx)}{2d} \\
 &= \frac{1}{2}(6a-b)b^2x + \frac{a(2a^2-5ab-2b^2) \coth(c+dx)}{2d} - \frac{a^2(2a+3b) \coth^3(c+dx)}{6d}
 \end{aligned}$$

Mathematica [A] time = 2.5041, size = 107, normalized size = 0.95

$$\frac{2 \sinh^6(c+dx) (\operatorname{acsch}^2(c+dx) + b)^3 (3b^2(2(6a-b)(c+dx) + b \sinh(2(c+dx))) - 4a^2 \coth(c+dx) (\operatorname{acsch}^2(c+dx) - 2))}{3d(2a + b \cosh(2(c+dx)) - b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] $(2*(b + a*\text{Csch}[c + d*x]^2)^3*\text{Sinh}[c + d*x]^6*(-4*a^2*\text{Coth}[c + d*x]*(-2*a + 9*b + a*\text{Csch}[c + d*x]^2) + 3*b^2*(2*(6*a - b)*(c + d*x) + b*\text{Sinh}[2*(c + d*x)])))/(3*d*(2*a - b + b*\text{Cosh}[2*(c + d*x)])^3)$

Maple [A] time = 0.039, size = 77, normalized size = 0.7

$$\frac{1}{d} \left(a^3 \left(\frac{2}{3} - \frac{(\text{csch}(dx+c))^2}{3} \right) \coth(dx+c) - 3a^2b \coth(dx+c) + 3ab^2(dx+c) + b^3 \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} - \frac{dx}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)`

[Out] $1/d*(a^3*(2/3-1/3*\text{csch}(d*x+c)^2)*\coth(d*x+c)-3*a^2*b*\coth(d*x+c)+3*a*b^2*(d*x+c)+b^3*(1/2*\cosh(d*x+c)*\sinh(d*x+c)-1/2*d*x-1/2*c))$

Maxima [A] time = 1.03046, size = 217, normalized size = 1.92

$$-\frac{1}{8}b^3 \left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d} \right) + 3ab^2x + \frac{4}{3}a^3 \left(\frac{3e^{-2dx-2c}}{d(3e^{-2dx-2c} - 3e^{-4dx-4c} + e^{-6dx-6c} - 1)} - \frac{1}{d(3e^{-2dx-2c} - 3e^{-4dx-4c} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $-1/8*b^3*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) + 3*a*b^2*x + 4/3*a^3*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1))) + 6*a^2*b/(d*(e^{(-2*d*x - 2*c)} - 1))$

Fricas [B] time = 1.7636, size = 668, normalized size = 5.91

$$3b^3 \cosh(dx+c)^5 + 15b^3 \cosh(dx+c) \sinh(dx+c)^4 + (16a^3 - 72a^2b - 9b^3) \cosh(dx+c)^3 - 4(4a^3 - 18a^2b - 3(6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $\frac{1}{24}*(3*b^3*\cosh(d*x + c)^5 + 15*b^3*\cosh(d*x + c)*\sinh(d*x + c)^4 + (16*a^3 - 72*a^2*b - 9*b^3)*\cosh(d*x + c)^3 - 4*(4*a^3 - 18*a^2*b - 3*(6*a*b^2 - b^3)*d*x)*\sinh(d*x + c)^3 + 3*(10*b^3*\cosh(d*x + c)^3 + (16*a^3 - 72*a^2*b - 9*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 6*(8*a^3 - 12*a^2*b - b^3)*\cosh(d*x + c) + 12*(4*a^3 - 18*a^2*b - 3*(6*a*b^2 - b^3)*d*x - (4*a^3 - 18*a^2*b - 3*(6*a*b^2 - b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(d*\sinh(d*x + c)^3 + 3*(d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [A] time = 1.36781, size = 217, normalized size = 1.92

$$\frac{b^3 e^{(2dx+2c)}}{8d} + \frac{(6ab^2 - b^3)(dx + c)}{2d} - \frac{(12ab^2 e^{(2dx+2c)} - 2b^3 e^{(2dx+2c)} + b^3) e^{(-2dx-2c)}}{8d} - \frac{2(9a^2 b e^{(4dx+4c)} + 6a^3 e^{(2dx+2c)} - 1)}{3d e^{(2dx+2c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{8}b^3 e^{(2d*x + 2*c)}/d + \frac{1}{2}*(6*a*b^2 - b^3)*(d*x + c)/d - \frac{1}{8}*(12*a*b^2 * e^{(2*d*x + 2*c)} - 2*b^3 * e^{(2*d*x + 2*c)} + b^3) * e^{(-2*d*x - 2*c)}/d - \frac{2}{3}*(9*a^2*b * e^{(4*d*x + 4*c)} + 6*a^3 * e^{(2*d*x + 2*c)} - 18*a^2*b * e^{(2*d*x + 2*c)} - 2*a^3 + 9*a^2*b)/(d*(e^{(2*d*x + 2*c)} - 1)^3)$

$$3.28 \quad \int \frac{\sinh^7(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=109

$$\frac{(a^2 + ab + b^2) \cosh(c + dx)}{b^3 d} - \frac{a^3 \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{7/2} d \sqrt{a-b}} - \frac{(a + 2b) \cosh^3(c + dx)}{3b^2 d} + \frac{\cosh^5(c + dx)}{5bd}$$

[Out] $-\left(\frac{a^3 \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c + d*x]}{\sqrt{a-b}}\right]}{\sqrt{a-b}}\right) / \left(\sqrt{a-b} * b^{(7/2)*d}\right) + \left(\frac{(a^2 + a*b + b^2) * \text{Cosh}[c + d*x]}{b^3 * d}\right) - \left(\frac{(a + 2*b) * \text{Cosh}[c + d*x]^3}{3 * b^2 * d}\right) + \frac{\text{Cosh}[c + d*x]^5}{5 * b * d}$

Rubi [A] time = 0.149371, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3186, 390, 205}

$$\frac{(a^2 + ab + b^2) \cosh(c + dx)}{b^3 d} - \frac{a^3 \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{7/2} d \sqrt{a-b}} - \frac{(a + 2b) \cosh^3(c + dx)}{3b^2 d} + \frac{\cosh^5(c + dx)}{5bd}$$

Antiderivative was successfully verified.

[In] `Int[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]`

[Out] $-\left(\frac{a^3 \text{ArcTan}\left[\frac{\sqrt{b} \text{Cosh}[c + d*x]}{\sqrt{a-b}}\right]}{\sqrt{a-b}}\right) / \left(\sqrt{a-b} * b^{(7/2)*d}\right) + \left(\frac{(a^2 + a*b + b^2) * \text{Cosh}[c + d*x]}{b^3 * d}\right) - \left(\frac{(a + 2*b) * \text{Cosh}[c + d*x]^3}{3 * b^2 * d}\right) + \frac{\text{Cosh}[c + d*x]^5}{5 * b * d}$

Rule 3186

`Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]`

Rule 390

`Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]`

Rule 205

$\text{Int}[(a + b \cdot (x^2)^{-1}), x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-b+bx^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{a^2+ab+b^2}{b^3} + \frac{(a+2b)x^2}{b^2} - \frac{x^4}{b} + \frac{a^3}{b^3(a-b+bx^2)}\right) dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d} - \frac{(a+2b)\cosh^3(c+dx)}{3b^2d} + \frac{\cosh^5(c+dx)}{5bd} - \frac{a^3 \text{Subst}\left(\int \frac{1}{a-b+bx^2}\right)}{d} \\ &= -\frac{a^3 \tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-bb^7/2d}} + \frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d} - \frac{(a+2b)\cosh^3(c+dx)}{3b^2d} + \frac{\cosh^5(c+dx)}{5bd} \end{aligned}$$

Mathematica [C] time = 0.868052, size = 165, normalized size = 1.51

$$30\sqrt{b}(8a^2+6ab+5b^2)\cosh(c+dx) - \frac{240a^3\left(\tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) + \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}} - 5b^{3/2}(4a+5b)\cosh(3(c+dx))}{240b^{7/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2), x]

[Out] ((-240*a^3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b] + 30*Sqrt[b]*(8*a^2 + 6*a*b + 5*b^2)*Cosh[c + d*x] - 5*b^(3/2)*(4*a + 5*b)*Cosh[3*(c + d*x)] + 3*b^(5/2)*Cosh[5*(c + d*x)])/(240*b^(7/2)*d)

Maple [B] time = 0.044, size = 448, normalized size = 4.1

$$\frac{1}{5bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-5} - \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-4} + \frac{a}{2db^2} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} + \frac{3}{8bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(dx+c)^7/(a+b*\sinh(dx+c)^2), x)$

[Out] $\frac{1}{5}d/b/(\tanh(1/2*d*x+1/2*c)+1)^5 - \frac{1}{2}d/b/(\tanh(1/2*d*x+1/2*c)+1)^4 + \frac{1}{2}d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^{2*a+3/8} - \frac{1}{3}d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^{3*a+1/12} - \frac{1}{d/b^3}(\tanh(1/2*d*x+1/2*c)+1)^{a^2+1/2} - \frac{1}{d/b^2}(\tanh(1/2*d*x+1/2*c)+1)^{a+3/8} - \frac{1}{5}d/b/(\tanh(1/2*d*x+1/2*c)-1)^5 - \frac{1}{2}d/b/(\tanh(1/2*d*x+1/2*c)-1)^4 + \frac{1}{2}d/b^2/(\tanh(1/2*d*x+1/2*c)-1)^{2*a+3/8} - \frac{1}{3}d/b^2/(\tanh(1/2*d*x+1/2*c)-1)^{3*a-1/12} - \frac{1}{d/b^3}(\tanh(1/2*d*x+1/2*c)-1)^{a^2-1/2} - \frac{1}{d/b^2}(\tanh(1/2*d*x+1/2*c)-1)^{a-3/8} - \frac{1}{d*a^3/b^3}(\tanh(1/2*d*x+1/2*c)-1)^{-1} \arctan\left(\frac{1}{4} \frac{2*\tanh(1/2*d*x+1/2*c)^{2*a-2*a+4*b}}{(a*b-b^2)^{1/2}}\right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(3b^2e^{10dx+10c} + 3b^2 - 5(4abe^{8c} + 5b^2e^{8c}))e^{8dx} + 30(8a^2e^{6c} + 6abe^{6c} + 5b^2e^{6c})e^{6dx} + 30(8a^2e^{4c} + 6abe^{4c} + 5b^2e^{4c})e^{4dx} - 5(4a^2e^{2c} + 6abe^{2c} + 5b^2e^{2c})e^{2dx} - 5(4a^2e^c + 6abe^c + 5b^2e^c)e^c}{480b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(dx+c)^7/(a+b*\sinh(dx+c)^2), x, \text{algorithm}="maxima")$

[Out] $\frac{1}{480}(3b^2e^{10dx+10c} + 3b^2 - 5(4a^2e^{8c} + 5b^2e^{8c}))e^{8dx} + 30(8a^2e^{6c} + 6a^2e^{6c} + 5b^2e^{6c})e^{6dx} + 30(8a^2e^{4c} + 6a^2e^{4c} + 5b^2e^{4c})e^{4dx} - 5(4a^2e^{2c} + 5b^2e^{2c})e^{2dx} - 5(4a^2e^c + 5b^2e^c)e^c / (b^3d) - \frac{1}{128} \int \frac{256(a^3e^{3dx+3c} - a^3e^{dx+c})}{(b^4e^{4dx+4c} + b^4 + 2(2a^3b^3e^{2c} - b^4e^{2c}))e^{2dx}} dx$

Fricas [B] time = 2.28412, size = 7640, normalized size = 70.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(dx+c)^7/(a+b*\sinh(dx+c)^2), x, \text{algorithm}="fricas")$

```
[Out] [1/480*(3*(a*b^3 - b^4)*cosh(d*x + c)^10 + 30*(a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + 3*(a*b^3 - b^4)*sinh(d*x + c)^10 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^8 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4 - 27*(a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^3 - (4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b^3 - 15*b^4 - 14*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*(a*b^3 - b^4)*cosh(d*x + c)^5 - 70*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^3 + 45*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^4 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^6 - 35*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b^3 - 15*b^4 + 45*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a*b^3 - 3*b^4 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^7 - 7*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 15*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^3 + 3*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2 + 5*(27*(a*b^3 - b^4)*cosh(d*x + c)^8 - 28*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 90*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 4*a^2*b^2 - a*b^3 + 5*b^4 + 36*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 240*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3*sinh(d*x + c)^5)*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 10*(3*(a*b^3 - b^4)*cosh(d*x + c)^9 - 4*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^7 + 18*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 12*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^3 - (4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c))/(a*b^4 - b^5)*d*cosh(d*x + c)^5 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a*b^4 - b^5)*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*(a*b^4 - b^5)*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a*b^4 - b^5)*d*sinh(d*x + c)^5), 1/480*(3*(a*b^3 - b^4)*cosh(d*x + c)^10 + 30*(a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + 3*(a*b^3 - b^4)*sinh(d*x + c)^10 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^8 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4 - 27*(a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^3 - (4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*
```

$$\begin{aligned}
& a^3b^3 - 15b^4 - 14(4a^2b^2 + a^3b^3 - 5b^4)\cosh(dx + c)^2 \sinh(dx + c)^6 + 4(189(a^3b^3 - b^4)\cosh(dx + c)^5 - 70(4a^2b^2 + a^3b^3 - 5b^4)\cosh(dx + c)^3 + 45(8a^3b - 2a^2b^2 - a^3b^3 - 5b^4)\cosh(dx + c))\sinh(dx + c)^5 + 30(8a^3b - 2a^2b^2 - a^3b^3 - 5b^4)\cosh(dx + c)^4 + 10(63(a^3b^3 - b^4)\cosh(dx + c)^6 - 35(4a^2b^2 + a^3b^3 - 5b^4)\cosh(dx + c)^4 + 24a^3b - 6a^2b^2 - 3a^3b^3 - 15b^4 + 45(8a^3b - 2a^2b^2 - a^3b^3 - 5b^4)\cosh(dx + c)^2)\sinh(dx + c)^4 + 3a^3b^3 - 3b^4 + 40(9(a^3b^3 - b^4)\cosh(dx + c)^7 - 7(4a^2b^2 + a^3b^3 - 5b^4)\cosh(dx + c)^5 + 15(8a^3b - 2a^2b^2 - a^3b^3 - 5b^4)\cosh(dx + c)^3 + 3(8a^3b - 2a^2b^2 - a^3b^3 - 5b^4)\cosh(dx + c))\sinh(dx + c)^3 - 5(4a^2b^2 + a^3b^3 - 5b^4)\cosh(dx + c)^2 + 5(27(a^3b^3 - b^4)\cosh(dx + c)^8 - 28(4a^2b^2 + a^3b^3 - 5b^4)\cosh(dx + c)^6 + 90(8a^3b - 2a^2b^2 - a^3b^3 - 5b^4)\cosh(dx + c)^4 - 4a^2b^2 - a^3b^3 + 5b^4 + 36(8a^3b - 2a^2b^2 - a^3b^3 - 5b^4)\cosh(dx + c)^2)\sinh(dx + c)^2 - 480(a^3\cosh(dx + c)^5 + 5a^3\cosh(dx + c)^4\sinh(dx + c) + 10a^3\cosh(dx + c)^3\sinh(dx + c)^2 + 10a^3\cosh(dx + c)^2\sinh(dx + c)^3 + 5a^3\cosh(dx + c)\sinh(dx + c)^4 + a^3\sinh(dx + c)^5)\sqrt{ab - b^2}\arctan(-1/2(b\cosh(dx + c)^3 + 3b\cosh(dx + c)\sinh(dx + c)^2 + b\sinh(dx + c)^3 + (4a - 3b)\cosh(dx + c) + (3b\cosh(dx + c)^2 + 4a - 3b)\sinh(dx + c))/\sqrt{ab - b^2}) + 480(a^3\cosh(dx + c)^5 + 5a^3\cosh(dx + c)^4\sinh(dx + c) + 10a^3\cosh(dx + c)^3\sinh(dx + c)^2 + 10a^3\cosh(dx + c)^2\sinh(dx + c)^3 + 5a^3\cosh(dx + c)\sinh(dx + c)^4 + a^3\sinh(dx + c)^5)\sqrt{ab - b^2}\arctan(-1/2\sqrt{ab - b^2}(\cosh(dx + c) + \sinh(dx + c))/(a - b)) + 10(3(a^3b^3 - b^4)\cosh(dx + c)^9 - 4(4a^2b^2 + a^3b^3 - 5b^4)\cosh(dx + c)^7 + 18(8a^3b - 2a^2b^2 - a^3b^3 - 5b^4)\cosh(dx + c)^5 + 12(8a^3b - 2a^2b^2 - a^3b^3 - 5b^4)\cosh(dx + c)^3 - (4a^2b^2 + a^3b^3 - 5b^4)\cosh(dx + c))\sinh(dx + c))/((a^4b - b^5)d\cosh(dx + c)^5 + 5(a^4b - b^5)d\cosh(dx + c)^4\sinh(dx + c) + 10(a^4b - b^5)d\cosh(dx + c)^3\sinh(dx + c)^2 + 10(a^4b - b^5)d\cosh(dx + c)^2\sinh(dx + c)^3 + 5(a^4b - b^5)d\cosh(dx + c)\sinh(dx + c)^4 + (a^4b - b^5)d\sinh(dx + c)^5)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)**7/(a+b*sinh(dx+c)**2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.29 \quad \int \frac{\sinh^6(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=121

$$-\frac{a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^3 d \sqrt{a-b}} + \frac{x(8a^2 + 4ab + 3b^2)}{8b^3} - \frac{(4a + 3b) \sinh(c+dx) \cosh(c+dx)}{8b^2 d} + \frac{\sinh^3(c+dx) \cosh(c+dx)}{4bd}$$

[Out] $((8*a^2 + 4*a*b + 3*b^2)*x)/(8*b^3) - (a^{(5/2)}*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b^3*d) - ((4*a + 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*b*d)$

Rubi [A] time = 0.235035, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3187, 470, 578, 522, 206, 208}

$$-\frac{a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^3 d \sqrt{a-b}} + \frac{x(8a^2 + 4ab + 3b^2)}{8b^3} - \frac{(4a + 3b) \sinh(c+dx) \cosh(c+dx)}{8b^2 d} + \frac{\sinh^3(c+dx) \cosh(c+dx)}{4bd}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2), x]

[Out] $((8*a^2 + 4*a*b + 3*b^2)*x)/(8*b^3) - (a^{(5/2)}*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b^3*d) - ((4*a + 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*b*d)$

Rule 3187

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)]/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 470

Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(

$b*n*(b*c - a*d)*(p + 1)$, Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 578

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3(a-(a-b)x^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{\text{Subst}\left(\int \frac{x^2(3a+(a+3b)x^2)}{(1-x^2)^2(a+(-a+b)x^2)} dx, x, \tanh(c+dx)\right)}{4bd} \\
&= -\frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{\text{Subst}\left(\int \frac{-a(4a+3b)+}{(1-x^2)}\right)}{4bd} \\
&= -\frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{a^3 \text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\right)}{4bd} \\
&= \frac{(8a^2+4ab+3b^2)x}{8b^3} - \frac{a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}b^3d} - \frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d}
\end{aligned}$$

Mathematica [A] time = 0.475127, size = 97, normalized size = 0.8

$$\frac{4(8a^2+4ab+3b^2)(c+dx) - \frac{32a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - 8b(a+b)\sinh(2(c+dx)) + b^2\sinh(4(c+dx))}{32b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]

[Out] (4*(8*a^2 + 4*a*b + 3*b^2)*(c + d*x) - (32*a^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b] - 8*b*(a + b)*Sinh[2*(c + d*x)] + b^2*Sinh[4*(c + d*x)]/(32*b^3*d)

Maple [B] time = 0.08, size = 670, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)

```
[Out] -1/4/d/b/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/
b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+1/8/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b^2
/(tanh(1/2*d*x+1/2*c)+1)*a-3/8/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/d/b^3*ln(tanh(
1/2*d*x+1/2*c)+1)*a^2+1/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+3/8/d/b*ln(tanh
(1/2*d*x+1/2*c)+1)+1/4/d/b/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/b/(tanh(1/2*d*x+
1/2*c)-1)^3-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1/8/d/b/(tanh(1/2*d*x+1/2
*c)-1)^2-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-3/8/d/b/(tanh(1/2*d*x+1/2*c)-1
)-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-
1)-3/8/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a^3/b^3/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
))+1/d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arcta
nh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a^3/b^3/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05375, size = 4313, normalized size = 35.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh
(d*x + c)^8 + 8*(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b + b^2)
*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b - 2*b^2)*sinh(d*x + c)^
6 + 8*(7*b^2*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5
+ 2*(35*b^2*cosh(d*x + c)^4 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*x - 60*(a*b + b^
```

```

2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 +
  4*a*b + 3*b^2)*d*x*cosh(d*x + c) - 20*(a*b + b^2)*cosh(d*x + c)^3)*sinh(d*
x + c)^3 + 8*(a*b + b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8
*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b + b^2)*cosh(d*x + c)^4
+ 2*a*b + 2*b^2)*sinh(d*x + c)^2 + 32*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x
+ c)^3*sinh(d*x + c) + 6*a^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^2*cosh(
d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4)*sqrt(a/(a - b))*log((b^2*co
sh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 +
  2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*
sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b
^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b
- b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 -
3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d
*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d
*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + b)) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 +
  4*a*b + 3*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c)^5 + 2*(a*
b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4), 1/64*(b^2*cosh(
d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 8*
(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b + b^2)*cosh(d*x + c)^6
+ 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b - 2*b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cos
h(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cos
h(d*x + c)^4 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*x - 60*(a*b + b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 + 4*a*b + 3*b^2)*
d*x*cosh(d*x + c) - 20*(a*b + b^2)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a*
b + b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8*a^2 + 4*a*b + 3
*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b + b^2)*cosh(d*x + c)^4 + 2*a*b + 2*b^2)
*sinh(d*x + c)^2 - 64*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^3*sinh(d*x
+ c) + 6*a^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^2*cosh(d*x + c)*sinh(d*
x + c)^3 + a^2*sinh(d*x + c)^4)*sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-
a/(a - b))/a) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*
x*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c)^5 + 2*(a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^3*d*cosh(d*x + c)*
sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [B] time = 1.20858, size = 296, normalized size = 2.45

$$\frac{a^3 \arctan\left(\frac{be^{2dx+2c}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b^3d} + \frac{(8a^2+4ab+3b^2)(dx+c)}{8b^3d} - \frac{(48a^2e^{4dx+4c}+24abe^{4dx+4c}+18b^2e^{4dx+4c}-8abe^{2dx+2c}-8b^2e^{2dx+2c}+b^2)e^{-4dx-4c}}{64b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] $-a^3 \arctan\left(\frac{1}{2} \frac{b e^{2 d x+2 c}+2 a-b}{\sqrt{-a^2+a b}}\right) / \left(\sqrt{-a^2+a b}\right) / \left(b^3 d\right) + \frac{1}{8} \frac{\left(8 a^2+4 a b+3 b^2\right)\left(d x+c\right)}{b^3 d} - \frac{1}{64} \frac{\left(48 a^2 e^{4 d x+4 c}+24 a b e^{4 d x+4 c}+18 b^2 e^{4 d x+4 c}-8 a b e^{2 d x+2 c}-8 b^2 e^{2 d x+2 c}+b^2\right) e^{-4 d x-4 c}}{b^3 d} + \frac{1}{64} \frac{\left(b d e^{4 d x+4 c}-8 a d e^{2 d x+2 c}-8 b d e^{2 d x+2 c}\right)}{b^2 d^2}$

$$3.30 \quad \int \frac{\sinh^5(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=79

$$\frac{a^2 \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{5/2} d \sqrt{a-b}} - \frac{(a+b) \cosh(c+dx)}{b^2 d} + \frac{\cosh^3(c+dx)}{3bd}$$

[Out] (a^2*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(5/2)*d) - ((a + b)*Cosh[c + d*x])/(b^2*d) + Cosh[c + d*x]^3/(3*b*d)

Rubi [A] time = 0.108028, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3186, 390, 205}

$$\frac{a^2 \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{5/2} d \sqrt{a-b}} - \frac{(a+b) \cosh(c+dx)}{b^2 d} + \frac{\cosh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2), x]

[Out] (a^2*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(5/2)*d) - ((a + b)*Cosh[c + d*x])/(b^2*d) + Cosh[c + d*x]^3/(3*b*d)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-bx^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{a+b}{b^2} + \frac{x^2}{b} + \frac{a^2}{b^2(a-bx^2)}\right) dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{(a+b)\cosh(c+dx)}{b^2d} + \frac{\cosh^3(c+dx)}{3bd} + \frac{a^2 \text{Subst}\left(\int \frac{1}{a-bx^2} dx, x, \cosh(c+dx)\right)}{b^2d} \\ &= \frac{a^2 \tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}b^{5/2}d} - \frac{(a+b)\cosh(c+dx)}{b^2d} + \frac{\cosh^3(c+dx)}{3bd} \end{aligned}$$

Mathematica [C] time = 0.434318, size = 134, normalized size = 1.7

$$\frac{12a^2 \left(\tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) + \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) \right)}{\sqrt{a-b}} - 3\sqrt{b}(4a+3b)\cosh(c+dx) + b^{3/2}\cosh(3(c+dx))}{12b^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2), x]

[Out] ((12*a^2*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b] - 3*Sqrt[b]*(4*a + 3*b)*Cosh[c + d*x] + b^(3/2)*Cosh[3*(c + d*x)]/(12*b^(5/2)*d)

Maple [B] time = 0.033, size = 227, normalized size = 2.9

$$\frac{1}{3bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-3} - \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} - \frac{a}{db^2} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)`

[Out] $\frac{1}{3} \frac{d}{b} \frac{1}{(\tanh(1/2*d*x+1/2*c)+1)^3} - \frac{1}{2} \frac{d}{b} \frac{1}{(\tanh(1/2*d*x+1/2*c)+1)^2} - \frac{1}{d} \frac{1}{b^2} \frac{1}{(\tanh(1/2*d*x+1/2*c)+1)*a} - \frac{1}{2} \frac{d}{b} \frac{1}{(\tanh(1/2*d*x+1/2*c)+1)} - \frac{1}{3} \frac{d}{b} \frac{1}{(\tanh(1/2*d*x+1/2*c)-1)^3} - \frac{1}{2} \frac{d}{b} \frac{1}{(\tanh(1/2*d*x+1/2*c)-1)^2} + \frac{1}{d} \frac{1}{b^2} \frac{1}{(\tanh(1/2*d*x+1/2*c)-1)*a} + \frac{1}{2} \frac{d}{b} \frac{1}{(\tanh(1/2*d*x+1/2*c)-1)} + \frac{1}{d} \frac{a^2}{b^2} \frac{1}{(a*b-b^2)^{1/2}} \arctan\left(\frac{1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)}{(a*b-b^2)^{1/2}}\right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\left(4ae^{4c} + 3be^{4c}\right)e^{4dx} + 3\left(4ae^{2c} + 3be^{2c}\right)e^{2dx} - be^{6dx+6c} - b\right)e^{-3dx-3c}}{24b^2d} + \frac{1}{32} \int \frac{64\left(a^2e^{3dx+3c} - b^3e^{4dx+4c} + b^3 + 2\left(2ab^2e^{2c} - b^3e^{2c}\right)e^{2dx}\right)}{b^3e^{4dx+4c} + b^3 + 2\left(2ab^2e^{2c} - b^3e^{2c}\right)e^{2dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")`

[Out] $-\frac{1}{24} \left(3 \left(4 a e^{4 c} + 3 b e^{4 c}\right) e^{4 d x} + 3 \left(4 a e^{2 c} + 3 b e^{2 c}\right) e^{2 d x} - b e^{6 d x+6 c} - b\right) e^{-3 d x-3 c} / \left(b^2 d\right) + \frac{1}{32} \int \left(64 \left(a^2 e^{3 d x+3 c} - b^3 e^{4 d x+4 c} + b^3 + 2 \left(2 a b^2 e^{2 c} - b^3 e^{2 c}\right) e^{2 d x}\right) / \left(b^3 e^{4 d x+4 c} + b^3 + 2 \left(2 a b^2 e^{2 c} - b^3 e^{2 c}\right) e^{2 d x}\right)\right) dx$

Fricas [B] time = 1.96975, size = 3993, normalized size = 50.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")`

[Out] $\left[\frac{1}{24} \left(\left(a b^2 - b^3\right) \cosh(d x + c)^6 + 6 \left(a b^2 - b^3\right) \cosh(d x + c) \sinh(d x + c)^5 + \left(a b^2 - b^3\right) \sinh(d x + c)^6 - 3 \left(4 a^2 b - a b^2 - 3 b^3\right) \cosh(d x + c)^4 - 3 \left(4 a^2 b - a b^2 - 3 b^3 - 5 \left(a b^2 - b^3\right) \cosh(d x + c)^2\right) \sinh(d x + c)^4 + 4 \left(5 \left(a b^2 - b^3\right) \cosh(d x + c)^3 - 3 \left(4 a^2 b - a b^2 - 3 b^3\right) \cosh(d x + c)\right) \sinh(d x + c)^3 + a b^2 - b^3 - 3 \left(4 a^2 b - a b^2 - 3 b^3\right) \cosh(d x + c)^2 + 3 \left(5 \left(a b^2 - b^3\right) \cosh(d x + c)^4 - 4 a^2 b + a b^2 + 3 b^3 - 6 \left(4 a^2 b - a b^2 - 3 b^3\right) \cosh(d x + c)^2\right) \sinh(d x + c)^2\right)$

$$\begin{aligned}
& 2 - 12*(a^2*\cosh(d*x + c)^3 + 3*a^2*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a^2*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^2*\sinh(d*x + c)^3)*\sqrt{-a*b + b^2}*\log((b \\
& *\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - \\
& 2*(2*a - 3*b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a + 3*b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a - 3*b)*\cosh(d*x + c))*\sinh(d*x + c) \\
& - 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + \\
& (3*\cosh(d*x + c)^2 + 1)*\sinh(d*x + c) + \cosh(d*x + c))*\sqrt{-a*b + b^2} + b \\
&)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + \\
& b)) + 6*((a*b^2 - b^3)*\cosh(d*x + c)^5 - 2*(4*a^2*b - a*b^2 - 3*b^3)*\cosh(d*x + c)^3 - (4*a^2*b - a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/((a*b^3 - b^4)*d*\cosh(d*x + c)^3 + 3*(a*b^3 - b^4)*d*\cosh(d*x + c)^2*\sinh(d*x + c) \\
& + 3*(a*b^3 - b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a*b^3 - b^4)*d*\sinh(d*x + c)^3), 1/24*((a*b^2 - b^3)*\cosh(d*x + c)^6 + 6*(a*b^2 - b^3)*\cosh(d*x + c)*\sinh(d*x + c)^5 + (a*b^2 - b^3)*\sinh(d*x + c)^6 - 3*(4*a^2*b - a*b^2 - \\
& 3*b^3)*\cosh(d*x + c)^4 - 3*(4*a^2*b - a*b^2 - 3*b^3 - 5*(a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(5*(a*b^2 - b^3)*\cosh(d*x + c)^3 - 3*(4*a^2*b - a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a*b^2 - b^3 - 3*(4*a^2*b - a*b^2 - 3*b^3)*\cosh(d*x + c)^2 + 3*(5*(a*b^2 - b^3)*\cosh(d*x + c)^4 - \\
& 4*a^2*b + a*b^2 + 3*b^3 - 6*(4*a^2*b - a*b^2 - 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 24*(a^2*\cosh(d*x + c)^3 + 3*a^2*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a^2*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^2*\sinh(d*x + c)^3)*\sqrt{a*b - b^2}*\arctan(-1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 + (4*a - 3*b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - 3*b)*\sinh(d*x + c))/\sqrt{a*b - b^2}) - 24*(a^2*\cosh(d*x + c)^3 + 3*a^2*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a^2*\cosh(d*x + c)*\sinh(d*x + c)^2 + a^2*\sinh(d*x + c)^3)*\sqrt{a*b - b^2}*\arctan(-1/2*\sqrt{a*b - b^2}*(\cosh(d*x + c) + \sinh(d*x + c)))/(a - b)) + 6*((a*b^2 - b^3)*\cosh(d*x + c)^5 - 2*(4*a^2*b - a*b^2 - 3*b^3)*\cosh(d*x + c)^3 - (4*a^2*b - a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/((a*b^3 - b^4)*d*\cosh(d*x + c)^3 + 3*(a*b^3 - b^4)*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a*b^3 - b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a*b^3 - b^4)*d*\sinh(d*x + c)^3)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.31 \quad \int \frac{\sinh^4(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=79

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^2 d \sqrt{a-b}} - \frac{x(2a+b)}{2b^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2bd}$$

[Out] $-\frac{(2a+b)x}{2b^2} + \frac{a^{3/2} \text{ArcTanh}[\frac{\sqrt{a-b} \text{Tanh}[c+dx]}{\sqrt{a}}]}{\sqrt{a}} / (\sqrt{a-b} b^2 d) + \frac{\text{Cosh}[c+dx] \text{Sinh}[c+dx]}{2bd}$

Rubi [A] time = 0.121325, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3187, 470, 522, 206, 208}

$$\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{b^2 d \sqrt{a-b}} - \frac{x(2a+b)}{2b^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]

[Out] $-\frac{(2a+b)x}{2b^2} + \frac{a^{3/2} \text{ArcTanh}[\frac{\sqrt{a-b} \text{Tanh}[c+dx]}{\sqrt{a}}]}{\sqrt{a}} / (\sqrt{a-b} b^2 d) + \frac{\text{Cosh}[c+dx] \text{Sinh}[c+dx]}{2bd}$

Rule 3187

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 470

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,

$x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[m - n + 1, n] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 522

$\text{Int}[\left(\frac{(e_+) + (f_+)(x_+)^{n_+}}{((a_+) + (b_+)(x_+)^{n_+})((c_+) + (d_+)(x_+)^{n_+})}\right), x_Symbol] \rightarrow \text{Dist}[\frac{b*e - a*f}{b*c - a*d}, \text{Int}[1/(a + b*x^n), x], x] - \text{Dist}[\frac{d*e - c*f}{b*c - a*d}, \text{Int}[1/(c + d*x^n), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x]$

Rule 206

$\text{Int}[\left(\frac{(a_+) + (b_+)(x_+)^2}{(x_+)^{-1}}\right)^{-1}, x_Symbol] \rightarrow \text{Simp}[\frac{1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2]]}{\text{Rt}[a, 2]*\text{Rt}[-b, 2]}, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 208

$\text{Int}[\left(\frac{(a_+) + (b_+)(x_+)^2}{(x_+)^{-1}}\right)^{-1}, x_Symbol] \rightarrow \text{Simp}[\frac{\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]]}{a}, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\sinh^4(c + dx)}{a + b \sinh^2(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2(a-(a-b)x^2)} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2bd} - \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1-x^2)(a+(-a+b)x^2)} dx, x, \tanh(c + dx)\right)}{2bd} \\ &= \frac{\cosh(c + dx) \sinh(c + dx)}{2bd} + \frac{a^2 \text{Subst}\left(\int \frac{1}{a+(-a+b)x^2} dx, x, \tanh(c + dx)\right)}{b^2d} - \frac{(2a + b) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{2bd} \\ &= -\frac{(2a + b)x}{2b^2} + \frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}b^2d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2bd} \end{aligned}$$

Mathematica [A] time = 0.24996, size = 71, normalized size = 0.9

$$\frac{\frac{4a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - 2(2a + b)(c + dx) + b \sinh(2(c + dx))}{4b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2), x]

[Out] $(-2*(2*a + b)*(c + d*x) + (4*a^{3/2}*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b] + b*Sinh[2*(c + d*x)])/(4*b^2*d)$

Maple [B] time = 0.043, size = 454, normalized size = 5.8

$$-\frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} + \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{a}{db^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{2bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2), x)

[Out] $-1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/2/d/b*\ln(\tanh(1/2*d*x+1/2*c)+1)+1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/2/d/b*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d*a^2/b^2/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/d*a^2/b/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/d*a^2/b^2/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-1/d*a^2/b/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.12917, size = 2191, normalized size = 27.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x + \\ & c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a + b)*d*x - 3*b*cosh(d*x \\ & + c)^2)*sinh(d*x + c)^2 - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d* \\ & x + c) + a*sinh(d*x + c)^2)*sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^ \\ & 2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos \\ & h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8* \\ & a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s \\ & inh(d*x + c) - 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c) \\ & *sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/ \\ & (a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d \\ & *x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b) \\ & *sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x \\ & + c) + b) + 4*(2*(2*a + b)*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d* \\ & x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + \\ & b^2*d*sinh(d*x + c)^2), -1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x \\ & + c)^4 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a \\ & + b)*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 8*(a*cosh(d*x + c)^2 + 2* \\ & a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*sqrt(-a/(a - b))*arctan(\\ & 1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^ \\ & 2 + 2*a - b)*sqrt(-a/(a - b))/a) + 4*(2*(2*a + b)*d*x*cosh(d*x + c) - b*cos \\ & h(d*x + c)^3)*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x \\ & + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [B] time = 1.23783, size = 181, normalized size = 2.29

$$\frac{a^2 \arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b^2d} - \frac{(dx+c)(2a+b)}{2b^2d} + \frac{e^{(2dx+2c)}}{8bd} + \frac{(4ae^{(2dx+2c)}+2be^{(2dx+2c)}-b)e^{(-2dx-2c)}}{8b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] a^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*b^2*d) - 1/2*(d*x + c)*(2*a + b)/(b^2*d) + 1/8*e^(2*d*x + 2*c)/(b*d) + 1/8*(4*a*e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c) - b)*e^(-2*d*x - 2*c)/(b^2*d)

$$3.32 \quad \int \frac{\sinh^3(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=56

$$\frac{\cosh(c+dx)}{bd} - \frac{a \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{3/2}d\sqrt{a-b}}$$

[Out] -((a*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(3/2)*d)) + Cosh[c + d*x]/(b*d)

Rubi [A] time = 0.0851327, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3186, 388, 205}

$$\frac{\cosh(c+dx)}{bd} - \frac{a \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{3/2}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2), x]

[Out] -((a*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(3/2)*d)) + Cosh[c + d*x]/(b*d)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 388

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)} dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+bx^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{\cosh(c+dx)}{bd} - \frac{a \text{Subst}\left(\int \frac{1}{a-b+bx^2} dx, x, \cosh(c+dx)\right)}{bd} \\ &= -\frac{a \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}b^{3/2}d} + \frac{\cosh(c+dx)}{bd} \end{aligned}$$

Mathematica [C] time = 0.233537, size = 107, normalized size = 1.91

$$\frac{\sqrt{b} \cosh(c+dx) - \frac{a \left(\tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) + \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) \right)}{\sqrt{a-b}}}{b^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2), x]

[Out] (-((a*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b]) + Sqrt[b]*Cosh[c + d*x])/(b^(3/2)*d)

Maple [B] time = 0.03, size = 98, normalized size = 1.8

$$-\frac{1}{bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-1} + \frac{1}{bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{a}{bd} \arctan\left(\frac{1}{4} \left(2 (\tanh(1/2 dx + c/2))^2 a - 2a + 4b \right) \frac{1}{\sqrt{ab -}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2), x)

[Out] $-1/d/b/(\tanh(1/2*d*x+1/2*c)-1)+1/d/b/(\tanh(1/2*d*x+1/2*c)+1)-1/d*a/b/(a*b-b^2)^{(1/2)*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^{(1/2)})}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(e^{(2dx+2c)} + 1)e^{-dx-c}}{2bd} - \frac{1}{8} \int \frac{16(ae^{(3dx+3c)} - ae^{(dx+c)})}{b^2e^{(4dx+4c)} + b^2 + 2(2abe^{(2c)} - b^2e^{(2c)})e^{(2dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")`

[Out] $1/2*(e^{(2*d*x + 2*c)} + 1)*e^{-d*x - c}/(b*d) - 1/8*integrate(16*(a*e^{(3*d*x + 3*c)} - a*e^{(d*x + c)})/(b^2*e^{(4*d*x + 4*c)} + b^2 + 2*(2*a*b*e^{(2*c)} - b^2*e^{(2*c)})*e^{(2*d*x)}), x)$

Fricas [B] time = 1.91584, size = 1894, normalized size = 33.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")`

[Out] $[1/2*((a*b - b^2)*\cosh(d*x + c)^2 + 2*(a*b - b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b - b^2)*\sinh(d*x + c)^2 - \sqrt{-a*b + b^2}*(a*\cosh(d*x + c) + a*\sinh(d*x + c))*\log((b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a - 3*b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a + 3*b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a - 3*b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 + 1)*\sinh(d*x + c) + \cosh(d*x + c))*\sqrt{-a*b + b^2} + b)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b) + a*b - b^2)/((a*b^2 - b^3)*d*\cosh(d*x + c) + (a*b^2 - b^3)*d*\sinh(d*x + c)), 1/2*((a*b - b^2)*\cosh(d*x + c)^2 + 2*(a*b - b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (a*b - b^2)*\sinh(d*x + c)^2 - 2*\sqrt{a*b - b^2}*(a*\cosh(d*x + c) + a*\sinh(d*x + c))*\arctan(-1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 + (4*a - 3*b)*\cosh(d*x +$

$$c) + (3*b*\cosh(d*x + c)^2 + 4*a - 3*b)*\sinh(d*x + c))/\sqrt{a*b - b^2}) + 2$$

$$*\sqrt{a*b - b^2}*(a*\cosh(d*x + c) + a*\sinh(d*x + c))*\arctan(-1/2*\sqrt{a*b -$$

$$b^2)*(\cosh(d*x + c) + \sinh(d*x + c))/(a - b)) + a*b - b^2)/((a*b^2 - b^3)*$$

$$d*\cosh(d*x + c) + (a*b^2 - b^3)*d*\sinh(d*x + c))]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.33 \quad \int \frac{\sinh^2(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=50

$$\frac{x}{b} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a-b}}$$

[Out] x/b - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b*d)

Rubi [A] time = 0.0871054, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3171, 3181, 208}

$$\frac{x}{b} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2), x]

[Out] x/b - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b*d)

Rule 3171

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])^2/((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)^2]), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a + b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3181

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2^(-1), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{x}{b} - \frac{a \int \frac{1}{a+b\sinh^2(c+dx)} dx}{b} \\ &= \frac{x}{b} - \frac{a \operatorname{Subst}\left(\int \frac{1}{a-(a-b)x^2} dx, x, \tanh(c+dx)\right)}{bd} \\ &= \frac{x}{b} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}bd} \end{aligned}$$

Mathematica [A] time = 0.131342, size = 50, normalized size = 1.

$$-\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + c + dx$$

$$bd$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2), x]
```

```
[Out] (c + d*x - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a -
b])/(b*d)
```

Maple [B] time = 0.033, size = 312, normalized size = 6.2

$$\frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right) - \frac{a}{bd} \operatorname{Artanh}\left(a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \frac{1}{\sqrt{(2\sqrt{-b}(a-b) + a - 2b)a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2), x)
```

```
[Out] 1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a/b/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2))+1/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01965, size = 1122, normalized size = 22.44

$$\left[2 dx + \sqrt{\frac{a}{a-b}} \log \left(\frac{b^2 \cosh(dx+c)^4 + 4b^2 \cosh(dx+c) \sinh(dx+c)^3 + b^2 \sinh(dx+c)^4 + 2(2ab-b^2) \cosh(dx+c)^2 + 2(3b^2 \cosh(dx+c)^2 + 2ab-b^2) \sinh(dx+c)^2}{b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 + 2(2a-b) \cosh(dx+c)^2 + 2(3b \cosh(dx+c)^2 + 2a-b) \sinh(dx+c)^2} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)
)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) +
4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)
+ (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*
(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)
^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/(
```

```
b*d), (d*x - sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a/(a - b))/a))/(b*d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26475, size = 88, normalized size = 1.76

$$-\frac{a \arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}bd} + \frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
a*b)*b*d) + (d*x + c)/(b*d)
```

$$3.34 \quad \int \frac{\sinh(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{bd}\sqrt{a-b}}$$

[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(Sqrt[a - b]*Sqrt[b]*d)

Rubi [A] time = 0.0468252, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3186, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{bd}\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(Sqrt[a - b]*Sqrt[b]*d)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 205

Int[((a_.) + (b_.)*(x_.)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\int \frac{\sinh(c + dx)}{a + b \sinh^2(c + dx)} dx = \frac{\text{Subst}\left(\int \frac{1}{a - b + bx^2} dx, x, \cosh(c + dx)\right)}{d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b} \cosh(c + dx)}{\sqrt{a - b}}\right)}{\sqrt{a - b} \sqrt{bd}}$$

Mathematica [C] time = 0.12318, size = 91, normalized size = 2.28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) + \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{bd}\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2), x]

[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[b]*d)

Maple [A] time = 0.016, size = 51, normalized size = 1.3

$$\frac{1}{d} \arctan\left(\frac{1}{4} \left(2 (\tanh(1/2 dx + c/2))^2 a - 2a + 4b\right) \frac{1}{\sqrt{ab - b^2}}\right) \frac{1}{\sqrt{ab - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2), x)

[Out] 1/d/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx + c)}{b \sinh(dx + c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)

Fricas [B] time = 2.01602, size = 1293, normalized size = 32.32

$$\left[\frac{\sqrt{-ab + b^2} \log\left(\frac{b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 - 2(2a-3b) \cosh(dx+c)^2 + 2(3b \cosh(dx+c)^2 - 2a+3b) \sinh(dx+c)^2 + 4(b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 + 2(2a-b) \cosh(dx+c)^2 + 2(3b \cosh(dx+c)^2 - 2a+3b) \sinh(dx+c)^2 + b \sinh(dx+c)^4)}{b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 + 2(2a-b) \cosh(dx+c)^2 + 2(3b \cosh(dx+c)^2 - 2a+3b) \sinh(dx+c)^2 + b \sinh(dx+c)^4}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b))/((a*b - b^2)*d), (sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - sqrt(a*b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)))/((a*b - b^2)*d)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.35 \quad \int \frac{1}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}\sqrt{a-b}}$$

[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)

Rubi [A] time = 0.0270021, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3181, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^(-1), x]

[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)

Rule 3181

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^2]^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\int \frac{1}{a + b \sinh^2(c + dx)} dx = \frac{\text{Subst}\left(\int \frac{1}{a - (a-b)x^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}}$$

Mathematica [A] time = 0.0709516, size = 40, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-1),x]

[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)

Maple [B] time = 0.042, size = 267, normalized size = 6.7

$$\frac{1}{d} \text{Arctanh}\left(a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \frac{1}{\sqrt{(2\sqrt{-b(a-b)} + a - 2b)a}}\right) \frac{1}{\sqrt{(2\sqrt{-b(a-b)} + a - 2b)a}} - \frac{b}{d} \text{Arctanh}\left(a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(d*x+c)^2),x)

[Out] 1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)-1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)+b-1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)-1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.92759, size = 1068, normalized size = 26.7

$$\log\left(\frac{b^2 \cosh(dx+c)^4 + 4b^2 \cosh(dx+c) \sinh(dx+c)^3 + b^2 \sinh(dx+c)^4 + 2(2ab-b^2) \cosh(dx+c)^2 + 2(3b^2 \cosh(dx+c)^2 + 2ab-b^2) \sinh(dx+c)^2 + 8a^2 - 8ab + b^2 + 4a^2}{b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 + 2(2a-b) \cosh(dx+c)^2 + 2(3b \cosh(dx+c)^2 + 2ab-b^2) \sinh(dx+c)^2 + 8a^2 - 8ab + b^2 + 4a^2}\right)$$

$2\sqrt{a^2 - abd}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] [1/2*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b))/(sqrt(a^2 - a*b)*d), -sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b))/(a^2 - a*b)*d)]

Sympy [A] time = 118.484, size = 3859, normalized size = 96.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned} & 3*d + 16*a*b**2*d*\sqrt{-a*b + b**2}) + 3*a**2*\sqrt{-a*b + b**2}*\sqrt{1 - 2* \\ & b/a + 2*\sqrt{-a*b + b**2}/a}*\log(\sqrt{1 - 2*b/a + 2*\sqrt{-a*b + b**2}/a} + \\ & \tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*\sqrt{-a*b + b**2}) + 3 \\ & 2*a**2*b**2*d - 24*a**2*b*d*\sqrt{-a*b + b**2}) - 16*a*b**3*d + 16*a*b**2*d*s \\ & \sqrt{-a*b + b**2}) - 28*a*b**2*\sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b**2}/a}*\log(- \\ & \sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b**2}/a} + \tanh(c/2 + d*x/2))/(2*a**4*d - 18 \\ & *a**3*b*d + 8*a**3*d*\sqrt{-a*b + b**2}) + 32*a**2*b**2*d - 24*a**2*b*d*\sqrt{ \\ & -a*b + b**2}) - 16*a*b**3*d + 16*a*b**2*d*\sqrt{-a*b + b**2}) + 28*a*b**2*sqr \\ & t(1 - 2*b/a - 2*\sqrt{-a*b + b**2}/a)*\log(\sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b** \\ & 2)/a} + \tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*\sqrt{-a*b + b \\ & **2}) + 32*a**2*b**2*d - 24*a**2*b*d*\sqrt{-a*b + b**2}) - 16*a*b**3*d + 16*a* \\ & b**2*d*\sqrt{-a*b + b**2}) - 4*a*b**2*\sqrt{1 - 2*b/a + 2*\sqrt{-a*b + b**2}/a} \\ &)*\log(-\sqrt{1 - 2*b/a + 2*\sqrt{-a*b + b**2}/a} + \tanh(c/2 + d*x/2))/(2*a**4 \\ & *d - 18*a**3*b*d + 8*a**3*d*\sqrt{-a*b + b**2}) + 32*a**2*b**2*d - 24*a**2*b* \\ & d*\sqrt{-a*b + b**2}) - 16*a*b**3*d + 16*a*b**2*d*\sqrt{-a*b + b**2}) + 4*a*b* \\ & *2*\sqrt{1 - 2*b/a + 2*\sqrt{-a*b + b**2}/a}*\log(\sqrt{1 - 2*b/a + 2*\sqrt{-a*b \\ & + b**2}/a} + \tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*\sqrt{-a \\ & *b + b**2}) + 32*a**2*b**2*d - 24*a**2*b*d*\sqrt{-a*b + b**2}) - 16*a*b**3*d + \\ & 16*a*b**2*d*\sqrt{-a*b + b**2}) + 20*a*b*\sqrt{-a*b + b**2}*\sqrt{1 - 2*b/a - \\ & 2*\sqrt{-a*b + b**2}/a}*\log(-\sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b**2}/a} + \tanh \\ & (c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*\sqrt{-a*b + b**2}) + 32*a* \\ & *2*b**2*d - 24*a**2*b*d*\sqrt{-a*b + b**2}) - 16*a*b**3*d + 16*a*b**2*d*\sqrt{ \\ & -a*b + b**2}) - 20*a*b*\sqrt{-a*b + b**2}*\sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b** \\ & 2)/a}*\log(\sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b**2}/a} + \tanh(c/2 + d*x/2))/(2*a \\ & **4*d - 18*a**3*b*d + 8*a**3*d*\sqrt{-a*b + b**2}) + 32*a**2*b**2*d - 24*a**2 \\ & *b*d*\sqrt{-a*b + b**2}) - 16*a*b**3*d + 16*a*b**2*d*\sqrt{-a*b + b**2}) + 4*a \\ & *b*\sqrt{-a*b + b**2}*\sqrt{1 - 2*b/a + 2*\sqrt{-a*b + b**2}/a}*\log(-\sqrt{1 - \\ & 2*b/a + 2*\sqrt{-a*b + b**2}/a} + \tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d \\ & + 8*a**3*d*\sqrt{-a*b + b**2}) + 32*a**2*b**2*d - 24*a**2*b*d*\sqrt{-a*b + b* \\ & **2}) - 16*a*b**3*d + 16*a*b**2*d*\sqrt{-a*b + b**2}) - 4*a*b*\sqrt{-a*b + b**2} \\ &)*\sqrt{1 - 2*b/a + 2*\sqrt{-a*b + b**2}/a}*\log(\sqrt{1 - 2*b/a + 2*\sqrt{-a*b \\ & + b**2}/a} + \tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*\sqrt{-a* \\ & b + b**2}) + 32*a**2*b**2*d - 24*a**2*b*d*\sqrt{-a*b + b**2}) - 16*a*b**3*d + \\ & 16*a*b**2*d*\sqrt{-a*b + b**2}) + 16*b**3*\sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b** \\ & 2)/a}*\log(-\sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b**2}/a} + \tanh(c/2 + d*x/2))/(2* \\ & a**4*d - 18*a**3*b*d + 8*a**3*d*\sqrt{-a*b + b**2}) + 32*a**2*b**2*d - 24*a** \\ & 2*b*d*\sqrt{-a*b + b**2}) - 16*a*b**3*d + 16*a*b**2*d*\sqrt{-a*b + b**2}) - 16 \\ & *b**3*\sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b**2}/a}*\log(\sqrt{1 - 2*b/a - 2*\sqrt{- \\ & a*b + b**2}/a} + \tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*\sqrt{ \\ & -a*b + b**2}) + 32*a**2*b**2*d - 24*a**2*b*d*\sqrt{-a*b + b**2}) - 16*a*b**3* \\ & d + 16*a*b**2*d*\sqrt{-a*b + b**2}) - 16*b**2*\sqrt{-a*b + b**2}*\sqrt{1 - 2*b \\ & /a - 2*\sqrt{-a*b + b**2}/a}*\log(-\sqrt{1 - 2*b/a - 2*\sqrt{-a*b + b**2}/a} + \\ & \tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*\sqrt{-a*b + b**2}) + 3 \\ & 2*a**2*b**2*d - 24*a**2*b*d*\sqrt{-a*b + b**2}) - 16*a*b**3*d + 16*a*b**2*d*s \\ & \sqrt{-a*b + b**2}) + 16*b**2*\sqrt{-a*b + b**2}*\sqrt{1 - 2*b/a - 2*\sqrt{-a*b} \end{aligned}$$

```
+ b**2)/a)*log(sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))
/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24
*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)),
True))
```

Giac [A] time = 1.25799, size = 63, normalized size = 1.58

$$\frac{\arctan\left(\frac{be^{(2dx+2c)+2a-b}}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+abd}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))
/(sqrt(-a^2 + a*b)*d)
```


$$3.36 \quad \int \frac{\operatorname{csch}(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=60

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*d)) - ArcTanh[Cosh[c + d*x]]/(a*d)

Rubi [A] time = 0.0742426, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3186, 391, 206, 205}

$$-\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2), x]

[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*d)) - ArcTanh[Cosh[c + d*x]]/(a*d)

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c+dx)\right)}{ad} - \frac{b \operatorname{Subst}\left(\int \frac{1}{a-b+bx^2} dx, x, \cosh(c+dx)\right)}{ad} \\ &= \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-bd}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad} \end{aligned}$$

Mathematica [C] time = 0.209569, size = 124, normalized size = 2.07

$$\frac{\sqrt{a-b} \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \sqrt{b} \tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) - \sqrt{b} \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2), x]
```

```
[Out] (-(Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]) - S
qrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + Sqrt[a
- b]*Log[Tanh[(c + d*x)/2]])/(a*Sqrt[a - b]*d)
```

Maple [A] time = 0.049, size = 74, normalized size = 1.2

$$\frac{1}{da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \frac{b}{da} \arctan\left(\frac{1}{4} \left(2 (\tanh(1/2 dx + c/2))^2 a - 2a + 4b\right) \frac{1}{\sqrt{ab-b^2}}\right) \frac{1}{\sqrt{ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x)`

[Out] $1/d/a*\ln(\tanh(1/2*d*x+1/2*c))-1/d/a*b/(a*b-b^2)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{-c}\right)}{ad} + \frac{\log\left(\left(e^{(dx+c)}-1\right)e^{-c}\right)}{ad} - 2 \int \frac{be^{(3dx+3c)} - be^{(dx+c)}}{abe^{(4dx+4c)} + ab + 2\left(2a^2e^{(2c)} - abe^{(2c)}\right)e^{(2dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")`

[Out] $-\log\left(\left(e^{(dx+c)}+1\right)e^{-c}\right)/(a*d) + \log\left(\left(e^{(dx+c)}-1\right)e^{-c}\right)/(a*d) - 2*\int\left(\left(b*e^{(3*d*x+3*c)} - b*e^{(d*x+c)}\right)/\left(a*b*e^{(4*d*x+4*c)} + a*b + 2*(2*a^2*e^{(2*c)} - a*b*e^{(2*c)})*e^{(2*d*x)}\right)\right), x$

Fricas [B] time = 2.01415, size = 1544, normalized size = 25.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")`

[Out] $[1/2*(\sqrt{-b/(a-b)}*\log((b*\cosh(d*x+c)^4 + 4*b*\cosh(d*x+c)*\sinh(d*x+c)^3 + b*\sinh(d*x+c)^4 - 2*(2*a-3*b)*\cosh(d*x+c)^2 + 2*(3*b*\cosh(d*x+c)^2 - 2*a+3*b)*\sinh(d*x+c)^2 + 4*(b*\cosh(d*x+c)^3 - (2*a-3*b)*\cosh(d*x+c))*\sinh(d*x+c) - 4*((a-b)*\cosh(d*x+c)^3 + 3*(a-b)*\cosh(d*x+c)*\sinh(d*x+c)^2 + (a-b)*\sinh(d*x+c)^3 + (a-b)*\cosh(d*x+c) + (3*(a-b)*\cosh(d*x+c)^2 + a-b)*\sinh(d*x+c))*\sqrt{-b/(a-b)} + b)/(b*\cosh(d*x+c)^4 + 4*b*\cosh(d*x+c)*\sinh(d*x+c)^3 + b*\sinh(d*x+c)^4 + 2*(2*a-b)*\cosh(d*x+c)^2 + 2*(3*b*\cosh(d*x+c)^2 + 2*a-b)*\sinh(d*x+c)^2 + 4*(b*\cosh(d*x+c)^3 + (2*a-b)*\cosh(d*x+c))*\sinh(d*x+c) + b) - 2*\log(\cosh(d*x+c) + \sinh(d*x+c) + 1) + 2*\log(\cosh(d*x+c) + \sinh(d*x+c) - 1))/(a*d), -(\sqrt{b/(a-b)}*\arctan(1/2*\sqrt{b/(a-b)}*(\cosh(d*x+c) + \sinh(d*x+c))))/(a*d)$

```
+ c) + sinh(d*x + c))) - sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3
*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x
+ c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b)
+ log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh(d*x +
c) - 1))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c + dx)}{a + b \sinh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sinh(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.37 \quad \int \frac{\operatorname{csch}^2(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=57

$$-\frac{b \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2} d \sqrt{a-b}} - \frac{\operatorname{coth}(c+dx)}{ad}$$

[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a - b]*d))
- Coth[c + d*x]/(a*d)

Rubi [A] time = 0.0808976, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3187, 453, 208}

$$-\frac{b \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2} d \sqrt{a-b}} - \frac{\operatorname{coth}(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2), x]

[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a - b]*d))
- Coth[c + d*x]/(a*d)

Rule 3187

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (

LtQ[n, 0] && GtQ[m + n, -1]) && !ILtQ[p, -1]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{x^2(a-(a-b)x^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{coth}(c+dx)}{ad} - \frac{b \operatorname{Subst}\left(\int \frac{1}{a+(-a+b)x^2} dx, x, \tanh(c+dx)\right)}{ad} \\ &= -\frac{b \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}d} - \frac{\operatorname{coth}(c+dx)}{ad} \end{aligned}$$

Mathematica [A] time = 0.292172, size = 57, normalized size = 1.

$$-\frac{b \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - \frac{\sqrt{a} \operatorname{coth}(c+dx)}{a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2), x]

[Out] (-((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b]) - Sqrt[a]*Coth[c + d*x])/(a^(3/2)*d)

Maple [B] time = 0.061, size = 319, normalized size = 5.6

$$-\frac{1}{2da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{b}{da} \operatorname{Artanh}\left(a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \frac{1}{\sqrt{(2\sqrt{-b(a-b)} + a - 2b)a}}\right) \frac{1}{\sqrt{(2\sqrt{-b(a-b)} + a - 2b)a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2+1/d*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21384, size = 1661, normalized size = 29.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - 4*a^2 + 4*a*b)/((a^3 - a^2*b)*d*cosh(d*x + c)^2
```

```

+ 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^2*b)*d*sinh(d*x
+ c)^2 - (a^3 - a^2*b)*d), ((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(
-a^2 + a*b)/(a^2 - a*b)) - 2*a^2 + 2*a*b)/((a^3 - a^2*b)*d*cosh(d*x + c)^2
+ 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^2*b)*d*sinh(d*x
+ c)^2 - (a^3 - a^2*b)*d)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30799, size = 100, normalized size = 1.75

$$-\frac{b \arctan\left(\frac{be^{2dx+2c}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}ad} - \frac{2}{ad(e^{2dx+2c}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
a*b)*a*d) - 2/(a*d*(e^(2*d*x + 2*c) - 1))
```


$$3.38 \quad \int \frac{\operatorname{csch}^3(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=88

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2 d \sqrt{a-b}} + \frac{(a+2b) \tanh^{-1}(\cosh(c+dx))}{2a^2 d} - \frac{\coth(c+dx) \operatorname{csch}(c+dx)}{2ad}$$

[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^2*Sqrt[a - b]*d) + ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d)

Rubi [A] time = 0.123493, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3186, 414, 522, 206, 205}

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2 d \sqrt{a-b}} + \frac{(a+2b) \tanh^{-1}(\cosh(c+dx))}{2a^2 d} - \frac{\coth(c+dx) \operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2), x]

[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^2*Sqrt[a - b]*d) + ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]]

```
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^3(c+dx)}{a+b \sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+bx^2)} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{\operatorname{Subst}\left(\int \frac{a+b+bx^2}{(1-x^2)(a-b+bx^2)} dx, x, \cosh(c+dx)\right)}{2ad} \\ &= -\frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{a-b+bx^2} dx, x, \cosh(c+dx)\right)}{a^2d} + \frac{(a+2b) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c+dx)\right)}{2ad} \\ &= \frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2\sqrt{a-bd}} + \frac{(a+2b) \tanh^{-1}(\cosh(c+dx))}{2a^2d} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} \end{aligned}$$

Mathematica [C] time = 0.693815, size = 201, normalized size = 2.28

$$\frac{\operatorname{csch}^4(c+dx)(2a+b \cosh(2(c+dx))-b)\left(2a\sqrt{a-b} \cosh(c+dx)-2 \sinh^2(c+dx)\right)\left(2b^{3/2} \tan^{-1}\left(\frac{\sqrt{b-i}\sqrt{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{8a^2d\sqrt{a-b}\left(\operatorname{acsch}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]

[Out] -((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^4*(2*a*Sqrt[a - b]*Cosh[c + d*x] - 2*(2*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + 2*b^(3/2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]) - Sqrt[a - b]*(a + 2*b)*Log[Tanh[(c + d*x)/2]])*Sinh[c + d*x]^2)/(8*a^2*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))

Maple [A] time = 0.062, size = 133, normalized size = 1.5

$$\frac{1}{8da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{1}{8da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-2} - \frac{1}{2da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - \frac{b}{da^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b^2}{da^2} \arctan\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)

[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/8/d/a/tanh(1/2*d*x+1/2*c)^-2-1/2/d/a*ln(tanh(1/2*d*x+1/2*c))-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))+1/d/a^2*b^2/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{(3dx+3c)} + e^{(dx+c)}}{ade^{(4dx+4c)} - 2ade^{(2dx+2c)} + ad} + \frac{(a+2b)\log\left(\left(e^{(dx+c)} + 1\right)e^{(-c)}\right)}{2a^2d} - \frac{(a+2b)\log\left(\left(e^{(dx+c)} - 1\right)e^{(-c)}\right)}{2a^2d} + 8 \int \frac{1}{4(a^2be^{(4dx+4c)} - 2ade^{(2dx+2c)} + ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a + 2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*(b^2*e^(3*d*x + 3*c) - b^2*e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)

Fricas [B] time = 2.81937, size = 4867, normalized size = 55.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d*x + c)^3 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b) + 2*a*cosh(d*x + c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(a^2*d*cosh(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(d*x + c)^4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d*x + c)^3 - 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh(d*x + c))) + 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 +$$

```

4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b) + 2*a*cosh(d*x + c) - ((a + 2*
b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2*b)*
sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x + c
)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*b)*c
osh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c) +
1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3
+ (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)
*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3
- (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sin
h(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(a^2*d*cosh(d*
x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(d*x + c)^4 -
2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(
d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*sinh(d*x + c))
]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.39 \quad \int \frac{\operatorname{csch}^4(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=78

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2} d \sqrt{a-b}} + \frac{(a+b) \operatorname{coth}(c+dx)}{a^2 d} - \frac{\operatorname{coth}^3(c+dx)}{3ad}$$

[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]*d) + ((a + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)

Rubi [A] time = 0.120393, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3187, 461, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2} d \sqrt{a-b}} + \frac{(a+b) \operatorname{coth}(c+dx)}{a^2 d} - \frac{\operatorname{coth}^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2), x]

[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]*d) + ((a + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])
```

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^4(a-(a-b)x^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^4} + \frac{-a-b}{a^2x^2} + \frac{b^2}{a^2(a-(a-b)x^2)}\right) dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{(a+b)\operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}^3(c+dx)}{3ad} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{a-(a-b)x^2} dx, x, \tanh(c+dx)\right)}{a^2d} \\ &= \frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}} + \frac{(a+b)\operatorname{coth}(c+dx)}{a^2d} - \frac{\operatorname{coth}^3(c+dx)}{3ad} \end{aligned}$$

Mathematica [A] time = 0.693455, size = 126, normalized size = 1.62

$$\frac{\operatorname{csch}^2(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\sqrt{a}\sqrt{a-b}\operatorname{coth}(c+dx)\left(\operatorname{acsch}^2(c+dx)-2a-3b\right)-3b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)\right)}{6a^{5/2}d\sqrt{a-b}\left(\operatorname{acsch}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2), x]

[Out] -((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(-3*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(-2*a - 3*b + a*Csch[c + d*x]^2)))/(6*a^(5/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))

Maple [B] time = 0.071, size = 401, normalized size = 5.1

$$-\frac{1}{24da}\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 + \frac{3}{8da}\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + \frac{b}{2da^2}\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{24da}\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-3} + \frac{3}{8da}\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)+1/2/d/a^2*tanh(
1/2*d*x+1/2*c)*b-1/24/d/a/tanh(1/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)
+1/2/d*b/a^2/tanh(1/2*d*x+1/2*c)+1/d*b^2/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-
1/d*b^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^2/a^2/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))-1/d*b^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3784, size = 4710, normalized size = 60.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 + 8*a^3 + 4*a^2*b - 1
2*a*b^2 - 24*(a^3 - a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - a*b^2 - 3*(a^2*b - a
*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cos
h(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 +
3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4
```


$$\begin{aligned}
& * (5*b^2*\cosh(d*x + c)^3 - 3*b^2*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(5*b^2*\cosh(d*x + c)^4 - 6*b^2*\cosh(d*x + c)^2 + b^2)*\sinh(d*x + c)^2 - b^2 + 6*(b^2*\cosh(d*x + c)^5 - 2*b^2*\cosh(d*x + c)^3 + b^2*\cosh(d*x + c))*\sinh(d*x + c) \\
&)*\sqrt{a^2 - a*b}*\log((b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*\sinh(d*x + c)^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*b^2*\cosh(d*x + c)^2 + 2*a*b - b^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*\cosh(d*x + c)^3 + (2*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{a^2 - a*b})/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) + 48*((a^2*b - a*b^2)*\cosh(d*x + c)^3 - (a^3 - a*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4 - a^3*b)*d*\cosh(d*x + c)^6 + 6*(a^4 - a^3*b)*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + (a^4 - a^3*b)*d*\sinh(d*x + c)^6 - 3*(a^4 - a^3*b)*d*\cosh(d*x + c)^4 + 3*(5*(a^4 - a^3*b)*d*\cosh(d*x + c)^2 - (a^4 - a^3*b)*d)*\sinh(d*x + c)^4 + 3*(a^4 - a^3*b)*d*\cosh(d*x + c)^2 + 4*(5*(a^4 - a^3*b)*d*\cosh(d*x + c)^3 - 3*(a^4 - a^3*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(5*(a^4 - a^3*b)*d*\cosh(d*x + c)^4 - 6*(a^4 - a^3*b)*d*\cosh(d*x + c)^2 + (a^4 - a^3*b)*d)*\sinh(d*x + c)^2 - (a^4 - a^3*b)*d + 6*((a^4 - a^3*b)*d*\cosh(d*x + c)^5 - 2*(a^4 - a^3*b)*d*\cosh(d*x + c)^3 + (a^4 - a^3*b)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/3*(6*(a^2*b - a*b^2)*\cosh(d*x + c)^4 + 24*(a^2*b - a*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + 6*(a^2*b - a*b^2)*\sinh(d*x + c)^4 + 4*a^3 + 2*a^2*b - 6*a*b^2 - 12*(a^3 - a*b^2)*\cosh(d*x + c)^2 - 12*(a^3 - a*b^2 - 3*(a^2*b - a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 3*(b^2*\cosh(d*x + c)^6 + 6*b^2*\cosh(d*x + c)*\sinh(d*x + c)^5 + b^2*\sinh(d*x + c)^6 - 3*b^2*\cosh(d*x + c)^4 + 3*(5*b^2*\cosh(d*x + c)^2 - b^2)*\sinh(d*x + c)^4 + 3*b^2*\cosh(d*x + c)^2 + 4*(5*b^2*\cosh(d*x + c)^3 - 3*b^2*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(5*b^2*\cosh(d*x + c)^4 - 6*b^2*\cosh(d*x + c)^2 + b^2)*\sinh(d*x + c)^2 - b^2 + 6*(b^2*\cosh(d*x + c)^5 - 2*b^2*\cosh(d*x + c)^3 + b^2*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a^2 + a*b}*\arctan(-1/2*(b*\cosh(d*x + c))^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-a^2 + a*b}/(a^2 - a*b)) + 24*((a^2*b - a*b^2)*\cosh(d*x + c)^3 - (a^3 - a*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4 - a^3*b)*d*\cosh(d*x + c)^6 + 6*(a^4 - a^3*b)*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + (a^4 - a^3*b)*d*\sinh(d*x + c)^6 - 3*(a^4 - a^3*b)*d*\cosh(d*x + c)^4 + 3*(5*(a^4 - a^3*b)*d*\cosh(d*x + c)^2 - (a^4 - a^3*b)*d)*\sinh(d*x + c)^4 + 3*(a^4 - a^3*b)*d*\cosh(d*x + c)^2 + 4*(5*(a^4 - a^3*b)*d*\cosh(d*x + c)^3 - 3*(a^4 - a^3*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(5*(a^4 - a^3*b)*d*\cosh(d*x + c)^4 - 6*(a^4 - a^3*b)*d*\cosh(d*x + c)^2 + (a^4 - a^3*b)*d)*\sinh(d*x + c)^2 - (a^4 - a^3*b)*d + 6*((a^4 - a^3*b)*d*\cosh(d*x + c)^5 - 2*(a^4 - a^3*b)*d*\cosh(d*x + c)^3 + (a^4 - a^3*b)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [A] time = 1.42493, size = 159, normalized size = 2.04

$$\frac{b^2 \arctan\left(\frac{be^{2dx+2c}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a^2d} + \frac{2(3be^{4dx+4c} - 6ae^{2dx+2c} - 6be^{2dx+2c} + 2a + 3b)}{3a^2d(e^{2dx+2c} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] b^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a^2*d) + 2/3*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 2*a + 3*b)/(a^2*d*(e^(2*d*x + 2*c) - 1)^3)

$$3.40 \quad \int \frac{\operatorname{csch}^5(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=130

$$\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3 d \sqrt{a-b}} - \frac{(3a^2 + 4ab + 8b^2) \tanh^{-1}(\cosh(c+dx))}{8a^3 d} + \frac{(3a + 4b) \coth(c+dx) \operatorname{csch}(c+dx)}{8a^2 d} - \frac{\coth(c+dx)}{4a d}$$

[Out] $-\left(\frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Cosh}[c + d*x]}{\sqrt{a-b}}\right]}{a^3 \sqrt{a-b}}\right) - \left(\frac{(3*a^2 + 4*a*b + 8*b^2) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{8*a^3*d} + \frac{(3*a + 4*b) \operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x]}{8*a^2*d} - \frac{\operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x]^3}{4*a*d}\right)$

Rubi [A] time = 0.203545, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3186, 414, 527, 522, 206, 205}

$$\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3 d \sqrt{a-b}} - \frac{(3a^2 + 4ab + 8b^2) \tanh^{-1}(\cosh(c+dx))}{8a^3 d} + \frac{(3a + 4b) \coth(c+dx) \operatorname{csch}(c+dx)}{8a^2 d} - \frac{\coth(c+dx)}{4a d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^5/(a + b*\operatorname{Sinh}[c + d*x]^2), x]$

[Out] $-\left(\frac{b^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \operatorname{Cosh}[c + d*x]}{\sqrt{a-b}}\right]}{a^3 \sqrt{a-b}}\right) - \left(\frac{(3*a^2 + 4*a*b + 8*b^2) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{8*a^3*d} + \frac{(3*a + 4*b) \operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x]}{8*a^2*d} - \frac{\operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x]^3}{4*a*d}\right)$

Rule 3186

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - b*ff^2*x^2)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x]] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 414

$\operatorname{Int}[(a_. + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*x*(a + b*x^n)^{(p+1})*(c + d*x^n)^{(q+1})/(a*n*(p+1)*(b*c -$

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f
_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(
n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^3(a-b+bx^2)} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{coth}(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int \frac{3a+b+3bx^2}{(1-x^2)^2(a-b+bx^2)} dx, x, \cosh(c+dx)\right)}{4ad} \\
&= \frac{(3a+4b)\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{8a^2d} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{\operatorname{Subst}\left(\int \frac{3a^2+ab+4b^2+bx^3}{(1-x^2)(a-b+bx^2)} dx, x, \cosh(c+dx)\right)}{4ad} \\
&= \frac{(3a+4b)\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{8a^2d} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{a-b+bx^2} dx, x, \cosh(c+dx)\right)}{a^3d} \\
&= -\frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3\sqrt{a-bd}} - \frac{(3a^2+4ab+8b^2)\tanh^{-1}(\cosh(c+dx))}{8a^3d} + \frac{(3a+4b)\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{8a^2d}
\end{aligned}$$

Mathematica [C] time = 6.28295, size = 574, normalized size = 4.42

$$\frac{(3a^2+4ab+8b^2)\operatorname{csch}^2(c+dx)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)(2a+b\cosh(2(c+dx))-b)}{16a^3d(\operatorname{acsch}^2(c+dx)+b)} - \frac{b^{5/2}\operatorname{csch}^2(c+dx)(2a+b\cosh(2(c+dx)))}{16a^3d(\operatorname{acsch}^2(c+dx)+b)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^5/(a + b*Sinh[c + d*x]^2), x]

[Out] $-(b^{5/2}\operatorname{ArcTan}[(\operatorname{Sech}[(c+d*x)/2]*(\operatorname{Sqrt}[b]*\operatorname{Cosh}[(c+d*x)/2] - I*\operatorname{Sqrt}[a]*\operatorname{Sinh}[(c+d*x)/2]))/\operatorname{Sqrt}[a-b]*(2*a-b+b*\operatorname{Cosh}[2*(c+d*x)])*\operatorname{Csch}[c+d*x]^2)/(2*a^3*\operatorname{Sqrt}[a-b]*d*(b+a*\operatorname{Csch}[c+d*x]^2)) - (b^{5/2}\operatorname{ArcTan}[(\operatorname{Sech}[(c+d*x)/2]*(\operatorname{Sqrt}[b]*\operatorname{Cosh}[(c+d*x)/2] + I*\operatorname{Sqrt}[a]*\operatorname{Sinh}[(c+d*x)/2]))/\operatorname{Sqrt}[a-b]*(2*a-b+b*\operatorname{Cosh}[2*(c+d*x)])*\operatorname{Csch}[c+d*x]^2)/(2*a^3*\operatorname{Sqrt}[a-b]*d*(b+a*\operatorname{Csch}[c+d*x]^2)) + ((3*a+4*b)*(2*a-b+b*\operatorname{Cosh}[2*(c+d*x)])*\operatorname{Csch}[(c+d*x)/2]^2*\operatorname{Csch}[c+d*x]^2)/(64*a^2*d*(b+a*\operatorname{Csch}[c+d*x]^2)) - ((2*a-b+b*\operatorname{Cosh}[2*(c+d*x)])*\operatorname{Csch}[(c+d*x)/2]^4*\operatorname{Csch}[c+d*x]^2)/(128*a*d*(b+a*\operatorname{Csch}[c+d*x]^2)) + ((3*a^2+4*a*b+8*b^2)*(2*a-b+b*\operatorname{Cosh}[2*(c+d*x)])*\operatorname{Csch}[c+d*x]^2*\operatorname{Log}[\operatorname{Tanh}[(c+d*x)/2]])/(16*a^3*d*(b+a*\operatorname{Csch}[c+d*x]^2)) + ((3*a+4*b)*(2*a-b+b*\operatorname{Cosh}[2*(c+d*x)])*\operatorname{Csch}[c+d*x]^2*\operatorname{Sech}[(c+d*x)/2]^2)/(64*a^2*d*(b+a*\operatorname{Csch}[c+d*x]^2)) + ((2*a-b+b*\operatorname{Cosh}[2*(c+d*x)])*\operatorname{Csch}[c+d*x]^2*\operatorname{Sech}[(c+d*x)/2]^4)/(128*a*d*(b+a*\operatorname{Csch}[c+d*x]^2))$

ch[c + d*x]^2))

Maple [A] time = 0.066, size = 232, normalized size = 1.8

$$\frac{1}{64da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^4 - \frac{1}{8da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{b}{8da^2} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{1}{64da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-4} + \frac{1}{8da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)

[Out] 1/64/d/a*tanh(1/2*d*x+1/2*c)^4-1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/8/d/a^2*tanh(1/2*d*x+1/2*c)^2*b-1/64/d/a/tanh(1/2*d*x+1/2*c)^4+1/8/d/a/tanh(1/2*d*x+1/2*c)^2+1/8/d/a^2/tanh(1/2*d*x+1/2*c)^2*b+3/8/d/a*ln(tanh(1/2*d*x+1/2*c))+1/2/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))+1/d/a^3*ln(tanh(1/2*d*x+1/2*c))*b^2-1/d/a^3*b^3/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(3ae^{7c} + 4be^{7c})e^{7dx} - (11ae^{5c} + 4be^{5c})e^{5dx} - (11ae^{3c} + 4be^{3c})e^{3dx} + (3ae^c + 4be^c)e^{dx}}{4(a^2de^{8dx+8c} - 4a^2de^{6dx+6c} + 6a^2de^{4dx+4c} - 4a^2de^{2dx+2c} + a^2d)} - \frac{(3a^2 + 4ab + 8b^2)}{4(a^2 + 4ab + 8b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/4*((3*a*e^(7*c) + 4*b*e^(7*c))*e^(7*d*x) - (11*a*e^(5*c) + 4*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) + 4*b*e^(3*c))*e^(3*d*x) + (3*a*e^c + 4*b*e^c)*e^(d*x))/(a^2*d*e^(8*d*x + 8*c) - 4*a^2*d*e^(6*d*x + 6*c) + 6*a^2*d*e^(4*d*x + 4*c) - 4*a^2*d*e^(2*d*x + 2*c) + a^2*d) - 1/8*(3*a^2 + 4*a*b + 8*b^2)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d) + 1/8*(3*a^2 + 4*a*b + 8*b^2)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 32*integrate(1/16*(b^3*e^(3*d*x + 3*c) - b^3*e^(d*x + c))/(a^3*b*e^(4*d*x + 4*c) + a^3*b + 2*(2*a^4*e^(2*c) - a^3*b*e^(2*c))*e^(2*d*x)), x)

Fricas [B] time = 3.03656, size = 14357, normalized size = 110.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/8*(2*(3*a^2 + 4*a*b)*\cosh(d*x + c)^7 + 14*(3*a^2 + 4*a*b)*\cosh(d*x + c)* \\ & \sinh(d*x + c)^6 + 2*(3*a^2 + 4*a*b)*\sinh(d*x + c)^7 - 2*(11*a^2 + 4*a*b)*\cosh(d*x + c)^5 + 2*(21*(3*a^2 + 4*a*b)*\cosh(d*x + c)^2 - 11*a^2 - 4*a*b)*\sinh(d*x + c)^5 + 10*(7*(3*a^2 + 4*a*b)*\cosh(d*x + c)^3 - (11*a^2 + 4*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 2*(11*a^2 + 4*a*b)*\cosh(d*x + c)^3 + 2*(35*(3*a^2 + 4*a*b)*\cosh(d*x + c)^4 - 10*(11*a^2 + 4*a*b)*\cosh(d*x + c)^2 - 11*a^2 - 4*a*b)*\sinh(d*x + c)^3 + 2*(21*(3*a^2 + 4*a*b)*\cosh(d*x + c)^5 - 10*(11*a^2 + 4*a*b)*\cosh(d*x + c)^3 - 3*(11*a^2 + 4*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 4*(b^2*\cosh(d*x + c)^8 + 8*b^2*\cosh(d*x + c)*\sinh(d*x + c)^7 + b^2*\sinh(d*x + c)^8 - 4*b^2*\cosh(d*x + c)^6 + 4*(7*b^2*\cosh(d*x + c)^2 - b^2)*\sinh(d*x + c)^6 + 6*b^2*\cosh(d*x + c)^4 + 8*(7*b^2*\cosh(d*x + c)^3 - 3*b^2*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*b^2*\cosh(d*x + c)^4 - 30*b^2*\cosh(d*x + c)^2 + 3*b^2)*\sinh(d*x + c)^4 - 4*b^2*\cosh(d*x + c)^2 + 8*(7*b^2*\cosh(d*x + c)^5 - 10*b^2*\cosh(d*x + c)^3 + 3*b^2*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*b^2*\cosh(d*x + c)^6 - 15*b^2*\cosh(d*x + c)^4 + 9*b^2*\cosh(d*x + c)^2 - b^2)*\sinh(d*x + c)^2 + b^2 + 8*(b^2*\cosh(d*x + c)^7 - 3*b^2*\cosh(d*x + c)^5 + 3*b^2*\cosh(d*x + c)^3 - b^2*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-b/(a - b)}*\log((b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a - 3*b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a + 3*b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a - 3*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a - b)*\cosh(d*x + c)^3 + 3*(a - b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a - b)*\sinh(d*x + c)^3 + (a - b)*\cosh(d*x + c) + (3*(a - b)*\cosh(d*x + c)^2 + a - b)*\sinh(d*x + c))*\sqrt{-b/(a - b)} + b)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) + 2*(3*a^2 + 4*a*b)*\cosh(d*x + c) - ((3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^8 + 8*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*\sinh(d*x + c)^8 - 4*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^4 + 2*(35*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^2 + 9*a^2 + 12*a*b + 24*b^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^5 - 10*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^3 + 3*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b + \end{aligned}$$

$$\begin{aligned}
& 8*b^2)*\cosh(d*x + c)^2 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^6 - 15 \\
& *(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d \\
& *x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*\sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2 \\
& + 8*((3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*\co \\
& sh(d*x + c)^5 + 3*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^3 - (3*a^2 + 4*a*b \\
& + 8*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + \\
& 1) + ((3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^8 + 8*(3*a^2 + 4*a*b + 8*b^2)*\c \\
& osh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*\sinh(d*x + c)^8 - 4* \\
& (3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*\cosh \\
& (d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2 + 4*a*b \\
& + 8*b^2)*\cosh(d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c))*\sinh(d* \\
& x + c)^5 + 6*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^4 + 2*(35*(3*a^2 + 4*a*b \\
& + 8*b^2)*\cosh(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^2 + 9* \\
& a^2 + 12*a*b + 24*b^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*\cosh(\\
& d*x + c)^5 - 10*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^3 + 3*(3*a^2 + 4*a*b \\
& + 8*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d* \\
& x + c)^2 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^6 - 15*(3*a^2 + 4*a*b \\
& + 8*b^2)*\cosh(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^2 - 3*a \\
& ^2 - 4*a*b - 8*b^2)*\sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2 + 8*((3*a^2 + 4 \\
& *a*b + 8*b^2)*\cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^5 + \\
& 3*(3*a^2 + 4*a*b + 8*b^2)*\cosh(d*x + c)^3 - (3*a^2 + 4*a*b + 8*b^2)*\cosh(d \\
& *x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(7*(3*a^ \\
& 2 + 4*a*b)*\cosh(d*x + c)^6 - 5*(11*a^2 + 4*a*b)*\cosh(d*x + c)^4 - 3*(11*a^2 \\
& + 4*a*b)*\cosh(d*x + c)^2 + 3*a^2 + 4*a*b)*\sinh(d*x + c))/(a^3*d*\cosh(d*x + \\
& c)^8 + 8*a^3*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^3*d*\sinh(d*x + c)^8 - 4*a \\
& ^3*d*\cosh(d*x + c)^6 + 6*a^3*d*\cosh(d*x + c)^4 + 4*(7*a^3*d*\cosh(d*x + c)^2 \\
& - a^3*d)*\sinh(d*x + c)^6 - 4*a^3*d*\cosh(d*x + c)^2 + 8*(7*a^3*d*\cosh(d*x + \\
& c)^3 - 3*a^3*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*a^3*d*\cosh(d*x + c)^ \\
& 4 - 30*a^3*d*\cosh(d*x + c)^2 + 3*a^3*d)*\sinh(d*x + c)^4 + a^3*d + 8*(7*a^3* \\
& d*\cosh(d*x + c)^5 - 10*a^3*d*\cosh(d*x + c)^3 + 3*a^3*d*\cosh(d*x + c))*\sinh(\\
& d*x + c)^3 + 4*(7*a^3*d*\cosh(d*x + c)^6 - 15*a^3*d*\cosh(d*x + c)^4 + 9*a^3* \\
& d*\cosh(d*x + c)^2 - a^3*d)*\sinh(d*x + c)^2 + 8*(a^3*d*\cosh(d*x + c)^7 - 3*a \\
& ^3*d*\cosh(d*x + c)^5 + 3*a^3*d*\cosh(d*x + c)^3 - a^3*d*\cosh(d*x + c))*\sinh(\\
& d*x + c)), 1/8*(2*(3*a^2 + 4*a*b)*\cosh(d*x + c)^7 + 14*(3*a^2 + 4*a*b)*\cosh \\
& (d*x + c)*\sinh(d*x + c)^6 + 2*(3*a^2 + 4*a*b)*\sinh(d*x + c)^7 - 2*(11*a^2 + \\
& 4*a*b)*\cosh(d*x + c)^5 + 2*(21*(3*a^2 + 4*a*b)*\cosh(d*x + c)^2 - 11*a^2 - \\
& 4*a*b)*\sinh(d*x + c)^5 + 10*(7*(3*a^2 + 4*a*b)*\cosh(d*x + c)^3 - (11*a^2 + \\
& 4*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 2*(11*a^2 + 4*a*b)*\cosh(d*x + c)^3 \\
& + 2*(35*(3*a^2 + 4*a*b)*\cosh(d*x + c)^4 - 10*(11*a^2 + 4*a*b)*\cosh(d*x + c) \\
& ^2 - 11*a^2 - 4*a*b)*\sinh(d*x + c)^3 + 2*(21*(3*a^2 + 4*a*b)*\cosh(d*x + c)^ \\
& 5 - 10*(11*a^2 + 4*a*b)*\cosh(d*x + c)^3 - 3*(11*a^2 + 4*a*b)*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^2 - 8*(b^2*\cosh(d*x + c)^8 + 8*b^2*\cosh(d*x + c)*\sinh(d*x + \\
& c)^7 + b^2*\sinh(d*x + c)^8 - 4*b^2*\cosh(d*x + c)^6 + 4*(7*b^2*\cosh(d*x + c) \\
& ^2 - b^2)*\sinh(d*x + c)^6 + 6*b^2*\cosh(d*x + c)^4 + 8*(7*b^2*\cosh(d*x + c)^ \\
& 3 - 3*b^2*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*b^2*\cosh(d*x + c)^4 - 30*b
\end{aligned}$$

$$\begin{aligned}
& ^2\cosh(dx + c)^2 + 3b^2)\sinh(dx + c)^4 - 4b^2\cosh(dx + c)^2 + 8(7* \\
& b^2\cosh(dx + c)^5 - 10b^2\cosh(dx + c)^3 + 3b^2\cosh(dx + c))\sinh(dx \\
& x + c)^3 + 4(7b^2\cosh(dx + c)^6 - 15b^2\cosh(dx + c)^4 + 9b^2\cosh(d \\
& *x + c)^2 - b^2)\sinh(dx + c)^2 + b^2 + 8(b^2\cosh(dx + c)^7 - 3b^2\cos \\
& h(dx + c)^5 + 3b^2\cosh(dx + c)^3 - b^2\cosh(dx + c))\sinh(dx + c))*sq \\
& rt(b/(a - b))\arctan(1/2\sqrt{b/(a - b)}(\cosh(dx + c) + \sinh(dx + c))) + \\
& 8(b^2\cosh(dx + c)^8 + 8b^2\cosh(dx + c)\sinh(dx + c)^7 + b^2\sinh(dx \\
& x + c)^8 - 4b^2\cosh(dx + c)^6 + 4(7b^2\cosh(dx + c)^2 - b^2)\sinh(dx \\
& + c)^6 + 6b^2\cosh(dx + c)^4 + 8(7b^2\cosh(dx + c)^3 - 3b^2\cosh(dx \\
& + c))\sinh(dx + c)^5 + 2(35b^2\cosh(dx + c)^4 - 30b^2\cosh(dx + c)^2 \\
& + 3b^2)\sinh(dx + c)^4 - 4b^2\cosh(dx + c)^2 + 8(7b^2\cosh(dx + c)^ \\
& 5 - 10b^2\cosh(dx + c)^3 + 3b^2\cosh(dx + c))\sinh(dx + c)^3 + 4(7b^ \\
& 2\cosh(dx + c)^6 - 15b^2\cosh(dx + c)^4 + 9b^2\cosh(dx + c)^2 - b^2)*s \\
& inh(dx + c)^2 + b^2 + 8(b^2\cosh(dx + c)^7 - 3b^2\cosh(dx + c)^5 + 3b \\
& ^2\cosh(dx + c)^3 - b^2\cosh(dx + c))\sinh(dx + c))*\sqrt{b/(a - b)}\arct \\
& an(1/2*(b\cosh(dx + c)^3 + 3b\cosh(dx + c)\sinh(dx + c)^2 + b\sinh(dx \\
& + c)^3 + (4a - 3b)\cosh(dx + c) + (3b\cosh(dx + c)^2 + 4a - 3b)\sinh \\
& (dx + c))*\sqrt{b/(a - b)})/b + 2*(3a^2 + 4a*b)\cosh(dx + c) - ((3a^2 + \\
& 4a*b + 8b^2)\cosh(dx + c)^8 + 8*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)*s \\
& inh(dx + c)^7 + (3a^2 + 4a*b + 8b^2)\sinh(dx + c)^8 - 4*(3a^2 + 4a*b \\
& + 8b^2)\cosh(dx + c)^6 + 4*(7*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^2 - \\
& 3a^2 - 4a*b - 8b^2)\sinh(dx + c)^6 + 8*(7*(3a^2 + 4a*b + 8b^2)\cosh(\\
& dx + c)^3 - 3*(3a^2 + 4a*b + 8b^2)\cosh(dx + c))\sinh(dx + c)^5 + 6*(\\
& 3a^2 + 4a*b + 8b^2)\cosh(dx + c)^4 + 2*(35*(3a^2 + 4a*b + 8b^2)\cosh \\
& (dx + c)^4 - 30*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^2 + 9a^2 + 12a*b + \\
& 24b^2)\sinh(dx + c)^4 + 8*(7*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^5 - 1 \\
& 0*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^3 + 3*(3a^2 + 4a*b + 8b^2)\cosh(\\
& dx + c))\sinh(dx + c)^3 - 4*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^2 + 4*(\\
& 7*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^6 - 15*(3a^2 + 4a*b + 8b^2)\cosh \\
& (dx + c)^4 + 9*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^2 - 3a^2 - 4a*b - 8 \\
& *b^2)\sinh(dx + c)^2 + 3a^2 + 4a*b + 8b^2 + 8*((3a^2 + 4a*b + 8b^2)* \\
& cosh(dx + c)^7 - 3*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^5 + 3*(3a^2 + 4* \\
& a*b + 8b^2)\cosh(dx + c)^3 - (3a^2 + 4a*b + 8b^2)\cosh(dx + c))\sinh(\\
& dx + c))*\log(\cosh(dx + c) + \sinh(dx + c) + 1) + ((3a^2 + 4a*b + 8b^2) \\
& *cosh(dx + c)^8 + 8*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)\sinh(dx + c)^7 \\
& + (3a^2 + 4a*b + 8b^2)\sinh(dx + c)^8 - 4*(3a^2 + 4a*b + 8b^2)\cosh(\\
& dx + c)^6 + 4*(7*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^2 - 3a^2 - 4a*b - \\
& 8b^2)\sinh(dx + c)^6 + 8*(7*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^3 - 3* \\
& (3a^2 + 4a*b + 8b^2)\cosh(dx + c))\sinh(dx + c)^5 + 6*(3a^2 + 4a*b + \\
& 8b^2)\cosh(dx + c)^4 + 2*(35*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^4 - 3 \\
& 0*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^2 + 9a^2 + 12a*b + 24b^2)\sinh(d \\
& *x + c)^4 + 8*(7*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^5 - 10*(3a^2 + 4a* \\
& b + 8b^2)\cosh(dx + c)^3 + 3*(3a^2 + 4a*b + 8b^2)\cosh(dx + c))\sinh(\\
& dx + c)^3 - 4*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^2 + 4*(7*(3a^2 + 4a* \\
& b + 8b^2)\cosh(dx + c)^6 - 15*(3a^2 + 4a*b + 8b^2)\cosh(dx + c)^4 + 9
\end{aligned}$$

```

*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x
+ c)^2 + 3*a^2 + 4*a*b + 8*b^2 + 8*((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^7
- 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 + 4*a*b + 8*b^2)*co
sh(d*x + c)^3 - (3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(c
osh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(3*a^2 + 4*a*b)*cosh(d*x + c)^6 -
5*(11*a^2 + 4*a*b)*cosh(d*x + c)^4 - 3*(11*a^2 + 4*a*b)*cosh(d*x + c)^2 + 3
*a^2 + 4*a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^8 + 8*a^3*d*cosh(d*x + c)
*sinh(d*x + c)^7 + a^3*d*sinh(d*x + c)^8 - 4*a^3*d*cosh(d*x + c)^6 + 6*a^3*
d*cosh(d*x + c)^4 + 4*(7*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^6 - 4
*a^3*d*cosh(d*x + c)^2 + 8*(7*a^3*d*cosh(d*x + c)^3 - 3*a^3*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(35*a^3*d*cosh(d*x + c)^4 - 30*a^3*d*cosh(d*x + c)^2
+ 3*a^3*d)*sinh(d*x + c)^4 + a^3*d + 8*(7*a^3*d*cosh(d*x + c)^5 - 10*a^3*d*
cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*d*cosh(
d*x + c)^6 - 15*a^3*d*cosh(d*x + c)^4 + 9*a^3*d*cosh(d*x + c)^2 - a^3*d)*si
nh(d*x + c)^2 + 8*(a^3*d*cosh(d*x + c)^7 - 3*a^3*d*cosh(d*x + c)^5 + 3*a^3*
d*cosh(d*x + c)^3 - a^3*d*cosh(d*x + c))*sinh(d*x + c))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.41 \quad \int \frac{\operatorname{csch}^6(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=110

$$-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2} d \sqrt{a-b}} - \frac{(a^2 + ab + b^2) \operatorname{coth}(c+dx)}{a^3 d} + \frac{(2a+b) \operatorname{coth}^3(c+dx)}{3a^2 d} - \frac{\operatorname{coth}^5(c+dx)}{5ad}$$

[Out] $-\left(\frac{b^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tanh}[c+d*x]}{\sqrt{a}}\right]}{a^{7/2} \operatorname{Sqrt}[a-b]}\right) / (a^{7/2} \operatorname{Sqrt}[a-b] * d)$
 $) - \left(\frac{(a^2 + a*b + b^2) \operatorname{Coth}[c+d*x]}{a^3 * d}\right) + \left(\frac{(2*a + b) \operatorname{Coth}[c+d*x]^3}{3*a^2 * d}\right) - \frac{\operatorname{Coth}[c+d*x]^5}{5*a*d}$

Rubi [A] time = 0.135537, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3187, 461, 208}

$$-\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2} d \sqrt{a-b}} - \frac{(a^2 + ab + b^2) \operatorname{coth}(c+dx)}{a^3 d} + \frac{(2a+b) \operatorname{coth}^3(c+dx)}{3a^2 d} - \frac{\operatorname{coth}^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c+d*x]^6 / (a + b \operatorname{Sinh}[c+d*x]^2), x]$

[Out] $-\left(\frac{b^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tanh}[c+d*x]}{\sqrt{a}}\right]}{a^{7/2} \operatorname{Sqrt}[a-b]}\right) / (a^{7/2} \operatorname{Sqrt}[a-b] * d)$
 $) - \left(\frac{(a^2 + a*b + b^2) \operatorname{Coth}[c+d*x]}{a^3 * d}\right) + \left(\frac{(2*a + b) \operatorname{Coth}[c+d*x]^3}{3*a^2 * d}\right) - \frac{\operatorname{Coth}[c+d*x]^5}{5*a*d}$

Rule 3187

$\operatorname{Int}[\sin[(e_.) + (f_.) * (x_)]^{(m_)} * ((a_.) + (b_.) * \sin[(e_.) + (f_.) * (x_)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m+1)} / f, \operatorname{Subst}[\operatorname{Int}[(x^m * (a + (a+b) * ff^2 * x^2)^p] / (1 + ff^2 * x^2)^{(m/2 + p + 1)}], x], x, \operatorname{Tan}[e + f*x] / ff], x]\} /;$ $\operatorname{FreeQ}\{a, b, e, f, x\} \ \&\& \ \operatorname{IntegerQ}[m/2] \ \&\& \ \operatorname{IntegerQ}[p]$

Rule 461

$\operatorname{Int}[\left(\frac{(e_.) * (x_)}{(a_.) + (b_.) * (x_)}\right)^{(m_)} * \left(\frac{(c_.) + (d_.) * (x_)}{(c_.) + (d_.) * (x_)}\right)^{(n_)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[\left(\frac{(e*x)^m * (a + b*x^n)^p}{(c + d*x^n)}\right), x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, m, x\} \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[n, 0]$

&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] || !RationalQ[m])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^6(c+dx)}{a+b \sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{x^6(a-(a-b)x^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^6} + \frac{-2a-b}{a^2x^4} + \frac{a^2+ab+b^2}{a^3x^2} + \frac{b^3}{a^3(-a+(a-b)x^2)}\right) dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{(a^2+ab+b^2) \operatorname{coth}(c+dx)}{a^3d} + \frac{(2a+b) \operatorname{coth}^3(c+dx)}{3a^2d} - \frac{\operatorname{coth}^5(c+dx)}{5ad} + \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{-a+(a-b)x^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2} \sqrt{a-bd}} - \frac{(a^2+ab+b^2) \operatorname{coth}(c+dx)}{a^3d} + \frac{(2a+b) \operatorname{coth}^3(c+dx)}{3a^2d} - \frac{\operatorname{coth}^5(c+dx)}{5ad} \end{aligned}$$

Mathematica [A] time = 1.45808, size = 155, normalized size = 1.41

$$\frac{\operatorname{csch}^2(c+dx)(2a+b \cosh(2(c+dx))-b) \left(\sqrt{a} \sqrt{a-b} \operatorname{coth}(c+dx) (3a^2 \operatorname{csch}^4(c+dx) + 8a^2 - a(4a+5b) \operatorname{csch}^2(c+dx))\right)}{30a^{7/2}d \sqrt{a-b} (a \operatorname{csch}^2(c+dx) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^6/(a + b*Sinh[c + d*x]^2), x]

[Out] -((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(15*b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(8*a^2 + 10*a*b + 15*b^2 - a*(4*a + 5*b)*Csch[c + d*x]^2 + 3*a^2*Csch[c + d*x]^4))/(30*a^(7/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))

Maple [B] time = 0.074, size = 519, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)`

[Out]
$$-1/160/d/a*\tanh(1/2*d*x+1/2*c)^5+5/96/d/a*\tanh(1/2*d*x+1/2*c)^3+1/24/d/a^2*\tanh(1/2*d*x+1/2*c)^3*b-5/16/d/a*\tanh(1/2*d*x+1/2*c)-3/8/d/a^2*\tanh(1/2*d*x+1/2*c)*b-1/2/d/a^3*b^2*\tanh(1/2*d*x+1/2*c)-1/160/d/a/\tanh(1/2*d*x+1/2*c)^5+5/96/d/a/\tanh(1/2*d*x+1/2*c)^3+1/24/d/a^2/\tanh(1/2*d*x+1/2*c)^3*b-5/16/d/a/\tanh(1/2*d*x+1/2*c)-3/8/d*b/a^2/\tanh(1/2*d*x+1/2*c)-1/2/d/a^3/\tanh(1/2*d*x+1/2*c)*b^2-1/d*b^3/a^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^4/a^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^3/a^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b^4/a^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 2.78102, size = 10761, normalized size = 97.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")`

[Out]
$$[-1/30*(60*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^8 + 480*(a^2*b^2 - a*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + 60*(a^2*b^2 - a*b^3)*\sinh(d*x + c)^8 - 120*(a^3*b + a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^6 - 120*(a^3*b + a^2*b^2 - 2*a*b^3 - 14*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 240*(14*(a^2*b^2 - a*$$

$$\begin{aligned}
& b^3) \cosh(dx + c)^3 - 3(a^3b + a^2b^2 - 2ab^3) \cosh(dx + c) \sinh(dx + c)^5 + 40(8a^4 - a^3b + 2a^2b^2 - 9ab^3) \cosh(dx + c)^4 + 40(105(a^2b^2 - ab^3) \cosh(dx + c)^4 + 8a^4 - a^3b + 2a^2b^2 - 9ab^3 \\
& - 45(a^3b + a^2b^2 - 2ab^3) \cosh(dx + c)^2) \sinh(dx + c)^4 + 32a^4 + 8a^3b + 20a^2b^2 - 60ab^3 + 160(21(a^2b^2 - ab^3) \cosh(dx + c)^5 - 15(a^3b + a^2b^2 - 2ab^3) \cosh(dx + c)^3 + (8a^4 - a^3b + 2a^2b^2 - 9ab^3) \cosh(dx + c) \sinh(dx + c)^3 - 40(4a^4 + a^3b + a^2b^2 - 6ab^3) \cosh(dx + c)^2 + 40(42(a^2b^2 - ab^3) \cosh(dx + c)^6 - 45(a^3b + a^2b^2 - 2ab^3) \cosh(dx + c)^4 - 4a^4 - a^3b - a^2b^2 + 6ab^3 + 6(8a^4 - a^3b + 2a^2b^2 - 9ab^3) \cosh(dx + c)^2) \sinh(dx + c)^2 - 15(b^3 \cosh(dx + c)^{10} + 10b^3 \cosh(dx + c) \sinh(dx + c)^9 + b^3 \sinh(dx + c)^{10} - 5b^3 \cosh(dx + c)^8 + 10b^3 \cosh(dx + c)^6 + 5(9b^3 \cosh(dx + c)^2 - b^3) \sinh(dx + c)^8 + 40(3b^3 \cosh(dx + c)^3 - b^3 \cosh(dx + c)) \sinh(dx + c)^7 - 10b^3 \cosh(dx + c)^4 + 10(21b^3 \cosh(dx + c)^4 - 14b^3 \cosh(dx + c)^2 + b^3) \sinh(dx + c)^6 + 4(63b^3 \cosh(dx + c)^5 - 70b^3 \cosh(dx + c)^3 + 15b^3 \cosh(dx + c)) \sinh(dx + c)^5 + 5b^3 \cosh(dx + c)^2 + 10(21b^3 \cosh(dx + c)^6 - 35b^3 \cosh(dx + c)^4 + 15b^3 \cosh(dx + c)^2 - b^3) \sinh(dx + c)^4 + 40(3b^3 \cosh(dx + c)^7 - 7b^3 \cosh(dx + c)^5 + 5b^3 \cosh(dx + c)^3 - b^3 \cosh(dx + c)) \sinh(dx + c)^3 - b^3 + 5(9b^3 \cosh(dx + c)^8 - 28b^3 \cosh(dx + c)^6 + 30b^3 \cosh(dx + c)^4 - 12b^3 \cosh(dx + c)^2 + b^3) \sinh(dx + c)^2 + 10(b^3 \cosh(dx + c)^9 - 4b^3 \cosh(dx + c)^7 + 6b^3 \cosh(dx + c)^5 - 4b^3 \cosh(dx + c)^3 + b^3 \cosh(dx + c)) \sinh(dx + c)) \sqrt{a^2 - ab} \log((b^2 \cosh(dx + c)^4 + 4b^2 \cosh(dx + c) \sinh(dx + c)^3 + b^2 \sinh(dx + c)^4 + 2(2ab - b^2) \cosh(dx + c)^2 + 2(3b^2 \cosh(dx + c)^2 + 2ab - b^2) \sinh(dx + c)^2 + 8a^2 - 8ab + b^2 + 4(b^2 \cosh(dx + c)^3 + (2ab - b^2) \cosh(dx + c)) \sinh(dx + c) + 4(b \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + 2a - b) \sqrt{a^2 - ab})) / (b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 + 2(2a - b) \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 + 2a - b) \sinh(dx + c)^2 + 4(b \cosh(dx + c)^3 + (2a - b) \cosh(dx + c)) \sinh(dx + c) + b) + 80(6(a^2b^2 - ab^3) \cosh(dx + c)^7 - 9(a^3b + a^2b^2 - 2ab^3) \cosh(dx + c)^5 + 2(8a^4 - a^3b + 2a^2b^2 - 9ab^3) \cosh(dx + c)^3 - (4a^4 + a^3b + a^2b^2 - 6ab^3) \cosh(dx + c)) \sinh(dx + c)) / ((a^5 - a^4b) d \cosh(dx + c)^{10} + 10(a^5 - a^4b) d \cosh(dx + c) \sinh(dx + c)^9 + (a^5 - a^4b) d \sinh(dx + c)^{10} - 5(a^5 - a^4b) d \cosh(dx + c)^8 + 5(9(a^5 - a^4b) d \cosh(dx + c)^2 - (a^5 - a^4b) d) \sinh(dx + c)^8 + 10(a^5 - a^4b) d \cosh(dx + c)^6 + 40(3(a^5 - a^4b) d \cosh(dx + c)^3 - (a^5 - a^4b) d \cosh(dx + c)) \sinh(dx + c)^7 + 10(21(a^5 - a^4b) d \cosh(dx + c)^4 - 14(a^5 - a^4b) d \cosh(dx + c)^2 + (a^5 - a^4b) d) \sinh(dx + c)^6 - 10(a^5 - a^4b) d \cosh(dx + c)^4 + 4(63(a^5 - a^4b) d \cosh(dx + c)^5 - 70(a^5 - a^4b) d \cosh(dx + c)^3 + 15(a^5 - a^4b) d \cosh(dx + c)) \sinh(dx + c)^5 + 10(21(a^5 - a^4b) d \cosh(dx + c)^6 - 35(a^5 - a^4b) d \cosh(dx + c)^4 + 15(a^5 - a^4b) d \cosh(dx + c)^2 - (a^5 - a^4b) d) \sinh(dx + c)^4 + 5(a^5 - a^4b) d \cosh(dx + c)^2 + 40(3(a^5
\end{aligned}$$

$$\begin{aligned}
& 5 - a^4*b)*d*\cosh(d*x + c)^7 - 7*(a^5 - a^4*b)*d*\cosh(d*x + c)^5 + 5*(a^5 - \\
& a^4*b)*d*\cosh(d*x + c)^3 - (a^5 - a^4*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& + 5*(9*(a^5 - a^4*b)*d*\cosh(d*x + c)^8 - 28*(a^5 - a^4*b)*d*\cosh(d*x + c)^6 \\
& + 30*(a^5 - a^4*b)*d*\cosh(d*x + c)^4 - 12*(a^5 - a^4*b)*d*\cosh(d*x + c)^2 \\
& + (a^5 - a^4*b)*d)*\sinh(d*x + c)^2 - (a^5 - a^4*b)*d + 10*((a^5 - a^4*b)*d* \\
& \cosh(d*x + c)^9 - 4*(a^5 - a^4*b)*d*\cosh(d*x + c)^7 + 6*(a^5 - a^4*b)*d*\cos \\
& h(d*x + c)^5 - 4*(a^5 - a^4*b)*d*\cosh(d*x + c)^3 + (a^5 - a^4*b)*d*\cosh(d*x \\
& + c))*\sinh(d*x + c)), -1/15*(30*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^8 + 240*(a \\
& ^2*b^2 - a*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + 30*(a^2*b^2 - a*b^3)*\sinh(d \\
& *x + c)^8 - 60*(a^3*b + a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^6 - 60*(a^3*b + a^ \\
& 2*b^2 - 2*a*b^3 - 14*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 1 \\
& 20*(14*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^3 - 3*(a^3*b + a^2*b^2 - 2*a*b^3)*\co \\
& sh(d*x + c))*\sinh(d*x + c)^5 + 20*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*\cos \\
& h(d*x + c)^4 + 20*(105*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^4 + 8*a^4 - a^3*b + \\
& 2*a^2*b^2 - 9*a*b^3 - 45*(a^3*b + a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^2)*\sinh(\\
& d*x + c)^4 + 16*a^4 + 4*a^3*b + 10*a^2*b^2 - 30*a*b^3 + 80*(21*(a^2*b^2 - a \\
& *b^3)*\cosh(d*x + c)^5 - 15*(a^3*b + a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^3 + (8 \\
& *a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 20*(4* \\
& a^4 + a^3*b + a^2*b^2 - 6*a*b^3)*\cosh(d*x + c)^2 + 20*(42*(a^2*b^2 - a*b^3) \\
& *\cosh(d*x + c)^6 - 45*(a^3*b + a^2*b^2 - 2*a*b^3)*\cosh(d*x + c)^4 - 4*a^4 - \\
& a^3*b - a^2*b^2 + 6*a*b^3 + 6*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*\cosh(d \\
& *x + c)^2)*\sinh(d*x + c)^2 - 15*(b^3*\cosh(d*x + c)^10 + 10*b^3*\cosh(d*x + c \\
&)*\sinh(d*x + c)^9 + b^3*\sinh(d*x + c)^10 - 5*b^3*\cosh(d*x + c)^8 + 10*b^3*\c \\
& osh(d*x + c)^6 + 5*(9*b^3*\cosh(d*x + c)^2 - b^3)*\sinh(d*x + c)^8 + 40*(3*b^ \\
& 3*\cosh(d*x + c)^3 - b^3*\cosh(d*x + c))*\sinh(d*x + c)^7 - 10*b^3*\cosh(d*x + \\
& c)^4 + 10*(21*b^3*\cosh(d*x + c)^4 - 14*b^3*\cosh(d*x + c)^2 + b^3)*\sinh(d*x \\
& + c)^6 + 4*(63*b^3*\cosh(d*x + c)^5 - 70*b^3*\cosh(d*x + c)^3 + 15*b^3*\cosh(d \\
& *x + c))*\sinh(d*x + c)^5 + 5*b^3*\cosh(d*x + c)^2 + 10*(21*b^3*\cosh(d*x + c) \\
& ^6 - 35*b^3*\cosh(d*x + c)^4 + 15*b^3*\cosh(d*x + c)^2 - b^3)*\sinh(d*x + c)^4 \\
& + 40*(3*b^3*\cosh(d*x + c)^7 - 7*b^3*\cosh(d*x + c)^5 + 5*b^3*\cosh(d*x + c)^ \\
& 3 - b^3*\cosh(d*x + c))*\sinh(d*x + c)^3 - b^3 + 5*(9*b^3*\cosh(d*x + c)^8 - 2 \\
& 8*b^3*\cosh(d*x + c)^6 + 30*b^3*\cosh(d*x + c)^4 - 12*b^3*\cosh(d*x + c)^2 + b \\
& ^3)*\sinh(d*x + c)^2 + 10*(b^3*\cosh(d*x + c)^9 - 4*b^3*\cosh(d*x + c)^7 + 6*b \\
& ^3*\cosh(d*x + c)^5 - 4*b^3*\cosh(d*x + c)^3 + b^3*\cosh(d*x + c))*\sinh(d*x + \\
& c))*\sqrt{-a^2 + a*b}*\arctan(-1/2*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sin \\
& h(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-a^2 + a*b}/(a^2 - a*b)) + 4 \\
& 0*(6*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^7 - 9*(a^3*b + a^2*b^2 - 2*a*b^3)*\cosh \\
& (d*x + c)^5 + 2*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^3 - (4* \\
& a^4 + a^3*b + a^2*b^2 - 6*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5 - a^4* \\
& b)*d*\cosh(d*x + c)^10 + 10*(a^5 - a^4*b)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + \\
& (a^5 - a^4*b)*d*\sinh(d*x + c)^10 - 5*(a^5 - a^4*b)*d*\cosh(d*x + c)^8 + 5*(9 \\
& *(a^5 - a^4*b)*d*\cosh(d*x + c)^2 - (a^5 - a^4*b)*d)*\sinh(d*x + c)^8 + 10*(a \\
& ^5 - a^4*b)*d*\cosh(d*x + c)^6 + 40*(3*(a^5 - a^4*b)*d*\cosh(d*x + c)^3 - (a^ \\
& 5 - a^4*b)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 10*(21*(a^5 - a^4*b)*d*\cosh(d \\
& *x + c)^4 - 14*(a^5 - a^4*b)*d*\cosh(d*x + c)^2 + (a^5 - a^4*b)*d)*\sinh(d*x
\end{aligned}$$

```
+ c)^6 - 10*(a^5 - a^4*b)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - a^4*b)*d*cosh(d*x + c)^5 - 70*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + 15*(a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - a^4*b)*d*cosh(d*x + c)^6 - 35*(a^5 - a^4*b)*d*cosh(d*x + c)^4 + 15*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5 - a^4*b)*d)*sinh(d*x + c)^4 + 5*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + 40*(3*(a^5 - a^4*b)*d*cosh(d*x + c)^7 - 7*(a^5 - a^4*b)*d*cosh(d*x + c)^5 + 5*(a^5 - a^4*b)*d*cosh(d*x + c)^3 - (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*(a^5 - a^4*b)*d*cosh(d*x + c)^8 - 28*(a^5 - a^4*b)*d*cosh(d*x + c)^6 + 30*(a^5 - a^4*b)*d*cosh(d*x + c)^4 - 12*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + (a^5 - a^4*b)*d)*sinh(d*x + c)^2 - (a^5 - a^4*b)*d + 10*((a^5 - a^4*b)*d*cosh(d*x + c)^9 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^7 + 6*(a^5 - a^4*b)*d*cosh(d*x + c)^5 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49275, size = 289, normalized size = 2.63

$$\frac{b^3 \arctan\left(\frac{be^{2dx+2c}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a^3d} - \frac{2\left(15b^2e^{8dx+8c} - 30abe^{6dx+6c} - 60b^2e^{6dx+6c} + 80a^2e^{4dx+4c} + 70abe^{4dx+4c} + 90b^2e^{2dx+2c}\right)}{15a^3d\left(e^{2dx+2c} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a^3*d) - 2/15*(15*b^2*e^(8*d*x + 8*c) - 30*a*b*e^(6*d*x + 6*c) - 60*b^2*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x + 4*c) + 70*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 40*a^2*e^(2*d*x + 2*c) - 50*a*b*e^(2*d*x + 2*c) - 60*b^2*e^(2*d*x + 2*c) + 8*a^2 + 10*a*b + 15*b^2)/(a^3*d*(e^(2*d*x + 2*c) - 1)^5)
```


$$3.42 \quad \int \frac{\sinh^4(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=102

$$-\frac{\sqrt{a}(2a-3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2b^2d(a-b)^{3/2}} - \frac{a \tanh(c+dx)}{2bd(a-b)\left(a-(a-b) \tanh^2(c+dx)\right)} + \frac{x}{b^2}$$

[Out] x/b^2 - (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*(a - b)^(3/2)*b^2*d) - (a*Tanh[c + d*x])/(2*(a - b)*b*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.169097, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3187, 470, 522, 206, 208}

$$-\frac{\sqrt{a}(2a-3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2b^2d(a-b)^{3/2}} - \frac{a \tanh(c+dx)}{2bd(a-b)\left(a-(a-b) \tanh^2(c+dx)\right)} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] x/b^2 - (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*(a - b)^(3/2)*b^2*d) - (a*Tanh[c + d*x])/(2*(a - b)*b*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)]/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + q))
```

$(p + 1)*(c + d*x^n)^{(q + 1)}/(b*n*(b*c - a*d)*(p + 1)), x] + \text{Dist}[e^{(2*n)}/(b*n*(b*c - a*d)*(p + 1)), \text{Int}[(e*x)^{(m - 2*n)}*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^q*\text{Simp}[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, q\}, x \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[m - n + 1, n] \&\& \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 522

$\text{Int}[(e_ + (f_)*(x_)^{(n_)})/((a_ + (b_)*(x_)^{(n_)})*((c_ + (d_)*(x_)^{(n_)})^{n_}))], x_Symbol] := \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^n), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[1/(c + d*x^n), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x]$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}], x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}], x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \&\& \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\sinh^4(c + dx)}{(a + b \sinh^2(c + dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a-(a-b)x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{a \tanh(c + dx)}{2(a-b)bd(a - (a-b)\tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{a+(a-2b)x^2}{(1-x^2)(a+(-a+b)x^2)} dx, x, \tanh(c + dx)\right)}{2(a-b)bd} \\ &= -\frac{a \tanh(c + dx)}{2(a-b)bd(a - (a-b)\tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{b^2d} - \frac{(a(2a-3b))}{2(a-b)bd} \\ &= \frac{x}{b^2} - \frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2(a-b)^{3/2}b^2d} - \frac{a \tanh(c + dx)}{2(a-b)bd(a - (a-b)\tanh^2(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.832878, size = 99, normalized size = 0.97

$$\frac{\frac{\sqrt{a}(2a-3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}} + \frac{ab \sinh(2(c+dx))}{(a-b)(2a+b \cosh(2(c+dx))-b)} - 2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] $-(2*(c + d*x) + (\text{Sqrt}[a]*(2*a - 3*b)*\text{ArcTan}h[(\text{Sqrt}[a - b]*\text{Tan}h[c + d*x])/S\text{qrt}[a]])/(a - b)^{(3/2)} + (a*b*\text{Sinh}[2*(c + d*x)])/((a - b)*(2*a - b + b*\text{Cosh}[2*(c + d*x)])))/(2*b^2*d)$

Maple [B] time = 0.051, size = 798, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)

[Out] $1/d/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/d/b^2*\ln(\tanh(1/2*d*x+1/2*c)-1)-1/d*a/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*\tanh(1/2*d*x+1/2*c)^3-1/d*a/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*\tanh(1/2*d*x+1/2*c)-1/d*a^2/b^2/(a-b)/((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)}))+1/d*a^2/b/(a-b)/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)}))+1/d*a^2/b^2/(a-b)/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}))+1/d*a^2/b/(a-b)/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}))+3/2/d*a/b/(a-b)/((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)}))-3/2/d*a/(a-b)/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)}))-3/2/d*a/b/(a-b)/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}))-3/2/d*a/(a-b)/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}))$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.55959, size = 4199, normalized size = 41.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(4*(a*b - b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b - b^2)*d*x*cosh(d*x + c)* \\ & sinh(d*x + c)^3 + 4*(a*b - b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b - b^2)*d*x + 4 \\ & *(2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x + c)^2 + 4*(6*(a*b - \\ & b^2)*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*sinh(\\ & d*x + c)^2 + ((2*a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(2*a*b - 3*b^2)*cosh(d*x \\ & + c)*sinh(d*x + c)^3 + (2*a*b - 3*b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 8*a*b + \\ & 3*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a^2 - 8* \\ & a*b + 3*b^2)*sinh(d*x + c)^2 + 2*a*b - 3*b^2 + 4*((2*a*b - 3*b^2)*cosh(d*x \\ & + c)^3 + (4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)*sqrt(a/(a - \\ & b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*si \\ & nh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 \\ & + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c) \\ & ^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^2)*cosh(d*x + \\ & c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + \\ & c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(\\ & d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 \\ & + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 \\ & + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*a*b + 8*(2*(a*b - b^2)*d \\ & *x*cosh(d*x + c)^3 + (2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x + \\ & c))*sinh(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^4 + 4*(a*b^3 - b^4)*d*co \\ & sh(d*x + c)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*sinh(d*x + c)^4 + 2*(2*a^2*b^2 \\ & - 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^2 \end{aligned}$$

$$\begin{aligned}
& + (2a^2b^2 - 3ab^3 + b^4)d \sinh(dx + c)^2 + (ab^3 - b^4)d + 4((a \\
& *b^3 - b^4)d \cosh(dx + c)^3 + (2a^2b^2 - 3ab^3 + b^4)d \cosh(dx + c) \\
&) \sinh(dx + c), \frac{1}{2}(2(ab - b^2)d^2x \cosh(dx + c)^4 + 8(ab - b^2)d^2x \\
& * \cosh(dx + c) \sinh(dx + c)^3 + 2(ab - b^2)d^2x \sinh(dx + c)^4 + 2(ab \\
& b - b^2)d^2x + 2(2(2a^2 - 3ab + b^2)d^2x + 2a^2 - ab) \cosh(dx + c)^2 \\
& + 2(6(ab - b^2)d^2x \cosh(dx + c)^2 + 2(2a^2 - 3ab + b^2)d^2x + 2a^2 - ab) \sinh(dx + c)^2 \\
& - ((2ab - 3b^2) \cosh(dx + c)^4 + 4(2ab - 3b^2) \cosh(dx + c) \sinh(dx + c)^3 \\
& + (2ab - 3b^2) \sinh(dx + c)^4 + 2(4a^2 - 8ab + 3b^2) \cosh(dx + c)^2 \\
& + 2(3(2ab - 3b^2) \cosh(dx + c))^2 + 4a^2 - 8ab + 3b^2) \sinh(dx + c)^2 + 2ab - 3b^2 \\
& + 4((2ab - 3b^2) \cosh(dx + c)^3 + (4a^2 - 8ab + 3b^2) \cosh(dx + c)) \sinh(dx + c) \\
&) \sqrt{-a/(a - b)} \arctan\left(\frac{1}{2}(b \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + 2a - b) \sqrt{-a/(a - b)}\right) / a \\
& + 2ab + 4(2(ab - b^2)d^2x \cosh(dx + c)^3 + (2(2a^2 - 3ab + b^2)d^2x + 2a^2 - ab) \cosh(dx + c)) \sinh(dx + c) \\
&) / ((ab^3 - b^4)d \cosh(dx + c)^4 + 4(ab^3 - b^4)d \cosh(dx + c) \sinh(dx + c)^3 \\
& + (ab^3 - b^4)d \sinh(dx + c)^4 + 2(2a^2b^2 - 3ab^3 + b^4)d \cosh(dx + c)^2 + 2(3(ab^3 - b^4)d \cosh(dx + c)^2 \\
& + (2a^2b^2 - 3ab^3 + b^4)d) \sinh(dx + c)^2 + (ab^3 - b^4)d + 4((ab^3 - b^4)d \cosh(dx + c)^3 \\
& + (2a^2b^2 - 3ab^3 + b^4)d \cosh(dx + c)) \sinh(dx + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)**4/(a+b*sinh(dx+c)**2)**2,x)

[Out] Timed out

Giac [A] time = 1.28282, size = 228, normalized size = 2.24

$$-\frac{(2a^2 - 3ab) \arctan\left(\frac{be^{(2dx+2c)} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{2(ab^2d - b^3d)\sqrt{-a^2 + ab}} + \frac{2a^2e^{(2dx+2c)} - abe^{(2dx+2c)} + ab}{(ab^2d - b^3d)(be^{(4dx+4c)} + 4ae^{(2dx+2c)} - 2be^{(2dx+2c)} + b)} + \frac{dx + c}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^4/(a+b*sinh(dx+c)^2)^2,x, algorithm="giac")

```
[Out] -1/2*(2*a^2 - 3*a*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a
*b))/((a*b^2*d - b^3*d)*sqrt(-a^2 + a*b)) + (2*a^2*e^(2*d*x + 2*c) - a*b*e^
(2*d*x + 2*c) + a*b)/((a*b^2*d - b^3*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x +
2*c) - 2*b*e^(2*d*x + 2*c) + b)) + (d*x + c)/(b^2*d)
```

$$3.43 \quad \int \frac{\sinh^3(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=90

$$\frac{(a-2b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{2b^{3/2}d(a-b)^{3/2}} - \frac{a \cosh(c+dx)}{2bd(a-b)(a+b \cosh^2(c+dx)-b)}$$

[Out] ((a - 2*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*(a - b)^(3/2)*b^(3/2)*d) - (a*Cosh[c + d*x])/(2*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2))

Rubi [A] time = 0.112523, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3186, 385, 205}

$$\frac{(a-2b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{2b^{3/2}d(a-b)^{3/2}} - \frac{a \cosh(c+dx)}{2bd(a-b)(a+b \cosh^2(c+dx)-b)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((a - 2*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*(a - b)^(3/2)*b^(3/2)*d) - (a*Cosh[c + d*x])/(2*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2))

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 385

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +

p, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^3(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+bx^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{a \cosh(c+dx)}{2(a-b)bd(a-b+b\cosh^2(c+dx))} + \frac{(a-2b) \text{Subst}\left(\int \frac{1}{a-b+bx^2} dx, x, \cosh(c+dx)\right)}{2(a-b)bd} \\ &= \frac{(a-2b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}b^{3/2}d} - \frac{a \cosh(c+dx)}{2(a-b)bd(a-b+b\cosh^2(c+dx))} \end{aligned}$$

Mathematica [C] time = 0.596782, size = 141, normalized size = 1.57

$$\frac{-\frac{2a\sqrt{b} \cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)} + \frac{(a-2b)\left(\tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) + \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{3/2}}}{2b^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2, x]

[Out] (((a - 2*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(3/2) - (2*a*Sqrt[b]*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*b^(3/2)*d)

Maple [B] time = 0.032, size = 341, normalized size = 3.8

$$16 \frac{(\tanh(1/2 dx + c/2))^2 a}{d(16 ab - 16 b^2) \left((\tanh(1/2 dx + c/2))^4 a - 2 (\tanh(1/2 dx + c/2))^2 a + 4 (\tanh(1/2 dx + c/2))^2 b + a \right)} - 32 \frac{a}{d(16 ab -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)`

[Out] $16/d/(16*a*b-16*b^2)/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)^2-32/d/(16*a*b-16*b^2)/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^2*b-16/d/(16*a*b-16*b^2)/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a+8/d/(16*a*b-16*b^2)/(a*b-b^2)^{(1/2)}*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^{(1/2)})*a-16/d/(16*a*b-16*b^2)/(a*b-b^2)^{(1/2)}*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^{(1/2)})*b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ae^{(3dx+3c)} + ae^{(dx+c)}}{ab^2d - b^3d + (ab^2de^{(4c)} - b^3de^{(4c)})e^{(4dx)} + 2(2a^2bde^{(2c)} - 3ab^2de^{(2c)} + b^3de^{(2c)})e^{(2dx)}} + \frac{1}{8} \int \frac{8((ae^{(3c)} - 2b^3e^{(4c)})e^{(3dx)} - (ae^{(3c)} - 2b^3e^{(4c)})e^{(3dx)} - (ae^{(3c)} - 2b^3e^{(4c)})e^{(3dx)} - (ae^{(3c)} - 2b^3e^{(4c)})e^{(3dx)}))}{ab^2 - b^3 + (ab^2e^{(4c)} - b^3e^{(4c)})e^{(2dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $-(a*e^{(3*d*x + 3*c)} + a*e^{(d*x + c)})/(a*b^2*d - b^3*d + (a*b^2*d*e^{(4*c)} - b^3*d*e^{(4*c)})e^{(4*d*x)} + 2*(2*a^2*b*d*e^{(2*c)} - 3*a*b^2*d*e^{(2*c)} + b^3*d*e^{(2*c)})e^{(2*d*x)}) + 1/8*integrate(8*((a*e^{(3*c)} - 2*b^3*e^{(4*c)})e^{(3*d*x)} - (a*e^{(3*c)} - 2*b^3*e^{(4*c)})e^{(3*d*x)} - (a*e^{(3*c)} - 2*b^3*e^{(4*c)})e^{(3*d*x)} - (a*e^{(3*c)} - 2*b^3*e^{(4*c)})e^{(3*d*x)})))/(a*b^2 - b^3 + (a*b^2*e^{(4*c)} - b^3*e^{(4*c)})e^{(2*d*x)} + 2*(2*a^2*b*e^{(2*c)} - 3*a*b^2*e^{(2*c)} + b^3*e^{(2*c)})e^{(2*d*x)}), x)$

Fricas [B] time = 2.43195, size = 4536, normalized size = 50.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $[-1/4*(4*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*sinh(d*x + c)^3 + ((a*b - 2*b^2)*cosh(d*x + c)^3 + 12*(a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 4*(a*b - 2*b^2)*sinh(d*x + c)^3 + ((a*b - 2*b^2)*cosh(d*x + c)^3 + 12*(a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 4*(a*b - 2*b^2)*sinh(d*x + c)^3)]/8$

$$\begin{aligned}
& d*x + c)^4 + 4*(a*b - 2*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a*b - 2*b^2)* \\
& \sinh(d*x + c)^4 + 2*(2*a^2 - 5*a*b + 2*b^2)*\cosh(d*x + c)^2 + 2*(3*(a*b - 2 \\
& *b^2)*\cosh(d*x + c)^2 + 2*a^2 - 5*a*b + 2*b^2)*\sinh(d*x + c)^2 + a*b - 2*b^ \\
& 2 + 4*((a*b - 2*b^2)*\cosh(d*x + c)^3 + (2*a^2 - 5*a*b + 2*b^2)*\cosh(d*x + c \\
&))*\sinh(d*x + c))*\sqrt{-a*b + b^2}*\log((b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + \\
& c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a - 3*b)*\cosh(d*x + c)^2 + 2* \\
& (3*b*\cosh(d*x + c)^2 - 2*a + 3*b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - \\
& (2*a - 3*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x \\
& + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 + 1)*\sinh(d*x + \\
& c) + \cosh(d*x + c))*\sqrt{-a*b + b^2} + b)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d* \\
& x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + \\
& 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + \\
& (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) + 4*(a^2*b - a*b^2)*\cosh(d*x + \\
& c) + 4*(a^2*b - a*b^2 + 3*(a^2*b - a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c))/ \\
& ((a^2*b^3 - 2*a*b^4 + b^5)*d*\cosh(d*x + c)^4 + 4*(a^2*b^3 - 2*a*b^4 + b^5)* \\
& d*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2*b^3 - 2*a*b^4 + b^5)*d*\sinh(d*x + c) \\
& ^4 + 2*(2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*\cosh(d*x + c)^2 + 2*(3*(a^ \\
& 2*b^3 - 2*a*b^4 + b^5)*d*\cosh(d*x + c)^2 + (2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 \\
& - b^5)*d)*\sinh(d*x + c)^2 + (a^2*b^3 - 2*a*b^4 + b^5)*d + 4*((a^2*b^3 - 2* \\
& a*b^4 + b^5)*d*\cosh(d*x + c)^3 + (2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d* \\
& \cosh(d*x + c))*\sinh(d*x + c)), -1/2*(2*(a^2*b - a*b^2)*\cosh(d*x + c)^3 + 6* \\
& (a^2*b - a*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*(a^2*b - a*b^2)*\sinh(d*x \\
& + c)^3 - ((a*b - 2*b^2)*\cosh(d*x + c)^4 + 4*(a*b - 2*b^2)*\cosh(d*x + c)*\sin \\
& h(d*x + c)^3 + (a*b - 2*b^2)*\sinh(d*x + c)^4 + 2*(2*a^2 - 5*a*b + 2*b^2)*\co \\
& sh(d*x + c)^2 + 2*(3*(a*b - 2*b^2)*\cosh(d*x + c)^2 + 2*a^2 - 5*a*b + 2*b^2) \\
& *\sinh(d*x + c)^2 + a*b - 2*b^2 + 4*((a*b - 2*b^2)*\cosh(d*x + c)^3 + (2*a^2 \\
& - 5*a*b + 2*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a*b - b^2}*\arctan(-1/2* \\
& (b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 \\
& + (4*a - 3*b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - 3*b)*\sinh(d*x + \\
& c))/\sqrt{a*b - b^2}) + ((a*b - 2*b^2)*\cosh(d*x + c)^4 + 4*(a*b - 2*b^2)*\cos \\
& h(d*x + c)*\sinh(d*x + c)^3 + (a*b - 2*b^2)*\sinh(d*x + c)^4 + 2*(2*a^2 - 5*a \\
& *b + 2*b^2)*\cosh(d*x + c)^2 + 2*(3*(a*b - 2*b^2)*\cosh(d*x + c)^2 + 2*a^2 - \\
& 5*a*b + 2*b^2)*\sinh(d*x + c)^2 + a*b - 2*b^2 + 4*((a*b - 2*b^2)*\cosh(d*x + \\
& c)^3 + (2*a^2 - 5*a*b + 2*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a*b - b^2} \\
&)*\arctan(-1/2*\sqrt{a*b - b^2}*(\cosh(d*x + c) + \sinh(d*x + c))/(a - b)) + 2* \\
& (a^2*b - a*b^2)*\cosh(d*x + c) + 2*(a^2*b - a*b^2 + 3*(a^2*b - a*b^2)*\cosh(d \\
& *x + c)^2)*\sinh(d*x + c))/((a^2*b^3 - 2*a*b^4 + b^5)*d*\cosh(d*x + c)^4 + 4* \\
& (a^2*b^3 - 2*a*b^4 + b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2*b^3 - 2*a* \\
& b^4 + b^5)*d*\sinh(d*x + c)^4 + 2*(2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d* \\
& \cosh(d*x + c)^2 + 2*(3*(a^2*b^3 - 2*a*b^4 + b^5)*d*\cosh(d*x + c)^2 + (2*a^3 \\
& *b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d)*\sinh(d*x + c)^2 + (a^2*b^3 - 2*a*b^4 + \\
& b^5)*d + 4*((a^2*b^3 - 2*a*b^4 + b^5)*d*\cosh(d*x + c)^3 + (2*a^3*b^2 - 5*a \\
& ^2*b^3 + 4*a*b^4 - b^5)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)`

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.44 \quad \int \frac{\sinh^2(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$$

Optimal. Leaf size=84

$$\frac{\sinh(c+dx) \cosh(c+dx)}{2d(a-b)(a+b \sinh^2(c+dx))} - \frac{\tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{ad}(a-b)^{3/2}}$$

[Out] -ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a - b)^(3/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.0861296, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3173, 12, 3181, 208}

$$\frac{\sinh(c+dx) \cosh(c+dx)}{2d(a-b)(a+b \sinh^2(c+dx))} - \frac{\tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{ad}(a-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2), x]

[Out] -ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a - b)^(3/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))

Rule 3173

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^2]^(p_))*((A_) + (B_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x] * (a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*(a + b)*(p + 1)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^2(c + dx)}{(a + b \sinh^2(c + dx))^2} dx &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a - b)d(a + b \sinh^2(c + dx))} - \frac{\int \frac{a}{a + b \sinh^2(c + dx)} dx}{2a(a - b)} \\
 &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a - b)d(a + b \sinh^2(c + dx))} - \frac{\int \frac{1}{a + b \sinh^2(c + dx)} dx}{2(a - b)} \\
 &= \frac{\cosh(c + dx) \sinh(c + dx)}{2(a - b)d(a + b \sinh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{1}{a - (a - b)x^2} dx, x, \tanh(c + dx)\right)}{2(a - b)d} \\
 &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a - b} \tanh(c + dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a - b)^{3/2}d} + \frac{\cosh(c + dx) \sinh(c + dx)}{2(a - b)d(a + b \sinh^2(c + dx))}
 \end{aligned}$$

Mathematica [A] time = 0.444132, size = 81, normalized size = 0.96

$$\frac{\frac{\sinh(2(c + dx))}{(a - b)(2a + b \cosh(2(c + dx)) - b)} - \frac{\tanh^{-1}\left(\frac{\sqrt{a - b} \tanh(c + dx)}{\sqrt{a}}\right)}{\sqrt{a}(a - b)^{3/2}}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (-(ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a - b)^(3/2)))) + Sinh[2*(c + d*x)]/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)

Maple [B] time = 0.041, size = 428, normalized size = 5.1

$$\frac{1}{d(a-b)} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^3 \left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^4 a - 2 (\tanh(1/2 dx + c/2))^2 a + 4 (\tanh(1/2 dx + c/2))^2 b + a \right)^{-1} + \frac{1}{d(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)-1/2/d/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b+1/2/d/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.39192, size = 3564, normalized size = 42.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

```
[Out] [-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c)^2 + (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b - 2*a*b^2 + 2*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)]]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [A] time = 1.40178, size = 184, normalized size = 2.19

$$-\frac{\arctan\left(\frac{be^{2dx+2c}+2a-b}{2\sqrt{-a^2+ab}}\right)}{2\sqrt{-a^2+ab}(ad-bd)} - \frac{2ae^{2dx+2c} - be^{2dx+2c} + b}{(abd - b^2d)(be^{4dx+4c} + 4ae^{2dx+2c} - 2be^{2dx+2c} + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-\frac{1}{2} \arctan\left(\frac{1}{2} \frac{b e^{2 d x+2 c}+2 a-b}{\sqrt{-a^2+a b}}\right) / \left(\sqrt{-a^2+a b}\right) \left(a d-b d\right) - \left(2 a e^{2 d x+2 c}-b e^{2 d x+2 c}+b\right) / \left(\left(a b d-b^2 d\right)\left(b e^{4 d x+4 c}+4 a e^{2 d x+2 c}-2 b e^{2 d x+2 c}+b\right)\right)$

$$3.45 \quad \int \frac{\sinh(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=81

$$\frac{\cosh(c+dx)}{2d(a-b)\left(a+b \cosh^2(c+dx)-b\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{bd}(a-b)^{3/2}}$$

[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*(a - b)^(3/2)*Sqrt[b]*d) + Cosh[c + d*x]/(2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2))

Rubi [A] time = 0.0654572, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3186, 199, 205}

$$\frac{\cosh(c+dx)}{2d(a-b)\left(a+b \cosh^2(c+dx)-b\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{bd}(a-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*(a - b)^(3/2)*Sqrt[b]*d) + Cosh[c + d*x]/(2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2))

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 199

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin

ator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\sinh(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{\cosh(c+dx)}{2(a-b)d(a-b+b\cosh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a-b+bx^2} dx, x, \cosh(c+dx)\right)}{2(a-b)d} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}\sqrt{bd}} + \frac{\cosh(c+dx)}{2(a-b)d(a-b+b\cosh^2(c+dx))} \end{aligned}$$

Mathematica [C] time = 0.34833, size = 130, normalized size = 1.6

$$\frac{2\cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)} + \frac{\tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) + \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{b}(a-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2, x]

[Out] ((ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/((a - b)^(3/2)*Sqrt[b]) + (2*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)

Maple [B] time = 0.027, size = 256, normalized size = 3.2

$$-\frac{1}{d(a-b)} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 \left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^4 a - 2(\tanh(1/2 dx + c/2))^2 a + 4(\tanh(1/2 dx + c/2))^2 b + a \right)^{-1} + 2 \frac{1}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)`

[Out]
$$-1/d/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}/(a-b)*\tanh(1/2*d*x+1/2*c)^{2+2/d/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}/(a-b)/a*\tanh(1/2*d*x+1/2*c)^{2*b+1/d/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}/(a-b)+1/2/d/(a-b)/(a*b-b^2)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^{2*a-2*a+4*b})/(a*b-b^2)^{(1/2))})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{(3dx+3c)} + e^{(dx+c)}}{abd - b^2d + (abde^{(4c)} - b^2de^{(4c)})e^{(4dx)} + 2(2a^2de^{(2c)} - 3abde^{(2c)} + b^2de^{(2c)})e^{(2dx)}} + \frac{1}{2} \int \frac{2}{ab - b^2 + (abe^{(4c)} - b^2e^{(4c)})e^{(2dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out]
$$(e^{(3*d*x + 3*c)} + e^{(d*x + c)})/(a*b*d - b^2*d + (a*b*d*e^{(4*c)} - b^2*d*e^{(4*c)})*e^{(4*d*x)} + 2*(2*a^2*d*e^{(2*c)} - 3*a*b*d*e^{(2*c)} + b^2*d*e^{(2*c)})*e^{(2*d*x)}) + 1/2*integrate(2*(e^{(3*d*x + 3*c)} - e^{(d*x + c)})/(a*b - b^2 + (a*b*e^{(4*c)} - b^2*e^{(4*c)})*e^{(4*d*x)} + 2*(2*a^2*e^{(2*c)} - 3*a*b*e^{(2*c)} + b^2*e^{(2*c)})*e^{(2*d*x)}), x)$$

Fricas [B] time = 2.39528, size = 3999, normalized size = 49.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out]
$$[1/4*(4*(a*b - b^2)*\cosh(d*x + c)^3 + 12*(a*b - b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + 4*(a*b - b^2)*\sinh(d*x + c)^3 + (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c)*\sinh(d*x + c) + b)*\sqrt{-a*b + b^2}*\log((b*\cosh(d*x$$

$$\begin{aligned}
& + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - \\
& 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 \\
& + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh \\
& (d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d \\
& *x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh \\
& (d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2* \\
& a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 \\
& + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(\\
& a*b - b^2)*cosh(d*x + c) + 4*(3*(a*b - b^2)*cosh(d*x + c)^2 + a*b - b^2)*si \\
& nh(d*x + c))/((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 - 2* \\
& a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d* \\
& sinh(d*x + c)^4 + 2*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*d*cosh(d*x + c)^2 \\
& + 2*(3*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (2*a^3*b - 5*a^2*b^2 \\
& + 4*a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*d + 4*((a^2 \\
& *b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - \\
& b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a*b - b^2)*cosh(d*x + c)^3 + \\
& 6*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a*b - b^2)*sinh(d*x + c)^3 \\
& + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c) \\
& ^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d \\
& *x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + \\
& b)*sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh \\
& (d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x \\
& + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - (b*cosh(d*x + c)^4 + \\
& 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d \\
& *x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d \\
& *x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(a*b - b^2)*arc \\
& tan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*(a*b \\
& - b^2)*cosh(d*x + c) + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + a*b - b^2)*sinh(d \\
& *x + c))/((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 - 2*a*b^ \\
& 3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d*sinh \\
& (d*x + c)^4 + 2*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2 \\
& *(3*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (2*a^3*b - 5*a^2*b^2 + 4* \\
& a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*d + 4*((a^2*b^2 \\
& - 2*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4) \\
& *d*cosh(d*x + c))*sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.46 \quad \int \frac{1}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=95

$$\frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}} - \frac{b \sinh(c+dx) \cosh(c+dx)}{2ad(a-b)(a+b \sinh^2(c+dx))}$$

[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^(3/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*a*(a - b)*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.0694698, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3184, 12, 3181, 208}

$$\frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}} - \frac{b \sinh(c+dx) \cosh(c+dx)}{2ad(a-b)(a+b \sinh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^(-2), x]

[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^(3/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*a*(a - b)*d*(a + b*Sinh[c + d*x]^2))

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a + b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^2]^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a + b \sinh^2(c + dx))^2} dx &= -\frac{b \cosh(c + dx) \sinh(c + dx)}{2a(a - b)d(a + b \sinh^2(c + dx))} - \frac{\int \frac{-2a+b}{a+b \sinh^2(c+dx)} dx}{2a(a - b)} \\
 &= -\frac{b \cosh(c + dx) \sinh(c + dx)}{2a(a - b)d(a + b \sinh^2(c + dx))} + \frac{(2a - b) \int \frac{1}{a+b \sinh^2(c+dx)} dx}{2a(a - b)} \\
 &= -\frac{b \cosh(c + dx) \sinh(c + dx)}{2a(a - b)d(a + b \sinh^2(c + dx))} + \frac{(2a - b) \text{Subst}\left(\int \frac{1}{a-(a-b)x^2} dx, x, \tanh(c + dx)\right)}{2a(a - b)d} \\
 &= \frac{(2a - b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a - b)^{3/2}d} - \frac{b \cosh(c + dx) \sinh(c + dx)}{2a(a - b)d(a + b \sinh^2(c + dx))}
 \end{aligned}$$

Mathematica [A] time = 0.302547, size = 96, normalized size = 1.01

$$\frac{(2a - b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a - b)^{3/2}} - \frac{b \sinh(2(c + dx))}{2ad(a - b)(2a + b \cosh(2(c + dx)) - b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-2),x]

[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^(3/2)*d) - (b*Sinh[2*(c + d*x)])/(2*a*(a - b)*d*(2*a - b + b*Cosh[2*(c + d*x)]))

Maple [B] time = 0.046, size = 749, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a+b*\sinh(d*x+c))^2,x)$

[Out]
$$\begin{aligned} & -1/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*b/a/(a-b)*\tanh(1/2*d*x+1/2*c)^3-1/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*b/a/(a-b)*\tanh(1/2*d*x+1/2*c)+1/d/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b-1/d/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b-1/2/d/(a-b)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(a-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2+1/2/d/(a-b)*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/(a-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2 \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(a+b*\sinh(d*x+c))^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [B] time = 2.43671, size = 3767, normalized size = 39.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c)^2 + ((2*a*b - b^2)*cosh(d*x + c)^4 + 4*(2*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b - b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(d*x + c))^3 + (4*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a^2*b - 2*a*b^2 + 2*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c)^2 - ((2*a*b - b^2)*cosh(d*x + c)^4 + 4*(2*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b - b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(d*x + c)^3 + (4*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)]]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [A] time = 1.21252, size = 196, normalized size = 2.06

$$\frac{(2a - b) \arctan\left(\frac{be^{(2dx+2c)} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{2(a^2d - abd)\sqrt{-a^2 + ab}} + \frac{2ae^{(2dx+2c)} - be^{(2dx+2c)} + b}{(a^2d - abd)(be^{(4dx+4c)} + 4ae^{(2dx+2c)} - 2be^{(2dx+2c)} + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*(2*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^2*d - a*b*d)*sqrt(-a^2 + a*b)) + (2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((a^2*d - a*b*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b))

$$3.47 \quad \int \frac{\operatorname{csch}(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=110

$$-\frac{\sqrt{b}(3a-2b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2d(a-b)^{3/2}} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{b \cosh(c+dx)}{2ad(a-b)(a+b \cosh^2(c+dx)-b)}$$

[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*a^2*(a - b)^(3/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^2*d) - (b*Cosh[c + d*x])/(2*a*(a - b)*d*(a - b + b*Cosh[c + d*x]^2))

Rubi [A] time = 0.149327, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3186, 414, 522, 206, 205}

$$-\frac{\sqrt{b}(3a-2b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2d(a-b)^{3/2}} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{b \cosh(c+dx)}{2ad(a-b)(a+b \cosh^2(c+dx)-b)}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*a^2*(a - b)^(3/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^2*d) - (b*Cosh[c + d*x])/(2*a*(a - b)*d*(a - b + b*Cosh[c + d*x]^2))

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(
n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\sinh^2(c+dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^2} dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b \cosh(c+dx)}{2a(a-b)d(a-b+b\cosh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{-2a+b+bx^2}{(1-x^2)(a-b+bx^2)} dx, x, \cosh(c+dx)\right)}{2a(a-b)d}$$

$$= -\frac{b \cosh(c+dx)}{2a(a-b)d(a-b+b\cosh^2(c+dx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c+dx)\right)}{a^2d} - \frac{((3a-2b)b)}{2a(a-b)d(a-b+b\cosh^2(c+dx))}$$

$$= -\frac{(3a-2b)\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{b \cosh(c+dx)}{2a(a-b)d(a-b+b\cosh^2(c+dx))}$$

Mathematica [C] time = 0.644036, size = 176, normalized size = 1.6

$$\frac{\frac{2ab \cosh(c+dx)}{(a-b)(2a+b \cosh(2(c+dx))-b)} + \frac{\sqrt{b}(2b-3a) \tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{b}(2b-3a) \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + 2 \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(3/2) + (Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(3/2) - (2*a*b*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])) + 2*Log[Tanh[(c + d*x)/2]]/(2*a^2*d)

Maple [B] time = 0.062, size = 350, normalized size = 3.2

$$\frac{1}{da^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b}{d(a-b)a} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 \left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^4 a - 2(\tanh(1/2 dx + c/2))^2 a + 4(\tanh(1/2 dx + c/2))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d/a^2*ln(tanh(1/2*d*x+1/2*c))+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)/a*tanh(1/2*d*x+1/2*c)^2*b-2/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2-1/d/a*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)-3/2/d/a*b/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))+1/d/a^2*b^2/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{be^{3dx+3c} + be^{dx+c}}{a^2bd - ab^2d + (a^2bde^{4c} - ab^2de^{4c})e^{4dx} + 2(2a^3de^{2c} - 3a^2bde^{2c} + ab^2de^{2c})e^{2dx}} - \frac{\log\left(\left(e^{dx+c} + 1\right)e^{-c}\right)}{a^2d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(4*c)
- a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(2*a^3*d*e^(2*c) - 3*a^2*b*d*e^(2*c) + a*b
^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((e^(
d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) - 2*b^2*e^(
3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^3*b - a^2*b^2 + (a^3*
b*e^(4*c) - a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 3*a^3*b*e^(2*c)
+ a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.91061, size = 6178, normalized size = 56.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) - ((3*a*b - 2*b^2)*cosh(d*x + c)^4 +
4*(3*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x
+ c)^4 + 2*(6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*
cosh(d*x + c)^2 + 6*a^2 - 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a*b - 2*b^2 +
4*((3*a*b - 2*b^2)*cosh(d*x + c)^3 + (6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)
*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2
*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a
- b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh
(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a -
b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d
*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x
+ c) + b)) + 4*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cos
h(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh
(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b +
b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
- 4*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^2
```

$$\begin{aligned}
& + 2*(3*(a*b - b^2)*\cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*\sinh(d*x + c)^2 \\
& + a*b - b^2 + 4*((a*b - b^2)*\cosh(d*x + c)^3 + (2*a^2 - 3*a*b + b^2)*\cosh(d \\
& *x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 4*(3*a*b*c \\
& \cosh(d*x + c)^2 + a*b)*\sinh(d*x + c))/((a^3*b - a^2*b^2)*d*\cosh(d*x + c)^4 + \\
& 4*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^3*b - a^2*b^2)*d* \\
& \sinh(d*x + c)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*d*\cosh(d*x + c)^2 + 2*(3*(a \\
& ^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 + (2*a^4 - 3*a^3*b + a^2*b^2)*d)*\sinh(d*x \\
& + c)^2 + (a^3*b - a^2*b^2)*d + 4*((a^3*b - a^2*b^2)*d*\cosh(d*x + c)^3 + (2 \\
& *a^4 - 3*a^3*b + a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/2*(2*a*b*\cosh \\
& (d*x + c)^3 + 6*a*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*a*b*\sinh(d*x + c)^3 + \\
& 2*a*b*\cosh(d*x + c) + ((3*a*b - 2*b^2)*\cosh(d*x + c)^4 + 4*(3*a*b - 2*b^2) \\
& *\cosh(d*x + c)*\sinh(d*x + c)^3 + (3*a*b - 2*b^2)*\sinh(d*x + c)^4 + 2*(6*a^2 \\
& - 7*a*b + 2*b^2)*\cosh(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*\cosh(d*x + c)^2 + \\
& 6*a^2 - 7*a*b + 2*b^2)*\sinh(d*x + c)^2 + 3*a*b - 2*b^2 + 4*((3*a*b - 2*b^2) \\
& *\cosh(d*x + c)^3 + (6*a^2 - 7*a*b + 2*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*sq \\
& rt(b/(a - b))*\arctan(1/2*sqrt(b/(a - b))*(\cosh(d*x + c) + \sinh(d*x + c))) - \\
& ((3*a*b - 2*b^2)*\cosh(d*x + c)^4 + 4*(3*a*b - 2*b^2)*\cosh(d*x + c)*\sinh(d* \\
& x + c)^3 + (3*a*b - 2*b^2)*\sinh(d*x + c)^4 + 2*(6*a^2 - 7*a*b + 2*b^2)*\cosh \\
& (d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*\cosh(d*x + c)^2 + 6*a^2 - 7*a*b + 2*b^2) \\
& *\sinh(d*x + c)^2 + 3*a*b - 2*b^2 + 4*((3*a*b - 2*b^2)*\cosh(d*x + c)^3 + (6* \\
& a^2 - 7*a*b + 2*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*sqrt(b/(a - b))*\arctan(1 \\
& /2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c) \\
& ^3 + (4*a - 3*b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - 3*b)*\sinh(d*x \\
& + c))*sqrt(b/(a - b))/b) + 2*((a*b - b^2)*\cosh(d*x + c)^4 + 4*(a*b - b^2)* \\
& \cosh(d*x + c)*\sinh(d*x + c)^3 + (a*b - b^2)*\sinh(d*x + c)^4 + 2*(2*a^2 - 3* \\
& a*b + b^2)*\cosh(d*x + c)^2 + 2*(3*(a*b - b^2)*\cosh(d*x + c)^2 + 2*a^2 - 3*a \\
& *b + b^2)*\sinh(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*\cosh(d*x + c)^3 + (2 \\
& *a^2 - 3*a*b + b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(\\
& d*x + c) + 1) - 2*((a*b - b^2)*\cosh(d*x + c)^4 + 4*(a*b - b^2)*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^3 + (a*b - b^2)*\sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*c \\
& \cosh(d*x + c)^2 + 2*(3*(a*b - b^2)*\cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*si \\
& nh(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*\cosh(d*x + c)^3 + (2*a^2 - 3*a*b \\
& + b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1 \\
&) + 2*(3*a*b*\cosh(d*x + c)^2 + a*b)*\sinh(d*x + c))/((a^3*b - a^2*b^2)*d*cos \\
& h(d*x + c)^4 + 4*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^3*b \\
& - a^2*b^2)*d*\sinh(d*x + c)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*d*\cosh(d*x + \\
& c)^2 + 2*(3*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 + (2*a^4 - 3*a^3*b + a^2*b^ \\
& 2)*d)*\sinh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 4*((a^3*b - a^2*b^2)*d*\cosh(d \\
& *x + c)^3 + (2*a^4 - 3*a^3*b + a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


$$3.48 \quad \int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=142

$$\frac{(2a^2 - 4ab + 3b^2) \tanh(c + dx)}{2a^2d(a - b)(a - (a - b) \tanh^2(c + dx))} - \frac{b(4a - 3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a - b)^{3/2}} - \frac{\operatorname{coth}(c + dx)}{ad(a - (a - b) \tanh^2(c + dx))}$$

[Out] -((4*a - 3*b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*(a - b)^(3/2)*d) - Coth[c + d*x]/(a*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((2*a^2 - 4*a*b + 3*b^2)*Tanh[c + d*x])/(2*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.153389, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3187, 462, 385, 208}

$$\frac{(2a^2 - 4ab + 3b^2) \tanh(c + dx)}{2a^2d(a - b)(a - (a - b) \tanh^2(c + dx))} - \frac{b(4a - 3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a - b)^{3/2}} - \frac{\operatorname{coth}(c + dx)}{ad(a - (a - b) \tanh^2(c + dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]

[Out] -((4*a - 3*b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*(a - b)^(3/2)*d) - Coth[c + d*x]/(a*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((2*a^2 - 4*a*b + 3*b^2)*Tanh[c + d*x])/(2*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3187

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)]/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 462

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 385

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c + dx)}{(a + b \sinh^2(c + dx))^2} dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^2(a-(a-b)x^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{\operatorname{coth}(c + dx)}{ad(a - (a - b) \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{a-3b+ax^2}{(a+(-a+b)x^2)^2} dx, x, \tanh(c + dx)\right)}{ad}$$

$$= -\frac{\operatorname{coth}(c + dx)}{ad(a - (a - b) \tanh^2(c + dx))} + \frac{(2a^2 - 4ab + 3b^2) \tanh(c + dx)}{2a^2(a - b)d(a - (a - b) \tanh^2(c + dx))} - \frac{((4a - 3b)b)}{2a^2(a - b)d(a - (a - b) \tanh^2(c + dx))}$$

$$= -\frac{(4a - 3b)b \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a - b)^{3/2}d} - \frac{\operatorname{coth}(c + dx)}{ad(a - (a - b) \tanh^2(c + dx))} + \frac{(2a^2 - 4ab + 3b^2) \tanh(c + dx)}{2a^2(a - b)d(a - (a - b) \tanh^2(c + dx))}$$

Mathematica [A] time = 0.811308, size = 170, normalized size = 1.2

$$\frac{\operatorname{csch}^5(c + dx)(2a + b \cosh(2(c + dx)) - b) \left(2\sqrt{a}\sqrt{a-b} \cosh(c + dx) (4a^2 + b(2a - 3b) \cosh(2(c + dx)) - 6ab + 3b^2) - 2\right)}{16a^{5/2}d(a - b)^{3/2} (\operatorname{acsch}^2(c + dx) + b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]

[Out] $-\left(\left(2a - b + b \cosh[2(c + d*x)]\right) \operatorname{Csch}[c + d*x]^5 \left(2\sqrt{a} \sqrt{a - b} \operatorname{Cosh}[c + d*x] \left(4a^2 - 6ab + 3b^2 + (2a - 3b)b \cosh[2(c + d*x)]\right) - 2b \left(-4a + 3b\right) \operatorname{ArcTanh}\left[\frac{\sqrt{a - b} \operatorname{Tanh}[c + d*x]}{\sqrt{a}}\right] \left(2a - b + b \cosh[2(c + d*x)]\right) \operatorname{Sinh}[c + d*x]\right)\right) / \left(16a^{5/2} (a - b)^{3/2} d (b + a \operatorname{Csch}[c + d*x]^2)^2\right)$

Maple [B] time = 0.076, size = 810, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)

[Out] $-\frac{1}{2} \frac{d}{a^2} \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right) - \frac{1}{2} \frac{d}{a^2} \frac{1}{\operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)} + \frac{1}{d} \frac{a^2 b^2}{\left(\operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right)^4 a - 2 \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 a + 4 \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 b + a} (a - b) \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)^3 + \frac{1}{d} \frac{a^2 b^2}{\left(\operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right)^4 a - 2 \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 a + 4 \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 b + a} (a - b) \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right) - \frac{2}{d} \frac{(a - b) b / a}{\left(\left(2(-b(a - b))^{1/2} + a - 2b\right) a\right)^{1/2} \operatorname{arctanh}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} + a - 2b\right) a\right)^{1/2}\right) + 2/d} (a - b) / a / \left(-b(a - b)\right)^{1/2} / \left(\left(2(-b(a - b))^{1/2} + a - 2b\right) a\right)^{1/2} \operatorname{arctanh}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} + a - 2b\right) a\right)^{1/2}\right) + 2/d} (a - b) / a / \left(-b(a - b)\right)^{1/2} / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2} \operatorname{arctan}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2}\right) + 2/d} (a - b) / a / \left(-b(a - b)\right)^{1/2} / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2} \operatorname{arctan}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2}\right) * b^2 + 2/d} (a - b) * b / a / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2} \operatorname{arctan}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2}\right) + 2/d} (a - b) / a / \left(-b(a - b)\right)^{1/2} / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2} \operatorname{arctan}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2}\right) * b^2 + 3/2} d} a^2 b^2 / (a - b) / \left(\left(2(-b(a - b))^{1/2} + a - 2b\right) a\right)^{1/2} \operatorname{arctanh}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} + a - 2b\right) a\right)^{1/2}\right) - 3/2} d} a^2 b^3 / (a - b) / \left(-b(a - b)\right)^{1/2} / \left(\left(2(-b(a - b))^{1/2} + a - 2b\right) a\right)^{1/2} \operatorname{arctanh}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} + a - 2b\right) a\right)^{1/2}\right) - 3/2} d} a^2 b^2 / (a - b) / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2} \operatorname{arctan}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2}\right) * a^{1/2} \operatorname{arctan}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2}\right) - 3/2} d} a^2 b^3 / (a - b) / \left(-b(a - b)\right)^{1/2} / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2} \operatorname{arctan}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2}\right) * \operatorname{arctan}\left(a \operatorname{tanh}\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) / \left(\left(2(-b(a - b))^{1/2} - a + 2b\right) a\right)^{1/2}\right)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62142, size = 6962, normalized size = 49.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 16*(4*a^3*b - 7*
a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(4*a^3*b - 7*a^2*b^2 +
3*a*b^3)*sinh(d*x + c)^4 + 8*a^3*b - 20*a^2*b^2 + 12*a*b^3 + 8*(4*a^4 - 11
*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d*x + c)^2 + 8*(4*a^4 - 11*a^3*b + 10*a
^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 - ((4*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 6*(4*a*b^2 - 3*b^3)*cosh(
d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - 3*b^3)*sinh(d*x + c)^6 + (16*a^2*b -
24*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + (16*a^2*b - 24*a*b^2 + 9*b^3 + 15*(4*a*
b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - 3*b^3)*cosh
(d*x + c)^3 + (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
- 4*a*b^2 + 3*b^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2 + (15*(4*
a*b^2 - 3*b^3)*cosh(d*x + c)^4 - 16*a^2*b + 24*a*b^2 - 9*b^3 + 6*(16*a^2*b
- 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - 3*b^
3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^3 - (16*
a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log
((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x
+ c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
- b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(
d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*
(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)
^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) +
16*((4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 11*a^3*b + 1
0*a^2*b^2 - 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 2*a^4*b^2 + a^
3*b^3)*d*cosh(d*x + c)^6 + 6*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^5 + (a^5*b - 2*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^6 + (4*a^6
- 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^4 + (15*(a^5*b - 2*a^4
```

$$\begin{aligned}
& *b^2 + a^3*b^3)*d*\cosh(d*x + c)^2 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3) \\
& *b^3)*d)*\sinh(d*x + c)^4 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*\cos \\
& h(d*x + c)^2 + 4*(5*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^3 + (4*a^ \\
& 6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (\\
& 15*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^4 + 6*(4*a^6 - 11*a^5*b + \\
& 10*a^4*b^2 - 3*a^3*b^3)*d*\cosh(d*x + c)^2 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 \\
& - 3*a^3*b^3)*d)*\sinh(d*x + c)^2 - (a^5*b - 2*a^4*b^2 + a^3*b^3)*d + 2*(3*(a \\
& ^5*b - 2*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^5 + 2*(4*a^6 - 11*a^5*b + 10*a^ \\
& 4*b^2 - 3*a^3*b^3)*d*\cosh(d*x + c)^3 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a \\
& ^3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/2*(2*(4*a^3*b - 7*a^2*b^2 + 3*a \\
& *b^3)*\cosh(d*x + c)^4 + 8*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)*\sin \\
& h(d*x + c)^3 + 2*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*\sinh(d*x + c)^4 + 4*a^3*b \\
& - 10*a^2*b^2 + 6*a*b^3 + 4*(4*a^4 - 11*a^3*b + 10*a^2*b^2 - 3*a*b^3)*\cosh(d \\
& *x + c)^2 + 4*(4*a^4 - 11*a^3*b + 10*a^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 7*a^2 \\
& *b^2 + 3*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - ((4*a*b^2 - 3*b^3)*\cosh(\\
& d*x + c)^6 + 6*(4*a*b^2 - 3*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^5 + (4*a*b^2 - \\
& 3*b^3)*\sinh(d*x + c)^6 + (16*a^2*b - 24*a*b^2 + 9*b^3)*\cosh(d*x + c)^4 + (\\
& 16*a^2*b - 24*a*b^2 + 9*b^3 + 15*(4*a*b^2 - 3*b^3)*\cosh(d*x + c)^2)*\sinh(d* \\
& x + c)^4 + 4*(5*(4*a*b^2 - 3*b^3)*\cosh(d*x + c)^3 + (16*a^2*b - 24*a*b^2 + \\
& 9*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*a*b^2 + 3*b^3 - (16*a^2*b - 24*a* \\
& b^2 + 9*b^3)*\cosh(d*x + c)^2 + (15*(4*a*b^2 - 3*b^3)*\cosh(d*x + c)^4 - 16*a \\
& ^2*b + 24*a*b^2 - 9*b^3 + 6*(16*a^2*b - 24*a*b^2 + 9*b^3)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - 3*b^3)*\cosh(d*x + c)^5 + 2*(16*a^2*b - 24 \\
& *a*b^2 + 9*b^3)*\cosh(d*x + c)^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*\cosh(d*x + \\
& c))*\sinh(d*x + c))*\sqrt{-a^2 + a*b}*\arctan(-1/2*(b*\cosh(d*x + c)^2 + 2*b*\co \\
& sh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-a^2 + a*b})/(\\
& a^2 - a*b)) + 8*((4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^3 + (4*a^4 - \\
& 11*a^3*b + 10*a^2*b^2 - 3*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5*b - 2 \\
& *a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^6 + 6*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*\c \\
& osh(d*x + c)*\sinh(d*x + c)^5 + (a^5*b - 2*a^4*b^2 + a^3*b^3)*d*\sinh(d*x + c \\
&)^6 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*\cosh(d*x + c)^4 + (15*(\\
& a^5*b - 2*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^2 + (4*a^6 - 11*a^5*b + 10*a^4 \\
& *b^2 - 3*a^3*b^3)*d)*\sinh(d*x + c)^4 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a \\
& ^3*b^3)*d*\cosh(d*x + c)^2 + 4*(5*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + \\
& c)^3 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*\cosh(d*x + c))*\sinh(d \\
& *x + c)^3 + (15*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^4 + 6*(4*a^6 \\
& - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*\cosh(d*x + c)^2 - (4*a^6 - 11*a^5*b \\
& + 10*a^4*b^2 - 3*a^3*b^3)*d)*\sinh(d*x + c)^2 - (a^5*b - 2*a^4*b^2 + a^3*b^3 \\
&)*d + 2*(3*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*\cosh(d*x + c)^5 + 2*(4*a^6 - 11* \\
& a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*\cosh(d*x + c)^3 - (4*a^6 - 11*a^5*b + 10* \\
& a^4*b^2 - 3*a^3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [A] time = 1.48533, size = 312, normalized size = 2.2

$$\frac{(4ab - 3b^2) \arctan\left(\frac{be^{(2dx+2c)} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{2(a^3d - a^2bd)\sqrt{-a^2 + ab}} - \frac{4abe^{(4dx+4c)} - 3b^2e^{(4dx+4c)} + 8a^2e^{(2dx+2c)} - 14abe^{(2dx+2c)} + 6b^2e^{(2dx+2c)} + 2a}{(a^3d - a^2bd)(be^{(6dx+6c)} + 4ae^{(4dx+4c)} - 3be^{(4dx+4c)} - 4ae^{(2dx+2c)} + 3be^{(2dx+2c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-1/2*(4*a*b - 3*b^2)*\arctan(1/2*(b*e^{(2*d*x + 2*c)} + 2*a - b)/\sqrt{-a^2 + a*b})/((a^3*d - a^2*b*d)*\sqrt{-a^2 + a*b}) - (4*a*b*e^{(4*d*x + 4*c)} - 3*b^2*e^{(4*d*x + 4*c)} + 8*a^2*e^{(2*d*x + 2*c)} - 14*a*b*e^{(2*d*x + 2*c)} + 6*b^2*e^{(2*d*x + 2*c)} + 2*a*b - 3*b^2)/((a^3*d - a^2*b*d)*(b*e^{(6*d*x + 6*c)} + 4*a*e^{(4*d*x + 4*c)} - 3*b*e^{(4*d*x + 4*c)} - 4*a*e^{(2*d*x + 2*c)} + 3*b*e^{(2*d*x + 2*c)} - b))$

$$3.49 \quad \int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=161

$$\frac{b^{3/2}(5a-4b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3d(a-b)^{3/2}} - \frac{b(a-2b) \cosh(c+dx)}{2a^2d(a-b)(a+b \cosh^2(c+dx)-b)} + \frac{(a+4b) \tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{\coth(c+dx)}{2ad(a+b \sinh^2(c+dx))}$$

[Out] ((5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*a^3*(a - b)^(3/2)*d) + ((a + 4*b)*ArcTanh[Cosh[c + d*x]]/(2*a^3*d) - ((a - 2*b)*b*Cosh[c + d*x])/(2*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cosh[c + d*x]^2))

Rubi [A] time = 0.272916, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3186, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}(5a-4b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3d(a-b)^{3/2}} - \frac{b(a-2b) \cosh(c+dx)}{2a^2d(a-b)(a+b \cosh^2(c+dx)-b)} + \frac{(a+4b) \tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{\coth(c+dx)}{2ad(a+b \sinh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*a^3*(a - b)^(3/2)*d) + ((a + 4*b)*ArcTanh[Cosh[c + d*x]]/(2*a^3*d) - ((a - 2*b)*b*Cosh[c + d*x])/(2*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cosh[c + d*x]^2))

Rule 3186

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]

```

Rule 527

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

```

Rule 522

```

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 205

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+bx^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad(a-b+b\cosh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{a+b+3bx^2}{(1-x^2)(a-b+bx^2)^2} dx, x, \cosh(c+dx)\right)}{2ad} \\
&= -\frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d(a-b+b\cosh^2(c+dx))} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad(a-b+b\cosh^2(c+dx))} - \frac{\operatorname{Subst}\left(\int \frac{-2(}{(1-x^2)(a-b+bx^2)^2} dx, x, \cosh(c+dx)\right)}{2ad} \\
&= -\frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d(a-b+b\cosh^2(c+dx))} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad(a-b+b\cosh^2(c+dx))} + \frac{((5a-4b)b^2)}{2ad} \\
&= \frac{(5a-4b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3(a-b)^{3/2}d} + \frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{(a-2b)b}{2a^2(a-b)d(a-b+b\cosh^2(c+dx))}
\end{aligned}$$

Mathematica [C] time = 1.4059, size = 350, normalized size = 2.17

$$\operatorname{csch}^3(c+dx)(2a+b\cosh(2(c+dx)))-b\left(\frac{8ab^2\coth(c+dx)}{a-b}+\frac{4b^{3/2}(5a-4b)\operatorname{csch}(c+dx)(2a+b\cosh(2(c+dx)))-b\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^3*((8*a*b^2*Coth[c + d*x])/(a - b) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^(3/2) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^(3/2) - a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[(c + d*x)/2]^2*Csch[c + d*x] - 4*(a + 4*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Log[Tanh[(c + d*x)/2]] - a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Sech[(c + d*x)/2]^2))/(32*a^3*d*(b + a*Csch[c + d*x]^2)^2)

Maple [B] time = 0.079, size = 415, normalized size = 2.6

$$\frac{1}{8da^2} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{1}{8da^2} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-2} - \frac{1}{2da^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) - 2 \frac{\ln(\tanh(1/2 dx + c/2))b}{da^3} - \frac{b^2}{da^2(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)

[Out] $\frac{1}{8} \frac{1}{d} \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 / a^2 - \frac{1}{8} \frac{1}{d} \frac{1}{a^2} \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^{-2} - \frac{1}{2} \frac{1}{d} \frac{1}{a^2} \ln\left(\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) - \frac{2}{d} \frac{1}{a^3} \ln\left(\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)\right) * b - \frac{1}{d} \frac{1}{a^2} \frac{b^2}{(a-b)}$
 $+ \frac{\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 * a - 2 * \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 * a + 4 * \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 * b + a}{(a-b) * \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 + 2/d * b^3/a^3} / (\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 * a - 2 * \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 * a + 4 * \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 * b + a) / (a-b) * \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 + 1/d * b^2/a^2 / (\tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^4 * a - 2 * \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 * a + 4 * \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)^2 * b + a) / (a-b) + 5/2/d/a^2 * b^2/(a-b) / (a*b-b^2)^(1/2) * arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2)) - 2/d*b^3/a^3/(a-b) / (a*b-b^2)^(1/2) * arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(abe^{7c} - 2b^2e^{7c})e^{7dx} + (4a^2e^{5c} - 5abe^{5c} + 2b^2e^{5c})e^{5dx} + (4a^2e^{3c} - 5abe^{3c} + 2b^2e^{3c})e^{3dx}}{a^3bd - a^2b^2d + (a^3bde^{8c} - a^2b^2de^{8c})e^{8dx} + 4(a^4de^{6c} - 2a^3bde^{6c} + a^2b^2de^{6c})e^{6dx} - 2(4a^4de^{4c} - 7a^3bde^{4c} + 4a^2b^2de^{4c})e^{4dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $-\left(\frac{a*b*e^{7*c} - 2*b^2*e^{7*c}}{a^3*b*d - a^2*b^2*d + (a^3*b*d*e^{8*c} - a^2*b^2*d*e^{8*c})e^{8*d*x} + 4*(a^4*d*e^{6*c} - 2*a^3*b*d*e^{6*c} + a^2*b^2*d*e^{6*c})e^{6*d*x} - 2*(4*a^4*d*e^{4*c} - 7*a^3*b*d*e^{4*c} + 3*a^2*b^2*d*e^{4*c})e^{4*d*x} + 4*(a^4*d*e^{2*c} - 2*a^3*b*d*e^{2*c} + a^2*b^2*d*e^{2*c})e^{2*d*x}\right) * e^{(7*d*x)}$
 $+ (4*a^2*e^{5*c} - 5*a*b*e^{5*c} + 2*b^2*e^{5*c}) * e^{(5*d*x)}$
 $+ (4*a^2*e^{3*c} - 5*a*b*e^{3*c} + 2*b^2*e^{3*c}) * e^{(3*d*x)}$
 $+ \frac{1}{a^3*d} \ln\left(\frac{(e^{d*x+c} + 1) * e^{-c}}{(e^{d*x+c} - 1) * e^{-c}}\right) + 8 * \int \frac{(5*a*b^2*e^{3*c} - 4*b^3*e^{3*c}) * e^{(3*d*x)} - (5*a*b^2*e^c - 4*b^3*e^c) * e^{(d*x)}}{(a^4*b - a^3*b^2 + (a^4*b*e^{4*c} - a^3*b^2*e^{4*c}) * e^{(4*d*x)})} dx$

) + 2*(2*a^5*e^(2*c) - 3*a^4*b*e^(2*c) + a^3*b^2*e^(2*c))*e^(2*d*x)), x)

Fricas [B] time = 3.66935, size = 19038, normalized size = 118.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [-1/4*(4*(a^2*b - 2*a*b^2)*cosh(d*x + c)^7 + 28*(a^2*b - 2*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(a^2*b - 2*a*b^2)*sinh(d*x + c)^7 + 4*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^5 + 4*(4*a^3 - 5*a^2*b + 2*a*b^2 + 21*(a^2*b - 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b - 2*a*b^2)*cosh(d*x + c)^3 + (4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 4*(35*(a^2*b - 2*a*b^2)*cosh(d*x + c)^4 + 4*a^3 - 5*a^2*b + 2*a*b^2 + 10*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(a^2*b - 2*a*b^2)*cosh(d*x + c)^5 + 10*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((5*a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8*(5*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 20*a^2*b + 31*a*b^2 - 12*b^3 + 30*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^3 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^2 - 4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 5*a^2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^3 + (5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*co

$$\begin{aligned}
& \text{sh}(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) + 4*(a^2*b - 2*a*b^2)*\cosh(d*x + c) - 2*((a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^8 + 8*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b + 3*a*b^2 - 4*b^3)*\sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\cosh(d*x + c)^4 + 2*(35*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b^2 - 12*b^3 + 30*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^5 + 10*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^6 + 15*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^4 + a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^7 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\cosh(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 2*((a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^8 + 8*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b + 3*a*b^2 - 4*b^3)*\sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\cosh(d*x + c)^4 + 2*(35*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b^2 - 12*b^3 + 30*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^5 + 10*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^6 + 15*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^4 + a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*\cosh(d*x + c)^7 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\cosh(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 4*(7*(a^2*b - 2*a*b^2)*\cosh(d*x + c)^6 + 5*(4*a^3 - 5*a^2*b + 2*a*b^2)*\cosh(d*x + c)^4 + a^2*b - 2*a*b^2 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^4*b - a^3*b^2)*d*\cosh(d*x + c)^8 + 8*(a^4*b - a^3*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^4*b - a^3*b^2)*d*\sinh(d*x + c)^8 + 4*(a^5 - 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^4*b - a^3*b^2)*d*\cosh(d*x + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*\sinh(d*x + c)^6 - 2*(4*a^5 - 7*a^4*b +
\end{aligned}$$

$$\begin{aligned}
& 3a^3b^2*d*cosh(dx + c)^4 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(dx + c)^3 + 3 \\
& *(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(dx + c))*sinh(dx + c)^5 + 2*(35*(a^4*b \\
& - a^3*b^2)*d*cosh(dx + c)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(dx + c) \\
& ^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d)*sinh(dx + c)^4 + 4*(a^5 - 2*a^4*b + \\
& a^3*b^2)*d*cosh(dx + c)^2 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(dx + c)^5 + 10* \\
& (a^5 - 2*a^4*b + a^3*b^2)*d*cosh(dx + c)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^2) \\
& *d*cosh(dx + c))*sinh(dx + c)^3 + 4*(7*(a^4*b - a^3*b^2)*d*cosh(dx + c)^6 \\
& + 15*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(dx + c)^4 - 3*(4*a^5 - 7*a^4*b + 3 \\
& *a^3*b^2)*d*cosh(dx + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*sinh(dx + c)^2 \\
& + (a^4*b - a^3*b^2)*d + 8*((a^4*b - a^3*b^2)*d*cosh(dx + c)^7 + 3*(a^5 - 2 \\
& *a^4*b + a^3*b^2)*d*cosh(dx + c)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d*cosh(\\
& dx + c)^3 + (a^5 - 2*a^4*b + a^3*b^2)*d*cosh(dx + c))*sinh(dx + c)), -1/ \\
& 2*(2*(a^2*b - 2*a*b^2)*cosh(dx + c)^7 + 14*(a^2*b - 2*a*b^2)*cosh(dx + c) \\
& *sinh(dx + c)^6 + 2*(a^2*b - 2*a*b^2)*sinh(dx + c)^7 + 2*(4*a^3 - 5*a^2*b \\
& + 2*a*b^2)*cosh(dx + c)^5 + 2*(4*a^3 - 5*a^2*b + 2*a*b^2 + 21*(a^2*b - 2* \\
& a*b^2)*cosh(dx + c)^2)*sinh(dx + c)^5 + 10*(7*(a^2*b - 2*a*b^2)*cosh(dx \\
& + c)^3 + (4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(dx + c))*sinh(dx + c)^4 + 2*(4* \\
& a^3 - 5*a^2*b + 2*a*b^2)*cosh(dx + c)^3 + 2*(35*(a^2*b - 2*a*b^2)*cosh(dx \\
& + c)^4 + 4*a^3 - 5*a^2*b + 2*a*b^2 + 10*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d \\
& *x + c)^2)*sinh(dx + c)^3 + 2*(21*(a^2*b - 2*a*b^2)*cosh(dx + c)^5 + 10*(\\
& 4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(dx + c)^3 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)* \\
& cosh(dx + c))*sinh(dx + c)^2 - ((5*a*b^2 - 4*b^3)*cosh(dx + c)^8 + 8*(5* \\
& a*b^2 - 4*b^3)*cosh(dx + c)*sinh(dx + c)^7 + (5*a*b^2 - 4*b^3)*sinh(dx + \\
& c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(dx + c)^6 + 4*(5*a^2*b - 9*a*b^ \\
& 2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(dx + c)^2)*sinh(dx + c)^6 + 8*(7*(5* \\
& a*b^2 - 4*b^3)*cosh(dx + c)^3 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(dx + c \\
&))*sinh(dx + c)^5 - 2*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(dx + c)^4 + 2*(\\
& 35*(5*a*b^2 - 4*b^3)*cosh(dx + c)^4 - 20*a^2*b + 31*a*b^2 - 12*b^3 + 30*(5 \\
& *a^2*b - 9*a*b^2 + 4*b^3)*cosh(dx + c)^2)*sinh(dx + c)^4 + 8*(7*(5*a*b^2 \\
& - 4*b^3)*cosh(dx + c)^5 + 10*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(dx + c)^3 - \\
& (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(dx + c))*sinh(dx + c)^3 + 5*a*b^2 - \\
& 4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(dx + c)^2 + 4*(7*(5*a*b^2 - 4*b \\
& ^3)*cosh(dx + c)^6 + 15*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(dx + c)^4 + 5*a^ \\
& 2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(dx + c)^2)*s \\
& inh(dx + c)^2 + 8*((5*a*b^2 - 4*b^3)*cosh(dx + c)^7 + 3*(5*a^2*b - 9*a*b^ \\
& 2 + 4*b^3)*cosh(dx + c)^5 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(dx + c)^3 \\
& + (5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(dx + c))*sinh(dx + c))*sqrt(b/(a - b) \\
&)*arctan(1/2*sqrt(b/(a - b))*(cosh(dx + c) + sinh(dx + c))) + ((5*a*b^2 - \\
& 4*b^3)*cosh(dx + c)^8 + 8*(5*a*b^2 - 4*b^3)*cosh(dx + c)*sinh(dx + c)^7 \\
& + (5*a*b^2 - 4*b^3)*sinh(dx + c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d \\
& *x + c)^6 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(dx + c \\
&)^2)*sinh(dx + c)^6 + 8*(7*(5*a*b^2 - 4*b^3)*cosh(dx + c)^3 + 3*(5*a^2*b \\
& - 9*a*b^2 + 4*b^3)*cosh(dx + c))*sinh(dx + c)^5 - 2*(20*a^2*b - 31*a*b^2 \\
& + 12*b^3)*cosh(dx + c)^4 + 2*(35*(5*a*b^2 - 4*b^3)*cosh(dx + c)^4 - 20*a^ \\
& 2*b + 31*a*b^2 - 12*b^3 + 30*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(dx + c)^2)*s
\end{aligned}$$

$$\begin{aligned}
& \operatorname{inh}(d*x + c)^4 + 8*(7*(5*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^5 + 10*(5*a^2*b - 9*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^3 - (20*a^2*b - 31*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)) * \operatorname{sinh}(d*x + c)^3 + 5*a*b^2 - 4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^2 + 4*(7*(5*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^6 + 15*(5*a^2*b - 9*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^4 + 5*a^2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)^2) * \operatorname{sinh}(d*x + c)^2 + 8*((5*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^7 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^5 - (20*a^2*b - 31*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)^3 + (5*a^2*b - 9*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)) * \operatorname{sinh}(d*x + c)) * \operatorname{sqrt}(b/(a - b)) * \operatorname{arctan}(1/2*(b*\operatorname{cosh}(d*x + c)^3 + 3*b*\operatorname{cosh}(d*x + c)*\operatorname{sinh}(d*x + c)^2 + b*\operatorname{sinh}(d*x + c)^3 + (4*a - 3*b)*\operatorname{cosh}(d*x + c) + (3*b*\operatorname{cosh}(d*x + c)^2 + 4*a - 3*b)*\operatorname{sinh}(d*x + c)) * \operatorname{sqrt}(b/(a - b)))/b) + 2*(a^2*b - 2*a*b^2)*\operatorname{cosh}(d*x + c) - ((a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^8 + 8*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)*\operatorname{sinh}(d*x + c)^7 + (a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{sinh}(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^2) * \operatorname{sinh}(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)) * \operatorname{sinh}(d*x + c)^5 - 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)^4 + 2*(35*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b^2 - 12*b^3 + 30*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^2) * \operatorname{sinh}(d*x + c)^4 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^5 + 10*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)) * \operatorname{sinh}(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^6 + 15*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^4 + a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)^2) * \operatorname{sinh}(d*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^7 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^5 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)) * \operatorname{sinh}(d*x + c)) * \log(\operatorname{cosh}(d*x + c) + \operatorname{sinh}(d*x + c) + 1) + ((a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^8 + 8*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)*\operatorname{sinh}(d*x + c)^7 + (a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{sinh}(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^2) * \operatorname{sinh}(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)) * \operatorname{sinh}(d*x + c)^5 - 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)^4 + 2*(35*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b^2 - 12*b^3 + 30*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^2) * \operatorname{sinh}(d*x + c)^4 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^5 + 10*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)) * \operatorname{sinh}(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^6 + 15*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\operatorname{cosh}(d*x + c)^4 + a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*\operatorname{cosh}(d*x + c)^2) * \operatorname{sinh}(d*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*\operatorname{cosh}(d*x + c)^7
\end{aligned}$$

$$\begin{aligned}
& + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - \\
& 25*a*b^2 + 12*b^3)*\cosh(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*\cos \\
& h(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(7*(a \\
& ^2*b - 2*a*b^2)*\cosh(d*x + c)^6 + 5*(4*a^3 - 5*a^2*b + 2*a*b^2)*\cosh(d*x + \\
& c)^4 + a^2*b - 2*a*b^2 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)*\cosh(d*x + c)^2)*\sin \\
& h(d*x + c))/((a^4*b - a^3*b^2)*d*\cosh(d*x + c)^8 + 8*(a^4*b - a^3*b^2)*d*\co \\
& sh(d*x + c)*\sinh(d*x + c)^7 + (a^4*b - a^3*b^2)*d*\sinh(d*x + c)^8 + 4*(a^5 \\
& - 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^4*b - a^3*b^2)*d*\cosh(d*x \\
& + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*\sinh(d*x + c)^6 - 2*(4*a^5 - 7*a^4*b \\
& + 3*a^3*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^4*b - a^3*b^2)*d*\cosh(d*x + c)^3 + \\
& 3*(a^5 - 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^4* \\
& b - a^3*b^2)*d*\cosh(d*x + c)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*d*\cosh(d*x + \\
& c)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d)*\sinh(d*x + c)^4 + 4*(a^5 - 2*a^4*b \\
& + a^3*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^4*b - a^3*b^2)*d*\cosh(d*x + c)^5 + 1 \\
& 0*(a^5 - 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^ \\
& 2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^4*b - a^3*b^2)*d*\cosh(d*x + c \\
&)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)^4 - 3*(4*a^5 - 7*a^4*b + \\
& 3*a^3*b^2)*d*\cosh(d*x + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*\sinh(d*x + c)^ \\
& 2 + (a^4*b - a^3*b^2)*d + 8*((a^4*b - a^3*b^2)*d*\cosh(d*x + c)^7 + 3*(a^5 - \\
& 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d*\cos \\
& h(d*x + c)^3 + (a^5 - 2*a^4*b + a^3*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

```
[Out] Exception raised: TypeError
```


$$3.50 \quad \int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=174

$$\frac{b^2(6a-5b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a-b)^{3/2}} + \frac{(2a^2+ab-5b^2) \operatorname{coth}(c+dx)}{2a^3d(a-b)} - \frac{(2a-5b) \operatorname{coth}^3(c+dx)}{6a^2d(a-b)} - \frac{b \operatorname{csch}^3(c+dx)}{2ad(a-b)(a-(a-b) \tanh(c+dx))}$$

[Out] ((6*a - 5*b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(a - b)^(3/2)*d) + ((2*a^2 + a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(a - b)*d) - ((2*a - 5*b)*Coth[c + d*x]^3)/(6*a^2*(a - b)*d) - (b*Csch[c + d*x]^3*Sech[c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.206368, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3187, 468, 570, 208}

$$\frac{b^2(6a-5b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a-b)^{3/2}} + \frac{(2a^2+ab-5b^2) \operatorname{coth}(c+dx)}{2a^3d(a-b)} - \frac{(2a-5b) \operatorname{coth}^3(c+dx)}{6a^2d(a-b)} - \frac{b \operatorname{csch}^3(c+dx)}{2ad(a-b)(a-(a-b) \tanh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((6*a - 5*b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(a - b)^(3/2)*d) + ((2*a^2 + a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(a - b)*d) - ((2*a - 5*b)*Coth[c + d*x]^3)/(6*a^2*(a - b)*d) - (b*Csch[c + d*x]^3*Sech[c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3187

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)]/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e*x)^(m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] := Int[ExpandIntegrand[(g*x)^(m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}^4(c + dx)}{(a + b \sinh^2(c + dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{x^4(a-(a-b)x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= -\frac{b \operatorname{csch}^3(c + dx) \operatorname{sech}(c + dx)}{2a(a-b)d(a - (a-b) \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(2a-5b+(-2a+b)x^2)}{x^4(a+(-a+b)x^2)} dx, x, \tanh(c + dx)\right)}{2a(a-b)d} \\ &= -\frac{b \operatorname{csch}^3(c + dx) \operatorname{sech}(c + dx)}{2a(a-b)d(a - (a-b) \tanh^2(c + dx))} + \frac{\operatorname{Subst}\left(\int \left(\frac{2a-5b}{ax^4} + \frac{-2a^2-ab+5b^2}{a^2x^2} + \frac{(6a-5b)b^2}{a^2(a-(a-b)x^2)}\right) dx, x, \tanh(c + dx)\right)}{2a(a-b)d} \\ &= \frac{(2a^2 + ab - 5b^2) \operatorname{coth}(c + dx)}{2a^3(a-b)d} - \frac{(2a-5b) \operatorname{coth}^3(c + dx)}{6a^2(a-b)d} - \frac{b \operatorname{csch}^3(c + dx) \operatorname{sech}(c + dx)}{2a(a-b)d(a - (a-b) \tanh^2(c + dx))} \\ &= \frac{(6a-5b)b^2 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^{3/2}d} + \frac{(2a^2 + ab - 5b^2) \operatorname{coth}(c + dx)}{2a^3(a-b)d} - \frac{(2a-5b) \operatorname{coth}^3(c + dx)}{6a^2(a-b)d} \end{aligned}$$

Mathematica [A] time = 1.25862, size = 210, normalized size = 1.21

$$\operatorname{csch}^4(c+dx)(2a+b\cosh(2(c+dx))-b) \left(-2a^{3/2} \coth(c+dx) \operatorname{csch}^2(c+dx)(2a+b\cosh(2(c+dx))-b) - \frac{3\sqrt{ab^3} \sinh(2(c+dx))}{a-b} \right)$$

$$24a^{7/2}d \left(\operatorname{acsch}^2(c+dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^4*((3*(6*a - 5*b)*b^2*ArcTan[h[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c + d*x])])]/(a - b)^(3/2) + 4*Sqrt[a]*(a + 3*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d*x] - 2*a^(3/2)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csch[c + d*x]^2 - (3*Sqrt[a]*b^3*Sinh[2*(c + d*x)]/(a - b)))/(24*a^(7/2)*d*(b + a*Csch[c + d*x]^2)^2)

Maple [B] time = 0.089, size = 890, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)

[Out] -1/24/d/a^2*tanh(1/2*d*x+1/2*c)^3+3/8/d/a^2*tanh(1/2*d*x+1/2*c)+1/d/a^3*tanh(1/2*d*x+1/2*c)*b-1/24/d/a^2/tanh(1/2*d*x+1/2*c)^3+3/8/d/a^2/tanh(1/2*d*x+1/2*c)+1/d/a^3/tanh(1/2*d*x+1/2*c)*b-1/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)+3/d/a^2*b^2/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d/a^2*b^3/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d/a^2*b^2/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/d/a^2*b^3/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/2/d*b^3/a^3/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+5/2/d*b^4/a^3/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))

$$\begin{aligned} & \sqrt{a-2b} \operatorname{arctanh}\left(\frac{a \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{2(-b(a-b)) + a - 2b}}\right) \\ & + \frac{5}{2} \frac{d^3 b^3}{a^3 (a-b)} \sqrt{2(-b(a-b)) + a - 2b} \\ & + \frac{5}{2} \frac{d^4 b^4}{a^3 (a-b)} \sqrt{-b(a-b)} \operatorname{arctan}\left(\frac{a \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{2(-b(a-b)) - a + 2b}}\right) \\ & + \frac{5}{2} \frac{d^3 b^3}{a^3 (a-b)} \sqrt{-b(a-b)} \operatorname{arctan}\left(\frac{a \tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{2(-b(a-b)) - a + 2b}}\right) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 3.16269, size = 16662, normalized size = 95.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & \left[\frac{1}{12} (12(6a^3b^2 - 11a^2b^3 + 5ab^4) \cosh(dx+c)^8 + 96(6a^3b^2 - 11a^2b^3 + 5ab^4) \cosh(dx+c) \sinh(dx+c)^7 + 12(6a^3b^2 - 11a^2b^3 + 5ab^4) \sinh(dx+c)^8 + 24(6a^4b - 23a^3b^2 + 27a^2b^3 - 10ab^4) \cosh(dx+c)^6 + 24(6a^4b - 23a^3b^2 + 27a^2b^3 - 10ab^4 + 14(6a^3b^2 - 11a^2b^3 + 5ab^4) \cosh(dx+c)^2) \sinh(dx+c)^6 + 48(14(6a^3b^2 - 11a^2b^3 + 5ab^4) \cosh(dx+c)^3 + 3(6a^4b - 23a^3b^2 + 27a^2b^3 - 10ab^4) \cosh(dx+c)) \sinh(dx+c)^5 + 16a^4b + 16a^3b^2 - 92a^2b^3 + 60ab^4 - 8(24a^5 - 14a^4b - 89a^3b^2 + 124a^2b^3 - 45ab^4) \cosh(dx+c)^4 - 8(24a^5 - 14a^4b - 89a^3b^2 + 124a^2b^3 - 45ab^4) \cosh(dx+c)^2 + 8(8a^5 - 2a^4b - 47a^3b^2 + 71a^2b^3 - 30ab^4) \cosh(dx+c)^2 + 8(42(6a^3b^2 - 11a^2b^3 + 5ab^4) \cosh(dx+c) \right. \end{aligned}$$

$$\begin{aligned}
&^6 + 8a^5 - 2a^4b - 47a^3b^2 + 71a^2b^3 - 30ab^4 + 45(6a^4b - 2 \\
&3a^3b^2 + 27a^2b^3 - 10ab^4) \cosh(dx + c)^4 - 6(24a^5 - 14a^4b - \\
&89a^3b^2 + 124a^2b^3 - 45ab^4) \cosh(dx + c)^2 \sinh(dx + c)^2 + 3 \\
&((6a^3b^3 - 5b^4) \cosh(dx + c)^{10} + 10(6a^3b^3 - 5b^4) \cosh(dx + c) \sinh(dx + c)^9 \\
&+ (6a^3b^3 - 5b^4) \sinh(dx + c)^{10} + (24a^2b^2 - 50a^3b^3 \\
&+ 25b^4) \cosh(dx + c)^8 + (24a^2b^2 - 50a^3b^3 + 25b^4 + 45(6a^3b^3 \\
&- 5b^4) \cosh(dx + c)^2) \sinh(dx + c)^8 + 8(15(6a^3b^3 - 5b^4) \cosh(dx \\
&+ c)^3 + (24a^2b^2 - 50a^3b^3 + 25b^4) \cosh(dx + c)) \sinh(dx + c)^7 \\
&- 2(36a^2b^2 - 60a^3b^3 + 25b^4) \cosh(dx + c)^6 + 2(105(6a^3b^3 - 5b^4) \\
&b^4) \cosh(dx + c)^4 - 36a^2b^2 + 60a^3b^3 - 25b^4 + 14(24a^2b^2 - 50 \\
&a^3b^3 + 25b^4) \cosh(dx + c)^2 \sinh(dx + c)^6 + 4(63(6a^3b^3 - 5b^4) \\
&\cosh(dx + c)^5 + 14(24a^2b^2 - 50a^3b^3 + 25b^4) \cosh(dx + c)^3 - 3 \\
&(36a^2b^2 - 60a^3b^3 + 25b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(36a^2 \\
&b^2 - 60a^3b^3 + 25b^4) \cosh(dx + c)^4 + 2(105(6a^3b^3 - 5b^4) \cosh(dx \\
&+ c)^6 + 35(24a^2b^2 - 50a^3b^3 + 25b^4) \cosh(dx + c)^4 + 36a^2b^2 \\
&- 60a^3b^3 + 25b^4 - 15(36a^2b^2 - 60a^3b^3 + 25b^4) \cosh(dx + c)^2 \\
&) \sinh(dx + c)^4 - 6a^3b^3 + 5b^4 + 8(15(6a^3b^3 - 5b^4) \cosh(dx + c) \\
&^7 + 7(24a^2b^2 - 50a^3b^3 + 25b^4) \cosh(dx + c)^5 - 5(36a^2b^2 - 6 \\
&0a^3b^3 + 25b^4) \cosh(dx + c)^3 + (36a^2b^2 - 60a^3b^3 + 25b^4) \cosh(dx \\
&+ c)) \sinh(dx + c)^3 - (24a^2b^2 - 50a^3b^3 + 25b^4) \cosh(dx + c)^2 \\
&+ (45(6a^3b^3 - 5b^4) \cosh(dx + c)^8 + 28(24a^2b^2 - 50a^3b^3 + 25b^4) \\
&^4) \cosh(dx + c)^6 - 30(36a^2b^2 - 60a^3b^3 + 25b^4) \cosh(dx + c)^4 - \\
&24a^2b^2 + 50a^3b^3 - 25b^4 + 12(36a^2b^2 - 60a^3b^3 + 25b^4) \cosh(dx \\
&+ c)^2 \sinh(dx + c)^2 + 2(5(6a^3b^3 - 5b^4) \cosh(dx + c)^9 + 4(2 \\
&4a^2b^2 - 50a^3b^3 + 25b^4) \cosh(dx + c)^7 - 6(36a^2b^2 - 60a^3b^3 + \\
&25b^4) \cosh(dx + c)^5 + 4(36a^2b^2 - 60a^3b^3 + 25b^4) \cosh(dx + c) \\
&^3 - (24a^2b^2 - 50a^3b^3 + 25b^4) \cosh(dx + c)) \sinh(dx + c)) \sqrt{a^2 \\
&- ab} \log((b^2 \cosh(dx + c)^4 + 4b^2 \cosh(dx + c) \sinh(dx + c)^3 + b \\
&^2 \sinh(dx + c)^4 + 2(2ab - b^2) \cosh(dx + c)^2 + 2(3b^2 \cosh(dx + \\
&c)^2 + 2ab - b^2) \sinh(dx + c)^2 + 8a^2 - 8ab + b^2 + 4(b^2 \cosh(dx \\
&+ c)^3 + (2ab - b^2) \cosh(dx + c)) \sinh(dx + c) - 4(b \cosh(dx + c)^2 \\
&+ 2b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + 2a - b) \sqrt{a^2 \\
&- ab}) / (b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx \\
&+ c)^4 + 2(2a - b) \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 + 2a - b) \sinh(dx \\
&+ c)^2 + 4(b \cosh(dx + c)^3 + (2a - b) \cosh(dx + c)) \sinh(dx + \\
&c) + b)) + 16(6(6a^3b^2 - 11a^2b^3 + 5ab^4) \cosh(dx + c)^7 + 9(6 \\
&a^4b - 23a^3b^2 + 27a^2b^3 - 10ab^4) \cosh(dx + c)^5 - 2(24a^5 - \\
&14a^4b - 89a^3b^2 + 124a^2b^3 - 45ab^4) \cosh(dx + c)^3 + (8a^5 - \\
&2a^4b - 47a^3b^2 + 71a^2b^3 - 30ab^4) \cosh(dx + c)) \sinh(dx + c) \\
&/ ((a^6b - 2a^5b^2 + a^4b^3) d \cosh(dx + c)^{10} + 10(a^6b - 2a^5b^2 \\
&+ a^4b^3) d \cosh(dx + c) \sinh(dx + c)^9 + (a^6b - 2a^5b^2 + a^4b^3) \\
&d \sinh(dx + c)^{10} + (4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) d \cosh(dx \\
&+ c)^8 + (45(a^6b - 2a^5b^2 + a^4b^3) d \cosh(dx + c)^2 + (4a^7 - 13 \\
&a^6b + 14a^5b^2 - 5a^4b^3) d) \sinh(dx + c)^8 - 2(6a^7 - 17a^6b + \\
&16a^5b^2 - 5a^4b^3) d \cosh(dx + c)^6 + 8(15(a^6b - 2a^5b^2 + a^4
\end{aligned}$$

$$\begin{aligned}
& *b^3)*d*\cosh(d*x + c)^3 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*\cos \\
& h(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*\cosh(d \\
& *x + c)^4 + 14*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*\cosh(d*x + c)^ \\
& 2 - (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*\sinh(d*x + c)^6 + 2*(6*a \\
& ^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*\cosh(d*x + c)^4 + 4*(63*(a^6*b - \\
& 2*a^5*b^2 + a^4*b^3)*d*\cosh(d*x + c)^5 + 14*(4*a^7 - 13*a^6*b + 14*a^5*b^2 \\
& - 5*a^4*b^3)*d*\cosh(d*x + c)^3 - 3*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b \\
& ^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*b^3) \\
& *d*\cosh(d*x + c)^6 + 35*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*\cosh(\\
& d*x + c)^4 - 15*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*\cosh(d*x + c) \\
& ^2 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*\sinh(d*x + c)^4 - (4*a^ \\
& 7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*\cosh(d*x + c)^2 + 8*(15*(a^6*b - 2 \\
& *a^5*b^2 + a^4*b^3)*d*\cosh(d*x + c)^7 + 7*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - \\
& 5*a^4*b^3)*d*\cosh(d*x + c)^5 - 5*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3 \\
&)*d*\cosh(d*x + c)^3 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*\cosh(d* \\
& x + c))*\sinh(d*x + c)^3 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*\cosh(d*x + c) \\
& ^8 + 28*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*\cosh(d*x + c)^6 - 30* \\
& (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*\cosh(d*x + c)^4 + 12*(6*a^7 - \\
& 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*\cosh(d*x + c)^2 - (4*a^7 - 13*a^6*b + \\
& 14*a^5*b^2 - 5*a^4*b^3)*d)*\sinh(d*x + c)^2 - (a^6*b - 2*a^5*b^2 + a^4*b^3) \\
& *d + 2*(5*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*\cosh(d*x + c)^9 + 4*(4*a^7 - 13*a \\
& ^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*\cosh(d*x + c)^7 - 6*(6*a^7 - 17*a^6*b + 16 \\
& *a^5*b^2 - 5*a^4*b^3)*d*\cosh(d*x + c)^5 + 4*(6*a^7 - 17*a^6*b + 16*a^5*b^2 \\
& - 5*a^4*b^3)*d*\cosh(d*x + c)^3 - (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3 \\
&)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/6*(6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4 \\
&)*\cosh(d*x + c)^8 + 48*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*\cosh(d*x + c)*\sin \\
& h(d*x + c)^7 + 6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*\sinh(d*x + c)^8 + 12*(6 \\
& *a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*\cosh(d*x + c)^6 + 12*(6*a^4*b \\
& - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4 + 14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4 \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 24*(14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b \\
& ^4)*\cosh(d*x + c)^3 + 3*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*\cosh \\
& (d*x + c))*\sinh(d*x + c)^5 + 8*a^4*b + 8*a^3*b^2 - 46*a^2*b^3 + 30*a*b^4 - \\
& 4*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*\cosh(d*x + c)^4 \\
& - 4*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4 - 105*(6*a^3* \\
& b^2 - 11*a^2*b^3 + 5*a*b^4)*\cosh(d*x + c)^4 - 45*(6*a^4*b - 23*a^3*b^2 + 27 \\
& *a^2*b^3 - 10*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 16*(21*(6*a^3*b^2 - \\
& 11*a^2*b^3 + 5*a*b^4)*\cosh(d*x + c)^5 + 15*(6*a^4*b - 23*a^3*b^2 + 27*a^2* \\
& b^3 - 10*a*b^4)*\cosh(d*x + c)^3 - (24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2 \\
& *b^3 - 45*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a^5 - 2*a^4*b - 47*a \\
& ^3*b^2 + 71*a^2*b^3 - 30*a*b^4)*\cosh(d*x + c)^2 + 4*(42*(6*a^3*b^2 - 11*a^2 \\
& *b^3 + 5*a*b^4)*\cosh(d*x + c)^6 + 8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 \\
& - 30*a*b^4 + 45*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*\cosh(d*x + \\
& c)^4 - 6*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*\cosh(d*x \\
& + c)^2)*\sinh(d*x + c)^2 - 3*((6*a*b^3 - 5*b^4)*\cosh(d*x + c)^10 + 10*(6*a* \\
& b^3 - 5*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (6*a*b^3 - 5*b^4)*\sinh(d*x + c
\end{aligned}$$

$$\begin{aligned}
&)^{10} + (24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)^8 + (24a^2b^2 - 50ab^3 + 25b^4 + 45(6ab^3 - 5b^4) \cosh(dx + c)^2) \sinh(dx + c)^8 + 8(15(6ab^3 - 5b^4) \cosh(dx + c)^3 + (24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)) \sinh(dx + c)^7 - 2(36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)^6 + 2(105(6ab^3 - 5b^4) \cosh(dx + c)^4 - 36a^2b^2 + 60ab^3 - 25b^4 + 14(24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)^2) \sinh(dx + c)^6 + 4(63(6ab^3 - 5b^4) \cosh(dx + c)^5 + 14(24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)^3 - 3(36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)^4 + 2(105(6ab^3 - 5b^4) \cosh(dx + c)^6 + 35(24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)^4 + 36a^2b^2 - 60ab^3 + 25b^4 - 15(36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 - 6ab^3 + 5b^4 + 8(15(6ab^3 - 5b^4) \cosh(dx + c)^7 + 7(24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)^5 - 5(36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)^3 + (36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)) \sinh(dx + c)^3 - (24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)^2 + (45(6ab^3 - 5b^4) \cosh(dx + c)^8 + 28(24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)^6 - 30(36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)^4 - 24a^2b^2 + 50ab^3 - 25b^4 + 12(36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 2(5(6ab^3 - 5b^4) \cosh(dx + c)^9 + 4(24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)^7 - 6(36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)^5 + 4(36a^2b^2 - 60ab^3 + 25b^4) \cosh(dx + c)^3 - (24a^2b^2 - 50ab^3 + 25b^4) \cosh(dx + c)) \sinh(dx + c)) \sqrt{-a^2 + ab} \arctan(-1/2(b \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + 2a - b) \sqrt{-a^2 + ab}) / (a^2 - ab) + 8(6(6a^3b^2 - 11a^2b^3 + 5ab^4) \cosh(dx + c)^7 + 9(6a^4b - 23a^3b^2 + 27a^2b^3 - 10ab^4) \cosh(dx + c)^5 - 2(24a^5 - 14a^4b - 89a^3b^2 + 124a^2b^3 - 45ab^4) \cosh(dx + c)^3 + (8a^5 - 2a^4b - 47a^3b^2 + 71a^2b^3 - 30ab^4) \cosh(dx + c)) \sinh(dx + c) / ((a^6b - 2a^5b^2 + a^4b^3) d \cosh(dx + c)^{10} + 10(a^6b - 2a^5b^2 + a^4b^3) d \cosh(dx + c) \sinh(dx + c)^9 + (a^6b - 2a^5b^2 + a^4b^3) d \sinh(dx + c)^{10} + (4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) d \cosh(dx + c)^8 + (45(a^6b - 2a^5b^2 + a^4b^3) d \cosh(dx + c)^2 + (4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) d) \sinh(dx + c)^8 - 2(6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) d \cosh(dx + c)^6 + 8(15(a^6b - 2a^5b^2 + a^4b^3) d \cosh(dx + c)^3 + (4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) d \cosh(dx + c)) \sinh(dx + c)^7 + 2(105(a^6b - 2a^5b^2 + a^4b^3) d \cosh(dx + c)^4 + 14(4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) d \cosh(dx + c)^2 - (6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) d) \sinh(dx + c)^6 + 2(6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) d \cosh(dx + c)^4 + 4(63(a^6b - 2a^5b^2 + a^4b^3) d \cosh(dx + c)^5 + 14(4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) d \cosh(dx + c)^3 - 3(6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(105(a^6b - 2a^5b^2 + a^4b^3) d \cosh(dx + c)^6 + 35(4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) d \cosh(dx + c)^4 - 15(6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) d \cosh(dx + c)^2 + (6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) d) \sinh(dx + c)^4 - (
\end{aligned}$$

$$4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)^2 + 8 * (15 * (a^6b - 2a^5b^2 + a^4b^3) * d * \cosh(dx + c)^7 + 7 * (4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)^5 - 5 * (6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)^3 + (6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) * d * \cosh(dx + c) * \sinh(dx + c)^3 + (45 * (a^6b - 2a^5b^2 + a^4b^3) * d * \cosh(dx + c)^8 + 28 * (4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)^6 - 30 * (6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)^4 + 12 * (6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)^2 - (4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) * d) * \sinh(dx + c)^2 - (a^6b - 2a^5b^2 + a^4b^3) * d + 2 * (5 * (a^6b - 2a^5b^2 + a^4b^3) * d * \cosh(dx + c)^9 + 4 * (4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)^7 - 6 * (6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)^5 + 4 * (6a^7 - 17a^6b + 16a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)^3 - (4a^7 - 13a^6b + 14a^5b^2 - 5a^4b^3) * d * \cosh(dx + c)) * \sinh(dx + c))]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)**4/(a+b*sinh(dx+c)**2)**2,x)

[Out] Timed out

Giac [A] time = 1.50775, size = 301, normalized size = 1.73

$$\frac{(6ab^2 - 5b^3) \arctan\left(\frac{be^{(2dx+2c)+2a-b}}{2\sqrt{-a^2+ab}}\right)}{2(a^4d - a^3bd)\sqrt{-a^2 + ab}} + \frac{2ab^2e^{(2dx+2c)} - b^3e^{(2dx+2c)} + b^3}{(a^4d - a^3bd)(be^{(4dx+4c)} + 4ae^{(2dx+2c)} - 2be^{(2dx+2c)} + b)} + \frac{4(3be^{(4dx+4c)} - 3ae^{(2dx+2c)} + b^3)}{3a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^4/(a+b*sinh(dx+c)^2)^2,x, algorithm="giac")

[Out] $\frac{1}{2} * (6 * a * b^2 - 5 * b^3) * \arctan(1/2 * (b * e^{(2 * d * x + 2 * c)} + 2 * a - b) / \sqrt{-a^2 + a * b}) / ((a^4 * d - a^3 * b * d) * \sqrt{-a^2 + a * b}) + (2 * a * b^2 * e^{(2 * d * x + 2 * c)} - b^3 * e^{(2 * d * x + 2 * c)} + b^3) / ((a^4 * d - a^3 * b * d) * (b * e^{(4 * d * x + 4 * c)} + 4 * a * e^{(2 * d * x + 2 * c)} - 2 * b * e^{(2 * d * x + 2 * c)} + b)) + 4/3 * (3 * b * e^{(4 * d * x + 4 * c)} - 3 * a * e^{(2 * d * x + 2 * c)} - 6 * b * e^{(2 * d * x + 2 * c)} + a + 3 * b) / (a^3 * d * (e^{(2 * d * x + 2 * c)} - 1)^3)$

$$3.51 \quad \int \frac{\sinh^4(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=124

$$\frac{\tanh^3(c+dx)}{4d(a-b)(a-(a-b)\tanh^2(c+dx))^2} - \frac{3 \tanh(c+dx)}{8d(a-b)^2(a-(a-b)\tanh^2(c+dx))} + \frac{3 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{ad}(a-b)^{5/2}}$$

[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^(5/2)*d) + Tanh[c + d*x]^3/(4*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - (3*Tanh[c + d*x])/(8*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.123065, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3187, 288, 208}

$$\frac{\tanh^3(c+dx)}{4d(a-b)(a-(a-b)\tanh^2(c+dx))^2} - \frac{3 \tanh(c+dx)}{8d(a-b)^2(a-(a-b)\tanh^2(c+dx))} + \frac{3 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{ad}(a-b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^(5/2)*d) + Tanh[c + d*x]^3/(4*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - (3*Tanh[c + d*x])/(8*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3187

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 288

Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^

$n*(m - n + 1)/(b*n*(p + 1)), \text{Int}[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]$
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
 LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 208

$\text{Int}[(a_ + (b_ .)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/$
 $\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rubi steps

$$\int \frac{\sinh^4(c + dx)}{(a + b \sinh^2(c + dx))^3} dx = \frac{\text{Subst}\left(\int \frac{x^4}{(a - (a-b)x^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\tanh^3(c + dx)}{4(a - b)d(a - (a - b)\tanh^2(c + dx))^2} - \frac{3 \text{Subst}\left(\int \frac{x^2}{(a + (-a+b)x^2)^2} dx, x, \tanh(c + dx)\right)}{4(a - b)d}$$

$$= \frac{\tanh^3(c + dx)}{4(a - b)d(a - (a - b)\tanh^2(c + dx))^2} - \frac{3 \tanh(c + dx)}{8(a - b)^2d(a - (a - b)\tanh^2(c + dx))} + \frac{3 \text{Subst}\left(\int \frac{x^0}{a - (a-b)x^2} dx, x, \tanh(c + dx)\right)}{8(a - b)^2d(a - (a - b)\tanh^2(c + dx))}$$

$$= \frac{3 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a-b)^{5/2}d} + \frac{\tanh^3(c + dx)}{4(a - b)d(a - (a - b)\tanh^2(c + dx))^2} - \frac{3 \tanh(c + dx)}{8(a - b)^2d(a - (a - b)\tanh^2(c + dx))}$$

Mathematica [A] time = 1.3691, size = 104, normalized size = 0.84

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}} + \frac{\sinh(2(c+dx))(2a-5b)\cosh(2(c+dx))-8a+5b}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2}}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a - b)^(5/2)) + ((-8*a + 5*b + (2*a - 5*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)]/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)]^2)))/(8*d)

Maple [B] time = 0.052, size = 768, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\sinh(dx+c)^4 / (a+b\sinh(dx+c)^2)^3, x)$

[Out]
$$-3/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7+11/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^5*a-5/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^5*b+11/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3*a-5/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3*b-3/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)+3/8/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b-3/8/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(dx+c)^4 / (a+b\sinh(dx+c)^2)^3, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [B] time = 2.94201, size = 11786, normalized size = 95.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*\cosh(d*x + c)^6 + 2 \\ & 4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*\cosh(d*x + c)*\sinh(d*x + c) \\ & ^5 + 4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*\sinh(d*x + c)^6 + 8*a^3 \\ & *b^2 - 28*a^2*b^3 + 20*a*b^4 + 4*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2 \\ & *b^3 + 15*a*b^4)*\cosh(d*x + c)^4 + 4*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61* \\ & a^2*b^3 + 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*\cosh(\\ & d*x + c)^2)*\sinh(d*x + c)^4 + 16*(5*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5* \\ & a*b^4)*\cosh(d*x + c)^3 + (16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15 \\ & *a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a^4*b - 40*a^3*b^2 + 47*a^2*b \\ & ^3 - 15*a*b^4)*\cosh(d*x + c)^2 + 4*(8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15* \\ & a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*\cosh(d*x + c)^4 + \\ & 6*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*\cosh(d*x + c)^2 \\ &)*\sinh(d*x + c)^2 - 3*(b^4*\cosh(d*x + c)^8 + 8*b^4*\cosh(d*x + c)*\sinh(d*x + \\ & c)^7 + b^4*\sinh(d*x + c)^8 + 4*(2*a*b^3 - b^4)*\cosh(d*x + c)^6 + 4*(7*b^4* \\ & \cosh(d*x + c)^2 + 2*a*b^3 - b^4)*\sinh(d*x + c)^6 + 8*(7*b^4*\cosh(d*x + c)^3 \\ & + 3*(2*a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^3 \\ & + 3*b^4)*\cosh(d*x + c)^4 + 2*(35*b^4*\cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^3 \\ & + 3*b^4 + 30*(2*a*b^3 - b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + b^4 + 8*(\\ & 7*b^4*\cosh(d*x + c)^5 + 10*(2*a*b^3 - b^4)*\cosh(d*x + c)^3 + (8*a^2*b^2 - 8 \\ & *a*b^3 + 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*\cosh(d*x \\ & + c)^2 + 4*(7*b^4*\cosh(d*x + c)^6 + 15*(2*a*b^3 - b^4)*\cosh(d*x + c)^4 + 2 \\ & *a*b^3 - b^4 + 3*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + \\ & c)^2 + 8*(b^4*\cosh(d*x + c)^7 + 3*(2*a*b^3 - b^4)*\cosh(d*x + c)^5 + (8*a^2* \\ & b^2 - 8*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + (2*a*b^3 - b^4)*\cosh(d*x + c))*\sin \\ & h(d*x + c))*\sqrt{a^2 - a*b}*\log((b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)* \\ & \sinh(d*x + c)^3 + b^2*\sinh(d*x + c)^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2 \\ & *(3*b^2*\cosh(d*x + c)^2 + 2*a*b - b^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 \\ & + 4*(b^2*\cosh(d*x + c)^3 + (2*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4 \\ & *(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + \\ & 2*a - b)*\sqrt{a^2 - a*b})/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x \\ & + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x \\ & + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(\\ & d*x + c))*\sinh(d*x + c) + b)) + 8*(3*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5* \\ & a*b^4)*\cosh(d*x + c)^5 + 2*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + \\ & 15*a*b^4)*\cosh(d*x + c)^3 + (8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)* \\ & \cosh(d*x + c))*\sinh(d*x + c))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d* \end{aligned}$$

$$\begin{aligned}
& \cosh(dx + c)^8 + 8*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(dx + \\
& c)*\sinh(dx + c)^7 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\sinh(dx + \\
& c)^8 + 4*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(dx \\
& x + c)^6 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(dx + c)^2 \\
& + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d)*\sinh(dx + c) \\
& ^6 + 2*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a \\
& *b^7)*d*\cosh(dx + c)^4 + 8*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d* \\
& \cosh(dx + c)^3 + 3*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7) \\
& *d*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(35*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 \\
& - a*b^7)*d*\cosh(dx + c)^4 + 30*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2* \\
& b^6 + a*b^7)*d*\cosh(dx + c)^2 + (8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41* \\
& a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d)*\sinh(dx + c)^4 + 4*(2*a^5*b^3 - 7*a^4*b \\
& ^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(dx + c)^2 + 8*(7*(a^4*b^4 - 3*a \\
& ^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(dx + c)^5 + 10*(2*a^5*b^3 - 7*a^4*b^4 + \\
& 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(dx + c)^3 + (8*a^6*b^2 - 32*a^5*b^3 \\
& + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*\cosh(dx + c))*\sinh(dx \\
& x + c)^3 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(dx + c)^6 \\
& + 15*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(dx + \\
& c)^4 + 3*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3 \\
& *a*b^7)*d*\cosh(dx + c)^2 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 \\
& + a*b^7)*d)*\sinh(dx + c)^2 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d + \\
& 8*((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(dx + c)^7 + 3*(2*a^5* \\
& b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(dx + c)^5 + (8*a^6 \\
& *b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*\cosh(\\
& dx + c)^3 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh \\
& (dx + c))*\sinh(dx + c)), -1/8*(2*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a \\
& *b^4)*\cosh(dx + c)^6 + 12*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*co \\
& sh(dx + c)*\sinh(dx + c)^5 + 2*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^ \\
& 4)*\sinh(dx + c)^6 + 4*a^3*b^2 - 14*a^2*b^3 + 10*a*b^4 + 2*(16*a^5 - 72*a^4 \\
& *b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*\cosh(dx + c)^4 + 2*(16*a^5 - 72* \\
& a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21 \\
& *a^2*b^3 - 5*a*b^4)*\cosh(dx + c)^2)*\sinh(dx + c)^4 + 8*(5*(8*a^4*b - 24*a \\
& ^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*\cosh(dx + c)^3 + (16*a^5 - 72*a^4*b + 102*a \\
& ^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*\cosh(dx + c))*\sinh(dx + c)^3 + 2*(8*a^4*b \\
& - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)*\cosh(dx + c)^2 + 2*(8*a^4*b - 40*a^ \\
& 3*b^2 + 47*a^2*b^3 - 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a \\
& *b^4)*\cosh(dx + c)^4 + 6*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 1 \\
& 5*a*b^4)*\cosh(dx + c)^2)*\sinh(dx + c)^2 + 3*(b^4*\cosh(dx + c)^8 + 8*b^4* \\
& \cosh(dx + c)*\sinh(dx + c)^7 + b^4*\sinh(dx + c)^8 + 4*(2*a*b^3 - b^4)*cos \\
& h(dx + c)^6 + 4*(7*b^4*\cosh(dx + c)^2 + 2*a*b^3 - b^4)*\sinh(dx + c)^6 + \\
& 8*(7*b^4*\cosh(dx + c)^3 + 3*(2*a*b^3 - b^4)*\cosh(dx + c))*\sinh(dx + c)^5 \\
& + 2*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*\cosh(dx + c)^4 + 2*(35*b^4*\cosh(dx + c \\
&)^4 + 8*a^2*b^2 - 8*a*b^3 + 3*b^4 + 30*(2*a*b^3 - b^4)*\cosh(dx + c)^2)*\sin \\
& h(dx + c)^4 + b^4 + 8*(7*b^4*\cosh(dx + c)^5 + 10*(2*a*b^3 - b^4)*\cosh(dx \\
& + c)^3 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*\cosh(dx + c))*\sinh(dx + c)^3 + 4*
\end{aligned}$$

$$\begin{aligned}
& (2*a*b^3 - b^4)*\cosh(d*x + c)^2 + 4*(7*b^4*\cosh(d*x + c)^6 + 15*(2*a*b^3 - \\
& b^4)*\cosh(d*x + c)^4 + 2*a*b^3 - b^4 + 3*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*\cosh \\
& (d*x + c)^2)*\sinh(d*x + c)^2 + 8*(b^4*\cosh(d*x + c)^7 + 3*(2*a*b^3 - b^4)*c \\
& \cosh(d*x + c)^5 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + (2*a*b^3 - \\
& b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a^2 + a*b}*\arctan(-1/2*(b*\cosh(d* \\
& x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*s \\
& \sqrt{-a^2 + a*b}/(a^2 - a*b)) + 4*(3*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5* \\
& a*b^4)*\cosh(d*x + c)^5 + 2*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + \\
& 15*a*b^4)*\cosh(d*x + c)^3 + (8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)* \\
& \cosh(d*x + c))*\sinh(d*x + c))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d* \\
& \cosh(d*x + c)^8 + 8*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + \\
& c)*\sinh(d*x + c)^7 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\sinh(d*x + \\
& c)^8 + 4*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(d* \\
& x + c)^6 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^2 \\
& + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d)*\sinh(d*x + c) \\
& ^6 + 2*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a \\
& *b^7)*d*\cosh(d*x + c)^4 + 8*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d* \\
& \cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7) \\
& *d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 \\
& - a*b^7)*d*\cosh(d*x + c)^4 + 30*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2* \\
& b^6 + a*b^7)*d*\cosh(d*x + c)^2 + (8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41* \\
& a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d)*\sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 7*a^4*b \\
& ^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - 3*a \\
& ^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^5 + 10*(2*a^5*b^3 - 7*a^4*b^4 + \\
& 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^3 + (8*a^6*b^2 - 32*a^5*b^3 \\
& + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*\cosh(d*x + c))*\sinh(d* \\
& x + c)^3 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^6 \\
& + 15*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + \\
& c)^4 + 3*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3 \\
& *a*b^7)*d*\cosh(d*x + c)^2 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 \\
& + a*b^7)*d)*\sinh(d*x + c)^2 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d + \\
& 8*((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*\cosh(d*x + c)^7 + 3*(2*a^5* \\
& b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh(d*x + c)^5 + (8*a^6 \\
& *b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*\cosh(\\
& d*x + c)^3 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*\cosh \\
& (d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.56499, size = 386, normalized size = 3.11

$$\frac{3 \arctan\left(\frac{be^{2dx+2c}+2a-b}{2\sqrt{-a^2+ab}}\right)}{8(a^2d-2abd+b^2d)\sqrt{-a^2+ab}} - \frac{8a^2be^{6dx+6c} - 16ab^2e^{6dx+6c} + 5b^3e^{6dx+6c} + 16a^3e^{4dx+4c} - 56a^2be^{4dx+4c} + 4b^4e^{4dx+4c}}{4(a^2b^2d - 2ab^3d + b^4d)(be^{4dx+4c} + 4a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{3}{8} \arctan\left(\frac{1}{2} \frac{b e^{2 d x+2 c}+2 a-b}{\sqrt{-a^2+a b}}\right) / \left(\left(a^2 d-2 a b d+b^2 d\right) \sqrt{-a^2+a b}\right) - \frac{1}{4} \frac{\left(8 a^2 b e^{6 d x+6 c}-16 a b^2 e^{6 d x+6 c}+5 b^3 e^{6 d x+6 c}+16 a^3 e^{4 d x+4 c}-56 a^2 b e^{4 d x+4 c}+4 b^4 e^{4 d x+4 c}\right)}{\left(a^2 b^2 d-2 a b^3 d+b^4 d\right)\left(b e^{4 d x+4 c}+4 a^3\right)}$

$$3.52 \quad \int \frac{\sinh^3(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^3} dx$$

Optimal. Leaf size=135

$$\frac{(a-4b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{8b^{3/2}d(a-b)^{5/2}} + \frac{(a-4b) \cosh(c+dx)}{8bd(a-b)^2(a+b \cosh^2(c+dx)-b)} - \frac{a \cosh(c+dx)}{4bd(a-b)(a+b \cosh^2(c+dx)-b)^2}$$

[Out] ((a - 4*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*b^(3/2)*d) - (a*Cosh[c + d*x])/(4*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2)^2) + ((a - 4*b)*Cosh[c + d*x])/(8*(a - b)^2*b*d*(a - b + b*Cosh[c + d*x]^2))

Rubi [A] time = 0.141105, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3186, 385, 199, 205}

$$\frac{(a-4b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{8b^{3/2}d(a-b)^{5/2}} + \frac{(a-4b) \cosh(c+dx)}{8bd(a-b)^2(a+b \cosh^2(c+dx)-b)} - \frac{a \cosh(c+dx)}{4bd(a-b)(a+b \cosh^2(c+dx)-b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((a - 4*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*b^(3/2)*d) - (a*Cosh[c + d*x])/(4*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2)^2) + ((a - 4*b)*Cosh[c + d*x])/(8*(a - b)^2*b*d*(a - b + b*Cosh[c + d*x]^2))

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^(m - 1)/2*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 385

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b

c(n*(p + 1) + 1)/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^3(c + dx)}{(a + b \sinh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+bx^2)^3} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{a \cosh(c + dx)}{4(a-b)bd(a-b+b \cosh^2(c + dx))^2} + \frac{(a-4b) \text{Subst}\left(\int \frac{1}{(a-b+bx^2)^2} dx, x, \cosh(c + dx)\right)}{4(a-b)bd} \\ &= -\frac{a \cosh(c + dx)}{4(a-b)bd(a-b+b \cosh^2(c + dx))^2} + \frac{(a-4b) \cosh(c + dx)}{8(a-b)^2bd(a-b+b \cosh^2(c + dx))} + \frac{(a-4b)}{8(a-b)^2bd} \\ &= \frac{(a-4b) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}b^{3/2}d} - \frac{a \cosh(c + dx)}{4(a-b)bd(a-b+b \cosh^2(c + dx))^2} + \frac{(a-4b)}{8(a-b)^2bd} \end{aligned}$$

Mathematica [C] time = 1.22242, size = 170, normalized size = 1.26

$$\frac{2\sqrt{b} \cosh(c+dx)(-2a^2+b(a-4b) \cosh(2(c+dx))-5ab+4b^2)}{(a-b)^2(2a+b \cosh(2(c+dx))-b)^2} + \frac{(a-4b)\left(\tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) + \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{5/2}}}{8b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (((a - 4*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(5/2)
+ (2*Sqrt[b]*Cosh[c + d*x]*(-2*a^2 - 5*a*b + 4*b^2 + (a - 4*b)*b*Cosh[2*(c
+ d*x)])))/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*b^(3/2)*d)
```

Maple [B] time = 0.042, size = 961, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/d/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^4+1/2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+2/d/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^4+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
2*a^2+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*a-4/d/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a^2/b/(a^2-2*a*b+b^2)-1/2/d/(
tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)^2*a/(a^2-2*a*b+b^2)+1/8/d/b/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*
(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))*a-1/2/d/(a^2-2*a*b+b^2
)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(
1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(abe^{7c} - 4b^2e^{7c})e^{7dx} - (4a^2e^{5c} + 9abe^{5c})}{4(a^2b^3d - 2ab^4d + b^5d + (a^2b^3de^{8c} - 2ab^4de^{8c} + b^5de^{8c})e^{8dx})} + 4(2a^3b^2de^{6c} - 5a^2b^3de^{6c} + 4ab^4de^{6c} - b^5de^{6c})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{4}((a*b*e^{7*c} - 4*b^2*e^{7*c})*e^{7*d*x} - (4*a^2*e^{5*c} + 9*a*b*e^{5*c} - 4*b^2*e^{5*c})*e^{5*d*x} - (4*a^2*e^{3*c} + 9*a*b*e^{3*c} - 4*b^2*e^{3*c})*e^{3*d*x} + (a*b*e^c - 4*b^2*e^c)*e^{d*x}) / (a^2*b^3*d - 2*a*b^4*d + b^5*d + (a^2*b^3*d*e^{8*c} - 2*a*b^4*d*e^{8*c} + b^5*d*e^{8*c})*e^{8*d*x} + 4*(2*a^3*b^2*d*e^{6*c} - 5*a^2*b^3*d*e^{6*c} + 4*a*b^4*d*e^{6*c} - b^5*d*e^{6*c})*e^{6*d*x} + 2*(8*a^4*b*d*e^{4*c} - 24*a^3*b^2*d*e^{4*c} + 27*a^2*b^3*d*e^{4*c} - 14*a*b^4*d*e^{4*c} + 3*b^5*d*e^{4*c})*e^{4*d*x} + 4*(2*a^3*b^2*d*e^{2*c} - 5*a^2*b^3*d*e^{2*c} + 4*a*b^4*d*e^{2*c} - b^5*d*e^{2*c})*e^{2*d*x}) + 1/8*integrate(2*((a*e^{3*c} - 4*b*e^{3*c})*e^{3*d*x} - (a*e^c - 4*b*e^c)*e^{d*x}) / (a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2*e^{4*c} - 2*a*b^3*e^{4*c} + b^4*e^{4*c})*e^{4*d*x} + 2*(2*a^3*b*e^{2*c} - 5*a^2*b^2*e^{2*c} + 4*a*b^3*e^{2*c} - b^4*e^{2*c})*e^{2*d*x}), x)$

Fricas [B] time = 2.96323, size = 14074, normalized size = 104.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $[1/16*(4*(a^2*b^2 - 5*a*b^3 + 4*b^4)*\cosh(d*x + c)^7 + 28*(a^2*b^2 - 5*a*b^3 + 4*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 4*(a^2*b^2 - 5*a*b^3 + 4*b^4)*\sinh(d*x + c)^7 - 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*\cosh(d*x + c)^5 - 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4 - 21*(a^2*b^2 - 5*a*b^3 + 4*b^4))*\cosh(d*x + c)^2*\sinh(d*x + c)^5 + 20*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*\cosh(d*x + c)^3 - (4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*\cosh(d*x + c)^3 + 4*(35*(a^2*b^2 - 5*a*b^3 + 4*b^4)*\cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 13*a*b^3 - 4*b^4 - 10*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 4*(21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*\cosh(d*x + c)^5 - 10*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*\cosh(d*x + c)^3 - 3*(4*a^3*b + 5*$

$$\begin{aligned}
& 6 + 15*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*\cosh(d*x + c)^4 \\
& + 3*(8*a^5*b^2 - 32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7) \\
& *d*\cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d) \\
& *sinh(d*x + c)^2 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d + 8*((a^3*b^4 - \\
& 3*a^2*b^5 + 3*a*b^6 - b^7)*d*\cosh(d*x + c)^7 + 3*(2*a^4*b^3 - 7*a^3*b^4 + 9 \\
& *a^2*b^5 - 5*a*b^6 + b^7)*d*\cosh(d*x + c)^5 + (8*a^5*b^2 - 32*a^4*b^3 + 51* \\
& a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*\cosh(d*x + c)^3 + (2*a^4*b^3 - 7 \\
& *a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*\cosh(d*x + c))*sinh(d*x + c)), 1/8* \\
& (2*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^7 + 14*(a^2*b^2 - 5*a*b^3 + 4* \\
& b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2*b^2 - 5*a*b^3 + 4*b^4)*sinh(d*x \\
& + c)^7 - 2*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^5 - 2*(4 \\
& *a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4 - 21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh \\
& (d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + \\
& c)^3 - (4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + \\
& c)^4 - 2*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + 2*(35*(\\
& a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 13*a*b^3 \\
& - 4*b^4 - 10*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^2)*sin \\
& h(d*x + c)^3 + 2*(21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^5 - 10*(4*a^ \\
& 3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 - 3*(4*a^3*b + 5*a^2*b^ \\
& 2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((a*b^2 - 4*b^3)*cos \\
& h(d*x + c)^8 + 8*(a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - 4 \\
& *b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 4*(\\
& 2*a^2*b - 9*a*b^2 + 4*b^3 + 7*(a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c \\
&)^6 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)* \\
& cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*c \\
& osh(d*x + c)^4 + 2*(35*(a*b^2 - 4*b^3)*cosh(d*x + c)^4 + 8*a^3 - 40*a^2*b + \\
& 35*a*b^2 - 12*b^3 + 30*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d \\
& *x + c)^4 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b - 9*a*b^2 + \\
& 4*b^3)*cosh(d*x + c)^3 + (8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + \\
& c))*sinh(d*x + c)^3 + a*b^2 - 4*b^3 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d* \\
& x + c)^2 + 4*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b - 9*a*b^2 + 4 \\
& *b^3)*cosh(d*x + c)^4 + 2*a^2*b - 9*a*b^2 + 4*b^3 + 3*(8*a^3 - 40*a^2*b + 3 \\
& 5*a*b^2 - 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 - 4*b^3)*cos \\
& h(d*x + c)^7 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + (8*a^3 - 40* \\
& a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + (2*a^2*b - 9*a*b^2 + 4*b^3)*co \\
& sh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3 \\
& + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(\\
& d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2) \\
&) - ((a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8*(a*b^2 - 4*b^3)*cosh(d*x + c)*sinh \\
& (d*x + c)^7 + (a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b - 9*a*b^2 + 4*b^ \\
& 3)*cosh(d*x + c)^6 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3 + 7*(a*b^2 - 4*b^3)*cosh(\\
& d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(2*a \\
& ^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 - 40*a^2* \\
& b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 - 4*b^3)*cosh(d*x + c \\
&)^4 + 8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3 + 30*(2*a^2*b - 9*a*b^2 + 4*b^3)
\end{aligned}$$

```

*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^5 +
10*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + (8*a^3 - 40*a^2*b + 35*a*b
^2 - 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - 4*b^3 + 4*(2*a^2*b -
9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 1
5*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 2*a^2*b - 9*a*b^2 + 4*b^3 +
3*(8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 8*((a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d
*x + c)^5 + (8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + (2*a^2
*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2)*arctan(
-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*(a^2*b^2
- 5*a*b^3 + 4*b^4)*cosh(d*x + c) + 2*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*
x + c)^6 - 5*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^4 + a^2
*b^2 - 5*a*b^3 + 4*b^4 - 3*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x
+ c)^8 + 8*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)*sinh(d*x +
c)^7 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*sinh(d*x + c)^8 + 4*(2*a^4*
b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^3*
b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4
+ 9*a^2*b^5 - 5*a*b^6 + b^7)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^2 - 32*a^4*b^
3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a
^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^3 + 3*(2*a^4*b^3 - 7*a^
3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35
*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 -
7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + (8*a^5*b^2 - 32*
a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d)*sinh(d*x + c)^4 +
4*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + 8
*(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^5 + 10*(2*a^4*b^3
- 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^3 + (8*a^5*b^2 -
32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 4*(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)
^6 + 15*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)
^4 + 3*(8*a^5*b^2 - 32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7
)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d
)*sinh(d*x + c)^2 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d + 8*((a^3*b^4 -
3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^7 + 3*(2*a^4*b^3 - 7*a^3*b^4 +
9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^5 + (8*a^5*b^2 - 32*a^4*b^3 + 51
*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^3 + (2*a^4*b^3 -
7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.53 \quad \int \frac{\sinh^2(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=139

$$-\frac{(4a-b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a-b)^{5/2}} + \frac{(2a+b) \sinh(c+dx) \cosh(c+dx)}{8ad(a-b)^2(a+b \sinh^2(c+dx))} + \frac{\sinh(c+dx) \cosh(c+dx)}{4d(a-b)(a+b \sinh^2(c+dx))^2}$$

[Out] $-\left(\frac{(4a-b) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tanh}[c+dx]}{\sqrt{a}}\right]}{8a^{3/2}d(a-b)^{5/2}} + \frac{(2a+b) \operatorname{Cosh}[c+dx] \operatorname{Sinh}[c+dx]}{8ad(a-b)^2(a+b \operatorname{Sinh}[c+dx]^2)} + \frac{\operatorname{Sinh}[c+dx] \operatorname{Cosh}[c+dx]}{4d(a-b)(a+b \operatorname{Sinh}[c+dx]^2)}\right)$

Rubi [A] time = 0.156531, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3173, 12, 3181, 208}

$$-\frac{(4a-b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a-b)^{5/2}} + \frac{(2a+b) \sinh(c+dx) \cosh(c+dx)}{8ad(a-b)^2(a+b \sinh^2(c+dx))} + \frac{\sinh(c+dx) \cosh(c+dx)}{4d(a-b)(a+b \sinh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sinh}[c+dx]^2/(a+b \operatorname{Sinh}[c+dx]^2)^3, x]$

[Out] $-\left(\frac{(4a-b) \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tanh}[c+dx]}{\sqrt{a}}\right]}{8a^{3/2}d(a-b)^{5/2}} + \frac{(2a+b) \operatorname{Cosh}[c+dx] \operatorname{Sinh}[c+dx]}{8ad(a-b)^2(a+b \operatorname{Sinh}[c+dx]^2)} + \frac{\operatorname{Sinh}[c+dx] \operatorname{Cosh}[c+dx]}{4d(a-b)(a+b \operatorname{Sinh}[c+dx]^2)}\right)$

Rule 3173

$\operatorname{Int}[(a + b \sin(e + f x))^2]^p \operatorname{Int}[(A + B \sin(e + f x) + (f x)^2), x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[(A b - a B) \operatorname{Cos}[e + f x] \operatorname{Sin}[e + f x] (a + b \operatorname{Sin}[e + f x]^2)^{p+1}] / (2 a f (a + b) (p + 1)), x] - \operatorname{Dist}[1 / (2 a (a + b) (p + 1)), \operatorname{Int}[(a + b \operatorname{Sin}[e + f x]^2)^{p+1} \operatorname{Simp}[a B - A (2 a (p + 1) + b (2 p + 3)) + 2 (A b - a B) (p + 2) \operatorname{Sin}[e + f x]^2, x], x] /; \operatorname{FreeQ}\{a, b, e, f, A, B\}, x] \&\& \operatorname{LtQ}[p, -1] \&\& \operatorname{NeQ}[a + b, 0]$

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3181

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^2(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d(a+b\sinh^2(c+dx))^2} - \frac{\int \frac{a-2a\sinh^2(c+dx)}{(a+b\sinh^2(c+dx))^2} dx}{4a(a-b)} \\
 &= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d(a+b\sinh^2(c+dx))^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d(a+b\sinh^2(c+dx))} - \frac{\int \frac{a(4a-b)}{a+b\sinh^2(c+dx)}}{8a^2(a-b)^2} \\
 &= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d(a+b\sinh^2(c+dx))^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d(a+b\sinh^2(c+dx))} - \frac{(4a-b)\int \frac{1}{a+b\sinh^2(c+dx)}}{8a(a-b)^2} \\
 &= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d(a+b\sinh^2(c+dx))^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d(a+b\sinh^2(c+dx))} - \frac{(4a-b)\text{Subst}\left[\frac{1}{a+b\sinh^2(c+dx)}, \frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right]}{8a^2(a-b)^2} \\
 &= -\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^{5/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d(a+b\sinh^2(c+dx))^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d(a+b\sinh^2(c+dx))}
 \end{aligned}$$

Mathematica [A] time = 1.34558, size = 121, normalized size = 0.87

$$\frac{\sinh(2(c+dx))(8a^2+b(2a+b)\cosh(2(c+dx))-4ab-b^2)}{a(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} - \frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}}$$

8d

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-(((4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a - b)^(5/2))) + ((8*a^2 - 4*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x)]*Sinh[2*(c + d*x)])/(a*(a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)]^2)))/(8*d)
```

Maple [B] time = 0.049, size = 1408, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*b-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a+9/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b^2-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*a+9/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b-1/2/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+b+1/2/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)+1/2/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+b+1/8/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8
```

$$\frac{d}{dx} \frac{1}{(a^2 - 2ab + b^2)^{1/2}} \frac{1}{a} \frac{1}{(-b(a-b))^{1/2}} \frac{1}{((2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}} \operatorname{arctanh}\left(\frac{a \tanh(1/2 dx + 1/2 c)}{(2(-b(a-b))^{1/2} + a - 2b)a}\right) b^2 - 1/8$$

$$\frac{d}{dx} \frac{1}{(a^2 - 2ab + b^2)^{1/2}} \frac{1}{b} \frac{1}{a} \frac{1}{((2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}} \operatorname{arctan}\left(\frac{a \tanh(1/2 dx + 1/2 c)}{(2(-b(a-b))^{1/2} - a + 2b)a}\right) - 1/8$$

$$\frac{d}{dx} \frac{1}{(a^2 - 2ab + b^2)^{1/2}} \frac{1}{a} \frac{1}{(-b(a-b))^{1/2}} \frac{1}{((2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}} \operatorname{arctan}\left(\frac{a \tanh(1/2 dx + 1/2 c)}{(2(-b(a-b))^{1/2} - a + 2b)a}\right) b^2$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 3.00344, size = 12358, normalized size = 88.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*\cosh(d*x + c)^6 + 24*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^5 + 4*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*\sinh(d*x + c)^6 + 8*a^3*b^2 - 4*a^2*b^3 - 4*a*b^4 + 4*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^4 + 4*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 15*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 16*(5*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*\cosh(d*x + c)^3 + (16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^2 + 4*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^4 + 15*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*\cosh(d*x + c)^4 + 6*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + ((4*a*b^3 - b^4)*\cosh(d*x + c)^8 + 8*(4*a*b^3 - b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (4*a*b^3 - b^4)*\sinh(d*x + c)^8 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4)*\cosh(d*x + c)^6 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4 + 7*(4*a*b^3 - b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(4*a*b^3 - b^4)*\cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 6*a*b^3 \end{aligned}$$

$$\begin{aligned}
& + b^4) * \cosh(d*x + c)) * \sinh(d*x + c)^5 + 2*(32*a^3*b - 40*a^2*b^2 + 20*a*b^3 \\
& - 3*b^4) * \cosh(d*x + c)^4 + 2*(35*(4*a*b^3 - b^4) * \cosh(d*x + c)^4 + 32*a^3*b \\
& - 40*a^2*b^2 + 20*a*b^3 - 3*b^4 + 30*(8*a^2*b^2 - 6*a*b^3 + b^4) * \cosh(d*x \\
& + c)^2) * \sinh(d*x + c)^4 + 4*a*b^3 - b^4 + 8*(7*(4*a*b^3 - b^4) * \cosh(d*x + \\
& c)^5 + 10*(8*a^2*b^2 - 6*a*b^3 + b^4) * \cosh(d*x + c)^3 + (32*a^3*b - 40*a^2*b \\
& - 20*a*b^3 - 3*b^4) * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 6*a \\
& *b^3 + b^4) * \cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - b^4) * \cosh(d*x + c)^6 + 15*(8 \\
& *a^2*b^2 - 6*a*b^3 + b^4) * \cosh(d*x + c)^4 + 8*a^2*b^2 - 6*a*b^3 + b^4 + 3*(3 \\
& 2*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + \\
& 8*((4*a*b^3 - b^4) * \cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 6*a*b^3 + b^4) * \cosh(d \\
& x + c)^5 + (32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4) * \cosh(d*x + c)^3 + (8 \\
& *a^2*b^2 - 6*a*b^3 + b^4) * \cosh(d*x + c)) * \sinh(d*x + c)) * \sqrt{a^2 - a*b} * \log(\\
& (b^2 * \cosh(d*x + c)^4 + 4*b^2 * \cosh(d*x + c) * \sinh(d*x + c)^3 + b^2 * \sinh(d*x + \\
& c)^4 + 2*(2*a*b - b^2) * \cosh(d*x + c)^2 + 2*(3*b^2 * \cosh(d*x + c)^2 + 2*a*b \\
& - b^2) * \sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2 * \cosh(d*x + c)^3 + (2 \\
& *a*b - b^2) * \cosh(d*x + c)) * \sinh(d*x + c) - 4*(b * \cosh(d*x + c)^2 + 2*b * \cosh(d \\
& *x + c) * \sinh(d*x + c) + b * \sinh(d*x + c)^2 + 2*a - b) * \sqrt{a^2 - a*b})) / (b * \co \\
& sh(d*x + c)^4 + 4*b * \cosh(d*x + c) * \sinh(d*x + c)^3 + b * \sinh(d*x + c)^4 + 2*(\\
& 2*a - b) * \cosh(d*x + c)^2 + 2*(3*b * \cosh(d*x + c)^2 + 2*a - b) * \sinh(d*x + c)^ \\
& 2 + 4*(b * \cosh(d*x + c)^3 + (2*a - b) * \cosh(d*x + c)) * \sinh(d*x + c) + b)) + 8 \\
& *(3*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4) * \cosh(d*x + c)^5 + 2*(16*a^5 - 24*a^4*b \\
& + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4) * \cosh(d*x + c)^3 + (16*a^4*b - 20*a^3*b^2 \\
& + a^2*b^3 + 3*a*b^4) * \cosh(d*x + c)) * \sinh(d*x + c)) / ((a^5*b^3 - 3*a^4*b^4 + \\
& 3*a^3*b^5 - a^2*b^6) * d * \cosh(d*x + c)^8 + 8*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^ \\
& 5 - a^2*b^6) * d * \cosh(d*x + c) * \sinh(d*x + c)^7 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3 \\
& *b^5 - a^2*b^6) * d * \sinh(d*x + c)^8 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - \\
& 5*a^3*b^5 + a^2*b^6) * d * \cosh(d*x + c)^6 + 4*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b \\
& b^5 - a^2*b^6) * d * \cosh(d*x + c)^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a \\
& ^3*b^5 + a^2*b^6) * d) * \sinh(d*x + c)^6 + 2*(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 \\
& - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6) * d * \cosh(d*x + c)^4 + 8*(7*(a^5*b^3 - \\
& 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6) * d * \cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 7*a^5 * \\
& b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6) * d * \cosh(d*x + c)) * \sinh(d*x + c)^5 + 2 \\
& *(35*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6) * d * \cosh(d*x + c)^4 + 30*(2 \\
& *a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6) * d * \cosh(d*x + c)^2 + \\
& (8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6) * d \\
&) * \sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2 \\
& *b^6) * d * \cosh(d*x + c)^2 + 8*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6) * d \\
& * \cosh(d*x + c)^5 + 10*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2 \\
& *b^6) * d * \cosh(d*x + c)^3 + (8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + \\
& 17*a^3*b^5 - 3*a^2*b^6) * d * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 4*(7*(a^5*b^3 - \\
& 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6) * d * \cosh(d*x + c)^6 + 15*(2*a^6*b^2 - 7*a^5 * \\
& b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6) * d * \cosh(d*x + c)^4 + 3*(8*a^7*b - 32 \\
& *a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6) * d * \cosh(d*x + c) \\
& ^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6) * d) * \sinh(d*x \\
& + c)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6) * d + 8*((a^5*b^3 - 3*a^
\end{aligned}$$

$$\begin{aligned}
&4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*\cosh(d*x + c)^7 + 3*(2*a^6*b^2 - 7*a^5*b^3 + \\
&9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*\cosh(d*x + c)^5 + (8*a^7*b - 32*a^6*b^2 \\
&+ 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*\cosh(d*x + c)^3 + (2 \\
&*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*\cosh(d*x + c))*\si \\
&\text{nh}(d*x + c)), -1/8*(2*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*\cosh(d*x + c)^6 + 12* \\
&(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^5 + 2*(4*a^3*b^ \\
&2 - 5*a^2*b^3 + a*b^4)*\sinh(d*x + c)^6 + 4*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + \\
&2*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^4 + 2 \\
&*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 15*(4*a^3*b^2 - 5*a \\
&^2*b^3 + a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(5*(4*a^3*b^2 - 5*a^2* \\
&b^3 + a*b^4)*\cosh(d*x + c)^3 + (16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - \\
&3*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 2*(16*a^4*b - 20*a^3*b^2 + a^2*b \\
&^3 + 3*a*b^4)*\cosh(d*x + c)^2 + 2*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^ \\
&4 + 15*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*\cosh(d*x + c)^4 + 6*(16*a^5 - 24*a^4 \\
&*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - ((\\
&4*a*b^3 - b^4)*\cosh(d*x + c)^8 + 8*(4*a*b^3 - b^4)*\cosh(d*x + c)*\sinh(d*x + \\
&c)^7 + (4*a*b^3 - b^4)*\sinh(d*x + c)^8 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4)*\cos \\
&\text{h}(d*x + c)^6 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4 + 7*(4*a*b^3 - b^4)*\cosh(d*x + \\
&c)^2)*\sinh(d*x + c)^6 + 8*(7*(4*a*b^3 - b^4)*\cosh(d*x + c)^3 + 3*(8*a^2*b^2 \\
&- 6*a*b^3 + b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(32*a^3*b - 40*a^2*b^2 \\
&+ 20*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 2*(35*(4*a*b^3 - b^4)*\cosh(d*x + c)^ \\
&4 + 32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4 + 30*(8*a^2*b^2 - 6*a*b^3 + b^ \\
&4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*a*b^3 - b^4 + 8*(7*(4*a*b^3 - b^4)* \\
&\cosh(d*x + c)^5 + 10*(8*a^2*b^2 - 6*a*b^3 + b^4)*\cosh(d*x + c)^3 + (32*a^3* \\
&b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a^ \\
&2*b^2 - 6*a*b^3 + b^4)*\cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - b^4)*\cosh(d*x + c) \\
&^6 + 15*(8*a^2*b^2 - 6*a*b^3 + b^4)*\cosh(d*x + c)^4 + 8*a^2*b^2 - 6*a*b^3 + \\
&b^4 + 3*(32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*\cosh(d*x + c)^2)*\sinh(d \\
&*x + c)^2 + 8*((4*a*b^3 - b^4)*\cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 6*a*b^3 + b \\
&^4)*\cosh(d*x + c)^5 + (32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*\cosh(d*x + \\
&c)^3 + (8*a^2*b^2 - 6*a*b^3 + b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\text{sqrt}(-a^2 \\
&+ a*b)*\text{arctan}(-1/2*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + \\
&b*\sinh(d*x + c)^2 + 2*a - b)*\text{sqrt}(-a^2 + a*b)/(a^2 - a*b)) + 4*(3*(4*a^3*b^ \\
&2 - 5*a^2*b^3 + a*b^4)*\cosh(d*x + c)^5 + 2*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + \\
&5*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^3 + (16*a^4*b - 20*a^3*b^2 + a^2*b^3 + \\
&3*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - \\
&a^2*b^6)*d*\cosh(d*x + c)^8 + 8*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)* \\
&d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^ \\
&6)*d*\sinh(d*x + c)^8 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a \\
&^2*b^6)*d*\cosh(d*x + c)^6 + 4*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6 \\
&)*d*\cosh(d*x + c)^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2* \\
&b^6)*d)*\sinh(d*x + c)^6 + 2*(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 \\
&+ 17*a^3*b^5 - 3*a^2*b^6)*d*\cosh(d*x + c)^4 + 8*(7*(a^5*b^3 - 3*a^4*b^4 + \\
&3*a^3*b^5 - a^2*b^6)*d*\cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b \\
&^4 - 5*a^3*b^5 + a^2*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^5*b^3
\end{aligned}$$

$$\begin{aligned}
& - 3a^4b^4 + 3a^3b^5 - a^2b^6) * d * \cosh(dx + c)^4 + 30 * (2a^6b^2 - 7a^5b^3 + 9a^4b^4 - 5a^3b^5 + a^2b^6) * d * \cosh(dx + c)^2 + (8a^7b - 32a^6b^2 + 51a^5b^3 - 41a^4b^4 + 17a^3b^5 - 3a^2b^6) * d * \sinh(dx + c)^4 + 4 * (2a^6b^2 - 7a^5b^3 + 9a^4b^4 - 5a^3b^5 + a^2b^6) * d * \cosh(dx + c)^2 + 8 * (7 * (a^5b^3 - 3a^4b^4 + 3a^3b^5 - a^2b^6) * d * \cosh(dx + c)^5 + 10 * (2a^6b^2 - 7a^5b^3 + 9a^4b^4 - 5a^3b^5 + a^2b^6) * d * \cosh(dx + c)^3 + (8a^7b - 32a^6b^2 + 51a^5b^3 - 41a^4b^4 + 17a^3b^5 - 3a^2b^6) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4 * (7 * (a^5b^3 - 3a^4b^4 + 3a^3b^5 - a^2b^6) * d * \cosh(dx + c)^6 + 15 * (2a^6b^2 - 7a^5b^3 + 9a^4b^4 - 5a^3b^5 + a^2b^6) * d * \cosh(dx + c)^4 + 3 * (8a^7b - 32a^6b^2 + 51a^5b^3 - 41a^4b^4 + 17a^3b^5 - 3a^2b^6) * d * \cosh(dx + c)^2 + (2a^6b^2 - 7a^5b^3 + 9a^4b^4 - 5a^3b^5 + a^2b^6) * d) * \sinh(dx + c)^2 + (a^5b^3 - 3a^4b^4 + 3a^3b^5 - a^2b^6) * d + 8 * ((a^5b^3 - 3a^4b^4 + 3a^3b^5 - a^2b^6) * d * \cosh(dx + c)^7 + 3 * (2a^6b^2 - 7a^5b^3 + 9a^4b^4 - 5a^3b^5 + a^2b^6) * d * \cosh(dx + c)^5 + (8a^7b - 32a^6b^2 + 51a^5b^3 - 41a^4b^4 + 17a^3b^5 - 3a^2b^6) * d * \cosh(dx + c)^3 + (2a^6b^2 - 7a^5b^3 + 9a^4b^4 - 5a^3b^5 + a^2b^6) * d * \cosh(dx + c)) * \sinh(dx + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)**2/(a+b*sinh(dx+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.52648, size = 378, normalized size = 2.72

$$\frac{(4a - b) \arctan\left(\frac{be^{2dx+2c} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{8(a^3d - 2a^2bd + ab^2d)\sqrt{-a^2 + ab}} - \frac{4ab^2e^{6dx+6c} - b^3e^{6dx+6c} + 16a^3e^{4dx+4c} - 8a^2be^{4dx+4c} - 2ab^2e^{4dx+4c} + 3b^3}{4(a^3bd - 2a^2b^2d + ab^3d)(be^{4dx+4c} + 4a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^2/(a+b*sinh(dx+c)^2)^3,x, algorithm="giac")

[Out] -1/8*(4*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(a^3*d - 2*a^2*b*d + a*b^2*d)*sqrt(-a^2 + a*b) - 1/4*(4*a*b^2*e^(6*d*x + 6

$$\begin{aligned}
 & *c) - b^3 e^{(6dx + 6c)} + 16a^3 e^{(4dx + 4c)} - 8a^2 b e^{(4dx + 4c)} \\
 &) - 2ab^2 e^{(4dx + 4c)} + 3b^3 e^{(4dx + 4c)} + 16a^2 b e^{(2dx + 2c)} \\
 & *c) - 4ab^2 e^{(2dx + 2c)} - 3b^3 e^{(2dx + 2c)} + 2ab^2 + b^3) / ((a^3 b d - 2a^2 b^2 d + a b^3 d) * (b e^{(4dx + 4c)} + 4a e^{(2dx + 2c)} - 2 \\
 & *b e^{(2dx + 2c)} + b)^2)
 \end{aligned}$$

$$3.54 \quad \int \frac{\sinh(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^3} dx$$

Optimal. Leaf size=118

$$\frac{3 \cosh(c+dx)}{8d(a-b)^2(a+b \cosh^2(c+dx)-b)} + \frac{\cosh(c+dx)}{4d(a-b)(a+b \cosh^2(c+dx)-b)^2} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{8\sqrt{bd}(a-b)^{5/2}}$$

[Out] (3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*Sqrt[b]*d) + Cosh[c + d*x]/(4*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) + (3*Cosh[c + d*x])/(8*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))

Rubi [A] time = 0.0831678, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3186, 199, 205}

$$\frac{3 \cosh(c+dx)}{8d(a-b)^2(a+b \cosh^2(c+dx)-b)} + \frac{\cosh(c+dx)}{4d(a-b)(a+b \cosh^2(c+dx)-b)^2} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{8\sqrt{bd}(a-b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*Sqrt[b]*d) + Cosh[c + d*x]/(4*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) + (3*Cosh[c + d*x])/(8*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 199

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer

$Q[2*p] \parallel (n == 2 \ \&\& \ \text{IntegerQ}[4*p]) \parallel (n == 2 \ \&\& \ \text{IntegerQ}[3*p]) \parallel \text{Denominator}[p + 1/n] < \text{Denominator}[p]$

Rule 205

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\sinh(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{\cosh(c+dx)}{4(a-b)d(a-b+b\cosh^2(c+dx))^2} + \frac{3 \text{Subst}\left(\int \frac{1}{(a-b+bx^2)^2} dx, x, \cosh(c+dx)\right)}{4(a-b)d} \\ &= \frac{\cosh(c+dx)}{4(a-b)d(a-b+b\cosh^2(c+dx))^2} + \frac{3 \cosh(c+dx)}{8(a-b)^2d(a-b+b\cosh^2(c+dx))} + \frac{3 \text{Subst}\left(\int \frac{1}{a-b+bx^2} dx, x, \cosh(c+dx)\right)}{4(a-b)d} \\ &= \frac{3 \tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}\sqrt{bd}} + \frac{\cosh(c+dx)}{4(a-b)d(a-b+b\cosh^2(c+dx))^2} + \frac{3 \cosh(c+dx)}{8(a-b)^2d(a-b+b\cosh^2(c+dx))} \end{aligned}$$

Mathematica [C] time = 0.746678, size = 149, normalized size = 1.26

$$\frac{2 \cosh(c+dx)(10a+3b \cosh(2(c+dx))-7b)}{(a-b)^2(2a+b \cosh(2(c+dx))-b)^2} + \frac{3 \left(\tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) + \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) \right)}{\sqrt{b}(a-b)^{5/2}}$$

$8d$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/((a - b)^(5/2)*Sqrt[b]) + (2*Cosh[c + d*x]*(10*a - 7*b + 3*b*Cosh[2*(c + d*x)]))/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)]^(2)))/(8*d)

Maple [B] time = 0.034, size = 964, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(dx+c)/(a+b\sinh(dx+c))^2)^3, x$

[Out]
$$-5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^6+4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^6*b-2/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^6*b^2+15/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4-23/2/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4+14/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4-4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4*b^3-15/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^2*a+8/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^2-2/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^2*b^2+5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)-1/2/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*b+3/8/d/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(20ae^{(5c)} - 11be^{(5c)})e^{(5dx)} + 4(a^2b^2d - 2ab^3d + b^4d + (a^2b^2de^{(8c)} - 2ab^3de^{(8c)} + b^4de^{(8c)})e^{(8dx)} + 4(2a^3bde^{(6c)} - 5a^2b^2de^{(6c)} + 4ab^3de^{(6c)} - b^4de^{(6c)}))}{(a^2-2ab+b^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(dx+c)/(a+b\sinh(dx+c))^2)^3, x, \text{algorithm}="maxima"$

```
[Out] 1/4*((20*a*e^(5*c) - 11*b*e^(5*c))*e^(5*d*x) + (20*a*e^(3*c) - 11*b*e^(3*c))
)*e^(3*d*x) + 3*b*e^(7*d*x + 7*c) + 3*b*e^(d*x + c))/(a^2*b^2*d - 2*a*b^3*d
+ b^4*d + (a^2*b^2*d*e^(8*c) - 2*a*b^3*d*e^(8*c) + b^4*d*e^(8*c))*e^(8*d*x
) + 4*(2*a^3*b*d*e^(6*c) - 5*a^2*b^2*d*e^(6*c) + 4*a*b^3*d*e^(6*c) - b^4*d*
e^(6*c))*e^(6*d*x) + 2*(8*a^4*d*e^(4*c) - 24*a^3*b*d*e^(4*c) + 27*a^2*b^2*d
*e^(4*c) - 14*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3*b*d*e
^(2*c) - 5*a^2*b^2*d*e^(2*c) + 4*a*b^3*d*e^(2*c) - b^4*d*e^(2*c))*e^(2*d*x)
) + 1/2*integrate(3/2*(e^(3*d*x + 3*c) - e^(d*x + c))/(a^2*b - 2*a*b^2 + b^
3 + (a^2*b*e^(4*c) - 2*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^3*e
^(2*c) - 5*a^2*b*e^(2*c) + 4*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.86219, size = 12081, normalized size = 102.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*(a*b^2 - b^3)*cosh(d*x + c)^7 + 84*(a*b^2 - b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + 12*(a*b^2 - b^3)*sinh(d*x + c)^7 + 4*(20*a^2*b - 31*a*b^2 +
11*b^3)*cosh(d*x + c)^5 + 4*(20*a^2*b - 31*a*b^2 + 11*b^3 + 63*(a*b^2 - b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(21*(a*b^2 - b^3)*cosh(d*x + c)^3
+ (20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(20*a^2
*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 4*(105*(a*b^2 - b^3)*cosh(d*x + c
)^4 + 20*a^2*b - 31*a*b^2 + 11*b^3 + 10*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^3 + 4*(63*(a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(20
*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 3*(20*a^2*b - 31*a*b^2 + 11*b
^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*
x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c
)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*co
sh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 -
8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b
^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cos
h(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*
cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b
^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x
+ c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x
+ c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b + b^2)*log(
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
- 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(
```

$$\begin{aligned}
& d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) \\
&) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 \\
& + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + \\
& b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c) \\
&)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d* \\
& x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) \\
& + b)) + 12*(a*b^2 - b^3)*cosh(d*x + c) + 4*(21*(a*b^2 - b^3)*cosh(d*x + c)^6 \\
& + 5*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 3*a*b^2 - 3*b^3 + 3* \\
& (20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^3*b^3 - \\
& 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^8 + 8*(a^3*b^3 - 3*a^2*b^4 + 3*a \\
& *b^5 - b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 \\
& - b^6)*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 \\
& + b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*co \\
& sh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh \\
& (d*x + c)^6 + 2*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 \\
& - 3*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*c \\
& osh(d*x + c)^3 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*co \\
& sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)* \\
& d*cosh(d*x + c)^4 + 30*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)* \\
& d*cosh(d*x + c)^2 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a* \\
& b^5 - 3*b^6)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5* \\
& a*b^5 + b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6) \\
& *d*cosh(d*x + c)^5 + 10*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6) \\
& *d*cosh(d*x + c)^3 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a \\
& *b^5 - 3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b^3 - 3*a^2*b^4 \\
& + 3*a*b^5 - b^6)*d*cosh(d*x + c)^6 + 15*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 \\
& - 5*a*b^5 + b^6)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - \\
& 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cosh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 \\
& + 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 - 3*a^2*b^4 + 3* \\
& a*b^5 - b^6)*d + 8*((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^7 \\
& + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d*x + c)^5 \\
& + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cos \\
& h(d*x + c)^3 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d \\
& *x + c))*sinh(d*x + c)), 1/8*(6*(a*b^2 - b^3)*cosh(d*x + c)^7 + 42*(a*b^2 - \\
& b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 6*(a*b^2 - b^3)*sinh(d*x + c)^7 + 2*(\\
& 20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 2*(20*a^2*b - 31*a*b^2 + 11 \\
& *b^3 + 63*(a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(21*(a*b^2 - \\
& b^3)*cosh(d*x + c)^3 + (20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d \\
& *x + c)^4 + 2*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 2*(105*(a*b^ \\
& 2 - b^3)*cosh(d*x + c)^4 + 20*a^2*b - 31*a*b^2 + 11*b^3 + 10*(20*a^2*b - 31 \\
& *a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*(a*b^2 - b^3)*cos \\
& h(d*x + c)^5 + 10*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 3*(20*a^ \\
& 2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + \\
& c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a* \\
& b - b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^6 + 8*(7*b^2*cosh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x \\
& + c)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + \\
& c)^4 + 30*(2*a*b - b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x \\
& + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a \\
& ^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d \\
& *x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3 \\
& *(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b \\
& ^2 + 8*(b^2*cosh(d*x + c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8* \\
& a*b + 3*b^2)*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))* \\
& sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x \\
& + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c \\
&)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - 3*(b^2*cosh(d*x + c)^8 + \\
& 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2 \\
&)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 \\
& + 8*(7*b^2*cosh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 \\
& + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + \\
& 30*(2*a*b - b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 \\
& + 8*(7*b^2*cosh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8* \\
& a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c) \\
& ^2 + 4*(7*b^2*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 \\
& - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8* \\
& (b^2*cosh(d*x + c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3 \\
& *b^2)*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a* \\
& b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b \\
&)) + 6*(a*b^2 - b^3)*cosh(d*x + c) + 2*(21*(a*b^2 - b^3)*cosh(d*x + c)^6 + \\
& 5*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 3*a*b^2 - 3*b^3 + 3*(20* \\
& a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^3*b^3 - 3*a^ \\
& 2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^8 + 8*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 \\
& - b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - \\
& b^6)*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b \\
& ^6)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d \\
& *x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh(d*x \\
& + c)^6 + 2*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 - 3* \\
& b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(\\
& d*x + c)^3 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d \\
& *x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*co \\
& sh(d*x + c)^4 + 30*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*co \\
& sh(d*x + c)^2 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 \\
& - 3*b^6)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^ \\
& 5 + b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*c \\
& osh(d*x + c)^5 + 10*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*c \\
& osh(d*x + c)^3 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 \\
& - 3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3* \\
& a*b^5 - b^6)*d*cosh(d*x + c)^6 + 15*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5* \\
& a*b^5 + b^6)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*
\end{aligned}$$

$$a^2b^4 + 17ab^5 - 3b^6)d \cosh(dx + c)^2 + (2a^4b^2 - 7a^3b^3 + 9a^2b^4 - 5ab^5 + b^6)d) \sinh(dx + c)^2 + (a^3b^3 - 3a^2b^4 + 3ab^5 - b^6)d + 8((a^3b^3 - 3a^2b^4 + 3ab^5 - b^6)d \cosh(dx + c)^7 + 3(2a^4b^2 - 7a^3b^3 + 9a^2b^4 - 5ab^5 + b^6)d \cosh(dx + c)^5 + (8a^5b - 32a^4b^2 + 51a^3b^3 - 41a^2b^4 + 17ab^5 - 3b^6)d \cosh(dx + c)^3 + (2a^4b^2 - 7a^3b^3 + 9a^2b^4 - 5ab^5 + b^6)d \cosh(dx + c)) \sinh(dx + c)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.55 \quad \int \frac{1}{\left(a+b \sinh^2(c+dx)\right)^3} dx$$

Optimal. Leaf size=154

$$\frac{(8a^2 - 8ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{5/2}} - \frac{3b(2a-b) \sinh(c+dx) \cosh(c+dx)}{8a^2d(a-b)^2(a+b \sinh^2(c+dx))} - \frac{b \sinh(c+dx) \cosh(c+dx)}{4ad(a-b)(a+b \sinh^2(c+dx))^2}$$

[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^(5/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(4*a*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2) - (3*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*a^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.154646, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3184, 3173, 12, 3181, 208}

$$\frac{(8a^2 - 8ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{5/2}} - \frac{3b(2a-b) \sinh(c+dx) \cosh(c+dx)}{8a^2d(a-b)^2(a+b \sinh^2(c+dx))} - \frac{b \sinh(c+dx) \cosh(c+dx)}{4ad(a-b)(a+b \sinh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^(-3), x]

[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^(5/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(4*a*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2) - (3*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*a^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))

Rule 3184

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a + b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;

FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \sinh^2(c + dx))^3} dx &= -\frac{b \cosh(c + dx) \sinh(c + dx)}{4a(a - b)d (a + b \sinh^2(c + dx))^2} - \frac{\int \frac{-4a + 3b + 2b \sinh^2(c + dx)}{(a + b \sinh^2(c + dx))^2} dx}{4a(a - b)} \\
&= -\frac{b \cosh(c + dx) \sinh(c + dx)}{4a(a - b)d (a + b \sinh^2(c + dx))^2} - \frac{3(2a - b)b \cosh(c + dx) \sinh(c + dx)}{8a^2(a - b)^2d (a + b \sinh^2(c + dx))} - \frac{\int \frac{-8a^2 + 8ab - b^2 \sinh^2(c + dx)}{a + b \sinh^2(c + dx)} dx}{8a^2(a - b)} \\
&= -\frac{b \cosh(c + dx) \sinh(c + dx)}{4a(a - b)d (a + b \sinh^2(c + dx))^2} - \frac{3(2a - b)b \cosh(c + dx) \sinh(c + dx)}{8a^2(a - b)^2d (a + b \sinh^2(c + dx))} + \frac{(8a^2 - 8ab - b^2 \sinh^2(c + dx))}{8a^2(a - b)} \\
&= -\frac{b \cosh(c + dx) \sinh(c + dx)}{4a(a - b)d (a + b \sinh^2(c + dx))^2} - \frac{3(2a - b)b \cosh(c + dx) \sinh(c + dx)}{8a^2(a - b)^2d (a + b \sinh^2(c + dx))} + \frac{(8a^2 - 8ab - b^2 \sinh^2(c + dx))}{8a^2(a - b)} \\
&= \frac{(8a^2 - 8ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c + dx)}{\sqrt{a}}\right)}{8a^{5/2}(a - b)^{5/2}d} - \frac{b \cosh(c + dx) \sinh(c + dx)}{4a(a - b)d (a + b \sinh^2(c + dx))^2} - \frac{3(2a - b)b \cosh(c + dx) \sinh(c + dx)}{8a^2(a - b)^2d (a + b \sinh^2(c + dx))}
\end{aligned}$$

Mathematica [A] time = 1.21876, size = 132, normalized size = 0.86

$$\frac{(8a^2 - 8ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{ab} \sinh(2(c+dx))(-16a^2 + 3b(b-2a) \cosh(2(c+dx)) + 16ab - 3b^2)}{(a-b)^2(2a+b \cosh(2(c+dx)) - b)^2}$$

$$8a^{5/2}d$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-3), x]

[Out] (((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a - b)^(5/2) + (Sqrt[a]*b*(-16*a^2 + 16*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2)/(8*a^(5/2)*d)

Maple [B] time = 0.053, size = 1768, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(d*x+c)^2)^3, x)

[Out] -2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*b+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b-29/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b^2+3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b-29/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2+3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)

$$\begin{aligned} &) * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (-b * (a - b))^{1/2} + a - 2 * b) * a)^{1/2}) - 1/d / (\\ & a^{2 - 2 * a * b + b^2} * b / a / ((2 * (-b * (a - b))^{1/2} + a - 2 * b) * a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2 * \\ & d * x + 1/2 * c) / ((2 * (-b * (a - b))^{1/2} + a - 2 * b) * a)^{1/2}) + 3/8 / d / a^2 / (a^{2 - 2 * a * b + b^2} / \\ & ((2 * (-b * (a - b))^{1/2} + a - 2 * b) * a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (-b * \\ & (a - b))^{1/2} + a - 2 * b) * a)^{1/2}) * b^2 - 1/d / (a^{2 - 2 * a * b + b^2} / (-b * (a - b))^{1/2} / ((2 * \\ & (-b * (a - b))^{1/2} + a - 2 * b) * a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (-b * (a - b) \\ &))^{1/2} + a - 2 * b) * a)^{1/2}) * b + 1/d / (a^{2 - 2 * a * b + b^2} / a / (-b * (a - b))^{1/2} / ((2 * (-b * \\ & (a - b))^{1/2} + a - 2 * b) * a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (-b * (a - b))^{1/2} \\ & (1/2) + a - 2 * b) * a)^{1/2}) * b^2 - 3/8 / d / a^2 / (a^{2 - 2 * a * b + b^2} / (-b * (a - b))^{1/2} / ((2 * (-b * \\ & (a - b))^{1/2} + a - 2 * b) * a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (-b * (a - b)) \\ & ^{1/2} + a - 2 * b) * a)^{1/2}) * b^3 - 1/d / (a^{2 - 2 * a * b + b^2} / ((2 * (-b * (a - b))^{1/2} - a + 2 * b) \\ & * a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (-b * (a - b))^{1/2} - a + 2 * b) * a)^{1/2} \\ &) + 1/d / (a^{2 - 2 * a * b + b^2} * b / a / ((2 * (-b * (a - b))^{1/2} - a + 2 * b) * a)^{1/2} * \operatorname{arctan}(a * \tan \\ & h(1/2 * d * x + 1/2 * c) / ((2 * (-b * (a - b))^{1/2} - a + 2 * b) * a)^{1/2}) - 3/8 / d / a^2 / (a^{2 - 2 * a * b \\ & + b^2} / ((2 * (-b * (a - b))^{1/2} - a + 2 * b) * a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 \\ & * (-b * (a - b))^{1/2} - a + 2 * b) * a)^{1/2}) * b^2 - 1/d / (a^{2 - 2 * a * b + b^2} / (-b * (a - b))^{1/2} \\ & / ((2 * (-b * (a - b))^{1/2} - a + 2 * b) * a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (-b * \\ & (a - b))^{1/2} - a + 2 * b) * a)^{1/2}) * b + 1/d / (a^{2 - 2 * a * b + b^2} / a / (-b * (a - b))^{1/2} / ((2 * \\ & (-b * (a - b))^{1/2} - a + 2 * b) * a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (-b * (a - b) \\ &)^{1/2} - a + 2 * b) * a)^{1/2}) * b^2 - 3/8 / d / a^2 / (a^{2 - 2 * a * b + b^2} / (-b * (a - b))^{1/2} / ((2 \\ & * (-b * (a - b))^{1/2} - a + 2 * b) * a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c) / ((2 * (-b * (a - b) \\ &))^{1/2} - a + 2 * b) * a)^{1/2}) * b^3 \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.96866, size = 13509, normalized size = 87.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c))^3,x, algorithm="fricas")

```
[Out] [1/16*(4*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24
*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^
5 + 4*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 24*a^
3*b^2 - 36*a^2*b^3 + 12*a*b^4 + 12*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2
*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 12*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a
^2*b^3 + 3*a*b^4 + 5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x
+ c)^2)*sinh(d*x + c)^4 + 16*(5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b
^4)*cosh(d*x + c)^3 + 3*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*
b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3
- 9*a*b^4)*cosh(d*x + c)^2 + 4*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b
^4 + 15*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 18*
(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^2*b
^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^2*b^2 - 8*a*b^3
+ 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos
h(d*x + c)^6 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 -
8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 - 8*a*
b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*
a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d
*x + c)^4 + 64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4 + 30*(16*a^
3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a
^2*b^2 - 8*a*b^3 + 3*b^4 + 8*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)
^5 + 10*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (64*a^
4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(
8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b - 24*a^2*b^2 +
14*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^
4 + 3*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(1
6*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (64*a^4 - 128*a^
3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (16*a^3*b - 24*a^2*
b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((
b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b -
b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a
*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*
x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2
*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 8*
(3*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 6*(16*a^
5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (40*a^4
*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6
*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 - 3*
```

$$\begin{aligned}
& a^5 b^3 + 3 a^4 b^4 - a^3 b^5) * d * \cosh(d * x + c) * \sinh(d * x + c)^7 + (a^6 b^2 - \\
& 3 a^5 b^3 + 3 a^4 b^4 - a^3 b^5) * d * \sinh(d * x + c)^8 + 4 * (2 a^7 b - 7 a^6 b^2 - \\
& 2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d * \cosh(d * x + c)^6 + 4 * (7 * (a^6 b^2 - 3 a^5 b^3 + 3 a^4 b^4 - a^3 b^5) * d * \cosh(d * x + c)^2 + (2 a^7 b - 7 a^6 b^2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d) * \sinh(d * x + c)^6 + 2 * (8 a^8 - 32 a^7 b + 51 a^6 b^2 - 41 a^5 b^3 + 17 a^4 b^4 - 3 a^3 b^5) * d * \cosh(d * x + c)^4 + 8 * (7 * (a^6 b^2 - 3 a^5 b^3 + 3 a^4 b^4 - a^3 b^5) * d * \cosh(d * x + c)^3 + 3 * (2 a^7 b - 7 a^6 b^2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d * \cosh(d * x + c)) * \sinh(d * x + c)^5 + 2 * (35 * (a^6 b^2 - 3 a^5 b^3 + 3 a^4 b^4 - a^3 b^5) * d * \cosh(d * x + c)^4 + 30 * (2 a^7 b - 7 a^6 b^2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d * \cosh(d * x + c)^2 + (8 a^8 - 32 a^7 b + 51 a^6 b^2 - 41 a^5 b^3 + 17 a^4 b^4 - 3 a^3 b^5) * d) * \sinh(d * x + c)^4 + 4 * (2 a^7 b - 7 a^6 b^2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d * \cosh(d * x + c)^2 + 8 * (7 * (a^6 b^2 - 3 a^5 b^3 + 3 a^4 b^4 - a^3 b^5) * d * \cosh(d * x + c)^5 + 10 * (2 a^7 b - 7 a^6 b^2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d * \cosh(d * x + c)^3 + (8 a^8 - 32 a^7 b + 51 a^6 b^2 - 41 a^5 b^3 + 17 a^4 b^4 - 3 a^3 b^5) * d * \cosh(d * x + c)) * \sinh(d * x + c)^3 + 4 * (7 * (a^6 b^2 - 3 a^5 b^3 + 3 a^4 b^4 - a^3 b^5) * d * \cosh(d * x + c)^6 + 15 * (2 a^7 b - 7 a^6 b^2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d * \cosh(d * x + c)^4 + 3 * (8 a^8 - 32 a^7 b + 51 a^6 b^2 - 41 a^5 b^3 + 17 a^4 b^4 - 3 a^3 b^5) * d * \cosh(d * x + c)^2 + (2 a^7 b - 7 a^6 b^2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d) * \sinh(d * x + c)^2 + (a^6 b^2 - 3 a^5 b^3 + 3 a^4 b^4 - a^3 b^5) * d + 8 * ((a^6 b^2 - 3 a^5 b^3 + 3 a^4 b^4 - a^3 b^5) * d * \cosh(d * x + c)^7 + 3 * (2 a^7 b - 7 a^6 b^2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d * \cosh(d * x + c)^5 + (8 a^8 - 32 a^7 b + 51 a^6 b^2 - 41 a^5 b^3 + 17 a^4 b^4 - 3 a^3 b^5) * d * \cosh(d * x + c)^3 + (2 a^7 b - 7 a^6 b^2 + 9 a^5 b^3 - 5 a^4 b^4 + a^3 b^5) * d * \cosh(d * x + c)) * \sinh(d * x + c)), 1/8 * (2 * (8 a^4 b - 16 a^3 b^2 + 11 a^2 b^3 - 3 a * b^4) * \cosh(d * x + c)^6 + 12 * (8 a^4 b - 16 a^3 b^2 + 11 a^2 b^3 - 3 a * b^4) * \cosh(d * x + c) * \sinh(d * x + c)^5 + 2 * (8 a^4 b - 16 a^3 b^2 + 11 a^2 b^3 - 3 a * b^4) * \sinh(d * x + c)^6 + 12 a^3 b^2 - 18 a^2 b^3 + 6 a * b^4 + 6 * (16 a^5 - 40 a^4 b + 38 a^3 b^2 - 17 a^2 b^3 + 3 a * b^4) * \cosh(d * x + c)^4 + 6 * (16 a^5 - 40 a^4 b + 38 a^3 b^2 - 17 a^2 b^3 + 3 a * b^4) * \cosh(d * x + c)^2 * \sinh(d * x + c)^4 + 8 * (5 * (8 a^4 b - 16 a^3 b^2 + 11 a^2 b^3 - 3 a * b^4) * \cosh(d * x + c)^3 + 3 * (16 a^5 - 40 a^4 b + 38 a^3 b^2 - 17 a^2 b^3 + 3 a * b^4) * \cosh(d * x + c)) * \sinh(d * x + c)^3 + 2 * (40 a^4 b - 80 a^3 b^2 + 49 a^2 b^3 - 9 a * b^4) * \cosh(d * x + c)^2 + 2 * (40 a^4 b - 80 a^3 b^2 + 49 a^2 b^3 - 9 a * b^4 + 15 * (8 a^4 b - 16 a^3 b^2 + 11 a^2 b^3 - 3 a * b^4) * \cosh(d * x + c)^4 + 18 * (16 a^5 - 40 a^4 b + 38 a^3 b^2 - 17 a^2 b^3 + 3 a * b^4) * \cosh(d * x + c)^2) * \sinh(d * x + c)^2 - ((8 a^2 b^2 - 8 a * b^3 + 3 b^4) * \cosh(d * x + c)^8 + 8 * (8 a^2 b^2 - 8 a * b^3 + 3 b^4) * \cosh(d * x + c) * \sinh(d * x + c)^7 + (8 a^2 b^2 - 8 a * b^3 + 3 b^4) * \sinh(d * x + c)^8 + 4 * (16 a^3 b - 24 a^2 b^2 + 14 a * b^3 - 3 b^4) * \cosh(d * x + c)^6 + 4 * (16 a^3 b - 24 a^2 b^2 + 14 a * b^3 - 3 b^4 + 7 * (8 a^2 b^2 - 8 a * b^3 + 3 b^4) * \cosh(d * x + c)^2) * \sinh(d * x + c)^6 + 8 * (7 * (8 a^2 b^2 - 8 a * b^3 + 3 b^4) * \cosh(d * x + c)^3 + 3 * (16 a^3 b - 24 a^2 b^2 + 14 a * b^3 - 3 b^4) * \cosh(d * x + c)) * \sinh(d * x + c)^5 + 2 * (64 a^4 - 128 a^3 b + 112 a^2 b^2 - 48 a * b^3 + 9 b^4) * \cosh(d * x + c)^4 + 2 * (35 * (8 a^2 b^2 - 8 a * b^3 + 3 b^4) * \cosh(d * x + c)^4 + 64
\end{aligned}$$

$$\begin{aligned}
& a^4 - 128a^3b + 112a^2b^2 - 48ab^3 + 9b^4 + 30(16a^3b - 24a^2b^2 + 14ab^3 - 3b^4) \cosh(dx + c)^2 \sinh(dx + c)^4 + 8a^2b^2 - 8ab^3 + 3b^4 + 8(7(8a^2b^2 - 8ab^3 + 3b^4) \cosh(dx + c)^5 + 10(16a^3b - 24a^2b^2 + 14ab^3 - 3b^4) \cosh(dx + c)^3 + (64a^4 - 128a^3b + 112a^2b^2 - 48ab^3 + 9b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(16a^3b - 24a^2b^2 + 14ab^3 - 3b^4) \cosh(dx + c)^2 + 4(7(8a^2b^2 - 8ab^3 + 3b^4) \cosh(dx + c)^6 + 15(16a^3b - 24a^2b^2 + 14ab^3 - 3b^4) \cosh(dx + c)^4 + 16a^3b - 24a^2b^2 + 14ab^3 - 3b^4 + 3(64a^4 - 128a^3b + 112a^2b^2 - 48ab^3 + 9b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((8a^2b^2 - 8ab^3 + 3b^4) \cosh(dx + c)^7 + 3(16a^3b - 24a^2b^2 + 14ab^3 - 3b^4) \cosh(dx + c)^5 + (64a^4 - 128a^3b + 112a^2b^2 - 48ab^3 + 9b^4) \cosh(dx + c)^3 + (16a^3b - 24a^2b^2 + 14ab^3 - 3b^4) \cosh(dx + c)) \sinh(dx + c) \sqrt{-a^2 + ab} \arctan(-1/2(b \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + 2a - b) \sqrt{-a^2 + ab} / (a^2 - ab)) + 4(3(8a^4b - 16a^3b^2 + 11a^2b^3 - 3ab^4) \cosh(dx + c)^5 + 6(16a^5 - 40a^4b + 38a^3b^2 - 17a^2b^3 + 3ab^4) \cosh(dx + c)^3 + (40a^4b - 80a^3b^2 + 49a^2b^3 - 9ab^4) \cosh(dx + c)) \sinh(dx + c) / ((a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d \cosh(dx + c)^8 + 8(a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d \cosh(dx + c) \sinh(dx + c)^7 + (a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d \sinh(dx + c)^8 + 4(2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d \cosh(dx + c)^6 + 4(7(a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d \cosh(dx + c)^2 + (2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d) \sinh(dx + c)^6 + 2(8a^8 - 32a^7b + 51a^6b^2 - 41a^5b^3 + 17a^4b^4 - 3a^3b^5) d \cosh(dx + c)^4 + 8(7(a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d \cosh(dx + c)^3 + 3(2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d \cosh(dx + c)^4 + 30(2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d \cosh(dx + c)^2 + (8a^8 - 32a^7b + 51a^6b^2 - 41a^5b^3 + 17a^4b^4 - 3a^3b^5) d) \sinh(dx + c)^4 + 4(2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d \cosh(dx + c)^2 + 8(7(a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d \cosh(dx + c)^5 + 10(2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d \cosh(dx + c)^3 + (8a^8 - 32a^7b + 51a^6b^2 - 41a^5b^3 + 17a^4b^4 - 3a^3b^5) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d \cosh(dx + c)^6 + 15(2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d \cosh(dx + c)^4 + 3(8a^8 - 32a^7b + 51a^6b^2 - 41a^5b^3 + 17a^4b^4 - 3a^3b^5) d \cosh(dx + c)^2 + (2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d) \sinh(dx + c)^2 + (a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d + 8((a^6b^2 - 3a^5b^3 + 3a^4b^4 - a^3b^5) d \cosh(dx + c)^7 + 3(2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d \cosh(dx + c)^5 + (8a^8 - 32a^7b + 51a^6b^2 - 41a^5b^3 + 17a^4b^4 - 3a^3b^5) d \cosh(dx + c)^3 + (2a^7b - 7a^6b^2 + 9a^5b^3 - 5a^4b^4 + a^3b^5) d \cosh(dx + c)) \sinh(dx + c)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.4622, size = 413, normalized size = 2.68

$$\frac{(8a^2 - 8ab + 3b^2) \arctan\left(\frac{be^{(2dx+2c)} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{8(a^4d - 2a^3bd + a^2b^2d)\sqrt{-a^2 + ab}} + \frac{8a^2be^{(6dx+6c)} - 8ab^2e^{(6dx+6c)} + 3b^3e^{(6dx+6c)} + 48a^3e^{(4dx+4c)} - 72a^2be^{(4dx+4c)}}{4(a^4d - 2a^3bd + a^2b^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{8} \frac{(8a^2 - 8ab + 3b^2) \arctan\left(\frac{be^{(2dx+2c)} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{(a^4d - 2a^3bd + a^2b^2d)\sqrt{-a^2 + ab}} + \frac{1}{4} \frac{(8a^2 * b * e^{(6dx+6c)} - 8a * b^2 * e^{(6dx+6c)} + 3b^3 * e^{(6dx+6c)} + 48a^3 * e^{(4dx+4c)} - 72a^2 * b * e^{(4dx+4c)} + 42a * b^2 * e^{(4dx+4c)} - 9b^3 * e^{(4dx+4c)} + 40a^2 * b * e^{(2dx+2c)} - 40a * b^2 * e^{(2dx+2c)} + 9b^3 * e^{(2dx+2c)} + 6a * b^2 - 3b^3)}{(a^4d - 2a^3bd + a^2b^2d) * (be^{(4dx+4c)} + 4a * e^{(2dx+2c)} - 2b * e^{(2dx+2c)} + b)^2}$

$$3.56 \quad \int \frac{\operatorname{csch}(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^3} dx$$

Optimal. Leaf size=166

$$\frac{\sqrt{b}(15a^2 - 20ab + 8b^2) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3d(a-b)^{5/2}} - \frac{b(7a-4b) \cosh(c+dx)}{8a^2d(a-b)^2(a+b \cosh^2(c+dx)-b)} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3d} - \frac{1}{4ad}$$

[Out] $-(\operatorname{Sqrt}[b]*(15*a^2 - 20*a*b + 8*b^2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Cosh}[c + d*x])/(\operatorname{Sqrt}[a - b])])/(8*a^3*(a - b)^{(5/2)*d}) - \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/(a^3*d) - (b*\operatorname{Cosh}[c + d*x])/(4*a*(a - b)*d*(a - b + b*\operatorname{Cosh}[c + d*x]^2)^2) - ((7*a - 4*b)*b*\operatorname{Cosh}[c + d*x])/(8*a^2*(a - b)^2*d*(a - b + b*\operatorname{Cosh}[c + d*x]^2))$

Rubi [A] time = 0.266536, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3186, 414, 527, 522, 206, 205}

$$\frac{\sqrt{b}(15a^2 - 20ab + 8b^2) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3d(a-b)^{5/2}} - \frac{b(7a-4b) \cosh(c+dx)}{8a^2d(a-b)^2(a+b \cosh^2(c+dx)-b)} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3d} - \frac{1}{4ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]/(a + b*\operatorname{Sinh}[c + d*x]^2)^3, x]$

[Out] $-(\operatorname{Sqrt}[b]*(15*a^2 - 20*a*b + 8*b^2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Cosh}[c + d*x])/(\operatorname{Sqrt}[a - b])])/(8*a^3*(a - b)^{(5/2)*d}) - \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/(a^3*d) - (b*\operatorname{Cosh}[c + d*x])/(4*a*(a - b)*d*(a - b + b*\operatorname{Cosh}[c + d*x]^2)^2) - ((7*a - 4*b)*b*\operatorname{Cosh}[c + d*x])/(8*a^2*(a - b)^2*d*(a - b + b*\operatorname{Cosh}[c + d*x]^2))$

Rule 3186

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] :> \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - b*ff^2*x^2)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x]] /;$ $\operatorname{FreeQ}\{a, b, e, f, p, x\} \ \&\& \ \operatorname{IntegerQ}[(m-1)/2]$

Rule 414

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]

```

Rule 527

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

```

Rule 522

```

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 205

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{b \cosh(c+dx)}{4a(a-b)d(a-b+b\cosh^2(c+dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{-4a+b+3bx^2}{(1-x^2)(a-b+bx^2)^2} dx, x, \cosh(c+dx)\right)}{4a(a-b)d} \\
&= -\frac{b \cosh(c+dx)}{4a(a-b)d(a-b+b\cosh^2(c+dx))^2} - \frac{(7a-4b)b \cosh(c+dx)}{8a^2(a-b)^2d(a-b+b\cosh^2(c+dx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c+dx)\right)}{4a(a-b)d} \\
&= -\frac{b \cosh(c+dx)}{4a(a-b)d(a-b+b\cosh^2(c+dx))^2} - \frac{(7a-4b)b \cosh(c+dx)}{8a^2(a-b)^2d(a-b+b\cosh^2(c+dx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c+dx)\right)}{4a(a-b)d} \\
&= -\frac{\sqrt{b}(15a^2-20ab+8b^2) \tan^{-1}\left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3d} - \frac{b \cosh(c+dx)}{4a(a-b)d}
\end{aligned}$$

Mathematica [C] time = 3.42551, size = 237, normalized size = 1.43

$$\frac{\sqrt{b}(15a^2-20ab+8b^2) \tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{b}(15a^2-20ab+8b^2) \tan^{-1}\left(\frac{\sqrt{b+i\sqrt{a}} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{8a^2b \cosh(c+dx)}{(a-b)(2a+b \cosh(2(c+dx))-b)^2} + \frac{2ab(7a-b)}{(a-b)^2(2a-b)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] -((Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(5/2) + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(5/2) + (8*a^2*b*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2) + (2*a*(7*a - 4*b)*b*Cosh[c + d*x])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])) - 8*Log[Tanh[(c + d*x)/2]]/(8*a^3*d)

Maple [B] time = 0.069, size = 1145, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{csch}(d*x+c)/(a+b*\sinh(d*x+c)^2)^3,x)$

[Out] $\frac{1}{d/a^3} \ln(\tanh(1/2*d*x+1/2*c)) + \frac{9}{4} \frac{d}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} (a^2-2*a*b+b^2) \tanh(1/2*d*x+1/2*c)^{6*b-7} \frac{d}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} \frac{a}{(a^2-2*a*b+b^2) \tanh(1/2*d*x+1/2*c)^{6*b^2+4}} \frac{d}{a^2*b^3} \frac{3}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} (a^2-2*a*b+b^2) \tanh(1/2*d*x+1/2*c)^{6-27/4} \frac{d}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} \frac{b}{(a^2-2*a*b+b^2) \tanh(1/2*d*x+1/2*c)^4} \frac{45}{2} \frac{d}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} \frac{a*b^2}{(a^2-2*a*b+b^2) \tanh(1/2*d*x+1/2*c)^4} \frac{30}{d} \frac{d}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} \frac{a^2}{(a^2-2*a*b+b^2) \tanh(1/2*d*x+1/2*c)^4} \frac{b^3+12}{d/a^3} \frac{b^4}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} (a^2-2*a*b+b^2) \tanh(1/2*d*x+1/2*c)^4 + \frac{27}{4} \frac{d}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} \frac{b}{(a^2-2*a*b+b^2) \tanh(1/2*d*x+1/2*c)^2} \frac{17}{d} \frac{d}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} \frac{a}{(a^2-2*a*b+b^2) \tanh(1/2*d*x+1/2*c)^2} \frac{b^2+8}{d/a^2} \frac{b^3}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} \frac{b+3}{2} \frac{d}{a*b^2} \frac{d}{(\tanh(1/2*d*x+1/2*c)^{4*a-2} \tanh(1/2*d*x+1/2*c)^{2*a+4} \tanh(1/2*d*x+1/2*c)^{2*b+a})^2} \frac{a*b}{(a^2-2*a*b+b^2)-15/8} \frac{d}{a*b} \frac{(a^2-2*a*b+b^2)}{(a*b-b^2)^{(1/2)}} \arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^{2*a-2*a+4*b})/(a*b-b^2)^{(1/2)}) + \frac{5}{2} \frac{d}{a^2*b^2} \frac{d}{(a^2-2*a*b+b^2)} \frac{(a*b-b^2)^{(1/2)}}{(a*b-b^2)^{(1/2)}} \arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^{2*a-2*a+4*b})/(a*b-b^2)^{(1/2)}) - \frac{1}{d/a^3} \frac{b^3}{(a^2-2*a*b+b^2)} \frac{(a*b-b^2)^{(1/2)}}{(a*b-b^2)^{(1/2)}} \arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^{2*a-2*a+4*b})/(a*b-b^2)^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\text{csch}(d*x+c)/(a+b*\sinh(d*x+c)^2)^3,x, \text{algorithm}="maxima")$

[Out] $-\frac{1}{4} * ((7*a*b^2*e^{(7*c)} - 4*b^3*e^{(7*c)}) * e^{(7*d*x)} + (36*a^2*b*e^{(5*c)} - 31*a*b^2*e^{(5*c)} + 4*b^3*e^{(5*c)}) * e^{(5*d*x)} + (36*a^2*b*e^{(3*c)} - 31*a*b^2*e^{(3*c)} + 4*b^3*e^{(3*c)}) * e^{(3*d*x)} + (7*a*b^2*e^c - 4*b^3*e^c) * e^{(d*x)}) / (a^4*b$

$$\begin{aligned} &^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^{(8*c)} - 2*a^3*b^3*d*e^{(8*c)} + \\ &a^2*b^4*d*e^{(8*c)})*e^{(8*d*x)} + 4*(2*a^5*b*d*e^{(6*c)} - 5*a^4*b^2*d*e^{(6*c)} \\ &+ 4*a^3*b^3*d*e^{(6*c)} - a^2*b^4*d*e^{(6*c)})*e^{(6*d*x)} + 2*(8*a^6*d*e^{(4*c)} - \\ &24*a^5*b*d*e^{(4*c)} + 27*a^4*b^2*d*e^{(4*c)} - 14*a^3*b^3*d*e^{(4*c)} + 3*a^2*b \\ &^4*d*e^{(4*c)})*e^{(4*d*x)} + 4*(2*a^5*b*d*e^{(2*c)} - 5*a^4*b^2*d*e^{(2*c)} + 4*a^ \\ &3*b^3*d*e^{(2*c)} - a^2*b^4*d*e^{(2*c)})*e^{(2*d*x)} - \log((e^{(d*x + c)} + 1)*e^{(\\ &-c)})/(a^3*d) + \log((e^{(d*x + c)} - 1)*e^{(-c)})/(a^3*d) - 2*\text{integrate}(1/8*((15 \\ &*a^2*b*e^{(3*c)} - 20*a*b^2*e^{(3*c)} + 8*b^3*e^{(3*c)})*e^{(3*d*x)} - (15*a^2*b*e^{ \\ &c} - 20*a*b^2*e^c + 8*b^3*e^c)*e^{(d*x)})/(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5* \\ &b*e^{(4*c)} - 2*a^4*b^2*e^{(4*c)} + a^3*b^3*e^{(4*c)})*e^{(4*d*x)} + 2*(2*a^6*e^{(2* \\ &c)} - 5*a^5*b*e^{(2*c)} + 4*a^4*b^2*e^{(2*c)} - a^3*b^3*e^{(2*c)})*e^{(2*d*x)}), x) \end{aligned}$$

Fricas [B] time = 4.05571, size = 22814, normalized size = 137.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")`

[Out]
$$\begin{aligned} &[-1/16*(4*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^7 + 28*(7*a^2*b^2 - 4*a*b^3)* \\ &\cosh(d*x + c)*\sinh(d*x + c)^6 + 4*(7*a^2*b^2 - 4*a*b^3)*\sinh(d*x + c)^7 + 4 \\ &*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*\cosh(d*x + c)^5 + 4*(36*a^3*b - 31*a^2*b \\ &^2 + 4*a*b^3 + 21*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + \\ &20*(7*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^3 + (36*a^3*b - 31*a^2*b^2 + 4*a* \\ &b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*c \\ &osh(d*x + c)^3 + 4*(35*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^4 + 36*a^3*b - 3 \\ &1*a^2*b^2 + 4*a*b^3 + 10*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*\cosh(d*x + c)^2) \\ &*\sinh(d*x + c)^3 + 4*(21*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^5 + 10*(36*a^3 \\ &*b - 31*a^2*b^2 + 4*a*b^3)*\cosh(d*x + c)^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a \\ &*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cos \\ &h(d*x + c)^8 + 8*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)*\sinh(d*x + c \\ &)^7 + (15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^ \\ &2*b^2 + 36*a*b^3 - 8*b^4)*\cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a \\ &*b^3 - 8*b^4 + 7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^2)*\sinh(d*x \\ &+ c)^6 + 8*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^3 + 3*(30*a^3*b \\ &- 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(120*a \\ &^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*\cosh(d*x + c)^4 + 2*(35* \\ &(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269 \\ &*a^2*b^2 - 124*a*b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^ \\ &4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7* \\ &(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2 \end{aligned}$$

$$\begin{aligned}
& + 36*a*b^3 - 8*b^4)*\cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - \\
& 124*a*b^3 + 24*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2* \\
& b^2 + 36*a*b^3 - 8*b^4)*\cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b \\
& ^4)*\cosh(d*x + c)^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*\cosh(d* \\
& x + c)^4 + 30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3* \\
& b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8* \\
& ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2 \\
& + 36*a*b^3 - 8*b^4)*\cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - \\
& 124*a*b^3 + 24*b^4)*\cosh(d*x + c)^3 + (30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - \\
& 8*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-b/(a - b))*\log((b*\cosh(d*x + c))^ \\
& 4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a - 3*b)*c \\
& osh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a + 3*b)*\sinh(d*x + c)^2 + 4*(b \\
& *\cosh(d*x + c)^3 - (2*a - 3*b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*((a - b)*co \\
& sh(d*x + c)^3 + 3*(a - b)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a - b)*\sinh(d*x \\
& + c)^3 + (a - b)*\cosh(d*x + c) + (3*(a - b)*\cosh(d*x + c)^2 + a - b)*\sinh(d \\
& *x + c))*\sqrt{-b/(a - b)) + b)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(\\
& d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(\\
& d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*co \\
& sh(d*x + c))*\sinh(d*x + c) + b)) + 4*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c) + \\
& 16*((a^2*b^2 - 2*a*b^3 + b^4)*\cosh(d*x + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4) \\
& *\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*\sinh(d*x + c)^8 \\
& + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^6 + 4*(2*a^3*b - 5* \\
& a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh \\
& (d*x + c)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(d*x + c)^3 + 3*(2*a^3*b - \\
& 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(8*a^4 - 24* \\
& a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 2*(35*(a^2*b^2 - 2 \\
& *a*b^3 + b^4)*\cosh(d*x + c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + \\
& 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(d*x \\
& + c)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^3 + (8*a^4 \\
& - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^2 + 4*(7*(a^2*b^2 - \\
& 2*a*b^3 + b^4)*\cosh(d*x + c)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)* \\
& \cosh(d*x + c)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b \\
& + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^ \\
& 2*b^2 - 2*a*b^3 + b^4)*\cosh(d*x + c)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - \\
& b^4)*\cosh(d*x + c)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)* \\
& \cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c))*\sinh \\
& (d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - 16*((a^2*b^2 - 2*a*b^3 \\
& + b^4)*\cosh(d*x + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(d*x + c)*\sinh(d*x \\
& + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*\sinh(d*x + c)^8 + 4*(2*a^3*b - 5*a^2*b^ \\
& 2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 \\
& + 7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^2 \\
& *b^2 - 2*a*b^3 + b^4)*\cosh(d*x + c)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - \\
& b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14
\end{aligned}$$

$$\begin{aligned}
& *a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(d*x \\
& + c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5 \\
& *a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + a^2*b^2 - 2*a* \\
& b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(d*x + c)^5 + 10*(2*a^3*b - \\
& 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 \\
& - 14*a*b^3 + 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(2*a^3*b - 5*a^2*b^ \\
& 2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(d* \\
& x + c)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^4 + 2*a^3 \\
& *b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^ \\
& 3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)* \\
& \cosh(d*x + c)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c)^5 + \\
& (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + (2*a^ \\
& 3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x \\
& + c) + \sinh(d*x + c) - 1) + 4*(7*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^6 + 5 \\
& *(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*\cosh(d*x + c)^4 + 7*a^2*b^2 - 4*a*b^3 + \\
& 3*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^5*b \\
& ^2 - 2*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^8 + 8*(a^5*b^2 - 2*a^4*b^3 + a^3* \\
& b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*si \\
& nh(d*x + c)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + \\
& c)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^2 + (2*a^6*b - \\
& 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x + c)^6 + 2*(8*a^7 - 24*a^6*b + \\
& 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 2 \\
& *a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^3 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 \\
& - a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^5*b^2 - 2*a^4*b^3 + \\
& a^3*b^4)*d*\cosh(d*x + c)^4 + 30*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4) \\
& *d*\cosh(d*x + c)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^ \\
& 4)*d)*\sinh(d*x + c)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cos \\
& h(d*x + c)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^5 + 10* \\
& (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^3 + (8*a^7 - 24 \\
& *a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c \\
&)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^6 + 15*(2*a^6*b \\
& - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^4 + 3*(8*a^7 - 24*a^6*b \\
& + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^2 + (2*a^6*b - 5*a^5 \\
& *b^2 + 4*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x + c)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3 \\
& *b^4)*d + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^7 + 3*(2*a^6*b \\
& - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^5 + (8*a^7 - 24*a^6*b + \\
& 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^3 + (2*a^6*b - 5*a^5* \\
& b^2 + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/8*(2*(7*a^2* \\
& b^2 - 4*a*b^3)*\cosh(d*x + c)^7 + 14*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)*sin \\
& h(d*x + c)^6 + 2*(7*a^2*b^2 - 4*a*b^3)*\sinh(d*x + c)^7 + 2*(36*a^3*b - 31*a \\
& ^2*b^2 + 4*a*b^3)*\cosh(d*x + c)^5 + 2*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3 + 21 \\
& *(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 10*(7*(7*a^2*b^2 \\
& - 4*a*b^3)*\cosh(d*x + c)^3 + (36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*\cosh(d*x + c \\
&))*\sinh(d*x + c)^4 + 2*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*\cosh(d*x + c)^3 + \\
& 2*(35*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^4 + 36*a^3*b - 31*a^2*b^2 + 4*a*b
\end{aligned}$$

$$\begin{aligned}
&^3 + 10*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 \\
&+ 2*(21*(7*a^2*b^2 - 4*a*b^3)*\cosh(d*x + c)^5 + 10*(36*a^3*b - 31*a^2*b^2 + \\
&4*a*b^3)*\cosh(d*x + c)^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*\cosh(d*x + \\
&c))*\sinh(d*x + c)^2 + ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^8 + 8* \\
&(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (15*a^2*b^2 \\
&- 20*a*b^3 + 8*b^4)*\sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 \\
&- 8*b^4)*\cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 7* \\
&(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(15 \\
&*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^3 + 3*(30*a^3*b - 55*a^2*b^2 + 3 \\
&6*a*b^3 - 8*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(120*a^4 - 280*a^3*b + \\
&269*a^2*b^2 - 124*a*b^3 + 24*b^4)*\cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 20* \\
&a*b^3 + 8*b^4)*\cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a* \\
&b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*\cosh(d*x + c)^ \\
&2)*\sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7*(15*a^2*b^2 - 20* \\
&a*b^3 + 8*b^4)*\cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b \\
&^4)*\cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b \\
&^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - \\
&8*b^4)*\cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c) \\
&^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*\cosh(d*x + c)^4 + 30*a^3 \\
&*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3*b + 269*a^2*b^2 - \\
&124*a*b^3 + 24*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 20 \\
&*a*b^3 + 8*b^4)*\cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b \\
&^4)*\cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b \\
&^4)*\cosh(d*x + c)^3 + (30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*\cosh(d*x + \\
&c))*\sinh(d*x + c))*\sqrt{b/(a - b)}*\arctan(1/2*\sqrt{b/(a - b)}*(\cosh(d*x + \\
&c) + \sinh(d*x + c))) - ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^8 + 8 \\
&*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (15*a^2*b^ \\
&2 - 20*a*b^3 + 8*b^4)*\sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 \\
&- 8*b^4)*\cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 7 \\
&*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(1 \\
&5*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c)^3 + 3*(30*a^3*b - 55*a^2*b^2 + \\
&36*a*b^3 - 8*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(120*a^4 - 280*a^3*b + \\
&269*a^2*b^2 - 124*a*b^3 + 24*b^4)*\cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 20 \\
&*a*b^3 + 8*b^4)*\cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a \\
&*b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*\cosh(d*x + c) \\
&^2)*\sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7*(15*a^2*b^2 - 20 \\
&*a*b^3 + 8*b^4)*\cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8* \\
&b^4)*\cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24* \\
&b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - \\
&8*b^4)*\cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*\cosh(d*x + c \\
&)^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*\cosh(d*x + c)^4 + 30*a^ \\
&3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3*b + 269*a^2*b^2 \\
&- 124*a*b^3 + 24*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 2 \\
&0*a*b^3 + 8*b^4)*\cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8* \\
&b^4)*\cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*
\end{aligned}$$

$$\begin{aligned}
& b^4) \cosh(dx + c)^3 + (30a^3b - 55a^2b^2 + 36ab^3 - 8b^4) \cosh(dx + c) \sinh(dx + c) \sqrt{b/(a-b)} \arctan(1/2(b \cosh(dx + c)^3 + 3b \cosh(dx + c) \sinh(dx + c)^2 + b \sinh(dx + c)^3 + (4a - 3b) \cosh(dx + c) + (3b \cosh(dx + c)^2 + 4a - 3b) \sinh(dx + c)) \sqrt{b/(a-b)})/b) + 2(7a^2b^2 - 4ab^3) \cosh(dx + c) + 8((a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^8 + 8(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c) \sinh(dx + c)^7 + (a^2b^2 - 2ab^3 + b^4) \sinh(dx + c)^8 + 4(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^6 + 4(2a^3b - 5a^2b^2 + 4ab^3 - b^4 + 7(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^3 + 3(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4) \cosh(dx + c)^4 + 2(35(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^4 + 8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4 + 30(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + a^2b^2 - 2ab^3 + b^4 + 8(7(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^5 + 10(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^3 + (8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^2 + 4(7(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^6 + 15(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^4 + 2a^3b - 5a^2b^2 + 4ab^3 - b^4 + 3(8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^7 + 3(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^5 + (8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4) \cosh(dx + c)^3 + (2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)) \sinh(dx + c)) \log(\cosh(dx + c) + \sinh(dx + c) + 1) - 8((a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^8 + 8(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c) \sinh(dx + c)^7 + (a^2b^2 - 2ab^3 + b^4) \sinh(dx + c)^8 + 4(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^6 + 4(2a^3b - 5a^2b^2 + 4ab^3 - b^4 + 7(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^2) \sinh(dx + c)^6 + 8(7(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^3 + 3(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4) \cosh(dx + c)^4 + 2(35(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^4 + 8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4 + 30(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + a^2b^2 - 2ab^3 + b^4 + 8(7(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^5 + 10(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^3 + (8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^2 + 4(7(a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^6 + 15(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^4 + 2a^3b - 5a^2b^2 + 4ab^3 - b^4 + 3(8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8((a^2b^2 - 2ab^3 + b^4) \cosh(dx + c)^7 + 3(2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)^5 + (8a^4 - 24a^3b + 27a^2b^2 - 14ab^3 + 3b^4) \cosh(dx + c)^3 + (2a^3b - 5a^2b^2 + 4ab^3 - b^4) \cosh(dx + c)) \sinh(dx + c)) \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 2(7(7a^2b^2 - 4ab^3) \cosh(dx + c)^6 + 5(36a^3b - 31a^2b^2 + 4ab^3) \cosh
\end{aligned}$$

$$\begin{aligned} & (d*x + c)^4 + 7*a^2*b^2 - 4*a*b^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*\cos \\ & h(d*x + c)^2*\sinh(d*x + c))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + \\ & c)^8 + 8*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + \\ & (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*\sinh(d*x + c)^8 + 4*(2*a^6*b - 5*a^5*b^2 \\ & + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3* \\ & b^4)*d*\cosh(d*x + c)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*\sin \\ & h(d*x + c)^6 + 2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d \\ & *\cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^3 + \\ & 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + \\ & c)^5 + 2*(35*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*\cosh(d*x + c)^4 + 30*(2*a^6*b \\ & - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^2 + (8*a^7 - 24*a^6*b \\ & + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d)*\sinh(d*x + c)^4 + 4*(2*a^6*b - 5* \\ & a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^ \\ & 3 + a^3*b^4)*d*\cosh(d*x + c)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3* \\ & b^4)*d*\cosh(d*x + c)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^ \\ & 3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b \\ & ^4)*d*\cosh(d*x + c)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*\co \\ & sh(d*x + c)^4 + 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)* \\ & d*\cosh(d*x + c)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x \\ & + c)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d + 8*((a^5*b^2 - 2*a^4*b^3 + a^3 \\ & *b^4)*d*\cosh(d*x + c)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*c \\ & osh(d*x + c)^5 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d \\ & *\cosh(d*x + c)^3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + \\ & c))*\sinh(d*x + c))] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.57 \quad \int \frac{\operatorname{csch}^2(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=215

$$\frac{3b(8a^2 - 12ab + 5b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a-b)^{5/2}} - \frac{(4a-5b)(2a-3b) \operatorname{coth}(c+dx)}{8a^3d(a-b)^2} - \frac{b \operatorname{coth}(c+dx) (-(4a-b) \tanh^2(c+dx))}{8a^2d(a-b)^2 (a - (a-b) \tanh^2(c+dx))}$$

[Out] $(-3*b*(8*a^2 - 12*a*b + 5*b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tanh}[c + d*x])/\operatorname{Sqrt}[a]])/(8*a^{(7/2)}*(a - b)^{(5/2)}*d) - ((4*a - 5*b)*(2*a - 3*b)*\operatorname{Coth}[c + d*x])/(8*a^3*(a - b)^2*d) - (b*\operatorname{Csch}[c + d*x]*\operatorname{Sech}[c + d*x]^3)/(4*a*(a - b)*d*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2)^2) - (b*\operatorname{Coth}[c + d*x]*(4*a - 5*b - (4*a - b)*\operatorname{Tanh}[c + d*x]^2))/(8*a^2*(a - b)^2*d*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2))$

Rubi [A] time = 0.287169, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3187, 468, 577, 453, 208}

$$\frac{3b(8a^2 - 12ab + 5b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a-b)^{5/2}} - \frac{(4a-5b)(2a-3b) \operatorname{coth}(c+dx)}{8a^3d(a-b)^2} - \frac{b \operatorname{coth}(c+dx) (-(4a-b) \tanh^2(c+dx))}{8a^2d(a-b)^2 (a - (a-b) \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^2/(a + b*\operatorname{Sinh}[c + d*x]^2)^3, x]$

[Out] $(-3*b*(8*a^2 - 12*a*b + 5*b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tanh}[c + d*x])/\operatorname{Sqrt}[a]])/(8*a^{(7/2)}*(a - b)^{(5/2)}*d) - ((4*a - 5*b)*(2*a - 3*b)*\operatorname{Coth}[c + d*x])/(8*a^3*(a - b)^2*d) - (b*\operatorname{Csch}[c + d*x]*\operatorname{Sech}[c + d*x]^3)/(4*a*(a - b)*d*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2)^2) - (b*\operatorname{Coth}[c + d*x]*(4*a - 5*b - (4*a - b)*\operatorname{Tanh}[c + d*x]^2))/(8*a^2*(a - b)^2*d*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2))$

Rule 3187

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m+1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + (a+b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^{(m/2+p+1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]] /; \operatorname{FreeQ}\{a, b, e, f\}, x] \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e*x)^(m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(g*x)^(m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{x^2(a-(a-b)x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{b\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)(4a-5b+(-4a+b)x^2)}{x^2(a+(-a+b)x^2)^2} dx, x, \tanh(c+dx)\right)}{4a(a-b)d} \\
&= -\frac{b\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{b\coth(c+dx)\left(4a-5b-(4a-b)\tanh^2(c+dx)\right)}{8a^2(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)} \\
&= -\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3(a-b)^2d} - \frac{b\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{b\coth(c+dx)}{8a^2(a-b)d} \\
&= -\frac{3b(8a^2-12ab+5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^{5/2}d} - \frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3(a-b)^2d} - \frac{b\coth(c+dx)}{4a(a-b)d}
\end{aligned}$$

Mathematica [A] time = 1.81292, size = 225, normalized size = 1.05

$$\frac{\operatorname{csch}^6(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\frac{4a^{3/2}b^2\sinh(2(c+dx))}{a-b} - \frac{3b(8a^2-12ab+5b^2)(2a+b\cosh(2(c+dx))-b)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}}\right) + \sqrt{a-b}}{64a^{7/2}d\left(\operatorname{acsch}^2(c+dx)+b\right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3, x]

[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^6*((-3*b*(8*a^2 - 12*a*b + 5*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c + d*x)])^2)/(a - b)^(5/2) - 8*Sqrt[a]*(2*a - b + b*Cosh[2*(c + d*x)])^2*Coth[c + d*x] + (4*a^(3/2)*b^2*Sinh[2*(c + d*x)])/(a - b) + (Sqrt[a]*(10*a - 7*b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a - b)^2))/(64*a^(7/2)*d*(b + a*Csch[c + d*x]^2)^3)

Maple [B] time = 0.089, size = 1850, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{csch}(d*x+c)^2/(a+b*\sinh(d*x+c))^2)^3,x$

[Out]
$$\begin{aligned} & -1/2/d/a^3*\tanh(1/2*d*x+1/2*c)-1/2/d/a^3/\tanh(1/2*d*x+1/2*c)+3/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2 \\ & /a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7-9/4/d/a^2*b^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2) \\ & *\tanh(1/2*d*x+1/2*c)^7-3/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^5 \\ & *b^2+49/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^5-7/d/a^3*b^4 \\ & /(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^5-3/d/(\tanh(1/2*d*x+1/2*c)^4*a \\ & -2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2) \\ & *\tanh(1/2*d*x+1/2*c)^3*b^2+49/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*\tanh(1/2*d \\ & *x+1/2*c)^3-7/d/a^3*b^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3+3/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)-9/4/d/a^2*b^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)-3/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+9/2/d/a^2/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-15/8/d/a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-9/2/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^3+15/8/d/a^3*b^4/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-9/2/d/a^2/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2+15/8/d/a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2) \end{aligned}$$

$$\frac{1}{2} \arctan\left(\frac{a \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)}{\left(\left(2(-b(a-b))^{1/2} - a + 2b\right) a\right)^{1/2}}\right) b^2 - \frac{9}{2} \frac{d}{a^2} \frac{(a^2 - 2ab + b^2)}{(-b(a-b))^{1/2}} \frac{1}{\left(\left(2(-b(a-b))^{1/2} - a + 2b\right) a\right)^{1/2}} \arctan\left(\frac{a \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)}{\left(\left(2(-b(a-b))^{1/2} - a + 2b\right) a\right)^{1/2}}\right) b^3 + \frac{15}{8} \frac{d}{a^3} \frac{b^4}{(a^2 - 2ab + b^2)} \frac{1}{(-b(a-b))^{1/2}} \frac{1}{\left(\left(2(-b(a-b))^{1/2} - a + 2b\right) a\right)^{1/2}} \arctan\left(\frac{a \tanh\left(\frac{1}{2} d x + \frac{1}{2} c\right)}{\left(\left(2(-b(a-b))^{1/2} - a + 2b\right) a\right)^{1/2}}\right)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 3.58571, size = 21173, normalized size = 98.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(12*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*\cosh(d*x + c)^8 \\ & + 96*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*\cosh(d*x + c)*\sinh(d*x \\ & + c)^7 + 12*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*\sinh(d*x + c)^8 \\ & + 24*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*\cosh(d*x \\ & + c)^6 + 24*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5 + \\ & 14*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*\cosh(d*x + c)^2)*\sinh(d*x \\ & + c)^6 + 32*a^4*b^2 - 136*a^3*b^3 + 164*a^2*b^4 - 60*a*b^5 + 48*(14*(8*a^4*b^2 \\ & - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*\cosh(d*x + c)^3 + 3*(24*a^5*b - \\ & 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*\cosh(d*x + c))*\sinh(d*x + \\ & c)^5 + 8*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 4 \\ & 5*a*b^5)*\cosh(d*x + c)^4 + 8*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 \\ & + 238*a^2*b^4 - 45*a*b^5 + 105*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a \\ & *b^5)*\cosh(d*x + c)^4 + 45*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 \\ & + 10*a*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 32*(21*(8*a^4*b^2 - 20*a^3*b^3 \\ & + 17*a^2*b^4 - 5*a*b^5)*\cosh(d*x + c)^5 + 15*(24*a^5*b - 76*a^4*b^2 + 9 \end{aligned}$$

$$\begin{aligned}
& 1*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*\cosh(d*x + c)^3 + (64*a^6 - 296*a^5*b + \\
& 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^3 + 8*(32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5) \\
& *\cosh(d*x + c)^2 + 8*(42*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*\co \\
& sh(d*x + c)^6 + 32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b \\
& ^5 + 45*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*\cosh(d \\
& *x + c)^4 + 6*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 \\
& - 45*a*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 3*((8*a^2*b^3 - 12*a*b^4 + \\
& 5*b^5)*\cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*\cosh(d*x + c)*s \\
& inh(d*x + c)^9 + (8*a^2*b^3 - 12*a*b^4 + 5*b^5)*\sinh(d*x + c)^10 + (64*a^3* \\
& b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c)^8 + (64*a^3*b^2 - 136 \\
& *a^2*b^3 + 100*a*b^4 - 25*b^5 + 45*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*\cosh(d*x \\
& + c)^2)*\sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*\cosh(d*x + c \\
&)^3 + (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c))*\sinh(d \\
& *x + c)^7 + 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*c \\
& osh(d*x + c)^6 + 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b \\
& ^5 + 105*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*\cosh(d*x + c)^4 + 14*(64*a^3*b^2 - \\
& 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63* \\
& (8*a^2*b^3 - 12*a*b^4 + 5*b^5)*\cosh(d*x + c)^5 + 14*(64*a^3*b^2 - 136*a^2*b \\
& ^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c)^3 + 3*(64*a^4*b - 192*a^3*b^2 + 224* \\
& a^2*b^3 - 120*a*b^4 + 25*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 8*a^2*b^3 + \\
& 12*a*b^4 - 5*b^5 - 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25 \\
& *b^5)*\cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*\cosh(d*x + c) \\
& ^6 - 64*a^4*b + 192*a^3*b^2 - 224*a^2*b^3 + 120*a*b^4 - 25*b^5 + 35*(64*a^3 \\
& *b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c)^4 + 15*(64*a^4*b - 1 \\
& 92*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^4 + 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*\cosh(d*x + c)^7 + 7*(64*a^3*b^2 \\
& - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c)^5 + 5*(64*a^4*b - 192*a^ \\
& 3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*\cosh(d*x + c)^3 - (64*a^4*b - 192 \\
& *a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& - (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c)^2 + (45*(8 \\
& *a^2*b^3 - 12*a*b^4 + 5*b^5)*\cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 136*a^2*b^3 \\
& + 100*a*b^4 - 25*b^5)*\cosh(d*x + c)^6 - 64*a^3*b^2 + 136*a^2*b^3 - 100*a*b \\
& ^4 + 25*b^5 + 30*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5 \\
&)*\cosh(d*x + c)^4 - 12*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + \\
& 25*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(8*a^2*b^3 - 12*a*b^4 + 5*b \\
& ^5)*\cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\cos \\
& h(d*x + c)^7 + 6*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5 \\
&)*\cosh(d*x + c)^5 - 4*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 2 \\
& 5*b^5)*\cosh(d*x + c)^3 - (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\co \\
& sh(d*x + c))*\sinh(d*x + c))*\sqrt{a^2 - a*b}*\log((b^2*\cosh(d*x + c)^4 + 4*b^ \\
& 2*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*\sinh(d*x + c)^4 + 2*(2*a*b - b^2)*\cos \\
& h(d*x + c)^2 + 2*(3*b^2*\cosh(d*x + c)^2 + 2*a*b - b^2)*\sinh(d*x + c)^2 + 8* \\
& a^2 - 8*a*b + b^2 + 4*(b^2*\cosh(d*x + c)^3 + (2*a*b - b^2)*\cosh(d*x + c))*s \\
& inh(d*x + c) + 4*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*s
\end{aligned}$$

$$\begin{aligned}
& \sinh(dx + c)^2 + 2a - b) \sqrt{a^2 - a*b}) / (b \cosh(dx + c)^4 + 4b \cosh(dx \\
& x + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 + 2*(2a - b) \cosh(dx + c)^2 + \\
& 2*(3b \cosh(dx + c)^2 + 2a - b) \sinh(dx + c)^2 + 4*(b \cosh(dx + c)^3 + \\
& (2a - b) \cosh(dx + c)) \sinh(dx + c) + b)) + 16*(6*(8a^4*b^2 - 20a^3*b^3 \\
& + 17a^2*b^4 - 5a*b^5) \cosh(dx + c)^7 + 9*(24a^5*b - 76a^4*b^2 + 91a \\
& ^3*b^3 - 49a^2*b^4 + 10a*b^5) \cosh(dx + c)^5 + 2*(64a^6 - 296a^5*b + 5 \\
& 48a^4*b^2 - 509a^3*b^3 + 238a^2*b^4 - 45a*b^5) \cosh(dx + c)^3 + (32a^5 \\
& *b - 144a^4*b^2 + 219a^3*b^3 - 137a^2*b^4 + 30a*b^5) \cosh(dx + c)) \sinh \\
& (dx + c)) / ((a^7*b^2 - 3a^6*b^3 + 3a^5*b^4 - a^4*b^5) * d \cosh(dx + c)^1 \\
& 0 + 10*(a^7*b^2 - 3a^6*b^3 + 3a^5*b^4 - a^4*b^5) * d \cosh(dx + c) \sinh(dx \\
& + c)^9 + (a^7*b^2 - 3a^6*b^3 + 3a^5*b^4 - a^4*b^5) * d \sinh(dx + c)^10 + \\
& (8a^8*b - 29a^7*b^2 + 39a^6*b^3 - 23a^5*b^4 + 5a^4*b^5) * d \cosh(dx + c \\
&)^8 + (45*(a^7*b^2 - 3a^6*b^3 + 3a^5*b^4 - a^4*b^5) * d \cosh(dx + c)^2 + (\\
& 8a^8*b - 29a^7*b^2 + 39a^6*b^3 - 23a^5*b^4 + 5a^4*b^5) * d) \sinh(dx + c \\
&)^8 + 2*(8a^9 - 36a^8*b + 65a^7*b^2 - 59a^6*b^3 + 27a^5*b^4 - 5a^4*b^5 \\
&) * d \cosh(dx + c)^6 + 8*(15*(a^7*b^2 - 3a^6*b^3 + 3a^5*b^4 - a^4*b^5) * d \\
& \cosh(dx + c)^3 + (8a^8*b - 29a^7*b^2 + 39a^6*b^3 - 23a^5*b^4 + 5a^4*b^5 \\
&) * d \cosh(dx + c)) \sinh(dx + c)^7 + 2*(105*(a^7*b^2 - 3a^6*b^3 + 3a^5 \\
& *b^4 - a^4*b^5) * d \cosh(dx + c)^4 + 14*(8a^8*b - 29a^7*b^2 + 39a^6*b^3 - \\
& 23a^5*b^4 + 5a^4*b^5) * d \cosh(dx + c)^2 + (8a^9 - 36a^8*b + 65a^7*b^2 \\
& - 59a^6*b^3 + 27a^5*b^4 - 5a^4*b^5) * d) \sinh(dx + c)^6 - 2*(8a^9 - 36a^ \\
& ^8*b + 65a^7*b^2 - 59a^6*b^3 + 27a^5*b^4 - 5a^4*b^5) * d \cosh(dx + c)^4 \\
& + 4*(63*(a^7*b^2 - 3a^6*b^3 + 3a^5*b^4 - a^4*b^5) * d \cosh(dx + c)^5 + 14* \\
& (8a^8*b - 29a^7*b^2 + 39a^6*b^3 - 23a^5*b^4 + 5a^4*b^5) * d \cosh(dx + c \\
&)^3 + 3*(8a^9 - 36a^8*b + 65a^7*b^2 - 59a^6*b^3 + 27a^5*b^4 - 5a^4*b^5 \\
&) * d \cosh(dx + c)) \sinh(dx + c)^5 + 2*(105*(a^7*b^2 - 3a^6*b^3 + 3a^5 \\
& *b^4 - a^4*b^5) * d \cosh(dx + c)^6 + 35*(8a^8*b - 29a^7*b^2 + 39a^6*b^3 - 2 \\
& 3a^5*b^4 + 5a^4*b^5) * d \cosh(dx + c)^4 + 15*(8a^9 - 36a^8*b + 65a^7*b^2 \\
& - 59a^6*b^3 + 27a^5*b^4 - 5a^4*b^5) * d \cosh(dx + c)^2 - (8a^9 - 36a^ \\
& ^8*b + 65a^7*b^2 - 59a^6*b^3 + 27a^5*b^4 - 5a^4*b^5) * d) \sinh(dx + c)^4 \\
& - (8a^8*b - 29a^7*b^2 + 39a^6*b^3 - 23a^5*b^4 + 5a^4*b^5) * d \cosh(dx + \\
& c)^2 + 8*(15*(a^7*b^2 - 3a^6*b^3 + 3a^5*b^4 - a^4*b^5) * d \cosh(dx + c)^7 \\
& + 7*(8a^8*b - 29a^7*b^2 + 39a^6*b^3 - 23a^5*b^4 + 5a^4*b^5) * d \cosh(dx \\
& x + c)^5 + 5*(8a^9 - 36a^8*b + 65a^7*b^2 - 59a^6*b^3 + 27a^5*b^4 - 5a \\
& ^4*b^5) * d \cosh(dx + c)^3 - (8a^9 - 36a^8*b + 65a^7*b^2 - 59a^6*b^3 + 2 \\
& 7a^5*b^4 - 5a^4*b^5) * d \cosh(dx + c)) \sinh(dx + c)^3 + (45*(a^7*b^2 - 3a \\
& ^6*b^3 + 3a^5*b^4 - a^4*b^5) * d \cosh(dx + c)^8 + 28*(8a^8*b - 29a^7*b^2 \\
& + 39a^6*b^3 - 23a^5*b^4 + 5a^4*b^5) * d \cosh(dx + c)^6 + 30*(8a^9 - 36a^ \\
& ^8*b + 65a^7*b^2 - 59a^6*b^3 + 27a^5*b^4 - 5a^4*b^5) * d \cosh(dx + c)^4 \\
& - 12*(8a^9 - 36a^8*b + 65a^7*b^2 - 59a^6*b^3 + 27a^5*b^4 - 5a^4*b^5) \\
& * d \cosh(dx + c)^2 - (8a^8*b - 29a^7*b^2 + 39a^6*b^3 - 23a^5*b^4 + 5a^ \\
& ^4*b^5) * d) \sinh(dx + c)^2 - (a^7*b^2 - 3a^6*b^3 + 3a^5*b^4 - a^4*b^5) * d + \\
& 2*(5*(a^7*b^2 - 3a^6*b^3 + 3a^5*b^4 - a^4*b^5) * d \cosh(dx + c)^9 + 4*(8a^ \\
& ^8*b - 29a^7*b^2 + 39a^6*b^3 - 23a^5*b^4 + 5a^4*b^5) * d \cosh(dx + c)^7 \\
& + 6*(8a^9 - 36a^8*b + 65a^7*b^2 - 59a^6*b^3 + 27a^5*b^4 - 5a^4*b^5) *
\end{aligned}$$

$$\begin{aligned} & d \cdot \cosh(dx + c)^5 - 4 \cdot (8a^9 - 36a^8b + 65a^7b^2 - 59a^6b^3 + 27a^5b^4 - 5a^4b^5) \cdot d \cdot \cosh(dx + c)^3 - (8a^8b - 29a^7b^2 + 39a^6b^3 - 2 \\ & 3a^5b^4 + 5a^4b^5) \cdot d \cdot \cosh(dx + c) \cdot \sinh(dx + c), -1/8 \cdot (6 \cdot (8a^4b^2 - 20a^3b^3 + 17a^2b^4 - 5ab^5) \cdot \cosh(dx + c)^8 + 48 \cdot (8a^4b^2 - 20a^ \\ & 3b^3 + 17a^2b^4 - 5ab^5) \cdot \cosh(dx + c) \cdot \sinh(dx + c)^7 + 6 \cdot (8a^4b^2 - 20a^3b^3 + 17a^2b^4 - 5ab^5) \cdot \sinh(dx + c)^8 + 12 \cdot (24a^5b - 76a^ \\ & 4b^2 + 91a^3b^3 - 49a^2b^4 + 10ab^5) \cdot \cosh(dx + c)^6 + 12 \cdot (24a^5b - 76a^4b^2 + 91a^3b^3 - 49a^2b^4 + 10ab^5 + 14 \cdot (8a^4b^2 - 20a^3 \\ & b^3 + 17a^2b^4 - 5ab^5) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^6 + 16 \cdot a^4b^2 - 68a^3b^3 + 82a^2b^4 - 30ab^5 + 24 \cdot (14 \cdot (8a^4b^2 - 20a^3b^3 + 17a \\ & a^2b^4 - 5ab^5) \cdot \cosh(dx + c)^3 + 3 \cdot (24a^5b - 76a^4b^2 + 91a^3b^3 - 49a^2b^4 + 10ab^5) \cdot \cosh(dx + c) \cdot \sinh(dx + c)^5 + 4 \cdot (64a^6 - 296a^ \\ & 5b + 548a^4b^2 - 509a^3b^3 + 238a^2b^4 - 45ab^5) \cdot \cosh(dx + c)^4 + 4 \cdot (64a^6 - 296a^5b + 548a^4b^2 - 509a^3b^3 + 238a^2b^4 - 45ab^ \\ & 5 + 105 \cdot (8a^4b^2 - 20a^3b^3 + 17a^2b^4 - 5ab^5) \cdot \cosh(dx + c)^4 + 4 \\ & 5 \cdot (24a^5b - 76a^4b^2 + 91a^3b^3 - 49a^2b^4 + 10ab^5) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^4 + 16 \cdot (21 \cdot (8a^4b^2 - 20a^3b^3 + 17a^2b^4 - 5ab^ \\ & 5) \cdot \cosh(dx + c)^5 + 15 \cdot (24a^5b - 76a^4b^2 + 91a^3b^3 - 49a^2b^4 + 10ab^5) \cdot \cosh(dx + c)^3 + (64a^6 - 296a^5b + 548a^4b^2 - 509a^3b^3 \\ & + 238a^2b^4 - 45ab^5) \cdot \cosh(dx + c) \cdot \sinh(dx + c)^3 + 4 \cdot (32a^5b - 1 \\ & 44a^4b^2 + 219a^3b^3 - 137a^2b^4 + 30ab^5) \cdot \cosh(dx + c)^2 + 4 \cdot (42 \cdot \\ & (8a^4b^2 - 20a^3b^3 + 17a^2b^4 - 5ab^5) \cdot \cosh(dx + c)^6 + 32a^5b - 144a^4b^2 + 219a^3b^3 - 137a^2b^4 + 30ab^5 + 45 \cdot (24a^5b - 76a^ \\ & 4b^2 + 91a^3b^3 - 49a^2b^4 + 10ab^5) \cdot \cosh(dx + c)^4 + 6 \cdot (64a^6 - 2 \\ & 96a^5b + 548a^4b^2 - 509a^3b^3 + 238a^2b^4 - 45ab^5) \cdot \cosh(dx + c \\ &)^2) \cdot \sinh(dx + c)^2 - 3 \cdot ((8a^2b^3 - 12ab^4 + 5b^5) \cdot \cosh(dx + c)^{10} + \\ & 10 \cdot (8a^2b^3 - 12ab^4 + 5b^5) \cdot \cosh(dx + c) \cdot \sinh(dx + c)^9 + (8a^2b \\ & ^3 - 12ab^4 + 5b^5) \cdot \sinh(dx + c)^{10} + (64a^3b^2 - 136a^2b^3 + 100a \\ & ab^4 - 25b^5) \cdot \cosh(dx + c)^8 + (64a^3b^2 - 136a^2b^3 + 100ab^4 - 25 \\ & b^5 + 45 \cdot (8a^2b^3 - 12ab^4 + 5b^5) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^8 + \\ & 8 \cdot (15 \cdot (8a^2b^3 - 12ab^4 + 5b^5) \cdot \cosh(dx + c)^3 + (64a^3b^2 - 136a^ \\ & ^2b^3 + 100ab^4 - 25b^5) \cdot \cosh(dx + c) \cdot \sinh(dx + c)^7 + 2 \cdot (64a^4b - \\ & 192a^3b^2 + 224a^2b^3 - 120ab^4 + 25b^5) \cdot \cosh(dx + c)^6 + 2 \cdot (64a^ \\ & 4b - 192a^3b^2 + 224a^2b^3 - 120ab^4 + 25b^5 + 105 \cdot (8a^2b^3 - 12a \\ & ab^4 + 5b^5) \cdot \cosh(dx + c)^4 + 14 \cdot (64a^3b^2 - 136a^2b^3 + 100ab^4 - \\ & 25b^5) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^6 + 4 \cdot (63 \cdot (8a^2b^3 - 12ab^4 + 5 \\ & b^5) \cdot \cosh(dx + c)^5 + 14 \cdot (64a^3b^2 - 136a^2b^3 + 100ab^4 - 25b^5) \cdot \\ & \cosh(dx + c)^3 + 3 \cdot (64a^4b - 192a^3b^2 + 224a^2b^3 - 120ab^4 + 25b \\ & b^5) \cdot \cosh(dx + c) \cdot \sinh(dx + c)^5 - 8a^2b^3 + 12ab^4 - 5b^5 - 2 \cdot (64a^ \\ & a^4b - 192a^3b^2 + 224a^2b^3 - 120ab^4 + 25b^5) \cdot \cosh(dx + c)^4 + 2 \\ & \cdot (105 \cdot (8a^2b^3 - 12ab^4 + 5b^5) \cdot \cosh(dx + c)^6 - 64a^4b + 192a^3b^ \\ & ^2 - 224a^2b^3 + 120ab^4 - 25b^5 + 35 \cdot (64a^3b^2 - 136a^2b^3 + 100a \\ & ab^4 - 25b^5) \cdot \cosh(dx + c)^4 + 15 \cdot (64a^4b - 192a^3b^2 + 224a^2b^3 - \\ & - 120ab^4 + 25b^5) \cdot \cosh(dx + c)^2) \cdot \sinh(dx + c)^4 + 8 \cdot (15 \cdot (8a^2b^3 - \\ & 12ab^4 + 5b^5) \cdot \cosh(dx + c)^7 + 7 \cdot (64a^3b^2 - 136a^2b^3 + 100ab^ \end{aligned}$$

$$\begin{aligned}
& 4 - 25*b^5)*\cosh(d*x + c)^5 + 5*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120 \\
& *a*b^4 + 25*b^5)*\cosh(d*x + c)^3 - (64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - \\
& 120*a*b^4 + 25*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^3 - (64*a^3*b^2 - 136*a^2* \\
& b^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c)^2 + (45*(8*a^2*b^3 - 12*a*b^4 + 5*b \\
& ^5)*\cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\co \\
& sh(d*x + c)^6 - 64*a^3*b^2 + 136*a^2*b^3 - 100*a*b^4 + 25*b^5 + 30*(64*a^4* \\
& b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*\cosh(d*x + c)^4 - 12*(6 \\
& 4*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^2 + 2*(5*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*\cosh(d*x + c)^9 + 4*(\\
& 64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c)^7 + 6*(64*a^4* \\
& b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*\cosh(d*x + c)^5 - 4*(64 \\
& *a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*\cosh(d*x + c)^3 - \\
& (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*\cosh(d*x + c))*\sinh(d*x + c \\
&))*\sqrt{-a^2 + a*b)*\arctan(-1/2*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh \\
& (d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-a^2 + a*b)/(a^2 - a*b)) + 8* \\
& (6*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*\cosh(d*x + c)^7 + 9*(24* \\
& a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*\cosh(d*x + c)^5 + \\
& 2*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5) \\
& *\cosh(d*x + c)^3 + (32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30 \\
& *a*b^5)*\cosh(d*x + c))*\sinh(d*x + c))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a \\
& ^4*b^5)*d*\cosh(d*x + c)^10 + 10*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5) \\
& *d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b \\
& ^5)*d*\sinh(d*x + c)^10 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + \\
& 5*a^4*b^5)*d*\cosh(d*x + c)^8 + (45*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b \\
& ^5)*d*\cosh(d*x + c)^2 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5 \\
& *a^4*b^5)*d)*\sinh(d*x + c)^8 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^ \\
& 3 + 27*a^5*b^4 - 5*a^4*b^5)*d*\cosh(d*x + c)^6 + 8*(15*(a^7*b^2 - 3*a^6*b^3 \\
& + 3*a^5*b^4 - a^4*b^5)*d*\cosh(d*x + c)^3 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b \\
& ^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^7 \\
& *b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*\cosh(d*x + c)^4 + 14*(8*a^8*b - 2 \\
& 9*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*\cosh(d*x + c)^2 + (8*a^9 \\
& - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d)*\sinh(d*x \\
& + c)^6 - 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^ \\
& 4*b^5)*d*\cosh(d*x + c)^4 + 4*(63*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5 \\
&)*d*\cosh(d*x + c)^5 + 14*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + \\
& 5*a^4*b^5)*d*\cosh(d*x + c)^3 + 3*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^ \\
& 3 + 27*a^5*b^4 - 5*a^4*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^7* \\
& b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*\cosh(d*x + c)^6 + 35*(8*a^8*b - 29 \\
& *a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*\cosh(d*x + c)^4 + 15*(8*a \\
& ^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*\cosh(d* \\
& x + c)^2 - (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4 \\
& *b^5)*d)*\sinh(d*x + c)^4 - (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 \\
& + 5*a^4*b^5)*d*\cosh(d*x + c)^2 + 8*(15*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a \\
& ^4*b^5)*d*\cosh(d*x + c)^7 + 7*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b \\
& ^4 + 5*a^4*b^5)*d*\cosh(d*x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a
\end{aligned}$$

$$\begin{aligned} &^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*\cosh(d*x + c)^3 - (8*a^9 - 36*a^8*b + 65 \\ &a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*\cosh(d*x + c))*\sinh(d*x + \\ &c)^3 + (45*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*\cosh(d*x + c)^8 + \\ &28*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*\cosh(d*x \\ &+ c)^6 + 30*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a \\ &^4*b^5)*d*\cosh(d*x + c)^4 - 12*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 \\ &+ 27*a^5*b^4 - 5*a^4*b^5)*d*\cosh(d*x + c)^2 - (8*a^8*b - 29*a^7*b^2 + 39*a^ \\ &6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d)*\sinh(d*x + c)^2 - (a^7*b^2 - 3*a^6*b^3 + \\ &3*a^5*b^4 - a^4*b^5)*d + 2*(5*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)* \\ &d*\cosh(d*x + c)^9 + 4*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a \\ &^4*b^5)*d*\cosh(d*x + c)^7 + 6*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + \\ &27*a^5*b^4 - 5*a^4*b^5)*d*\cosh(d*x + c)^5 - 4*(8*a^9 - 36*a^8*b + 65*a^7*b \\ &^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*\cosh(d*x + c)^3 - (8*a^8*b - 29 \\ &a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*\cosh(d*x + c))*\sinh(d*x + \\ &c))] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [A] time = 1.5999, size = 455, normalized size = 2.12

$$\frac{3(8a^2b - 12ab^2 + 5b^3) \arctan\left(\frac{be^{(2dx+2c)+2a-b}}{2\sqrt{-a^2+ab}}\right)}{8(a^5d - 2a^4bd + a^3b^2d)\sqrt{-a^2+ab}} - \frac{16a^2b^2e^{(6dx+6c)} - 20ab^3e^{(6dx+6c)} + 7b^4e^{(6dx+6c)} + 80a^3be^{(4dx+4c)}}{4(a^5d -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$-3/8*(8*a^2*b - 12*a*b^2 + 5*b^3)*\arctan(1/2*(b*e^{(2*d*x + 2*c)} + 2*a - b)/\sqrt{-a^2 + a*b})/((a^5*d - 2*a^4*b*d + a^3*b^2*d)*\sqrt{-a^2 + a*b}) - 1/4*(16*a^2*b^2*e^{(6*d*x + 6*c)} - 20*a*b^3*e^{(6*d*x + 6*c)} + 7*b^4*e^{(6*d*x + 6$$

$$\begin{aligned}
& *c) + 80*a^3*b*e^{(4*d*x + 4*c)} - 136*a^2*b^2*e^{(4*d*x + 4*c)} + 86*a*b^3*e^{(4*d*x + 4*c)} - 21*b^4*e^{(4*d*x + 4*c)} + 64*a^2*b^2*e^{(2*d*x + 2*c)} - 76*a*b^3*e^{(2*d*x + 2*c)} + 21*b^4*e^{(2*d*x + 2*c)} + 10*a*b^3 - 7*b^4)/((a^5*d - 2*a^4*b*d + a^3*b^2*d)*(b*e^{(4*d*x + 4*c)} + 4*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + b)^2) - 2/(a^3*d*(e^{(2*d*x + 2*c)} - 1))
\end{aligned}$$

$$3.58 \quad \int \frac{\operatorname{csch}^3(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=224

$$\frac{b^{3/2} (35a^2 - 56ab + 24b^2) \tan^{-1} \left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}} \right)}{8a^4 d(a-b)^{5/2}} - \frac{b(a-4b)(4a-3b) \cosh(c+dx)}{8a^3 d(a-b)^2 (a+b \cosh^2(c+dx) - b)} - \frac{b(2a-3b) \cosh(c+dx)}{4a^2 d(a-b) (a+b \cosh^2(c+dx))}$$

[Out] (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*a^4*(a - b)^(5/2)*d) + ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d) - ((2*a - 3*b)*b*Cosh[c + d*x])/(4*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) - ((a - 4*b)*(4*a - 3*b)*b*Cosh[c + d*x])/(8*a^3*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cosh[c + d*x]^2)^2)

Rubi [A] time = 0.43874, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3186, 414, 527, 522, 206, 205}

$$\frac{b^{3/2} (35a^2 - 56ab + 24b^2) \tan^{-1} \left(\frac{\sqrt{b} \cosh(c+dx)}{\sqrt{a-b}} \right)}{8a^4 d(a-b)^{5/2}} - \frac{b(a-4b)(4a-3b) \cosh(c+dx)}{8a^3 d(a-b)^2 (a+b \cosh^2(c+dx) - b)} - \frac{b(2a-3b) \cosh(c+dx)}{4a^2 d(a-b) (a+b \cosh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*a^4*(a - b)^(5/2)*d) + ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d) - ((2*a - 3*b)*b*Cosh[c + d*x])/(4*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) - ((a - 4*b)*(4*a - 3*b)*b*Cosh[c + d*x])/(8*a^3*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cosh[c + d*x]^2)^2)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +

$f*x]/ff], x]] /; \text{FreeQ}\{a, b, e, f, p\}, x\} \&\& \text{IntegerQ}[(m - 1)/2]$

Rule 414

$\text{Int}[(a_ + (b_.)*(x_)^{(n_)})^{(p_)}*((c_ + (d_.)*(x_)^{(n_)})^{(q_)}), x_Symbol]$
 $:= -\text{Simp}[(b*x*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q + 1)})/(a*n*(p + 1)*(b*c - a*d)), x]$
 $+ \text{Dist}[1/(a*n*(p + 1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p + 1)}*(c + d*x^n)^q*\text{Simp}[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x]$
 $/; \text{FreeQ}\{a, b, c, d, n, q\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& !(!\text{IntegerQ}[p] \&\& \text{IntegerQ}[q] \&\& \text{LtQ}[q, -1]) \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 527

$\text{Int}[(a_ + (b_.)*(x_)^{(n_)})^{(p_)}*((c_ + (d_.)*(x_)^{(n_)})^{(q_)}*((e_ + (f_.)*(x_)^{(n_)}), x_Symbol]$
 $:= -\text{Simp}[(b*e - a*f)*x*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q + 1)})/(a*n*(b*c - a*d)*(p + 1)), x]$
 $+ \text{Dist}[1/(a*n*(b*c - a*d)*(p + 1)), \text{Int}[(a + b*x^n)^{(p + 1)}*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x]$
 $/; \text{FreeQ}\{a, b, c, d, e, f, n, q\}, x\} \&\& \text{LtQ}[p, -1]$

Rule 522

$\text{Int}[(e_ + (f_.)*(x_)^{(n_)})/((a_ + (b_.)*(x_)^{(n_)})*((c_ + (d_.)*(x_)^{(n_)}), x_Symbol]$
 $:= \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^n), x], x]$
 $- \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[1/(c + d*x^n), x], x]$
 $/; \text{FreeQ}\{a, b, c, d, e, f, n\}, x]$

Rule 206

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol]$
 $:= \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x]$
 $/; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

Rule 205

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x_Symbol]$
 $:= \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x]$
 $/; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+bx^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad(a-b+b\cosh^2(c+dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{a+b+5bx^2}{(1-x^2)(a-b+bx^2)^3} dx, x, \cosh(c+dx)\right)}{2ad} \\
&= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d(a-b+b\cosh^2(c+dx))^2} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad(a-b+b\cosh^2(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{a+b-5bx^2}{(1-x^2)(a-b+bx^2)^3} dx, x, \cosh(c+dx)\right)}{2ad} \\
&= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d(a-b+b\cosh^2(c+dx))^2} - \frac{(a-4b)(4a-3b)b\cosh(c+dx)}{8a^3(a-b)^2d(a-b+b\cosh^2(c+dx))} - \frac{\operatorname{csch}(c+dx)}{2ad} \\
&= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d(a-b+b\cosh^2(c+dx))^2} - \frac{(a-4b)(4a-3b)b\cosh(c+dx)}{8a^3(a-b)^2d(a-b+b\cosh^2(c+dx))} - \frac{\operatorname{csch}(c+dx)}{2ad} \\
&= \frac{b^{3/2}(35a^2-56ab+24b^2)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{5/2}d} + \frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d} - \frac{\operatorname{csch}(c+dx)}{4a^2d}
\end{aligned}$$

Mathematica [C] time = 2.94302, size = 419, normalized size = 1.87

$$\operatorname{csch}^5(c+dx)(2a+b\cosh(2(c+dx))-b) \left(\frac{8a^2b^2\operatorname{coth}(c+dx)}{a-b} + \frac{b^{3/2}(35a^2-56ab+24b^2)\operatorname{csch}(c+dx)(2a+b\cosh(2(c+dx))-b)^2\tan^{-1}\left(\frac{\sqrt{b-i\sqrt{a}}\tan\left(\frac{c+dx}{2}\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^5*((8*a^2*b^2*Coth[c + d*x])/((a - b) + (2*a*(11*a - 8*b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d*x]))/(a - b)^2 + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])^2*Csch[c + d*x])/(a - b)^(5/2) + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqr

$$\frac{t[b] + I\sqrt{a}\operatorname{Tanh}\left[\frac{c + d*x}{2}\right]/\sqrt{a - b}*(2*a - b + b*\operatorname{Cosh}[2*(c + d*x)])^2*\operatorname{Csch}[c + d*x]}{(a - b)^{5/2} - a*(2*a - b + b*\operatorname{Cosh}[2*(c + d*x)])^2*\operatorname{Csch}\left[\frac{c + d*x}{2}\right]^2*\operatorname{Csch}[c + d*x] - 4*(a + 6*b)*(2*a - b + b*\operatorname{Cosh}[2*(c + d*x)])^2*\operatorname{Csch}[c + d*x]*\operatorname{Log}\left[\operatorname{Tanh}\left[\frac{c + d*x}{2}\right]\right] - a*(2*a - b + b*\operatorname{Cosh}[2*(c + d*x)])^2*\operatorname{Csch}[c + d*x]*\operatorname{Sech}\left[\frac{c + d*x}{2}\right]^2)}{(64*a^4*d*(b + a*\operatorname{Csch}[c + d*x]^2)^3)}$$

Maple [B] time = 0.091, size = 1225, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\operatorname{csch}(d*x+c))^3/(a+b*\sinh(d*x+c))^2 dx, x$

[Out] $\frac{1}{8}d*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2/a^3 - \frac{1}{8}d/a^3/\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2 - \frac{1}{2}d/a^3*\ln\left(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right) - \frac{3}{d/a^4*\ln\left(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)\right)*b - \frac{13}{4}d/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/a}{(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^6*b^2 + 10/d/a^2*b^3/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^6 - 6/d*b^4/a^3/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^6 + 39/4/d/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/a*b^2/(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4 - 67/2/d/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/a^2/(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4*b^3 + 46/d/a^3*b^4/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4 - 20/d*b^5/a^4/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^4 - 39/4/d/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/a/(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b^2 + 26/d/a^2*b^3/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2 - 14/d*b^4/a^3/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/(a^2 - 2*a*b + b^2)*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2 + 13/4/d/a*b^2/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/(a^2 - 2*a*b + b^2) - 5/2/d*b^3/a^2/(\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right))^4*a - 2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a + 4*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*b + a)^2/(a^2 - 2*a*b + b^2) + 35/8/d/a^2*b^2/(a^2 - 2*a*b + b^2)/(a*b - b^2)^{1/2}*\arctan\left(\frac{1}{4}*(2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a - 2*a + 4*b)/(a*b - b^2)^{1/2}\right) - 7/d/a^3*b^3/(a^2 - 2*a*b + b^2)/(a*b - b^2)^{1/2}*\arctan\left(\frac{1}{4}*(2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a - 2*a + 4*b)/(a*b - b^2)^{1/2}\right) + 3/d*b^4/a^4/(a^2 - 2*a*b + b^2)/(a*b - b^2)^{1/2}*\operatorname{arctan}\left(\frac{1}{4}*(2*\operatorname{tanh}\left(\frac{1}{2}d*x+\frac{1}{2}c\right)^2*a - 2*a + 4*b)/(a*b - b^2)^{1/2}\right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$-1/4*((4*a^2*b^2*e^{(11*c)} - 19*a*b^3*e^{(11*c)} + 12*b^4*e^{(11*c)})e^{(11*d*x)} + (32*a^3*b*e^{(9*c)} - 128*a^2*b^2*e^{(9*c)} + 129*a*b^3*e^{(9*c)} - 36*b^4*e^{(9*c)})e^{(9*d*x)} + 2*(32*a^4*e^{(7*c)} - 80*a^3*b*e^{(7*c)} + 94*a^2*b^2*e^{(7*c)} - 55*a*b^3*e^{(7*c)} + 12*b^4*e^{(7*c)})e^{(7*d*x)} + 2*(32*a^4*e^{(5*c)} - 80*a^3*b*e^{(5*c)} + 94*a^2*b^2*e^{(5*c)} - 55*a*b^3*e^{(5*c)} + 12*b^4*e^{(5*c)})e^{(5*d*x)} + (32*a^3*b*e^{(3*c)} - 128*a^2*b^2*e^{(3*c)} + 129*a*b^3*e^{(3*c)} - 36*b^4*e^{(3*c)})e^{(3*d*x)} + (4*a^2*b^2*e^c - 19*a*b^3*e^c + 12*b^4*e^c)e^{(d*x)})/(a^5*b^2*d - 2*a^4*b^3*d + a^3*b^4*d + (a^5*b^2*d*e^{(12*c)} - 2*a^4*b^3*d*e^{(12*c)} + a^3*b^4*d*e^{(12*c)})e^{(12*d*x)} + 2*(4*a^6*b*d*e^{(10*c)} - 11*a^5*b^2*d*e^{(10*c)} + 10*a^4*b^3*d*e^{(10*c)} - 3*a^3*b^4*d*e^{(10*c)})e^{(10*d*x)} + (16*a^7*d*e^{(8*c)} - 64*a^6*b*d*e^{(8*c)} + 95*a^5*b^2*d*e^{(8*c)} - 62*a^4*b^3*d*e^{(8*c)} + 15*a^3*b^4*d*e^{(8*c)})e^{(8*d*x)} - 4*(8*a^7*d*e^{(6*c)} - 28*a^6*b*d*e^{(6*c)} + 37*a^5*b^2*d*e^{(6*c)} - 22*a^4*b^3*d*e^{(6*c)} + 5*a^3*b^4*d*e^{(6*c)})e^{(6*d*x)} + (16*a^7*d*e^{(4*c)} - 64*a^6*b*d*e^{(4*c)} + 95*a^5*b^2*d*e^{(4*c)} - 62*a^4*b^3*d*e^{(4*c)} + 15*a^3*b^4*d*e^{(4*c)})e^{(4*d*x)} + 2*(4*a^6*b*d*e^{(2*c)} - 11*a^5*b^2*d*e^{(2*c)} + 10*a^4*b^3*d*e^{(2*c)} - 3*a^3*b^4*d*e^{(2*c)})e^{(2*d*x)}) + 1/2*(a + 6*b)*log((e^{(d*x + c)} + 1)*e^{(-c)})/(a^4*d) - 1/2*(a + 6*b)*log((e^{(d*x + c)} - 1)*e^{(-c)})/(a^4*d) + 8*integrate(1/32*((35*a^2*b^2*e^{(3*c)} - 56*a*b^3*e^c + 24*b^4*e^c)*e^{(3*d*x)} - (35*a^2*b^2*e^c - 56*a*b^3*e^c + 24*b^4*e^c)*e^{(d*x)})/(a^6*b - 2*a^5*b^2 + a^4*b^3 + (a^6*b*e^{(4*c)} - 2*a^5*b^2*e^{(4*c)} + a^4*b^3*e^{(4*c)})e^{(4*d*x)} + 2*(2*a^7*e^{(2*c)} - 5*a^6*b*e^{(2*c)} + 4*a^5*b^2*e^{(2*c)} - a^4*b^3*e^{(2*c)})e^{(2*d*x)}), x)$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)`

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.59 \quad \int \frac{\operatorname{csch}^4(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=259

$$\frac{b^2(48a^2 - 80ab + 35b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a-b)^{5/2}} - \frac{(8a^2 - 52ab + 35b^2) \operatorname{coth}^3(c+dx)}{24a^3d(a-b)^2} + \frac{(-4a^2b + 8a^3 - 45ab^2 + 35b^3)}{8a^4d(a-b)^2}$$

[Out] (b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(9/2)*(a - b)^(5/2)*d) + ((8*a^3 - 4*a^2*b - 45*a*b^2 + 35*b^3)*Cot h[c + d*x])/(8*a^4*(a - b)^2*d) - ((8*a^2 - 52*a*b + 35*b^2)*Coth[c + d*x]^3)/(24*a^3*(a - b)^2*d) - (b*Csch[c + d*x]^3*Sech[c + d*x]^3)/(4*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((10*a - 7*b)*b*Csch[c + d*x]^3*Sech[c + d*x])/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.337981, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3187, 468, 577, 570, 208}

$$\frac{b^2(48a^2 - 80ab + 35b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a-b)^{5/2}} - \frac{(8a^2 - 52ab + 35b^2) \operatorname{coth}^3(c+dx)}{24a^3d(a-b)^2} + \frac{(-4a^2b + 8a^3 - 45ab^2 + 35b^3)}{8a^4d(a-b)^2}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(9/2)*(a - b)^(5/2)*d) + ((8*a^3 - 4*a^2*b - 45*a*b^2 + 35*b^3)*Cot h[c + d*x])/(8*a^4*(a - b)^2*d) - ((8*a^2 - 52*a*b + 35*b^2)*Coth[c + d*x]^3)/(24*a^3*(a - b)^2*d) - (b*Csch[c + d*x]^3*Sech[c + d*x]^3)/(4*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((10*a - 7*b)*b*Csch[c + d*x]^3*Sech[c + d*x])/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3187

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)]/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p]/(1 + ff^2*x^2)^(m/2 + p + 1),

$x], x, \text{Tan}[e + f*x]/ff], x]] /; \text{FreeQ}\{a, b, e, f\}, x\} \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[p]$

Rule 468

$\text{Int}[(e_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_)^{(n_*)})^{(q_*)}, x_Symbol] \rightarrow -\text{Simp}[(c*b - a*d)*(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)}/(a*b*e*n*(p+1)), x] + \text{Dist}[1/(a*b*n*(p+1)), \text{Int}[(e*x)^m*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-2)}*\text{Simp}[c*(c*b*n*(p+1) + (c*b - a*d)*(m+1)) + d*(c*b*n*(p+1) + (c*b - a*d)*(m + n*(q-1) + 1))*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, m\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[q, 1] \&\& \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 577

$\text{Int}[(g_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_)^{(n_*)})^{(q_*)}*((e_*) + (f_*)*(x_)^{(n_*)}), x_Symbol] \rightarrow -\text{Simp}[(b*e - a*f)*(g*x)^{(m+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^q/(a*b*g*n*(p+1)), x] + \text{Dist}[1/(a*b*n*(p+1)), \text{Int}[(g*x)^m*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)}*\text{Simp}[c*(b*e*n*(p+1) + (b*e - a*f)*(m+1)) + d*(b*e*n*(p+1) + (b*e - a*f)*(m + n*q + 1))*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m\}, x\} \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[q, 0] \&\& !(\text{EqQ}[q, 1] \&\& \text{SimplerQ}[b*c - a*d, b*e - a*f])$

Rule 570

$\text{Int}[(g_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_)^{(n_*)})^{(q_*)}*((e_*) + (f_*)*(x_)^{(n_*)})^{(r_*)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, n\}, x\} \&\& \text{IGtQ}[p, -2] \&\& \text{IGtQ}[q, 0] \&\& \text{IGtQ}[r, 0]$

Rule 208

$\text{Int}[(a_*) + (b_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^4}{x^4(a-(a-b)x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2(4a-7b+(-4a+b)x^2)}{x^4(a+(-a+b)x^2)^2} dx, x, \tanh(c+dx)\right)}{4a(a-b)d} \\
&= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(10a-7b)b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{8a^2(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{S}{S} \\
&= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(10a-7b)b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{8a^2(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{S}{S} \\
&= \frac{(8a^3-4a^2b-45ab^2+35b^3)\operatorname{coth}(c+dx)}{8a^4(a-b)^2d} - \frac{(8a^2-52ab+35b^2)\operatorname{coth}^3(c+dx)}{24a^3(a-b)^2d} - \frac{4a(a-b)^2d}{4a(a-b)^2d} \\
&= \frac{b^2(48a^2-80ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^{5/2}d} + \frac{(8a^3-4a^2b-45ab^2+35b^3)\operatorname{coth}(c+dx)}{8a^4(a-b)^2d}
\end{aligned}$$

Mathematica [A] time = 2.76237, size = 167, normalized size = 0.64

$$\frac{3b^2(48a^2-80ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}} + \sqrt{a}\left(\frac{3b^3\sinh(2(c+dx))(-32a^2+b(11b-14a)\cosh(2(c+dx))+40ab-11b^2)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} - 8\operatorname{coth}(c+dx)\right)\operatorname{acsch}(c+dx)$$

$$24a^{9/2}d$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((3*b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a - b)^(5/2) + Sqrt[a]*(-8*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*x]^2) + (3*b^3*(-32*a^2 + 40*a*b - 11*b^2 + b*(-14*a + 11*b))*Cosh[2*(c + d*x)]*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)]^2)))/(24*a^(9/2)*d)

Maple [B] time = 0.111, size = 1930, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{csch}(d*x+c)^4/(a+b*\sinh(d*x+c)^2)^3,x)$

[Out]
$$\begin{aligned} & 13/4/d*b^4/a^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)+35/8/d*b^4/a^4/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-35/8/d*b^4/a^4/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+13/4/d*b^4/a^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7+11/d*b^5/a^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^5+11/d*b^5/a^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3-69/4/d/a^3*b^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^5-69/4/d/a^3*b^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3-4/d/a^2*b^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)-10/d/a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+10/d/a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-4/d/a^2*b^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7+4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^5+4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3+6/d/a^2/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})*b^2-6/d/a^2/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})*b^2-35/8/d*b^5/a^4/(a^2-2*a*b+b^2)/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})*b^2-35/8/d*b^5/a^4/(a^2-2*a*b+b^2)/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+10/d/a^3*b^4/(a^2-2*a*b+b^2)/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+10/d/a^3*b^4/(a^2-2*a*b+b^2)/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}) \end{aligned}$$

$$\frac{((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}-6/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^{1/2}}{((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c))} \frac{((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}}{((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c))} \frac{b^3-6/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^{1/2}}{((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}} \frac{b^3+3/2/d/a^4*\operatorname{tanh}(1/2*d*x+1/2*c)*b+3/2/d/a^4/\operatorname{tanh}(1/2*d*x+1/2*c)*b+3/8/d/a^3*\operatorname{tanh}(1/2*d*x+1/2*c)+3/8/d/a^3/\operatorname{tanh}(1/2*d*x+1/2*c)-1/24/d/a^3*\operatorname{tanh}(1/2*d*x+1/2*c)^3-1/24/d/a^3/\operatorname{tanh}(1/2*d*x+1/2*c)^3}{1}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [A] time = 1.60946, size = 518, normalized size = 2.

$$\frac{(48a^2b^2 - 80ab^3 + 35b^4) \arctan\left(\frac{be^{(2dx+2c)+2a-b}}{2\sqrt{-a^2+ab}}\right)}{8(a^6d - 2a^5bd + a^4b^2d)\sqrt{-a^2 + ab}} + \frac{24a^2b^3e^{(6dx+6c)} - 32ab^4e^{(6dx+6c)} + 11b^5e^{(6dx+6c)} + 112a^3b^2e^{(4dx+4c)}}{4(a^6d - 2a^5bd + a^4b^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{8} \cdot \frac{(48a^2b^2 - 80a^3b + 35b^4) \arctan\left(\frac{1}{2} \cdot \frac{b e^{(2dx+2c)} + 2a - b}{\sqrt{-a^2 + ab}}\right) + \frac{1}{4} \cdot (24a^2b^3e^{(6dx+6c)} - 32a^3b^4e^{(6dx+6c)} + 11b^5e^{(6dx+6c)} + 112a^3b^2e^{(4dx+4c)} - 200a^2b^3e^{(4dx+4c)} + 130a^3b^4e^{(4dx+4c)} - 33b^5e^{(4dx+4c)} + 88a^2b^3e^{(2dx+2c)} - 112a^3b^4e^{(2dx+2c)} + 33b^5e^{(2dx+2c)} + 14a^3b^4 - 11b^5)}{(a^6d - 2a^5bd + a^4b^2d) \cdot (b e^{(4dx+4c)} + 4a e^{(2dx+2c)} - 2b e^{(2dx+2c)} + b)^2} + \frac{2}{3} \cdot \frac{(9b^3e^{(4dx+4c)} - 6a e^{(2dx+2c)} - 18b e^{(2dx+2c)} + 2a + 9b)}{(a^4d \cdot (e^{(2dx+2c)} - 1)^3)}$

$$3.60 \quad \int \frac{1}{1+\sinh^2(x)} dx$$

Optimal. Leaf size=2

tanh(x)

[Out] Tanh[x]

Rubi [A] time = 0.0163758, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3175, 3767, 8}

tanh(x)

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^2)^(-1), x]

[Out] Tanh[x]

Rule 3175

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1 + \sinh^2(x)} dx &= \int \operatorname{sech}^2(x) dx \\ &= i \operatorname{Subst}\left(\int 1 dx, x, -i \tanh(x)\right) \\ &= \tanh(x) \end{aligned}$$

Mathematica [A] time = 0.0036714, size = 2, normalized size = 1.

$\tanh(x)$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^2)^(-1), x]

[Out] Tanh[x]

Maple [B] time = 0.016, size = 17, normalized size = 8.5

$$2 \frac{\tanh(x/2)}{(\tanh(x/2))^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+sinh(x)^2), x)

[Out] 2*tanh(1/2*x)/(tanh(1/2*x)^2+1)

Maxima [B] time = 1.02647, size = 14, normalized size = 7.

$$\frac{2}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^2), x, algorithm="maxima")

[Out] $2/(e^{-2*x} + 1)$

Fricas [B] time = 1.75827, size = 70, normalized size = 35.

$$-\frac{2}{\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^2),x, algorithm="fricas")`

[Out] $-2/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2 + 1)$

Sympy [B] time = 0.894588, size = 14, normalized size = 7.

$$\frac{2 \tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)**2),x)`

[Out] $2*\tanh(x/2)/(\tanh(x/2)**2 + 1)$

Giac [B] time = 1.29848, size = 14, normalized size = 7.

$$-\frac{2}{e^{(2*x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^2),x, algorithm="giac")`

[Out] $-2/(e^{(2*x)} + 1)$

$$3.61 \quad \int \frac{1}{\left(1+\sinh^2(x)\right)^2} dx$$

Optimal. Leaf size=11

$$\tanh(x) - \frac{\tanh^3(x)}{3}$$

[Out] Tanh[x] - Tanh[x]^3/3

Rubi [A] time = 0.0190848, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3175, 3767}

$$\tanh(x) - \frac{\tanh^3(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^2)^(-2), x]

[Out] Tanh[x] - Tanh[x]^3/3

Rule 3175

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p, x_Symbol] :> Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^n, x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(1 + \sinh^2(x))^2} dx &= \int \operatorname{sech}^4(x) dx \\
&= i \operatorname{Subst} \left(\int (1 + x^2) dx, x, -i \tanh(x) \right) \\
&= \tanh(x) - \frac{\tanh^3(x)}{3}
\end{aligned}$$

Mathematica [A] time = 0.0027172, size = 17, normalized size = 1.55

$$\frac{2 \tanh(x)}{3} + \frac{1}{3} \tanh(x) \operatorname{sech}^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^2)^(-2), x]

[Out] (2*Tanh[x])/3 + (Sech[x]^2*Tanh[x])/3

Maple [B] time = 0.015, size = 36, normalized size = 3.3

$$-2 \frac{-(\tanh(x/2))^5 - 2/3 (\tanh(x/2))^3 - \tanh(x/2)}{((\tanh(x/2))^2 + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+sinh(x)^2)^2, x)

[Out] -2*(-tanh(1/2*x)^5-2/3*tanh(1/2*x)^3-tanh(1/2*x))/(tanh(1/2*x)^2+1)^3

Maxima [B] time = 1.03861, size = 66, normalized size = 6.

$$\frac{4e^{(-2x)}}{3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1} + \frac{4}{3(3e^{(-2x)} + 3e^{(-4x)} + e^{(-6x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="maxima")

[Out] $4e^{-2x}/(3e^{-2x} + 3e^{-4x} + e^{-6x} + 1) + 4/3/(3e^{-2x} + 3e^{-4x} + e^{-6x} + 1)$

Fricas [B] time = 1.79279, size = 286, normalized size = 26.

$$\frac{8(2 \cosh(x) + \sinh(x))}{3(\cosh(x)^5 + 5 \cosh(x) \sinh(x)^4 + \sinh(x)^5 + (10 \cosh(x)^2 + 3) \sinh(x)^3 + 3 \cosh(x)^3 + (10 \cosh(x)^3 + 9 \cosh(x) \sinh(x)^2 + 5 \cosh(x)^4 + 9 \cosh(x)^2 + 2) \sinh(x) + 4 \cosh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="fricas")

[Out] $-8/3*(2*\cosh(x) + \sinh(x))/(\cosh(x)^5 + 5*\cosh(x)*\sinh(x)^4 + \sinh(x)^5 + (10*\cosh(x)^2 + 3)*\sinh(x)^3 + 3*\cosh(x)^3 + (10*\cosh(x)^3 + 9*\cosh(x))*\sinh(x)^2 + (5*\cosh(x)^4 + 9*\cosh(x)^2 + 2)*\sinh(x) + 4*\cosh(x))$

Sympy [B] time = 4.90601, size = 104, normalized size = 9.45

$$\frac{6 \tanh^5\left(\frac{x}{2}\right)}{3 \tanh^6\left(\frac{x}{2}\right) + 9 \tanh^4\left(\frac{x}{2}\right) + 9 \tanh^2\left(\frac{x}{2}\right) + 3} + \frac{4 \tanh^3\left(\frac{x}{2}\right)}{3 \tanh^6\left(\frac{x}{2}\right) + 9 \tanh^4\left(\frac{x}{2}\right) + 9 \tanh^2\left(\frac{x}{2}\right) + 3} + \frac{6 \tanh\left(\frac{x}{2}\right)}{3 \tanh^6\left(\frac{x}{2}\right) + 9 \tanh^4\left(\frac{x}{2}\right) + 9 \tanh^2\left(\frac{x}{2}\right) + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)**2)**2,x)

[Out] $6*\tanh(x/2)**5/(3*\tanh(x/2)**6 + 9*\tanh(x/2)**4 + 9*\tanh(x/2)**2 + 3) + 4*\tanh(x/2)**3/(3*\tanh(x/2)**6 + 9*\tanh(x/2)**4 + 9*\tanh(x/2)**2 + 3) + 6*\tanh(x/2)/(3*\tanh(x/2)**6 + 9*\tanh(x/2)**4 + 9*\tanh(x/2)**2 + 3)$

Giac [A] time = 1.27509, size = 24, normalized size = 2.18

$$\frac{4(3e^{2x} + 1)}{3(e^{2x} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*x) + 1)/(e^(2*x) + 1)^3
```

$$3.62 \quad \int \frac{1}{\left(1+\sinh^2(x)\right)^3} dx$$

Optimal. Leaf size=19

$$\frac{\tanh^5(x)}{5} - \frac{2 \tanh^3(x)}{3} + \tanh(x)$$

[Out] Tanh[x] - (2*Tanh[x]^3)/3 + Tanh[x]^5/5

Rubi [A] time = 0.0201132, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3175, 3767}

$$\frac{\tanh^5(x)}{5} - \frac{2 \tanh^3(x)}{3} + \tanh(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^2)^(-3), x]

[Out] Tanh[x] - (2*Tanh[x]^3)/3 + Tanh[x]^5/5

Rule 3175

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-p_), x_Symbol] :> Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1 + \sinh^2(x))^3} dx &= \int \operatorname{sech}^6(x) dx \\ &= i \operatorname{Subst} \left(\int (1 + 2x^2 + x^4) dx, x, -i \tanh(x) \right) \\ &= \tanh(x) - \frac{2 \tanh^3(x)}{3} + \frac{\tanh^5(x)}{5} \end{aligned}$$

Mathematica [A] time = 0.0029111, size = 27, normalized size = 1.42

$$\frac{8 \tanh(x)}{15} + \frac{1}{5} \tanh(x) \operatorname{sech}^4(x) + \frac{4}{15} \tanh(x) \operatorname{sech}^2(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^2)^(-3), x]

[Out] (8*Tanh[x])/15 + (4*Sech[x]^2*Tanh[x])/15 + (Sech[x]^4*Tanh[x])/5

Maple [B] time = 0.019, size = 52, normalized size = 2.7

$$-2 \frac{1}{((\tanh(x/2))^2 + 1)^5} \left(-(\tanh(x/2))^9 - 4/3 (\tanh(x/2))^7 - \frac{58 (\tanh(x/2))^5}{15} - 4/3 (\tanh(x/2))^3 - \tanh(x/2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+sinh(x)^2)^3, x)

[Out] -2*(-tanh(1/2*x)^9-4/3*tanh(1/2*x)^7-58/15*tanh(1/2*x)^5-4/3*tanh(1/2*x)^3-tanh(1/2*x))/(tanh(1/2*x)^2+1)^5

Maxima [B] time = 1.04492, size = 150, normalized size = 7.89

$$\frac{16 e^{-2x}}{3(5 e^{-2x} + 10 e^{-4x} + 10 e^{-6x} + 5 e^{-8x} + e^{-10x} + 1)} + \frac{32 e^{-4x}}{3(5 e^{-2x} + 10 e^{-4x} + 10 e^{-6x} + 5 e^{-8x} + e^{-10x} + 1)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="maxima")

[Out] $\frac{16}{3}e^{-2x}/(5e^{-2x} + 10e^{-4x} + 10e^{-6x} + 5e^{-8x} + e^{-10x} + 1) + \frac{32}{3}e^{-4x}/(5e^{-2x} + 10e^{-4x} + 10e^{-6x} + 5e^{-8x} + e^{-10x} + 1) + \frac{16}{15}/(5e^{-2x} + 10e^{-4x} + 10e^{-6x} + 5e^{-8x} + e^{-10x} + 1)$

Fricas [B] time = 1.78008, size = 628, normalized size = 33.05

$$\frac{15(\cosh(x)^8 + 8 \cosh(x) \sinh(x)^7 + \sinh(x)^8 + (28 \cosh(x)^2 + 5) \sinh(x)^6 + 5 \cosh(x)^6 + 2(28 \cosh(x)^3 + 15 \cosh(x) \sinh(x)^2) \sinh(x)^4 + 10 \cosh(x)^4 + 4(14 \cosh(x)^5 + 25 \cosh(x)^3 + 10 \cosh(x) \sinh(x)^2 + 11 \cosh(x)^2 + 2(4 \cosh(x)^7 + 15 \cosh(x)^5 + 20 \cosh(x)^3 + 9 \cosh(x) \sinh(x) + 5) \sinh(x) + 5)}{15 \cosh(x)^8 + 8 \cosh(x) \sinh(x)^7 + \sinh(x)^8 + (28 \cosh(x)^2 + 5) \sinh(x)^6 + 5 \cosh(x)^6 + 2(28 \cosh(x)^3 + 15 \cosh(x) \sinh(x)^2) \sinh(x)^4 + 10 \cosh(x)^4 + 4(14 \cosh(x)^5 + 25 \cosh(x)^3 + 10 \cosh(x) \sinh(x)^2 + 11 \cosh(x)^2 + 2(4 \cosh(x)^7 + 15 \cosh(x)^5 + 20 \cosh(x)^3 + 9 \cosh(x) \sinh(x) + 5) \sinh(x) + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="fricas")

[Out] $\frac{-16}{15} \frac{(11 \cosh(x)^2 + 18 \cosh(x) \sinh(x) + 11 \sinh(x)^2 + 5)}{(\cosh(x)^8 + 8 \cosh(x) \sinh(x)^7 + \sinh(x)^8 + (28 \cosh(x)^2 + 5) \sinh(x)^6 + 5 \cosh(x)^6 + 2(28 \cosh(x)^3 + 15 \cosh(x) \sinh(x)^2) \sinh(x)^4 + 10 \cosh(x)^4 + 4(14 \cosh(x)^5 + 25 \cosh(x)^3 + 10 \cosh(x) \sinh(x)^2 + 11 \cosh(x)^2 + 2(4 \cosh(x)^7 + 15 \cosh(x)^5 + 20 \cosh(x)^3 + 9 \cosh(x) \sinh(x) + 5) \sinh(x) + 5)}$

Sympy [B] time = 18.1921, size = 260, normalized size = 13.68

$$\frac{30 \tanh^9\left(\frac{x}{2}\right)}{15 \tanh^{10}\left(\frac{x}{2}\right) + 75 \tanh^8\left(\frac{x}{2}\right) + 150 \tanh^6\left(\frac{x}{2}\right) + 150 \tanh^4\left(\frac{x}{2}\right) + 75 \tanh^2\left(\frac{x}{2}\right) + 15} + \frac{15 \tanh^{10}\left(\frac{x}{2}\right) + 75 \tanh^8\left(\frac{x}{2}\right) + 150 \tanh^6\left(\frac{x}{2}\right) + 150 \tanh^4\left(\frac{x}{2}\right) + 75 \tanh^2\left(\frac{x}{2}\right) + 15}{15 \tanh^{10}\left(\frac{x}{2}\right) + 75 \tanh^8\left(\frac{x}{2}\right) + 150 \tanh^6\left(\frac{x}{2}\right) + 150 \tanh^4\left(\frac{x}{2}\right) + 75 \tanh^2\left(\frac{x}{2}\right) + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)**2)**3,x)

[Out] $\frac{30 \tanh(x/2)**9}{(15 \tanh(x/2)**10 + 75 \tanh(x/2)**8 + 150 \tanh(x/2)**6 + 150 \tanh(x/2)**4 + 75 \tanh(x/2)**2 + 15)} + \frac{40 \tanh(x/2)**7}{(15 \tanh(x/2)**10 + 75 \tanh(x/2)**8 + 150 \tanh(x/2)**6 + 150 \tanh(x/2)**4 + 75 \tanh(x/2)**2 + 15)} + \frac{116 \tanh(x/2)**5}{(15 \tanh(x/2)**10 + 75 \tanh(x/2)**8 + 150 \tanh(x/2)**6 + 150 \tanh(x/2)**4 + 75 \tanh(x/2)**2 + 15)} + \frac{40 \tanh(x/2)**3}{(15 \tanh(x/2)**10 + 75 \tanh(x/2)**8 + 150 \tanh(x/2)**6 + 150 \tanh(x/2)**4 + 75 \tanh(x/2)**2 + 15)}$

$$\frac{1}{2}^{10} + 75 \tanh(x/2)^8 + 150 \tanh(x/2)^6 + 150 \tanh(x/2)^4 + 75 \tanh(x/2)^2 + 15) + 30 \tanh(x/2) / (15 \tanh(x/2)^{10} + 75 \tanh(x/2)^8 + 150 \tanh(x/2)^6 + 150 \tanh(x/2)^4 + 75 \tanh(x/2)^2 + 15)$$

Giac [A] time = 1.27453, size = 32, normalized size = 1.68

$$-\frac{16(10e^{4x} + 5e^{2x} + 1)}{15(e^{2x} + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="giac")

[Out] -16/15*(10*e^(4*x) + 5*e^(2*x) + 1)/(e^(2*x) + 1)^5

3.63

$$\int \frac{1}{1-\sinh^2(x)} dx$$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{\sqrt{2}}$$

[Out] ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2]

Rubi [A] time = 0.0127272, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3181, 206}

$$\frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^2)^(-1), x]

[Out] ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2]

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1 - \sinh^2(x)} dx = \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \tanh(x) \right) \\ = \frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{\sqrt{2}}$$

Mathematica [F] time = 0.0215643, size = 0, normalized size = 0.

$$\int \frac{1}{1 - \sinh^2(x)} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - Sinh[x]^2)^(-1), x]

[Out] Integrate[(1 - Sinh[x]^2)^(-1), x]

Maple [B] time = 0.016, size = 40, normalized size = 2.7

$$\frac{\sqrt{2}}{2} \text{Artanh} \left(\frac{\sqrt{2}}{4} (2 \tanh(x/2) - 2) \right) + \frac{\sqrt{2}}{2} \text{Artanh} \left(\frac{\sqrt{2}}{4} (2 \tanh(x/2) + 2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-sinh(x)^2), x)

[Out] 1/2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+1/2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))

Maxima [B] time = 1.53046, size = 82, normalized size = 5.47

$$\frac{1}{4} \sqrt{2} \log \left(-\frac{\sqrt{2} - e^{(-x)} + 1}{\sqrt{2} + e^{(-x)} - 1} \right) - \frac{1}{4} \sqrt{2} \log \left(-\frac{\sqrt{2} - e^{(-x)} - 1}{\sqrt{2} + e^{(-x)} + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^2),x, algorithm="maxima")

[Out] 1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1))

Fricas [B] time = 1.84293, size = 215, normalized size = 14.33

$$\frac{1}{4} \sqrt{2} \log \left(-\frac{3(2\sqrt{2}-3) \cosh(x)^2 - 4(3\sqrt{2}-4) \cosh(x) \sinh(x) + 3(2\sqrt{2}-3) \sinh(x)^2 - 2\sqrt{2} + 3}{\cosh(x)^2 + \sinh(x)^2 - 3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3))

Sympy [B] time = 3.56741, size = 209, normalized size = 13.93

$$\frac{12\sqrt{2} \log\left(\tanh\left(\frac{x}{2}\right) - 1 + \sqrt{2}\right)}{48 + 34\sqrt{2}} + \frac{17 \log\left(\tanh\left(\frac{x}{2}\right) - 1 + \sqrt{2}\right)}{48 + 34\sqrt{2}} + \frac{12\sqrt{2} \log\left(\tanh\left(\frac{x}{2}\right) + 1 + \sqrt{2}\right)}{48 + 34\sqrt{2}} + \frac{17 \log\left(\tanh\left(\frac{x}{2}\right) + 1 + \sqrt{2}\right)}{48 + 34\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)**2),x)

[Out] 12*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(48 + 34*sqrt(2)) + 17*log(tanh(x/2) - 1 + sqrt(2))/(48 + 34*sqrt(2)) + 12*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(48 + 34*sqrt(2)) + 17*log(tanh(x/2) + 1 + sqrt(2))/(48 + 34*sqrt(2)) - 17*log(tanh(x/2) - sqrt(2) - 1)/(48 + 34*sqrt(2)) - 12*sqrt(2)*log(tanh(x/2) - sqrt(2) - 1)/(48 + 34*sqrt(2)) - 17*log(tanh(x/2) - sqrt(2) + 1)/(48 + 34*sqrt(2)) - 12*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)/(48 + 34*sqrt(2))

Giac [B] time = 1.26601, size = 50, normalized size = 3.33

$$-\frac{1}{4} \sqrt{2} \log \left(\frac{|-4 \sqrt{2} + 2 e^{(2x)} - 6|}{|4 \sqrt{2} + 2 e^{(2x)} - 6|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^2),x, algorithm="giac")

[Out] -1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x) - 6))

$$3.64 \quad \int \frac{1}{(1 - \sinh^2(x))^2} dx$$

Optimal. Leaf size=37

$$\frac{3 \tanh^{-1}(\sqrt{2} \tanh(x))}{4\sqrt{2}} + \frac{\sinh(x) \cosh(x)}{4(1 - \sinh^2(x))}$$

[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + (Cosh[x]*Sinh[x])/(4*(1 - Sinh[x]^2))

Rubi [A] time = 0.0268138, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {3184, 12, 3181, 206}

$$\frac{3 \tanh^{-1}(\sqrt{2} \tanh(x))}{4\sqrt{2}} + \frac{\sinh(x) \cosh(x)}{4(1 - \sinh^2(x))}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^2)^(-2), x]

[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + (Cosh[x]*Sinh[x])/(4*(1 - Sinh[x]^2))

Rule 3184

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a + b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181


```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(-1), x_Symbol] := With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(1 - \sinh^2(x))^2} dx &= \frac{\cosh(x) \sinh(x)}{4(1 - \sinh^2(x))} - \frac{1}{4} \int -\frac{3}{1 - \sinh^2(x)} dx \\ &= \frac{\cosh(x) \sinh(x)}{4(1 - \sinh^2(x))} + \frac{3}{4} \int \frac{1}{1 - \sinh^2(x)} dx \\ &= \frac{\cosh(x) \sinh(x)}{4(1 - \sinh^2(x))} + \frac{3}{4} \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \tanh(x) \right) \\ &= \frac{3 \tanh^{-1}(\sqrt{2} \tanh(x))}{4\sqrt{2}} + \frac{\cosh(x) \sinh(x)}{4(1 - \sinh^2(x))} \end{aligned}$$

Mathematica [A] time = 0.128136, size = 35, normalized size = 0.95

$$\frac{3 \tanh^{-1}(\sqrt{2} \tanh(x))}{4\sqrt{2}} - \frac{\sinh(2x)}{4(\cosh(2x) - 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(-2), x]
```

```
[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) - Sinh[2*x]/(4*(-3 + Cosh[2*x]))
```

Maple [B] time = 0.02, size = 92, normalized size = 2.5

$$-\left(-\frac{1}{4} \tanh\left(\frac{x}{2}\right) - \frac{1}{4}\right) \left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 - 2 \tanh\left(\frac{x}{2}\right) - 1\right)^{-1} + \frac{3\sqrt{2}}{8} \text{Artanh}\left(\frac{\sqrt{2}}{4} (2 \tanh(x/2) - 2)\right) - \left(-\frac{1}{4} \tanh\left(\frac{x}{2}\right) + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1-sinh(x)^2)^2,x)`

[Out] $-(1/4*\tanh(1/2*x)-1/4)/(\tanh(1/2*x)^2-2*\tanh(1/2*x)-1)+3/8*2^{(1/2)}*\operatorname{arctanh}(1/4*(2*\tanh(1/2*x)-2)*2^{(1/2)})-(-1/4*\tanh(1/2*x)+1/4)/(\tanh(1/2*x)^2+2*\tanh(1/2*x)-1)+3/8*2^{(1/2)}*\operatorname{arctanh}(1/4*(2*\tanh(1/2*x)+2)*2^{(1/2)})$

Maxima [B] time = 1.54868, size = 117, normalized size = 3.16

$$\frac{3}{16} \sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \frac{3}{16} \sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) - \frac{3e^{(-2x)}-1}{2(6e^{(-2x)}-e^{(-4x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^2)^2,x, algorithm="maxima")`

[Out] $3/16*\sqrt{2}*\log(-(\sqrt{2}-e^{(-x)}+1)/(\sqrt{2}+e^{(-x)}-1))-3/16*\sqrt{2}*\log(-(\sqrt{2}-e^{(-x)}-1)/(\sqrt{2}+e^{(-x)}+1))-1/2*(3*e^{(-2*x)}-1)/(6*e^{(-2*x)}-e^{(-4*x)}-1)$

Fricas [B] time = 1.8539, size = 729, normalized size = 19.7

$$\frac{24 \cosh(x)^2 - 3(\sqrt{2} \cosh(x)^4 + 4\sqrt{2} \cosh(x) \sinh(x)^3 + \sqrt{2} \sinh(x)^4 + 6(\sqrt{2} \cosh(x)^2 - \sqrt{2}) \sinh(x)^2 - 6\sqrt{2} \cosh(x) \sinh(x) + 24 \sinh(x)^2 - 8)}{16(\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 6(\cosh(x)^2 - 1) \sinh(x)^2 - 6 \cosh(x)^2 + 4(\cosh(x)^3 - 3 \cosh(x)))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^2)^2,x, algorithm="fricas")`

[Out] $-1/16*(24*\cosh(x)^2 - 3*(\sqrt{2}*\cosh(x)^4 + 4*\sqrt{2}*\cosh(x)*\sinh(x)^3 + \sqrt{2}*\sinh(x)^4 + 6*(\sqrt{2}*\cosh(x)^2 - \sqrt{2})*\sinh(x)^2 - 6*\sqrt{2}*\cosh(x)*\sinh(x) + 24*\sinh(x)^2 - 8)/(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 6*(\cosh(x)^2 - 1)*\sinh(x)^2 - 6*\cosh(x)^2 + 4*(\cosh(x)^3 - 3*\cosh(x)))$

*sinh(x) + 1)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\sinh(x) - 1)^2 (\sinh(x) + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)**2)**2,x)

[Out] Integral(1/((sinh(x) - 1)**2*(sinh(x) + 1)**2), x)

Giac [B] time = 1.2771, size = 84, normalized size = 2.27

$$-\frac{3}{16} \sqrt{2} \log \left(\frac{|-4\sqrt{2} + 2e^{(2x)} - 6|}{|4\sqrt{2} + 2e^{(2x)} - 6|} \right) - \frac{3e^{(2x)} - 1}{2(e^{(4x)} - 6e^{(2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="giac")

[Out] -3/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x) - 6)) - 1/2*(3*e^(2*x) - 1)/(e^(4*x) - 6*e^(2*x) + 1)

$$3.65 \quad \int \frac{1}{(1 - \sinh^2(x))^3} dx$$

Optimal. Leaf size=55

$$\frac{19 \tanh^{-1}(\sqrt{2} \tanh(x))}{32\sqrt{2}} + \frac{9 \sinh(x) \cosh(x)}{32(1 - \sinh^2(x))} + \frac{\sinh(x) \cosh(x)}{8(1 - \sinh^2(x))^2}$$

[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + (Cosh[x]*Sinh[x])/(8*(1 - Sinh[x]^2)^2) + (9*Cosh[x]*Sinh[x])/(32*(1 - Sinh[x]^2))

Rubi [A] time = 0.0581055, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3184, 3173, 12, 3181, 206}

$$\frac{19 \tanh^{-1}(\sqrt{2} \tanh(x))}{32\sqrt{2}} + \frac{9 \sinh(x) \cosh(x)}{32(1 - \sinh^2(x))} + \frac{\sinh(x) \cosh(x)}{8(1 - \sinh^2(x))^2}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^2)^(-3), x]

[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + (Cosh[x]*Sinh[x])/(8*(1 - Sinh[x]^2)^2) + (9*Cosh[x]*Sinh[x])/(32*(1 - Sinh[x]^2))

Rule 3184

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a + b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*(a + b)*(p + 1)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
```

eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(1 - \sinh^2(x))^3} dx &= \frac{\cosh(x) \sinh(x)}{8(1 - \sinh^2(x))^2} - \frac{1}{8} \int \frac{-7 - 2 \sinh^2(x)}{(1 - \sinh^2(x))^2} dx \\
 &= \frac{\cosh(x) \sinh(x)}{8(1 - \sinh^2(x))^2} + \frac{9 \cosh(x) \sinh(x)}{32(1 - \sinh^2(x))} - \frac{1}{32} \int \frac{19}{1 - \sinh^2(x)} dx \\
 &= \frac{\cosh(x) \sinh(x)}{8(1 - \sinh^2(x))^2} + \frac{9 \cosh(x) \sinh(x)}{32(1 - \sinh^2(x))} + \frac{19}{32} \int \frac{1}{1 - \sinh^2(x)} dx \\
 &= \frac{\cosh(x) \sinh(x)}{8(1 - \sinh^2(x))^2} + \frac{9 \cosh(x) \sinh(x)}{32(1 - \sinh^2(x))} + \frac{19}{32} \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \tanh(x) \right) \\
 &= \frac{19 \tanh^{-1}(\sqrt{2} \tanh(x))}{32\sqrt{2}} + \frac{\cosh(x) \sinh(x)}{8(1 - \sinh^2(x))^2} + \frac{9 \cosh(x) \sinh(x)}{32(1 - \sinh^2(x))}
 \end{aligned}$$

Mathematica [A] time = 0.177785, size = 51, normalized size = 0.93

$$\frac{19 \tanh^{-1}(\sqrt{2} \tanh(x))}{32\sqrt{2}} - \frac{9 \sinh(2x)}{32(\cosh(2x) - 3)} + \frac{\sinh(2x)}{4(\cosh(2x) - 3)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sinh[x]^2)^(-3), x]

[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + Sinh[2*x]/(4*(-3 + Cosh[2*x])^2) - (9*Sinh[2*x])/(32*(-3 + Cosh[2*x]))

Maple [B] time = 0.023, size = 124, normalized size = 2.3

$$-\frac{1}{4} \left(-\frac{13}{8} \left(\tanh\left(\frac{x}{2}\right) \right)^3 + \frac{11}{8} \left(\tanh\left(\frac{x}{2}\right) \right)^2 + \frac{31}{8} \tanh\left(\frac{x}{2}\right) + \frac{11}{8} \right) \left(\left(\tanh\left(\frac{x}{2}\right) \right)^2 - 2 \tanh\left(\frac{x}{2}\right) - 1 \right)^{-2} + \frac{19\sqrt{2}}{64} \operatorname{Arctanh}\left(\frac{\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-sinh(x)^2)^3,x)

[Out] -1/4*(-13/8*tanh(1/2*x)^3+11/8*tanh(1/2*x)^2+31/8*tanh(1/2*x)+11/8)/(tanh(1/2*x)^2-2*tanh(1/2*x)-1)^2+19/64*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))-1/4*(-13/8*tanh(1/2*x)^3-11/8*tanh(1/2*x)^2+31/8*tanh(1/2*x)-11/8)/(tanh(1/2*x)^2+2*tanh(1/2*x)-1)^2+19/64*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))

Maxima [B] time = 1.56568, size = 150, normalized size = 2.73

$$\frac{19}{128} \sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \frac{19}{128} \sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) - \frac{89e^{(-2x)}-171e^{(-4x)}+19e^{(-6x)}-9}{16(12e^{(-2x)}-38e^{(-4x)}+12e^{(-6x)}-e^{(-8x)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="maxima")

[Out] 19/128*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 19/128*sqrt(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 1/16*(89*e^(-2*x) - 171*e^(-4*x) + 19*e^(-6*x) - 9)/(12*e^(-2*x) - 38*e^(-4*x) + 12*e^(-6*x) - e^(-8*x) - 1)

Fricas [B] time = 2.06877, size = 1936, normalized size = 35.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="fricas")

[Out]
$$-1/128*(152*\cosh(x)^6 + 912*\cosh(x)*\sinh(x)^5 + 152*\sinh(x)^6 + 456*(5*\cosh(x)^2 - 3)*\sinh(x)^4 - 1368*\cosh(x)^4 + 608*(5*\cosh(x)^3 - 9*\cosh(x))*\sinh(x)^3 + 8*(285*\cosh(x)^4 - 1026*\cosh(x)^2 + 89)*\sinh(x)^2 + 712*\cosh(x)^2 - 19*(\sqrt{2}*\cosh(x)^8 + 8*\sqrt{2}*\cosh(x)*\sinh(x)^7 + \sqrt{2}*\sinh(x)^8 + 4*(7*\sqrt{2}*\cosh(x)^2 - 3*\sqrt{2}))*\sinh(x)^6 - 12*\sqrt{2}*\cosh(x)^6 + 8*(7*\sqrt{2}*\cosh(x)^3 - 9*\sqrt{2}*\cosh(x))*\sinh(x)^5 + 2*(35*\sqrt{2}*\cosh(x)^4 - 90*\sqrt{2}*\cosh(x)^2 + 19*\sqrt{2}))*\sinh(x)^4 + 38*\sqrt{2}*\cosh(x)^4 + 8*(7*\sqrt{2}*\cosh(x)^5 - 30*\sqrt{2}*\cosh(x)^3 + 19*\sqrt{2}*\cosh(x))*\sinh(x)^3 + 4*(7*\sqrt{2}*\cosh(x)^6 - 45*\sqrt{2}*\cosh(x)^4 + 57*\sqrt{2}*\cosh(x)^2 - 3*\sqrt{2})*\sinh(x)^2 - 12*\sqrt{2}*\cosh(x)^2 + 8*(\sqrt{2}*\cosh(x)^7 - 9*\sqrt{2}*\cosh(x)^5 + 19*\sqrt{2}*\cosh(x)^3 - 3*\sqrt{2}*\cosh(x))*\sinh(x) + \sqrt{2}*\log(-(3*(2*\sqrt{2} - 3)*\cosh(x)^2 - 4*(3*\sqrt{2} - 4)*\cosh(x)*\sinh(x) + 3*(2*\sqrt{2} - 3)*\sinh(x)^2 - 2*\sqrt{2} + 3)/(\cosh(x)^2 + \sinh(x)^2 - 3)) + 16*(57*\cosh(x)^5 - 342*\cosh(x)^3 + 89*\cosh(x))*\sinh(x) - 72)/(\cosh(x)^8 + 8*\cosh(x)*\sinh(x)^7 + \sinh(x)^8 + 4*(7*\cosh(x)^2 - 3)*\sinh(x)^6 - 12*\cosh(x)^6 + 8*(7*\cosh(x)^3 - 9*\cosh(x))*\sinh(x)^5 + 2*(35*\cosh(x)^4 - 90*\cosh(x)^2 + 19)*\sinh(x)^4 + 38*\cosh(x)^4 + 8*(7*\cosh(x)^5 - 30*\cosh(x)^3 + 19*\cosh(x))*\sinh(x)^3 + 4*(7*\cosh(x)^6 - 45*\cosh(x)^4 + 57*\cosh(x)^2 - 3)*\sinh(x)^2 - 12*\cosh(x)^2 + 8*(\cosh(x)^7 - 9*\cosh(x)^5 + 19*\cosh(x)^3 - 3*\cosh(x))*\sinh(x) + 1)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)**2)**3,x)

[Out] Timed out

Giac [A] time = 1.27688, size = 100, normalized size = 1.82

$$-\frac{19}{128}\sqrt{2}\log\left(\frac{|-4\sqrt{2} + 2e^{(2x)} - 6|}{|4\sqrt{2} + 2e^{(2x)} - 6|}\right) - \frac{19e^{(6x)} - 171e^{(4x)} + 89e^{(2x)} - 9}{16(e^{(4x)} - 6e^{(2x)} + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -19/128*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x) - 6)) - 1/16*(19*e^(6*x) - 171*e^(4*x) + 89*e^(2*x) - 9)/(e^(4*x) - 6*e^(2*x) + 1)^2
```


3.66 $\int \sinh^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=130

$$\frac{(a-b)(a+3b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{8b^{3/2}f} + \frac{\cosh(e+fx)(a+b \cosh^2(e+fx)-b)^{3/2}}{4bf} - \frac{(a+3b) \cosh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8bf}$$

[Out] $-\frac{((a-b)(a+3b) \operatorname{ArcTanh}[\frac{\sqrt{b} \operatorname{Cosh}[e+fx]}{\sqrt{a+b \operatorname{Cosh}[e+fx]^2}}])}{(8b^{3/2}f)} - \frac{((a+3b) \operatorname{Cosh}[e+fx] \sqrt{a+b \operatorname{Cosh}[e+fx]^2})}{(8b^*f)} + \frac{(\operatorname{Cosh}[e+fx] * (a-b+b \operatorname{Cosh}[e+fx]^2)^{3/2})}{(4b^*f)}$

Rubi [A] time = 0.148348, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3186, 388, 195, 217, 206}

$$\frac{(a-b)(a+3b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{8b^{3/2}f} + \frac{\cosh(e+fx)(a+b \cosh^2(e+fx)-b)^{3/2}}{4bf} - \frac{(a+3b) \cosh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8bf}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sinh}[e+fx]^3 \sqrt{a+b \operatorname{Sinh}[e+fx]^2}, x]$

[Out] $-\frac{((a-b)(a+3b) \operatorname{ArcTanh}[\frac{\sqrt{b} \operatorname{Cosh}[e+fx]}{\sqrt{a+b \operatorname{Cosh}[e+fx]^2}}])}{(8b^{3/2}f)} - \frac{((a+3b) \operatorname{Cosh}[e+fx] \sqrt{a+b \operatorname{Cosh}[e+fx]^2})}{(8b^*f)} + \frac{(\operatorname{Cosh}[e+fx] * (a-b+b \operatorname{Cosh}[e+fx]^2)^{3/2})}{(4b^*f)}$

Rule 3186

$\operatorname{Int}[\sin[(e_.) + (f_.) * (x_)]^{(m_.)} * ((a_.) + (b_.) * \sin[(e_.) + (f_.) * (x_)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e+fx], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1-ff^2*x^2)^{(m-1)/2} * (a+b-b*ff^2*x^2)^p, x], x, \operatorname{Cos}[e+fx]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 388

$\operatorname{Int}[((a_.) + (b_.) * (x_))^{(n_.)} * ((c_.) + (d_.) * (x_))^{(n_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x*(a+b*x^n)^{(p+1}) / (b*(n*(p+1)+1)), x] - \operatorname{Dist}[(a*d - b*c*(n*($

$p + 1) + 1)) / (b*(n*(p + 1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 195

$\text{Int}[(a_) + (b_.)*(x_)^(n_)^(p_), x_Symbol] := \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^(p - 1), x], x] /;$ FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] := \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \sinh^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx &= \frac{\text{Subst}\left(\int (1 - x^2) \sqrt{a - b + bx^2} dx, x, \cosh(e + fx)\right)}{f} \\ &= \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^{3/2}}{4bf} - \frac{(a + 3b) \text{Subst}\left(\int \sqrt{a - b + bx^2} dx\right)}{4bf} \\ &= -\frac{(a + 3b) \cosh(e + fx) \sqrt{a - b + b \cosh^2(e + fx)}}{8bf} + \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^{3/2}}{4bf} \\ &= -\frac{(a + 3b) \cosh(e + fx) \sqrt{a - b + b \cosh^2(e + fx)}}{8bf} + \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^{3/2}}{4bf} \\ &= -\frac{(a - b)(a + 3b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e + fx)}{\sqrt{a - b + b \cosh^2(e + fx)}}\right)}{8b^{3/2}f} - \frac{(a + 3b) \cosh(e + fx) \sqrt{a - b + b \cosh^2(e + fx)}}{8bf} \end{aligned}$$

Mathematica [A] time = 0.651416, size = 114, normalized size = 0.88

$$\frac{\frac{(b-a)(a+3b) \log\left(\sqrt{2a+b \cosh(2(e+fx))-b} + \sqrt{2}\sqrt{b} \cosh(e+fx)\right)}{b^{3/2}} + \frac{\cosh(e+fx)\sqrt{4a+2b \cosh(2(e+fx))-2b(a+b \cosh(2(e+fx))-4b)}}{2b}}{8f}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((Cosh[e + f*x]*(a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])/(2*b) + ((-a + b)*(a + 3*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])]/b^(3/2))/(8*f)

Maple [B] time = 0.114, size = 339, normalized size = 2.6

$$\frac{1}{16 f \cosh(fx + e)} \sqrt{\left(a + b (\sinh(fx + e))^2\right) (\cosh(fx + e))^2} \left(4 \sqrt{b (\cosh(fx + e))^4 + (a - b) (\cosh(fx + e))^2} b^{5/2} (\cosh(fx + e))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(5/2)*cosh(f*x+e)^2-10*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(5/2)+2*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(3/2)-ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*a^2*b-2*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*b^2+3*b^3*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \sinh(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^3, x)
```

Fricas [B] time = 3.89635, size = 7794, normalized size = 59.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(2*((a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*
cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sin
h(f*x + e)^2 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2
+ 2*a*b - 3*b^2)*sinh(f*x + e)^4)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a
^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b
)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e
)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x
+ e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x +
e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x
+ e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 +
(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2
- b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*co
sh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh
(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f
*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*
x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x
+ e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3
*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) +
4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e
))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*
sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*s
inh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*((
a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e
)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2
+ 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b - 3
*b^2)*sinh(f*x + e)^4)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*
```

$$\begin{aligned}
& \sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b)*\sinh(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)} + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2) - \sqrt{2}*(b^2*\cosh(f*x + e)^6 + 6*b^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + b^2*\sinh(f*x + e)^6 + (2*a*b - 7*b^2)*\cosh(f*x + e)^4 + (15*b^2*\cosh(f*x + e)^2 + 2*a*b - 7*b^2)*\sinh(f*x + e)^4 + 4*(5*b^2*\cosh(f*x + e)^3 + (2*a*b - 7*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 + (2*a*b - 7*b^2)*\cosh(f*x + e)^2 + (15*b^2*\cosh(f*x + e)^4 + 6*(2*a*b - 7*b^2)*\cosh(f*x + e)^2 + 2*a*b - 7*b^2)*\sinh(f*x + e)^2 + b^2 + 2*(3*b^2*\cosh(f*x + e)^5 + 2*(2*a*b - 7*b^2)*\cosh(f*x + e)^3 + (2*a*b - 7*b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b^2*f*\cosh(f*x + e)^4 + 4*b^2*f*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*b^2*f*\cosh(f*x + e)^2*\sinh(f*x + e)^2 + 4*b^2*f*\cosh(f*x + e)*\sinh(f*x + e)^3 + b^2*f*\sinh(f*x + e)^4), 1/64*(4*(a^2 + 2*a*b - 3*b^2)*\cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*\cosh(f*x + e)^2*\sinh(f*x + e)^2 + 4*(a^2 + 2*a*b - 3*b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a^2 + 2*a*b - 3*b^2)*\sinh(f*x + e)^4)*\sqrt{-b}*\arctan(\sqrt{2}*(a*\cosh(f*x + e)^2 + 2*a*\cosh(f*x + e)*\sinh(f*x + e) + a*\sinh(f*x + e)^2 + b))*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a*b*\cosh(f*x + e)^4 + 4*a*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*b*\sinh(f*x + e)^4 + (3*a*b - b^2)*\cosh(f*x + e)^2 + (6*a*b*\cosh(f*x + e)^2 + 3*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(2*a*b*\cosh(f*x + e)^3 + (3*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e))) + 4*((a^2 + 2*a*b - 3*b^2)*\cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*\cosh(f*x + e)^2*\sinh(f*x + e)^2 + 4*(a^2 + 2*a*b - 3*b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a^2 + 2*a*b - 3*b^2)*\sinh(f*x + e)^4)*\sqrt{-b}*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1))*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) + \sqrt{2}*(b^2*\cosh(f*x + e)^6 + 6*b^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + b^2*\sinh(f*x + e)^6 + (2*a*b - 7*b^2)*\cosh(f*x + e)^4 + (15*b^2*\cosh(f*x + e)^2 + 2*a*b - 7*b^2)*\sinh(f*x + e)^4 + 4*(5*b^2*\cosh(f*x + e)^3 + (2*a*b - 7*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 + (2*a*b - 7*b^2)*\cosh(f*x + e)^2 + (15*b^2*\cosh(f*x + e)^4 + 6*(2*a*b - 7*b^2)*\cosh(f*x + e)^2 + 2*a*b - 7*b^2)*\sinh(f*x + e)^2 + b^2 + 2*(3*b^2*\cosh(f*x + e)^5 + 2*(2*a*b - 7*b^2)*\cosh(f*x + e)^3 + (2*a*b - 7*b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b^2*f*\cosh(f*x + e)^4 + 4
\end{aligned}$$

```
*b^2*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + b^2*f*sinh(f*x + e)^4]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \sinh^3(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^3, x)
```

3.67 $\int \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=82

$$\frac{\cosh(e + fx) \sqrt{a + b \cosh^2(e + fx) - b}}{2f} + \frac{(a - b) \tanh^{-1} \left(\frac{\sqrt{b} \cosh(e + fx)}{\sqrt{a + b \cosh^2(e + fx) - b}} \right)}{2\sqrt{b}f}$$

[Out] ((a - b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*Sqrt[b]*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*f)

Rubi [A] time = 0.0702846, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3186, 195, 217, 206}

$$\frac{\cosh(e + fx) \sqrt{a + b \cosh^2(e + fx) - b}}{2f} + \frac{(a - b) \tanh^{-1} \left(\frac{\sqrt{b} \cosh(e + fx)}{\sqrt{a + b \cosh^2(e + fx) - b}} \right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((a - b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*Sqrt[b]*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*f)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 195

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],

Denominator[p]])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x \ \&\& \ !\text{GtQ}[a, 0]$

Rule 206

$\text{Int}[(a_) + (b_.)(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx &= \frac{\text{Subst}\left(\int \sqrt{a - b + bx^2} dx, x, \cosh(e + fx)\right)}{f} \\ &= \frac{\cosh(e + fx) \sqrt{a - b + b \cosh^2(e + fx)}}{2f} + \frac{(a - b) \text{Subst}\left(\int \frac{1}{\sqrt{a - b + bx^2}} dx, x, \cosh(e + fx)\right)}{2f} \\ &= \frac{\cosh(e + fx) \sqrt{a - b + b \cosh^2(e + fx)}}{2f} + \frac{(a - b) \text{Subst}\left(\int \frac{1}{1 - bx^2} dx, x, \frac{\cosh(e + fx)}{\sqrt{a - b + b \cosh^2(e + fx)}}\right)}{2f} \\ &= \frac{(a - b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e + fx)}{\sqrt{a - b + b \cosh^2(e + fx)}}\right)}{2\sqrt{b}f} + \frac{\cosh(e + fx) \sqrt{a - b + b \cosh^2(e + fx)}}{2f} \end{aligned}$$

Mathematica [A] time = 0.161826, size = 97, normalized size = 1.18

$$\frac{\cosh(e + fx) \sqrt{2a + b \cosh(2(e + fx)) - b}}{2\sqrt{2}f} + \frac{(a - b) \log\left(\sqrt{2a + b \cosh(2(e + fx)) - b} + \sqrt{2}\sqrt{b} \cosh(e + fx)\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*f) + ((a - b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

$/(2*\text{Sqrt}[b]*f)$

Maple [B] time = 0.066, size = 200, normalized size = 2.4

$$\frac{1}{4f \cosh(fx + e)} \sqrt{\left(a + b(\sinh(fx + e))^2\right) (\cosh(fx + e))^2 \left(2\sqrt{b(\cosh(fx + e))^4 + (a - b)(\cosh(fx + e))^2} \sqrt{b} + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2), x)`

[Out] $\frac{1}{4} * ((a + b * \sinh(f * x + e)^2) * \cosh(f * x + e)^2)^{(1/2)} * (2 * (b * \cosh(f * x + e)^4 + (a - b) * \cosh(f * x + e)^2)^{(1/2)} * b^{(1/2)} + a * \ln(1/2 * (2 * b * \cosh(f * x + e)^2 + 2 * (b * \cosh(f * x + e)^4 + (a - b) * \cosh(f * x + e)^2)^{(1/2)} * b^{(1/2)} + a - b) / b^{(1/2)}) - b * \ln(1/2 * (2 * b * \cosh(f * x + e)^2 + 2 * (b * \cosh(f * x + e)^4 + (a - b) * \cosh(f * x + e)^2)^{(1/2)} * b^{(1/2)} + a - b) / b^{(1/2)}) / b^{(1/2)} / \cosh(f * x + e) / (a + b * \sinh(f * x + e)^2)^{(1/2)} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2), x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e), x)`

Fricas [B] time = 3.46369, size = 5562, normalized size = 67.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2), x, algorithm="fricas")`

[Out]
$$\begin{aligned} & [-1/8*((a - b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (\\ & a - b)*\sinh(f*x + e)^2)*\sqrt{b}*\log((a^2*b*\cosh(f*x + e)^8 + 8*a^2*b*\cosh(f \\ & *x + e)*\sinh(f*x + e)^7 + a^2*b*\sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*\cosh(f*x \\ & + e)^6 + 2*(14*a^2*b*\cosh(f*x + e)^2 + a^3 + a^2*b)*\sinh(f*x + e)^6 + 4*(14 \\ & *a^2*b*\cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (\\ & 9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^4 + (70*a^2*b*\cosh(f*x + e)^4 + 9*a^ \\ & 2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4 \\ & *(14*a^2*b*\cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*\cosh(f*x + e)^3 + (9*a^2*b - \\ & 4*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*\cos \\ & h(f*x + e)^2 + 2*(14*a^2*b*\cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*\cosh(f*x + e) \\ & ^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x \\ & + e)^2 - \sqrt{2}*(a^2*\cosh(f*x + e)^6 + 6*a^2*\cosh(f*x + e)*\sinh(f*x + e)^5 \\ & + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x + e)^4 + 3*(5*a^2*\cosh(f*x + e)^2 + \\ & a^2)*\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e))*\sin \\ & h(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e)^2 + (15*a^2*\cosh(f*x + e)^4 + 18 \\ & *a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(3*a^2*\cosh(f \\ & *x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + \\ & e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f \\ & *x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(2*a^2*b* \\ & \cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*\cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^ \\ & 3)*\cosh(f*x + e)^3 + (3*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f \\ & x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + \\ & e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e \\ &)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + ((a - b)*\cosh(f \\ & *x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 \\ &)*\sqrt{b}*\log(-(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*s \\ & inh(f*x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b \\ &)*\sinh(f*x + e)^2 - \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e \\ &) + \sinh(f*x + e)^2 - 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^ \\ & 2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + \\ & e)^2))} + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/ \\ & (\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) - \sqrt{2} \\ & *(b*\cosh(f*x + e)^2 + 2*b*\cosh(f*x + e)*\sinh(f*x + e) + b*\sinh(f*x + e)^2 \\ & + b)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e) \\ & ^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*f*\cosh(f*x + e)^ \\ & 2 + 2*b*f*\cosh(f*x + e)*\sinh(f*x + e) + b*f*\sinh(f*x + e)^2), -1/8*(2*((a - \\ & b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh \\ & (f*x + e)^2)*\sqrt{-b}*\arctan(\sqrt{2}*(a*\cosh(f*x + e)^2 + 2*a*\cosh(f*x + e)* \\ & \sinh(f*x + e) + a*\sinh(f*x + e)^2 + b)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b \\ & *\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e \\ &) + \sinh(f*x + e)^2)))/(a*b*\cosh(f*x + e)^4 + 4*a*b*\cosh(f*x + e)*\sinh(f*x + \\ & e)^3 + a*b*\sinh(f*x + e)^4 + (3*a*b - b^2)*\cosh(f*x + e)^2 + (6*a*b*\cosh(f \\ & *x + e)^2 + 3*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(2*a*b*\cosh(f*x + e)^3 + \\ & (3*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e))) + 2*((a - b)*\cosh(f*x + e)^2 \\ & + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2)*\sqrt{-b} \end{aligned}$$

```
*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*c
osh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*
(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)
^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) -
sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x +
e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x
+ e)^2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e), x)

3.68 $\int \operatorname{csch}(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=84

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{f} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{f}$$

[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/f

Rubi [A] time = 0.105076, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3186, 402, 217, 206, 377}

$$\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{f} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/f

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 402

Int[((a_.) + (b_.)*(x_)^2)^(p_.)/((c_.) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&

GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{1-x^2} dx, x, \cosh(e+fx)\right)}{f} \\ &= -\frac{a \operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}} dx, x, \cosh(e+fx)\right)}{f} + \frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}} dx, x, \cosh(e+fx)\right)}{f} \\ &= -\frac{a \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{f} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{f} \\ &= -\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{f} \end{aligned}$$

Mathematica [A] time = 0.123564, size = 97, normalized size = 1.15

$$\frac{\sqrt{b} \log\left(\sqrt{2a+b \cosh(2(e+fx))} - b + \sqrt{2}\sqrt{b} \cosh(e+fx)\right) - \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a} \cosh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))} - b}\right)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] $(-\text{Sqrt}[a] \cdot \text{ArcTanh}[\frac{\text{Sqrt}[2] \cdot \text{Sqrt}[a] \cdot \text{Cosh}[e + f \cdot x]}{\text{Sqrt}[2 \cdot a - b + b \cdot \text{Cosh}[2 \cdot (e + f \cdot x)]}]) + \text{Sqrt}[b] \cdot \text{Log}[\text{Sqrt}[2] \cdot \text{Sqrt}[b] \cdot \text{Cosh}[e + f \cdot x] + \text{Sqrt}[2 \cdot a - b + b \cdot \text{Cosh}[2 \cdot (e + f \cdot x)]]]) / f$

Maple [B] time = 0.098, size = 174, normalized size = 2.1

$$\frac{1}{2f \cosh(fx + e)} \sqrt{(a + b(\sinh(fx + e))^2)(\cosh(fx + e))^2} \left(\sqrt{b} \ln \left(\frac{1}{2} \left(2b(\cosh(fx + e))^2 + 2\sqrt{b(\cosh(fx + e))^4 + a} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2), x)

[Out] $\frac{1}{2} \cdot ((a + b \cdot \sinh(f \cdot x + e)^2) \cdot \cosh(f \cdot x + e)^2)^{(1/2)} \cdot (b^{(1/2)} \cdot \ln(1/2 \cdot (2 \cdot b \cdot \cosh(f \cdot x + e)^2 + 2 \cdot (b \cdot \cosh(f \cdot x + e)^4 + (a - b) \cdot \cosh(f \cdot x + e)^2)^{(1/2)} \cdot b^{(1/2)} + a - b) / b^{(1/2)}) - a^{(1/2)} \cdot \ln((a + b) \cdot \cosh(f \cdot x + e)^2 + 2 \cdot a^{(1/2)} \cdot (b \cdot \cosh(f \cdot x + e)^4 + (a - b) \cdot \cosh(f \cdot x + e)^2)^{(1/2)} + a - b) / \sinh(f \cdot x + e)^2) / \cosh(f \cdot x + e) / (a + b \cdot \sinh(f \cdot x + e)^2)^{(1/2)} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \operatorname{csch}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e), x)

Fricas [B] time = 3.26454, size = 11636, normalized size = 138.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(\sqrt{b}*\log((a^2*b*\cosh(f*x + e)^8 + 8*a^2*b*\cosh(f*x + e)*\sinh(f*x + e)^7 + a^2*b*\sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*\cosh(f*x + e)^6 + 2*(14*a^2*b*\cosh(f*x + e)^2 + a^3 + a^2*b)*\sinh(f*x + e)^6 + 4*(14*a^2*b*\cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^4 + (70*a^2*b*\cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*a^2*b*\cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*\cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*\cosh(f*x + e)^2 + 2*(14*a^2*b*\cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*\cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + \sqrt{2}*(a^2*\cosh(f*x + e)^6 + 6*a^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x + e)^4 + 3*(5*a^2*\cosh(f*x + e)^2 + a^2)*\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e))*\sinh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e)^2 + (15*a^2*\cosh(f*x + e)^4 + 18*a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(3*a^2*\cosh(f*x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)} + 4*(2*a^2*b*\cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*\cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + 2*\sqrt{a}*\log(-((a + b)*\cosh(f*x + e)^4 + 4*(a + b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a + b)*\sinh(f*x + e)^4 + 2*(3*a - b)*\cosh(f*x + e)^2 + 2*(3*(a + b)*\cosh(f*x + e)^2 + 3*a - b)*\sinh(f*x + e)^2 - 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1))*\sqrt{a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)} + 4*((a + b)*\cosh(f*x + e)^3 + (3*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + a + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 - 1)*\sinh(f*x + e)^2 - 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 - \cosh(f*x + e))*\sinh(f*x + e) + 1)) + \sqrt{b}*\log(-(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b)*\sinh(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)} + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/f, 1/4*(4*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1))*s$$

$$\begin{aligned} & \text{qrt}(-a) * \text{sqrt}((b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) / (b * \cosh(f*x + e)^4 \\ & + 4 * b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2 * (2*a - b) * \cosh(f*x + e)^2 + 2 * (3 * b * \cosh(f*x + e)^2 + 2*a - b) * \sinh(f*x + e)^2 + 4 * (b * \cosh(f*x + e)^3 + (2*a - b) * \cosh(f*x + e) * \sinh(f*x + e) + b)) + \text{sqrt}(b) * \log((a \\ & ^2 * b * \cosh(f*x + e)^8 + 8 * a^2 * b * \cosh(f*x + e) * \sinh(f*x + e)^7 + a^2 * b * \sinh(f*x + e)^8 + 2 * (a^3 + a^2 * b) * \cosh(f*x + e)^6 + 2 * (14 * a^2 * b * \cosh(f*x + e)^2 + a^3 + a^2 * b) * \sinh(f*x + e)^6 + 4 * (14 * a^2 * b * \cosh(f*x + e)^3 + 3 * (a^3 + a^2 * b) * \cosh(f*x + e)) * \sinh(f*x + e)^5 + (9 * a^2 * b - 4 * a * b^2 + b^3) * \cosh(f*x + e)^4 + (70 * a^2 * b * \cosh(f*x + e)^4 + 9 * a^2 * b - 4 * a * b^2 + b^3 + 30 * (a^3 + a^2 * b) * \cosh(f*x + e)^2) * \sinh(f*x + e)^4 + 4 * (14 * a^2 * b * \cosh(f*x + e)^5 + 10 * (a^3 + a^2 * b) * \cosh(f*x + e)^3 + (9 * a^2 * b - 4 * a * b^2 + b^3) * \cosh(f*x + e)) * \sinh(f*x + e)^3 + b^3 + 2 * (3 * a * b^2 - b^3) * \cosh(f*x + e)^2 + 2 * (14 * a^2 * b * \cosh(f*x + e)^6 + 15 * (a^3 + a^2 * b) * \cosh(f*x + e)^4 + 3 * a * b^2 - b^3 + 3 * (9 * a^2 * b - 4 * a * b^2 + b^3) * \cosh(f*x + e)^2) * \sinh(f*x + e)^2 + \text{sqrt}(2) * (a^2 * \cosh(f*x + e)^6 + 6 * a^2 * \cosh(f*x + e) * \sinh(f*x + e)^5 + a^2 * \sinh(f*x + e)^6 + 3 * a^2 * \cosh(f*x + e)^4 + 3 * (5 * a^2 * \cosh(f*x + e)^2 + a^2) * \sinh(f*x + e)^4 + 4 * (5 * a^2 * \cosh(f*x + e)^3 + 3 * a^2 * \cosh(f*x + e)) * \sinh(f*x + e)^3 + (4 * a * b - b^2) * \cosh(f*x + e)^2 + (15 * a^2 * \cosh(f*x + e)^4 + 18 * a^2 * \cosh(f*x + e)^2 + 4 * a * b - b^2) * \sinh(f*x + e)^2 + b^2 + 2 * (3 * a^2 * \cosh(f*x + e)^5 + 6 * a^2 * \cosh(f*x + e)^3 + (4 * a * b - b^2) * \cosh(f*x + e)) * \sinh(f*x + e)) * \text{sqrt}(b) * \text{sqrt}((b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 4 * (2 * a^2 * b * \cosh(f*x + e)^7 + 3 * (a^3 + a^2 * b) * \cosh(f*x + e)^5 + (9 * a^2 * b - 4 * a * b^2 + b^3) * \cosh(f*x + e)^3 + (3 * a * b^2 - b^3) * \cosh(f*x + e)) * \sinh(f*x + e)) / (\cosh(f*x + e)^6 + 6 * \cosh(f*x + e)^5 * \sinh(f*x + e) + 15 * \cosh(f*x + e)^4 * \sinh(f*x + e)^2 + 20 * \cosh(f*x + e)^3 * \sinh(f*x + e)^3 + 15 * \cosh(f*x + e)^2 * \sinh(f*x + e)^4 + 6 * \cosh(f*x + e) * \sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + \text{sqrt}(b) * \log(-(b * \cosh(f*x + e)^4 + 4 * b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2 * (a - b) * \cosh(f*x + e)^2 + 2 * (3 * b * \cosh(f*x + e)^2 + a - b) * \sinh(f*x + e)^2 + \text{sqrt}(2) * (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2 - 1)) * \text{sqrt}(b) * \text{sqrt}((b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 4 * (b * \cosh(f*x + e)^3 + (a - b) * \cosh(f*x + e)) * \sinh(f*x + e) + b) / (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2))) / f, -1/2 * (\text{sqrt}(-b) * \arctan(\text{sqrt}(2) * (a * \cosh(f*x + e)^2 + 2 * a * \cosh(f*x + e) * \sinh(f*x + e) + a * \sinh(f*x + e)^2 + b)) * \text{sqrt}(-b) * \text{sqrt}((b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) / (a * b * \cosh(f*x + e)^4 + 4 * a * b * \cosh(f*x + e) * \sinh(f*x + e)^3 + a * b * \sinh(f*x + e)^4 + (3 * a * b - b^2) * \cosh(f*x + e)^2 + (6 * a * b * \cosh(f*x + e)^2 + 3 * a * b - b^2) * \sinh(f*x + e)^2 + b^2 + 2 * (2 * a * b * \cosh(f*x + e)^3 + (3 * a * b - b^2) * \cosh(f*x + e)) * \sinh(f*x + e))) + \text{sqrt}(-b) * \arctan(\text{sqrt}(2) * (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2 - 1)) * \text{sqrt}(-b) * \text{sqrt}((b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) / (b * \cosh(f*x + e)^4 + 4 * b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2 * (2*a - b$$


```

)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(
b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(a)*
log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (
a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*
x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*
x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
+ e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*co
sh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1))/f,
1/2*(2*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh
(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f
*x + e) + b)) - sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x +
e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
+ b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2)))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f
*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*co
sh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)
^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) - sqrt(-b)*arctan(sqrt(2)
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sq
rt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*cosh(f*x + e)^4
+ 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(
f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(
f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)))/f]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \operatorname{csch}(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x), x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")`

[Out] Exception raised: RuntimeError

3.69 $\int \operatorname{csch}^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=88

$$\frac{(a - b) \tanh^{-1} \left(\frac{\sqrt{a} \cosh(e + fx)}{\sqrt{a + b \cosh^2(e + fx) - b}} \right)}{2\sqrt{af}} - \frac{\coth(e + fx) \operatorname{csch}(e + fx) \sqrt{a + b \cosh^2(e + fx) - b}}{2f}$$

[Out] ((a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*Sqrt[a]*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x])/(2*f)

Rubi [A] time = 0.118046, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3186, 378, 377, 206}

$$\frac{(a - b) \tanh^{-1} \left(\frac{\sqrt{a} \cosh(e + fx)}{\sqrt{a + b \cosh^2(e + fx) - b}} \right)}{2\sqrt{af}} - \frac{\coth(e + fx) \operatorname{csch}(e + fx) \sqrt{a + b \cosh^2(e + fx) - b}}{2f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*Sqrt[a]*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x])/(2*f)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 378

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre

$eQ[\{a, b, c, d, n, p\}, x] \ \&\& \ NeQ[b*c - a*d, 0] \ \&\& \ EqQ[n*(p + q + 1) + 1, 0]$
 $\ \&\& \ GtQ[q, 0] \ \&\& \ NeQ[p, -1]$

Rule 377

$Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] \ :> \ Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] \ /; \ FreeQ[\{a, b, c, d\}, x] \ \&\& \ NeQ[b*c - a*d, 0] \ \&\& \ EqQ[n*p + 1, 0] \ \&\& \ IntegerQ[n]$

Rule 206

$Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] \ :> \ Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] \ /; \ FreeQ[\{a, b\}, x] \ \&\& \ NegQ[a/b] \ \&\& \ (GtQ[a, 0] \ || \ LtQ[b, 0])$

Rubi steps

$$\int \operatorname{csch}^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{(1-x^2)^2} dx, x, \cosh(e + fx)\right)}{f}$$

$$= -\frac{\sqrt{a-b+b \cosh^2(e + fx)} \coth(e + fx) \operatorname{csch}(e + fx)}{2f} + \frac{(a-b) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(e + fx)\right)}{2f}$$

$$= -\frac{\sqrt{a-b+b \cosh^2(e + fx)} \coth(e + fx) \operatorname{csch}(e + fx)}{2f} + \frac{(a-b) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(e + fx)\right)}{2f}$$

$$= \frac{(a-b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e + fx)}{\sqrt{a-b+b \cosh^2(e + fx)}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a-b+b \cosh^2(e + fx)} \coth(e + fx)}{2f}$$

Mathematica [A] time = 0.293015, size = 104, normalized size = 1.18

$$\frac{2(a-b) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a} \cosh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right) - \sqrt{2}\sqrt{a} \coth(e+fx) \operatorname{csch}(e+fx) \sqrt{2a+b \cosh(2(e+fx))-b}}{4\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2], x]

```
[Out] (2*(a - b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e + f*x]*Csch[e + f*x])/(4*Sqrt[a]*f)
```

Maple [B] time = 0.09, size = 230, normalized size = 2.6

$$\frac{1}{4 (\sinh (fx + e))^2 \cosh (fx + e) f} \sqrt{(a + b (\sinh (fx + e))^2) (\cosh (fx + e))^2} \left(a \ln \left(\frac{1}{(\sinh (fx + e))^2} \left((a + b) (\cosh (fx + e))^2 + \dots \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(a*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/a^(1/2)/sinh(f*x+e)^2/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh (fx + e)^2 + a} \operatorname{csch}(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^3, x)
```

Fricas [B] time = 2.27941, size = 3421, normalized size = 38.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 +
(a - b)*sinh(f*x + e)^4 - 2*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*
x + e)^2 - a + b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 - (a - b)*co
sh(f*x + e))*sinh(f*x + e) + a - b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 +
4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a
- b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x +
e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/
(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3
*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2
*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^4 + 4*a*f*cosh(f*x + e)*s
inh(f*x + e)^3 + a*f*sinh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 + 2*(3*a*f*cos
h(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 4*(a*f*cosh(f*x + e)^3 - a*f*cos
h(f*x + e))*sinh(f*x + e)), -1/2*(((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cos
h(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f*x + e)^4 - 2*(a - b)*cosh(f*x +
e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 - a + b)*sinh(f*x + e)^2 + 4*((a - b)*
cosh(f*x + e)^3 - (a - b)*cosh(f*x + e))*sinh(f*x + e) + a - b)*sqrt(-a)*ar
ctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*cosh
(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*
a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2
+ 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqr
t(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)
^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e
)^4 + 4*a*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*f*sinh(f*x + e)^4 - 2*a*f*cos
h(f*x + e)^2 + 2*(3*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 4*(a
*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*sinh(f*x + e)]]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [B] time = 1.37309, size = 938, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out]
$$-(a - b) \arctan\left(\frac{-1/2(\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b) - \sqrt{b}}{\sqrt{-a}}\right) / (\sqrt{-a}f) + 2\left(\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b\right)^3 a + (\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b)^3 b + 5(\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b)^2 a \sqrt{b} - 3(\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b)^2 b^{3/2} + 4(\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b) a^2 - 9(\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b) a b + 3(\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b) b^2 - 4a^2 \sqrt{b} + 3a b^{3/2} - b^{5/2}) / (((\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b)^2 - 2(\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e}} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b)) \sqrt{b} - 4a + b)^{2f}$$

3.70 $\int \operatorname{csch}^5(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=144

$$\frac{(a-b)(3a+b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{8a^{3/2}f} - \frac{\operatorname{coth}(e+fx) \operatorname{csch}^3(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{4af} + \frac{(3a+b) \operatorname{coth}(e+fx)}{4af}$$

[Out] $-\frac{(a-b)(3a+b) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cosh}[e+fx]}{\sqrt{a+b \operatorname{Cosh}[e+fx]^2}}\right]}{(8a^{3/2}f)} + \frac{(3a+b) \sqrt{a-b+b \operatorname{Cosh}[e+fx]^2} \operatorname{Coth}[e+fx] \operatorname{CsCh}[e+fx]}{(8af)} - \frac{(a-b+b \operatorname{Cosh}[e+fx]^2)^{3/2} \operatorname{Coth}[e+fx] \operatorname{CsCh}[e+fx]^3}{(4af)}$

Rubi [A] time = 0.150017, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3186, 382, 378, 377, 206}

$$\frac{(a-b)(3a+b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{8a^{3/2}f} - \frac{\operatorname{coth}(e+fx) \operatorname{csch}^3(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{4af} + \frac{(3a+b) \operatorname{coth}(e+fx)}{4af}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{CsCh}[e+fx]^5 \sqrt{a+b \operatorname{Sinh}[e+fx]^2}, x]$

[Out] $-\frac{(a-b)(3a+b) \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cosh}[e+fx]}{\sqrt{a+b \operatorname{Cosh}[e+fx]^2}}\right]}{(8a^{3/2}f)} + \frac{(3a+b) \sqrt{a-b+b \operatorname{Cosh}[e+fx]^2} \operatorname{Coth}[e+fx] \operatorname{CsCh}[e+fx]}{(8af)} - \frac{(a-b+b \operatorname{Cosh}[e+fx]^2)^{3/2} \operatorname{Coth}[e+fx] \operatorname{CsCh}[e+fx]^3}{(4af)}$

Rule 3186

$\operatorname{Int}[\sin[(e_.) + (f_.) (x_)]^{(m_.)} ((a_.) + (b_.) \sin[(e_.) + (f_.) (x_)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e+fx], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1-ff^2 x^2)^{(m-1)/2} (a+b-bff^2 x^2)^p, x], x, \operatorname{Cos}[e+fx]/ff], x] \;/; \operatorname{FreeQ}\{a, b, e, f, p\}, x \ \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 382

$\operatorname{Int}[(a_.) + (b_.) (x_)]^{(n_.)} ((c_.) + (d_.) (x_)]^{(q_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b x (a + b x^n)^{(p+1}) (c + d x^n)^{(q+1)}) / (a^n (p+1) (b c -$

$a*d)), x] + \text{Dist}[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p + 1)}*(c + d*x^n)^q, x], x] /;$
 $\text{FreeQ}\{a, b, c, d, n, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[n*(p + q + 2) + 1, 0] \ \&\& \ (\text{LtQ}[p, -1] \ || \ !\text{LtQ}[q, -1]) \ \&\& \ \text{NeQ}[p, -1]$

Rule 378

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \ :> \ -\text{Simp}[(x*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^q)/(a*n*(p + 1)), x] - \text{Dist}[(c*q)/(a*(p + 1)), \text{Int}[(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q - 1)}, x], x] /;$
 $\text{FreeQ}\{a, b, c, d, n, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[n*(p + q + 1) + 1, 0] \ \&\& \ \text{GtQ}[q, 0] \ \&\& \ \text{NeQ}[p, -1]$

Rule 377

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}]/((c_ + (d_)*(x_)^{(n_)}), x_Symbol] \ :> \ \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] /;$
 $\text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{EqQ}[n*p + 1, 0] \ \&\& \ \text{IntegerQ}[n]$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}], x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$
 $\text{FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^5(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{(1-x^2)^3} dx, x, \cosh(e+fx)\right)}{f} \\
&= -\frac{(a-b+b \cosh^2(e+fx))^{3/2} \coth(e+fx) \operatorname{csch}^3(e+fx)}{4af} - \frac{(3a+b) \operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{(1-x^2)^3} dx, x, \cosh(e+fx)\right)}{4af} \\
&= \frac{(3a+b) \sqrt{a-b+b \cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{8af} - \frac{(a-b+b \cosh^2(e+fx))^{3/2} \coth(e+fx) \operatorname{csch}^3(e+fx)}{8af} \\
&= \frac{(3a+b) \sqrt{a-b+b \cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{8af} - \frac{(a-b+b \cosh^2(e+fx))^{3/2} \coth(e+fx) \operatorname{csch}^3(e+fx)}{8af} \\
&= -\frac{(a-b)(3a+b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a+b) \sqrt{a-b+b \cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{8af}
\end{aligned}$$

Mathematica [A] time = 0.538088, size = 129, normalized size = 0.9

$$\frac{(-6a^2 + 4ab + 2b^2) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a} \cosh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right) - \sqrt{2}\sqrt{a} \coth(e+fx) \operatorname{csch}(e+fx) \sqrt{2a+b \cosh(2(e+fx))-b} (2a \operatorname{csch}^3(e+fx) \coth(e+fx) \sqrt{a-b+b \cosh^2(e+fx)})}{16a^{3/2}f}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((-6*a^2 + 4*a*b + 2*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e + f*x]*Csch[e + f*x]*(-3*a + b + 2*a*Csch[e + f*x]^2))/(16*a^(3/2)*f)

Maple [B] time = 0.099, size = 381, normalized size = 2.7

$$\frac{1}{16 (\sinh(fx+e))^4 \cosh(fx+e) f} \sqrt{(a+b(\sinh(fx+e))^2)(\cosh(fx+e))^2} \left(6 \sqrt{(a+b(\sinh(fx+e))^2)(\cosh(fx+e))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] $\frac{1}{16} \left((a+b \sinh(fx+e))^2 \cosh(fx+e)^2 \right)^{1/2} \left(6 \left((a+b \sinh(fx+e))^2 \cosh(fx+e)^2 \right)^{1/2} \sinh(fx+e)^2 a^{5/2} - 3 a^3 \ln \left((a+b \cosh(fx+e)^2 + 2 a^{1/2}) (b \cosh(fx+e)^4 + (a-b) \cosh(fx+e)^2)^{1/2} + a - b \right) / \sinh(fx+e)^2 \sinh(fx+e)^4 + 2 b \ln \left((a+b \cosh(fx+e)^2 + 2 a^{1/2}) (b \cosh(fx+e)^4 + (a-b) \cosh(fx+e)^2)^{1/2} + a - b \right) / \sinh(fx+e)^2 \sinh(fx+e)^4 a^2 + \ln \left((a+b \cosh(fx+e)^2 + 2 a^{1/2}) (b \cosh(fx+e)^4 + (a-b) \cosh(fx+e)^2)^{1/2} + a - b \right) / \sinh(fx+e)^2 \sinh(fx+e)^4 a - 2 b \left((a+b \sinh(fx+e))^2 \cosh(fx+e)^2 \right)^{1/2} \sinh(fx+e)^2 a^{3/2} - 4 \left((a+b \sinh(fx+e))^2 \cosh(fx+e)^2 \right)^{1/2} a^{5/2} \right) / \sinh(fx+e)^4 a^{5/2} / \cosh(fx+e) / (a+b \sinh(fx+e))^2)^{1/2} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx+e)^2 + a} \operatorname{csch}(fx+e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^5, x)`

Fricas [B] time = 4.09698, size = 8303, normalized size = 57.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] $[-1/16 \left((3a^2 - 2ab - b^2) \cosh(fx+e)^8 + 8(3a^2 - 2ab - b^2) \cosh(fx+e) \sinh(fx+e)^7 + (3a^2 - 2ab - b^2) \sinh(fx+e)^8 - 4(3a^2 - 2ab - b^2) \cosh(fx+e)^6 + 4(7(3a^2 - 2ab - b^2) \cosh(fx+e)^2 - 3a^2 + 2ab + b^2) \sinh(fx+e)^6 + 8(7(3a^2 - 2ab - b^2) \cosh(fx+e)^3 - 3(3a^2 - 2ab - b^2) \cosh(fx+e)) \sinh(fx+e)^5 + 6(3a^2 - 2ab - b^2) \cosh(fx+e)^4 + 2(35(3a^2 - 2ab - b^2) \cosh(fx+e)^3 - 3(3a^2 - 2ab - b^2) \cosh(fx+e)) \sinh(fx+e)^4 + 4(35(3a^2 - 2ab - b^2) \cosh(fx+e)^2 - 3(3a^2 - 2ab - b^2) \cosh(fx+e)) \sinh(fx+e)^4 + 8(35(3a^2 - 2ab - b^2) \cosh(fx+e) \sinh(fx+e)^3 - 3(3a^2 - 2ab - b^2) \sinh(fx+e)^3) \right) / \sinh(fx+e)^4 a^{5/2} / \cosh(fx+e) / (a+b \sinh(fx+e))^2)^{1/2} / f$

$$\begin{aligned}
& x + e)^4 - 30*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^2 + 9*a^2 - 6*a*b - 3*b^2 \\
&)*\sinh(f*x + e)^4 + 8*(7*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^5 - 10*(3*a^2 \\
& - 2*a*b - b^2)*\cosh(f*x + e)^3 + 3*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e))*\sin \\
& h(f*x + e)^3 - 4*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^2 + 4*(7*(3*a^2 - 2*a* \\
& b - b^2)*\cosh(f*x + e)^6 - 15*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^4 + 9*(3* \\
& a^2 - 2*a*b - b^2)*\cosh(f*x + e)^2 - 3*a^2 + 2*a*b + b^2)*\sinh(f*x + e)^2 + \\
& 3*a^2 - 2*a*b - b^2 + 8*((3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^7 - 3*(3*a^2 \\
& - 2*a*b - b^2)*\cosh(f*x + e)^5 + 3*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^3 - \\
& (3*a^2 - 2*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{a}*\log(-((a + b)*c \\
& osh(f*x + e)^4 + 4*(a + b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a + b)*\sinh(f*x \\
& + e)^4 + 2*(3*a - b)*\cosh(f*x + e)^2 + 2*(3*(a + b)*\cosh(f*x + e)^2 + 3*a \\
& - b)*\sinh(f*x + e)^2 + 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f* \\
& x + e) + \sinh(f*x + e)^2 + 1))*\sqrt{a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x \\
& + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f \\
& *x + e)^2)) + 4*((a + b)*\cosh(f*x + e)^3 + (3*a - b)*\cosh(f*x + e))*\sinh(f* \\
& x + e) + a + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f \\
& *x + e)^4 + 2*(3*\cosh(f*x + e)^2 - 1)*\sinh(f*x + e)^2 - 2*\cosh(f*x + e)^2 + \\
& 4*(\cosh(f*x + e)^3 - \cosh(f*x + e))*\sinh(f*x + e) + 1)) - 2*\sqrt{2}*((3*a^ \\
& 2 - a*b)*\cosh(f*x + e)^6 + 6*(3*a^2 - a*b)*\cosh(f*x + e)*\sinh(f*x + e)^5 + \\
& (3*a^2 - a*b)*\sinh(f*x + e)^6 - (11*a^2 - a*b)*\cosh(f*x + e)^4 + (15*(3*a^2 \\
& - a*b)*\cosh(f*x + e)^2 - 11*a^2 + a*b)*\sinh(f*x + e)^4 + 4*(5*(3*a^2 - a*b) \\
&)*\cosh(f*x + e)^3 - (11*a^2 - a*b)*\cosh(f*x + e))*\sinh(f*x + e)^3 - (11*a^2 \\
& - a*b)*\cosh(f*x + e)^2 + (15*(3*a^2 - a*b)*\cosh(f*x + e)^4 - 6*(11*a^2 - a \\
& *b)*\cosh(f*x + e)^2 - 11*a^2 + a*b)*\sinh(f*x + e)^2 + 3*a^2 - a*b + 2*(3*(3 \\
& *a^2 - a*b)*\cosh(f*x + e)^5 - 2*(11*a^2 - a*b)*\cosh(f*x + e)^3 - (11*a^2 - \\
& a*b)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e) \\
&)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x \\
& + e)^2)))/(a^2*f*\cosh(f*x + e)^8 + 8*a^2*f*\cosh(f*x + e)*\sinh(f*x + e)^7 + \\
& a^2*f*\sinh(f*x + e)^8 - 4*a^2*f*\cosh(f*x + e)^6 + 6*a^2*f*\cosh(f*x + e)^4 + \\
& 4*(7*a^2*f*\cosh(f*x + e)^2 - a^2*f)*\sinh(f*x + e)^6 + 8*(7*a^2*f*\cosh(f*x \\
& + e)^3 - 3*a^2*f*\cosh(f*x + e))*\sinh(f*x + e)^5 - 4*a^2*f*\cosh(f*x + e)^2 + \\
& 2*(35*a^2*f*\cosh(f*x + e)^4 - 30*a^2*f*\cosh(f*x + e)^2 + 3*a^2*f)*\sinh(f*x \\
& + e)^4 + 8*(7*a^2*f*\cosh(f*x + e)^5 - 10*a^2*f*\cosh(f*x + e)^3 + 3*a^2*f*c \\
& osh(f*x + e))*\sinh(f*x + e)^3 + a^2*f + 4*(7*a^2*f*\cosh(f*x + e)^6 - 15*a^2 \\
& *f*\cosh(f*x + e)^4 + 9*a^2*f*\cosh(f*x + e)^2 - a^2*f)*\sinh(f*x + e)^2 + 8*(\\
& a^2*f*\cosh(f*x + e)^7 - 3*a^2*f*\cosh(f*x + e)^5 + 3*a^2*f*\cosh(f*x + e)^3 - \\
& a^2*f*\cosh(f*x + e))*\sinh(f*x + e)), 1/8*((3*a^2 - 2*a*b - b^2)*\cosh(f*x \\
& + e)^8 + 8*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^7 + (3*a^2 - 2 \\
& *a*b - b^2)*\sinh(f*x + e)^8 - 4*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^6 + 4*(\\
& 7*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^2 - 3*a^2 + 2*a*b + b^2)*\sinh(f*x + e \\
&)^6 + 8*(7*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^3 - 3*(3*a^2 - 2*a*b - b^2)* \\
& cosh(f*x + e))*\sinh(f*x + e)^5 + 6*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^4 + \\
& 2*(35*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^4 - 30*(3*a^2 - 2*a*b - b^2)*\cosh \\
& (f*x + e)^2 + 9*a^2 - 6*a*b - 3*b^2)*\sinh(f*x + e)^4 + 8*(7*(3*a^2 - 2*a*b \\
& - b^2)*\cosh(f*x + e)^5 - 10*(3*a^2 - 2*a*b - b^2)*\cosh(f*x + e)^3 + 3*(3*a^
\end{aligned}$$

$$\begin{aligned}
& 2 - 2ab - b^2) \cosh(fx + e) \sinh(fx + e)^3 - 4(3a^2 - 2ab - b^2) \cosh(fx + e)^2 + 4(7(3a^2 - 2ab - b^2) \cosh(fx + e)^6 - 15(3a^2 - 2ab - b^2) \cosh(fx + e)^4 + 9(3a^2 - 2ab - b^2) \cosh(fx + e)^2 - 3a^2 + 2ab + b^2) \sinh(fx + e)^2 + 3a^2 - 2ab - b^2 + 8((3a^2 - 2ab - b^2) \cosh(fx + e)^7 - 3(3a^2 - 2ab - b^2) \cosh(fx + e)^5 + 3(3a^2 - 2ab - b^2) \cosh(fx + e)^3 - (3a^2 - 2ab - b^2) \cosh(fx + e)) \sinh(fx + e) \sqrt{-a} \arctan(\sqrt{2}(\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 + 1) \sqrt{-a} \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)}) / (b \cosh(fx + e)^4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 + 2(2a - b) \cosh(fx + e)^2 + 2(3b \cosh(fx + e)^2 + 2a - b) \sinh(fx + e)^2 + 4(b \cosh(fx + e)^3 + (2a - b) \cosh(fx + e)) \sinh(fx + e) + b) + \sqrt{2}((3a^2 - ab) \cosh(fx + e)^6 + 6(3a^2 - ab) \cosh(fx + e) \sinh(fx + e)^5 + (3a^2 - ab) \sinh(fx + e)^6 - (11a^2 - ab) \cosh(fx + e)^4 + (15(3a^2 - ab) \cosh(fx + e)^2 - 11a^2 + ab) \sinh(fx + e)^4 + 4(5(3a^2 - ab) \cosh(fx + e)^3 - (11a^2 - ab) \cosh(fx + e) \sinh(fx + e)^3 - (11a^2 - ab) \cosh(fx + e)^2 + (15(3a^2 - ab) \cosh(fx + e)^4 - 6(11a^2 - ab) \cosh(fx + e)^2 - 11a^2 + ab) \sinh(fx + e)^2 + 3a^2 - ab + 2(3(3a^2 - ab) \cosh(fx + e)^5 - 2(11a^2 - ab) \cosh(fx + e)^3 - (11a^2 - ab) \cosh(fx + e)) \sinh(fx + e)) \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)}) / (a^2 f \cosh(fx + e)^8 + 8a^2 f \cosh(fx + e) \sinh(fx + e)^7 + a^2 f \sinh(fx + e)^8 - 4a^2 f \cosh(fx + e)^6 + 6a^2 f \cosh(fx + e)^4 + 4(7a^2 f \cosh(fx + e)^2 - a^2 f) \sinh(fx + e)^6 + 8(7a^2 f \cosh(fx + e)^3 - 3a^2 f \cosh(fx + e)) \sinh(fx + e)^5 - 4a^2 f \cosh(fx + e)^2 + 2(35a^2 f \cosh(fx + e)^4 - 30a^2 f \cosh(fx + e)^2 + 3a^2 f) \sinh(fx + e)^4 + 8(7a^2 f \cosh(fx + e)^5 - 10a^2 f \cosh(fx + e)^3 + 3a^2 f \cosh(fx + e)) \sinh(fx + e)^3 + a^2 f + 4(7a^2 f \cosh(fx + e)^6 - 15a^2 f \cosh(fx + e)^4 + 9a^2 f \cosh(fx + e)^2 - a^2 f) \sinh(fx + e)^2 + 8(a^2 f \cosh(fx + e)^7 - 3a^2 f \cosh(fx + e)^5 + 3a^2 f \cosh(fx + e)^3 - a^2 f \cosh(fx + e)) \sinh(fx + e)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [B] time = 1.57355, size = 3086, normalized size = 21.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*a^2 - 2*a*b - b^2)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a)/(sqrt(-a)*a*f) - 1/2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2 - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a*b - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*b^2 - 21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a^2*sqrt(b) - 18*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(3/2) + 7*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*b^(5/2) - 44*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3 - 121*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^2*b + 122*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2 - 21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*b^3 - 292*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^3*sqrt(b) + 559*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2) - 270*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2) + 35*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*b^(7/2) - 176*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^4 + 872*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^3*b - 823*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*b^2 + 290*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*b^3 - 35*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^4 + 528*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^4*sqrt(b) - 936*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^3*b^(3/2) + 577*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b
```

$$\begin{aligned}
& *e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}})^{2*a^2*b^{(5/2)}} - 158*(\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})}^{2*a*b^{(7/2)}} + 21*(\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})}^{2*b^{(9/2)}} + 192*(\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})}^{a^5} - 656*(\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})}^{a^4*b} + 580*(\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})}^{a^3*b^2} - 211*(\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})}^{a^2*b^3} + 38*(\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})}^{a*b^4} - 7*(\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})}^{b^5} - 192*a^5*\sqrt{b} + 304*a^4*b^{(3/2)} - 180*a^3*b^{(5/2)} + 37*a^2*b^{(7/2)} - 2*a*b^{(9/2)} + b^{(11/2)})/(((\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})}^2 - 2*(\sqrt{b}*e^{(2*f*x + 2*e) - \sqrt{b*e^{(4*f*x + 4*e) + 4*a*e^{(2*f*x + 2*e) - 2*b*e^{(2*f*x + 2*e) + b}}})})*\sqrt{b} - 4*a + b)^{4*a*f}
\end{aligned}$$

3.71 $\int \sinh^4(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=300

$$\frac{(a - 4b) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) - (2a^2 + 3ab - 8b^2) \tanh(e + fx) \sqrt{a}}{15bf \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} - \frac{(2a^2 + 3ab - 8b^2) \tanh(e + fx) \sqrt{a}}{15b^2 f}$$

```
[Out] ((a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f)
+ (Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) + ((
*a^2 + 3*a*b - 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*
x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) - ((a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a +
b*Sinh[e + f*x]^2))/a]) - ((2*a^2 + 3*a*b - 8*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2]*Tanh[e + f*x])/(15*b^2*f)
```

Rubi [A] time = 0.327036, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3188, 478, 582, 531, 418, 492, 411}

$$\frac{(2a^2 + 3ab - 8b^2) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15b^2 f} + \frac{(2a^2 + 3ab - 8b^2) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} E\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{15b^2 f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2], x]
```

```
[Out] ((a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f)
+ (Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) + ((
*a^2 + 3*a*b - 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*
x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) - ((a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a +
b*Sinh[e + f*x]^2))/a]) - ((2*a^2 + 3*a*b - 8*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2]*Tanh[e + f*x])/(15*b^2*f)
```

Rule 3188


```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x] /; FreeQ[{a, b, c,
d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1))*x^n, x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

$a*d, 0] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c] \ \&\& \ !\text{SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^{(3/2)}, x_Symbol] \ :> \ \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned} \int \sinh^4(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{x^4 \sqrt{a + bx^2}}{\sqrt{1 + x^2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\cosh(e + fx) \sinh^3(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{5f} - \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right)}{5f} \\ &= \frac{(a - 4b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} + \frac{\cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} \\ &= \frac{(a - 4b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} + \frac{\cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} \\ &= \frac{(a - 4b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} + \frac{\cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} \\ &= \frac{(a - 4b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} + \frac{\cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} \end{aligned}$$

Mathematica [C] time = 1.3688, size = 210, normalized size = 0.7

$$\frac{-32ia(a^2 + ab - 2b^2) \sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} \operatorname{EllipticF}\left(i(e + fx), \frac{b}{a}\right) + \sqrt{2b} \sinh(2(e + fx)) (8a^2 + 4b(4a - 7b) \cosh(2(e + fx)))}{240b^2 f \sqrt{2a + b \cosh(2(e + fx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] $((16*I)*a*(2*a^2 + 3*a*b - 8*b^2)*\text{Sqrt}[(2*a - b + b*\text{Cosh}[2*(e + f*x)])]/a)*\text{EllipticE}[I*(e + f*x), b/a] - (32*I)*a*(a^2 + a*b - 2*b^2)*\text{Sqrt}[(2*a - b + b*\text{Cosh}[2*(e + f*x)])]/a*\text{EllipticF}[I*(e + f*x), b/a] + \text{Sqrt}[2]*b*(8*a^2 - 48*a*b + 25*b^2 + 4*(4*a - 7*b)*b*\text{Cosh}[2*(e + f*x)] + 3*b^2*\text{Cosh}[4*(e + f*x)])*\text{Sinh}[2*(e + f*x)]/(240*b^2*f*\text{Sqrt}[2*a - b + b*\text{Cosh}[2*(e + f*x)])]$

Maple [A] time = 0.086, size = 512, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] $1/15*(3*(-1/a*b)^{(1/2)}*b^2*\sinh(f*x+e)^7+4*(-1/a*b)^{(1/2)}*a*b*\sinh(f*x+e)^5 - (-1/a*b)^{(1/2)}*b^2*\sinh(f*x+e)^5+(-1/a*b)^{(1/2)}*a^2*\sinh(f*x+e)^3-4*(-1/a*b)^{(1/2)}*b^2*\sinh(f*x+e)^3+a^2*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})+7*a*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*b-8*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*b^2-2*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*a^2-3*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*a*b+8*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*b^2+(-1/a*b)^{(1/2)}*a^2*\sinh(f*x+e)-4*(-1/a*b)^{(1/2)}*a*b*\sinh(f*x+e)/b/(-1/a*b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a \sinh(fx + e)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh^2(fx + e) + a} \sinh^4(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \sinh^4(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)

3.72 $\int \sinh^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=177

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}} + 1\text{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{3bf\sqrt{a + b\sinh^2(e + fx)}} + \frac{\sinh(e + fx) \cosh(e + fx) \sqrt{a + b\sinh^2(e + fx)}}{3f} - \frac{i(a-2b)\sqrt{a + b\sinh^2(e + fx)}}{3bf\sqrt{a + b\sinh^2(e + fx)}}$$

[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((I/3)*(a - 2*b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.213308, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$\frac{\sinh(e + fx) \cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}} + 1F\left(ie + ifx \middle| \frac{b}{a}\right)}{3bf\sqrt{a + b\sinh^2(e + fx)}} - \frac{i(a-2b)\sqrt{a + b\sinh^2(e + fx)}}{3bf\sqrt{\frac{b\sinh^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((I/3)*(a - 2*b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3170

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^2]^(p_))*((A_) + (B_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] := -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sinh[e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p + 2*b*p))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[p, 0]

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]^2)/Sqrt[(a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_.)]^2], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2], x_Symbol] := Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2], x_Symbol] := Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2], x_Symbol] := Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2], x_Symbol] := Simp[(1*El
lipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= \frac{\cosh(e+fx) \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f} - \frac{1}{3} \int \frac{a-(a-2b) \sinh^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx \\
&= \frac{\cosh(e+fx) \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f} + \frac{(a-2b) \int \sqrt{a+b \sinh^2(e+fx)} dx}{3b} \\
&= \frac{\cosh(e+fx) \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f} + \frac{\left((a-2b) \sqrt{a+b \sinh^2(e+fx)} \right)}{3b \sqrt{1+\frac{b \sinh^2(e+fx)}{a}}} \\
&= \frac{\cosh(e+fx) \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f} - \frac{i(a-2b)E\left(ie+ifx \left| \frac{b}{a} \right. \right)}{3bf \sqrt{1+\frac{b \sinh^2(e+fx)}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.84108, size = 170, normalized size = 0.96

$$\frac{2i\sqrt{2a(a-b)}\sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \text{EllipticF}\left(ie+fx, \frac{b}{a}\right) + b \sinh(2(e+fx))(2a+b \cosh(2(e+fx))-b) - 2i\sqrt{2a(a-2b)} \sqrt{a+b \sinh^2(e+fx)}}{6bf\sqrt{4a+2b \cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-2*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.065, size = 343, normalized size = 1.9

$$-\frac{1}{3f \cosh(fx+e)} \left(-\sqrt{\frac{b}{a}} b \sinh(fx+e) (\cosh(fx+e))^4 + \left(-\sqrt{\frac{b}{a}} a + \sqrt{\frac{b}{a}} b \right) (\cosh(fx+e))^2 \sinh(fx+e) + 2a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out]
$$\begin{aligned} & -1/3*(-(-1/a*b)^{(1/2)}*b*\sinh(f*x+e)*\cosh(f*x+e)^4+(-(-1/a*b)^{(1/2)}*a+(-1/a*b)^{(1/2)}*b)*\cosh(f*x+e)^2*\sinh(f*x+e)+2*a*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)} \\ & *(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-2* \\ & (b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)}) \\ & *b-(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)}) \\ & *a+2*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)}) \\ & *b)/(-1/a*b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a \sinh^2(fx + e)}^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh^2(fx + e) + a \sinh^2(fx + e)}^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \sinh^2(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)`

3.73 $\int \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=60

$$\frac{i\sqrt{a + b \sinh^2(e + fx)} E\left(ie + ifx \middle| \frac{b}{a}\right)}{f\sqrt{\frac{b \sinh^2(e + fx)}{a} + 1}}$$

[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rubi [A] time = 0.0363792, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3178, 3177}

$$\frac{i\sqrt{a + b \sinh^2(e + fx)} E\left(ie + ifx \middle| \frac{b}{a}\right)}{f\sqrt{\frac{b \sinh^2(e + fx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Dist[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a + b \sinh^2(e + fx)} dx = \frac{\sqrt{a + b \sinh^2(e + fx)} \int \sqrt{1 + \frac{b \sinh^2(e+fx)}{a}} dx}{\sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}}$$

$$= -\frac{iE\left(ie + ifx \left| \frac{b}{a} \right. \right) \sqrt{a + b \sinh^2(e + fx)}}{f \sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.0960923, size = 69, normalized size = 1.15

$$\frac{ia \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} E\left(i(e+fx) \left| \frac{b}{a} \right. \right)}{f \sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a]/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.065, size = 140, normalized size = 2.3

$$\frac{1}{f \cosh(fx + e)} \sqrt{\frac{a + b (\sinh(fx + e))^2}{a}} \sqrt{(\cosh(fx + e))^2} \left(a \text{EllipticF}\left(\sinh(fx + e) \sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) - b \text{EllipticF}\left(\sinh(fx + e) \sqrt{\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2)))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh (fx + e)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh (fx + e)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2 (e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh (fx + e)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.74 $\int \operatorname{csch}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=199

$$\frac{b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{af \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{\tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{\operatorname{coth}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f}$$

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rubi [A] time = 0.181818, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3188, 475, 21, 422, 418, 492, 411}

$$\frac{\tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{\operatorname{coth}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} F\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{af \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2], x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^(p/2)]
```

p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 475

Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1) + b*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplifierQ[c + d*x, a + b*x])

Rule 422

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a]

Rule 418

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[

$d/c, 2] * \text{Sqrt}[c + d*x^2] * \text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned} \int \text{csch}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \text{sech}(e + fx)}\right) \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1+x^2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= -\frac{\coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{\left(\sqrt{\cosh^2(e + fx) \text{sech}(e + fx)}\right) \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1+x^2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= -\frac{\coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{\left(b\sqrt{\cosh^2(e + fx) \text{sech}(e + fx)}\right) \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1+x^2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= -\frac{\coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{\left(b\sqrt{\cosh^2(e + fx) \text{sech}(e + fx)}\right) \text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1+x^2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= -\frac{\coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{bF\left(\tan^{-1}(\sinh(e + fx)) \mid 1 - \frac{b}{a}\right) \text{sech}(e + fx)}{af\sqrt{\frac{\text{sech}^2(e+fx)(a+b)}{a}}} \\ &= -\frac{\coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{E\left(\tan^{-1}(\sinh(e + fx)) \mid 1 - \frac{b}{a}\right) \text{sech}(e + fx)}{f\sqrt{\frac{\text{sech}^2(e+fx)(a+b)}{a}}} \end{aligned}$$

Mathematica [C] time = 0.593719, size = 151, normalized size = 0.76

$$\frac{2i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}} \text{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2} \coth(e+fx)(-2a-b\cosh(2(e+fx))+b) - 2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}{2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(2*f*S


```
qrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.072, size = 160, normalized size = 0.8

$$\frac{1}{\cosh(fx + e) \sinh(fx + e) f} \left(b \sqrt{\frac{b (\cosh(fx + e))^2}{a} + \frac{a - b}{a} \sqrt{(\cosh(fx + e))^2 \sinh(fx + e)}} \operatorname{EllipticE} \left(\sinh(fx + e) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] (b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+(-(-1/a*b)^(1/2)*a+(-1/a*b)^(1/2)*b)*cosh(f*x+e)^2/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \operatorname{csch}(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\sqrt{b \sinh(fx + e)^2 + a} \operatorname{csch}(fx + e)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \operatorname{csch}(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)`

3.75 $\int \operatorname{csch}^4(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=276

$$\frac{b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{3af \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} - \frac{(2a - b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af}$$

```
[Out] ((2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e + f
*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + ((2*a - b)*Ellipti
cE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/ (3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF
[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/ (3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a - b)*Sqr
t[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rubi [A] time = 0.281847, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3188, 475, 583, 531, 418, 492, 411}

$$\frac{(2a - b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af} + \frac{(2a - b) \operatorname{coth}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af} - \frac{\operatorname{coth}(e + fx) \operatorname{csch}^2(e + fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2], x]
```

```
[Out] ((2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e + f
*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + ((2*a - b)*Ellipti
cE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/ (3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF
[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/ (3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a - b)*Sqr
t[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
```

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 475

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1) + b*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4 \sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{f} \\ &= -\frac{\operatorname{coth}(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right)}{3f} \\ &= \frac{(2a-b) \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3af} - \frac{\operatorname{coth}(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f} \\ &= \frac{(2a-b) \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3af} - \frac{\operatorname{coth}(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f} \\ &= \frac{(2a-b) \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3af} - \frac{\operatorname{coth}(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f} \\ &= \frac{(2a-b) \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3af} - \frac{\operatorname{coth}(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f} \end{aligned}$$

Mathematica [C] time = 3.02826, size = 208, normalized size = 0.75

$$\frac{-4ia(a-b) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \frac{\operatorname{coth}(e+fx) \operatorname{csch}^2(e+fx) (4(2a^2-4ab+b^2) \cosh(2(e+fx)) - (2a-b)(8a-b \cosh(4(e+fx))))}{2\sqrt{2}}}{6af \sqrt{2a+b \cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2], x]

```
[Out] (((4*(2*a^2 - 4*a*b + b^2)*Cosh[2*(e + f*x)] - (2*a - b)*(8*a - 3*b - b*Cos
h[4*(e + f*x)])))*Coth[e + f*x]*Csch[e + f*x]^2)/(2*sqrt[2]) + (2*I)*a*(2*a
- b)*sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a*EllipticE[I*(e + f*x), b/a] -
(4*I)*a*(a - b)*sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a*EllipticF[I*(e + f*
x), b/a]/(6*a*f*sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.121, size = 436, normalized size = 1.6

$$\frac{1}{3a(\sinh(fx+e))^3 \cosh(fx+e)f} \left(2\sqrt{\frac{b}{a}}ab(\sinh(fx+e))^6 - \sqrt{\frac{b}{a}}b^2(\sinh(fx+e))^6 + b\sqrt{\frac{a+b(\sinh(fx+e))^2}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+b*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2), (a/b)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2*s
inh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b*sinh(f*x+e)^3+((a+b*sinh(f*x+
e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (
a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-(-1/a*b)^(
1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2-2*(-1/a*b)^(1/2)*a*
b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2)/a/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x
+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx+e) + a} \operatorname{csch}(fx+e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh(fx + e)^2 + a} \operatorname{csch}(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \operatorname{csch}(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)

3.76 $\int \sinh^3(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=177

$$\frac{(a-b)^2(a+5b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{16b^{3/2}f} + \frac{\cosh(e+fx) (a+b \cosh^2(e+fx)-b)^{5/2}}{6bf} - \frac{(a+5b) \cosh(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{24bf}$$

[Out] -((a - b)^2*(a + 5*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(16*b^(3/2)*f) - ((a - b)*(a + 5*b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(16*b*f) - ((a + 5*b)*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(24*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(5/2))/(6*b*f)

Rubi [A] time = 0.181481, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3186, 388, 195, 217, 206}

$$\frac{(a-b)^2(a+5b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{16b^{3/2}f} + \frac{\cosh(e+fx) (a+b \cosh^2(e+fx)-b)^{5/2}}{6bf} - \frac{(a+5b) \cosh(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{24bf}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] -((a - b)^2*(a + 5*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(16*b^(3/2)*f) - ((a - b)*(a + 5*b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(16*b*f) - ((a + 5*b)*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(24*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(5/2))/(6*b*f)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 388


```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p
+ 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sinh^3(e+fx) (a+b\sinh^2(e+fx))^{3/2} dx &= -\frac{\text{Subst}\left(\int (1-x^2)(a-b+bx^2)^{3/2} dx, x, \cosh(e+fx)\right)}{f} \\
&= \frac{\cosh(e+fx)(a-b+b\cosh^2(e+fx))^{5/2}}{6bf} - \frac{(a+5b)\text{Subst}\left(\int (a-b+bx^2)^{3/2} dx, x, \cosh(e+fx)\right)}{6bf} \\
&= -\frac{(a+5b)\cosh(e+fx)(a-b+b\cosh^2(e+fx))^{3/2}}{24bf} + \frac{\cosh(e+fx)(a-b+b\cosh^2(e+fx))^{5/2}}{24bf} \\
&= -\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf} - \frac{(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf} \\
&= -\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf} - \frac{(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf} \\
&= -\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf}
\end{aligned}$$

Mathematica [A] time = 0.582482, size = 151, normalized size = 0.85

$$\frac{\sqrt{2}\sqrt{b}\sqrt{2a+b\cosh(2(e+fx))-b}\left((6a^2-51ab+37b^2)\cosh(e+fx)+b((7a-8b)\cosh(3(e+fx))+b\cosh(5(e+fx)))\right)}{192b^{3/2}f}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (Sqrt[2]*Sqrt[b]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*((6*a^2 - 51*a*b + 37*b^2)*Cosh[e + f*x] + b*((7*a - 8*b)*Cosh[3*(e + f*x)] + b*Cosh[5*(e + f*x)])) - 12*(a - b)^2*(a + 5*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/(192*b^(3/2)*f)

Maple [B] time = 0.091, size = 483, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out]
$$\frac{1}{96} \left((a+b \sinh(fx+e))^2 \cosh(fx+e)^2 \right)^{1/2} (16b^{7/2} (b \cosh(fx+e))^4 + (a-b) \cosh(fx+e)^2)^{1/2} \cosh(fx+e)^4 + 4b^{5/2} (b \cosh(fx+e))^4 + (a-b) \cosh(fx+e)^2)^{1/2} (-13b+7a) \cosh(fx+e)^2 + 66b^{7/2} (b \cosh(fx+e))^4 + (a-b) \cosh(fx+e)^2)^{1/2} - 72a (b \cosh(fx+e))^4 + (a-b) \cosh(fx+e)^2)^{1/2} b^{5/2} + 6a^2 (b \cosh(fx+e))^4 + (a-b) \cosh(fx+e)^2)^{1/2} b^{3/2} - 3 \ln \left(\frac{1}{2} (2b \cosh(fx+e))^2 + 2(b \cosh(fx+e))^4 + (a-b) \cosh(fx+e)^2)^{1/2} b^{1/2} + a - b \right) / b^{1/2} \left(a^3 b - 9 \ln \left(\frac{1}{2} (2b \cosh(fx+e))^2 + 2(b \cosh(fx+e))^4 + (a-b) \cosh(fx+e)^2)^{1/2} b^{1/2} + a - b \right) / b^{1/2} \right) a^2 b^2 + 27b^3 a \ln \left(\frac{1}{2} (2b \cosh(fx+e))^2 + 2(b \cosh(fx+e))^4 + (a-b) \cosh(fx+e)^2)^{1/2} b^{1/2} + a - b \right) / b^{1/2} \right) - 15b^4 \ln \left(\frac{1}{2} (2b \cosh(fx+e))^2 + 2(b \cosh(fx+e))^4 + (a-b) \cosh(fx+e)^2)^{1/2} b^{1/2} + a - b \right) / b^{1/2} \right) / b^{5/2} / \cosh(fx+e) / (a+b \sinh(fx+e))^2)^{1/2} / f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx+e)^2 + a \right)^{3/2} \sinh(fx+e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^3, x)`

Fricas [B] time = 3.80683, size = 11580, normalized size = 65.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out]
$$\frac{1}{384} (6((a^3 + 3a^2b - 9ab^2 + 5b^3) \cosh(fx+e)^6 + 6(a^3 + 3a^2b - 9ab^2 + 5b^3) \cosh(fx+e)^5 \sinh(fx+e) + 15(a^3 + 3a^2b - 9ab^2 + 5b^3) \cosh(fx+e)^4 \sinh(fx+e)^2 + 20(a^3 + 3a^2b - 9ab^2 + 5b^3) \cosh(fx+e)^3 \sinh(fx+e)^3 + 15(a^3 + 3a^2b - 9ab^2 + 5b^3) \cosh(fx+e)^2 \sinh(fx+e)^4 + 6(a^3 + 3a^2b - 9ab^2 + 5b^3) \cosh(fx+e) \sinh(fx+e)^5 + (a^3 + 3a^2b - 9ab^2 + 5b^3) \sinh$$

$$\begin{aligned}
& (f*x + e)^6 * \text{sqrt}(b) * \log((a^2*b*\cosh(f*x + e)^8 + 8*a^2*b*\cosh(f*x + e)*\sinh(f*x + e)^7 + a^2*b*\sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*\cosh(f*x + e)^6 + 2*(14*a^2*b*\cosh(f*x + e)^2 + a^3 + a^2*b)*\sinh(f*x + e)^6 + 4*(14*a^2*b*\cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^4 + (70*a^2*b*\cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*a^2*b*\cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*\cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*\cosh(f*x + e)^2 + 2*(14*a^2*b*\cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*\cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 - \text{sqrt}(2)*(a^2*\cosh(f*x + e)^6 + 6*a^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x + e)^4 + 3*(5*a^2*\cosh(f*x + e)^2 + a^2)*\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e))*\sinh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e)^2 + (15*a^2*\cosh(f*x + e)^4 + 18*a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(3*a^2*\cosh(f*x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e)) * \text{sqrt}(b) * \text{sqrt}((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(2*a^2*b*\cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*\cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + 6*((a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^6 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\sinh(f*x + e)^6)*\text{sqrt}(b) * \log(-(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b)*\sinh(f*x + e)^2 - \text{sqrt}(2)*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)) * \text{sqrt}(b) * \text{sqrt}((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + \text{sqrt}(2)*(b^3*\cosh(f*x + e)^10 + 10*b^3*\cosh(f*x + e)*\sinh(f*x + e)^9 + b^3*\sinh(f*x + e)^10 + (7*a*b^2 - 8*b^3)*\cosh(f*x + e)^8 + (45*b^3*\cosh(f*x + e)^2 + 7*a*b^2 - 8*b^3)*\sinh(f*x + e)^8 + 8*(15*b^3*\cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^7 + (6*a^2*b - 51*a*b^2 + 37*b^3)*\cosh(f*x + e)^6 + (210*b^3*\cosh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b^3 + 28*(7*a*b^2 - 8*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 2*(126*b^3*\cosh(f*x + e)^5 + 28*(7*a*b^2 - 8*b^3)*\cosh(f*x + e)^3 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (6*a^2*b - 51*a*b^2 + 37*b^3)*\cosh(f*x + e)^4 + (210*b^3*\cosh(f*x + e)^6 + 70*(7*a*b^2 - 8*b^3)*\cosh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b
\end{aligned}$$

$$\begin{aligned}
&^3 + 15*(6*a^2*b - 51*a*b^2 + 37*b^3)*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 4* \\
&(30*b^3*\cosh(f*x + e)^7 + 14*(7*a*b^2 - 8*b^3)*\cosh(f*x + e)^5 + 5*(6*a^2*b \\
&- 51*a*b^2 + 37*b^3)*\cosh(f*x + e)^3 + (6*a^2*b - 51*a*b^2 + 37*b^3)*\cosh(\\
&f*x + e))*\sinh(f*x + e)^3 + b^3 + (7*a*b^2 - 8*b^3)*\cosh(f*x + e)^2 + (45*b \\
&^3*\cosh(f*x + e)^8 + 28*(7*a*b^2 - 8*b^3)*\cosh(f*x + e)^6 + 15*(6*a^2*b - 5 \\
&1*a*b^2 + 37*b^3)*\cosh(f*x + e)^4 + 7*a*b^2 - 8*b^3 + 6*(6*a^2*b - 51*a*b^2 \\
&+ 37*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 2*(5*b^3*\cosh(f*x + e)^9 + 4* \\
&(7*a*b^2 - 8*b^3)*\cosh(f*x + e)^7 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*\cosh(f* \\
&x + e)^5 + 2*(6*a^2*b - 51*a*b^2 + 37*b^3)*\cosh(f*x + e)^3 + (7*a*b^2 - 8*b \\
&^3)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e) \\
&^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + \\
&e)^2)))/(b^2*f*\cosh(f*x + e)^6 + 6*b^2*f*\cosh(f*x + e)^5*\sinh(f*x + e) + 1 \\
&5*b^2*f*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*b^2*f*\cosh(f*x + e)^3*\sinh(f*x \\
&+ e)^3 + 15*b^2*f*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*b^2*f*\cosh(f*x + e)* \\
&\sinh(f*x + e)^5 + b^2*f*\sinh(f*x + e)^6), 1/384*(12*((a^3 + 3*a^2*b - 9*a*b \\
&^2 + 5*b^3)*\cosh(f*x + e)^6 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x \\
&+ e)^5*\sinh(f*x + e) + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^4 \\
&*\sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^3*\sin \\
&h(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^2*\sinh(f* \\
&x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)*\sinh(f*x + e)^ \\
&5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\sinh(f*x + e)^6)*\sqrt{-b}*\arctan(\sqrt{ \\
&2)*(a*\cosh(f*x + e)^2 + 2*a*\cosh(f*x + e)*\sinh(f*x + e) + a*\sinh(f*x + e)^ \\
&2 + b)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cos \\
&h(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a*b*\cosh(\\
&f*x + e)^4 + 4*a*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*b*\sinh(f*x + e)^4 + (3 \\
&*a*b - b^2)*\cosh(f*x + e)^2 + (6*a*b*\cosh(f*x + e)^2 + 3*a*b - b^2)*\sinh(f* \\
&x + e)^2 + b^2 + 2*(2*a*b*\cosh(f*x + e)^3 + (3*a*b - b^2)*\cosh(f*x + e))*\si \\
&nh(f*x + e))) + 12*((a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^6 + 6*(\\
&a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*(a^3 + \\
&3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*(a^3 + 3*a^ \\
&2*b - 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*(a^3 + 3*a^2*b \\
&- 9*a*b^2 + 5*b^3)*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a \\
&*b^2 + 5*b^3)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5* \\
&b^3)*\sinh(f*x + e)^6)*\sqrt{-b}*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x \\
&+ e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 \\
&+ b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x \\
&+ e) + \sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x \\
&+ e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x \\
&+ e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f* \\
&x + e))*\sinh(f*x + e) + b)) + \sqrt{2}*(b^3*\cosh(f*x + e)^10 + 10*b^3*\cosh(f \\
&*x + e)*\sinh(f*x + e)^9 + b^3*\sinh(f*x + e)^10 + (7*a*b^2 - 8*b^3)*\cosh(f*x \\
&+ e)^8 + (45*b^3*\cosh(f*x + e)^2 + 7*a*b^2 - 8*b^3)*\sinh(f*x + e)^8 + 8*(1 \\
&5*b^3*\cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^7 + \\
&(6*a^2*b - 51*a*b^2 + 37*b^3)*\cosh(f*x + e)^6 + (210*b^3*\cosh(f*x + e)^4 + \\
&6*a^2*b - 51*a*b^2 + 37*b^3 + 28*(7*a*b^2 - 8*b^3)*\cosh(f*x + e)^2)*\sinh(f*
\end{aligned}$$

```

x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^
3 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (6*a^2
*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*x + e)^6 + 70*(7*
a*b^2 - 8*b^3)*cosh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b^3 + 15*(6*a^2*b
- 51*a*b^2 + 37*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(30*b^3*cosh(f*x
+ e)^7 + 14*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b - 51*a*b^2 + 37*
b^3)*cosh(f*x + e)^3 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*
x + e)^3 + b^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)^2 + (45*b^3*cosh(f*x + e)^
8 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b - 51*a*b^2 + 37*b^3)
*cosh(f*x + e)^4 + 7*a*b^2 - 8*b^3 + 6*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f
*x + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*a*b^2 - 8*b^3)
*cosh(f*x + e)^7 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^5 + 2*(6*a
^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(co
sh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*c
osh(f*x + e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*cosh(f*x
+ e)^4*sinh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*b^2*
f*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*sinh(f*x + e)^5 +
b^2*f*sinh(f*x + e)^6)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \sinh(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^3, x)

3.77 $\int \sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=121

$$\frac{3(a-b) \cosh(e+fx) \sqrt{a+b \cosh^2(e+fx)-b}}{8f} + \frac{\cosh(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{4f} + \frac{3(a-b)^2 \tanh^{-1}\left(\frac{\sqrt{bc}}{\sqrt{a+bc}}\right)}{8\sqrt{bf}}$$

[Out] (3*(a - b)^2*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(8*Sqrt[b]*f) + (3*(a - b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(8*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*f)

Rubi [A] time = 0.096654, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3186, 195, 217, 206}

$$\frac{3(a-b) \cosh(e+fx) \sqrt{a+b \cosh^2(e+fx)-b}}{8f} + \frac{\cosh(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{4f} + \frac{3(a-b)^2 \tanh^{-1}\left(\frac{\sqrt{bc}}{\sqrt{a+bc}}\right)}{8\sqrt{bf}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (3*(a - b)^2*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(8*Sqrt[b]*f) + (3*(a - b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(8*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*f)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 195

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&

IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
Denominator[p]])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{\text{Subst}\left(\int (a - b + bx^2)^{3/2} dx, x, \cosh(e + fx)\right)}{f} \\ &= \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^{3/2}}{4f} + \frac{(3(a - b)) \text{Subst}\left(\int \sqrt{a - b + bx^2} dx, x, \cosh(e + fx)\right)}{4f} \\ &= \frac{3(a - b) \cosh(e + fx) \sqrt{a - b + b \cosh^2(e + fx)}}{8f} + \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^{3/2}}{4f} \\ &= \frac{3(a - b) \cosh(e + fx) \sqrt{a - b + b \cosh^2(e + fx)}}{8f} + \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^{3/2}}{4f} \\ &= \frac{3(a - b)^2 \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e + fx)}{\sqrt{a - b + b \cosh^2(e + fx)}}\right)}{8\sqrt{b}f} + \frac{3(a - b) \cosh(e + fx) \sqrt{a - b + b \cosh^2(e + fx)}}{8f} \end{aligned}$$

Mathematica [A] time = 0.312135, size = 111, normalized size = 0.92

$$\frac{\frac{1}{2} \cosh(e + fx) (5a + b \cosh(2(e + fx)) - 4b) \sqrt{4a + 2b \cosh(2(e + fx)) - 2b} + \frac{3(a - b)^2 \log(\sqrt{2a + b \cosh(2(e + fx)) - b} + \sqrt{2} \sqrt{b} \cosh(e + fx))}{\sqrt{b}}}{8f}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2), x]


```
[Out] ((Cosh[e + f*x]*(5*a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cosh
[2*(e + f*x)])]/2 + (3*(a - b)^2*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2
*a - b + b*Cosh[2*(e + f*x)])])/Sqrt[b])/(8*f)
```

Maple [B] time = 0.071, size = 336, normalized size = 2.8

$$\frac{1}{16 f \cosh(fx + e)} \sqrt{\left(a + b (\sinh(fx + e))^2\right) (\cosh(fx + e))^2} \left(4 b^{3/2} \sqrt{b (\cosh(fx + e))^4 + (a - b) (\cosh(fx + e))^2} \left(\cosh(fx + e)\right)^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*b^(3/2)*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2-10*b^(3/2)*(b*cosh(f*x+e)^4+(a-b)*
cosh(f*x+e)^2)^(1/2)+10*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/
2)+3*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2
))*b^(1/2)+a-b)/b^(1/2))*a^2-6*b*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e
)^4+(a-b)*cosh(f*x+e)^2)^(1/2))*b^(1/2)+a-b)/b^(1/2))+3*b^2*ln(1/2*(2*b*cosh
(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2))*b^(1/2)+a-b)/b^(1/2
)))/b^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e), x)
```

Fricas [B] time = 2.92628, size = 7737, normalized size = 63.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/64*(6*((a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2*\sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*\sinh(f*x + e)^4)*\sqrt{b}*\log((a^2*b*\cosh(f*x + e)^8 + 8*a^2*b*\cosh(f*x + e)*\sinh(f*x + e)^7 + a^2*b*\sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*\cosh(f*x + e)^6 + 2*(14*a^2*b*\cosh(f*x + e)^2 + a^3 + a^2*b)*\sinh(f*x + e)^6 + 4*(14*a^2*b*\cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^4 + (70*a^2*b*\cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*a^2*b*\cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*\cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*\cosh(f*x + e)^2 + 2*(14*a^2*b*\cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*\cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + \sqrt{2}*(a^2*\cosh(f*x + e)^6 + 6*a^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x + e)^4 + 3*(5*a^2*\cosh(f*x + e)^2 + a^2)*\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e))*\sinh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e)^2 + (15*a^2*\cosh(f*x + e)^4 + 18*a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(3*a^2*\cosh(f*x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(2*a^2*b*\cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*\cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + 6*((a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2*\sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*\sinh(f*x + e)^4)*\sqrt{b}*\log(-(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b)*\sinh(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + \sqrt{2}*(b^2*\cosh(f*x + e)^6 + 6*b^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + b^2*\sinh(f*x + e)^6 + (10*a*b - 7*b^2)*\cosh(f*x + e)^4 + (15*b^2*\cosh(f*x + e)^2 + 10*a*b - 7*b^2)*\sinh(f*x + e)^4 + 4*(5*b^2*\cosh(f*x + e)^3 + (10*a*b - 7*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 + (10*a*b - 7*b^2)*\cosh(f*x + e)^2 + (15*b^2$$

```

*cosh(f*x + e)^4 + 6*(10*a*b - 7*b^2)*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*sin
h(f*x + e)^2 + b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 7*b^2)*cosh(f*x
+ e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^
3*sinh(f*x + e) + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x +
e)*sinh(f*x + e)^3 + b*f*sinh(f*x + e)^4), -1/64*(12*((a^2 - 2*a*b + b^2)*c
osh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*c
osh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4)*sqrt(-b
)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*s
inh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
)))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*
x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b -
b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh
(f*x + e))*sinh(f*x + e))) + 12*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 4*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*cos
h(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x
+ e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cos
h(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b
))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*(b^2*cosh(f*
x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (10*
a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*si
nh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e))*
sinh(f*x + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^
4 + 6*(10*a*b - 7*b^2)*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*sinh(f*x + e)^2 +
b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 7*b^2)*cosh(f*x + e)^3 + (10*a
*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^3*sinh(f*x + e)
+ 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x + e)*sinh(f*x + e
)^3 + b*f*sinh(f*x + e)^4)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e), x)
```

3.78 $\int \operatorname{csch}(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$

Optimal. Leaf size=127

$$-\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{f} + \frac{b \cosh(e+fx) \sqrt{a+b \cosh^2(e+fx)-b}}{2f} + \frac{\sqrt{b}(3a-b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2f}$$

[Out] $-\left(\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cosh}[e + f*x]}{\sqrt{a - b + b \operatorname{Cosh}[e + f*x]^2}}\right]}{f}\right) + \left(\frac{(3*a - b) \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Cosh}[e + f*x]}{\sqrt{a - b + b \operatorname{Cosh}[e + f*x]^2}}\right]}{2*f}\right) + \left(\frac{b \operatorname{Cosh}[e + f*x] \sqrt{a - b + b \operatorname{Cosh}[e + f*x]^2}}{2*f}\right)$

Rubi [A] time = 0.16497, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3186, 416, 523, 217, 206, 377}

$$-\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{f} + \frac{b \cosh(e+fx) \sqrt{a+b \cosh^2(e+fx)-b}}{2f} + \frac{\sqrt{b}(3a-b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[e + f*x] * (a + b * \operatorname{Sinh}[e + f*x]^2)^{(3/2)}, x]$

[Out] $-\left(\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a} \operatorname{Cosh}[e + f*x]}{\sqrt{a - b + b \operatorname{Cosh}[e + f*x]^2}}\right]}{f}\right) + \left(\frac{(3*a - b) \sqrt{b} \operatorname{ArcTanh}\left[\frac{\sqrt{b} \operatorname{Cosh}[e + f*x]}{\sqrt{a - b + b \operatorname{Cosh}[e + f*x]^2}}\right]}{2*f}\right) + \left(\frac{b \operatorname{Cosh}[e + f*x] \sqrt{a - b + b \operatorname{Cosh}[e + f*x]^2}}{2*f}\right)$

Rule 3186

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)} * ((a_.) + (b_.) * \sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2} * (a + b - b*ff^2*x^2)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, p\}, x\} \ \&\& \ \operatorname{IntegerQ}[(m-1)/2]$

Rule 416

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)} * ((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Simp}[(d*x*(a + b*x^n)^{(p+1}) * (c + d*x^n)^{(q-1)}) / (b*(n*(p+q) + 1)),$

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x
_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^{3/2}}{1-x^2} dx, x, \cosh(e+fx)\right)}{f} \\
&= \frac{b \cosh(e+fx) \sqrt{a-b+b \cosh^2(e+fx)}}{2f} + \frac{\operatorname{Subst}\left(\int \frac{-(a-b)(2a-b)-(3a-b)bx^2}{(1-x^2)\sqrt{a-b+bx^2}} dx\right)}{2f} \\
&= \frac{b \cosh(e+fx) \sqrt{a-b+b \cosh^2(e+fx)}}{2f} - \frac{a^2 \operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}} dx\right)}{f} \\
&= \frac{b \cosh(e+fx) \sqrt{a-b+b \cosh^2(e+fx)}}{2f} - \frac{a^2 \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{f} \\
&= -\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{f} + \frac{(3a-b)\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{2f}
\end{aligned}$$

Mathematica [A] time = 0.553553, size = 136, normalized size = 1.07

$$\frac{-4a^{3/2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a} \cosh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right) + b \cosh(e+fx) \sqrt{4a+2b \cosh(2(e+fx))-2b} - 2\sqrt{b}(b-3a) \log\left(\sqrt{2a+b \cosh(2(e+fx))}\right)}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] $(-4a^{3/2} \operatorname{ArcTanh}[(\sqrt{2} \sqrt{a} \operatorname{Cosh}[e + f*x]) / \sqrt{2a - b + b \operatorname{Cosh}[2(e + f*x)]}] + b \operatorname{Cosh}[e + f*x] \sqrt{4a - 2b + 2b \operatorname{Cosh}[2(e + f*x)]} - 2 \sqrt{b} (-3a + b) \operatorname{Log}[\sqrt{2} \sqrt{b} \operatorname{Cosh}[e + f*x] + \sqrt{2a - b + b \operatorname{Cosh}[2(e + f*x)]}]) / (4f)$

Maple [B] time = 0.086, size = 268, normalized size = 2.1

$$-\frac{1}{4f \cosh(fx+e)} \sqrt{(a+b(\sinh(fx+e))^2)(\cosh(fx+e))^2} \left(2a^{3/2} \ln\left(\frac{(a+b)(\cosh(fx+e))^2 + 2\sqrt{a}\sqrt{b}(\cosh(fx+e))}{(\cosh(fx+e))^2}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*a^(3/2)*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/(cosh(f*x+e)^2-1))+b^(3/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-3*b^(1/2)*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-2*b*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{csch}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e), x)
```

Fricas [B] time = 3.68536, size = 14642, normalized size = 115.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^
```


$$\begin{aligned}
& 2*b - 4*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3) \\
& *\cosh(f*x + e)^2 + 2*(14*a^2*b*\cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*\cosh(f* \\
& x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sin \\
& h(f*x + e)^2 - \sqrt{2}*(a^2*\cosh(f*x + e)^6 + 6*a^2*\cosh(f*x + e)*\sinh(f*x \\
& + e)^5 + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x + e)^4 + 3*(5*a^2*\cosh(f*x + \\
& e)^2 + a^2)*\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e \\
&))*\sinh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e)^2 + (15*a^2*\cosh(f*x + e)^ \\
& 4 + 18*a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(3*a^2* \\
& \cosh(f*x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e))*\sinh \\
& (f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\\
& \cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)} + 4*(2* \\
& a^2*b*\cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*\cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^ \\
& 2 + b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(c \\
& osh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(\\
& f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f \\
& *x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) - 4*(a*\cosh \\
& (f*x + e)^2 + 2*a*\cosh(f*x + e)*\sinh(f*x + e) + a*\sinh(f*x + e)^2)*\sqrt{a}* \\
& \log(-((a + b)*\cosh(f*x + e)^4 + 4*(a + b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (\\
& a + b)*\sinh(f*x + e)^4 + 2*(3*a - b)*\cosh(f*x + e)^2 + 2*(3*(a + b)*\cosh(f* \\
& x + e)^2 + 3*a - b)*\sinh(f*x + e)^2 - 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f \\
& *x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1))*\sqrt{a}*\sqrt{(b*\cosh(f*x + e)^ \\
& 2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f* \\
& x + e) + \sinh(f*x + e)^2)} + 4*((a + b)*\cosh(f*x + e)^3 + (3*a - b)*\cosh(f* \\
& x + e))*\sinh(f*x + e) + a + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x \\
& + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 - 1)*\sinh(f*x + e)^2 - 2*\co \\
& sh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 - \cosh(f*x + e))*\sinh(f*x + e) + 1)) + (\\
& (3*a - b)*\cosh(f*x + e)^2 + 2*(3*a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (3*a \\
& - b)*\sinh(f*x + e)^2)*\sqrt{b}*\log(-(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\s \\
& inh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cos \\
& h(f*x + e)^2 + a - b)*\sinh(f*x + e)^2 - \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f \\
& *x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^ \\
& 2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f* \\
& x + e) + \sinh(f*x + e)^2)} + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))* \\
& \sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(\\
& f*x + e)^2)) - \sqrt{2}*(b*\cosh(f*x + e)^2 + 2*b*\cosh(f*x + e)*\sinh(f*x + e) \\
& + b*\sinh(f*x + e)^2 + b)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a \\
& - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} \\
& / (f*\cosh(f*x + e)^2 + 2*f*\cosh(f*x + e)*\sinh(f*x + e) + f*\sinh(f*x + e)^2), \\
& 1/8*(8*(a*\cosh(f*x + e)^2 + 2*a*\cosh(f*x + e)*\sinh(f*x + e) + a*\sinh(f*x + \\
& e)^2)*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x \\
& + e) + \sinh(f*x + e)^2 + 1))*\sqrt{-a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + \\
& e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f* \\
& x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(\\
& f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b \\
&)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*
\end{aligned}$$

$$\begin{aligned}
& x + e) + b)) - ((3a - b)\cosh(f*x + e)^2 + 2*(3a - b)\cosh(f*x + e)\sinh(f*x + e) + (3a - b)\sinh(f*x + e)^2)\sqrt{b}\log((a^2*b\cosh(f*x + e)^8 + 8*a^2*b\cosh(f*x + e)\sinh(f*x + e)^7 + a^2*b\sinh(f*x + e)^8 + 2*(a^3 + a^2*b)\cosh(f*x + e)^6 + 2*(14*a^2*b\cosh(f*x + e)^2 + a^3 + a^2*b)\sinh(f*x + e)^6 + 4*(14*a^2*b\cosh(f*x + e)^3 + 3*(a^3 + a^2*b)\cosh(f*x + e))\sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)\cosh(f*x + e)^4 + (70*a^2*b\cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)\cosh(f*x + e)^2)\sinh(f*x + e)^4 + 4*(14*a^2*b\cosh(f*x + e)^5 + 10*(a^3 + a^2*b)\cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)\cosh(f*x + e))\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)\cosh(f*x + e)^2 + 2*(14*a^2*b\cosh(f*x + e)^6 + 15*(a^3 + a^2*b)\cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)\cosh(f*x + e)^2)\sinh(f*x + e)^2 - \sqrt{2}*(a^2*\cosh(f*x + e)^6 + 6*a^2*\cosh(f*x + e)\sinh(f*x + e)^5 + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x + e)^4 + 3*(5*a^2*\cosh(f*x + e)^2 + a^2)\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e))\sinh(f*x + e)^3 + (4*a*b - b^2)\cosh(f*x + e)^2 + (15*a^2*\cosh(f*x + e)^4 + 18*a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)\sinh(f*x + e)^2 + b^2 + 2*(3*a^2*\cosh(f*x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4*a*b - b^2)\cosh(f*x + e))\sinh(f*x + e))\sqrt{b}\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(2*a^2*b\cosh(f*x + e)^7 + 3*(a^3 + a^2*b)\cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)\cosh(f*x + e)^3 + (3*a*b^2 - b^3)\cosh(f*x + e))\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) - ((3a - b)\cosh(f*x + e)^2 + 2*(3a - b)\cosh(f*x + e)\sinh(f*x + e) + (3a - b)\sinh(f*x + e)^2)\sqrt{b}\log(-(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(a - b)\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b)\sinh(f*x + e)^2 - \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)\sinh(f*x + e) + \sinh(f*x + e)^2 - 1))\sqrt{b}\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(b*\cosh(f*x + e)^3 + (a - b)\cosh(f*x + e)\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)\sinh(f*x + e) + \sinh(f*x + e)^2)) + \sqrt{2}*(b*\cosh(f*x + e)^2 + 2*b*\cosh(f*x + e)\sinh(f*x + e) + b*\sinh(f*x + e)^2 + b)\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)\sinh(f*x + e) + \sinh(f*x + e)^2)))/(f*\cosh(f*x + e)^2 + 2*f*\cosh(f*x + e)\sinh(f*x + e) + f*\sinh(f*x + e)^2), -1/8*(2*((3a - b)\cosh(f*x + e)^2 + 2*(3a - b)\cosh(f*x + e)\sinh(f*x + e) + (3a - b)\sinh(f*x + e)^2)\sqrt{-b}\arctan(\sqrt{2}*(a*\cosh(f*x + e)^2 + 2*a*\cosh(f*x + e)\sinh(f*x + e) + a*\sinh(f*x + e)^2 + b)\sqrt{-b}\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a*b*\cosh(f*x + e)^4 + 4*a*b*\cosh(f*x + e)\sinh(f*x + e)^3 + a*b*\sinh(f*x + e)^4 + (3*a*b - b^2)\cosh(f*x + e)^2 + (6*a*b*\cosh(f*x + e)^2 + 3*a*b - b^2)\sinh(f*x + e)^2 + b^2 + 2*(2*a*b*\cosh(f*x + e)^3 + (3*a*b - b^2)\cosh(f*x + e))\sinh(f*x + e))) + 2*((3a - b)\cosh(f*x + e)^2 + 2*(3a - b)\cosh(f*x + e)\sinh(f*x + e) + (3a - b)
\end{aligned}$$

```

*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)
)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^
3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^
2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x +
e))*sinh(f*x + e) + b)) - 4*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x
+ e) + a*sinh(f*x + e)^2)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b
)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cos
h(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*
sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
+ 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*
cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x
+ e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x
+ e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cos
h(f*x + e))*sinh(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x
+ e)*sinh(f*x + e) + b*sinh(f*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) +
f*sinh(f*x + e)^2), 1/8*(8*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x
+ e) + a*sinh(f*x + e)^2)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f
*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f
*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cos
h(f*x + e))*sinh(f*x + e) + b)) - 2*((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b
)*cosh(f*x + e)*sinh(f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(
sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x +
e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*c
osh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4
+ (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sin
h(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e)
)*sinh(f*x + e))) - 2*((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e
)*sinh(f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(
f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(b*cosh(f*x +
e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 + b)*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +

```

```
e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x +
e)*sinh(f*x + e) + f*sinh(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^{\frac{3}{2}} \operatorname{csch}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e), x)
```

3.79 $\int \operatorname{csch}^3(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=130

$$\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{f} + \frac{\sqrt{a}(a-3b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2f} - \frac{a \coth(e+fx) \operatorname{csch}(e+fx) \sqrt{a+b \cosh^2(e+fx)}}{2f}$$

[Out] (Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/f - (a*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x])/(2*f)

Rubi [A] time = 0.168235, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3186, 413, 523, 217, 206, 377}

$$\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{f} + \frac{\sqrt{a}(a-3b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2f} - \frac{a \coth(e+fx) \operatorname{csch}(e+fx) \sqrt{a+b \cosh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/f - (a*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x])/(2*f)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 413

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1))

```
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^{3/2}}{(1-x^2)^2} dx, x, \cosh(e+fx)\right)}{f} \\
&= -\frac{a\sqrt{a-b+b \cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{2f} - \frac{\operatorname{Subst}\left(\int \frac{-(a-b+bx^2)^{3/2}}{(1-x^2)^2} dx, x, \cosh(e+fx)\right)}{2f} \\
&= -\frac{a\sqrt{a-b+b \cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{2f} + \frac{(a(a-3b)) \operatorname{Subst}\left(\int \frac{-(a-b+bx^2)^{3/2}}{(1-x^2)^2} dx, x, \cosh(e+fx)\right)}{2f} \\
&= -\frac{a\sqrt{a-b+b \cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{2f} + \frac{(a(a-3b)) \operatorname{Subst}\left(\int \frac{-(a-b+bx^2)^{3/2}}{(1-x^2)^2} dx, x, \cosh(e+fx)\right)}{2f} \\
&= \frac{\sqrt{a}(a-3b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{2f} + \frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{f}
\end{aligned}$$

Mathematica [A] time = 0.71128, size = 143, normalized size = 1.1

$$\frac{4b^{3/2} \log\left(\sqrt{2a+b \cosh(2(e+fx))} - b + \sqrt{2}\sqrt{b} \cosh(e+fx)\right) + 2\sqrt{a}(a-3b) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a} \cosh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))}-b}\right) - a \coth(e+fx)}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (2*Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] - a*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]*Coth[e + f*x]*Csch[e + f*x] + 4*b^(3/2)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])/(4*f)

Maple [B] time = 0.097, size = 297, normalized size = 2.3

$$\frac{1}{4 (\sinh(fx+e))^2 \cosh(fx+e) f} \sqrt{(a+b(\sinh(fx+e))^2)(\cosh(fx+e))^2} \left(2b^{3/2} \ln\left(\frac{1}{2} \frac{2b(\cosh(fx+e))^2 + 2\sqrt{a-b+b \cosh^2(fx+e)}}{\cosh(fx+e)}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*b^(3/2)*ln(1/2*(2*b*cosh(f
*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))
*sinh(f*x+e)^2+a^(3/2)*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(
a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-3*a^(1/2)*b*ln(
((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+
a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-2*a*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(
1/2))/sinh(f*x+e)^2/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{csch}(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^3, x)
```

Fricas [B] time = 4.75341, size = 17554, normalized size = 135.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
e)^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 +
4*(b*cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log((a^
2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*
x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 +
a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b
)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^
4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*
```


$$\begin{aligned}
& \cosh(f*x + e)^2 * \sinh(f*x + e)^4 + 4*(14*a^2*b*\cosh(f*x + e)^5 + 10*(a^3 + \\
& a^2*b)*\cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x \\
& + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*\cosh(f*x + e)^2 + 2*(14*a^2*b*\cosh(f*x + e) \\
&)^6 + 15*(a^3 + a^2*b)*\cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b \\
& ^2 + b^3)*\cosh(f*x + e)^2*\sinh(f*x + e)^2 + \sqrt{2}*(a^2*\cosh(f*x + e)^6 + \\
& 6*a^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x \\
& + e)^4 + 3*(5*a^2*\cosh(f*x + e)^2 + a^2)*\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f \\
& *x + e)^3 + 3*a^2*\cosh(f*x + e))*\sinh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + \\
& e)^2 + (15*a^2*\cosh(f*x + e)^4 + 18*a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)*\sin \\
& h(f*x + e)^2 + b^2 + 2*(3*a^2*\cosh(f*x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4* \\
& a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + \\
& b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + \\
& e) + \sinh(f*x + e)^2))} + 4*(2*a^2*b*\cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*\cosh(\\
& f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - b^3)*\co \\
& sh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + \\
& e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e) \\
& ^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \\
& \sinh(f*x + e)^6)) - ((a - 3*b)*\cosh(f*x + e)^4 + 4*(a - 3*b)*\cosh(f*x + e) \\
& *\sinh(f*x + e)^3 + (a - 3*b)*\sinh(f*x + e)^4 - 2*(a - 3*b)*\cosh(f*x + e)^2 \\
& + 2*(3*(a - 3*b)*\cosh(f*x + e)^2 - a + 3*b)*\sinh(f*x + e)^2 + 4*((a - 3*b)* \\
& \cosh(f*x + e)^3 - (a - 3*b)*\cosh(f*x + e))*\sinh(f*x + e) + a - 3*b)*\sqrt{a} \\
& * \log(-((a + b)*\cosh(f*x + e)^4 + 4*(a + b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + \\
& (a + b)*\sinh(f*x + e)^4 + 2*(3*a - b)*\cosh(f*x + e)^2 + 2*(3*(a + b)*\cosh(f \\
& *x + e)^2 + 3*a - b)*\sinh(f*x + e)^2 - 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(\\
& f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1))*\sqrt{a}*\sqrt{(b*\cosh(f*x + e) \\
& ^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f \\
& *x + e) + \sinh(f*x + e)^2))} + 4*((a + b)*\cosh(f*x + e)^3 + (3*a - b)*\cosh(f \\
& *x + e))*\sinh(f*x + e) + a + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x \\
& + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 - 1)*\sinh(f*x + e)^2 - 2*\c \\
& osh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 - \cosh(f*x + e))*\sinh(f*x + e) + 1)) + \\
& (b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 \\
& - 2*b*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 - b)*\sinh(f*x + e)^2 + 4*(b* \\
& \cosh(f*x + e)^3 - b*\cosh(f*x + e))*\sinh(f*x + e) + b)*\sqrt{b}*\log(- (b*\cosh(\\
& f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(a - \\
& b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b)*\sinh(f*x + e)^2 + \sqrt{ \\
& 2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1) \\
&)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x \\
& + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(b*\cosh(f*x \\
& + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cos \\
& h(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) - 2*\sqrt{2}*(a*\cosh(f*x + e)^2 \\
& + 2*a*\cosh(f*x + e)*\sinh(f*x + e) + a*\sinh(f*x + e)^2 + a)*\sqrt{(b*\cosh(f* \\
& x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)* \\
& \sinh(f*x + e) + \sinh(f*x + e)^2)))/(f*\cosh(f*x + e)^4 + 4*f*\cosh(f*x + e)*\s \\
& inh(f*x + e)^3 + f*\sinh(f*x + e)^4 - 2*f*\cosh(f*x + e)^2 + 2*(3*f*\cosh(f*x \\
& + e)^2 - f)*\sinh(f*x + e)^2 + 4*(f*\cosh(f*x + e)^3 - f*\cosh(f*x + e))*\sinh(
\end{aligned}$$

$$\begin{aligned}
& f*x + e) + f), -1/4*(2*((a - 3*b)*\cosh(f*x + e)^4 + 4*(a - 3*b)*\cosh(f*x + \\
& e)*\sinh(f*x + e)^3 + (a - 3*b)*\sinh(f*x + e)^4 - 2*(a - 3*b)*\cosh(f*x + e)^ \\
& 2 + 2*(3*(a - 3*b)*\cosh(f*x + e)^2 - a + 3*b)*\sinh(f*x + e)^2 + 4*((a - 3*b) \\
&)*\cosh(f*x + e)^3 - (a - 3*b)*\cosh(f*x + e))*\sinh(f*x + e) + a - 3*b)*\sqrt{ \\
& -a)*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(\\
& f*x + e)^2 + 1)*\sqrt{-a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a \\
& - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(\\
& b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + \\
& 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + \\
& e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) \\
& - (b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e) \\
& ^4 - 2*b*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 - b)*\sinh(f*x + e)^2 + 4* \\
& (b*\cosh(f*x + e)^3 - b*\cosh(f*x + e))*\sinh(f*x + e) + b)*\sqrt{b}*\log((a^2*b \\
& *\cosh(f*x + e)^8 + 8*a^2*b*\cosh(f*x + e)*\sinh(f*x + e)^7 + a^2*b*\sinh(f*x + \\
& e)^8 + 2*(a^3 + a^2*b)*\cosh(f*x + e)^6 + 2*(14*a^2*b*\cosh(f*x + e)^2 + a^3 \\
& + a^2*b)*\sinh(f*x + e)^6 + 4*(14*a^2*b*\cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*\c \\
& osh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^4 + \\
& (70*a^2*b*\cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*\cos \\
& h(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*a^2*b*\cosh(f*x + e)^5 + 10*(a^3 + a^2 \\
& *b)*\cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e \\
&)^3 + b^3 + 2*(3*a*b^2 - b^3)*\cosh(f*x + e)^2 + 2*(14*a^2*b*\cosh(f*x + e)^6 \\
& + 15*(a^3 + a^2*b)*\cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 \\
& + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + \sqrt{2}*(a^2*\cosh(f*x + e)^6 + 6* \\
& a^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x + \\
& e)^4 + 3*(5*a^2*\cosh(f*x + e)^2 + a^2)*\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f*x \\
& + e)^3 + 3*a^2*\cosh(f*x + e))*\sinh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e) \\
& ^2 + (15*a^2*\cosh(f*x + e)^4 + 18*a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)*\sinh(f \\
& *x + e)^2 + b^2 + 2*(3*a^2*\cosh(f*x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4*a*b \\
& - b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*s \\
& inh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) \\
& + \sinh(f*x + e)^2)) + 4*(2*a^2*b*\cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*\cosh(f*x \\
& + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - b^3)*\cosh(\\
& f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) \\
& + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 \\
& + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \si \\
& nh(f*x + e)^6)) - (b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + \\
& b*\sinh(f*x + e)^4 - 2*b*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 - b)*\sinh(\\
& f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 - b*\cosh(f*x + e))*\sinh(f*x + e) + b)*\sqrt{ \\
& t(b)*\log(-(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f \\
& *x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b)*\sin \\
& h(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + s \\
& inh(f*x + e)^2 - 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2 \\
& *a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2) \\
&) + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(\\
& f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 2*\sqrt{2}*
\end{aligned}$$

$$\begin{aligned}
& (a \cosh(fx + e)^2 + 2a \cosh(fx + e) \sinh(fx + e) + a \sinh(fx + e)^2 + \\
& a) \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - \\
& 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)) / (f \cosh(fx + e)^4 + 4 \\
& * f \cosh(fx + e) \sinh(fx + e)^3 + f \sinh(fx + e)^4 - 2f \cosh(fx + e)^2 \\
& + 2*(3f \cosh(fx + e)^2 - f) \sinh(fx + e)^2 + 4*(f \cosh(fx + e)^3 - f \cosh \\
& (fx + e)) \sinh(fx + e) + f), -1/4*(2*(b \cosh(fx + e)^4 + 4b \cosh(fx \\
& + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 - 2b \cosh(fx + e)^2 + 2*(3b \cosh \\
& (fx + e)^2 - b) \sinh(fx + e)^2 + 4*(b \cosh(fx + e)^3 - b \cosh(fx + e)) \\
& * \sinh(fx + e) + b) \sqrt{-b} \arctan(\sqrt{2}*(a \cosh(fx + e)^2 + 2a \cosh(f \\
& * x + e) \sinh(fx + e) + a \sinh(fx + e)^2 + b) \sqrt{-b} \sqrt{(b \cosh(fx + \\
& e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh \\
& (fx + e) + \sinh(fx + e)^2)) / (a b \cosh(fx + e)^4 + 4a b \cosh(fx + e) \sinh \\
& (fx + e)^3 + a b \sinh(fx + e)^4 + (3a b - b^2) \cosh(fx + e)^2 + (6a a \\
& b \cosh(fx + e)^2 + 3a a b - b^2) \sinh(fx + e)^2 + b^2 + 2*(2a a b \cosh(fx \\
& + e)^3 + (3a a b - b^2) \cosh(fx + e)) \sinh(fx + e)) + 2*(b \cosh(fx + e)^ \\
& 4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 - 2b \cosh(fx + \\
& e)^2 + 2*(3b \cosh(fx + e)^2 - b) \sinh(fx + e)^2 + 4*(b \cosh(fx + e)^3 - \\
& b \cosh(fx + e)) \sinh(fx + e) + b) \sqrt{-b} \arctan(\sqrt{2}*(\cosh(fx + e) \\
& ^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 - 1) \sqrt{-b} \sqrt{(b \cosh \\
& (fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx \\
& + e) \sinh(fx + e) + \sinh(fx + e)^2)) / (b \cosh(fx + e)^4 + 4b \cosh(fx \\
& + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 + 2*(2a - b) \cosh(fx + e)^2 + 2* \\
& (3b \cosh(fx + e)^2 + 2a - b) \sinh(fx + e)^2 + 4*(b \cosh(fx + e)^3 + (2 \\
& * a - b) \cosh(fx + e)) \sinh(fx + e) + b)) + ((a - 3b) \cosh(fx + e)^4 + 4 \\
& * (a - 3b) \cosh(fx + e) \sinh(fx + e)^3 + (a - 3b) \sinh(fx + e)^4 - 2*(a \\
& - 3b) \cosh(fx + e)^2 + 2*(3*(a - 3b) \cosh(fx + e)^2 - a + 3b) \sinh(fx \\
& + e)^2 + 4*((a - 3b) \cosh(fx + e)^3 - (a - 3b) \cosh(fx + e)) \sinh(fx \\
& + e) + a - 3b) \sqrt{a} \log(-((a + b) \cosh(fx + e)^4 + 4*(a + b) \cosh(fx \\
& + e) \sinh(fx + e)^3 + (a + b) \sinh(fx + e)^4 + 2*(3a - b) \cosh(fx + e) \\
& ^2 + 2*(3*(a + b) \cosh(fx + e)^2 + 3a - b) \sinh(fx + e)^2 - 2\sqrt{2}*(\cosh \\
& (fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 + 1) \sqrt{a} \sqrt{(b \cosh \\
& (fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh \\
& (fx + e) + \sinh(fx + e)^2)) + 4*((a + b) \cosh(fx + e)^3 + (3a - b) \cosh(fx + e)) \sinh \\
& (fx + e) + a + b) / (\cosh(fx + e)^4 + 4 \cosh(fx + e) \sinh(fx + e)^3 + \sinh(fx + e)^4 + 2*(3 \cosh(fx + e)^2 - \\
& 1) \sinh(fx + e)^2 - 2 \cosh(fx + e)^2 + 4*(\cosh(fx + e)^3 - \cosh(fx + e) \\
&) \sinh(fx + e) + 1)) + 2\sqrt{2}*(a \cosh(fx + e)^2 + 2a \cosh(fx + e) \sinh \\
& (fx + e) + a \sinh(fx + e)^2 + a) \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e) \\
& ^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx \\
& + e)^2)) / (f \cosh(fx + e)^4 + 4f \cosh(fx + e) \sinh(fx + e)^3 + f \sinh \\
& (fx + e)^4 - 2f \cosh(fx + e)^2 + 2*(3f \cosh(fx + e)^2 - f) \sinh(fx + \\
& e)^2 + 4*(f \cosh(fx + e)^3 - f \cosh(fx + e)) \sinh(fx + e) + f), -1/2*((\\
& a - 3b) \cosh(fx + e)^4 + 4*(a - 3b) \cosh(fx + e) \sinh(fx + e)^3 + (a - \\
& 3b) \sinh(fx + e)^4 - 2*(a - 3b) \cosh(fx + e)^2 + 2*(3*(a - 3b) \cosh(f \\
& * x + e)^2 - a + 3b) \sinh(fx + e)^2 + 4*((a - 3b) \cosh(fx + e)^3 - (a -
\end{aligned}$$

```

3*b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt(-a)*arctan(sqrt(2)*(cosh(
f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + (b*cosh(f*x + e)^4 + 4
*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*cosh(f*x + e)^2
+ 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 - b*cos
h(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2
+ 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh
(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x +
e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a
*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + (b*cosh
(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*c
osh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*
x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(cos
h(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(a*cosh(f*x
+ e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x
+ e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 - 2*f*cosh(f*x + e)^2 + 2*(3*f*cos
h(f*x + e)^2 - f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 - f*cosh(f*x + e))
*sinh(f*x + e) + f)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{csch}(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^3, x)

3.80 $\int \operatorname{csch}^5(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=135

$$\frac{3(a-b)^2 \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{8\sqrt{a}f} - \frac{\coth(e+fx) \operatorname{csch}^3(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{4f} + \frac{3(a-b) \coth(e+fx)}{4f}$$

[Out] $(-3*(a - b)^2 * \operatorname{ArcTanh}[(\operatorname{Sqrt}[a] * \operatorname{Cosh}[e + f*x]) / \operatorname{Sqrt}[a - b + b * \operatorname{Cosh}[e + f*x]^2]]) / (8 * \operatorname{Sqrt}[a] * f) + (3 * (a - b) * \operatorname{Sqrt}[a - b + b * \operatorname{Cosh}[e + f*x]^2] * \operatorname{Coth}[e + f*x] * \operatorname{Csch}[e + f*x]) / (8 * f) - ((a - b + b * \operatorname{Cosh}[e + f*x]^2)^{(3/2)} * \operatorname{Coth}[e + f*x] * \operatorname{Csch}[e + f*x]^3) / (4 * f)$

Rubi [A] time = 0.140604, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3186, 378, 377, 206}

$$\frac{3(a-b)^2 \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{8\sqrt{a}f} - \frac{\coth(e+fx) \operatorname{csch}^3(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{4f} + \frac{3(a-b) \coth(e+fx)}{4f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[e + f*x]^5 * (a + b * \operatorname{Sinh}[e + f*x]^2)^{(3/2)}, x]$

[Out] $(-3*(a - b)^2 * \operatorname{ArcTanh}[(\operatorname{Sqrt}[a] * \operatorname{Cosh}[e + f*x]) / \operatorname{Sqrt}[a - b + b * \operatorname{Cosh}[e + f*x]^2]]) / (8 * \operatorname{Sqrt}[a] * f) + (3 * (a - b) * \operatorname{Sqrt}[a - b + b * \operatorname{Cosh}[e + f*x]^2] * \operatorname{Coth}[e + f*x] * \operatorname{Csch}[e + f*x]) / (8 * f) - ((a - b + b * \operatorname{Cosh}[e + f*x]^2)^{(3/2)} * \operatorname{Coth}[e + f*x] * \operatorname{Csch}[e + f*x]^3) / (4 * f)$

Rule 3186

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)} * ((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2} * (a + b - b*ff^2*x^2)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 378

$\operatorname{Int}(((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)} * ((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(x*(a + b*x^n)^{(p+1)} * (c + d*x^n)^q) / (a*n*(p+1)), x] - \operatorname{Dist}[(c$

q)/(a(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^5(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a-bx^2)^{3/2}}{(1-x^2)^3} dx, x, \cosh(e + fx)\right)}{f} \\ &= -\frac{(a - b + b \cosh^2(e + fx))^{3/2} \coth(e + fx) \operatorname{csch}^3(e + fx)}{4f} - \frac{(3(a - b)) S}{4f} \\ &= \frac{3(a - b) \sqrt{a - b + b \cosh^2(e + fx)} \coth(e + fx) \operatorname{csch}(e + fx)}{8f} - \frac{(a - b + b \cosh^2(e + fx))^{3/2} \coth(e + fx) \operatorname{csch}^3(e + fx)}{4f} \\ &= \frac{3(a - b) \sqrt{a - b + b \cosh^2(e + fx)} \coth(e + fx) \operatorname{csch}(e + fx)}{8f} - \frac{(a - b + b \cosh^2(e + fx))^{3/2} \coth(e + fx) \operatorname{csch}^3(e + fx)}{4f} \\ &= -\frac{3(a - b)^2 \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e + fx)}{\sqrt{a - b + b \cosh^2(e + fx)}}\right)}{8\sqrt{a}f} + \frac{3(a - b) \sqrt{a - b + b \cosh^2(e + fx)} \coth(e + fx) \operatorname{csch}(e + fx)}{8f} \end{aligned}$$

Mathematica [A] time = 0.630421, size = 123, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{a} \coth(e + fx) \operatorname{csch}(e + fx) \sqrt{2a + b \cosh(2(e + fx)) - b} (-2a \operatorname{csch}^2(e + fx) + 3a - 5b) - 6(a - b)^2 \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a} \cosh(e + fx)}{\sqrt{2a + b \cosh(2(e + fx)) - b}}\right)}{16\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (-6*(a - b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e + f*x]*Csch[e + f*x]*(3*a - 5*b - 2*a*Csch[e + f*x]^2))/(16*Sqrt[a]*f)

Maple [B] time = 0.095, size = 379, normalized size = 2.8

$$\frac{1}{16 (\sinh(fx + e))^4 \cosh(fx + e) f} \sqrt{(a + b (\sinh(fx + e))^2) (\cosh(fx + e))^2} \left(3a^2 \ln \left(\frac{(a + b) (\cosh(fx + e))^2 + 2\sqrt{a + b (\sinh(fx + e))^2} \cosh(fx + e)}{\dots} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] -1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(3*a^2*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4-6*a*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4+3*b^2*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4-6*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^2+10*b*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*a^(1/2)*sinh(f*x+e)^2+4*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/a^(1/2)/sinh(f*x+e)^4/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{csch}(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^5, x)

Fricas [B] time = 4.11192, size = 8128, normalized size = 60.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/16*(3*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 2*(35*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 30*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 3*a^2 - 6*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 10*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 - 15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 9*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + a^2 - 2*a*b + b^2 + 8*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^7 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-(a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*((3*a^2 - 5*a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - 5*a*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b)*sinh(f*x + e)^6 - (11*a^2 - 5*a*b)*cosh(f*x + e)^4 + (15*(3*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*a*b)*sinh(f*x + e)^4 + 4*(5*(3*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e)^2 + (15*(3*a^2 - 5*a*b)*cosh(f*x + e)^4 - 6*(11*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*a*b)*sinh(f*x + e)^2 + 3*a^2 - 5*a*b + 2*(3*(3*a^2 - 5*a*b)*cosh(f*x + e)^5 - 2*(11*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^8 + 8*a*f*cosh(f*x + e)*sinh(f*x + e)^7 + a*f*sinh(f*x + e)^8 - 4*a*f*cosh(f*x + e)^6 + 4*(7*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^6

$$\begin{aligned}
& + 6*a*f*cosh(f*x + e)^4 + 8*(7*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a*f*cosh(f*x + e)^4 - 30*a*f*cosh(f*x + e)^2 + 3*a*f) \\
&)*sinh(f*x + e)^4 - 4*a*f*cosh(f*x + e)^2 + 8*(7*a*f*cosh(f*x + e)^5 - 10*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f*x + e)^6 - 15*a*f*cosh(f*x + e)^4 + 9*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 8*(a*f*cosh(f*x + e)^7 - 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(3*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 2*(35*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 30*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 3*a^2 - 6*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 10*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 - 15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 9*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + a^2 - 2*a*b + b^2 + 8*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^7 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2 - 5*a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - 5*a*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b)*sinh(f*x + e)^6 - (11*a^2 - 5*a*b)*cosh(f*x + e)^4 + (15*(3*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*a*b)*sinh(f*x + e)^4 + 4*(5*(3*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e)^2 + (15*(3*a^2 - 5*a*b)*cosh(f*x + e)^4 - 6*(11*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*a*b)*sinh(f*x + e)^2 + 3*a^2 - 5*a*b + 2*(3*(3*a^2 - 5*a*b)*cosh(f*x + e)^5 - 2*(11*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^8 + 8*a*f*cosh(f*x + e)*sinh(f*x + e)^7 + a*f*sinh(f*x + e)^8 - 4*a*f*cosh(f*x + e)^6 + 4*(7*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 8*(7*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a*f*cosh(f*x + e)^4 - 30*a*f*cosh(f*x + e)^2 + 3*a*f)*sinh(f*x + e)^4 - 4*a*f*cosh(f*x + e)^2 + 8*(7*a*f*cosh(f*x + e)^5 - 10*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f*x + e)^6 - 15*a*f*cosh(f*x + e)^4 + 9*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 8*(a*f*cosh(f*x + e)^7 - 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e)
\end{aligned}$$

```
) *sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2), x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{csch}(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^5, x)
```

3.81 $\int \operatorname{csch}^7(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=199

$$\frac{(a-b)^2(5a+b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{16a^{3/2}f} - \frac{\coth(e+fx) \operatorname{csch}^5(e+fx) (a+b \cosh^2(e+fx)-b)^{5/2}}{6af} + \frac{(5a+b) \coth(e+fx) \operatorname{csch}^3(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{6af}$$

[Out] ((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(16*a^(3/2)*f) - ((a - b)*(5*a + b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x])/(16*a*f) + ((5*a + b)*(a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth[e + f*x]*Csch[e + f*x]^3)/(24*a*f) - ((a - b + b*Cosh[e + f*x]^2)^(5/2)*Coth[e + f*x]*Csch[e + f*x]^5)/(6*a*f)

Rubi [A] time = 0.199041, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3186, 382, 378, 377, 206}

$$\frac{(a-b)^2(5a+b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{16a^{3/2}f} - \frac{\coth(e+fx) \operatorname{csch}^5(e+fx) (a+b \cosh^2(e+fx)-b)^{5/2}}{6af} + \frac{(5a+b) \coth(e+fx) \operatorname{csch}^3(e+fx) (a+b \cosh^2(e+fx)-b)^{3/2}}{6af}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(16*a^(3/2)*f) - ((a - b)*(5*a + b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x])/(16*a*f) + ((5*a + b)*(a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth[e + f*x]*Csch[e + f*x]^3)/(24*a*f) - ((a - b + b*Cosh[e + f*x]^2)^(5/2)*Coth[e + f*x]*Csch[e + f*x]^5)/(6*a*f)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 382

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)], I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]

```

Rule 378

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
&& GtQ[q, 0] && NeQ[p, -1]

```

Rule 377

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^7(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^{3/2}}{(1-x^2)^4} dx, x, \cosh(e+fx)\right)}{f} \\
&= -\frac{(a-b+b \cosh^2(e+fx))^{5/2} \coth(e+fx) \operatorname{csch}^5(e+fx)}{6af} + \frac{(5a+b) \operatorname{Subst}\left(\int \frac{(a-b+bx^2)^{3/2}}{(1-x^2)^4} dx, x, \cosh(e+fx)\right)}{6af} \\
&= \frac{(5a+b)(a-b+b \cosh^2(e+fx))^{3/2} \coth(e+fx) \operatorname{csch}^3(e+fx)}{24af} - \frac{(a-b) \operatorname{Subst}\left(\int \frac{(a-b+bx^2)^{3/2}}{(1-x^2)^4} dx, x, \cosh(e+fx)\right)}{24af} \\
&= -\frac{(a-b)(5a+b) \sqrt{a-b+b \cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{16af} + \frac{(5a+b) \operatorname{Subst}\left(\int \frac{(a-b+bx^2)^{3/2}}{(1-x^2)^4} dx, x, \cosh(e+fx)\right)}{16af} \\
&= -\frac{(a-b)(5a+b) \sqrt{a-b+b \cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{16af} + \frac{(5a+b) \operatorname{Subst}\left(\int \frac{(a-b+bx^2)^{3/2}}{(1-x^2)^4} dx, x, \cosh(e+fx)\right)}{16af} \\
&= \frac{(a-b)^2(5a+b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{16a^{3/2}f} - \frac{(a-b)(5a+b) \sqrt{a-b+b \cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{16af}
\end{aligned}$$

Mathematica [A] time = 1.01245, size = 174, normalized size = 0.87

$$\frac{(a-b)^2(5a+b) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a} \cosh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right)}{a^{3/2}} - \frac{\coth(e+fx) \operatorname{csch}^5(e+fx) \sqrt{a+\frac{1}{2}b \cosh(2(e+fx))-\frac{b}{2}} (-4(25a^2-36ab+3b^2) \cosh(2(e+fx)) + (15a^2-22ab+3b^2) \cosh(4(e+fx)))}{24a}$$

16f

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])/a^(3/2) - (Sqrt[a - b/2 + (b*Cosh[2*(e + f*x)])/2] * (149*a^2 - 122*a*b + 9*b^2 - 4*(25*a^2 - 36*a*b + 3*b^2)*Cosh[2*(e + f*x)] + (15*a^2 - 22*a*b + 3*b^2)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^5)/(24*a))/(16*f)

Maple [B] time = 0.122, size = 569, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out]
$$-1/96*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*(30*a^{(7/2)}*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*\sinh(f*x+e)^4-44*b*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*\sinh(f*x+e)^4*a^{(5/2)}-15*a^4*\ln(((a+b)*\cosh(f*x+e)^2+2*a^{(1/2)}*(b*\cosh(f*x+e)^4+(a-b)*\cosh(f*x+e)^2)^{(1/2)}+a-b)/\sinh(f*x+e)^2)*\sinh(f*x+e)^6+27*a^3*b*\ln(((a+b)*\cosh(f*x+e)^2+2*a^{(1/2)}*(b*\cosh(f*x+e)^4+(a-b)*\cosh(f*x+e)^2)^{(1/2)}+a-b)/\sinh(f*x+e)^2)*\sinh(f*x+e)^6-9*b^2*\ln(((a+b)*\cosh(f*x+e)^2+2*a^{(1/2)}*(b*\cosh(f*x+e)^4+(a-b)*\cosh(f*x+e)^2)^{(1/2)}+a-b)/\sinh(f*x+e)^2)*\sinh(f*x+e)^6*a^2-3*b^3*\ln(((a+b)*\cosh(f*x+e)^2+2*a^{(1/2)}*(b*\cosh(f*x+e)^4+(a-b)*\cosh(f*x+e)^2)^{(1/2)}+a-b)/\sinh(f*x+e)^2)*\sinh(f*x+e)^6*a-20*a^{(7/2)}*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*\sinh(f*x+e)^2+6*b^2*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*\sinh(f*x+e)^4*a^{(3/2)}+28*b*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*\sinh(f*x+e)^2*a^{(5/2)}+16*a^{(7/2)}*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)})/\sinh(f*x+e)^6/a^{(5/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e)^2 + a)^{\frac{3}{2}} \operatorname{csch}(fx + e)^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^7, x)`

Fricas [B] time = 10.1742, size = 18294, normalized size = 91.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/96*(3*((5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^{12} + 12*(5*a^3 - 9*a^2*b \\ & + 3*a*b^2 + b^3)*\cosh(f*x + e)*\sinh(f*x + e)^{11} + (5*a^3 - 9*a^2*b \\ & + 3*a*b^2 + b^3)*\sinh(f*x + e)^{12} - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cos \\ & h(f*x + e)^{10} - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 11*(5*a^3 - 9*a^2*b + \\ & 3*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^{10} + 20*(11*(5*a^3 - 9*a^2*b \\ & + 3*a*b^2 + b^3)*\cosh(f*x + e)^3 - 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh \\ & (f*x + e))*\sinh(f*x + e)^9 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x \\ & + e)^8 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^4 + 5*a^3 - \\ & 9*a^2*b + 3*a*b^2 + b^3 - 18*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + \\ & e)^2)*\sinh(f*x + e)^8 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + \\ & e)^5 - 30*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^3 + 5*(5*a^3 - 9 \\ & *a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^7 - 20*(5*a^3 - 9*a^2*b \\ & + 3*a*b^2 + b^3)*\cosh(f*x + e)^6 + 4*(231*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^ \\ & 3)*\cosh(f*x + e)^6 - 315*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^4 \\ & - 25*a^3 + 45*a^2*b - 15*a*b^2 - 5*b^3 + 105*(5*a^3 - 9*a^2*b + 3*a*b^2 + b \\ & ^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + \\ & b^3)*\cosh(f*x + e)^7 - 63*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^5 \\ & + 35*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^3 - 5*(5*a^3 - 9*a^2*b \\ & + 3*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + 15*(5*a^3 - 9*a^2*b + 3 \\ & *a*b^2 + b^3)*\cosh(f*x + e)^4 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\co \\ & sh(f*x + e)^8 - 84*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^6 + 70*(\\ & 5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3*a*b^ \\ & 2 + b^3 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + \\ & e)^4 + 20*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^9 - 36*(5*a^3 \\ & - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^7 + 42*(5*a^3 - 9*a^2*b + 3*a*b^2 \\ & + b^3)*\cosh(f*x + e)^5 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e \\ &)^3 + 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + \\ & 5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(\\ & f*x + e)^2 + 6*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^{10} - 45* \\ & (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^8 + 70*(5*a^3 - 9*a^2*b + 3 \\ & *a*b^2 + b^3)*\cosh(f*x + e)^6 - 50*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f \\ & *x + e)^4 - 5*a^3 + 9*a^2*b - 3*a*b^2 - b^3 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 \\ & + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 12*((5*a^3 - 9*a^2*b + 3*a*b^2 + \\ & b^3)*\cosh(f*x + e)^{11} - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^ \\ & 9 + 10*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^7 - 10*(5*a^3 - 9*a^ \\ & 2*b + 3*a*b^2 + b^3)*\cosh(f*x + e)^5 + 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)* \\ & \cosh(f*x + e)^3 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x \\ & + e))*\sqrt{a}*\log(-((a + b)*\cosh(f*x + e)^4 + 4*(a + b)*\cosh(f*x + e)*\sinh \\ & (f*x + e)^3 + (a + b)*\sinh(f*x + e)^4 + 2*(3*a - b)*\cosh(f*x + e)^2 + 2*(3* \\ & (a + b)*\cosh(f*x + e)^2 + 3*a - b)*\sinh(f*x + e)^2 + 2*\sqrt{2}*(\cosh(f*x + \\ & e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1))*\sqrt{a}*\sqrt{(b \\ & *\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f \end{aligned}$$

$$\begin{aligned}
& *x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*((a + b) * \cosh(f*x + e)^3 + (3 \\
& *a - b) * \cosh(f*x + e)) * \sinh(f*x + e) + a + b) / (\cosh(f*x + e)^4 + 4 * \cosh(f*x \\
& + e) * \sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2 * (3 * \cosh(f*x + e)^2 - 1) * \sinh(f*x \\
& x + e)^2 - 2 * \cosh(f*x + e)^2 + 4 * (\cosh(f*x + e)^3 - \cosh(f*x + e)) * \sinh(f*x \\
& + e) + 1)) - 2 * \sqrt{2} * ((15*a^3 - 22*a^2*b + 3*a*b^2) * \cosh(f*x + e)^{10} + 1 \\
& 0 * (15*a^3 - 22*a^2*b + 3*a*b^2) * \cosh(f*x + e) * \sinh(f*x + e)^9 + (15*a^3 - 2 \\
& 2*a^2*b + 3*a*b^2) * \sinh(f*x + e)^{10} - (85*a^3 - 122*a^2*b + 9*a*b^2) * \cosh(f \\
& *x + e)^8 - (85*a^3 - 122*a^2*b + 9*a*b^2 - 45 * (15*a^3 - 22*a^2*b + 3*a*b^2 \\
&) * \cosh(f*x + e)^2) * \sinh(f*x + e)^8 + 8 * (15 * (15*a^3 - 22*a^2*b + 3*a*b^2) * \cosh \\
& sh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2) * \cosh(f*x + e)) * \sinh(f*x + e) \\
& ^7 + 2 * (99*a^3 - 50*a^2*b + 3*a*b^2) * \cosh(f*x + e)^6 + 2 * (105 * (15*a^3 - 22* \\
& a^2*b + 3*a*b^2) * \cosh(f*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 - 14 * (85*a^3 \\
& - 122*a^2*b + 9*a*b^2) * \cosh(f*x + e)^2) * \sinh(f*x + e)^6 + 4 * (63 * (15*a^3 - \\
& 22*a^2*b + 3*a*b^2) * \cosh(f*x + e)^5 - 14 * (85*a^3 - 122*a^2*b + 9*a*b^2) * \cosh \\
& h(f*x + e)^3 + 3 * (99*a^3 - 50*a^2*b + 3*a*b^2) * \cosh(f*x + e)) * \sinh(f*x + e) \\
& ^5 + 2 * (99*a^3 - 50*a^2*b + 3*a*b^2) * \cosh(f*x + e)^4 + 2 * (105 * (15*a^3 - 22* \\
& a^2*b + 3*a*b^2) * \cosh(f*x + e)^6 - 35 * (85*a^3 - 122*a^2*b + 9*a*b^2) * \cosh(f \\
& *x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 + 15 * (99*a^3 - 50*a^2*b + 3*a*b^2) * \\
& \cosh(f*x + e)^2) * \sinh(f*x + e)^4 + 8 * (15 * (15*a^3 - 22*a^2*b + 3*a*b^2) * \cosh \\
& (f*x + e)^7 - 7 * (85*a^3 - 122*a^2*b + 9*a*b^2) * \cosh(f*x + e)^5 + 5 * (99*a^3 \\
& - 50*a^2*b + 3*a*b^2) * \cosh(f*x + e)^3 + (99*a^3 - 50*a^2*b + 3*a*b^2) * \cosh(\\
& f*x + e)) * \sinh(f*x + e)^3 + 15*a^3 - 22*a^2*b + 3*a*b^2 - (85*a^3 - 122*a^2 \\
& *b + 9*a*b^2) * \cosh(f*x + e)^2 + (45 * (15*a^3 - 22*a^2*b + 3*a*b^2) * \cosh(f*x \\
& + e)^8 - 28 * (85*a^3 - 122*a^2*b + 9*a*b^2) * \cosh(f*x + e)^6 + 30 * (99*a^3 - 5 \\
& 0*a^2*b + 3*a*b^2) * \cosh(f*x + e)^4 - 85*a^3 + 122*a^2*b - 9*a*b^2 + 12 * (99* \\
& a^3 - 50*a^2*b + 3*a*b^2) * \cosh(f*x + e)^2) * \sinh(f*x + e)^2 + 2 * (5 * (15*a^3 - \\
& 22*a^2*b + 3*a*b^2) * \cosh(f*x + e)^9 - 4 * (85*a^3 - 122*a^2*b + 9*a*b^2) * \cosh \\
& h(f*x + e)^7 + 6 * (99*a^3 - 50*a^2*b + 3*a*b^2) * \cosh(f*x + e)^5 + 4 * (99*a^3 \\
& - 50*a^2*b + 3*a*b^2) * \cosh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2) * \cosh \\
& (f*x + e)) * \sinh(f*x + e) * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a \\
& - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) \\
& / (a^2 * f * \cosh(f*x + e)^{12} + 12 * a^2 * f * \cosh(f*x + e) * \sinh(f*x + e)^{11} + a^2 * f * \\
& \sinh(f*x + e)^{12} - 6 * a^2 * f * \cosh(f*x + e)^{10} + 15 * a^2 * f * \cosh(f*x + e)^8 + 6 * \\
& (11 * a^2 * f * \cosh(f*x + e)^2 - a^2 * f) * \sinh(f*x + e)^{10} + 20 * (11 * a^2 * f * \cosh(f*x \\
& + e)^3 - 3 * a^2 * f * \cosh(f*x + e)) * \sinh(f*x + e)^9 - 20 * a^2 * f * \cosh(f*x + e)^6 \\
& + 15 * (33 * a^2 * f * \cosh(f*x + e)^4 - 18 * a^2 * f * \cosh(f*x + e)^2 + a^2 * f) * \sinh(f* \\
& x + e)^8 + 24 * (33 * a^2 * f * \cosh(f*x + e)^5 - 30 * a^2 * f * \cosh(f*x + e)^3 + 5 * a^2 * \\
& f * \cosh(f*x + e)) * \sinh(f*x + e)^7 + 15 * a^2 * f * \cosh(f*x + e)^4 + 4 * (231 * a^2 * f * \\
& \cosh(f*x + e)^6 - 315 * a^2 * f * \cosh(f*x + e)^4 + 105 * a^2 * f * \cosh(f*x + e)^2 - 5 \\
& * a^2 * f) * \sinh(f*x + e)^6 + 24 * (33 * a^2 * f * \cosh(f*x + e)^7 - 63 * a^2 * f * \cosh(f*x \\
& + e)^5 + 35 * a^2 * f * \cosh(f*x + e)^3 - 5 * a^2 * f * \cosh(f*x + e)) * \sinh(f*x + e)^5 \\
& - 6 * a^2 * f * \cosh(f*x + e)^2 + 15 * (33 * a^2 * f * \cosh(f*x + e)^8 - 84 * a^2 * f * \cosh(f* \\
& x + e)^6 + 70 * a^2 * f * \cosh(f*x + e)^4 - 20 * a^2 * f * \cosh(f*x + e)^2 + a^2 * f) * \sinh \\
& h(f*x + e)^4 + 20 * (11 * a^2 * f * \cosh(f*x + e)^9 - 36 * a^2 * f * \cosh(f*x + e)^7 + 42 \\
& * a^2 * f * \cosh(f*x + e)^5 - 20 * a^2 * f * \cosh(f*x + e)^3 + 3 * a^2 * f * \cosh(f*x + e)) *
\end{aligned}$$

$$\begin{aligned}
& \sinh(f*x + e)^3 + a^2*f + 6*(11*a^2*f*cosh(f*x + e)^10 - 45*a^2*f*cosh(f*x \\
& + e)^8 + 70*a^2*f*cosh(f*x + e)^6 - 50*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cos \\
& h(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 12*(a^2*f*cosh(f*x + e)^11 - 5*a^2* \\
& f*cosh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^7 - 10*a^2*f*cosh(f*x + e)^5 + 5 \\
& *a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), -1/48*(3*((5* \\
& a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^12 + 12*(5*a^3 - 9*a^2*b + 3*a \\
& *b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (5*a^3 - 9*a^2*b + 3*a*b^2 + b \\
& ^3)*sinh(f*x + e)^12 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^10 \\
& - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 \\
&)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b \\
& ^3)*cosh(f*x + e)^3 - 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*si \\
& nh(f*x + e)^9 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 + 15*(\\
& 33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3* \\
& a*b^2 + b^3 - 18*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f* \\
& x + e)^8 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 - 30*(5 \\
& *a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 + 5*(5*a^3 - 9*a^2*b + 3*a* \\
& b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^7 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + \\
& b^3)*cosh(f*x + e)^6 + 4*(231*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + \\
& e)^6 - 315*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 - 25*a^3 + 45 \\
& *a^2*b - 15*a*b^2 - 5*b^3 + 105*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x \\
& + e)^2)*sinh(f*x + e)^6 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x \\
& + e)^7 - 63*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 + 35*(5*a^3 \\
& - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + \\
& b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3) \\
& *cosh(f*x + e)^4 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 \\
& - 84*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 70*(5*a^3 - 9*a^2 \\
& *b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 20* \\
& (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 20*(11 \\
& *(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^9 - 36*(5*a^3 - 9*a^2*b + \\
& 3*a*b^2 + b^3)*cosh(f*x + e)^7 + 42*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(\\
& f*x + e)^5 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(5*a^ \\
& 3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 5*a^3 - 9*a^2 \\
& *b + 3*a*b^2 + b^3 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2 + \\
& 6*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^10 - 45*(5*a^3 - 9*a^ \\
& 2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 + 70*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3) \\
& *cosh(f*x + e)^6 - 50*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 - 5 \\
& *a^3 + 9*a^2*b - 3*a*b^2 - b^3 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(\\
& f*x + e)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f* \\
& x + e)^11 - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^9 + 10*(5*a^3 \\
& - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 - 10*(5*a^3 - 9*a^2*b + 3*a*b^2 \\
& + b^3)*cosh(f*x + e)^5 + 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e) \\
& ^3 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(- \\
& a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f \\
& *x + e)^2 + 1))*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - \\
& b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b
\end{aligned}$$

$$\begin{aligned}
& * \cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + \\
& 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + \\
& e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) \\
& + \sqrt{2}*((15*a^3 - 22*a^2*b + 3*a*b^2)*\cosh(f*x + e)^{10} + 10*(15*a^3 - 22 \\
& *a^2*b + 3*a*b^2)*\cosh(f*x + e)*\sinh(f*x + e)^9 + (15*a^3 - 22*a^2*b + 3*a* \\
& b^2)*\sinh(f*x + e)^{10} - (85*a^3 - 122*a^2*b + 9*a*b^2)*\cosh(f*x + e)^8 - (8 \\
& 5*a^3 - 122*a^2*b + 9*a*b^2 - 45*(15*a^3 - 22*a^2*b + 3*a*b^2)*\cosh(f*x + e \\
&)^2)*\sinh(f*x + e)^8 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*\cosh(f*x + e)^3 \\
& - (85*a^3 - 122*a^2*b + 9*a*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^7 + 2*(99*a^3 \\
& - 50*a^2*b + 3*a*b^2)*\cosh(f*x + e)^6 + 2*(105*(15*a^3 - 22*a^2*b + 3*a*b^ \\
& 2)*\cosh(f*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 - 14*(85*a^3 - 122*a^2*b + \\
& 9*a*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 4*(63*(15*a^3 - 22*a^2*b + 3*a \\
& *b^2)*\cosh(f*x + e)^5 - 14*(85*a^3 - 122*a^2*b + 9*a*b^2)*\cosh(f*x + e)^3 + \\
& 3*(99*a^3 - 50*a^2*b + 3*a*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(99*a^3 \\
& - 50*a^2*b + 3*a*b^2)*\cosh(f*x + e)^4 + 2*(105*(15*a^3 - 22*a^2*b + 3*a*b^ \\
& 2)*\cosh(f*x + e)^6 - 35*(85*a^3 - 122*a^2*b + 9*a*b^2)*\cosh(f*x + e)^4 + 99 \\
& *a^3 - 50*a^2*b + 3*a*b^2 + 15*(99*a^3 - 50*a^2*b + 3*a*b^2)*\cosh(f*x + e)^ \\
& 2)*\sinh(f*x + e)^4 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*\cosh(f*x + e)^7 - \\
& 7*(85*a^3 - 122*a^2*b + 9*a*b^2)*\cosh(f*x + e)^5 + 5*(99*a^3 - 50*a^2*b + 3 \\
& *a*b^2)*\cosh(f*x + e)^3 + (99*a^3 - 50*a^2*b + 3*a*b^2)*\cosh(f*x + e))*\sinh \\
& (f*x + e)^3 + 15*a^3 - 22*a^2*b + 3*a*b^2 - (85*a^3 - 122*a^2*b + 9*a*b^2)* \\
& \cosh(f*x + e)^2 + (45*(15*a^3 - 22*a^2*b + 3*a*b^2)*\cosh(f*x + e)^8 - 28*(8 \\
& 5*a^3 - 122*a^2*b + 9*a*b^2)*\cosh(f*x + e)^6 + 30*(99*a^3 - 50*a^2*b + 3*a* \\
& b^2)*\cosh(f*x + e)^4 - 85*a^3 + 122*a^2*b - 9*a*b^2 + 12*(99*a^3 - 50*a^2*b \\
& + 3*a*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 2*(5*(15*a^3 - 22*a^2*b + 3* \\
& a*b^2)*\cosh(f*x + e)^9 - 4*(85*a^3 - 122*a^2*b + 9*a*b^2)*\cosh(f*x + e)^7 + \\
& 6*(99*a^3 - 50*a^2*b + 3*a*b^2)*\cosh(f*x + e)^5 + 4*(99*a^3 - 50*a^2*b + 3 \\
& *a*b^2)*\cosh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2)*\cosh(f*x + e))*\sin \\
& h(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f* \\
& x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a^2*f*\cosh(f \\
& *x + e)^{12} + 12*a^2*f*\cosh(f*x + e)*\sinh(f*x + e)^{11} + a^2*f*\sinh(f*x + e)^ \\
& 12 - 6*a^2*f*\cosh(f*x + e)^{10} + 15*a^2*f*\cosh(f*x + e)^8 + 6*(11*a^2*f*\cosh \\
& (f*x + e)^2 - a^2*f)*\sinh(f*x + e)^{10} + 20*(11*a^2*f*\cosh(f*x + e)^3 - 3*a^ \\
& 2*f*\cosh(f*x + e))*\sinh(f*x + e)^9 - 20*a^2*f*\cosh(f*x + e)^6 + 15*(33*a^2* \\
& f*\cosh(f*x + e)^4 - 18*a^2*f*\cosh(f*x + e)^2 + a^2*f)*\sinh(f*x + e)^8 + 24* \\
& (33*a^2*f*\cosh(f*x + e)^5 - 30*a^2*f*\cosh(f*x + e)^3 + 5*a^2*f*\cosh(f*x + e \\
&))*\sinh(f*x + e)^7 + 15*a^2*f*\cosh(f*x + e)^4 + 4*(231*a^2*f*\cosh(f*x + e)^ \\
& 6 - 315*a^2*f*\cosh(f*x + e)^4 + 105*a^2*f*\cosh(f*x + e)^2 - 5*a^2*f)*\sinh(f \\
& *x + e)^6 + 24*(33*a^2*f*\cosh(f*x + e)^7 - 63*a^2*f*\cosh(f*x + e)^5 + 35*a^ \\
& 2*f*\cosh(f*x + e)^3 - 5*a^2*f*\cosh(f*x + e))*\sinh(f*x + e)^5 - 6*a^2*f*\cosh \\
& (f*x + e)^2 + 15*(33*a^2*f*\cosh(f*x + e)^8 - 84*a^2*f*\cosh(f*x + e)^6 + 70* \\
& a^2*f*\cosh(f*x + e)^4 - 20*a^2*f*\cosh(f*x + e)^2 + a^2*f)*\sinh(f*x + e)^4 + \\
& 20*(11*a^2*f*\cosh(f*x + e)^9 - 36*a^2*f*\cosh(f*x + e)^7 + 42*a^2*f*\cosh(f* \\
& x + e)^5 - 20*a^2*f*\cosh(f*x + e)^3 + 3*a^2*f*\cosh(f*x + e))*\sinh(f*x + e)^ \\
& 3 + a^2*f + 6*(11*a^2*f*\cosh(f*x + e)^{10} - 45*a^2*f*\cosh(f*x + e)^8 + 70*a^
\end{aligned}$$

```
2*f*cosh(f*x + e)^6 - 50*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 -
a^2*f)*sinh(f*x + e)^2 + 12*(a^2*f*cosh(f*x + e)^11 - 5*a^2*f*cosh(f*x + e
)^9 + 10*a^2*f*cosh(f*x + e)^7 - 10*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*
x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{csch}(fx + e)^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^7, x)
```

$$3.82 \quad \int \sinh^4(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=367

$$\frac{(a^2 - 11ab + 8b^2) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) - 2(a - 2b)(a^2 + 4ab - 4b^2)}{35bf \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

```
[Out] ((a^2 - 11*a*b + 8*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b*f) + (2*(4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]*Sinh[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2])/(7*f) + (2*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 11*a*b + 8*b^2)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(35*b^2*f)
```

Rubi [A] time = 0.470834, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3188, 477, 582, 531, 418, 492, 411}

$$\frac{2(a - 2b)(a^2 + 4ab - 4b^2) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35b^2f} + \frac{(a^2 - 11ab + 8b^2) \sinh(e + fx) \cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a^2 - 11*a*b + 8*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b*f) + (2*(4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]*Sinh[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2])/(7*f) + (2*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 11*a*b + 8*b^2)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(35*b^2*f)
```

$$\frac{(a - 2b)(a^2 + 4ab - 4b^2)\sqrt{a + b\sinh[e + fx]^2}\tanh[e + fx]}{(35b^2f)}$$

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^ (q_.), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^(m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^ (q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m
- n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt
[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
```

$eQ[\{a, b, c, d\}, x] \&\& \text{PosQ}[d/c] \&\& \text{PosQ}[b/a] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

Rule 492

$\text{Int}[(x_)^2/(\text{Sqrt}[(a_)+(b_)*(x_)^2]*\text{Sqrt}[(c_)+(d_)*(x_)^2]), x_Symbol] \text{ :> } \text{Simp}[(x*\text{Sqrt}[a + b*x^2])/(b*\text{Sqrt}[c + d*x^2]), x] - \text{Dist}[c/b, \text{Int}[\text{Sqrt}[a + b*x^2]/(c + d*x^2)^{(3/2)}, x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_)+(b_)*(x_)^2]/((c_)+(d_)*(x_)^2)^{(3/2)}, x_Symbol] \text{ :> } \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] \text{ /; } \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned}
\int \sinh^4(e+fx) (a+b\sinh^2(e+fx))^{3/2} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{b \cosh(e+fx) \sinh^5(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{7f} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4(a+bx^2)^{3/2}}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{2(4a-3b) \cosh(e+fx) \sinh^3(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{35f} + \frac{b \cosh(e+fx) \sinh^5(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{7f} \\
&= \frac{(a^2-11ab+8b^2) \cosh(e+fx) \sinh(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2(4a-3b) \cosh(e+fx) \sinh^3(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{35f} \\
&= \frac{(a^2-11ab+8b^2) \cosh(e+fx) \sinh(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2(4a-3b) \cosh(e+fx) \sinh^3(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{35f} \\
&= \frac{(a^2-11ab+8b^2) \cosh(e+fx) \sinh(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2(4a-3b) \cosh(e+fx) \sinh^3(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{35f} \\
&= \frac{(a^2-11ab+8b^2) \cosh(e+fx) \sinh(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2(4a-3b) \cosh(e+fx) \sinh^3(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{35f}
\end{aligned}$$

Mathematica [C] time = 2.87267, size = 262, normalized size = 0.71

$$-64ia(3a^2b + 2a^3 - 13ab^2 + 8b^3) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2}b \sinh(2(e+fx)) (b(144a^2 - 480ab + 80a^2b + 299b^2) \cosh[2(e+fx)] + 2(26a - 27b)b^2 \cosh[4(e+fx)])$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((128*I)*a*(a^3 + 2*a^2*b - 12*a*b^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] - (64*I)*a*(2*a^3 + 3*a^2*b - 13*a*b^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(32*a^3 - 496*a^2*b + 684*a*b^2 - 250*b^3 + b*(144*a^2 - 480*a*b + 299*b^2)*Cosh[2*(e + f*x)] + 2*(26*a - 27*b)*b^2*Cosh[4*(e + f*x)])

$$+ 5*b^3*\text{Cosh}[6*(e + f*x)]*\text{Sinh}[2*(e + f*x)]/(2240*b^2*f*\text{Sqrt}[2*a - b + b*\text{Cosh}[2*(e + f*x)])]$$

Maple [A] time = 0.074, size = 743, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(f*x+e)^4*(a+b*\sinh(f*x+e)^2)^{(3/2)}, x)$

[Out] $\frac{1}{35}*(5*(-1/a*b)^{(1/2)}*b^3*\sinh(f*x+e)*\cosh(f*x+e)^8+(13*(-1/a*b)^{(1/2)}*a*b^2-21*(-1/a*b)^{(1/2)}*b^3)*\cosh(f*x+e)^6*\sinh(f*x+e)+(9*(-1/a*b)^{(1/2)}*a^2*b-43*(-1/a*b)^{(1/2)}*a*b^2+35*(-1/a*b)^{(1/2)}*b^3)*\cosh(f*x+e)^4*\sinh(f*x+e)+((-1/a*b)^{(1/2)}*a^3-20*(-1/a*b)^{(1/2)}*a^2*b+38*(-1/a*b)^{(1/2)}*a*b^2-19*(-1/a*b)^{(1/2)}*b^3)*\cosh(f*x+e)^2*\sinh(f*x+e)+(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*a^3+15*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*a^2*b-32*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*a*b^2+16*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*b^3-2*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*a^3-4*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*a^2*b+24*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*a*b^2-16*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*b^3)/b/(-1/a*b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \sinh(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(f*x+e)^4*(a+b*\sinh(f*x+e)^2)^{(3/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^6 + a \sinh(fx + e)^4\right) \sqrt{b \sinh(fx + e)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral((b*sinh(f*x + e)^6 + a*sinh(f*x + e)^4)*sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}} \sinh(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^4, x)

3.83 $\int \sinh^2(e + fx) \left(a + b \sinh^2(e + fx)\right)^{3/2} dx$

Optimal. Leaf size=236

$$\frac{ia(3a - 4b)(a - b)\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1} \text{EllipticF}\left(ie + ifx, \frac{b}{a}\right) - i(3a^2 - 13ab + 8b^2)\sqrt{a + b \sinh^2(e + fx)}E\left(ie + ifx \left|\frac{b}{a}\right.\right)}{15bf\sqrt{a + b \sinh^2(e + fx)} - 15bf\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1}}$$

```
[Out] ((3*a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f)
+ (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*f) - ((I
/15)*(3*a^2 - 13*a*b + 8*b^2)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e
+ f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/15)*a*(3*a - 4*b)*(
a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sq
rt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.325431, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$-\frac{i(3a^2 - 13ab + 8b^2)\sqrt{a + b \sinh^2(e + fx)}E\left(ie + ifx \left|\frac{b}{a}\right.\right)}{15bf\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1}} + \frac{\sinh(e + fx) \cosh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{5f} + \frac{(3a - 4b)(a - b)\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1} \text{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{15bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] ((3*a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f)
+ (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*f) - ((I
/15)*(3*a^2 - 13*a*b + 8*b^2)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e
+ f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/15)*a*(3*a - 4*b)*(
a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sq
rt[a + b*Sinh[e + f*x]^2])
```

Rule 3170

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)])^(2), x_Symbol] := -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
```

```
f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
2*b*p))*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]^2)/Sqrt[(a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_.)]^2], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Ssin[e + f*x]^2], x],
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Ssin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2], x_Symbol] := Dist[Sqrt[a
+ b*Ssin[e + f*x]^2]/Sqrt[1 + (b*Ssin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Ssin[e +
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2], x_Symbol] := Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)]/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2], x_Symbol] := Dist[Sqrt[
1 + (b*Ssin[e + f*x]^2)/a]/Sqrt[a + b*Ssin[e + f*x]^2], Int[1/Sqrt[1 + (b*Ssin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2], x_Symbol] := Simp[(1*El
lipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh^2(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{\cosh(e + fx) \sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{5f} - \frac{1}{5} \int (a - (3a - 4b) \sinh^2(e + fx))^{3/2} dx \\
&= \frac{(3a - 4b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15f} + \frac{\cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{5} \\
&= \frac{(3a - 4b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15f} + \frac{\cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{5} \\
&= \frac{(3a - 4b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15f} + \frac{\cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{5} \\
&= \frac{(3a - 4b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15f} + \frac{\cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{5}
\end{aligned}$$

Mathematica [A] time = 1.36271, size = 213, normalized size = 0.9

$$\frac{16ia(3a^2 - 7ab + 4b^2) \sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} \text{EllipticF}\left(i(e + fx), \frac{b}{a}\right) + \sqrt{2}b \sinh(2(e + fx)) (48a^2 + 4b(9a - 7b) \cosh(2(e + fx)))}{240bf \sqrt{2a + b \cosh(2(e + fx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((-16*I)*a*(3*a^2 - 13*a*b + 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 7*a*b + 4*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48*a^2 - 68*a*b + 25*b^2 + 4*(9*a - 7*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x)])*Sinh[2*(e + f*x)]/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.071, size = 535, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out]
$$\begin{aligned} & -1/15*(-3*(-1/a*b)^{(1/2)}*b^2*\sinh(f*x+e)*\cosh(f*x+e)^6+(-9*(-1/a*b)^{(1/2)}*a \\ & *b+10*(-1/a*b)^{(1/2)}*b^2)*\cosh(f*x+e)^4*\sinh(f*x+e)+(-6*(-1/a*b)^{(1/2)}*a^2+ \\ & 13*(-1/a*b)^{(1/2)}*a*b-7*(-1/a*b)^{(1/2)}*b^2)*\cosh(f*x+e)^2*\sinh(f*x+e)+9*a^2 \\ & *(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x \\ & +e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-17*a*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh \\ & (f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b+8*(b/ \\ & a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)* \\ & (-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^2-3*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f \\ & *x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a^2+13*(b/ \\ & a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)* \\ & (-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a*b-8*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f \\ & *x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^2)/(-1/a \\ & *b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^{\frac{3}{2}} \sinh^2(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\left(b \sinh^4(fx + e) + a \sinh^2(fx + e) \right) \sqrt{b \sinh^2(fx + e) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] integral((b*sinh(f*x + e)^4 + a*sinh(f*x + e)^2)*sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \sinh(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^2, x)

3.84 $\int (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=174

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a} + 1}\text{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{3f\sqrt{a + b\sinh^2(e + fx)}} + \frac{b\sinh(e + fx)\cosh(e + fx)\sqrt{a + b\sinh^2(e + fx)}}{3f} - \frac{2i(2a - b)\sqrt{a + b\sinh^2(e + fx)}}{3f}$$

[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.193697, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{b\sinh(e + fx)\cosh(e + fx)\sqrt{a + b\sinh^2(e + fx)}}{3f} + \frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a} + 1}F\left(ie + ifx \middle| \frac{b}{a}\right)}{3f\sqrt{a + b\sinh^2(e + fx)}} - \frac{2i(2a - b)\sqrt{a + b\sinh^2(e + fx)}}{3f\sqrt{\frac{b\sinh^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3180

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && GtQ[p, 1]

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{1}{3} \int \frac{a(3a - b) + 2(2a - b)b \sinh^2(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx \\
&= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{1}{3}(a(a - b)) \int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx \\
&= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{\left(2(2a - b) \sqrt{a + b \sinh^2(e + fx)}\right) \int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx}{3 \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}} \\
&= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{2i(2a - b)E\left(ie + ifx \left| \frac{b}{a} \right.\right) \sqrt{a + b \sinh^2(e + fx)}}{3f \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.679463, size = 169, normalized size = 0.97

$$\frac{2i\sqrt{2a(a-b)}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)-4i\sqrt{2a(2a-b)}}{6f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.069, size = 416, normalized size = 2.4

$$\frac{1}{3f \cosh(fx + e)} \left(\sqrt{-\frac{b}{a}} b^2 \sinh(fx + e) (\cosh(fx + e))^4 + \left(\sqrt{-\frac{b}{a}} ab - \sqrt{-\frac{b}{a}} b^2 \right) (\cosh(fx + e))^2 \sinh(fx + e) + 3a^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] $\frac{1}{3} \left((-1/a*b)^{(1/2)} * b^2 * \sinh(f*x+e) * \cosh(f*x+e)^4 + (-1/a*b)^{(1/2)} * a*b - (-1/a*b)^{(1/2)} * b^2 * \cosh(f*x+e)^2 * \sinh(f*x+e) + 3*a^2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) \right) - 5*a * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^2 + 2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a*b - 2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^2 \right) / (-1/a*b)^{(1/2)} / \cosh(f*x+e) / (a+b*sinh(f*x+e)^2)^{(1/2)} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.85 $\int \operatorname{csch}^2(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=204

$$\frac{2b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{(a + b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f}$$

[Out] -((a*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - ((a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f

Rubi [A] time = 0.211493, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3188, 474, 531, 418, 492, 411}

$$\frac{(a + b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{a \operatorname{coth}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{2b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] -((a*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - ((a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f

Rule 3188

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^(p/2)]

p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 474

Int[((e_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right) \operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{x^2 \sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{a \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right)}{f} \\
&= -\frac{a \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{\left(2ab \sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right)}{f} \\
&= -\frac{a \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{2bF\left(\tan^{-1}(\sinh(e+fx))\right) - \frac{b}{a} \operatorname{arctanh}\left(\frac{\sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f \sqrt{\operatorname{sech}^2(e+fx)}} \\
&= -\frac{a \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{f} - \frac{(a+b)E\left(\tan^{-1}(\sinh(e+fx))\right) - \frac{b}{a} \operatorname{arctanh}\left(\frac{\sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f \sqrt{\operatorname{sech}^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 1.04665, size = 155, normalized size = 0.76

$$\frac{a \left(-2i(a-b) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2} \operatorname{coth}(e+fx) (2a+b \cosh(2(e+fx))-b) + 2i(a+b) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \right)}{2f \sqrt{2a+b \cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] $-(a*(\operatorname{Sqrt}[2]*(2*a - b + b*\operatorname{Cosh}[2*(e + f*x)]))*\operatorname{Coth}[e + f*x] + (2*I)*(a + b)*\operatorname{Sqrt}[(2*a - b + b*\operatorname{Cosh}[2*(e + f*x)])]/a)*\operatorname{EllipticE}[I*(e + f*x), b/a] - (2*I)*(a - b)*\operatorname{Sqrt}[(2*a - b + b*\operatorname{Cosh}[2*(e + f*x)])]/a)*\operatorname{EllipticF}[I*(e + f*x), b/a])/ (2*f*\operatorname{Sqrt}[2*a - b + b*\operatorname{Cosh}[2*(e + f*x)])]$

Maple [A] time = 0.079, size = 243, normalized size = 1.2

$$\frac{1}{\cosh(fx+e) \sinh(fx+e) f} \left(\sinh(fx+e) \sqrt{(\cosh(fx+e))^2} \sqrt{\frac{b(\cosh(fx+e))^2}{a} + \frac{a-b}{a} b} \left(a \operatorname{EllipticF}\left(\sinh(fx+e), \frac{b}{a}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out] $(\sinh(f*x+e)*(\cosh(f*x+e)^2)^{(1/2)}*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*b*(a*E$
 $llipticF(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-b*EllipticF(\sinh(f*x+e)*(-$
 $1/a*b)^{(1/2)},(a/b)^{(1/2)})+EllipticE(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2))$
 $*a+b*EllipticE(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)))-(-1/a*b)^{(1/2)}*a*b*$
 $\cosh(f*x+e)^4+(-(-1/a*b)^{(1/2)}*a^2+(-1/a*b)^{(1/2)}*a*b)*\cosh(f*x+e)^2)/\sinh($
 $f*x+e)/(-1/a*b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(x+e) + a \right)^{\frac{3}{2}} \operatorname{csch}^2(x+e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \operatorname{csch}^2(x+e) \sinh^2(x+e) + a \operatorname{csch}^2(x+e)\right) \sqrt{b \sinh^2(x+e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*csch(f*x + e)^2*sinh(f*x + e)^2 + a*csch(f*x + e)^2)*sqrt(b*sin`
`h(f*x + e)^2 + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{csch}(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^2, x)`

3.86 $\int \operatorname{csch}^4(e + fx) \left(a + b \sinh^2(e + fx)\right)^{3/2} dx$

Optimal. Leaf size=267

$$\frac{b(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}$$

[Out] (2*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (a*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)

Rubi [A] time = 0.305941, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3188, 474, 583, 531, 418, 492, 411}

$$\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{a\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (2*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (a*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)

Rule 3188

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 474

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \operatorname{csch}^4(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx = \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{(a + bx^2)^{3/2}}{x^4 \sqrt{1 + x^2}} dx, x, \sinh(e + fx)\right)}{f}$$

$$= -\frac{a \operatorname{coth}(e + fx) \operatorname{csch}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right)}{3f}$$

$$= \frac{2(a - 2b) \operatorname{coth}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{a \operatorname{coth}(e + fx) \operatorname{csch}^2(e + fx)}{3f}$$

$$= \frac{2(a - 2b) \operatorname{coth}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{a \operatorname{coth}(e + fx) \operatorname{csch}^2(e + fx)}{3f}$$

$$= \frac{2(a - 2b) \operatorname{coth}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{a \operatorname{coth}(e + fx) \operatorname{csch}^2(e + fx)}{3f}$$

$$= \frac{2(a - 2b) \operatorname{coth}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{a \operatorname{coth}(e + fx) \operatorname{csch}^2(e + fx)}{3f}$$

Mathematica [C] time = 4.06128, size = 213, normalized size = 0.8

$$\frac{-2i(2a^2 - 5ab + 3b^2) \sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} \operatorname{EllipticF}\left(i(e + fx), \frac{b}{a}\right) + \frac{\operatorname{coth}(e + fx) \operatorname{csch}^2(e + fx) (2(2a^2 - 7ab + 4b^2) \cosh(2(e + fx)) - 8a^2 + b(a - 2b))}{\sqrt{2}}}{6f \sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] (((-8*a^2 + 13*a*b - 6*b^2 + 2*(2*a^2 - 7*a*b + 4*b^2)*Cosh[2*(e + f*x)] +
(a - 2*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (4*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a]/(6*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.087, size = 454, normalized size = 1.7

$$\frac{1}{3 (\sinh (fx + e))^3 \cosh (fx + e) f} \left(2 \sqrt{-\frac{b}{a}} ab (\sinh (fx + e))^6 - 4 \sqrt{-\frac{b}{a}} b^2 (\sinh (fx + e))^6 + b \sqrt{\frac{a + b (\sinh (fx + e))}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+
b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2), (a/b)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2
*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b*sinh(f*x+e)^3+4*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-3*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2-5*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2)/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{csch} (fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \operatorname{csch}(fx + e)^4 \sinh(fx + e)^2 + a \operatorname{csch}(fx + e)^4\right) \sqrt{b \sinh(fx + e)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral((b*csch(f*x + e)^4*sinh(f*x + e)^2 + a*csch(f*x + e)^4)*sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}} \operatorname{csch}(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^4, x)

3.87 $\int (a + b \sinh^2(c + dx))^{5/2} dx$

Optimal. Leaf size=232

$$\frac{4ia(a-b)(2a-b)\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}\operatorname{EllipticF}\left(ic+idx,\frac{b}{a}\right)}{15d\sqrt{a+b\sinh^2(c+dx)}} - \frac{i(23a^2-23ab+8b^2)\sqrt{a+b\sinh^2(c+dx)}E\left(ic+idx\left|\frac{b}{a}\right.\right)}{15d\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}}$$

[Out] (4*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x]*Sqrt[a + b*Sinh[c + d*x]^2])/(15*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^(3/2))/(5*d) - ((I/15)*(23*a^2 - 23*a*b + 8*b^2)*EllipticE[I*c + I*d*x, b/a]*Sqrt[a + b*Sinh[c + d*x]^2])/(d*Sqrt[1 + (b*Sinh[c + d*x]^2)/a]) + (((4*I)/15)*a*(a - b)*(2*a - b)*EllipticF[I*c + I*d*x, b/a]*Sqrt[1 + (b*Sinh[c + d*x]^2)/a])/(d*Sqrt[a + b*Sinh[c + d*x]^2])

Rubi [A] time = 0.303784, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3180, 3170, 3172, 3178, 3177, 3183, 3182}

$$-\frac{i(23a^2-23ab+8b^2)\sqrt{a+b\sinh^2(c+dx)}E\left(ic+idx\left|\frac{b}{a}\right.\right)}{15d\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}} + \frac{b\sinh(c+dx)\cosh(c+dx)(a+b\sinh^2(c+dx))^{3/2}}{5d} + \dots$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^(5/2), x]

[Out] (4*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x]*Sqrt[a + b*Sinh[c + d*x]^2])/(15*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^(3/2))/(5*d) - ((I/15)*(23*a^2 - 23*a*b + 8*b^2)*EllipticE[I*c + I*d*x, b/a]*Sqrt[a + b*Sinh[c + d*x]^2])/(d*Sqrt[1 + (b*Sinh[c + d*x]^2)/a]) + (((4*I)/15)*a*(a - b)*(2*a - b)*EllipticF[I*c + I*d*x, b/a]*Sqrt[1 + (b*Sinh[c + d*x]^2)/a])/(d*Sqrt[a + b*Sinh[c + d*x]^2])

Rule 3180

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[1/(2*p), Int[(a + b*Sinh[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b

)*(2*p - 1)*Sin[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && GtQ[p, 1]

Rule 3170

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p + 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[p, 0]

Rule 3172

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)]^2)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3178

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3177

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rule 3183

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Simp[(1*EllipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^2(c + dx))^{5/2} dx &= \frac{b \cosh(c + dx) \sinh(c + dx) (a + b \sinh^2(c + dx))^{3/2}}{5d} + \frac{1}{5} \int \sqrt{a + b \sinh^2(c + dx)} (a + b \sinh^2(c + dx))^{3/2} dx \\
&= \frac{4(2a - b)b \cosh(c + dx) \sinh(c + dx) \sqrt{a + b \sinh^2(c + dx)}}{15d} + \frac{b \cosh(c + dx) \sinh(c + dx) (a + b \sinh^2(c + dx))^{3/2}}{5d} \\
&= \frac{4(2a - b)b \cosh(c + dx) \sinh(c + dx) \sqrt{a + b \sinh^2(c + dx)}}{15d} + \frac{b \cosh(c + dx) \sinh(c + dx) (a + b \sinh^2(c + dx))^{3/2}}{5d} \\
&= \frac{4(2a - b)b \cosh(c + dx) \sinh(c + dx) \sqrt{a + b \sinh^2(c + dx)}}{15d} + \frac{b \cosh(c + dx) \sinh(c + dx) (a + b \sinh^2(c + dx))^{3/2}}{5d} \\
&= \frac{4(2a - b)b \cosh(c + dx) \sinh(c + dx) \sqrt{a + b \sinh^2(c + dx)}}{15d} + \frac{b \cosh(c + dx) \sinh(c + dx) (a + b \sinh^2(c + dx))^{3/2}}{5d}
\end{aligned}$$

Mathematica [A] time = 1.39443, size = 208, normalized size = 0.9

$$\frac{64ia(2a^2 - 3ab + b^2) \sqrt{\frac{2a+b \cosh(2(c+dx))-b}{a}} \text{EllipticF}\left(i(c+dx), \frac{b}{a}\right) + \sqrt{2}b \sinh(2(c+dx)) (88a^2 + 28b(2a-b) \cosh(2(c+dx)) + 240d\sqrt{2a+b \cosh(2(c+dx))})}{240d\sqrt{2a+b \cosh(2(c+dx))}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^(5/2),x]

[Out] ((-16*I)*a*(23*a^2 - 23*a*b + 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(c + d*x)])]/a + (64*I)*a*(2*a^2 - 3*a*b + b^2)*Sqrt[(2*a - b + b*Cosh[2*(c + d*x)])]/a)*EllipticE[I*(c + d*x), b/a] + (64*I)*a*(2*a^2 - 3*a*b + b^2)*Sqrt[(2*a - b + b*Cosh[2*(c + d*x)])]/a)*EllipticF[I*(c + d*x), b/a] + Sqrt[2]*b*(88*a^2 - 88*a*b + 25*b^2 + 28*(2*a - b)*b*Cosh[2*(c + d*x)] + 3*b^2*Cosh[4*(c + d*x)])*Sinh[2*(c + d*x)]/(240*d*Sqrt[2*a - b + b*Cosh[2*(c + d*x)]])

Maple [B] time = 0.085, size = 609, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(d*x+c)^2)^(5/2),x)`

[Out] $\frac{1}{15} \cdot (3 \cdot (-1/a \cdot b)^{1/2} \cdot b^3 \cdot \sinh(d \cdot x + c) \cdot \cosh(d \cdot x + c)^6 + (14 \cdot (-1/a \cdot b)^{1/2} \cdot a \cdot b^2 - 10 \cdot (-1/a \cdot b)^{1/2} \cdot b^3) \cdot \cosh(d \cdot x + c)^4 \cdot \sinh(d \cdot x + c) + (11 \cdot (-1/a \cdot b)^{1/2} \cdot a^2 \cdot b - 18 \cdot (-1/a \cdot b)^{1/2} \cdot a \cdot b^2 + 7 \cdot (-1/a \cdot b)^{1/2} \cdot b^3) \cdot \cosh(d \cdot x + c)^2 \cdot \sinh(d \cdot x + c) + 15 \cdot a^3 \cdot (b/a \cdot \cosh(d \cdot x + c)^2 + (a-b)/a)^{1/2} \cdot (\cosh(d \cdot x + c)^2)^{1/2} \cdot \text{EllipticF}(\sinh(d \cdot x + c) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) - 34 \cdot a^2 \cdot b \cdot (b/a \cdot \cosh(d \cdot x + c)^2 + (a-b)/a)^{1/2} \cdot (\cosh(d \cdot x + c)^2)^{1/2} \cdot \text{EllipticF}(\sinh(d \cdot x + c) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) + 27 \cdot (b/a \cdot \cosh(d \cdot x + c)^2 + (a-b)/a)^{1/2} \cdot (\cosh(d \cdot x + c)^2)^{1/2} \cdot \text{EllipticF}(\sinh(d \cdot x + c) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot a \cdot b^2 - 8 \cdot (b/a \cdot \cosh(d \cdot x + c)^2 + (a-b)/a)^{1/2} \cdot (\cosh(d \cdot x + c)^2)^{1/2} \cdot \text{EllipticF}(\sinh(d \cdot x + c) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot b^3 + 23 \cdot a^2 \cdot b \cdot (b/a \cdot \cosh(d \cdot x + c)^2 + (a-b)/a)^{1/2} \cdot (\cosh(d \cdot x + c)^2)^{1/2} \cdot \text{EllipticE}(\sinh(d \cdot x + c) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) - 23 \cdot (b/a \cdot \cosh(d \cdot x + c)^2 + (a-b)/a)^{1/2} \cdot (\cosh(d \cdot x + c)^2)^{1/2} \cdot \text{EllipticE}(\sinh(d \cdot x + c) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot a \cdot b^2 + 8 \cdot (b/a \cdot \cosh(d \cdot x + c)^2 + (a-b)/a)^{1/2} \cdot (\cosh(d \cdot x + c)^2)^{1/2} \cdot \text{EllipticE}(\sinh(d \cdot x + c) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot b^3) / (-1/a \cdot b)^{1/2} / \cosh(d \cdot x + c) / (a + b \cdot \sinh(d \cdot x + c)^2)^{1/2} / d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(d*x + c)^2 + a)^(5/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2 \sinh(dx + c)^4 + 2ab \sinh(dx + c)^2 + a^2\right) \sqrt{b \sinh(dx + c)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="fricas")`

[Out] `integral((b^2*sinh(d*x + c)^4 + 2*a*b*sinh(d*x + c)^2 + a^2)*sqrt(b*sinh(d*x + c)^2 + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)**2)**(5/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="giac")`

[Out] `integrate((b*sinh(d*x + c)^2 + a)^(5/2), x)`

$$3.88 \quad \int \sqrt{1 + \sinh^2(x)} dx$$

Optimal. Leaf size=11

$$\sqrt{\cosh^2(x)} \tanh(x)$$

[Out] Sqrt[Cosh[x]^2]*Tanh[x]

Rubi [A] time = 0.0239073, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {3176, 3207, 2637}

$$\sqrt{\cosh^2(x)} \tanh(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sinh[x]^2], x]

[Out] Sqrt[Cosh[x]^2]*Tanh[x]

Rule 3176

```
Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p, x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^n)^p, x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p]]/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^m]) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{1 + \sinh^2(x)} dx &= \int \sqrt{\cosh^2(x)} dx \\
 &= \left(\sqrt{\cosh^2(x)} \operatorname{sech}(x) \right) \int \cosh(x) dx \\
 &= \sqrt{\cosh^2(x)} \tanh(x)
 \end{aligned}$$

Mathematica [A] time = 0.0065058, size = 11, normalized size = 1.

$$\sqrt{\cosh^2(x)} \tanh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Sinh[x]^2], x]

[Out] Sqrt[Cosh[x]^2]*Tanh[x]

Maple [A] time = 0.049, size = 14, normalized size = 1.3

$$\frac{\sinh(x)}{\cosh(x)} \sqrt{(\cosh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+sinh(x)^2)^(1/2), x)

[Out] (cosh(x)^2)^(1/2)*sinh(x)/cosh(x)

Maxima [A] time = 1.55792, size = 15, normalized size = 1.36

$$-\frac{1}{2} e^{(-x)} + \frac{1}{2} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="maxima")

[Out] -1/2*e^(-x) + 1/2*e^x

Fricas [A] time = 1.87428, size = 12, normalized size = 1.09

sinh(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="fricas")

[Out] sinh(x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sinh^2(x) + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sinh(x)**2)**(1/2),x)

[Out] Integral(sqrt(sinh(x)**2 + 1), x)

Giac [A] time = 1.25695, size = 15, normalized size = 1.36

$$-\frac{1}{2}e^{(-x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*e^(-x) + 1/2*e^x

$$3.89 \quad \int \sqrt{-1 - \sinh^2(x)} dx$$

Optimal. Leaf size=13

$$\sqrt{-\cosh^2(x)} \tanh(x)$$

[Out] Sqrt[-Cosh[x]^2]*Tanh[x]

Rubi [A] time = 0.0256116, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3176, 3207, 2637}

$$\sqrt{-\cosh^2(x)} \tanh(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 - Sinh[x]^2], x]

[Out] Sqrt[-Cosh[x]^2]*Tanh[x]

Rule 3176

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p], x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^n)^p], x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{-1 - \sinh^2(x)} dx &= \int \sqrt{-\cosh^2(x)} dx \\
&= \left(\sqrt{-\cosh^2(x)} \operatorname{sech}(x) \right) \int \cosh(x) dx \\
&= \sqrt{-\cosh^2(x)} \tanh(x)
\end{aligned}$$

Mathematica [A] time = 0.0053552, size = 13, normalized size = 1.

$$\sqrt{-\cosh^2(x)} \tanh(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 - Sinh[x]^2], x]

[Out] Sqrt[-Cosh[x]^2]*Tanh[x]

Maple [A] time = 0.033, size = 15, normalized size = 1.2

$$-\cosh(x) \sinh(x) \frac{1}{\sqrt{-(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1-sinh(x)^2)^(1/2), x)

[Out] -cosh(x)*sinh(x)/(-cosh(x)^2)^(1/2)

Maxima [B] time = 1.55258, size = 34, normalized size = 2.62

$$-\frac{e^{(-2x)}}{2\sqrt{-e^{(-2x)}}} + \frac{1}{2\sqrt{-e^{(-2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="maxima")

[Out] $-1/2*e^{(-2*x)}/\sqrt{-e^{(-2*x)}} + 1/2/\sqrt{-e^{(-2*x)}}$

Fricas [C] time = 1.7685, size = 38, normalized size = 2.92

$$\frac{1}{2} (i e^{(2x)} - i) e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="fricas")

[Out] $1/2*(I*e^{(2*x)} - I)*e^{(-x)}$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sinh^2(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-sinh(x)**2)**(1/2),x)

[Out] Integral(sqrt(-sinh(x)**2 - 1), x)

Giac [C] time = 1.30061, size = 15, normalized size = 1.15

$$-\frac{1}{2}i e^{(-x)} + \frac{1}{2}i e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="giac")

[Out] $-1/2*I*e^{(-x)} + 1/2*I*e^x$

$$3.90 \quad \int \sqrt{1 - \sinh^2(x)} dx$$

Optimal. Leaf size=11

$$-iE(ix| - 1)$$

[Out] (-I)*EllipticE[I*x, -1]

Rubi [A] time = 0.0102307, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3177}

$$-iE(ix| - 1)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Sinh[x]^2], x]

[Out] (-I)*EllipticE[I*x, -1]

Rule 3177

Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a] * EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\int \sqrt{1 - \sinh^2(x)} dx = -iE(ix| - 1)$$

Mathematica [A] time = 0.024372, size = 11, normalized size = 1.

$$-iE(ix| - 1)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - Sinh[x]^2], x]

[Out] $(-I)*\text{EllipticE}[I*x, -1]$

Maple [B] time = 0.07, size = 51, normalized size = 4.6

$$\frac{2 \text{EllipticF}(\sinh(x), i) - \text{EllipticE}(\sinh(x), i)}{\cosh(x)} \sqrt{-(-1 + (\sinh(x))^2) (\cosh(x))^2} \sqrt{(\cosh(x))^2} \frac{1}{\sqrt{1 - (\sinh(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-sinh(x)^2)^(1/2),x)`

[Out] $(-(-1+\sinh(x)^2)*\cosh(x)^2)^{(1/2)}*(\cosh(x)^2)^{(1/2)}*(2*\text{EllipticF}(\sinh(x),I)-\text{EllipticE}(\sinh(x),I))/(1-\sinh(x)^4)^{(1/2)}/\cosh(x)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sinh(x)^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-sinh(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(-sinh(x)^2 + 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-\sinh(x)^2 + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-sinh(x)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(-sinh(x)^2 + 1), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{1 - \sinh^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sinh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sinh(x)^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sinh(x)^2 + 1), x)
```

3.91 $\int \sqrt{-1 + \sinh^2(x)} dx$

Optimal. Leaf size=33

$$-\frac{i\sqrt{\sinh^2(x) - 1}E(ix| - 1)}{\sqrt{1 - \sinh^2(x)}}$$

[Out] `((-I)*EllipticE[I*x, -1]*Sqrt[-1 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]`

Rubi [A] time = 0.0202611, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3178, 3177}

$$-\frac{i\sqrt{\sinh^2(x) - 1}E(ix| - 1)}{\sqrt{1 - \sinh^2(x)}}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[-1 + Sinh[x]^2], x]`

[Out] `((-I)*EllipticE[I*x, -1]*Sqrt[-1 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]`

Rule 3178

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]`

Rule 3177

`Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]`

Rubi steps

$$\int \sqrt{-1 + \sinh^2(x)} dx = \frac{\sqrt{-1 + \sinh^2(x)} \int \sqrt{1 - \sinh^2(x)} dx}{\sqrt{1 - \sinh^2(x)}}$$

$$= -\frac{iE(ix| -1)\sqrt{-1 + \sinh^2(x)}}{\sqrt{1 - \sinh^2(x)}}$$

Mathematica [A] time = 0.0329341, size = 33, normalized size = 1.

$$\frac{i\sqrt{3 - \cosh(2x)}E(ix| -1)}{\sqrt{\cosh(2x) - 3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + Sinh[x]^2],x]

[Out] (I*Sqrt[3 - Cosh[2*x]]*EllipticE[I*x, -1])/Sqrt[-3 + Cosh[2*x]]

Maple [A] time = 0.064, size = 61, normalized size = 1.9

$$\frac{i\text{EllipticE}(i \sinh(x), i)}{\cosh(x)} \sqrt{(-1 + (\sinh(x))^2)} (\cosh(x))^2 \sqrt{(\cosh(x))^2} \sqrt{1 - (\sinh(x))^2} \frac{1}{\sqrt{(\sinh(x))^4 - 1}} \frac{1}{\sqrt{-1 + (\sinh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+sinh(x)^2)^(1/2),x)

[Out] I*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(1-sinh(x)^2)^(1/2)*EllipticE(I*sinh(x),I)/(sinh(x)^4-1)^(1/2)/cosh(x)/(-1+sinh(x)^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sinh(x)^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(x)^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\sinh(x)^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sinh(x)^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sinh^2(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(x)**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sinh(x)^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(x)^2 - 1), x)
```

3.92 $\int \sqrt{a + b \sinh^2(x)} dx$

Optimal. Leaf size=42

$$\frac{i\sqrt{a + b \sinh^2(x)} E\left(ix \left| \frac{b}{a} \right. \right)}{\sqrt{\frac{b \sinh^2(x)}{a} + 1}}$$

[Out] ((-I)*EllipticE[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a]

Rubi [A] time = 0.0317896, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3178, 3177}

$$\frac{i\sqrt{a + b \sinh^2(x)} E\left(ix \left| \frac{b}{a} \right. \right)}{\sqrt{\frac{b \sinh^2(x)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sinh[x]^2], x]

[Out] ((-I)*EllipticE[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a]

Rule 3178

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)]/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a + b \sinh^2(x)} dx = \frac{\sqrt{a + b \sinh^2(x)} \int \sqrt{1 + \frac{b \sinh^2(x)}{a}} dx}{\sqrt{1 + \frac{b \sinh^2(x)}{a}}}$$

$$= -\frac{iE\left(ix \left| \frac{b}{a} \right. \right) \sqrt{a + b \sinh^2(x)}}{\sqrt{1 + \frac{b \sinh^2(x)}{a}}}$$

Mathematica [A] time = 0.0439838, size = 54, normalized size = 1.29

$$-\frac{ia\sqrt{\frac{2a+b\cosh(2x)-b}{a}}E\left(ix\left|\frac{b}{a}\right.\right)}{\sqrt{2a+b\cosh(2x)-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sinh[x]^2],x]

[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticE[I*x, b/a])/Sqrt[2*a - b + b*Cosh[2*x]]

Maple [B] time = 0.067, size = 109, normalized size = 2.6

$$\frac{1}{\cosh(x)} \sqrt{\frac{a + b(\sinh(x))^2}{a}} \sqrt{(\cosh(x))^2} \left(a \operatorname{EllipticF}\left(\sinh(x) \sqrt{\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) - b \operatorname{EllipticF}\left(\sinh(x) \sqrt{\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) + b \operatorname{EllipticE}\left(\sinh(x) \sqrt{\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(x)^2)^(1/2),x)

[Out] ((a+b*sinh(x)^2)/a)^(1/2)*(cosh(x)^2)^(1/2)*(a*EllipticF(sinh(x)*(-1/a*b)^(1/2), (a/b)^(1/2))-b*EllipticF(sinh(x)*(-1/a*b)^(1/2), (a/b)^(1/2))+b*EllipticE(sinh(x)*(-1/a*b)^(1/2), (a/b)^(1/2)))/(-1/a*b)^(1/2)/cosh(x)/(a+b*sinh(x)^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(x)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh(x)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(x)^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(x)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sinh(x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(x)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^2 + a), x)
```

3.93 $\int (1 + \sinh^2(x))^{3/2} dx$

Optimal. Leaf size=29

$$\frac{1}{3} \cosh^2(x)^{3/2} \tanh(x) + \frac{2}{3} \sqrt{\cosh^2(x)} \tanh(x)$$

[Out] (2*Sqrt[Cosh[x]^2]*Tanh[x])/3 + ((Cosh[x]^2)^(3/2)*Tanh[x])/3

Rubi [A] time = 0.026434, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$, Rules used = {3176, 3203, 3207, 2637}

$$\frac{1}{3} \cosh^2(x)^{3/2} \tanh(x) + \frac{2}{3} \sqrt{\cosh^2(x)} \tanh(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^2)^(3/2), x]

[Out] (2*Sqrt[Cosh[x]^2]*Tanh[x])/3 + ((Cosh[x]^2)^(3/2)*Tanh[x])/3

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3203

Int[((b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] && !IntegerQ[p] && GtQ[p, 1]

Rule 3207

Int[(u_)*((b_)*sin[(e_) + (f_)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^n)^FracPart[p]]/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_)*(trig_)[e + f*x])^(m_) /;

FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (1 + \sinh^2(x))^{3/2} dx &= \int \cosh^2(x)^{3/2} dx \\ &= \frac{1}{3} \cosh^2(x)^{3/2} \tanh(x) + \frac{2}{3} \int \sqrt{\cosh^2(x)} dx \\ &= \frac{1}{3} \cosh^2(x)^{3/2} \tanh(x) + \frac{1}{3} \left(2\sqrt{\cosh^2(x)\operatorname{sech}(x)} \right) \int \cosh(x) dx \\ &= \frac{2}{3} \sqrt{\cosh^2(x)} \tanh(x) + \frac{1}{3} \cosh^2(x)^{3/2} \tanh(x) \end{aligned}$$

Mathematica [A] time = 0.0203646, size = 23, normalized size = 0.79

$$\frac{1}{12} (9 \sinh(x) + \sinh(3x)) \sqrt{\cosh^2(x) \operatorname{sech}(x)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^2)^(3/2), x]

[Out] (Sqrt[Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))/12

Maple [A] time = 0.041, size = 21, normalized size = 0.7

$$\frac{\sinh(x) \left((\sinh(x))^2 + 3 \right) \sqrt{(\cosh(x))^2}}{3 \cosh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+sinh(x)^2)^(3/2), x)

[Out] $\frac{1}{3} * (\cosh(x)^2)^{(1/2)} * \sinh(x) * (\sinh(x)^2 + 3) / \cosh(x)$

Maxima [A] time = 1.5582, size = 31, normalized size = 1.07

$$\frac{1}{24} e^{(3x)} - \frac{3}{8} e^{(-x)} - \frac{1}{24} e^{(-3x)} + \frac{3}{8} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+sinh(x)^2)^(3/2),x, algorithm="maxima")`

[Out] $\frac{1}{24} * e^{(3*x)} - \frac{3}{8} * e^{(-x)} - \frac{1}{24} * e^{(-3*x)} + \frac{3}{8} * e^x$

Fricas [A] time = 1.78809, size = 62, normalized size = 2.14

$$\frac{1}{12} \sinh(x)^3 + \frac{1}{4} (\cosh(x)^2 + 3) \sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+sinh(x)^2)^(3/2),x, algorithm="fricas")`

[Out] $\frac{1}{12} * \sinh(x)^3 + \frac{1}{4} * (\cosh(x)^2 + 3) * \sinh(x)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+sinh(x)**2)**(3/2),x)`

[Out] Timed out

Giac [A] time = 1.22696, size = 34, normalized size = 1.17

$$-\frac{1}{24} (9e^{2x} + 1)e^{-3x} + \frac{1}{24} e^{3x} + \frac{3}{8} e^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="giac")

[Out] -1/24*(9*e^(2*x) + 1)*e^(-3*x) + 1/24*e^(3*x) + 3/8*e^x

3.94 $\int (-1 - \sinh^2(x))^{3/2} dx$

Optimal. Leaf size=33

$$\frac{1}{3}(-\cosh^2(x))^{3/2} \tanh(x) - \frac{2}{3}\sqrt{-\cosh^2(x)} \tanh(x)$$

[Out] $(-2*\text{Sqrt}[-\text{Cosh}[x]^2]*\text{Tanh}[x])/3 + ((-\text{Cosh}[x]^2)^{(3/2)}*\text{Tanh}[x])/3$

Rubi [A] time = 0.0293963, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3176, 3203, 3207, 2637}

$$\frac{1}{3}(-\cosh^2(x))^{3/2} \tanh(x) - \frac{2}{3}\sqrt{-\cosh^2(x)} \tanh(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(-1 - \text{Sinh}[x]^2)^{(3/2)}, x]$

[Out] $(-2*\text{Sqrt}[-\text{Cosh}[x]^2]*\text{Tanh}[x])/3 + ((-\text{Cosh}[x]^2)^{(3/2)}*\text{Tanh}[x])/3$

Rule 3176

$\text{Int}[(u_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u*(a*\cos[e + f*x]^2)^p], x] /; \text{FreeQ}\{a, b, e, f, p\}, x] \&\& \text{EqQ}[a + b, 0]$

Rule 3203

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]^2)^{(p_.)}, x_Symbol] \rightarrow -\text{Simp}[(\text{Cot}[e + f*x]*(b*\sin[e + f*x]^2)^p)/(2*f*p), x] + \text{Dist}[(b*(2*p - 1))/(2*p), \text{Int}[(b*\sin[e + f*x]^2)^{(p - 1)}, x], x] /; \text{FreeQ}\{b, e, f\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{GtQ}[p, 1]$

Rule 3207

$\text{Int}[(u_.)*((b_.)*\sin[(e_.) + (f_.)*(x_)]^n)^{(p_.)}, x_Symbol] \rightarrow \text{With}\{\{ff = \text{FreeFactors}[\sin[e + f*x], x]\}, \text{Dist}[(b*ff^n)^{\text{IntPart}[p]}*(b*\sin[e + f*x]^n)^{\text{FracPart}[p]}]/(\sin[e + f*x]/ff)^{(n*\text{FracPart}[p])}, \text{Int}[\text{ActivateTrig}[u*(\sin[e + f*x]/ff)^{(n*p)}, x], x]\} /; \text{FreeQ}\{b, e, f, n, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{IntegerQ}[n] \&\& (\text{EqQ}[u, 1] \parallel \text{MatchQ}[u, ((d_.)*(trig_)[e + f*x])^{(m_.)} /;$

FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (-1 - \sinh^2(x))^{3/2} dx &= \int (-\cosh^2(x))^{3/2} dx \\ &= \frac{1}{3} (-\cosh^2(x))^{3/2} \tanh(x) - \frac{2}{3} \int \sqrt{-\cosh^2(x)} dx \\ &= \frac{1}{3} (-\cosh^2(x))^{3/2} \tanh(x) - \frac{1}{3} \left(2\sqrt{-\cosh^2(x)} \operatorname{sech}(x) \right) \int \cosh(x) dx \\ &= -\frac{2}{3} \sqrt{-\cosh^2(x)} \tanh(x) + \frac{1}{3} (-\cosh^2(x))^{3/2} \tanh(x) \end{aligned}$$

Mathematica [A] time = 0.0063351, size = 25, normalized size = 0.76

$$-\frac{1}{12} (9 \sinh(x) + \sinh(3x)) \sqrt{-\cosh^2(x)} \operatorname{sech}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 - Sinh[x]^2)^(3/2), x]

[Out] -(Sqrt[-Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))/12

Maple [A] time = 0.043, size = 21, normalized size = 0.6

$$\frac{\cosh(x) \sinh(x) \left((\cosh(x))^2 + 2 \right)}{3} \frac{1}{\sqrt{-(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1-sinh(x)^2)^(3/2), x)

[Out] $\frac{1}{3} \cosh(x) \sinh(x) (\cosh(x)^2 + 2) / (-\cosh(x)^2)^{1/2}$

Maxima [B] time = 1.54951, size = 72, normalized size = 2.18

$$\frac{3e^{-2x}}{8(-e^{-2x})^{\frac{3}{2}}} - \frac{3e^{-4x}}{8(-e^{-2x})^{\frac{3}{2}}} - \frac{e^{-6x}}{24(-e^{-2x})^{\frac{3}{2}}} + \frac{1}{24(-e^{-2x})^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{3}{8} e^{-2x} / (-e^{-2x})^{3/2} - \frac{3}{8} e^{-4x} / (-e^{-2x})^{3/2} - \frac{1}{24} e^{-6x} / (-e^{-2x})^{3/2} + \frac{1}{24} / (-e^{-2x})^{3/2}$

Fricas [C] time = 1.80619, size = 81, normalized size = 2.45

$$\frac{1}{24} (-ie^{6x} - 9ie^{4x} + 9ie^{2x} + i)e^{-3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="fricas")

[Out] $\frac{1}{24} (-Ie^{6x} - 9Ie^{4x} + 9Ie^{2x} + I)e^{-3x}$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-sinh(x)**2)**(3/2),x)

[Out] Timed out

Giac [C] time = 1.19878, size = 34, normalized size = 1.03

$$\frac{1}{24}i(9e^{2x} + 1)e^{-3x} - \frac{1}{24}ie^{3x} - \frac{3}{8}ie^x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="giac")

[Out] 1/24*I*(9*e^(2*x) + 1)*e^(-3*x) - 1/24*I*e^(3*x) - 3/8*I*e^x

3.95 $\int (1 - \sinh^2(x))^{3/2} dx$

Optimal. Leaf size=45

$$\frac{2}{3}i\text{EllipticF}(ix, -1) - 2iE(ix| -1) - \frac{1}{3} \sinh(x)\sqrt{1 - \sinh^2(x)} \cosh(x)$$

[Out] $(-2*I)*\text{EllipticE}[I*x, -1] + ((2*I)/3)*\text{EllipticF}[I*x, -1] - (\text{Cosh}[x]*\text{Sinh}[x]*\text{Sqrt}[1 - \text{Sinh}[x]^2])/3$

Rubi [A] time = 0.0635546, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3180, 3172, 3177, 3182}

$$\frac{2}{3}iF(ix| -1) - 2iE(ix| -1) - \frac{1}{3} \sinh(x)\sqrt{1 - \sinh^2(x)} \cosh(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - \text{Sinh}[x]^2)^{(3/2)}, x]$

[Out] $(-2*I)*\text{EllipticE}[I*x, -1] + ((2*I)/3)*\text{EllipticF}[I*x, -1] - (\text{Cosh}[x]*\text{Sinh}[x]*\text{Sqrt}[1 - \text{Sinh}[x]^2])/3$

Rule 3180

$\text{Int}[(a + (b \cdot \sin[e + f \cdot x])^2)^p, x_Symbol] \rightarrow -\text{Simp}[b \cdot \cos[e + f \cdot x] \cdot \sin[e + f \cdot x] \cdot (a + b \cdot \sin[e + f \cdot x]^2)^{p-1} / (2 \cdot f \cdot p), x] + \text{Dist}[1 / (2 \cdot p), \text{Int}[(a + b \cdot \sin[e + f \cdot x]^2)^{p-2} \cdot \text{Simp}[a \cdot (b + 2 \cdot a \cdot p) + b \cdot (2 \cdot a + b) \cdot (2 \cdot p - 1) \cdot \sin[e + f \cdot x]^2, x], x], x] /;$ FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && GtQ[p, 1]

Rule 3172

$\text{Int}[(A + (B \cdot \sin[e + f \cdot x])^2) / \text{Sqrt}[a + (b \cdot \sin[e + f \cdot x])^2], x_Symbol] \rightarrow \text{Dist}[B/b, \text{Int}[\text{Sqrt}[a + b \cdot \sin[e + f \cdot x]^2], x], x] + \text{Dist}[(A \cdot b - a \cdot B) / b, \text{Int}[1 / \text{Sqrt}[a + b \cdot \sin[e + f \cdot x]^2], x], x] /;$ FreeQ[{a, b, e, f, A, B}, x]

Rule 3177

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(1*El
lipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned} \int (1 - \sinh^2(x))^{3/2} dx &= -\frac{1}{3} \cosh(x) \sinh(x) \sqrt{1 - \sinh^2(x)} + \frac{1}{3} \int \frac{4 - 6 \sinh^2(x)}{\sqrt{1 - \sinh^2(x)}} dx \\ &= -\frac{1}{3} \cosh(x) \sinh(x) \sqrt{1 - \sinh^2(x)} - \frac{2}{3} \int \frac{1}{\sqrt{1 - \sinh^2(x)}} dx + 2 \int \sqrt{1 - \sinh^2(x)} dx \\ &= -2iE(ix| -1) + \frac{2}{3}iF(ix| -1) - \frac{1}{3} \cosh(x) \sinh(x) \sqrt{1 - \sinh^2(x)} \end{aligned}$$

Mathematica [A] time = 0.0688293, size = 45, normalized size = 1.

$$\frac{1}{12} (8i\text{EllipticF}(ix, -1) - 24iE(ix| -1) - \sinh(2x)\sqrt{6 - 2\cosh(2x)})$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(3/2), x]
```

```
[Out] ((-24*I)*EllipticE[I*x, -1] + (8*I)*EllipticF[I*x, -1] - Sqrt[6 - 2*Cosh[2*
x]]*Sinh[2*x])/12
```

Maple [A] time = 0.077, size = 103, normalized size = 2.3

$$\frac{1}{3 \cosh(x)} \sqrt{-(-1 + (\sinh(x))^2) (\cosh(x))^2} \left(\sinh(x) (\cosh(x))^4 + 10 \sqrt{-(\cosh(x))^2} + 2 \sqrt{(\cosh(x))^2} \text{EllipticF}(\sinh(x)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-sinh(x)^2)^(3/2),x)`

[Out] $\frac{1}{3} * (-(-1 + \sinh(x)^2) * \cosh(x)^2)^{1/2} * (\sinh(x) * \cosh(x)^4 + 10 * (-\cosh(x)^2 + 2)^{1/2} * (\cosh(x)^2)^{1/2} * \text{EllipticF}(\sinh(x), I) - 6 * (-\cosh(x)^2 + 2)^{1/2} * (\cosh(x)^2)^{1/2} * \text{EllipticE}(\sinh(x), I) - 2 * \cosh(x)^2 * \sinh(x)) / (1 - \sinh(x)^4)^{1/2} / \cosh(x) / (1 - \sinh(x)^2)^{1/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (-\sinh(x)^2 + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-sinh(x)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((-sinh(x)^2 + 1)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\sinh(x)^2 + 1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-sinh(x)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral((-sinh(x)^2 + 1)^(3/2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-sinh(x)**2)**(3/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (-\sinh(x)^2 + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="giac")

[Out] integrate((-sinh(x)^2 + 1)^(3/2), x)

3.96 $\int (-1 + \sinh^2(x))^{3/2} dx$

Optimal. Leaf size=87

$$\frac{2i\sqrt{1 - \sinh^2(x)}\text{EllipticF}(ix, -1)}{3\sqrt{\sinh^2(x) - 1}} + \frac{1}{3}\sinh(x)\sqrt{\sinh^2(x) - 1}\cosh(x) + \frac{2i\sqrt{\sinh^2(x) - 1}E(ix| - 1)}{\sqrt{1 - \sinh^2(x)}}$$

[Out] (((2*I)/3)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2] + (Cosh[x]*Sinh[x]*Sqrt[-1 + Sinh[x]^2])/3 + ((2*I)*EllipticE[I*x, -1]*Sqrt[-1 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]

Rubi [A] time = 0.0855633, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{1}{3}\sinh(x)\sqrt{\sinh^2(x) - 1}\cosh(x) + \frac{2i\sqrt{1 - \sinh^2(x)}F(ix| - 1)}{3\sqrt{\sinh^2(x) - 1}} + \frac{2i\sqrt{\sinh^2(x) - 1}E(ix| - 1)}{\sqrt{1 - \sinh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sinh[x]^2)^(3/2), x]

[Out] (((2*I)/3)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2] + (Cosh[x]*Sinh[x]*Sqrt[-1 + Sinh[x]^2])/3 + ((2*I)*EllipticE[I*x, -1]*Sqrt[-1 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]

Rule 3180

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && GtQ[p, 1]

Rule 3172

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)]^2)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ

[{a, b, e, f, A, B}, x]

Rule 3178

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3177

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rule 3183

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Simp[(1*EllipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
 \int (-1 + \sinh^2(x))^{3/2} dx &= \frac{1}{3} \cosh(x) \sinh(x) \sqrt{-1 + \sinh^2(x)} + \frac{1}{3} \int \frac{4 - 6 \sinh^2(x)}{\sqrt{-1 + \sinh^2(x)}} dx \\
 &= \frac{1}{3} \cosh(x) \sinh(x) \sqrt{-1 + \sinh^2(x)} - \frac{2}{3} \int \frac{1}{\sqrt{-1 + \sinh^2(x)}} dx - 2 \int \sqrt{-1 + \sinh^2(x)} dx \\
 &= \frac{1}{3} \cosh(x) \sinh(x) \sqrt{-1 + \sinh^2(x)} - \frac{\left(2\sqrt{1 - \sinh^2(x)}\right) \int \frac{1}{\sqrt{1 - \sinh^2(x)}} dx}{3\sqrt{-1 + \sinh^2(x)}} - \frac{\left(2\sqrt{-1 + \sinh^2(x)}\right) \int \sqrt{-1 + \sinh^2(x)} dx}{\sqrt{1 - \sinh^2(x)}} \\
 &= \frac{2iF(ix| -1)\sqrt{1 - \sinh^2(x)}}{3\sqrt{-1 + \sinh^2(x)}} + \frac{1}{3} \cosh(x) \sinh(x) \sqrt{-1 + \sinh^2(x)} + \frac{2iE(ix| -1)\sqrt{-1 + \sinh^2(x)}}{\sqrt{1 - \sinh^2(x)}}
 \end{aligned}$$

Mathematica [A] time = 0.12193, size = 78, normalized size = 0.9

$$\frac{8i\sqrt{3 - \cosh(2x)}\text{EllipticF}(ix, -1) + \frac{\sinh(4x) - 6\sinh(2x)}{\sqrt{2}} - 24i\sqrt{3 - \cosh(2x)}E(ix| - 1)}{12\sqrt{\cosh(2x) - 3}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sinh[x]^2)^(3/2), x]

[Out] ((-24*I)*Sqrt[3 - Cosh[2*x]]*EllipticE[I*x, -1] + (8*I)*Sqrt[3 - Cosh[2*x]]*EllipticF[I*x, -1] + (-6*Sinh[2*x] + Sinh[4*x])/Sqrt[2])/(12*Sqrt[-3 + Cosh[2*x]])

Maple [A] time = 0.079, size = 106, normalized size = 1.2

$$\frac{1}{3 \cosh(x)} \sqrt{(-1 + (\sinh(x))^2) (\cosh(x))^2} \left(\sinh(x) (\cosh(x))^4 + 2i \sqrt{(\cosh(x))^2} \sqrt{-(\cosh(x))^2 + 2} \text{EllipticF}(i \sinh(x), \sqrt{-(\cosh(x))^2 + 2}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+sinh(x)^2)^(3/2), x)

[Out] 1/3*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(sinh(x)*cosh(x)^4+2*I*(cosh(x)^2)^(1/2)*(-cosh(x)^2+2)^(1/2)*EllipticF(I*sinh(x), I)-6*I*(cosh(x)^2)^(1/2)*(-cosh(x)^2+2)^(1/2)*EllipticE(I*sinh(x), I)-2*cosh(x)^2*sinh(x))/(sinh(x)^4-1)^(1/2)/cosh(x)/(-1+sinh(x)^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (\sinh(x)^2 - 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sinh(x)^2)^(3/2), x, algorithm="maxima")

[Out] integrate((sinh(x)^2 - 1)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\sinh(x)^2 - 1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="fricas")

[Out] integral((sinh(x)^2 - 1)^(3/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sinh(x)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (\sinh(x)^2 - 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="giac")

[Out] integrate((sinh(x)^2 - 1)^(3/2), x)

3.97 $\int (a + b \sinh^2(x))^{3/2} dx$

Optimal. Leaf size=123

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(x)}{a}+1}\text{EllipticF}\left(ix, \frac{b}{a}\right)}{3\sqrt{a+b\sinh^2(x)}} + \frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\sinh^2(x)} - \frac{2i(2a-b)\sqrt{a+b\sinh^2(x)}E\left(ix\left|\frac{b}{a}\right.\right)}{3\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

[Out] (b*Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2])/3 - (((2*I)/3)*(2*a - b)*EllipticE[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a] + ((I/3)*a*(a - b)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]

Rubi [A] time = 0.162822, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\sinh^2(x)} + \frac{ia(a-b)\sqrt{\frac{b\sinh^2(x)}{a}+1}F\left(ix\left|\frac{b}{a}\right.\right)}{3\sqrt{a+b\sinh^2(x)}} - \frac{2i(2a-b)\sqrt{a+b\sinh^2(x)}E\left(ix\left|\frac{b}{a}\right.\right)}{3\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[x]^2)^(3/2), x]

[Out] (b*Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2])/3 - (((2*I)/3)*(2*a - b)*EllipticE[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a] + ((I/3)*a*(a - b)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]

Rule 3180

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && GtQ[p, 1]

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^2(x))^{3/2} dx &= \frac{1}{3} b \cosh(x) \sinh(x) \sqrt{a + b \sinh^2(x)} + \frac{1}{3} \int \frac{a(3a - b) + 2(2a - b)b \sinh^2(x)}{\sqrt{a + b \sinh^2(x)}} dx \\
&= \frac{1}{3} b \cosh(x) \sinh(x) \sqrt{a + b \sinh^2(x)} - \frac{1}{3} (a(a - b)) \int \frac{1}{\sqrt{a + b \sinh^2(x)}} dx + \frac{1}{3} (2(2a - b)) \int \sqrt{a + b \sinh^2(x)} dx \\
&= \frac{1}{3} b \cosh(x) \sinh(x) \sqrt{a + b \sinh^2(x)} + \frac{\left(2(2a - b) \sqrt{a + b \sinh^2(x)}\right) \int \sqrt{1 + \frac{b \sinh^2(x)}{a}} dx}{3 \sqrt{1 + \frac{b \sinh^2(x)}{a}}} - \frac{a(a - b) \int \frac{1}{\sqrt{a + b \sinh^2(x)}} dx}{3 \sqrt{a + b \sinh^2(x)}} \\
&= \frac{1}{3} b \cosh(x) \sinh(x) \sqrt{a + b \sinh^2(x)} - \frac{2i(2a - b) E\left(ix, \frac{b}{a}\right) \sqrt{a + b \sinh^2(x)}}{3 \sqrt{1 + \frac{b \sinh^2(x)}{a}}} + \frac{ia(a - b) F\left(ix, \frac{b}{a}\right)}{3 \sqrt{a + b \sinh^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.378756, size = 132, normalized size = 1.07

$$\frac{4ia(a - b) \sqrt{\frac{2a + b \cosh(2x) - b}{a}} \text{EllipticF}\left(ix, \frac{b}{a}\right) + \sqrt{2} b \sinh(2x) (2a + b \cosh(2x) - b) - 8ia(2a - b) \sqrt{\frac{2a + b \cosh(2x) - b}{a}} E\left(ix, \frac{b}{a}\right)}{12 \sqrt{2a + b \cosh(2x) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[x]^2)^(3/2), x]

[Out] ((-8*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticE[I*x, b/a] + (4*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticF[I*x, b/a] + Sqrt[2]*b*(2*a - b + b*Cosh[2*x])*Sinh[2*x])/(12*Sqrt[2*a - b + b*Cosh[2*x]])

Maple [B] time = 0.07, size = 329, normalized size = 2.7

$$\frac{1}{3 \cosh(x)} \left(\sqrt{\frac{b}{a}} b^2 \sinh(x) (\cosh(x))^4 + \left(\sqrt{\frac{b}{a}} ab - \sqrt{\frac{b}{a}} b^2 \right) (\cosh(x))^2 \sinh(x) + 3a^2 \sqrt{\frac{b(\cosh(x))^2}{a} + \frac{a-b}{a}} \sqrt{\cosh(x)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(x)^2)^(3/2), x)

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(x)*cosh(x)^4+((-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)
)*b^2)*cosh(x)^2*sinh(x)+3*a^2*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1
/2)*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))-5*a*b*(b/a*cosh(x)^2+(a-b
)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))+
2*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2+4*a*b*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1
/2)*EllipticE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*(b/a*cosh(x)^2+(a-b)/a
)^(1/2)*(cosh(x)^2)^(1/2)*EllipticE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
)/(-1/a*b)^(1/2)/cosh(x)/(a+b*sinh(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(x)^2 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(x)^2 + a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(x)^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(x)^2 + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x)^2 + a)^(3/2), x)
```


$$3.98 \quad \int \frac{\sinh^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=83

$$\frac{\cosh(e+fx)\sqrt{a+b \cosh^2(e+fx)-b}}{2bf} - \frac{(a+b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2b^{3/2}f}$$

[Out] $-\left(\frac{(a+b)\text{ArcTanh}\left[\frac{\sqrt{b}\text{Cosh}[e+f*x]}{\sqrt{a-b+b*\text{Cosh}[e+f*x]^2}}\right]}{(2*b^{3/2}*f)} + \frac{\text{Cosh}[e+f*x]*\text{Sqrt}[a-b+b*\text{Cosh}[e+f*x]^2]}{(2*b*f)}\right)$

Rubi [A] time = 0.0992391, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3186, 388, 217, 206}

$$\frac{\cosh(e+fx)\sqrt{a+b \cosh^2(e+fx)-b}}{2bf} - \frac{(a+b) \tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[e+f*x]^3/\text{Sqrt}[a+b*\text{Sinh}[e+f*x]^2], x]$

[Out] $-\left(\frac{(a+b)\text{ArcTanh}\left[\frac{\sqrt{b}\text{Cosh}[e+f*x]}{\sqrt{a-b+b*\text{Cosh}[e+f*x]^2}}\right]}{(2*b^{3/2}*f)} + \frac{\text{Cosh}[e+f*x]*\text{Sqrt}[a-b+b*\text{Cosh}[e+f*x]^2]}{(2*b*f)}\right)$

Rule 3186

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - b*ff^2*x^2)^p, x], x, \text{Cos}[e + f*x]/ff], x] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[(m-1)/2]$

Rule 388

$\text{Int}[(a_. + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{(p+1)})/(b*(n*(p+1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(b*(n*(p+1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n*(p+1) + 1, 0]$

Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned} \int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a-bx^2}} dx, x, \cosh(e+fx)\right)}{f} \\ &= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2}} dx, x, \cosh(e+fx)\right)}{2bf} \\ &= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf} - \frac{(a+b)\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2bf} \\ &= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf} \end{aligned}$$

Mathematica [A] time = 0.269282, size = 98, normalized size = 1.18

$$\frac{\cosh(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{2\sqrt{2}bf} - \frac{(a+b)\log\left(\sqrt{2a+b\cosh(2(e+fx))-b} + \sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*b*f) - ((a + b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])]/(2*b^(3/2)*f)

Maple [B] time = 0.109, size = 204, normalized size = 2.5

$$\frac{1}{4f \cosh(fx + e)} \sqrt{\left(a + b(\sinh(fx + e))^2\right) (\cosh(fx + e))^2} \left(2b^{3/2} \sqrt{b(\cosh(fx + e))^4 + (a - b)(\cosh(fx + e))^2} - b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2), x)`

[Out] `1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*b^(3/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)-b^2*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-b*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^3}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2), x, algorithm="maxima")`

[Out] `integrate(sinh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [B] time = 2.5622, size = 5576, normalized size = 67.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2), x, algorithm="fricas")`

[Out] `[1/8*(((a + b)*cosh(f*x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*`

$$\begin{aligned}
& x + e) \sinh(f*x + e)^7 + a^2*b*\sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*\cosh(f*x + \\
& e)^6 + 2*(14*a^2*b*\cosh(f*x + e)^2 + a^3 + a^2*b)*\sinh(f*x + e)^6 + 4*(14* \\
& a^2*b*\cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9 \\
& *a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^4 + (70*a^2*b*\cosh(f*x + e)^4 + 9*a^2 \\
& *b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4* \\
& (14*a^2*b*\cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*\cosh(f*x + e)^3 + (9*a^2*b - 4 \\
& *a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*\cosh \\
& (f*x + e)^2 + 2*(14*a^2*b*\cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*\cosh(f*x + e)^ \\
& 4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + \\
& e)^2 - \sqrt{2}*(a^2*\cosh(f*x + e)^6 + 6*a^2*\cosh(f*x + e)*\sinh(f*x + e)^5 \\
& + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x + e)^4 + 3*(5*a^2*\cosh(f*x + e)^2 + \\
& a^2)*\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e))*\sinh \\
& (f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e)^2 + (15*a^2*\cosh(f*x + e)^4 + 18* \\
& a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(3*a^2*\cosh(f* \\
& x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + \\
& e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f* \\
& x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(2*a^2*b*c \\
& osh(f*x + e)^7 + 3*(a^3 + a^2*b)*\cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3 \\
&)*\cosh(f*x + e)^3 + (3*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x \\
& + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e \\
&)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e \\
&)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + ((a + b)*\cosh(f* \\
& x + e)^2 + 2*(a + b)*\cosh(f*x + e)*\sinh(f*x + e) + (a + b)*\sinh(f*x + e)^2) \\
& *\sqrt{b}*\log(-(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\si \\
& nh(f*x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b) \\
& *\sinh(f*x + e)^2 - \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) \\
& + \sinh(f*x + e)^2 - 1))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 \\
& + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e \\
&)^2))} + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/((\c \\
& osh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + \sqrt{2} \\
& *(b*\cosh(f*x + e)^2 + 2*b*\cosh(f*x + e)*\sinh(f*x + e) + b*\sinh(f*x + e)^2 \\
& + b)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^ \\
& 2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))}/(b^2*f*\cosh(f*x + e) \\
& ^2 + 2*b^2*f*\cosh(f*x + e)*\sinh(f*x + e) + b^2*f*\sinh(f*x + e)^2), 1/8*(2*(\\
& (a + b)*\cosh(f*x + e)^2 + 2*(a + b)*\cosh(f*x + e)*\sinh(f*x + e) + (a + b)*s \\
& inh(f*x + e)^2)*\sqrt{-b}*\arctan(\sqrt{2}*(a*\cosh(f*x + e)^2 + 2*a*\cosh(f*x + \\
& e)*\sinh(f*x + e) + a*\sinh(f*x + e)^2 + b)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 \\
& + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x \\
& + e) + \sinh(f*x + e)^2))}/(a*b*\cosh(f*x + e)^4 + 4*a*b*\cosh(f*x + e)*\sinh(f \\
& *x + e)^3 + a*b*\sinh(f*x + e)^4 + (3*a*b - b^2)*\cosh(f*x + e)^2 + (6*a*b*co \\
& sh(f*x + e)^2 + 3*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(2*a*b*\cosh(f*x + e) \\
& ^3 + (3*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e))) + 2*((a + b)*\cosh(f*x + e \\
&)^2 + 2*(a + b)*\cosh(f*x + e)*\sinh(f*x + e) + (a + b)*\sinh(f*x + e)^2)*\sqrt{ \\
& (-b)*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh \\
& (f*x + e)^2 - 1))*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a}
\end{aligned}$$

```

- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/
(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
+ 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
+ e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)
) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f
*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cos
h(f*x + e)^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2)
]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^3}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2), x, algorithm="giac")

[Out] integrate(sinh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.99 \quad \int \frac{\sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{\sqrt{b}f}$$

[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[b]*f)

Rubi [A] time = 0.0505043, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3186, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[b]*f)

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 217

```
Int[1/Sqrt[(a_.) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}} dx, x, \cosh(e+fx)\right)}{f} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{b}f} \end{aligned}$$

Mathematica [A] time = 0.10851, size = 49, normalized size = 1.2

$$\frac{\log\left(\sqrt{2a+b\cosh(2(e+fx))}-b+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2], x]
```

```
[Out] Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/(S
qrt[b]*f)
```

Maple [B] time = 0.054, size = 108, normalized size = 2.6

$$\frac{1}{2f\cosh(fx+e)}\sqrt{\left(a+b(\sinh(fx+e))^2\right)\left(\cosh(fx+e)\right)^2}\ln\left(\frac{1}{2}\left(2b(\cosh(fx+e))^2+2\sqrt{b(\cosh(fx+e))^4+(a-b)^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2), x)
```

```
[Out] 1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(
b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))/b^(1/2)/co
sh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.36186, size = 4286, normalized size = 104.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x +
e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2
*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x +
e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2
+ b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^
3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*
x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cos
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(
14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3
+ 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(
a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x +
e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x +
e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)
^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*
cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt(
```


$$\begin{aligned}
& (b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2) + 4(2a^2 b \cosh(fx + e)^7 + 3(a^3 + a^2 b) \cosh(fx + e)^5 + (9a^2 b - 4a b^2 + b^3) \cosh(fx + e)^3 + (3a b^2 - b^3) \cosh(fx + e) \sinh(fx + e)) / (\cosh(fx + e)^6 + 6 \cosh(fx + e)^5 \sinh(fx + e) + 15 \cosh(fx + e)^4 \sinh(fx + e)^2 + 20 \cosh(fx + e)^3 \sinh(fx + e)^3 + 15 \cosh(fx + e)^2 \sinh(fx + e)^4 + 6 \cosh(fx + e) \sinh(fx + e)^5 + \sinh(fx + e)^6) + \sqrt{b} \log(-b \cosh(fx + e)^4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 + 2(a - b) \cosh(fx + e)^2 + 2(3b \cosh(fx + e)^2 + a - b) \sinh(fx + e)^2 + \sqrt{2} (\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 - 1) \sqrt{b} \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)}) + 4(b \cosh(fx + e)^3 + (a - b) \cosh(fx + e) \sinh(fx + e) + b) / (\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)) / (b f), -1/2(\sqrt{-b} \arctan(\sqrt{2}(a \cosh(fx + e)^2 + 2a \cosh(fx + e) \sinh(fx + e) + a \sinh(fx + e)^2 + b) \sqrt{-b} \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)}) / (a b \cosh(fx + e)^4 + 4a b \cosh(fx + e) \sinh(fx + e)^3 + a b \sinh(fx + e)^4 + (3a b - b^2) \cosh(fx + e)^2 + (6a b \cosh(fx + e)^2 + 3a b - b^2) \sinh(fx + e)^2 + b^2 + 2(2a b \cosh(fx + e)^3 + (3a b - b^2) \cosh(fx + e) \sinh(fx + e))) + \sqrt{-b} \arctan(\sqrt{2}(\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 - 1) \sqrt{-b} \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)}) / (b \cosh(fx + e)^4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 + 2(2a - b) \cosh(fx + e)^2 + 2(3b \cosh(fx + e)^2 + 2a - b) \sinh(fx + e)^2 + 4(b \cosh(fx + e)^3 + (2a - b) \cosh(fx + e) \sinh(fx + e) + b))) / (b f)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

$$3.100 \quad \int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=42

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[a]*f))

Rubi [A] time = 0.0802426, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3186, 377, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[a]*f))

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 377

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)/((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}} dx, x, \cosh(e+fx)\right)}{f} \\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{f} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{\sqrt{a}f} \end{aligned}$$

Mathematica [A] time = 0.17785, size = 49, normalized size = 1.17

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a} \cosh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/(Sqrt[a]*f))

Maple [B] time = 0.069, size = 113, normalized size = 2.7

$$-\frac{1}{2f \cosh(fx+e)} \sqrt{(a+b(\sinh(fx+e))^2)(\cosh(fx+e))^2} \ln\left(\frac{1}{(\sinh(fx+e))^2} \left((a+b)(\cosh(fx+e))^2 + 2\sqrt{a}\sqrt{b}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/a^(1/2)*ln(((a+b)*cosh(f*x+e)
)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^
2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.07714, size = 1561, normalized size = 37.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^
3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*co
sh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*co
sh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh
(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 -
2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)
)/(sqrt(a)*f), sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2)))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
```

```
b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))
*sinh(f*x + e) + b))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2), x)
```

```
[Out] Integral(csch(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [B] time = 1.4009, size = 143, normalized size = 3.4

$$\frac{2 \arctan\left(-\frac{\sqrt{b}e^{(-2fx-2e)} - \sqrt{4ae^{(-2fx-2e)} - 2be^{(-2fx-2e)} + be^{(-4fx-4e)} + b - \sqrt{b}}}{2\sqrt{-a}}\right)}{\sqrt{-a}f} - \frac{2\sqrt{-a} \arctan\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2), x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(b)*e^(-2*f*x - 2*e) - sqrt(4*a*e^(-2*f*x - 2*e) - 2*b*e
^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b) - sqrt(b))/sqrt(-a))/sqrt(-a)*f
- 2*sqrt(-a)*arctan(sqrt(-a)/sqrt(b))/(a*f)
```

$$3.101 \quad \int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=89

$$\frac{(a+b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2a^{3/2}f} - \frac{\coth(e+fx) \operatorname{csch}(e+fx) \sqrt{a+b \cosh^2(e+fx)-b}}{2af}$$

[Out] ((a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*a^(3/2)*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x])/(2*a*f)

Rubi [A] time = 0.114886, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3186, 382, 377, 206}

$$\frac{(a+b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2a^{3/2}f} - \frac{\coth(e+fx) \operatorname{csch}(e+fx) \sqrt{a+b \cosh^2(e+fx)-b}}{2af}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*a^(3/2)*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x])/(2*a*f)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 382

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -

```
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}} dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{2af} + \frac{(a+b) \operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2af}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{2af} + \frac{(a+b) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2af}$$

$$= \frac{(a+b) \tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\sqrt{a-b+b\cosh^2(e+fx)} \coth(e+fx) \operatorname{csch}(e+fx)}{2af}$$

Mathematica [A] time = 0.318659, size = 102, normalized size = 1.15

$$\frac{2(a+b) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right) - \sqrt{2}\sqrt{a} \coth(e+fx) \operatorname{csch}(e+fx) \sqrt{2a+b\cosh(2(e+fx))-b}}{4a^{3/2}f}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] (2*(a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e + f*x]*Csch[e + f*x])/(4*a^(3/2)*f)

Maple [B] time = 0.091, size = 232, normalized size = 2.6

$$\frac{1}{4 (\sinh (fx + e))^2 \cosh (fx + e) f} \sqrt{(a + b (\sinh (fx + e))^2) (\cosh (fx + e))^2} \left(\ln \left(\frac{1}{(\sinh (fx + e))^2} \left((a + b) (\cosh (fx + e))^2 \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2*a^2+ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*b*sinh(f*x+e)^2*a-2*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2/a^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)^3}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(csch(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [B] time = 2.56671, size = 3468, normalized size = 38.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 +
(a + b)*sinh(f*x + e)^4 - 2*(a + b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x
+ e)^2 - a - b)*sinh(f*x + e)^2 + 4*((a + b)*cosh(f*x + e)^3 - (a + b)*cos
h(f*x + e))*sinh(f*x + e) + a + b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 +
4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a
- b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e
)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(
(a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(
cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*
cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)
^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2*
a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e)
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e
)*sinh(f*x + e)^3 + a^2*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f
+ 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x
+ e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((a + b)*cosh(f*x + e)^
4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 - 2*(
a + b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 - a - b)*sinh(f*x + e
)^2 + 4*((a + b)*cosh(f*x + e)^3 - (a + b)*cosh(f*x + e))*sinh(f*x + e) + a
+ b)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)
*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x
+ e) + b)) + sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e)
+ a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(
(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*f*sinh
(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2
- a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*
sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.102 \quad \int \frac{\sinh^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=229

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} - \frac{2(a+b) \tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3b^2f}$$

[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f)

Rubi [A] time = 0.217968, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3188, 479, 531, 418, 492, 411}

$$\frac{2(a+b) \tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3b^2f} + \frac{2(a+b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f)

Rule 3188

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 479

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}}{3bf} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(a\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}}{3bf} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{F\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)}{3bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{2(a+b)E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 0.951974, size = 168, normalized size = 0.73

$$\frac{-2i\sqrt{2a(2a+b)}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b) + 4i\sqrt{2a}(a+b\cosh(2(e+fx)))}{6b^2f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((4*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.068, size = 344, normalized size = 1.5

$$\frac{1}{3b\cosh(fx+e)f} \left(\sqrt{\frac{b}{a}} b \sinh(fx+e) (\cosh(fx+e))^4 + \left(\sqrt{\frac{b}{a}} a - \sqrt{\frac{b}{a}} b \right) (\cosh(fx+e))^2 \sinh(fx+e) + a \sqrt{\frac{b}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] $\frac{1}{3} * ((-1/a*b)^{(1/2)} * b * \sinh(f*x+e) * \cosh(f*x+e)^4 + ((-1/a*b)^{(1/2)} * a - (-1/a*b)^{(1/2)} * b) * \cosh(f*x+e)^2 * \sinh(f*x+e) + a * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) + 2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b - 2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a - 2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b) / b / (-1/a*b)^{(1/2)} / \cosh(f*x+e) / (a+b*\sinh(f*x+e)^2)^{(1/2)} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sinh(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2), x, algorithm="giac")

[Out] integrate(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.103 \quad \int \frac{\sinh^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=128

$$\frac{ia\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1} \text{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{bf\sqrt{a + b \sinh^2(e + fx)}} - \frac{i\sqrt{a + b \sinh^2(e + fx)} E\left(ie + ifx \left|\frac{b}{a}\right.\right)}{bf\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1}}$$

[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.135813, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3172, 3178, 3177, 3183, 3182}

$$\frac{ia\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1} F\left(ie + ifx \left|\frac{b}{a}\right.\right)}{bf\sqrt{a + b \sinh^2(e + fx)}} - \frac{i\sqrt{a + b \sinh^2(e + fx)} E\left(ie + ifx \left|\frac{b}{a}\right.\right)}{bf\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + (I*a*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3172

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)^2])/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3178

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)]/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(1*El
lipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\int \sqrt{a+b\sinh^2(e+fx)} dx}{b} - \frac{a \int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}} dx}{b} \\ &= \frac{\sqrt{a+b\sinh^2(e+fx)} \int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}} dx}{b\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} - \frac{\left(a\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\right) \int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} dx}{b\sqrt{a+b\sinh^2(e+fx)}} \\ &= -\frac{iE\left(ie+ifx\left|\frac{b}{a}\right.\right)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} + \frac{iaF\left(ie+ifx\left|\frac{b}{a}\right.\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{bf\sqrt{a+b\sinh^2(e+fx)}} \end{aligned}$$

Mathematica [A] time = 0.256913, size = 89, normalized size = 0.7

$$\frac{i\sqrt{2a+b\cosh(2(e+fx))-b}\left(E\left(i(e+fx)\left|\frac{b}{a}\right.\right)-\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)\right)}{bf\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-I)*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*(EllipticE[I*(e + f*x), b/a] - EllipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])

Maple [A] time = 0.066, size = 113, normalized size = 0.9

$$-\frac{1}{f \cosh(fx + e)} \sqrt{\frac{a + b(\sinh(fx + e))^2}{a}} \sqrt{(\cosh(fx + e))^2} \left(\text{EllipticF}\left(\sinh(fx + e) \sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) - \text{EllipticE}\left(\sinh(fx + e) \sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] -1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^2}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh(fx + e)^2}{\sqrt{b \sinh(fx + e)^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^2}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.104 \quad \int \frac{1}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=60

$$-\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1 \operatorname{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{f\sqrt{a + b \sinh^2(e + fx)}}$$

[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.0362947, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1F\left(ie + ifx \left| \frac{b}{a} \right. \right)}{f\sqrt{a + b \sinh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3183

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Simp[(1*EllipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx = \frac{\sqrt{1 + \frac{b \sinh^2(e+fx)}{a}} \int \frac{1}{\sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}} dx}{\sqrt{a + b \sinh^2(e + fx)}}$$

$$= -\frac{iF\left(ie + ifx \left| \frac{b}{a} \right. \right) \sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}}{f \sqrt{a + b \sinh^2(e + fx)}}$$

Mathematica [A] time = 0.0811603, size = 68, normalized size = 1.13

$$\frac{i \sqrt{\frac{2a + b \cosh(2(e+fx)) - b}{a}} \text{EllipticF}\left(i(e + fx), \frac{b}{a}\right)}{f \sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-1)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a])/ (f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)])]

Maple [A] time = 0.047, size = 86, normalized size = 1.4

$$\frac{1}{f \cosh(fx + e)} \sqrt{\frac{a + b (\sinh(fx + e))^2}{a}} \sqrt{(\cosh(fx + e))^2} \text{EllipticF}\left(\sinh(fx + e) \sqrt{\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) \frac{1}{\sqrt{\frac{b}{a}}} \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{1}{\sqrt{b \sinh^2(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.105 \quad \int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=134

$$\frac{\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{af} - \frac{\operatorname{coth}(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{af} - \frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

[Out] -(((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*f))

Rubi [A] time = 0.148971, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3188, 480, 12, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{af} - \frac{\operatorname{coth}(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{af} - \frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] -(((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*f))

Rule 3188

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\operatorname{coth}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{bx^2}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{af} \\
&= -\frac{\operatorname{coth}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(b\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{af} \\
&= -\frac{\operatorname{coth}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\sqrt{a+b\sinh^2(e+fx)}\operatorname{tanh}(e+fx)}{af} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{af} \\
&= -\frac{\operatorname{coth}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} - \frac{E\left(\tan^{-1}(\sinh(e+fx))\right)\left[1-\frac{b}{a}\right]\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 0.514988, size = 150, normalized size = 1.12

$$\frac{2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2}\operatorname{coth}(e+fx)(-2a-b\cosh(2(e+fx))+b) - 2ia\sqrt{\frac{2a+b\cosh(2(e+fx))}{a}}}{2af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] + (2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a*EllipticF[I*(e + f*x), b/a]/(2*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)])]

Maple [A] time = 0.079, size = 189, normalized size = 1.4

$$-\frac{1}{a\sinh(fx+e)\cosh(fx+e)f}\left(\sinh(fx+e)\sqrt{(\cosh(fx+e))^2}\sqrt{\frac{b(\cosh(fx+e))^2}{a} + \frac{a-b}{a}b}\left(\operatorname{EllipticF}\left(\sinh(fx+e), \frac{b}{a}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] `-(sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))+(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*cosh(f*x+e)^2)/a/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx+e)^2}{\sqrt{b \sinh(fx+e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}(fx+e)^2}{\sqrt{b \sinh(fx+e)^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)`

[Out] `Integral(csch(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)^2}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")`

[Out] `integrate(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)`

$$3.106 \quad \int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=267

$$\frac{b \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1 - \frac{b}{a}\right)}{3a^2 f \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} - \frac{2(a+b) \tanh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3a^2 f}$$

[Out] (2*(a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) + (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)

Rubi [A] time = 0.283546, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3188, 480, 583, 531, 418, 492, 411}

$$\frac{2(a+b) \tanh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3a^2 f} + \frac{2(a+b) \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3a^2 f} - \frac{b \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3a^2 f \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (2*(a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) + (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^ (q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
+ 1))/(a*c*e^(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
+ b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^ (q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g^(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
+ 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

$a*d, 0] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c] \ \&\& \ \text{!SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^{(3/2)}, x_Symbol] \ :> \ \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned} \int \frac{\text{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\text{sech}(e+fx)}\right) \text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\ &= -\frac{\coth(e+fx)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cosh^2(e+fx)\text{sech}(e+fx)}\right) \text{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{3f} \\ &= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} \\ &= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} \\ &= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} \\ &= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} \end{aligned}$$

Mathematica [C] time = 3.79533, size = 201, normalized size = 0.75

$$\frac{-2ia(2a+b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \frac{\coth(e+fx)\text{csch}^2(e+fx)((4a^2-2ab-4b^2)\cosh(2(e+fx))-8a^2+b(a+b)\cosh(4(e+fx)))}{\sqrt{2}}}{6a^2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] (((-8*a^2 + a*b + 3*b^2 + (4*a^2 - 2*a*b - 4*b^2)*Cosh[2*(e + f*x)] + b*(a + b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (4*I)*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] - (2*I)*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a]/(6*a^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.098, size = 456, normalized size = 1.7

$$\frac{1}{3a^2(\sinh(fx+e))^3 \cosh(fx+e)} f \left(2\sqrt{-\frac{b}{a}} ab (\sinh(fx+e))^6 + 2\sqrt{-\frac{b}{a}} b^2 (\sinh(fx+e))^6 + b\sqrt{\frac{a+b(\sinh(fx+e))}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+3*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2)/a^2/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx+e)^4}{\sqrt{b \sinh(fx+e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\text{csch}(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{csch}(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.107 \quad \int \frac{\sinh^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{b^{3/2} f} - \frac{a \cosh(e+fx)}{bf(a-b)\sqrt{a+b \cosh^2(e+fx)-b}}$$

[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(3/2)*f) - (a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rubi [A] time = 0.112454, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3186, 385, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{b^{3/2} f} - \frac{a \cosh(e+fx)}{bf(a-b)\sqrt{a+b \cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(3/2)*f) - (a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 385

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b

c(n*(p + 1) + 1)/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^3(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+bx^2)^{3/2}} dx, x, \cosh(e + fx)\right)}{f} \\
 &= -\frac{a \cosh(e + fx)}{(a-b)bf\sqrt{a-b+b \cosh^2(e + fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}} dx, x, \cosh(e + fx)\right)}{bf} \\
 &= -\frac{a \cosh(e + fx)}{(a-b)bf\sqrt{a-b+b \cosh^2(e + fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{bf} \\
 &= \frac{\tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a-b+b \cosh^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a \cosh(e + fx)}{(a-b)bf\sqrt{a-b+b \cosh^2(e + fx)}}
 \end{aligned}$$

Mathematica [A] time = 0.422344, size = 98, normalized size = 1.18

$$\frac{\log\left(\sqrt{2a + b \cosh(2(e + fx)) - b} + \sqrt{2}\sqrt{b} \cosh(e + fx)\right)}{b^{3/2}f} - \frac{\sqrt{2}a \cosh(e + fx)}{bf(a-b)\sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] -((Sqrt[2]*a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]) + Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/(b^(3/2)*f)

Maple [A] time = 0.109, size = 146, normalized size = 1.8

$$\frac{1}{f \cosh(fx + e)} \sqrt{(a + b(\sinh(fx + e))^2)(\cosh(fx + e))^2} \left(\frac{1}{2} \ln \left(\left(\frac{a}{2} + \frac{b}{2} + b(\sinh(fx + e))^2 \right) \frac{1}{\sqrt{b}} + \sqrt{(a + b(\sinh(fx + e))^2)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(3/2)*ln((1/2*a+1/2*b+b*sinh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))-a/b*cosh(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^3}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(sinh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [B] time = 3.16763, size = 7602, normalized size = 91.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out]
$$\begin{aligned} & [1/4*((a*b - b^2)*\cosh(f*x + e)^4 + 4*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*\cosh(f*x + e)^2 + 2*(3*(a*b - b^2)*\cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*\sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b - b^2)*\cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\log((a^2*b*\cosh(f*x + e)^8 + 8*a^2*b*\cosh(f*x + e)*\sinh(f*x + e)^7 + a^2*b*\sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*\cosh(f*x + e)^6 + 2*(14*a^2*b*\cosh(f*x + e)^2 + a^3 + a^2*b)*\sinh(f*x + e)^6 + 4*(14*a^2*b*\cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^4 + (70*a^2*b*\cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*a^2*b*\cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*\cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*\cosh(f*x + e)^2 + 2*(14*a^2*b*\cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*\cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + \sqrt{2}*(a^2*\cosh(f*x + e)^6 + 6*a^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + a^2*\sinh(f*x + e)^6 + 3*a^2*\cosh(f*x + e)^4 + 3*(5*a^2*\cosh(f*x + e)^2 + a^2)*\sinh(f*x + e)^4 + 4*(5*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e))*\sinh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e)^2 + (15*a^2*\cosh(f*x + e)^4 + 18*a^2*\cosh(f*x + e)^2 + 4*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 2*(3*a^2*\cosh(f*x + e)^5 + 6*a^2*\cosh(f*x + e)^3 + (4*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(2*a^2*b*\cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*\cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + ((a*b - b^2)*\cosh(f*x + e)^4 + 4*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*\cosh(f*x + e)^2 + 2*(3*(a*b - b^2)*\cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*\sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b - b^2)*\cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\log(-(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b)*\sinh(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) - 4*\sqrt{2}*(a*b*\cosh(f*x + e)^2 + 2*a*b*\cosh(f*x + e)*\sinh(f*x + e) + a*b*\sinh(f*x + e)^2 + a*b)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)} \end{aligned}$$

```

/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*
b^3 - b^4)*f*cosh(f*x + e)^4 + 4*(a*b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e
)^3 + (a*b^3 - b^4)*f*sinh(f*x + e)^4 + 2*(2*a^2*b^2 - 3*a*b^3 + b^4)*f*cos
h(f*x + e)^2 + 2*(3*(a*b^3 - b^4)*f*cosh(f*x + e)^2 + (2*a^2*b^2 - 3*a*b^3
+ b^4)*f)*sinh(f*x + e)^2 + (a*b^3 - b^4)*f + 4*((a*b^3 - b^4)*f*cosh(f*x +
e)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(
((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3
+ (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2
*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*
b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x +
e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f
*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b))*sqrt(-b)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*si
nh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*
b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x
+ e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + ((a*b - b^2)*cosh(f
*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(
f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^2)*cos
h(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b -
b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))
*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2 - 1))*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh
(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e)
+ b)) + 2*sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e)
+ a*b*sinh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)))/((a*b^3 - b^4)*f*cosh(f*x + e)^4 + 4*(a*b^3 - b^4)*f*cosh(f*x + e)*sin
h(f*x + e)^3 + (a*b^3 - b^4)*f*sinh(f*x + e)^4 + 2*(2*a^2*b^2 - 3*a*b^3 + b
^4)*f*cosh(f*x + e)^2 + 2*(3*(a*b^3 - b^4)*f*cosh(f*x + e)^2 + (2*a^2*b^2 -
3*a*b^3 + b^4)*f)*sinh(f*x + e)^2 + (a*b^3 - b^4)*f + 4*((a*b^3 - b^4)*f*c
osh(f*x + e)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)
)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^3}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```


$$3.108 \quad \int \frac{\sinh(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=36

$$\frac{\cosh(e+fx)}{f(a-b)\sqrt{a+b \cosh^2(e+fx)-b}}$$

[Out] Cosh[e + f*x]/((a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rubi [A] time = 0.0526382, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3186, 191}

$$\frac{\cosh(e+fx)}{f(a-b)\sqrt{a+b \cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] Cosh[e + f*x]/((a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sinh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx = \frac{\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{3/2}} dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)}{(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.15519, size = 43, normalized size = 1.19

$$\frac{\sqrt{2} \cosh(e+fx)}{f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (Sqrt[2]*Cosh[e + f*x])/((a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.051, size = 32, normalized size = 0.9

$$\frac{\cosh(fx+e)}{(a-b)f} \frac{1}{\sqrt{a+b(\sinh(fx+e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2), x)

[Out] cosh(f*x+e)/(a-b)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [B] time = 1.60599, size = 319, normalized size = 8.86

$$\frac{b^2 e^{(-6fx-6e)} + 2ab - b^2 + (8a^2 - 8ab + 3b^2)e^{(-2fx-2e)} + 3(2ab - b^2)e^{(-4fx-4e)}}{2(a^2 - ab)\left(2(2a - b)e^{(-2fx-2e)} + be^{(-4fx-4e)} + b\right)^{\frac{3}{2}}f} + \frac{b^2 + 3(2ab - b^2)e^{(-2fx-2e)} + (8a^2 - 8ab + 3b^2)e^{(-4fx-4e)}}{2(a^2 - ab)\left(2(2a - b)e^{(-2fx-2e)} + be^{(-4fx-4e)} + b\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{2}(b^2 e^{-6fx-6e} + 2ab - b^2 + (8a^2 - 8ab + 3b^2)e^{-2fx-2e} + 3(2ab - b^2)e^{-4fx-4e}) / ((a^2 - ab)(2(2a - b)e^{-2fx-2e} + b e^{-4fx-4e} + b)^{3/2} f) + \frac{1}{2}(b^2 + 3(2ab - b^2)e^{-2fx-2e} + (8a^2 - 8ab + 3b^2)e^{-4fx-4e} + (2ab - b^2)e^{-6fx-6e}) / ((a^2 - ab)(2(2a - b)e^{-2fx-2e} + b e^{-4fx-4e} + b)^{3/2} f)$

Fricas [B] time = 2.17215, size = 738, normalized size = 20.5

$$\sqrt{2}(\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 + 1)$$

$$(ab - b^2)f \cosh(fx + e)^4 + 4(ab - b^2)f \cosh(fx + e) \sinh(fx + e)^3 + (ab - b^2)f \sinh(fx + e)^4 + 2(2a^2 - 3ab + b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] $\sqrt{2}(\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 + 1) \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)} / ((ab - b^2) f \cosh(fx + e)^4 + 4(ab - b^2) f \cosh(fx + e) \sinh(fx + e)^3 + (ab - b^2) f \sinh(fx + e)^4 + 2(2a^2 - 3ab + b^2) f \cosh(fx + e)^2 + 2(3(ab - b^2) f \cosh(fx + e)^2 + (2a^2 - 3ab + b^2) f) \sinh(fx + e)^2 + (ab - b^2) f + 4((ab - b^2) f \cosh(fx + e)^3 + (2a^2 - 3ab + b^2) f \cosh(fx + e)) \sinh(fx + e)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [B] time = 1.22891, size = 207, normalized size = 5.75

$$-\frac{\sqrt{b}f}{256(ab^4 - b^5)} + \frac{\frac{(a^3f - a^2bf)e^{(2fx+2e)}}{a^4b^3 - 2a^3b^4 + a^2b^5} + \frac{a^3f - a^2bf}{a^4b^3 - 2a^3b^4 + a^2b^5}}{256\sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] $-\frac{1}{256}\sqrt{b}f/(a*b^4 - b^5) + \frac{1}{256}*((a^3*f - a^2*b*f)*e^{(2*f*x + 2*e)})/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5) + (a^3*f - a^2*b*f)/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5)/\sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b}$

$$3.109 \quad \int \frac{\operatorname{csch}(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{a^{3/2}f} - \frac{b \cosh(e+fx)}{af(a-b)\sqrt{a+b \cosh^2(e+fx)-b}}$$

[Out] $-(\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cosh}[e+f*x])/\operatorname{Sqrt}[a-b+b*\operatorname{Cosh}[e+f*x]^2]]/(a^{(3/2)}*f)) - (b*\operatorname{Cosh}[e+f*x])/(a*(a-b)*f*\operatorname{Sqrt}[a-b+b*\operatorname{Cosh}[e+f*x]^2])$

Rubi [A] time = 0.10844, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3186, 382, 377, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{a^{3/2}f} - \frac{b \cosh(e+fx)}{af(a-b)\sqrt{a+b \cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[e+f*x]/(a+b*\operatorname{Sinh}[e+f*x]^2)^{(3/2)}, x]$

[Out] $-(\operatorname{ArcTanh}[(\operatorname{Sqrt}[a]*\operatorname{Cosh}[e+f*x])/\operatorname{Sqrt}[a-b+b*\operatorname{Cosh}[e+f*x]^2]]/(a^{(3/2)}*f)) - (b*\operatorname{Cosh}[e+f*x])/(a*(a-b)*f*\operatorname{Sqrt}[a-b+b*\operatorname{Cosh}[e+f*x]^2])$

Rule 3186

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e+f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1-ff^2*x^2)^{(m-1)/2}*(a+b-b*ff^2*x^2)^p, x], x, \operatorname{Cos}[e+f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 382

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*x*(a+b*x^n)^{(p+1)}*(c+d*x^n)^{(q+1)})/(a*n*(p+1)*(b*c-a*d)), x] + \operatorname{Dist}[(b*c+n*(p+1)*(b*c-a*d))/(a*n*(p+1)*(b*c-a*d)), I$

```
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x]
&& NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{csch}(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^{3/2}} dx, x, \cosh(e+fx)\right)}{f} \\ &= -\frac{b \cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}} dx, x, \cosh(e+fx)\right)}{af} \\ &= -\frac{b \cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{af} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{3/2}f} - \frac{b \cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} \end{aligned}$$

Mathematica [A] time = 0.400944, size = 98, normalized size = 1.17

$$\frac{\frac{\sqrt{2}\sqrt{ab}\cosh(e+fx)}{(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}} - \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (-ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] - (Sqrt[2]*Sqrt[a]*b*Cosh[e + f*x])/((a - b)*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]))/(a^(3/2)*f)

Maple [B] time = 0.123, size = 154, normalized size = 1.8

$$\frac{1}{f \cosh(fx + e)} \sqrt{(a + b(\sinh(fx + e))^2)(\cosh(fx + e))^2} \left(-\frac{b(\cosh(fx + e))^2}{a(a - b)} \frac{1}{\sqrt{(a + b(\sinh(fx + e))^2)(\cosh(fx + e))^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/a*b*cosh(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)-1/2/a^(3/2)*ln((2*a+(a+b)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [B] time = 2.58476, size = 4082, normalized size = 48.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^(3/2)),x, algorithm="fricas")

[Out]
$$\frac{1}{2} \left(\left((a*b - b^2) \cosh(f*x + e)^4 + 4*(a*b - b^2) \cosh(f*x + e) \sinh(f*x + e)^3 + (a*b - b^2) \sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2) \cosh(f*x + e)^2 + 2*(3*(a*b - b^2) \cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2) \sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b - b^2) \cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2) \cosh(f*x + e)) \sinh(f*x + e) \right) \sqrt{a} \log(-((a + b) \cosh(f*x + e)^4 + 4*(a + b) \cosh(f*x + e) \sinh(f*x + e)^3 + (a + b) \sinh(f*x + e)^4 + 2*(3*a - b) \cosh(f*x + e)^2 + 2*(3*(a + b) \cosh(f*x + e)^2 + 3*a - b) \sinh(f*x + e)^2 - 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e)^2 + 1)) \sqrt{a} \sqrt{(b \cosh(f*x + e)^2 + b \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2*\cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e)^2)} + 4*((a + b) \cosh(f*x + e)^3 + (3*a - b) \cosh(f*x + e)) \sinh(f*x + e) + a + b) / (\cosh(f*x + e)^4 + 4*\cosh(f*x + e) \sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 - 1) \sinh(f*x + e)^2 - 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 - \cosh(f*x + e)) \sinh(f*x + e) + 1) - 2*\sqrt{2}*(a*b \cosh(f*x + e)^2 + 2*a*b \cosh(f*x + e) \sinh(f*x + e) + a*b \sinh(f*x + e)^2 + a*b) \sqrt{(b \cosh(f*x + e)^2 + b \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2*\cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e)^2)} \right) / ((a^3*b - a^2*b^2)*f*\cosh(f*x + e)^4 + 4*(a^3*b - a^2*b^2)*f*\cosh(f*x + e) \sinh(f*x + e)^3 + (a^3*b - a^2*b^2)*f*\sinh(f*x + e)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*f*\cosh(f*x + e)^2 + 2*(3*(a^3*b - a^2*b^2)*f*\cosh(f*x + e)^2 + (2*a^4 - 3*a^3*b + a^2*b^2)*f) \sinh(f*x + e)^2 + (a^3*b - a^2*b^2)*f + 4*((a^3*b - a^2*b^2)*f*\cosh(f*x + e)^3 + (2*a^4 - 3*a^3*b + a^2*b^2)*f*\cosh(f*x + e)) \sinh(f*x + e)), ((a*b - b^2) \cosh(f*x + e)^4 + 4*(a*b - b^2) \cosh(f*x + e) \sinh(f*x + e)^3 + (a*b - b^2) \sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2) \cosh(f*x + e)^2 + 2*(3*(a*b - b^2) \cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2) \sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b - b^2) \cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2) \cosh(f*x + e)) \sinh(f*x + e)) \sqrt{(-a) \arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e)^2 + 1)) \sqrt{(-a) \sqrt{(b \cosh(f*x + e)^2 + b \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2*\cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e)^2)}}} / (b \cosh(f*x + e)^4 + 4*b \cosh(f*x + e) \sinh(f*x + e)^3 + b \sinh(f*x + e)^4 + 2*(2*a - b) \cosh(f*x + e)^2 + 2*(3*b \cosh(f*x + e)^2 + 2*a - b) \sinh(f*x + e)^2 + 4*(b \cosh(f*x + e)^3 + (2*a - b) \cosh(f*x + e)) \sinh(f*x + e) + b)) - \sqrt{2}*(a*b \cosh(f*x + e)^2 + 2*a*b \cosh(f*x + e) \sinh(f*x + e) + a*b \sinh(f*x + e)^2 + a*b) \sqrt{(b \cosh(f*x + e)^2 + b \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2*\cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e)^2)} \right) / ((a^3*b - a^2*b^2)*f*\cosh(f*x + e)^4 + 4*(a^3*b - a^2*b^2)*f*\cosh(f*x + e) \sinh(f*x + e)^3 + (a^3*b - a^2*b^2)*f*\sinh(f*x + e)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*f*\cosh(f*x + e)^2 + 2*(3*(a^3*b - a^2*b^2)*f*\cosh(f*x + e)^2 + (2*a^4 - 3*a^3*b + a^2*b^2)*f) \sinh(f*x + e)^2 + (a^3*b - a^2*b^2)*f + 4*((a^3*b - a^2*b^2)*f*\cosh(f*x + e)^3 + (2*a^4 - 3*a^3*b + a^2*b^2)*f*\cosh(f*x + e)$$

e))*sinh(f*x + e))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [B] time = 1.4239, size = 331, normalized size = 3.94

$$\frac{a\sqrt{b}f}{8(ab^3 - b^4)} - \frac{\frac{(a^4bf - a^3b^2f)e^{(2fx+2e)}}{a^4b^3 - 2a^3b^4 + a^2b^5} + \frac{a^4bf - a^3b^2f}{a^4b^3 - 2a^3b^4 + a^2b^5}}{8\sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b}} + \frac{2 \arctan\left(-\frac{\sqrt{be^{(2fx+2e)}} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b}}{2\sqrt{-a}}\right)}{\sqrt{-a}af}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] 1/8*a*sqrt(b)*f/(a*b^3 - b^4) - 1/8*((a^4*b*f - a^3*b^2*f)*e^(2*f*x + 2*e)/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5) + (a^4*b*f - a^3*b^2*f)/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))/sqrt(-a)*a*f)

$$3.110 \quad \int \frac{\operatorname{csch}^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=139

$$-\frac{b(a-3b) \cosh(e+fx)}{2a^2 f(a-b) \sqrt{a+b \cosh^2(e+fx)-b}} + \frac{(a+3b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2a^{5/2} f} - \frac{\coth(e+fx) \operatorname{csch}(e+fx)}{2af \sqrt{a+b \cosh^2(e+fx)-b}}$$

[Out] ((a + 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*a^(5/2)*f) - ((a - 3*b)*b*Cosh[e + f*x])/(2*a^2*(a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x])/(2*a*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rubi [A] time = 0.181834, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3186, 414, 527, 12, 377, 206}

$$-\frac{b(a-3b) \cosh(e+fx)}{2a^2 f(a-b) \sqrt{a+b \cosh^2(e+fx)-b}} + \frac{(a+3b) \tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{2a^{5/2} f} - \frac{\coth(e+fx) \operatorname{csch}(e+fx)}{2af \sqrt{a+b \cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((a + 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/(2*a^(5/2)*f) - ((a - 3*b)*b*Cosh[e + f*x])/(2*a^2*(a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x])/(2*a*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+bx^2)^{3/2}} dx, x, \cosh(e+fx)\right)}{f} \\
&= -\frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{a+b+2bx^2}{(1-x^2)(a-b+bx^2)^{3/2}} dx, x, \cosh(e+fx)\right)}{2af} \\
&= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{a}{(1-x^2)(a-b+bx^2)^{3/2}} dx, x, \cosh(e+fx)\right)}{2af} \\
&= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{(a+3b)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^{3/2}} dx, x, \cosh(e+fx)\right)}{2af} \\
&= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{(a+3b)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^{3/2}} dx, x, \cosh(e+fx)\right)}{2af} \\
&= \frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.71249, size = 134, normalized size = 0.96

$$\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{5/2}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)(2a^2+b(a-3b)\cosh(2(e+fx))-3ab+3b^2)}{a^2(a-b)\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (((a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])/a^(5/2) - ((2*a^2 - 3*a*b + 3*b^2 + (a - 3*b)*b*Cosh[2*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x])/(a^2*(a - b)*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])/(2*f)

Maple [B] time = 0.132, size = 251, normalized size = 1.8

$$\frac{1}{f \cosh(fx + e)} \sqrt{(a + b(\sinh(fx + e))^2)(\cosh(fx + e))^2} \left(-\frac{1}{2a^2(\sinh(fx + e))^2} \sqrt{(a + b(\sinh(fx + e))^2)(\cosh(fx + e))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2), x)`

[Out] $((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*(-1/2/a^2/\sinh(f*x+e)^2*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}+1/4/a^{(3/2)}*\ln((2*a+(a+b)*\sinh(f*x+e)^2+2*a^{(1/2)}*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)})/\sinh(f*x+e)^2+3/4/a^{(5/2)}*b*\ln((2*a+(a+b)*\sinh(f*x+e)^2+2*a^{(1/2)}*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)})/\sinh(f*x+e)^2)+b^2/a^2*\cosh(f*x+e)^2/(a-b)/((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)})/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)^3}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="maxima")`

[Out] `integrate(csch(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)`

Fricas [B] time = 4.79229, size = 10539, normalized size = 75.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="fricas")`

```
[Out] [1/4*((a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 8*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b + 2*a*b^2 - 3*b^3)*sinh(f*x + e)^8 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3 + 7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^4 + 2*(35*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 - 4*a^3 - 5*a^2*b + 18*a*b^2 - 9*b^3 + 30*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 10*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^3 - (4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b + 2*a*b^2 - 3*b^3 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^2 + 4*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 15*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + a^3 + a^2*b - 5*a*b^2 + 3*b^3 - 3*(4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 3*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^5 - (4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a^2*b - 3*a*b^2)*cosh(f*x + e)^6 + 6*(a^2*b - 3*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b - 3*a*b^2)*sinh(f*x + e)^6 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + (4*a^3 - 5*a^2*b + 3*a*b^2 + 15*(a^2*b - 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(a^2*b - 3*a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b - 3*a*b^2 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^2 + (15*(a^2*b - 3*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 5*a^2*b + 3*a*b^2 + 6*(4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a^2*b - 3*a*b^2)*cosh(f*x + e)^5 + 2*(4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - a^3*b^2)*f*cosh(f*x + e)^8 + 8*(a^4*b - a^3*b^2)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4*b - a^3*b^2)*f*sinh(f*x + e)^8 + 4*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^6 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x + e)^6 - 2*(4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^3 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)
```

$$\begin{aligned}
& 2)*f)*\sinh(f*x + e)^4 + 4*(a^5 - 2*a^4*b + a^3*b^2)*f*\cosh(f*x + e)^2 + 8*(\\
& 7*(a^4*b - a^3*b^2)*f*\cosh(f*x + e)^5 + 10*(a^5 - 2*a^4*b + a^3*b^2)*f*\cosh \\
& (f*x + e)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*\cosh(f*x + e))*\sinh(f*x + e)^ \\
& 3 + 4*(7*(a^4*b - a^3*b^2)*f*\cosh(f*x + e)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2) \\
& *f*\cosh(f*x + e)^4 - 3*(4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*\cosh(f*x + e)^2 + (a \\
& ^5 - 2*a^4*b + a^3*b^2)*f)*\sinh(f*x + e)^2 + (a^4*b - a^3*b^2)*f + 8*((a^4* \\
& b - a^3*b^2)*f*\cosh(f*x + e)^7 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*\cosh(f*x + e \\
&)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*\cosh(f*x + e)^3 + (a^5 - 2*a^4*b + a^ \\
& 3*b^2)*f*\cosh(f*x + e))*\sinh(f*x + e)), -1/2*(((a^2*b + 2*a*b^2 - 3*b^3)*co \\
& sh(f*x + e)^8 + 8*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)*\sinh(f*x + e)^7 + \\
& (a^2*b + 2*a*b^2 - 3*b^3)*\sinh(f*x + e)^8 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b \\
& ^3)*cosh(f*x + e)^6 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3 + 7*(a^2*b + 2*a*b^2 \\
& - 3*b^3)*cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3) \\
& *cosh(f*x + e)^3 + 3*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e))*\sinh(f* \\
& x + e)^5 - 2*(4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^4 + 2*(35*(\\
& a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 - 4*a^3 - 5*a^2*b + 18*a*b^2 - 9*b \\
& ^3 + 30*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*\sinh(f*x + e)^4 + \\
& 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 10*(a^3 + a^2*b - 5*a*b^2 \\
& + 3*b^3)*cosh(f*x + e)^3 - (4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + \\
& e))*\sinh(f*x + e)^3 + a^2*b + 2*a*b^2 - 3*b^3 + 4*(a^3 + a^2*b - 5*a*b^2 + \\
& 3*b^3)*cosh(f*x + e)^2 + 4*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 1 \\
& 5*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + a^3 + a^2*b - 5*a*b^2 + \\
& 3*b^3 - 3*(4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^2)*\sinh(f*x + \\
& e)^2 + 8*((a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 3*(a^3 + a^2*b - 5*a \\
& *b^2 + 3*b^3)*cosh(f*x + e)^5 - (4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f \\
& *x + e)^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e))*\sinh(f*x + e))*s \\
& \text{qrt}(-a)*\arctan(\text{sqrt}(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*\sinh(f*x + e) + s \\
& \text{inh}(f*x + e)^2 + 1)*\text{sqrt}(-a)*\text{sqrt}((b*cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + \\
& 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 \\
&))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e) \\
& ^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*\sinh(f \\
& *x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*\sinh(f*x + e) + \\
& b)) + \text{sqrt}(2)*((a^2*b - 3*a*b^2)*cosh(f*x + e)^6 + 6*(a^2*b - 3*a*b^2)*cos \\
& h(f*x + e)*\sinh(f*x + e)^5 + (a^2*b - 3*a*b^2)*\sinh(f*x + e)^6 + (4*a^3 - 5 \\
& *a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + (4*a^3 - 5*a^2*b + 3*a*b^2 + 15*(a^2*b \\
& - 3*a*b^2)*cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(5*(a^2*b - 3*a*b^2)*cosh(f \\
& *x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e))*\sinh(f*x + e)^3 + a^ \\
& 2*b - 3*a*b^2 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^2 + (15*(a^2*b - \\
& 3*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 5*a^2*b + 3*a*b^2 + 6*(4*a^3 - 5*a^2*b + \\
& 3*a*b^2)*cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 2*(3*(a^2*b - 3*a*b^2)*cosh(f* \\
& x + e)^5 + 2*(4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b \\
& + 3*a*b^2)*cosh(f*x + e))*\sinh(f*x + e))*\text{sqrt}((b*cosh(f*x + e)^2 + b*\sinh(\\
& f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*\sinh(f*x + e) + si \\
& nh(f*x + e)^2)))/((a^4*b - a^3*b^2)*f*\cosh(f*x + e)^8 + 8*(a^4*b - a^3*b^2) \\
& *f*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^4*b - a^3*b^2)*f*\sinh(f*x + e)^8 + 4*
\end{aligned}$$

```
(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^6 + 4*(7*(a^4*b - a^3*b^2)*f*cosh
(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x + e)^6 - 2*(4*a^5 - 7*a
^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e
)^3 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*
(a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f
*x + e)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f)*sinh(f*x + e)^4 + 4*(a^5 - 2*a
^4*b + a^3*b^2)*f*cosh(f*x + e)^2 + 8*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^
5 + 10*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^3 - (4*a^5 - 7*a^4*b + 3*a
^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*
x + e)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^4 - 3*(4*a^5 - 7*a^
4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x
+ e)^2 + (a^4*b - a^3*b^2)*f + 8*((a^4*b - a^3*b^2)*f*cosh(f*x + e)^7 + 3*(
a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*
f*cosh(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)^3}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(csch(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.111 \quad \int \frac{\sinh^6(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=341

$$\frac{(4a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3b^2f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}-\frac{(8a^2-3ab-2b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)}$$

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
(3*(a - b)*b^2*f) + ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]]
, 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*b^3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((4*a - b)*EllipticF[ArcTa
n[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a
- b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 -
3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*b^3*f)
```

Rubi [A] time = 0.340662, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3188, 470, 582, 531, 418, 492, 411}

$$\frac{(8a^2-3ab-2b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)}+\frac{(8a^2-3ab-2b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3b^3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
(3*(a - b)*b^2*f) + ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]]
, 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*b^3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((4*a - b)*EllipticF[ArcTa
n[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a
- b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 -
3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*b^3*f)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1) + 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
  := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
  + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
  a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^6}{\sqrt{1+x^2}(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^2(3a+4a)}{\sqrt{1+x^2}\sqrt{a}} dx, x, \sinh(e+fx)\right)}{(a-b)bf} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b) \cosh(e+fx) \sinh(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{3(a-b)b^2f} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b) \cosh(e+fx) \sinh(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{3(a-b)b^2f} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b) \cosh(e+fx) \sinh(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{3(a-b)b^2f} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b) \cosh(e+fx) \sinh(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{3(a-b)b^2f} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b) \cosh(e+fx) \sinh(e+fx) \sqrt{a+b\sinh^2(e+fx)}}{3(a-b)b^2f}
\end{aligned}$$

Mathematica [C] time = 1.25201, size = 211, normalized size = 0.62

$$\frac{-2i\sqrt{2a}(8a^2 - 7ab - b^2) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - b \sinh(2(e+fx))(-8a^2 + b(b-a) \cosh(2(e+fx)))}{6b^3 f(a-b) \sqrt{4a + 2b \cosh(2(e+fx))} - 2b}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((2*I)*Sqrt[2]*a*(8*a^2 - 3*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(8*a^2 - 7*a*b - b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] - b*(-8*a^2 + 3*a*b - b^2 + b*(-a + b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]/(6*(a - b)*b^3*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.102, size = 500, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2), x)

[Out] 1/3*((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+4*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))-2*a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^2+3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)-(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/b^2/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^6}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sinh(fx + e)^2 + a} \sinh(fx + e)^6}{b^2 \sinh(fx + e)^4 + 2ab \sinh(fx + e)^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^6}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

$$3.112 \quad \int \frac{\sinh^4(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=256

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{bf(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{(2a-b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{b^2f(a-b)}$$

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/((a - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*
Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/((
(a - b)*b^2*f)
```

Rubi [A] time = 0.235018, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3188, 470, 531, 418, 492, 411}

$$\frac{(2a-b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{b^2f(a-b)} - \frac{(2a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{b^2f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/((a - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*
Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/((
(a - b)*b^2*f)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```


Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{a \cosh(e+fx) \sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a+(2a-b)}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{(a-b)bf} \\
&= -\frac{a \cosh(e+fx) \sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(a\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{(a-b)bf} \\
&= -\frac{a \cosh(e+fx) \sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{F\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right) \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&= -\frac{a \cosh(e+fx) \sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right) \operatorname{sech}(e+fx)}{(a-b)b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 1.05016, size = 156, normalized size = 0.61

$$\frac{a\left(4i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - 2i(2a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right) - \sqrt{2}b\sinh(2(e+fx))\right)}{2b^2f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (a*((-2*I)*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] + (4*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]/(2*(a - b)*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.093, size = 313, normalized size = 1.2

$$-\frac{1}{b(a-b)\cosh(fx+e)f}\left(\sqrt{-\frac{b}{a}}\sinh(fx+e)(\cosh(fx+e))^2+a\sqrt{\frac{b(\cosh(fx+e))^2}{a}+\frac{a-b}{a}}\sqrt{(\cosh(fx+e))^2}\right)\text{EllipticE}\left(\frac{\sinh(fx+e)}{\cosh(fx+e)},\frac{a-b}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out]
$$-1/b*((-1/a*b)^{(1/2)}*a*\sinh(f*x+e)*\cosh(f*x+e)^2+a*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b-2*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a+(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b)/(a-b)/(-1/a*b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx+e)^4}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh(fx+e)^2+a}\sinh(fx+e)^4}{b^2\sinh(fx+e)^4+2ab\sinh(fx+e)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^4}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

$$3.113 \quad \int \frac{\sinh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=173

$$\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1 \operatorname{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{bf\sqrt{a + b \sinh^2(e+fx)}} + \frac{\sinh(e+fx) \cosh(e+fx)}{f(a-b)\sqrt{a + b \sinh^2(e+fx)}} + \frac{i\sqrt{a + b \sinh^2(e+fx)} E\left(ie + ifx \left|\frac{b}{a}\right.\right)}{bf(a-b)\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1}$$

[Out] (Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) - (I*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.212076, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{\sinh(e+fx) \cosh(e+fx)}{f(a-b)\sqrt{a + b \sinh^2(e+fx)}} - \frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1 F\left(ie + ifx \left|\frac{b}{a}\right.\right)}{bf\sqrt{a + b \sinh^2(e+fx)}} + \frac{i\sqrt{a + b \sinh^2(e+fx)} E\left(ie + ifx \left|\frac{b}{a}\right.\right)}{bf(a-b)\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) - (I*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3173

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*(a + b)*(p + 1)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x] /; Fr

eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]

Rule 3172

Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3178

Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3177

Int[Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rule 3183

Int[1/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*EllipticF[e + f*x, -(b/a)]/(Sqrt[a]*f)), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{a+a\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx}{a(a-b)} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}} dx}{b} - \frac{\int \sqrt{a+b\sinh^2(e+fx)} dx}{(a-b)b} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\sqrt{a+b\sinh^2(e+fx)} \int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}} dx}{(a-b)b\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} + \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{(a-b)b} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{iE\left(ie+ifx\left|\frac{b}{a}\right.\right)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} - \frac{iF\left(ie+ifx\left|\frac{b}{a}\right.\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.481106, size = 151, normalized size = 0.87

$$\frac{-i\sqrt{2}(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)+b\sinh(2(e+fx))}{bf(a-b)\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*Sinh[2*(e + f*x)]/((a - b)*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.084, size = 127, normalized size = 0.7

$$-\frac{1}{(a-b)\cosh(fx+e)f}\left(-\sqrt{\frac{b}{a}}\sinh(fx+e)(\cosh(fx+e))^2+\sqrt{\frac{b(\cosh(fx+e))^2}{a}+\frac{a-b}{a}}\sqrt{(\cosh(fx+e))^2}\text{EllipticE}\left(\sqrt{\frac{b}{a}}\sinh(fx+e),\frac{b}{a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out] `-((-1/a*b)^(1/2)*sinh(f*x+e)*cosh(f*x+e)^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(fx + e)}{(b \sinh^2(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \sinh^2(fx + e)}{b^2 \sinh^4(fx + e) + 2ab \sinh^2(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^2}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.114 \quad \int \frac{1}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=115

$$\frac{b \sinh(e+fx) \cosh(e+fx)}{af(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{i\sqrt{a+b \sinh^2(e+fx)}E\left(ie+ifx \left| \frac{b}{a} \right. \right)}{af(a-b)\sqrt{\frac{b \sinh^2(e+fx)}{a}+1}}$$

[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rubi [A] time = 0.0616497, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3184, 21, 3178, 3177}

$$\frac{b \sinh(e+fx) \cosh(e+fx)}{af(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{i\sqrt{a+b \sinh^2(e+fx)}E\left(ie+ifx \left| \frac{b}{a} \right. \right)}{af(a-b)\sqrt{\frac{b \sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]

[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rule 3184

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a + b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]

Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \sinh^2(e + fx))^{3/2}} dx &= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} - \frac{\int \frac{-a - b \sinh^2(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx}{a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} + \frac{\int \sqrt{a + b \sinh^2(e + fx)} dx}{a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} + \frac{\sqrt{a + b \sinh^2(e + fx)} \int \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}} dx}{a(a - b)\sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} - \frac{iE\left(ie + ifx \left| \frac{b}{a} \right. \right) \sqrt{a + b \sinh^2(e + fx)}}{a(a - b)f\sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.159245, size = 100, normalized size = 0.87

$$\frac{-\sqrt{2}b \sinh(2(e + fx)) - 2ia\sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} E\left(ie + ifx \left| \frac{b}{a} \right. \right)}{2af(a - b)\sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]

[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.078, size = 252, normalized size = 2.2

$$\frac{1}{a(a-b)\cosh(fx+e)f} \left(-\sqrt{-\frac{b}{a}}b\sinh(fx+e)(\cosh(fx+e))^2 + a\sqrt{\frac{b(\cosh(fx+e))^2}{a} + \frac{a-b}{a}}\sqrt{(\cosh(fx+e))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^2+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2)) - (b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b + (b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b/a/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh(fx + e)^2 + a}}{b^2 \sinh(fx + e)^4 + 2ab \sinh(fx + e)^2 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)

$$3.115 \quad \int \frac{\operatorname{csch}^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=290

$$\frac{b \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{a^2 f(a-b) \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{(a-2b) \tanh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{a^2 f(a-b)}$$

```
[Out] -((b*Coth[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - ((a - 2*b)
*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)*f) - ((a - 2*b)*El
lipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
(b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)
/a]) + ((a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2*(a - b)*f
)
```

Rubi [A] time = 0.310008, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3188, 472, 583, 531, 418, 492, 411}

$$\frac{(a-2b) \tanh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{a^2 f(a-b)} - \frac{(a-2b) \operatorname{coth}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{a^2 f(a-b)} - \frac{b \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{a^2 f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] -((b*Coth[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - ((a - 2*b)
*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)*f) - ((a - 2*b)*El
lipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
(b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)
/a]) + ((a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2*(a - b)*f
)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
```

```

:> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

```

Rule 411

```

Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/
(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{b \coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a-2b-t}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{a(a-b)f} \\
&= -\frac{b \coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b) \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a-2b-t}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{a^2(a-b)f} \\
&= -\frac{b \coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b) \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f} - \frac{\left(b\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a-2b-t}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{a^2(a-b)f} \\
&= -\frac{b \coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b) \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f} - \frac{bF\left(\operatorname{arctanh}\left(\frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)\right)}{a^2(a-b)f} \\
&= -\frac{b \coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b) \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f} - \frac{(a-b)\operatorname{arctanh}\left(\frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{a^2(a-b)f}
\end{aligned}$$

Mathematica [C] time = 1.26166, size = 185, normalized size = 0.64

$$\frac{i\sqrt{2a(a-b)}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\coth(e+fx)\left(-\left(2a^2+b(a-2b)\cosh(2(e+fx))-3ab+2b^2\right)\right)}{a^2f(a-b)\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] $-\left(\left(2a^2 - 3ab + 2b^2 + (a - 2b)b\cosh[2(e + fx)]\right)\coth[e + fx] - I\sqrt{2}a(a - 2b)\sqrt{(2a - b + b\cosh[2(e + fx)])}/a\right)\text{EllipticE}[I(e + fx), b/a] + I\sqrt{2}a(a - b)\sqrt{(2a - b + b\cosh[2(e + fx)])}/a\text{EllipticF}[I(e + fx), b/a]/(a^2(a - b)f\sqrt{4a - 2b + 2b\cosh[2(e + fx)]})$

Maple [A] time = 0.104, size = 284, normalized size = 1.

$$-\frac{1}{a^2 \sinh(fx + e) (a - b) \cosh(fx + e) f} \left(\sinh(fx + e) \sqrt{(\cosh(fx + e))^2} \sqrt{\frac{b(\cosh(fx + e))^2}{a} + \frac{a - b}{a}} \left(2a \text{EllipticF} \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] $-(\sinh(fx+e)(\cosh(fx+e)^2)^{(1/2)}(b/a\cosh(fx+e)^2+(a-b)/a)^{(1/2)}b(2a\text{EllipticF}(\sinh(fx+e)(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-2b\text{EllipticF}(\sinh(fx+e)(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-\text{EllipticE}(\sinh(fx+e)(-1/a*b)^{(1/2)},(a/b)^{(1/2)})+2b\text{EllipticE}(\sinh(fx+e)(-1/a*b)^{(1/2)},(a/b)^{(1/2)}))+((-1/a*b)^{(1/2)}a^2b-2(-1/a*b)^{(1/2)}b^2)\cosh(fx+e)^4+((-1/a*b)^{(1/2)}a^2-2(-1/a*b)^{(1/2)}ab+2(-1/a*b)^{(1/2)}b^2)\cosh(fx+e)^2/a^2/\sinh(fx+e)/(-1/a*b)^{(1/2)}/(a-b)/\cosh(fx+e)/(a+b\sinh(fx+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{csch}(fx + e)^2}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \operatorname{csch}(fx + e)^2}{b^2 \sinh^4(fx + e) + 2ab \sinh^2(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)^2}{(b \sinh^2(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

$$3.116 \quad \int \frac{\sinh^5(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=143

$$-\frac{a(3a-5b) \cosh(e+fx)}{3b^2 f(a-b)^2 \sqrt{a+b \cosh^2(e+fx)-b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{b^{5/2} f} - \frac{a \sinh^2(e+fx) \cosh(e+fx)}{3bf(a-b)(a+b \cosh^2(e+fx)-b)^{3/2}}$$

[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(5/2)*f) - (a*(3*a - 5*b)*Cosh[e + f*x])/(3*(a - b)^2*b^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2]) - (a*Cosh[e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*b*f*(a - b + b*Cos h[e + f*x]^2)^(3/2))

Rubi [A] time = 0.166095, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3186, 413, 385, 217, 206}

$$-\frac{a(3a-5b) \cosh(e+fx)}{3b^2 f(a-b)^2 \sqrt{a+b \cosh^2(e+fx)-b}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{b^{5/2} f} - \frac{a \sinh^2(e+fx) \cosh(e+fx)}{3bf(a-b)(a+b \cosh^2(e+fx)-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(5/2)*f) - (a*(3*a - 5*b)*Cosh[e + f*x])/(3*(a - b)^2*b^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2]) - (a*Cosh[e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*b*f*(a - b + b*Cos h[e + f*x]^2)^(3/2))

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a-b+bx^2)^{5/2}} dx, x, \cosh(e+fx)\right)}{f} \\
&= -\frac{a \cosh(e+fx) \sinh^2(e+fx)}{3(a-b)bf(a-b+b\cosh^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a+3b+3(a-b)x^2}{(a-b+bx^2)^{3/2}} dx, x, \cosh(e+fx)\right)}{3(a-b)bf} \\
&= -\frac{a(3a-5b) \cosh(e+fx)}{3(a-b)^2 b^2 f \sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a \cosh(e+fx) \sinh^2(e+fx)}{3(a-b)bf(a-b+b\cosh^2(e+fx))^{3/2}} + \dots \\
&= -\frac{a(3a-5b) \cosh(e+fx)}{3(a-b)^2 b^2 f \sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a \cosh(e+fx) \sinh^2(e+fx)}{3(a-b)bf(a-b+b\cosh^2(e+fx))^{3/2}} + \dots \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b} \cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{5/2} f} - \frac{a(3a-5b) \cosh(e+fx)}{3(a-b)^2 b^2 f \sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a \cosh(e+fx) \sinh^2(e+fx)}{3(a-b)bf(a-b+b\cosh^2(e+fx))^{3/2}} + \dots
\end{aligned}$$

Mathematica [A] time = 0.871048, size = 130, normalized size = 0.91

$$\frac{\log(\sqrt{2a+b} \cosh(2(e+fx))-b+\sqrt{2}\sqrt{b} \cosh(e+fx))}{b^{5/2}} - \frac{2\sqrt{2}a \cosh(e+fx)(3a^2+b(2a-3b) \cosh(2(e+fx))-7ab+3b^2)}{3b^2(a-b)^2(2a+b \cosh(2(e+fx))-b)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]

[Out] ((-2*Sqrt[2]*a*Cosh[e + f*x]*(3*a^2 - 7*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(e + f*x)]))/((3*(a - b)^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)) + Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/b^(5/2)))/f

Maple [A] time = 0.16, size = 230, normalized size = 1.6

$$\frac{1}{f \cosh(fx + e)} \sqrt{\left(a + b(\sinh(fx + e))^2\right) (\cosh(fx + e))^2} \left(\frac{1}{2} \ln \left(\left(\frac{a}{2} + \frac{b}{2} + b(\sinh(fx + e))^2 \right) \frac{1}{\sqrt{b}} + \sqrt{\left(a + b(\sinh(fx + e))^2\right)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(5/2)*ln((1/2*a+1/2*b+b*sinh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))+1/3*a^2/b^2*(2*b*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)^2/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(a+b*sinh(f*x+e)^2)/(a^2-2*a*b+b^2)-2*a/b^2*cosh(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^5}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(sinh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [B] time = 6.88764, size = 18590, normalized size = 130.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

```
[Out] [1/12*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)
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$$\begin{aligned}
& + 7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^2*\sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\log(- (b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a - b)*\sinh(f*x + e))^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)} + 4*(b*\cosh(f*x + e)^3 + (a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) - 8*\sqrt{2}*((2*a^2*b^2 - 3*a*b^3)*\cosh(f*x + e)^6 + 6*(2*a^2*b^2 - 3*a*b^3)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (2*a^2*b^2 - 3*a*b^3)*\sinh(f*x + e)^6 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*\cosh(f*x + e)^4 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3 + 5*(2*a^2*b^2 - 3*a*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 2*a^2*b^2 - 3*a*b^3 + 4*(5*(2*a^2*b^2 - 3*a*b^3)*\cosh(f*x + e)^3 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*\cosh(f*x + e)^2 + 3*(5*(2*a^2*b^2 - 3*a*b^3)*\cosh(f*x + e)^4 + 2*a^3*b - 4*a^2*b^2 + a*b^3 + 6*(2*a^3*b - 4*a^2*b^2 + a*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 6*((2*a^2*b^2 - 3*a*b^3)*\cosh(f*x + e)^5 + 2*(2*a^3*b - 4*a^2*b^2 + a*b^3)*\cosh(f*x + e)^3 + (2*a^3*b - 4*a^2*b^2 + a*b^3)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a^2*b^5 - 2*a*b^6 + b^7)*f*\cosh(f*x + e)^8 + 8*(a^2*b^5 - 2*a*b^6 + b^7)*f*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^2*b^5 - 2*a*b^6 + b^7)*f*\sinh(f*x + e)^8 + 4*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*\cosh(f*x + e)^6 + 4*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*\cosh(f*x + e)^2 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f)*\sinh(f*x + e)^6 + 2*(8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*\cosh(f*x + e)^4 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*\cosh(f*x + e)^3 + 3*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(35*(a^2*b^5 - 2*a*b^6 + b^7)*f*\cosh(f*x + e)^4 + 30*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*\cosh(f*x + e)^2 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f)*\sinh(f*x + e)^4 + 4*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*\cosh(
\end{aligned}$$

$$\begin{aligned}
& f*x + e)^2 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^5 + 10*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^3 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^6 + 15*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e)^2 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f)*sinh(f*x + e)^2 + (a^2*b^5 - 2*a*b^6 + b^7)*f + 8*((a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^7 + 3*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^5 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e)^3 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e))*sinh(f*x + e), -1/6*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a
\end{aligned}$$

$$\begin{aligned}
& ^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^5 + 1 \\
& 0*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b \\
& + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(2*a^3 \\
& *b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + \\
& b^4)*\cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + \\
& e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2* \\
& b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 8*((a^2*b^2 - 2* \\
& a*b^3 + b^4)*\cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh \\
& (f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + \\
& e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e))*\sinh(f*x + e) \\
& *sqrt(-b)*\arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + \\
& sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 \\
& + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e) \\
& ^2)))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + \\
& e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh \\
& (f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) \\
& + b)) + 4*sqrt(2)*((2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b^2 - \\
& 3*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b^2 - 3*a*b^3)*sinh(f*x + e \\
&)^6 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 3*(2*a^3*b - 4*a^2* \\
& b^2 + a*b^3 + 5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2* \\
& a^2*b^2 - 3*a*b^3 + 4*(5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^3 + 3*(2*a^3*b \\
& - 4*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a^3*b - 4*a^2*b \\
& ^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^4 + \\
& 2*a^3*b - 4*a^2*b^2 + a*b^3 + 6*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e) \\
& ^2)*sinh(f*x + e)^2 + 6*((2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^5 + 2*(2*a^3*b \\
& - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh \\
& (f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a \\
& - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/ \\
& ((a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^8 + 8*(a^2*b^5 - 2*a*b^6 + b^7)* \\
& f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^5 - 2*a*b^6 + b^7)*f*sinh(f*x + e) \\
& ^8 + 4*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^6 + 4*(7*(a^ \\
& 2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^2 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 \\
& - b^7)*f)*sinh(f*x + e)^6 + 2*(8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a* \\
& b^6 + 3*b^7)*f*cosh(f*x + e)^4 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x \\
& + e)^3 + 3*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e))*sinh(f* \\
& x + e)^5 + 2*(35*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^4 + 30*(2*a^3*b^ \\
& 4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^2 + (8*a^4*b^3 - 24*a^3*b^4 \\
& + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b^4 - 5*a^2* \\
& b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^2 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*c \\
& osh(f*x + e)^5 + 10*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e) \\
& ^3 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + \\
& e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^6 + 15 \\
& *(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 - \\
& 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e)^2 + (2*a^3*b^4 \\
& - 5*a^2*b^5 + 4*a*b^6 - b^7)*f)*sinh(f*x + e)^2 + (a^2*b^5 - 2*a*b^6 + b^7
\end{aligned}$$

```
) * f + 8 * ((a^2 * b^5 - 2 * a * b^6 + b^7) * f * cosh(f * x + e)^7 + 3 * (2 * a^3 * b^4 - 5 * a^2 * b^5 + 4 * a * b^6 - b^7) * f * cosh(f * x + e)^5 + (8 * a^4 * b^3 - 24 * a^3 * b^4 + 27 * a^2 * b^5 - 14 * a * b^6 + 3 * b^7) * f * cosh(f * x + e)^3 + (2 * a^3 * b^4 - 5 * a^2 * b^5 + 4 * a * b^6 - b^7) * f * cosh(f * x + e)) * sinh(f * x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2), x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^5}{(b \sinh(fx + e)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2), x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

$$3.117 \quad \int \frac{\sinh^3(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=87

$$\frac{\sinh^2(e+fx) \cosh(e+fx)}{3f(a-b)(a+b \cosh^2(e+fx)-b)^{3/2}} - \frac{2 \cosh(e+fx)}{3f(a-b)^2 \sqrt{a+b \cosh^2(e+fx)-b}}$$

[Out] $(-2*\text{Cosh}[e + f*x])/(3*(a - b)^2*f*\text{Sqrt}[a - b + b*\text{Cosh}[e + f*x]^2]) + (\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x]^2)/(3*(a - b)*f*(a - b + b*\text{Cosh}[e + f*x]^2)^{(3/2)})$

Rubi [A] time = 0.10722, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3186, 378, 191}

$$\frac{\sinh^2(e+fx) \cosh(e+fx)}{3f(a-b)(a+b \cosh^2(e+fx)-b)^{3/2}} - \frac{2 \cosh(e+fx)}{3f(a-b)^2 \sqrt{a+b \cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[e + f*x]^3/(a + b*\text{Sinh}[e + f*x]^2)^{(5/2)}, x]$

[Out] $(-2*\text{Cosh}[e + f*x])/(3*(a - b)^2*f*\text{Sqrt}[a - b + b*\text{Cosh}[e + f*x]^2]) + (\text{Cosh}[e + f*x]*\text{Sinh}[e + f*x]^2)/(3*(a - b)*f*(a - b + b*\text{Cosh}[e + f*x]^2)^{(3/2)})$

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol]
:> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

$\text{Int}[(a + b \cdot x^n)^p, x_Symbol] \rightarrow \text{Simp}[(x \cdot (a + b \cdot x^n)^{p+1})/a, x] /; \text{FreeQ}\{a, b, n, p, x\} \ \&\& \ \text{EqQ}[1/n + p + 1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sinh^3(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+bx^2)^{5/2}} dx, x, \cosh(e + fx)\right)}{f} \\ &= \frac{\cosh(e + fx) \sinh^2(e + fx)}{3(a-b)f(a-b+b \cosh^2(e + fx))^{3/2}} - \frac{2 \text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{3/2}} dx, x, \cosh(e + fx)\right)}{3(a-b)f} \\ &= -\frac{2 \cosh(e + fx)}{3(a-b)^2 f \sqrt{a-b+b \cosh^2(e + fx)}} + \frac{\cosh(e + fx) \sinh^2(e + fx)}{3(a-b)f(a-b+b \cosh^2(e + fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.328655, size = 67, normalized size = 0.77

$$\frac{\sqrt{2} \cosh(e + fx)((a - 3b) \cosh(2(e + fx)) - 5a + 3b)}{3f(a - b)^2(2a + b \cosh(2(e + fx)) - b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] (Sqrt[2]*Cosh[e + f*x]*(-5*a + 3*b + (a - 3*b)*Cosh[2*(e + f*x)]))/(3*(a - b)^2*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))

Maple [A] time = 0.077, size = 58, normalized size = 0.7

$$\frac{(a(\sinh(fx + e))^2 - 3b(\sinh(fx + e))^2 - 2a) \cosh(fx + e)}{3(a-b)^2 f} (a + b(\sinh(fx + e))^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2), x)

[Out] $\frac{1}{3}*(a*\sinh(f*x+e)^2-3*b*\sinh(f*x+e)^2-2*a)*\cosh(f*x+e)/(a-b)^2/(a+b*\sinh(f*x+e)^2)^{(3/2)}/f$

Maxima [B] time = 1.77411, size = 1251, normalized size = 14.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out]
$$\begin{aligned} & -1/12*(b^4*e^{(-10*f*x - 10*e)} - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^{(-2*f*x - 2*e)} + 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^{(-4*f*x - 4*e)} + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^{(-6*f*x - 6*e)} + 5*(2*a*b^3 - b^4)*e^{(-8*f*x - 8*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f}) - 1/4*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^{(-2*f*x - 2*e)} + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^{(-4*f*x - 4*e)} + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^{(-6*f*x - 6*e)} + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^{(-8*f*x - 8*e)} + (2*a*b^3 - b^4)*e^{(-10*f*x - 10*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f}) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^{(-2*f*x - 2*e)} + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^{(-4*f*x - 4*e)} + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^{(-6*f*x - 6*e)} + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^{(-8*f*x - 8*e)} + (2*a^2*b^2 - 2*a*b^3 + b^4)*e^{(-10*f*x - 10*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f}) - 1/12*(b^4 + 5*(2*a*b^3 - b^4)*e^{(-2*f*x - 2*e)} + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^{(-4*f*x - 4*e)} + 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^{(-6*f*x - 6*e)} - (16*a^4 - 32*a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^{(-8*f*x - 8*e)} - (4*a^3*b - 6*a^2*b^2 + b^4)*e^{(-10*f*x - 10*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f}) \end{aligned}$$

Fricas [B] time = 3.41105, size = 2873, normalized size = 33.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")`

```
[Out] 1/3*sqrt(2)*((a - 3*b)*cosh(f*x + e)^6 + 6*(a - 3*b)*cosh(f*x + e)*sinh(f*x
+ e)^5 + (a - 3*b)*sinh(f*x + e)^6 - 3*(3*a - b)*cosh(f*x + e)^4 + 3*(5*(a
- 3*b)*cosh(f*x + e)^2 - 3*a + b)*sinh(f*x + e)^4 + 4*(5*(a - 3*b)*cosh(f*
x + e)^3 - 3*(3*a - b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a - b)*cosh(f*
x + e)^2 + 3*(5*(a - 3*b)*cosh(f*x + e)^4 - 6*(3*a - b)*cosh(f*x + e)^2 - 3
*a + b)*sinh(f*x + e)^2 + 6*((a - 3*b)*cosh(f*x + e)^5 - 2*(3*a - b)*cosh(f
*x + e)^3 - (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt((b*cosh(
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2))/((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x +
e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*
b^2 - 2*a*b^3 + b^4)*f*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^2
+ (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^2*b^2 -
2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f
*x + e)^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + (8
*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*
a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^2*b^2 - 2*a*
b^3 + b^4)*f*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*c
osh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(
f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^
6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^4 + 3*(8*a^4 -
24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^2 + (2*a^3*b - 5
*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*f
+ 8*((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 +
4*a*b^3 - b^4)*f*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b
^3 + 3*b^4)*f*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cos
h(f*x + e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33895, size = 699, normalized size = 8.03

$$\frac{a - 3b}{3\left(a^2 b^{\frac{3}{2}} f - 2ab^{\frac{5}{2}} f + b^{\frac{7}{2}} f\right)} + \frac{\left(\left(\frac{a^7 b^2 f^3 - 5a^6 b^3 f^3 + 7a^5 b^4 f^3 - 3a^4 b^5 f^3}{a^8 b^2 f^4 - 4a^7 b^3 f^4 + 6a^6 b^4 f^4 - 4a^5 b^5 f^4 + a^4 b^6 f^4}\right)e^{(2fx+2e)} - \frac{3(3a^7 b^2 f^3 - 7a^6 b^3 f^3 + 5a^5 b^4 f^3 - a^4 b^5 f^3)}{a^8 b^2 f^4 - 4a^7 b^3 f^4 + 6a^6 b^4 f^4 - 4a^5 b^5 f^4 + a^4 b^6 f^4}\right)e^{(2fx+2e)}}{3\left(b e^{(4fx+4e)} + 4ae^{(2fx+2e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/3*(a - 3*b)/(a^2*b^{(3/2)}*f - 2*a*b^{(5/2)}*f + b^{(7/2)}*f) + 1/3*(((a^7*b^2*f^3 - 5*a^6*b^3*f^3 + 7*a^5*b^4*f^3 - 3*a^4*b^5*f^3)*e^{(2*f*x + 2*e)}/(a^8*b^2*f^4 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4) - 3*(3*a^7*b^2*f^3 - 7*a^6*b^3*f^3 + 5*a^5*b^4*f^3 - a^4*b^5*f^3)/(a^8*b^2*f^4 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4))*e^{(2*f*x + 2*e)} - 3*(3*a^7*b^2*f^3 - 7*a^6*b^3*f^3 + 5*a^5*b^4*f^3 - a^4*b^5*f^3)/(a^8*b^2*f^4 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4))*e^{(2*f*x + 2*e)} + (a^7*b^2*f^3 - 5*a^6*b^3*f^3 + 7*a^5*b^4*f^3 - 3*a^4*b^5*f^3)/(a^8*b^2*f^4 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4))/(b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b)^{(3/2)} \end{aligned}$$

$$3.118 \quad \int \frac{\sinh(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{5/2}} dx$$

Optimal. Leaf size=79

$$\frac{2 \cosh(e+fx)}{3f(a-b)^2 \sqrt{a+b \cosh^2(e+fx)-b}} + \frac{\cosh(e+fx)}{3f(a-b) \left(a+b \cosh^2(e+fx)-b\right)^{3/2}}$$

[Out] Cosh[e + f*x]/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2)) + (2*Cosh[e + f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rubi [A] time = 0.0675662, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3186, 192, 191}

$$\frac{2 \cosh(e+fx)}{3f(a-b)^2 \sqrt{a+b \cosh^2(e+fx)-b}} + \frac{\cosh(e+fx)}{3f(a-b) \left(a+b \cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] Cosh[e + f*x]/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2)) + (2*Cosh[e + f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rule 3186

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 192

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sinh(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx = \frac{\text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{5/2}} dx, x, \cosh(e + fx)\right)}{f}$$

$$= \frac{\cosh(e + fx)}{3(a-b)f(a-b+b \cosh^2(e + fx))^{3/2}} + \frac{2 \text{Subst}\left(\int \frac{1}{(a-b+bx^2)^{3/2}} dx, x, \cosh(e + fx)\right)}{3(a-b)f}$$

$$= \frac{\cosh(e + fx)}{3(a-b)f(a-b+b \cosh^2(e + fx))^{3/2}} + \frac{2 \cosh(e + fx)}{3(a-b)^2 f \sqrt{a-b+b \cosh^2(e + fx)}}$$

Mathematica [A] time = 0.179514, size = 63, normalized size = 0.8

$$\frac{2\sqrt{2} \cosh(e + fx)(3a + b \cosh(2(e + fx)) - 2b)}{3f(a-b)^2(2a + b \cosh(2(e + fx)) - b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]
```

```
[Out] (2*Sqrt[2]*Cosh[e + f*x]*(3*a - 2*b + b*Cosh[2*(e + f*x)]))/(3*(a - b)^2*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [A] time = 0.066, size = 57, normalized size = 0.7

$$\frac{(2b(\sinh(fx + e))^2 + 3a - b) \cosh(fx + e)}{(3a^2 - 6ab + 3b^2)f} (a + b(\sinh(fx + e))^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2), x)
```

[Out] $\frac{1}{3} \cdot (2 \cdot b \cdot \sinh(f \cdot x + e))^2 + 3 \cdot a - b) \cdot \cosh(f \cdot x + e) / (a + b \cdot \sinh(f \cdot x + e))^2)^{3/2} / (a^2 - 2 \cdot a \cdot b + b^2) / f$

Maxima [B] time = 1.67485, size = 655, normalized size = 8.29

$$\frac{2a^2b^2 - 2ab^3 + b^4 + 5(4a^3b - 6a^2b^2 + 4ab^3 - b^4)e^{(-2fx-2e)} + 2(24a^4 - 48a^3b + 49a^2b^2 - 25ab^3 + 5b^4)e^{(-4fx-4e)} + 3(a^4 - 2a^3b + a^2b^2)(2(2a - b)e^{(-2fx-2e)} + b^4)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] $\frac{1}{3} \cdot (2 \cdot a^2 \cdot b^2 - 2 \cdot a \cdot b^3 + b^4 + 5 \cdot (4 \cdot a^3 \cdot b - 6 \cdot a^2 \cdot b^2 + 4 \cdot a \cdot b^3 - b^4) \cdot e^{(-2 \cdot f \cdot x - 2 \cdot e)} + 2 \cdot (24 \cdot a^4 - 48 \cdot a^3 \cdot b + 49 \cdot a^2 \cdot b^2 - 25 \cdot a \cdot b^3 + 5 \cdot b^4) \cdot e^{(-4 \cdot f \cdot x - 4 \cdot e)} + 10 \cdot (6 \cdot a^3 \cdot b - 9 \cdot a^2 \cdot b^2 + 5 \cdot a \cdot b^3 - b^4) \cdot e^{(-6 \cdot f \cdot x - 6 \cdot e)} + 5 \cdot (4 \cdot a^2 \cdot b^2 - 4 \cdot a \cdot b^3 + b^4) \cdot e^{(-8 \cdot f \cdot x - 8 \cdot e)} + (2 \cdot a \cdot b^3 - b^4) \cdot e^{(-10 \cdot f \cdot x - 10 \cdot e)}) / ((a^4 - 2 \cdot a^3 \cdot b + a^2 \cdot b^2) \cdot (2 \cdot (2 \cdot a - b) \cdot e^{(-2 \cdot f \cdot x - 2 \cdot e)} + b \cdot e^{(-4 \cdot f \cdot x - 4 \cdot e)} + b)^{5/2} \cdot f) + \frac{1}{3} \cdot (2 \cdot a \cdot b^3 - b^4 + 5 \cdot (4 \cdot a^2 \cdot b^2 - 4 \cdot a \cdot b^3 + b^4) \cdot e^{(-2 \cdot f \cdot x - 2 \cdot e)} + 10 \cdot (6 \cdot a^3 \cdot b - 9 \cdot a^2 \cdot b^2 + 5 \cdot a \cdot b^3 - b^4) \cdot e^{(-4 \cdot f \cdot x - 4 \cdot e)} + 2 \cdot (24 \cdot a^4 - 48 \cdot a^3 \cdot b + 49 \cdot a^2 \cdot b^2 - 25 \cdot a \cdot b^3 + 5 \cdot b^4) \cdot e^{(-6 \cdot f \cdot x - 6 \cdot e)} + 5 \cdot (4 \cdot a^3 \cdot b - 6 \cdot a^2 \cdot b^2 + 4 \cdot a \cdot b^3 - b^4) \cdot e^{(-8 \cdot f \cdot x - 8 \cdot e)} + (2 \cdot a^2 \cdot b^2 - 2 \cdot a \cdot b^3 + b^4) \cdot e^{(-10 \cdot f \cdot x - 10 \cdot e)}) / ((a^4 - 2 \cdot a^3 \cdot b + a^2 \cdot b^2) \cdot (2 \cdot (2 \cdot a - b) \cdot e^{(-2 \cdot f \cdot x - 2 \cdot e)} + b \cdot e^{(-4 \cdot f \cdot x - 4 \cdot e)} + b)^{5/2} \cdot f)$

Fricas [B] time = 3.32509, size = 2789, normalized size = 35.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")`

[Out] $\frac{2}{3} \cdot \sqrt{2} \cdot (b \cdot \cosh(f \cdot x + e))^6 + 6 \cdot b \cdot \cosh(f \cdot x + e) \cdot \sinh(f \cdot x + e)^5 + b \cdot \sinh(f \cdot x + e)^6 + 3 \cdot (2 \cdot a - b) \cdot \cosh(f \cdot x + e)^4 + 3 \cdot (5 \cdot b \cdot \cosh(f \cdot x + e)^2 + 2 \cdot a - b) \cdot \sinh(f \cdot x + e)^4 + 4 \cdot (5 \cdot b \cdot \cosh(f \cdot x + e)^3 + 3 \cdot (2 \cdot a - b) \cdot \cosh(f \cdot x + e)) \cdot \sinh(f \cdot x + e)^3 + 3 \cdot (2 \cdot a - b) \cdot \cosh(f \cdot x + e)^2 + 3 \cdot (5 \cdot b \cdot \cosh(f \cdot x + e)^4 + 6 \cdot (2 \cdot a - b) \cdot \cosh(f \cdot x + e)^2 + 2 \cdot a - b) \cdot \sinh(f \cdot x + e)^2 + 6 \cdot (b \cdot \cosh(f \cdot x + e))^5 + 2 \cdot (2 \cdot a - b) \cdot \cosh(f \cdot x + e)^3 + (2 \cdot a - b) \cdot \cosh(f \cdot x + e)) \cdot \sinh(f \cdot x + e) + b) \cdot \sinh(f \cdot x + e)$

$$\sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)} / ((a^2 b^2 - 2ab^3 + b^4) * f * \cosh(fx + e)^8 + 8(a^2 b^2 - 2ab^3 + b^4) * f * \cosh(fx + e) * \sinh(fx + e)^7 + (a^2 b^2 - 2ab^3 + b^4) * f * \sinh(fx + e)^8 + 4(2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f * \cosh(fx + e)^6 + 4(7(a^2 b^2 - 2ab^3 + b^4) * f * \cosh(fx + e)^2 + (2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f) * \sinh(fx + e)^6 + 2(8a^4 - 24a^3 b + 27a^2 b^2 - 14ab^3 + 3b^4) * f * \cosh(fx + e)^4 + 8(7(a^2 b^2 - 2ab^3 + b^4) * f * \cosh(fx + e)^3 + 3(2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f * \cosh(fx + e)) * \sinh(fx + e)^5 + 2(35(a^2 b^2 - 2ab^3 + b^4) * f * \cosh(fx + e)^4 + 30(2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f * \cosh(fx + e)^2 + (8a^4 - 24a^3 b + 27a^2 b^2 - 14ab^3 + 3b^4) * f) * \sinh(fx + e)^4 + 4(2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f * \cosh(fx + e)^2 + 8(7(a^2 b^2 - 2ab^3 + b^4) * f * \cosh(fx + e)^5 + 10(2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f * \cosh(fx + e)^3 + (8a^4 - 24a^3 b + 27a^2 b^2 - 14ab^3 + 3b^4) * f * \cosh(fx + e)) * \sinh(fx + e)^3 + 4(7(a^2 b^2 - 2ab^3 + b^4) * f * \cosh(fx + e)^6 + 15(2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f * \cosh(fx + e)^4 + 3(8a^4 - 24a^3 b + 27a^2 b^2 - 14ab^3 + 3b^4) * f * \cosh(fx + e)^2 + (2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f) * \sinh(fx + e)^2 + (a^2 b^2 - 2ab^3 + b^4) * f + 8((a^2 b^2 - 2ab^3 + b^4) * f * \cosh(fx + e)^7 + 3(2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f * \cosh(fx + e)^5 + (8a^4 - 24a^3 b + 27a^2 b^2 - 14ab^3 + 3b^4) * f * \cosh(fx + e)^3 + (2a^3 b - 5a^2 b^2 + 4ab^3 - b^4) * f * \cosh(fx + e)) * \sinh(fx + e))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [B] time = 1.37829, size = 622, normalized size = 7.87

$$2 \left(\left(\frac{(a^6 b^3 f - 2a^5 b^4 f + a^4 b^5 f) e^{(2fx+2e)}}{a^8 b^2 f^2 - 4a^7 b^3 f^2 + 6a^6 b^4 f^2 - 4a^5 b^5 f^2 + a^4 b^6 f^2} + \frac{3(2a^7 b^2 f - 5a^6 b^3 f + 4a^5 b^4 f - a^4 b^5 f)}{a^8 b^2 f^2 - 4a^7 b^3 f^2 + 6a^6 b^4 f^2 - 4a^5 b^5 f^2 + a^4 b^6 f^2} \right) e^{(2fx+2e)} + \frac{3(2a^7 b^2 f - 5a^6 b^3 f + 4a^5 b^4 f - a^4 b^5 f)}{a^8 b^2 f^2 - 4a^7 b^3 f^2 + 6a^6 b^4 f^2 - 4a^5 b^5 f^2 + a^4 b^6 f^2} \right) \sqrt{3 \left(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 b e^{(2fx+2e)} + b \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out]
$$\frac{2}{3} \left(\frac{((a^6 b^3 f - 2 a^5 b^4 f + a^4 b^5 f) e^{2 f x + 2 e})}{(a^8 b^2 f^2 - 4 a^7 b^3 f^2 + 6 a^6 b^4 f^2 - 4 a^5 b^5 f^2 + a^4 b^6 f^2)} + 3 \frac{(2 a^7 b^2 f - 5 a^6 b^3 f + 4 a^5 b^4 f - a^4 b^5 f)}{(a^8 b^2 f^2 - 4 a^7 b^3 f^2 + 6 a^6 b^4 f^2 - 4 a^5 b^5 f^2 + a^4 b^6 f^2)} e^{2 f x + 2 e} + 3 \frac{(2 a^7 b^2 f - 5 a^6 b^3 f + 4 a^5 b^4 f - a^4 b^5 f)}{(a^8 b^2 f^2 - 4 a^7 b^3 f^2 + 6 a^6 b^4 f^2 - 4 a^5 b^5 f^2 + a^4 b^6 f^2)} e^{2 f x + 2 e} + \frac{(a^6 b^3 f - 2 a^5 b^4 f + a^4 b^5 f)}{(a^8 b^2 f^2 - 4 a^7 b^3 f^2 + 6 a^6 b^4 f^2 - 4 a^5 b^5 f^2 + a^4 b^6 f^2)} \right) / (b e^{4 f x + 4 e} + 4 a e^{2 f x + 2 e} - 2 b e^{2 f x + 2 e} + b)^{3/2} - \frac{2}{3} (a^2 \sqrt{b} f - 2 a b^{3/2} f + b^{5/2}) f$$

$$3.119 \quad \int \frac{\operatorname{csch}(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=136

$$\frac{b(5a-3b) \cosh(e+fx)}{3a^2 f(a-b)^2 \sqrt{a+b \cosh^2(e+fx)-b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{a^{5/2} f} - \frac{b \cosh(e+fx)}{3af(a-b)(a+b \cosh^2(e+fx)-b)^{3/2}}$$

[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(5/2)*f)) - (b*Cosh[e + f*x])/(3*a*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2)) - ((5*a - 3*b)*b*Cosh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rubi [A] time = 0.171386, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3186, 414, 527, 12, 377, 206}

$$\frac{b(5a-3b) \cosh(e+fx)}{3a^2 f(a-b)^2 \sqrt{a+b \cosh^2(e+fx)-b}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a+b \cosh^2(e+fx)-b}}\right)}{a^{5/2} f} - \frac{b \cosh(e+fx)}{3af(a-b)(a+b \cosh^2(e+fx)-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(5/2)*f)) - (b*Cosh[e + f*x])/(3*a*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2)) - ((5*a - 3*b)*b*Cosh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2])

Rule 3186

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)^{5/2}} dx, x, \cosh(e+fx)\right)}{f} \\
&= -\frac{b \cosh(e+fx)}{3a(a-b)f(a-b+b\cosh^2(e+fx))^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{-3a+b+2bx^2}{(1-x^2)(a-b+bx^2)^{3/2}} dx, x, \cosh(e+fx)\right)}{3a(a-b)f} \\
&= -\frac{b \cosh(e+fx)}{3a(a-b)f(a-b+b\cosh^2(e+fx))^{3/2}} - \frac{(5a-3b)b \cosh(e+fx)}{3a^2(a-b)^2 f \sqrt{a-b+b\cosh^2(e+fx)}} \\
&= -\frac{b \cosh(e+fx)}{3a(a-b)f(a-b+b\cosh^2(e+fx))^{3/2}} - \frac{(5a-3b)b \cosh(e+fx)}{3a^2(a-b)^2 f \sqrt{a-b+b\cosh^2(e+fx)}} \\
&= -\frac{b \cosh(e+fx)}{3a(a-b)f(a-b+b\cosh^2(e+fx))^{3/2}} - \frac{(5a-3b)b \cosh(e+fx)}{3a^2(a-b)^2 f \sqrt{a-b+b\cosh^2(e+fx)}} \\
&= -\frac{\operatorname{tanh}^{-1}\left(\frac{\sqrt{a} \cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{5/2} f} - \frac{b \cosh(e+fx)}{3a(a-b)f(a-b+b\cosh^2(e+fx))^{3/2}} - \frac{(5a-3b)b \cosh(e+fx)}{3a^2(a-b)^2 f \sqrt{a-b+b\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.797738, size = 130, normalized size = 0.96

$$\frac{\sqrt{2}b \cosh(e+fx)(-12a^2+b(3b-5a) \cosh(2(e+fx))+13ab-3b^2)}{3a^2(a-b)^2(2a+b \cosh(2(e+fx))-b)^{3/2}} - \frac{\operatorname{tanh}^{-1}\left(\frac{\sqrt{2}\sqrt{a} \cosh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right)}{a^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] (-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/a^(5/2)) + (Sqrt[2]*b*Cosh[e + f*x]*(-12*a^2 + 13*a*b - 3*b^2 + b*(-5*a + 3*b)*Cosh[2*(e + f*x)]))/(3*a^2*(a - b)^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)))/f

Maple [A] time = 0.176, size = 236, normalized size = 1.7

$$\frac{1}{f \cosh(fx + e)} \sqrt{\left(a + b (\sinh(fx + e))^2\right) (\cosh(fx + e))^2} \left(\frac{b \left(2b (\sinh(fx + e))^2 + 3a - b\right) (\cosh(fx + e))^2}{3a \left(a + b (\sinh(fx + e))^2\right) (a^2 - 2ab + b^2)} \frac{1}{\sqrt{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)`

[Out] $((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*(-1/3/a*b*(2*b*\sinh(f*x+e)^2+3*a-b)*\cosh(f*x+e)^2/((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}/(a+b*\sinh(f*x+e)^2)/(a^2-2*a*b+b^2)-1/a^2*b*\cosh(f*x+e)^2/(a-b)/((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}-1/2/a^{(5/2)}*\ln((2*a+(a+b)*\sinh(f*x+e)^2+2*a^{(1/2)}*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)})/\sinh(f*x+e)^2)/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)`

Fricas [B] time = 5.34341, size = 12269, normalized size = 90.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")`

```
[Out] [1/6*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 +
b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x +
e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b
- 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)
*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^
3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4
- 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^
2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b
^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh
(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh
(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8
*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x +
e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*
b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8
*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*
b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*
b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))
*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x
+ e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^
2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(co
sh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1))*sqrt(a
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x +
e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4
*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1
)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))
*sinh(f*x + e) + 1)) - 2*sqrt(2)*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6
*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (5*a^2*b^2 - 3*a*b^3
)*sinh(f*x + e)^6 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 3*(8*
a^3*b - 7*a^2*b^2 + a*b^3 + 5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f
*x + e)^4 + 5*a^2*b^2 - 3*a*b^3 + 4*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^
3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(8*a
^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 3*a*b^3)*cosh
(f*x + e)^4 + 8*a^3*b - 7*a^2*b^2 + a*b^3 + 6*(8*a^3*b - 7*a^2*b^2 + a*b^3)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^
5 + 2*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (8*a^3*b - 7*a^2*b^2
+ a*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^8 + 8*(a^5*b
^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 2*a^
4*b^3 + a^3*b^4)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a
^3*b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x
+ e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^6 +
```

$$\begin{aligned}
& 2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*\cosh(f*x + e)^4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*\cosh(f*x + e)^3 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(35*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*\cosh(f*x + e)^4 + 30*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*\cosh(f*x + e)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f)*\sinh(f*x + e)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*\cosh(f*x + e)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*\cosh(f*x + e)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*\cosh(f*x + e)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*\cosh(f*x + e)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*\cosh(f*x + e)^4 + 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*\cosh(f*x + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*\sinh(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*\cosh(f*x + e)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*\cosh(f*x + e)^5 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*\cosh(f*x + e)^3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*\cosh(f*x + e))*\sinh(f*x + e)), 1/3*(3*((a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*\sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*\cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*\cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-a}*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1))*\sqrt{-a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) - \sqrt{2}*((5*a^2*b^2 - 3*a*b^3)*\cosh(f*x + e)^6 + 6*(5*a^2*b^2 - 3*a*b^3)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (5*a^2*b^2 - 3*a*b^3)*\sinh(f*x + e)^6 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*\cosh(f*x + e)^4 + 3
\end{aligned}$$

```

*(8*a^3*b - 7*a^2*b^2 + a*b^3 + 5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 5*a^2*b^2 - 3*a*b^3 + 4*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^4 + 8*a^3*b - 7*a^2*b^2 + a*b^3 + 6*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^5 + 2*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^8 + 8*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^6 + 2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(3*5*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^4 + 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^5 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [B] time = 1.51809, size = 857, normalized size = 6.3

$$\frac{5ab - 3b^2}{3\left(a^4\sqrt{b}f - 2a^3b^{\frac{3}{2}}f + a^2b^{\frac{5}{2}}f\right)} - \frac{\left(\frac{(5a^{13}b^4f^3 - 13a^{12}b^5f^3 + 11a^{11}b^6f^3 - 3a^{10}b^7f^3)e^{(2fx+2e)}}{a^{16}b^2f^4 - 4a^{15}b^3f^4 + 6a^{14}b^4f^4 - 4a^{13}b^5f^4 + a^{12}b^6f^4} + \frac{3(8a^{14}b^3f^3 - 23a^{13}b^4f^3 + 23a^{12}b^5f^3 - 9a^{11}b^6f^3 + a^{10}b^7f^3)}{a^{16}b^2f^4 - 4a^{15}b^3f^4 + 6a^{14}b^4f^4 - 4a^{13}b^5f^4 + a^{12}b^6f^4}\right)}{3\left(be^{(4fx+2e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] $\frac{1}{3} \cdot \frac{(5ab - 3b^2) \sqrt{b} f - 2a^3 b^{\frac{3}{2}} f + a^2 b^{\frac{5}{2}} f}{(a^4 \sqrt{b} f - 2a^3 b^{\frac{3}{2}} f + a^2 b^{\frac{5}{2}} f)} - \frac{1}{3} \cdot \frac{\left(\frac{(5a^{13}b^4f^3 - 13a^{12}b^5f^3 + 11a^{11}b^6f^3 - 3a^{10}b^7f^3)e^{(2fx+2e)}}{a^{16}b^2f^4 - 4a^{15}b^3f^4 + 6a^{14}b^4f^4 - 4a^{13}b^5f^4 + a^{12}b^6f^4} + \frac{3(8a^{14}b^3f^3 - 23a^{13}b^4f^3 + 23a^{12}b^5f^3 - 9a^{11}b^6f^3 + a^{10}b^7f^3)}{a^{16}b^2f^4 - 4a^{15}b^3f^4 + 6a^{14}b^4f^4 - 4a^{13}b^5f^4 + a^{12}b^6f^4} \right)}{3 \left(be^{(4fx+2e)} \right)}$

$$3.120 \quad \int \frac{\sinh^6(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=344

$$\frac{2(2a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3b^2f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{(8a^2-13ab+3b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)^2}$$

```
[Out] -(a*Cosh[e + f*x]*Sinh[e + f*x]^3)/(3*(a - b)*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*a*(2*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*b^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*b^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*(2*a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((8*a^2 - 13*a*b + 3*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)^2*b^3*f)
```

Rubi [A] time = 0.366433, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3188, 470, 578, 531, 418, 492, 411}

$$\frac{(8a^2-13ab+3b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)^2} - \frac{(8a^2-13ab+3b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3b^3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2), x]
```

```
[Out] -(a*Cosh[e + f*x]*Sinh[e + f*x]^3)/(3*(a - b)*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*a*(2*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*b^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*b^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*(2*a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((8*a^2 - 13*a*b + 3*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)^2*b^3*f)
```

)]/(3*(a - b)^2*b^3*f)

Rule 3188

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 470

Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 578

Int[((g_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((e_.) + (f_.)*(x_.)^(n_.)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]

Rule 531

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((e_.) + (f_.)*(x_.)^(n_.)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_.) + (b_.)*(x_.)^2]*Sqrt[(c_.) + (d_.)*(x_.)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
  :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
  + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
  a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^6(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^6}{\sqrt{1+x^2}(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{3(a-b)bf(a+b\sinh^2(e+fx))^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^2(3a+c)}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{3(a-b)bf} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{3(a-b)bf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2a(2a-3b) \cosh(e+fx) \sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^2(3a+c)}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{3(a-b)bf} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{3(a-b)bf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2a(2a-3b) \cosh(e+fx) \sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2a(2a-3b) \cosh(e+fx) \sinh(e+fx)\right)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{3(a-b)bf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2a(2a-3b) \cosh(e+fx) \sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{2(2a-3b) \cosh(e+fx) \sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} \\
&= -\frac{a \cosh(e+fx) \sinh^3(e+fx)}{3(a-b)bf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2a(2a-3b) \cosh(e+fx) \sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a^2-6ab+3b^2) \cosh(e+fx) \sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 2.03492, size = 207, normalized size = 0.6

$$\frac{a \left(2ia (8a^2 - 17ab + 9b^2) \left(\frac{2a+b \cosh(2(e+fx))-b}{a} \right)^{3/2} \operatorname{EllipticF} \left(i(e+fx), \frac{b}{a} \right) + \sqrt{2}b \sinh(2(e+fx)) (-8a^2 + b(7b-5a) \cosh(2(e+fx))) \right)}{6b^3 f(a-b)^2 (2a+b \cosh(2(e+fx)))}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]

[Out] (a*((-2*I)*a*(8*a^2 - 13*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f*x)]))/a)^(3/2)*EllipticE[I*(e + f*x), b/a] + (2*I)*a*(8*a^2 - 17*a*b + 9*b^2)*((2*a - b + b*Cosh[2*(e + f*x)]))/a^(3/2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-8*a^2 + 17*a*b - 7*b^2 + b*(-5*a + 7*b)*Cosh[2*(e + f*x)]*Sinh[2*(e + f*x)]))/(6*(a - b)^2*b^3*f*(2*a - b + b*Cosh[2*(e + f*x)]^(3/2))

Maple [B] time = 0.13, size = 868, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] -1/3*((5*(-1/a*b)^(1/2)*a^2*b-7*(-1/a*b)^(1/2)*a*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+(4*(-1/a*b)^(1/2)*a^3-11*(-1/a*b)^(1/2)*a^2*b+7*(-1/a*b)^(1/2)*a*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(4*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-7*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-3*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3-11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+10*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3+21*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-16*(b/a

*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/b^2/cosh(f*x+e)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^6}{(b \sinh(fx + e)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh(fx + e)^2 + a} \sinh(fx + e)^6}{b^3 \sinh(fx + e)^6 + 3ab^2 \sinh(fx + e)^4 + 3a^2b \sinh(fx + e)^2 + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^6}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)

$$3.121 \quad \int \frac{\sinh^4(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=244

$$\frac{(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3abf(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{2\sqrt{a}(a-2b)\cosh(e+fx)E\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3b^{3/2}f(a-b)^2\sqrt{a+b \sinh^2(e+fx)}}$$

[Out] $-(a*\operatorname{Cosh}[e+f*x]*\operatorname{Sinh}[e+f*x])/(3*(a-b)*b*f*(a+b*\operatorname{Sinh}[e+f*x]^2)^{(3/2)}) + (2*\operatorname{Sqrt}[a]*(a-2*b)*\operatorname{Cosh}[e+f*x]*\operatorname{EllipticE}[\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[e+f*x])/\operatorname{Sqrt}[a]], 1-a/b])/(3*(a-b)^2*b^{(3/2)}*f*\operatorname{Sqrt}[(a*\operatorname{Cosh}[e+f*x]^2)/(a+b*\operatorname{Sinh}[e+f*x]^2)]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2]) - ((a-3*b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+f*x]], 1-b/a]*\operatorname{Sech}[e+f*x]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])/(3*a*(a-b)^2*b*f*\operatorname{Sqrt}[(\operatorname{Sech}[e+f*x]^2*(a+b*\operatorname{Sinh}[e+f*x]^2))/a])$

Rubi [A] time = 0.249229, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3188, 470, 525, 418, 411}

$$\frac{2\sqrt{a}(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\left[1-\frac{a}{b}\right]\right)}{3b^{3/2}f(a-b)^2\sqrt{a+b \sinh^2(e+fx)}\sqrt{\frac{a \cosh^2(e+fx)}{a+b \sinh^2(e+fx)}}} - \frac{a \sinh(e+fx) \cosh(e+fx)}{3bf(a-b)(a+b \sinh^2(e+fx))^{3/2}} - \frac{(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3abf(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sinh}[e+f*x]^4/(a+b*\operatorname{Sinh}[e+f*x]^2)^{(5/2)}, x]$

[Out] $-(a*\operatorname{Cosh}[e+f*x]*\operatorname{Sinh}[e+f*x])/(3*(a-b)*b*f*(a+b*\operatorname{Sinh}[e+f*x]^2)^{(3/2)}) + (2*\operatorname{Sqrt}[a]*(a-2*b)*\operatorname{Cosh}[e+f*x]*\operatorname{EllipticE}[\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[e+f*x])/\operatorname{Sqrt}[a]], 1-a/b])/(3*(a-b)^2*b^{(3/2)}*f*\operatorname{Sqrt}[(a*\operatorname{Cosh}[e+f*x]^2)/(a+b*\operatorname{Sinh}[e+f*x]^2)]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2]) - ((a-3*b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+f*x]], 1-b/a]*\operatorname{Sech}[e+f*x]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])/(3*a*(a-b)^2*b*f*\operatorname{Sqrt}[(\operatorname{Sech}[e+f*x]^2*(a+b*\operatorname{Sinh}[e+f*x]^2))/a])$

Rule 3188

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_)]^{(m_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e+f*x], x]\}, \operatorname{Dist}[(ff^{(m+} p_.)$

1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 470

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 525

Int[((e_) + (f_)*(x_)^2)/(Sqrt[(a_) + (b_)*(x_)^2]*((c_) + (d_)*(x_)^2)^(3/2)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] && PosQ[d/c]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{a \cosh(e+fx) \sinh(e+fx)}{3(a-b)bf(a+b\sinh^2(e+fx))^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a+(2a-bx^2)}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{3(a-b)bf} \\
&= -\frac{a \cosh(e+fx) \sinh(e+fx)}{3(a-b)bf(a+b\sinh^2(e+fx))^{3/2}} - \frac{\left((a-3b)\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a+(2a-bx^2)}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{3(a-b)^2bf} \\
&= -\frac{a \cosh(e+fx) \sinh(e+fx)}{3(a-b)bf(a+b\sinh^2(e+fx))^{3/2}} + \frac{2\sqrt{a}(a-2b) \cosh(e+fx) E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right)}{3(a-b)^2b^{3/2}f\sqrt{\frac{a \cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 1.60918, size = 198, normalized size = 0.81

$$\frac{-ia(2a^2 - 5ab + 3b^2) \left(\frac{2a+b \cosh(2(e+fx))-b}{a}\right)^{3/2} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \sqrt{2}b \sinh(2(e+fx))(-a^2 - b(a-2b) \cosh(2(e+fx)))}{3b^2f(a-b)^2(2a+b \cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)]))/a)^(3/2)*EllipticE[I*(e + f*x), b/a] - I*a*(2*a^2 - 5*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f*x)]))/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-a^2 + 4*a*b - 2*b^2 - (a - 2*b)*b*Cosh[2*(e + f*x)]*Sinh[2*(e + f*x)])/(3*(a - b)^2*b^2*f*(2*a - b + b*Cosh[2*(e + f*x)]))^(3/2)

Maple [B] time = 0.107, size = 659, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)`

[Out] $\frac{1}{3} \cdot (2 \cdot (-1/a \cdot b)^{1/2} \cdot a \cdot b \cdot \sinh(f \cdot x + e)^5 - 4 \cdot (-1/a \cdot b)^{1/2} \cdot b^2 \cdot \sinh(f \cdot x + e)^5 + ((a + b \cdot \sinh(f \cdot x + e)^2)/a)^{1/2} \cdot (\cosh(f \cdot x + e)^2)^{1/2} \cdot \text{EllipticF}(\sinh(f \cdot x + e) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot a \cdot b \cdot \sinh(f \cdot x + e)^2 - ((a + b \cdot \sinh(f \cdot x + e)^2)/a)^{1/2} \cdot (\cosh(f \cdot x + e)^2)^{1/2} \cdot \text{EllipticF}(\sinh(f \cdot x + e) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot b^2 \cdot \sinh(f \cdot x + e)^2 - 2 \cdot ((a + b \cdot \sinh(f \cdot x + e)^2)/a)^{1/2} \cdot (\cosh(f \cdot x + e)^2)^{1/2} \cdot \text{EllipticE}(\sinh(f \cdot x + e) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot a \cdot b \cdot \sinh(f \cdot x + e)^2 + 4 \cdot ((a + b \cdot \sinh(f \cdot x + e)^2)/a)^{1/2} \cdot (\cosh(f \cdot x + e)^2)^{1/2} \cdot \text{EllipticE}(\sinh(f \cdot x + e) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot b^2 \cdot \sinh(f \cdot x + e)^2 + (-1/a \cdot b)^{1/2} \cdot a^2 \cdot \sinh(f \cdot x + e)^3 - (-1/a \cdot b)^{1/2} \cdot a \cdot b \cdot \sinh(f \cdot x + e)^3 - 4 \cdot (-1/a \cdot b)^{1/2} \cdot b^2 \cdot \sinh(f \cdot x + e)^3 + a^2 \cdot ((a + b \cdot \sinh(f \cdot x + e)^2)/a)^{1/2} \cdot (\cosh(f \cdot x + e)^2)^{1/2} \cdot \text{EllipticF}(\sinh(f \cdot x + e) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) - a \cdot ((a + b \cdot \sinh(f \cdot x + e)^2)/a)^{1/2} \cdot (\cosh(f \cdot x + e)^2)^{1/2} \cdot \text{EllipticF}(\sinh(f \cdot x + e) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot b - 2 \cdot ((a + b \cdot \sinh(f \cdot x + e)^2)/a)^{1/2} \cdot (\cosh(f \cdot x + e)^2)^{1/2} \cdot \text{EllipticE}(\sinh(f \cdot x + e) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot a^2 + 4 \cdot ((a + b \cdot \sinh(f \cdot x + e)^2)/a)^{1/2} \cdot (\cosh(f \cdot x + e)^2)^{1/2} \cdot \text{EllipticE}(\sinh(f \cdot x + e) \cdot (-1/a \cdot b)^{1/2}, (a/b)^{1/2}) \cdot a \cdot b + (-1/a \cdot b)^{1/2} \cdot a^2 \cdot \sinh(f \cdot x + e) - 3 \cdot (-1/a \cdot b)^{1/2} \cdot a \cdot b \cdot \sinh(f \cdot x + e) / (-1/a \cdot b)^{1/2} / (a + b \cdot \sinh(f \cdot x + e)^2)^{3/2} / (a - b)^{1/2} / b / \cosh(f \cdot x + e) / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^4(fx + e)}{(b \sinh^2(fx + e) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \sinh^4(fx + e)}{b^3 \sinh^6(fx + e) + 3ab^2 \sinh^4(fx + e) + 3a^2b \sinh^2(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4/(b^3*sinh(f*x + e)^6 +
3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx + e)^4}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```


$$3.122 \quad \int \frac{\sinh^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=241

$$\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1 \operatorname{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{3bf(a-b)\sqrt{a+b \sinh^2(e+fx)}} + \frac{(a+b) \sinh(e+fx) \cosh(e+fx)}{3af(a-b)^2\sqrt{a+b \sinh^2(e+fx)}} + \frac{\sinh(e+fx) \cosh(e+fx)}{3f(a-b)(a+b \sinh^2(e+fx))^{3/2}} +$$

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) +
((a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Sinh[e +
f*x]^2]) + ((I/3)*(a + b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e +
f*x]^2])/(a*(a - b)^2*b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) - ((I/3)*Ellipti
cF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/((a - b)*b*f*Sqrt[a +
b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.325695, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{(a+b) \sinh(e+fx) \cosh(e+fx)}{3af(a-b)^2\sqrt{a+b \sinh^2(e+fx)}} + \frac{\sinh(e+fx) \cosh(e+fx)}{3f(a-b)(a+b \sinh^2(e+fx))^{3/2}} - \frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1F\left(ie + ifx \left| \frac{b}{a} \right.\right)}{3bf(a-b)\sqrt{a+b \sinh^2(e+fx)}} + \frac{i(a+b)\sqrt{a+b \sinh^2(e+fx)}}{3af(a-b)^2\sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) +
((a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Sinh[e +
f*x]^2]) + ((I/3)*(a + b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e +
f*x]^2])/(a*(a - b)^2*b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) - ((I/3)*Ellipti
cF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/((a - b)*b*f*Sqrt[a +
b*Sinh[e + f*x]^2])
```

Rule 3173

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^2]^(p_))*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)^2], x_Symbol] := -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
```

$$\frac{(a + b \sin[e + f x])^{p+1}}{(2 a f (a + b) (p + 1))} x - \text{Dist}\left[\frac{1}{(2 a (a + b) (p + 1))}, \text{Int}\left[(a + b \sin[e + f x])^{p+1} \text{Simp}[a B - A (2 a (p + 1) + b (2 p + 3)) + 2 (A b - a B) (p + 2) \sin[e + f x]^2, x], x\right], x\right] /;$$

$$\text{FreeQ}\{a, b, e, f, A, B\}, x \} \&\& \text{LtQ}[p, -1] \&\& \text{NeQ}[a + b, 0]$$

Rule 3172

$$\text{Int}\left[\frac{(A + B \sin[e + f x]) \sqrt{a + b \sin[e + f x]^2}}{\sqrt{a + b \sin[e + f x]^2}}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}\left[\frac{B}{b}, \text{Int}\left[\sqrt{a + b \sin[e + f x]^2}, x\right], x\right] + \text{Dist}\left[\frac{A b - a B}{b}, \text{Int}\left[\frac{1}{\sqrt{a + b \sin[e + f x]^2}}, x\right], x\right] /;$$

$$\text{FreeQ}\{a, b, e, f, A, B\}, x \}$$

Rule 3178

$$\text{Int}\left[\sqrt{a + b \sin[e + f x]^2} \sqrt{1 + (b \sin[e + f x]^2)/a}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}\left[\sqrt{a + b \sin[e + f x]^2} / \sqrt{1 + (b \sin[e + f x]^2)/a}, \text{Int}\left[\sqrt{1 + (b \sin[e + f x]^2)/a}, x\right], x\right] /;$$

$$\text{FreeQ}\{a, b, e, f\}, x \} \&\& !\text{GtQ}[a, 0]$$

Rule 3177

$$\text{Int}\left[\sqrt{a + b \sin[e + f x]^2} \text{EllipticE}[e + f x, -(b/a)], x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{\sqrt{a + b \sin[e + f x]^2} \text{EllipticE}[e + f x, -(b/a)]}{f}, x\right] /;$$

$$\text{FreeQ}\{a, b, e, f\}, x \} \&\& \text{GtQ}[a, 0]$$

Rule 3183

$$\text{Int}\left[\frac{1}{\sqrt{a + b \sin[e + f x]^2}} \sqrt{1 + (b \sin[e + f x]^2)/a}, x_{\text{Symbol}}\right] \rightarrow \text{Dist}\left[\frac{\sqrt{1 + (b \sin[e + f x]^2)/a}}{\sqrt{a + b \sin[e + f x]^2}}, \text{Int}\left[\frac{1}{\sqrt{1 + (b \sin[e + f x]^2)/a}}, x\right], x\right] /;$$

$$\text{FreeQ}\{a, b, e, f\}, x \} \&\& !\text{GtQ}[a, 0]$$

Rule 3182

$$\text{Int}\left[\frac{1}{\sqrt{a + b \sin[e + f x]^2}} \text{EllipticF}[e + f x, -(b/a)], x_{\text{Symbol}}\right] \rightarrow \text{Simp}\left[\frac{\text{EllipticF}[e + f x, -(b/a)]}{\sqrt{a + b \sin[e + f x]^2}}, x\right] /;$$

$$\text{FreeQ}\{a, b, e, f\}, x \} \&\& \text{GtQ}[a, 0]$$

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\int \frac{a-a\sinh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx}{3a(a-b)} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{2a^2+a(a+b)\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx}{3a^2(a-b)} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}} dx}{3(a-b)} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left((a+b)\sqrt{a+b\sinh^2(e+fx)}\right)}{3(a-b)} + \dots \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{i(a+b)E\left(i\sqrt{a+b\sinh^2(e+fx)}\right)}{3a(a-b)}
\end{aligned}$$

Mathematica [A] time = 1.40341, size = 187, normalized size = 0.78

$$\frac{-2ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\text{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2}b\sinh(2(e+fx))(4a^2+b(a+b)\cosh(2(e+fx))-ab)}{6abf(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ((2*I)*a^2*(a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(4*a^2 - a*b - b^2 + b*(a + b)*Cosh[2*(e + f*x)]*Sinh[2*(e + f*x)]/(6*a*(a - b)^2*b*f*(2*a - b + b*Cosh[2*(e + f*x)]))^(3/2))

Maple [B] time = 0.114, size = 598, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)`

[Out]
$$-1/3 * ((-1/a*b)^{1/2} * a*b - (-1/a*b)^{1/2} * b^2) * \sinh(f*x+e) * \cosh(f*x+e)^4 + (-2 * (-1/a*b)^{1/2} * a^2 + (-1/a*b)^{1/2} * a*b + (-1/a*b)^{1/2} * b^2) * \cosh(f*x+e)^2 * \sinh(f*x+e) + (\cosh(f*x+e)^2)^{1/2} * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{1/2} * b * (a * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{1/2}, (a/b)^{1/2}) - b * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{1/2}, (a/b)^{1/2})) + \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{1/2}, (a/b)^{1/2}) * a + b * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{1/2}, (a/b)^{1/2})) * \cosh(f*x+e)^2 + a^2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{1/2} * (\cosh(f*x+e)^2)^{1/2} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{1/2}, (a/b)^{1/2}) - 2 * a * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{1/2} * (\cosh(f*x+e)^2)^{1/2} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{1/2}, (a/b)^{1/2}) * b + (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{1/2} * (\cosh(f*x+e)^2)^{1/2} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{1/2}, (a/b)^{1/2}) * b^2 + (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{1/2} * (\cosh(f*x+e)^2)^{1/2} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{1/2}, (a/b)^{1/2}) * a^2 - (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{1/2} * (\cosh(f*x+e)^2)^{1/2} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{1/2}, (a/b)^{1/2}) * b^2) / (-1/a*b)^{1/2} / (a+b*\sinh(f*x+e)^2)^{3/2} / (a-b)^2 / a / \cosh(f*x+e) / f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx+e)^2}{(b \sinh(fx+e)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \sinh^2(fx + e)}{b^3 \sinh^6(fx + e) + 3ab^2 \sinh^4(fx + e) + 3a^2b \sinh^2(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh^2(fx + e)}{\left(b \sinh^2(fx + e) + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)

$$3.123 \quad \int \frac{1}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=251

$$\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1 \operatorname{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{3af(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{2b(2a-b) \sinh(e+fx) \cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b \sinh^2(e+fx)}} - \frac{2i(2a-b)\sqrt{a+b \sinh^2(e+fx)}E\left(ie + ifx, \frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1}$$

[Out] $-(b*\operatorname{Cosh}[e + f*x]*\operatorname{Sinh}[e + f*x])/(3*a*(a - b)*f*(a + b*\operatorname{Sinh}[e + f*x]^2)^{(3/2)}) - (2*(2*a - b)*b*\operatorname{Cosh}[e + f*x]*\operatorname{Sinh}[e + f*x])/(3*a^2*(a - b)^2*f*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*\operatorname{EllipticE}[I*e + I*f*x, b/a]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(a^2*(a - b)^2*f*\operatorname{Sqrt}[1 + (b*\operatorname{Sinh}[e + f*x]^2)/a]) + ((I/3)*\operatorname{EllipticF}[I*e + I*f*x, b/a]*\operatorname{Sqrt}[1 + (b*\operatorname{Sinh}[e + f*x]^2)/a])/(a*(a - b)*f*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])$

Rubi [A] time = 0.279692, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{2b(2a-b) \sinh(e+fx) \cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b \sinh^2(e+fx)}} - \frac{2i(2a-b)\sqrt{a+b \sinh^2(e+fx)}E\left(ie + ifx, \frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1} - \frac{b \sinh(e+fx) \cosh(e+fx)}{3af(a-b)(a+b \sinh^2(e+fx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Sinh}[e + f*x]^2)^{-5/2}, x]$

[Out] $-(b*\operatorname{Cosh}[e + f*x]*\operatorname{Sinh}[e + f*x])/(3*a*(a - b)*f*(a + b*\operatorname{Sinh}[e + f*x]^2)^{(3/2)}) - (2*(2*a - b)*b*\operatorname{Cosh}[e + f*x]*\operatorname{Sinh}[e + f*x])/(3*a^2*(a - b)^2*f*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*\operatorname{EllipticE}[I*e + I*f*x, b/a]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(a^2*(a - b)^2*f*\operatorname{Sqrt}[1 + (b*\operatorname{Sinh}[e + f*x]^2)/a]) + ((I/3)*\operatorname{EllipticF}[I*e + I*f*x, b/a]*\operatorname{Sqrt}[1 + (b*\operatorname{Sinh}[e + f*x]^2)/a])/(a*(a - b)*f*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])$

Rule 3184

$\operatorname{Int}[(a + b*\sin[(e + f*x)*(x)]^2)^{(p)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[e + f*x]*\operatorname{Sin}[e + f*x]*(a + b*\operatorname{Sin}[e + f*x]^2)^{(p+1)})/(2*a*f*(p+1)*(a +$

b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*SIN[e + f*x]^2)^(p + 1) *Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*SIN[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]

Rule 3173

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x] * (a + b*SIN[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*(a + b)*(p + 1)), Int[(a + b*SIN[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*SIN[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]

Rule 3172

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)]^2)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*SIN[e + f*x]^2], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*SIN[e + f*x]^2], x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3178

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[a + b*SIN[e + f*x]^2]/Sqrt[1 + (b*SIN[e + f*x]^2)/a], Int[Sqrt[1 + (b*SIN[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3177

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a] * EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rule 3183

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[1 + (b*SIN[e + f*x]^2)/a]/Sqrt[a + b*SIN[e + f*x]^2], Int[1/Sqrt[1 + (b*SIN[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(1 * EllipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \sinh^2(e + fx))^{5/2}} dx &= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{\int \frac{-3a+2b+b \sinh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx}{3a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{\int \frac{-a(3a-b)}{\sqrt{a+b \sinh^2(e+fx)}} dx}{3a} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{\int \frac{-a(3a-b)}{\sqrt{a+b \sinh^2(e+fx)}} dx}{3a} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} + \frac{(2(2a - b)b \cosh(e + fx) \sinh(e + fx))}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{2i(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 1.31988, size = 190, normalized size = 0.76

$$\frac{ia^2(a - b) \left(\frac{2a + b \cosh(2(e + fx)) - b}{a} \right)^{3/2} \text{EllipticF} \left(i(e + fx), \frac{b}{a} \right) + \sqrt{2}b \sinh(2(e + fx)) (-5a^2 + b(b - 2a) \cosh(2(e + fx)) + 5ab)}{3a^2 f (a - b)^2 (2a + b \cosh(2(e + fx)) - b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2), x]

[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b)*Cosh[2*(e + f*x)]*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cosh[2*(e + f*x)]^(3/2))

Maple [A] time = 0.198, size = 406, normalized size = 1.6

$$\frac{1}{f \cosh(fx + e)} \sqrt{\left(a + b(\sinh(fx + e))^2\right) (\cosh(fx + e))^2} \left(-\frac{\sinh(fx + e)}{3ab(a-b)} \sqrt{\left(a + b(\sinh(fx + e))^2\right) (\cosh(fx + e))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] $((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*(-1/3/a/b/(a-b)*\sinh(f*x+e)*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}/(\sinh(f*x+e)^2+a/b)^{2-2/3*b*\cosh(f*x+e)^2/a^2/(a-b)^2*\sinh(f*x+e)*(2*a-b)/((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^{(1/2)}*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}/((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-2/3*b*(2*a-b)/a^2/(a-b)^2/(-1/a*b)^{(1/2)}*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}/((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*(\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})))/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh(fx + e)^2 + a}}{b^3 \sinh(fx + e)^6 + 3ab^2 \sinh(fx + e)^4 + 3a^2b \sinh(fx + e)^2 + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*
x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

$$3.124 \quad \int \frac{\operatorname{csch}^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=385

$$\frac{2b(3a-2b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3a^3 f(a-b)^2 \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{(3a^2-13ab+8b^2)\tanh(e+fx)}{3a^3 f(a-b)^2}$$

```
[Out] -(b*Coth[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(3*a - 2*b)*b*Coth[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((3*a^2 - 13*a*b + 8*b^2)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f) - ((3*a^2 - 13*a*b + 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(3*a - 2*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a^2 - 13*a*b + 8*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^3*(a - b)^2*f)
```

Rubi [A] time = 0.456218, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {3188, 472, 579, 583, 531, 418, 492, 411}

$$\frac{(3a^2-13ab+8b^2)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^3 f(a-b)^2} - \frac{(3a^2-13ab+8b^2)\coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^3 f(a-b)^2} - \frac{(3a^2-13ab+8b^2)\operatorname{csch}^2(e+fx)}{3a^3 f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]
```

```
[Out] -(b*Coth[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(3*a - 2*b)*b*Coth[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((3*a^2 - 13*a*b + 8*b^2)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f) - ((3*a^2 - 13*a*b + 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(3*a - 2*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a^2 - 13*a*b + 8*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^3*(a - b)^2*f)
```

$$(3a^3(a-b)^2f\sqrt{(\operatorname{Sech}[e+fx]^2(a+b\sinh[e+fx]^2))/a}) + ((3a^2 - 13ab + 8b^2)\sqrt{a+b\sinh[e+fx]^2}\tanh[e+fx])/(3a^3(a-b)^2f)$$

Rule 3188

```
Int[sin[(e_.) + (f_.)(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 472

```
Int[((e_.)(x_))^(m_.)*((a_) + (b_.)(x_)^(n_))^(p_)*((c_) + (d_.)(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)(x_))^(m_.)*((a_) + (b_.)(x_)^(n_))^(p_)*((c_) + (d_.)(x_)^(n_))^(q_)*((e_) + (f_.)(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)(x_))^(m_)*((a_) + (b_.)(x_)^(n_))^(p_.)*((c_) + (d_.)(x_)^(n_))^(q_.)*((e_) + (f_.)(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)(x_)^(n_))^(p_.)*((c_) + (d_.)(x_)^(n_))^(q_.)*((e_) + (
```

```
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{b \coth(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{3a-}{x^2\sqrt{1+x^2}}\right)}{3a(a-b)f} \\
&= -\frac{b \coth(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(3a-2b)b \coth(e+fx)}{3a^2(a-b)^2 f \sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{3a-}{x^2\sqrt{1+x^2}}\right)}{3a(a-b)f} \\
&= -\frac{b \coth(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(3a-2b)b \coth(e+fx)}{3a^2(a-b)^2 f \sqrt{a+b\sinh^2(e+fx)}} - \frac{(3a^2-13ab)}{3a(a-b)f} \\
&= -\frac{b \coth(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(3a-2b)b \coth(e+fx)}{3a^2(a-b)^2 f \sqrt{a+b\sinh^2(e+fx)}} - \frac{(3a^2-13ab)}{3a(a-b)f} \\
&= -\frac{b \coth(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(3a-2b)b \coth(e+fx)}{3a^2(a-b)^2 f \sqrt{a+b\sinh^2(e+fx)}} - \frac{(3a^2-13ab)}{3a(a-b)f} \\
&= -\frac{b \coth(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(3a-2b)b \coth(e+fx)}{3a^2(a-b)^2 f \sqrt{a+b\sinh^2(e+fx)}} - \frac{(3a^2-13ab)}{3a(a-b)f}
\end{aligned}$$

Mathematica [C] time = 2.3105, size = 234, normalized size = 0.61

$$\frac{i\left(4a^2\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\left((3a^2-7ab+4b^2)\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + (-3a^2+13ab-8b^2)E\left(i(e+fx)\left|\frac{b}{a}\right.\right)\right) + 2i\sqrt{2}\right)}{12a^3f(a-}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ((I/12)*(4*a^2*((2*a - b + b*Cosh[2*(e + f*x)]))/a)^(3/2)*((-3*a^2 + 13*a*b - 8*b^2)*EllipticE[I*(e + f*x), b/a] + (3*a^2 - 7*a*b + 4*b^2)*EllipticF[I*

```
(e + f*x), b/a] + (2*I)*Sqrt[2]*(3*(a - b)^2*(2*a - b + b*Cosh[2*(e + f*x)]
)^2*Coth[e + f*x] - 2*a*(a - b)*b^2*Sinh[2*(e + f*x)] - (7*a - 5*b)*b^2*(2
*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])))/(a^3*(a - b)^2*f*(2*a -
b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [A] time = 0.128, size = 747, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2), x)
```

```
[Out] -1/3*((b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b^2*(9*Elliptic
icF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^2-17*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2), (a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(
1/2))*b^2-3*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^2+13*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b-8*EllipticE(sinh(f*x+e)*(-
1/a*b)^(1/2), (a/b)^(1/2))*b^2)*sinh(f*x+e)*cosh(f*x+e)^2+(b/a*cosh(f*x+e)^2
+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(9*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2), (a/b)^(1/2))*a^3-26*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*
a^2*b+25*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b^2-8*Elliptic
F(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^3-3*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2), (a/b)^(1/2))*a^3+16*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1
/2))*a^2*b-21*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b^2+8*Ell
ipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^3)*sinh(f*x+e)+(3*(-1/a*b)
^(1/2)*a^2*b^2-13*(-1/a*b)^(1/2)*a*b^3+8*(-1/a*b)^(1/2)*b^4)*cosh(f*x+e)^6+
(6*(-1/a*b)^(1/2)*a^3*b-26*(-1/a*b)^(1/2)*a^2*b^2+38*(-1/a*b)^(1/2)*a*b^3-1
6*(-1/a*b)^(1/2)*b^4)*cosh(f*x+e)^4+(3*(-1/a*b)^(1/2)*a^4-12*(-1/a*b)^(1/2)
*a^3*b+26*(-1/a*b)^(1/2)*a^2*b^2-25*(-1/a*b)^(1/2)*a*b^3+8*(-1/a*b)^(1/2)*b
^4)*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/(-1/a*b)^(1/2)/sinh(f*
x+e)/a^3/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)^2}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \operatorname{csch}(fx + e)^2}{b^3 \sinh^6(fx + e) + 3ab^2 \sinh^4(fx + e) + 3a^2b \sinh^2(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx + e)^2}{\left(b \sinh^2(fx + e) + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")


```
[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

$$3.125 \quad \int \frac{1}{\sqrt{1+\sinh^2(x)}} dx$$

Optimal. Leaf size=14

$$\frac{\cosh(x) \tan^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}$$

[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[Cosh[x]^2]

Rubi [A] time = 0.0240236, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {3176, 3207, 3770}

$$\frac{\cosh(x) \tan^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + Sinh[x]^2],x]

[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[Cosh[x]^2]

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p, x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^n)^p, x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x]^m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1 + \sinh^2(x)}} dx &= \int \frac{1}{\sqrt{\cosh^2(x)}} dx \\ &= \frac{\cosh(x) \int \operatorname{sech}(x) dx}{\sqrt{\cosh^2(x)}} \\ &= \frac{\tan^{-1}(\sinh(x)) \cosh(x)}{\sqrt{\cosh^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.0108901, size = 19, normalized size = 1.36

$$\frac{2 \cosh(x) \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sinh[x]^2], x]
```

```
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[Cosh[x]^2]
```

Maple [A] time = 0.05, size = 15, normalized size = 1.1

$$\frac{\arctan(\sinh(x))}{\cosh(x)} \sqrt{(\cosh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^(1/2), x)
```

```
[Out] (cosh(x)^2)^(1/2)*arctan(sinh(x))/cosh(x)
```

Maxima [A] time = 1.65678, size = 7, normalized size = 0.5

$$2 \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="maxima")

[Out] 2*arctan(e^x)

Fricas [A] time = 2.03552, size = 39, normalized size = 2.79

$$2 \arctan(\cosh(x) + \sinh(x))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="fricas")

[Out] 2*arctan(cosh(x) + sinh(x))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh^2(x) + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)**2)**(1/2),x)

[Out] Integral(1/sqrt(sinh(x)**2 + 1), x)

Giac [A] time = 1.14593, size = 7, normalized size = 0.5

$$2 \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)
```

$$3.126 \quad \int \frac{1}{\sqrt{1-\sinh^2(x)}} dx$$

Optimal. Leaf size=11

$$-i\text{EllipticF}(ix, -1)$$

[Out] (-I)*EllipticF[I*x, -1]

Rubi [A] time = 0.0108332, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3182}

$$-iF(ix|-1)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - Sinh[x]^2], x]

[Out] (-I)*EllipticF[I*x, -1]

Rule 3182

Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*EllipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\int \frac{1}{\sqrt{1-\sinh^2(x)}} dx = -iF(ix|-1)$$

Mathematica [A] time = 0.0398722, size = 11, normalized size = 1.

$$-i\text{EllipticF}(ix, -1)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - Sinh[x]^2],x]

[Out] (-I)*EllipticF[I*x, -1]

Maple [A] time = 0.147, size = 41, normalized size = 3.7

$$\frac{\text{EllipticF}(\sinh(x), i)}{\cosh(x)} \sqrt{-(-1 + (\sinh(x))^2) (\cosh(x))^2} \sqrt{(\cosh(x))^2} \frac{1}{\sqrt{1 - (\sinh(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-sinh(x)^2)^(1/2),x)

[Out] (-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)/(1-sinh(x)^4)^(1/2)*EllipticF(sinh(x),I)/cosh(x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sinh(x)^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(-sinh(x)^2 + 1), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{-\sinh(x)^2 + 1}}{\sinh(x)^2 - 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="fricas")

[Out] `integral(-sqrt(-sinh(x)^2 + 1)/(sinh(x)^2 - 1), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1 - \sinh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)**2)**(1/2),x)`

[Out] `Integral(1/sqrt(1 - sinh(x)**2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sinh(x)^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="giac")`

[Out] `integrate(1/sqrt(-sinh(x)^2 + 1), x)`

$$3.127 \quad \int \frac{1}{\sqrt{-1+\sinh^2(x)}} dx$$

Optimal. Leaf size=33

$$-\frac{i\sqrt{1-\sinh^2(x)}\text{EllipticF}(ix, -1)}{\sqrt{\sinh^2(x)-1}}$$

[Out] ((-I)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2]

Rubi [A] time = 0.021624, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3183, 3182}

$$-\frac{i\sqrt{1-\sinh^2(x)}F(ix|-1)}{\sqrt{\sinh^2(x)-1}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 + Sinh[x]^2], x]

[Out] ((-I)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2]

Rule 3183

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Simp[(1*EllipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\int \frac{1}{\sqrt{-1 + \sinh^2(x)}} dx = \frac{\sqrt{1 - \sinh^2(x)} \int \frac{1}{\sqrt{1 - \sinh^2(x)}} dx}{\sqrt{-1 + \sinh^2(x)}} = -\frac{iF(ix| -1)\sqrt{1 - \sinh^2(x)}}{\sqrt{-1 + \sinh^2(x)}}$$

Mathematica [A] time = 0.0414932, size = 33, normalized size = 1.

$$-\frac{i\sqrt{3 - \cosh(2x)}\text{EllipticF}(ix, -1)}{\sqrt{\cosh(2x) - 3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-1 + Sinh[x]^2], x]

[Out] ((-I)*Sqrt[3 - Cosh[2*x]]*EllipticF[I*x, -1])/Sqrt[-3 + Cosh[2*x]]

Maple [A] time = 0.068, size = 61, normalized size = 1.9

$$\frac{-i\text{EllipticF}(i\sinh(x), i)}{\cosh(x)} \sqrt{(-1 + (\sinh(x))^2) (\cosh(x))^2} \sqrt{(\cosh(x))^2} \sqrt{1 - (\sinh(x))^2} \frac{1}{\sqrt{(\sinh(x))^4 - 1}} \frac{1}{\sqrt{-1 + (\sinh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-1+sinh(x)^2)^(1/2), x)

[Out] -I*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(1-sinh(x)^2)^(1/2)/(sinh(x)^4-1)^(1/2)*EllipticF(I*sinh(x), I)/cosh(x)/(-1+sinh(x)^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh(x)^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(sinh(x)^2 - 1), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{1}{\sqrt{\sinh(x)^2 - 1}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(1/sqrt(sinh(x)^2 - 1), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh^2(x) - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+sinh(x)**2)**(1/2),x)`

[Out] `Integral(1/sqrt(sinh(x)**2 - 1), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh(x)^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sinh(x)^2 - 1), x)
```

$$3.128 \quad \int \frac{1}{\sqrt{-1-\sinh^2(x)}} dx$$

Optimal. Leaf size=16

$$\frac{\cosh(x) \tan^{-1}(\sinh(x))}{\sqrt{-\cosh^2(x)}}$$

[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[-Cosh[x]^2]

Rubi [A] time = 0.0237084, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3176, 3207, 3770}

$$\frac{\cosh(x) \tan^{-1}(\sinh(x))}{\sqrt{-\cosh^2(x)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-1 - Sinh[x]^2],x]

[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[-Cosh[x]^2]

Rule 3176

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p], x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^n)^p], x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1 - \sinh^2(x)}} dx &= \int \frac{1}{\sqrt{-\cosh^2(x)}} dx \\ &= \frac{\cosh(x) \int \operatorname{sech}(x) dx}{\sqrt{-\cosh^2(x)}} \\ &= \frac{\tan^{-1}(\sinh(x)) \cosh(x)}{\sqrt{-\cosh^2(x)}} \end{aligned}$$

Mathematica [A] time = 0.0079731, size = 21, normalized size = 1.31

$$\frac{2 \cosh(x) \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Sinh[x]^2], x]
```

```
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[-Cosh[x]^2]
```

Maple [B] time = 0.04, size = 34, normalized size = 2.1

$$-\frac{\cosh(x)}{\sinh(x)} \sqrt{-(\sinh(x))^2} \operatorname{Artanh}\left(\frac{1}{\sqrt{-(\sinh(x))^2}}\right) \frac{1}{\sqrt{-(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-sinh(x)^2)^(1/2), x)
```

```
[Out] -cosh(x)*(-sinh(x)^2)^(1/2)*arctanh(1/(-sinh(x)^2)^(1/2))/sinh(x)/(-cosh(x)
^2)^(1/2)
```

Maxima [C] time = 1.92463, size = 7, normalized size = 0.44

$$-2i \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="maxima")

[Out] -2*I*arctan(e^x)

Fricas [C] time = 1.77536, size = 39, normalized size = 2.44

$$\log(e^x + i) - \log(e^x - i)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="fricas")

[Out] log(e^x + I) - log(e^x - I)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sinh^2(x) - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1-sinh(x)**2)**(1/2),x)

[Out] Integral(1/sqrt(-sinh(x)**2 - 1), x)

Giac [C] time = 1.15006, size = 7, normalized size = 0.44

$$-2i \arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*arctan(e^x)
```


$$3.129 \quad \int \frac{1}{\sqrt{a+b \sinh^2(x)}} dx$$

Optimal. Leaf size=42

$$-\frac{i\sqrt{\frac{b \sinh^2(x)}{a}} + 1 \operatorname{EllipticF}\left(ix, \frac{b}{a}\right)}{\sqrt{a + b \sinh^2(x)}}$$

[Out] `((-I)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]`

Rubi [A] time = 0.0329238, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b \sinh^2(x)}{a}} + 1F\left(ix \left| \frac{b}{a} \right. \right)}{\sqrt{a + b \sinh^2(x)}}$$

Antiderivative was successfully verified.

[In] `Int[1/Sqrt[a + b*Sinh[x]^2], x]`

[Out] `((-I)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]`

Rule 3183

`Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]`

Rule 3182

`Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(1*EllipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]`

Rubi steps

$$\int \frac{1}{\sqrt{a + b \sinh^2(x)}} dx = \frac{\sqrt{1 + \frac{b \sinh^2(x)}{a}} \int \frac{1}{\sqrt{1 + \frac{b \sinh^2(x)}{a}}} dx}{\sqrt{a + b \sinh^2(x)}} = \frac{iF\left(ix \left| \frac{b}{a} \right. \right) \sqrt{1 + \frac{b \sinh^2(x)}{a}}}{\sqrt{a + b \sinh^2(x)}}$$

Mathematica [A] time = 0.0507961, size = 53, normalized size = 1.26

$$\frac{i \sqrt{\frac{2a + b \cosh(2x) - b}{a}} \text{EllipticF}\left(ix, \frac{b}{a}\right)}{\sqrt{2a + b \cosh(2x) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*Sinh[x]^2], x]

[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticF[I*x, b/a])/Sqrt[2*a - b + b*Cosh[2*x]]

Maple [A] time = 0.059, size = 63, normalized size = 1.5

$$\frac{1}{\cosh(x)} \sqrt{\frac{a + b (\sinh(x))^2}{a}} \sqrt{(\cosh(x))^2} \text{EllipticF}\left(\sinh(x) \sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) \frac{1}{\sqrt{-\frac{b}{a}}} \frac{1}{\sqrt{a + b (\sinh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(x)^2)^(1/2), x)

[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(x)^2)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x)*(-1/a*b)^(1/2), (a/b)^(1/2))/cosh(x)/(a+b*sinh(x)^2)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sinh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*sinh(x)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b \sinh(x)^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(b*sinh(x)^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \sinh^2(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(x)**2)**(1/2),x)

[Out] Integral(1/sqrt(a + b*sinh(x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sinh(x)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*sinh(x)^2 + a), x)

$$3.130 \quad \int (d \sinh(e + fx))^m (a + b \sinh^2(e + fx))^p dx$$

Optimal. Leaf size=128

$$\frac{d \cosh(e + fx) (-\sinh^2(e + fx))^{\frac{1-m}{2}} (d \sinh(e + fx))^{m-1} (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a-b} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{1-m}{2}, -p; \right)}{f}$$

[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*(d*Sinh[e + f*x])^(-1 + m)*(-Sinh[e + f*x]^2)^((1 - m)/2))/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)

Rubi [A] time = 0.116103, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3189, 430, 429}

$$\frac{d \cosh(e + fx) (-\sinh^2(e + fx))^{\frac{1-m}{2}} (d \sinh(e + fx))^{m-1} (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a-b} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{1-m}{2}, -p; \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*(d*Sinh[e + f*x])^(-1 + m)*(-Sinh[e + f*x]^2)^((1 - m)/2))/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)

Rule 3189

Int[((d_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sinh[e + f*x])^(2*FracPart[(m - 1)/2]))/(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
  Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d \sinh(e + fx))^m (a + b \sinh^2(e + fx))^p dx = \frac{\left(d(d \sinh(e + fx))^{2\left(-\frac{1}{2} + \frac{m}{2}\right)} (-\sinh^2(e + fx))^{\frac{1-m}{2}}\right) \text{Subst}\left(\int (1-x^2)^{\frac{1}{2}(-\frac{1}{2} + \frac{m}{2})} dx, x, \frac{d \sinh(e + fx)}{a + b \sinh^2(e + fx)}\right)}{f}$$

$$= \frac{\left(d(a - b + b \cosh^2(e + fx))^p \left(1 + \frac{b \cosh^2(e + fx)}{a - b}\right)^{-p} (d \sinh(e + fx))^{2\left(-\frac{1}{2} + \frac{m}{2}\right)}\right)}{f}$$

$$= \frac{dF_1\left(\frac{1}{2}; \frac{1-m}{2}, -p; \frac{3}{2}; \cosh^2(e + fx), -\frac{b \cosh^2(e + fx)}{a - b}\right) \cosh(e + fx) (a - b + b \cosh^2(e + fx))^p}{f}$$

Mathematica [F] time = 8.65666, size = 0, normalized size = 0.

$$\int (d \sinh(e + fx))^m (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.641, size = 0, normalized size = 0.

$$\int (d \sinh(fx + e))^m (a + b (\sinh(fx + e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \left(d \sinh(fx + e) \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \left(d \sinh(fx + e)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*sinh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \left(d \sinh(fx + e) \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)

$$3.131 \quad \int \sinh^5(e + fx) \left(a + b \sinh^2(e + fx) \right)^p dx$$

Optimal. Leaf size=226

$$\frac{(3a^2 + 4ab(p+1) + 4b^2(p^2 + 3p + 2)) \cosh(e + fx) (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a-b} + 1 \right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \cosh^2(e + fx)}{a-b}\right)}{b^2 f (2p+3)(2p+5)}$$

```
[Out] -(((3*a + 2*b*(2 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/
(b^2*f*(3 + 2*p)*(5 + 2*p))) + ((3*a^2 + 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^
2))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*Cosh[e + f*x]^2)/(a - b))]/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Co
sh[e + f*x]^2)/(a - b))^p) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1
+ p)*Sinh[e + f*x]^2)/(b*f*(5 + 2*p))
```

Rubi [A] time = 0.25908, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3186, 416, 388, 246, 245}

$$\frac{(3a^2 + 4ab(p+1) + 4b^2(p^2 + 3p + 2)) \cosh(e + fx) (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a-b} + 1 \right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \cosh^2(e + fx)}{a-b}\right)}{b^2 f (2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a + 2*b*(2 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/
(b^2*f*(3 + 2*p)*(5 + 2*p))) + ((3*a^2 + 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^
2))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*Cosh[e + f*x]^2)/(a - b))]/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Co
sh[e + f*x]^2)/(a - b))^p) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1
+ p)*Sinh[e + f*x]^2)/(b*f*(5 + 2*p))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simp
lify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \sinh^5(e+fx) (a+b\sinh^2(e+fx))^p dx &= \frac{\text{Subst}\left(\int (1-x^2)^2 (a-b+bx^2)^p dx, x, \cosh(e+fx)\right)}{f} \\
&= \frac{\cosh(e+fx) (a-b+b\cosh^2(e+fx))^{1+p} \sinh^2(e+fx)}{bf(5+2p)} + \frac{\text{Subst}\left(\int (a-b+bx^2)^p dx, x, \cosh(e+fx)\right)}{bf(5+2p)} \\
&= -\frac{(3a+2b(2+p)) \cosh(e+fx) (a-b+b\cosh^2(e+fx))^{1+p}}{b^2 f(3+2p)(5+2p)} + \frac{\cosh(e+fx) (a-b+b\cosh^2(e+fx))^{1+p}}{b^2 f(3+2p)(5+2p)} \\
&= -\frac{(3a+2b(2+p)) \cosh(e+fx) (a-b+b\cosh^2(e+fx))^{1+p}}{b^2 f(3+2p)(5+2p)} + \frac{\cosh(e+fx) (a-b+b\cosh^2(e+fx))^{1+p}}{b^2 f(3+2p)(5+2p)} \\
&= -\frac{(3a+2b(2+p)) \cosh(e+fx) (a-b+b\cosh^2(e+fx))^{1+p}}{b^2 f(3+2p)(5+2p)} + \frac{(3a^2+4ab+4b^2) \cosh(e+fx) (a-b+b\cosh^2(e+fx))^{1+p}}{b^2 f(3+2p)(5+2p)}
\end{aligned}$$

Mathematica [F] time = 11.4338, size = 0, normalized size = 0.

$$\int \sinh^5(e+fx) (a+b\sinh^2(e+fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]

[Out] Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.629, size = 0, normalized size = 0.

$$\int (\sinh(fx+e))^5 (a+b(\sinh(fx+e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p, x)

[Out] int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p, x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^p \sinh (fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh (fx + e)^2 + a\right)^p \sinh (fx + e)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")

[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^p \sinh (fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

$$3.132 \quad \int \sinh^3(e + fx) \left(a + b \sinh^2(e + fx) \right)^p dx$$

Optimal. Leaf size=137

$$\frac{\cosh(e + fx) \left(a + b \cosh^2(e + fx) - b \right)^{p+1}}{bf(2p + 3)} - \frac{(a + 2b(p + 1)) \cosh(e + fx) \left(a + b \cosh^2(e + fx) - b \right)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p}}{bf(2p + 3)}$$

[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a + 2*b*(1 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))]/(b*f*(3 + 2*p)*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)

Rubi [A] time = 0.122933, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3186, 388, 246, 245}

$$\frac{\cosh(e + fx) \left(a + b \cosh^2(e + fx) - b \right)^{p+1}}{bf(2p + 3)} - \frac{(a + 2b(p + 1)) \cosh(e + fx) \left(a + b \cosh^2(e + fx) - b \right)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p}}{bf(2p + 3)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a + 2*b*(1 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))]/(b*f*(3 + 2*p)*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 388

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(

$p + 1) + 1)) / (b * (n * (p + 1) + 1))$, Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 246

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]) / (1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

Rule 245

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \sinh^3(e + fx) (a + b \sinh^2(e + fx))^p dx &= -\frac{\text{Subst}\left(\int (1 - x^2) (a - b + bx^2)^p dx, x, \cosh(e + fx)\right)}{f} \\ &= \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^{1+p}}{bf(3 + 2p)} - \frac{(a + 2b(1 + p)) \text{Subst}\left(\int (a - b + b x^2)^p dx, x, \cosh(e + fx)\right)}{bf(3 + 2p)} \\ &= \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^{1+p}}{bf(3 + 2p)} - \frac{(a + 2b(1 + p)) (a - b + b \cosh^2(e + fx))^p}{bf(3 + 2p)} \\ &= \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^{1+p}}{bf(3 + 2p)} - \frac{(a + 2b(1 + p)) \cosh(e + fx) (a - b + b \cosh^2(e + fx))^p}{bf(3 + 2p)} \end{aligned}$$

Mathematica [F] time = 15.0967, size = 0, normalized size = 0.

$$\int \sinh^3(e + fx) (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.436, size = 0, normalized size = 0.

$$\int (\sinh(fx + e))^3 (a + b(\sinh(fx + e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)

[Out] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e)^2 + a)^p \sinh(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \sinh(fx + e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")

[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^p \sinh^3(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)

3.133 $\int \sinh(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=78

$$\frac{\cosh(e + fx) (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p} {}_2F_1 \left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \cosh^2(e + fx)}{a - b} \right)}{f}$$

[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)

Rubi [A] time = 0.057892, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3186, 246, 245}

$$\frac{\cosh(e + fx) (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p} {}_2F_1 \left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \cosh^2(e + fx)}{a - b} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 246

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !LtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \sinh(e + fx) (a + b \sinh^2(e + fx))^p dx &= \frac{\text{Subst}\left(\int (a - b + bx^2)^p dx, x, \cosh(e + fx)\right)}{f} \\ &= \frac{\left((a - b + b \cosh^2(e + fx))^p \left(1 + \frac{b \cosh^2(e + fx)}{a - b}\right)^{-p}\right) \text{Subst}\left(\int \left(1 + \frac{bx^2}{a - b}\right)^p dx, x, \cosh(e + fx)\right)}{f} \\ &= \frac{\cosh(e + fx) (a - b + b \cosh^2(e + fx))^p \left(1 + \frac{b \cosh^2(e + fx)}{a - b}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{b \cosh^2(e + fx)}{a - b}\right)}{f} \end{aligned}$$

Mathematica [A] time = 0.207017, size = 78, normalized size = 1.

$$\frac{\cosh(e + fx) (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \cosh^2(e + fx)}{a - b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int \sinh(fx + e) \left(a + b (\sinh(fx + e))^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

[Out] `int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \sinh(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

3.134 $\int \operatorname{csch}(e + fx) \left(a + b \sinh^2(e + fx) \right)^p dx$

Optimal. Leaf size=88

$$\frac{\cosh(e + fx) \left(a + b \cosh^2(e + fx) - b \right)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; 1, -p; \frac{3}{2}; \cosh^2(e + fx), -\frac{b \cosh^2(e + fx)}{a - b} \right)}{f}$$

[Out] -((AppellF1[1/2, 1, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p))

Rubi [A] time = 0.0878811, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3186, 430, 429}

$$\frac{\cosh(e + fx) \left(a + b \cosh^2(e + fx) - b \right)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; 1, -p; \frac{3}{2}; \cosh^2(e + fx), -\frac{b \cosh^2(e + fx)}{a - b} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] -((AppellF1[1/2, 1, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p))

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol]
:> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{csch}(e+fx) (a+b \sinh^2(e+fx))^p dx = -\frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^p}{1-x^2} dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left((a-b+b \cosh^2(e+fx))^p \left(1 + \frac{b \cosh^2(e+fx)}{a-b}\right)^{-p}\right) \operatorname{Subst}\left(\int \frac{\left(1 + \frac{bx^2}{a-b}\right)^p}{1-x^2} dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{F_1\left(\frac{1}{2}; 1, -p; \frac{3}{2}; \cosh^2(e+fx), -\frac{b \cosh^2(e+fx)}{a-b}\right) \cosh(e+fx) (a-b+b \cosh^2(e+fx))^p}{f}$$

Mathematica [F] time = 4.1886, size = 0, normalized size = 0.

$$\int \operatorname{csch}(e+fx) (a+b \sinh^2(e+fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]

[Out] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int \operatorname{csch}(fx+e) \left(a+b (\sinh(fx+e))^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p, x)

[Out] `int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{csch}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \operatorname{csch}(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{csch}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

3.135 $\int \operatorname{csch}^3(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=87

$$\frac{\cosh(e + fx) (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; 2, -p; \frac{3}{2}; \cosh^2(e + fx), -\frac{b \cosh^2(e + fx)}{a - b} \right)}{f}$$

[Out] (AppellF1[1/2, 2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))] *Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)

Rubi [A] time = 0.0968984, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3186, 430, 429}

$$\frac{\cosh(e + fx) (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; 2, -p; \frac{3}{2}; \cosh^2(e + fx), -\frac{b \cosh^2(e + fx)}{a - b} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[1/2, 2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))] *Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)

Rule 3186

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 430

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(e+fx) (a+b \sinh^2(e+fx))^p dx = \frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^p}{(1-x^2)^2} dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\left((a-b+b \cosh^2(e+fx))^p \left(1 + \frac{b \cosh^2(e+fx)}{a-b}\right)^{-p}\right) \operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)^p}{(1-x^2)^2} dx, x,\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, -p; \frac{3}{2}; \cosh^2(e+fx), -\frac{b \cosh^2(e+fx)}{a-b}\right) \cosh(e+fx) (a-b+b \cosh^2(e+fx))^p}{f}$$

Mathematica [F] time = 7.89471, size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(e+fx) (a+b \sinh^2(e+fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.294, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(fx+e))^3 (a+b(\sinh(fx+e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)

[Out] `int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{csch}(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \operatorname{csch}(fx + e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^p \operatorname{csch} (fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

3.136 $\int \operatorname{csch}^5(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=88

$$\frac{\cosh(e + fx) (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; 3, -p; \frac{3}{2}; \cosh^2(e + fx), -\frac{b \cosh^2(e + fx)}{a - b} \right)}{f}$$

[Out] -((AppellF1[1/2, 3, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p))

Rubi [A] time = 0.0970843, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3186, 430, 429}

$$\frac{\cosh(e + fx) (a + b \cosh^2(e + fx) - b)^p \left(\frac{b \cosh^2(e + fx)}{a - b} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; 3, -p; \frac{3}{2}; \cosh^2(e + fx), -\frac{b \cosh^2(e + fx)}{a - b} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] -((AppellF1[1/2, 3, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p))

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol]
:> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx) (a+b \sinh^2(e+fx))^p dx = -\frac{\operatorname{Subst}\left(\int \frac{(a-b+bx^2)^p}{(1-x^2)^3} dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left((a-b+b \cosh^2(e+fx))^p \left(1 + \frac{b \cosh^2(e+fx)}{a-b}\right)^{-p}\right) \operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)^p}{(1-x^2)^3} dx, x\right)}{f}$$

$$= -\frac{F_1\left(\frac{1}{2}; 3, -p; \frac{3}{2}; \cosh^2(e+fx), -\frac{b \cosh^2(e+fx)}{a-b}\right) \cosh(e+fx) (a-b+b \cosh^2(e+fx))^p}{f}$$

Mathematica [F] time = 15.0046, size = 0, normalized size = 0.

$$\int \operatorname{csch}^5(e+fx) (a+b \sinh^2(e+fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(fx+e))^5 (a+b(\sinh(fx+e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)

[Out] `int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{csch}(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \operatorname{csch}(fx + e)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^p \operatorname{csch} (fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

3.137 $\int \sinh^4(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=103

$$\frac{\sinh^4(e + fx) \sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{5}{2}; \frac{1}{2}, -p; \frac{7}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{5f}$$

[Out] (AppellF1[5/2, 1/2, -p, 7/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(5*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.109966, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3188, 511, 510}

$$\frac{\sinh^4(e + fx) \sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{5}{2}; \frac{1}{2}, -p; \frac{7}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{5f}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[5/2, 1/2, -p, 7/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(5*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3188

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;

FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \sinh^4(e + fx) (a + b \sinh^2(e + fx))^p dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{x^4 (a + bx^2)^p}{\sqrt{1 + x^2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} (a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e + fx)}{a}\right)^{-p}\right)}{f} \\ &= \frac{F_1\left(\frac{5}{2}; \frac{1}{2}, -p; \frac{7}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a}\right) \sqrt{\cosh^2(e + fx) \sinh^4(e + fx)}}{5f} \end{aligned}$$

Mathematica [F] time = 10.2633, size = 0, normalized size = 0.

$$\int \sinh^4(e + fx) (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$\int (\sinh(fx + e))^4 (a + b (\sinh(fx + e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^p \sinh^4(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh^2(fx + e) + a\right)^p \sinh^4(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \sinh(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)

3.138 $\int \sinh^2(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=101

$$\frac{\tanh^3(e + fx) \operatorname{sech}^2(e + fx)^p (a + b \sinh^2(e + fx))^p \left(1 - \frac{(a-b) \tanh^2(e+fx)}{a}\right)^{-p} F_1\left(\frac{3}{2}; p+2, -p; \frac{5}{2}; \tanh^2(e + fx), \frac{(a-b) \tanh^2(e+fx)}{a}\right)}{3f}$$

[Out] (AppellF1[3/2, 2 + p, -p, 5/2, Tanh[e + f*x]^2, ((a - b)*Tanh[e + f*x]^2)/a] * (Sech[e + f*x]^2)^p * (a + b*Sinh[e + f*x]^2)^p * Tanh[e + f*x]^3) / (3*f*(1 - ((a - b)*Tanh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.192478, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3174, 511, 510}

$$\frac{\tanh^3(e + fx) \operatorname{sech}^2(e + fx)^p (a + b \sinh^2(e + fx))^p \left(1 - \frac{(a-b) \tanh^2(e+fx)}{a}\right)^{-p} F_1\left(\frac{3}{2}; p+2, -p; \frac{5}{2}; \tanh^2(e + fx), \frac{(a-b) \tanh^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

[In] Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[3/2, 2 + p, -p, 5/2, Tanh[e + f*x]^2, ((a - b)*Tanh[e + f*x]^2)/a] * (Sech[e + f*x]^2)^p * (a + b*Sinh[e + f*x]^2)^p * Tanh[e + f*x]^3) / (3*f*(1 - ((a - b)*Tanh[e + f*x]^2)/a)^p)

Rule 3174

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(ff*(a + b*Sinh[e + f*x]^2)^p*(Sec[e + f*x]^2)^p)/(f*(a + (a + b)*Tan[e + f*x]^2)^p), Subst[Int[((a + (a + b)*ff^2*x^2)^p*(A + (A + B)*ff^2*x^2))/(1 + ff^2*x^2)^(p + 2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, A, B}, x] && !IntegerQ[p]

Rule 511

Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x

```

n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

```

Rule 510

```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

```

Rubi steps

$$\begin{aligned}
 \int \sinh^2(e + fx) (a + b \sinh^2(e + fx))^p dx &= \frac{\left(\operatorname{sech}^2(e + fx)^p (a + b \sinh^2(e + fx))^p (a - (a - b) \tanh^2(e + fx))^{-p}\right) \operatorname{Subst}\left(\int \frac{1}{u} du, u, a + b \sinh^2(e + fx)\right)}{f} \\
 &= \frac{\left(\operatorname{sech}^2(e + fx)^p (a + b \sinh^2(e + fx))^p (a - (a - b) \tanh^2(e + fx))^{-p} (a + b \sinh^2(e + fx))\right)}{f} \\
 &= \frac{F_1\left(\frac{3}{2}; 2 + p, -p; \frac{5}{2}; \tanh^2(e + fx), \frac{(a-b) \tanh^2(e+fx)}{a}\right) \operatorname{sech}^2(e + fx)^p (a + b \sinh^2(e + fx))^p}{3f}
 \end{aligned}$$

Mathematica [B] time = 0.744882, size = 250, normalized size = 2.48

$$\frac{2^{-p-2} \operatorname{csch}(2(e + fx)) \sqrt{-\frac{b \sinh^2(e+fx)}{a}} \sqrt{\frac{b \cosh^2(e+fx)}{b-a}} (2a + b \cosh(2(e + fx)) - b)^{p+1} \left((p+1)(2a + b \cosh(2(e + fx)) - b) \right)}{b^2 f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```

[Out] (2^(-2 - p)*Sqrt[(b*Cosh[e + f*x]^2)/(-a + b)]*(2*a - b + b*Cosh[2*(e + f*x)])^(1 + p)*(-2*a*(2 + p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a - b + b*Cosh[2*(e + f*x)])/(2*a), (2*a - b + b*Cosh[2*(e + f*x)])/(2*(a - b))] + (1 + p)*AppellF1[2 + p, 1/2, 1/2, 3 + p, (2*a - b + b*Cosh[2*(e + f*x)])/(2*a), (2*a - b + b*Cosh[2*(e + f*x)])/(2*(a - b))]*(2*a - b + b*Cosh[2*(e + f*x)])*Csch[2*(e + f*x)]*Sqrt[-((b*Sinh[e + f*x]^2)/a)]]/(b^2*f*(1 + p)*(2 + p)

```

)

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int (\sinh(fx + e))^2 (a + b(\sinh(fx + e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)

[Out] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e)^2 + a)^p \sinh(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \sinh(fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")

[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^p \sinh^2(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)

3.139 $\int \operatorname{csch}^2(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=99

$$\frac{\sqrt{\cosh^2(e + fx) \operatorname{csch}(e + fx) \operatorname{sech}(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(-\frac{1}{2}; \frac{1}{2}, -p; \frac{1}{2}; -\sinh^2(e + fx), - \right)}{f}$$

[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)] *Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p))

Rubi [A] time = 0.104391, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3188, 511, 510}

$$\frac{\sqrt{\cosh^2(e + fx) \operatorname{csch}(e + fx) \operatorname{sech}(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(-\frac{1}{2}; \frac{1}{2}, -p; \frac{1}{2}; -\sinh^2(e + fx), - \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)] *Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p))

Rule 3188

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;

FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(e + fx) (a + b \sinh^2(e + fx))^p dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} \right) \operatorname{Subst} \left(\int \frac{(a + bx^2)^p}{x^2 \sqrt{1 + x^2}} dx, x, \sinh(e + fx) \right)}{f} \\ &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} (a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e + fx)}{a} \right)^{-p} \right)}{f} \\ &= \frac{F_1 \left(-\frac{1}{2}; \frac{1}{2}, -p; \frac{1}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right) \sqrt{\cosh^2(e + fx) \operatorname{csch}(e + fx)}}{f} \end{aligned}$$

Mathematica [F] time = 5.55103, size = 0, normalized size = 0.

$$\int \operatorname{csch}^2(e + fx) (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(fx + e))^2 (a + b (\sinh(fx + e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^p \operatorname{csch}^2(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sinh^2(fx + e) + a\right)^p \operatorname{csch}^2(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{csch}(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)`

3.140 $\int \mathbf{csch}^4(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=103

$$\frac{\sqrt{\cosh^2(e + fx) \operatorname{csch}^3(e + fx) \operatorname{sech}(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p}}}{3f} F_1 \left(-\frac{3}{2}; \frac{1}{2}, -p; -\frac{1}{2}; -\sinh^2(e + fx) \right)$$

[Out] -(AppellF1[-3/2, 1/2, -p, -1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(3*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.104743, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3188, 511, 510}

$$\frac{\sqrt{\cosh^2(e + fx) \operatorname{csch}^3(e + fx) \operatorname{sech}(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p}}}{3f} F_1 \left(-\frac{3}{2}; \frac{1}{2}, -p; -\frac{1}{2}; -\sinh^2(e + fx) \right)$$

Antiderivative was successfully verified.

[In] Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] -(AppellF1[-3/2, 1/2, -p, -1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(3*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3188

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 511

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;

FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(e + fx) (a + b \sinh^2(e + fx))^p dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} \right) \operatorname{Subst} \left(\int \frac{(a + bx^2)^p}{x^4 \sqrt{1 + x^2}} dx, x, \sinh(e + fx) \right)}{f} \\ &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} (a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e + fx)}{a} \right)^{-p} \right)}{f} \\ &= -\frac{F_1 \left(-\frac{3}{2}; \frac{1}{2}, -p; -\frac{1}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right) \sqrt{\cosh^2(e + fx) \operatorname{csch}^3}}{3f} \end{aligned}$$

Mathematica [F] time = 10.5505, size = 0, normalized size = 0.

$$\int \operatorname{csch}^4(e + fx) (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.249, size = 0, normalized size = 0.

$$\int (\operatorname{csch}(fx + e))^4 (a + b (\sinh(fx + e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{csch}(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \operatorname{csch}(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{csch}(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)

3.141 $\int \sinh^4(c + dx) (a + b \sinh^3(c + dx)) dx$

Optimal. Leaf size=106

$$\frac{a \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{3a \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3ax}{8} + \frac{b \cosh^7(c + dx)}{7d} - \frac{3b \cosh^5(c + dx)}{5d} + \frac{b \cosh^3(c + dx)}{d}$$

[Out] (3*a*x)/8 - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/d - (3*b*Cosh[c + d*x]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d) - (3*a*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)

Rubi [A] time = 0.0943742, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3220, 2635, 8, 2633}

$$\frac{a \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{3a \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3ax}{8} + \frac{b \cosh^7(c + dx)}{7d} - \frac{3b \cosh^5(c + dx)}{5d} + \frac{b \cosh^3(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]

[Out] (3*a*x)/8 - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/d - (3*b*Cosh[c + d*x]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d) - (3*a*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (a*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)

Rule 3220

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^p_., x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sinh[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sinh[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned}
 \int \sinh^4(c + dx) (a + b \sinh^3(c + dx)) dx &= \int (a \sinh^4(c + dx) + b \sinh^7(c + dx)) dx \\
 &= a \int \sinh^4(c + dx) dx + b \int \sinh^7(c + dx) dx \\
 &= \frac{a \cosh(c + dx) \sinh^3(c + dx)}{4d} - \frac{1}{4}(3a) \int \sinh^2(c + dx) dx - \frac{b \operatorname{Subst}\left(\int (1 - x^2)^{\frac{n-1}{2}} dx, x, \cos(c + dx)\right)}{d} \\
 &= -\frac{b \cosh(c + dx)}{d} + \frac{b \cosh^3(c + dx)}{d} - \frac{3b \cosh^5(c + dx)}{5d} + \frac{b \cosh^7(c + dx)}{7d} \\
 &= \frac{3ax}{8} - \frac{b \cosh(c + dx)}{d} + \frac{b \cosh^3(c + dx)}{d} - \frac{3b \cosh^5(c + dx)}{5d} + \frac{b \cosh^7(c + dx)}{7d}
 \end{aligned}$$

Mathematica [A] time = 0.102702, size = 81, normalized size = 0.76

$$\frac{-560a \sinh(2(c + dx)) + 70a \sinh(4(c + dx)) + 840ac + 840adx - 1225b \cosh(c + dx) + 245b \cosh(3(c + dx)) - 49b \cosh(5(c + dx)) + 5b \cosh(7(c + dx)) - 560a \sinh(2(c + dx)) + 70a \sinh(4(c + dx))}{2240d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3), x]

[Out] (840*a*c + 840*a*d*x - 1225*b*Cosh[c + d*x] + 245*b*Cosh[3*(c + d*x)] - 49*b*Cosh[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] - 560*a*Sinh[2*(c + d*x)] + 70*a*Sinh[4*(c + d*x)])/(2240*d)

Maple [A] time = 0.016, size = 82, normalized size = 0.8

$$\frac{1}{d} \left(b \left(-\frac{16}{35} + \frac{(\sinh(dx + c))^6}{7} - \frac{6(\sinh(dx + c))^4}{35} + \frac{8(\sinh(dx + c))^2}{35} \right) \cosh(dx + c) + a \left(\frac{(\sinh(dx + c))^3}{4} - \frac{3 \sinh(dx + c)}{8} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x)`

[Out] $\frac{1}{d}*(b*(-16/35+1/7*\sinh(d*x+c)^6-6/35*\sinh(d*x+c)^4+8/35*\sinh(d*x+c)^2)*\cosh(d*x+c)+a*((1/4*\sinh(d*x+c)^3-3/8*\sinh(d*x+c))*\cosh(d*x+c)+3/8*d*x+3/8*c))$

Maxima [A] time = 1.19747, size = 221, normalized size = 2.08

$$\frac{1}{64} a \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{4480} b \left(\frac{(49e^{(-2dx-2c)} - 245e^{(-4dx-4c)} + 1225e^{(-6dx-6c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")`

[Out] $\frac{1}{64}a*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) - \frac{1}{4480}b*((49*e^{(-2*d*x - 2*c)} - 245*e^{(-4*d*x - 4*c)} + 1225*e^{(-6*d*x - 6*c)} - 5)*e^{(7*d*x + 7*c)}/d + (1225*e^{(-d*x - c)} - 245*e^{(-3*d*x - 3*c)} + 49*e^{(-5*d*x - 5*c)} - 5*e^{(-7*d*x - 7*c)})/d)$

Fricas [A] time = 1.93103, size = 539, normalized size = 5.08

$$5b \cosh(dx+c)^7 + 35b \cosh(dx+c) \sinh(dx+c)^6 - 49b \cosh(dx+c)^5 + 280a \cosh(dx+c) \sinh(dx+c)^3 + 35(5b \cosh(dx+c)^3 - 7b \cosh(dx+c)) \sinh(dx+c)^4 + 245b \cosh(dx+c)^3 + 840a dx + 35(3b \cosh(dx+c)^5 - 14b \cosh(dx+c)^3 + 21b \cosh(dx+c)) \sinh(dx+c)^2 - 1225b \cosh(dx+c) + 280(a \cosh(dx+c)^3 - 4a \cosh(dx+c)) \sinh(dx+c) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")`

[Out] $\frac{1}{2240}*(5*b*\cosh(d*x + c)^7 + 35*b*\cosh(d*x + c)*\sinh(d*x + c)^6 - 49*b*\cosh(d*x + c)^5 + 280*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + 35*(5*b*\cosh(d*x + c)^3 - 7*b*\cosh(d*x + c))*\sinh(d*x + c)^4 + 245*b*\cosh(d*x + c)^3 + 840*a*d*x + 35*(3*b*\cosh(d*x + c)^5 - 14*b*\cosh(d*x + c)^3 + 21*b*\cosh(d*x + c))*\sinh(d*x + c)^2 - 1225*b*\cosh(d*x + c) + 280*(a*\cosh(d*x + c)^3 - 4*a*\cosh(d*x + c))*\sinh(d*x + c))/d$

Sympy [A] time = 8.10677, size = 192, normalized size = 1.81

$$\left\{ \frac{3ax \sinh^4(c+dx)}{8} - \frac{3ax \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3ax \cosh^4(c+dx)}{8} + \frac{5a \sinh^3(c+dx) \cosh(c+dx)}{8d} - \frac{3a \sinh(c+dx) \cosh^3(c+dx)}{8d} + \frac{b \sinh^6(c+dx)}{d} \right\} x (a + b \sinh^3(c)) \sinh^4(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**3), x)

[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)*sinh(c)**4, True))

Giac [A] time = 1.21197, size = 213, normalized size = 2.01

$$\frac{1680(dx+c)a + 5be^{(7dx+7c)} - 49be^{(5dx+5c)} + 70ae^{(4dx+4c)} + 245be^{(3dx+3c)} - 560ae^{(2dx+2c)} - 1225be^{(dx+c)} - (1225be^{(6dx+6c)} - 560ae^{(5dx+5c)} - 245b*e^{(4dx+4c)} + 70a*e^{(3dx+3c)} + 49b*e^{(2dx+2c)} - 5b)*e^{(-7dx-7c)}}{4480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3), x, algorithm="giac")

[Out] 1/4480*(1680*(d*x + c)*a + 5*b*e^(7*d*x + 7*c) - 49*b*e^(5*d*x + 5*c) + 70*a*e^(4*d*x + 4*c) + 245*b*e^(3*d*x + 3*c) - 560*a*e^(2*d*x + 2*c) - 1225*b*e^(d*x + c) - (1225*b*e^(6*d*x + 6*c) - 560*a*e^(5*d*x + 5*c) - 245*b*e^(4*d*x + 4*c) + 70*a*e^(3*d*x + 3*c) + 49*b*e^(2*d*x + 2*c) - 5*b)*e^(-7*d*x - 7*c))/d

3.142 $\int \sinh^3(c + dx) (a + b \sinh^3(c + dx)) dx$

Optimal. Leaf size=99

$$\frac{a \cosh^3(c + dx)}{3d} - \frac{a \cosh(c + dx)}{d} + \frac{b \sinh^5(c + dx) \cosh(c + dx)}{6d} - \frac{5b \sinh^3(c + dx) \cosh(c + dx)}{24d} + \frac{5b \sinh(c + dx) \cosh(c + dx)}{16d}$$

[Out] $(-5*b*x)/16 - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^3)/(3*d) + (5*b*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)$

Rubi [A] time = 0.108472, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3220, 2633, 2635, 8}

$$\frac{a \cosh^3(c + dx)}{3d} - \frac{a \cosh(c + dx)}{d} + \frac{b \sinh^5(c + dx) \cosh(c + dx)}{6d} - \frac{5b \sinh^3(c + dx) \cosh(c + dx)}{24d} + \frac{5b \sinh(c + dx) \cosh(c + dx)}{16d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^3*(a + b*\text{Sinh}[c + d*x]^3), x]$

[Out] $(-5*b*x)/16 - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^3)/(3*d) + (5*b*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)$

Rule 3220

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^{m*(a + b*\sin[e + f*x]^n)^p}, x], x] /; \text{FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{IntegersQ}[m, p] \ \&\& \ (\text{EqQ}[n, 4] \ || \ \text{GtQ}[p, 0] \ || \ (\text{EqQ}[p, -1] \ \&\& \ \text{IntegerQ}[n]))$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n-1)/2}], x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x \ \&\& \ \text{IGtQ}[(n-1)/2, 0]$

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x]
)*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
 \int \sinh^3(c + dx) (a + b \sinh^3(c + dx)) dx &= i \int (-ia \sinh^3(c + dx) - ib \sinh^6(c + dx)) dx \\
 &= a \int \sinh^3(c + dx) dx + b \int \sinh^6(c + dx) dx \\
 &= \frac{b \cosh(c + dx) \sinh^5(c + dx)}{6d} - \frac{1}{6}(5b) \int \sinh^4(c + dx) dx - \frac{a \operatorname{Subst}\left(\int (1 - u^2) du, u = \sinh(c + dx)\right)}{6d} \\
 &= -\frac{a \cosh(c + dx)}{d} + \frac{a \cosh^3(c + dx)}{3d} - \frac{5b \cosh(c + dx) \sinh^3(c + dx)}{24d} + \frac{b \cosh^5(c + dx)}{6d} \\
 &= -\frac{a \cosh(c + dx)}{d} + \frac{a \cosh^3(c + dx)}{3d} + \frac{5b \cosh(c + dx) \sinh(c + dx)}{16d} - \frac{5b \cosh^5(c + dx)}{6d} \\
 &= -\frac{5bx}{16} - \frac{a \cosh(c + dx)}{d} + \frac{a \cosh^3(c + dx)}{3d} + \frac{5b \cosh(c + dx) \sinh(c + dx)}{16d}
 \end{aligned}$$

Mathematica [A] time = 0.129329, size = 66, normalized size = 0.67

$$\frac{-144a \cosh(c + dx) + 16a \cosh(3(c + dx)) + b(45 \sinh(2(c + dx)) - 9 \sinh(4(c + dx)) + \sinh(6(c + dx)) - 60c - 60dx)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3), x]
```

```
[Out] (-144*a*Cosh[c + d*x] + 16*a*Cosh[3*(c + d*x)] + b*(-60*c - 60*d*x + 45*Sinh[2*(c + d*x)] - 9*Sinh[4*(c + d*x)] + Sinh[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.015, size = 72, normalized size = 0.7

$$\frac{1}{d} \left(b \left(\left(\frac{(\sinh(dx + c))^5}{6} - \frac{5(\sinh(dx + c))^3}{24} + \frac{5 \sinh(dx + c)}{16} \right) \cosh(dx + c) - \frac{5dx}{16} - \frac{5c}{16} \right) + a \left(-\frac{2}{3} + \frac{(\sinh(dx + c))^2}{3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x)`

[Out] $1/d*(b*((1/6*\sinh(d*x+c)^5-5/24*\sinh(d*x+c)^3+5/16*\sinh(d*x+c))*\cosh(d*x+c)-5/16*d*x-5/16*c)+a*(-2/3+1/3*\sinh(d*x+c)^2)*\cosh(d*x+c))$

Maxima [A] time = 1.11576, size = 193, normalized size = 1.95

$$-\frac{1}{384} b \left(\frac{(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e^{(-6dx-6c)}}{d} \right) + \frac{1}{24} a \left(\frac{e^{(3dx+3c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")`

[Out] $-1/384*b*((9*e^{(-2*d*x - 2*c)} - 45*e^{(-4*d*x - 4*c)} - 1)*e^{(6*d*x + 6*c)}/d + 120*(d*x + c)/d + (45*e^{(-2*d*x - 2*c)} - 9*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})/d) + 1/24*a*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d)$

Fricas [A] time = 1.88429, size = 375, normalized size = 3.79

$$3 b \cosh(dx + c) \sinh(dx + c)^5 + 8 a \cosh(dx + c)^3 + 24 a \cosh(dx + c) \sinh(dx + c)^2 + 2 (5 b \cosh(dx + c)^3 - 9 b \cosh(dx + c) \sinh(dx + c)^2 + 15 b \cosh(dx + c) \sinh(dx + c) - 3 a \cosh(dx + c)^3 + 24 a \cosh(dx + c) \sinh(dx + c) - 3 a \sinh(dx + c)^2) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")`

[Out] $1/96*(3*b*\cosh(d*x + c)*\sinh(d*x + c)^5 + 8*a*\cosh(d*x + c)^3 + 24*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*(5*b*\cosh(d*x + c)^3 - 9*b*\cosh(d*x + c))*\sinh(d*x + c)^3 - 30*b*d*x - 72*a*\cosh(d*x + c) + 3*(b*\cosh(d*x + c)^5 - 6*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + 15*b*\cosh(d*x + c))*\sinh(d*x + c))/d$

Sympy [A] time = 4.72493, size = 194, normalized size = 1.96

$$\frac{\left(\frac{a \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2a \cosh^3(c+dx)}{3d} + \frac{5bx \sinh^6(c+dx)}{16} - \frac{15bx \sinh^4(c+dx) \cosh^2(c+dx)}{16} + \frac{15bx \sinh^2(c+dx) \cosh^4(c+dx)}{16} - \frac{5bx \cosh^6(c+dx)}{16} \right)}{x(a + b \sinh^3(c)) \sinh^3(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3), x)

[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d) + 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)*sinh(c)**3, True))

Giac [A] time = 1.15322, size = 177, normalized size = 1.79

$$\frac{120(dx + c)b - be^{(6dx+6c)} + 9be^{(4dx+4c)} - 16ae^{(3dx+3c)} - 45be^{(2dx+2c)} + 144ae^{(dx+c)} + (144ae^{(5dx+5c)} + 45be^{(4dx+4c)})}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3), x, algorithm="giac")

[Out] -1/384*(120*(d*x + c)*b - b*e^(6*d*x + 6*c) + 9*b*e^(4*d*x + 4*c) - 16*a*e^(3*d*x + 3*c) - 45*b*e^(2*d*x + 2*c) + 144*a*e^(d*x + c) + (144*a*e^(5*d*x + 5*c) + 45*b*e^(4*d*x + 4*c) - 16*a*e^(3*d*x + 3*c) - 9*b*e^(2*d*x + 2*c) + b)*e^(-6*d*x - 6*c))/d

3.143 $\int \sinh^2(c + dx) (a + b \sinh^3(c + dx)) dx$

Optimal. Leaf size=70

$$\frac{a \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{ax}{2} + \frac{b \cosh^5(c + dx)}{5d} - \frac{2b \cosh^3(c + dx)}{3d} + \frac{b \cosh(c + dx)}{d}$$

[Out] $-(a*x)/2 + (b*\text{Cosh}[c + d*x])/d - (2*b*\text{Cosh}[c + d*x]^3)/(3*d) + (b*\text{Cosh}[c + d*x]^5)/(5*d) + (a*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*d)$

Rubi [A] time = 0.0708939, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3220, 2635, 8, 2633}

$$\frac{a \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{ax}{2} + \frac{b \cosh^5(c + dx)}{5d} - \frac{2b \cosh^3(c + dx)}{3d} + \frac{b \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^2*(a + b*\text{Sinh}[c + d*x]^3), x]$

[Out] $-(a*x)/2 + (b*\text{Cosh}[c + d*x])/d - (2*b*\text{Cosh}[c + d*x]^3)/(3*d) + (b*\text{Cosh}[c + d*x]^5)/(5*d) + (a*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(2*d)$

Rule 3220

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^{m*(a + b*\sin[e + f*x]^n)^p}, x], x] /; \text{FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{IntegersQ}[m, p] \ \&\& \ (\text{EqQ}[n, 4] \ || \ \text{GtQ}[p, 0] \ || \ (\text{EqQ}[p, -1] \ \&\& \ \text{IntegerQ}[n]))$

Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^{2*(n-1)})/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2633

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rubi steps

$$\begin{aligned} \int \sinh^2(c + dx)(a + b \sinh^3(c + dx)) dx &= - \int (-a \sinh^2(c + dx) - b \sinh^5(c + dx)) dx \\ &= a \int \sinh^2(c + dx) dx + b \int \sinh^5(c + dx) dx \\ &= \frac{a \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{1}{2} a \int 1 dx + \frac{b \operatorname{Subst}\left(\int (1 - 2x^2 + x^4) dx, x\right)}{d} \\ &= -\frac{ax}{2} + \frac{b \cosh(c + dx)}{d} - \frac{2b \cosh^3(c + dx)}{3d} + \frac{b \cosh^5(c + dx)}{5d} + \frac{a \cosh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.0954682, size = 79, normalized size = 1.13

$$\frac{a(-c - dx)}{2d} + \frac{a \sinh(2(c + dx))}{4d} + \frac{5b \cosh(c + dx)}{8d} - \frac{5b \cosh(3(c + dx))}{48d} + \frac{b \cosh(5(c + dx))}{80d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3), x]

[Out] (a*(-c - d*x))/(2*d) + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x)])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[2*(c + d*x)])/(4*d)

Maple [A] time = 0.013, size = 60, normalized size = 0.9

$$\frac{1}{d} \left(b \left(\frac{8}{15} + \frac{(\sinh(dx + c))^4}{5} - \frac{4(\sinh(dx + c))^2}{15} \right) \cosh(dx + c) + a \left(\frac{\cosh(dx + c) \sinh(dx + c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3), x)

[Out] 1/d*(b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))

Maxima [A] time = 1.11831, size = 162, normalized size = 2.31

$$-\frac{1}{8}a\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + \frac{1}{480}b\left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)

Fricas [A] time = 2.0538, size = 302, normalized size = 4.31

$$\frac{3b \cosh(dx+c)^5 + 15b \cosh(dx+c) \sinh(dx+c)^4 - 25b \cosh(dx+c)^3 - 120adx + 120a \cosh(dx+c) \sinh(dx+c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cosh(d*x + c)^3 - 120*a*d*x + 120*a*cosh(d*x + c)*sinh(d*x + c) + 15*(2*b*cosh(d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^2 + 150*b*cosh(d*x + c))/d

Sympy [A] time = 2.30679, size = 117, normalized size = 1.67

$$\left\{ \begin{array}{l} \frac{ax \sinh^2(c+dx)}{2} - \frac{ax \cosh^2(c+dx)}{2} + \frac{a \sinh(c+dx) \cosh(c+dx)}{2d} + \frac{b \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{4b \sinh^2(c+dx) \cosh^3(c+dx)}{3d} + \frac{8b \cosh^5(c+dx)}{15d} \\ x(a + b \sinh^3(c)) \sinh^2(c) \end{array} \right. \text{for } \text{ot}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)

[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x)*cosh(c + d*x)/(2*d) + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d

```
*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*
(a + b*sinh(c)**3)*sinh(c)**2, True))
```

Giac [A] time = 1.16479, size = 149, normalized size = 2.13

$$\frac{240(dx+c)a - 3be^{5dx+5c} + 25be^{3dx+3c} - 60ae^{2dx+2c} - 150be^{dx+c} - (150be^{4dx+4c} - 60ae^{3dx+3c} - 25be^{2dx+2c})}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/480*(240*(d*x + c)*a - 3*b*e^(5*d*x + 5*c) + 25*b*e^(3*d*x + 3*c) - 60*a
*e^(2*d*x + 2*c) - 150*b*e^(d*x + c) - (150*b*e^(4*d*x + 4*c) - 60*a*e^(3*d
*x + 3*c) - 25*b*e^(2*d*x + 2*c) + 3*b)*e^(-5*d*x - 5*c))/d
```

3.144 $\int \sinh(c + dx) (a + b \sinh^3(c + dx)) dx$

Optimal. Leaf size=60

$$\frac{a \cosh(c + dx)}{d} + \frac{b \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{3b \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3bx}{8}$$

[Out] (3*b*x)/8 + (a*Cosh[c + d*x])/d - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)

Rubi [A] time = 0.0684057, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {3220, 2638, 2635, 8}

$$\frac{a \cosh(c + dx)}{d} + \frac{b \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{3b \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3bx}{8}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3),x]

[Out] (3*b*x)/8 + (a*Cosh[c + d*x])/d - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)

Rule 3220

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^p, x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 2638

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sinh[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sinh[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \sinh(c + dx) (a + b \sinh^3(c + dx)) dx &= - \left(i \int (ia \sinh(c + dx) + ib \sinh^4(c + dx)) dx \right) \\
&= a \int \sinh(c + dx) dx + b \int \sinh^4(c + dx) dx \\
&= \frac{a \cosh(c + dx)}{d} + \frac{b \cosh(c + dx) \sinh^3(c + dx)}{4d} - \frac{1}{4}(3b) \int \sinh^2(c + dx) dx \\
&= \frac{a \cosh(c + dx)}{d} - \frac{3b \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b \cosh(c + dx) \sinh^3(c + dx)}{4d} \\
&= \frac{3bx}{8} + \frac{a \cosh(c + dx)}{d} - \frac{3b \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b \cosh(c + dx) \sinh^3(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.125911, size = 45, normalized size = 0.75

$$\frac{32a \cosh(c + dx) + b(12(c + dx) - 8 \sinh(2(c + dx)) + \sinh(4(c + dx)))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3), x]

[Out] (32*a*Cosh[c + d*x] + b*(12*(c + d*x) - 8*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(32*d)

Maple [A] time = 0.014, size = 50, normalized size = 0.8

$$\frac{1}{d} \left(b \left(\left(\frac{(\sinh(dx + c))^3}{4} - \frac{3 \sinh(dx + c)}{8} \right) \cosh(dx + c) + \frac{3dx}{8} + \frac{3c}{8} \right) + a \cosh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3), x)

[Out] $1/d*(b*((1/4*\sinh(d*x+c)^3-3/8*\sinh(d*x+c))*\cosh(d*x+c)+3/8*d*x+3/8*c)+a*\cosh(d*x+c))$

Maxima [A] time = 1.15149, size = 100, normalized size = 1.67

$$\frac{1}{64} b \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{a \cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")`

[Out] $1/64*b*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) + a*\cosh(d*x + c)/d$

Fricas [A] time = 1.80175, size = 171, normalized size = 2.85

$$\frac{b \cosh(dx+c) \sinh(dx+c)^3 + 3bdx + 8a \cosh(dx+c) + (b \cosh(dx+c)^3 - 4b \cosh(dx+c)) \sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")`

[Out] $1/8*(b*\cosh(d*x + c)*\sinh(d*x + c)^3 + 3*b*d*x + 8*a*\cosh(d*x + c) + (b*\cosh(d*x + c)^3 - 4*b*\cosh(d*x + c))*\sinh(d*x + c))/d$

Sympy [A] time = 1.24827, size = 121, normalized size = 2.02

$$\left\{ \begin{array}{l} \frac{a \cosh(c+dx)}{d} + \frac{3bx \sinh^4(c+dx)}{8} - \frac{3bx \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3bx \cosh^4(c+dx)}{8} + \frac{5b \sinh^3(c+dx) \cosh(c+dx)}{8d} - \frac{3b \sinh(c+dx) \cosh^3(c+dx)}{8d} \\ x(a + b \sinh^3(c)) \sinh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3),x)`


```
[Out] Piecewise((a*cosh(c + d*x)/d + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)*sinh(c), True))
```

Giac [A] time = 1.17231, size = 113, normalized size = 1.88

$$\frac{24(dx + c)b + be^{4dx+4c} - 8be^{2dx+2c} + 32ae^{dx+c} + (32ae^{3dx+3c} + 8be^{2dx+2c} - b)e^{-4dx-4c}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*b + b*e^(4*d*x + 4*c) - 8*b*e^(2*d*x + 2*c) + 32*a*e^(d*x + c) + (32*a*e^(3*d*x + 3*c) + 8*b*e^(2*d*x + 2*c) - b)*e^(-4*d*x - 4*c)) /d
```

3.145 $\int (a + b \sinh^3(c + dx)) dx$

Optimal. Leaf size=32

$$ax + \frac{b \cosh^3(c + dx)}{3d} - \frac{b \cosh(c + dx)}{d}$$

[Out] a*x - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0211492, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2633}

$$ax + \frac{b \cosh^3(c + dx)}{3d} - \frac{b \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[a + b*Sinh[c + d*x]^3,x]

[Out] a*x - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int (a + b \sinh^3(c + dx)) dx &= ax + b \int \sinh^3(c + dx) dx \\ &= ax - \frac{b \operatorname{Subst}\left(\int (1 - x^2) dx, x, \cosh(c + dx)\right)}{d} \\ &= ax - \frac{b \cosh(c + dx)}{d} + \frac{b \cosh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0105271, size = 34, normalized size = 1.06

$$ax - \frac{3b \cosh(c + dx)}{4d} + \frac{b \cosh(3(c + dx))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Sinh[c + d*x]^3,x]

[Out] a*x - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d)

Maple [A] time = 0.005, size = 28, normalized size = 0.9

$$ax + \frac{b \cosh(dx + c)}{d} \left(-\frac{2}{3} + \frac{(\sinh(dx + c))^2}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b*sinh(d*x+c)^3,x)

[Out] a*x+b/d*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)

Maxima [A] time = 1.15456, size = 80, normalized size = 2.5

$$ax + \frac{1}{24} b \left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="maxima")

[Out] a*x + 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)

Fricas [A] time = 1.90505, size = 128, normalized size = 4.

$$\frac{b \cosh(dx + c)^3 + 3b \cosh(dx + c) \sinh(dx + c)^2 + 12adx - 9b \cosh(dx + c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="fricas")

[Out] $1/12*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + 12*a*d*x - 9*b*\cosh(d*x + c))/d$

Sympy [A] time = 0.549777, size = 41, normalized size = 1.28

$$ax + b \left(\begin{cases} \frac{\sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2 \cosh^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x \sinh^3(c) & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*sinh(d*x+c)**3,x)`

[Out] `a*x + b*Piecewise((sinh(c + d*x)**2*cosh(c + d*x)/d - 2*cosh(c + d*x)**3/(3*d), Ne(d, 0)), (x*sinh(c)**3, True))`

Giac [A] time = 1.12415, size = 72, normalized size = 2.25

$$ax - \frac{\left(9e^{(2dx+2c)} - 1\right)e^{(-3dx-3c)} - e^{(3dx+3c)} + 9e^{(dx+c)}\right)b}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(a+b*sinh(d*x+c)^3,x, algorithm="giac")`

[Out] `a*x - 1/24*((9*e^(2*d*x + 2*c) - 1)*e^(-3*d*x - 3*c) - e^(3*d*x + 3*c) + 9*e^(d*x + c))*b/d`

3.146 $\int \operatorname{csch}(c + dx) (a + b \sinh^3(c + dx)) dx$

Optimal. Leaf size=40

$$-\frac{a \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{bx}{2}$$

[Out] $-(b*x)/2 - (a*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d + (b*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(2*d)$

Rubi [A] time = 0.0559113, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$, Rules used = {3220, 3770, 2635, 8}

$$-\frac{a \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{bx}{2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Sinh}[c + d*x]^3), x]$

[Out] $-(b*x)/2 - (a*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d + (b*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(2*d)$

Rule 3220

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e + f*x]^{m*(a + b*\sin[e + f*x]^n)^p}, x], x] /;$ FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rule 2635

$\operatorname{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \operatorname{Dist}[(b^2*(n-1))/n, \operatorname{Int}[(b*\operatorname{Sin}[c + d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}(c+dx) (a+b \sinh^3(c+dx)) dx &= i \int (-i a \operatorname{csch}(c+dx) - i b \sinh^2(c+dx)) dx \\
&= a \int \operatorname{csch}(c+dx) dx + b \int \sinh^2(c+dx) dx \\
&= -\frac{a \tanh^{-1}(\cosh(c+dx))}{d} + \frac{b \cosh(c+dx) \sinh(c+dx)}{2d} - \frac{1}{2} b \int 1 dx \\
&= -\frac{bx}{2} - \frac{a \tanh^{-1}(\cosh(c+dx))}{d} + \frac{b \cosh(c+dx) \sinh(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0542062, size = 72, normalized size = 1.8

$$\frac{a \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} + \frac{b(-c-dx)}{2d} + \frac{b \sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3),x]

[Out] (b*(-c - d*x))/(2*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (d*x)/2]])/d + (b*Sinh[2*(c + d*x)])/(4*d)

Maple [A] time = 0.036, size = 40, normalized size = 1.

$$\frac{1}{d} \left(-2 a \operatorname{Artanh}(e^{dx+c}) + b \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x)

[Out] $1/d*(-2*a*\operatorname{arctanh}(\exp(d*x+c))+b*(1/2*\cosh(d*x+c)*\sinh(d*x+c)-1/2*d*x-1/2*c))$

Maxima [A] time = 1.1445, size = 68, normalized size = 1.7

$$-\frac{1}{8}b\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + \frac{a \log\left(\tanh\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")`

[Out] $-1/8*b*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) + a*\log(\tanh(1/2*d*x + 1/2*c))/d$

Fricas [B] time = 1.99806, size = 721, normalized size = 18.02

$$4 b d x \cosh (d x + c)^2 - b \cosh (d x + c)^4 - 4 b \cosh (d x + c) \sinh (d x + c)^3 - b \sinh (d x + c)^4 + 2 (2 b d x - 3 b \cosh (d x +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")`

[Out] $-1/8*(4*b*d*x*\cosh(d*x + c)^2 - b*\cosh(d*x + c)^4 - 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 - b*\sinh(d*x + c)^4 + 2*(2*b*d*x - 3*b*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - 8*(a*\cosh(d*x + c)^2 + 2*a*\cosh(d*x + c)*\sinh(d*x + c) + a*\sinh(d*x + c)^2)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 4*(2*b*d*x*\cosh(d*x + c) - b*\cosh(d*x + c)^3)*\sinh(d*x + c) + b)/(d*\cosh(d*x + c)^2 + 2*d*\cosh(d*x + c)*\sinh(d*x + c) + d*\sinh(d*x + c)^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3),x)

[Out] Timed out

Giac [A] time = 1.13918, size = 97, normalized size = 2.42

$$-\frac{(dx+c)b}{2d} + \frac{be^{2dx+2c}}{8d} - \frac{be^{-2dx-2c}}{8d} - \frac{a \log(e^{(dx+c)} + 1)}{d} + \frac{a \log(|e^{(dx+c)} - 1|)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] $-\frac{1}{2}(d*x + c)*b/d + \frac{1}{8}b*e^{(2*d*x + 2*c)}/d - \frac{1}{8}b*e^{(-2*d*x - 2*c)}/d - a$
 $*\log(e^{(d*x + c)} + 1)/d + a*\log(\text{abs}(e^{(d*x + c)} - 1))/d$

3.147 $\int \operatorname{csch}^2(c + dx) (a + b \sinh^3(c + dx)) dx$

Optimal. Leaf size=24

$$\frac{b \cosh(c + dx)}{d} - \frac{a \coth(c + dx)}{d}$$

[Out] (b*Cosh[c + d*x])/d - (a*Coth[c + d*x])/d

Rubi [A] time = 0.0500271, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3220, 3767, 8, 2638}

$$\frac{b \cosh(c + dx)}{d} - \frac{a \coth(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]

[Out] (b*Cosh[c + d*x])/d - (a*Coth[c + d*x])/d

Rule 3220

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^ (p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) (a + b \sinh^3(c + dx)) dx &= - \int (-\operatorname{acsch}^2(c + dx) - b \sinh(c + dx)) dx \\ &= a \int \operatorname{csch}^2(c + dx) dx + b \int \sinh(c + dx) dx \\ &= \frac{b \cosh(c + dx)}{d} - \frac{(ia) \operatorname{Subst}(\int 1 dx, x, -i \coth(c + dx))}{d} \\ &= \frac{b \cosh(c + dx)}{d} - \frac{a \coth(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.0291625, size = 35, normalized size = 1.46

$$-\frac{a \coth(c + dx)}{d} + \frac{b \sinh(c) \sinh(dx)}{d} + \frac{b \cosh(c) \cosh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3), x]
```

```
[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Coth[c + d*x])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.036, size = 23, normalized size = 1.

$$\frac{-\coth(dx + c) a + b \cosh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3), x)
```

```
[Out] 1/d*(-coth(d*x+c)*a+b*cosh(d*x+c))
```

Maxima [A] time = 1.13484, size = 63, normalized size = 2.62

$$\frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) + \frac{2a}{d(e^{(-2dx-2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))

Fricas [A] time = 1.95052, size = 103, normalized size = 4.29

$$-\frac{a \cosh(dx + c) - (b \cosh(dx + c) + a) \sinh(dx + c)}{d \sinh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] -(a*cosh(d*x + c) - (b*cosh(d*x + c) + a)*sinh(d*x + c))/(d*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)

[Out] Timed out

Giac [B] time = 1.20409, size = 84, normalized size = 3.5

$$\frac{be^{(dx+c)}}{2d} + \frac{be^{(2dx+2c)} - 4ae^{(dx+c)} - b}{2d(e^{(3dx+3c)} - e^{(dx+c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*b*e^(d*x + c)/d + 1/2*(b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c) - b)/(d*(e^(3*d*x + 3*c) - e^(d*x + c)))
```

3.148 $\int \operatorname{csch}^3(c + dx) (a + b \sinh^3(c + dx)) dx$

Optimal. Leaf size=39

$$\frac{a \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + bx$$

[Out] b*x + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rubi [A] time = 0.0626266, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3220, 3768, 3770}

$$\frac{a \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + bx$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3), x]

[Out] b*x + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rule 3220

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx) (a+b \sinh^3(c+dx)) dx &= -\left(i \int (ib + i a \operatorname{csch}^3(c+dx)) dx\right) \\
&= bx + a \int \operatorname{csch}^3(c+dx) dx \\
&= bx - \frac{a \coth(c+dx) \operatorname{csch}(c+dx)}{2d} - \frac{1}{2} a \int \operatorname{csch}(c+dx) dx \\
&= bx + \frac{a \tanh^{-1}(\cosh(c+dx))}{2d} - \frac{a \coth(c+dx) \operatorname{csch}(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0129128, size = 63, normalized size = 1.62

$$-\frac{a \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a \operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + bx$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3), x]

[Out] b*x - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d)

Maple [A] time = 0.04, size = 37, normalized size = 1.

$$\frac{1}{d} \left(a \left(-\frac{\operatorname{csch}(dx+c) \coth(dx+c)}{2} + \operatorname{Artanh}(e^{dx+c}) \right) + (dx+c)b \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3), x)

[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+(d*x+c)*b)

Maxima [B] time = 1.09883, size = 123, normalized size = 3.15

$$bx + \frac{1}{2} a \left(\frac{\log(e^{-dx-c} + 1)}{d} - \frac{\log(e^{-dx-c} - 1)}{d} + \frac{2(e^{-dx-c} + e^{-3dx-3c})}{d(2e^{-2dx-2c} - e^{-4dx-4c} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] b*x + 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.12639, size = 1413, normalized size = 36.23

$$2 b d x \cosh (d x + c)^4 + 2 b d x \sinh (d x + c)^4 - 4 b d x \cosh (d x + c)^2 - 2 a \cosh (d x + c)^3 + 2 (4 b d x \cosh (d x + c) - a) \sinh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x*cosh(d*x + c)^4 + 2*b*d*x*sinh(d*x + c)^4 - 4*b*d*x*cosh(d*x + c)^2 - 2*a*cosh(d*x + c)^3 + 2*(4*b*d*x*cosh(d*x + c) - a)*sinh(d*x + c)^3 + 2*b*d*x + 2*(6*b*d*x*cosh(d*x + c)^2 - 2*b*d*x - 3*a*cosh(d*x + c))*sinh(d*x + c)^2 - 2*a*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(4*b*d*x*cosh(d*x + c)^3 - 4*b*d*x*cosh(d*x + c) - 3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3),x)
```

[Out] Timed out

Giac [B] time = 1.16116, size = 108, normalized size = 2.77

$$\frac{(dx+c)b}{d} + \frac{a \log(e^{(dx+c)} + 1)}{2d} - \frac{a \log(|e^{(dx+c)} - 1|)}{2d} - \frac{ae^{(3dx+3c)} + ae^{(dx+c)}}{d(e^{(2dx+2c)} - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] (d*x + c)*b/d + 1/2*a*log(e^(d*x + c) + 1)/d - 1/2*a*log(abs(e^(d*x + c) - 1))/d - (a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(d*(e^(2*d*x + 2*c) - 1)^2)

3.149 $\int \operatorname{csch}^4(c + dx) (a + b \sinh^3(c + dx)) dx$

Optimal. Leaf size=41

$$-\frac{a \operatorname{coth}^3(c + dx)}{3d} + \frac{a \operatorname{coth}(c + dx)}{d} - \frac{b \tanh^{-1}(\cosh(c + dx))}{d}$$

[Out] $-\left(\frac{b \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) + \frac{a \operatorname{Coth}[c + d*x]}{d} - \frac{a \operatorname{Coth}[c + d*x]^3}{(3*d)}$

Rubi [A] time = 0.0571616, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3220, 3770, 3767}

$$-\frac{a \operatorname{coth}^3(c + dx)}{3d} + \frac{a \operatorname{coth}(c + dx)}{d} - \frac{b \tanh^{-1}(\cosh(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^4*(a + b*\operatorname{Sinh}[c + d*x]^3), x]$

[Out] $-\left(\frac{b \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) + \frac{a \operatorname{Coth}[c + d*x]}{d} - \frac{a \operatorname{Coth}[c + d*x]^3}{(3*d)}$

Rule 3220

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e + f*x]^m*(a + b*\sin[e + f*x]^n)^p, x], x] /;$ FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(c+dx) (a+b \sinh^3(c+dx)) dx &= \int (b \operatorname{csch}(c+dx) + a \operatorname{csch}^4(c+dx)) dx \\
&= a \int \operatorname{csch}^4(c+dx) dx + b \int \operatorname{csch}(c+dx) dx \\
&= -\frac{b \tanh^{-1}(\cosh(c+dx))}{d} + \frac{(ia) \operatorname{Subst}\left(\int (1+x^2) dx, x, -i \coth(c+dx)\right)}{d} \\
&= -\frac{b \tanh^{-1}(\cosh(c+dx))}{d} + \frac{a \coth(c+dx)}{d} - \frac{a \coth^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0302817, size = 76, normalized size = 1.85

$$\frac{2a \coth(c+dx)}{3d} - \frac{a \coth(c+dx) \operatorname{csch}^2(c+dx)}{3d} + \frac{b \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{b \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3), x]

[Out] (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d

Maple [A] time = 0.043, size = 36, normalized size = 0.9

$$\frac{1}{d} \left(a \left(\frac{2}{3} - \frac{(\operatorname{csch}(dx+c))^2}{3} \right) \coth(dx+c) - 2b \operatorname{Artanh}(e^{dx+c}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3), x)

[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*b*arctanh(exp(d*x+c)))

Maxima [B] time = 1.16159, size = 177, normalized size = 4.32

$$-b \left(\frac{\log(e^{-dx-c} + 1)}{d} - \frac{\log(e^{-dx-c} - 1)}{d} \right) + \frac{4}{3} a \left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a*(3*e^(-2*d*x
- 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)
) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.02867, size = 1773, normalized size = 43.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*cosh(d*x + c)^2 + 24*a*cosh(d*x + c)*sinh(d*x + c) + 12*a*sinh(d
*x + c)^2 + 3*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*si
nh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*
cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*
x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*si
nh(d*x + c) - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(b*cosh(d*x + c
)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x
+ c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)
^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh
(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)
^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log(cosh(d*x
+ c) + sinh(d*x + c) - 1) - 4*a)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*si
nh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x +
c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 +
d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3),x)

[Out] Timed out

Giac [A] time = 1.1937, size = 88, normalized size = 2.15

$$-\frac{b \log(e^{(dx+c)} + 1)}{d} + \frac{b \log(|e^{(dx+c)} - 1|)}{d} - \frac{4(3ae^{(2dx+2c)} - a)}{3d(e^{(2dx+2c)} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] -b*log(e^(d*x + c) + 1)/d + b*log(abs(e^(d*x + c) - 1))/d - 4/3*(3*a*e^(2*d*x + 2*c) - a)/(d*(e^(2*d*x + 2*c) - 1)^3)

3.150 $\int \sinh^3(c + dx) (a + b \sinh^3(c + dx))^2 dx$

Optimal. Leaf size=192

$$\frac{a^2 \cosh^3(c + dx)}{3d} - \frac{a^2 \cosh(c + dx)}{d} + \frac{ab \sinh^5(c + dx) \cosh(c + dx)}{3d} - \frac{5ab \sinh^3(c + dx) \cosh(c + dx)}{12d} + \frac{5ab \sinh(c + dx)}{12d}$$

[Out] $(-5*a*b*x)/8 - (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x])/d + (a^2*Cosh[c + d*x]^3)/(3*d) - (4*b^2*Cosh[c + d*x]^3)/(3*d) + (6*b^2*Cosh[c + d*x]^5)/(5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[c + d*x]^9)/(9*d) + (5*a*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) - (5*a*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(12*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(3*d)$

Rubi [A] time = 0.179161, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3220, 2633, 2635, 8}

$$\frac{a^2 \cosh^3(c + dx)}{3d} - \frac{a^2 \cosh(c + dx)}{d} + \frac{ab \sinh^5(c + dx) \cosh(c + dx)}{3d} - \frac{5ab \sinh^3(c + dx) \cosh(c + dx)}{12d} + \frac{5ab \sinh(c + dx)}{12d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^3*(a + b*\text{Sinh}[c + d*x]^3)^2, x]$

[Out] $(-5*a*b*x)/8 - (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x])/d + (a^2*Cosh[c + d*x]^3)/(3*d) - (4*b^2*Cosh[c + d*x]^3)/(3*d) + (6*b^2*Cosh[c + d*x]^5)/(5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[c + d*x]^9)/(9*d) + (5*a*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) - (5*a*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(12*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(3*d)$

Rule 3220

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^m*(a + b*\sin[e + f*x]^n)^p, x], x] /; \text{FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{IntegersQ}[m, p] \ \&\& \ (\text{EqQ}[n, 4] \ || \ \text{GtQ}[p, 0] \ || \ (\text{EqQ}[p, -1] \ \&\& \ \text{IntegerQ}[n]))$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n-1)/2}, x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x$

&& IGtQ[(n - 1)/2, 0]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
 \int \sinh^3(c + dx) (a + b \sinh^3(c + dx))^2 dx &= i \int (-ia^2 \sinh^3(c + dx) - 2iab \sinh^6(c + dx) - ib^2 \sinh^9(c + dx)) dx \\
 &= a^2 \int \sinh^3(c + dx) dx + (2ab) \int \sinh^6(c + dx) dx + b^2 \int \sinh^9(c + dx) dx \\
 &= \frac{ab \cosh(c + dx) \sinh^5(c + dx)}{3d} - \frac{1}{3}(5ab) \int \sinh^4(c + dx) dx - \frac{a^2 \text{Subst}\left(\int \sinh^3(u) du\right)}{3d} \\
 &= -\frac{a^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh(c + dx)}{d} + \frac{a^2 \cosh^3(c + dx)}{3d} - \frac{4b^2 \cosh^3(c + dx)}{3d} \\
 &= -\frac{a^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh(c + dx)}{d} + \frac{a^2 \cosh^3(c + dx)}{3d} - \frac{4b^2 \cosh^3(c + dx)}{3d} \\
 &= -\frac{5}{8}abx - \frac{a^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh(c + dx)}{d} + \frac{a^2 \cosh^3(c + dx)}{3d} - \frac{4b^2 \cosh^3(c + dx)}{3d}
 \end{aligned}$$

Mathematica [A] time = 0.683032, size = 125, normalized size = 0.65

$$\frac{-1890(32a^2 - 21b^2) \cosh(c + dx) + 420(16a^2 - 21b^2) \cosh(3(c + dx)) + b(-840a(-45 \sinh(2(c + dx)) + 9 \sinh(4(c + dx)))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-1890*(32*a^2 - 21*b^2)*Cosh[c + d*x] + 420*(16*a^2 - 21*b^2)*Cosh[3*(c +
d*x)] + b*(2268*b*Cosh[5*(c + d*x)] - 405*b*Cosh[7*(c + d*x)] + 35*b*Cosh[9
*(c + d*x)] - 840*a*(60*c + 60*d*x - 45*Sinh[2*(c + d*x)] + 9*Sinh[4*(c + d
```

x)] - Sinh[6(c + d*x)])))/(80640*d)

Maple [A] time = 0.02, size = 128, normalized size = 0.7

$$\frac{1}{d} \left(b^2 \left(\frac{128}{315} + \frac{(\sinh(dx+c))^8}{9} - \frac{8(\sinh(dx+c))^6}{63} + \frac{16(\sinh(dx+c))^4}{105} - \frac{64(\sinh(dx+c))^2}{315} \right) \cosh(dx+c) + 2ab \left(\left(\frac{1}{6}\sinh(dx+c)^5 - \frac{5}{24}\sinh(dx+c)^3 + \frac{5}{16}\sinh(dx+c) \right) \cosh(dx+c) - \frac{5}{16}d^2x - \frac{5}{16}c \right) + a^2 \left(-\frac{2}{3} + \frac{1}{3}\sinh(dx+c)^2 \right) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x)

[Out] 1/d*(b^2*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))

Maxima [A] time = 1.18855, size = 367, normalized size = 1.91

$$-\frac{1}{161280} b^2 \left(\frac{(405 e^{(-2dx-2c)} - 2268 e^{(-4dx-4c)} + 8820 e^{(-6dx-6c)} - 39690 e^{(-8dx-8c)} - 35) e^{(9dx+9c)}}{d} - \frac{39690 e^{(-dx-c)} - 8820 e^{(-3dx-3c)} + 2268 e^{(-5dx-5c)} - 405 e^{(-7dx-7c)} + 35 e^{(-9dx-9c)}}{d} - \frac{1}{192} a b \left(\frac{9 e^{(-2dx-2c)} - 45 e^{(-4dx-4c)} - 1}{d} e^{(6dx+6c)} + \frac{120(dx+c)}{d} + \frac{45 e^{(-2dx-2c)} - 9 e^{(-4dx-4c)} + e^{(-6dx-6c)}}{d} + \frac{1}{24} a^2 \left(\frac{e^{(3dx+3c)}}{d} - 9 e^{(dx+c)} \right) \cosh(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")

[Out] -1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)

Fricas [B] time = 1.91839, size = 973, normalized size = 5.07

$$35 b^2 \cosh(dx+c)^9 + 315 b^2 \cosh(dx+c) \sinh(dx+c)^8 - 405 b^2 \cosh(dx+c)^7 + 5040 ab \cosh(dx+c) \sinh(dx+c)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] $\frac{1}{80640} \cdot (35 \cdot b^2 \cdot \cosh(d \cdot x + c)^9 + 315 \cdot b^2 \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^8 - 405 \cdot b^2 \cdot \cosh(d \cdot x + c)^7 + 5040 \cdot a \cdot b \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^5 + 2268 \cdot b^2 \cdot \cosh(d \cdot x + c)^5 + 105 \cdot (28 \cdot b^2 \cdot \cosh(d \cdot x + c)^3 - 27 \cdot b^2 \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^6 + 315 \cdot (14 \cdot b^2 \cdot \cosh(d \cdot x + c)^5 - 45 \cdot b^2 \cdot \cosh(d \cdot x + c)^3 + 36 \cdot b^2 \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^4 - 50400 \cdot a \cdot b \cdot d \cdot x + 420 \cdot (16 \cdot a^2 - 21 \cdot b^2) \cdot \cosh(d \cdot x + c)^3 + 3360 \cdot (5 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^3 - 9 \cdot a \cdot b \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^3 + 315 \cdot (4 \cdot b^2 \cdot \cosh(d \cdot x + c)^7 - 27 \cdot b^2 \cdot \cosh(d \cdot x + c)^5 + 72 \cdot b^2 \cdot \cosh(d \cdot x + c)^3 + 4 \cdot (16 \cdot a^2 - 21 \cdot b^2) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^2 - 1890 \cdot (32 \cdot a^2 - 21 \cdot b^2) \cdot \cosh(d \cdot x + c) + 5040 \cdot (a \cdot b \cdot \cosh(d \cdot x + c)^5 - 6 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^3 + 15 \cdot a \cdot b \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)) / d$

Sympy [A] time = 31.0145, size = 325, normalized size = 1.69

$$\left\{ \begin{array}{l} \frac{a^2 \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2a^2 \cosh^3(c+dx)}{3d} + \frac{5abx \sinh^6(c+dx)}{8} - \frac{15abx \sinh^4(c+dx) \cosh^2(c+dx)}{8} + \frac{15abx \sinh^2(c+dx) \cosh^4(c+dx)}{8} - \frac{5abx \cosh^6(c+dx)}{8} \\ x(a + b \sinh^3(c))^2 \sinh^3(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)

[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/(3*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*cosh(c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b**2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c)**3, True))

Giac [A] time = 1.26917, size = 369, normalized size = 1.92

$$100800(dx + c)ab - 35b^2e^{(9dx+9c)} + 405b^2e^{(7dx+7c)} - 840abe^{(6dx+6c)} - 2268b^2e^{(5dx+5c)} + 7560abe^{(4dx+4c)} - 6720a^2e^{(3dx+3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/161280*(100800*(d*x + c)*a*b - 35*b^2*e^(9*d*x + 9*c) + 405*b^2*e^(7*d*x
+ 7*c) - 840*a*b*e^(6*d*x + 6*c) - 2268*b^2*e^(5*d*x + 5*c) + 7560*a*b*e^(
4*d*x + 4*c) - 6720*a^2*e^(3*d*x + 3*c) + 8820*b^2*e^(3*d*x + 3*c) - 37800*
a*b*e^(2*d*x + 2*c) + 60480*a^2*e^(d*x + c) - 39690*b^2*e^(d*x + c) + (3780
0*a*b*e^(7*d*x + 7*c) - 7560*a*b*e^(5*d*x + 5*c) - 2268*b^2*e^(4*d*x + 4*c)
+ 840*a*b*e^(3*d*x + 3*c) + 405*b^2*e^(2*d*x + 2*c) - 35*b^2 + 1890*(32*a^
2 - 21*b^2)*e^(8*d*x + 8*c) - 420*(16*a^2 - 21*b^2)*e^(6*d*x + 6*c))*e^(-9*
d*x - 9*c))/d
```

3.151 $\int \sinh^2(c + dx) (a + b \sinh^3(c + dx))^2 dx$

Optimal. Leaf size=180

$$\frac{a^2 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{a^2 x}{2} + \frac{2ab \cosh^5(c + dx)}{5d} - \frac{4ab \cosh^3(c + dx)}{3d} + \frac{2ab \cosh(c + dx)}{d} + \frac{b^2 \sinh^7(c + dx) \cosh(c + dx)}{8d}$$

[Out] $-(a^2 x)/2 + (35*b^2 x)/128 + (2*a*b*Cosh[c + d*x])/d - (4*a*b*Cosh[c + d*x]^3)/(3*d) + (2*a*b*Cosh[c + d*x]^5)/(5*d) + (a^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (35*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) - (7*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)$

Rubi [A] time = 0.154268, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3220, 2635, 8, 2633}

$$\frac{a^2 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{a^2 x}{2} + \frac{2ab \cosh^5(c + dx)}{5d} - \frac{4ab \cosh^3(c + dx)}{3d} + \frac{2ab \cosh(c + dx)}{d} + \frac{b^2 \sinh^7(c + dx) \cosh(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]

[Out] $-(a^2 x)/2 + (35*b^2 x)/128 + (2*a*b*Cosh[c + d*x])/d - (4*a*b*Cosh[c + d*x]^3)/(3*d) + (2*a*b*Cosh[c + d*x]^5)/(5*d) + (a^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (35*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) - (7*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)$

Rule 3220

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^p, x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_.), x_Symbol] :> -Simp[(b*cos[c + d*x]*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c

+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned}
 \int \sinh^2(c + dx) (a + b \sinh^3(c + dx))^2 dx &= - \int (-a^2 \sinh^2(c + dx) - 2ab \sinh^5(c + dx) - b^2 \sinh^8(c + dx)) dx \\
 &= a^2 \int \sinh^2(c + dx) dx + (2ab) \int \sinh^5(c + dx) dx + b^2 \int \sinh^8(c + dx) dx \\
 &= \frac{a^2 \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{b^2 \cosh(c + dx) \sinh^7(c + dx)}{8d} - \frac{1}{2} a^2 \int 1 dx \\
 &= -\frac{a^2 x}{2} + \frac{2ab \cosh(c + dx)}{d} - \frac{4ab \cosh^3(c + dx)}{3d} + \frac{2ab \cosh^5(c + dx)}{5d} + \frac{a^2}{5d} \\
 &= -\frac{a^2 x}{2} + \frac{2ab \cosh(c + dx)}{d} - \frac{4ab \cosh^3(c + dx)}{3d} + \frac{2ab \cosh^5(c + dx)}{5d} + \frac{a^2}{5d} \\
 &= -\frac{a^2 x}{2} + \frac{2ab \cosh(c + dx)}{d} - \frac{4ab \cosh^3(c + dx)}{3d} + \frac{2ab \cosh^5(c + dx)}{5d} + \frac{a^2}{5d} \\
 &= -\frac{a^2 x}{2} + \frac{35b^2 x}{128} + \frac{2ab \cosh(c + dx)}{d} - \frac{4ab \cosh^3(c + dx)}{3d} + \frac{2ab \cosh^5(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] time = 0.129859, size = 133, normalized size = 0.74

$$\frac{3840a^2 \sinh(2(c + dx)) - 7680a^2c - 7680a^2dx + 19200ab \cosh(c + dx) - 3200ab \cosh(3(c + dx)) + 384ab \cosh(5(c + dx))}{1}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]

[Out] (-7680*a^2*c + 4200*b^2*c - 7680*a^2*d*x + 4200*b^2*d*x + 19200*a*b*Cosh[c + d*x] - 3200*a*b*Cosh[3*(c + d*x)] + 384*a*b*Cosh[5*(c + d*x)] + 3840*a^2*

$$\frac{\text{Sinh}[2*(c + d*x)] - 3360*b^2*\text{Sinh}[2*(c + d*x)] + 840*b^2*\text{Sinh}[4*(c + d*x)] - 160*b^2*\text{Sinh}[6*(c + d*x)] + 15*b^2*\text{Sinh}[8*(c + d*x)]}{(15360*d)}$$

Maple [A] time = 0.021, size = 122, normalized size = 0.7

$$\frac{1}{d} \left(b^2 \left(\left(\frac{(\sinh(dx+c))^7}{8} - \frac{7(\sinh(dx+c))^5}{48} + \frac{35(\sinh(dx+c))^3}{192} - \frac{35\sinh(dx+c)}{128} \right) \cosh(dx+c) + \frac{35dx}{128} + \frac{35c}{128} \right) + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x)

[Out] 1/d*(b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+2*a*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))

Maxima [A] time = 1.15235, size = 320, normalized size = 1.78

$$-\frac{1}{8} a^2 \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) - \frac{1}{6144} b^2 \left(\frac{(32e^{(-2dx-2c)} - 168e^{(-4dx-4c)} + 672e^{(-6dx-6c)} - 3)e^{(8dx+8c)}}{d} - \frac{1680(dx+c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")

[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/6144*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)

Fricas [A] time = 1.90907, size = 734, normalized size = 4.08

$$15b^2 \cosh(dx+c) \sinh(dx+c)^7 + 48ab \cosh(dx+c)^5 + 240ab \cosh(dx+c) \sinh(dx+c)^4 + 15(7b^2 \cosh(dx+c)^3 - \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] $\frac{1}{1920} \cdot (15 \cdot b^2 \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^7 + 48 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^5 + 240 \cdot a \cdot b \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^4 + 15 \cdot (7 \cdot b^2 \cdot \cosh(d \cdot x + c)^3 - 8 \cdot b^2 \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^5 - 400 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^3 + 5 \cdot (21 \cdot b^2 \cdot \cosh(d \cdot x + c)^5 - 80 \cdot b^2 \cdot \cosh(d \cdot x + c)^3 + 84 \cdot b^2 \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^3 - 15 \cdot (64 \cdot a^2 - 35 \cdot b^2) \cdot d \cdot x + 2400 \cdot a \cdot b \cdot \cosh(d \cdot x + c) + 240 \cdot (2 \cdot a \cdot b \cdot \cosh(d \cdot x + c)^3 - 5 \cdot a \cdot b \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^2 + 15 \cdot (b^2 \cdot \cosh(d \cdot x + c)^7 - 8 \cdot b^2 \cdot \cosh(d \cdot x + c)^5 + 28 \cdot b^2 \cdot \cosh(d \cdot x + c)^3 + 8 \cdot (8 \cdot a^2 - 7 \cdot b^2) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)) / d$

Sympy [A] time = 15.3381, size = 340, normalized size = 1.89

$$\left\{ \begin{array}{l} \frac{a^2 x \sinh^2(c+dx)}{2} - \frac{a^2 x \cosh^2(c+dx)}{2} + \frac{a^2 \sinh(c+dx) \cosh(c+dx)}{2d} + \frac{2ab \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{8ab \sinh^2(c+dx) \cosh^3(c+dx)}{3d} + \frac{16ab \cosh^5(c+dx)}{15d} \\ x \left(a + b \sinh^3(c) \right)^2 \sinh^2(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)

[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*d) + 35*b**2*x*sinh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c)**2, True))

Giac [A] time = 1.25071, size = 317, normalized size = 1.76

$$\frac{15 b^2 e^{(8dx+8c)} - 160 b^2 e^{(6dx+6c)} + 384 a b e^{(5dx+5c)} + 840 b^2 e^{(4dx+4c)} - 3200 a b e^{(3dx+3c)} + 3840 a^2 e^{(2dx+2c)} - 3360 b^2 e^{(2dx+2c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(15*b^2*e^(8*d*x + 8*c) - 160*b^2*e^(6*d*x + 6*c) + 384*a*b*e^(5*d*x + 5*c) + 840*b^2*e^(4*d*x + 4*c) - 3200*a*b*e^(3*d*x + 3*c) + 3840*a^2*e^(2*d*x + 2*c) - 3360*b^2*e^(2*d*x + 2*c) + 19200*a*b*e^(d*x + c) - 240*(64*a^2 - 35*b^2)*(d*x + c) + (19200*a*b*e^(7*d*x + 7*c) - 3200*a*b*e^(5*d*x + 5*c) - 840*b^2*e^(4*d*x + 4*c) + 384*a*b*e^(3*d*x + 3*c) + 160*b^2*e^(2*d*x + 2*c) - 15*b^2 - 480*(8*a^2 - 7*b^2)*e^(6*d*x + 6*c))*e^(-8*d*x - 8*c))/d
```

3.152 $\int \sinh(c + dx) \left(a + b \sinh^3(c + dx) \right)^2 dx$

Optimal. Leaf size=130

$$\frac{a^2 \cosh(c + dx)}{d} + \frac{ab \sinh^3(c + dx) \cosh(c + dx)}{2d} - \frac{3ab \sinh(c + dx) \cosh(c + dx)}{4d} + \frac{3abx}{4} + \frac{b^2 \cosh^7(c + dx)}{7d} - \frac{3b^2 \cosh^5(c + dx)}{5d}$$

```
[Out] (3*a*b*x)/4 + (a^2*Cosh[c + d*x])/d - (b^2*Cosh[c + d*x])/d + (b^2*Cosh[c +
d*x]^3)/d - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d) -
(3*a*b*Cosh[c + d*x]*Sinh[c + d*x])/(4*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x
]^3)/(2*d)
```

Rubi [A] time = 0.118934, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3220, 2638, 2635, 8, 2633}

$$\frac{a^2 \cosh(c + dx)}{d} + \frac{ab \sinh^3(c + dx) \cosh(c + dx)}{2d} - \frac{3ab \sinh(c + dx) \cosh(c + dx)}{4d} + \frac{3abx}{4} + \frac{b^2 \cosh^7(c + dx)}{7d} - \frac{3b^2 \cosh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*a*b*x)/4 + (a^2*Cosh[c + d*x])/d - (b^2*Cosh[c + d*x])/d + (b^2*Cosh[c +
d*x]^3)/d - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d) -
(3*a*b*Cosh[c + d*x]*Sinh[c + d*x])/(4*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x
]^3)/(2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^ (p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)^(n_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sinh(c + dx) (a + b \sinh^3(c + dx))^2 dx &= -\left(i \int (ia^2 \sinh(c + dx) + 2iab \sinh^4(c + dx) + ib^2 \sinh^7(c + dx)) dx\right) \\
&= a^2 \int \sinh(c + dx) dx + (2ab) \int \sinh^4(c + dx) dx + b^2 \int \sinh^7(c + dx) dx \\
&= \frac{a^2 \cosh(c + dx)}{d} + \frac{ab \cosh(c + dx) \sinh^3(c + dx)}{2d} - \frac{1}{2}(3ab) \int \sinh^2(c + dx) dx \\
&= \frac{a^2 \cosh(c + dx)}{d} - \frac{b^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh^3(c + dx)}{d} - \frac{3b^2 \cosh^5(c + dx)}{5d} \\
&= \frac{3abx}{4} + \frac{a^2 \cosh(c + dx)}{d} - \frac{b^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh^3(c + dx)}{d} - \frac{3b^2 \cosh^5(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.40963, size = 92, normalized size = 0.71

$$\frac{35(64a^2 - 35b^2) \cosh(c + dx) + b(140a(12(c + dx) - 8 \sinh(2(c + dx)) + \sinh(4(c + dx))) + 245b \cosh(3(c + dx)) - 49b \cosh(5(c + dx)))}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (35*(64*a^2 - 35*b^2)*Cosh[c + d*x] + b*(245*b*Cosh[3*(c + d*x)] - 49*b*Cos
h[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] + 140*a*(12*(c + d*x) - 8*Sinh[2*(c
+ d*x)] + Sinh[4*(c + d*x)])))/(2240*d)
```

Maple [A] time = 0.019, size = 96, normalized size = 0.7

$$\frac{1}{d} \left(b^2 \left(-\frac{16}{35} + \frac{(\sinh(dx+c))^6}{7} - \frac{6(\sinh(dx+c))^4}{35} + \frac{8(\sinh(dx+c))^2}{35} \right) \cosh(dx+c) + 2ab \left(\left(\frac{1}{4} (\sinh(dx+c))^3 - 3 \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x)

[Out] 1/d*(b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^2*cosh(d*x+c))

Maxima [A] time = 1.07251, size = 243, normalized size = 1.87

$$\frac{1}{32} ab \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{4480} b^2 \left(\frac{(49e^{(-2dx-2c)} - 245e^{(-4dx-4c)} + 1225e^{(-6dx-6c)} - 5)e^{(7dx+7c)}}{d} + (1225e^{(-dx-c)} - 245e^{(-3dx-3c)} + 49e^{(-5dx-5c)} - 5e^{(-7dx-7c)})/d \right) + a^2 \cosh(dx+c)/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")

[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + a^2*cosh(d*x + c)/d

Fricas [A] time = 1.90338, size = 594, normalized size = 4.57

$$5b^2 \cosh(dx+c)^7 + 35b^2 \cosh(dx+c) \sinh(dx+c)^6 - 49b^2 \cosh(dx+c)^5 + 560ab \cosh(dx+c) \sinh(dx+c)^3 + 2a^2 \cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] $\frac{1}{2240} \cdot (5b^2 \cosh(dx + c)^7 + 35b^2 \cosh(dx + c) \sinh(dx + c)^6 - 49b^2 \cosh(dx + c)^5 + 560ab \cosh(dx + c) \sinh(dx + c)^3 + 245b^2 \cosh(dx + c)^3 + 35(5b^2 \cosh(dx + c)^3 - 7b^2 \cosh(dx + c)) \sinh(dx + c)^4 + 1680abd + 35(3b^2 \cosh(dx + c)^5 - 14b^2 \cosh(dx + c)^3 + 21b^2 \cosh(dx + c)) \sinh(dx + c)^2 + 35(64a^2 - 35b^2) \cosh(dx + c) + 560(ab \cosh(dx + c)^3 - 4ab \cosh(dx + c)) \sinh(dx + c)) / d$

Sympy [A] time = 8.4291, size = 219, normalized size = 1.68

$$\left\{ \frac{a^2 \cosh(c+dx)}{d} + \frac{3abx \sinh^4(c+dx)}{4} - \frac{3abx \sinh^2(c+dx) \cosh^2(c+dx)}{2} + \frac{3abx \cosh^4(c+dx)}{4} + \frac{5ab \sinh^3(c+dx) \cosh(c+dx)}{4d} - \frac{3ab \sinh(c+dx) \cosh^3(c+dx)}{4d} \right\} x (a + b \sinh^3(c))^2 \sinh(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)

[Out] Piecewise((a**2*cosh(c + d*x)/d + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) + b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c), True))

Giac [A] time = 1.20251, size = 266, normalized size = 2.05

$$3360(dx + c)ab + 5b^2e^{(7dx+7c)} - 49b^2e^{(5dx+5c)} + 140abe^{(4dx+4c)} + 245b^2e^{(3dx+3c)} - 1120abe^{(2dx+2c)} + 2240a^2e^{(dx+c)} - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")

[Out] $\frac{1}{4480} \cdot (3360(dx + c)ab + 5b^2e^{(7dx+7c)} - 49b^2e^{(5dx+5c)} + 140a^2b^2e^{(4dx+4c)} + 245b^2e^{(3dx+3c)} - 1120a^2b^2e^{(2dx+2c)} + 2240a^2e^{(dx+c)} - 1225b^2e^{(dx+c)} + (1120a^2b^2e^{(5dx+5c)} + 245b^2e^{(4dx+4c)} - 140a^2b^2e^{(3dx+3c)} - 49b^2e^{(2dx+2c)} + 5b^2 + 35(64a^2 - 35b^2)e^{(6dx+6c)})e^{(-7dx-7c)}) / d$

3.153 $\int (a + b \sinh^3(c + dx))^2 dx$

Optimal. Leaf size=114

$$a^2x + \frac{2ab \cosh^3(c + dx)}{3d} - \frac{2ab \cosh(c + dx)}{d} + \frac{b^2 \sinh^5(c + dx) \cosh(c + dx)}{6d} - \frac{5b^2 \sinh^3(c + dx) \cosh(c + dx)}{24d} + \frac{5b^2 \sinh(c + dx)}{24d}$$

[Out] $a^2x - (5b^2x)/16 - (2ab \cosh(c + dx))/d + (2ab \cosh(c + dx)^3)/(3d) + (5b^2 \cosh(c + dx) \sinh(c + dx))/(16d) - (5b^2 \cosh(c + dx) \sinh(c + dx)^3)/(24d) + (b^2 \cosh(c + dx) \sinh(c + dx)^5)/(6d)$

Rubi [A] time = 0.0832559, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3213, 2633, 2635, 8}

$$a^2x + \frac{2ab \cosh^3(c + dx)}{3d} - \frac{2ab \cosh(c + dx)}{d} + \frac{b^2 \sinh^5(c + dx) \cosh(c + dx)}{6d} - \frac{5b^2 \sinh^3(c + dx) \cosh(c + dx)}{24d} + \frac{5b^2 \sinh(c + dx)}{24d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^3)^2,x]

[Out] $a^2x - (5b^2x)/16 - (2ab \cosh(c + dx))/d + (2ab \cosh(c + dx)^3)/(3d) + (5b^2 \cosh(c + dx) \sinh(c + dx))/(16d) - (5b^2 \cosh(c + dx) \sinh(c + dx)^3)/(24d) + (b^2 \cosh(c + dx) \sinh(c + dx)^5)/(6d)$

Rule 3213

Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sinh[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sinh[c + d*x])^(n - 1), x], x]

$+ d*x])^{(n - 2), x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned}
 \int (a + b \sinh^3(c + dx))^2 dx &= \int (a^2 + 2ab \sinh^3(c + dx) + b^2 \sinh^6(c + dx)) dx \\
 &= a^2x + (2ab) \int \sinh^3(c + dx) dx + b^2 \int \sinh^6(c + dx) dx \\
 &= a^2x + \frac{b^2 \cosh(c + dx) \sinh^5(c + dx)}{6d} - \frac{1}{6} (5b^2) \int \sinh^4(c + dx) dx - \frac{(2ab) \text{Subst} \left(\int (1 - \right.}{ \\
 &= a^2x - \frac{2ab \cosh(c + dx)}{d} + \frac{2ab \cosh^3(c + dx)}{3d} - \frac{5b^2 \cosh(c + dx) \sinh^3(c + dx)}{24d} + \frac{b^2 \cosh}{ \\
 &= a^2x - \frac{2ab \cosh(c + dx)}{d} + \frac{2ab \cosh^3(c + dx)}{3d} + \frac{5b^2 \cosh(c + dx) \sinh(c + dx)}{16d} - \frac{5b^2 \cosh}{ \\
 &= a^2x - \frac{5b^2x}{16} - \frac{2ab \cosh(c + dx)}{d} + \frac{2ab \cosh^3(c + dx)}{3d} + \frac{5b^2 \cosh(c + dx) \sinh(c + dx)}{16d} - \frac{5}{
 \end{aligned}$$

Mathematica [A] time = 0.146479, size = 94, normalized size = 0.82

$$\frac{192a^2c + 192a^2dx - 288ab \cosh(c + dx) + 32ab \cosh(3(c + dx)) + 45b^2 \sinh(2(c + dx)) - 9b^2 \sinh(4(c + dx)) + b^2 \sinh(6(c + dx))}{192d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^3)^2,x]

[Out] (192*a^2*c - 60*b^2*c + 192*a^2*d*x - 60*b^2*d*x - 288*a*b*Cosh[c + d*x] + 32*a*b*Cosh[3*(c + d*x)] + 45*b^2*Sinh[2*(c + d*x)] - 9*b^2*Sinh[4*(c + d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)

Maple [A] time = 0.017, size = 85, normalized size = 0.8

$$\frac{1}{d} \left(b^2 \left(\left(\frac{(\sinh(dx + c))^5}{6} - \frac{5(\sinh(dx + c))^3}{24} + \frac{5 \sinh(dx + c)}{16} \right) \cosh(dx + c) - \frac{5dx}{16} - \frac{5c}{16} \right) + 2ab \left(-\frac{2}{3} + \frac{1}{3} \sinh(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(d*x+c))^3)^2,x`

[Out] $\frac{1}{d} * (b^2 * ((\frac{1}{6} * \sinh(d*x+c)^5 - \frac{5}{24} * \sinh(d*x+c)^3 + \frac{5}{16} * \sinh(d*x+c)) * \cosh(d*x+c) - \frac{5}{16} * d*x - \frac{5}{16} * c) + 2 * a * b * (-\frac{2}{3} + \frac{1}{3} * \sinh(d*x+c)^2) * \cosh(d*x+c) + a^2 * (d*x+c))$

Maxima [A] time = 1.14157, size = 204, normalized size = 1.79

$$a^2x - \frac{1}{384} b^2 \left(\frac{(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e^{(-6dx-6c)}}{d} \right) + \frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c))^3)^2,x, algorithm="maxima")`

[Out] $a^2x - \frac{1}{384} b^2 * ((9e^{(-2*d*x - 2*c)} - 45e^{(-4*d*x - 4*c)} - 1) * e^{(6*d*x + 6*c)} / d + 120 * (d*x + c) / d + (45e^{(-2*d*x - 2*c)} - 9e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)}) / d) + \frac{1}{12} * a * b * (e^{(3*d*x + 3*c)} / d - 9e^{(d*x + c)} / d - 9e^{(-d*x - c)} / d + e^{(-3*d*x - 3*c)} / d)$

Fricas [A] time = 1.98662, size = 421, normalized size = 3.69

$$3b^2 \cosh(dx+c) \sinh(dx+c)^5 + 16ab \cosh(dx+c)^3 + 48ab \cosh(dx+c) \sinh(dx+c)^2 + 2(5b^2 \cosh(dx+c)^3 - 9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c))^3)^2,x, algorithm="fricas")`

[Out] $\frac{1}{96} * (3 * b^2 * \cosh(d*x + c) * \sinh(d*x + c)^5 + 16 * a * b * \cosh(d*x + c)^3 + 48 * a * b * \cosh(d*x + c) * \sinh(d*x + c)^2 + 2 * (5 * b^2 * \cosh(d*x + c)^3 - 9 * b^2 * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 6 * (16 * a^2 - 5 * b^2) * d * x - 144 * a * b * \cosh(d*x + c) + 3 * (b^2 * \cosh(d*x + c)^5 - 6 * b^2 * \cosh(d*x + c)^3 + 15 * b^2 * \cosh(d*x + c)) * \sinh(d*x + c)) / d$

Sympy [A] time = 5.48219, size = 212, normalized size = 1.86

$$\frac{\left\{ a^2 x + \frac{2ab \sinh^2(c+dx) \cosh(c+dx)}{d^2} - \frac{4ab \cosh^3(c+dx)}{3d} + \frac{5b^2 x \sinh^6(c+dx)}{16} - \frac{15b^2 x \sinh^4(c+dx) \cosh^2(c+dx)}{16} + \frac{15b^2 x \sinh^2(c+dx) \cosh^4(c+dx)}{16} \right\}}{x(a + b \sinh^3(c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)**3)**2,x)

[Out] Piecewise((a**2*x + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 4*a*b*cosh(c + d*x)**3/(3*d) + 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b**2*x*cosh(c + d*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2, True))

Giac [A] time = 1.12821, size = 212, normalized size = 1.86

$$\frac{b^2 e^{(6dx+6c)} - 9b^2 e^{(4dx+4c)} + 32abe^{(3dx+3c)} + 45b^2 e^{(2dx+2c)} - 288abe^{(dx+c)} + 24(16a^2 - 5b^2)(dx+c) - (288abe^{(5dx+5c)} + \dots)}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")

[Out] 1/384*(b^2*e^(6*d*x + 6*c) - 9*b^2*e^(4*d*x + 4*c) + 32*a*b*e^(3*d*x + 3*c) + 45*b^2*e^(2*d*x + 2*c) - 288*a*b*e^(d*x + c) + 24*(16*a^2 - 5*b^2)*(d*x + c) - (288*a*b*e^(5*d*x + 5*c) + 45*b^2*e^(4*d*x + 4*c) - 32*a*b*e^(3*d*x + 3*c) - 9*b^2*e^(2*d*x + 2*c) + b^2)*e^(-6*d*x - 6*c))/d

3.154 $\int \operatorname{csch}(c + dx) \left(a + b \sinh^3(c + dx) \right)^2 dx$

Optimal. Leaf size=88

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{ab \sinh(c + dx) \cosh(c + dx)}{d} - abx + \frac{b^2 \cosh^5(c + dx)}{5d} - \frac{2b^2 \cosh^3(c + dx)}{3d} + \frac{b^2 \cosh(c + dx)}{d}$$

[Out] $-(a*b*x) - (a^2*ArcTanh[Cosh[c + d*x]])/d + (b^2*Cosh[c + d*x])/d - (2*b^2*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x])/d$

Rubi [A] time = 0.101747, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3220, 3770, 2635, 8, 2633}

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{ab \sinh(c + dx) \cosh(c + dx)}{d} - abx + \frac{b^2 \cosh^5(c + dx)}{5d} - \frac{2b^2 \cosh^3(c + dx)}{3d} + \frac{b^2 \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csch}[c + d*x]*(a + b*\text{Sinh}[c + d*x]^3)^2, x]$

[Out] $-(a*b*x) - (a^2*ArcTanh[Cosh[c + d*x]])/d + (b^2*Cosh[c + d*x])/d - (2*b^2*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x])/d$

Rule 3220

$\text{Int}[\sin[(e_.) + (f_.)*(x_)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)})^{(p_.)}, x_Symbol] := \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^m*(a + b*\sin[e + f*x]^n)^p, x], x] /; \text{FreeQ}\{a, b, e, f\}, x] \ \&\& \ \text{IntegersQ}[m, p] \ \&\& \ (\text{EqQ}[n, 4] \ || \ \text{GtQ}[p, 0] \ || \ (\text{EqQ}[p, -1] \ \&\& \ \text{IntegerQ}[n]))$

Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_)], x_Symbol] := -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2635

$\text{Int}[(b_.*\sin[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] := -\text{Simp}[(b*\text{Cos}[c + d*x])*(b*\text{Sin}[c + d*x])^{(n-1)}]/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c$

+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c + dx) (a + b \sinh^3(c + dx))^2 dx &= i \int (-ia^2 \operatorname{csch}(c + dx) - 2iab \sinh^2(c + dx) - ib^2 \sinh^5(c + dx)) dx \\ &= a^2 \int \operatorname{csch}(c + dx) dx + (2ab) \int \sinh^2(c + dx) dx + b^2 \int \sinh^5(c + dx) dx \\ &= -\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{ab \cosh(c + dx) \sinh(c + dx)}{d} - (ab) \int 1 dx + \frac{b^2}{d} \int \sinh^4(c + dx) dx \\ &= -abx - \frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2 \cosh(c + dx)}{d} - \frac{2b^2 \cosh^3(c + dx)}{3d} + \frac{b^2 \sinh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.301818, size = 96, normalized size = 1.09

$$\frac{120a \left(b \sinh(2(c + dx)) - 2 \left(-a \log \left(\sinh \left(\frac{1}{2}(c + dx) \right) \right) + a \log \left(\cosh \left(\frac{1}{2}(c + dx) \right) \right) + bc + bdx \right) \right) + 150b^2 \cosh(c + dx) - 2b^2 \sinh^5(c + dx)}{240d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]

[Out] (150*b^2*Cosh[c + d*x] - 25*b^2*Cosh[3*(c + d*x)] + 3*b^2*Cosh[5*(c + d*x)] + 120*a*(-2*(b*c + b*d*x + a*Log[Cosh[(c + d*x)/2]] - a*Log[Sinh[(c + d*x)/2]]) + b*Sinh[2*(c + d*x)])/(240*d)

Maple [A] time = 0.056, size = 76, normalized size = 0.9

$$\frac{1}{d} \left(-2a^2 \operatorname{Artanh}(e^{dx+c}) + 2ab \left(\frac{1}{2} \cosh(dx+c) \sinh(dx+c) - \frac{1}{2} dx - \frac{c}{2} \right) + b^2 \left(\frac{8}{15} + \frac{(\sinh(dx+c))^4}{5} - \frac{4(\sinh(dx+c))^3}{15} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x)`

[Out] $\frac{1}{d}(-2a^2\operatorname{arctanh}(\exp(dx+c))+2ab*(\frac{1}{2}\cosh(dx+c)*\sinh(dx+c)-\frac{1}{2}dx-\frac{1}{2}c)+b^2*(\frac{8}{15}+\frac{1}{5}\sinh(dx+c)^4-\frac{4}{15}\sinh(dx+c)^2)*\cosh(dx+c))$

Maxima [A] time = 1.17573, size = 189, normalized size = 2.15

$$-\frac{1}{4}ab\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + \frac{1}{480}b^2\left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d}\right) + a^2\log(\tanh(\frac{1}{2}dx + \frac{1}{2}c))/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")`

[Out] $-1/4*a*b*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) + 1/480*b^2*(3*e^{(5*d*x + 5*c)}/d - 25*e^{(3*d*x + 3*c)}/d + 150*e^{(d*x + c)}/d + 150*e^{(-d*x - c)}/d - 25*e^{(-3*d*x - 3*c)}/d + 3*e^{(-5*d*x - 5*c)}/d) + a^2*\log(\tanh(1/2*d*x + 1/2*c))/d$

Fricas [B] time = 2.16028, size = 2851, normalized size = 32.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")`

[Out] $\frac{1}{480}(3b^2\cosh(dx+c)^{10} + 30b^2\cosh(dx+c)\sinh(dx+c)^9 + 3b^2\sinh(dx+c)^{10} - 25b^2\cosh(dx+c)^8 - 480abdx\cosh(dx+c)^5 + 120ab\cosh(dx+c)^7 + 5(27b^2\cosh(dx+c)^2 - 5b^2)\sinh(dx+c)^8 + 150b^2\cosh(dx+c)^6 + 40(9b^2\cosh(dx+c)^3 - 5b^2\cosh(dx+c) + 3ab)\sinh(dx+c)^7 + 10(63b^2\cosh(dx+c)^4 - 70b^2\cosh(dx+c)^2 + 84ab\cosh(dx+c) + 15b^2)\sinh(dx+c)^6 + 150b^2\cosh(dx+c)^4 + 4(189b^2\cosh(dx+c)^5 - 350b^2\cosh(dx+c)^3 - 120abdx + 630ab\cosh(dx+c)^2 + 225b^2\cosh(dx+c))\sinh(dx+c)^5 - 120ab\cosh(dx+c)^3 + 10(63b^2\cosh(dx+c)^6 - 175b^2\cosh(dx+c)^4 - 240abdx\cosh(dx+c) + 420ab\cosh(dx+c)^3 + 225b^2\cosh(dx+c)$

```

c)^2 + 15*b^2)*sinh(d*x + c)^4 - 25*b^2*cosh(d*x + c)^2 + 40*(9*b^2*cosh(d*
x + c)^7 - 35*b^2*cosh(d*x + c)^5 - 120*a*b*d*x*cosh(d*x + c)^2 + 105*a*b*c
osh(d*x + c)^4 + 75*b^2*cosh(d*x + c)^3 + 15*b^2*cosh(d*x + c) - 3*a*b)*sin
h(d*x + c)^3 + 5*(27*b^2*cosh(d*x + c)^8 - 140*b^2*cosh(d*x + c)^6 - 960*a*
b*d*x*cosh(d*x + c)^3 + 504*a*b*cosh(d*x + c)^5 + 450*b^2*cosh(d*x + c)^4 +
180*b^2*cosh(d*x + c)^2 - 72*a*b*cosh(d*x + c) - 5*b^2)*sinh(d*x + c)^2 +
3*b^2 - 480*(a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^4*sinh(d*x + c) + 10
*a^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^2*cosh(d*x + c)^2*sinh(d*x + c)
^3 + 5*a^2*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*sinh(d*x + c)^5)*log(cosh(d*
x + c) + sinh(d*x + c) + 1) + 480*(a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c
)^4*sinh(d*x + c) + 10*a^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^2*cosh(d*
x + c)^2*sinh(d*x + c)^3 + 5*a^2*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*sinh(d
*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(3*b^2*cosh(d*x + c)
^9 - 20*b^2*cosh(d*x + c)^7 - 240*a*b*d*x*cosh(d*x + c)^4 + 84*a*b*cosh(d*x
+ c)^6 + 90*b^2*cosh(d*x + c)^5 + 60*b^2*cosh(d*x + c)^3 - 36*a*b*cosh(d*x
+ c)^2 - 5*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh
(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*d*cos
h(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d
*x + c)^5)

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)

[Out] Timed out

Giac [B] time = 1.25366, size = 240, normalized size = 2.73

$$-\frac{(dx+c)ab}{d} - \frac{a^2 \log(e^{(dx+c)} + 1)}{d} + \frac{a^2 \log(|e^{(dx+c)} - 1|)}{d} + \frac{(150b^2e^{(4dx+4c)} - 120abe^{(3dx+3c)} - 25b^2e^{(2dx+2c)} + 3b^2)e^{(-5a}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")

```
[Out] -(d*x + c)*a*b/d - a^2*log(e^(d*x + c) + 1)/d + a^2*log(abs(e^(d*x + c) - 1))/d + 1/480*(150*b^2*e^(4*d*x + 4*c) - 120*a*b*e^(3*d*x + 3*c) - 25*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-5*d*x - 5*c)/d + 1/480*(3*b^2*d^4*e^(5*d*x + 5*c) - 25*b^2*d^4*e^(3*d*x + 3*c) + 120*a*b*d^4*e^(2*d*x + 2*c) + 150*b^2*d^4*e^(d*x + c))/d^5
```

3.155 $\int \operatorname{csch}^2(c + dx) \left(a + b \sinh^3(c + dx) \right)^2 dx$

Optimal. Leaf size=82

$$-\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{2ab \cosh(c + dx)}{d} + \frac{b^2 \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{3b^2 \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3b^2 x}{8}$$

[Out] (3*b^2*x)/8 + (2*a*b*Cosh[c + d*x])/d - (a^2*Coth[c + d*x])/d - (3*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)

Rubi [A] time = 0.0979725, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3220, 3767, 8, 2638, 2635}

$$-\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{2ab \cosh(c + dx)}{d} + \frac{b^2 \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{3b^2 \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3b^2 x}{8}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]

[Out] (3*b^2*x)/8 + (2*a*b*Cosh[c + d*x])/d - (a^2*Coth[c + d*x])/d - (3*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)

Rule 3220

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^p, x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^2(c + dx) (a + b \sinh^3(c + dx))^2 dx &= - \int (-a^2 \operatorname{csch}^2(c + dx) - 2ab \sinh(c + dx) - b^2 \sinh^4(c + dx)) dx \\
 &= a^2 \int \operatorname{csch}^2(c + dx) dx + (2ab) \int \sinh(c + dx) dx + b^2 \int \sinh^4(c + dx) dx \\
 &= \frac{2ab \cosh(c + dx)}{d} + \frac{b^2 \cosh(c + dx) \sinh^3(c + dx)}{4d} - \frac{1}{4} (3b^2) \int \sinh^2(c + dx) dx \\
 &= \frac{2ab \cosh(c + dx)}{d} - \frac{a^2 \operatorname{coth}(c + dx)}{d} - \frac{3b^2 \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b^2 \sinh^4(c + dx)}{32d} \\
 &= \frac{3b^2 x}{8} + \frac{2ab \cosh(c + dx)}{d} - \frac{a^2 \operatorname{coth}(c + dx)}{d} - \frac{3b^2 \cosh(c + dx) \sinh(c + dx)}{8d}
 \end{aligned}$$

Mathematica [A] time = 0.284753, size = 92, normalized size = 1.12

$$-\frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{2ab \sinh(c) \sinh(dx)}{d} + \frac{2ab \cosh(c) \cosh(dx)}{d} + \frac{3b^2(c + dx)}{8d} - \frac{b^2 \sinh(2(c + dx))}{4d} + \frac{b^2 \sinh(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]

[Out] (3*b^2*(c + d*x))/(8*d) + (2*a*b*Cosh[c]*Cosh[d*x])/d - (a^2*Coth[c + d*x])/d + (2*a*b*Sinh[c]*Sinh[d*x])/d - (b^2*Sinh[2*(c + d*x)]/(4*d) + (b^2*Sinh[4*(c + d*x)]/(32*d)

Maple [A] time = 0.046, size = 65, normalized size = 0.8

$$\frac{1}{d} \left(-a^2 \operatorname{coth}(dx + c) + 2ab \cosh(dx + c) + b^2 \left(\left(\frac{\sinh(dx + c)}{4} - \frac{3 \sinh(dx + c)}{8} \right) \cosh(dx + c) + \frac{3dx}{8} + \frac{3c}{8} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x)`

[Out] $1/d*(-a^2*\coth(d*x+c)+2*a*b*\cosh(d*x+c)+b^2*((1/4*\sinh(d*x+c)^3-3/8*\sinh(d*x+c))*\cosh(d*x+c)+3/8*d*x+3/8*c))$

Maxima [A] time = 1.07606, size = 153, normalized size = 1.87

$$\frac{1}{64} b^2 \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) + ab \left(\frac{e^{dx+c}}{d} + \frac{e^{-dx-c}}{d} \right) + \frac{2a^2}{d(e^{-2dx-2c}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")`

[Out] $1/64*b^2*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) + a*b*(e^{(d*x + c)}/d + e^{(-d*x - c)}/d) + 2*a^2/(d*(e^{(-2*d*x - 2*c)} - 1))$

Fricas [A] time = 1.91237, size = 360, normalized size = 4.39

$$\frac{b^2 \cosh(dx+c)^5 + 5b^2 \cosh(dx+c) \sinh(dx+c)^4 - 9b^2 \cosh(dx+c)^3 + (10b^2 \cosh(dx+c)^3 - 27b^2 \cosh(dx+c)) \sinh(dx+c)^2 - 8(8a^2 - b^2) \cosh(dx+c) + 8(3b^2 dx + 16ab \cosh(dx+c) + 8a^2) \sinh(dx+c)}{64d \sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")`

[Out] $1/64*(b^2*\cosh(d*x+c)^5 + 5*b^2*\cosh(d*x+c)*\sinh(d*x+c)^4 - 9*b^2*\cosh(d*x+c)^3 + (10*b^2*\cosh(d*x+c)^3 - 27*b^2*\cosh(d*x+c))*\sinh(d*x+c)^2 - 8*(8*a^2 - b^2)*\cosh(d*x+c) + 8*(3*b^2*d*x + 16*a*b*\cosh(d*x+c) + 8*a^2)*\sinh(d*x+c))/(d*\sinh(d*x+c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)

[Out] Timed out

Giac [B] time = 1.22934, size = 224, normalized size = 2.73

$$\frac{3(dx+c)b^2}{8d} + \frac{(64abe^{5dx+5c} - 64abe^{3dx+3c} - 9b^2e^{2dx+2c} + b^2 - 8(16a^2 - b^2)e^{4dx+4c})e^{(-4dx-4c)}}{64d(e^{(dx+c)} + 1)(e^{(dx+c)} - 1)} + \frac{b^2d^3e^{(4dx+4c)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")

[Out] 3/8*(d*x + c)*b^2/d + 1/64*(64*a*b*e^(5*d*x + 5*c) - 64*a*b*e^(3*d*x + 3*c) - 9*b^2*e^(2*d*x + 2*c) + b^2 - 8*(16*a^2 - b^2)*e^(4*d*x + 4*c))*e^(-4*d*x - 4*c)/(d*(e^(d*x + c) + 1)*(e^(d*x + c) - 1)) + 1/64*(b^2*d^3*e^(4*d*x + 4*c) - 8*b^2*d^3*e^(2*d*x + 2*c) + 64*a*b*d^3*e^(d*x + c))/d^4

3.156 $\int \operatorname{csch}^3(c + dx) (a + b \sinh^3(c + dx))^2 dx$

Optimal. Leaf size=77

$$\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + 2abx + \frac{b^2 \cosh^3(c + dx)}{3d} - \frac{b^2 \cosh(c + dx)}{d}$$

[Out] $2*a*b*x + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)$

Rubi [A] time = 0.105334, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3220, 3768, 3770, 2633}

$$\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + 2abx + \frac{b^2 \cosh^3(c + dx)}{3d} - \frac{b^2 \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\operatorname{Csch}[c + d*x]^3*(a + b*\operatorname{Sinh}[c + d*x]^3)^2, x]$

[Out] $2*a*b*x + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)$

Rule 3220

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^{m*(a + b*\sin[e + f*x]^n)}^p, x], x] /; \text{FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{IntegersQ}[m, p] \ \&\& \ (\text{EqQ}[n, 4] \ || \ \text{GtQ}[p, 0] \ || \ (\text{EqQ}[p, -1] \ \&\& \ \text{IntegerQ}[n]))$

Rule 3768

$\text{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\cos[c + d*x]*(b*\csc[c + d*x])^{(n-1)})/(d*(n-1)), x] + \text{Dist}[(b^2*(n-2))/(n-1), \text{Int}[(b*\csc[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\cos[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2633

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c + dx) (a + b \sinh^3(c + dx))^2 dx &= - \left(i \int (2iab + ia^2 \operatorname{csch}^3(c + dx) + ib^2 \sinh^3(c + dx)) dx \right) \\ &= 2abx + a^2 \int \operatorname{csch}^3(c + dx) dx + b^2 \int \sinh^3(c + dx) dx \\ &= 2abx - \frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} - \frac{1}{2} a^2 \int \operatorname{csch}(c + dx) dx - \frac{b^2 \operatorname{Subst}}{2d} \\ &= 2abx + \frac{a^2 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{b^2 \cosh(c + dx)}{d} + \frac{b^2 \cosh^3(c + dx)}{3d} - \end{aligned}$$

Mathematica [A] time = 0.0305111, size = 105, normalized size = 1.36

$$-\frac{a^2 \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a^2 \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a^2 \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{2d} + 2abx - \frac{3b^2 \cosh(c + dx)}{4d} + \frac{b^2 \cosh(3(c + dx))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]

[Out] $2*a*b*x - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)])/(12*d) - (a^2*Csch[(c + d*x)/2]^2)/(8*d) - (a^2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d)$

Maple [A] time = 0.059, size = 63, normalized size = 0.8

$$\frac{1}{d} \left(a^2 \left(-\frac{\operatorname{csch}(dx + c) \coth(dx + c)}{2} + \operatorname{Artanh}(e^{dx+c}) \right) + 2ab(dx + c) + b^2 \left(-\frac{2}{3} + \frac{(\sinh(dx + c))^2}{3} \right) \cosh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x)

[Out] $1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(\exp(d*x+c)))+2*a*b*(d*x+c)+b^2*(-2/3+1/3*\sinh(d*x+c)^2)*\cosh(d*x+c)$

Maxima [B] time = 1.22024, size = 205, normalized size = 2.66

$$2abx + \frac{1}{24}b^2\left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d}\right) + \frac{1}{2}a^2\left(\frac{\log(e^{(-dx-c)}+1)}{d} - \frac{\log(e^{(-dx-c)}-1)}{d}\right) + \frac{2(e^{(-dx-c)}}{d(2e^{(-2dx-2c)}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")`

[Out] $2*a*b*x + 1/24*b^2*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d) + 1/2*a^2*(\log(e^{(-d*x - c)} + 1)/d - \log(e^{(-d*x - c)} - 1)/d + 2*(e^{(-d*x - c)} + e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1)))$

Fricas [B] time = 2.20708, size = 4246, normalized size = 55.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")`

[Out] $1/24*(b^2*\cosh(d*x + c)^{10} + 10*b^2*\cosh(d*x + c)*\sinh(d*x + c)^9 + b^2*\sinh(d*x + c)^{10} + 48*a*b*d*x*\cosh(d*x + c)^7 - 11*b^2*\cosh(d*x + c)^8 - 96*a*b*d*x*\cosh(d*x + c)^5 + (45*b^2*\cosh(d*x + c)^2 - 11*b^2)*\sinh(d*x + c)^8 + 8*(15*b^2*\cosh(d*x + c)^3 + 6*a*b*d*x - 11*b^2*\cosh(d*x + c))*\sinh(d*x + c)^7 + 48*a*b*d*x*\cosh(d*x + c)^3 - 2*(12*a^2 - 5*b^2)*\cosh(d*x + c)^6 + 2*(105*b^2*\cosh(d*x + c)^4 + 168*a*b*d*x*\cosh(d*x + c) - 154*b^2*\cosh(d*x + c)^2 - 12*a^2 + 5*b^2)*\sinh(d*x + c)^6 + 4*(63*b^2*\cosh(d*x + c)^5 + 252*a*b*d*x*\cosh(d*x + c)^2 - 154*b^2*\cosh(d*x + c)^3 - 24*a*b*d*x - 3*(12*a^2 - 5*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(12*a^2 - 5*b^2)*\cosh(d*x + c)^4 + 2*(105*b^2*\cosh(d*x + c)^6 + 840*a*b*d*x*\cosh(d*x + c)^3 - 385*b^2*\cosh(d*x + c)^4 - 240*a*b*d*x*\cosh(d*x + c) - 15*(12*a^2 - 5*b^2)*\cosh(d*x + c)^2 - 12*a^2 + 5*b^2)*\sinh(d*x + c)^4 - 11*b^2*\cosh(d*x + c)^2 + 8*(15*b^2*\cosh(d*x + c)^7 + 210*a*b*d*x*\cosh(d*x + c)^4 - 77*b^2*\cosh(d*x + c)^5 - 120*a*b*d*x*\cosh(d*x + c)^2 + 6*a*b*d*x - 5*(12*a^2 - 5*b^2)*\cosh(d*x + c)^3 - (12$

```

*a^2 - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (45*b^2*cosh(d*x + c)^8 + 10
08*a*b*d*x*cosh(d*x + c)^5 - 308*b^2*cosh(d*x + c)^6 - 960*a*b*d*x*cosh(d*x
+ c)^3 + 144*a*b*d*x*cosh(d*x + c) - 30*(12*a^2 - 5*b^2)*cosh(d*x + c)^4 -
12*(12*a^2 - 5*b^2)*cosh(d*x + c)^2 - 11*b^2)*sinh(d*x + c)^2 + b^2 + 12*(
a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6 + a^2*sinh(d*x +
c)^7 - 2*a^2*cosh(d*x + c)^5 + (21*a^2*cosh(d*x + c)^2 - 2*a^2)*sinh(d*x +
c)^5 + a^2*cosh(d*x + c)^3 + 5*(7*a^2*cosh(d*x + c)^3 - 2*a^2*cosh(d*x + c)
)*sinh(d*x + c)^4 + (35*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(d*x + c)^2 + a^2)
*sinh(d*x + c)^3 + (21*a^2*cosh(d*x + c)^5 - 20*a^2*cosh(d*x + c)^3 + 3*a^2
*cosh(d*x + c))*sinh(d*x + c)^2 + (7*a^2*cosh(d*x + c)^6 - 10*a^2*cosh(d*x
+ c)^4 + 3*a^2*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
+ c) + 1) - 12*(a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6
+ a^2*sinh(d*x + c)^7 - 2*a^2*cosh(d*x + c)^5 + (21*a^2*cosh(d*x + c)^2 - 2
*a^2)*sinh(d*x + c)^5 + a^2*cosh(d*x + c)^3 + 5*(7*a^2*cosh(d*x + c)^3 - 2*
a^2*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(
d*x + c)^2 + a^2)*sinh(d*x + c)^3 + (21*a^2*cosh(d*x + c)^5 - 20*a^2*cosh(d
*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (7*a^2*cosh(d*x + c)^6 -
10*a^2*cosh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*
x + c) + sinh(d*x + c) - 1) + 2*(5*b^2*cosh(d*x + c)^9 + 168*a*b*d*x*cosh(d
*x + c)^6 - 44*b^2*cosh(d*x + c)^7 - 240*a*b*d*x*cosh(d*x + c)^4 + 72*a*b*d
*x*cosh(d*x + c)^2 - 6*(12*a^2 - 5*b^2)*cosh(d*x + c)^5 - 4*(12*a^2 - 5*b^2
)*cosh(d*x + c)^3 - 11*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7
+ 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 - 2*d*cosh(d*x + c
)^5 + (21*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3
- 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5
- 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d*cosh(d*x
+ c)^6 - 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)

[Out] Timed out

Giac [B] time = 1.2288, size = 244, normalized size = 3.17

$$\frac{2(dx+c)ab}{d} + \frac{a^2 \log(e^{(dx+c)} + 1)}{2d} - \frac{a^2 \log(|e^{(dx+c)} - 1|)}{2d} + \frac{b^2 d^2 e^{(3dx+3c)} - 9b^2 d^2 e^{(dx+c)}}{24d^3} - \frac{(11b^2 e^{(2dx+2c)} - b^2 + 3(8a^2 + 3b^2))e^{(dx+c)}}{24d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")

[Out] 2*(d*x + c)*a*b/d + 1/2*a^2*log(e^(d*x + c) + 1)/d - 1/2*a^2*log(abs(e^(d*x + c) - 1))/d + 1/24*(b^2*d^2*e^(3*d*x + 3*c) - 9*b^2*d^2*e^(d*x + c))/d^3 - 1/24*(11*b^2*e^(2*d*x + 2*c) - b^2 + 3*(8*a^2 + 3*b^2)*e^(6*d*x + 6*c) + (24*a^2 - 19*b^2)*e^(4*d*x + 4*c))*e^(-3*d*x - 3*c)/(d*(e^(d*x + c) + 1)^2*(e^(d*x + c) - 1)^2)

3.157 $\int \operatorname{csch}^4(c + dx) \left(a + b \sinh^3(c + dx)\right)^2 dx$

Optimal. Leaf size=76

$$-\frac{a^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{a^2 \operatorname{coth}(c + dx)}{d} - \frac{2ab \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{b^2 x}{2}$$

[Out] $-(b^2*x)/2 - (2*a*b*ArcTanh[Cosh[c + d*x]])/d + (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)$

Rubi [A] time = 0.0910175, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3220, 3770, 3767, 2635, 8}

$$-\frac{a^2 \operatorname{coth}^3(c + dx)}{3d} + \frac{a^2 \operatorname{coth}(c + dx)}{d} - \frac{2ab \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{b^2 x}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csch}[c + d*x]^4*(a + b*\text{Sinh}[c + d*x]^3)^2, x]$

[Out] $-(b^2*x)/2 - (2*a*b*ArcTanh[Cosh[c + d*x]])/d + (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)$

Rule 3220

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^m*(a + b*\sin[e + f*x]^n)^p, x], x] /;$ FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /;$ FreeQ[{c, d}, x]

Rule 3767

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(c + dx) (a + b \sinh^3(c + dx))^2 dx &= \int (2ab \operatorname{csch}(c + dx) + a^2 \operatorname{csch}^4(c + dx) + b^2 \sinh^2(c + dx)) dx \\
&= a^2 \int \operatorname{csch}^4(c + dx) dx + (2ab) \int \operatorname{csch}(c + dx) dx + b^2 \int \sinh^2(c + dx) dx \\
&= -\frac{2ab \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2 \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{1}{2} b^2 \int 1 dx + \\
&= -\frac{b^2 x}{2} - \frac{2ab \tanh^{-1}(\cosh(c + dx))}{d} + \frac{a^2 \operatorname{coth}(c + dx)}{d} - \frac{a^2 \operatorname{coth}^3(c + dx)}{3d} +
\end{aligned}$$

Mathematica [A] time = 0.398911, size = 81, normalized size = 1.07

$$\frac{3b \left(b \sinh(2(c + dx)) - 2 \left(-4a \log \left(\sinh \left(\frac{1}{2}(c + dx) \right) \right) + 4a \log \left(\cosh \left(\frac{1}{2}(c + dx) \right) \right) + bc + bdx \right) \right) - 4a^2 \operatorname{coth}(c + dx) \left(\operatorname{csch} \right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-4*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + 3*b*(-2*(b*c + b*d*x + 4*a*Log[Cosh[(c + d*x)/2]] - 4*a*Log[Sinh[(c + d*x)/2]]) + b*Sinh[2*(c + d*x)])/(12*d)
```

Maple [A] time = 0.069, size = 65, normalized size = 0.9

$$\frac{1}{d} \left(a^2 \left(\frac{2}{3} - \frac{(\operatorname{csch}(dx + c))^2}{3} \right) \operatorname{coth}(dx + c) - 4ab \operatorname{Artanh}(e^{dx+c}) + b^2 \left(\frac{\cosh(dx + c) \sinh(dx + c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x)`

[Out] $1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-4*a*b*arctanh(\exp(d*x+c))+b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))$

Maxima [B] time = 1.25967, size = 230, normalized size = 3.03

$$-\frac{1}{8}b^2\left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d}\right) - 2ab\left(\frac{\log(e^{-dx-c}+1)}{d} - \frac{\log(e^{-dx-c}-1)}{d}\right) + \frac{4}{3}a^2\left(\frac{3e^{-2dx-2c}}{d(3e^{-2dx-2c}-3e^{-4dx-4c})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")`

[Out] $-1/8*b^2*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 2*a*b*(\log(e^{(-d*x - c) + 1})/d - \log(e^{(-d*x - c) - 1})/d) + 4/3*a^2*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)))$

Fricas [B] time = 2.08082, size = 4551, normalized size = 59.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")`

[Out] $1/24*(3*b^2*cosh(d*x + c)^{10} + 30*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^2*sinh(d*x + c)^{10} - 3*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^8 - 3*(4*b^2*d*x - 45*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^8 + 24*(15*b^2*cosh(d*x + c)^3 - (4*b^2*d*x + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 6*(6*b^2*d*x + b^2)*cosh(d*x + c)^6 + 6*(105*b^2*cosh(d*x + c)^4 + 6*b^2*d*x - 14*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 12*(63*b^2*cosh(d*x + c)^5 - 14*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^3 + 3*(6*b^2*d*x + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 6*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c)^4 + 6*(105*b^2*cosh(d*x + c)^6 - 35*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^4 - 6*b^2*d*x + 15*(6*b^2*d*x + b^2)*cosh(d*x + c)^2 - 16*a^2 + b^2)*sinh(d*x + c)^4 + 24$

```

*(15*b^2*cosh(d*x + c)^7 - 7*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^5 + 5*(6*b^2
*d*x + b^2)*cosh(d*x + c)^3 - (6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + (12*b^2*d*x + 32*a^2 - 9*b^2)*cosh(d*x + c)^2 + (135*b^2*cos
h(d*x + c)^8 - 84*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^6 + 90*(6*b^2*d*x + b^2
)*cosh(d*x + c)^4 + 12*b^2*d*x - 36*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c
)^2 + 32*a^2 - 9*b^2)*sinh(d*x + c)^2 + 3*b^2 - 48*(a*b*cosh(d*x + c)^8 + 8
*a*b*cosh(d*x + c)*sinh(d*x + c)^7 + a*b*sinh(d*x + c)^8 - 3*a*b*cosh(d*x +
c)^6 + (28*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^6 + 3*a*b*cosh(d*x +
c)^4 + 2*(28*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*sinh(d*x + c)^5 +
(70*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^4 -
a*b*cosh(d*x + c)^2 + 4*(14*a*b*cosh(d*x + c)^5 - 15*a*b*cosh(d*x + c)^3 +
3*a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (28*a*b*cosh(d*x + c)^6 - 45*a*b*co
sh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^2 + 2*(4*a*b*co
sh(d*x + c)^7 - 9*a*b*cosh(d*x + c)^5 + 6*a*b*cosh(d*x + c)^3 - a*b*cosh(d*
x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 48*(a*b*cos
h(d*x + c)^8 + 8*a*b*cosh(d*x + c)*sinh(d*x + c)^7 + a*b*sinh(d*x + c)^8 -
3*a*b*cosh(d*x + c)^6 + (28*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^6 +
3*a*b*cosh(d*x + c)^4 + 2*(28*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*si
nh(d*x + c)^5 + (70*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2 + 3*a*b)*s
inh(d*x + c)^4 - a*b*cosh(d*x + c)^2 + 4*(14*a*b*cosh(d*x + c)^5 - 15*a*b*c
osh(d*x + c)^3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (28*a*b*cosh(d*x +
c)^6 - 45*a*b*cosh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)
^2 + 2*(4*a*b*cosh(d*x + c)^7 - 9*a*b*cosh(d*x + c)^5 + 6*a*b*cosh(d*x + c)
^3 - a*b*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) -
1) + 2*(15*b^2*cosh(d*x + c)^9 - 12*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^7 + 1
8*(6*b^2*d*x + b^2)*cosh(d*x + c)^5 - 12*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*
x + c)^3 + (12*b^2*d*x + 32*a^2 - 9*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*c
osh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 3*
d*cosh(d*x + c)^6 + (28*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(28*d*
cosh(d*x + c)^3 - 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 3*d*cosh(d*x + c)^4
+ (70*d*cosh(d*x + c)^4 - 45*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 4*(
14*d*cosh(d*x + c)^5 - 15*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
c)^3 - d*cosh(d*x + c)^2 + (28*d*cosh(d*x + c)^6 - 45*d*cosh(d*x + c)^4 +
18*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 2*(4*d*cosh(d*x + c)^7 - 9*d*co
sh(d*x + c)^5 + 6*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**2,x)

[Out] Timed out

Giac [B] time = 1.25275, size = 219, normalized size = 2.88

$$-\frac{(dx+c)b^2}{2d} + \frac{b^2e^{(2dx+2c)}}{8d} - \frac{2ab\log(e^{(dx+c)}+1)}{d} + \frac{2ab\log(|e^{(dx+c)}-1|)}{d} - \frac{(3b^2e^{(6dx+6c)} - 3b^2 + 3(32a^2 - 3b^2)e^{(4dx+c)})}{24d(e^{(dx+c)}+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")

[Out]
$$-1/2*(d*x + c)*b^2/d + 1/8*b^2*e^{(2*d*x + 2*c)}/d - 2*a*b*\log(e^{(d*x + c)} + 1)/d + 2*a*b*\log(\text{abs}(e^{(d*x + c)} - 1))/d - 1/24*(3*b^2*e^{(6*d*x + 6*c)} - 3*b^2 + 3*(32*a^2 - 3*b^2)*e^{(4*d*x + 4*c)} - (32*a^2 - 9*b^2)*e^{(2*d*x + 2*c)})*e^{(-2*d*x - 2*c)}/(d*(e^{(d*x + c)} + 1)^3*(e^{(d*x + c)} - 1)^3)$$

3.158 $\int \operatorname{csch}^5(c + dx) (a + b \sinh^3(c + dx))^2 dx$

Optimal. Leaf size=90

$$-\frac{3a^2 \tanh^{-1}(\cosh(c + dx))}{8d} - \frac{a^2 \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{3a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{8d} - \frac{2ab \coth(c + dx)}{d} + \frac{b^2 \coth(c + dx)}{d}$$

[Out] $(-3a^2 \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]])/(8d) + (b^2 \operatorname{Cosh}[c + dx])/d - (2ab \operatorname{Coth}[c + dx])/d + (3a^2 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx])/(8d) - (a^2 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^3)/(4d)$

Rubi [A] time = 0.132429, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3220, 3767, 8, 3768, 3770, 2638}

$$-\frac{3a^2 \tanh^{-1}(\cosh(c + dx))}{8d} - \frac{a^2 \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{3a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{8d} - \frac{2ab \coth(c + dx)}{d} + \frac{b^2 \coth(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + dx]^5 (a + b \operatorname{Sinh}[c + dx]^3)^2, x]$

[Out] $(-3a^2 \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]])/(8d) + (b^2 \operatorname{Cosh}[c + dx])/d - (2ab \operatorname{Coth}[c + dx])/d + (3a^2 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx])/(8d) - (a^2 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^3)/(4d)$

Rule 3220

$\operatorname{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)} ((a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e + f*x]^{m*} (a + b \sin[e + f*x]^{n*})^p, x], x] /;$ FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^5(c + dx) (a + b \sinh^3(c + dx))^2 dx &= i \int (-2iab \operatorname{csch}^2(c + dx) - ia^2 \operatorname{csch}^5(c + dx) - ib^2 \sinh(c + dx)) dx \\
 &= a^2 \int \operatorname{csch}^5(c + dx) dx + (2ab) \int \operatorname{csch}^2(c + dx) dx + b^2 \int \sinh(c + dx) dx \\
 &= \frac{b^2 \cosh(c + dx)}{d} - \frac{a^2 \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} - \frac{1}{4} (3a^2) \int \operatorname{csch}^3(c + dx) dx \\
 &= \frac{b^2 \cosh(c + dx)}{d} - \frac{2ab \coth(c + dx)}{d} + \frac{3a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{8d} - \frac{a^2}{8d} \int \operatorname{csch}^3(c + dx) dx \\
 &= -\frac{3a^2 \tanh^{-1}(\cosh(c + dx))}{8d} + \frac{b^2 \cosh(c + dx)}{d} - \frac{2ab \coth(c + dx)}{d} + \frac{3a^2}{8d} \int \operatorname{csch}^3(c + dx) dx
 \end{aligned}$$

Mathematica [A] time = 0.0380619, size = 149, normalized size = 1.66

$$-\frac{a^2 \operatorname{csch}^4\left(\frac{1}{2}(c + dx)\right)}{64d} + \frac{3a^2 \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right)}{32d} + \frac{a^2 \operatorname{sech}^4\left(\frac{1}{2}(c + dx)\right)}{64d} + \frac{3a^2 \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right)}{32d} + \frac{3a^2 \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^2,x]

[Out] $(b^2 \cosh[c] \cosh[dx])/d - (2ab \coth[c + dx])/d + (3a^2 \operatorname{csch}[(c + dx)/2]^2)/(32d) - (a^2 \operatorname{csch}[(c + dx)/2]^4)/(64d) + (3a^2 \operatorname{Log}[\operatorname{Tanh}[(c + dx)/2]])/(8d) + (3a^2 \operatorname{sech}[(c + dx)/2]^2)/(32d) + (a^2 \operatorname{sech}[(c + dx)/2]^4)/(64d) + (b^2 \sinh[c] \sinh[dx])/d$

Maple [A] time = 0.109, size = 66, normalized size = 0.7

$$\frac{1}{d} \left(a^2 \left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4} + \frac{3 \operatorname{csch}(dx+c)}{8} \right) \coth(dx+c) - \frac{3 \operatorname{Artanh}(e^{dx+c})}{4} \right) - 2ab \coth(dx+c) + b^2 \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x)`

[Out] $1/d * (a^2 * ((-1/4 * \operatorname{csch}(d*x+c)^3 + 3/8 * \operatorname{csch}(d*x+c)) * \coth(d*x+c) - 3/4 * \operatorname{arctanh}(\exp(d*x+c))) - 2*a*b * \coth(d*x+c) + b^2 * \cosh(d*x+c))$

Maxima [B] time = 1.12204, size = 254, normalized size = 2.82

$$\frac{1}{2} b^2 \left(\frac{e^{dx+c}}{d} + \frac{e^{-dx-c}}{d} \right) - \frac{1}{8} a^2 \left(\frac{3 \log(e^{-dx-c} + 1)}{d} - \frac{3 \log(e^{-dx-c} - 1)}{d} + \frac{2(3e^{-dx-c} - 11e^{-3dx-3c} - 11e^{-5dx-5c} + 3e^{-7dx-7c})}{d(4e^{-2dx-2c} - 6e^{-4dx-4c} + 4e^{-6dx-6c} - e^{-8dx-8c} - 1)} \right) + 4ab/(d(e^{-2dx-2c} - 1))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")`

[Out] $1/2*b^2*(e^{(d*x + c)}/d + e^{(-d*x - c)}/d) - 1/8*a^2*(3*\log(e^{(-d*x - c)} + 1)/d - 3*\log(e^{(-d*x - c)} - 1)/d + 2*(3*e^{(-d*x - c)} - 11*e^{(-3*d*x - 3*c)} - 11*e^{(-5*d*x - 5*c)} + 3*e^{(-7*d*x - 7*c)})/(d*(4*e^{(-2*d*x - 2*c)} - 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} - e^{(-8*d*x - 8*c)} - 1))) + 4*a*b/(d*(e^{(-2*d*x - 2*c)} - 1))$

Fricas [B] time = 2.26968, size = 5495, normalized size = 61.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")

[Out] $\frac{1}{8}(4b^2\cosh(dx+c)^{10} + 40b^2\cosh(dx+c)\sinh(dx+c)^9 + 4b^2\sinh(dx+c)^{10} - 32ab\cosh(dx+c)^7 + 6(a^2 - 2b^2)\cosh(dx+c)^8 + 6(30b^2\cosh(dx+c)^2 + a^2 - 2b^2)\sinh(dx+c)^8 + 16(30b^2\cosh(dx+c)^3 - 2ab + 3(a^2 - 2b^2)\cosh(dx+c))\sinh(dx+c)^7 + 96ab\cosh(dx+c)^5 - 2(11a^2 - 4b^2)\cosh(dx+c)^6 + 2(420b^2\cosh(dx+c)^4 - 112ab\cosh(dx+c) + 84(a^2 - 2b^2)\cosh(dx+c)^2 - 11a^2 + 4b^2)\sinh(dx+c)^6 + 12(84b^2\cosh(dx+c)^5 - 56ab\cosh(dx+c)^2 + 28(a^2 - 2b^2)\cosh(dx+c)^3 + 8ab - (11a^2 - 4b^2)\cosh(dx+c))\sinh(dx+c)^5 - 96ab\cosh(dx+c)^3 - 2(11a^2 - 4b^2)\cosh(dx+c)^4 + 2(420b^2\cosh(dx+c)^6 - 560ab\cosh(dx+c)^3 + 210(a^2 - 2b^2)\cosh(dx+c)^4 + 240ab\cosh(dx+c) - 15(11a^2 - 4b^2)\cosh(dx+c)^2 - 11a^2 + 4b^2)\sinh(dx+c)^4 + 8(60b^2\cosh(dx+c))^7 - 140ab\cosh(dx+c)^4 + 42(a^2 - 2b^2)\cosh(dx+c)^5 + 120ab\cosh(dx+c)^2 - 5(11a^2 - 4b^2)\cosh(dx+c)^3 - 12ab - (11a^2 - 4b^2)\cosh(dx+c))\sinh(dx+c)^3 + 32ab\cosh(dx+c) + 6(a^2 - 2b^2)\cosh(dx+c)^2 + 6(30b^2\cosh(dx+c)^8 - 112ab\cosh(dx+c)^5 + 28(a^2 - 2b^2)\cosh(dx+c)^6 + 160ab\cosh(dx+c)^3 - 5(11a^2 - 4b^2)\cosh(dx+c)^4 - 48ab\cosh(dx+c) - 2(11a^2 - 4b^2)\cosh(dx+c)^2 + a^2 - 2b^2)\sinh(dx+c)^2 + 4b^2 - 3(a^2\cosh(dx+c))^9 + 9a^2\cosh(dx+c)\sinh(dx+c)^8 + a^2\sinh(dx+c)^9 - 4a^2\cosh(dx+c)^7 + 4(9a^2\cosh(dx+c)^2 - a^2)\sinh(dx+c)^7 + 6a^2\cosh(dx+c)^5 + 28(3a^2\cosh(dx+c)^3 - a^2\cosh(dx+c))\sinh(dx+c)^6 + 6(21a^2\cosh(dx+c)^4 - 14a^2\cosh(dx+c)^2 + a^2)\sinh(dx+c)^5 - 4a^2\cosh(dx+c)^3 + 2(63a^2\cosh(dx+c)^5 - 70a^2\cosh(dx+c)^3 + 15a^2\cosh(dx+c))\sinh(dx+c)^4 + 4(21a^2\cosh(dx+c)^6 - 35a^2\cosh(dx+c)^4 + 15a^2\cosh(dx+c)^2 - a^2)\sinh(dx+c)^3 + a^2\cosh(dx+c) + 12(3a^2\cosh(dx+c))^7 - 7a^2\cosh(dx+c)^5 + 5a^2\cosh(dx+c)^3 - a^2\cosh(dx+c))\sinh(dx+c)^2 + (9a^2\cosh(dx+c))^8 - 28a^2\cosh(dx+c)^6 + 30a^2\cosh(dx+c)^4 - 12a^2\cosh(dx+c)^2 + a^2)\sinh(dx+c))\log(\cosh(dx+c) + \sinh(dx+c) + 1) + 3(a^2\cosh(dx+c))^9 + 9a^2\cosh(dx+c)\sinh(dx+c)^8 + a^2\sinh(dx+c)^9 - 4a^2\cosh(dx+c)^7 + 4(9a^2\cosh(dx+c)^2 - a^2)\sinh(dx+c)^7 + 6a^2\cosh(dx+c)^5 + 28(3a^2\cosh(dx+c)^3 - a^2\cosh(dx+c))\sinh(dx+c)^6 + 6(21a^2\cosh(dx+c)^4 - 14a^2\cosh(dx+c)^2 + a^2)\sinh(dx+c)^5 - 4a^2\cosh(dx+c)^3 + 2(63a^2\cosh(dx+c)^5 - 70a^2\cosh(dx+c)^3 + 15a^2\cosh(dx+c))\sinh(dx+c)^4 + 4(21a^2\cosh(dx+c)^6 - 35a^2\cosh(dx+c)^4 + 15a^2\cosh(dx+c)^2 - a^2)\sinh(dx+c)^3 + a^2\cosh(dx+c) + 12(3a^2\cosh(dx+c))^7 - 7a^2\cosh(dx+c)^5 + 5a^2\cosh(dx+c)^3 - a^2\cosh(dx+c))\sinh(dx+c)^2 + (9a^2\cosh(dx+c))^8 - 28a^2\cosh(dx+c)^6 + 30a^2\cosh(dx+c)^4 - 12a^2\cosh(dx+c)^2 + a^2)\sinh(dx+c))\log(\cosh(dx+c) + \sinh(dx+c) - 1) + 4(10b^2\cosh(dx+c)^9 - 56ab\cosh(dx+c)^6 + 12(a^2 - 2b^2)\cosh(dx+c)$

$$c)^7 + 120*a*b*cosh(d*x + c)^4 - 3*(11*a^2 - 4*b^2)*cosh(d*x + c)^5 - 72*a*b*cosh(d*x + c)^2 - 2*(11*a^2 - 4*b^2)*cosh(d*x + c)^3 + 8*a*b + 3*(a^2 - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 4*(9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 - 4*d*cosh(d*x + c)^3 + 4*(21*d*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 12*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (9*d*cosh(d*x + c)^8 - 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**2,x)

[Out] Timed out

Giac [B] time = 1.26709, size = 246, normalized size = 2.73

$$\frac{b^2 e^{(dx+c)}}{2d} + \frac{b^2 e^{(-dx-c)}}{2d} - \frac{3a^2 \log(e^{(dx+c)} + 1)}{8d} + \frac{3a^2 \log(|e^{(dx+c)} - 1|)}{8d} + \frac{3a^2 e^{(7dx+7c)} - 16abe^{(6dx+6c)} - 11a^2 e^{(5dx+5c)} + 4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")

[Out] $\frac{1}{2}b^2e^{(dx+c)}/d + \frac{1}{2}b^2e^{(-dx-c)}/d - \frac{3}{8}a^2\log(e^{(dx+c)} + 1)/d + \frac{3}{8}a^2\log(\text{abs}(e^{(dx+c)} - 1))/d + \frac{1}{4}(3a^2e^{(7dx+7c)} - 16a^2be^{(6dx+6c)} - 11a^2e^{(5dx+5c)} + 4) - 16a^2be^{(6dx+6c)} - 11a^2e^{(5dx+5c)} + 48a^2be^{(4dx+4c)} - 16a^2e^{(3dx+3c)} - 48a^2be^{(2dx+2c)} + 3a^2e^{(dx+c)} + 16a^2b)/(d*(e^{(2dx+2c)} - 1)^4)$

3.159 $\int \operatorname{csch}^6(c + dx) (a + b \sinh^3(c + dx))^2 dx$

Optimal. Leaf size=88

$$-\frac{a^2 \coth^5(c + dx)}{5d} + \frac{2a^2 \coth^3(c + dx)}{3d} - \frac{a^2 \coth(c + dx)}{d} + \frac{ab \tanh^{-1}(\cosh(c + dx))}{d} - \frac{ab \coth(c + dx) \operatorname{csch}(c + dx)}{d}$$

[Out] $b^2 x + (a*b*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d - (a^2*\operatorname{Coth}[c + d*x])/d + (2*a^2*\operatorname{Cot}h[c + d*x]^3)/(3*d) - (a^2*\operatorname{Coth}[c + d*x]^5)/(5*d) - (a*b*\operatorname{Coth}[c + d*x]*\operatorname{Cs}ch[c + d*x])/d$

Rubi [A] time = 0.102543, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3220, 3768, 3770, 3767}

$$-\frac{a^2 \coth^5(c + dx)}{5d} + \frac{2a^2 \coth^3(c + dx)}{3d} - \frac{a^2 \coth(c + dx)}{d} + \frac{ab \tanh^{-1}(\cosh(c + dx))}{d} - \frac{ab \coth(c + dx) \operatorname{csch}(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cs}ch[c + d*x]^6*(a + b*\operatorname{S}inh[c + d*x]^3)^2, x]$

[Out] $b^2 x + (a*b*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/d - (a^2*\operatorname{Coth}[c + d*x])/d + (2*a^2*\operatorname{Cot}h[c + d*x]^3)/(3*d) - (a^2*\operatorname{Coth}[c + d*x]^5)/(5*d) - (a*b*\operatorname{Coth}[c + d*x]*\operatorname{Cs}ch[c + d*x])/d$

Rule 3220

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e + f*x]^{m*(a + b*\sin[e + f*x]^n)^p}, x], x] /;$ $\operatorname{FreeQ}\{a, b, e, f\}, x \ \&\& \ \operatorname{IntegersQ}[m, p] \ \&\& \ (\operatorname{EqQ}[n, 4] \ || \ \operatorname{GtQ}[p, 0] \ || \ (\operatorname{EqQ}[p, -1] \ \&\& \ \operatorname{IntegerQ}[n]))$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\operatorname{Cos}[c + d*x])*(b*\operatorname{Csc}[c + d*x])^{(n-1)}]/(d*(n-1)), x] + \operatorname{Dist}[(b^2*(n-2))/(n-1), \operatorname{Int}[(b*\operatorname{Csc}[c + d*x])^{(n-2)}, x], x] /;$ $\operatorname{FreeQ}\{b, c, d\}, x \ \&\& \ \operatorname{GtQ}[n, 1] \ \&\& \ \operatorname{IntegerQ}[2*n]$

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^6(c + dx) (a + b \sinh^3(c + dx))^2 dx &= - \int (-b^2 - 2abcsch^3(c + dx) - a^2csch^6(c + dx)) dx \\ &= b^2x + a^2 \int \operatorname{csch}^6(c + dx) dx + (2ab) \int \operatorname{csch}^3(c + dx) dx \\ &= b^2x - \frac{ab \operatorname{coth}(c + dx)\operatorname{csch}(c + dx)}{d} - (ab) \int \operatorname{csch}(c + dx) dx - \frac{(ia^2) \operatorname{Subst}}{d} \\ &= b^2x + \frac{ab \tanh^{-1}(\cosh(c + dx))}{d} - \frac{a^2 \operatorname{coth}(c + dx)}{d} + \frac{2a^2 \operatorname{coth}^3(c + dx)}{3d} - \frac{a}{d} \end{aligned}$$

Mathematica [B] time = 0.941217, size = 197, normalized size = 2.24

$$16 \left(-16a^2 \tanh\left(\frac{1}{2}(c + dx)\right) - 12a^2 \sinh^6\left(\frac{1}{2}(c + dx)\right) \operatorname{csch}^5(c + dx) - 19a^2 \sinh^4\left(\frac{1}{2}(c + dx)\right) \operatorname{csch}^3(c + dx) - 15ab \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-256*a^2*Coth[(c + d*x)/2] - 240*a*b*Csch[(c + d*x)/2]^2 + 19*a^2*Csch[(c + d*x)/2]^4*Sinh[c + d*x] - 3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x] + 16*(60*b^2*c + 60*b^2*d*x - 60*a*b*Log[Tanh[(c + d*x)/2]] - 15*a*b*Sech[(c + d*x)/2]^2 - 19*a^2*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - 12*a^2*Csch[c + d*x]^5*Sinh[(c + d*x)/2]^6 - 16*a^2*Tanh[(c + d*x)/2]))/(960*d)
```

Maple [A] time = 0.108, size = 73, normalized size = 0.8

$$\frac{1}{d} \left(a^2 \left(-\frac{8}{15} - \frac{(\operatorname{csch}(dx + c))^4}{5} + \frac{4(\operatorname{csch}(dx + c))^2}{15} \right) \operatorname{coth}(dx + c) + 2ab \left(-\frac{1}{2} \operatorname{csch}(dx + c) \operatorname{coth}(dx + c) + \operatorname{Artanh}(e^d) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x)`

[Out] $1/d*(a^2*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(\exp(d*x+c)))+b^2*(d*x+c))$

Maxima [B] time = 1.1531, size = 409, normalized size = 4.65

$$b^2x + ab \left(\frac{\log(e^{-dx-c} + 1)}{d} - \frac{\log(e^{-dx-c} - 1)}{d} + \frac{2(e^{-dx-c} + e^{-3dx-3c})}{d(2e^{-2dx-2c} - e^{-4dx-4c} - 1)} \right) - \frac{16}{15} a^2 \left(\frac{1}{d(5e^{-2dx-2c} - 10e^{-4dx-4c} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")`

[Out] $b^2x + a*b*(\log(e^{-d*x - c} + 1)/d - \log(e^{-d*x - c} - 1)/d + 2*(e^{-d*x - c} + e^{-3*d*x - 3*c})/(d*(2*e^{-2*d*x - 2*c} - e^{-4*d*x - 4*c} - 1))) - 16/15*a^2*(5*e^{-2*d*x - 2*c}/(d*(5*e^{-2*d*x - 2*c} - 10*e^{-4*d*x - 4*c} + 10*e^{-6*d*x - 6*c} - 5*e^{-8*d*x - 8*c} + e^{-10*d*x - 10*c} - 1)) - 10*e^{-4*d*x - 4*c}/(d*(5*e^{-2*d*x - 2*c} - 10*e^{-4*d*x - 4*c} + 10*e^{-6*d*x - 6*c} - 5*e^{-8*d*x - 8*c} + e^{-10*d*x - 10*c} - 1)) - 1/(d*(5*e^{-2*d*x - 2*c} - 10*e^{-4*d*x - 4*c} + 10*e^{-6*d*x - 6*c} - 5*e^{-8*d*x - 8*c} + e^{-10*d*x - 10*c} - 1)))$

Fricas [B] time = 2.31189, size = 6268, normalized size = 71.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")`

[Out] $1/15*(15*b^2*d*x*cosh(d*x + c)^10 + 15*b^2*d*x*sinh(d*x + c)^10 - 75*b^2*d*x*cosh(d*x + c)^8 - 30*a*b*cosh(d*x + c)^9 + 150*b^2*d*x*cosh(d*x + c)^6 + 30*(5*b^2*d*x*cosh(d*x + c) - a*b)*sinh(d*x + c)^9 + 60*a*b*cosh(d*x + c)^7 + 15*(45*b^2*d*x*cosh(d*x + c)^2 - 5*b^2*d*x - 18*a*b*cosh(d*x + c))*sinh(d*x + c)^8 + 60*(30*b^2*d*x*cosh(d*x + c)^3 - 10*b^2*d*x*cosh(d*x + c) - 18*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^7 + 30*(105*b^2*d*x*cosh(d*x + c)$

$$\begin{aligned}
&^4 - 70*b^2*d*x*cosh(d*x + c)^2 - 84*a*b*cosh(d*x + c)^3 + 5*b^2*d*x + 14*a \\
&*b*cosh(d*x + c))*sinh(d*x + c)^6 + 60*(63*b^2*d*x*cosh(d*x + c)^5 - 70*b^2 \\
&*d*x*cosh(d*x + c)^3 - 63*a*b*cosh(d*x + c)^4 + 15*b^2*d*x*cosh(d*x + c) + \\
&21*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^5 - 60*a*b*cosh(d*x + c)^3 - 10*(15*b \\
&^2*d*x + 16*a^2)*cosh(d*x + c)^4 + 10*(315*b^2*d*x*cosh(d*x + c)^6 - 525*b^ \\
&2*d*x*cosh(d*x + c)^4 - 378*a*b*cosh(d*x + c)^5 + 225*b^2*d*x*cosh(d*x + c) \\
&^2 + 210*a*b*cosh(d*x + c)^3 - 15*b^2*d*x - 16*a^2)*sinh(d*x + c)^4 - 15*b^ \\
&2*d*x + 20*(90*b^2*d*x*cosh(d*x + c)^7 - 210*b^2*d*x*cosh(d*x + c)^5 - 126* \\
&a*b*cosh(d*x + c)^6 + 150*b^2*d*x*cosh(d*x + c)^3 + 105*a*b*cosh(d*x + c)^4 \\
&- 3*a*b - 2*(15*b^2*d*x + 16*a^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*a*b* \\
&cosh(d*x + c) + 5*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^2 + 5*(135*b^2*d*x*co \\
&sh(d*x + c)^8 - 420*b^2*d*x*cosh(d*x + c)^6 - 216*a*b*cosh(d*x + c)^7 + 450 \\
&*b^2*d*x*cosh(d*x + c)^4 + 252*a*b*cosh(d*x + c)^5 + 15*b^2*d*x - 36*a*b*co \\
&sh(d*x + c) - 12*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^2 + 16*a^2)*sinh(d*x + \\
&c)^2 - 16*a^2 + 15*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x + \\
&c)^9 + a*b*sinh(d*x + c)^10 - 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x + \\
&c)^2 - a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x + \\
&c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 - 1 \\
&4*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 - 10*a*b*cosh(d*x + c)^4 + 4*(\\
&63*a*b*cosh(d*x + c)^5 - 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sin \\
&h(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 - 35*a*b*cosh(d*x + c)^4 + 15*a*b \\
&*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b \\
&*cosh(d*x + c)^7 - 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 - a*b*cosh \\
&(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 - 28*a*b*cosh(d*x + c \\
&)^6 + 30*a*b*cosh(d*x + c)^4 - 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^ \\
&2 - a*b + 10*(a*b*cosh(d*x + c)^9 - 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x \\
&+ c)^5 - 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*log(cosh \\
&(d*x + c) + sinh(d*x + c) + 1) - 15*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x \\
&+ c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^10 - 5*a*b*cosh(d*x + c)^8 + 5*(9 \\
&*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(\\
&3*a*b*cosh(d*x + c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cos \\
&h(d*x + c)^4 - 14*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 - 10*a*b*cosh(\\
&d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5 - 70*a*b*cosh(d*x + c)^3 + 15*a*b*co \\
&sh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 - 35*a*b*cosh(d*x \\
&+ c)^4 + 15*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + \\
&c)^2 + 40*(3*a*b*cosh(d*x + c)^7 - 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + \\
&c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 - 28* \\
&a*b*cosh(d*x + c)^6 + 30*a*b*cosh(d*x + c)^4 - 12*a*b*cosh(d*x + c)^2 + a*b \\
&)*sinh(d*x + c)^2 - a*b + 10*(a*b*cosh(d*x + c)^9 - 4*a*b*cosh(d*x + c)^7 + \\
&6*a*b*cosh(d*x + c)^5 - 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d* \\
&x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(15*b^2*d*x*cosh(d*x + \\
&c)^9 - 60*b^2*d*x*cosh(d*x + c)^7 - 27*a*b*cosh(d*x + c)^8 + 90*b^2*d*x*cos \\
&h(d*x + c)^5 + 42*a*b*cosh(d*x + c)^6 - 18*a*b*cosh(d*x + c)^2 - 4*(15*b^2* \\
&d*x + 16*a^2)*cosh(d*x + c)^3 + 3*a*b + (15*b^2*d*x + 16*a^2)*cosh(d*x + c) \\
&)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 +
\end{aligned}$$

$$\begin{aligned}
& d \sinh(dx + c)^{10} - 5d \cosh(dx + c)^8 + 5(9d \cosh(dx + c)^2 - d) \sinh(dx + c)^8 \\
& + 40(3d \cosh(dx + c)^3 - d \cosh(dx + c)) \sinh(dx + c)^7 + 10d \cosh(dx + c)^6 \\
& + 10(21d \cosh(dx + c)^4 - 14d \cosh(dx + c)^2 + d) \sinh(dx + c)^6 + 4(63d \cosh(dx + c)^5 \\
& - 70d \cosh(dx + c)^3 + 15d \cosh(dx + c)) \sinh(dx + c)^5 - 10d \cosh(dx + c)^4 \\
& + 10(21d \cosh(dx + c)^6 - 35d \cosh(dx + c)^4 + 15d \cosh(dx + c)^2 - d) \sinh(dx + c)^4 \\
& + 40(3d \cosh(dx + c)^7 - 7d \cosh(dx + c)^5 + 5d \cosh(dx + c)^3 - d \cosh(dx + c)) \sinh(dx + c)^3 \\
& + 5d \cosh(dx + c)^2 + 5(9d \cosh(dx + c)^8 - 28d \cosh(dx + c)^6 + 30d \cosh(dx + c)^4 \\
& - 12d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 10(d \cosh(dx + c)^9 - 4d \cosh(dx + c)^7 \\
& + 6d \cosh(dx + c)^5 - 4d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c) - d
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)**6*(a+b*sinh(dx+c)**3)**2,x)

[Out] Timed out

Giac [A] time = 1.24754, size = 197, normalized size = 2.24

$$\frac{(dx + c)b^2}{d} + \frac{ab \log(e^{(dx+c)} + 1)}{d} - \frac{ab \log(|e^{(dx+c)} - 1|)}{d} - \frac{2(15abe^{(9dx+9c)} - 30abe^{(7dx+7c)} + 80a^2e^{(4dx+4c)} + 30abe^{(3dx+3c)} - 40a^2e^{(2dx+2c)} - 15a^2b)}{15d(e^{(2dx+2c)} - 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^6*(a+b*sinh(dx+c)^3)^2,x, algorithm="giac")

[Out] (dx + c)*b^2/d + a*b*log(e^(dx + c) + 1)/d - a*b*log(abs(e^(dx + c) - 1))/d - 2/15*(15*a*b*e^(9*d*x + 9*c) - 30*a*b*e^(7*d*x + 7*c) + 80*a^2*e^(4*d*x + 4*c) + 30*a*b*e^(3*d*x + 3*c) - 40*a^2*e^(2*d*x + 2*c) - 15*a*b*e^(d*x + c) + 8*a^2)/(d*(e^(2*d*x + 2*c) - 1)^5)

3.160 $\int \operatorname{csch}^7(c + dx) (a + b \sinh^3(c + dx))^2 dx$

Optimal. Leaf size=133

$$\frac{5a^2 \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{a^2 \coth(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a^2 \coth(c + dx) \operatorname{csch}^3(c + dx)}{24d} - \frac{5a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{16d}$$

[Out] (5*a^2*ArcTanh[Cosh[c + d*x]])/(16*d) - (b^2*ArcTanh[Cosh[c + d*x]])/d + (2*a*b*Coth[c + d*x])/d - (2*a*b*Coth[c + d*x]^3)/(3*d) - (5*a^2*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)

Rubi [A] time = 0.174964, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3220, 3770, 3767, 3768}

$$\frac{5a^2 \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{a^2 \coth(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a^2 \coth(c + dx) \operatorname{csch}^3(c + dx)}{24d} - \frac{5a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{16d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]

[Out] (5*a^2*ArcTanh[Cosh[c + d*x]])/(16*d) - (b^2*ArcTanh[Cosh[c + d*x]])/d + (2*a*b*Coth[c + d*x])/d - (2*a*b*Coth[c + d*x]^3)/(3*d) - (5*a^2*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)

Rule 3220

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^p_.], x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^7(c + dx) (a + b \sinh^3(c + dx))^2 dx &= - \left(i \int (ib^2 \operatorname{csch}(c + dx) + 2iab \operatorname{csch}^4(c + dx) + ia^2 \operatorname{csch}^7(c + dx)) dx \right) \\
 &= a^2 \int \operatorname{csch}^7(c + dx) dx + (2ab) \int \operatorname{csch}^4(c + dx) dx + b^2 \int \operatorname{csch}(c + dx) dx \\
 &= -\frac{b^2 \tanh^{-1}(\cosh(c + dx))}{d} - \frac{a^2 \coth(c + dx) \operatorname{csch}^5(c + dx)}{6d} - \frac{1}{6} (5a^2) \int \operatorname{csch}^6(c + dx) dx \\
 &= -\frac{b^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{2ab \coth(c + dx)}{d} - \frac{2ab \coth^3(c + dx)}{3d} + \frac{5a^2}{6d} \int \operatorname{csch}^5(c + dx) dx \\
 &= -\frac{b^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{2ab \coth(c + dx)}{d} - \frac{2ab \coth^3(c + dx)}{3d} - \frac{5a^2}{6d} \int \operatorname{csch}^4(c + dx) dx \\
 &= \frac{5a^2 \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{b^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{2ab \coth(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.0525052, size = 235, normalized size = 1.77

$$-\frac{a^2 \operatorname{csch}^6\left(\frac{1}{2}(c + dx)\right)}{384d} + \frac{a^2 \operatorname{csch}^4\left(\frac{1}{2}(c + dx)\right)}{64d} - \frac{5a^2 \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right)}{64d} - \frac{a^2 \operatorname{sech}^6\left(\frac{1}{2}(c + dx)\right)}{384d} - \frac{a^2 \operatorname{sech}^4\left(\frac{1}{2}(c + dx)\right)}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]

[Out] (4*a*b*Coth[c + d*x])/(3*d) - (5*a^2*Csch[(c + d*x)/2]^2)/(64*d) + (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[(c + d*x)/2]^6)/(384*d) - (2*a*b*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b^2*Log[Cosh[c/2 + (d*x)/2]])/d + (b^2*Log[Sinh[c/2 + (d*x)/2]])/d - (5*a^2*Log[Tanh[(c + d*x)/2]])/(16*d) - (5*a^2*Sech[(c + d*x)/2]^2)/(64*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d) - (a^2*Sech

$$[(c + d*x)/2]^6/(384*d)$$

Maple [A] time = 0.099, size = 90, normalized size = 0.7

$$\frac{1}{d} \left(a^2 \left(\left(-\frac{(\operatorname{csch}(dx+c))^5}{6} + \frac{5(\operatorname{csch}(dx+c))^3}{24} - \frac{5\operatorname{csch}(dx+c)}{16} \right) \coth(dx+c) + \frac{5 \operatorname{Artanh}(e^{dx+c})}{8} \right) + 2ab \left(\frac{2}{3} - \frac{1}{3} \coth(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x)

[Out] 1/d*(a^2*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x+c)+5/8*arctanh(exp(d*x+c)))+2*a*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*b^2*arctanh(exp(d*x+c)))

Maxima [B] time = 1.19719, size = 427, normalized size = 3.21

$$\frac{1}{48} a^2 \left(\frac{15 \log(e^{-dx-c} + 1)}{d} - \frac{15 \log(e^{-dx-c} - 1)}{d} + \frac{2(15e^{-dx-c} - 85e^{-3dx-3c} + 198e^{-5dx-5c} + 198e^{-7dx-7c} - 85e^{-9dx-9c} + 15e^{-11dx-11c})}{d(6e^{-2dx-2c} - 15e^{-4dx-4c} + 20e^{-6dx-6c} - 15e^{-8dx-8c} + 6e^{-10dx-10c} - e^{-12dx-12c} - 1)} \right) - b^2 \left(\frac{\log(e^{-dx-c} + 1)}{d} - \frac{\log(e^{-dx-c} - 1)}{d} + \frac{8/3 a b (3e^{-2dx-2c} - 1)}{d(3e^{-2dx-2c} - 3e^{-4dx-4c} + e^{-6dx-6c} - 1)} - \frac{1}{d(3e^{-2dx-2c} - 3e^{-4dx-4c} + e^{-6dx-6c} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")

[Out] 1/48*a^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - b^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 8/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))))

Fricas [B] time = 2.28431, size = 9393, normalized size = 70.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")`

[Out]
$$-1/48*(30*a^2*\cosh(d*x + c)^{11} + 330*a^2*\cosh(d*x + c)*\sinh(d*x + c)^{10} + 30*a^2*\sinh(d*x + c)^{11} - 170*a^2*\cosh(d*x + c)^9 + 384*a*b*\cosh(d*x + c)^8 + 10*(165*a^2*\cosh(d*x + c)^2 - 17*a^2)*\sinh(d*x + c)^9 + 396*a^2*\cosh(d*x + c)^7 + 6*(825*a^2*\cosh(d*x + c)^3 - 255*a^2*\cosh(d*x + c) + 64*a*b)*\sinh(d*x + c)^8 - 1280*a*b*\cosh(d*x + c)^6 + 12*(825*a^2*\cosh(d*x + c)^4 - 510*a^2*\cosh(d*x + c)^2 + 256*a*b*\cosh(d*x + c) + 33*a^2)*\sinh(d*x + c)^7 + 396*a^2*\cosh(d*x + c)^5 + 4*(3465*a^2*\cosh(d*x + c)^5 - 3570*a^2*\cosh(d*x + c)^3 + 2688*a*b*\cosh(d*x + c)^2 + 693*a^2*\cosh(d*x + c) - 320*a*b)*\sinh(d*x + c)^6 + 1536*a*b*\cosh(d*x + c)^4 + 12*(1155*a^2*\cosh(d*x + c)^6 - 1785*a^2*\cosh(d*x + c)^4 + 1792*a*b*\cosh(d*x + c)^3 + 693*a^2*\cosh(d*x + c)^2 - 640*a*b*\cosh(d*x + c) + 33*a^2)*\sinh(d*x + c)^5 - 170*a^2*\cosh(d*x + c)^3 + 12*(825*a^2*\cosh(d*x + c)^7 - 1785*a^2*\cosh(d*x + c)^5 + 2240*a*b*\cosh(d*x + c)^4 + 1155*a^2*\cosh(d*x + c)^3 - 1600*a*b*\cosh(d*x + c)^2 + 165*a^2*\cosh(d*x + c) + 128*a*b)*\sinh(d*x + c)^4 - 768*a*b*\cosh(d*x + c)^2 + 2*(2475*a^2*\cosh(d*x + c)^8 - 7140*a^2*\cosh(d*x + c)^6 + 10752*a*b*\cosh(d*x + c)^5 + 6930*a^2*\cosh(d*x + c)^4 - 12800*a*b*\cosh(d*x + c)^3 + 1980*a^2*\cosh(d*x + c)^2 + 3072*a*b*\cosh(d*x + c) - 85*a^2)*\sinh(d*x + c)^3 + 30*a^2*\cosh(d*x + c) + 6*(275*a^2*\cosh(d*x + c)^9 - 1020*a^2*\cosh(d*x + c)^7 + 1792*a*b*\cosh(d*x + c)^6 + 1386*a^2*\cosh(d*x + c)^5 - 3200*a*b*\cosh(d*x + c)^4 + 660*a^2*\cosh(d*x + c)^3 + 1536*a*b*\cosh(d*x + c)^2 - 85*a^2*\cosh(d*x + c) - 128*a*b)*\sinh(d*x + c)^2 + 128*a*b - 3*((5*a^2 - 16*b^2)*\cosh(d*x + c)^{12} + 12*(5*a^2 - 16*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^{11} + (5*a^2 - 16*b^2)*\sinh(d*x + c)^{12} - 6*(5*a^2 - 16*b^2)*\cosh(d*x + c)^{10} + 6*(11*(5*a^2 - 16*b^2)*\cosh(d*x + c)^2 - 5*a^2 + 16*b^2)*\sinh(d*x + c)^{10} + 20*(11*(5*a^2 - 16*b^2)*\cosh(d*x + c)^3 - 3*(5*a^2 - 16*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 15*(5*a^2 - 16*b^2)*\cosh(d*x + c)^8 + 15*(33*(5*a^2 - 16*b^2)*\cosh(d*x + c)^4 - 18*(5*a^2 - 16*b^2)*\cosh(d*x + c)^2 + 5*a^2 - 16*b^2)*\sinh(d*x + c)^8 + 24*(33*(5*a^2 - 16*b^2)*\cosh(d*x + c)^5 - 30*(5*a^2 - 16*b^2)*\cosh(d*x + c)^3 + 5*(5*a^2 - 16*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 20*(5*a^2 - 16*b^2)*\cosh(d*x + c)^6 + 4*(231*(5*a^2 - 16*b^2)*\cosh(d*x + c)^6 - 315*(5*a^2 - 16*b^2)*\cosh(d*x + c)^4 + 105*(5*a^2 - 16*b^2)*\cosh(d*x + c)^2 - 25*a^2 + 80*b^2)*\sinh(d*x + c)^6 + 24*(33*(5*a^2 - 16*b^2)*\cosh(d*x + c)^7 - 63*(5*a^2 - 16*b^2)*\cosh(d*x + c)^5 + 35*(5*a^2 - 16*b^2)*\cosh(d*x + c)^3 - 5*(5*a^2 - 16*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 15*(5*a^2 - 16*b^2)*\cosh(d*x + c)^4 + 15*(33*(5*a^2 - 16*b^2)*\cosh(d*x + c)^8 - 84*(5*a^2 - 16*b^2)*\cosh(d*x + c)^6 + 70*(5*a^2 - 16*b^2)*\cosh(d*x + c)^4 - 20*(5*a^2 - 16*b^2)*\cosh(d*x + c)^2 + 5*a^2 - 16*b^2)*\sinh(d*x + c)^4 + 20*(11*(5*a^2 - 16*b^2)*\cosh(d*x + c)^9 - 36*(5*a^2 - 16*b^2)*\cosh(d*x + c)^7 + 42*(5*a^2 - 16*b^2)*\cosh(d*x + c)^5 - 20*(5*a^2 - 16*b^2)*\cosh(d*x + c)^3 + 3*(5*a^2 - 16*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 6*(5*a^2 - 16*b^2)*\cosh(d*x + c)^2 + 6*(11*(5*a^2 - 16$$

$$\begin{aligned}
& *b^2) * \cosh(dx + c)^{10} - 45*(5a^2 - 16b^2) * \cosh(dx + c)^8 + 70*(5a^2 - \\
& 16b^2) * \cosh(dx + c)^6 - 50*(5a^2 - 16b^2) * \cosh(dx + c)^4 + 15*(5a^2 - \\
& 16b^2) * \cosh(dx + c)^2 - 5a^2 + 16b^2) * \sinh(dx + c)^2 + 5a^2 - 16b^2 \\
& + 12*((5a^2 - 16b^2) * \cosh(dx + c)^{11} - 5*(5a^2 - 16b^2) * \cosh(dx + c) \\
& ^9 + 10*(5a^2 - 16b^2) * \cosh(dx + c)^7 - 10*(5a^2 - 16b^2) * \cosh(dx + c) \\
&)^5 + 5*(5a^2 - 16b^2) * \cosh(dx + c)^3 - (5a^2 - 16b^2) * \cosh(dx + c) * \\
& \sinh(dx + c)) * \log(\cosh(dx + c) + \sinh(dx + c) + 1) + 3*((5a^2 - 16b^2) \\
& * \cosh(dx + c)^{12} + 12*(5a^2 - 16b^2) * \cosh(dx + c) * \sinh(dx + c)^{11} + (5 \\
& a^2 - 16b^2) * \sinh(dx + c)^{12} - 6*(5a^2 - 16b^2) * \cosh(dx + c)^{10} + 6*(\\
& 11*(5a^2 - 16b^2) * \cosh(dx + c)^2 - 5a^2 + 16b^2) * \sinh(dx + c)^{10} + 20 \\
& *(11*(5a^2 - 16b^2) * \cosh(dx + c)^3 - 3*(5a^2 - 16b^2) * \cosh(dx + c)) * \sinh(dx + c)^9 \\
& + 15*(5a^2 - 16b^2) * \cosh(dx + c)^8 + 15*(33*(5a^2 - 16b^2) * \cosh(dx + c)^4 \\
& - 18*(5a^2 - 16b^2) * \cosh(dx + c)^2 + 5a^2 - 16b^2) * \sinh(dx + c)^8 + 24*(33*(5a^2 - 16b^2) \\
& * \cosh(dx + c)^5 - 30*(5a^2 - 16b^2) * \cosh(dx + c)^3 + 5*(5a^2 - 16b^2) * \cosh(dx + c)) * \sinh(dx + c)^7 \\
& - 20*(5a^2 - 16b^2) * \cosh(dx + c)^6 + 4*(231*(5a^2 - 16b^2) * \cosh(dx + c) \\
&)^6 - 315*(5a^2 - 16b^2) * \cosh(dx + c)^4 + 105*(5a^2 - 16b^2) * \cosh(dx \\
& + c)^2 - 25a^2 + 80b^2) * \sinh(dx + c)^6 + 24*(33*(5a^2 - 16b^2) * \cosh(dx \\
& + c)^7 - 63*(5a^2 - 16b^2) * \cosh(dx + c)^5 + 35*(5a^2 - 16b^2) * \cosh(dx \\
& + c)^3 - 5*(5a^2 - 16b^2) * \cosh(dx + c)) * \sinh(dx + c)^5 + 15*(5a^2 - \\
& 16b^2) * \cosh(dx + c)^4 + 15*(33*(5a^2 - 16b^2) * \cosh(dx + c)^8 - 84*(5a^2 \\
& - 16b^2) * \cosh(dx + c)^6 + 70*(5a^2 - 16b^2) * \cosh(dx + c)^4 - 20*(5a^2 \\
& - 16b^2) * \cosh(dx + c)^2 + 5a^2 - 16b^2) * \sinh(dx + c)^4 + 20*(11*(\\
& 5a^2 - 16b^2) * \cosh(dx + c)^9 - 36*(5a^2 - 16b^2) * \cosh(dx + c)^7 + 42* \\
& (5a^2 - 16b^2) * \cosh(dx + c)^5 - 20*(5a^2 - 16b^2) * \cosh(dx + c)^3 + 3* \\
& (5a^2 - 16b^2) * \cosh(dx + c)) * \sinh(dx + c)^3 - 6*(5a^2 - 16b^2) * \cosh(dx \\
& + c)^2 + 6*(11*(5a^2 - 16b^2) * \cosh(dx + c)^{10} - 45*(5a^2 - 16b^2) * \cosh(dx + c)^8 \\
& + 70*(5a^2 - 16b^2) * \cosh(dx + c)^6 - 50*(5a^2 - 16b^2) * \cosh(dx + c)^4 \\
& + 15*(5a^2 - 16b^2) * \cosh(dx + c)^2 - 5a^2 + 16b^2) * \sinh(dx + c)^2 + 5a^2 - 16b^2 \\
& + 12*((5a^2 - 16b^2) * \cosh(dx + c)^{11} - 5*(5a^2 - 16b^2) * \cosh(dx + c)^9 \\
& + 10*(5a^2 - 16b^2) * \cosh(dx + c)^7 - 10*(5a^2 - 16b^2) * \cosh(dx + c)^5 \\
& + 5*(5a^2 - 16b^2) * \cosh(dx + c)^3 - (5a^2 - 16b^2) * \cosh(dx + c)) * \sinh(dx + c)) * \log(\cosh(dx + c) \\
& + \sinh(dx + c) - 1) + 6*(55a^2 * \cosh(dx + c)^{10} - 255a^2 * \cosh(dx + c)^8 + 512a * b * \cosh(dx + c)^7 \\
& + 462a^2 * \cosh(dx + c)^6 - 1280a * b * \cosh(dx + c)^5 + 330a^2 * \cosh(dx + c)^4 \\
& + 1024a * b * \cosh(dx + c)^3 - 85a^2 * \cosh(dx + c)^2 - 256a * b * \cosh(dx + c) + 5a^2) * \sinh(dx + c)) / (d * \cosh(dx + c)^{12} \\
& + 12d * \cosh(dx + c) * \sinh(dx + c)^{11} + d * \sinh(dx + c)^{12} - 6d * \cosh(dx + c)^{10} + 6*(\\
& 11d * \cosh(dx + c)^2 - d) * \sinh(dx + c)^{10} + 20*(11d * \cosh(dx + c)^3 - 3d \\
& * \cosh(dx + c)) * \sinh(dx + c)^9 + 15d * \cosh(dx + c)^8 + 15*(33d * \cosh(dx \\
& + c)^4 - 18d * \cosh(dx + c)^2 + d) * \sinh(dx + c)^8 + 24*(33d * \cosh(dx + c) \\
& ^5 - 30d * \cosh(dx + c)^3 + 5d * \cosh(dx + c)) * \sinh(dx + c)^7 - 20d * \cosh(dx + c)^6 \\
& + 4*(231d * \cosh(dx + c)^6 - 315d * \cosh(dx + c)^4 + 105d * \cosh(dx + c)^2 - 5d) * \sinh(dx + c)^6 \\
& + 24*(33d * \cosh(dx + c)^7 - 63d * \cosh(dx + c)^5 + 35d * \cosh(dx + c)^3 - 5d * \cosh(dx + c)) * \sinh(dx + c)^5 + 15d
\end{aligned}$$


```
*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21541, size = 281, normalized size = 2.11

$$\frac{(5a^2 - 16b^2) \log(e^{dx+c} + 1)}{16d} - \frac{(5a^2 - 16b^2) \log(|e^{dx+c} - 1|)}{16d} - \frac{15a^2 e^{(11dx+11c)} - 85a^2 e^{(9dx+9c)} + 192abe^{(8dx+8c)} + 198a^2 e^{(7dx+7c)} - 640a^2 b e^{(6dx+6c)} + 198a^2 e^{(5dx+5c)} + 768a^2 b e^{(4dx+4c)} - 85a^2 e^{(3dx+3c)} - 384a^2 b e^{(2dx+2c)} + 15a^2 e^{(dx+c)} + 64a^2 b}{d(e^{2dx+2c} - 1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/16*(5*a^2 - 16*b^2)*log(e^(d*x + c) + 1)/d - 1/16*(5*a^2 - 16*b^2)*log(abs(e^(d*x + c) - 1))/d - 1/24*(15*a^2*e^(11*d*x + 11*c) - 85*a^2*e^(9*d*x + 9*c) + 192*a*b*e^(8*d*x + 8*c) + 198*a^2*e^(7*d*x + 7*c) - 640*a*b*e^(6*d*x + 6*c) + 198*a^2*e^(5*d*x + 5*c) + 768*a*b*e^(4*d*x + 4*c) - 85*a^2*e^(3*d*x + 3*c) - 384*a*b*e^(2*d*x + 2*c) + 15*a^2*e^(d*x + c) + 64*a*b)/(d*(e^(2*d*x + 2*c) - 1)^6)
```

3.161 $\int \sinh^2(c + dx) (a + b \sinh^3(c + dx))^3 dx$

Optimal. Leaf size=291

$$\frac{3a^2b \cosh^5(c + dx)}{5d} - \frac{2a^2b \cosh^3(c + dx)}{d} + \frac{3a^2b \cosh(c + dx)}{d} + \frac{a^3 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{a^3x}{2} + \frac{3ab^2 \sinh^7(c + dx)}{8d}$$

[Out] $-(a^3x)/2 + (105*a*b^2*x)/128 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c + d*x])/d - (2*a^2*b*Cosh[c + d*x]^3)/d + (5*b^3*Cosh[c + d*x]^3)/(3*d) + (3*a^2*b*Cosh[c + d*x]^5)/(5*d) - (2*b^3*Cosh[c + d*x]^5)/d + (10*b^3*Cosh[c + d*x]^7)/(7*d) - (5*b^3*Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(11*d) + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (105*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(64*d) - (7*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(16*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)$

Rubi [A] time = 0.208646, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3220, 2635, 8, 2633}

$$\frac{3a^2b \cosh^5(c + dx)}{5d} - \frac{2a^2b \cosh^3(c + dx)}{d} + \frac{3a^2b \cosh(c + dx)}{d} + \frac{a^3 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{a^3x}{2} + \frac{3ab^2 \sinh^7(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] $-(a^3x)/2 + (105*a*b^2*x)/128 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c + d*x])/d - (2*a^2*b*Cosh[c + d*x]^3)/d + (5*b^3*Cosh[c + d*x]^3)/(3*d) + (3*a^2*b*Cosh[c + d*x]^5)/(5*d) - (2*b^3*Cosh[c + d*x]^5)/d + (10*b^3*Cosh[c + d*x]^7)/(7*d) - (5*b^3*Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(11*d) + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (105*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(64*d) - (7*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(16*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)$

Rule 3220

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^p, x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt

$Q[p, 0] \parallel (\text{EqQ}[p, -1] \ \&\& \ \text{IntegerQ}[n])$

Rule 2635

$\text{Int}[(b \cdot \sin[c + d \cdot x])^n, x_Symbol] \rightarrow -\text{Simp}[(b \cdot \cos[c + d \cdot x]) \cdot (b \cdot \sin[c + d \cdot x])^{n-1} / (d \cdot n), x] + \text{Dist}[(b^2)^{n-1} / n, \text{Int}[(b \cdot \sin[c + d \cdot x])^{n-2}, x], x] /;$ $\text{FreeQ}\{b, c, d, x\} \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2 \cdot n]$

Rule 8

$\text{Int}[a \cdot x, x_Symbol] \rightarrow \text{Simp}[a \cdot x, x] /;$ $\text{FreeQ}[a, x]$

Rule 2633

$\text{Int}[\sin[c + d \cdot x]^n, x_Symbol] \rightarrow -\text{Dist}[d^{-1}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n-1)/2}, x], x], x, \cos[c + d \cdot x]], x] /;$ $\text{FreeQ}\{c, d, x\} \ \&\& \ \text{IGtQ}[(n-1)/2, 0]$

Rubi steps

$$\begin{aligned} \int \sinh^2(c + dx) (a + b \sinh^3(c + dx))^3 dx &= - \int (-a^3 \sinh^2(c + dx) - 3a^2b \sinh^5(c + dx) - 3ab^2 \sinh^8(c + dx) - b^3 \sinh^{11}(c + dx)) dx \\ &= a^3 \int \sinh^2(c + dx) dx + (3a^2b) \int \sinh^5(c + dx) dx + (3ab^2) \int \sinh^8(c + dx) dx + b^3 \int \sinh^{11}(c + dx) dx \\ &= \frac{a^3 \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{3ab^2 \cosh(c + dx) \sinh^7(c + dx)}{8d} - \frac{1}{2} a^3 \int \sinh^2(c + dx) dx \\ &= -\frac{a^3 x}{2} + \frac{3a^2b \cosh(c + dx)}{d} - \frac{b^3 \cosh(c + dx)}{d} - \frac{2a^2b \cosh^3(c + dx)}{d} + \frac{5b^3 \cosh^5(c + dx)}{5d} \\ &= -\frac{a^3 x}{2} + \frac{3a^2b \cosh(c + dx)}{d} - \frac{b^3 \cosh(c + dx)}{d} - \frac{2a^2b \cosh^3(c + dx)}{d} + \frac{5b^3 \cosh^5(c + dx)}{5d} \\ &= -\frac{a^3 x}{2} + \frac{3a^2b \cosh(c + dx)}{d} - \frac{b^3 \cosh(c + dx)}{d} - \frac{2a^2b \cosh^3(c + dx)}{d} + \frac{5b^3 \cosh^5(c + dx)}{5d} \\ &= -\frac{a^3 x}{2} + \frac{105}{128} ab^2 x + \frac{3a^2b \cosh(c + dx)}{d} - \frac{b^3 \cosh(c + dx)}{d} - \frac{2a^2b \cosh^3(c + dx)}{d} + \frac{5b^3 \cosh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.476869, size = 194, normalized size = 0.67

$$\frac{-27720a(64a^2 - 105b^2)(c + dx) + 110880a(8a^2 - 21b^2)\sinh(2(c + dx)) - 20790b(77b^2 - 320a^2)\cosh(c + dx) + 3465b^3\cosh^3(c + dx)}{128}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] $(-27720*a*(64*a^2 - 105*b^2)*(c + d*x) - 20790*b*(-320*a^2 + 77*b^2)*\text{Cosh}[c + d*x] + 34650*b*(-32*a^2 + 11*b^2)*\text{Cosh}[3*(c + d*x)] - 2079*b*(-64*a^2 + 55*b^2)*\text{Cosh}[5*(c + d*x)] + 27225*b^3*\text{Cosh}[7*(c + d*x)] - 4235*b^3*\text{Cosh}[9*(c + d*x)] + 315*b^3*\text{Cosh}[11*(c + d*x)] + 110880*a*(8*a^2 - 21*b^2)*\text{Sinh}[2*(c + d*x)] + 582120*a*b^2*\text{Sinh}[4*(c + d*x)] - 110880*a*b^2*\text{Sinh}[6*(c + d*x)] + 10395*a*b^2*\text{Sinh}[8*(c + d*x)])/(3548160*d)$

Maple [A] time = 0.06, size = 188, normalized size = 0.7

$$\frac{1}{d} \left(b^3 \left(-\frac{256}{693} + \frac{(\sinh(dx+c))^{10}}{11} - \frac{10(\sinh(dx+c))^8}{99} + \frac{80(\sinh(dx+c))^6}{693} - \frac{32(\sinh(dx+c))^4}{231} + \frac{128(\sinh(dx+c))}{693} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x)

[Out] $1/d*(b^3*(-256/693+1/11*\sinh(d*x+c)^{10}-10/99*\sinh(d*x+c)^8+80/693*\sinh(d*x+c)^6-32/231*\sinh(d*x+c)^4+128/693*\sinh(d*x+c)^2)*\cosh(d*x+c)+3*a*b^2*((1/8*\sinh(d*x+c)^7-7/48*\sinh(d*x+c)^5+35/192*\sinh(d*x+c)^3-35/128*\sinh(d*x+c))*\cosh(d*x+c)+35/128*d*x+35/128*c)+3*a^2*b*(8/15+1/5*\sinh(d*x+c)^4-4/15*\sinh(d*x+c)^2)*\cosh(d*x+c)+a^3*(1/2*\cosh(d*x+c)*\sinh(d*x+c)-1/2*d*x-1/2*c)$

Maxima [A] time = 1.09023, size = 522, normalized size = 1.79

$$-\frac{1}{8} a^3 \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) - \frac{1}{1419264} b^3 \left(\frac{(847 e^{(-2dx-2c)} - 5445 e^{(-4dx-4c)} + 22869 e^{(-6dx-6c)} - 76230 e^{(-8dx-8c)} + 320166 e^{(-10dx-10c)} - 63) e^{(11dx+11c)}}{d} + (320166 e^{(-dx-c)} - 76230 e^{(-3dx-3c)} + 22869 e^{(-5dx-5c)} - 5445 e^{(-7dx-7c)} + 847 e^{(-9dx-9c)} - 63 e^{(-11dx-11c)})/d - 1/ \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] $-1/8*a^3*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/1419264*b^3*((847*e^{(-2*d*x - 2*c)} - 5445*e^{(-4*d*x - 4*c)} + 22869*e^{(-6*d*x - 6*c)} - 76230*e^{(-8*d*x - 8*c)} + 320166*e^{(-10*d*x - 10*c)} - 63)*e^{(11*d*x + 11*c)}/d + (320166*e^{(-d*x - c)} - 76230*e^{(-3*d*x - 3*c)} + 22869*e^{(-5*d*x - 5*c)} - 5445*e^{(-7*d*x - 7*c)} + 847*e^{(-9*d*x - 9*c)} - 63*e^{(-11*d*x - 11*c)})/d) - 1/$

$$2048*a*b^2*((32*e^{(-2*d*x - 2*c)} - 168*e^{(-4*d*x - 4*c)} + 672*e^{(-6*d*x - 6*c)} - 3)*e^{(8*d*x + 8*c)}/d - 1680*(d*x + c)/d - (672*e^{(-2*d*x - 2*c)} - 168*e^{(-4*d*x - 4*c)} + 32*e^{(-6*d*x - 6*c)} - 3*e^{(-8*d*x - 8*c)})/d) + 1/160*a^2*b*(3*e^{(5*d*x + 5*c)}/d - 25*e^{(3*d*x + 3*c)}/d + 150*e^{(d*x + c)}/d + 150*e^{(-d*x - c)}/d - 25*e^{(-3*d*x - 3*c)}/d + 3*e^{(-5*d*x - 5*c)}/d)$$

Fricas [B] time = 1.93065, size = 1546, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] $1/3548160*(315*b^3*\cosh(d*x + c)^{11} + 3465*b^3*\cosh(d*x + c)*\sinh(d*x + c)^{10} - 4235*b^3*\cosh(d*x + c)^9 + 83160*a*b^2*\cosh(d*x + c)*\sinh(d*x + c)^7 + 27225*b^3*\cosh(d*x + c)^7 + 3465*(15*b^3*\cosh(d*x + c)^3 - 11*b^3*\cosh(d*x + c))*\sinh(d*x + c)^8 + 1155*(126*b^3*\cosh(d*x + c)^5 - 308*b^3*\cosh(d*x + c)^3 + 165*b^3*\cosh(d*x + c))*\sinh(d*x + c)^6 + 2079*(64*a^2*b - 55*b^3)*\cosh(d*x + c)^5 + 83160*(7*a*b^2*\cosh(d*x + c)^3 - 8*a*b^2*\cosh(d*x + c))*\sinh(d*x + c)^5 + 3465*(30*b^3*\cosh(d*x + c)^7 - 154*b^3*\cosh(d*x + c)^5 + 275*b^3*\cosh(d*x + c)^3 + 3*(64*a^2*b - 55*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 34650*(32*a^2*b - 11*b^3)*\cosh(d*x + c)^3 + 27720*(21*a*b^2*\cosh(d*x + c)^5 - 80*a*b^2*\cosh(d*x + c)^3 + 84*a*b^2*\cosh(d*x + c))*\sinh(d*x + c)^3 - 27720*(64*a^3 - 105*a*b^2)*d*x + 3465*(5*b^3*\cosh(d*x + c)^9 - 44*b^3*\cosh(d*x + c)^7 + 165*b^3*\cosh(d*x + c)^5 + 6*(64*a^2*b - 55*b^3)*\cosh(d*x + c)^3 - 30*(32*a^2*b - 11*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 20790*(320*a^2*b - 77*b^3)*\cosh(d*x + c) + 27720*(3*a*b^2*\cosh(d*x + c)^7 - 24*a*b^2*\cosh(d*x + c)^5 + 84*a*b^2*\cosh(d*x + c)^3 + 8*(8*a^3 - 21*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/d$

Sympy [A] time = 83.4585, size = 498, normalized size = 1.71

$$\left\{ \frac{a^3 x \sinh^2(c+dx)}{2} - \frac{a^3 x \cosh^2(c+dx)}{2} + \frac{a^3 \sinh(c+dx) \cosh(c+dx)}{2d} + \frac{3a^2 b \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{4a^2 b \sinh^2(c+dx) \cosh^3(c+dx)}{d} + \frac{8a^2 b \cosh^5(c+dx)}{5d} \right\} x (a + b \sinh^3(c))^3 \sinh^2(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)

```
[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/(5*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 105*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x)**8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + b**3*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b**3*sinh(c + d*x)**8*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**6*cosh(c + d*x)**5/(3*d) - 32*b**3*sinh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c + d*x)**2*cosh(c + d*x)**9/(63*d) - 256*b**3*cosh(c + d*x)**11/(693*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**3*sinh(c)**2, True))
```

Giac [A] time = 1.44249, size = 545, normalized size = 1.87

$$315b^3e^{(11dx+11c)} - 4235b^3e^{(9dx+9c)} + 10395ab^2e^{(8dx+8c)} + 27225b^3e^{(7dx+7c)} - 110880ab^2e^{(6dx+6c)} + 133056a^2be^{(5dx+5c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/7096320*(315*b^3*e^(11*d*x + 11*c) - 4235*b^3*e^(9*d*x + 9*c) + 10395*a*b^2*e^(8*d*x + 8*c) + 27225*b^3*e^(7*d*x + 7*c) - 110880*a*b^2*e^(6*d*x + 6*c) + 133056*a^2*b*e^(5*d*x + 5*c) - 114345*b^3*e^(5*d*x + 5*c) + 582120*a*b^2*e^(4*d*x + 4*c) - 1108800*a^2*b*e^(3*d*x + 3*c) + 381150*b^3*e^(3*d*x + 3*c) + 887040*a^3*e^(2*d*x + 2*c) - 2328480*a*b^2*e^(2*d*x + 2*c) + 6652800*a^2*b*e^(d*x + c) - 1600830*b^3*e^(d*x + c) - 55440*(64*a^3 - 105*a*b^2)*(d*x + c) - (582120*a*b^2*e^(7*d*x + 7*c) - 110880*a*b^2*e^(5*d*x + 5*c) - 27225*b^3*e^(4*d*x + 4*c) + 10395*a*b^2*e^(3*d*x + 3*c) + 4235*b^3*e^(2*d*x + 2*c) - 315*b^3 - 20790*(320*a^2*b - 77*b^3)*e^(10*d*x + 10*c) + 110880*(8*a^3 - 21*a*b^2)*e^(9*d*x + 9*c) + 34650*(32*a^2*b - 11*b^3)*e^(8*d*x + 8*c) - 2079*(64*a^2*b - 55*b^3)*e^(6*d*x + 6*c))*e^(-11*d*x - 11*c))/d
```

3.162 $\int \sinh(c + dx) \left(a + b \sinh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=267

$$\frac{3a^2b \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{9a^2b \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{9}{8}a^2bx + \frac{a^3 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh^7(c + dx)}{7d}$$

[Out] (9*a^2*b*x)/8 - (63*b^3*x)/256 + (a^3*Cosh[c + d*x])/d - (3*a*b^2*Cosh[c + d*x])/d + (3*a*b^2*Cosh[c + d*x]^3)/d - (9*a*b^2*Cosh[c + d*x]^5)/(5*d) + (3*a*b^2*Cosh[c + d*x]^7)/(7*d) - (9*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (63*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + (3*a^2*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d) - (21*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(128*d) + (21*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(160*d) - (9*b^3*Cosh[c + d*x]*Sinh[c + d*x]^7)/(80*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^9)/(10*d)

Rubi [A] time = 0.222053, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3220, 2638, 2635, 8, 2633}

$$\frac{3a^2b \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{9a^2b \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{9}{8}a^2bx + \frac{a^3 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] (9*a^2*b*x)/8 - (63*b^3*x)/256 + (a^3*Cosh[c + d*x])/d - (3*a*b^2*Cosh[c + d*x])/d + (3*a*b^2*Cosh[c + d*x]^3)/d - (9*a*b^2*Cosh[c + d*x]^5)/(5*d) + (3*a*b^2*Cosh[c + d*x]^7)/(7*d) - (9*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (63*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + (3*a^2*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d) - (21*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(128*d) + (21*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(160*d) - (9*b^3*Cosh[c + d*x]*Sinh[c + d*x]^7)/(80*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^9)/(10*d)

Rule 3220

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^p_., x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 2635

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2633

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rubi steps

$$\begin{aligned}
 \int \sinh(c + dx) (a + b \sinh^3(c + dx))^3 dx &= -\left(i \int (ia^3 \sinh(c + dx) + 3ia^2b \sinh^4(c + dx) + 3iab^2 \sinh^7(c + dx) + ib^3 \sinh^{10}(c + dx)) dx\right) \\
 &= a^3 \int \sinh(c + dx) dx + (3a^2b) \int \sinh^4(c + dx) dx + (3ab^2) \int \sinh^7(c + dx) dx \\
 &= \frac{a^3 \cosh(c + dx)}{d} + \frac{3a^2b \cosh(c + dx) \sinh^3(c + dx)}{4d} + \frac{b^3 \cosh(c + dx) \sinh^9(c + dx)}{10d} \\
 &= \frac{a^3 \cosh(c + dx)}{d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh^3(c + dx)}{d} - \frac{9ab^2 \cosh^5(c + dx)}{5d} \\
 &= \frac{9}{8}a^2bx + \frac{a^3 \cosh(c + dx)}{d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh^3(c + dx)}{d} - \frac{9ab^2 \cosh^5(c + dx)}{5d} \\
 &= \frac{9}{8}a^2bx + \frac{a^3 \cosh(c + dx)}{d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh^3(c + dx)}{d} - \frac{9ab^2 \cosh^5(c + dx)}{5d} \\
 &= \frac{9}{8}a^2bx + \frac{a^3 \cosh(c + dx)}{d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh^3(c + dx)}{d} - \frac{9ab^2 \cosh^5(c + dx)}{5d} \\
 &= \frac{9}{8}a^2bx - \frac{63b^3x}{256} + \frac{a^3 \cosh(c + dx)}{d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{3ab^2 \cosh^3(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.459003, size = 184, normalized size = 0.69

$$\frac{1120a(64a^2 - 105b^2) \cosh(c + dx) + b(-53760a^2 \sinh(2(c + dx)) + 6720a^2 \sinh(4(c + dx)) + 80640a^2c + 80640a^2dx - 80640ab^2c - 17640ab^2dx + 23520ab^2 \cosh(3(c + dx)) - 4704ab^2 \cosh(5(c + dx)) + 480ab^2 \cosh(7(c + dx)) - 53760a^2 \sinh(2(c + dx)) + 14700b^2 \sinh(2(c + dx)) + 6720a^2 \sinh(4(c + dx)) - 4200b^2 \sinh(4(c + dx)) + 1050b^2 \sinh(6(c + dx)) - 175b^2 \sinh(8(c + dx)) + 14b^2 \sinh(10(c + dx)))}{(71680d)}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] (1120*a*(64*a^2 - 105*b^2)*Cosh[c + d*x] + b*(80640*a^2*c - 17640*b^2*c + 80640*a^2*d*x - 17640*b^2*d*x + 23520*a*b*Cosh[3*(c + d*x)] - 4704*a*b*Cosh[5*(c + d*x)] + 480*a*b*Cosh[7*(c + d*x)] - 53760*a^2*Sinh[2*(c + d*x)] + 14700*b^2*Sinh[2*(c + d*x)] + 6720*a^2*Sinh[4*(c + d*x)] - 4200*b^2*Sinh[4*(c + d*x)] + 1050*b^2*Sinh[6*(c + d*x)] - 175*b^2*Sinh[8*(c + d*x)] + 14*b^2*Sinh[10*(c + d*x)])/(71680*d)

Maple [A] time = 0.023, size = 168, normalized size = 0.6

$$\frac{1}{d} \left(b^3 \left(\left(\frac{(\sinh(dx+c))^9}{10} - \frac{9(\sinh(dx+c))^7}{80} + \frac{21(\sinh(dx+c))^5}{160} - \frac{21(\sinh(dx+c))^3}{128} + \frac{63 \sinh(dx+c)}{256} \right) \cosh(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x)

[Out] 1/d*(b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/128*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+3*a*b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^3*cosh(d*x+c))

Maxima [A] time = 1.14679, size = 429, normalized size = 1.61

$$\frac{3}{64} a^2 b \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) - \frac{1}{20480} b^3 \left(\frac{(25e^{-2dx-2c} - 150e^{-4dx-4c} + 600e^{-6dx-6c} - 150e^{-8dx-8c} + 25e^{-10dx-10c}) \cosh(dx+c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + a^3*cosh(d*x + c)/d
```

Fricas [A] time = 2.00611, size = 1193, normalized size = 4.47

$$35b^3 \cosh(dx + c) \sinh(dx + c)^9 + 120ab^2 \cosh(dx + c)^7 + 840ab^2 \cosh(dx + c) \sinh(dx + c)^6 - 1176ab^2 \cosh(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/17920*(35*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 120*a*b^2*cosh(d*x + c)^7 + 840*a*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 1176*a*b^2*cosh(d*x + c)^5 + 70*(6*b^3*cosh(d*x + c)^3 - 5*b^3*cosh(d*x + c))*sinh(d*x + c)^7 + 5880*a*b^2*cosh(d*x + c)^3 + 7*(126*b^3*cosh(d*x + c)^5 - 350*b^3*cosh(d*x + c)^3 + 225*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 840*(5*a*b^2*cosh(d*x + c)^3 - 7*a*b^2*cosh(d*x + c))*sinh(d*x + c)^4 + 70*(6*b^3*cosh(d*x + c)^7 - 35*b^3*cosh(d*x + c)^5 + 75*b^3*cosh(d*x + c)^3 + 12*(8*a^2*b - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 630*(32*a^2*b - 7*b^3)*d*x + 840*(3*a*b^2*cosh(d*x + c)^5 - 14*a*b^2*cosh(d*x + c)^3 + 21*a*b^2*cosh(d*x + c))*sinh(d*x + c)^2 + 280*(64*a^3 - 105*a*b^2)*cosh(d*x + c) + 35*(b^3*cosh(d*x + c)^9 - 10*b^3*cosh(d*x + c)^7 + 45*b^3*cosh(d*x + c)^5 + 24*(8*a^2*b - 5*b^3)*cosh(d*x + c)^3 - 6*(128*a^2*b - 35*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 97.2029, size = 496, normalized size = 1.86

$$\left\{ \begin{array}{l} \frac{a^3 \cosh(c+dx)}{d} + \frac{9a^2bx \sinh^4(c+dx)}{8} - \frac{9a^2bx \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{9a^2bx \cosh^4(c+dx)}{8} + \frac{15a^2b \sinh^3(c+dx) \cosh(c+dx)}{8d} - \frac{9a^2b \sinh(c+dx) \cosh^3(c+dx)}{8d} \\ x \left(a + b \sinh^3(c) \right)^3 \sinh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*a**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 3*a*b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 6*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 24*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 48*a*b**2*cosh(c + d*x)**7/(35*d) + 63*b**3*x*sinh(c + d*x)**10/256 - 315*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 315*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 315*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b**3*x*cosh(c + d*x)**10/256 + 193*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21*b**3*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 147*b**3*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 63*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**3*sinh(c), True))
```

Giac [A] time = 1.47622, size = 474, normalized size = 1.78

$$14b^3e^{(10dx+10c)} - 175b^3e^{(8dx+8c)} + 480ab^2e^{(7dx+7c)} + 1050b^3e^{(6dx+6c)} - 4704ab^2e^{(5dx+5c)} + 6720a^2be^{(4dx+4c)} - 4200b^3e^{(4dx+4c)} + 23520a^2be^{(3dx+3c)} - 53760a^2be^{(2dx+2c)} + 14700b^3e^{(2dx+2c)} + 71680a^3e^{(dx+c)} - 117600a^2be^{(dx+c)} + 5040(32a^2b - 7b^3)(dx+c) + (23520a^2be^{(7dx+7c)} - 4704a^2be^{(5dx+5c)} - 1050b^3e^{(4dx+4c)} + 480a^2be^{(3dx+3c)} + 175b^3e^{(2dx+2c)} - 14b^3 + 1120(64a^3 - 105a^2b^2)e^{(9dx+9c)} + 420(128a^2b - 35b^3)e^{(8dx+8c)} - 840(8a^2b - 5b^3)e^{(6dx+6c)})e^{(-10dx-10c)}/d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/143360*(14*b^3*e^(10*d*x + 10*c) - 175*b^3*e^(8*d*x + 8*c) + 480*a*b^2*e^(7*d*x + 7*c) + 1050*b^3*e^(6*d*x + 6*c) - 4704*a*b^2*e^(5*d*x + 5*c) + 6720*a^2*b*e^(4*d*x + 4*c) - 4200*b^3*e^(4*d*x + 4*c) + 23520*a*b^2*e^(3*d*x + 3*c) - 53760*a^2*b*e^(2*d*x + 2*c) + 14700*b^3*e^(2*d*x + 2*c) + 71680*a^3*e^(d*x + c) - 117600*a*b^2*e^(d*x + c) + 5040*(32*a^2*b - 7*b^3)*(d*x + c) + (23520*a*b^2*e^(7*d*x + 7*c) - 4704*a*b^2*e^(5*d*x + 5*c) - 1050*b^3*e^(4*d*x + 4*c) + 480*a*b^2*e^(3*d*x + 3*c) + 175*b^3*e^(2*d*x + 2*c) - 14*b^3 + 1120*(64*a^3 - 105*a*b^2)*e^(9*d*x + 9*c) + 420*(128*a^2*b - 35*b^3)*e^(8*d*x + 8*c) - 840*(8*a^2*b - 5*b^3)*e^(6*d*x + 6*c))*e^(-10*d*x - 10*c))/d
```

3.163 $\int (a + b \sinh^3(c + dx))^3 dx$

Optimal. Leaf size=204

$$\frac{a^2 b \cosh^3(c + dx)}{d} - \frac{3a^2 b \cosh(c + dx)}{d} + a^3 x + \frac{ab^2 \sinh^5(c + dx) \cosh(c + dx)}{2d} - \frac{5ab^2 \sinh^3(c + dx) \cosh(c + dx)}{8d} + \frac{15a^2 b \cosh^3(c + dx)}{d} - \frac{3a^2 b \cosh(c + dx)}{d} + a^3 x + \frac{ab^2 \sinh^5(c + dx) \cosh(c + dx)}{2d} - \frac{5ab^2 \sinh^3(c + dx) \cosh(c + dx)}{8d} + \frac{15a^2 b \cosh^3(c + dx)}{d}$$

```
[Out] a^3*x - (15*a*b^2*x)/16 - (3*a^2*b*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x])/d
+ (a^2*b*Cosh[c + d*x]^3)/d - (4*b^3*Cosh[c + d*x]^3)/(3*d) + (6*b^3*Cosh[
c + d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(
9*d) + (15*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*a*b^2*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(8*d) + (a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(2*d)
```

Rubi [A] time = 0.127754, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3213, 2633, 2635, 8}

$$\frac{a^2 b \cosh^3(c + dx)}{d} - \frac{3a^2 b \cosh(c + dx)}{d} + a^3 x + \frac{ab^2 \sinh^5(c + dx) \cosh(c + dx)}{2d} - \frac{5ab^2 \sinh^3(c + dx) \cosh(c + dx)}{8d} + \frac{15a^2 b \cosh^3(c + dx)}{d} - \frac{3a^2 b \cosh(c + dx)}{d} + a^3 x + \frac{ab^2 \sinh^5(c + dx) \cosh(c + dx)}{2d} - \frac{5ab^2 \sinh^3(c + dx) \cosh(c + dx)}{8d} + \frac{15a^2 b \cosh^3(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^3, x]
```

```
[Out] a^3*x - (15*a*b^2*x)/16 - (3*a^2*b*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x])/d
+ (a^2*b*Cosh[c + d*x]^3)/d - (4*b^3*Cosh[c + d*x]^3)/(3*d) + (6*b^3*Cosh[
c + d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(
9*d) + (15*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*a*b^2*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(8*d) + (a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(2*d)
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :=
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^3(c + dx))^3 dx &= \int (a^3 + 3a^2b \sinh^3(c + dx) + 3ab^2 \sinh^6(c + dx) + b^3 \sinh^9(c + dx)) dx \\
&= a^3x + (3a^2b) \int \sinh^3(c + dx) dx + (3ab^2) \int \sinh^6(c + dx) dx + b^3 \int \sinh^9(c + dx) dx \\
&= a^3x + \frac{ab^2 \cosh(c + dx) \sinh^5(c + dx)}{2d} - \frac{1}{2} (5ab^2) \int \sinh^4(c + dx) dx - \frac{(3a^2b) \text{Subst}(\int)}{3d} \\
&= a^3x - \frac{3a^2b \cosh(c + dx)}{d} + \frac{b^3 \cosh(c + dx)}{d} + \frac{a^2b \cosh^3(c + dx)}{d} - \frac{4b^3 \cosh^3(c + dx)}{3d} + \\
&= a^3x - \frac{3a^2b \cosh(c + dx)}{d} + \frac{b^3 \cosh(c + dx)}{d} + \frac{a^2b \cosh^3(c + dx)}{d} - \frac{4b^3 \cosh^3(c + dx)}{3d} + \\
&= a^3x - \frac{15}{16} ab^2x - \frac{3a^2b \cosh(c + dx)}{d} + \frac{b^3 \cosh(c + dx)}{d} + \frac{a^2b \cosh^3(c + dx)}{d} - \frac{4b^3 \cosh^3(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.251226, size = 159, normalized size = 0.78

$$5670b(7b^2 - 32a^2) \cosh(c + dx) + 1260(16a^2b - 7b^3) \cosh(3(c + dx)) + 80640a^3c + 80640a^3dx + 56700ab^2 \sinh(2(c + dx))$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (80640*a^3*c - 75600*a*b^2*c + 80640*a^3*d*x - 75600*a*b^2*d*x + 5670*b*(-3
2*a^2 + 7*b^2)*Cosh[c + d*x] + 1260*(16*a^2*b - 7*b^3)*Cosh[3*(c + d*x)] +
2268*b^3*Cosh[5*(c + d*x)] - 405*b^3*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(c +
d*x)] + 56700*a*b^2*Sinh[2*(c + d*x)] - 11340*a*b^2*Sinh[4*(c + d*x)] + 12
60*a*b^2*Sinh[6*(c + d*x)])/(80640*d)
```

Maple [A] time = 0.02, size = 141, normalized size = 0.7

$$\frac{1}{d} \left(b^3 \left(\frac{128}{315} + \frac{(\sinh(dx+c))^8}{9} - \frac{8(\sinh(dx+c))^6}{63} + \frac{16(\sinh(dx+c))^4}{105} - \frac{64(\sinh(dx+c))^2}{315} \right) \cosh(dx+c) + 3ab^2 \left(\left(\frac{1}{6} \sinh(dx+c)^5 - \frac{5}{24} \sinh(dx+c)^3 + \frac{5}{16} \sinh(dx+c) \right) \cosh(dx+c) - \frac{5}{16} dx - \frac{5}{16} c \right) + 3a^2 b \left(-\frac{2}{3} + \frac{1}{3} \sinh(dx+c)^2 \right) \cosh(dx+c) + a^3 (dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(d*x+c)^3)^3,x)

[Out] 1/d*(b^3*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+3*a^2*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(d*x+c)

Maxima [A] time = 1.17724, size = 378, normalized size = 1.85

$$a^3 x - \frac{1}{161280} b^3 \left(\frac{(405 e^{(-2dx-2c)} - 2268 e^{(-4dx-4c)} + 8820 e^{(-6dx-6c)} - 39690 e^{(-8dx-8c)} - 35) e^{(9dx+9c)}}{d} - \frac{39690 e^{(-dx-c)} - 8820 e^{(-3dx-3c)} + 2268 e^{(-5dx-5c)} - 405 e^{(-7dx-7c)} + 35 e^{(-9dx-9c)}}{d} - \frac{1}{128} a b^2 \left(\frac{(9 e^{(-2dx-2c)} - 45 e^{(-4dx-4c)} - 1) e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{(45 e^{(-2dx-2c)} - 9 e^{(-4dx-4c)} + e^{(-6dx-6c)})}{d} + \frac{1}{8} a^2 b \left(\frac{e^{(3dx+3c)}}{d} - \frac{9 e^{(dx+c)}}{d} - \frac{9 e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] a^3*x - 1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/8*a^2*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)

Fricas [B] time = 1.81282, size = 1017, normalized size = 4.99

$$35 b^3 \cosh(dx+c)^9 + 315 b^3 \cosh(dx+c) \sinh(dx+c)^8 - 405 b^3 \cosh(dx+c)^7 + 7560 ab^2 \cosh(dx+c) \sinh(dx+c)^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] $\frac{1}{80640} (35b^3 \cosh(dx+c)^9 + 315b^3 \cosh(dx+c) \sinh(dx+c)^8 - 405b^3 \cosh(dx+c)^7 + 7560a^2b^2 \cosh(dx+c) \sinh(dx+c)^5 + 2268b^3 \cosh(dx+c)^5 + 105(28b^3 \cosh(dx+c)^3 - 27b^3 \cosh(dx+c)) \sinh(dx+c)^6 + 315(14b^3 \cosh(dx+c)^5 - 45b^3 \cosh(dx+c)^3 + 36b^3 \cosh(dx+c)) \sinh(dx+c)^4 + 1260(16a^2b - 7b^3) \cosh(dx+c)^3 + 5040(5a^2b^2 \cosh(dx+c)^3 - 9a^2b^2 \cosh(dx+c)) \sinh(dx+c)^3 + 5040(16a^3 - 15a^2b^2) dx + 315(4b^3 \cosh(dx+c)^7 - 27b^3 \cosh(dx+c)^5 + 72b^3 \cosh(dx+c)^3 + 12(16a^2b - 7b^3) \cosh(dx+c)) \sinh(dx+c)^2 - 5670(32a^2b - 7b^3) \cosh(dx+c) + 7560(a^2b^2 \cosh(dx+c)^5 - 6a^2b^2 \cosh(dx+c)^3 + 15a^2b^2 \cosh(dx+c)) \sinh(dx+c)) / d$

Sympy [A] time = 33.4784, size = 340, normalized size = 1.67

$$\left\{ \begin{array}{l} a^3x + \frac{3a^2b \sinh^2(c+dx) \cosh(c+dx)}{d^3} - \frac{2a^2b \cosh^3(c+dx)}{d} + \frac{15ab^2x \sinh^6(c+dx)}{16} - \frac{45ab^2x \sinh^4(c+dx) \cosh^2(c+dx)}{16} + \frac{45ab^2x \sinh^2(c+dx) \cosh^4(c+dx)}{16} \\ x(a + b \sinh^3(c)) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)**3)**3,x)

[Out] Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*b*cosh(c + d*x)**3/d + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a*b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + b**3*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**3*sinh(c + d*x)**6*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**3*cosh(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**3, True))

Giac [A] time = 1.16286, size = 405, normalized size = 1.99

$$35b^3e^{(9dx+9c)} - 405b^3e^{(7dx+7c)} + 1260ab^2e^{(6dx+6c)} + 2268b^3e^{(5dx+5c)} - 11340ab^2e^{(4dx+4c)} + 20160a^2be^{(3dx+3c)} - 8820$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{161280} \cdot (35 \cdot b^3 \cdot e^{(9 \cdot d \cdot x + 9 \cdot c)} - 405 \cdot b^3 \cdot e^{(7 \cdot d \cdot x + 7 \cdot c)} + 1260 \cdot a \cdot b^2 \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)} + 2268 \cdot b^3 \cdot e^{(5 \cdot d \cdot x + 5 \cdot c)} - 11340 \cdot a \cdot b^2 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 20160 \cdot a^2 \cdot b \cdot e^{(3 \cdot d \cdot x + 3 \cdot c)} - 8820 \cdot b^3 \cdot e^{(3 \cdot d \cdot x + 3 \cdot c)} + 56700 \cdot a \cdot b^2 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - 181440 \cdot a^2 \cdot b \cdot e^{(d \cdot x + c)} + 39690 \cdot b^3 \cdot e^{(d \cdot x + c)} + 10080 \cdot (16 \cdot a^3 - 15 \cdot a \cdot b^2) \cdot (d \cdot x + c) - (56700 \cdot a \cdot b^2 \cdot e^{(7 \cdot d \cdot x + 7 \cdot c)} - 11340 \cdot a \cdot b^2 \cdot e^{(5 \cdot d \cdot x + 5 \cdot c)} - 2268 \cdot b^3 \cdot e^{(4 \cdot d \cdot x + 4 \cdot c)} + 1260 \cdot a \cdot b^2 \cdot e^{(3 \cdot d \cdot x + 3 \cdot c)} + 405 \cdot b^3 \cdot e^{(2 \cdot d \cdot x + 2 \cdot c)} - 35 \cdot b^3 + 5670 \cdot (32 \cdot a^2 \cdot b - 7 \cdot b^3) \cdot e^{(8 \cdot d \cdot x + 8 \cdot c)} - 1260 \cdot (16 \cdot a^2 \cdot b - 7 \cdot b^3) \cdot e^{(6 \cdot d \cdot x + 6 \cdot c)}) \cdot e^{(-9 \cdot d \cdot x - 9 \cdot c)}) / d$

3.164 $\int \operatorname{csch}(c + dx) \left(a + b \sinh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=201

$$\frac{3a^2b \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{3}{2}a^2bx - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3ab^2 \cosh^5(c + dx)}{5d} - \frac{2ab^2 \cosh^3(c + dx)}{d} + \frac{3ab^2 \cosh(c + dx)}{d}$$

[Out] $(-3*a^2*b*x)/2 + (35*b^3*x)/128 - (a^3*ArcTanh[Cosh[c + d*x]])/d + (3*a*b^2*Cosh[c + d*x])/d - (2*a*b^2*Cosh[c + d*x]^3)/d + (3*a*b^2*Cosh[c + d*x]^5)/(5*d) + (3*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (35*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) - (7*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)$

Rubi [A] time = 0.188718, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3220, 3770, 2635, 8, 2633}

$$\frac{3a^2b \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{3}{2}a^2bx - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3ab^2 \cosh^5(c + dx)}{5d} - \frac{2ab^2 \cosh^3(c + dx)}{d} + \frac{3ab^2 \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] $(-3*a^2*b*x)/2 + (35*b^3*x)/128 - (a^3*ArcTanh[Cosh[c + d*x]])/d + (3*a*b^2*Cosh[c + d*x])/d - (2*a*b^2*Cosh[c + d*x]^3)/d + (3*a*b^2*Cosh[c + d*x]^5)/(5*d) + (3*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (35*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) - (7*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)$

Rule 3220

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] *(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}(c + dx) (a + b \sinh^3(c + dx))^3 dx &= i \int (-ia^3 \operatorname{csch}(c + dx) - 3ia^2b \sinh^2(c + dx) - 3iab^2 \sinh^5(c + dx) - ib^3 \sinh^8(c + dx)) dx \\
 &= a^3 \int \operatorname{csch}(c + dx) dx + (3a^2b) \int \sinh^2(c + dx) dx + (3ab^2) \int \sinh^5(c + dx) dx + b^3 \int \sinh^8(c + dx) dx \\
 &= -\frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3a^2b \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{b^3 \cosh(c + dx) \sinh^3(c + dx)}{3d} \\
 &= -\frac{3}{2}a^2bx - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3ab^2 \cosh(c + dx)}{d} - \frac{2ab^2 \cosh^3(c + dx)}{d} \\
 &= -\frac{3}{2}a^2bx - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3ab^2 \cosh(c + dx)}{d} - \frac{2ab^2 \cosh^3(c + dx)}{d} \\
 &= -\frac{3}{2}a^2bx - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3ab^2 \cosh(c + dx)}{d} - \frac{2ab^2 \cosh^3(c + dx)}{d} \\
 &= -\frac{3}{2}a^2bx + \frac{35b^3x}{128} - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3ab^2 \cosh(c + dx)}{d} - \frac{2ab^2 \cosh^3(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.21892, size = 158, normalized size = 0.79

$$11520a^2b \sinh(2(c + dx)) - 23040a^2bc - 23040a^2bdx + 15360a^3 \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right) + 28800ab^2 \cosh(c + dx) - 4800$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] $(-23040*a^2*b*c + 4200*b^3*c - 23040*a^2*b*d*x + 4200*b^3*d*x + 28800*a*b^2*\text{Cosh}[c + d*x] - 4800*a*b^2*\text{Cosh}[3*(c + d*x)] + 576*a*b^2*\text{Cosh}[5*(c + d*x)] + 15360*a^3*\text{Log}[\text{Tanh}[(c + d*x)/2]] + 11520*a^2*b*\text{Sinh}[2*(c + d*x)] - 3360*b^3*\text{Sinh}[2*(c + d*x)] + 840*b^3*\text{Sinh}[4*(c + d*x)] - 160*b^3*\text{Sinh}[6*(c + d*x)] + 15*b^3*\text{Sinh}[8*(c + d*x)])/(15360*d)$

Maple [A] time = 0.072, size = 138, normalized size = 0.7

$$\frac{1}{d} \left(-2a^3 \text{Artanh}(e^{dx+c}) + 3a^2b \left(\frac{1}{2} \cosh(dx+c) \sinh(dx+c) - \frac{1}{2} dx - \frac{c}{2} \right) + 3ab^2 \left(\frac{8}{15} + \frac{1}{5} (\sinh(dx+c))^4 - \frac{4}{5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x)

[Out] $\frac{1}{d} * (-2*a^3*\text{arctanh}(\exp(d*x+c)) + 3*a^2*b*(\frac{1}{2}*\cosh(d*x+c)*\sinh(d*x+c) - \frac{1}{2}*d*x - \frac{1}{2}*c) + 3*a*b^2*(\frac{8}{15} + \frac{1}{5}*\sinh(d*x+c)^4 - \frac{4}{15}*\sinh(d*x+c)^2)*\cosh(d*x+c) + b^3*((\frac{1}{8}*\sinh(d*x+c)^7 - \frac{7}{48}*\sinh(d*x+c)^5 + \frac{35}{192}*\sinh(d*x+c)^3 - \frac{35}{128}*\sinh(d*x+c))*\cosh(d*x+c) + \frac{35}{128}*d*x + \frac{35}{128}*c)$

Maxima [A] time = 1.18444, size = 347, normalized size = 1.73

$$-\frac{3}{8}a^2b \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) - \frac{1}{6144}b^3 \left(\frac{(32e^{(-2dx-2c)} - 168e^{(-4dx-4c)} + 672e^{(-6dx-6c)} - 3)e^{(8dx+8c)}}{d} - \frac{1680(dx+c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] $-3/8*a^2*b*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/6144*b^3*((32*e^{(-2*d*x - 2*c)} - 168*e^{(-4*d*x - 4*c)} + 672*e^{(-6*d*x - 6*c)} - 3)*e^{(8*d*x + 8*c)}/d - 1680*(d*x + c)/d - (672*e^{(-2*d*x - 2*c)} - 168*e^{(-4*d*x - 4*c)} + 32*e^{(-6*d*x - 6*c)} - 3*e^{(-8*d*x - 8*c)})/d + 1/160*a*b^2*(3*e^{(5*d*x + 5*c)}/d - 25*e^{(3*d*x + 3*c)}/d + 150*e^{(d*x + c)}/d + 150*e^{(-d*x - c)}/d -$

$$25e^{(-3dx - 3c)/d} + 3e^{(-5dx - 5c)/d} + a^3 \log(\tanh(1/2dx + 1/2c))/d$$

Fricas [B] time = 2.07054, size = 7191, normalized size = 35.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)*(a+b*sinh(dx+c))^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/30720*(15*b^3*\cosh(dx + c)^{16} + 240*b^3*\cosh(dx + c)*\sinh(dx + c)^{15} + \\ & 15*b^3*\sinh(dx + c)^{16} - 160*b^3*\cosh(dx + c)^{14} + 576*a*b^2*\cosh(dx + \\ & c)^{13} + 840*b^3*\cosh(dx + c)^{12} + 40*(45*b^3*\cosh(dx + c)^2 - 4*b^3)*\sinh \\ & (dx + c)^{14} - 4800*a*b^2*\cosh(dx + c)^{11} + 16*(525*b^3*\cosh(dx + c)^3 - \\ & 140*b^3*\cosh(dx + c) + 36*a*b^2)*\sinh(dx + c)^{13} + 4*(6825*b^3*\cosh(dx + \\ & c)^4 - 3640*b^3*\cosh(dx + c)^2 + 1872*a*b^2*\cosh(dx + c) + 210*b^3)*\sinh \\ & (dx + c)^{12} + 28800*a*b^2*\cosh(dx + c)^9 + 16*(4095*b^3*\cosh(dx + c)^5 - \\ & 3640*b^3*\cosh(dx + c)^3 + 2808*a*b^2*\cosh(dx + c)^2 + 630*b^3*\cosh(dx + \\ & c) - 300*a*b^2)*\sinh(dx + c)^{11} - 240*(192*a^2*b - 35*b^3)*dx*\cosh(dx + \\ & c)^8 + 480*(24*a^2*b - 7*b^3)*\cosh(dx + c)^{10} + 8*(15015*b^3*\cosh(dx + c) \\ &)^6 - 20020*b^3*\cosh(dx + c)^4 + 20592*a*b^2*\cosh(dx + c)^3 + 6930*b^3*\co \\ & sh(dx + c)^2 - 6600*a*b^2*\cosh(dx + c) + 1440*a^2*b - 420*b^3)*\sinh(dx + \\ & c)^{10} + 28800*a*b^2*\cosh(dx + c)^7 + 80*(2145*b^3*\cosh(dx + c)^7 - 4004* \\ & b^3*\cosh(dx + c)^5 + 5148*a*b^2*\cosh(dx + c)^4 + 2310*b^3*\cosh(dx + c)^3 \\ & - 3300*a*b^2*\cosh(dx + c)^2 + 360*a*b^2 + 60*(24*a^2*b - 7*b^3)*\cosh(dx \\ & + c))*\sinh(dx + c)^9 + 6*(32175*b^3*\cosh(dx + c)^8 - 80080*b^3*\cosh(dx + \\ & c)^6 + 123552*a*b^2*\cosh(dx + c)^5 + 69300*b^3*\cosh(dx + c)^4 - 132000*a \\ & *b^2*\cosh(dx + c)^3 + 43200*a*b^2*\cosh(dx + c) - 40*(192*a^2*b - 35*b^3)* \\ & dx + 3600*(24*a^2*b - 7*b^3)*\cosh(dx + c)^2)*\sinh(dx + c)^8 - 4800*a*b^2 \\ & *\cosh(dx + c)^5 + 48*(3575*b^3*\cosh(dx + c)^9 - 11440*b^3*\cosh(dx + c)^7 \\ & + 20592*a*b^2*\cosh(dx + c)^6 + 13860*b^3*\cosh(dx + c)^5 - 33000*a*b^2*\co \\ & sh(dx + c)^4 + 21600*a*b^2*\cosh(dx + c)^2 - 40*(192*a^2*b - 35*b^3)*dx*c \\ & osh(dx + c) + 1200*(24*a^2*b - 7*b^3)*\cosh(dx + c)^3 + 600*a*b^2)*\sinh(dx \\ & x + c)^7 - 840*b^3*\cosh(dx + c)^4 - 480*(24*a^2*b - 7*b^3)*\cosh(dx + c)^6 \\ & + 24*(5005*b^3*\cosh(dx + c)^{10} - 20020*b^3*\cosh(dx + c)^8 + 41184*a*b^2* \\ & \cosh(dx + c)^7 + 32340*b^3*\cosh(dx + c)^6 - 92400*a*b^2*\cosh(dx + c)^5 + \\ & 100800*a*b^2*\cosh(dx + c)^3 - 280*(192*a^2*b - 35*b^3)*dx*\cosh(dx + c)^ \\ & 2 + 4200*(24*a^2*b - 7*b^3)*\cosh(dx + c)^4 + 8400*a*b^2*\cosh(dx + c) - 48 \\ & 0*a^2*b + 140*b^3)*\sinh(dx + c)^6 + 576*a*b^2*\cosh(dx + c)^3 + 16*(4095*b \\ & ^3*\cosh(dx + c)^{11} - 20020*b^3*\cosh(dx + c)^9 + 46332*a*b^2*\cosh(dx + c) \\ & ^8 + 41580*b^3*\cosh(dx + c)^7 - 138600*a*b^2*\cosh(dx + c)^6 + 226800*a*b^ \end{aligned}$$

$$\begin{aligned}
& 2*\cosh(d*x + c)^4 - 840*(192*a^2*b - 35*b^3)*d*x*\cosh(d*x + c)^3 + 7560*(24*a^2*b - 7*b^3)*\cosh(d*x + c)^5 + 37800*a*b^2*\cosh(d*x + c)^2 - 300*a*b^2 - \\
& 180*(24*a^2*b - 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 160*b^3*\cosh(d*x + c)^2 + 20*(1365*b^3*\cosh(d*x + c)^12 - 8008*b^3*\cosh(d*x + c)^10 + 20592*a \\
& *b^2*\cosh(d*x + c)^9 + 20790*b^3*\cosh(d*x + c)^8 - 79200*a*b^2*\cosh(d*x + c)^7 + 181440*a*b^2*\cosh(d*x + c)^5 - 840*(192*a^2*b - 35*b^3)*d*x*\cosh(d*x \\
& + c)^4 + 5040*(24*a^2*b - 7*b^3)*\cosh(d*x + c)^6 + 50400*a*b^2*\cosh(d*x + c)^3 - 1200*a*b^2*\cosh(d*x + c) - 42*b^3 - 360*(24*a^2*b - 7*b^3)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^4 + 16*(525*b^3*\cosh(d*x + c)^13 - 3640*b^3*\cosh(d*x + c)^11 + 10296*a*b^2*\cosh(d*x + c)^10 + 11550*b^3*\cosh(d*x + c)^9 - 49500*a \\
& *b^2*\cosh(d*x + c)^8 + 151200*a*b^2*\cosh(d*x + c)^6 - 840*(192*a^2*b - 35*b^3)*d*x*\cosh(d*x + c)^5 + 3600*(24*a^2*b - 7*b^3)*\cosh(d*x + c)^7 + 63000*a \\
& *b^2*\cosh(d*x + c)^4 - 3000*a*b^2*\cosh(d*x + c)^2 - 210*b^3*\cosh(d*x + c) - 600*(24*a^2*b - 7*b^3)*\cosh(d*x + c)^3 + 36*a*b^2)*\sinh(d*x + c)^3 - 15*b^ \\
& 3 + 8*(225*b^3*\cosh(d*x + c)^14 - 1820*b^3*\cosh(d*x + c)^12 + 5616*a*b^2*\cosh(d*x + c)^11 + 6930*b^3*\cosh(d*x + c)^10 - 33000*a*b^2*\cosh(d*x + c)^9 + \\
& 129600*a*b^2*\cosh(d*x + c)^7 - 840*(192*a^2*b - 35*b^3)*d*x*\cosh(d*x + c)^6 + 2700*(24*a^2*b - 7*b^3)*\cosh(d*x + c)^8 + 75600*a*b^2*\cosh(d*x + c)^5 - \\
& 6000*a*b^2*\cosh(d*x + c)^3 - 630*b^3*\cosh(d*x + c)^2 - 900*(24*a^2*b - 7*b^3)*\cosh(d*x + c)^4 + 216*a*b^2*\cosh(d*x + c) + 20*b^3)*\sinh(d*x + c)^2 - 30 \\
& 720*(a^3*\cosh(d*x + c)^8 + 8*a^3*\cosh(d*x + c)^7*\sinh(d*x + c) + 28*a^3*\cosh(d*x + c)^6*\sinh(d*x + c)^2 + 56*a^3*\cosh(d*x + c)^5*\sinh(d*x + c)^3 + 70* \\
& a^3*\cosh(d*x + c)^4*\sinh(d*x + c)^4 + 56*a^3*\cosh(d*x + c)^3*\sinh(d*x + c)^5 + 28*a^3*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 8*a^3*\cosh(d*x + c)*\sinh(d*x + \\
& c)^7 + a^3*\sinh(d*x + c)^8)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 30720 \\
& *(a^3*\cosh(d*x + c)^8 + 8*a^3*\cosh(d*x + c)^7*\sinh(d*x + c) + 28*a^3*\cosh(d*x + c)^6*\sinh(d*x + c)^2 + 56*a^3*\cosh(d*x + c)^5*\sinh(d*x + c)^3 + 70*a^3 \\
& *\cosh(d*x + c)^4*\sinh(d*x + c)^4 + 56*a^3*\cosh(d*x + c)^3*\sinh(d*x + c)^5 + 28*a^3*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 8*a^3*\cosh(d*x + c)*\sinh(d*x + c) \\
& ^7 + a^3*\sinh(d*x + c)^8)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 16*(15*b \\
& ^3*\cosh(d*x + c)^15 - 140*b^3*\cosh(d*x + c)^13 + 468*a*b^2*\cosh(d*x + c)^12 + 630*b^3*\cosh(d*x + c)^11 - 3300*a*b^2*\cosh(d*x + c)^10 + 16200*a*b^2*\cosh \\
& h(d*x + c)^8 - 120*(192*a^2*b - 35*b^3)*d*x*\cosh(d*x + c)^7 + 300*(24*a^2*b \\
& - 7*b^3)*\cosh(d*x + c)^9 + 12600*a*b^2*\cosh(d*x + c)^6 - 1500*a*b^2*\cosh(d \\
& *x + c)^4 - 210*b^3*\cosh(d*x + c)^3 - 180*(24*a^2*b - 7*b^3)*\cosh(d*x + c)^ \\
& 5 + 108*a*b^2*\cosh(d*x + c)^2 + 20*b^3*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^8 + 8*d*\cosh(d*x + c)^7*\sinh(d*x + c) + 28*d*\cosh(d*x + c)^6*\sinh \\
& h(d*x + c)^2 + 56*d*\cosh(d*x + c)^5*\sinh(d*x + c)^3 + 70*d*\cosh(d*x + c)^4*\sinh(d*x + c)^4 + 56*d*\cosh(d*x + c)^3*\sinh(d*x + c)^5 + 28*d*\cosh(d*x + c) \\
& ^2*\sinh(d*x + c)^6 + 8*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + d*\sinh(d*x + c)^8)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)

[Out] Timed out

Giac [A] time = 1.40701, size = 427, normalized size = 2.12

$$-\frac{a^3 \log(e^{(dx+c)} + 1)}{d} + \frac{a^3 \log(|e^{(dx+c)} - 1|)}{d} - \frac{(192a^2b - 35b^3)(dx+c)}{128d} + \frac{(28800ab^2e^{(7dx+7c)} - 4800ab^2e^{(5dx+5c)} - 8400ab^2e^{(4dx+4c)} + 576a^2b^2e^{(3dx+3c)} + 160b^3e^{(2dx+2c)} - 15b^3 - 480(24a^2b - 7b^3)e^{(6dx+6c)})e^{(-8dx-8c)}}{d} + \frac{1}{30720} \frac{(15b^3d^7e^{(8dx+8c)} - 160b^3d^7e^{(6dx+6c)} + 576a^2b^2d^7e^{(5dx+5c)} + 840b^3d^7e^{(4dx+4c)} - 4800a^2b^2d^7e^{(3dx+3c)} + 11520a^2b^2d^7e^{(2dx+2c)} - 3360b^3d^7e^{(2dx+2c)} + 28800a^2b^2d^7e^{(dx+c)})}{d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")

[Out] $-a^3 \log(e^{(dx+c)} + 1)/d + a^3 \log(\text{abs}(e^{(dx+c)} - 1))/d - 1/128 * (192 * a^2 * b - 35 * b^3) * (d * x + c) / d + 1/30720 * (28800 * a * b^2 * e^{(7 * d * x + 7 * c)} - 4800 * a * b^2 * e^{(5 * d * x + 5 * c)} - 8400 * b^3 * e^{(4 * d * x + 4 * c)} + 576 * a * b^2 * e^{(3 * d * x + 3 * c)} + 160 * b^3 * e^{(2 * d * x + 2 * c)} - 15 * b^3 - 480 * (24 * a^2 * b - 7 * b^3) * e^{(6 * d * x + 6 * c)}) * e^{(-8 * d * x - 8 * c)} / d + 1/30720 * (15 * b^3 * d^7 * e^{(8 * d * x + 8 * c)} - 160 * b^3 * d^7 * e^{(6 * d * x + 6 * c)} + 576 * a * b^2 * d^7 * e^{(5 * d * x + 5 * c)} + 840 * b^3 * d^7 * e^{(4 * d * x + 4 * c)} - 4800 * a * b^2 * d^7 * e^{(3 * d * x + 3 * c)} + 11520 * a^2 * b^2 * d^7 * e^{(2 * d * x + 2 * c)} - 3360 * b^3 * d^7 * e^{(2 * d * x + 2 * c)} + 28800 * a * b^2 * d^7 * e^{(d * x + c)}) / d^8$

3.165 $\int \operatorname{csch}^2(c + dx) \left(a + b \sinh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=152

$$\frac{3a^2b \cosh(c + dx)}{d} - \frac{a^3 \coth(c + dx)}{d} + \frac{3ab^2 \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{9ab^2 \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{9}{8}ab^2x + \dots$$

[Out] $(9*a*b^2*x)/8 + (3*a^2*b*\operatorname{Cosh}[c + d*x])/d - (b^3*\operatorname{Cosh}[c + d*x])/d + (b^3*\operatorname{Cosh}[c + d*x]^3)/d - (3*b^3*\operatorname{Cosh}[c + d*x]^5)/(5*d) + (b^3*\operatorname{Cosh}[c + d*x]^7)/(7*d) - (a^3*\operatorname{Coth}[c + d*x])/d - (9*a*b^2*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(8*d) + (3*a*b^2*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x]^3)/(4*d)$

Rubi [A] time = 0.138335, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3220, 3767, 8, 2638, 2635, 2633}

$$\frac{3a^2b \cosh(c + dx)}{d} - \frac{a^3 \coth(c + dx)}{d} + \frac{3ab^2 \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{9ab^2 \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{9}{8}ab^2x + \dots$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^2*(a + b*\operatorname{Sinh}[c + d*x]^3)^3, x]$

[Out] $(9*a*b^2*x)/8 + (3*a^2*b*\operatorname{Cosh}[c + d*x])/d - (b^3*\operatorname{Cosh}[c + d*x])/d + (b^3*\operatorname{Cosh}[c + d*x]^3)/d - (3*b^3*\operatorname{Cosh}[c + d*x]^5)/(5*d) + (b^3*\operatorname{Cosh}[c + d*x]^7)/(7*d) - (a^3*\operatorname{Coth}[c + d*x])/d - (9*a*b^2*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(8*d) + (3*a*b^2*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x]^3)/(4*d)$

Rule 3220

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e + f*x]^{m*(a + b*\sin[e + f*x]^n)^p}, x], x] /;$ FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2638

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]`

Rule 2635

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] *(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 2633

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^2(c + dx) (a + b \sinh^3(c + dx))^3 dx &= - \int (-a^3 \operatorname{csch}^2(c + dx) - 3a^2b \sinh(c + dx) - 3ab^2 \sinh^4(c + dx) - b^3 \sinh^7(c + dx)) dx \\
 &= a^3 \int \operatorname{csch}^2(c + dx) dx + (3a^2b) \int \sinh(c + dx) dx + (3ab^2) \int \sinh^4(c + dx) dx - b^3 \int \sinh^7(c + dx) dx \\
 &= \frac{3a^2b \cosh(c + dx)}{d} + \frac{3ab^2 \cosh(c + dx) \sinh^3(c + dx)}{4d} - \frac{1}{4} (9ab^2) \int \sinh^2(c + dx) dx - b^3 \int \sinh^6(c + dx) dx \\
 &= \frac{3a^2b \cosh(c + dx)}{d} - \frac{b^3 \cosh(c + dx)}{d} + \frac{b^3 \cosh^3(c + dx)}{d} - \frac{3b^3 \cosh^5(c + dx)}{5d} \\
 &= \frac{9}{8} ab^2 x + \frac{3a^2b \cosh(c + dx)}{d} - \frac{b^3 \cosh(c + dx)}{d} + \frac{b^3 \cosh^3(c + dx)}{d} - \frac{3b^3 \cosh^5(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] time = 1.14364, size = 140, normalized size = 0.92

$$\frac{35b(192a^2 - 35b^2) \cosh(c + dx) - 1120a^3 \tanh\left(\frac{1}{2}(c + dx)\right) - 1120a^3 \coth\left(\frac{1}{2}(c + dx)\right) - 1680ab^2 \sinh(2(c + dx)) + 210a^3 \cosh^2(c + dx) - 2240ab^2 \cosh(c + dx) \sinh^2(c + dx) + 1120b^3 \sinh^4(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] (2520*a*b^2*c + 2520*a*b^2*d*x + 35*b*(192*a^2 - 35*b^2)*Cosh[c + d*x] + 24*5*b^3*Cosh[3*(c + d*x)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*x)] - 1120*a^3*Coth[(c + d*x)/2] - 1680*a*b^2*Sinh[2*(c + d*x)] + 210*a*b^2*Sinh[4*(c + d*x)] - 1120*a^3*Tanh[(c + d*x)/2])/(2240*d)

Maple [A] time = 0.06, size = 111, normalized size = 0.7

$$\frac{1}{d} \left(-a^3 \coth(dx + c) + 3a^2b \cosh(dx + c) + 3ab^2 \left(\frac{1}{4} (\sinh(dx + c))^3 - \frac{3}{8} \sinh(dx + c) \right) \cosh(dx + c) + \frac{3}{8} dx + 3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x)

[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*cosh(d*x+c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c))

Maxima [A] time = 1.20085, size = 297, normalized size = 1.95

$$\frac{3}{64} ab^2 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{4480} b^3 \left(\frac{(49e^{(-2dx-2c)} - 245e^{(-4dx-4c)} + 1225e^{(-6dx-6c)} - 5)e^{(7dx+7c)}}{d} + (1225e^{(-dx-c)} - 245e^{(-3dx-3c)} + 49e^{(-5dx-5c)} - 5e^{(-7dx-7c)})/d + 3/2*a^2*b*(e^{(dx+c)}/d + e^{(-dx-c)}/d) + 2*a^3/(d*(e^{(-2dx-2c)} - 1)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))

Fricas [B] time = 1.87147, size = 809, normalized size = 5.32

$$20b^3 \cosh(dx + c) \sinh(dx + c)^7 + 105ab^2 \cosh(dx + c)^5 + 525ab^2 \cosh(dx + c) \sinh(dx + c)^4 - 945ab^2 \cosh(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] 1/2240*(20*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 105*a*b^2*cosh(d*x + c)^5 + 525*a*b^2*cosh(d*x + c)*sinh(d*x + c)^4 - 945*a*b^2*cosh(d*x + c)^3 + 2*(70*b^3*cosh(d*x + c)^3 - 81*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*b^3*cosh(d*x + c)^5 - 135*b^3*cosh(d*x + c)^3 + 147*b^3*cosh(d*x + c))*sinh(d*x + c)^3 + 105*(10*a*b^2*cosh(d*x + c)^3 - 27*a*b^2*cosh(d*x + c))*sinh(d*x + c)^2 - 280*(8*a^3 - 3*a*b^2)*cosh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^7 - 81*b^3*cosh(d*x + c)^5 + 294*b^3*cosh(d*x + c)^3 + 1260*a*b^2*d*x + 1120*a^3 + 105*(32*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)

[Out] Timed out

Giac [B] time = 1.44803, size = 410, normalized size = 2.7

$$\frac{9(dx + c)ab^2}{8d} - \frac{(1890ab^2e^{(5dx+5c)} + 294b^3e^{(4dx+4c)} - 210ab^2e^{(3dx+3c)} - 54b^3e^{(2dx+2c)} + 5b^3 - 35(192a^2b - 35b^3)e^{(8dx+8c)})}{4480d(e^{(dx+c)} + 1)(e^{(dx+c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")

[Out] 9/8*(d*x + c)*a*b^2/d - 1/4480*(1890*a*b^2*e^(5*d*x + 5*c) + 294*b^3*e^(4*d*x + 4*c) - 210*a*b^2*e^(3*d*x + 3*c) - 54*b^3*e^(2*d*x + 2*c) + 5*b^3 - 35

$$\begin{aligned} &*(192*a^2*b - 35*b^3)*e^{(8*d*x + 8*c)} + 560*(16*a^3 - 3*a*b^2)*e^{(7*d*x + 7*c)} \\ &+ 210*(32*a^2*b - 7*b^3)*e^{(6*d*x + 6*c)})*e^{(-7*d*x - 7*c)}/(d*(e^{(d*x + c)} + 1)*(e^{(d*x + c)} - 1)) + 1/4480*(5*b^3*d^6*e^{(7*d*x + 7*c)} - 49*b^3*d^6*e^{(5*d*x + 5*c)} \\ &+ 210*a*b^2*d^6*e^{(4*d*x + 4*c)} + 245*b^3*d^6*e^{(3*d*x + 3*c)} - 1680*a*b^2*d^6*e^{(2*d*x + 2*c)} + 6720*a^2*b*d^6*e^{(d*x + c)} - 1225*b^3*d^6*e^{(d*x + c)})/d^7 \end{aligned}$$

3.166 $\int \operatorname{csch}^3(c + dx) \left(a + b \sinh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=156

$$3a^2bx + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{ab^2 \cosh^3(c + dx)}{d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{b^3 \sinh^5(c + dx)}{6d}$$

[Out] $3a^2bx - (5b^3x)/16 + (a^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]])/(2d) - (3ab^2 \operatorname{Cosh}[c + dx])/d + (a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx])/(2d) + (5b^3 \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c + dx])/(16d) - (5b^3 \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c + dx]^3)/(24d) + (b^3 \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c + dx]^5)/(6d)$

Rubi [A] time = 0.168601, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3220, 3768, 3770, 2633, 2635, 8}

$$3a^2bx + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{ab^2 \cosh^3(c + dx)}{d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{b^3 \sinh^5(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + dx]^3 (a + b \operatorname{Sinh}[c + dx]^3)^3, x]$

[Out] $3a^2bx - (5b^3x)/16 + (a^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]])/(2d) - (3ab^2 \operatorname{Cosh}[c + dx])/d + (a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx])/(2d) + (5b^3 \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c + dx])/(16d) - (5b^3 \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c + dx]^3)/(24d) + (b^3 \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c + dx]^5)/(6d)$

Rule 3220

$\operatorname{Int}[\sin[(e_.) + (f_.) (x_)]^{(m_.)} ((a_.) + (b_.) \sin[(e_.) + (f_.) (x_)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e + f x]^m (a + b \sin[e + f x]^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, e, f\}, x] \&\& \operatorname{IntegersQ}[m, p] \&\& (\operatorname{EqQ}[n, 4] \mid \mid \operatorname{GtQ}[p, 0] \mid \mid (\operatorname{EqQ}[p, -1] \&\& \operatorname{IntegerQ}[n]))$

Rule 3768

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.) (x_)] (b_.))^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b \operatorname{Cos}[c + dx]) (b \operatorname{Csc}[c + dx])^{(n-1)}] / (d (n-1)), x] + \operatorname{Dist}[(b^2 (n-2)) / (n-1), \operatorname{Int}[(b \operatorname{Csc}[c + dx])^{(n-2)}, x], x] /; \operatorname{FreeQ}\{b, c, d\}, x] \&\& \operatorname{GtQ}[n, 1] \&\& \operatorname{IntegerQ}[2n]$

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x])*(b*sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^3(c + dx) (a + b \sinh^3(c + dx))^3 dx &= -\left(i \int (3ia^2b + ia^3 \operatorname{csch}^3(c + dx) + 3iab^2 \sinh^3(c + dx) + ib^3 \sinh^6(c + dx)) dx\right) \\
 &= 3a^2bx + a^3 \int \operatorname{csch}^3(c + dx) dx + (3ab^2) \int \sinh^3(c + dx) dx + b^3 \int \sinh^6(c + dx) dx \\
 &= 3a^2bx - \frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b^3 \cosh(c + dx) \sinh^5(c + dx)}{6d} - \frac{b^6 \cosh^6(c + dx)}{6d} \\
 &= 3a^2bx + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{ab^2 \cosh^3(c + dx)}{d} \\
 &= 3a^2bx + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{ab^2 \cosh^3(c + dx)}{d} \\
 &= 3a^2bx - \frac{5b^3x}{16} + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{3ab^2 \cosh(c + dx)}{d} + \frac{ab^2 \cosh^3(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] time = 3.70225, size = 150, normalized size = 0.96

$$576a^2bc + 576a^2bdx - 24a^3 \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right) - 24a^3 \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right) - 96a^3 \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right) - 432ab^2 \cosh(c + dx)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] (576*a^2*b*c - 60*b^3*c + 576*a^2*b*d*x - 60*b^3*d*x - 432*a*b^2*Cosh[c + d*x] + 48*a*b^2*Cosh[3*(c + d*x)] - 24*a^3*Csch[(c + d*x)/2]^2 - 96*a^3*Log[Tanh[(c + d*x)/2]] - 24*a^3*Sech[(c + d*x)/2]^2 + 45*b^3*Sinh[2*(c + d*x)] - 9*b^3*Sinh[4*(c + d*x)] + b^3*Sinh[6*(c + d*x)])/(192*d)

Maple [A] time = 0.066, size = 115, normalized size = 0.7

$$\frac{1}{d} \left(a^3 \left(-\frac{\operatorname{csch}(dx+c) \operatorname{coth}(dx+c)}{2} + \operatorname{Artanh}(e^{dx+c}) \right) + 3a^2b(dx+c) + 3ab^2 \left(-\frac{2}{3} + \frac{1}{3} (\sinh(dx+c))^2 \right) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x)

[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(d*x+c)+3*a*b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c))

Maxima [A] time = 1.17251, size = 329, normalized size = 2.11

$$3a^2bx - \frac{1}{384}b^3 \left(\frac{(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e^{(-6dx-6c)}}{d} \right) + \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] 3*a^2*b*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/8*a*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))

Fricas [B] time = 2.61754, size = 9546, normalized size = 61.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] $\frac{1}{384}(b^3 \cosh(dx+c)^{16} + 16b^3 \cosh(dx+c) \sinh(dx+c)^{15} + b^3 \sinh(dx+c)^{16} - 11b^3 \cosh(dx+c)^{14} + 48a^2 b^2 \cosh(dx+c)^{13} + 64b^3 \cosh(dx+c)^{12} + (120b^3 \cosh(dx+c)^2 - 11b^3) \sinh(dx+c)^{14} - 528a^2 b^2 \cosh(dx+c)^{11} + 2(280b^3 \cosh(dx+c)^3 - 77b^3 \cosh(dx+c) + 24a^2 b^2) \sinh(dx+c)^{13} + (1820b^3 \cosh(dx+c)^4 - 1001b^3 \cosh(dx+c)^2 + 624a^2 b^2 \cosh(dx+c) + 64b^3) \sinh(dx+c)^{12} + 4(1092b^3 \cosh(dx+c)^5 - 1001b^3 \cosh(dx+c)^3 + 936a^2 b^2 \cosh(dx+c)^2 + 192b^3 \cosh(dx+c) - 132a^2 b^2) \sinh(dx+c)^{11} - 48(48a^2 b - 5b^3) dx \cosh(dx+c)^8 - 3(33b^3 - 8(48a^2 b - 5b^3) dx) \cosh(dx+c)^{10} + (8008b^3 \cosh(dx+c)^6 - 11011b^3 \cosh(dx+c)^4 + 13728a^2 b^2 \cosh(dx+c)^3 + 4224b^3 \cosh(dx+c)^2 - 5808a^2 b^2 \cosh(dx+c) - 99b^3 + 24(48a^2 b - 5b^3) dx) \sinh(dx+c)^{10} - 96(4a^3 - 5a^2 b^2) \cosh(dx+c)^9 + 2(5720b^3 \cosh(dx+c)^7 - 11011b^3 \cosh(dx+c)^5 + 17160a^2 b^2 \cosh(dx+c)^4 + 7040b^3 \cosh(dx+c)^3 - 14520a^2 b^2 \cosh(dx+c)^2 - 192a^3 + 240a^2 b^2 - 15(33b^3 - 8(48a^2 b - 5b^3) dx) \cosh(dx+c)) \sinh(dx+c)^9 + 3(4290b^3 \cosh(dx+c)^8 - 11011b^3 \cosh(dx+c)^6 + 20592a^2 b^2 \cosh(dx+c)^5 + 10560b^3 \cosh(dx+c)^4 - 29040a^2 b^2 \cosh(dx+c)^3 - 16(48a^2 b - 5b^3) dx - 45(33b^3 - 8(48a^2 b - 5b^3) dx) \cosh(dx+c)^2 - 288(4a^3 - 5a^2 b^2) \cosh(dx+c)) \sinh(dx+c)^8 - 528a^2 b^2 \cosh(dx+c)^5 - 96(4a^3 - 5a^2 b^2) \cosh(dx+c)^7 + 8(1430b^3 \cosh(dx+c)^9 - 4719b^3 \cosh(dx+c)^7 + 10296a^2 b^2 \cosh(dx+c)^6 + 6336b^3 \cosh(dx+c)^5 - 21780a^2 b^2 \cosh(dx+c)^4 - 48(48a^2 b - 5b^3) dx \cosh(dx+c) - 45(33b^3 - 8(48a^2 b - 5b^3) dx) \cosh(dx+c)^3 - 48a^3 + 60a^2 b^2 - 432(4a^3 - 5a^2 b^2) \cosh(dx+c)^2) \sinh(dx+c)^7 - 64b^3 \cosh(dx+c)^4 + 3(33b^3 + 8(48a^2 b - 5b^3) dx) \cosh(dx+c)^6 + (8008b^3 \cosh(dx+c)^{10} - 33033b^3 \cosh(dx+c)^8 + 82368a^2 b^2 \cosh(dx+c)^7 + 59136b^3 \cosh(dx+c)^6 - 243936a^2 b^2 \cosh(dx+c)^5 - 1344(48a^2 b - 5b^3) dx \cosh(dx+c)^2 - 630(33b^3 - 8(48a^2 b - 5b^3) dx) \cosh(dx+c)^4 - 8064(4a^3 - 5a^2 b^2) \cosh(dx+c)^3 + 99b^3 + 24(48a^2 b - 5b^3) dx - 672(4a^3 - 5a^2 b^2) \cosh(dx+c)) \sinh(dx+c)^6 + 48a^2 b^2 \cosh(dx+c)^3 + 2(2184b^3 \cosh(dx+c)^{11} - 11011b^3 \cosh(dx+c)^9 + 30888a^2 b^2 \cosh(dx+c)^8 + 25344b^3 \cosh(dx+c)^7 - 121968a^2 b^2 \cosh(dx+c)^6 - 1344(48a^2 b - 5b^3) dx \cosh(dx+c)^3 - 378(33b^3 - 8(48a^2 b - 5b^3) dx) \cosh(dx+c)^5 - 6048(4a^3 - 5a^2 b^2) \cosh(dx+c)^4 - 264a^2 b^2$

$$\begin{aligned}
& - 1008*(4*a^3 - 5*a*b^2)*\cosh(d*x + c)^2 + 9*(33*b^3 + 8*(48*a^2*b - 5*b^3) \\
& *d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 11*b^3*\cosh(d*x + c)^2 + (1820*b^3*c \\
& \cosh(d*x + c)^{12} - 11011*b^3*\cosh(d*x + c)^{10} + 34320*a*b^2*\cosh(d*x + c)^9 \\
& + 31680*b^3*\cosh(d*x + c)^8 - 174240*a*b^2*\cosh(d*x + c)^7 - 3360*(48*a^2*b \\
& - 5*b^3)*d*x*\cosh(d*x + c)^4 - 630*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*\cos \\
& h(d*x + c)^6 - 12096*(4*a^3 - 5*a*b^2)*\cosh(d*x + c)^5 - 2640*a*b^2*\cosh(d* \\
& x + c) - 3360*(4*a^3 - 5*a*b^2)*\cosh(d*x + c)^3 - 64*b^3 + 45*(33*b^3 + 8*(\\
& 48*a^2*b - 5*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(140*b^3*\cosh(d \\
& *x + c)^{13} - 1001*b^3*\cosh(d*x + c)^{11} + 3432*a*b^2*\cosh(d*x + c)^{10} + 3520 \\
& *b^3*\cosh(d*x + c)^9 - 21780*a*b^2*\cosh(d*x + c)^8 - 672*(48*a^2*b - 5*b^3) \\
& *d*x*\cosh(d*x + c)^5 - 90*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*\cosh(d*x + c) \\
& ^7 - 2016*(4*a^3 - 5*a*b^2)*\cosh(d*x + c)^6 - 1320*a*b^2*\cosh(d*x + c)^2 - \\
& 840*(4*a^3 - 5*a*b^2)*\cosh(d*x + c)^4 - 64*b^3*\cosh(d*x + c) + 15*(33*b^3 + \\
& 8*(48*a^2*b - 5*b^3)*d*x)*\cosh(d*x + c)^3 + 12*a*b^2)*\sinh(d*x + c)^3 - b^ \\
& 3 + (120*b^3*\cosh(d*x + c)^{14} - 1001*b^3*\cosh(d*x + c)^{12} + 3744*a*b^2*\cosh \\
& (d*x + c)^{11} + 4224*b^3*\cosh(d*x + c)^{10} - 29040*a*b^2*\cosh(d*x + c)^9 - 13 \\
& 44*(48*a^2*b - 5*b^3)*d*x*\cosh(d*x + c)^6 - 135*(33*b^3 - 8*(48*a^2*b - 5*b \\
& ^3)*d*x)*\cosh(d*x + c)^8 - 3456*(4*a^3 - 5*a*b^2)*\cosh(d*x + c)^7 - 5280*a* \\
& b^2*\cosh(d*x + c)^3 - 2016*(4*a^3 - 5*a*b^2)*\cosh(d*x + c)^5 - 384*b^3*\cosh \\
& (d*x + c)^2 + 45*(33*b^3 + 8*(48*a^2*b - 5*b^3)*d*x)*\cosh(d*x + c)^4 + 144* \\
& a*b^2*\cosh(d*x + c) + 11*b^3)*\sinh(d*x + c)^2 + 192*(a^3*\cosh(d*x + c)^{10} + \\
& 10*a^3*\cosh(d*x + c)*\sinh(d*x + c)^9 + a^3*\sinh(d*x + c)^{10} - 2*a^3*\cosh(d \\
& *x + c)^8 + a^3*\cosh(d*x + c)^6 + (45*a^3*\cosh(d*x + c)^2 - 2*a^3)*\sinh(d*x \\
& + c)^8 + 8*(15*a^3*\cosh(d*x + c)^3 - 2*a^3*\cosh(d*x + c))*\sinh(d*x + c)^7 \\
& + (210*a^3*\cosh(d*x + c)^4 - 56*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x + c)^6 \\
& + 2*(126*a^3*\cosh(d*x + c)^5 - 56*a^3*\cosh(d*x + c)^3 + 3*a^3*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^5 + 5*(42*a^3*\cosh(d*x + c)^6 - 28*a^3*\cosh(d*x + c)^4 + 3* \\
& a^3*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(30*a^3*\cosh(d*x + c)^7 - 28*a^3*c \\
& \cosh(d*x + c)^5 + 5*a^3*\cosh(d*x + c)^3)*\sinh(d*x + c)^3 + (45*a^3*\cosh(d*x \\
& + c)^8 - 56*a^3*\cosh(d*x + c)^6 + 15*a^3*\cosh(d*x + c)^4)*\sinh(d*x + c)^2 + \\
& 2*(5*a^3*\cosh(d*x + c)^9 - 8*a^3*\cosh(d*x + c)^7 + 3*a^3*\cosh(d*x + c)^5)* \\
& \sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - 192*(a^3*\cosh(d*x + \\
& c)^{10} + 10*a^3*\cosh(d*x + c)*\sinh(d*x + c)^9 + a^3*\sinh(d*x + c)^{10} - 2*a^ \\
& 3*\cosh(d*x + c)^8 + a^3*\cosh(d*x + c)^6 + (45*a^3*\cosh(d*x + c)^2 - 2*a^3)* \\
& \sinh(d*x + c)^8 + 8*(15*a^3*\cosh(d*x + c)^3 - 2*a^3*\cosh(d*x + c))*\sinh(d*x \\
& + c)^7 + (210*a^3*\cosh(d*x + c)^4 - 56*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x \\
& + c)^6 + 2*(126*a^3*\cosh(d*x + c)^5 - 56*a^3*\cosh(d*x + c)^3 + 3*a^3*\cosh(\\
& d*x + c))*\sinh(d*x + c)^5 + 5*(42*a^3*\cosh(d*x + c)^6 - 28*a^3*\cosh(d*x + c) \\
&)^4 + 3*a^3*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(30*a^3*\cosh(d*x + c)^7 - \\
& 28*a^3*\cosh(d*x + c)^5 + 5*a^3*\cosh(d*x + c)^3)*\sinh(d*x + c)^3 + (45*a^3*c \\
& \cosh(d*x + c)^8 - 56*a^3*\cosh(d*x + c)^6 + 15*a^3*\cosh(d*x + c)^4)*\sinh(d*x \\
& + c)^2 + 2*(5*a^3*\cosh(d*x + c)^9 - 8*a^3*\cosh(d*x + c)^7 + 3*a^3*\cosh(d*x \\
& + c)^5)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(8*b^3*c \\
& \cosh(d*x + c)^{15} - 77*b^3*\cosh(d*x + c)^{13} + 312*a*b^2*\cosh(d*x + c)^{12} + 384 \\
& *b^3*\cosh(d*x + c)^{11} - 2904*a*b^2*\cosh(d*x + c)^{10} - 192*(48*a^2*b - 5*b^3
\end{aligned}$$


```
)d*x*cosh(d*x + c)^7 - 15*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^9 - 432*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^8 - 1320*a*b^2*cosh(d*x + c)^4 - 336*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^6 - 128*b^3*cosh(d*x + c)^3 + 9*(33*b^3 + 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^5 + 72*a*b^2*cosh(d*x + c)^2 + 11*b^3*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 - 2*d*cosh(d*x + c)^8 + (45*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(d*x + c)^7 + d*cosh(d*x + c)^6 + (210*d*cosh(d*x + c)^4 - 56*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d*x + c)^5 - 56*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 5*(42*d*cosh(d*x + c)^6 - 28*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x + c)^7 - 28*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3)*sinh(d*x + c)^3 + (45*d*cosh(d*x + c)^8 - 56*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4)*sinh(d*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 - 8*d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c)^5)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45249, size = 428, normalized size = 2.74

$$\frac{a^3 \log(e^{(dx+c)} + 1)}{2d} - \frac{a^3 \log(|e^{(dx+c)} - 1|)}{2d} + \frac{(48a^2b - 5b^3)(dx + c)}{16d} - \frac{(45b^3e^{(8dx+8c)} - 99b^3e^{(6dx+6c)} + 528ab^2e^{(5dx+5c)})}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/2*a^3*log(e^(d*x + c) + 1)/d - 1/2*a^3*log(abs(e^(d*x + c) - 1))/d + 1/16*(48*a^2*b - 5*b^3)*(d*x + c)/d - 1/384*(45*b^3*e^(8*d*x + 8*c) - 99*b^3*e^(6*d*x + 6*c) + 528*a*b^2*e^(5*d*x + 5*c) + 64*b^3*e^(4*d*x + 4*c) - 48*a*b^2*e^(3*d*x + 3*c) - 11*b^3*e^(2*d*x + 2*c) + b^3 + 48*(8*a^3 + 9*a*b^2)*e^(d*x + c))
```

$$\begin{aligned} & (9dx + 9c) + 48(8a^3 - 19ab^2)e^{(7dx + 7c)}e^{(-6dx - 6c)} / (d \\ & (e^{(dx + c)} + 1)^2(e^{(dx + c)} - 1)^2) + 1/384(b^3d^5e^{(6dx + 6c)} - \\ & 9b^3d^5e^{(4dx + 4c)} + 48ab^2d^5e^{(3dx + 3c)} + 45b^3d^5e^{(2 \\ & dx + 2c)} - 432ab^2d^5e^{(dx + c)})/d^6 \end{aligned}$$

3.167 $\int \operatorname{csch}^4(c + dx) (a + b \sinh^3(c + dx))^3 dx$

Optimal. Leaf size=129

$$-\frac{3a^2b \tanh^{-1}(\cosh(c + dx))}{d} - \frac{a^3 \coth^3(c + dx)}{3d} + \frac{a^3 \coth(c + dx)}{d} + \frac{3ab^2 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{3}{2}ab^2x + \frac{b^3 \cos}{2}$$

[Out] $(-3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cosh[c + d*x]])/d + (b^3*Cosh[c + d*x])/d - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) + (a^3*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)$

Rubi [A] time = 0.120472, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3220, 3770, 3767, 2635, 8, 2633}

$$-\frac{3a^2b \tanh^{-1}(\cosh(c + dx))}{d} - \frac{a^3 \coth^3(c + dx)}{3d} + \frac{a^3 \coth(c + dx)}{d} + \frac{3ab^2 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{3}{2}ab^2x + \frac{b^3 \cos}{2}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csch}[c + d*x]^4*(a + b*\text{Sinh}[c + d*x]^3)^3, x]$

[Out] $(-3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cosh[c + d*x]])/d + (b^3*Cosh[c + d*x])/d - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) + (a^3*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)$

Rule 3220

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^{m*(a + b*\sin[e + f*x]^n)^p}, x], x] /; \text{FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{IntegersQ}[m, p] \ \&\& \ (\text{EqQ}[n, 4] \ || \ \text{GtQ}[p, 0] \ || \ (\text{EqQ}[p, -1] \ \&\& \ \text{IntegerQ}[n]))$

Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^4(c + dx) (a + b \sinh^3(c + dx))^3 dx &= \int (3a^2 b \operatorname{csch}(c + dx) + a^3 \operatorname{csch}^4(c + dx) + 3ab^2 \sinh^2(c + dx) + b^3 \sinh^5(c + dx)) dx \\ &= a^3 \int \operatorname{csch}^4(c + dx) dx + (3a^2 b) \int \operatorname{csch}(c + dx) dx + (3ab^2) \int \sinh^2(c + dx) dx + b^3 \int \sinh^5(c + dx) dx \\ &= -\frac{3a^2 b \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3ab^2 \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{1}{2} (3ab^2) \int \sinh^4(c + dx) dx \\ &= -\frac{3}{2} ab^2 x - \frac{3a^2 b \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^3 \cosh(c + dx)}{d} - \frac{2b^3 \cosh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.444961, size = 169, normalized size = 1.31

$$\frac{720a^2 b \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right) + 80a^3 \tanh\left(\frac{1}{2}(c + dx)\right) + 80a^3 \coth\left(\frac{1}{2}(c + dx)\right) + 80a^3 \sinh^4\left(\frac{1}{2}(c + dx)\right) \operatorname{csch}^3(c + dx)}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^3,x]
```

[Out] $(-360*a*b^2*c - 360*a*b^2*d*x + 150*b^3*\text{Cosh}[c + d*x] - 25*b^3*\text{Cosh}[3*(c + d*x)] + 3*b^3*\text{Cosh}[5*(c + d*x)] + 80*a^3*\text{Coth}[(c + d*x)/2] + 720*a^2*b*\text{Log}[\text{Tanh}[(c + d*x)/2]] + 80*a^3*\text{Csch}[c + d*x]^3*\text{Sinh}[(c + d*x)/2]^4 - 5*a^3*\text{Csch}[(c + d*x)/2]^4*\text{Sinh}[c + d*x] + 180*a*b^2*\text{Sinh}[2*(c + d*x)] + 80*a^3*\text{Tanh}[(c + d*x)/2])/(240*d)$

Maple [A] time = 0.077, size = 101, normalized size = 0.8

$\frac{1}{d} \left(a^3 \left(\frac{2}{3} - \frac{(\text{csch}(dx+c))^2}{3} \right) \coth(dx+c) - 6a^2b \text{Artanh}(e^{dx+c}) + 3ab^2 (1/2 \cosh(dx+c) \sinh(dx+c) - 1/2 dx - c/2) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x)`

[Out] $1/d*(a^3*(2/3-1/3*\text{csch}(d*x+c)^2)*\coth(d*x+c)-6*a^2*b*\text{arctanh}(\exp(d*x+c))+3*a*b^2*(1/2*\cosh(d*x+c)*\sinh(d*x+c)-1/2*d*x-1/2*c)+b^3*(8/15+1/5*\sinh(d*x+c)^4-4/15*\sinh(d*x+c)^2)*\cosh(d*x+c))$

Maxima [B] time = 1.20844, size = 351, normalized size = 2.72

$-\frac{3}{8}ab^2 \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) + \frac{1}{480}b^3 \left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \dots \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")`

[Out] $-3/8*a*b^2*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) + 1/480*b^3*(3*e^{(5*d*x + 5*c)}/d - 25*e^{(3*d*x + 3*c)}/d + 150*e^{(d*x + c)}/d + 150*e^{(-d*x - c)}/d - 25*e^{(-3*d*x - 3*c)}/d + 3*e^{(-5*d*x - 5*c)}/d - 3*a^2*b*(\log(e^{(-d*x - c)} + 1)/d - \log(e^{(-d*x - c)} - 1)/d) + 4/3*a^3*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)))$

Fricas [B] time = 2.43673, size = 10166, normalized size = 78.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] $\frac{1}{480} \cdot (3b^3 \cosh(dx+c)^{16} + 48b^3 \cosh(dx+c) \sinh(dx+c)^{15} + 3b^3 \sinh(dx+c)^{16} - 34b^3 \cosh(dx+c)^{14} + 180ab^2 \cosh(dx+c)^{13} + 234b^3 \cosh(dx+c)^{12} + 2(180b^3 \cosh(dx+c)^2 - 17b^3) \sinh(dx+c)^{14} + 4(420b^3 \cosh(dx+c)^3 - 119b^3 \cosh(dx+c) + 45ab^2) \sinh(dx+c)^{13} - 378b^3 \cosh(dx+c)^{10} + 26(210b^3 \cosh(dx+c)^4 - 19b^3 \cosh(dx+c)^2 + 90ab^2 \cosh(dx+c) + 9b^3) \sinh(dx+c)^{12} - 180(4ab^2 dx + 3ab^2) \cosh(dx+c)^{11} + 4(3276b^3 \cosh(dx+c)^5 - 3094b^3 \cosh(dx+c)^3 - 180ab^2 dx + 3510ab^2 \cosh(dx+c)^2 + 702b^3 \cosh(dx+c) - 135ab^2) \sinh(dx+c)^{11} + 2(12012b^3 \cosh(dx+c)^6 - 17017b^3 \cosh(dx+c)^4 + 25740ab^2 \cosh(dx+c)^3 + 7722b^3 \cosh(dx+c)^2 - 189b^3 - 990(4ab^2 dx + 3ab^2) \cosh(dx+c)) \sinh(dx+c)^{10} + 360(6ab^2 dx + ab^2) \cosh(dx+c)^9 + 4(8580b^3 \cosh(dx+c)^7 - 17017b^3 \cosh(dx+c)^5 + 32175ab^2 \cosh(dx+c)^4 + 12870b^3 \cosh(dx+c)^3 + 540ab^2 dx - 945b^3 \cosh(dx+c) + 90ab^2 - 2475(4ab^2 dx + 3ab^2) \cosh(dx+c)^2) \sinh(dx+c)^9 + 378b^3 \cosh(dx+c)^6 + 6(6435b^3 \cosh(dx+c)^8 - 17017b^3 \cosh(dx+c)^6 + 38610ab^2 \cosh(dx+c)^5 + 19305b^3 \cosh(dx+c)^4 - 2835b^3 \cosh(dx+c)^2 - 4950(4ab^2 dx + 3ab^2) \cosh(dx+c)^3 + 540(6ab^2 dx + ab^2) \cosh(dx+c)) \sinh(dx+c)^8 - 120(18ab^2 dx + 16a^3 - 3ab^2) \cosh(dx+c)^7 + 24(1430b^3 \cosh(dx+c)^9 - 4862b^3 \cosh(dx+c)^7 + 12870ab^2 \cosh(dx+c)^6 + 7722b^3 \cosh(dx+c)^5 - 1890b^3 \cosh(dx+c)^3 - 90ab^2 dx - 2475(4ab^2 dx + 3ab^2) \cosh(dx+c)^4 - 80a^3 + 15ab^2 + 540(6ab^2 dx + ab^2) \cosh(dx+c)^2) \sinh(dx+c)^7 - 234b^3 \cosh(dx+c)^4 + 6(4004b^3 \cosh(dx+c)^{10} - 17017b^3 \cosh(dx+c)^8 + 51480ab^2 \cosh(dx+c)^7 + 36036b^3 \cosh(dx+c)^6 - 13230b^3 \cosh(dx+c)^4 - 13860(4ab^2 dx + 3ab^2) \cosh(dx+c)^5 + 5040(6ab^2 dx + ab^2) \cosh(dx+c)^3 + 63b^3 - 140(18ab^2 dx + 16a^3 - 3ab^2) \cosh(dx+c)) \sinh(dx+c)^6 + 180ab^2 \cosh(dx+c)^3 + 20(36ab^2 dx + 32a^3 - 27ab^2) \cosh(dx+c)^5 + 4(3276b^3 \cosh(dx+c)^{11} - 17017b^3 \cosh(dx+c)^9 + 57915ab^2 \cosh(dx+c)^8 + 46332b^3 \cosh(dx+c)^7 - 23814b^3 \cosh(dx+c)^5 - 20790(4ab^2 dx + 3ab^2) \cosh(dx+c)^6 + 180ab^2 dx + 11340(6ab^2 dx + ab^2) \cosh(dx+c)^4 + 567b^3 \cosh(dx+c) + 160a^3 - 135ab^2 - 630(18ab^2 dx + 16a^3 - 3ab^2) \cosh(dx+c)^2) \sinh(dx+c)^5 + 34b^3 \cosh(dx+c)^2 + 2(2730b^3 \cosh(dx+c)^{12} - 17017b^3 \cosh(dx+c)^{10} + 64350ab^2 \cosh(dx+c)^9 + 57915b^3 \cosh(dx+c)^8 - 39690b^3 \cosh(dx+c)^6 - 29700(4ab^2 dx + 3ab^2) \cosh(dx+c)^7 + 22680(6ab^2 dx + ab^2) \cosh(dx+c)^5 + 2835b^3 \cosh(dx+c)^2 - 2100(18ab^2 dx + 16a^3 - 3ab^2) \cosh(dx+c)^3 - 117b^3 + 50(36ab^2 dx + 32a^3 - 27ab^2) \cosh(dx+c)) \sinh(dx+c)^4 + 4(420b^3 \cosh(dx+c)^{13} - 3094$

$$\begin{aligned}
& b^3 \cosh(dx + c)^{11} + 12870 a b^2 \cosh(dx + c)^{10} + 12870 b^3 \cosh(dx + c)^9 - 11340 b^3 \cosh(dx + c)^7 - 7425 (4 a b^2 dx + 3 a b^2) \cosh(dx + c)^8 + 7560 (6 a b^2 dx + a b^2) \cosh(dx + c)^6 + 1890 b^3 \cosh(dx + c)^3 - 1050 (18 a b^2 dx + 16 a^3 - 3 a b^2) \cosh(dx + c)^4 - 234 b^3 \cosh(dx + c) + 45 a b^2 + 50 (36 a b^2 dx + 32 a^3 - 27 a b^2) \cosh(dx + c)^2 \\
& * \sinh(dx + c)^3 - 3 b^3 + 2 (180 b^3 \cosh(dx + c)^{14} - 1547 b^3 \cosh(dx + c)^{12} + 7020 a b^2 \cosh(dx + c)^{11} + 7722 b^3 \cosh(dx + c)^{10} - 8505 b^3 \cosh(dx + c)^8 - 4950 (4 a b^2 dx + 3 a b^2) \cosh(dx + c)^9 + 6480 (6 a b^2 dx + a b^2) \cosh(dx + c)^7 + 2835 b^3 \cosh(dx + c)^4 - 1260 (18 a b^2 dx + 16 a^3 - 3 a b^2) \cosh(dx + c)^5 - 702 b^3 \cosh(dx + c)^2 + 270 a b^2 \cosh(dx + c) + 100 (36 a b^2 dx + 32 a^3 - 27 a b^2) \cosh(dx + c)^3 + 17 b^3) \sinh(dx + c)^2 - 1440 (a^2 b \cosh(dx + c)^{11} + 11 a^2 b \cosh(dx + c) \sinh(dx + c)^{10} + a^2 b \sinh(dx + c)^{11} - 3 a^2 b \cosh(dx + c)^9 + 3 a^2 b \cosh(dx + c)^7 + (55 a^2 b \cosh(dx + c)^2 - 3 a^2 b) \sinh(dx + c)^9 + 3 (55 a^2 b \cosh(dx + c)^3 - 9 a^2 b \cosh(dx + c)) \sinh(dx + c)^8 - a^2 b \cosh(dx + c)^5 + 3 (110 a^2 b \cosh(dx + c)^4 - 36 a^2 b \cosh(dx + c)^2 + a^2 b) \sinh(dx + c)^7 + 21 (22 a^2 b \cosh(dx + c)^5 - 12 a^2 b \cosh(dx + c)^3 + a^2 b \cosh(dx + c)) \sinh(dx + c)^6 + (462 a^2 b \cosh(dx + c)^6 - 378 a^2 b \cosh(dx + c)^4 + 63 a^2 b \cosh(dx + c)^2 - a^2 b) \sinh(dx + c)^5 + (330 a^2 b \cosh(dx + c)^7 - 378 a^2 b \cosh(dx + c)^5 + 105 a^2 b \cosh(dx + c)^3 - 5 a^2 b \cosh(dx + c)) \sinh(dx + c)^4 + (165 a^2 b \cosh(dx + c)^8 - 252 a^2 b \cosh(dx + c)^6 + 105 a^2 b \cosh(dx + c)^4 - 10 a^2 b \cosh(dx + c)^2) \sinh(dx + c)^3 + (55 a^2 b \cosh(dx + c)^9 - 108 a^2 b \cosh(dx + c)^7 + 63 a^2 b \cosh(dx + c)^5 - 10 a^2 b \cosh(dx + c)^3) \sinh(dx + c)^2 + (11 a^2 b \cosh(dx + c)^{10} - 27 a^2 b \cosh(dx + c)^8 + 21 a^2 b \cosh(dx + c)^6 - 5 a^2 b \cosh(dx + c)^4) \sinh(dx + c) * \log(\cosh(dx + c) + \sinh(dx + c) + 1) + 1440 (a^2 b \cosh(dx + c)^{11} + 11 a^2 b \cosh(dx + c) \sinh(dx + c)^{10} + a^2 b \sinh(dx + c)^{11} - 3 a^2 b \cosh(dx + c)^9 + 3 a^2 b \cosh(dx + c)^7 + (55 a^2 b \cosh(dx + c)^2 - 3 a^2 b) \sinh(dx + c)^9 + 3 (55 a^2 b \cosh(dx + c)^3 - 9 a^2 b \cosh(dx + c)) \sinh(dx + c)^8 - a^2 b \cosh(dx + c)^5 + 3 (110 a^2 b \cosh(dx + c)^4 - 36 a^2 b \cosh(dx + c)^2 + a^2 b) \sinh(dx + c)^7 + 21 (22 a^2 b \cosh(dx + c)^5 - 12 a^2 b \cosh(dx + c)^3 + a^2 b \cosh(dx + c)) \sinh(dx + c)^6 + (462 a^2 b \cosh(dx + c)^6 - 378 a^2 b \cosh(dx + c)^4 + 63 a^2 b \cosh(dx + c)^2 - a^2 b) \sinh(dx + c)^5 + (330 a^2 b \cosh(dx + c)^7 - 378 a^2 b \cosh(dx + c)^5 + 105 a^2 b \cosh(dx + c)^3 - 5 a^2 b \cosh(dx + c)) \sinh(dx + c)^4 + (165 a^2 b \cosh(dx + c)^8 - 252 a^2 b \cosh(dx + c)^6 + 105 a^2 b \cosh(dx + c)^4 - 10 a^2 b \cosh(dx + c)^2) \sinh(dx + c)^3 + (55 a^2 b \cosh(dx + c)^9 - 108 a^2 b \cosh(dx + c)^7 + 63 a^2 b \cosh(dx + c)^5 - 10 a^2 b \cosh(dx + c)^3) \sinh(dx + c)^2 + (11 a^2 b \cosh(dx + c)^{10} - 27 a^2 b \cosh(dx + c)^8 + 21 a^2 b \cosh(dx + c)^6 - 5 a^2 b \cosh(dx + c)^4) \sinh(dx + c) * \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 4 (12 b^3 \cosh(dx + c)^{15} - 119 b^3 \cosh(dx + c)^{13} + 585 a b^2 \cosh(dx + c)^{12} + 702 b^3 \cosh(dx + c)^{11} - 945 b^3 \cosh(dx + c)^9 - 495 (4 a b^2 dx + 3 a b^2) \cosh(dx + c)^{10} + 810 (6 a b^2 dx + a b^2) \cosh(dx + c)^8 + 567 b^3 \cosh(dx + c)
\end{aligned}$$

)⁵ - 210*(18*a*b²*d*x + 16*a³ - 3*a*b²)*cosh(d*x + c)⁶ - 234*b³*cosh(d*x + c)³ + 135*a*b²*cosh(d*x + c)² + 25*(36*a*b²*d*x + 32*a³ - 27*a*b²)*cosh(d*x + c)⁴ + 17*b³*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)¹¹ + 11*d*cosh(d*x + c)*sinh(d*x + c)¹⁰ + d*sinh(d*x + c)¹¹ - 3*d*cosh(d*x + c)⁹ + (55*d*cosh(d*x + c)² - 3*d)*sinh(d*x + c)⁹ + 3*(55*d*cosh(d*x + c)³ - 9*d*cosh(d*x + c))*sinh(d*x + c)⁸ + 3*d*cosh(d*x + c)⁷ + 3*(110*d*cosh(d*x + c)⁴ - 36*d*cosh(d*x + c)² + d)*sinh(d*x + c)⁷ + 21*(22*d*cosh(d*x + c)⁵ - 12*d*cosh(d*x + c)³ + d*cosh(d*x + c))*sinh(d*x + c)⁶ - d*cosh(d*x + c)⁵ + (462*d*cosh(d*x + c)⁶ - 378*d*cosh(d*x + c)⁴ + 63*d*cosh(d*x + c)² - d)*sinh(d*x + c)⁵ + (330*d*cosh(d*x + c)⁷ - 378*d*cosh(d*x + c)⁵ + 105*d*cosh(d*x + c)³ - 5*d*cosh(d*x + c))*sinh(d*x + c)⁴ + (165*d*cosh(d*x + c)⁸ - 252*d*cosh(d*x + c)⁶ + 105*d*cosh(d*x + c)⁴ - 10*d*cosh(d*x + c)²)*sinh(d*x + c)³ + (55*d*cosh(d*x + c)⁹ - 108*d*cosh(d*x + c)⁷ + 63*d*cosh(d*x + c)⁵ - 10*d*cosh(d*x + c)³)*sinh(d*x + c)² + (11*d*cosh(d*x + c)¹⁰ - 27*d*cosh(d*x + c)⁸ + 21*d*cosh(d*x + c)⁶ - 5*d*cosh(d*x + c)⁴)*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**3,x)

[Out] Timed out

Giac [B] time = 1.43691, size = 419, normalized size = 3.25

$$-\frac{3(dx+c)ab^2}{2d} - \frac{3a^2b \log(e^{(dx+c)} + 1)}{d} + \frac{3a^2b \log(|e^{(dx+c)} - 1|)}{d} + \frac{3b^3d^4e^{(5dx+5c)} - 25b^3d^4e^{(3dx+3c)} + 180ab^2d^4e^{(2dx+2c)}}{480d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")

[Out] -3/2*(d*x + c)*a*b²/d - 3*a²*b*log(e^(d*x + c) + 1)/d + 3*a²*b*log(abs(e^(d*x + c) - 1))/d + 1/480*(3*b³*d⁴*e^(5*d*x + 5*c) - 25*b³*d⁴*e^(3*d*x + 3*c) + 180*a*b²*d⁴*e^(2*d*x + 2*c) + 150*b³*d⁴*e^(d*x + c))/d⁵ + 1/

$$480*(150*b^3*e^(10*d*x + 10*c) - 180*a*b^2*e^(9*d*x + 9*c) - 475*b^3*e^(8*d*x + 8*c) + 528*b^3*e^(6*d*x + 6*c) - 234*b^3*e^(4*d*x + 4*c) + 180*a*b^2*e^(3*d*x + 3*c) + 34*b^3*e^(2*d*x + 2*c) - 3*b^3 - 60*(32*a^3 - 9*a*b^2)*e^(7*d*x + 7*c) + 20*(32*a^3 - 27*a*b^2)*e^(5*d*x + 5*c))*e^(-5*d*x - 5*c)/(d*(e^(d*x + c) + 1)^3*(e^(d*x + c) - 1)^3)$$

3.168 $\int \operatorname{csch}^5(c + dx) (a + b \sinh^3(c + dx))^3 dx$

Optimal. Leaf size=148

$$-\frac{3a^2b \operatorname{coth}(c + dx)}{d} - \frac{3a^3 \tanh^{-1}(\operatorname{cosh}(c + dx))}{8d} - \frac{a^3 \operatorname{coth}(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{3a^3 \operatorname{coth}(c + dx) \operatorname{csch}(c + dx)}{8d} + \frac{3ab}{8d}$$

```
[Out] (3*b^3*x)/8 - (3*a^3*ArcTanh[Cosh[c + d*x]])/(8*d) + (3*a*b^2*Cosh[c + d*x]
)/d - (3*a^2*b*Coth[c + d*x])/d + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d)
- (a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d) - (3*b^3*Cosh[c + d*x]*Sinh[c
+ d*x])/(8*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.172692, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3220, 3767, 8, 3768, 3770, 2638, 2635}

$$-\frac{3a^2b \operatorname{coth}(c + dx)}{d} - \frac{3a^3 \tanh^{-1}(\operatorname{cosh}(c + dx))}{8d} - \frac{a^3 \operatorname{coth}(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{3a^3 \operatorname{coth}(c + dx) \operatorname{csch}(c + dx)}{8d} + \frac{3ab}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (3*b^3*x)/8 - (3*a^3*ArcTanh[Cosh[c + d*x]])/(8*d) + (3*a*b^2*Cosh[c + d*x]
)/d - (3*a^2*b*Coth[c + d*x])/d + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d)
- (a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d) - (3*b^3*Cosh[c + d*x]*Sinh[c
+ d*x])/(8*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^ (p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n
)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^n, x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^5(c + dx) (a + b \sinh^3(c + dx))^3 dx &= i \int (-3ia^2 b \operatorname{csch}^2(c + dx) - ia^3 \operatorname{csch}^5(c + dx) - 3iab^2 \sinh(c + dx) - ib^3 \sinh^3(c + dx)) \operatorname{csch}^4(c + dx) dx \\
 &= a^3 \int \operatorname{csch}^5(c + dx) dx + (3a^2 b) \int \operatorname{csch}^2(c + dx) dx + (3ab^2) \int \sinh(c + dx) dx + (b^3) \int \sinh^3(c + dx) dx \\
 &= \frac{3ab^2 \cosh(c + dx)}{d} - \frac{a^3 \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{b^3 \cosh(c + dx) \sinh(c + dx)}{4d} \\
 &= \frac{3ab^2 \cosh(c + dx)}{d} - \frac{3a^2 b \coth(c + dx)}{d} + \frac{3a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{8d} \\
 &= \frac{3b^3 x}{8} - \frac{3a^3 \tanh^{-1}(\cosh(c + dx))}{8d} + \frac{3ab^2 \cosh(c + dx)}{d} - \frac{3a^2 b \coth(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] time = 6.13235, size = 218, normalized size = 1.47

$$-\frac{3a^2b \tanh\left(\frac{1}{2}(c+dx)\right)}{2d} - \frac{3a^2b \coth\left(\frac{1}{2}(c+dx)\right)}{2d} - \frac{a^3 \operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^3 \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a^3 \operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] (3*b^3*(c + d*x))/(8*d) + (3*a*b^2*Cosh[c + d*x])/d - (3*a^2*b*Coth[(c + d*x)/2])/(2*d) + (3*a^3*Csch[(c + d*x)/2]^2)/(32*d) - (a^3*Csch[(c + d*x)/2]^4)/(64*d) + (3*a^3*Log[Tanh[(c + d*x)/2]])/(8*d) + (3*a^3*Sech[(c + d*x)/2]^2)/(32*d) + (a^3*Sech[(c + d*x)/2]^4)/(64*d) - (b^3*Sinh[2*(c + d*x)])/(4*d) + (b^3*Sinh[4*(c + d*x)])/(32*d) - (3*a^2*b*Tanh[(c + d*x)/2])/(2*d)

Maple [A] time = 0.078, size = 108, normalized size = 0.7

$$\frac{1}{d} \left(a^3 \left(\left(-\frac{(\operatorname{csch}(dx+c))^3}{4} + \frac{3 \operatorname{csch}(dx+c)}{8} \right) \coth(dx+c) - \frac{3 \operatorname{Arctanh}(e^{dx+c})}{4} \right) - 3a^2b \coth(dx+c) + 3ab^2 \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x)

[Out] 1/d*(a^3*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d*x+c)))-3*a^2*b*coth(d*x+c)+3*a*b^2*cosh(d*x+c)+b^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))

Maxima [A] time = 1.16093, size = 344, normalized size = 2.32

$$\frac{1}{64} b^3 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{3}{2} ab^2 \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) - \frac{1}{8} a^3 \left(\frac{3 \log(e^{(-dx-c)} + 1)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c))/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) - 1/8*a^3*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c)))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1)) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 2.60223, size = 11867, normalized size = 80.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*cosh(d*x + c)^16 + 16*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + b^3*sinh(d*x + c)^16 - 12*b^3*cosh(d*x + c)^14 + 96*a*b^2*cosh(d*x + c)^13 + 12*(10*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^14 + 8*(70*b^3*cosh(d*x + c)^3 - 21*b^3*cosh(d*x + c) + 12*a*b^2)*sinh(d*x + c)^13 + 2*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^12 + 2*(910*b^3*cosh(d*x + c)^4 + 12*b^3*d*x - 546*b^3*cosh(d*x + c)^2 + 624*a*b^2*cosh(d*x + c) + 19*b^3)*sinh(d*x + c)^12 + 48*(a^3 - 6*a*b^2)*cosh(d*x + c)^11 + 24*(182*b^3*cosh(d*x + c)^5 - 182*b^3*cosh(d*x + c)^3 + 312*a*b^2*cosh(d*x + c)^2 + 2*a^3 - 12*a*b^2 + (12*b^3*d*x + 19*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 4*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^10 + 4*(2002*b^3*cosh(d*x + c)^6 - 3003*b^3*cosh(d*x + c)^4 + 6864*a*b^2*cosh(d*x + c)^3 - 24*b^3*d*x - 96*a^2*b - 11*b^3 + 33*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^2 + 132*(a^3 - 6*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^10 - 16*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^9 + 8*(1430*b^3*cosh(d*x + c)^7 - 3003*b^3*cosh(d*x + c)^5 + 8580*a*b^2*cosh(d*x + c)^4 + 55*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^3 - 22*a^3 + 24*a*b^2 + 330*(a^3 - 6*a*b^2)*cosh(d*x + c)^2 - 5*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 144*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^8 + 18*(715*b^3*cosh(d*x + c)^8 - 2002*b^3*cosh(d*x + c)^6 + 6864*a*b^2*cosh(d*x + c)^5 + 8*b^3*d*x + 55*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^4 + 440*(a^3 - 6*a*b^2)*cosh(d*x + c)^3 + 64*a^2*b - 10*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^2 - 8*(11*a^3 - 12*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 - 16*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^7 + 16*(715*b^3*cosh(d*x + c)^9 - 2574*b^3*cosh(d*x + c)^7 + 10296*a*b^2*cosh(d*x + c)^6 + 99*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^5 + 990*(a^3 - 6*a*b^2)*cosh(d*x + c)^4 - 30*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^3 - 11*a^3 + 12*a*b^2 - 36*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^2 + 72*(b^3*d*x + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)^6 + 4*(2002*b^3*cosh(d*x + c)^10 - 9009*b^3*cosh
```

$$\begin{aligned}
& (d*x + c)^8 + 41184*a*b^2*cosh(d*x + c)^7 + 462*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^6 + 5544*(a^3 - 6*a*b^2)*cosh(d*x + c)^5 - 24*b^3*d*x - 210*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^4 - 336*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^3 - 288*a^2*b + 11*b^3 + 1008*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^2 - 28*(11*a^3 - 12*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + 96*a*b^2*cosh(d*x + c)^3 + 48*(a^3 - 6*a*b^2)*cosh(d*x + c)^5 + 24*(182*b^3*cosh(d*x + c)^11 - 1001*b^3*cosh(d*x + c)^9 + 5148*a*b^2*cosh(d*x + c)^8 + 66*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^7 + 924*(a^3 - 6*a*b^2)*cosh(d*x + c)^6 - 42*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^5 - 84*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^4 + 336*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^3 + 2*a^3 - 12*a*b^2 - 14*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^2 - (24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + 12*b^3*cosh(d*x + c)^2 + 2*(12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c)^4 + 2*(910*b^3*cosh(d*x + c)^12 - 6006*b^3*cosh(d*x + c)^10 + 34320*a*b^2*cosh(d*x + c)^9 + 495*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^8 + 7920*(a^3 - 6*a*b^2)*cosh(d*x + c)^7 - 420*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^6 - 1008*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^5 + 12*b^3*d*x + 5040*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^4 - 280*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^3 + 192*a^2*b - 19*b^3 - 30*(24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)^2 + 120*(a^3 - 6*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + 8*(70*b^3*cosh(d*x + c)^13 - 546*b^3*cosh(d*x + c)^11 + 3432*a*b^2*cosh(d*x + c)^10 + 55*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^9 + 990*(a^3 - 6*a*b^2)*cosh(d*x + c)^8 - 60*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^7 - 168*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^6 + 1008*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^5 - 70*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^4 - 10*(24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)^3 + 12*a*b^2 + 60*(a^3 - 6*a*b^2)*cosh(d*x + c)^2 + (12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 - b^3 + 12*(10*b^3*cosh(d*x + c)^14 - 91*b^3*cosh(d*x + c)^12 + 624*a*b^2*cosh(d*x + c)^11 + 11*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^10 + 220*(a^3 - 6*a*b^2)*cosh(d*x + c)^9 - 15*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^8 - 48*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^7 + 336*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^6 - 28*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^5 - 5*(24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)^4 + 24*a*b^2*cosh(d*x + c) + 40*(a^3 - 6*a*b^2)*cosh(d*x + c)^3 + b^3 + (12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 24*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*sinh(d*x + c)^12 - 4*a^3*cosh(d*x + c)^10 + 6*a^3*cosh(d*x + c)^8 + 2*(33*a^3*cosh(d*x + c)^2 - 2*a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 4*a^3*cosh(d*x + c)^6 + 3*(165*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + 2*a^3)*sinh(d*x + c)^8 + 24*(33*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + 2*a^3*cosh(d*x + c))*sinh(d*x + c)^7 + a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c)^6 - 210*a^3*cosh(d*x + c)^4 + 42*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^6 + 24*(33*a^3*cosh(d*x + c)^7 - 42*a^3*cosh(d*x + c)^5 + 14*a^3*cosh(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c)^5 + (495*a^3*cosh(d*x + c)^8 - 840*a^3*cosh(d*x + c)^6 + 420*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 - 120*a^3*cosh(d*x + c)^7 + 84*a^3*co
\end{aligned}$$

$$\begin{aligned}
& \text{sh}(d*x + c)^5 - 20*a^3*\text{cosh}(d*x + c)^3 + a^3*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^3 \\
& + 6*(11*a^3*\text{cosh}(d*x + c)^{10} - 30*a^3*\text{cosh}(d*x + c)^8 + 28*a^3*\text{cosh}(d*x + \\
& c)^6 - 10*a^3*\text{cosh}(d*x + c)^4 + a^3*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^2 + 4*(3 \\
& *a^3*\text{cosh}(d*x + c)^{11} - 10*a^3*\text{cosh}(d*x + c)^9 + 12*a^3*\text{cosh}(d*x + c)^7 - 6 \\
& *a^3*\text{cosh}(d*x + c)^5 + a^3*\text{cosh}(d*x + c)^3)*\text{sinh}(d*x + c))*\log(\text{cosh}(d*x + c \\
&) + \text{sinh}(d*x + c) + 1) + 24*(a^3*\text{cosh}(d*x + c)^{12} + 12*a^3*\text{cosh}(d*x + c)*\text{si} \\
& \text{nh}(d*x + c)^{11} + a^3*\text{sinh}(d*x + c)^{12} - 4*a^3*\text{cosh}(d*x + c)^{10} + 6*a^3*\text{cosh} \\
& (d*x + c)^8 + 2*(33*a^3*\text{cosh}(d*x + c)^2 - 2*a^3)*\text{sinh}(d*x + c)^{10} + 20*(11* \\
& a^3*\text{cosh}(d*x + c)^3 - 2*a^3*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^9 - 4*a^3*\text{cosh}(d*x \\
& + c)^6 + 3*(165*a^3*\text{cosh}(d*x + c)^4 - 60*a^3*\text{cosh}(d*x + c)^2 + 2*a^3)*\text{sinh} \\
& (d*x + c)^8 + 24*(33*a^3*\text{cosh}(d*x + c)^5 - 20*a^3*\text{cosh}(d*x + c)^3 + 2*a^3*\text{c} \\
& \text{osh}(d*x + c))*\text{sinh}(d*x + c)^7 + a^3*\text{cosh}(d*x + c)^4 + 4*(231*a^3*\text{cosh}(d*x + \\
& c)^6 - 210*a^3*\text{cosh}(d*x + c)^4 + 42*a^3*\text{cosh}(d*x + c)^2 - a^3)*\text{sinh}(d*x + \\
& c)^6 + 24*(33*a^3*\text{cosh}(d*x + c)^7 - 42*a^3*\text{cosh}(d*x + c)^5 + 14*a^3*\text{cosh}(d* \\
& x + c)^3 - a^3*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^5 + (495*a^3*\text{cosh}(d*x + c)^8 - \\
& 840*a^3*\text{cosh}(d*x + c)^6 + 420*a^3*\text{cosh}(d*x + c)^4 - 60*a^3*\text{cosh}(d*x + c)^2 \\
& + a^3)*\text{sinh}(d*x + c)^4 + 4*(55*a^3*\text{cosh}(d*x + c)^9 - 120*a^3*\text{cosh}(d*x + c)^ \\
& 7 + 84*a^3*\text{cosh}(d*x + c)^5 - 20*a^3*\text{cosh}(d*x + c)^3 + a^3*\text{cosh}(d*x + c))*\text{si} \\
& \text{nh}(d*x + c)^3 + 6*(11*a^3*\text{cosh}(d*x + c)^{10} - 30*a^3*\text{cosh}(d*x + c)^8 + 28*a^ \\
& 3*\text{cosh}(d*x + c)^6 - 10*a^3*\text{cosh}(d*x + c)^4 + a^3*\text{cosh}(d*x + c)^2)*\text{sinh}(d*x \\
& + c)^2 + 4*(3*a^3*\text{cosh}(d*x + c)^{11} - 10*a^3*\text{cosh}(d*x + c)^9 + 12*a^3*\text{cosh}(d \\
& *x + c)^7 - 6*a^3*\text{cosh}(d*x + c)^5 + a^3*\text{cosh}(d*x + c)^3)*\text{sinh}(d*x + c))*\log \\
& (\text{cosh}(d*x + c) + \text{sinh}(d*x + c) - 1) + 8*(2*b^3*\text{cosh}(d*x + c)^{15} - 21*b^3*\text{co} \\
& \text{sh}(d*x + c)^{13} + 156*a*b^2*\text{cosh}(d*x + c)^{12} + 3*(12*b^3*d*x + 19*b^3)*\text{cosh}(\\
& d*x + c)^{11} + 66*(a^3 - 6*a*b^2)*\text{cosh}(d*x + c)^{10} - 5*(24*b^3*d*x + 96*a^2* \\
& b + 11*b^3)*\text{cosh}(d*x + c)^9 - 18*(11*a^3 - 12*a*b^2)*\text{cosh}(d*x + c)^8 + 144* \\
& (b^3*d*x + 8*a^2*b)*\text{cosh}(d*x + c)^7 - 14*(11*a^3 - 12*a*b^2)*\text{cosh}(d*x + c)^ \\
& 6 - 3*(24*b^3*d*x + 288*a^2*b - 11*b^3)*\text{cosh}(d*x + c)^5 + 36*a*b^2*\text{cosh}(d*x \\
& + c)^2 + 30*(a^3 - 6*a*b^2)*\text{cosh}(d*x + c)^4 + 3*b^3*\text{cosh}(d*x + c) + (12*b^ \\
& 3*d*x + 192*a^2*b - 19*b^3)*\text{cosh}(d*x + c)^3)*\text{sinh}(d*x + c))/(d*\text{cosh}(d*x + c \\
&)^{12} + 12*d*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^{11} + d*\text{sinh}(d*x + c)^{12} - 4*d*\text{cosh} \\
& (d*x + c)^{10} + 2*(33*d*\text{cosh}(d*x + c)^2 - 2*d)*\text{sinh}(d*x + c)^{10} + 20*(11*d*\text{co} \\
& \text{sh}(d*x + c)^3 - 2*d*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^9 + 6*d*\text{cosh}(d*x + c)^8 + \\
& 3*(165*d*\text{cosh}(d*x + c)^4 - 60*d*\text{cosh}(d*x + c)^2 + 2*d)*\text{sinh}(d*x + c)^8 + 24 \\
& *(33*d*\text{cosh}(d*x + c)^5 - 20*d*\text{cosh}(d*x + c)^3 + 2*d*\text{cosh}(d*x + c))*\text{sinh}(d*x \\
& + c)^7 - 4*d*\text{cosh}(d*x + c)^6 + 4*(231*d*\text{cosh}(d*x + c)^6 - 210*d*\text{cosh}(d*x + \\
& c)^4 + 42*d*\text{cosh}(d*x + c)^2 - d)*\text{sinh}(d*x + c)^6 + 24*(33*d*\text{cosh}(d*x + c)^ \\
& 7 - 42*d*\text{cosh}(d*x + c)^5 + 14*d*\text{cosh}(d*x + c)^3 - d*\text{cosh}(d*x + c))*\text{sinh}(d*x \\
& + c)^5 + d*\text{cosh}(d*x + c)^4 + (495*d*\text{cosh}(d*x + c)^8 - 840*d*\text{cosh}(d*x + c)^ \\
& 6 + 420*d*\text{cosh}(d*x + c)^4 - 60*d*\text{cosh}(d*x + c)^2 + d)*\text{sinh}(d*x + c)^4 + 4*(\\
& 55*d*\text{cosh}(d*x + c)^9 - 120*d*\text{cosh}(d*x + c)^7 + 84*d*\text{cosh}(d*x + c)^5 - 20*d* \\
& \text{cosh}(d*x + c)^3 + d*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^3 + 6*(11*d*\text{cosh}(d*x + c)^ \\
& 10 - 30*d*\text{cosh}(d*x + c)^8 + 28*d*\text{cosh}(d*x + c)^6 - 10*d*\text{cosh}(d*x + c)^4 + d \\
& *\text{cosh}(d*x + c)^2)*\text{sinh}(d*x + c)^2 + 4*(3*d*\text{cosh}(d*x + c)^{11} - 10*d*\text{cosh}(d*x \\
& + c)^9 + 12*d*\text{cosh}(d*x + c)^7 - 6*d*\text{cosh}(d*x + c)^5 + d*\text{cosh}(d*x + c)^3)*\text{s}
\end{aligned}$$

`inh(d*x + c))`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**3,x)`

[Out] Timed out

Giac [B] time = 1.4095, size = 475, normalized size = 3.21

$$\frac{3(dx+c)b^3}{8d} - \frac{3a^3 \log(e^{dx+c} + 1)}{8d} + \frac{3a^3 \log(|e^{dx+c} - 1|)}{8d} + \frac{b^3 d^3 e^{4dx+4c} - 8b^3 d^3 e^{2dx+2c} + 96ab^2 d^3 e^{dx+c}}{64d^4} + \frac{(96ab^2}{$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")`

[Out]
$$\frac{3}{8}(dx+c)b^3/d - \frac{3}{8}a^3 \log(e^{dx+c} + 1)/d + \frac{3}{8}a^3 \log(\text{abs}(e^{dx+c} - 1))/d + \frac{1}{64}(b^3 d^3 e^{4dx+4c} - 8b^3 d^3 e^{2dx+2c} + 96a^2 b^2 d^3 e^{dx+c})/d^4 + \frac{1}{64}(96a^2 b^2 e^{3dx+3c} + 12b^3 e^{2dx+2c} - b^3 + 48(a^3 + 2ab^2)e^{11dx+11c} - 8(48a^2 b - b^3)e^{10dx+10c} - 16(11a^3 + 24ab^2)e^{9dx+9c} + 3(384a^2 b - 11b^3)e^{8dx+8c} - 16(11a^3 - 36ab^2)e^{7dx+7c} - 4(288a^2 b - 13b^3)e^{6dx+6c} + 48(a^3 - 8ab^2)e^{5dx+5c} + 2(192a^2 b - 19b^3)e^{4dx+4c})e^{-4dx-4c}/(d(e^{dx+c} + 1)^4(e^{dx+c} - 1)^4)$$

3.169 $\int \operatorname{csch}^6(c + dx) \left(a + b \sinh^3(c + dx) \right)^3 dx$

Optimal. Leaf size=131

$$\frac{3a^2b \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{3a^2b \coth(c + dx)\operatorname{csch}(c + dx)}{2d} - \frac{a^3 \coth^5(c + dx)}{5d} + \frac{2a^3 \coth^3(c + dx)}{3d} - \frac{a^3 \coth(c + dx)}{d}$$

```
[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^3*Cosh[c + d*x])/d
+ (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x])/d + (2*a^3*Coth[c + d*x]
]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) - (3*a^2*b*Coth[c + d*x]*Csch[c +
d*x])/(2*d)
```

Rubi [A] time = 0.124965, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3220, 3768, 3770, 3767, 2633}

$$\frac{3a^2b \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{3a^2b \coth(c + dx)\operatorname{csch}(c + dx)}{2d} - \frac{a^3 \coth^5(c + dx)}{5d} + \frac{2a^3 \coth^3(c + dx)}{3d} - \frac{a^3 \coth(c + dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^3*Cosh[c + d*x])/d
+ (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x])/d + (2*a^3*Coth[c + d*x]
]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) - (3*a^2*b*Coth[c + d*x]*Csch[c +
d*x])/(2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol]
:> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] &&
IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol]
:> -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1),
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^6(c + dx) (a + b \sinh^3(c + dx))^3 dx &= - \int (-3ab^2 - 3a^2b \operatorname{csch}^3(c + dx) - a^3 \operatorname{csch}^6(c + dx) - b^3 \sinh^3(c + dx)) dx \\ &= 3ab^2x + a^3 \int \operatorname{csch}^6(c + dx) dx + (3a^2b) \int \operatorname{csch}^3(c + dx) dx + b^3 \int \sinh^3(c + dx) dx \\ &= 3ab^2x - \frac{3a^2b \coth(c + dx) \operatorname{csch}(c + dx)}{2d} - \frac{1}{2} (3a^2b) \int \operatorname{csch}(c + dx) dx - \frac{b^3}{3} \int \sinh^3(c + dx) dx \\ &= 3ab^2x + \frac{3a^2b \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{b^3 \cosh(c + dx)}{d} + \frac{b^3 \cosh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 1.84643, size = 225, normalized size = 1.72

$$\frac{1}{2}a \left(8 \left(-32a^2 \tanh\left(\frac{1}{2}(c + dx)\right) - 24a^2 \sinh^6\left(\frac{1}{2}(c + dx)\right) \operatorname{csch}^5(c + dx) - 38a^2 \sinh^4\left(\frac{1}{2}(c + dx)\right) \operatorname{csch}^3(c + dx) - 45ab \operatorname{sech}^3(c + dx) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] (-360*b^3*Cosh[c + d*x] + 40*b^3*Cosh[3*(c + d*x)] + (a*(-256*a^2*Coth[(c + d*x)/2] - 360*a*b*Csch[(c + d*x)/2]^2 + 19*a^2*Csch[(c + d*x)/2]^4*Sinh[c + d*x] - 3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x] + 8*(180*b*(2*b*(c + d*x) - a*Log[Tanh[(c + d*x)/2]]) - 45*a*b*Sech[(c + d*x)/2]^2 - 38*a^2*Csch[c +

$d*x]^3*\text{Sinh}[(c + d*x)/2]^4 - 24*a^2*\text{Csch}[c + d*x]^5*\text{Sinh}[(c + d*x)/2]^6 - 3$
 $2*a^2*\text{Tanh}[(c + d*x)/2]))/2)/(480*d)$

Maple [A] time = 0.085, size = 99, normalized size = 0.8

$$\frac{1}{d} \left(a^3 \left(-\frac{8}{15} - \frac{(\text{csch}(dx+c))^4}{5} + \frac{4(\text{csch}(dx+c))^2}{15} \right) \coth(dx+c) + 3a^2b \left(-\frac{1}{2} \text{csch}(dx+c) \coth(dx+c) + \text{Artanh} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x)`

[Out] `1/d*(a^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(`
`-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a*b^2*(d*x+c)+b^3*(-2/3`
`+1/3*sinh(d*x+c)^2)*cosh(d*x+c))`

Maxima [B] time = 1.08746, size = 493, normalized size = 3.76

$$3ab^2x + \frac{1}{24}b^3 \left(\frac{e^{3dx+3c}}{d} - \frac{9e^{dx+c}}{d} - \frac{9e^{-dx-c}}{d} + \frac{e^{-3dx-3c}}{d} \right) + \frac{3}{2}a^2b \left(\frac{\log(e^{-dx-c}+1)}{d} - \frac{\log(e^{-dx-c}-1)}{d} + \frac{2(e^{-dx-c}-1)}{d(2e^{-2dx-2c}-e^{-4dx-4c}-1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")`

[Out] `3*a*b^2*x + 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/`
`d + e^(-3*d*x - 3*c)/d) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x`
`- c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) -`
`e^(-4*d*x - 4*c) - 1))) - 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2`
`*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-`
`10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-`
`4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c`
`) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6`
`*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)))`

Fricas [B] time = 2.36594, size = 12388, normalized size = 94.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")

[Out] $\frac{1}{120} \cdot (5b^3 \cosh(dx+c)^{16} + 80b^3 \cosh(dx+c) \sinh(dx+c)^{15} + 5b^3 \sinh(dx+c)^{16} + 360ab^2 dx \cosh(dx+c)^{13} - 70b^3 \cosh(dx+c)^{14} - 1800a^2 b^2 dx \cosh(dx+c)^{11} + 10(60b^3 \cosh(dx+c)^2 - 7b^3) \sinh(dx+c)^{14} + 3600ab^2 dx \cosh(dx+c)^9 + 20(140b^3 \cosh(dx+c)^3 + 18a^2 b^2 dx - 49b^3 \cosh(dx+c)) \sinh(dx+c)^{13} - 10(36a^2 b - 23b^3) \cosh(dx+c)^{12} + 10(910b^3 \cosh(dx+c)^4 + 468ab^2 dx \cosh(dx+c) - 637b^3 \cosh(dx+c)^2 - 36a^2 b + 23b^3) \sinh(dx+c)^{12} + 40(546b^3 \cosh(dx+c)^5 + 702ab^2 dx \cosh(dx+c)^2 - 637b^3 \cosh(dx+c)^3 - 45ab^2 dx - 3(36a^2 b - 23b^3) \cosh(dx+c)) \sinh(dx+c)^{11} + 90(8a^2 b - 3b^3) \cosh(dx+c)^{10} + 10(4004b^3 \cosh(dx+c)^6 + 10296ab^2 dx \cosh(dx+c)^3 - 7007b^3 \cosh(dx+c)^4 - 1980ab^2 dx \cosh(dx+c) + 72a^2 b - 27b^3 - 66(36a^2 b - 23b^3) \cosh(dx+c)^2) \sinh(dx+c)^{10} + 20(2860b^3 \cosh(dx+c)^7 + 12870ab^2 dx \cosh(dx+c)^4 - 7007b^3 \cosh(dx+c)^5 - 4950ab^2 dx \cosh(dx+c)^2 + 180ab^2 dx - 110(36a^2 b - 23b^3) \cosh(dx+c)^3 + 45(8a^2 b - 3b^3) \cosh(dx+c)) \sinh(dx+c)^9 + 30(2145b^3 \cosh(dx+c)^8 + 15444ab^2 dx \cosh(dx+c)^5 - 7007b^3 \cosh(dx+c)^6 - 9900ab^2 dx \cosh(dx+c)^3 + 1080ab^2 dx \cosh(dx+c) - 165(36a^2 b - 23b^3) \cosh(dx+c)^4 + 135(8a^2 b - 3b^3) \cosh(dx+c)^2) \sinh(dx+c)^8 - 80(45ab^2 dx + 16a^3) \cosh(dx+c)^7 + 80(715b^3 \cosh(dx+c)^9 + 7722ab^2 dx \cosh(dx+c)^6 - 3003b^3 \cosh(dx+c)^7 - 7425ab^2 dx \cosh(dx+c)^4 + 1620ab^2 dx \cosh(dx+c)^2 - 99(36a^2 b - 23b^3) \cosh(dx+c)^5 - 45ab^2 dx + 135(8a^2 b - 3b^3) \cosh(dx+c)^3 - 16a^3) \sinh(dx+c)^7 - 90(8a^2 b - 3b^3) \cosh(dx+c)^6 + 10(4004b^3 \cosh(dx+c)^{10} + 61776ab^2 dx \cosh(dx+c)^7 - 21021b^3 \cosh(dx+c)^8 - 83160ab^2 dx \cosh(dx+c)^5 + 30240ab^2 dx \cosh(dx+c)^3 - 924(36a^2 b - 23b^3) \cosh(dx+c)^6 + 1890(8a^2 b - 3b^3) \cosh(dx+c)^4 - 72a^2 b + 27b^3 - 56(45ab^2 dx + 16a^3) \cosh(dx+c)) \sinh(dx+c)^6 + 40(45ab^2 dx + 16a^3) \cosh(dx+c)^5 + 20(1092b^3 \cosh(dx+c)^{11} + 23166ab^2 dx \cosh(dx+c)^8 - 7007b^3 \cosh(dx+c)^9 - 41580ab^2 dx \cosh(dx+c)^6 + 22680ab^2 dx \cosh(dx+c)^4 - 396(36a^2 b - 23b^3) \cosh(dx+c)^7 + 1134(8a^2 b - 3b^3) \cosh(dx+c)^5 + 90ab^2 dx + 32a^3 - 84(45ab^2 dx + 16a^3) \cosh(dx+c)^2 - 27(8a^2 b - 3b^3) \cosh(dx+c)) \sinh(dx+c)^5 + 70b^3 \cosh(dx+c)^2 + 10(36a^2 b - 23b^3) \cosh(dx+c)^4 + 10(910b^3 \cosh(dx+c)^{12} + 25740ab^2 dx \cosh(dx+c)^9 - 7007b^3 \cosh(dx+c)^{10} - 59400ab^2 dx \cosh(dx+c)^7 + 45360ab^2 dx \cosh(dx+c)^5 - 495(36a^2 b - 23b^3) \cosh(dx+c)^8 + 1890(8a^2 b - 3b^3) \cosh(dx+c)^6 - 280(45ab^2 dx + 16a^3) \cosh(dx+c)^3 + 36a^2 b - 23b^3 - 135(8a^2 b - 3b^3) \cosh(dx+c)^2 + 20(45ab^2 dx + 16a^3) \cosh(dx+c)) \sinh(dx+c)^4 - 8*$

$$\begin{aligned}
& (45ab^2dx + 16a^3)\cosh(dx + c)^3 + 8(350b^3\cosh(dx + c)^{13} + 128 \\
& 70ab^2dx\cosh(dx + c)^{10} - 3185b^3\cosh(dx + c)^{11} - 37125ab^2dx \\
& \cosh(dx + c)^8 + 37800ab^2dx\cosh(dx + c)^6 - 275(36a^2b - 23b^3) \\
&)\cosh(dx + c)^9 + 1350(8a^2b - 3b^3)\cosh(dx + c)^7 - 45ab^2dx - \\
& 350(45ab^2dx + 16a^3)\cosh(dx + c)^4 - 225(8a^2b - 3b^3)\cosh(dx \\
& *x + c)^3 - 16a^3 + 50(45ab^2dx + 16a^3)\cosh(dx + c)^2 + 5(36a^2 \\
& *b - 23b^3)\cosh(dx + c)\sinh(dx + c)^3 - 5b^3 + 2(300b^3\cosh(dx + \\
& c)^{14} + 14040ab^2dx\cosh(dx + c)^{11} - 3185b^3\cosh(dx + c)^{12} - 495 \\
& 00ab^2dx\cosh(dx + c)^9 + 64800ab^2dx\cosh(dx + c)^7 - 330(36a^2 \\
& *b - 23b^3)\cosh(dx + c)^{10} + 2025(8a^2b - 3b^3)\cosh(dx + c)^8 - 8 \\
& 40(45ab^2dx + 16a^3)\cosh(dx + c)^5 - 675(8a^2b - 3b^3)\cosh(dx \\
& + c)^4 + 200(45ab^2dx + 16a^3)\cosh(dx + c)^3 + 35b^3 + 30(36a^2 \\
& *b - 23b^3)\cosh(dx + c)^2 - 12(45ab^2dx + 16a^3)\cosh(dx + c)*si \\
& nh(dx + c)^2 + 180(a^2b\cosh(dx + c)^{13} + 13a^2b\cosh(dx + c)*sinh(d \\
& *x + c)^{12} + a^2b*sinh(dx + c)^{13} - 5a^2b\cosh(dx + c)^{11} + 10a^2b*c \\
& osh(dx + c)^9 + (78a^2b\cosh(dx + c)^2 - 5a^2b)*sinh(dx + c)^{11} + 11 \\
& *(26a^2b\cosh(dx + c)^3 - 5a^2b\cosh(dx + c))*sinh(dx + c)^{10} - 10a \\
& ^2b\cosh(dx + c)^7 + 5(143a^2b\cosh(dx + c)^4 - 55a^2b\cosh(dx + c \\
&)^2 + 2a^2b)*sinh(dx + c)^9 + 3(429a^2b\cosh(dx + c)^5 - 275a^2b*c \\
& osh(dx + c)^3 + 30a^2b\cosh(dx + c))*sinh(dx + c)^8 + 5a^2b\cosh(dx \\
& + c)^5 + 2(858a^2b\cosh(dx + c)^6 - 825a^2b\cosh(dx + c)^4 + 180a^ \\
& 2b\cosh(dx + c)^2 - 5a^2b)*sinh(dx + c)^7 + 2(858a^2b\cosh(dx + c \\
&)^7 - 1155a^2b\cosh(dx + c)^5 + 420a^2b\cosh(dx + c)^3 - 35a^2b\cosh \\
& (dx + c))*sinh(dx + c)^6 - a^2b\cosh(dx + c)^3 + (1287a^2b\cosh(dx + \\
& c)^8 - 2310a^2b\cosh(dx + c)^6 + 1260a^2b\cosh(dx + c)^4 - 210a^2b \\
& *cosh(dx + c)^2 + 5a^2b)*sinh(dx + c)^5 + 5(143a^2b\cosh(dx + c)^9 \\
& - 330a^2b\cosh(dx + c)^7 + 252a^2b\cosh(dx + c)^5 - 70a^2b\cosh(dx \\
& + c)^3 + 5a^2b\cosh(dx + c))*sinh(dx + c)^4 + (286a^2b\cosh(dx + c \\
&)^{10} - 825a^2b\cosh(dx + c)^8 + 840a^2b\cosh(dx + c)^6 - 350a^2b*cos \\
& h(dx + c)^4 + 50a^2b\cosh(dx + c)^2 - a^2b)*sinh(dx + c)^3 + (78a^2 \\
& b\cosh(dx + c)^{11} - 275a^2b\cosh(dx + c)^9 + 360a^2b\cosh(dx + c)^7 \\
& - 210a^2b\cosh(dx + c)^5 + 50a^2b\cosh(dx + c)^3 - 3a^2b\cosh(dx + \\
& c))*sinh(dx + c)^2 + (13a^2b\cosh(dx + c)^{12} - 55a^2b\cosh(dx + c)^ \\
& 10 + 90a^2b\cosh(dx + c)^8 - 70a^2b\cosh(dx + c)^6 + 25a^2b\cosh(dx \\
& *x + c)^4 - 3a^2b\cosh(dx + c)^2)*sinh(dx + c))*log(cosh(dx + c) + sinh \\
& (dx + c) + 1) - 180(a^2b\cosh(dx + c)^{13} + 13a^2b\cosh(dx + c)*sinh(\\
& dx + c)^{12} + a^2b*sinh(dx + c)^{13} - 5a^2b\cosh(dx + c)^{11} + 10a^2b* \\
& cosh(dx + c)^9 + (78a^2b\cosh(dx + c)^2 - 5a^2b)*sinh(dx + c)^{11} + 1 \\
& 1*(26a^2b\cosh(dx + c)^3 - 5a^2b\cosh(dx + c))*sinh(dx + c)^{10} - 10* \\
& a^2b\cosh(dx + c)^7 + 5(143a^2b\cosh(dx + c)^4 - 55a^2b\cosh(dx + \\
& c)^2 + 2a^2b)*sinh(dx + c)^9 + 3(429a^2b\cosh(dx + c)^5 - 275a^2b* \\
& cosh(dx + c)^3 + 30a^2b\cosh(dx + c))*sinh(dx + c)^8 + 5a^2b\cosh(dx \\
& *x + c)^5 + 2(858a^2b\cosh(dx + c)^6 - 825a^2b\cosh(dx + c)^4 + 180a \\
& ^2b\cosh(dx + c)^2 - 5a^2b)*sinh(dx + c)^7 + 2(858a^2b\cosh(dx + c \\
&)^7 - 1155a^2b\cosh(dx + c)^5 + 420a^2b\cosh(dx + c)^3 - 35a^2b*cos
\end{aligned}$$

$$\begin{aligned}
& h(dx + c) \sinh(dx + c)^6 - a^2 b \cosh(dx + c)^3 + (1287 a^2 b \cosh(dx + c)^8 - 2310 a^2 b \cosh(dx + c)^6 + 1260 a^2 b \cosh(dx + c)^4 - 210 a^2 b \cosh(dx + c)^2 + 5 a^2 b) \sinh(dx + c)^5 + 5(143 a^2 b \cosh(dx + c)^9 - 330 a^2 b \cosh(dx + c)^7 + 252 a^2 b \cosh(dx + c)^5 - 70 a^2 b \cosh(dx + c)^3 + 5 a^2 b \cosh(dx + c)) \sinh(dx + c)^4 + (286 a^2 b \cosh(dx + c)^{10} - 825 a^2 b \cosh(dx + c)^8 + 840 a^2 b \cosh(dx + c)^6 - 350 a^2 b \cosh(dx + c)^4 + 50 a^2 b \cosh(dx + c)^2 - a^2 b) \sinh(dx + c)^3 + (78 a^2 b \cosh(dx + c)^{11} - 275 a^2 b \cosh(dx + c)^9 + 360 a^2 b \cosh(dx + c)^7 - 210 a^2 b \cosh(dx + c)^5 + 50 a^2 b \cosh(dx + c)^3 - 3 a^2 b \cosh(dx + c)) \sinh(dx + c)^2 + (13 a^2 b \cosh(dx + c)^{12} - 55 a^2 b \cosh(dx + c)^{10} + 90 a^2 b \cosh(dx + c)^8 - 70 a^2 b \cosh(dx + c)^6 + 25 a^2 b \cosh(dx + c)^4 - 3 a^2 b \cosh(dx + c)^2) \sinh(dx + c) \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 4(20 b^3 \cosh(dx + c)^{15} + 1170 a b^2 d x \cosh(dx + c)^{12} - 245 b^3 \cosh(dx + c)^{13} - 4950 a b^2 d x \cosh(dx + c)^{10} + 8100 a b^2 d x \cosh(dx + c)^8 - 30(36 a^2 b - 23 b^3) \cosh(dx + c)^{11} + 225(8 a^2 b - 3 b^3) \cosh(dx + c)^9 - 140(45 a b^2 d x + 16 a^3) \cosh(dx + c)^6 - 135(8 a^2 b - 3 b^3) \cosh(dx + c)^5 + 50(45 a b^2 d x + 16 a^3) \cosh(dx + c)^4 + 35 b^3 \cosh(dx + c) + 10(36 a^2 b - 23 b^3) \cosh(dx + c)^3 - 6(45 a b^2 d x + 16 a^3) \cosh(dx + c)^2) \sinh(dx + c) / (d \cosh(dx + c))^{13} + 13 d \cosh(dx + c) \sinh(dx + c)^{12} + d \sinh(dx + c)^{13} - 5 d \cosh(dx + c)^{11} + (78 d \cosh(dx + c)^2 - 5 d) \sinh(dx + c)^{11} + 11(26 d \cosh(dx + c)^3 - 5 d \cosh(dx + c)) \sinh(dx + c)^{10} + 10 d \cosh(dx + c)^9 + 5(143 d \cosh(dx + c)^4 - 55 d \cosh(dx + c)^2 + 2 d) \sinh(dx + c)^9 + 3(429 d \cosh(dx + c)^5 - 275 d \cosh(dx + c)^3 + 30 d \cosh(dx + c)) \sinh(dx + c)^8 - 10 d \cosh(dx + c)^7 + 2(858 d \cosh(dx + c)^6 - 825 d \cosh(dx + c)^4 + 180 d \cosh(dx + c)^2 - 5 d) \sinh(dx + c)^7 + 2(858 d \cosh(dx + c)^7 - 1155 d \cosh(dx + c)^5 + 420 d \cosh(dx + c)^3 - 35 d \cosh(dx + c)) \sinh(dx + c)^6 + 5 d \cosh(dx + c)^5 + (1287 d \cosh(dx + c)^8 - 2310 d \cosh(dx + c)^6 + 1260 d \cosh(dx + c)^4 - 210 d \cosh(dx + c)^2 + 5 d) \sinh(dx + c)^5 + 5(143 d \cosh(dx + c)^9 - 330 d \cosh(dx + c)^7 + 252 d \cosh(dx + c)^5 - 70 d \cosh(dx + c)^3 + 5 d \cosh(dx + c)) \sinh(dx + c)^4 - d \cosh(dx + c)^3 + (286 d \cosh(dx + c)^{10} - 825 d \cosh(dx + c)^8 + 840 d \cosh(dx + c)^6 - 350 d \cosh(dx + c)^4 + 50 d \cosh(dx + c)^2 - d) \sinh(dx + c)^3 + (78 d \cosh(dx + c)^{11} - 275 d \cosh(dx + c)^9 + 360 d \cosh(dx + c)^7 - 210 d \cosh(dx + c)^5 + 50 d \cosh(dx + c)^3 - 3 d \cosh(dx + c)) \sinh(dx + c)^2 + (13 d \cosh(dx + c)^{12} - 55 d \cosh(dx + c)^{10} + 90 d \cosh(dx + c)^8 - 70 d \cosh(dx + c)^6 + 25 d \cosh(dx + c)^4 - 3 d \cosh(dx + c)^2) \sinh(dx + c)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**3)**3,x)

[Out] Timed out

Giac [B] time = 1.44204, size = 389, normalized size = 2.97

$$\frac{3(dx+c)ab^2}{d} + \frac{3a^2b \log(e^{dx+c} + 1)}{2d} - \frac{3a^2b \log(|e^{dx+c} - 1|)}{2d} + \frac{b^3d^2e^{(3dx+3c)} - 9b^3d^2e^{(dx+c)}}{24d^3} - \frac{(475b^3e^{(8dx+8c)} + 1280a^3e^{(7dx+7c)} - 640a^3e^{(5dx+5c)} + 128a^3e^{(3dx+3c)} - 70b^3e^{(2dx+2c)} + 5b^3 + 45(8a^2b + b^3)e^{(12dx+12c)} - 10(72a^2b + 23b^3)e^{(10dx+10c)} + 20(36a^2b - 25b^3)e^{(6dx+6c)} - 5(72a^2b - 55b^3)e^{(4dx+4c)})e^{(-3dx-3c)}}{d(e^{(dx+c)} + 1)^5(e^{(dx+c)} - 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")

[Out] 3*(d*x + c)*a*b^2/d + 3/2*a^2*b*log(e^(d*x + c) + 1)/d - 3/2*a^2*b*log(abs(e^(d*x + c) - 1))/d + 1/24*(b^3*d^2*e^(3*d*x + 3*c) - 9*b^3*d^2*e^(d*x + c))/d^3 - 1/120*(475*b^3*e^(8*d*x + 8*c) + 1280*a^3*e^(7*d*x + 7*c) - 640*a^3*e^(5*d*x + 5*c) + 128*a^3*e^(3*d*x + 3*c) - 70*b^3*e^(2*d*x + 2*c) + 5*b^3 + 45*(8*a^2*b + b^3)*e^(12*d*x + 12*c) - 10*(72*a^2*b + 23*b^3)*e^(10*d*x + 10*c) + 20*(36*a^2*b - 25*b^3)*e^(6*d*x + 6*c) - 5*(72*a^2*b - 55*b^3)*e^(4*d*x + 4*c))*e^(-3*d*x - 3*c)/(d*(e^(d*x + c) + 1)^5*(e^(d*x + c) - 1)^5)

3.170 $\int \operatorname{csch}^7(c + dx) (a + b \sinh^3(c + dx))^3 dx$

Optimal. Leaf size=166

$$-\frac{a^2 b \operatorname{coth}^3(c + dx)}{d} + \frac{3a^2 b \operatorname{coth}(c + dx)}{d} + \frac{5a^3 \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{a^3 \operatorname{coth}(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a^3 \operatorname{coth}(c + dx)}{24d}$$

[Out] $-(b^3 x)/2 + (5a^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]])/(16d) - (3a^2 b \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]])/d + (3a^2 b \operatorname{Coth}[c + dx])/d - (a^2 b \operatorname{Coth}[c + dx]^3)/d - (5a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx])/(16d) + (5a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^3)/(24d) - (a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^5)/(6d) + (b^3 \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c + dx])/(2d)$

Rubi [A] time = 0.197102, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3220, 3770, 3767, 3768, 2635, 8}

$$-\frac{a^2 b \operatorname{coth}^3(c + dx)}{d} + \frac{3a^2 b \operatorname{coth}(c + dx)}{d} + \frac{5a^3 \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{a^3 \operatorname{coth}(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a^3 \operatorname{coth}(c + dx)}{24d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + dx]^7 (a + b \operatorname{Sinh}[c + dx]^3)^3, x]$

[Out] $-(b^3 x)/2 + (5a^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]])/(16d) - (3a^2 b \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]])/d + (3a^2 b \operatorname{Coth}[c + dx])/d - (a^2 b \operatorname{Coth}[c + dx]^3)/d - (5a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx])/(16d) + (5a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^3)/(24d) - (a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^5)/(6d) + (b^3 \operatorname{Cosh}[c + dx] \operatorname{Sinh}[c + dx])/(2d)$

Rule 3220

$\operatorname{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)} ((a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandTrig}[\sin[e + f x]^m (a + b \sin[e + f x]^n)^p, x], x] /; \operatorname{FreeQ}\{a, b, e, f\}, x \ \&\& \operatorname{IntegersQ}[m, p] \ \&\& (\operatorname{EqQ}[n, 4] \ || \ \operatorname{GtQ}[p, 0] \ || \ (\operatorname{EqQ}[p, -1] \ \&\& \operatorname{IntegerQ}[n]))$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + dx]]/d, x] /; \operatorname{FreeQ}\{c, d\}, x]$

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^7(c + dx) (a + b \sinh^3(c + dx))^3 dx &= -\left(i \int (3iab^2 \operatorname{csch}(c + dx) + 3ia^2b \operatorname{csch}^4(c + dx) + ia^3 \operatorname{csch}^7(c + dx) + ib^3 \operatorname{csch}^7(c + dx)) dx\right. \\
 &= a^3 \int \operatorname{csch}^7(c + dx) dx + (3a^2b) \int \operatorname{csch}^4(c + dx) dx + (3ab^2) \int \operatorname{csch}(c + dx) dx \\
 &= -\frac{3ab^2 \tanh^{-1}(\cosh(c + dx))}{d} - \frac{a^3 \coth(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{b^3 \cosh(c + dx)}{d} \\
 &= -\frac{b^3 x}{2} - \frac{3ab^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3a^2b \coth(c + dx)}{d} - \frac{a^2b \coth^3(c + dx)}{d} \\
 &= -\frac{b^3 x}{2} - \frac{3ab^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3a^2b \coth(c + dx)}{d} - \frac{a^2b \coth^3(c + dx)}{d} \\
 &= -\frac{b^3 x}{2} + \frac{5a^3 \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{3ab^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{3a^2b \coth(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] time = 1.55176, size = 236, normalized size = 1.42

$$-384a^2b \tanh\left(\frac{1}{2}(c + dx)\right) - 384a^2b \coth\left(\frac{1}{2}(c + dx)\right) - 384a^2b \sinh^4\left(\frac{1}{2}(c + dx)\right) \operatorname{csch}^3(c + dx) - 6a^2 \operatorname{csch}^4\left(\frac{1}{2}(c + dx)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^3,x]

[Out] $-(192*b^3*c + 192*b^3*d*x - 384*a^2*b*Coth[(c + d*x)/2] + 30*a^3*Csch[(c + d*x)/2]^2 + a^3*Csch[(c + d*x)/2]^6 + 120*a^3*Log[Tanh[(c + d*x)/2]] - 1152*a*b^2*Log[Tanh[(c + d*x)/2]] + 30*a^3*Sech[(c + d*x)/2]^2 + 6*a^3*Sech[(c + d*x)/2]^4 + a^3*Sech[(c + d*x)/2]^6 - 384*a^2*b*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - 6*a^2*Csch[(c + d*x)/2]^4*(a - 4*b*Sinh[c + d*x]) - 96*b^3*Sinh[2*(c + d*x)] - 384*a^2*b*Tanh[(c + d*x)/2])/(384*d)$

Maple [A] time = 0.083, size = 119, normalized size = 0.7

$$\frac{1}{d} \left(a^3 \left(\left(-\frac{(\operatorname{csch}(dx+c))^5}{6} + \frac{5(\operatorname{csch}(dx+c))^3}{24} - \frac{5\operatorname{csch}(dx+c)}{16} \right) \coth(dx+c) + \frac{5 \operatorname{Arctanh}(e^{dx+c})}{8} \right) + 3a^2b \left(\frac{2}{3} - \frac{1}{3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x)

[Out] $1/d*(a^3*((-1/6*\operatorname{csch}(d*x+c)^5+5/24*\operatorname{csch}(d*x+c)^3-5/16*\operatorname{csch}(d*x+c))*\operatorname{coth}(d*x+c)+5/8*\operatorname{arctanh}(\exp(d*x+c)))+3*a^2*b*(2/3-1/3*\operatorname{csch}(d*x+c)^2)*\operatorname{coth}(d*x+c)-6*a*b^2*\operatorname{arctanh}(\exp(d*x+c))+b^3*(1/2*\operatorname{cosh}(d*x+c)*\operatorname{sinh}(d*x+c)-1/2*d*x-1/2*c))$

Maxima [B] time = 1.08364, size = 479, normalized size = 2.89

$$-\frac{1}{8}b^3\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + \frac{1}{48}a^3\left(\frac{15\log(e^{-dx-c}+1)}{d} - \frac{15\log(e^{-dx-c}-1)}{d} + \frac{2(15e^{-dx-c} - 85e^{-3dx-3c})}{d(6e^{-2dx-2c} - 15e^{-4dx-4c})} + \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")

[Out] $-1/8*b^3*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) + 1/48*a^3*(15*\log(e^{-d*x - c} + 1)/d - 15*\log(e^{-d*x - c} - 1)/d + 2*(15*e^{-d*x - c} - 85*e^{-3*d*x - 3*c}) + 198*e^{-5*d*x - 5*c} + 198*e^{-7*d*x - 7*c} - 85*e^{-9*d*x - 9*c} + 15*e^{-11*d*x - 11*c})/(d*(6*e^{-2*d*x - 2*c} - 15*e^{-4*d*x - 4*c}) + 20*e^{-6*d*x - 6*c} - 15*e^{-8*d*x - 8*c} + 6*e^{-10*d*x - 10*c} - e$

$$\begin{aligned} & ^{-12dx - 12c} - 1)) - 3ab^2(\log(e^{-dx - c} + 1)/d - \log(e^{-dx - c} - 1)/d) + 4a^2b(3e^{-2dx - 2c}/(d(3e^{-2dx - 2c} - 3e^{-4dx - 4c}) + e^{-6dx - 6c} - 1)) - 1/(d(3e^{-2dx - 2c} - 3e^{-4dx - 4c}) + e^{-6dx - 6c} - 1)) \end{aligned}$$

Fricas [B] time = 2.78234, size = 16188, normalized size = 97.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^7*(a+b*sinh(dx+c)^3)^3,x, algorithm="fricas")

[Out] $\frac{1}{48}(6b^3 \cosh(dx+c)^{16} + 96b^3 \cosh(dx+c) \sinh(dx+c)^{15} + 6b^3 \sinh(dx+c)^{16} - 30a^3 \cosh(dx+c)^{13} - 12(2b^3 dx + 3b^3) \cosh(dx+c)^{14} - 12(2b^3 dx - 60b^3 \cosh(dx+c)^2 + 3b^3) \sinh(dx+c)^{14} + 170a^3 \cosh(dx+c)^{11} + 6(560b^3 \cosh(dx+c)^3 - 5a^3 - 28(2b^3 dx + 3b^3) \cosh(dx+c)) \sinh(dx+c)^{13} + 12(12b^3 dx + 7b^3) \cosh(dx+c)^{12} + 6(1820b^3 \cosh(dx+c)^4 + 24b^3 dx - 65a^3 \cosh(dx+c) + 14b^3 - 182(2b^3 dx + 3b^3) \cosh(dx+c)^2) \sinh(dx+c)^{12} - 396a^3 \cosh(dx+c)^9 + 2(13104b^3 \cosh(dx+c)^5 - 1170a^3 \cosh(dx+c)^2 - 2184(2b^3 dx + 3b^3) \cosh(dx+c)^3 + 85a^3 + 72(12b^3 dx + 7b^3) \cosh(dx+c)) \sinh(dx+c)^{11} - 12(30b^3 dx + 48a^2 b + 7b^3) \cosh(dx+c)^{10} + 2(24024b^3 \cosh(dx+c)^6 - 4290a^3 \cosh(dx+c)^3 - 180b^3 dx - 6006(2b^3 dx + 3b^3) \cosh(dx+c)^4 + 935a^3 \cosh(dx+c) - 288a^2 b - 42b^3 + 396(12b^3 dx + 7b^3) \cosh(dx+c)^2) \sinh(dx+c)^{10} - 396a^3 \cosh(dx+c)^7 + 2(34320b^3 \cosh(dx+c)^7 - 10725a^3 \cosh(dx+c)^4 - 12012(2b^3 dx + 3b^3) \cosh(dx+c)^5 + 4675a^3 \cosh(dx+c)^2 + 1320(12b^3 dx + 7b^3) \cosh(dx+c)^3 - 198a^3 - 60(30b^3 dx + 48a^2 b + 7b^3) \cosh(dx+c)) \sinh(dx+c)^9 + 480(b^3 dx + 4a^2 b) \cosh(dx+c)^8 + 6(12870b^3 \cosh(dx+c)^8 - 6435a^3 \cosh(dx+c)^5 - 6006(2b^3 dx + 3b^3) \cosh(dx+c)^6 + 4675a^3 \cosh(dx+c)^3 + 80b^3 dx + 990(12b^3 dx + 7b^3) \cosh(dx+c)^4 - 594a^3 \cosh(dx+c) + 320a^2 b - 90(30b^3 dx + 48a^2 b + 7b^3) \cosh(dx+c)^2) \sinh(dx+c)^8 + 170a^3 \cosh(dx+c)^5 + 12(5720b^3 \cosh(dx+c)^9 - 4290a^3 \cosh(dx+c)^6 - 3432(2b^3 dx + 3b^3) \cosh(dx+c)^7 + 4675a^3 \cosh(dx+c)^4 + 792(12b^3 dx + 7b^3) \cosh(dx+c)^5 - 1188a^3 \cosh(dx+c)^2 - 120(30b^3 dx + 48a^2 b + 7b^3) \cosh(dx+c)^3 - 33a^3 + 320(b^3 dx + 4a^2 b) \cosh(dx+c)) \sinh(dx+c)^7 - 12(30b^3 dx + 192a^2 b - 7b^3) \cosh(dx+c)^6 + 12(4004b^3 \cosh(dx+c)^{10} - 4290a^3 \cosh(dx+c)^7 - 3003(2b^3 dx + 3b^3) \cosh(dx+c)^8 + 6545a^3 \cosh(dx+c)^5 + 924(12b^3 dx + 7b^3) \cosh(dx+c)^$

$$\begin{aligned}
& 6 - 2772*a^3*\cosh(d*x + c)^3 - 30*b^3*d*x - 210*(30*b^3*d*x + 48*a^2*b + 7* \\
& b^3)*\cosh(d*x + c)^4 - 231*a^3*\cosh(d*x + c) - 192*a^2*b + 7*b^3 + 1120*(b^ \\
& 3*d*x + 4*a^2*b)*\cosh(d*x + c)^2*\sinh(d*x + c)^6 - 30*a^3*\cosh(d*x + c)^3 \\
& + 2*(13104*b^3*\cosh(d*x + c)^11 - 19305*a^3*\cosh(d*x + c)^8 - 12012*(2*b^3* \\
& d*x + 3*b^3)*\cosh(d*x + c)^9 + 39270*a^3*\cosh(d*x + c)^6 + 4752*(12*b^3*d*x \\
& + 7*b^3)*\cosh(d*x + c)^7 - 24948*a^3*\cosh(d*x + c)^4 - 1512*(30*b^3*d*x + \\
& 48*a^2*b + 7*b^3)*\cosh(d*x + c)^5 - 4158*a^3*\cosh(d*x + c)^2 + 13440*(b^3*d \\
& *x + 4*a^2*b)*\cosh(d*x + c)^3 + 85*a^3 - 36*(30*b^3*d*x + 192*a^2*b - 7*b^3 \\
&)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 12*(12*b^3*d*x + 96*a^2*b - 7*b^3)*\cosh(\\
& d*x + c)^4 + 2*(5460*b^3*\cosh(d*x + c)^12 - 10725*a^3*\cosh(d*x + c)^9 - 600 \\
& 6*(2*b^3*d*x + 3*b^3)*\cosh(d*x + c)^10 + 28050*a^3*\cosh(d*x + c)^7 + 2970*(\\
& 12*b^3*d*x + 7*b^3)*\cosh(d*x + c)^8 - 24948*a^3*\cosh(d*x + c)^5 - 1260*(30* \\
& b^3*d*x + 48*a^2*b + 7*b^3)*\cosh(d*x + c)^6 - 6930*a^3*\cosh(d*x + c)^3 + 72 \\
& *b^3*d*x + 16800*(b^3*d*x + 4*a^2*b)*\cosh(d*x + c)^4 + 425*a^3*\cosh(d*x + c \\
&) + 576*a^2*b - 42*b^3 - 90*(30*b^3*d*x + 192*a^2*b - 7*b^3)*\cosh(d*x + c)^ \\
& 2)*\sinh(d*x + c)^4 + 2*(1680*b^3*\cosh(d*x + c)^13 - 4290*a^3*\cosh(d*x + c)^ \\
& 10 - 2184*(2*b^3*d*x + 3*b^3)*\cosh(d*x + c)^11 + 14025*a^3*\cosh(d*x + c)^8 \\
& + 1320*(12*b^3*d*x + 7*b^3)*\cosh(d*x + c)^9 - 16632*a^3*\cosh(d*x + c)^6 - 7 \\
& 20*(30*b^3*d*x + 48*a^2*b + 7*b^3)*\cosh(d*x + c)^7 - 6930*a^3*\cosh(d*x + c) \\
& ^4 + 13440*(b^3*d*x + 4*a^2*b)*\cosh(d*x + c)^5 + 850*a^3*\cosh(d*x + c)^2 - \\
& 120*(30*b^3*d*x + 192*a^2*b - 7*b^3)*\cosh(d*x + c)^3 - 15*a^3 + 24*(12*b^3* \\
& d*x + 96*a^2*b - 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 6*b^3 - 12*(2*b^3* \\
& d*x + 16*a^2*b - 3*b^3)*\cosh(d*x + c)^2 + 2*(360*b^3*\cosh(d*x + c)^14 - 117 \\
& 0*a^3*\cosh(d*x + c)^11 - 546*(2*b^3*d*x + 3*b^3)*\cosh(d*x + c)^12 + 4675*a^ \\
& 3*\cosh(d*x + c)^9 + 396*(12*b^3*d*x + 7*b^3)*\cosh(d*x + c)^10 - 7128*a^3*co \\
& sh(d*x + c)^7 - 270*(30*b^3*d*x + 48*a^2*b + 7*b^3)*\cosh(d*x + c)^8 - 4158* \\
& a^3*\cosh(d*x + c)^5 + 6720*(b^3*d*x + 4*a^2*b)*\cosh(d*x + c)^6 + 850*a^3*co \\
& sh(d*x + c)^3 - 12*b^3*d*x - 90*(30*b^3*d*x + 192*a^2*b - 7*b^3)*\cosh(d*x + \\
& c)^4 - 45*a^3*\cosh(d*x + c) - 96*a^2*b + 18*b^3 + 36*(12*b^3*d*x + 96*a^2* \\
& b - 7*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 3*((5*a^3 - 48*a*b^2)*\cosh(d* \\
& x + c)^14 + 14*(5*a^3 - 48*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^13 + (5*a^3 - \\
& 48*a*b^2)*\sinh(d*x + c)^14 - 6*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^12 - (30*a \\
& ^3 - 288*a*b^2 - 91*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^12 + \\
& 4*(91*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^3 - 18*(5*a^3 - 48*a*b^2)*\cosh(d*x + \\
& c))*\sinh(d*x + c)^11 + 15*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^10 + (1001*(5*a \\
& ^3 - 48*a*b^2)*\cosh(d*x + c)^4 + 75*a^3 - 720*a*b^2 - 396*(5*a^3 - 48*a*b^2 \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^10 + 2*(1001*(5*a^3 - 48*a*b^2)*\cosh(d*x + \\
& c)^5 - 660*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^3 + 75*(5*a^3 - 48*a*b^2)*\cosh \\
& (d*x + c))*\sinh(d*x + c)^9 - 20*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^8 + (3003* \\
& (5*a^3 - 48*a*b^2)*\cosh(d*x + c)^6 - 2970*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^ \\
& 4 - 100*a^3 + 960*a*b^2 + 675*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^8 + 8*(429*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^7 - 594*(5*a^3 - 48*a*b^2) \\
& *\cosh(d*x + c)^5 + 225*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^3 - 20*(5*a^3 - 48* \\
& a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 15*(5*a^3 - 48*a*b^2)*\cosh(d*x + c) \\
& ^6 + (3003*(5*a^3 - 48*a*b^2)*\cosh(d*x + c)^8 - 5544*(5*a^3 - 48*a*b^2)*cos
\end{aligned}$$

$$\begin{aligned}
& h(dx + c)^6 + 3150*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^4 + 75*a^3 - 720*a*b^2 \\
& - 560*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^2*\sinh(dx + c)^6 + 2*(1001*(5*a^3 \\
& - 48*a*b^2)*\cosh(dx + c)^9 - 2376*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^7 + 18 \\
& 90*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^5 - 560*(5*a^3 - 48*a*b^2)*\cosh(dx + c \\
&)^3 + 45*(5*a^3 - 48*a*b^2)*\cosh(dx + c))*\sinh(dx + c)^5 - 6*(5*a^3 - 48* \\
& a*b^2)*\cosh(dx + c)^4 + (1001*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^10 - 2970*(\\
& 5*a^3 - 48*a*b^2)*\cosh(dx + c)^8 + 3150*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^6 \\
& - 1400*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^4 - 30*a^3 + 288*a*b^2 + 225*(5*a^ \\
& 3 - 48*a*b^2)*\cosh(dx + c)^2*\sinh(dx + c)^4 + 4*(91*(5*a^3 - 48*a*b^2)*\c \\
& osh(dx + c)^11 - 330*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^9 + 450*(5*a^3 - 48* \\
& a*b^2)*\cosh(dx + c)^7 - 280*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^5 + 75*(5*a^3 \\
& - 48*a*b^2)*\cosh(dx + c)^3 - 6*(5*a^3 - 48*a*b^2)*\cosh(dx + c))*\sinh(dx \\
& + c)^3 + (5*a^3 - 48*a*b^2)*\cosh(dx + c)^2 + (91*(5*a^3 - 48*a*b^2)*\cosh(\\
& dx + c)^12 - 396*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^10 + 675*(5*a^3 - 48*a*b \\
& ^2)*\cosh(dx + c)^8 - 560*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^6 + 225*(5*a^3 - \\
& 48*a*b^2)*\cosh(dx + c)^4 + 5*a^3 - 48*a*b^2 - 36*(5*a^3 - 48*a*b^2)*\cosh(\\
& dx + c)^2)*\sinh(dx + c)^2 + 2*(7*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^13 - 36 \\
& *(5*a^3 - 48*a*b^2)*\cosh(dx + c)^11 + 75*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^ \\
& 9 - 80*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^7 + 45*(5*a^3 - 48*a*b^2)*\cosh(dx \\
& + c)^5 - 12*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^3 + (5*a^3 - 48*a*b^2)*\cosh(dx \\
& + c))*\sinh(dx + c))*\log(\cosh(dx + c) + \sinh(dx + c) + 1) - 3*((5*a^3 - \\
& 48*a*b^2)*\cosh(dx + c)^14 + 14*(5*a^3 - 48*a*b^2)*\cosh(dx + c)*\sinh(dx \\
& + c)^13 + (5*a^3 - 48*a*b^2)*\sinh(dx + c)^14 - 6*(5*a^3 - 48*a*b^2)*\cosh(dx \\
& + c)^12 - (30*a^3 - 288*a*b^2 - 91*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^2)*\s \\
& inh(dx + c)^12 + 4*(91*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^3 - 18*(5*a^3 - 48 \\
& *a*b^2)*\cosh(dx + c))*\sinh(dx + c)^11 + 15*(5*a^3 - 48*a*b^2)*\cosh(dx + \\
& c)^10 + (1001*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^4 + 75*a^3 - 720*a*b^2 - 396 \\
& *(5*a^3 - 48*a*b^2)*\cosh(dx + c)^2)*\sinh(dx + c)^10 + 2*(1001*(5*a^3 - 48 \\
& *a*b^2)*\cosh(dx + c)^5 - 660*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^3 + 75*(5*a^ \\
& 3 - 48*a*b^2)*\cosh(dx + c))*\sinh(dx + c)^9 - 20*(5*a^3 - 48*a*b^2)*\cosh(dx \\
& + c)^8 + (3003*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^6 - 2970*(5*a^3 - 48*a*b \\
& ^2)*\cosh(dx + c)^4 - 100*a^3 + 960*a*b^2 + 675*(5*a^3 - 48*a*b^2)*\cosh(dx \\
& + c)^2)*\sinh(dx + c)^8 + 8*(429*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^7 - 594* \\
& (5*a^3 - 48*a*b^2)*\cosh(dx + c)^5 + 225*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^3 \\
& - 20*(5*a^3 - 48*a*b^2)*\cosh(dx + c))*\sinh(dx + c)^7 + 15*(5*a^3 - 48*a* \\
& b^2)*\cosh(dx + c)^6 + (3003*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^8 - 5544*(5*a \\
& ^3 - 48*a*b^2)*\cosh(dx + c)^6 + 3150*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^4 + \\
& 75*a^3 - 720*a*b^2 - 560*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^2)*\sinh(dx + c)^ \\
& 6 + 2*(1001*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^9 - 2376*(5*a^3 - 48*a*b^2)*\co \\
& sh(dx + c)^7 + 1890*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^5 - 560*(5*a^3 - 48*a \\
& *b^2)*\cosh(dx + c)^3 + 45*(5*a^3 - 48*a*b^2)*\cosh(dx + c))*\sinh(dx + c)^ \\
& 5 - 6*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^4 + (1001*(5*a^3 - 48*a*b^2)*\cosh(dx \\
& + c)^10 - 2970*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^8 + 3150*(5*a^3 - 48*a*b^ \\
& 2)*\cosh(dx + c)^6 - 1400*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^4 - 30*a^3 + 288 \\
& *a*b^2 + 225*(5*a^3 - 48*a*b^2)*\cosh(dx + c)^2)*\sinh(dx + c)^4 + 4*(91*(5
\end{aligned}$$

$$\begin{aligned}
& a^3 - 48ab^2) \cosh(dx + c)^{11} - 330(5a^3 - 48ab^2) \cosh(dx + c)^9 \\
& + 450(5a^3 - 48ab^2) \cosh(dx + c)^7 - 280(5a^3 - 48ab^2) \cosh(dx \\
& + c)^5 + 75(5a^3 - 48ab^2) \cosh(dx + c)^3 - 6(5a^3 - 48ab^2) \cosh(\\
& dx + c) \sinh(dx + c)^3 + (5a^3 - 48ab^2) \cosh(dx + c)^2 + (91(5a^3 \\
& - 48ab^2) \cosh(dx + c)^{12} - 396(5a^3 - 48ab^2) \cosh(dx + c)^{10} + 6 \\
& 75(5a^3 - 48ab^2) \cosh(dx + c)^8 - 560(5a^3 - 48ab^2) \cosh(dx + c \\
&)^6 + 225(5a^3 - 48ab^2) \cosh(dx + c)^4 + 5a^3 - 48ab^2 - 36(5a^3 \\
& - 48ab^2) \cosh(dx + c)^2) \sinh(dx + c)^2 + 2(7(5a^3 - 48ab^2) \cos \\
& h(dx + c)^{13} - 36(5a^3 - 48ab^2) \cosh(dx + c)^{11} + 75(5a^3 - 48ab \\
& ^2) \cosh(dx + c)^9 - 80(5a^3 - 48ab^2) \cosh(dx + c)^7 + 45(5a^3 - 4 \\
& 8ab^2) \cosh(dx + c)^5 - 12(5a^3 - 48ab^2) \cosh(dx + c)^3 + (5a^3 - \\
& 48ab^2) \cosh(dx + c)) \sinh(dx + c) \log(\cosh(dx + c) + \sinh(dx + c) \\
& - 1) + 2(48b^3 \cosh(dx + c)^{15} - 195a^3 \cosh(dx + c)^{12} - 84(2b^3 dx \\
& x + 3b^3) \cosh(dx + c)^{13} + 935a^3 \cosh(dx + c)^{10} + 72(12b^3 dx + 7 \\
& *b^3) \cosh(dx + c)^{11} - 1782a^3 \cosh(dx + c)^8 - 60(30b^3 dx + 48a^2 \\
& *b + 7b^3) \cosh(dx + c)^9 - 1386a^3 \cosh(dx + c)^6 + 1920(b^3 dx + 4 \\
& a^2 b) \cosh(dx + c)^7 + 425a^3 \cosh(dx + c)^4 - 36(30b^3 dx + 192a^2 \\
& *b - 7b^3) \cosh(dx + c)^5 - 45a^3 \cosh(dx + c)^2 + 24(12b^3 dx + 96 \\
& a^2 b - 7b^3) \cosh(dx + c)^3 - 12(2b^3 dx + 16a^2 b - 3b^3) \cosh(dx \\
& + c) \sinh(dx + c)) / (d \cosh(dx + c)^{14} + 14d \cosh(dx + c) \sinh(dx + c \\
&)^{13} + d \sinh(dx + c)^{14} - 6d \cosh(dx + c)^{12} + (91d \cosh(dx + c)^2 - \\
& 6d) \sinh(dx + c)^{12} + 4(91d \cosh(dx + c)^3 - 18d \cosh(dx + c)) \sinh(\\
& dx + c)^{11} + 15d \cosh(dx + c)^{10} + (1001d \cosh(dx + c)^4 - 396d \cosh(\\
& dx + c)^2 + 15d) \sinh(dx + c)^{10} + 2(1001d \cosh(dx + c)^5 - 660d \cos \\
& h(dx + c)^3 + 75d \cosh(dx + c)) \sinh(dx + c)^9 - 20d \cosh(dx + c)^8 + \\
& (3003d \cosh(dx + c)^6 - 2970d \cosh(dx + c)^4 + 675d \cosh(dx + c)^2 - \\
& 20d) \sinh(dx + c)^8 + 8(429d \cosh(dx + c)^7 - 594d \cosh(dx + c)^5 + \\
& 225d \cosh(dx + c)^3 - 20d \cosh(dx + c)) \sinh(dx + c)^7 + 15d \cosh(dx \\
& x + c)^6 + (3003d \cosh(dx + c)^8 - 5544d \cosh(dx + c)^6 + 3150d \cosh(d \\
& *x + c)^4 - 560d \cosh(dx + c)^2 + 15d) \sinh(dx + c)^6 + 2(1001d \cosh(\\
& dx + c)^9 - 2376d \cosh(dx + c)^7 + 1890d \cosh(dx + c)^5 - 560d \cosh(d \\
& *x + c)^3 + 45d \cosh(dx + c)) \sinh(dx + c)^5 - 6d \cosh(dx + c)^4 + (10 \\
& 01d \cosh(dx + c)^{10} - 2970d \cosh(dx + c)^8 + 3150d \cosh(dx + c)^6 - 1 \\
& 400d \cosh(dx + c)^4 + 225d \cosh(dx + c)^2 - 6d) \sinh(dx + c)^4 + 4(9 \\
& 1d \cosh(dx + c)^{11} - 330d \cosh(dx + c)^9 + 450d \cosh(dx + c)^7 - 280 \\
& d \cosh(dx + c)^5 + 75d \cosh(dx + c)^3 - 6d \cosh(dx + c)) \sinh(dx + c) \\
& ^3 + d \cosh(dx + c)^2 + (91d \cosh(dx + c)^{12} - 396d \cosh(dx + c)^{10} + \\
& 675d \cosh(dx + c)^8 - 560d \cosh(dx + c)^6 + 225d \cosh(dx + c)^4 - 36 \\
& d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 2(7d \cosh(dx + c)^{13} - 36d \cos \\
& h(dx + c)^{11} + 75d \cosh(dx + c)^9 - 80d \cosh(dx + c)^7 + 45d \cosh(dx \\
& + c)^5 - 12d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**3,x)

[Out] Timed out

Giac [B] time = 1.46106, size = 455, normalized size = 2.74

$$-\frac{(dx+c)b^3}{2d} + \frac{b^3e^{2dx+2c}}{8d} + \frac{(5a^3-48ab^2)\log(e^{(dx+c)}+1)}{16d} - \frac{(5a^3-48ab^2)\log(|e^{(dx+c)}-1|)}{16d} - \frac{(15a^3e^{(13dx+13c)}+3b^3e^{(12dx+12c)}-85a^3e^{(11dx+11c)}+198a^3e^{(9dx+9c)}+198a^3e^{(7dx+7c)}-85a^3e^{(5dx+5c)}+15a^3e^{(3dx+3c)}+3b^3+18(16a^2b-b^3)e^{(10dx+10c)}-15(64a^2b-3b^3)e^{(8dx+8c)}+12(96a^2b-5b^3)e^{(6dx+6c)}-9(64a^2b-5b^3)e^{(4dx+4c)}+6(16a^2b-3b^3)e^{(2dx+2c)})e^{(-2dx-2c)}}{(d(e^{(dx+c)}+1))^6(e^{(dx+c)}-1)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")

[Out]
$$-1/2*(d*x + c)*b^3/d + 1/8*b^3*e^{(2*d*x + 2*c)}/d + 1/16*(5*a^3 - 48*a*b^2)*\log(e^{(d*x + c)} + 1)/d - 1/16*(5*a^3 - 48*a*b^2)*\log(\text{abs}(e^{(d*x + c)} - 1))/d - 1/24*(15*a^3*e^{(13*d*x + 13*c)} + 3*b^3*e^{(12*d*x + 12*c)} - 85*a^3*e^{(11*d*x + 11*c)} + 198*a^3*e^{(9*d*x + 9*c)} + 198*a^3*e^{(7*d*x + 7*c)} - 85*a^3*e^{(5*d*x + 5*c)} + 15*a^3*e^{(3*d*x + 3*c)} + 3*b^3 + 18*(16*a^2*b - b^3)*e^{(10*d*x + 10*c)} - 15*(64*a^2*b - 3*b^3)*e^{(8*d*x + 8*c)} + 12*(96*a^2*b - 5*b^3)*e^{(6*d*x + 6*c)} - 9*(64*a^2*b - 5*b^3)*e^{(4*d*x + 4*c)} + 6*(16*a^2*b - 3*b^3)*e^{(2*d*x + 2*c)})*e^{(-2*d*x - 2*c)}/(d*(e^{(d*x + c)} + 1)^6*(e^{(d*x + c)} - 1)^6)$$

$$3.171 \quad \int \frac{\sinh^6(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=328

$$\frac{2a^{4/3} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^2 d \sqrt{a^{2/3}+b^{2/3}}} - \frac{2(-1)^{2/3} a^{4/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2 d \sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} - \frac{2(-1)^{2/3} a^{4/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}(-1)^{5/6}}{\sqrt{\sqrt[3]{-1}a^{2/3}}}\right)}{3b^2 d \sqrt{\sqrt[3]{-1}a^{2/3}}}$$

[Out] $-\left(\frac{a x}{b^2}\right) - \left(2(-1)^{2/3} a^{4/3} \operatorname{ArcTan}\left[\left((-1)^{1/6} \left((-1)^{1/6} b^{1/3} + I a^{1/3} \operatorname{Tanh}\left[\frac{c+dx}{2}\right]\right)\right) / \sqrt{\left(-1\right)^{1/3} a^{2/3} - \left(-1\right)^{2/3} b^{2/3}}\right] - \left(2(-1)^{2/3} a^{4/3} \operatorname{ArcTan}\left[\left((-1)^{1/6} \left((-1)^{5/6} b^{1/3} + I a^{1/3} \operatorname{Tanh}\left[\frac{c+dx}{2}\right]\right)\right) / \sqrt{\left(-1\right)^{1/3} a^{2/3} - b^{2/3}}\right] - \left(2 a^{4/3} \operatorname{ArcTanh}\left[\frac{b^{1/3} - a^{1/3} \operatorname{Tanh}\left[\frac{c+dx}{2}\right]}{\sqrt{a^{2/3} + b^{2/3}}}\right] / \left(3 \sqrt{a^{2/3} + b^{2/3}}\right) b^{2/3} d - \operatorname{Cosh}[c+dx] / (b d) + \operatorname{Cosh}[c+dx]^3 / (3 b d)\right)$

Rubi [A] time = 0.738527, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3220, 2633, 3213, 2660, 618, 204}

$$\frac{2a^{4/3} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^2 d \sqrt{a^{2/3}+b^{2/3}}} - \frac{2(-1)^{2/3} a^{4/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2 d \sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} - \frac{2(-1)^{2/3} a^{4/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}(-1)^{5/6}}{\sqrt{\sqrt[3]{-1}a^{2/3}}}\right)}{3b^2 d \sqrt{\sqrt[3]{-1}a^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3), x]

[Out] $-\left(\frac{a x}{b^2}\right) - \left(2(-1)^{2/3} a^{4/3} \operatorname{ArcTan}\left[\left((-1)^{1/6} \left((-1)^{1/6} b^{1/3} + I a^{1/3} \operatorname{Tanh}\left[\frac{c+dx}{2}\right]\right)\right) / \sqrt{\left(-1\right)^{1/3} a^{2/3} - \left(-1\right)^{2/3} b^{2/3}}\right] - \left(2(-1)^{2/3} a^{4/3} \operatorname{ArcTan}\left[\left((-1)^{1/6} \left((-1)^{5/6} b^{1/3} + I a^{1/3} \operatorname{Tanh}\left[\frac{c+dx}{2}\right]\right)\right) / \sqrt{\left(-1\right)^{1/3} a^{2/3} - b^{2/3}}\right] - \left(2 a^{4/3} \operatorname{ArcTanh}\left[\frac{b^{1/3} - a^{1/3} \operatorname{Tanh}\left[\frac{c+dx}{2}\right]}{\sqrt{a^{2/3} + b^{2/3}}}\right] / \left(3 \sqrt{a^{2/3} + b^{2/3}}\right) b^{2/3} d - \operatorname{Cosh}[c+dx] / (b d) + \operatorname{Cosh}[c+dx]^3 / (3 b d)\right)$

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]
```

Rule 3213

```
Int[((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_), x_Symbol] := Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)} dx &= -\int \left(\frac{a}{b^2} - \frac{\sinh^3(c+dx)}{b} - \frac{a^2}{b^2(a+b\sinh^3(c+dx))} \right) dx \\
&= -\frac{ax}{b^2} + \frac{a^2 \int \frac{1}{a+b\sinh^3(c+dx)} dx}{b^2} + \frac{\int \sinh^3(c+dx) dx}{b} \\
&= -\frac{ax}{b^2} + \frac{a^2 \int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}(\sqrt[6]{-1}\sqrt[3]{a-i}\sqrt[3]{b}\sinh(c+dx))} + \frac{\sqrt[6]{-1}}{3a^{2/3}(\sqrt[6]{-1}\sqrt[3]{a+i}\sqrt[3]{b}\sinh(c+dx))} + \frac{\sqrt[6]{-1}}{3a^{2/3}(\sqrt[6]{-1}\sqrt[3]{a+(-1)^{5/6}\sqrt[3]{b}}} \right) dx}{b^2} \\
&= -\frac{ax}{b^2} - \frac{\cosh(c+dx)}{bd} + \frac{\cosh^3(c+dx)}{3bd} + \frac{(\sqrt[6]{-1}a^{4/3}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a-i}\sqrt[3]{b}\sinh(c+dx)} dx}{3b^2} + \frac{(\sqrt[6]{-1}a^{4/3}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a+i}\sqrt[3]{b}\sinh(c+dx)} dx}{3b^2} \\
&= -\frac{ax}{b^2} - \frac{\cosh(c+dx)}{bd} + \frac{\cosh^3(c+dx)}{3bd} - \frac{(2(-1)^{2/3}a^{4/3}) \text{Subst} \left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a-2}\sqrt[3]{bx}+\sqrt[6]{-1}\sqrt[3]{ax^2}} dx, x, \right)}{3b^2d} \\
&= -\frac{ax}{b^2} - \frac{\cosh(c+dx)}{bd} + \frac{\cosh^3(c+dx)}{3bd} + \frac{(4(-1)^{2/3}a^{4/3}) \text{Subst} \left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}-b^{2/3})-x^2} dx, x, -2\sqrt[3]{-1} \right)}{3b^2d} \\
&= -\frac{ax}{b^2} + \frac{2(-1)^{2/3}a^{4/3} \tan^{-1} \left(\frac{\sqrt[3]{b}(-1)^{2/3}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}} \right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}b^2d}} - \frac{2(-1)^{2/3}a^{4/3} \tan^{-1} \left(\frac{\sqrt[3]{-1}\sqrt[3]{b+(-1)^{2/3}\sqrt[3]{a}}}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}b^2d}} \right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}b^2d}}
\end{aligned}$$

Mathematica [C] time = 0.349243, size = 168, normalized size = 0.51

$$8a^2 \text{RootSum} \left[8\#1^3 a + \#1^6 b - 3\#1^4 b + 3\#1^2 b - b \&, \frac{2\#1 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right) + \#1 c + \#1 d}{\#1^4 b - 2\#1^2 b + 4\#1 a + b} \right]$$

$$12b^2d$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3), x]

[Out] (-12*a*c - 12*a*d*x - 9*b*Cosh[c + d*x] + b*Cosh[3*(c + d*x)] + 8*a^2*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &, (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) &])/(12*b^2*d)

Maple [C] time = 0.066, size = 259, normalized size = 0.8

$$\frac{1}{3bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-3} - \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} - \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{a}{db^2} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x)

[Out] 1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/3/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/3/d*a^2/b^2*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8a^2 \int \frac{e^{(3dx+3c)}}{b^3 e^{(6dx+6c)} - 3b^3 e^{(4dx+4c)} + 8ab^2 e^{(3dx+3c)} + 3b^3 e^{(2dx+2c)} - b^3} dx - \frac{(24 adxe^{(3dx+3c)} - be^{(6dx+6c)} + 9be^{(4dx+4c)} + 24b^2d)}{24b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] 8*a^2*integrate(e^(3*d*x + 3*c)/(b^3*e^(6*d*x + 6*c) - 3*b^3*e^(4*d*x + 4*c) + 8*a*b^2*e^(3*d*x + 3*c) + 3*b^3*e^(2*d*x + 2*c) - b^3), x) - 1/24*(24*a*d*x*e^(3*d*x + 3*c) - b*e^(6*d*x + 6*c) + 9*b*e^(4*d*x + 4*c) + 9*b*e^(2*d*x + 2*c) - b)*e^(-3*d*x - 3*c)/(b^2*d)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^6}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] integrate(sinh(d*x + c)^6/(b*sinh(d*x + c)^3 + a), x)

$$3.172 \quad \int \frac{\sinh^5(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=295

$$\frac{2a \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}+b^{2/3}}} + \frac{2a \tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} + \frac{2a \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

[Out] $-x/(2*b) + (2*a*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2])]/Sqrt[-((-1)^(2/3)*a^(2/3) - b^(2/3))]/(3*Sqrt[-((-1)^(2/3)*a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2])]/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - a^(1/3))*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(5/3)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)$

Rubi [A] time = 0.558601, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3220, 2635, 8, 2660, 618, 206, 204}

$$\frac{2a \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}+b^{2/3}}} + \frac{2a \tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} + \frac{2a \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3), x]

[Out] $-x/(2*b) + (2*a*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2])]/Sqrt[-((-1)^(2/3)*a^(2/3) - b^(2/3))]/(3*Sqrt[-((-1)^(2/3)*a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2])]/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - a^(1/3))*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(5/3)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)$

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)} dx &= -\left(i \int \left(\frac{i \sinh^2(c+dx)}{b} - \frac{ia \sinh^2(c+dx)}{b(a+b\sinh^3(c+dx))} \right) dx \right) \\
&= \frac{\int \sinh^2(c+dx) dx}{b} - \frac{a \int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)} dx}{b} \\
&= \frac{\cosh(c+dx) \sinh(c+dx)}{2bd} - \frac{\int 1 dx}{2b} + \frac{a \int \left(\frac{i}{3b^{2/3}(-i\sqrt[3]{a}-i\sqrt[3]{b} \sinh(c+dx))} + \frac{i}{3b^{2/3}(\sqrt[3]{-1}\sqrt[3]{a}-i\sqrt[3]{b} \sinh(c+dx))} \right) dx}{b} \\
&= -\frac{x}{2b} + \frac{\cosh(c+dx) \sinh(c+dx)}{2bd} + \frac{(ia) \int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b} \sinh(c+dx)} dx}{3b^{5/3}} + \frac{(ia) \int \frac{1}{\sqrt[3]{-1}\sqrt[3]{a}-i\sqrt[3]{b} \sinh(c+dx)} dx}{3b^{5/3}} \\
&= -\frac{x}{2b} + \frac{\cosh(c+dx) \sinh(c+dx)}{2bd} + \frac{(2a) \text{Subst} \left(\int \frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}x-i\sqrt[3]{ax^2}} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right) \right)}{3b^{5/3}d} \\
&= -\frac{x}{2b} + \frac{\cosh(c+dx) \sinh(c+dx)}{2bd} - \frac{(4a) \text{Subst} \left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}-b^{2/3})-x^2} dx, x, -2\sqrt[3]{b} + 2\sqrt[3]{-1}\sqrt[3]{b} \right)}{3b^{5/3}d} \\
&= -\frac{x}{2b} - \frac{2a \tan^{-1} \left(\frac{\sqrt[3]{b} + \sqrt[3]{-1}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} \right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}b^{5/3}d} - \frac{2a \tan^{-1} \left(\frac{\sqrt[3]{b} - (-1)^{2/3}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}} \right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}b^{5/3}d} + \frac{2a \tanh\left(\frac{1}{2}(c+dx)\right)}{3}
\end{aligned}$$

Mathematica [C] time = 0.316158, size = 299, normalized size = 1.01

$$-2a\text{RootSum} \left[8\#1^3a + \#1^6b - 3\#1^4b + 3\#1^2b - b\&, \frac{2\#1^4 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right) - 4}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} \right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3),x]

[Out] (-6*(c + d*x) - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &, (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) &] + 3*Sinh[2*(c + d*x)]/(12*b*d)

Maple [C] time = 0.06, size = 207, normalized size = 0.7

$$-\frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} + \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{1}{2bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) + \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-2} - \frac{1}{2bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x)

[Out] $-1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)-1/2/d/b*\ln(\tanh(1/2*d*x+1/2*c)+1)+1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)+1/2/d/b*\ln(\tanh(1/2*d*x+1/2*c)-1)+4/3/d*a/b*\sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R), _R=\text{RootOf}(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(4dx e^{(2dx+2c)} - e^{(4dx+4c)} + 1)e^{(-2dx-2c)}}{8bd} - \frac{1}{32} \int \frac{64(ae^{(5dx+5c)} - 2ae^{(3dx+3c)} + ae^{(dx+c)})}{b^2e^{(6dx+6c)} - 3b^2e^{(4dx+4c)} + 8abe^{(3dx+3c)} + 3b^2e^{(2dx+2c)} - b^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] $-1/8*(4*d*x*e^{(2*d*x + 2*c)} - e^{(4*d*x + 4*c)} + 1)*e^{(-2*d*x - 2*c)}/(b*d) - 1/32*\text{integrate}(64*(a*e^{(5*d*x + 5*c)} - 2*a*e^{(3*d*x + 3*c)} + a*e^{(d*x + c)})/(b^2*e^{(6*d*x + 6*c)} - 3*b^2*e^{(4*d*x + 4*c)} + 8*a*b*e^{(3*d*x + 3*c)} + 3*b^2*e^{(2*d*x + 2*c)} - b^2), x)$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^5}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] integrate(sinh(d*x + c)^5/(b*sinh(d*x + c)^3 + a), x)

$$3.173 \quad \int \frac{\sinh^4(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=303

$$\frac{2a^{2/3} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+b^{2/3}}} - \frac{2a^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} + \frac{2\sqrt[3]{-1}a^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

[Out] $(-2*a^{(2/3)}*ArcTan[(-1)^{(1/6)}*(-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2]])/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}]/(3*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}]*b^{(4/3)}*d) + (2*(-1)^{(1/3)}*a^{(2/3)}*ArcTan[(-1)^{(1/6)}*(-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2]])/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}]/(3*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}]*b^{(4/3)}*d) - (2*a^{(2/3)}*ArcTanh[(b^{(1/3)} - a^{(1/3)}*Tanh[(c + d*x)/2])/Sqrt[a^{(2/3)} + b^{(2/3)}]])/(3*Sqrt[a^{(2/3)} + b^{(2/3)}]*b^{(4/3)}*d) + Cosh[c + d*x]/(b*d)$

Rubi [A] time = 0.544964, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3220, 2638, 2660, 618, 204}

$$\frac{2a^{2/3} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+b^{2/3}}} - \frac{2a^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} + \frac{2\sqrt[3]{-1}a^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3), x]

[Out] $(-2*a^{(2/3)}*ArcTan[(-1)^{(1/6)}*(-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2]])/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}]/(3*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}]*b^{(4/3)}*d) + (2*(-1)^{(1/3)}*a^{(2/3)}*ArcTan[(-1)^{(1/6)}*(-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2]])/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}]/(3*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}]*b^{(4/3)}*d) - (2*a^{(2/3)}*ArcTanh[(b^{(1/3)} - a^{(1/3)}*Tanh[(c + d*x)/2])/Sqrt[a^{(2/3)} + b^{(2/3)}]])/(3*Sqrt[a^{(2/3)} + b^{(2/3)}]*b^{(4/3)}*d) + Cosh[c + d*x]/(b*d)$

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^ (p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)} dx &= \int \left(\frac{\sinh(c+dx)}{b} - \frac{a\sinh(c+dx)}{b(a+b\sinh^3(c+dx))} \right) dx \\
&= \frac{\int \sinh(c+dx) dx}{b} - \frac{a \int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)} dx}{b} \\
&= \frac{\cosh(c+dx)}{bd} + \frac{(ia) \int \left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx))} - \frac{1}{3\sqrt[3]{a}\sqrt[3]{b}} \right) dx}{b} \\
&= \frac{\cosh(c+dx)}{bd} - \frac{(ia^{2/3}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)} dx}{3b^{4/3}} + \frac{(\sqrt[6]{-1}a^{2/3}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)} dx}{3b^{4/3}} \\
&= \frac{\cosh(c+dx)}{bd} - \frac{(2a^{2/3}) \text{Subst} \left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[6]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right) \right)}{3b^{4/3}d} + \frac{(2\sqrt[3]{-1}a^{2/3}) \text{Subst} \left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[6]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right) \right)}{3b^{4/3}d} \\
&= \frac{\cosh(c+dx)}{bd} + \frac{(4a^{2/3}) \text{Subst} \left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3})-x^2} dx, x, 2\sqrt[3]{-1}\sqrt[3]{b} + 2\sqrt[6]{-1}\sqrt[3]{a} \tan\left(\frac{1}{2}(ic+idx)\right) \right)}{3b^{4/3}d} \\
&= -\frac{2\sqrt[3]{-1}a^{2/3} \tan^{-1} \left(\frac{\sqrt[3]{b}(-1)^{2/3}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}} \right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}b^{4/3}d}} - \frac{2a^{2/3} \tan^{-1} \left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} \right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}b^{4/3}d}}
\end{aligned}$$

Mathematica [C] time = 0.34527, size = 214, normalized size = 0.71

$$\frac{3 \cosh(c+dx) - a \text{RootSum} \left[8\#1^3 a + \#1^6 b - 3\#1^4 b + 3\#1^2 b - b \&, \frac{2\#1^2 \log(-\#1 \sinh(\frac{1}{2}(c+dx)) + \#1 \cosh(\frac{1}{2}(c+dx)) - \sinh(\frac{1}{2}(c+dx)) - \cosh(\frac{1}{2}(c+dx)))}{\sqrt[3]{-1}a^{2/3} - (-1)^{2/3}b^{2/3}} \right]}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3), x]

[Out] (3*Cosh[c + d*x] - a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &, (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) &]/(3*b*d)

Maple [C] time = 0.052, size = 128, normalized size = 0.4

$$\frac{1}{bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{1}{bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - 1 \right)^{-1} - \frac{2a}{3bd} \sum_{_R=\text{RootOf}(a_Z^6-3a_Z^4-8b_Z^3+3a_Z^2-a)} \frac{_R^3 - _R}{_R^5 a - 2 _R^3 a - 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x)

[Out] 1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d/b/(tanh(1/2*d*x+1/2*c)-1)-2/3/d*a/b*sum((
_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Root
Of(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(e^{(2dx+2c)} + 1)e^{(-dx-c)}}{2bd} - \frac{1}{16} \int \frac{64(ae^{(4dx+4c)} - ae^{(2dx+2c)})}{b^2e^{(6dx+6c)} - 3b^2e^{(4dx+4c)} + 8abe^{(3dx+3c)} + 3b^2e^{(2dx+2c)} - b^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/16*integrate(64*(a*e^(4*d*
x + 4*c) - a*e^(2*d*x + 2*c))/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c)
+ 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^4}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] integrate(sinh(d*x + c)^4/(b*sinh(d*x + c)^3 + a), x)

$$3.174 \quad \int \frac{\sinh^3(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=294

$$\frac{2\sqrt[3]{a} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+b^{2/3}}} + \frac{2(-1)^{2/3}\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} + \frac{2(-1)^{2/3}\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[6]{-1}((-1)^{5/6}\sqrt[3]{b-i})}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

[Out] x/b + (2*(-1)^(2/3)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2])]/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d) + (2*(-1)^(2/3)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2])]/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b*d) + (2*a^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b*d)

Rubi [A] time = 0.439897, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3220, 3213, 2660, 618, 204}

$$\frac{2\sqrt[3]{a} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+b^{2/3}}} + \frac{2(-1)^{2/3}\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} + \frac{2(-1)^{2/3}\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[6]{-1}((-1)^{5/6}\sqrt[3]{b-i})}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3), x]

[Out] x/b + (2*(-1)^(2/3)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2])]/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d) + (2*(-1)^(2/3)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2])]/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b*d) + (2*a^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b*d)

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_), x_Symbol] := Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)} dx &= i \int \left(-\frac{i}{b} + \frac{ia}{b(a+b\sinh^3(c+dx))} \right) dx \\
&= \frac{x}{b} - \frac{a \int \frac{1}{a+b\sinh^3(c+dx)} dx}{b} \\
&= \frac{x}{b} - \frac{a \int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} + \frac{\sqrt[6]{-1}}{3a^{2/3}(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx))} + \frac{\sqrt[6]{-1}}{3a^{2/3}(\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx))} \right) dx}{b} \\
&= \frac{x}{b} - \frac{(\sqrt[6]{-1}\sqrt[3]{a}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)} dx}{3b} - \frac{(\sqrt[6]{-1}\sqrt[3]{a}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)} dx}{3b} - \frac{(\sqrt[6]{-1}\sqrt[3]{a}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)} dx}{3b} \\
&= \frac{x}{b} + \frac{(2(-1)^{2/3}\sqrt[3]{a}) \operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{ax^2}} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3bd} + \frac{(2(-1)^{2/3}\sqrt[3]{a}) \operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{ax^2}} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3bd} \\
&= \frac{x}{b} - \frac{(4(-1)^{2/3}\sqrt[3]{a}) \operatorname{Subst}\left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}-b^{2/3})-x^2}} dx, x, -2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3bd} \\
&= \frac{x}{b} - \frac{2(-1)^{2/3}\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[3]{b}(-1)^{2/3}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}bd} + \frac{2(-1)^{2/3}\sqrt[3]{a} \tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}bd}
\end{aligned}$$

Mathematica [C] time = 0.230597, size = 145, normalized size = 0.49

$$\frac{-2a\operatorname{RootSum}\left[8\#1^3a + \#1^6b - 3\#1^4b + 3\#1^2b - b\&, \frac{2\#1 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right) + \#1}{\#1^4b - 2\#1^2b + 4\#1a + b}\right]}{3bd}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]

[Out] (3*c + 3*d*x - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) &])/(3*b*d)

Maple [C] time = 0.043, size = 129, normalized size = 0.4

$$\frac{1}{bd} \ln \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right) - \frac{1}{bd} \ln \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) - 1 \right) + \frac{a}{3bd} \sum_{_R=\text{RootOf}(a_Z^6-3a_Z^4-8b_Z^3+3a_Z^2-a)} \frac{_{R^4}-2_{R^3}a-4_{R^5}a-2_{R^3}a-4}{_{R^5}a-2_{R^3}a-4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x)

[Out] 1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/3/d*a/b*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-8a \int \frac{e^{(3dx+3c)}}{b^2 e^{(6dx+6c)} - 3b^2 e^{(4dx+4c)} + 8abe^{(3dx+3c)} + 3b^2 e^{(2dx+2c)} - b^2} dx + \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] -8*a*integrate(e^(3*d*x + 3*c)/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c) + 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x) + x/b

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^3}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")`

[Out] `integrate(sinh(d*x + c)^3/(b*sinh(d*x + c)^3 + a), x)`

$$3.175 \quad \int \frac{\sinh^2(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=262

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}+b^{2/3}}} - \frac{2 \tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} - \frac{2 \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

[Out] $(-2*\text{ArcTan}[\left((-1)^{(5/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)])/\text{Sqrt}[-\left((-1)^{(2/3)}*a^{(2/3)} - b^{(2/3)}\right)]/(3*\text{Sqrt}[-\left((-1)^{(2/3)}*a^{(2/3)} - b^{(2/3)}\right)]*b^{(2/3)}*d) - (2*\text{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)])/\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)]/(3*\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)]*b^{(2/3)}*d) - (2*\text{ArcTan}[\left(b^{(1/3)} - a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)])/\text{Sqrt}[a^{(2/3)} + b^{(2/3)}]/(3*\text{Sqrt}[a^{(2/3)} + b^{(2/3)}]*b^{(2/3)}*d)$

Rubi [A] time = 0.270745, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3220, 2660, 618, 206, 204}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}+b^{2/3}}} - \frac{2 \tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} - \frac{2 \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^2/(a + b*\text{Sinh}[c + d*x]^3), x]$

[Out] $(-2*\text{ArcTan}[\left((-1)^{(5/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)])/\text{Sqrt}[-\left((-1)^{(2/3)}*a^{(2/3)} - b^{(2/3)}\right)]/(3*\text{Sqrt}[-\left((-1)^{(2/3)}*a^{(2/3)} - b^{(2/3)}\right)]*b^{(2/3)}*d) - (2*\text{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)])/\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)]/(3*\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)]*b^{(2/3)}*d) - (2*\text{ArcTan}[\left(b^{(1/3)} - a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)])/\text{Sqrt}[a^{(2/3)} + b^{(2/3)}]/(3*\text{Sqrt}[a^{(2/3)} + b^{(2/3)}]*b^{(2/3)}*d)$

Rule 3220

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrig}[\sin[e + f*x]^{m*(a + b*\sin[e + f*x]^n)]$

x^p , $x]$, $x]$ /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 2660

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)} dx &= -\int \left(\frac{i}{3b^{2/3}(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} + \frac{i}{3b^{2/3}(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} + \frac{i}{3b^{2/3}((-1)^{5/6}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} \right) dx \\
&= \frac{i \int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)} dx}{3b^{2/3}} - \frac{i \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)} dx}{3b^{2/3}} - \frac{i \int \frac{1}{(-1)^{5/6}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)} dx}{3b^{2/3}} \\
&= -\frac{2 \operatorname{Subst}\left(\int \frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{bx}-i\sqrt[3]{ax^2}} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{bx}+\sqrt[6]{-1}\sqrt[3]{ax^2}} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d} \\
&= \frac{4 \operatorname{Subst}\left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}-b^{2/3})-x^2} dx, x, -2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d} + \frac{4 \operatorname{Subst}\left(\int \frac{1}{4(a^{2/3}+b^{2/3})-x^2} dx, x, 2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d} \\
&= \frac{2 \tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}b^{2/3}d} + \frac{2 \tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}b^{2/3}d} - \frac{2 \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt{a^{2/3}+b^{2/3}}b^{2/3}d}
\end{aligned}$$

Mathematica [C] time = 0.17752, size = 275, normalized size = 1.05

$$\operatorname{RootSum}\left[8\#1^3a + \#1^6b - 3\#1^4b + 3\#1^2b - b\&, \frac{2\#1^4 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right) - 4\#1^2 \log\left(\frac{\sqrt[3]{b} + \sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right) + 2 \tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right) - 2 \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{(b\#1 + 4a\#1^2 - 2b\#1^3 + b\#1^5) \& } / (6*d)
\right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^3), x]

[Out] RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) &]/(6*d)

Maple [C] time = 0.042, size = 78, normalized size = 0.3

$$-\frac{4}{3d} \sum_{R=\operatorname{RootOf}(aZ^6-3aZ^4-8bZ^3+3aZ^2-a)} \frac{-R^2}{-R^5a-2R^3a-4R^2b+Ra} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] -4/3/d*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),
_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^2}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx + c)^2}{b \sinh(dx + c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)

$$3.176 \quad \int \frac{\sinh(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=290

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{bd}\sqrt{a^{2/3}+b^{2/3}}} + \frac{2 \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{bd}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} - \frac{2\sqrt[3]{-1} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{bd}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

[Out] (2*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]]/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(1/3)*d) - (2*(-1)^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]]/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]]/(3*a^(1/3)*Sqrt[a^(2/3) + b^(2/3)]*b^(1/3)*d)

Rubi [A] time = 0.335756, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3220, 2660, 618, 204}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{bd}\sqrt{a^{2/3}+b^{2/3}}} + \frac{2 \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{bd}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} - \frac{2\sqrt[3]{-1} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{bd}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3), x]

[Out] (2*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]]/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(1/3)*d) - (2*(-1)^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]]/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]]/(3*a^(1/3)*Sqrt[a^(2/3) + b^(2/3)]*b^(1/3)*d)

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)} dx &= -\left(i \int \left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx))} \right) \right. \\
&= \frac{i \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)} dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{\sqrt[6]{-1} \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)} dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{5/6} \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)} dx}{3\sqrt[3]{a}\sqrt[3]{b}} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{bd}} - \frac{(2\sqrt[3]{-1}) \operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{bd}} \\
&= \frac{4 \operatorname{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2} dx, x, 2\sqrt[3]{-1}\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{bd}} + \frac{(4\sqrt[3]{-1}) \operatorname{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2} dx, x, 2\sqrt[3]{-1}\sqrt[3]{b}-2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{bd}} \\
&= \frac{2\sqrt[3]{-1} \tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\sqrt[3]{bd}} + \frac{2 \tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\sqrt[3]{bd}} + \frac{2 \tanh^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\sqrt[3]{bd}}
\end{aligned}$$

Mathematica [C] time = 0.236425, size = 199, normalized size = 0.69

$$\operatorname{RootSum}\left[8\#1^3a + \#1^6b - 3\#1^4b + 3\#1^2b - b\&, \frac{2\#1^2 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right) + \#1^2c + \#1^4b - 2\#1^2b}{\#1^4b - 2\#1^2b}\right]$$

3d

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3), x]

[Out] RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) &]/(3*d)

Maple [C] time = 0.038, size = 82, normalized size = 0.3

$$\frac{2}{3d} \sum_{\substack{_R = \operatorname{RootOf}(a_Z^6 - 3a_Z^4 - 8b_Z^3 + 3a_Z^2 - a) \\ _R^5 a - 2_R^3 a - 4_R^2 b + _R a}} \frac{_R^3 - _R}{_R^5 a - 2_R^3 a - 4_R^2 b + _R a} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - _R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 2/3/d*sum((_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**3),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx + c)}{b \sinh(dx + c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)

$$3.177 \quad \int \frac{1}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=280

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+b^{2/3}}} - \frac{2(-1)^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} - \frac{2(-1)^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

[Out] $(-2*(-1)^{(2/3)}*\text{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)]/\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}\right)])/(3*a^{(2/3)}*\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}\right)]*d) - (2*(-1)^{(2/3)}*\text{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)]/\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)])/(3*a^{(2/3)}*\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)]*d) - (2*\text{ArcTan}[\left(b^{(1/3)} - a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)]/\text{Sqrt}[a^{(2/3)} + b^{(2/3)}])/(3*a^{(2/3)}*\text{Sqrt}[a^{(2/3)} + b^{(2/3)}]*d)$

Rubi [A] time = 0.250961, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3213, 2660, 618, 204}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+b^{2/3}}} - \frac{2(-1)^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} - \frac{2(-1)^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^3)^(-1), x]

[Out] $(-2*(-1)^{(2/3)}*\text{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)]/\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}\right)])/(3*a^{(2/3)}*\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}\right)]*d) - (2*(-1)^{(2/3)}*\text{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)]/\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)])/(3*a^{(2/3)}*\text{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)]*d) - (2*\text{ArcTan}[\left(b^{(1/3)} - a^{(1/3)}*\text{Tanh}[(c + d*x)/2]\right)]/\text{Sqrt}[a^{(2/3)} + b^{(2/3)}])/(3*a^{(2/3)}*\text{Sqrt}[a^{(2/3)} + b^{(2/3)}]*d)$

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :=
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f,
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a + b \sinh^3(c + dx)} dx &= \int \left(\frac{\sqrt[6]{-1}}{3a^{2/3} (\sqrt[6]{-1} \sqrt[3]{a} - i \sqrt[3]{b} \sinh(c + dx))} + \frac{\sqrt[6]{-1}}{3a^{2/3} (\sqrt[6]{-1} \sqrt[3]{a} + \sqrt[6]{-1} \sqrt[3]{b} \sinh(c + dx))} + \frac{\sqrt[6]{-1}}{3a^{2/3} (\sqrt[6]{-1} \sqrt[3]{a} - i \sqrt[3]{b} \sinh(c + dx))} \right) dx \\
&= \frac{\sqrt[6]{-1} \int \frac{1}{\sqrt[6]{-1} \sqrt[3]{a} - i \sqrt[3]{b} \sinh(c + dx)} dx}{3a^{2/3}} + \frac{\sqrt[6]{-1} \int \frac{1}{\sqrt[6]{-1} \sqrt[3]{a} + \sqrt[6]{-1} \sqrt[3]{b} \sinh(c + dx)} dx}{3a^{2/3}} + \frac{\sqrt[6]{-1} \int \frac{1}{\sqrt[6]{-1} \sqrt[3]{a} - i \sqrt[3]{b} \sinh(c + dx)} dx}{3a^{2/3}} \\
&= -\frac{(2(-1)^{2/3}) \operatorname{Subst} \left(\int \frac{1}{\sqrt[6]{-1} \sqrt[3]{a} - 2 \sqrt[3]{b} x + \sqrt[6]{-1} \sqrt[3]{a} x^2} dx, x, \tan \left(\frac{1}{2}(ic + idx) \right) \right)}{3a^{2/3} d} - \frac{(2(-1)^{2/3}) \operatorname{Subst} \left(\int \frac{1}{\sqrt[6]{-1} \sqrt[3]{a} + 2 \sqrt[3]{b} x + \sqrt[6]{-1} \sqrt[3]{a} x^2} dx, x, \tan \left(\frac{1}{2}(ic + idx) \right) \right)}{3a^{2/3} d} \\
&= \frac{(4(-1)^{2/3}) \operatorname{Subst} \left(\int \frac{1}{-4(\sqrt[3]{-1} a^{2/3} - b^{2/3}) - x^2} dx, x, -2 \sqrt[3]{b} + 2 \sqrt[6]{-1} \sqrt[3]{a} \tan \left(\frac{1}{2}(ic + idx) \right) \right)}{3a^{2/3} d} + \frac{(4(-1)^{2/3}) \operatorname{Subst} \left(\int \frac{1}{-4(\sqrt[3]{-1} a^{2/3} + b^{2/3}) - x^2} dx, x, -2 \sqrt[3]{b} - 2 \sqrt[6]{-1} \sqrt[3]{a} \tan \left(\frac{1}{2}(ic + idx) \right) \right)}{3a^{2/3} d} \\
&= \frac{2(-1)^{2/3} \tan^{-1} \left(\frac{\sqrt[3]{b} - (-1)^{2/3} \sqrt[3]{a} \tanh \left(\frac{1}{2}(c + dx) \right)}{\sqrt{\sqrt[3]{-1} a^{2/3} - b^{2/3}}} \right)}{3a^{2/3} \sqrt{\sqrt[3]{-1} a^{2/3} - b^{2/3}}} - \frac{2(-1)^{2/3} \tan^{-1} \left(\frac{\sqrt[3]{-1} \sqrt[3]{b} + (-1)^{2/3} \sqrt[3]{a} \tanh \left(\frac{1}{2}(c + dx) \right)}{\sqrt{\sqrt[3]{-1} a^{2/3} - (-1)^{2/3} b^{2/3}}} \right)}{3a^{2/3} \sqrt{\sqrt[3]{-1} a^{2/3} - (-1)^{2/3} b^{2/3}}}
\end{aligned}$$

Mathematica [C] time = 0.173833, size = 131, normalized size = 0.47

$$2\text{RootSum}\left[8\#1^3a + \#1^6b - 3\#1^4b + 3\#1^2b - b\&, \frac{2\#1 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right) + \#1c + \#1}{\#1^4b - 2\#1^2b + 4\#1a + b}\right]$$

$$3d$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^3)^(-1), x]

[Out] (2*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) &])/(3*d)

Maple [C] time = 0.04, size = 87, normalized size = 0.3

$$\frac{1}{3d} \sum_{_R=\text{RootOf}(a_Z^6 - 3a_Z^4 - 8b_Z^3 + 3a_Z^2 - a)} \frac{-_R^4 + 2_R^2 - 1}{-_R^5 a - 2_R^3 a - 4_R^2 b + _R a} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - _R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(d*x+c)^3), x)

[Out] 1/3/d*sum((-_R^4+2*_R^2-1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R), _R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \sinh(dx + c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)^3), x, algorithm="maxima")

[Out] integrate(1/(b*sinh(d*x + c)^3 + a), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \sinh^3(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)**3),x)

[Out] Integral(1/(a + b*sinh(c + d*x)**3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \sinh(dx + c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] integrate(1/(b*sinh(d*x + c)^3 + a), x)

$$3.178 \quad \int \frac{\operatorname{csch}(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=286

$$\frac{2\sqrt[3]{b} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}+b^{2/3}}} + \frac{2\sqrt[3]{b} \tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} + \frac{2\sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

[Out] (2*b^(1/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3) - b^(2/3))]/(3*a*Sqrt[-((-1)^(2/3)*a^(2/3) - b^(2/3)]*d) + (2*b^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]/(3*a*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (2*b^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]]/(3*a*Sqrt[a^(2/3) + b^(2/3)]*d)

Rubi [A] time = 0.432414, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3220, 3770, 2660, 618, 206, 204}

$$\frac{2\sqrt[3]{b} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}+b^{2/3}}} + \frac{2\sqrt[3]{b} \tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} + \frac{2\sqrt[3]{b} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]

[Out] (2*b^(1/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3) - b^(2/3))]/(3*a*Sqrt[-((-1)^(2/3)*a^(2/3) - b^(2/3)]*d) + (2*b^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]/(3*a*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (2*b^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]]/(3*a*Sqrt[a^(2/3) + b^(2/3)]*d)

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)} dx &= i \int \left(-\frac{i\operatorname{csch}(c+dx)}{a} + \frac{ib\sinh^2(c+dx)}{a(a+b\sinh^3(c+dx))} \right) dx \\
&= \frac{\int \operatorname{csch}(c+dx) dx}{a} - \frac{b \int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)} dx}{a} \\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{b \int \left(\frac{i}{3b^{2/3}(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} + \frac{i}{3b^{2/3}(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} + \frac{i}{3b^{2/3}(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} \right) dx}{a} \\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(i\sqrt[3]{b}) \int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)} dx}{3a} + \frac{(i\sqrt[3]{b}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)} dx}{3a} + \frac{(i\sqrt[3]{b}) \int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)} dx}{3a} \\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{(2\sqrt[3]{b}) \operatorname{Subst} \left(\int \frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{bx}-i\sqrt[3]{ax^2}} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right) \right)}{3ad} + \frac{(2\sqrt[3]{b}) \operatorname{Subst} \left(\int \frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{bx}-i\sqrt[3]{ax^2}} dx, x, \tan\left(\frac{1}{2}(ic+idx)\right) \right)}{3ad} \\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} - \frac{(4\sqrt[3]{b}) \operatorname{Subst} \left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}-b^{2/3})-x^2} dx, x, -2\sqrt[3]{b} + 2\sqrt[6]{-1}\sqrt[3]{a} \tan\left(\frac{1}{2}(ic+idx)\right) \right)}{3ad} \\
&= -\frac{2\sqrt[3]{b} \tan^{-1} \left(\frac{\sqrt[3]{b} + \sqrt[3]{-1}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} \right)}{3a\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}d}} - \frac{2\sqrt[3]{b} \tan^{-1} \left(\frac{\sqrt[3]{b} - (-1)^{2/3}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}} \right)}{3a\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}d}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}
\end{aligned}$$

Mathematica [C] time = 0.254159, size = 295, normalized size = 1.03

$$6 \log \left(\tanh \left(\frac{1}{2}(c+dx) \right) \right) - b \operatorname{RootSum} \left[8\#1^3 a + \#1^6 b - 3\#1^4 b + 3\#1^2 b - b \&, \frac{2\#1^4 \log \left(-\#1 \sinh \left(\frac{1}{2}(c+dx) \right) + \#1 \cosh \left(\frac{1}{2}(c+dx) \right) - \sinh \left(\frac{1}{2}(c+dx) \right) \right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} \right]$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]

[Out] (6*Log[Tanh[(c + d*x)/2]] - b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &, (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) &])/(6*a*d)

Maple [C] time = 0.072, size = 100, normalized size = 0.4

$$\frac{1}{da} \ln \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) \right) + \frac{4b}{3da} \sum_{_R=\text{RootOf}(a_Z^6-3a_Z^4-8b_Z^3+3a_Z^2-a)} \frac{_{R}^2}{_{R}^5 a - 2_{R}^3 a - 4_{R}^2 b + _R a} \ln \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x)

[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c))+4/3/d/a*b*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad} + \frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad} - 2 \int \frac{be^{(5dx+5c)} - 2be^{(3dx+3c)} + be^{(dx+c)}}{abe^{(6dx+6c)} - 3abe^{(4dx+4c)} + 8a^2e^{(3dx+3c)} + 3abe^{(2dx+2c)} - ab^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d) - 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] integrate(csch(d*x + c)/(b*sinh(d*x + c)^3 + a), x)

$$3.179 \quad \int \frac{\operatorname{csch}^2(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=304

$$\frac{2b^{2/3} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+b^{2/3}}} - \frac{2b^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} + \frac{2\sqrt[3]{-1}b^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i\sqrt[3]{a}}\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

[Out] $(-2*b^{(2/3)}*ArcTan[((-1)^{(1/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2])]/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}])/(3*a^{(4/3)}*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}]*d) + (2*(-1)^{(1/3)}*b^{(2/3)}*ArcTan[((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2])]/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}])/(3*a^{(4/3)}*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}]*d) - (2*b^{(2/3)}*ArcTanh[(b^{(1/3)} - a^{(1/3)}*Tanh[(c + d*x)/2])/Sqrt[a^{(2/3)} + b^{(2/3)}])/(3*a^{(4/3)}*Sqrt[a^{(2/3)} + b^{(2/3)}]*d) - Coth[c + d*x]/(a*d)$

Rubi [A] time = 0.478469, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3220, 3767, 8, 2660, 618, 204}

$$\frac{2b^{2/3} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+b^{2/3}}} - \frac{2b^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i\sqrt[3]{a}} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} + \frac{2\sqrt[3]{-1}b^{2/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i\sqrt[3]{a}}\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3), x]

[Out] $(-2*b^{(2/3)}*ArcTan[((-1)^{(1/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2])]/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}])/(3*a^{(4/3)}*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}]*d) + (2*(-1)^{(1/3)}*b^{(2/3)}*ArcTan[((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2])]/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}])/(3*a^{(4/3)}*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}]*d) - (2*b^{(2/3)}*ArcTanh[(b^{(1/3)} - a^{(1/3)}*Tanh[(c + d*x)/2])/Sqrt[a^{(2/3)} + b^{(2/3)}])/(3*a^{(4/3)}*Sqrt[a^{(2/3)} + b^{(2/3)}]*d) - Coth[c + d*x]/(a*d)$

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)} dx &= -\int \left(-\frac{\operatorname{csch}^2(c+dx)}{a} + \frac{b\sinh(c+dx)}{a(a+b\sinh^3(c+dx))} \right) dx \\
&= \frac{\int \operatorname{csch}^2(c+dx) dx}{a} - \frac{b \int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)} dx}{a} \\
&\stackrel{(ib)}{=} \frac{\int \left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx))} - \frac{1}{3\sqrt[3]{a}\sqrt[3]{b}(\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx))} \right) dx}{a} \\
&= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{(ib^{2/3}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)} dx}{3a^{4/3}} + \frac{(\sqrt[6]{-1}b^{2/3}) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)} dx}{3a^{4/3}} \\
&= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{(2b^{2/3}) \operatorname{Subst} \left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2} dx, x, \tan \left(\frac{1}{2}(ic+idx) \right) \right)}{3a^{4/3}d} + \dots \\
&= \frac{\operatorname{coth}(c+dx)}{ad} + \frac{(4b^{2/3}) \operatorname{Subst} \left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3})-x^2} dx, x, 2\sqrt[3]{-1}\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan \left(\frac{1}{2}(ic+idx) \right) \right)}{3a^{4/3}d} \\
&= \frac{2\sqrt[3]{-1}b^{2/3} \tan^{-1} \left(\frac{\sqrt[3]{b}(-1)^{2/3}\sqrt[3]{a} \tanh \left(\frac{1}{2}(c+dx) \right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}} \right)}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}d}} - \frac{2b^{2/3} \tan^{-1} \left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a} \tanh \left(\frac{1}{2}(c+dx) \right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} \right)}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}d}}
\end{aligned}$$

Mathematica [C] time = 0.390087, size = 230, normalized size = 0.76

$$2b\operatorname{RootSum} \left[8\#1^3a + \#1^6b - 3\#1^4b + 3\#1^2b - b\&, \frac{2\#1^2 \log \left(-\#1 \sinh \left(\frac{1}{2}(c+dx) \right) + \#1 \cosh \left(\frac{1}{2}(c+dx) \right) - \sinh \left(\frac{1}{2}(c+dx) \right) - \cosh \left(\frac{1}{2}(c+dx) \right) \right) + \#1^4b}{\#1^4b} \right]$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]

[Out] $-(3*\operatorname{Coth}[(c+d*x)/2] + 2*b*\operatorname{RootSum}[-b + 3*b*\#1^2 + 8*a*\#1^3 - 3*b*\#1^4 + b*\#1^6 \&, (-c-d*x-2*\operatorname{Log}[-\operatorname{Cosh}[(c+d*x)/2] - \operatorname{Sinh}[(c+d*x)/2] + \operatorname{Cosh}[(c+d*x)/2]*\#1 - \operatorname{Sinh}[(c+d*x)/2]*\#1 + c*\#1^2 + d*x*\#1^2 + 2*\operatorname{Log}[-\operatorname{Cosh}[(c+d*x)/2] - \operatorname{Sinh}[(c+d*x)/2] + \operatorname{Cosh}[(c+d*x)/2]*\#1 - \operatorname{Sinh}[(c+d*x)/2]*\#1]*\#1^2)/(b + 4*a*\#1 - 2*b*\#1^2 + b*\#1^4) \&] + 3*\operatorname{Tanh}[(c+d*x)/2])/(6*a*d)$

Maple [C] time = 0.075, size = 123, normalized size = 0.4

$$-\frac{1}{2da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{2b}{3da} \sum_{_R=\text{RootOf}(a_Z^6-3a_Z^4-8b_Z^3+3a_Z^2-a)} \frac{_{R^3} - _R}{-_{R^5}a - 2_{R^3}a - 4_{R^2}b +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x)

[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-2/3/d/a*b*sum((_{R^3}-_{R^3})/(_{R^5}a-2_{R^3}a-4_{R^2}b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{ade^{2dx+2c}-ad} - 4 \int \frac{be^{4dx+4c} - be^{2dx+2c}}{abe^{6dx+6c} - 3abe^{4dx+4c} + 8a^2e^{3dx+3c} + 3abe^{2dx+2c} - ab} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d*x + 2*c))/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)^2}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] integrate(csch(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)

$$3.180 \quad \int \frac{\operatorname{csch}^3(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=322

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+b^{2/3}}} + \frac{2(-1)^{2/3}b \tanh^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} + \frac{2(-1)^{2/3}b \tanh^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

[Out] $(2*(-1)^{(2/3)}*b*\operatorname{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*\operatorname{Tanh}[(c + d*x)/2]\right)]/\operatorname{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}\right)])/(3*a^{(5/3)}*\operatorname{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}\right)*d] + (2*(-1)^{(2/3)}*b*\operatorname{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*\operatorname{Tanh}[(c + d*x)/2]\right)]/\operatorname{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)])/(3*a^{(5/3)}*\operatorname{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)*d] + \operatorname{ArcTan}[\operatorname{Cosh}[c + d*x]]/(2*a*d) + (2*b*\operatorname{ArcTanh}[(b^{(1/3)} - a^{(1/3)})*\operatorname{Tanh}[(c + d*x)/2]]/\operatorname{Sqrt}[a^{(2/3)} + b^{(2/3)}])/(3*a^{(5/3)}*\operatorname{Sqrt}[a^{(2/3)} + b^{(2/3)}]*d) - (\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(2*a*d)$

Rubi [A] time = 0.457326, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3220, 3768, 3770, 3213, 2660, 618, 204}

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+b^{2/3}}} + \frac{2(-1)^{2/3}b \tanh^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}} + \frac{2(-1)^{2/3}b \tanh^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^3/(a + b*\operatorname{Sinh}[c + d*x]^3), x]$

[Out] $(2*(-1)^{(2/3)}*b*\operatorname{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*\operatorname{Tanh}[(c + d*x)/2]\right)]/\operatorname{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}\right)])/(3*a^{(5/3)}*\operatorname{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - (-1)^{(2/3)}*b^{(2/3)}\right)*d] + (2*(-1)^{(2/3)}*b*\operatorname{ArcTan}[\left((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*\operatorname{Tanh}[(c + d*x)/2]\right)]/\operatorname{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)])/(3*a^{(5/3)}*\operatorname{Sqrt}[\left((-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}\right)*d] + \operatorname{ArcTan}[\operatorname{Cosh}[c + d*x]]/(2*a*d) + (2*b*\operatorname{ArcTanh}[(b^{(1/3)} - a^{(1/3)})*\operatorname{Tanh}[(c + d*x)/2]]/\operatorname{Sqrt}[a^{(2/3)} + b^{(2/3)}])/(3*a^{(5/3)}*\operatorname{Sqrt}[a^{(2/3)} + b^{(2/3)}]*d) - (\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(2*a*d)$

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3213

```
Int[((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_.)]^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_.) + (b_.)*(x_.)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^3(c+dx)} dx &= -\left(i \int \left(\frac{i\operatorname{csch}^3(c+dx)}{a} - \frac{ib}{a(a+b\sinh^3(c+dx))} \right) dx \right) \\
&= \frac{\int \operatorname{csch}^3(c+dx) dx}{a} - \frac{b \int \frac{1}{a+b\sinh^3(c+dx)} dx}{a} \\
&= -\frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{\int \operatorname{csch}(c+dx) dx}{2a} - \frac{b \int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}(\sqrt[6]{-1}\sqrt[3]{a-i}\sqrt[3]{b}\sinh(c+dx))} + \frac{\sqrt[6]{-1}}{3a^{2/3}(\sqrt[6]{-1}\sqrt[3]{a+i}\sqrt[3]{b}\sinh(c+dx))} \right) dx}{3a^{5/3}} \\
&= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{(\sqrt[6]{-1}b) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a-i}\sqrt[3]{b}\sinh(c+dx)} dx}{3a^{5/3}} - \frac{(\sqrt[6]{-1}b) \int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a+i}\sqrt[3]{b}\sinh(c+dx)} dx}{3a^{5/3}} \\
&= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} + \frac{(2(-1)^{2/3}b) \operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a-2}\sqrt[3]{bx+\sqrt[6]{-1}}-x} dx\right)}{3a^{5/3}d} - \frac{(2(-1)^{2/3}b) \operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a+2}\sqrt[3]{bx+\sqrt[6]{-1}}-x} dx\right)}{3a^{5/3}d} \\
&= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad} - \frac{\operatorname{coth}(c+dx)\operatorname{csch}(c+dx)}{2ad} - \frac{(4(-1)^{2/3}b) \operatorname{Subst}\left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}-b^{2/3})-x} dx\right)}{3a^{5/3}d} + \frac{(4(-1)^{2/3}b) \operatorname{Subst}\left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}+b^{2/3})-x} dx\right)}{3a^{5/3}d} \\
&= -\frac{2(-1)^{2/3}b \tan^{-1}\left(\frac{\sqrt[3]{b}(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}d} + \frac{2(-1)^{2/3}b \tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}d}
\end{aligned}$$

Mathematica [C] time = 0.497428, size = 178, normalized size = 0.55

$$\frac{16b\operatorname{RootSum}\left[8\#1^3a + \#1^6b - 3\#1^4b + 3\#1^2b - b\&, \frac{2\#1 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right) + \#1}{\#1^4b - 2\#1^2b + 4\#1a + b}\right]}{24ad}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^3), x]

[Out] -(16*b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &, (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) &] + 3*(Csch[(c + d*x)/2]^2 + 4*Log[Tanh[(c + d*x)/2]] + Sech[(c + d*x)/2]^2))/(24*a*d)

Maple [C] time = 0.089, size = 146, normalized size = 0.5

$$\frac{1}{8da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{1}{8da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^{-2} - \frac{1}{2da} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right) + \frac{b}{3da} \sum_{_R=\text{RootOf}(a_Z^6-3a_Z^4-8b_Z^3+3a_Z^3-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x)

[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/8/d/a/tanh(1/2*d*x+1/2*c)^-2-1/2/d/a*ln(tanh(1/2*d*x+1/2*c))+1/3/d/a*b*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-8b \int \frac{e^{(3dx+3c)}}{abe^{(6dx+6c)} - 3abe^{(4dx+4c)} + 8a^2e^{(3dx+3c)} + 3abe^{(2dx+2c)} - ab} dx - \frac{e^{(3dx+3c)} + e^{(dx+c)}}{ade^{(4dx+4c)} - 2ade^{(2dx+2c)} + ad} + \frac{\log\left(\left(\frac{e^{(dx+c)} + 1}{e^{(dx+c)} - 1}\right)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] -8*b*integrate(e^(3*d*x + 3*c)/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x) - (e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) - 1/2*log((e^(d*x + c) - 1)*e^(-c))/(a*d)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)^3}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")

[Out] integrate(csch(d*x + c)^3/(b*sinh(d*x + c)^3 + a), x)

$$3.181 \quad \int \frac{\operatorname{csch}^4(c+dx)}{a+b \sinh^3(c+dx)} dx$$

Optimal. Leaf size=317

$$\frac{2b^{4/3} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^2 d \sqrt{a^{2/3}+b^{2/3}}} - \frac{2b^{4/3} \tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2 d \sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} - \frac{2b^{4/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^2 d \sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

[Out] $(-2*b^{(4/3)}*ArcTan[((-1)^{(5/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2]))/Sqrt[-((-1)^{(2/3)}*a^{(2/3)} - b^{(2/3)})]/(3*a^2*Sqrt[-((-1)^{(2/3)}*a^{(2/3)} - b^{(2/3)})*d] - (2*b^{(4/3)}*ArcTan[((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2]))/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)})]/(3*a^2*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}]*d) + (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^{(4/3)}*ArcTanh[(b^{(1/3)} - a^{(1/3)}*Tanh[(c + d*x)/2])/Sqrt[a^{(2/3)} + b^{(2/3)})]/(3*a^2*Sqrt[a^{(2/3)} + b^{(2/3)}]*d) + Coth[c + d*x]/(a*d) - Coth[c + d*x]^3/(3*a*d)$

Rubi [A] time = 0.432501, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3220, 3770, 3767, 2660, 618, 206, 204}

$$\frac{2b^{4/3} \tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^2 d \sqrt{a^{2/3}+b^{2/3}}} - \frac{2b^{4/3} \tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2 d \sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} - \frac{2b^{4/3} \tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b+i}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^2 d \sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3), x]

[Out] $(-2*b^{(4/3)}*ArcTan[((-1)^{(5/6)}*((-1)^{(1/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2]))/Sqrt[-((-1)^{(2/3)}*a^{(2/3)} - b^{(2/3)})]/(3*a^2*Sqrt[-((-1)^{(2/3)}*a^{(2/3)} - b^{(2/3)})*d] - (2*b^{(4/3)}*ArcTan[((-1)^{(1/6)}*((-1)^{(5/6)}*b^{(1/3)} + I*a^{(1/3)}*Tanh[(c + d*x)/2]))/Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)})]/(3*a^2*Sqrt[(-1)^{(1/3)}*a^{(2/3)} - b^{(2/3)}]*d) + (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) - (2*b^{(4/3)}*ArcTanh[(b^{(1/3)} - a^{(1/3)}*Tanh[(c + d*x)/2])/Sqrt[a^{(2/3)} + b^{(2/3)})]/(3*a^2*Sqrt[a^{(2/3)} + b^{(2/3)}]*d) + Coth[c + d*x]/(a*d) - Coth[c + d*x]^3/(3*a*d)$

Rule 3220

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^ (p_.), x_Symbol] := Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || GtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)} dx &= \int \left(-\frac{b\operatorname{csch}(c+dx)}{a^2} + \frac{\operatorname{csch}^4(c+dx)}{a} - \frac{b^2\sinh^2(c+dx)}{a^2(-a-b\sinh^3(c+dx))} \right) dx \\
&= \frac{\int \operatorname{csch}^4(c+dx) dx}{a} - \frac{b \int \operatorname{csch}(c+dx) dx}{a^2} - \frac{b^2 \int \frac{\sinh^2(c+dx)}{-a-b\sinh^3(c+dx)} dx}{a^2} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{b^2 \int \left(-\frac{i}{3b^{2/3}(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} - \frac{i}{3b^{2/3}(\sqrt[3]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx))} - \frac{bx^{2/3}}{3b^{2/3}} \right) dx}{a^2} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{(ib^{4/3}) \int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)} dx}{3a^2} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{(2b^{4/3}) \operatorname{Subst} \left(\int \frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{bx}-i\sqrt[3]{b}} dx \right)}{3a^2 d} \\
&= \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} + \frac{(4b^{4/3}) \operatorname{Subst} \left(\int \frac{1}{-4(\sqrt[3]{-1}a^{2/3}-b^{2/3})} dx \right)}{3a^2 d} \\
&= \frac{2b^{4/3} \tan^{-1} \left(\frac{\sqrt[3]{b} + \sqrt[3]{-1}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} \right)}{3a^2 \sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}d}} + \frac{2b^{4/3} \tan^{-1} \left(\frac{\sqrt[3]{b} - (-1)^{2/3}\sqrt[3]{a} \tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}} \right)}{3a^2 \sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}d}} + \frac{b \tanh^{-1}(\cosh(c+dx))}{a^2 d}
\end{aligned}$$

Mathematica [C] time = 6.06156, size = 370, normalized size = 1.17

$$4b^2 \operatorname{RootSum} \left[8\#1^3 a + \#1^6 b - 3\#1^4 b + 3\#1^2 b - b\&, \frac{2\#1^4 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right) - 4\#1^4}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}} \right]$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3), x]

[Out] (8*a*Coth[(c + d*x)/2] - 24*b*Log[Tanh[(c + d*x)/2]] + 4*b^2*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &, (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) &] + 8*a*Csch[c + d*x]

$$\frac{-3\sinh\left(\frac{c+dx}{2}\right)^4 - (a\operatorname{csch}\left(\frac{c+dx}{2}\right)^4\sinh[c+dx])/2 + 8a\operatorname{Tanh}\left(\frac{c+dx}{2}\right)}{(24a^2d)}$$

Maple [C] time = 0.087, size = 178, normalized size = 0.6

$$-\frac{1}{24da}\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^3 + \frac{3}{8da}\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{24da}\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-3} + \frac{3}{8da}\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{b}{da^2}\ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x)

[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)-1/24/d/a/tanh(1/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))-4/3/d/a^2*b^2*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4(3e^{2dx+2c}-1)}{3(ade^{6dx+6c}-3ade^{4dx+4c}+3ade^{2dx+2c}-ad)} + \frac{b\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{a^2d} - \frac{b\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{a^2d} + 16\int\frac{1}{8(a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")

[Out] -4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*d*x + 4*c) + 3*a*d*e^(2*d*x + 2*c) - a*d) + b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 16*integrate(1/8*(b^2*e^(5*d*x + 5*c) - 2*b^2*e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^2*b*e^(6*d*x + 6*c) - 3*a^2*b*e^(4*d*x + 4*c) + 8*a^3*e^(3*d*x + 3*c) + 3*a^2*b*e^(2*d*x + 2*c) - a^2*b), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)^4}{b \sinh(dx+c)^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^4/(b*sinh(d*x + c)^3 + a), x)
```

$$3.182 \quad \int \frac{1}{1+\sinh^3(x)} dx$$

Optimal. Leaf size=139

$$-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - \frac{1}{3}\sqrt[6]{-1}\log\left(-\sqrt[6]{-1}\tanh\left(\frac{x}{2}\right) + (-1)^{5/6} + 1\right) + \frac{1}{3}\sqrt[6]{-1}\log\left(\sqrt[3]{-1}\tanh\left(\frac{x}{2}\right) + \sqrt[6]{-1} + 1\right) - \dots$$

[Out] $(-2*(-1)^{(1/6)}*\text{ArcTan}[(1 + (-1)^{(1/6)}*\text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(1/3)}]])/(3*\text{Sqrt}[1 - (-1)^{(1/3)}) - (\text{Sqrt}[2]*\text{ArcTanh}[(1 - \text{Tanh}[x/2])/ \text{Sqrt}[2]])/3 - ((-1)^{(1/6)}*\text{Log}[1 + (-1)^{(5/6)} - (-1)^{(1/6)}*\text{Tanh}[x/2]])/3 + ((-1)^{(1/6)}*\text{Log}[1 + (-1)^{(1/6)} + (-1)^{(1/3)}*\text{Tanh}[x/2]])/3$

Rubi [A] time = 0.187819, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 1.$, Rules used = {3213, 2660, 618, 204, 617, 206, 616, 31}

$$-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - \frac{1}{3}\sqrt[6]{-1}\log\left(-\sqrt[6]{-1}\tanh\left(\frac{x}{2}\right) + (-1)^{5/6} + 1\right) + \frac{1}{3}\sqrt[6]{-1}\log\left(\sqrt[3]{-1}\tanh\left(\frac{x}{2}\right) + \sqrt[6]{-1} + 1\right) - \dots$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^3)^(-1), x]

[Out] $(-2*(-1)^{(1/6)}*\text{ArcTan}[(1 + (-1)^{(1/6)}*\text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(1/3)}]])/(3*\text{Sqrt}[1 - (-1)^{(1/3)}) - (\text{Sqrt}[2]*\text{ArcTanh}[(1 - \text{Tanh}[x/2])/ \text{Sqrt}[2]])/3 - ((-1)^{(1/6)}*\text{Log}[1 + (-1)^{(5/6)} - (-1)^{(1/6)}*\text{Tanh}[x/2]])/3 + ((-1)^{(1/6)}*\text{Log}[1 + (-1)^{(1/6)} + (-1)^{(1/3)}*\text{Tanh}[x/2]])/3$

Rule 3213

Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*

e^{2*x^2} , x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 31

Int[((a_) + (b_.)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{1 + \sinh^3(x)} dx &= \int \left(\frac{\sqrt[6]{-1}}{3(\sqrt[6]{-1} - i \sinh(x))} + \frac{\sqrt[6]{-1}}{3(\sqrt[6]{-1} + \sqrt[6]{-1} \sinh(x))} + \frac{\sqrt[6]{-1}}{3(\sqrt[6]{-1} + (-1)^{5/6} \sinh(x))} \right) dx \\
&= \frac{1}{3} \sqrt[6]{-1} \int \frac{1}{\sqrt[6]{-1} - i \sinh(x)} dx + \frac{1}{3} \sqrt[6]{-1} \int \frac{1}{\sqrt[6]{-1} + \sqrt[6]{-1} \sinh(x)} dx + \frac{1}{3} \sqrt[6]{-1} \int \frac{1}{\sqrt[6]{-1} + (-1)^{5/6} \sinh(x)} dx \\
&= \frac{1}{3} (2\sqrt[6]{-1}) \text{Subst} \left(\int \frac{1}{\sqrt[6]{-1} - 2ix - \sqrt[6]{-1}x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right) + \frac{1}{3} (2\sqrt[6]{-1}) \text{Subst} \left(\int \frac{1}{\sqrt[6]{-1} + 2\sqrt[6]{-1}x - \sqrt[6]{-1}x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right) \\
&= -\left(\frac{2}{3} \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, 1 - \tanh\left(\frac{x}{2}\right) \right)\right) - \frac{1}{3} (4\sqrt[6]{-1}) \text{Subst} \left(\int \frac{1}{-4(1 - \sqrt[3]{-1}) - x^2} dx, x, -2i - \tanh\left(\frac{x}{2}\right) \right) \\
&= -\frac{2\sqrt[6]{-1} \tan^{-1} \left(\frac{i + \sqrt[6]{-1} \tanh\left(\frac{x}{2}\right)}{\sqrt{1 - \sqrt[3]{-1}}} \right)}{3\sqrt{1 - \sqrt[3]{-1}}} - \frac{1}{3} \sqrt{2} \tan^{-1} \left(\frac{1 - \tanh\left(\frac{x}{2}\right)}{\sqrt{2}} \right) - \frac{1}{3} \sqrt[6]{-1} \log \left(1 + (-1)^{5/6} - \sqrt[6]{-1} \tanh\left(\frac{x}{2}\right) \right)
\end{aligned}$$

Mathematica [A] time = 1.39943, size = 156, normalized size = 1.12

$$\frac{2 \tanh^{-1} \left(\frac{\tanh\left(\frac{x}{2}\right) - 1}{\sqrt{2}} \right) + i \sqrt{-1 - i\sqrt{3}} (\sqrt{3} + i) \tan^{-1} \left(\frac{2 + (1 - i\sqrt{3}) \tanh\left(\frac{x}{2}\right)}{\sqrt{-2 + 2i\sqrt{3}}} \right) + (-1 - i\sqrt{3}) \sqrt{-1 + i\sqrt{3}} \tan^{-1} \left(\frac{2 + (1 + i\sqrt{3}) \tanh\left(\frac{x}{2}\right)}{\sqrt{-2 - 2i\sqrt{3}}} \right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^3)^(-1), x]

[Out] (I*Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[(2 + (1 - I*Sqrt[3])*Tanh[x/2])/Sqrt[-2 + (2*I)*Sqrt[3]]] + (-1 - I*Sqrt[3])*Sqrt[-1 + I*Sqrt[3]]*ArcTan[(2 + (1 + I*Sqrt[3])*Tanh[x/2])/Sqrt[-2 - (2*I)*Sqrt[3]]] + 2*ArcTanh[(-1 + Tanh[x/2])/Sqrt[2]])/(3*Sqrt[2])

Maple [C] time = 0.029, size = 82, normalized size = 0.6

$$\frac{2}{3} \sum_{_R=\text{RootOf}(_Z^4+2_Z^3+2_Z^2-2_Z+1)} \frac{-_R^2 - _R + 1}{2_R^3 + 3_R^2 + 2_R - 1} \ln \left(\tanh\left(\frac{x}{2}\right) - _R \right) + \frac{\sqrt{2}}{3} \text{Arctanh} \left(\frac{\sqrt{2}}{4} (2 \tanh(x/2) - 2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+sinh(x)^3), x)

[Out] $\frac{2}{3} \sum \left(\frac{-R^2 - R + 1}{(2R^3 + 3R^2 + 2R - 1) \ln(\tanh(1/2 * x) - R)}, R = \text{RootOf}(Z^4 + 2Z^3 + 2Z^2 - 2Z + 1) \right) + \frac{1}{3} 2^{1/2} \operatorname{arctanh}\left(\frac{1}{4}(2 \tanh(1/2 * x) - 2) * 2^{1/2}\right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6} \sqrt{2} \log \left(-\frac{\sqrt{2} - e^x - 1}{\sqrt{2} + e^x + 1} \right) - \int \frac{2(e^{3x} - 4e^{2x} - e^x)}{3(e^{4x} - 2e^{3x} + 2e^{2x} + 2e^x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^3),x, algorithm="maxima")`

[Out] $\frac{1}{6} \sqrt{2} \log(-(\sqrt{2} - e^x - 1)/(\sqrt{2} + e^x + 1)) - \int \frac{2(e^{3x} - 4e^{2x} - e^x)}{3(e^{4x} - 2e^{3x} + 2e^{2x} + 2e^x + 1)} dx$

Fricas [B] time = 1.93521, size = 591, normalized size = 4.25

$$-\frac{1}{6} \sqrt{3} \log(-4(\sqrt{3} + 1)e^x + 4\sqrt{3} + 4e^{2x} + 8) + \frac{1}{6} \sqrt{3} \log(4(\sqrt{3} - 1)e^x - 4\sqrt{3} + 4e^{2x} + 8) + \frac{1}{6} \sqrt{2} \log\left(-\frac{2(\sqrt{2} - 1)}{e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^3),x, algorithm="fricas")`

[Out] $-\frac{1}{6} \sqrt{3} \log(-4(\sqrt{3} + 1)e^x + 4\sqrt{3} + 4e^{2x} + 8) + \frac{1}{6} \sqrt{3} \log(4(\sqrt{3} - 1)e^x - 4\sqrt{3} + 4e^{2x} + 8) + \frac{1}{6} \sqrt{2} \log\left(-\frac{2(\sqrt{2} - 1)}{e}\right) + \frac{2}{3} \operatorname{arctan}\left(\frac{-(\sqrt{3} + 1)e^x + \sqrt{3} - 1}{e^{2x} + 2e^x - 1}\right) + \frac{2}{3} \operatorname{arctan}\left(\frac{-(\sqrt{3} + 1)e^x + \sqrt{3} - 1}{e^{2x} + 2e^x - 1}\right) + \frac{2}{3} \operatorname{arctan}\left(\frac{-(\sqrt{3} - 1)e^x + 1}{e^{2x} + 2e^x - 1}\right) + \frac{2}{3} \operatorname{arctan}\left(\frac{-(\sqrt{3} - 1)e^x + 1}{e^{2x} + 2e^x - 1}\right)$

Sympy [B] time = 174.633, size = 1423, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)**3),x)

[Out] $167721543\sqrt{1 + \sqrt{3}}I \cdot \log(\tanh(x/2) - 1 + \sqrt{2}) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) + 118590990\sqrt{2}\sqrt{1 + \sqrt{3}}I \cdot \log(\tanh(x/2) - 1 + \sqrt{2}) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) + 193668156\sqrt{6}I \cdot \log(\tanh(x/2) - 1 + \sqrt{2}) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) + 273874160\sqrt{3}I \cdot \log(\tanh(x/2) - 1 + \sqrt{2}) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I) + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) + 167721543\sqrt{3}I\sqrt{1 + \sqrt{3}}I \cdot \log(\tanh(x/2) - 1 + \sqrt{2}) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I) + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) + 167721543\sqrt{3}I\sqrt{1 + \sqrt{3}}I \cdot \log(\tanh(x/2) - \sqrt{2} - 1) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I) + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) - 118590990\sqrt{6}I\sqrt{1 + \sqrt{3}}I \cdot \log(\tanh(x/2) - \sqrt{2} - 1) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I) + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) - 273874160\sqrt{3}I \cdot \log(\tanh(x/2) - \sqrt{2} - 1) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I) + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) - 193668156\sqrt{6}I \cdot \log(\tanh(x/2) - \sqrt{2} - 1) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I) + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) - 118590990\sqrt{2}\sqrt{1 + \sqrt{3}}I \cdot \log(\tanh(x/2) - \sqrt{2} - 1) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I) + 711545940\sqrt{3}I\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{6}I\sqrt{1 + \sqrt{3}}I + 821622480\sqrt{6}I + 1162008936\sqrt{3}I) - 167721543\sqrt{1 + \sqrt{3}}I \cdot \log(\tanh(x/2) - \sqrt{2} - 1) / (711545940\sqrt{1 + \sqrt{3}}I + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I) + 503164629\sqrt{2}\sqrt{1 + \sqrt{3}}I$

$\sqrt{2}*\sqrt{1 + \sqrt{3}*I} + 711545940*\sqrt{3}*I*\sqrt{1 + \sqrt{3}*I} + 503$
 $164629*\sqrt{6}*I*\sqrt{1 + \sqrt{3}*I} + 821622480*\sqrt{6}*I + 1162008936*\sqrt{3}*I$

Giac [A] time = 1.16328, size = 140, normalized size = 1.01

$$\frac{1}{6}(\sqrt{3} + i) \log(\sqrt{3} + (i+1)e^x - 1) + \frac{1}{6}(\sqrt{3} - i) \log(i\sqrt{3} + (i+1)e^x - i) - \frac{1}{6}(\sqrt{3} + i) \log(-i\sqrt{3} + (i+1)e^x - i) - \frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^3),x, algorithm="giac")

[Out] 1/6*(sqrt(3) + I)*log(sqrt(3) + (I + 1)*e^x - 1) + 1/6*(sqrt(3) - I)*log(I*sqrt(3) + (I + 1)*e^x - I) - 1/6*(sqrt(3) + I)*log(-I*sqrt(3) + (I + 1)*e^x - I) - 1/6*(sqrt(3) - I)*log(-sqrt(3) + (I + 1)*e^x - 1) + 1/6*sqrt(2)*log(1/2*abs(-2*sqrt(2) + 2*e^x + 2)/(sqrt(2) + e^x + 1))

$$3.183 \quad \int \frac{1}{1 - \sinh^3(x)} dx$$

Optimal. Leaf size=133

$$\frac{1}{3}\sqrt{2} \tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right) - \frac{1}{3}(-1)^{5/6} \log\left((-1)^{2/3} \tanh\left(\frac{x}{2}\right) + (-1)^{5/6} + 1\right) + \frac{1}{3}(-1)^{5/6} \log\left((-1)^{5/6} \tanh\left(\frac{x}{2}\right) + \sqrt[6]{-1} + 1\right)$$

[Out] (2*(-1)^(5/6)*ArcTan[(1 - (-1)^(5/6)*Tanh[x/2])/Sqrt[1 + (-1)^(2/3)]]/(3*Sqrt[1 + (-1)^(2/3)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]])/3 - ((-1)^(5/6)*Log[1 + (-1)^(5/6) + (-1)^(2/3)*Tanh[x/2]])/3 + ((-1)^(5/6)*Log[1 + (-1)^(1/6) + (-1)^(5/6)*Tanh[x/2]])/3

Rubi [A] time = 0.188952, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.8$, Rules used = {3213, 2660, 618, 204, 616, 31, 617, 206}

$$\frac{1}{3}\sqrt{2} \tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right) - \frac{1}{3}(-1)^{5/6} \log\left((-1)^{2/3} \tanh\left(\frac{x}{2}\right) + (-1)^{5/6} + 1\right) + \frac{1}{3}(-1)^{5/6} \log\left((-1)^{5/6} \tanh\left(\frac{x}{2}\right) + \sqrt[6]{-1} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^3)^(-1), x]

[Out] (2*(-1)^(5/6)*ArcTan[(1 - (-1)^(5/6)*Tanh[x/2])/Sqrt[1 + (-1)^(2/3)]]/(3*Sqrt[1 + (-1)^(2/3)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]])/3 - ((-1)^(5/6)*Log[1 + (-1)^(5/6) + (-1)^(2/3)*Tanh[x/2]])/3 + ((-1)^(5/6)*Log[1 + (-1)^(1/6) + (-1)^(5/6)*Tanh[x/2]])/3

Rule 3213

Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] := Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*

e^{2x^2} , x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 31

Int[((a_) + (b_.)*(x_))(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{1 - \sinh^3(x)} dx &= \int \left(\frac{(-1)^{5/6}}{3(-(-1)^{5/6} - i \sinh(x))} - \frac{(-1)^{5/6}}{3(-(-1)^{5/6} + \sqrt[6]{-1} \sinh(x))} - \frac{(-1)^{5/6}}{3(-(-1)^{5/6} + (-1)^{5/6} \sinh(x))} \right) dx \\
&= -\left(\frac{1}{3}(-1)^{5/6} \int \frac{1}{-(-1)^{5/6} - i \sinh(x)} dx \right) - \frac{1}{3}(-1)^{5/6} \int \frac{1}{-(-1)^{5/6} + \sqrt[6]{-1} \sinh(x)} dx - \frac{1}{3}(-1)^{5/6} \int \frac{1}{-(-1)^{5/6} + (-1)^{5/6} \sinh(x)} dx \\
&= -\left(\frac{1}{3} (2(-1)^{5/6}) \text{Subst} \left(\int \frac{1}{-(-1)^{5/6} - 2ix + (-1)^{5/6}x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right) \right) - \frac{1}{3} (2(-1)^{5/6}) \text{Subst} \left(\int \frac{1}{-(-1)^{5/6} + \sqrt[6]{-1} \sinh(x)} dx, x, \tanh\left(\frac{x}{2}\right) \right) \\
&= \frac{2}{3} \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, 1 + \tanh\left(\frac{x}{2}\right) \right) + \frac{1}{3}(-1)^{2/3} \text{Subst} \left(\int \frac{1}{-1 + \sqrt[6]{-1} + (-1)^{5/6}x} dx, x, \tanh\left(\frac{x}{2}\right) \right) \\
&= \frac{2(-1)^{5/6} \tan^{-1} \left(\frac{i(-1)^{5/6} \tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{2/3}}} \right)}{3\sqrt{1+(-1)^{2/3}}} + \frac{1}{3}\sqrt{2} \tanh^{-1} \left(\frac{1 + \tanh\left(\frac{x}{2}\right)}{\sqrt{2}} \right) - \frac{1}{3}(-1)^{5/6} \log \left(1 + (-1)^{5/6} + (-1)^{2/3} \right)
\end{aligned}$$

Mathematica [A] time = 1.21926, size = 156, normalized size = 1.17

$$\frac{2 \tanh^{-1} \left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}} \right) + \sqrt{-1+i\sqrt{3}} (1+i\sqrt{3}) \tan^{-1} \left(\frac{2+(-1-i\sqrt{3}) \tanh\left(\frac{x}{2}\right)}{\sqrt{-2-2i\sqrt{3}}} \right) + \sqrt{-1-i\sqrt{3}} (1-i\sqrt{3}) \tan^{-1} \left(\frac{2+i(\sqrt{3}+i) \tanh\left(\frac{x}{2}\right)}{\sqrt{-2+2i\sqrt{3}}} \right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sinh[x]^3)^(-1), x]

[Out] (Sqrt[-1 + I*Sqrt[3]]*(1 + I*Sqrt[3])*ArcTan[(2 + (-1 - I*Sqrt[3])*Tanh[x/2])/Sqrt[-2 - (2*I)*Sqrt[3]]] + Sqrt[-1 - I*Sqrt[3]]*(1 - I*Sqrt[3])*ArcTan[(2 + I*(I + Sqrt[3])*Tanh[x/2])/Sqrt[-2 + (2*I)*Sqrt[3]]] + 2*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]])/(3*Sqrt[2])

Maple [C] time = 0.029, size = 80, normalized size = 0.6

$$\frac{2}{3} \sum_{_R=\text{RootOf}(Z^4-2Z^3+2Z^2+2Z+1)} \frac{-_R^2 + _R + 1}{2_R^3 - 3_R^2 + 2_R + 1} \ln \left(\tanh\left(\frac{x}{2}\right) - _R \right) + \frac{\sqrt{2}}{3} \text{Arctanh} \left(\frac{\sqrt{2}}{4} (2 \tanh(x/2) + 2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-sinh(x)^3), x)

[Out] $2/3*\text{sum}((-R^2+_R+1)/(2*_R^3-3*_R^2+2*_R+1)*\ln(\tanh(1/2*x)-_R),_R=\text{RootOf}(_Z^4-2*_Z^3+2*_Z^2+2*_Z+1))+1/3*2^{(1/2)}*\text{arctanh}(1/4*(2*\tanh(1/2*x)+2)*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x+1}{\sqrt{2}+e^x-1}\right)+\int\frac{2(e^{3x}+4e^{2x}-e^x)}{3(e^{4x}+2e^{3x}+2e^{2x}-2e^x+1)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^3),x, algorithm="maxima")`

[Out] $-1/6*\text{sqrt}(2)*\log(-(\text{sqrt}(2)-e^x+1)/(\text{sqrt}(2)+e^x-1))+\text{integrate}(2/3*(e^{(3*x)}+4*e^{(2*x)}-e^x)/(e^{(4*x)}+2*e^{(3*x)}+2*e^{(2*x)}-2*e^x+1),x)$

Fricas [B] time = 1.85063, size = 590, normalized size = 4.44

$$-\frac{1}{6}\sqrt{3}\log(4(\sqrt{3}+1)e^x+4\sqrt{3}+4e^{2x}+8)+\frac{1}{6}\sqrt{3}\log(-4(\sqrt{3}-1)e^x-4\sqrt{3}+4e^{2x}+8)+\frac{1}{6}\sqrt{2}\log\left(\frac{2(\sqrt{2}-1)}{e^{2x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^3),x, algorithm="fricas")`

[Out] $-1/6*\text{sqrt}(3)*\log(4*(\text{sqrt}(3)+1)*e^x+4*\text{sqrt}(3)+4*e^{(2*x)}+8)+1/6*\text{sqrt}(3)*\log(-4*(\text{sqrt}(3)-1)*e^x-4*\text{sqrt}(3)+4*e^{(2*x)}+8)+1/6*\text{sqrt}(2)*\log((2*(\text{sqrt}(2)-1)*e^x-2*\text{sqrt}(2)+e^{(2*x)}+3)/(e^{(2*x)}-2*e^x-1))-2/3*\text{arctan}(-(\text{sqrt}(3)+1)*e^x+1/2*\text{sqrt}(-4*(\text{sqrt}(3)-1)*e^x-4*\text{sqrt}(3)+4*e^{(2*x)}+8)*(\text{sqrt}(3)+1)+1)+2/3*\text{arctan}(-(\text{sqrt}(3)-1)*e^x+\text{sqrt}((\text{sqrt}(3)+1)*e^x+\text{sqrt}(3)+e^{(2*x)}+2)*(\text{sqrt}(3)-1)-1)$

Sympy [B] time = 72.4185, size = 3742, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)**3),x)

[Out] 831721548108794611356*sqrt(6)*I*log(tanh(x/2) + 1 + sqrt(2))/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 3528695680360094803440*log(tanh(x/2) + 1 + sqrt(2))/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 2037293400944170367740*sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) + 1 + sqrt(2))/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 2881167958148549067306*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) + 1 + sqrt(2))/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 1176231893453364934480*sqrt(3)*I*log(tanh(x/2) + 1 + sqrt(2))/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 2495164644326383834068*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 2495164644326383834068*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 1176231893453364934480*sqrt(3)*I*log(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 2881167958148549067306*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 2037293400944170367740*sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 3528695680360094803

$*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) - 1440583979074274533653*\sqrt{1 - \sqrt{3}}*I)*\sqrt{1 + \sqrt{3}}*I)*\log(\tanh(x/2) - 1/2 + \sqrt{2}*\sqrt{1 - \sqrt{3}}*I)/2 + \sqrt{3}*I/2)/(-10586087041080284410320*\sqrt{2} + 14970987865958303004408 - 8643503874445647201918*\sqrt{2})*\sqrt{1 + \sqrt{3}}*I) - 3528695680360094803440*\sqrt{6}*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) + 1018646700472085183870*\sqrt{2}*\sqrt{1 - \sqrt{3}}*I)*\sqrt{1 + \sqrt{3}}*I)*\log(\tanh(x/2) - 1/2 + \sqrt{2}*\sqrt{1 - \sqrt{3}}*I)/2 + \sqrt{3}*I/2)/(-10586087041080284410320*\sqrt{2} + 14970987865958303004408 - 8643503874445647201918*\sqrt{2})*\sqrt{1 + \sqrt{3}}*I) - 3528695680360094803440*\sqrt{6}*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) - 2352463786906729868960*\sqrt{3}*I*\sqrt{1 - \sqrt{3}}*I)*\log(\tanh(x/2) - 1/2 + \sqrt{2}*\sqrt{1 - \sqrt{3}}*I)/2 + \sqrt{3}*I/2)/(-10586087041080284410320*\sqrt{2} + 14970987865958303004408 - 8643503874445647201918*\sqrt{2})*\sqrt{1 + \sqrt{3}}*I) - 3528695680360094803440*\sqrt{6}*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) - 1440583979074274533653*\sqrt{3}*I*\sqrt{1 - \sqrt{3}}*I)*\sqrt{1 + \sqrt{3}}*I)*\log(\tanh(x/2) - 1/2 + \sqrt{2}*\sqrt{1 - \sqrt{3}}*I)/2 + \sqrt{3}*I/2)/(-10586087041080284410320*\sqrt{2} + 14970987865958303004408 - 8643503874445647201918*\sqrt{2})*\sqrt{1 + \sqrt{3}}*I) - 3528695680360094803440*\sqrt{6}*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) + 831721548108794611356*\sqrt{6}*I*\sqrt{1 + \sqrt{3}}*I)*\log(\tanh(x/2) - 1/2 - \sqrt{3}*I/2 - \sqrt{2}*\sqrt{1 + \sqrt{3}}*I)/2)/(-10586087041080284410320*\sqrt{2} + 14970987865958303004408 - 8643503874445647201918*\sqrt{2})*\sqrt{1 + \sqrt{3}}*I) - 3528695680360094803440*\sqrt{6}*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) - 4074586801888340735480*\sqrt{2}*\log(\tanh(x/2) - 1/2 - \sqrt{3}*I/2 - \sqrt{2}*\sqrt{1 + \sqrt{3}}*I)/2)/(-10586087041080284410320*\sqrt{2} + 14970987865958303004408 - 8643503874445647201918*\sqrt{2})*\sqrt{1 + \sqrt{3}}*I) - 3528695680360094803440*\sqrt{6}*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) + 3528695680360094803440*\sqrt{1 + \sqrt{3}}*I)*\log(\tanh(x/2) - 1/2 - \sqrt{3}*I/2 - \sqrt{2}*\sqrt{1 + \sqrt{3}}*I)/2)/(-10586087041080284410320*\sqrt{2} + 14970987865958303004408 - 8643503874445647201918*\sqrt{2})*\sqrt{1 + \sqrt{3}}*I) - 3528695680360094803440*\sqrt{6}*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) - 2495164644326383834068*\sqrt{2}*\sqrt{1 + \sqrt{3}}*I)*\log(\tanh(x/2) - 1/2 - \sqrt{3}*I/2 - \sqrt{2}*\sqrt{1 + \sqrt{3}}*I)/2)/(-10586087041080284410320*\sqrt{2} + 14970987865958303004408 - 8643503874445647201918*\sqrt{2})*\sqrt{1 + \sqrt{3}}*I) - 3528695680360094803440*\sqrt{6}*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) - 1176231893453364934480*\sqrt{3}*I*\sqrt{1 + \sqrt{3}}*I)*\log(\tanh(x/2) - 1/2 - \sqrt{3}*I/2 - \sqrt{2}*\sqrt{1 + \sqrt{3}}*I)/2)/(-10586087041080284410320*\sqrt{2} + 14970987865958303004408 - 8643503874445647201918*\sqrt{2})*\sqrt{1 + \sqrt{3}}*I) - 3528695680360094803440*\sqrt{6}*I + 12223760405665022206440*\sqrt{1 + \sqrt{3}}*I) + 4990329288652767668136*\sqrt{3}*I) + 5762335916297098134612*\log(\tanh(x/2) - 1/2 - \sqrt{3}*I/2 - \sqrt{2}*\sqrt{1 + \sqrt{3}}*I)/2)/(-105860870410802$

```

84410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)
*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022
206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 5762335916
297098134612*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 + sqrt(2)*sqrt(1 + sqrt(3)*I
)/2)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 86435038
74445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)
*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*s
qrt(3)*I) + 1176231893453364934480*sqrt(3)*I*sqrt(1 + sqrt(3)*I)*log(tanh(x
/2) - 1/2 - sqrt(3)*I/2 + sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(-1058608704108028
4410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*
sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 122237604056650222
06440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 24951646443
26383834068*sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 +
sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(-10586087041080284410320*sqrt(2) + 1497098
7865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 352
8695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I)
+ 4990329288652767668136*sqrt(3)*I) - 3528695680360094803440*sqrt(1 + sqrt
(3)*I)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 + sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(-
10586087041080284410320*sqrt(2) + 14970987865958303004408 - 86435038744456
47201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 1
2223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)
*I) + 4074586801888340735480*sqrt(2)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 + sq
rt(2)*sqrt(1 + sqrt(3)*I)/2)/(-10586087041080284410320*sqrt(2) + 1497098786
5958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 352869
5680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) +
4990329288652767668136*sqrt(3)*I) - 831721548108794611356*sqrt(6)*I*sqrt(1
+ sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 + sqrt(2)*sqrt(1 + sqrt(3)*I
)/2)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 86435038
74445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)
*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*s
qrt(3)*I)

```

Giac [A] time = 1.18007, size = 146, normalized size = 1.1

$$-\frac{1}{6}(\sqrt{3}-i)\log(\sqrt{3}+(i+1)e^x+1)-\frac{1}{6}(\sqrt{3}+i)\log(i\sqrt{3}+(i+1)e^x+i)+\frac{1}{6}(\sqrt{3}-i)\log(-i\sqrt{3}+(i+1)e^x+i)+$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^3),x, algorithm="giac")

[Out] -1/6*(sqrt(3) - I)*log(sqrt(3) + (I + 1)*e^x + 1) - 1/6*(sqrt(3) + I)*log(I

```
*sqrt(3) + (I + 1)*e^x + I) + 1/6*(sqrt(3) - I)*log(-I*sqrt(3) + (I + 1)*e^
x + I) + 1/6*(sqrt(3) + I)*log(-sqrt(3) + (I + 1)*e^x + 1) - 1/6*sqrt(2)*lo
g(abs(-2*sqrt(2) + 2*e^x - 2)/abs(2*sqrt(2) + 2*e^x - 2))
```

3.184 $\int \sinh^4(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=111

$$\frac{(48a + 163b) \sinh(c + dx) \cosh^3(c + dx)}{192d} - \frac{(80a + 93b) \sinh(c + dx) \cosh(c + dx)}{128d} + \frac{1}{128} x(48a + 35b) + \frac{b \sinh(c + dx)}{8}$$

[Out] $((48*a + 35*b)*x)/128 - ((80*a + 93*b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(128*d)$
 $+ ((48*a + 163*b)*\text{Cosh}[c + d*x]^3*\text{Sinh}[c + d*x])/(192*d) - (25*b*\text{Cosh}[c +$
 $d*x]^5*\text{Sinh}[c + d*x])/(48*d) + (b*\text{Cosh}[c + d*x]^7*\text{Sinh}[c + d*x])/(8*d)$

Rubi [A] time = 0.142796, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3217, 1257, 1814, 1157, 385, 206}

$$\frac{(48a + 163b) \sinh(c + dx) \cosh^3(c + dx)}{192d} - \frac{(80a + 93b) \sinh(c + dx) \cosh(c + dx)}{128d} + \frac{1}{128} x(48a + 35b) + \frac{b \sinh(c + dx)}{8}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^4*(a + b*\text{Sinh}[c + d*x]^4), x]$

[Out] $((48*a + 35*b)*x)/128 - ((80*a + 93*b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(128*d)$
 $+ ((48*a + 163*b)*\text{Cosh}[c + d*x]^3*\text{Sinh}[c + d*x])/(192*d) - (25*b*\text{Cosh}[c +$
 $d*x]^5*\text{Sinh}[c + d*x])/(48*d) + (b*\text{Cosh}[c + d*x]^7*\text{Sinh}[c + d*x])/(8*d)$

Rule 3217

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \text{With}\{\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff^{(m + 1)}/f, \text{Subst}[\text{Int}[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \text{Tan}[e + f*x]/ff], x]\} /; \text{FreeQ}\{a, b, e, f\}, x\} \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[p]$

Rule 1257

$\text{Int}[(x_.)^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(q_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Simp}[((-d)^{(m/2 - 1)}*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^{(q + 1)}/(2*e^{(2*p + m/2)}*(q + 1)), x] + \text{Dist}[1/(2*e^{(2*p + m/2)}*(q + 1)), \text{Int}[(d + e*x^2)^{(q + 1)}*\text{ExpandToSum}[\text{Together}[(1*(2*e^{(2*p + m/2)}*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^{(m/2 - 1)}*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x], x] /; \text{FreeQ}\{a, b, c, d, e\}$

, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sinh^4(c + dx) (a + b \sinh^4(c + dx)) dx &= \frac{\text{Subst} \left(\int \frac{x^4(a-2ax^2+(a+b)x^4)}{(1-x^2)^5} dx, x, \tanh(c + dx) \right)}{d} \\
&= \frac{b \cosh^7(c + dx) \sinh(c + dx)}{8d} - \frac{\text{Subst} \left(\int \frac{b+8bx^2-8(a-b)x^4+8(a+b)x^6}{(1-x^2)^4} dx, x, \tanh(c + dx) \right)}{8d} \\
&= -\frac{25b \cosh^5(c + dx) \sinh(c + dx)}{48d} + \frac{b \cosh^7(c + dx) \sinh(c + dx)}{8d} + \frac{\text{Subst} \left(\int \frac{8a-8ax^2+8(a+b)x^4-8b}{(1-x^2)^3} dx, x, \tanh(c + dx) \right)}{8d} \\
&= \frac{(48a + 163b) \cosh^3(c + dx) \sinh(c + dx)}{192d} - \frac{25b \cosh^5(c + dx) \sinh(c + dx)}{48d} \\
&= -\frac{(80a + 93b) \cosh(c + dx) \sinh(c + dx)}{128d} + \frac{(48a + 163b) \cosh^3(c + dx) \sinh(c + dx)}{192d} \\
&= \frac{1}{128}(48a + 35b)x - \frac{(80a + 93b) \cosh(c + dx) \sinh(c + dx)}{128d} + \frac{(48a + 163b) \cosh^3(c + dx) \sinh(c + dx)}{192d}
\end{aligned}$$

Mathematica [A] time = 0.141628, size = 82, normalized size = 0.74

$$\frac{-96(8a + 7b) \sinh(2(c + dx)) + 24(4a + 7b) \sinh(4(c + dx)) + 1152ac + 1152adx - 32b \sinh(6(c + dx)) + 3b \sinh(8(c + dx))}{3072d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]

[Out] (1152*a*c + 840*b*c + 1152*a*d*x + 840*b*d*x - 96*(8*a + 7*b)*Sinh[2*(c + d*x)] + 24*(4*a + 7*b)*Sinh[4*(c + d*x)] - 32*b*Sinh[6*(c + d*x)] + 3*b*Sinh[8*(c + d*x)])/(3072*d)

Maple [A] time = 0.014, size = 98, normalized size = 0.9

$$\frac{1}{d} \left(b \left(\left(\frac{(\sinh(dx + c))^7}{8} - \frac{7(\sinh(dx + c))^5}{48} + \frac{35(\sinh(dx + c))^3}{192} - \frac{35 \sinh(dx + c)}{128} \right) \cosh(dx + c) + \frac{35 dx}{128} + \frac{35 c}{128} \right) + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x)`

[Out] $\frac{1}{d} * (b * ((\frac{1}{8} * \sinh(d*x+c)^7 - \frac{7}{48} * \sinh(d*x+c)^5 + \frac{35}{192} * \sinh(d*x+c)^3 - \frac{35}{128} * \sinh(d*x+c)) * \cosh(d*x+c) + \frac{35}{128} * d*x + \frac{35}{128} * c) + a * ((\frac{1}{4} * \sinh(d*x+c)^3 - \frac{3}{8} * \sinh(d*x+c)) * \cosh(d*x+c) + \frac{3}{8} * d*x + \frac{3}{8} * c))$

Maxima [A] time = 1.14058, size = 236, normalized size = 2.13

$$\frac{1}{64} a \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{6144} b \left(\frac{(32e^{(-2dx-2c)} - 168e^{(-4dx-4c)} + 672e^{(-6dx-6c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")`

[Out] $\frac{1}{64} * a * (24 * x + e^{(4 * d * x + 4 * c)} / d - 8 * e^{(2 * d * x + 2 * c)} / d + 8 * e^{(-2 * d * x - 2 * c)} / d - e^{(-4 * d * x - 4 * c)} / d) - \frac{1}{6144} * b * ((32 * e^{(-2 * d * x - 2 * c)} - 168 * e^{(-4 * d * x - 4 * c)} + 672 * e^{(-6 * d * x - 6 * c)} - 3) * e^{(8 * d * x + 8 * c)} / d - 1680 * (d * x + c) / d - (672 * e^{(-2 * d * x - 2 * c)} - 168 * e^{(-4 * d * x - 4 * c)} + 32 * e^{(-6 * d * x - 6 * c)} - 3 * e^{(-8 * d * x - 8 * c)}) / d)$

Fricas [A] time = 1.62985, size = 467, normalized size = 4.21

$$3b \cosh(dx+c) \sinh(dx+c)^7 + 3(7b \cosh(dx+c)^3 - 8b \cosh(dx+c)) \sinh(dx+c)^5 + (21b \cosh(dx+c)^5 - 80b \cosh(dx+c)) \sinh(dx+c)^3 + 12(4a + 7b) \cosh(dx+c) \sinh(dx+c)^3 + 3(48a + 35b) d x + 3(b \cosh(dx+c)^7 - 8b \cosh(dx+c)^5 + 4(4a + 7b) \cosh(dx+c)^3 - 8(8a + 7b) \cosh(dx+c)) \sinh(dx+c) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")`

[Out] $\frac{1}{384} * (3 * b * \cosh(d * x + c) * \sinh(d * x + c)^7 + 3 * (7 * b * \cosh(d * x + c)^3 - 8 * b * \cosh(d * x + c)) * \sinh(d * x + c)^5 + (21 * b * \cosh(d * x + c)^5 - 80 * b * \cosh(d * x + c)) * \sinh(d * x + c)^3 + 12 * (4 * a + 7 * b) * \cosh(d * x + c) * \sinh(d * x + c)^3 + 3 * (48 * a + 35 * b) * d * x + 3 * (b * \cosh(d * x + c)^7 - 8 * b * \cosh(d * x + c)^5 + 4 * (4 * a + 7 * b) * \cosh(d * x + c)^3 - 8 * (8 * a + 7 * b) * \cosh(d * x + c)) * \sinh(d * x + c)) / d$

Sympy [A] time = 14.1661, size = 306, normalized size = 2.76

$$\left\{ \begin{array}{l} \frac{3ax \sinh^4(c+dx)}{8} - \frac{3ax \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3ax \cosh^4(c+dx)}{8} + \frac{5a \sinh^3(c+dx) \cosh(c+dx)}{8d} - \frac{3a \sinh(c+dx) \cosh^3(c+dx)}{8d} + \frac{35bx \sinh^8(c+dx)}{128} \\ x(a + b \sinh^4(c)) \sinh^4(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**4), x)

[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 35*b*x*sinh(c + d*x)**8/128 - 35*b*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b*x*cosh(c + d*x)**8/128 + 93*b*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**4, True))

Giac [A] time = 1.20316, size = 258, normalized size = 2.32

$$48(dx+c)(48a+35b) + 3be^{(8dx+8c)} - 32be^{(6dx+6c)} + 96ae^{(4dx+4c)} + 168be^{(4dx+4c)} - 768ae^{(2dx+2c)} - 672be^{(2dx+2c)} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4), x, algorithm="giac")

[Out] 1/6144*(48*(d*x + c)*(48*a + 35*b) + 3*b*e^(8*d*x + 8*c) - 32*b*e^(6*d*x + 6*c) + 96*a*e^(4*d*x + 4*c) + 168*b*e^(4*d*x + 4*c) - 768*a*e^(2*d*x + 2*c) - 672*b*e^(2*d*x + 2*c) - (2400*a*e^(8*d*x + 8*c) + 1750*b*e^(8*d*x + 8*c) - 768*a*e^(6*d*x + 6*c) - 672*b*e^(6*d*x + 6*c) + 96*a*e^(4*d*x + 4*c) + 168*b*e^(4*d*x + 4*c) - 32*b*e^(2*d*x + 2*c) + 3*b)*e^(-8*d*x - 8*c))/d

3.185 $\int \sinh^3(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=67

$$\frac{(a + 3b) \cosh^3(c + dx)}{3d} - \frac{(a + b) \cosh(c + dx)}{d} + \frac{b \cosh^7(c + dx)}{7d} - \frac{3b \cosh^5(c + dx)}{5d}$$

[Out] -(((a + b)*Cosh[c + d*x])/d) + ((a + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b*Cosh[c + d*x]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d)

Rubi [A] time = 0.0661279, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3215, 1153}

$$\frac{(a + 3b) \cosh^3(c + dx)}{3d} - \frac{(a + b) \cosh(c + dx)}{d} + \frac{b \cosh^7(c + dx)}{7d} - \frac{3b \cosh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]

[Out] -(((a + b)*Cosh[c + d*x])/d) + ((a + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b*Cosh[c + d*x]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d)

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol]
:> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x]
/; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1153

```
Int[((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \sinh^3(c + dx) (a + b \sinh^4(c + dx)) dx = -\frac{\text{Subst}\left(\int (1 - x^2)(a + b - 2bx^2 + bx^4) dx, x, \cosh(c + dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(a\left(1 + \frac{b}{a}\right) - (a + 3b)x^2 + 3bx^4 - bx^6\right) dx, x, \cosh(c + dx)\right)}{d}$$

$$= -\frac{(a + b) \cosh(c + dx)}{d} + \frac{(a + 3b) \cosh^3(c + dx)}{3d} - \frac{3b \cosh^5(c + dx)}{5d} + \frac{b \cosh^7(c + dx)}{7d}$$

Mathematica [A] time = 0.0288673, size = 93, normalized size = 1.39

$$-\frac{3a \cosh(c + dx)}{4d} + \frac{a \cosh(3(c + dx))}{12d} - \frac{35b \cosh(c + dx)}{64d} + \frac{7b \cosh(3(c + dx))}{64d} - \frac{7b \cosh(5(c + dx))}{320d} + \frac{b \cosh(7(c + dx))}{448d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4), x]

[Out] (-3*a*Cosh[c + d*x])/(4*d) - (35*b*Cosh[c + d*x])/(64*d) + (a*Cosh[3*(c + d*x)])/(12*d) + (7*b*Cosh[3*(c + d*x)])/(64*d) - (7*b*Cosh[5*(c + d*x)])/(320*d) + (b*Cosh[7*(c + d*x)])/(448*d)

Maple [A] time = 0.014, size = 66, normalized size = 1.

$$\frac{1}{d} \left(b \left(-\frac{16}{35} + \frac{(\sinh(dx + c))^6}{7} - \frac{6(\sinh(dx + c))^4}{35} + \frac{8(\sinh(dx + c))^2}{35} \right) \cosh(dx + c) + a \left(-\frac{2}{3} + \frac{(\sinh(dx + c))^2}{3} \right) \cosh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4), x)

[Out] 1/d*(b*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))

Maxima [B] time = 1.12945, size = 212, normalized size = 3.16

$$-\frac{1}{4480} b \left(\frac{(49 e^{(-2 dx - 2 c)} - 245 e^{(-4 dx - 4 c)} + 1225 e^{(-6 dx - 6 c)} - 5) e^{(7 dx + 7 c)}}{d} + \frac{1225 e^{(-dx - c)} - 245 e^{(-3 dx - 3 c)} + 49 e^{(-5 dx - 5 c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] $-1/4480*b*((49*e^{(-2*d*x - 2*c)} - 245*e^{(-4*d*x - 4*c)} + 1225*e^{(-6*d*x - 6*c)} - 5)*e^{(7*d*x + 7*c)}/d + (1225*e^{(-d*x - c)} - 245*e^{(-3*d*x - 3*c)} + 49*e^{(-5*d*x - 5*c)} - 5*e^{(-7*d*x - 7*c)})/d) + 1/24*a*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d)$

Fricas [B] time = 1.6346, size = 439, normalized size = 6.55

$15 b \cosh(dx + c)^7 + 105 b \cosh(dx + c) \sinh(dx + c)^6 - 147 b \cosh(dx + c)^5 + 105 (5 b \cosh(dx + c)^3 - 7 b \cosh(dx + c) \sinh(dx + c))^2 + 35 (16 a + 21 b) \cosh(dx + c) \sinh(dx + c)^4 - 105 (3 b \cosh(dx + c)^5 - 14 b \cosh(dx + c)^3 + (16 a + 21 b) \cosh(dx + c)) \sinh(dx + c)^2 - 105 (48 a + 35 b) \cosh(dx + c) \sinh(dx + c) + 35 a \sinh(dx + c)^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out] $1/6720*(15*b*cosh(d*x + c)^7 + 105*b*cosh(d*x + c)*sinh(d*x + c)^6 - 147*b*cosh(d*x + c)^5 + 105*(5*b*cosh(d*x + c)^3 - 7*b*cosh(d*x + c))*sinh(d*x + c)^4 + 35*(16*a + 21*b)*cosh(d*x + c)^3 + 105*(3*b*cosh(d*x + c)^5 - 14*b*cosh(d*x + c)^3 + (16*a + 21*b)*cosh(d*x + c))*sinh(d*x + c)^2 - 105*(48*a + 35*b)*cosh(d*x + c)*sinh(d*x + c) + 35*a*sinh(d*x + c)^3)/d$

Sympy [A] time = 8.48735, size = 128, normalized size = 1.91

$$\left\{ \frac{a \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2a \cosh^3(c+dx)}{3d} + \frac{b \sinh^6(c+dx) \cosh(c+dx)}{d} - \frac{2b \sinh^4(c+dx) \cosh^3(c+dx)}{d} + \frac{8b \sinh^2(c+dx) \cosh^5(c+dx)}{5d} - \frac{16b \cosh^7(c+dx)}{35d} \right\} / (x (a + b \sinh^4(c)) \sinh^3(c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4),x)

[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d) + b*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**3, True))

Giac [B] time = 1.16218, size = 198, normalized size = 2.96

$$\frac{15 b e^{(7 d x+7 c)} - 147 b e^{(5 d x+5 c)} + 560 a e^{(3 d x+3 c)} + 735 b e^{(3 d x+3 c)} - 5040 a e^{(d x+c)} - 3675 b e^{(d x+c)} - (5040 a e^{(6 d x+6 c)} + 3675 b e^{(6 d x+6 c)} - 560 a e^{(4 d x+4 c)} - 735 b e^{(4 d x+4 c)} + 147 b e^{(2 d x+2 c)} - 15 b) e^{(-7 d x-7 c)}}{13440 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="giac")

[Out] 1/13440*(15*b*e^(7*d*x + 7*c) - 147*b*e^(5*d*x + 5*c) + 560*a*e^(3*d*x + 3*c) + 735*b*e^(3*d*x + 3*c) - 5040*a*e^(d*x + c) - 3675*b*e^(d*x + c) - (5040*a*e^(6*d*x + 6*c) + 3675*b*e^(6*d*x + 6*c) - 560*a*e^(4*d*x + 4*c) - 735*b*e^(4*d*x + 4*c) + 147*b*e^(2*d*x + 2*c) - 15*b)*e^(-7*d*x - 7*c))/d

3.186 $\int \sinh^2(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=83

$$\frac{(8a + 11b) \sinh(c + dx) \cosh(c + dx)}{16d} - \frac{1}{16}x(8a + 5b) + \frac{b \sinh(c + dx) \cosh^5(c + dx)}{6d} - \frac{13b \sinh(c + dx) \cosh^3(c + dx)}{24d}$$

[Out] $-\frac{(8a + 5b)x}{16} + \frac{(8a + 11b)\cosh[c + dx]\sinh[c + dx]}{16d} - \frac{13b\cosh[c + dx]^3\sinh[c + dx]}{24d} + \frac{b\cosh[c + dx]^5\sinh[c + dx]}{6d}$

Rubi [A] time = 0.0976626, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3217, 1257, 1157, 385, 206}

$$\frac{(8a + 11b) \sinh(c + dx) \cosh(c + dx)}{16d} - \frac{1}{16}x(8a + 5b) + \frac{b \sinh(c + dx) \cosh^5(c + dx)}{6d} - \frac{13b \sinh(c + dx) \cosh^3(c + dx)}{24d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]

[Out] $-\frac{(8a + 5b)x}{16} + \frac{(8a + 11b)\cosh[c + dx]\sinh[c + dx]}{16d} - \frac{13b\cosh[c + dx]^3\sinh[c + dx]}{24d} + \frac{b\cosh[c + dx]^5\sinh[c + dx]}{6d}$

Rule 3217

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1257

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}

, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 1157

```
Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sinh^2(c + dx) (a + b \sinh^4(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{x^2(a-2ax^2+(a+b)x^4)}{(1-x^2)^4} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b \cosh^5(c + dx) \sinh(c + dx)}{6d} + \frac{\text{Subst}\left(\int \frac{-b+6(a-b)x^2-6(a+b)x^4}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{6d} \\
&= -\frac{13b \cosh^3(c + dx) \sinh(c + dx)}{24d} + \frac{b \cosh^5(c + dx) \sinh(c + dx)}{6d} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{6d} \\
&= \frac{(8a + 11b) \cosh(c + dx) \sinh(c + dx)}{16d} - \frac{13b \cosh^3(c + dx) \sinh(c + dx)}{24d} + \frac{b \cosh^5(c + dx) \sinh(c + dx)}{6d} \\
&= -\frac{1}{16}(8a + 5b)x + \frac{(8a + 11b) \cosh(c + dx) \sinh(c + dx)}{16d} - \frac{13b \cosh^3(c + dx) \sinh(c + dx)}{24d} + \frac{b \cosh^5(c + dx) \sinh(c + dx)}{6d}
\end{aligned}$$

Mathematica [A] time = 0.0771824, size = 63, normalized size = 0.76

$$\frac{(48a + 45b) \sinh(2(c + dx)) - 96ac - 96adx - 9b \sinh(4(c + dx)) + b \sinh(6(c + dx)) - 60bc - 60bdx}{192d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4), x]

[Out] (-96*a*c - 60*b*c - 96*a*d*x - 60*b*d*x + (48*a + 45*b)*Sinh[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)

Maple [A] time = 0.014, size = 76, normalized size = 0.9

$$\frac{1}{d} \left(b \left(\left(\frac{(\sinh(dx + c))^5}{6} - \frac{5(\sinh(dx + c))^3}{24} + \frac{5 \sinh(dx + c)}{16} \right) \cosh(dx + c) - \frac{5dx}{16} - \frac{5c}{16} \right) + a \left(\frac{\cosh(dx + c) \sinh(dx + c)}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4), x)

[Out] 1/d*(b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))

Maxima [A] time = 1.11428, size = 165, normalized size = 1.99

$$-\frac{1}{8}a\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{384}b\left(\frac{(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e^{(-6dx-6c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] $-\frac{1}{8}a\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{384}b\left(\frac{(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} + e^{(-6dx-6c)}}{d}\right)$

Fricas [A] time = 1.60365, size = 292, normalized size = 3.52

$$\frac{3b \cosh(dx+c) \sinh(dx+c)^5 + 2(5b \cosh(dx+c)^3 - 9b \cosh(dx+c)) \sinh(dx+c)^3 - 6(8a+5b)dx + 3(b \cosh(dx+c)^5 - 6(8a+5b)dx + 3(b \cosh(dx+c)^3 - 9b \cosh(dx+c)) \sinh(dx+c)^3)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out] $\frac{1}{96}(3b \cosh(dx+c) \sinh(dx+c)^5 + 2(5b \cosh(dx+c)^3 - 9b \cosh(dx+c)) \sinh(dx+c)^3 - 6(8a+5b)dx + 3(b \cosh(dx+c)^5 - 6(8a+5b)dx + 3(b \cosh(dx+c)^3 - 9b \cosh(dx+c)) \sinh(dx+c)^3)$

Sympy [A] time = 4.93975, size = 206, normalized size = 2.48

$$\frac{\left\{ \begin{array}{l} \frac{ax \sinh^2(c+dx)}{2} - \frac{ax \cosh^2(c+dx)}{2} + \frac{a \sinh(c+dx) \cosh(c+dx)}{2d} + \frac{5bx \sinh^6(c+dx)}{16} - \frac{15bx \sinh^4(c+dx) \cosh^2(c+dx)}{16} + \frac{15bx \sinh^2(c+dx) \cosh^4(c+dx)}{16} \\ x(a + b \sinh^4(c)) \sinh^2(c) \end{array} \right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)

[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)**4

```
*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*x*
cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*sinh
(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)**5/(1
6*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**2, True))
```

Giac [A] time = 1.16648, size = 193, normalized size = 2.33

$$\frac{24(dx + c)(8a + 5b) - be^{(6dx+6c)} + 9be^{(4dx+4c)} - 48ae^{(2dx+2c)} - 45be^{(2dx+2c)} - (176ae^{(6dx+6c)} + 110be^{(6dx+6c)} - 48ae^{(6dx+6c)})}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/384*(24*(d*x + c)*(8*a + 5*b) - b*e^(6*d*x + 6*c) + 9*b*e^(4*d*x + 4*c)
- 48*a*e^(2*d*x + 2*c) - 45*b*e^(2*d*x + 2*c) - (176*a*e^(6*d*x + 6*c) + 11
0*b*e^(6*d*x + 6*c) - 48*a*e^(4*d*x + 4*c) - 45*b*e^(4*d*x + 4*c) + 9*b*e^(
2*d*x + 2*c) - b)*e^(-6*d*x - 6*c))/d
```

$$3.187 \quad \int \sinh(c + dx) \left(a + b \sinh^4(c + dx) \right) dx$$

Optimal. Leaf size=46

$$\frac{(a + b) \cosh(c + dx)}{d} + \frac{b \cosh^5(c + dx)}{5d} - \frac{2b \cosh^3(c + dx)}{3d}$$

[Out] ((a + b)*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c + d*x]^5)/(5*d)

Rubi [A] time = 0.0342356, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3215}

$$\frac{(a + b) \cosh(c + dx)}{d} + \frac{b \cosh^5(c + dx)}{5d} - \frac{2b \cosh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]

[Out] ((a + b)*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c + d*x]^5)/(5*d)

Rule 3215

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \sinh(c + dx) \left(a + b \sinh^4(c + dx) \right) dx &= \frac{\text{Subst} \left(\int \left(a + b - 2bx^2 + bx^4 \right) dx, x, \cosh(c + dx) \right)}{d} \\ &= \frac{(a + b) \cosh(c + dx)}{d} - \frac{2b \cosh^3(c + dx)}{3d} + \frac{b \cosh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.0223954, size = 69, normalized size = 1.5

$$\frac{a \sinh(c) \sinh(dx)}{d} + \frac{a \cosh(c) \cosh(dx)}{d} + \frac{5b \cosh(c + dx)}{8d} - \frac{5b \cosh(3(c + dx))}{48d} + \frac{b \cosh(5(c + dx))}{80d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]

[Out] (a*Cosh[c]*Cosh[d*x])/d + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x)])/ (48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[c]*Sinh[d*x])/d

Maple [A] time = 0.013, size = 44, normalized size = 1.

$$\frac{1}{d} \left(b \left(\frac{8}{15} + \frac{(\sinh(dx+c))^4}{5} - \frac{4(\sinh(dx+c))^2}{15} \right) \cosh(dx+c) + a \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x)

[Out] 1/d*(b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a*cosh(d*x+c))

Maxima [B] time = 1.05633, size = 131, normalized size = 2.85

$$\frac{1}{480} b \left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d} \right) + \frac{a \cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] 1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*cosh(d*x + c)/d

Fricas [B] time = 1.66974, size = 250, normalized size = 5.43

$$\frac{3b \cosh(dx+c)^5 + 15b \cosh(dx+c) \sinh(dx+c)^4 - 25b \cosh(dx+c)^3 + 15(2b \cosh(dx+c)^3 - 5b \cosh(dx+c)) \sinh(dx+c)^2 + 30(8a+5b) \cosh(dx+c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cosh(d*x + c)^3 + 15*(2*b*cosh(d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(8*a + 5*b)*cosh(d*x + c))/d

Sympy [A] time = 2.53825, size = 80, normalized size = 1.74

$$\begin{cases} \frac{a \cosh(c+dx)}{d} + \frac{b \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{4b \sinh^2(c+dx) \cosh^3(c+dx)}{3d} + \frac{8b \cosh^5(c+dx)}{15d} & \text{for } d \neq 0 \\ x(a + b \sinh^4(c)) \sinh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4),x)

[Out] Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c), True))

Giac [B] time = 1.18581, size = 132, normalized size = 2.87

$$\frac{3be^{(5dx+5c)} - 25be^{(3dx+3c)} + 240ae^{(dx+c)} + 150be^{(dx+c)} + (240ae^{(4dx+4c)} + 150be^{(4dx+4c)} - 25be^{(2dx+2c)} + 3b)e^{(-5dx-5c)}}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")

[Out] 1/480*(3*b*e^(5*d*x + 5*c) - 25*b*e^(3*d*x + 3*c) + 240*a*e^(d*x + c) + 150*b*e^(d*x + c) + (240*a*e^(4*d*x + 4*c) + 150*b*e^(4*d*x + 4*c) - 25*b*e^(2*d*x + 2*c) + 3*b)*e^(-5*d*x - 5*c))/d

3.188 $\int (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=52

$$ax + \frac{b \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{3b \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3bx}{8}$$

[Out] a*x + (3*b*x)/8 - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)

Rubi [A] time = 0.0339566, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2635, 8}

$$ax + \frac{b \sinh^3(c + dx) \cosh(c + dx)}{4d} - \frac{3b \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{3bx}{8}$$

Antiderivative was successfully verified.

[In] Int[a + b*Sinh[c + d*x]^4, x]

[Out] a*x + (3*b*x)/8 - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]
)*(b*Sinh[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sinh[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^4(c + dx)) dx &= ax + b \int \sinh^4(c + dx) dx \\
&= ax + \frac{b \cosh(c + dx) \sinh^3(c + dx)}{4d} - \frac{1}{4}(3b) \int \sinh^2(c + dx) dx \\
&= ax - \frac{3b \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b \cosh(c + dx) \sinh^3(c + dx)}{4d} + \frac{1}{8}(3b) \int 1 dx \\
&= ax + \frac{3bx}{8} - \frac{3b \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b \cosh(c + dx) \sinh^3(c + dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.0539226, size = 49, normalized size = 0.94

$$ax + \frac{3b(c + dx)}{8d} - \frac{b \sinh(2(c + dx))}{4d} + \frac{b \sinh(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[a + b*Sinh[c + d*x]^4,x]

[Out] a*x + (3*b*(c + d*x))/(8*d) - (b*Sinh[2*(c + d*x)])/(4*d) + (b*Sinh[4*(c + d*x)])/(32*d)

Maple [A] time = 0.005, size = 44, normalized size = 0.9

$$ax + \frac{b}{d} \left(\left(\frac{(\sinh(dx + c))^3}{4} - \frac{3 \sinh(dx + c)}{8} \right) \cosh(dx + c) + \frac{3dx}{8} + \frac{3c}{8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a+b*sinh(d*x+c)^4,x)

[Out] a*x+b/d*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)

Maxima [A] time = 1.16404, size = 89, normalized size = 1.71

$$\frac{1}{64} b \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="maxima")

[Out] $\frac{1}{64}b(24x + e^{(4dx+4c)})/d - 8e^{(2dx+2c)}/d + 8e^{(-2dx-2c)}/d - e^{(-4dx-4c)}/d + ax$

Fricas [A] time = 1.61935, size = 155, normalized size = 2.98

$$\frac{b \cosh(dx+c) \sinh(dx+c)^3 + (8a+3b)dx + (b \cosh(dx+c)^3 - 4b \cosh(dx+c)) \sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="fricas")

[Out] $\frac{1}{8}(b \cosh(dx+c) \sinh(dx+c)^3 + (8a+3b)dx + (b \cosh(dx+c)^3 - 4b \cosh(dx+c)) \sinh(dx+c))/d$

Sympy [A] time = 1.24768, size = 100, normalized size = 1.92

$$ax + b \begin{cases} \frac{3x \sinh^4(c+dx)}{8} - \frac{3x \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3x \cosh^4(c+dx)}{8} + \frac{5 \sinh^3(c+dx) \cosh(c+dx)}{8d} - \frac{3 \sinh(c+dx) \cosh^3(c+dx)}{8d} & \text{for } d \neq 0 \\ x \sinh^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a+b*sinh(d*x+c)**4,x)

[Out] $ax + b \text{Piecewise}((3x \sinh(c+dx)**4/8 - 3x \sinh(c+dx)**2 \cosh(c+dx)**2/4 + 3x \cosh(c+dx)**4/8 + 5 \sinh(c+dx)**3 \cosh(c+dx)/(8d) - 3 \sinh(c+dx) \cosh(c+dx)**3/(8d), \text{Ne}(d, 0)), (x \sinh(c)**4, \text{True}))$

Giac [A] time = 1.12907, size = 99, normalized size = 1.9

$$ax + \frac{(24dx - (18e^{(4dx+4c)} - 8e^{(2dx+2c)} + 1))e^{(-4dx-4c)} + 24c + e^{(4dx+4c)} - 8e^{(2dx+2c)}}{64d}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] a*x + 1/64*(24*d*x - (18*e^(4*d*x + 4*c) - 8*e^(2*d*x + 2*c) + 1)*e^(-4*d*x - 4*c) + 24*c + e^(4*d*x + 4*c) - 8*e^(2*d*x + 2*c))*b/d
```

3.189 $\int \operatorname{csch}(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=42

$$-\frac{a \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b \cosh^3(c + dx)}{3d} - \frac{b \cosh(c + dx)}{d}$$

[Out] $-\left(\frac{a \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) - \frac{b \operatorname{Cosh}[c + d*x]}{d} + \frac{b \operatorname{Cosh}[c + d*x]^3}{3*d}$

Rubi [A] time = 0.0433106, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3215, 1153, 206}

$$-\frac{a \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b \cosh^3(c + dx)}{3d} - \frac{b \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Sinh}[c + d*x]^4), x]$

[Out] $-\left(\frac{a \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) - \frac{b \operatorname{Cosh}[c + d*x]}{d} + \frac{b \operatorname{Cosh}[c + d*x]^3}{3*d}$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 1153

$\operatorname{Int}[\left(\frac{d}{e} + (e_.)*(x_.)^2\right)^{(q_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e\}, x\} \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \&\& \operatorname{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{IGtQ}[q, -2]$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c+dx) (a+b \sinh^4(c+dx)) dx &= -\frac{\operatorname{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{1-x^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(b-bx^2+\frac{a}{1-x^2}\right) dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{b \cosh(c+dx)}{d} + \frac{b \cosh^3(c+dx)}{3d} - \frac{a \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{a \tanh^{-1}(\cosh(c+dx))}{d} - \frac{b \cosh(c+dx)}{d} + \frac{b \cosh^3(c+dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0240189, size = 70, normalized size = 1.67

$$\frac{a \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{3b \cosh(c+dx)}{4d} + \frac{b \cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4), x]

[Out] (-3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (d*x)/2]])/d

Maple [A] time = 0.029, size = 36, normalized size = 0.9

$$\frac{1}{d} \left(-2a \operatorname{Artanh}(e^{dx+c}) + b \left(-\frac{2}{3} + \frac{(\sinh(dx+c))^2}{3} \right) \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4), x)

[Out] $1/d*(-2*a*\operatorname{arctanh}(\exp(d*x+c))+b*(-2/3+1/3*\sinh(d*x+c)^2)*\cosh(d*x+c))$

Maxima [A] time = 1.16071, size = 96, normalized size = 2.29

$$\frac{1}{24} b \left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d} \right) + \frac{a \log \left(\tanh \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")`

[Out] $1/24*b*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d) + a*\log(\tanh(1/2*d*x + 1/2*c))/d$

Fricas [B] time = 1.68997, size = 1100, normalized size = 26.19

$b \cosh(dx + c)^6 + 6b \cosh(dx + c) \sinh(dx + c)^5 + b \sinh(dx + c)^6 - 9b \cosh(dx + c)^4 + 3(5b \cosh(dx + c)^2 - 3b) \sinh(dx + c)^4$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")`

[Out] $1/24*(b*\cosh(d*x + c)^6 + 6*b*\cosh(d*x + c)*\sinh(d*x + c)^5 + b*\sinh(d*x + c)^6 - 9*b*\cosh(d*x + c)^4 + 3*(5*b*\cosh(d*x + c)^2 - 3*b)*\sinh(d*x + c)^4 + 4*(5*b*\cosh(d*x + c)^3 - 9*b*\cosh(d*x + c))*\sinh(d*x + c)^3 - 9*b*\cosh(d*x + c)^2 + 3*(5*b*\cosh(d*x + c)^4 - 18*b*\cosh(d*x + c)^2 - 3*b)*\sinh(d*x + c)^2 - 24*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 24*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 6*(b*\cosh(d*x + c)^5 - 6*b*\cosh(d*x + c)^3 - 3*b*\cosh(d*x + c))*\sinh(d*x + c) + b)/(d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + d*\sinh(d*x + c)^3)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4),x)`

[Out] Timed out

Giac [B] time = 1.18077, size = 126, normalized size = 3.

$$-\frac{(9be^{2dx+2c}-b)e^{-3dx-3c}}{24d} - \frac{a \log(e^{(dx+c)}+1)}{d} + \frac{a \log(|e^{(dx+c)}-1|)}{d} + \frac{bd^2e^{(3dx+3c)}-9bd^2e^{(dx+c)}}{24d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")`

[Out] $-1/24*(9*b*e^{(2*d*x+2*c)}-b)*e^{(-3*d*x-3*c)}/d - a*\log(e^{(d*x+c)}+1)/d + a*\log(\text{abs}(e^{(d*x+c)}-1))/d + 1/24*(b*d^2*e^{(3*d*x+3*c)}-9*b*d^2*e^{(d*x+c)})/d^3$

3.190 $\int \operatorname{csch}^2(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=39

$$-\frac{a \operatorname{coth}(c + dx)}{d} + \frac{b \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{bx}{2}$$

[Out] $-(b*x)/2 - (a*\operatorname{Coth}[c + d*x])/d + (b*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(2*d)$

Rubi [A] time = 0.0522995, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3217, 1259, 453, 206}

$$-\frac{a \operatorname{coth}(c + dx)}{d} + \frac{b \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{bx}{2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^2*(a + b*\operatorname{Sinh}[c + d*x]^4), x]$

[Out] $-(b*x)/2 - (a*\operatorname{Coth}[c + d*x])/d + (b*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(2*d)$

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol]
:> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
:> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^2(c + dx) (a + b \sinh^4(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a - 2ax^2 + (a+b)x^4}{x^2(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-2a + (2a+b)x^2}{x^2(1-x^2)} dx, x, \tanh(c + dx)\right)}{2d} \\ &= -\frac{a \coth(c + dx)}{d} + \frac{b \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{b \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{2d} \\ &= -\frac{bx}{2} - \frac{a \coth(c + dx)}{d} + \frac{b \cosh(c + dx) \sinh(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.119255, size = 45, normalized size = 1.15

$$-\frac{a \coth(c + dx)}{d} + \frac{b(-c - dx)}{2d} + \frac{b \sinh(2(c + dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4), x]
```

```
[Out] (b*(-c - d*x))/(2*d) - (a*Coth[c + d*x])/d + (b*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.03, size = 39, normalized size = 1.

$$\frac{1}{d} \left(-\coth(dx+c)a + b \left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x)

[Out] 1/d*(-coth(d*x+c)*a+b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))

Maxima [A] time = 1.1604, size = 73, normalized size = 1.87

$$-\frac{1}{8}b \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) + \frac{2a}{d(e^{(-2dx-2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))

Fricas [A] time = 1.6723, size = 185, normalized size = 4.74

$$\frac{b \cosh(dx+c)^3 + 3b \cosh(dx+c)\sinh(dx+c)^2 - (8a+b)\cosh(dx+c) - 4(bdx-2a)\sinh(dx+c)}{8d \sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out] 1/8*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a + b)*cosh(d*x + c) - 4*(b*d*x - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)`

[Out] Timed out

Giac [B] time = 1.15407, size = 124, normalized size = 3.18

$$-\frac{(dx+c)b}{2d} + \frac{be^{(2dx+2c)}}{8d} + \frac{be^{(4dx+4c)} - 16ae^{(2dx+2c)} - 2be^{(2dx+2c)} + b}{8d(e^{(4dx+4c)} - e^{(2dx+2c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")`

[Out] `-1/2*(d*x + c)*b/d + 1/8*b*e^(2*d*x + 2*c)/d + 1/8*(b*e^(4*d*x + 4*c) - 16*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)/(d*(e^(4*d*x + 4*c) - e^(2*d*x + 2*c)))`

3.191 $\int \operatorname{csch}^3(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=47

$$\frac{a \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b \cosh(c + dx)}{d}$$

[Out] (a*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*Cosh[c + d*x])/d - (a*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rubi [A] time = 0.0539296, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3215, 1157, 388, 206}

$$\frac{a \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b \cosh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]

[Out] (a*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*Cosh[c + d*x])/d - (a*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol]
:> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1157

```
Int[((d_.) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol]
:> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c + dx) (a + b \sinh^4(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-a-2b+2bx^2}{1-x^2} dx, x, \cosh(c + dx)\right)}{2d} \\ &= \frac{b \cosh(c + dx)}{d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{a \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \cosh(c + dx)\right)}{2d} \\ &= \frac{a \tanh^{-1}(\cosh(c + dx))}{2d} + \frac{b \cosh(c + dx)}{d} - \frac{a \coth(c + dx) \operatorname{csch}(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.0260933, size = 82, normalized size = 1.74

$$-\frac{\operatorname{acsch}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{\operatorname{asech}^2\left(\frac{1}{2}(c + dx)\right)}{8d} - \frac{a \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{2d} + \frac{b \sinh(c) \sinh(dx)}{d} + \frac{b \cosh(c) \cosh(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4), x]

[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sinh[c]*Sinh[d*x])/d

Maple [A] time = 0.04, size = 38, normalized size = 0.8

$$\frac{1}{d} \left(a \left(-\frac{\operatorname{csch}(dx+c) \operatorname{coth}(dx+c)}{2} + \operatorname{Artanh}(e^{dx+c}) \right) + b \cosh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x)

[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*cosh(d*x+c))

Maxima [B] time = 1.23746, size = 155, normalized size = 3.3

$$\frac{1}{2} b \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) + \frac{1}{2} a \left(\frac{\log(e^{(-dx-c)} + 1)}{d} - \frac{\log(e^{(-dx-c)} - 1)}{d} + \frac{2(e^{(-dx-c)} + e^{(-3dx-3c)})}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))

Fricas [B] time = 1.7422, size = 1867, normalized size = 39.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out] 1/2*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - (2*a + b)*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c)^3 - (2*a + b)*cosh(d*x + c)^2 + (15*b*cosh(d*x + c)^4 - 6*(2*a + b)*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + (a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)

$$\begin{aligned}
& x + c)^2 - a) \sinh(dx + c)^3 + 2*(5*a*\cosh(dx + c)^3 - 3*a*\cosh(dx + c)) \\
& * \sinh(dx + c)^2 + a*\cosh(dx + c) + (5*a*\cosh(dx + c)^4 - 6*a*\cosh(dx + \\
& c)^2 + a) \sinh(dx + c)) * \log(\cosh(dx + c) + \sinh(dx + c) + 1) - (a*\cosh(d \\
& *x + c)^5 + 5*a*\cosh(dx + c)*\sinh(dx + c)^4 + a*\sinh(dx + c)^5 - 2*a*\cos \\
& h(dx + c)^3 + 2*(5*a*\cosh(dx + c)^2 - a) \sinh(dx + c)^3 + 2*(5*a*\cosh(dx \\
& x + c)^3 - 3*a*\cosh(dx + c)) \sinh(dx + c)^2 + a*\cosh(dx + c) + (5*a*\cosh \\
& (dx + c)^4 - 6*a*\cosh(dx + c)^2 + a) \sinh(dx + c)) * \log(\cosh(dx + c) + s \\
& inh(dx + c) - 1) + 2*(3*b*\cosh(dx + c)^5 - 2*(2*a + b)*\cosh(dx + c)^3 - \\
& (2*a + b)*\cosh(dx + c)) \sinh(dx + c) + b)/(d*\cosh(dx + c)^5 + 5*d*\cosh(d \\
& *x + c)*\sinh(dx + c)^4 + d*\sinh(dx + c)^5 - 2*d*\cosh(dx + c)^3 + 2*(5*d* \\
& cosh(dx + c)^2 - d) \sinh(dx + c)^3 + 2*(5*d*\cosh(dx + c)^3 - 3*d*\cosh(dx \\
& x + c)) \sinh(dx + c)^2 + d*\cosh(dx + c) + (5*d*\cosh(dx + c)^4 - 6*d*\cosh \\
& (dx + c)^2 + d) \sinh(dx + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)**3*(a+b*sinh(dx+c)**4), x)

[Out] Timed out

Giac [B] time = 1.19865, size = 155, normalized size = 3.3

$$\frac{b(e^{dx+c} + e^{-dx-c})}{2d} + \frac{a \log(e^{dx+c} + e^{-dx-c} + 2)}{4d} - \frac{a \log(e^{dx+c} + e^{-dx-c} - 2)}{4d} - \frac{a(e^{dx+c} + e^{-dx-c})}{\left((e^{dx+c} + e^{-dx-c})^2 - 4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^3*(a+b*sinh(dx+c)^4), x, algorithm="giac")

[Out] 1/2*b*(e^(dx + c) + e^(-dx - c))/d + 1/4*a*log(e^(dx + c) + e^(-dx - c) + 2)/d - 1/4*a*log(e^(dx + c) + e^(-dx - c) - 2)/d - a*(e^(dx + c) + e^(-dx - c))/(((e^(dx + c) + e^(-dx - c))^2 - 4)*d)

3.192 $\int \operatorname{csch}^4(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=31

$$-\frac{a \operatorname{coth}^3(c + dx)}{3d} + \frac{a \operatorname{coth}(c + dx)}{d} + bx$$

[Out] $b*x + (a*\operatorname{Coth}[c + d*x])/d - (a*\operatorname{Coth}[c + d*x]^3)/(3*d)$

Rubi [A] time = 0.0498847, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3217, 1261, 207}

$$-\frac{a \operatorname{coth}^3(c + dx)}{3d} + \frac{a \operatorname{coth}(c + dx)}{d} + bx$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^4*(a + b*\operatorname{Sinh}[c + d*x]^4), x]$

[Out] $b*x + (a*\operatorname{Coth}[c + d*x])/d - (a*\operatorname{Coth}[c + d*x]^3)/(3*d)$

Rule 3217

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x] \;/; \operatorname{FreeQ}\{a, b, e, f\}, x\} \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 1261

$\operatorname{Int}[(f_.)*(x_.)^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(q_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] \;/; \operatorname{FreeQ}\{a, b, c, d, e, f, m, q\}, x\} \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \&\& \operatorname{IGtQ}[p, 0] \&\& \operatorname{IGtQ}[q, -2]$

Rule 207

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2]^{(-1)}, x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[-a, 2]]/(\operatorname{Rt}[-a, 2]*\operatorname{Rt}[b, 2]), x] \;/; \operatorname{FreeQ}\{a, b\}, x\} \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{LtQ}[a$

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^4(c+dx)(a+b\sinh^4(c+dx))dx &= \frac{\operatorname{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^4(1-x^2)}dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^4} - \frac{a}{x^2} - \frac{b}{-1+x^2}\right)dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{a \operatorname{coth}(c+dx)}{d} - \frac{a \operatorname{coth}^3(c+dx)}{3d} - \frac{b \operatorname{Subst}\left(\int \frac{1}{-1+x^2}dx, x, \tanh(c+dx)\right)}{d} \\
 &= bx + \frac{a \operatorname{coth}(c+dx)}{d} - \frac{a \operatorname{coth}^3(c+dx)}{3d}
 \end{aligned}$$

Mathematica [A] time = 0.0154378, size = 40, normalized size = 1.29

$$\frac{2a \operatorname{coth}(c+dx)}{3d} - \frac{a \operatorname{coth}(c+dx)\operatorname{csch}^2(c+dx)}{3d} + bx$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4), x]

[Out] b*x + (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)

Maple [A] time = 0.039, size = 33, normalized size = 1.1

$$\frac{1}{d} \left(a \left(\frac{2}{3} - \frac{(\operatorname{csch}(dx+c))^2}{3} \right) \operatorname{coth}(dx+c) + (dx+c)b \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4), x)

[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+(d*x+c)*b)

Maxima [B] time = 1.16455, size = 131, normalized size = 4.23

$$bx + \frac{4}{3} a \left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} - \frac{1}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] b*x + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))

Fricas [B] time = 1.67282, size = 333, normalized size = 10.74

$$\frac{2a \cosh(dx+c)^3 + 6a \cosh(dx+c) \sinh(dx+c)^2 + (3bdx - 2a) \sinh(dx+c)^3 - 6a \cosh(dx+c) - 3(3bdx - (3bdx - (3bdx - 3bdx - 2a) \sinh(dx+c)^2 - 2a) \cosh(dx+c))}{3(d \sinh(dx+c)^3 + 3(d \cosh(dx+c)^2 - d) \sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out] 1/3*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + (3*b*d*x - 2*a)*sinh(d*x + c)^3 - 6*a*cosh(d*x + c) - 3*(3*b*d*x - (3*b*d*x - 2*a)*cosh(d*x + c)^2 - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4),x)

[Out] Timed out

Giac [A] time = 1.16184, size = 61, normalized size = 1.97

$$\frac{(dx + c)b}{d} - \frac{4(3ae^{(2dx+2c)} - a)}{3d(e^{(2dx+2c)} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="giac")

[Out] (d*x + c)*b/d - 4/3*(3*a*e^(2*d*x + 2*c) - a)/(d*(e^(2*d*x + 2*c) - 1)^3)

3.193 $\int \operatorname{csch}^5(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=64

$$-\frac{(3a + 8b) \tanh^{-1}(\cosh(c + dx))}{8d} - \frac{a \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{3a \coth(c + dx) \operatorname{csch}(c + dx)}{8d}$$

[Out] $-\frac{((3*a + 8*b)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])}{(8*d)} + \frac{(3*a*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])}{(8*d)} - \frac{(a*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x]^3)}{(4*d)}$

Rubi [A] time = 0.0627252, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3215, 1157, 385, 206}

$$-\frac{(3a + 8b) \tanh^{-1}(\cosh(c + dx))}{8d} - \frac{a \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{3a \coth(c + dx) \operatorname{csch}(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^5*(a + b*\operatorname{Sinh}[c + d*x]^4), x]$

[Out] $-\frac{((3*a + 8*b)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])}{(8*d)} + \frac{(3*a*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])}{(8*d)} - \frac{(a*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x]^3)}{(4*d)}$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x]] \;/; \operatorname{FreeQ}[\{a, b, e, f, p\}, x] \ \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 1157

$\operatorname{Int}[(d_.) + (e_.)*(x_.)^2]^{(q_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{Qx = \operatorname{PolynomialQuotient}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]\}, -\operatorname{Simp}[(R*x*(d + e*x^2)^{(q+1)})/(2*d*(q+1)), x] + \operatorname{Dist}[1/(2*d*(q+1)), \operatorname{Int}[(d + e*x^2)^{(q+1)}*\operatorname{ExpandToSum}[2*d*(q+1)*Qx + R*(2*q+3), x], x]] \;/; \operatorname{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{LtQ}[q, -1]$

Rule 385

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> -Simp[(b*c - a*d)*x*(a + b*x^n)^(p + 1)/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^5(c + dx) (a + b \sinh^4(c + dx)) dx &= -\frac{\operatorname{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^3} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{a \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{-3a-4b+4bx^2}{(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{4d} \\ &= \frac{3a \coth(c + dx) \operatorname{csch}(c + dx)}{8d} - \frac{a \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} - \frac{(3a + 8b) \operatorname{csch}^2(c + dx)}{8d} \\ &= -\frac{(3a + 8b) \tanh^{-1}(\cosh(c + dx))}{8d} + \frac{3a \coth(c + dx) \operatorname{csch}(c + dx)}{8d} - \frac{a \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} \end{aligned}$$

Mathematica [B] time = 0.0291577, size = 139, normalized size = 2.17

$$-\frac{a \operatorname{csch}^4\left(\frac{1}{2}(c + dx)\right)}{64d} + \frac{3a \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right)}{32d} + \frac{a \operatorname{sech}^4\left(\frac{1}{2}(c + dx)\right)}{64d} + \frac{3a \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right)}{32d} + \frac{3a \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4), x]

[Out] (3*a*Csch[(c + d*x)/2]^2)/(32*d) - (a*Csch[(c + d*x)/2]^4)/(64*d) - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d + (3*a*Log[Tanh[(c + d*x)/2]])/(8*d) + (3*a*Sech[(c + d*x)/2]^2)/(32*d) + (a*Sech[(c + d*x)/2]^4)/(64*d)

]^4)/(64*d)

Maple [A] time = 0.039, size = 54, normalized size = 0.8

$$\frac{1}{d} \left(a \left(\left(-\frac{(\operatorname{csch}(dx+c))^3}{4} + \frac{3 \operatorname{csch}(dx+c)}{8} \right) \coth(dx+c) - \frac{3 \operatorname{Artanh}(e^{dx+c})}{4} \right) - 2b \operatorname{Artanh}(e^{dx+c}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x)

[Out] 1/d*(a*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d*x+c)))-2*b*arctanh(exp(d*x+c)))

Maxima [B] time = 1.04026, size = 235, normalized size = 3.67

$$-\frac{1}{8} a \left(\frac{3 \log(e^{-dx-c} + 1)}{d} - \frac{3 \log(e^{-dx-c} - 1)}{d} + \frac{2(3e^{-dx-c} - 11e^{-3dx-3c} - 11e^{-5dx-5c} + 3e^{-7dx-7c})}{d(4e^{-2dx-2c} - 6e^{-4dx-4c} + 4e^{-6dx-6c} - e^{-8dx-8c} - 1)} \right) - b \left(\frac{\log(e^{-dx-c} + 1)}{d} - \frac{\log(e^{-dx-c} - 1)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] -1/8*a*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)

Fricas [B] time = 1.86017, size = 3906, normalized size = 61.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")

```
[Out] 1/8*(6*a*cosh(d*x + c)^7 + 42*a*cosh(d*x + c)*sinh(d*x + c)^6 + 6*a*sinh(d*x + c)^7 - 22*a*cosh(d*x + c)^5 + 2*(63*a*cosh(d*x + c)^2 - 11*a)*sinh(d*x + c)^5 + 10*(21*a*cosh(d*x + c)^3 - 11*a*cosh(d*x + c))*sinh(d*x + c)^4 - 22*a*cosh(d*x + c)^3 + 2*(105*a*cosh(d*x + c)^4 - 110*a*cosh(d*x + c)^2 - 11*a)*sinh(d*x + c)^3 + 2*(63*a*cosh(d*x + c)^5 - 110*a*cosh(d*x + c)^3 - 33*a*cosh(d*x + c))*sinh(d*x + c)^2 + 6*a*cosh(d*x + c) - ((3*a + 8*b)*cosh(d*x + c)^8 + 8*(3*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)*sinh(d*x + c)^8 - 4*(3*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^6 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^3 - 3*(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + 8*b)*cosh(d*x + c)^4 + 2*(35*(3*a + 8*b)*cosh(d*x + c)^4 - 30*(3*a + 8*b)*cosh(d*x + c)^2 + 9*a + 24*b)*sinh(d*x + c)^4 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^5 - 10*(3*a + 8*b)*cosh(d*x + c)^3 + 3*(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a + 8*b)*cosh(d*x + c)^2 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^6 - 15*(3*a + 8*b)*cosh(d*x + c)^4 + 9*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^2 + 8*((3*a + 8*b)*cosh(d*x + c)^7 - 3*(3*a + 8*b)*cosh(d*x + c)^5 + 3*(3*a + 8*b)*cosh(d*x + c)^3 - (3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 3*a + 8*b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((3*a + 8*b)*cosh(d*x + c)^8 + 8*(3*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)*sinh(d*x + c)^8 - 4*(3*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^6 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^3 - 3*(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + 8*b)*cosh(d*x + c)^4 + 2*(35*(3*a + 8*b)*cosh(d*x + c)^4 - 30*(3*a + 8*b)*cosh(d*x + c)^2 + 9*a + 24*b)*sinh(d*x + c)^4 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^5 - 10*(3*a + 8*b)*cosh(d*x + c)^3 + 3*(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a + 8*b)*cosh(d*x + c)^2 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^6 - 15*(3*a + 8*b)*cosh(d*x + c)^4 + 9*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^2 + 8*((3*a + 8*b)*cosh(d*x + c)^7 - 3*(3*a + 8*b)*cosh(d*x + c)^5 + 3*(3*a + 8*b)*cosh(d*x + c)^3 - (3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 3*a + 8*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(21*a*cosh(d*x + c)^6 - 55*a*cosh(d*x + c)^4 - 33*a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4),x)

[Out] Timed out

Giac [B] time = 1.16487, size = 174, normalized size = 2.72

$$-\frac{(3a + 8b) \log(e^{(dx+c)} + e^{(-dx-c)} + 2)}{16d} + \frac{(3a + 8b) \log(e^{(dx+c)} + e^{(-dx-c)} - 2)}{16d} + \frac{3a(e^{(dx+c)} + e^{(-dx-c)})^3 - 20a(e^{(dx+c)} + e^{(-dx-c)})}{4\left(\left(e^{(dx+c)} + e^{(-dx-c)}\right)^2 - 4\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="giac")

[Out] -1/16*(3*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d + 1/16*(3*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d + 1/4*(3*a*(e^(d*x + c) + e^(-d*x - c))^3 - 20*a*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)^2*d)

3.194 $\int \operatorname{csch}^6(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=47

$$-\frac{(a+b)\operatorname{coth}(c+dx)}{d} - \frac{a\operatorname{coth}^5(c+dx)}{5d} + \frac{2a\operatorname{coth}^3(c+dx)}{3d}$$

[Out] -(((a + b)*Coth[c + d*x])/d) + (2*a*Coth[c + d*x]^3)/(3*d) - (a*Coth[c + d*x]^5)/(5*d)

Rubi [A] time = 0.0434493, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3217, 14}

$$-\frac{(a+b)\operatorname{coth}(c+dx)}{d} - \frac{a\operatorname{coth}^5(c+dx)}{5d} + \frac{2a\operatorname{coth}^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4),x]

[Out] -(((a + b)*Coth[c + d*x])/d) + (2*a*Coth[c + d*x]^3)/(3*d) - (a*Coth[c + d*x]^5)/(5*d)

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)]/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^6(c+dx) (a+b \sinh^4(c+dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^6} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^6} - \frac{2a}{x^4} + \frac{a+b}{x^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{(a+b) \operatorname{coth}(c+dx)}{d} + \frac{2a \operatorname{coth}^3(c+dx)}{3d} - \frac{a \operatorname{coth}^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.0342195, size = 71, normalized size = 1.51

$$-\frac{8a \operatorname{coth}(c+dx)}{15d} - \frac{a \operatorname{coth}(c+dx) \operatorname{csch}^4(c+dx)}{5d} + \frac{4a \operatorname{coth}(c+dx) \operatorname{csch}^2(c+dx)}{15d} - \frac{b \operatorname{coth}(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4), x]

[Out] (-8*a*Coth[c + d*x])/(15*d) - (b*Coth[c + d*x])/d + (4*a*Coth[c + d*x]*Csch[c + d*x]^2)/(15*d) - (a*Coth[c + d*x]*Csch[c + d*x]^4)/(5*d)

Maple [A] time = 0.036, size = 45, normalized size = 1.

$$\frac{1}{d} \left(a \left(-\frac{8}{15} - \frac{(\operatorname{csch}(dx+c))^4}{5} + \frac{4(\operatorname{csch}(dx+c))^2}{15} \right) \operatorname{coth}(dx+c) - b \operatorname{coth}(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4), x)

[Out] 1/d*(a*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-b*coth(d*x+c))

Maxima [B] time = 1.04262, size = 308, normalized size = 6.55

$$-\frac{16}{15} a \left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1)} - \frac{10}{d(5e^{(-2dx-2c)} - 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} - 5e^{(-8dx-8c)} + e^{(-10dx-10c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out]
$$-16/15*a*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) - 10*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) - 1/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1))) + 2*b/(d*(e^{(-2*d*x - 2*c)} - 1))$$

Fricas [B] time = 1.66127, size = 903, normalized size = 19.21

$$\frac{4 \left((4a + 15b) \cosh(dx + c)^4 - 16a \cosh(dx + c) \sinh(dx + c)^3 + (4a + 15b) \sinh(dx + c)^4 - 20(a + 3b) \cosh(dx + c)^2 + 2(3(4a + 15b) \cosh(dx + c)^2 - 10a - 30b) \sinh(dx + c)^2 - 8(2a \cosh(dx + c)^3 - 5a \cosh(dx + c)) \sinh(dx + c) + 40a + 45b \right)}{d \cosh(dx + c)^6 + 6d \cosh(dx + c) \sinh(dx + c)^5 + d \sinh(dx + c)^6 - 6d \cosh(dx + c)^4 + 3(5d \cosh(dx + c)^2 - 2d) \sinh(dx + c)^4 + 4(5d \cosh(dx + c)^3 - 4d \cosh(dx + c)) \sinh(dx + c)^3 + 15d \cosh(dx + c)^2 + 3(5d \cosh(dx + c)^4 - 12d \cosh(dx + c)^2 + 5d) \sinh(dx + c)^2 + 2(3d \cosh(dx + c)^5 - 8d \cosh(dx + c)^3 + 5d \cosh(dx + c)) \sinh(dx + c) - 10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out]
$$-4/15*((4*a + 15*b)*\cosh(d*x + c)^4 - 16*a*\cosh(d*x + c)*\sinh(d*x + c)^3 + (4*a + 15*b)*\sinh(d*x + c)^4 - 20*(a + 3*b)*\cosh(d*x + c)^2 + 2*(3*(4*a + 15*b)*\cosh(d*x + c)^2 - 10*a - 30*b)*\sinh(d*x + c)^2 - 8*(2*a*\cosh(d*x + c)^3 - 5*a*\cosh(d*x + c))*\sinh(d*x + c) + 40*a + 45*b)/(d*\cosh(d*x + c)^6 + 6*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + d*\sinh(d*x + c)^6 - 6*d*\cosh(d*x + c)^4 + 3*(5*d*\cosh(d*x + c)^2 - 2*d)*\sinh(d*x + c)^4 + 4*(5*d*\cosh(d*x + c)^3 - 4*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 15*d*\cosh(d*x + c)^2 + 3*(5*d*\cosh(d*x + c)^4 - 12*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^5 - 8*d*\cosh(d*x + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c) - 10*d)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4),x)

[Out] Timed out

Giac [B] time = 1.14916, size = 131, normalized size = 2.79

$$\frac{2 \left(15 b e^{(8 dx + 8 c)} - 60 b e^{(6 dx + 6 c)} + 80 a e^{(4 dx + 4 c)} + 90 b e^{(4 dx + 4 c)} - 40 a e^{(2 dx + 2 c)} - 60 b e^{(2 dx + 2 c)} + 8 a + 15 b \right)}{15 d \left(e^{(2 dx + 2 c)} - 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="giac")

[Out] -2/15*(15*b*e^(8*d*x + 8*c) - 60*b*e^(6*d*x + 6*c) + 80*a*e^(4*d*x + 4*c) + 90*b*e^(4*d*x + 4*c) - 40*a*e^(2*d*x + 2*c) - 60*b*e^(2*d*x + 2*c) + 8*a + 15*b)/(d*(e^(2*d*x + 2*c) - 1)^5)

3.195 $\int \operatorname{csch}^7(c + dx) (a + b \sinh^4(c + dx)) dx$

Optimal. Leaf size=92

$$\frac{(5a + 8b) \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{(5a + 8b) \coth(c + dx) \operatorname{csch}(c + dx)}{16d} - \frac{a \coth(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a \coth(c + dx)}{2d}$$

[Out] ((5*a + 8*b)*ArcTanh[Cosh[c + d*x]])/(16*d) - ((5*a + 8*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)

Rubi [A] time = 0.0837145, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3215, 1157, 385, 199, 206}

$$\frac{(5a + 8b) \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{(5a + 8b) \coth(c + dx) \operatorname{csch}(c + dx)}{16d} - \frac{a \coth(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a \coth(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4), x]

[Out] ((5*a + 8*b)*ArcTanh[Cosh[c + d*x]])/(16*d) - ((5*a + 8*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)

Rule 3215

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 1157

Int[((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -

$b*d*e + a*e^2, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{LtQ}[q, -1]$

Rule 385

$\text{Int}[\{(a_)+ (b_)*(x_)^{(n_)}\}^{(p_)}*((c_)+ (d_)*(x_)^{(n_)}), x_Symbol] \text{ :> } -\text{Simp}[\{(b*c - a*d)*x*(a + b*x^n)^{(p+1)}\}/(a*b*n*(p+1)), x] - \text{Dist}[\{a*d - b*c*(n*(p+1) + 1)\}/(a*b*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b, c, d, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{LtQ}[p, -1] \|\| \text{ILtQ}[1/n + p, 0])$

Rule 199

$\text{Int}[\{(a_)+ (b_)*(x_)^{(n_)}\}^{(p_)}, x_Symbol] \text{ :> } -\text{Simp}[(x*(a + b*x^n)^{(p+1)})/(a*n*(p+1)), x] + \text{Dist}[(n*(p+1) + 1)/(a*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& (\text{IntegerQ}[2*p] \|\| (n == 2 \&\& \text{IntegerQ}[4*p]) \|\| (n == 2 \&\& \text{IntegerQ}[3*p]) \|\| \text{Denominator}[p + 1/n] < \text{Denominator}[p])$

Rule 206

$\text{Int}[\{(a_)+ (b_)*(x_)^2\}^{(-1)}, x_Symbol] \text{ :> } \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \|\| \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \text{csch}^7(c + dx) (a + b \sinh^4(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^4} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{a \coth(c + dx) \text{csch}^5(c + dx)}{6d} - \frac{\text{Subst}\left(\int \frac{-5a-6b+6bx^2}{(1-x^2)^3} dx, x, \cosh(c + dx)\right)}{6d} \\ &= \frac{5a \coth(c + dx) \text{csch}^3(c + dx)}{24d} - \frac{a \coth(c + dx) \text{csch}^5(c + dx)}{6d} + \frac{(5a + 8b) \text{csch}^3(c + dx)}{16d} \\ &= -\frac{(5a + 8b) \coth(c + dx) \text{csch}(c + dx)}{16d} + \frac{5a \coth(c + dx) \text{csch}^3(c + dx)}{24d} - \frac{a \coth(c + dx) \text{csch}^5(c + dx)}{6d} \\ &= \frac{(5a + 8b) \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{(5a + 8b) \coth(c + dx) \text{csch}(c + dx)}{16d} + \frac{5a \coth(c + dx) \text{csch}^3(c + dx)}{24d} - \frac{a \coth(c + dx) \text{csch}^5(c + dx)}{6d} \end{aligned}$$

Mathematica [B] time = 0.035123, size = 199, normalized size = 2.16

$$-\frac{\operatorname{acsch}^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{\operatorname{acsch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5\operatorname{acsch}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{\operatorname{asech}^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{\operatorname{asech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5\operatorname{asech}^2\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4), x]

[Out] $(-5*a*\operatorname{Csch}[(c+d*x)/2]^2)/(64*d) - (b*\operatorname{Csch}[(c+d*x)/2]^2)/(8*d) + (a*\operatorname{Csch}[(c+d*x)/2]^4)/(64*d) - (a*\operatorname{Csch}[(c+d*x)/2]^6)/(384*d) - (5*a*\operatorname{Log}[\operatorname{Tanh}[(c+d*x)/2]])/(16*d) - (b*\operatorname{Log}[\operatorname{Tanh}[(c+d*x)/2]])/(2*d) - (5*a*\operatorname{Sech}[(c+d*x)/2]^2)/(64*d) - (b*\operatorname{Sech}[(c+d*x)/2]^2)/(8*d) - (a*\operatorname{Sech}[(c+d*x)/2]^4)/(64*d) - (a*\operatorname{Sech}[(c+d*x)/2]^6)/(384*d)$

Maple [A] time = 0.041, size = 78, normalized size = 0.9

$$\frac{1}{d} \left(a \left(\left(-\frac{(\operatorname{csch}(dx+c))^5}{6} + \frac{5(\operatorname{csch}(dx+c))^3}{24} - \frac{5\operatorname{csch}(dx+c)}{16} \right) \operatorname{coth}(dx+c) + \frac{5 \operatorname{Arctanh}(e^{dx+c})}{8} \right) + b \left(-\frac{\operatorname{csch}(dx+c)}{8} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4), x)

[Out] $1/d*(a*((-1/6*\operatorname{csch}(d*x+c)^5+5/24*\operatorname{csch}(d*x+c)^3-5/16*\operatorname{csch}(d*x+c))*\operatorname{coth}(d*x+c)+5/8*\operatorname{arctanh}(\exp(d*x+c)))+b*(-1/2*\operatorname{csch}(d*x+c)*\operatorname{coth}(d*x+c)+\operatorname{arctanh}(\exp(d*x+c))))$

Maxima [B] time = 1.05621, size = 362, normalized size = 3.93

$$\frac{1}{48} a \left(\frac{15 \log(e^{-dx-c} + 1)}{d} - \frac{15 \log(e^{-dx-c} - 1)}{d} \right) + \frac{2 \left(15 e^{-dx-c} - 85 e^{-3dx-3c} + 198 e^{-5dx-5c} + 198 e^{-7dx-7c} - 85 e^{-9dx-9c} \right)}{d \left(6 e^{-2dx-2c} - 15 e^{-4dx-4c} + 20 e^{-6dx-6c} - 15 e^{-8dx-8c} + 6 e^{-10dx-10c} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4), x, algorithm="maxima")

```
[Out] 1/48*a*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 1/2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 1.88138, size = 8357, normalized size = 90.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/48*(6*(5*a + 8*b)*cosh(d*x + c)^11 + 66*(5*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^10 + 6*(5*a + 8*b)*sinh(d*x + c)^11 - 2*(85*a + 72*b)*cosh(d*x + c)^9 + 2*(165*(5*a + 8*b)*cosh(d*x + c)^2 - 85*a - 72*b)*sinh(d*x + c)^9 + 18*(55*(5*a + 8*b)*cosh(d*x + c)^3 - (85*a + 72*b)*cosh(d*x + c))*sinh(d*x + c)^8 + 12*(33*a + 8*b)*cosh(d*x + c)^7 + 12*(165*(5*a + 8*b)*cosh(d*x + c)^4 - 6*(85*a + 72*b)*cosh(d*x + c)^2 + 33*a + 8*b)*sinh(d*x + c)^7 + 84*(33*(5*a + 8*b)*cosh(d*x + c)^5 - 2*(85*a + 72*b)*cosh(d*x + c)^3 + (33*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 12*(33*a + 8*b)*cosh(d*x + c)^5 + 12*(231*(5*a + 8*b)*cosh(d*x + c)^6 - 21*(85*a + 72*b)*cosh(d*x + c)^4 + 21*(33*a + 8*b)*cosh(d*x + c)^2 + 33*a + 8*b)*sinh(d*x + c)^5 + 12*(165*(5*a + 8*b)*cosh(d*x + c)^7 - 21*(85*a + 72*b)*cosh(d*x + c)^5 + 35*(33*a + 8*b)*cosh(d*x + c)^3 + 5*(33*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(85*a + 72*b)*cosh(d*x + c)^3 + 2*(495*(5*a + 8*b)*cosh(d*x + c)^8 - 84*(85*a + 72*b)*cosh(d*x + c)^6 + 210*(33*a + 8*b)*cosh(d*x + c)^4 + 60*(33*a + 8*b)*cosh(d*x + c)^2 - 85*a - 72*b)*sinh(d*x + c)^3 + 6*(55*(5*a + 8*b)*cosh(d*x + c)^9 - 12*(85*a + 72*b)*cosh(d*x + c)^7 + 42*(33*a + 8*b)*cosh(d*x + c)^5 + 20*(33*a + 8*b)*cosh(d*x + c)^3 - (85*a + 72*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 6*(5*a + 8*b)*cosh(d*x + c) - 3*((5*a + 8*b)*cosh(d*x + c)^12 + 12*(5*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a + 8*b)*sinh(d*x + c)^12 - 6*(5*a + 8*b)*cosh(d*x + c)^10 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)*sinh(d*x + c)^10 + 20*(11*(5*a + 8*b)*cosh(d*x + c)^3 - 3*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a + 8*b)*cosh(d*x + c)^8 + 15*(33*(5*a + 8*b)*cosh(d*x + c)^4 - 18*(5*a + 8*b)*cosh(d*x + c)^2 + 5*a + 8*b)*sinh(d*x + c)^8 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^5 - 30*(5*a + 8*b)*cosh(d*x + c)^3 + 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a + 8*b)*cosh(d*x + c)^6 + 4*(231*(5*a + 8*b)*cosh(d*x + c)^6 - 315*(5*a + 8*b)*cosh(d*x +
```

$$\begin{aligned}
& c)^4 + 105*(5*a + 8*b)*\cosh(d*x + c)^2 - 25*a - 40*b)*\sinh(d*x + c)^6 + 24* \\
& (33*(5*a + 8*b)*\cosh(d*x + c)^7 - 63*(5*a + 8*b)*\cosh(d*x + c)^5 + 35*(5*a \\
& + 8*b)*\cosh(d*x + c)^3 - 5*(5*a + 8*b)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 15* \\
& (5*a + 8*b)*\cosh(d*x + c)^4 + 15*(33*(5*a + 8*b)*\cosh(d*x + c)^8 - 84*(5*a \\
& + 8*b)*\cosh(d*x + c)^6 + 70*(5*a + 8*b)*\cosh(d*x + c)^4 - 20*(5*a + 8*b)*\co \\
& sh(d*x + c)^2 + 5*a + 8*b)*\sinh(d*x + c)^4 + 20*(11*(5*a + 8*b)*\cosh(d*x + \\
& c)^9 - 36*(5*a + 8*b)*\cosh(d*x + c)^7 + 42*(5*a + 8*b)*\cosh(d*x + c)^5 - 20 \\
& *(5*a + 8*b)*\cosh(d*x + c)^3 + 3*(5*a + 8*b)*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& - 6*(5*a + 8*b)*\cosh(d*x + c)^2 + 6*(11*(5*a + 8*b)*\cosh(d*x + c)^10 - 45* \\
& (5*a + 8*b)*\cosh(d*x + c)^8 + 70*(5*a + 8*b)*\cosh(d*x + c)^6 - 50*(5*a + 8* \\
& b)*\cosh(d*x + c)^4 + 15*(5*a + 8*b)*\cosh(d*x + c)^2 - 5*a - 8*b)*\sinh(d*x + \\
& c)^2 + 12*((5*a + 8*b)*\cosh(d*x + c)^11 - 5*(5*a + 8*b)*\cosh(d*x + c)^9 + \\
& 10*(5*a + 8*b)*\cosh(d*x + c)^7 - 10*(5*a + 8*b)*\cosh(d*x + c)^5 + 5*(5*a + \\
& 8*b)*\cosh(d*x + c)^3 - (5*a + 8*b)*\cosh(d*x + c))*\sinh(d*x + c) + 5*a + 8*b \\
&)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 3*((5*a + 8*b)*\cosh(d*x + c)^12 \\
& + 12*(5*a + 8*b)*\cosh(d*x + c)*\sinh(d*x + c)^11 + (5*a + 8*b)*\sinh(d*x + c) \\
& ^12 - 6*(5*a + 8*b)*\cosh(d*x + c)^10 + 6*(11*(5*a + 8*b)*\cosh(d*x + c)^2 - \\
& 5*a - 8*b)*\sinh(d*x + c)^10 + 20*(11*(5*a + 8*b)*\cosh(d*x + c)^3 - 3*(5*a + \\
& 8*b)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 15*(5*a + 8*b)*\cosh(d*x + c)^8 + 15* \\
& (33*(5*a + 8*b)*\cosh(d*x + c)^4 - 18*(5*a + 8*b)*\cosh(d*x + c)^2 + 5*a + 8* \\
& b)*\sinh(d*x + c)^8 + 24*(33*(5*a + 8*b)*\cosh(d*x + c)^5 - 30*(5*a + 8*b)*\co \\
& sh(d*x + c)^3 + 5*(5*a + 8*b)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 20*(5*a + 8* \\
& b)*\cosh(d*x + c)^6 + 4*(231*(5*a + 8*b)*\cosh(d*x + c)^6 - 315*(5*a + 8*b)*\c \\
& osh(d*x + c)^4 + 105*(5*a + 8*b)*\cosh(d*x + c)^2 - 25*a - 40*b)*\sinh(d*x + \\
& c)^6 + 24*(33*(5*a + 8*b)*\cosh(d*x + c)^7 - 63*(5*a + 8*b)*\cosh(d*x + c)^5 \\
& + 35*(5*a + 8*b)*\cosh(d*x + c)^3 - 5*(5*a + 8*b)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^5 + 15*(5*a + 8*b)*\cosh(d*x + c)^4 + 15*(33*(5*a + 8*b)*\cosh(d*x + c)^8 \\
& - 84*(5*a + 8*b)*\cosh(d*x + c)^6 + 70*(5*a + 8*b)*\cosh(d*x + c)^4 - 20*(5*a \\
& + 8*b)*\cosh(d*x + c)^2 + 5*a + 8*b)*\sinh(d*x + c)^4 + 20*(11*(5*a + 8*b)*\c \\
& osh(d*x + c)^9 - 36*(5*a + 8*b)*\cosh(d*x + c)^7 + 42*(5*a + 8*b)*\cosh(d*x + \\
& c)^5 - 20*(5*a + 8*b)*\cosh(d*x + c)^3 + 3*(5*a + 8*b)*\cosh(d*x + c))*\sinh(\\
& d*x + c)^3 - 6*(5*a + 8*b)*\cosh(d*x + c)^2 + 6*(11*(5*a + 8*b)*\cosh(d*x + c \\
&)^10 - 45*(5*a + 8*b)*\cosh(d*x + c)^8 + 70*(5*a + 8*b)*\cosh(d*x + c)^6 - 50 \\
& *(5*a + 8*b)*\cosh(d*x + c)^4 + 15*(5*a + 8*b)*\cosh(d*x + c)^2 - 5*a - 8*b)* \\
& \sinh(d*x + c)^2 + 12*((5*a + 8*b)*\cosh(d*x + c)^11 - 5*(5*a + 8*b)*\cosh(d*x \\
& + c)^9 + 10*(5*a + 8*b)*\cosh(d*x + c)^7 - 10*(5*a + 8*b)*\cosh(d*x + c)^5 + \\
& 5*(5*a + 8*b)*\cosh(d*x + c)^3 - (5*a + 8*b)*\cosh(d*x + c))*\sinh(d*x + c) + \\
& 5*a + 8*b)*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 6*(11*(5*a + 8*b)*\cosh \\
& (d*x + c)^10 - 3*(85*a + 72*b)*\cosh(d*x + c)^8 + 14*(33*a + 8*b)*\cosh(d*x + \\
& c)^6 + 10*(33*a + 8*b)*\cosh(d*x + c)^4 - (85*a + 72*b)*\cosh(d*x + c)^2 + 5 \\
& *a + 8*b)*\sinh(d*x + c))/(d*\cosh(d*x + c)^12 + 12*d*\cosh(d*x + c)*\sinh(d*x \\
& + c)^11 + d*\sinh(d*x + c)^12 - 6*d*\cosh(d*x + c)^10 + 6*(11*d*\cosh(d*x + c) \\
& ^2 - d)*\sinh(d*x + c)^10 + 20*(11*d*\cosh(d*x + c)^3 - 3*d*\cosh(d*x + c))*\si \\
& nh(d*x + c)^9 + 15*d*\cosh(d*x + c)^8 + 15*(33*d*\cosh(d*x + c)^4 - 18*d*\cosh \\
& (d*x + c)^2 + d)*\sinh(d*x + c)^8 + 24*(33*d*\cosh(d*x + c)^5 - 30*d*\cosh(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^3 + 5*d*\cosh(d*x + c))*\sinh(d*x + c)^7 - 20*d*\cosh(d*x + c)^6 + 4*(23 \\
& 1*d*\cosh(d*x + c)^6 - 315*d*\cosh(d*x + c)^4 + 105*d*\cosh(d*x + c)^2 - 5*d)* \\
& \sinh(d*x + c)^6 + 24*(33*d*\cosh(d*x + c)^7 - 63*d*\cosh(d*x + c)^5 + 35*d*\co \\
& sh(d*x + c)^3 - 5*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 15*d*\cosh(d*x + c)^4 + \\
& 15*(33*d*\cosh(d*x + c)^8 - 84*d*\cosh(d*x + c)^6 + 70*d*\cosh(d*x + c)^4 - 2 \\
& 0*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^4 + 20*(11*d*\cosh(d*x + c)^9 - 36*d* \\
& \cosh(d*x + c)^7 + 42*d*\cosh(d*x + c)^5 - 20*d*\cosh(d*x + c)^3 + 3*d*\cosh(d* \\
& x + c))*\sinh(d*x + c)^3 - 6*d*\cosh(d*x + c)^2 + 6*(11*d*\cosh(d*x + c)^10 - \\
& 45*d*\cosh(d*x + c)^8 + 70*d*\cosh(d*x + c)^6 - 50*d*\cosh(d*x + c)^4 + 15*d*c \\
& osh(d*x + c)^2 - d)*\sinh(d*x + c)^2 + 12*(d*\cosh(d*x + c)^11 - 5*d*\cosh(d*x \\
& + c)^9 + 10*d*\cosh(d*x + c)^7 - 10*d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)^3 \\
& - d*\cosh(d*x + c))*\sinh(d*x + c) + d)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4),x)

[Out] Timed out

Giac [B] time = 1.17426, size = 285, normalized size = 3.1

$$\frac{(5a + 8b) \log(e^{(dx+c)} + e^{(-dx-c)} + 2)}{32d} - \frac{(5a + 8b) \log(e^{(dx+c)} + e^{(-dx-c)} - 2)}{32d} - \frac{15a(e^{(dx+c)} + e^{(-dx-c)})^5 + 24b(e^{(dx+c)} + e^{(-dx-c)})^5}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="giac")

[Out] 1/32*(5*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/32*(5*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d - 1/24*(15*a*(e^(d*x + c) + e^(-d*x - c))^5 + 24*b*(e^(d*x + c) + e^(-d*x - c))^5 - 160*a*(e^(d*x + c) + e^(-d*x - c))^3 - 192*b*(e^(d*x + c) + e^(-d*x - c))^3 + 528*a*(e^(d*x + c) + e^(-d*x - c)) + 384*b*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3*d)

3.196 $\int \sinh^3(c + dx) (a + b \sinh^4(c + dx))^2 dx$

Optimal. Leaf size=120

$$\frac{2b(a+5b)\cosh^7(c+dx)}{7d} - \frac{2b(3a+5b)\cosh^5(c+dx)}{5d} + \frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d} - \frac{(a+b)^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d}$$

[Out] -(((a + b)^2*Cosh[c + d*x])/d) + ((a + b)*(a + 5*b)*Cosh[c + d*x]^3)/(3*d) - (2*b*(3*a + 5*b)*Cosh[c + d*x]^5)/(5*d) + (2*b*(a + 5*b)*Cosh[c + d*x]^7)/(7*d) - (5*b^2*Cosh[c + d*x]^9)/(9*d) + (b^2*Cosh[c + d*x]^11)/(11*d)

Rubi [A] time = 0.129451, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3215, 1153}

$$\frac{2b(a+5b)\cosh^7(c+dx)}{7d} - \frac{2b(3a+5b)\cosh^5(c+dx)}{5d} + \frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d} - \frac{(a+b)^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] -(((a + b)^2*Cosh[c + d*x])/d) + ((a + b)*(a + 5*b)*Cosh[c + d*x]^3)/(3*d) - (2*b*(3*a + 5*b)*Cosh[c + d*x]^5)/(5*d) + (2*b*(a + 5*b)*Cosh[c + d*x]^7)/(7*d) - (5*b^2*Cosh[c + d*x]^9)/(9*d) + (b^2*Cosh[c + d*x]^11)/(11*d)

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned} \int \sinh^3(c+dx) (a+b\sinh^4(c+dx))^2 dx &= -\frac{\text{Subst}\left(\int (1-x^2)(a+b-2bx^2+bx^4)^2 dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\text{Subst}\left(\int ((a+b)^2+(-a-5b)(a+b)x^2+2b(3a+5b)x^4-2b(a+5b)x^6+\dots) dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{(a+b)^2 \cosh(c+dx)}{d} + \frac{(a+b)(a+5b) \cosh^3(c+dx)}{3d} - \frac{2b(3a+5b) \cosh^5(c+dx)}{5d} + \dots \end{aligned}$$

Mathematica [A] time = 0.0635944, size = 207, normalized size = 1.72

$$-\frac{3a^2 \cosh(c+dx)}{4d} + \frac{a^2 \cosh(3(c+dx))}{12d} - \frac{35ab \cosh(c+dx)}{32d} + \frac{7ab \cosh(3(c+dx))}{32d} - \frac{7ab \cosh(5(c+dx))}{160d} + \frac{ab \cosh(7(c+dx))}{224d} - \dots$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] $(-3*a^2*\text{Cosh}[c + d*x])/(4*d) - (35*a*b*\text{Cosh}[c + d*x])/(32*d) - (231*b^2*\text{Cosh}[c + d*x])/(512*d) + (a^2*\text{Cosh}[3*(c + d*x)])/(12*d) + (7*a*b*\text{Cosh}[3*(c + d*x)])/(32*d) + (55*b^2*\text{Cosh}[3*(c + d*x)])/(512*d) - (7*a*b*\text{Cosh}[5*(c + d*x)])/(160*d) - (33*b^2*\text{Cosh}[5*(c + d*x)])/(1024*d) + (a*b*\text{Cosh}[7*(c + d*x)])/(224*d) + (55*b^2*\text{Cosh}[7*(c + d*x)])/(7168*d) - (11*b^2*\text{Cosh}[9*(c + d*x)])/(9216*d) + (b^2*\text{Cosh}[11*(c + d*x)])/(11264*d)$

Maple [A] time = 0.023, size = 132, normalized size = 1.1

$$\frac{1}{d} \left(b^2 \left(-\frac{256}{693} + \frac{(\sinh(dx+c))^{10}}{11} - \frac{10(\sinh(dx+c))^8}{99} + \frac{80(\sinh(dx+c))^6}{693} - \frac{32(\sinh(dx+c))^4}{231} + \frac{128(\sinh(dx+c))^2}{693} \right) + 2ab \cosh(dx+c) + a^2 \cosh^3(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x)

[Out] $1/d*(b^2*(-256/693+1/11*\sinh(d*x+c)^{10}-10/99*\sinh(d*x+c)^8+80/693*\sinh(d*x+c)^6-32/231*\sinh(d*x+c)^4+128/693*\sinh(d*x+c)^2)*\cosh(d*x+c)+2*a*b*(-16/35+1/7*\sinh(d*x+c)^6-6/35*\sinh(d*x+c)^4+8/35*\sinh(d*x+c)^2)*\cosh(d*x+c)+a^2*(-2/3+1/3*\sinh(d*x+c)^2)*\cosh(d*x+c)$

Maxima [B] time = 1.04615, size = 414, normalized size = 3.45

$$-\frac{1}{1419264} b^2 \left(\frac{(847 e^{(-2 dx - 2c)} - 5445 e^{(-4 dx - 4c)} + 22869 e^{(-6 dx - 6c)} - 76230 e^{(-8 dx - 8c)} + 320166 e^{(-10 dx - 10c)} - 63) e^{(11 dx + 11c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out]
$$-1/1419264*b^2*((847*e^{(-2*d*x - 2*c)} - 5445*e^{(-4*d*x - 4*c)} + 22869*e^{(-6*d*x - 6*c)} - 76230*e^{(-8*d*x - 8*c)} + 320166*e^{(-10*d*x - 10*c)} - 63)*e^{(11*d*x + 11*c)}/d + (320166*e^{(-d*x - c)} - 76230*e^{(-3*d*x - 3*c)} + 22869*e^{(-5*d*x - 5*c)} - 5445*e^{(-7*d*x - 7*c)} + 847*e^{(-9*d*x - 9*c)} - 63*e^{(-11*d*x - 11*c)})/d - 1/2240*a*b*((49*e^{(-2*d*x - 2*c)} - 245*e^{(-4*d*x - 4*c)} + 1225*e^{(-6*d*x - 6*c)} - 5)*e^{(7*d*x + 7*c)}/d + (1225*e^{(-d*x - c)} - 245*e^{(-3*d*x - 3*c)} + 49*e^{(-5*d*x - 5*c)} - 5*e^{(-7*d*x - 7*c)})/d) + 1/24*a^2*(e^{(3*d*x + 3*c)}/d - 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d + e^{(-3*d*x - 3*c)}/d)$$

Fricas [B] time = 1.63243, size = 1111, normalized size = 9.26

$$315 b^2 \cosh(dx + c)^{11} + 3465 b^2 \cosh(dx + c) \sinh(dx + c)^{10} - 4235 b^2 \cosh(dx + c)^9 + 3465 (15 b^2 \cosh(dx + c)^3 - 11 b^2 \cosh(dx + c) \sinh(dx + c)^8 + 495 (32 a b + 55 b^2) \cosh(dx + c)^7 + 1155 (126 b^2 \cosh(dx + c)^5 - 308 b^2 \cosh(dx + c)^3 + 3 (32 a b + 55 b^2) \cosh(dx + c) \sinh(dx + c)^6 - 693 (224 a b + 165 b^2) \cosh(dx + c)^5 + 3465 (30 b^2 \cosh(dx + c)^7 - 154 b^2 \cosh(dx + c)^5 + 5 (32 a b + 55 b^2) \cosh(dx + c)^3 - (224 a b + 165 b^2) \cosh(dx + c) \sinh(dx + c)^4 + 2310 (128 a^2 + 336 a b + 165 b^2) \cosh(dx + c)^3 + 3465 (5 b^2 \cosh(dx + c)^9 - 44 b^2 \cosh(dx + c)^7 + 3 (32 a b + 55 b^2) \cosh(dx + c)^5 - 2 (224 a b + 165 b^2) \cosh(dx + c)^3 + 2 (128 a^2 + 336 a b + 165 b^2) \cosh(dx + c) \sinh(dx + c)^2 - 6930 (384 a^2 + 560 a b + 231 b^2) \cosh(dx + c))/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out]
$$1/3548160*(315*b^2*\cosh(d*x + c)^{11} + 3465*b^2*\cosh(d*x + c)*\sinh(d*x + c)^{10} - 4235*b^2*\cosh(d*x + c)^9 + 3465*(15*b^2*\cosh(d*x + c)^3 - 11*b^2*\cosh(d*x + c)*\sinh(d*x + c)^8 + 495*(32*a*b + 55*b^2)*\cosh(d*x + c)^7 + 1155*(126*b^2*\cosh(d*x + c)^5 - 308*b^2*\cosh(d*x + c)^3 + 3*(32*a*b + 55*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^6 - 693*(224*a*b + 165*b^2)*\cosh(d*x + c)^5 + 3465*(30*b^2*\cosh(d*x + c)^7 - 154*b^2*\cosh(d*x + c)^5 + 5*(32*a*b + 55*b^2)*\cosh(d*x + c)^3 - (224*a*b + 165*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^4 + 2310*(128*a^2 + 336*a*b + 165*b^2)*\cosh(d*x + c)^3 + 3465*(5*b^2*\cosh(d*x + c)^9 - 44*b^2*\cosh(d*x + c)^7 + 3*(32*a*b + 55*b^2)*\cosh(d*x + c)^5 - 2*(224*a*b + 165*b^2)*\cosh(d*x + c)^3 + 2*(128*a^2 + 336*a*b + 165*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 - 6930*(384*a^2 + 560*a*b + 231*b^2)*\cosh(d*x + c))/d$$

Sympy [A] time = 74.8908, size = 280, normalized size = 2.33

$$\frac{\left\{ \frac{a^2 \sinh^2(c+dx) \cosh(c+dx)}{d} - \frac{2a^2 \cosh^3(c+dx)}{3d} + \frac{2ab \sinh^6(c+dx) \cosh(c+dx)}{d} - \frac{4ab \sinh^4(c+dx) \cosh^3(c+dx)}{d} + \frac{16ab \sinh^2(c+dx) \cosh^5(c+dx)}{5d} - \dots \right\}}{x \left(a + b \sinh^4(c) \right)^2 \sinh^3(c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**2,x)

[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/(3*d) + 2*a*b*sinh(c + d*x)**6*cosh(c + d*x)/d - 4*a*b*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 16*a*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 32*a*b*cosh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b**2*sinh(c + d*x)**8*cosh(c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**6*cosh(c + d*x)**5/(3*d) - 32*b**2*sinh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128*b**2*sinh(c + d*x)**2*cosh(c + d*x)**9/(63*d) - 256*b**2*cosh(c + d*x)**11/(693*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c)**3, True))

Giac [B] time = 1.31993, size = 443, normalized size = 3.69

$$\frac{315 b^2 e^{(11 dx+11 c)} - 4235 b^2 e^{(9 dx+9 c)} + 15840 a b e^{(7 dx+7 c)} + 27225 b^2 e^{(7 dx+7 c)} - 155232 a b e^{(5 dx+5 c)} - 114345 b^2 e^{(5 dx+5 c)} + \dots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] 1/7096320*(315*b^2*e^(11*d*x + 11*c) - 4235*b^2*e^(9*d*x + 9*c) + 15840*a*b*e^(7*d*x + 7*c) + 27225*b^2*e^(7*d*x + 7*c) - 155232*a*b*e^(5*d*x + 5*c) - 114345*b^2*e^(5*d*x + 5*c) + 295680*a^2*e^(3*d*x + 3*c) + 776160*a*b*e^(3*d*x + 3*c) + 381150*b^2*e^(3*d*x + 3*c) - 2661120*a^2*e^(d*x + c) - 3880800*a*b*e^(d*x + c) - 1600830*b^2*e^(d*x + c) - (2661120*a^2*e^(10*d*x + 10*c) + 3880800*a*b*e^(10*d*x + 10*c) + 1600830*b^2*e^(10*d*x + 10*c) - 295680*a^2*e^(8*d*x + 8*c) - 776160*a*b*e^(8*d*x + 8*c) - 381150*b^2*e^(8*d*x + 8*c) + 155232*a*b*e^(6*d*x + 6*c) + 114345*b^2*e^(6*d*x + 6*c) - 15840*a*b*e^(4*d*x + 4*c) - 27225*b^2*e^(4*d*x + 4*c) + 4235*b^2*e^(2*d*x + 2*c) - 315*b^2*e^(-11*d*x - 11*c))/d

3.197 $\int \sinh^2(c + dx) (a + b \sinh^4(c + dx))^2 dx$

Optimal. Leaf size=161

$$\frac{(128a^2 + 352ab + 193b^2) \sinh(c + dx) \cosh(c + dx)}{256d} - \frac{1}{256}x(128a^2 + 160ab + 63b^2) + \frac{b(160a + 513b) \sinh(c + dx) \cosh(c + dx)}{480d}$$

```
[Out] -((128*a^2 + 160*a*b + 63*b^2)*x)/256 + ((128*a^2 + 352*a*b + 193*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) - (b*(416*a + 447*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(384*d) + (b*(160*a + 513*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(480*d) - (41*b^2*Cosh[c + d*x]^7*Sinh[c + d*x])/(80*d) + (b^2*Cosh[c + d*x]^9*Sinh[c + d*x])/(10*d)
```

Rubi [A] time = 0.280985, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3217, 1257, 1814, 1157, 385, 206}

$$\frac{(128a^2 + 352ab + 193b^2) \sinh(c + dx) \cosh(c + dx)}{256d} - \frac{1}{256}x(128a^2 + 160ab + 63b^2) + \frac{b(160a + 513b) \sinh(c + dx) \cosh(c + dx)}{480d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((128*a^2 + 160*a*b + 63*b^2)*x)/256 + ((128*a^2 + 352*a*b + 193*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) - (b*(416*a + 447*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(384*d) + (b*(160*a + 513*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(480*d) - (41*b^2*Cosh[c + d*x]^7*Sinh[c + d*x])/(80*d) + (b^2*Cosh[c + d*x]^9*Sinh[c + d*x])/(10*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 1257

```
Int[(x_.)^(m_.)*((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
```

```

+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

```

Rule 1814

```

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]

```

Rule 1157

```

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

```

Rule 385

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \sinh^2(c + dx) (a + b \sinh^4(c + dx))^2 dx &= \frac{\text{Subst} \left(\int \frac{x^2(a-2ax^2+(a+b)x^4)^2}{(1-x^2)^6} dx, x, \tanh(c + dx) \right)}{d} \\
&= \frac{b^2 \cosh^9(c + dx) \sinh(c + dx)}{10d} + \frac{\text{Subst} \left(\int \frac{-b^2+10(a^2-b^2)x^2-10(3a^2+b^2)x^4+10(3a^2-b^2)x^6}{(1-x^2)^5} dx, x, \tanh(c + dx) \right)}{10d} \\
&= -\frac{41b^2 \cosh^7(c + dx) \sinh(c + dx)}{80d} + \frac{b^2 \cosh^9(c + dx) \sinh(c + dx)}{10d} - \frac{\text{Subst} \left(\int \frac{10b^2x^2-10(3a^2+b^2)x^4+10(3a^2-b^2)x^6}{(1-x^2)^4} dx, x, \tanh(c + dx) \right)}{10d} \\
&= \frac{b(160a + 513b) \cosh^5(c + dx) \sinh(c + dx)}{480d} - \frac{41b^2 \cosh^7(c + dx) \sinh(c + dx)}{80d} \\
&= -\frac{b(416a + 447b) \cosh^3(c + dx) \sinh(c + dx)}{384d} + \frac{b(160a + 513b) \cosh^5(c + dx) \sinh(c + dx)}{480d} \\
&= \frac{(128a^2 + 352ab + 193b^2) \cosh(c + dx) \sinh(c + dx)}{256d} - \frac{b(416a + 447b) \cosh^3(c + dx) \sinh(c + dx)}{384d} \\
&= -\frac{1}{256} (128a^2 + 160ab + 63b^2) x + \frac{(128a^2 + 352ab + 193b^2) \cosh(c + dx) \sinh(c + dx)}{256d}
\end{aligned}$$

Mathematica [A] time = 0.352497, size = 139, normalized size = 0.86

$$-\frac{60(128a^2 + 240ab + 105b^2) \sinh(2(c + dx)) + 15360a^2c + 15360a^2dx - 320ab \sinh(6(c + dx)) + 360b(8a + 5b) \sinh(8(c + dx))}{30720d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] $-\frac{(15360a^2c + 19200a^2dx + 19200ab \sinh(2(c + dx)) + 7560b^2 \sinh(4(c + dx)) - 60(128a^2 + 240ab + 105b^2) \sinh(2(c + dx)) + 360b(8a + 5b) \sinh(8(c + dx)) - 320ab \sinh(6(c + dx)) - 450b^2 \sinh(6(c + dx)) + 75b^2 \sinh(8(c + dx)) - 6b^2 \sinh(10(c + dx))}{30720d}$

Maple [A] time = 0.022, size = 148, normalized size = 0.9

$$\frac{1}{d} \left(b^2 \left(\frac{(\sinh(dx + c))^9}{10} - \frac{9(\sinh(dx + c))^7}{80} + \frac{21(\sinh(dx + c))^5}{160} - \frac{21(\sinh(dx + c))^3}{128} + \frac{63 \sinh(dx + c)}{256} \right) \cosh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x)`

[Out] $\frac{1}{d} * (b^2 * ((\frac{1}{10} * \sinh(d*x+c)^9 - \frac{9}{80} * \sinh(d*x+c)^7 + \frac{21}{160} * \sinh(d*x+c)^5 - \frac{21}{12} * \sinh(d*x+c)^3 + \frac{63}{256} * \sinh(d*x+c)) * \cosh(d*x+c) - \frac{63}{256} * d*x - \frac{63}{256} * c) + 2 * a * b * ((\frac{1}{6} * \sinh(d*x+c)^5 - \frac{5}{24} * \sinh(d*x+c)^3 + \frac{5}{16} * \sinh(d*x+c)) * \cosh(d*x+c) - \frac{5}{16} * d * x - \frac{5}{16} * c) + a^2 * (\frac{1}{2} * \cosh(d*x+c) * \sinh(d*x+c) - \frac{1}{2} * d * x - \frac{1}{2} * c))$

Maxima [A] time = 1.10221, size = 351, normalized size = 2.18

$$-\frac{1}{8} a^2 \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) - \frac{1}{20480} b^2 \left(\frac{(25e^{(-2dx-2c)} - 150e^{(-4dx-4c)} + 600e^{(-6dx-6c)} - 2100e^{(-8dx-8c)} - 2)e^{(10dx+10c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")`

[Out] $-\frac{1}{8} * a^2 * (4 * x - \frac{e^{(2 * d * x + 2 * c)}}{d} + \frac{e^{(-2 * d * x - 2 * c)}}{d}) - \frac{1}{20480} * b^2 * ((25 * e^{(-2 * d * x - 2 * c)} - 150 * e^{(-4 * d * x - 4 * c)} + 600 * e^{(-6 * d * x - 6 * c)} - 2100 * e^{(-8 * d * x - 8 * c)} - 2) * e^{(10 * d * x + 10 * c)} / d + 5040 * (d * x + c) / d + (2100 * e^{(-2 * d * x - 2 * c)} - 600 * e^{(-4 * d * x - 4 * c)} + 150 * e^{(-6 * d * x - 6 * c)} - 25 * e^{(-8 * d * x - 8 * c)} + 2 * e^{(-10 * d * x - 10 * c)}) / d - \frac{1}{192} * a * b * ((9 * e^{(-2 * d * x - 2 * c)} - 45 * e^{(-4 * d * x - 4 * c)} - 1) * e^{(6 * d * x + 6 * c)} / d + 120 * (d * x + c) / d + (45 * e^{(-2 * d * x - 2 * c)} - 9 * e^{(-4 * d * x - 4 * c)} + e^{(-6 * d * x - 6 * c)}) / d)$

Fricas [B] time = 1.65095, size = 802, normalized size = 4.98

$$\frac{15 b^2 \cosh(dx + c) \sinh(dx + c)^9 + 30 (6 b^2 \cosh(dx + c)^3 - 5 b^2 \cosh(dx + c)) \sinh(dx + c)^7 + 3 (126 b^2 \cosh(dx + c)^5 - 350 b^2 \cosh(dx + c)^3 + 5 (32 a b + 45 b^2) \cosh(dx + c)) \sinh(dx + c)^5 + 10 (18 b^2 \cosh(dx + c)^7 - 105 b^2 \cosh(dx + c)^5 + 5 (32 a b + 45 b^2) \cosh(dx + c)) \sinh(dx + c)^3 + 3 (6 b^2 \cosh(dx + c)^3 - 5 b^2 \cosh(dx + c)) \sinh(dx + c) + a^2 \cosh(dx + c) \sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")`

[Out] $\frac{1}{7680} * (15 * b^2 * \cosh(d * x + c) * \sinh(d * x + c)^9 + 30 * (6 * b^2 * \cosh(d * x + c)^3 - 5 * b^2 * \cosh(d * x + c)) * \sinh(d * x + c)^7 + 3 * (126 * b^2 * \cosh(d * x + c)^5 - 350 * b^2 * \cosh(d * x + c)^3 + 5 * (32 * a * b + 45 * b^2) * \cosh(d * x + c)) * \sinh(d * x + c)^5 + 10 * (18 * b^2 * \cosh(d * x + c)^7 - 105 * b^2 * \cosh(d * x + c)^5 + 5 * (32 * a * b + 45 * b^2) * \cosh(d * x + c)) * \sinh(d * x + c)^3 + 3 * (6 * b^2 * \cosh(d * x + c)^3 - 5 * b^2 * \cosh(d * x + c)) * \sinh(d * x + c) + a^2 * \cosh(d * x + c) * \sinh(d * x + c)) / d$

$$\frac{h(dx + c)^3 - 36(8ab + 5b^2)\cosh(dx + c)\sinh(dx + c)^3 - 30(128a^2 + 160ab + 63b^2)dx + 15(b^2\cosh(dx + c)^9 - 10b^2\cosh(dx + c)^7 + (32ab + 45b^2)\cosh(dx + c)^5 - 24(8ab + 5b^2)\cosh(dx + c)^3 + 2(128a^2 + 240ab + 105b^2)\cosh(dx + c)\sinh(dx + c))/d}{x(a + b\sinh^4(c))^2 \sinh^2(c)}$$

Sympy [A] time = 60.8387, size = 484, normalized size = 3.01

$$\left\{ \begin{array}{l} \frac{a^2x \sinh^2(c+dx)}{2} - \frac{a^2x \cosh^2(c+dx)}{2} + \frac{a^2 \sinh(c+dx) \cosh(c+dx)}{2d} + \frac{5abx \sinh^6(c+dx)}{8} - \frac{15abx \sinh^4(c+dx) \cosh^2(c+dx)}{8} + \frac{15abx \sinh^2(c+dx) \cosh^2(c+dx)}{8} \\ x(a + b\sinh^4(c))^2 \sinh^2(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)**2*(a+b*sinh(dx+c)**4)**2,x)

[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + 63*b**2*x*sinh(c + d*x)**10/256 - 315*b**2*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 315*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 315*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b**2*x*cosh(c + d*x)**10/256 + 193*b**2*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b**2*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21*b**2*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 147*b**2*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 63*b**2*sinh(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c)**2, True))

Giac [B] time = 1.34792, size = 428, normalized size = 2.66

$$6b^2e^{(10dx+10c)} - 75b^2e^{(8dx+8c)} + 320abe^{(6dx+6c)} + 450b^2e^{(6dx+6c)} - 2880abe^{(4dx+4c)} - 1800b^2e^{(4dx+4c)} + 7680a^2e^{(2dx+2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^2*(a+b*sinh(dx+c)^4)^2,x, algorithm="giac")

[Out] $\frac{1}{61440} \cdot (6b^2e^{(10dx + 10c)} - 75b^2e^{(8dx + 8c)} + 320ab e^{(6dx + 6c)} + 450b^2e^{(6dx + 6c)} - 2880ab e^{(4dx + 4c)} - 1800b^2e^{(4dx + 4c)} + 7680a^2e^{(2dx + 2c)} + 14400ab e^{(2dx + 2c)} + 6300b^2e^{(2dx + 2c)} - 240(128a^2 + 160ab + 63b^2)(dx + c) + (35072a^2e^{(10dx + 10c)} + 43840ab e^{(10dx + 10c)} + 17262b^2e^{(10dx + 10c)} - 7680a^2e^{(8dx + 8c)} - 14400ab e^{(8dx + 8c)} - 6300b^2e^{(8dx + 8c)} + 2880ab e^{(6dx + 6c)} + 1800b^2e^{(6dx + 6c)} - 320ab e^{(4dx + 4c)} - 450b^2e^{(4dx + 4c)} + 75b^2e^{(2dx + 2c)} - 6b^2e^{(-10dx - 10c)})/d$

3.198 $\int \sinh(c + dx) \left(a + b \sinh^4(c + dx) \right)^2 dx$

Optimal. Leaf size=92

$$\frac{2b(a+3b)\cosh^5(c+dx)}{5d} - \frac{4b(a+b)\cosh^3(c+dx)}{3d} + \frac{(a+b)^2\cosh(c+dx)}{d} + \frac{b^2\cosh^9(c+dx)}{9d} - \frac{4b^2\cosh^7(c+dx)}{7d}$$

[Out] ((a + b)^2*Cosh[c + d*x])/d - (4*b*(a + b)*Cosh[c + d*x]^3)/(3*d) + (2*b*(a + 3*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[c + d*x]^9)/(9*d)

Rubi [A] time = 0.0860677, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3215, 1090}

$$\frac{2b(a+3b)\cosh^5(c+dx)}{5d} - \frac{4b(a+b)\cosh^3(c+dx)}{3d} + \frac{(a+b)^2\cosh(c+dx)}{d} + \frac{b^2\cosh^9(c+dx)}{9d} - \frac{4b^2\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] ((a + b)^2*Cosh[c + d*x])/d - (4*b*(a + b)*Cosh[c + d*x]^3)/(3*d) + (2*b*(a + 3*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[c + d*x]^9)/(9*d)

Rule 3215

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 1090

Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0]

Rubi steps

$$\int \sinh(c + dx) (a + b \sinh^4(c + dx))^2 dx = \frac{\text{Subst}\left(\int (a + b - 2bx^2 + bx^4)^2 dx, x, \cosh(c + dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2 \left(1 + \frac{b(2a+b)}{a^2}\right) - 4ab \left(1 + \frac{b}{a}\right) x^2 + 2ab \left(1 + \frac{3b}{a}\right) x^4 - 4b^2 x^6 + b^2 x^8\right) dx, x, \cosh(c + dx)\right)}{d}$$

$$= \frac{(a + b)^2 \cosh(c + dx)}{d} - \frac{4b(a + b) \cosh^3(c + dx)}{3d} + \frac{2b(a + 3b) \cosh^5(c + dx)}{5d}$$

Mathematica [A] time = 0.0440235, size = 164, normalized size = 1.78

$$\frac{a^2 \sinh(c) \sinh(dx)}{d} + \frac{a^2 \cosh(c) \cosh(dx)}{d} + \frac{5ab \cosh(c + dx)}{4d} - \frac{5ab \cosh(3(c + dx))}{24d} + \frac{ab \cosh(5(c + dx))}{40d} + \frac{63b^2 \cosh(7(c + dx))}{128d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] (a^2*Cosh[c]*Cosh[d*x])/d + (5*a*b*Cosh[c + d*x])/(4*d) + (63*b^2*Cosh[c + d*x])/(128*d) - (5*a*b*Cosh[3*(c + d*x)])/(24*d) - (7*b^2*Cosh[3*(c + d*x)])/(64*d) + (a*b*Cosh[5*(c + d*x)])/(40*d) + (9*b^2*Cosh[5*(c + d*x)])/(320*d) - (9*b^2*Cosh[7*(c + d*x)])/(1792*d) + (b^2*Cosh[9*(c + d*x)])/(2304*d) + (a^2*Sinh[c]*Sinh[d*x])/d

Maple [A] time = 0.021, size = 100, normalized size = 1.1

$$\frac{1}{d} \left(b^2 \left(\frac{128}{315} + \frac{(\sinh(dx + c))^8}{9} - \frac{8(\sinh(dx + c))^6}{63} + \frac{16(\sinh(dx + c))^4}{105} - \frac{64(\sinh(dx + c))^2}{315} \right) \cosh(dx + c) + 2ab \left(\frac{8}{15} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x)

[Out] 1/d*(b^2*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*cosh(d*x+c)

Maxima [B] time = 1.04689, size = 305, normalized size = 3.32

$$-\frac{1}{161280} b^2 \left(\frac{(405 e^{(-2dx-2c)} - 2268 e^{(-4dx-4c)} + 8820 e^{(-6dx-6c)} - 39690 e^{(-8dx-8c)} - 35) e^{(9dx+9c)}}{d} - \frac{39690 e^{(-dx-c)} - 8820 e^{(-3dx-3c)} + 2268 e^{(-5dx-5c)} - 405 e^{(-7dx-7c)} + 35 e^{(-9dx-9c)}}{d} + \frac{1}{240} a b (3 e^{(5dx+5c)})/d - 25 e^{(3dx+3c)}/d + 150 e^{(dx+c)}/d + 150 e^{(-dx-c)}/d - 25 e^{(-3dx-3c)}/d + 3 e^{(-5dx-5c)}/d + a^2 \cosh(dx+c)/d \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out] -1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^2*cosh(d*x + c)/d

Fricas [B] time = 1.63248, size = 740, normalized size = 8.04

$$\frac{35 b^2 \cosh(dx+c)^9 + 315 b^2 \cosh(dx+c) \sinh(dx+c)^8 - 405 b^2 \cosh(dx+c)^7 + 105 (28 b^2 \cosh(dx+c)^3 - 27 b^2 \cosh(dx+c) \sinh(dx+c)^2) \cosh(dx+c)^5 + 105 (28 b^2 \cosh(dx+c)^3 - 27 b^2 \cosh(dx+c) \sinh(dx+c)^2) \sinh(dx+c)^5 - 405 b^2 \cosh(dx+c)^7 + 105 (28 b^2 \cosh(dx+c)^3 - 27 b^2 \cosh(dx+c) \sinh(dx+c)^2) \cosh(dx+c)^5 + 105 (28 b^2 \cosh(dx+c)^3 - 27 b^2 \cosh(dx+c) \sinh(dx+c)^2) \sinh(dx+c)^5}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out] 1/80640*(35*b^2*cosh(d*x + c)^9 + 315*b^2*cosh(d*x + c)*sinh(d*x + c)^8 - 405*b^2*cosh(d*x + c)^7 + 105*(28*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c)*sinh(d*x + c)^2)*cosh(d*x + c)^5 + 315*(14*b^2*cosh(d*x + c)^5 - 45*b^2*cosh(d*x + c)^3 + 4*(8*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 420*(40*a*b + 21*b^2)*cosh(d*x + c)^3 + 315*(4*b^2*cosh(d*x + c)^3 + 315*(4*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c)*sinh(d*x + c)^2)*cosh(d*x + c)^5 + 8*(8*a*b + 9*b^2)*cosh(d*x + c)^3 - 4*(40*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 630*(128*a^2 + 160*a*b + 63*b^2)*cosh(d*x + c))/d

Sympy [A] time = 43.7265, size = 204, normalized size = 2.22

$$\left\{ \frac{a^2 \cosh(c+dx)}{d} + \frac{2ab \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{8ab \sinh^2(c+dx) \cosh^3(c+dx)}{3d} + \frac{16ab \cosh^5(c+dx)}{15d} + \frac{b^2 \sinh^8(c+dx) \cosh(c+dx)}{d} - \frac{8b^2 \sinh^6(c+dx) \cosh(c+dx)}{d} \right\} x (a + b \sinh^4(c))^2 \sinh(c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**2,x)

[Out] Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*d) + b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*cosh(c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b**2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c), True))

Giac [B] time = 1.27739, size = 331, normalized size = 3.6

$$35 b^2 e^{(9dx+9c)} - 405 b^2 e^{(7dx+7c)} + 2016 abe^{(5dx+5c)} + 2268 b^2 e^{(5dx+5c)} - 16800 abe^{(3dx+3c)} - 8820 b^2 e^{(3dx+3c)} + 80640 a^2 e^{(2dx+2c)} + 35 b^2 e^{(-9dx-9c)}/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] 1/161280*(35*b^2*e^(9*d*x + 9*c) - 405*b^2*e^(7*d*x + 7*c) + 2016*a*b*e^(5*d*x + 5*c) + 2268*b^2*e^(5*d*x + 5*c) - 16800*a*b*e^(3*d*x + 3*c) - 8820*b^2*e^(3*d*x + 3*c) + 80640*a^2*e^(d*x + c) + 100800*a*b*e^(d*x + c) + 39690*b^2*e^(d*x + c) + (80640*a^2*e^(8*d*x + 8*c) + 100800*a*b*e^(8*d*x + 8*c) + 39690*b^2*e^(8*d*x + 8*c) - 16800*a*b*e^(6*d*x + 6*c) - 8820*b^2*e^(6*d*x + 6*c) + 2016*a*b*e^(4*d*x + 4*c) + 2268*b^2*e^(4*d*x + 4*c) - 405*b^2*e^(2*d*x + 2*c) + 35*b^2)*e^(-9*d*x - 9*c))/d

3.199 $\int (a + b \sinh^4(c + dx))^2 dx$

Optimal. Leaf size=125

$$\frac{1}{128}x(128a^2 + 96ab + 35b^2) + \frac{b(96a + 163b) \sinh(c + dx) \cosh^3(c + dx)}{192d} - \frac{b(160a + 93b) \sinh(c + dx) \cosh(c + dx)}{128d} +$$

```
[Out] ((128*a^2 + 96*a*b + 35*b^2)*x)/128 - (b*(160*a + 93*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(128*d) + (b*(96*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d
) - (25*b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^2*Cosh[c + d*x]^7*Si
nh[c + d*x])/(8*d)
```

Rubi [A] time = 0.161943, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3209, 1157, 1814, 385, 206}

$$\frac{1}{128}x(128a^2 + 96ab + 35b^2) + \frac{b(96a + 163b) \sinh(c + dx) \cosh^3(c + dx)}{192d} - \frac{b(160a + 93b) \sinh(c + dx) \cosh(c + dx)}{128d} +$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((128*a^2 + 96*a*b + 35*b^2)*x)/128 - (b*(160*a + 93*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(128*d) + (b*(96*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d
) - (25*b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^2*Cosh[c + d*x]^7*Si
nh[c + d*x])/(8*d)
```

Rule 3209

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^4)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
+ b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1157

```
Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
```

```
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^4(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^2}{(1-x^2)^5} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b^2 \cosh^7(c + dx) \sinh(c + dx)}{8d} - \frac{\text{Subst}\left(\int \frac{-8a^2+b^2+8(3a^2+b^2)x^2-8(3a-b)(a+b)x^4+8(a+b)^2x^6}{(1-x^2)^4} dx, x\right)}{8d} \\
&= -\frac{25b^2 \cosh^5(c + dx) \sinh(c + dx)}{48d} + \frac{b^2 \cosh^7(c + dx) \sinh(c + dx)}{8d} + \frac{\text{Subst}\left(\int \frac{48a^2+19b^2}{(1-x^2)^3} dx, x\right)}{8d} \\
&= \frac{b(96a + 163b) \cosh^3(c + dx) \sinh(c + dx)}{192d} - \frac{25b^2 \cosh^5(c + dx) \sinh(c + dx)}{48d} + \frac{b^2 \cosh^7(c + dx) \sinh(c + dx)}{8d} \\
&= -\frac{b(160a + 93b) \cosh(c + dx) \sinh(c + dx)}{128d} + \frac{b(96a + 163b) \cosh^3(c + dx) \sinh(c + dx)}{192d} \\
&= \frac{1}{128} (128a^2 + 96ab + 35b^2) x - \frac{b(160a + 93b) \cosh(c + dx) \sinh(c + dx)}{128d} + \frac{b(96a + 163b) \cosh^3(c + dx) \sinh(c + dx)}{192d}
\end{aligned}$$

Mathematica [A] time = 0.165761, size = 92, normalized size = 0.74

$$\frac{24(128a^2 + 96ab + 35b^2)(c + dx) - 96b(16a + 7b) \sinh(2(c + dx)) + 24b(8a + 7b) \sinh(4(c + dx)) - 32b^2 \sinh(6(c + dx)) + 3b^2 \sinh(8(c + dx))}{3072d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^4)^2,x]

[Out] (24*(128*a^2 + 96*a*b + 35*b^2)*(c + d*x) - 96*b*(16*a + 7*b)*Sinh[2*(c + d*x)] + 24*b*(8*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh[6*(c + d*x)] + 3*b^2*Sinh[8*(c + d*x)])/(3072*d)

Maple [A] time = 0.017, size = 111, normalized size = 0.9

$$\frac{1}{d} \left(b^2 \left(\left(\frac{(\sinh(dx + c))^7}{8} - \frac{7(\sinh(dx + c))^5}{48} + \frac{35(\sinh(dx + c))^3}{192} - \frac{35\sinh(dx + c)}{128} \right) \cosh(dx + c) + \frac{35dx}{128} + \frac{35c}{128} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(d*x+c)^4)^2,x)

[Out] $\frac{1}{d} * (b^2 * ((\frac{1}{8} * \sinh(d*x+c)^7 - \frac{7}{48} * \sinh(d*x+c)^5 + \frac{35}{192} * \sinh(d*x+c)^3 - \frac{35}{128} * \sinh(d*x+c)) * \cosh(d*x+c) + \frac{35}{128} * d*x + \frac{35}{128} * c) + 2 * a * b * ((\frac{1}{4} * \sinh(d*x+c)^3 - \frac{3}{8} * \sinh(d*x+c)) * \cosh(d*x+c) + \frac{3}{8} * d*x + \frac{3}{8} * c) + a^2 * (d*x+c))$

Maxima [A] time = 1.11707, size = 247, normalized size = 1.98

$$\frac{1}{32} ab \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) + a^2 x - \frac{1}{6144} b^2 \left(\frac{32e^{-2dx-2c} - 168e^{-4dx-4c} + 672e^{-6dx-6c} - 32e^{-8dx-8c}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out] $\frac{1}{32} * a * b * (24 * x + \frac{e^{4 * d * x + 4 * c}}{d} - \frac{8 * e^{2 * d * x + 2 * c}}{d} + \frac{8 * e^{-2 * d * x - 2 * c}}{d} - \frac{e^{-4 * d * x - 4 * c}}{d}) + a^2 * x - \frac{1}{6144} * b^2 * ((32 * e^{-2 * d * x - 2 * c} - 168 * e^{-4 * d * x - 4 * c} + 672 * e^{-6 * d * x - 6 * c} - 32 * e^{-8 * d * x - 8 * c}) / d - 1680 * (d * x + c) / d - (672 * e^{-2 * d * x - 2 * c} - 168 * e^{-4 * d * x - 4 * c} + 32 * e^{-6 * d * x - 6 * c} - 3 * e^{-8 * d * x - 8 * c}) / d)$

Fricas [A] time = 1.62932, size = 522, normalized size = 4.18

$$3b^2 \cosh(dx+c) \sinh(dx+c)^7 + 3(7b^2 \cosh(dx+c)^3 - 8b^2 \cosh(dx+c)) \sinh(dx+c)^5 + (21b^2 \cosh(dx+c)^5 - 80b^2 \cosh(dx+c)^3 + 12(8ab + 7b^2) \cosh(dx+c)) \sinh(dx+c)^3 + 3(128a^2 + 96ab + 35b^2) dx + 3(b^2 \cosh(dx+c)^7 - 8b^2 \cosh(dx+c)^5 + 4(8ab + 7b^2) \cosh(dx+c)^3 - 8(16ab + 7b^2) \cosh(dx+c)) \sinh(dx+c) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out] $\frac{1}{384} * (3 * b^2 * \cosh(d * x + c) * \sinh(d * x + c)^7 + 3 * (7 * b^2 * \cosh(d * x + c)^3 - 8 * b^2 * \cosh(d * x + c)) * \sinh(d * x + c)^5 + (21 * b^2 * \cosh(d * x + c)^5 - 80 * b^2 * \cosh(d * x + c)^3 + 12 * (8 * a * b + 7 * b^2) * \cosh(d * x + c)) * \sinh(d * x + c)^3 + 3 * (128 * a^2 + 96 * a * b + 35 * b^2) * d * x + 3 * (b^2 * \cosh(d * x + c)^7 - 8 * b^2 * \cosh(d * x + c)^5 + 4 * (8 * a * b + 7 * b^2) * \cosh(d * x + c)^3 - 8 * (16 * a * b + 7 * b^2) * \cosh(d * x + c)) * \sinh(d * x + c)) / d$

Sympy [A] time = 19.2012, size = 332, normalized size = 2.66

$$\left\{ \begin{array}{l} a^2x + \frac{3abx \sinh^4(c+dx)}{4} - \frac{3abx \sinh^2(c+dx) \cosh^2(c+dx)}{2} + \frac{3abx \cosh^4(c+dx)}{4} + \frac{5ab \sinh^3(c+dx) \cosh(c+dx)}{4d} - \frac{3ab \sinh(c+dx) \cosh^3(c+dx)}{4d} + \\ x(a + b \sinh^4(c))^2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)**4)**2,x)

[Out] Piecewise((a**2*x + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) + 35*b**2*x*sinh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2, True))

Giac [B] time = 1.18992, size = 319, normalized size = 2.55

$$3b^2e^{(8dx+8c)} - 32b^2e^{(6dx+6c)} + 192abe^{(4dx+4c)} + 168b^2e^{(4dx+4c)} - 1536abe^{(2dx+2c)} - 672b^2e^{(2dx+2c)} + 48(128a^2 + 96a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] 1/6144*(3*b^2*e^(8*d*x + 8*c) - 32*b^2*e^(6*d*x + 6*c) + 192*a*b*e^(4*d*x + 4*c) + 168*b^2*e^(4*d*x + 4*c) - 1536*a*b*e^(2*d*x + 2*c) - 672*b^2*e^(2*d*x + 2*c) + 48*(128*a^2 + 96*a*b + 35*b^2)*(d*x + c) - (6400*a^2*e^(8*d*x + 8*c) + 4800*a*b*e^(8*d*x + 8*c) + 1750*b^2*e^(8*d*x + 8*c) - 1536*a*b*e^(6*d*x + 6*c) - 672*b^2*e^(6*d*x + 6*c) + 192*a*b*e^(4*d*x + 4*c) + 168*b^2*e^(4*d*x + 4*c) - 32*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-8*d*x - 8*c))/d

3.200 $\int \operatorname{csch}(c + dx) \left(a + b \sinh^4(c + dx) \right)^2 dx$

Optimal. Leaf size=92

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b(2a + 3b) \cosh^3(c + dx)}{3d} - \frac{b(2a + b) \cosh(c + dx)}{d} + \frac{b^2 \cosh^7(c + dx)}{7d} - \frac{3b^2 \cosh^5(c + dx)}{5d}$$

[Out] $-\left(\frac{a^2 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) - \frac{b*(2*a + b)*\operatorname{Cosh}[c + d*x]}{d} + \frac{b*(2*a + 3*b)*\operatorname{Cosh}[c + d*x]^3}{(3*d)} - \frac{(3*b^2*\operatorname{Cosh}[c + d*x]^5)}{(5*d)} + \frac{b^2*\operatorname{Cosh}[c + d*x]^7}{(7*d)}$

Rubi [A] time = 0.092699, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3215, 1153, 206}

$$-\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b(2a + 3b) \cosh^3(c + dx)}{3d} - \frac{b(2a + b) \cosh(c + dx)}{d} + \frac{b^2 \cosh^7(c + dx)}{7d} - \frac{3b^2 \cosh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]*(a + b*\operatorname{Sinh}[c + d*x]^4)^2, x]$

[Out] $-\left(\frac{a^2 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) - \frac{b*(2*a + b)*\operatorname{Cosh}[c + d*x]}{d} + \frac{b*(2*a + 3*b)*\operatorname{Cosh}[c + d*x]^3}{(3*d)} - \frac{(3*b^2*\operatorname{Cosh}[c + d*x]^5)}{(5*d)} + \frac{b^2*\operatorname{Cosh}[c + d*x]^7}{(7*d)}$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x]] \;/; \operatorname{FreeQ}[\{a, b, e, f, p\}, x] \ \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 1153

$\operatorname{Int}[(d_.) + (e_.)*(x_.)^2]^{(q_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] \;/; \operatorname{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \ \&\& \operatorname{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{IGtQ}[q, -2]$

Rule 206

$\text{Int}[(a + (b \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[(\text{Rt}[-b, 2] \cdot x) / \text{Rt}[a, 2]]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] / ; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \text{csch}(c + dx) (a + b \sinh^4(c + dx))^2 dx &= -\frac{\text{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^2}{1-x^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \left(b(2a+b) - b(2a+3b)x^2 + 3b^2x^4 - b^2x^6 + \frac{a^2}{1-x^2}\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{b(2a+b)\cosh(c + dx)}{d} + \frac{b(2a+3b)\cosh^3(c + dx)}{3d} - \frac{3b^2\cosh^5(c + dx)}{5d} \\ &= -\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{d} - \frac{b(2a+b)\cosh(c + dx)}{d} + \frac{b(2a+3b)\cosh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0411324, size = 146, normalized size = 1.59

$$\frac{a^2 \log\left(\sinh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{a^2 \log\left(\cosh\left(\frac{c}{2} + \frac{dx}{2}\right)\right)}{d} - \frac{3ab \cosh(c + dx)}{2d} + \frac{ab \cosh(3(c + dx))}{6d} - \frac{35b^2 \cosh(c + dx)}{64d} + \frac{7b^2 \cosh(5(c + dx))}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] $(-3*a*b*\text{Cosh}[c + d*x])/(2*d) - (35*b^2*\text{Cosh}[c + d*x])/(64*d) + (a*b*\text{Cosh}[3*(c + d*x)])/(6*d) + (7*b^2*\text{Cosh}[3*(c + d*x)])/(64*d) - (7*b^2*\text{Cosh}[5*(c + d*x)])/(320*d) + (b^2*\text{Cosh}[7*(c + d*x)])/(448*d) - (a^2*\text{Log}[\text{Cosh}[c/2 + (d*x)/2]])/d + (a^2*\text{Log}[\text{Sinh}[c/2 + (d*x)/2]])/d$

Maple [A] time = 0.039, size = 82, normalized size = 0.9

$$\frac{1}{d} \left(-2a^2 \text{Artanh}(e^{dx+c}) + 2ab \left(-\frac{2}{3} + \frac{1}{3} (\sinh(dx+c))^2 \right) \cosh(dx+c) + b^2 \left(-\frac{16}{35} + \frac{(\sinh(dx+c))^6}{7} - \frac{6(\sinh(dx+c))^4}{35} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x)

[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c))

Maxima [B] time = 1.05125, size = 239, normalized size = 2.6

$$-\frac{1}{4480} b^2 \left(\frac{(49 e^{(-2dx-2c)} - 245 e^{(-4dx-4c)} + 1225 e^{(-6dx-6c)} - 5) e^{(7dx+7c)}}{d} + \frac{1225 e^{(-dx-c)} - 245 e^{(-3dx-3c)} + 49 e^{(-5dx-5c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out] -1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/12*a*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^2*log(tanh(1/2*d*x + 1/2*c))/d

Fricas [B] time = 1.92082, size = 4263, normalized size = 46.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out] 1/13440*(15*b^2*cosh(d*x + c)^14 + 210*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 15*b^2*sinh(d*x + c)^14 - 147*b^2*cosh(d*x + c)^12 + 21*(65*b^2*cosh(d*x + c)^2 - 7*b^2)*sinh(d*x + c)^12 + 84*(65*b^2*cosh(d*x + c)^3 - 21*b^2*cosh(d*x + c))*sinh(d*x + c)^11 + 35*(32*a*b + 21*b^2)*cosh(d*x + c)^10 + 7*(214*5*b^2*cosh(d*x + c)^4 - 1386*b^2*cosh(d*x + c)^2 + 160*a*b + 105*b^2)*sinh(d*x + c)^10 + 70*(429*b^2*cosh(d*x + c)^5 - 462*b^2*cosh(d*x + c)^3 + 5*(32*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 105*(96*a*b + 35*b^2)*cosh(d*x + c)^8 + 105*(429*b^2*cosh(d*x + c)^6 - 693*b^2*cosh(d*x + c)^4 + 15*(3*2*a*b + 21*b^2)*cosh(d*x + c)^2 - 96*a*b - 35*b^2)*sinh(d*x + c)^8 + 24*(21

$$\begin{aligned}
&45b^2 \cosh(dx+c)^7 - 4851b^2 \cosh(dx+c)^5 + 175(32ab + 21b^2) \cosh(dx+c)^3 - 35(96ab + 35b^2) \cosh(dx+c) \sinh(dx+c)^7 - 105(96ab + 35b^2) \cosh(dx+c)^6 + 21(2145b^2 \cosh(dx+c)^8 - 6468b^2 \cosh(dx+c)^6 + 350(32ab + 21b^2) \cosh(dx+c)^4 - 140(96ab + 35b^2) \cosh(dx+c)^2 - 480ab - 175b^2) \sinh(dx+c)^6 + 42(715b^2 \cosh(dx+c)^9 - 2772b^2 \cosh(dx+c)^7 + 210(32ab + 21b^2) \cosh(dx+c)^5 - 140(96ab + 35b^2) \cosh(dx+c)^3 - 15(96ab + 35b^2) \cosh(dx+c)) \sinh(dx+c)^5 + 35(32ab + 21b^2) \cosh(dx+c)^4 + 35(429b^2 \cosh(dx+c)^{10} - 2079b^2 \cosh(dx+c)^8 + 210(32ab + 21b^2) \cosh(dx+c)^6 - 210(96ab + 35b^2) \cosh(dx+c)^4 - 45(96ab + 35b^2) \cosh(dx+c)^2 + 32ab + 21b^2) \sinh(dx+c)^4 - 147b^2 \cosh(dx+c)^2 + 140(39b^2 \cosh(dx+c)^{11} - 231b^2 \cosh(dx+c)^9 + 30(32ab + 21b^2) \cosh(dx+c)^7 - 42(96ab + 35b^2) \cosh(dx+c)^5 - 15(96ab + 35b^2) \cosh(dx+c)^3 + (32ab + 21b^2) \cosh(dx+c)) \sinh(dx+c)^3 + 21(65b^2 \cosh(dx+c)^{12} - 462b^2 \cosh(dx+c)^{10} + 75(32ab + 21b^2) \cosh(dx+c)^8 - 140(96ab + 35b^2) \cosh(dx+c)^6 - 75(96ab + 35b^2) \cosh(dx+c)^4 + 10(32ab + 21b^2) \cosh(dx+c)^2 - 7b^2) \sinh(dx+c)^2 + 15b^2 - 13440(a^2 \cosh(dx+c)^7 + 7a^2 \cosh(dx+c)^6 \sinh(dx+c) + 21a^2 \cosh(dx+c)^5 \sinh(dx+c)^2 + 35a^2 \cosh(dx+c)^4 \sinh(dx+c)^3 + 35a^2 \cosh(dx+c)^3 \sinh(dx+c)^4 + 21a^2 \cosh(dx+c)^2 \sinh(dx+c)^5 + 7a^2 \cosh(dx+c) \sinh(dx+c)^6 + a^2 \sinh(dx+c)^7) \log(\cosh(dx+c) + \sinh(dx+c) + 1) + 13440(a^2 \cosh(dx+c)^7 + 7a^2 \cosh(dx+c)^6 \sinh(dx+c) + 21a^2 \cosh(dx+c)^5 \sinh(dx+c)^2 + 35a^2 \cosh(dx+c)^4 \sinh(dx+c)^3 + 35a^2 \cosh(dx+c)^3 \sinh(dx+c)^4 + 21a^2 \cosh(dx+c)^2 \sinh(dx+c)^5 + 7a^2 \cosh(dx+c) \sinh(dx+c)^6 + a^2 \sinh(dx+c)^7) \log(\cosh(dx+c) + \sinh(dx+c) - 1) + 14(15b^2 \cosh(dx+c)^{13} - 126b^2 \cosh(dx+c)^{11} + 25(32ab + 21b^2) \cosh(dx+c)^9 - 60(96ab + 35b^2) \cosh(dx+c)^7 - 45(96ab + 35b^2) \cosh(dx+c)^5 + 10(32ab + 21b^2) \cosh(dx+c)^3 - 21b^2 \cosh(dx+c)) \sinh(dx+c) / (d \cosh(dx+c)^7 + 7d \cosh(dx+c)^6 \sinh(dx+c) + 21d \cosh(dx+c)^5 \sinh(dx+c)^2 + 35d \cosh(dx+c)^4 \sinh(dx+c)^3 + 35d \cosh(dx+c)^3 \sinh(dx+c)^4 + 21d \cosh(dx+c)^2 \sinh(dx+c)^5 + 7d \cosh(dx+c) \sinh(dx+c)^6 + d \sinh(dx+c)^7)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)*(a+b*sinh(dx+c)**4)**2,x)

[Out] Timed out

Giac [B] time = 1.30251, size = 301, normalized size = 3.27

$$-\frac{a^2 \log(e^{(dx+c)} + 1)}{d} + \frac{a^2 \log(|e^{(dx+c)} - 1|)}{d} - \frac{(10080 abe^{(6dx+6c)} + 3675 b^2 e^{(6dx+6c)} - 1120 abe^{(4dx+4c)} - 735 b^2 e^{(4dx+4c)} + 147 b^2 e^{(2dx+2c)} - 15 b^2 e^{(-7dx-7c)})}{13440 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c))^4)^2,x, algorithm="giac")

[Out] $-a^2 \log(e^{(d*x + c)} + 1)/d + a^2 \log(\text{abs}(e^{(d*x + c)} - 1))/d - 1/13440 * (10080 * a * b * e^{(6*d*x + 6*c)} + 3675 * b^2 * e^{(6*d*x + 6*c)} - 1120 * a * b * e^{(4*d*x + 4*c)} - 735 * b^2 * e^{(4*d*x + 4*c)} + 147 * b^2 * e^{(2*d*x + 2*c)} - 15 * b^2 * e^{(-7*d*x - 7*c)})/d + 1/13440 * (15 * b^2 * d^6 * e^{(7*d*x + 7*c)} - 147 * b^2 * d^6 * e^{(5*d*x + 5*c)} + 1120 * a * b * d^6 * e^{(3*d*x + 3*c)} + 735 * b^2 * d^6 * e^{(3*d*x + 3*c)} - 10080 * a * b * d^6 * e^{(d*x + c)} - 3675 * b^2 * d^6 * e^{(d*x + c)})/d^7$

3.201 $\int \operatorname{csch}^2(c + dx) \left(a + b \sinh^4(c + dx) \right)^2 dx$

Optimal. Leaf size=103

$$-\frac{a^2 \coth(c + dx)}{d} + \frac{b(16a + 11b) \sinh(c + dx) \cosh(c + dx)}{16d} - \frac{1}{16}bx(16a + 5b) + \frac{b^2 \sinh(c + dx) \cosh^5(c + dx)}{6d} - \frac{13b^2 s}{6d}$$

[Out] $-(b*(16*a + 5*b)*x)/16 - (a^2*\operatorname{Coth}[c + d*x])/d + (b*(16*a + 11*b)*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(16*d) - (13*b^2*\operatorname{Cosh}[c + d*x]^3*\operatorname{Sinh}[c + d*x])/(24*d) + (b^2*\operatorname{Cosh}[c + d*x]^5*\operatorname{Sinh}[c + d*x])/(6*d)$

Rubi [A] time = 0.194466, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3217, 1259, 1805, 453, 206}

$$-\frac{a^2 \coth(c + dx)}{d} + \frac{b(16a + 11b) \sinh(c + dx) \cosh(c + dx)}{16d} - \frac{1}{16}bx(16a + 5b) + \frac{b^2 \sinh(c + dx) \cosh^5(c + dx)}{6d} - \frac{13b^2 s}{6d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^2*(a + b*\operatorname{Sinh}[c + d*x]^4)^2, x]$

[Out] $-(b*(16*a + 5*b)*x)/16 - (a^2*\operatorname{Coth}[c + d*x])/d + (b*(16*a + 11*b)*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(16*d) - (13*b^2*\operatorname{Cosh}[c + d*x]^3*\operatorname{Sinh}[c + d*x])/(24*d) + (b^2*\operatorname{Cosh}[c + d*x]^5*\operatorname{Sinh}[c + d*x])/(6*d)$

Rule 3217

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]] /; \operatorname{FreeQ}\{a, b, e, f\}, x\} \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 1259

$\operatorname{Int}[(x_.)^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(q_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[((-d)^{(m/2 - 1)}*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^{(q + 1)}/(2*e^{(2*p + m/2)}*(q + 1)), x] + \operatorname{Dist}[(-d)^{(m/2 - 1)}/(2*e^{(2*p)}*(q + 1)), \operatorname{Int}[x^m*(d + e*x^2)^{(q + 1)}*\operatorname{ExpandToSum}[\operatorname{Together}[(1*(2*(-d)^{-(m/2 + 1)}*e^{(2*p)}*(q + 1)*(a + b*x^2 + c*x^4))^p - ((c*d^2 - b*d*e + a*e$

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^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

```

Rule 1805

```

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]

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Rule 453

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Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

```

Rule 206

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^2(c+dx) (a+b \sinh^4(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^2}{x^2(1-x^2)^4} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^2 \cosh^5(c+dx) \sinh(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-6a^2+(18a^2+b^2)x^2-6(3a-b)(a+b)x^4+6b^2}{x^2(1-x^2)^3} dx, x, \tanh(c+dx)\right)}{6d} \\
&= -\frac{13b^2 \cosh^3(c+dx) \sinh(c+dx)}{24d} + \frac{b^2 \cosh^5(c+dx) \sinh(c+dx)}{6d} + \frac{a^2 \coth(c+dx)}{d} \\
&= \frac{b(16a+11b) \cosh(c+dx) \sinh(c+dx)}{16d} - \frac{13b^2 \cosh^3(c+dx) \sinh(c+dx)}{24d} \\
&= -\frac{a^2 \coth(c+dx)}{d} + \frac{b(16a+11b) \cosh(c+dx) \sinh(c+dx)}{16d} - \frac{13b^2 \cosh^3(c+dx) \sinh(c+dx)}{24d} \\
&= -\frac{1}{16}b(16a+5b)x - \frac{a^2 \coth(c+dx)}{d} + \frac{b(16a+11b) \cosh(c+dx) \sinh(c+dx)}{16d}
\end{aligned}$$

Mathematica [A] time = 0.306401, size = 77, normalized size = 0.75

$$\frac{b((96a+45b) \sinh(2(c+dx)) - 192ac - 192adx - 9b \sinh(4(c+dx)) + b \sinh(6(c+dx)) - 60bc - 60bdx) - 192a^2 \coth(c+dx)}{192d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] (-192*a^2*Coth[c + d*x] + b*(-192*a*c - 60*b*c - 192*a*d*x - 60*b*d*x + (96*a + 45*b)*Sinh[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)

Maple [A] time = 0.038, size = 91, normalized size = 0.9

$$\frac{1}{d} \left(-a^2 \coth(dx+c) + 2ab \left(\frac{1}{2} \cosh(dx+c) \sinh(dx+c) - \frac{1}{2} dx - \frac{c}{2} \right) + b^2 \left(\left(\frac{(\sinh(dx+c))^5}{6} - \frac{5(\sinh(dx+c))^3}{24} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x)

[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c))

Maxima [A] time = 1.08813, size = 197, normalized size = 1.91

$$-\frac{1}{4}ab\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{384}b^2\left(\frac{(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)} - 9e^{(-4dx-4c)} - 1}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out] -1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1))

Fricas [B] time = 1.70794, size = 562, normalized size = 5.46

$$b^2 \cosh(dx+c)^7 + 7b^2 \cosh(dx+c) \sinh(dx+c)^6 - 10b^2 \cosh(dx+c)^5 + 5(7b^2 \cosh(dx+c)^3 - 10b^2 \cosh(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out] 1/384*(b^2*cosh(d*x + c)^7 + 7*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 10*b^2*cosh(d*x + c)^5 + 5*(7*b^2*cosh(d*x + c)^3 - 10*b^2*cosh(d*x + c))*sinh(d*x + c)^4 + 6*(16*a*b + 9*b^2)*cosh(d*x + c)^3 + (21*b^2*cosh(d*x + c)^5 - 100*b^2*cosh(d*x + c)^3 + 18*(16*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(128*a^2 + 32*a*b + 15*b^2)*cosh(d*x + c) - 24*((16*a*b + 5*b^2)*d*x - 16*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [B] time = 1.29338, size = 273, normalized size = 2.65

$$-\frac{(16ab + 5b^2)(dx + c)}{16d} + \frac{(352abe^{(6dx+6c)} + 110b^2e^{(6dx+6c)} - 96abe^{(4dx+4c)} - 45b^2e^{(4dx+4c)} + 9b^2e^{(2dx+2c)} - b^2)e^{(-6dx+6c)}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out]
$$-1/16*(16*a*b + 5*b^2)*(d*x + c)/d + 1/384*(352*a*b*e^{(6*d*x + 6*c)} + 110*b^2*e^{(6*d*x + 6*c)} - 96*a*b*e^{(4*d*x + 4*c)} - 45*b^2*e^{(4*d*x + 4*c)} + 9*b^2*e^{(2*d*x + 2*c)} - b^2)*e^{(-6*d*x - 6*c)}/d - 2*a^2/(d*(e^{(2*d*x + 2*c)} - 1)) + 1/384*(b^2*d^2*e^{(6*d*x + 6*c)} - 9*b^2*d^2*e^{(4*d*x + 4*c)} + 96*a*b*d^2*e^{(2*d*x + 2*c)} + 45*b^2*d^2*e^{(2*d*x + 2*c)})/d^3$$

3.202 $\int \operatorname{csch}^3(c + dx) (a + b \sinh^4(c + dx))^2 dx$

Optimal. Leaf size=92

$$\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b(2a + b) \cosh(c + dx)}{d} + \frac{b^2 \cosh^5(c + dx)}{5d} - \frac{2b^2 \cosh^3(c + dx)}{3d}$$

[Out] (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(2*a + b)*Cosh[c + d*x])/d - (2*b^2*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rubi [A] time = 0.134862, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3215, 1157, 1810, 206}

$$\frac{a^2 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{b(2a + b) \cosh(c + dx)}{d} + \frac{b^2 \cosh^5(c + dx)}{5d} - \frac{2b^2 \cosh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(2*a + b)*Cosh[c + d*x])/d - (2*b^2*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)

Rule 3215

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 1157

Int[((d_.) + (e_.)*(x_)^2)^(q_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],

x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1810

Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c+dx) (a+b \sinh^4(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^2}{(1-x^2)^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{a^2 \coth(c+dx) \operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-a^2-4ab-2b^2+2b(2a+3b)x^2-6b^2x^4+2b^2x^6}{1-x^2} dx, x, \cosh(c+dx)\right)}{2d} \\ &= -\frac{a^2 \coth(c+dx) \operatorname{csch}(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int (-2b(2a+b) + 4b^2x^2 - 2b^2x^4) dx, x, \cosh(c+dx)\right)}{2d} \\ &= \frac{b(2a+b) \cosh(c+dx)}{d} - \frac{2b^2 \cosh^3(c+dx)}{3d} + \frac{b^2 \cosh^5(c+dx)}{5d} - \frac{a^2 \coth(c+dx) \operatorname{csch}(c+dx)}{2d} \\ &= \frac{a^2 \tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b(2a+b) \cosh(c+dx)}{d} - \frac{2b^2 \cosh^3(c+dx)}{3d} + \frac{b^2 \cosh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.0457128, size = 144, normalized size = 1.57

$$-\frac{a^2 \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2 \operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2 \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{2ab \sinh(c) \sinh(dx)}{d} + \frac{2ab \cosh(c) \cosh(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] (2*a*b*Cosh[c]*Cosh[d*x])/d + (5*b^2*Cosh[c + d*x])/(8*d) - (5*b^2*Cosh[3*(c + d*x)])/(48*d) + (b^2*Cosh[5*(c + d*x)])/(80*d) - (a^2*Csch[(c + d*x)/2])

$$\frac{1}{(8*d)} - \frac{(a^2*\text{Log}[\text{Tanh}[(c + d*x)/2]])}{(2*d)} - \frac{(a^2*\text{Sech}[(c + d*x)/2]^2)}{(8*d)} + \frac{(2*a*b*\text{Sinh}[c]*\text{Sinh}[d*x])}{d}$$

Maple [A] time = 0.048, size = 74, normalized size = 0.8

$$\frac{1}{d} \left(a^2 \left(-\frac{\text{csch}(dx+c) \coth(dx+c)}{2} + \text{Artanh}(e^{dx+c}) \right) + 2ab \cosh(dx+c) + b^2 \left(\frac{8}{15} + \frac{(\sinh(dx+c))^4}{5} - \frac{4(\sinh(dx+c))^2}{15} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x)

[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*cosh(d*x+c)+b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))

Maxima [B] time = 1.1179, size = 275, normalized size = 2.99

$$\frac{1}{480} b^2 \left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d} \right) + ab \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out] 1/480*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))

Fricas [B] time = 1.98212, size = 6047, normalized size = 65.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")


```
[Out] 1/480*(3*b^2*cosh(d*x + c)^14 + 42*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 3*b
^2*sinh(d*x + c)^14 - 31*b^2*cosh(d*x + c)^12 + (273*b^2*cosh(d*x + c)^2 -
31*b^2)*sinh(d*x + c)^12 + 12*(91*b^2*cosh(d*x + c)^3 - 31*b^2*cosh(d*x + c
))*sinh(d*x + c)^11 + (480*a*b + 203*b^2)*cosh(d*x + c)^10 + (3003*b^2*cosh
(d*x + c)^4 - 2046*b^2*cosh(d*x + c)^2 + 480*a*b + 203*b^2)*sinh(d*x + c)^1
0 + 2*(3003*b^2*cosh(d*x + c)^5 - 3410*b^2*cosh(d*x + c)^3 + 5*(480*a*b + 2
03*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(
d*x + c)^8 + (9009*b^2*cosh(d*x + c)^6 - 15345*b^2*cosh(d*x + c)^4 + 45*(48
0*a*b + 203*b^2)*cosh(d*x + c)^2 - 480*a^2 - 480*a*b - 175*b^2)*sinh(d*x +
c)^8 + 8*(1287*b^2*cosh(d*x + c)^7 - 3069*b^2*cosh(d*x + c)^5 + 15*(480*a*b
+ 203*b^2)*cosh(d*x + c)^3 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c))*s
inh(d*x + c)^7 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^6 + (9009*b^2*c
osh(d*x + c)^8 - 28644*b^2*cosh(d*x + c)^6 + 210*(480*a*b + 203*b^2)*cosh(d
*x + c)^4 - 140*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^2 - 480*a^2 - 480*
a*b - 175*b^2)*sinh(d*x + c)^6 + 2*(3003*b^2*cosh(d*x + c)^9 - 12276*b^2*co
sh(d*x + c)^7 + 126*(480*a*b + 203*b^2)*cosh(d*x + c)^5 - 140*(96*a^2 + 96*
a*b + 35*b^2)*cosh(d*x + c)^3 - 15*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c
))*sinh(d*x + c)^5 + (480*a*b + 203*b^2)*cosh(d*x + c)^4 + (3003*b^2*cosh(d*
x + c)^10 - 15345*b^2*cosh(d*x + c)^8 + 210*(480*a*b + 203*b^2)*cosh(d*x +
c)^6 - 350*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^4 - 75*(96*a^2 + 96*a*b
+ 35*b^2)*cosh(d*x + c)^2 + 480*a*b + 203*b^2)*sinh(d*x + c)^4 - 31*b^2*co
sh(d*x + c)^2 + 4*(273*b^2*cosh(d*x + c)^11 - 1705*b^2*cosh(d*x + c)^9 + 30
*(480*a*b + 203*b^2)*cosh(d*x + c)^7 - 70*(96*a^2 + 96*a*b + 35*b^2)*cosh(d
*x + c)^5 - 25*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^3 + (480*a*b + 203*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (273*b^2*cosh(d*x + c)^12 - 2046*b^2*
cosh(d*x + c)^10 + 45*(480*a*b + 203*b^2)*cosh(d*x + c)^8 - 140*(96*a^2 + 9
6*a*b + 35*b^2)*cosh(d*x + c)^6 - 75*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x +
c)^4 + 6*(480*a*b + 203*b^2)*cosh(d*x + c)^2 - 31*b^2)*sinh(d*x + c)^2 + 3*
b^2 + 240*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*
sinh(d*x + c)^9 - 2*a^2*cosh(d*x + c)^7 + 2*(18*a^2*cosh(d*x + c)^2 - a^2)*
sinh(d*x + c)^7 + a^2*cosh(d*x + c)^5 + 14*(6*a^2*cosh(d*x + c)^3 - a^2*cos
h(d*x + c))*sinh(d*x + c)^6 + (126*a^2*cosh(d*x + c)^4 - 42*a^2*cosh(d*x +
c)^2 + a^2)*sinh(d*x + c)^5 + (126*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x +
c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(42*a^2*cosh(d*x + c)^6 - 3
5*a^2*cosh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*a^2*
cosh(d*x + c)^7 - 21*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3)*sinh(d*x
+ c)^2 + (9*a^2*cosh(d*x + c)^8 - 14*a^2*cosh(d*x + c)^6 + 5*a^2*cosh(d*x +
c)^4)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 240*(a^2*cos
h(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 -
2*a^2*cosh(d*x + c)^7 + 2*(18*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 +
a^2*cosh(d*x + c)^5 + 14*(6*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d
*x + c)^6 + (126*a^2*cosh(d*x + c)^4 - 42*a^2*cosh(d*x + c)^2 + a^2)*sinh(d
*x + c)^5 + (126*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x + c)^3 + 5*a^2*cosh(
d*x + c))*sinh(d*x + c)^4 + 2*(42*a^2*cosh(d*x + c)^6 - 35*a^2*cosh(d*x + c
)^4 + 5*a^2*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*a^2*cosh(d*x + c)^7 -
```

$$21a^2 \cosh(dx+c)^5 + 5a^2 \cosh(dx+c)^3 \sinh(dx+c)^2 + (9a^2 \cosh(dx+c)^8 - 14a^2 \cosh(dx+c)^6 + 5a^2 \cosh(dx+c)^4) \sinh(dx+c) \log(\cosh(dx+c) + \sinh(dx+c) - 1) + 2(21b^2 \cosh(dx+c)^{13} - 186b^2 \cosh(dx+c)^{11} + 5(480ab + 203b^2) \cosh(dx+c)^9 - 20(96a^2 + 96ab + 35b^2) \cosh(dx+c)^7 - 15(96a^2 + 96ab + 35b^2) \cosh(dx+c)^5 + 2(480ab + 203b^2) \cosh(dx+c)^3 - 31b^2 \cosh(dx+c)) \sinh(dx+c) / (d \cosh(dx+c)^9 + 9d \cosh(dx+c) \sinh(dx+c)^8 + d \sinh(dx+c)^9 - 2d \cosh(dx+c)^7 + 2(18d \cosh(dx+c)^2 - d) \sinh(dx+c)^7 + 14(6d \cosh(dx+c)^3 - d \cosh(dx+c)) \sinh(dx+c)^6 + d \cosh(dx+c)^5 + (126d \cosh(dx+c)^4 - 42d \cosh(dx+c)^2 + d) \sinh(dx+c)^5 + (126d \cosh(dx+c)^5 - 70d \cosh(dx+c)^3 + 5d \cosh(dx+c)) \sinh(dx+c)^4 + 2(42d \cosh(dx+c)^6 - 35d \cosh(dx+c)^4 + 5d \cosh(dx+c)^2) \sinh(dx+c)^3 + 2(18d \cosh(dx+c)^7 - 21d \cosh(dx+c)^5 + 5d \cosh(dx+c)^3) \sinh(dx+c)^2 + (9d \cosh(dx+c)^8 - 14d \cosh(dx+c)^6 + 5d \cosh(dx+c)^4) \sinh(dx+c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)**3*(a+b*sinh(dx+c)**4)**2,x)

[Out] Timed out

Giac [B] time = 1.28386, size = 275, normalized size = 2.99

$$\frac{a^2 \log(e^{(dx+c)} + e^{(-dx-c)} + 2)}{4d} - \frac{a^2 \log(e^{(dx+c)} + e^{(-dx-c)} - 2)}{4d} - \frac{a^2(e^{(dx+c)} + e^{(-dx-c)})}{((e^{(dx+c)} + e^{(-dx-c)})^2 - 4)d} + \frac{3b^2 d^4 (e^{(dx+c)} + e^{(-dx-c)})^5 - 4}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^3*(a+b*sinh(dx+c)^4)^2,x, algorithm="giac")

[Out] 1/4*a^2*log(e^(dx+c) + e^(-dx-c) + 2)/d - 1/4*a^2*log(e^(dx+c) + e^(-dx-c) - 2)/d - a^2*(e^(dx+c) + e^(-dx-c))/(((e^(dx+c) + e^(-dx-c))^2 - 4)*d) + 1/480*(3*b^2*d^4*(e^(dx+c) + e^(-dx-c))^5 - 40*

$$b^2 d^4 (e^{dx+c} + e^{-dx-c})^3 + 480 a b d^4 (e^{dx+c} + e^{-dx-c}) + 240 b^2 d^4 (e^{dx+c} + e^{-dx-c}) / d^5$$

3.203 $\int \operatorname{csch}^4(c + dx) \left(a + b \sinh^4(c + dx) \right)^2 dx$

Optimal. Leaf size=91

$$-\frac{a^2 \coth^3(c + dx)}{3d} + \frac{a^2 \coth(c + dx)}{d} + \frac{1}{8}bx(16a + 3b) + \frac{b^2 \sinh(c + dx) \cosh^3(c + dx)}{4d} - \frac{5b^2 \sinh(c + dx) \cosh(c + dx)}{8d}$$

[Out] (b*(16*a + 3*b)*x)/8 + (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) - (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d)

Rubi [A] time = 0.164625, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3217, 1259, 1805, 1261, 207}

$$-\frac{a^2 \coth^3(c + dx)}{3d} + \frac{a^2 \coth(c + dx)}{d} + \frac{1}{8}bx(16a + 3b) + \frac{b^2 \sinh(c + dx) \cosh^3(c + dx)}{4d} - \frac{5b^2 \sinh(c + dx) \cosh(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] (b*(16*a + 3*b)*x)/8 + (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d) - (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d)

Rule 3217

Int[sin[(e_.) + (f_.)*(x_.)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(m/2 - 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e

```

^2)^p/(e^(m/2)*x^m)*(d + e*(2*q + 3)*x^2))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]

```

Rule 1805

```

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]

```

Rule 1261

```

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

```

Rule 207

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(c+dx) (a+b \sinh^4(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^2}{x^4(1-x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^2 \cosh^3(c+dx) \sinh(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{4a^2-12a^2x^2+(12a^2+8ab-b^2)x^4-4(a+b)^2x^6}{x^4(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{4d} \\
&= -\frac{5b^2 \cosh(c+dx) \sinh(c+dx)}{8d} + \frac{b^2 \cosh^3(c+dx) \sinh(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{4a^2-12a^2x^2+(12a^2+8ab-b^2)x^4-4(a+b)^2x^6}{x^4(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{4d} \\
&= -\frac{5b^2 \cosh(c+dx) \sinh(c+dx)}{8d} + \frac{b^2 \cosh^3(c+dx) \sinh(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{4a^2-12a^2x^2+(12a^2+8ab-b^2)x^4-4(a+b)^2x^6}{x^4(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{4d} \\
&= \frac{a^2 \coth(c+dx)}{d} - \frac{a^2 \coth^3(c+dx)}{3d} - \frac{5b^2 \cosh(c+dx) \sinh(c+dx)}{8d} + \frac{b^2 \cosh^3(c+dx) \sinh(c+dx)}{4d} \\
&= \frac{1}{8}b(16a+3b)x + \frac{a^2 \coth(c+dx)}{d} - \frac{a^2 \coth^3(c+dx)}{3d} - \frac{5b^2 \cosh(c+dx) \sinh(c+dx)}{8d} + \frac{b^2 \cosh^3(c+dx) \sinh(c+dx)}{4d}
\end{aligned}$$

Mathematica [A] time = 0.324689, size = 68, normalized size = 0.75

$$\frac{3b(64adx - 8b \sinh(2(c+dx))) + b \sinh(4(c+dx)) + 12bc + 12bdx - 32a^2 \coth(c+dx) (\operatorname{csch}^2(c+dx) - 2)}{96d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] (-32*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + 3*b*(12*b*c + 64*a*d*x + 12*b*d*x - 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(96*d)

Maple [A] time = 0.044, size = 75, normalized size = 0.8

$$\frac{1}{d} \left(a^2 \left(\frac{2}{3} - \frac{(\operatorname{csch}(dx+c))^2}{3} \right) \coth(dx+c) + 2ab(dx+c) + b^2 \left(\left(\frac{(\sinh(dx+c))^3}{4} - \frac{3 \sinh(dx+c)}{8} \right) \cosh(dx+c) + \frac{3dx}{8} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x)

[Out] $1/d*(a^2*(2/3-1/3*\operatorname{csch}(d*x+c)^2)*\operatorname{coth}(d*x+c)+2*a*b*(d*x+c)+b^2*((1/4*\sinh(d*x+c))^3-3/8*\sinh(d*x+c))*\operatorname{cosh}(d*x+c)+3/8*d*x+3/8*c)$

Maxima [A] time = 1.04836, size = 223, normalized size = 2.45

$$\frac{1}{64} b^2 \left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + 2abx + \frac{4}{3} a^2 \left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + e^{(-6dx-6c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")`

[Out] $1/64*b^2*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) + 2*a*b*x + 4/3*a^2*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)))$

Fricas [B] time = 1.74776, size = 761, normalized size = 8.36

$$3b^2 \cosh(dx+c)^7 + 21b^2 \cosh(dx+c) \sinh(dx+c)^6 - 33b^2 \cosh(dx+c)^5 + 15(7b^2 \cosh(dx+c)^3 - 11b^2 \cosh(dx+c) \sinh(dx+c)^2) \cosh(dx+c)^4 + (128a^2 + 81b^2) \cosh(dx+c)^3 + 8(3(16ab + 3b^2)d*x - 16a^2) \sinh(dx+c)^3 + 3(21b^2 \cosh(dx+c)^5 - 110b^2 \cosh(dx+c) \sinh(dx+c)^4 + (128a^2 + 81b^2) \cosh(dx+c) \sinh(dx+c)^2 - 3(128a^2 + 17b^2) \cosh(dx+c) - 24(3(16ab + 3b^2)d*x - (3(16ab + 3b^2)d*x - 16a^2) \cosh(dx+c)^2 - 16a^2) \sinh(dx+c)) / (d \sinh(dx+c)^3 + 3(d \cosh(dx+c)^2 - d) \sinh(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")`

[Out] $1/192*(3*b^2*\cosh(d*x + c)^7 + 21*b^2*\cosh(d*x + c)*\sinh(d*x + c)^6 - 33*b^2*\cosh(d*x + c)^5 + 15*(7*b^2*\cosh(d*x + c)^3 - 11*b^2*\cosh(d*x + c))*\sinh(d*x + c)^4 + (128*a^2 + 81*b^2)*\cosh(d*x + c)^3 + 8*(3*(16*a*b + 3*b^2)*d*x - 16*a^2)*\sinh(d*x + c)^3 + 3*(21*b^2*\cosh(d*x + c)^5 - 110*b^2*\cosh(d*x + c) \sinh(d*x + c)^4 + (128*a^2 + 81*b^2)*\cosh(d*x + c) \sinh(d*x + c)^2 - 3*(128*a^2 + 17*b^2)*\cosh(d*x + c) - 24*(3*(16*a*b + 3*b^2)*d*x - (3*(16*a*b + 3*b^2)*d*x - 16*a^2)*\cosh(d*x + c)^2 - 16*a^2)*\sinh(d*x + c)) / (d*\sinh(d*x + c)^3 + 3*(d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [A] time = 1.31744, size = 207, normalized size = 2.27

$$\frac{(16ab + 3b^2)(dx + c)}{8d} - \frac{(96abe^{4dx+4c} + 18b^2e^{4dx+4c} - 8b^2e^{2dx+2c} + b^2)e^{-4dx-4c}}{64d} + \frac{b^2de^{4dx+4c} - 8b^2de^{2dx+2c}}{64d^2} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] 1/8*(16*a*b + 3*b^2)*(d*x + c)/d - 1/64*(96*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) - 8*b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/d + 1/64*(b^2*d*e^(4*d*x + 4*c) - 8*b^2*d*e^(2*d*x + 2*c))/d^2 - 4/3*(3*a^2*e^(2*d*x + 2*c) - a^2)/(d*(e^(2*d*x + 2*c) - 1)^3)

3.204 $\int \operatorname{csch}^5(c + dx) \left(a + b \sinh^4(c + dx) \right)^2 dx$

Optimal. Leaf size=101

$$-\frac{a^2 \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{3a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{8d} - \frac{a(3a + 16b) \tanh^{-1}(\cosh(c + dx))}{8d} + \frac{b^2 \cosh^3(c + dx)}{3d}$$

[Out] $-(a*(3*a + 16*b)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(8*d) - (b^2*\operatorname{Cosh}[c + d*x])/d + (b^2*\operatorname{Cosh}[c + d*x]^3)/(3*d) + (3*a^2*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(8*d) - (a^2*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x]^3)/(4*d)$

Rubi [A] time = 0.151209, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 1153, 206}

$$-\frac{a^2 \coth(c + dx) \operatorname{csch}^3(c + dx)}{4d} + \frac{3a^2 \coth(c + dx) \operatorname{csch}(c + dx)}{8d} - \frac{a(3a + 16b) \tanh^{-1}(\cosh(c + dx))}{8d} + \frac{b^2 \cosh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^5*(a + b*\operatorname{Sinh}[c + d*x]^4)^2, x]$

[Out] $-(a*(3*a + 16*b)*\operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(8*d) - (b^2*\operatorname{Cosh}[c + d*x])/d + (b^2*\operatorname{Cosh}[c + d*x]^3)/(3*d) + (3*a^2*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x])/(8*d) - (a^2*\operatorname{Coth}[c + d*x]*\operatorname{Csch}[c + d*x]^3)/(4*d)$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] :> \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}[\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 1157

$\operatorname{Int}[(d_. + (e_.)*(x_.)^2)^{(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] :> \operatorname{With}[\{Qx = \operatorname{PolynomialQuotient}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]\}, -\operatorname{Simp}[(R*x*(d + e*x^2)^{(q+1)})/(2*d*(q+1)), x] + \operatorname{Dist}[1/(2*d*(q+1)), \operatorname{Int}[(d + e*x^2)^{(q+1)}*\operatorname{ExpandToSum}[2*d*(q+1)*Qx + R*(2*q+3), x],$

$x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{LtQ}[q, -1]$

Rule 1814

$\text{Int}[(Pq_)*((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \ :> \ \text{With}[\{Q = \text{PolynomialQuotient}[Pq, a + b*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + b*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + b*x^2, x], x, 1]\}, \text{Simp}[(a*g - b*f*x)*(a + b*x^2)^{(p + 1)}/(2*a*b*(p + 1)), x] + \text{Dist}[1/(2*a*(p + 1)), \text{Int}[(a + b*x^2)^{(p + 1)}*\text{ExpandToSum}[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{LtQ}[p, -1]$

Rule 1153

$\text{Int}(((d_) + (e_)*(x_)^2)^{(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^{(p_)}, x_Symbol] \ :> \ \text{Int}[\text{ExpandIntegrand}[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0] \ \&\& \ \text{NeQ}[cd^2 - bde + ae^2, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IGtQ}[q, -2]$

Rule 206

$\text{Int}(((a_) + (b_)*(x_)^2)^{(-1)}, x_Symbol] \ :> \ \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^5(c+dx) (a+b \sinh^4(c+dx))^2 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^2}{(1-x^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^2 \coth(c+dx) \operatorname{csch}^3(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{-(a+2b)(3a+2b)+4b(2a+3b)x^2-12b^2x^4}{(1-x^2)^2} dx, x, \cosh(c+dx)\right)}{4d} \\
&= \frac{3a^2 \coth(c+dx) \operatorname{csch}(c+dx)}{8d} - \frac{a^2 \coth(c+dx) \operatorname{csch}^3(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{-(a+2b)(3a+2b)+4b(2a+3b)x^2-12b^2x^4}{(1-x^2)^2} dx, x, \cosh(c+dx)\right)}{4d} \\
&= \frac{3a^2 \coth(c+dx) \operatorname{csch}(c+dx)}{8d} - \frac{a^2 \coth(c+dx) \operatorname{csch}^3(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{-(a+2b)(3a+2b)+4b(2a+3b)x^2-12b^2x^4}{(1-x^2)^2} dx, x, \cosh(c+dx)\right)}{4d} \\
&= -\frac{b^2 \cosh(c+dx)}{d} + \frac{b^2 \cosh^3(c+dx)}{3d} + \frac{3a^2 \coth(c+dx) \operatorname{csch}(c+dx)}{8d} - \frac{a^2 \coth(c+dx) \operatorname{csch}^3(c+dx)}{4d} \\
&= -\frac{a(3a+16b) \tanh^{-1}(\cosh(c+dx))}{8d} - \frac{b^2 \cosh(c+dx)}{d} + \frac{b^2 \cosh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0434006, size = 186, normalized size = 1.84

$$-\frac{a^2 \operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^2 \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a^2 \operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^2 \operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a^2 \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] $(-3*b^2*\operatorname{Cosh}[c + d*x])/(4*d) + (b^2*\operatorname{Cosh}[3*(c + d*x)])/(12*d) + (3*a^2*\operatorname{Csch}[(c + d*x)/2]^2)/(32*d) - (a^2*\operatorname{Csch}[(c + d*x)/2]^4)/(64*d) - (2*a*b*\operatorname{Log}[\operatorname{Cosh}[c/2 + (d*x)/2]])/d + (2*a*b*\operatorname{Log}[\operatorname{Sinh}[c/2 + (d*x)/2]])/d + (3*a^2*\operatorname{Log}[\operatorname{Tanh}[(c + d*x)/2]])/(8*d) + (3*a^2*\operatorname{Sech}[(c + d*x)/2]^2)/(32*d) + (a^2*\operatorname{Sech}[(c + d*x)/2]^4)/(64*d)$

Maple [A] time = 0.057, size = 79, normalized size = 0.8

$$\frac{1}{d} \left(a^2 \left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4} + \frac{3 \operatorname{csch}(dx+c)}{8} \right) \coth(dx+c) - \frac{3 \operatorname{Artanh}(e^{dx+c})}{4} \right) - 4ab \operatorname{Artanh}(e^{dx+c}) + b^2 \left(-\frac{2}{3} + \frac{\operatorname{csch}(dx+c)}{3} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x)

[Out] 1/d*(a^2*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d*x+c)))-4*a*b*arctanh(exp(d*x+c))+b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))

Maxima [B] time = 1.12611, size = 316, normalized size = 3.13

$$\frac{1}{24} b^2 \left(\frac{e^{(3dx+3c)}}{d} - \frac{9e^{(dx+c)}}{d} - \frac{9e^{(-dx-c)}}{d} + \frac{e^{(-3dx-3c)}}{d} \right) - \frac{1}{8} a^2 \left(\frac{3 \log(e^{(-dx-c)} + 1)}{d} - \frac{3 \log(e^{(-dx-c)} - 1)}{d} + \frac{2(3e^{(-dx-c)} - 1)}{d(4e^{(-2dx-2c)} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out] 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) - 1/8*a^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)

Fricas [B] time = 2.0716, size = 8741, normalized size = 86.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out] 1/24*(b^2*cosh(d*x + c)^14 + 14*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + b^2*sinh(d*x + c)^14 - 13*b^2*cosh(d*x + c)^12 + 13*(7*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^12 + 52*(7*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^11 + 3*(6*a^2 + 11*b^2)*cosh(d*x + c)^10 + (1001*b^2*cosh(d*x + c)^4 - 858*b^2*cosh(d*x + c)^2 + 18*a^2 + 33*b^2)*sinh(d*x + c)^10 + 2*(1001*b^2*cosh(d*x + c)^5 - 1430*b^2*cosh(d*x + c)^3 + 15*(6*a^2 + 11*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 3*(22*a^2 + 7*b^2)*cosh(d*x + c)^8 + 3*(1001*b^2*cosh(d*x + c)^6 - 2145*b^2*cosh(d*x + c)^4 + 45*(6*a^2 + 11*b^2)*cosh(d*x + c)^2 - 22*a^2 - 7*b^2)*sinh(d*x + c)^8 + 24*(143*b^2*cosh(d*x + c)^7 - 429

$$\begin{aligned}
& *b^2 \cosh(dx + c)^5 + 15*(6*a^2 + 11*b^2) \cosh(dx + c)^3 - (22*a^2 + 7*b^2) \cosh(dx + c) * \sinh(dx + c)^7 - 3*(22*a^2 + 7*b^2) \cosh(dx + c)^6 + 3* \\
& (1001*b^2 \cosh(dx + c)^8 - 4004*b^2 \cosh(dx + c)^6 + 210*(6*a^2 + 11*b^2) \cosh(dx + c)^4 - 28*(22*a^2 + 7*b^2) \cosh(dx + c)^2 - 22*a^2 - 7*b^2) * \sinh(dx + c)^6 + 2*(1001*b^2 \cosh(dx + c)^9 - 5148*b^2 \cosh(dx + c)^7 + 37 \\
& 8*(6*a^2 + 11*b^2) \cosh(dx + c)^5 - 84*(22*a^2 + 7*b^2) \cosh(dx + c)^3 - 9*(22*a^2 + 7*b^2) \cosh(dx + c) * \sinh(dx + c)^5 + 3*(6*a^2 + 11*b^2) \cosh(dx + c)^4 + (1001*b^2 \cosh(dx + c)^10 - 6435*b^2 \cosh(dx + c)^8 + 630*(\\
& 6*a^2 + 11*b^2) \cosh(dx + c)^6 - 210*(22*a^2 + 7*b^2) \cosh(dx + c)^4 - 45*(22*a^2 + 7*b^2) \cosh(dx + c)^2 + 18*a^2 + 33*b^2) * \sinh(dx + c)^4 - 13*b^2 \cosh(dx + c)^2 + 4*(91*b^2 \cosh(dx + c)^11 - 715*b^2 \cosh(dx + c)^9 + \\
& 90*(6*a^2 + 11*b^2) \cosh(dx + c)^7 - 42*(22*a^2 + 7*b^2) \cosh(dx + c)^5 - 15*(22*a^2 + 7*b^2) \cosh(dx + c)^3 + 3*(6*a^2 + 11*b^2) \cosh(dx + c) * \sinh(dx + c)^3 + (91*b^2 \cosh(dx + c)^12 - 858*b^2 \cosh(dx + c)^10 + 135* \\
& (6*a^2 + 11*b^2) \cosh(dx + c)^8 - 84*(22*a^2 + 7*b^2) \cosh(dx + c)^6 - 45*(22*a^2 + 7*b^2) \cosh(dx + c)^4 + 18*(6*a^2 + 11*b^2) \cosh(dx + c)^2 - 13*b^2) * \sinh(dx + c)^2 + b^2 - 3*((3*a^2 + 16*a*b) \cosh(dx + c)^11 + 11*(3 \\
& *a^2 + 16*a*b) \cosh(dx + c) * \sinh(dx + c)^10 + (3*a^2 + 16*a*b) * \sinh(dx + c)^11 - 4*(3*a^2 + 16*a*b) \cosh(dx + c)^9 + (55*(3*a^2 + 16*a*b) \cosh(dx + c)^2 - 12*a^2 - 64*a*b) * \sinh(dx + c)^9 + 3*(55*(3*a^2 + 16*a*b) \cosh(dx + c)^3 - 12*(3*a^2 + 16*a*b) \cosh(dx + c) * \sinh(dx + c)^8 + 6*(3*a^2 + 16*a*b) \cosh(dx + c)^7 + 6*(55*(3*a^2 + 16*a*b) \cosh(dx + c)^4 - 24*(3*a^2 + 16*a*b) \cosh(dx + c)^2 + 3*a^2 + 16*a*b) * \sinh(dx + c)^7 + 42*(11*(3*a^2 + 16*a*b) \cosh(dx + c)^5 - 8*(3*a^2 + 16*a*b) \cosh(dx + c)^3 + (3*a^2 + 16*a*b) \cosh(dx + c) * \sinh(dx + c)^6 - 4*(3*a^2 + 16*a*b) \cosh(dx + c)^5 + 2*(231*(3*a^2 + 16*a*b) \cosh(dx + c)^6 - 252*(3*a^2 + 16*a*b) \cosh(dx + c)^4 + 63*(3*a^2 + 16*a*b) \cosh(dx + c)^2 - 6*a^2 - 32*a*b) * \sinh(dx + c)^5 + 2*(165*(3*a^2 + 16*a*b) \cosh(dx + c)^7 - 252*(3*a^2 + 16*a*b) \cosh(dx + c)^5 + 105*(3*a^2 + 16*a*b) \cosh(dx + c)^3 - 10*(3*a^2 + 16*a*b) \cosh(dx + c) * \sinh(dx + c)^4 + (3*a^2 + 16*a*b) \cosh(dx + c)^3 + (165*(3*a^2 + 16*a*b) \cosh(dx + c)^8 - 336*(3*a^2 + 16*a*b) \cosh(dx + c)^6 + 210*(3*a^2 + 16*a*b) \cosh(dx + c)^4 - 40*(3*a^2 + 16*a*b) \cosh(dx + c)^2 + 3*a^2 + 16*a*b) * \sinh(dx + c)^3 + (55*(3*a^2 + 16*a*b) \cosh(dx + c)^9 - 144*(3*a^2 + 16*a*b) \cosh(dx + c)^7 + 126*(3*a^2 + 16*a*b) \cosh(dx + c)^5 - 40*(3*a^2 + 16*a*b) \cosh(dx + c)^3 + 3*(3*a^2 + 16*a*b) \cosh(dx + c) * \sinh(dx + c)^2 + (11*(3*a^2 + 16*a*b) \cosh(dx + c)^10 - 36*(3*a^2 + 16*a*b) \cosh(dx + c)^8 + 42*(3*a^2 + 16*a*b) \cosh(dx + c)^6 - 20*(3*a^2 + 16*a*b) \cosh(dx + c)^4 + 3*(3*a^2 + 16*a*b) \cosh(dx + c)^2) * \sinh(dx + c) * \log(\cosh(dx + c) + \sinh(dx + c) + 1) + 3*((3*a^2 + 16*a*b) \cosh(dx + c)^11 + 11*(3*a^2 + 16*a*b) \cosh(dx + c) * \sinh(dx + c)^10 + (3*a^2 + 16*a*b) * \sinh(dx + c)^11 - 4*(3*a^2 + 16*a*b) \cosh(dx + c)^9 + (55*(3*a^2 + 16*a*b) \cosh(dx + c)^2 - 12*a^2 - 64*a*b) * \sinh(dx + c)^9 + 3*(55*(3*a^2 + 16*a*b) \cosh(dx + c)^3 - 12*(3*a^2 + 16*a*b) \cosh(dx + c) * \sinh(dx + c)^8 + 6*(3*a^2 + 16*a*b) \cosh(dx + c)^7 + 6*(55*(3*a^2 + 16*a*b) \cosh(dx + c)^4 - 24*(3*a^2 + 16*a*b) \cosh(dx + c)^2 + 3*a^2 + 16*a*b) * \sinh(dx + c)^7 + 42*(11*(
\end{aligned}$$

$$\begin{aligned}
& 3a^2 + 16ab) \cosh(dx + c)^5 - 8(3a^2 + 16ab) \cosh(dx + c)^3 + (3a^2 + 16ab) \cosh(dx + c) \sinh(dx + c)^6 - 4(3a^2 + 16ab) \cosh(dx + c)^5 + 2(231(3a^2 + 16ab) \cosh(dx + c)^6 - 252(3a^2 + 16ab) \cosh(dx + c)^4 + 63(3a^2 + 16ab) \cosh(dx + c)^2 - 6a^2 - 32ab) \sinh(dx + c)^5 + 2(165(3a^2 + 16ab) \cosh(dx + c)^7 - 252(3a^2 + 16ab) \cosh(dx + c)^5 + 105(3a^2 + 16ab) \cosh(dx + c)^3 - 10(3a^2 + 16ab) \cosh(dx + c) \sinh(dx + c)^4 + (3a^2 + 16ab) \cosh(dx + c)^3 + (165(3a^2 + 16ab) \cosh(dx + c)^8 - 336(3a^2 + 16ab) \cosh(dx + c)^6 + 210(3a^2 + 16ab) \cosh(dx + c)^4 - 40(3a^2 + 16ab) \cosh(dx + c)^2 + 3a^2 + 16ab) \sinh(dx + c)^3 + (55(3a^2 + 16ab) \cosh(dx + c)^9 - 144(3a^2 + 16ab) \cosh(dx + c)^7 + 126(3a^2 + 16ab) \cosh(dx + c)^5 - 40(3a^2 + 16ab) \cosh(dx + c)^3 + 3(3a^2 + 16ab) \cosh(dx + c)) \sinh(dx + c)^2 + (11(3a^2 + 16ab) \cosh(dx + c)^{10} - 36(3a^2 + 16ab) \cosh(dx + c)^8 + 42(3a^2 + 16ab) \cosh(dx + c)^6 - 20(3a^2 + 16ab) \cosh(dx + c)^4 + 3(3a^2 + 16ab) \cosh(dx + c)^2) \sinh(dx + c) \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 2(7b^2 \cosh(dx + c)^{13} - 78b^2 \cosh(dx + c)^{11} + 15(6a^2 + 11b^2) \cosh(dx + c)^9 - 12(22a^2 + 7b^2) \cosh(dx + c)^7 - 9(22a^2 + 7b^2) \cosh(dx + c)^5 + 6(6a^2 + 11b^2) \cosh(dx + c)^3 - 13b^2 \cosh(dx + c)) \sinh(dx + c) / (d \cosh(dx + c)^{11} + 11d \cosh(dx + c) \sinh(dx + c)^{10} + d \sinh(dx + c)^{11} - 4d \cosh(dx + c)^9 + (55d \cosh(dx + c)^2 - 4d) \sinh(dx + c)^9 + 3(55d \cosh(dx + c)^3 - 12d \cosh(dx + c)) \sinh(dx + c)^8 + 6d \cosh(dx + c)^7 + 6(55d \cosh(dx + c)^4 - 24d \cosh(dx + c)^2 + d) \sinh(dx + c)^7 + 42(11d \cosh(dx + c)^5 - 8d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^6 - 4d \cosh(dx + c)^5 + 2(231d \cosh(dx + c)^6 - 252d \cosh(dx + c)^4 + 63d \cosh(dx + c)^2 - 2d) \sinh(dx + c)^5 + 2(165d \cosh(dx + c)^7 - 252d \cosh(dx + c)^5 + 105d \cosh(dx + c)^3 - 10d \cosh(dx + c)) \sinh(dx + c)^4 + d \cosh(dx + c)^3 + (165d \cosh(dx + c)^8 - 336d \cosh(dx + c)^6 + 210d \cosh(dx + c)^4 - 40d \cosh(dx + c)^2 + d) \sinh(dx + c)^3 + (55d \cosh(dx + c)^9 - 144d \cosh(dx + c)^7 + 126d \cosh(dx + c)^5 - 40d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^2 + (11d \cosh(dx + c)^{10} - 36d \cosh(dx + c)^8 + 42d \cosh(dx + c)^6 - 20d \cosh(dx + c)^4 + 3d \cosh(dx + c)^2) \sinh(dx + c)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)**5*(a+b*sinh(dx+c)**4)**2,x)

[Out] Timed out

Giac [B] time = 1.31755, size = 262, normalized size = 2.59

$$\frac{(3a^2 + 16ab) \log(e^{dx+c} + e^{-dx-c} + 2)}{16d} + \frac{(3a^2 + 16ab) \log(e^{dx+c} + e^{-dx-c} - 2)}{16d} + \frac{b^2 d^2 (e^{dx+c} + e^{-dx-c})^3 - 12b^2 c}{24d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out]
$$-1/16*(3*a^2 + 16*a*b)*\log(e^{d*x + c} + e^{-d*x - c} + 2)/d + 1/16*(3*a^2 + 16*a*b)*\log(e^{d*x + c} + e^{-d*x - c} - 2)/d + 1/24*(b^2*d^2*(e^{d*x + c} + e^{-d*x - c})^3 - 12*b^2*d^2*(e^{d*x + c} + e^{-d*x - c}))/d^3 + 1/4*(3*a^2*(e^{d*x + c} + e^{-d*x - c})^3 - 20*a^2*(e^{d*x + c} + e^{-d*x - c}))/((e^{d*x + c} + e^{-d*x - c})^2 - 4)^2*d$$

3.205 $\int \operatorname{csch}^6(c + dx) (a + b \sinh^4(c + dx))^2 dx$

Optimal. Leaf size=84

$$-\frac{a^2 \operatorname{coth}^5(c + dx)}{5d} + \frac{2a^2 \operatorname{coth}^3(c + dx)}{3d} - \frac{a(a + 2b) \operatorname{coth}(c + dx)}{d} + \frac{b^2 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{b^2 x}{2}$$

[Out] $-(b^2 x)/2 - (a(a + 2b) \operatorname{Coth}[c + d x])/d + (2 a^2 \operatorname{Coth}[c + d x]^3)/(3 d) - (a^2 \operatorname{Coth}[c + d x]^5)/(5 d) + (b^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(2 d)$

Rubi [A] time = 0.145736, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3217, 1259, 1802, 207}

$$-\frac{a^2 \operatorname{coth}^5(c + dx)}{5d} + \frac{2a^2 \operatorname{coth}^3(c + dx)}{3d} - \frac{a(a + 2b) \operatorname{coth}(c + dx)}{d} + \frac{b^2 \sinh(c + dx) \cosh(c + dx)}{2d} - \frac{b^2 x}{2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d x]^6 (a + b \operatorname{Sinh}[c + d x]^4)^2, x]$

[Out] $-(b^2 x)/2 - (a(a + 2b) \operatorname{Coth}[c + d x])/d + (2 a^2 \operatorname{Coth}[c + d x]^3)/(3 d) - (a^2 \operatorname{Coth}[c + d x]^5)/(5 d) + (b^2 \operatorname{Cosh}[c + d x] \operatorname{Sinh}[c + d x])/(2 d)$

Rule 3217

$\operatorname{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)} ((a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m (a + 2 a ff^2 x^2 + (a + b) ff^4 x^4)^p]/(1 + ff^2 x^2)^{(m/2 + 2 p + 1)}, x], x, \operatorname{Tan}[e + f x]/ff], x]\} /;$ $\operatorname{FreeQ}\{a, b, e, f, x\} \ \&\amp; \ \operatorname{IntegerQ}[m/2] \ \&\amp; \ \operatorname{IntegerQ}[p]$

Rule 1259

$\operatorname{Int}[(x_.)^{(m_.)} ((d_.) + (e_.)(x_.)^2)^{(q_.)} ((a_.) + (b_.)(x_.)^2 + (c_.)(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[((-d)^{(m/2 - 1)} (c d^2 - b d e + a e^2)^p x (d + e x^2)^{(q + 1)})/(2 e^{(2 p + m/2)} (q + 1)), x] + \operatorname{Dist}[(-d)^{(m/2 - 1)}/(2 e^{(2 p)} (q + 1)), \operatorname{Int}[x^m (d + e x^2)^{(q + 1)} \operatorname{ExpandToSum}[\operatorname{Together}[(1 + (2(-d)^{-(m/2 + 1)} e^{(2 p)} (q + 1) (a + b x^2 + c x^4))^p - ((c d^2 - b d e + a e^2)^p / (e^{(m/2)} x^m)) (d + e (2 q + 3) x^2))]/(d + e x^2)], x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, e, x\} \ \&\amp; \operatorname{NeQ}[b^2 - 4 a c, 0] \ \&\amp; \operatorname{IGtQ}[p, 0] \ \&\amp; \operatorname{ILtQ}[q, -1]$

&& ILtQ[m/2, 0]

Rule 1802

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \operatorname{csch}^6(c+dx) (a+b \sinh^4(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^2}{x^6(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{b^2 \cosh(c+dx) \sinh(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-2a^2+6a^2x^2-2a(3a+2b)x^4+(2a^2+4ab+b^2)}{x^6(1-x^2)} dx, x, \tanh(c+dx)\right)}{2d} \\
 &= \frac{b^2 \cosh(c+dx) \sinh(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \left(-\frac{2a^2}{x^6} + \frac{4a^2}{x^4} - \frac{2a(3a+2b)}{x^2} - \frac{b^2}{-1+x^2}\right) dx, x, \tanh(c+dx)\right)}{2d} \\
 &= -\frac{a(a+2b) \operatorname{coth}(c+dx)}{d} + \frac{2a^2 \operatorname{coth}^3(c+dx)}{3d} - \frac{a^2 \operatorname{coth}^5(c+dx)}{5d} + \frac{b^2 \operatorname{coth}(c+dx)}{2d} \\
 &= -\frac{b^2 x}{2} - \frac{a(a+2b) \operatorname{coth}(c+dx)}{d} + \frac{2a^2 \operatorname{coth}^3(c+dx)}{3d} - \frac{a^2 \operatorname{coth}^5(c+dx)}{5d} + \dots
 \end{aligned}$$

Mathematica [A] time = 0.635186, size = 67, normalized size = 0.8

$$\frac{15b^2(\sinh(2(c+dx)) - 2(c+dx)) - 4a \operatorname{coth}(c+dx) (3a \operatorname{csch}^4(c+dx) - 4a \operatorname{csch}^2(c+dx) + 8a + 30b)}{60d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] $(-4*a*Coth[c + d*x]*(8*a + 30*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*x]^4) + 15*b^2*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(60*d)$

Maple [A] time = 0.049, size = 74, normalized size = 0.9

$$\frac{1}{d} \left(a^2 \left(-\frac{8}{15} - \frac{(\operatorname{csch}(dx+c))^4}{5} + \frac{4(\operatorname{csch}(dx+c))^2}{15} \right) \operatorname{coth}(dx+c) - 2ab \operatorname{coth}(dx+c) + b^2 \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x)`

[Out] $1/d*(a^2*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-2*a*b*cot h(d*x+c)+b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))$

Maxima [B] time = 1.06529, size = 360, normalized size = 4.29

$$-\frac{1}{8} b^2 \left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d} \right) - \frac{16}{15} a^2 \left(\frac{5e^{-2dx-2c}}{d(5e^{-2dx-2c} - 10e^{-4dx-4c} + 10e^{-6dx-6c} - 5e^{-8dx-8c} + e^{-10dx-10c} - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")`

[Out] $-1/8*b^2*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 16/15*a^2*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) - 10*e^{(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) - 1/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1))) + 4*a*b/(d*(e^{(-2*d*x - 2*c)} - 1))$

Fricas [B] time = 1.71391, size = 1185, normalized size = 14.11

$$15b^2 \cosh(dx+c)^7 + 105b^2 \cosh(dx+c) \sinh(dx+c)^6 - (64a^2 + 240ab + 75b^2) \cosh(dx+c)^5 - 4(15b^2 dx - 16a^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out] $\frac{1}{120}*(15*b^2*cosh(d*x + c)^7 + 105*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - (64*a^2 + 240*a*b + 75*b^2)*cosh(d*x + c)^5 - 4*(15*b^2*d*x - 16*a^2 - 60*a*b)*sinh(d*x + c)^5 + 5*(105*b^2*cosh(d*x + c)^3 - (64*a^2 + 240*a*b + 75*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 5*(64*a^2 + 144*a*b + 27*b^2)*cosh(d*x + c)^3 + 20*(15*b^2*d*x - 2*(15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^2 - 16*a^2 - 60*a*b)*sinh(d*x + c)^3 + 5*(63*b^2*cosh(d*x + c)^5 - 2*(64*a^2 + 240*a*b + 75*b^2)*cosh(d*x + c)^3 + 3*(64*a^2 + 144*a*b + 27*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 5*(128*a^2 + 96*a*b + 15*b^2)*cosh(d*x + c) - 20*((15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^4 + 30*b^2*d*x - 3*(15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^2 - 32*a^2 - 120*a*b)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [B] time = 1.27883, size = 234, normalized size = 2.79

$$-\frac{(dx+c)b^2}{2d} + \frac{b^2e^{(2dx+2c)}}{8d} + \frac{(2b^2e^{(2dx+2c)} - b^2)e^{(-2dx-2c)}}{8d} - \frac{4(15abe^{(8dx+8c)} - 60abe^{(6dx+6c)} + 40a^2e^{(4dx+4c)} + 90abe^{(2dx+2c)})}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] $-\frac{1}{2}*(d*x + c)*b^2/d + \frac{1}{8}*b^2*e^{(2*d*x + 2*c)}/d + \frac{1}{8}*(2*b^2*e^{(2*d*x + 2*c)} - b^2)*e^{(-2*d*x - 2*c)}/d - \frac{4}{15}*(15*a*b*e^{(8*d*x + 8*c)} - 60*a*b*e^{(6*d$

$$\begin{aligned} & *x + 6*c) + 40*a^2*e^{(4*d*x + 4*c)} + 90*a*b*e^{(4*d*x + 4*c)} - 20*a^2*e^{(2*d \\ & *x + 2*c)} - 60*a*b*e^{(2*d*x + 2*c)} + 4*a^2 + 15*a*b)/(d*(e^{(2*d*x + 2*c)} - \\ & 1)^5) \end{aligned}$$

3.206 $\int \operatorname{csch}^7(c + dx) (a + b \sinh^4(c + dx))^2 dx$

Optimal. Leaf size=111

$$-\frac{a^2 \operatorname{coth}(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a^2 \operatorname{coth}(c + dx) \operatorname{csch}^3(c + dx)}{24d} + \frac{a(5a + 16b) \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{a(5a + 16b) \operatorname{coth}(c + dx)}{16d}$$

[Out] (a*(5*a + 16*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*Cosh[c + d*x])/d - (a*(5*a + 16*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)

Rubi [A] time = 0.174123, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 388, 206}

$$-\frac{a^2 \operatorname{coth}(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a^2 \operatorname{coth}(c + dx) \operatorname{csch}^3(c + dx)}{24d} + \frac{a(5a + 16b) \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{a(5a + 16b) \operatorname{coth}(c + dx)}{16d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] (a*(5*a + 16*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*Cosh[c + d*x])/d - (a*(5*a + 16*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)

Rule 3215

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 1157

Int[((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],

```
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^7(c+dx) (a+b \sinh^4(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^2}{(1-x^2)^4} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^2 \coth(c+dx) \operatorname{csch}^5(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-5a^2-12ab-6b^2+6b(2a+3b)x^2-18b^2x^4}{(1-x^2)^3} dx, x, \cosh(c+dx)\right)}{6d} \\
&= \frac{5a^2 \coth(c+dx) \operatorname{csch}^3(c+dx)}{24d} - \frac{a^2 \coth(c+dx) \operatorname{csch}^5(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{5a^2+12ab+6b^2+6b(2a+3b)x^2+18b^2x^4}{(1-x^2)^3} dx, x, \cosh(c+dx)\right)}{6d} \\
&= -\frac{a(5a+16b) \coth(c+dx) \operatorname{csch}(c+dx)}{16d} + \frac{5a^2 \coth(c+dx) \operatorname{csch}^3(c+dx)}{24d} \\
&= \frac{b^2 \cosh(c+dx)}{d} - \frac{a(5a+16b) \coth(c+dx) \operatorname{csch}(c+dx)}{16d} + \frac{5a^2 \coth(c+dx) \operatorname{csch}^3(c+dx)}{24d} \\
&= \frac{a(5a+16b) \tanh^{-1}(\cosh(c+dx))}{16d} + \frac{b^2 \cosh(c+dx)}{d} - \frac{a(5a+16b) \coth(c+dx)}{16d}
\end{aligned}$$

Mathematica [B] time = 0.0355098, size = 240, normalized size = 2.16

$$-\frac{a^2 \operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a^2 \operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a^2 \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a^2 \operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a^2 \operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]

[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (5*a^2*Csch[(c + d*x)/2]^2)/(64*d) - (a*b*Csch[(c + d*x)/2]^2)/(4*d) + (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[(c + d*x)/2]^6)/(384*d) - (5*a^2*Log[Tanh[(c + d*x)/2]])/(16*d) - (a*b*Log[Tanh[(c + d*x)/2]])/d - (5*a^2*Sech[(c + d*x)/2]^2)/(64*d) - (a*b*Sech[(c + d*x)/2]^2)/(4*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d) - (a^2*Sech[(c + d*x)/2]^6)/(384*d) + (b^2*Sinh[c]*Sinh[d*x])/d

Maple [A] time = 0.048, size = 92, normalized size = 0.8

$$\frac{1}{d} \left(a^2 \left(\left(-\frac{(\operatorname{csch}(dx+c))^5}{6} + \frac{5(\operatorname{csch}(dx+c))^3}{24} - \frac{5\operatorname{csch}(dx+c)}{16} \right) \coth(dx+c) + \frac{5 \operatorname{Artanh}\left(e^{dx+c}\right)}{8} \right) + 2ab \left(-\frac{1}{2} \operatorname{csch} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x+c)+5/8*arctanh(exp(d*x+c)))+2*a*b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b^2*cosh(d*x+c)
```

Maxima [B] time = 1.04059, size = 404, normalized size = 3.64

$$\frac{1}{2} b^2 \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) + \frac{1}{48} a^2 \left(\frac{15 \log(e^{(-dx-c)} + 1)}{d} - \frac{15 \log(e^{(-dx-c)} - 1)}{d} + \frac{2(15 e^{(-dx-c)} - 85 e^{(-3dx-3c)} + 198 e^{(-5dx-5c)} + 198 e^{(-7dx-7c)} - 85 e^{(-9dx-9c)} + 15 e^{(-11dx-11c)})}{d(6 e^{(-2dx-2c)} - 15 e^{(-4dx-4c)} + 20 e^{(-6dx-6c)} - 15 e^{(-8dx-8c)} + 6 e^{(-10dx-10c)} - e^{(-12dx-12c)} - 1)} \right) + a*b*(\log(e^{(-dx-c)} + 1)/d - \log(e^{(-dx-c)} - 1)/d + 2*(e^{(-dx-c)} + e^{(-3dx-3c)})/(d*(2*e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.11855, size = 11957, normalized size = 107.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(24*b^2*cosh(d*x + c)^14 + 336*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 24*b^2*sinh(d*x + c)^14 - 6*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^12 + 6*(364*b^2*cosh(d*x + c)^2 - 5*a^2 - 16*a*b - 20*b^2)*sinh(d*x + c)^12 + 24*(364*b^2*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 + 2*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^10 + 2*(12012*b^2*c
```


$$\begin{aligned}
& \text{osh}(d*x + c)^4 - 198*(5*a^2 + 16*a*b + 20*b^2)*\text{cosh}(d*x + c)^2 + 85*a^2 + 1 \\
& 44*a*b + 108*b^2)*\text{sinh}(d*x + c)^{10} + 4*(12012*b^2*\text{cosh}(d*x + c)^5 - 330*(5* \\
& a^2 + 16*a*b + 20*b^2)*\text{cosh}(d*x + c)^3 + 5*(85*a^2 + 144*a*b + 108*b^2)*\text{cos} \\
& h(d*x + c))*\text{sinh}(d*x + c)^9 - 12*(33*a^2 + 16*a*b + 10*b^2)*\text{cosh}(d*x + c)^8 \\
& + 6*(12012*b^2*\text{cosh}(d*x + c)^6 - 495*(5*a^2 + 16*a*b + 20*b^2)*\text{cosh}(d*x + \\
& c)^4 + 15*(85*a^2 + 144*a*b + 108*b^2)*\text{cosh}(d*x + c)^2 - 66*a^2 - 32*a*b - \\
& 20*b^2)*\text{sinh}(d*x + c)^8 + 48*(1716*b^2*\text{cosh}(d*x + c)^7 - 99*(5*a^2 + 16*a*b \\
& + 20*b^2)*\text{cosh}(d*x + c)^5 + 5*(85*a^2 + 144*a*b + 108*b^2)*\text{cosh}(d*x + c)^3 \\
& - 2*(33*a^2 + 16*a*b + 10*b^2)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^7 - 12*(33*a^2 \\
& + 16*a*b + 10*b^2)*\text{cosh}(d*x + c)^6 + 12*(6006*b^2*\text{cosh}(d*x + c)^8 - 462*(5 \\
& *a^2 + 16*a*b + 20*b^2)*\text{cosh}(d*x + c)^6 + 35*(85*a^2 + 144*a*b + 108*b^2)*\text{c} \\
& \text{osh}(d*x + c)^4 - 28*(33*a^2 + 16*a*b + 10*b^2)*\text{cosh}(d*x + c)^2 - 33*a^2 - 1 \\
& 6*a*b - 10*b^2)*\text{sinh}(d*x + c)^6 + 24*(2002*b^2*\text{cosh}(d*x + c)^9 - 198*(5*a^2 \\
& + 16*a*b + 20*b^2)*\text{cosh}(d*x + c)^7 + 21*(85*a^2 + 144*a*b + 108*b^2)*\text{cosh}(\\
& d*x + c)^5 - 28*(33*a^2 + 16*a*b + 10*b^2)*\text{cosh}(d*x + c)^3 - 3*(33*a^2 + 16 \\
& *a*b + 10*b^2)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^5 + 2*(85*a^2 + 144*a*b + 108*b \\
& ^2)*\text{cosh}(d*x + c)^4 + 2*(12012*b^2*\text{cosh}(d*x + c)^{10} - 1485*(5*a^2 + 16*a*b \\
& + 20*b^2)*\text{cosh}(d*x + c)^8 + 210*(85*a^2 + 144*a*b + 108*b^2)*\text{cosh}(d*x + c)^ \\
& 6 - 420*(33*a^2 + 16*a*b + 10*b^2)*\text{cosh}(d*x + c)^4 - 90*(33*a^2 + 16*a*b + \\
& 10*b^2)*\text{cosh}(d*x + c)^2 + 85*a^2 + 144*a*b + 108*b^2)*\text{sinh}(d*x + c)^4 + 8*(\\
& 1092*b^2*\text{cosh}(d*x + c)^{11} - 165*(5*a^2 + 16*a*b + 20*b^2)*\text{cosh}(d*x + c)^9 + \\
& 30*(85*a^2 + 144*a*b + 108*b^2)*\text{cosh}(d*x + c)^7 - 84*(33*a^2 + 16*a*b + 10 \\
& *b^2)*\text{cosh}(d*x + c)^5 - 30*(33*a^2 + 16*a*b + 10*b^2)*\text{cosh}(d*x + c)^3 + (85 \\
& *a^2 + 144*a*b + 108*b^2)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^3 - 6*(5*a^2 + 16*a* \\
& b + 20*b^2)*\text{cosh}(d*x + c)^2 + 6*(364*b^2*\text{cosh}(d*x + c)^{12} - 66*(5*a^2 + 16* \\
& a*b + 20*b^2)*\text{cosh}(d*x + c)^{10} + 15*(85*a^2 + 144*a*b + 108*b^2)*\text{cosh}(d*x + \\
& c)^8 - 56*(33*a^2 + 16*a*b + 10*b^2)*\text{cosh}(d*x + c)^6 - 30*(33*a^2 + 16*a*b \\
& + 10*b^2)*\text{cosh}(d*x + c)^4 + 2*(85*a^2 + 144*a*b + 108*b^2)*\text{cosh}(d*x + c)^2 \\
& - 5*a^2 - 16*a*b - 20*b^2)*\text{sinh}(d*x + c)^2 + 24*b^2 + 3*((5*a^2 + 16*a*b)* \\
& \text{cosh}(d*x + c)^{13} + 13*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)*\text{sinh}(d*x + c)^{12} + (5* \\
& a^2 + 16*a*b)*\text{sinh}(d*x + c)^{13} - 6*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^{11} + 6*(1 \\
& 3*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^2 - 5*a^2 - 16*a*b)*\text{sinh}(d*x + c)^{11} + 22* \\
& (13*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^3 - 3*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c))*\text{si} \\
& nh(d*x + c)^{10} + 15*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^9 + 5*(143*(5*a^2 + 16*a \\
& *b)*\text{cosh}(d*x + c)^4 - 66*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^2 + 15*a^2 + 48*a*b \\
&)*\text{sinh}(d*x + c)^9 + 9*(143*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^5 - 110*(5*a^2 + \\
& 16*a*b)*\text{cosh}(d*x + c)^3 + 15*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^ \\
& 8 - 20*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*\text{cosh}(d*x \\
& + c)^6 - 495*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^4 + 135*(5*a^2 + 16*a*b)*\text{cosh}(d \\
& *x + c)^2 - 25*a^2 - 80*a*b)*\text{sinh}(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*\text{cosh} \\
& (d*x + c)^7 - 693*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^5 + 315*(5*a^2 + 16*a*b)*\text{c} \\
& \text{osh}(d*x + c)^3 - 35*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c))*\text{sinh}(d*x + c)^6 + 15*(5 \\
& *a^2 + 16*a*b)*\text{cosh}(d*x + c)^5 + 3*(429*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^8 - \\
& 924*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^6 + 630*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^4 \\
& - 140*(5*a^2 + 16*a*b)*\text{cosh}(d*x + c)^2 + 25*a^2 + 80*a*b)*\text{sinh}(d*x + c)^5
\end{aligned}$$

$$\begin{aligned}
& + 5*(143*(5*a^2 + 16*a*b)*\cosh(d*x + c)^9 - 396*(5*a^2 + 16*a*b)*\cosh(d*x + c)^7 + 378*(5*a^2 + 16*a*b)*\cosh(d*x + c)^5 - 140*(5*a^2 + 16*a*b)*\cosh(d*x + c)^3 + 15*(5*a^2 + 16*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 6*(5*a^2 + 16*a*b)*\cosh(d*x + c)^3 + 2*(143*(5*a^2 + 16*a*b)*\cosh(d*x + c)^10 - 495*(5*a^2 + 16*a*b)*\cosh(d*x + c)^8 + 630*(5*a^2 + 16*a*b)*\cosh(d*x + c)^6 - 350*(5*a^2 + 16*a*b)*\cosh(d*x + c)^4 + 75*(5*a^2 + 16*a*b)*\cosh(d*x + c)^2 - 15*a^2 - 48*a*b)*\sinh(d*x + c)^3 + 6*(13*(5*a^2 + 16*a*b)*\cosh(d*x + c)^11 - 55*(5*a^2 + 16*a*b)*\cosh(d*x + c)^9 + 90*(5*a^2 + 16*a*b)*\cosh(d*x + c)^7 - 70*(5*a^2 + 16*a*b)*\cosh(d*x + c)^5 + 25*(5*a^2 + 16*a*b)*\cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (5*a^2 + 16*a*b)*\cosh(d*x + c) + (13*(5*a^2 + 16*a*b)*\cosh(d*x + c)^12 - 66*(5*a^2 + 16*a*b)*\cosh(d*x + c)^10 + 135*(5*a^2 + 16*a*b)*\cosh(d*x + c)^8 - 140*(5*a^2 + 16*a*b)*\cosh(d*x + c)^6 + 75*(5*a^2 + 16*a*b)*\cosh(d*x + c)^4 - 18*(5*a^2 + 16*a*b)*\cosh(d*x + c)^2 + 5*a^2 + 16*a*b)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - 3*((5*a^2 + 16*a*b)*\cosh(d*x + c)^13 + 13*(5*a^2 + 16*a*b)*\cosh(d*x + c)*\sinh(d*x + c)^12 + (5*a^2 + 16*a*b)*\sinh(d*x + c)^13 - 6*(5*a^2 + 16*a*b)*\cosh(d*x + c)^11 + 6*(13*(5*a^2 + 16*a*b)*\cosh(d*x + c)^2 - 5*a^2 - 16*a*b)*\sinh(d*x + c)^11 + 22*(13*(5*a^2 + 16*a*b)*\cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^10 + 15*(5*a^2 + 16*a*b)*\cosh(d*x + c)^9 + 5*(143*(5*a^2 + 16*a*b)*\cosh(d*x + c)^4 - 66*(5*a^2 + 16*a*b)*\cosh(d*x + c)^2 + 15*a^2 + 48*a*b)*\sinh(d*x + c)^9 + 9*(143*(5*a^2 + 16*a*b)*\cosh(d*x + c)^5 - 110*(5*a^2 + 16*a*b)*\cosh(d*x + c)^3 + 15*(5*a^2 + 16*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^8 - 20*(5*a^2 + 16*a*b)*\cosh(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*\cosh(d*x + c)^6 - 495*(5*a^2 + 16*a*b)*\cosh(d*x + c)^4 + 135*(5*a^2 + 16*a*b)*\cosh(d*x + c)^2 - 25*a^2 - 80*a*b)*\sinh(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*\cosh(d*x + c)^7 - 693*(5*a^2 + 16*a*b)*\cosh(d*x + c)^5 + 315*(5*a^2 + 16*a*b)*\cosh(d*x + c)^3 - 35*(5*a^2 + 16*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^6 + 15*(5*a^2 + 16*a*b)*\cosh(d*x + c)^5 + 3*(429*(5*a^2 + 16*a*b)*\cosh(d*x + c)^8 - 924*(5*a^2 + 16*a*b)*\cosh(d*x + c)^6 + 630*(5*a^2 + 16*a*b)*\cosh(d*x + c)^4 - 140*(5*a^2 + 16*a*b)*\cosh(d*x + c)^2 + 25*a^2 + 80*a*b)*\sinh(d*x + c)^5 + 5*(143*(5*a^2 + 16*a*b)*\cosh(d*x + c)^9 - 396*(5*a^2 + 16*a*b)*\cosh(d*x + c)^7 + 378*(5*a^2 + 16*a*b)*\cosh(d*x + c)^5 - 140*(5*a^2 + 16*a*b)*\cosh(d*x + c)^3 + 15*(5*a^2 + 16*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 6*(5*a^2 + 16*a*b)*\cosh(d*x + c)^3 + 2*(143*(5*a^2 + 16*a*b)*\cosh(d*x + c)^10 - 495*(5*a^2 + 16*a*b)*\cosh(d*x + c)^8 + 630*(5*a^2 + 16*a*b)*\cosh(d*x + c)^6 - 350*(5*a^2 + 16*a*b)*\cosh(d*x + c)^4 + 75*(5*a^2 + 16*a*b)*\cosh(d*x + c)^2 - 15*a^2 - 48*a*b)*\sinh(d*x + c)^3 + 6*(13*(5*a^2 + 16*a*b)*\cosh(d*x + c)^11 - 55*(5*a^2 + 16*a*b)*\cosh(d*x + c)^9 + 90*(5*a^2 + 16*a*b)*\cosh(d*x + c)^7 - 70*(5*a^2 + 16*a*b)*\cosh(d*x + c)^5 + 25*(5*a^2 + 16*a*b)*\cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (5*a^2 + 16*a*b)*\cosh(d*x + c) + (13*(5*a^2 + 16*a*b)*\cosh(d*x + c)^12 - 66*(5*a^2 + 16*a*b)*\cosh(d*x + c)^10 + 135*(5*a^2 + 16*a*b)*\cosh(d*x + c)^8 - 140*(5*a^2 + 16*a*b)*\cosh(d*x + c)^6 + 75*(5*a^2 + 16*a*b)*\cosh(d*x + c)^4 - 18*(5*a^2 + 16*a*b)*\cosh(d*x + c)^2 + 5*a^2 + 16*a*b)*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 4*(84*b^2*\cosh(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^{13} - 18(5a^2 + 16ab + 20b^2)\cosh(dx + c)^{11} + 5(85a^2 + 144ab \\
& * b + 108b^2)\cosh(dx + c)^9 - 24(33a^2 + 16ab + 10b^2)\cosh(dx + c) \\
& ^7 - 18(33a^2 + 16ab + 10b^2)\cosh(dx + c)^5 + 2(85a^2 + 144ab + \\
& 108b^2)\cosh(dx + c)^3 - 3(5a^2 + 16ab + 20b^2)\cosh(dx + c))\sinh(\\
& dx + c)/(d\cosh(dx + c)^{13} + 13d\cosh(dx + c)\sinh(dx + c)^{12} + d\sin \\
& h(dx + c)^{13} - 6d\cosh(dx + c)^{11} + 6(13d\cosh(dx + c)^2 - d)\sinh(dx \\
& x + c)^{11} + 22(13d\cosh(dx + c)^3 - 3d\cosh(dx + c))\sinh(dx + c)^{10} \\
& + 15d\cosh(dx + c)^9 + 5(143d\cosh(dx + c)^4 - 66d\cosh(dx + c)^2 + \\
& 3d)\sinh(dx + c)^9 + 9(143d\cosh(dx + c)^5 - 110d\cosh(dx + c)^3 + 1 \\
& 5d\cosh(dx + c))\sinh(dx + c)^8 - 20d\cosh(dx + c)^7 + 4(429d\cosh(d \\
& * x + c)^6 - 495d\cosh(dx + c)^4 + 135d\cosh(dx + c)^2 - 5d)\sinh(dx + \\
& c)^7 + 4(429d\cosh(dx + c)^7 - 693d\cosh(dx + c)^5 + 315d\cosh(dx + \\
& c)^3 - 35d\cosh(dx + c))\sinh(dx + c)^6 + 15d\cosh(dx + c)^5 + 3(429 \\
& * d\cosh(dx + c)^8 - 924d\cosh(dx + c)^6 + 630d\cosh(dx + c)^4 - 140d* \\
& cosh(dx + c)^2 + 5d)\sinh(dx + c)^5 + 5(143d\cosh(dx + c)^9 - 396d*c \\
& osh(dx + c)^7 + 378d\cosh(dx + c)^5 - 140d\cosh(dx + c)^3 + 15d\cosh(\\
& dx + c))\sinh(dx + c)^4 - 6d\cosh(dx + c)^3 + 2(143d\cosh(dx + c)^{10} \\
& - 495d\cosh(dx + c)^8 + 630d\cosh(dx + c)^6 - 350d\cosh(dx + c)^4 + \\
& 75d\cosh(dx + c)^2 - 3d)\sinh(dx + c)^3 + 6(13d\cosh(dx + c)^{11} - 55 \\
& * d\cosh(dx + c)^9 + 90d\cosh(dx + c)^7 - 70d\cosh(dx + c)^5 + 25d*cos \\
& h(dx + c)^3 - 3d\cosh(dx + c))\sinh(dx + c)^2 + d\cosh(dx + c) + (13d \\
& * \cosh(dx + c)^{12} - 66d\cosh(dx + c)^{10} + 135d\cosh(dx + c)^8 - 140d*c \\
& osh(dx + c)^6 + 75d\cosh(dx + c)^4 - 18d\cosh(dx + c)^2 + d)\sinh(dx \\
& + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)**7*(a+b*sinh(dx+c)**4)**2,x)

[Out] Timed out

Giac [B] time = 1.27771, size = 338, normalized size = 3.05

$$\frac{b^2(e^{(dx+c)} + e^{-dx-c})}{2d} + \frac{(5a^2 + 16ab) \log(e^{(dx+c)} + e^{-dx-c} + 2)}{32d} - \frac{(5a^2 + 16ab) \log(e^{(dx+c)} + e^{-dx-c} - 2)}{32d} - \frac{15a^2(e^{dx-c} + e^{-dx-c})}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*(e^(d*x + c) + e^(-d*x - c))/d + 1/32*(5*a^2 + 16*a*b)*log(e^(d*x +
c) + e^(-d*x - c) + 2)/d - 1/32*(5*a^2 + 16*a*b)*log(e^(d*x + c) + e^(-d*x
- c) - 2)/d - 1/24*(15*a^2*(e^(d*x + c) + e^(-d*x - c))^5 + 48*a*b*(e^(d*x
+ c) + e^(-d*x - c))^5 - 160*a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 384*a*b*
(e^(d*x + c) + e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) + e^(-d*x - c)) + 768
*a*b*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3*
d)
```

3.207 $\int \sinh^5(c + dx) (a + b \sinh^4(c + dx))^3 dx$

Optimal. Leaf size=220

$$\frac{b(3a^2 + 45ab + 70b^2) \cosh^9(c + dx)}{9d} - \frac{4b(3a^2 + 15ab + 14b^2) \cosh^7(c + dx)}{7d} + \frac{(a + b)(a^2 + 17ab + 28b^2) \cosh^5(c + dx)}{5d}$$

[Out] $((a + b)^3 \text{Cosh}[c + d*x])/d - (2*(a + b)^2*(a + 4*b)*\text{Cosh}[c + d*x]^3)/(3*d)$
 $+ ((a + b)*(a^2 + 17*a*b + 28*b^2)*\text{Cosh}[c + d*x]^5)/(5*d) - (4*b*(3*a^2 + 15*a*b + 14*b^2)*\text{Cosh}[c + d*x]^7)/(7*d) + (b*(3*a^2 + 45*a*b + 70*b^2)*\text{Cosh}[c + d*x]^9)/(9*d) - (2*b^2*(9*a + 28*b)*\text{Cosh}[c + d*x]^11)/(11*d) + (b^2*(3*a + 28*b)*\text{Cosh}[c + d*x]^13)/(13*d) - (8*b^3*\text{Cosh}[c + d*x]^15)/(15*d) + (b^3*\text{Cosh}[c + d*x]^17)/(17*d)$

Rubi [A] time = 0.218072, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3215, 1153}

$$\frac{b(3a^2 + 45ab + 70b^2) \cosh^9(c + dx)}{9d} - \frac{4b(3a^2 + 15ab + 14b^2) \cosh^7(c + dx)}{7d} + \frac{(a + b)(a^2 + 17ab + 28b^2) \cosh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^5*(a + b*\text{Sinh}[c + d*x]^4)^3, x]$

[Out] $((a + b)^3 \text{Cosh}[c + d*x])/d - (2*(a + b)^2*(a + 4*b)*\text{Cosh}[c + d*x]^3)/(3*d)$
 $+ ((a + b)*(a^2 + 17*a*b + 28*b^2)*\text{Cosh}[c + d*x]^5)/(5*d) - (4*b*(3*a^2 + 15*a*b + 14*b^2)*\text{Cosh}[c + d*x]^7)/(7*d) + (b*(3*a^2 + 45*a*b + 70*b^2)*\text{Cosh}[c + d*x]^9)/(9*d) - (2*b^2*(9*a + 28*b)*\text{Cosh}[c + d*x]^11)/(11*d) + (b^2*(3*a + 28*b)*\text{Cosh}[c + d*x]^13)/(13*d) - (8*b^3*\text{Cosh}[c + d*x]^15)/(15*d) + (b^3*\text{Cosh}[c + d*x]^17)/(17*d)$

Rule 3215

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] :> \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m - 1)/2}*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \text{Cos}[e + f*x]/ff], x]] /; \text{FreeQ}\{a, b, e, f, p, x\} \&\& \text{IntegerQ}[(m - 1)/2]$

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
 x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned} \int \sinh^5(c + dx) (a + b \sinh^4(c + dx))^3 dx &= \frac{\text{Subst}\left(\int (1 - x^2)^2 (a + b - 2bx^2 + bx^4)^3 dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int ((a + b)^3 - 2(a + b)^2(a + 4b)x^2 + (a + b)(a^2 + 17ab + 28b^2)x^4 - \dots) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{(a + b)^3 \cosh(c + dx)}{d} - \frac{2(a + b)^2(a + 4b) \cosh^3(c + dx)}{3d} + \frac{(a + b)(a^2 + 17ab + 28b^2) \cosh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 2.26629, size = 288, normalized size = 1.31

$$\frac{1531530(48384a^2b + 20480a^3 + 41184ab^2 + 12155b^3) \cosh(c + dx) - 2042040(8064a^2b + 2560a^3 + 7722ab^2 + 2431b^3) \cosh^3(c + dx) + 627314688a^3 \cosh^5(c + dx) + 4234374144a^2b \cosh^5(c + dx) + 5256210960ab^2 \cosh^5(c + dx) + 1895421528b^3 \cosh^5(c + dx) - 756138240a^2b \cosh^7(c + dx) - 1501774560ab^2 \cosh^7(c + dx) - 676936260b^3 \cosh^7(c + dx) + 65345280a^2b \cosh^9(c + dx) + 318558240ab^2 \cosh^9(c + dx) + 202502300b^3 \cosh^9(c + dx) - 43439760a^2b \cosh^{11}(c + dx) - 47338200ab^2 \cosh^{11}(c + dx) + 2827440ab^2 \cosh^{13}(c + dx) + 8011080b^3 \cosh^{13}(c + dx) - 867867b^3 \cosh^{15}(c + dx) + 45045b^3 \cosh^{17}(c + dx)}{(50185175040d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (1531530*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*Cosh[c + d*x]
- 2042040*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*Cosh[3*(c + d*x)]
+ 627314688*a^3*Cosh[5*(c + d*x)] + 4234374144*a^2*b*Cosh[5*(c + d*x)] + 5
256210960*a*b^2*Cosh[5*(c + d*x)] + 1895421528*b^3*Cosh[5*(c + d*x)] - 7561
38240*a^2*b*Cosh[7*(c + d*x)] - 1501774560*a*b^2*Cosh[7*(c + d*x)] - 676936
260*b^3*Cosh[7*(c + d*x)] + 65345280*a^2*b*Cosh[9*(c + d*x)] + 318558240*a*
b^2*Cosh[9*(c + d*x)] + 202502300*b^3*Cosh[9*(c + d*x)] - 43439760*a*b^2*Co
sh[11*(c + d*x)] - 47338200*b^3*Cosh[11*(c + d*x)] + 2827440*a*b^2*Cosh[13*
(c + d*x)] + 8011080*b^3*Cosh[13*(c + d*x)] - 867867*b^3*Cosh[15*(c + d*x)]
+ 45045*b^3*Cosh[17*(c + d*x)])/(50185175040*d)
```

Maple [A] time = 0.096, size = 258, normalized size = 1.2

$$\frac{1}{d} \left(b^3 \left(\frac{32768}{109395} + \frac{(\sinh(dx + c))^{16}}{17} - \frac{16(\sinh(dx + c))^{14}}{255} + \frac{224(\sinh(dx + c))^{12}}{3315} - \frac{896(\sinh(dx + c))^{10}}{12155} + \frac{1792(\sinh(dx + c))^{8}}{2187} - \frac{128(\sinh(dx + c))^{6}}{11025} + \frac{128(\sinh(dx + c))^{4}}{11025} - \frac{128(\sinh(dx + c))^{2}}{11025} + \frac{128}{11025} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(dx+c)^5*(a+b*\sinh(dx+c)^4)^3,x)$

[Out] $\frac{1}{d}*(b^3*(\frac{32768}{109395}+\frac{1}{17}*\sinh(dx+c)^{16}-\frac{16}{255}*\sinh(dx+c)^{14}+\frac{224}{3315}*\sinh(dx+c)^{12}-\frac{896}{12155}*\sinh(dx+c)^{10}+\frac{1792}{21879}*\sinh(dx+c)^8-\frac{2048}{21879}*\sinh(dx+c)^6+\frac{4096}{36465}*\sinh(dx+c)^4-\frac{16384}{109395}*\sinh(dx+c)^2)*\cosh(dx+c)+3*a*b^2*(\frac{1024}{3003}+\frac{1}{13}*\sinh(dx+c)^{12}-\frac{12}{143}*\sinh(dx+c)^{10}+\frac{40}{429}*\sinh(dx+c)^8-\frac{320}{3003}*\sinh(dx+c)^6+\frac{128}{1001}*\sinh(dx+c)^4-\frac{512}{3003}*\sinh(dx+c)^2)*\cosh(dx+c)+3*a^2*b*(\frac{128}{315}+\frac{1}{9}*\sinh(dx+c)^8-\frac{8}{63}*\sinh(dx+c)^6+\frac{16}{105}*\sinh(dx+c)^4-\frac{64}{315}*\sinh(dx+c)^2)*\cosh(dx+c)+a^3*(\frac{8}{15}+\frac{1}{5}*\sinh(dx+c)^4-\frac{4}{15}*\sinh(dx+c)^2)*\cosh(dx+c))$

Maxima [B] time = 1.07614, size = 810, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(dx+c)^5*(a+b*\sinh(dx+c)^4)^3,x, \text{algorithm}="maxima")$

[Out]
$$\begin{aligned} & -1/14338621440*b^3*((123981*e^{(-2*d*x - 2*c)} - 1144440*e^{(-4*d*x - 4*c)} + 6762600*e^{(-6*d*x - 6*c)} - 28928900*e^{(-8*d*x - 8*c)} + 96705180*e^{(-10*d*x - 10*c)} - 270774504*e^{(-12*d*x - 12*c)} + 709171320*e^{(-14*d*x - 14*c)} - 2659392450*e^{(-16*d*x - 16*c)} - 6435)*e^{(17*d*x + 17*c)}/d - (2659392450*e^{(-d*x - c)} - 709171320*e^{(-3*d*x - 3*c)} + 270774504*e^{(-5*d*x - 5*c)} - 96705180*e^{(-7*d*x - 7*c)} + 28928900*e^{(-9*d*x - 9*c)} - 6762600*e^{(-11*d*x - 11*c)} + 1144440*e^{(-13*d*x - 13*c)} - 123981*e^{(-15*d*x - 15*c)} + 6435*e^{(-17*d*x - 17*c)})/d - 1/8200192*a*b^2*((3549*e^{(-2*d*x - 2*c)} - 26026*e^{(-4*d*x - 4*c)} + 122694*e^{(-6*d*x - 6*c)} - 429429*e^{(-8*d*x - 8*c)} + 1288287*e^{(-10*d*x - 10*c)} - 5153148*e^{(-12*d*x - 12*c)} - 231)*e^{(13*d*x + 13*c)}/d - (5153148*e^{(-d*x - c)} - 1288287*e^{(-3*d*x - 3*c)} + 429429*e^{(-5*d*x - 5*c)} - 122694*e^{(-7*d*x - 7*c)} + 26026*e^{(-9*d*x - 9*c)} - 3549*e^{(-11*d*x - 11*c)} + 231)*e^{(-13*d*x - 13*c)})/d - 1/53760*a^2*b*((405*e^{(-2*d*x - 2*c)} - 2268*e^{(-4*d*x - 4*c)} + 8820*e^{(-6*d*x - 6*c)} - 39690*e^{(-8*d*x - 8*c)} - 35)*e^{(9*d*x + 9*c)}/d - (39690*e^{(-d*x - c)} - 8820*e^{(-3*d*x - 3*c)} + 2268*e^{(-5*d*x - 5*c)} - 405*e^{(-7*d*x - 7*c)} + 35)*e^{(-9*d*x - 9*c)})/d + 1/480*a^3*(3*e^{(5*d*x + 5*c)}/d - 25*e^{(3*d*x + 3*c)}/d + 150*e^{(d*x + c)}/d + 150*e^{(-d*x - c)}/d - 25*e^{(-3*d*x - 3*c)}/d + 3*e^{(-5*d*x - 5*c)}/d) \end{aligned}$$

Fricas [B] time = 1.78734, size = 2898, normalized size = 13.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out]
$$\frac{1}{50185175040} \cdot (45045b^3 \cosh(dx+c)^{17} + 765765b^3 \cosh(dx+c) \sinh(dx+c)^{16} - 867867b^3 \cosh(dx+c)^{15} + 765765(40b^3 \cosh(dx+c)^3 - 17b^3 \cosh(dx+c)) \sinh(dx+c)^{14} + 471240(6ab^2 + 17b^3) \cosh(dx+c)^{13} + 255255(1092b^3 \cosh(dx+c)^5 - 1547b^3 \cosh(dx+c)^3 + 24(6ab^2 + 17b^3) \cosh(dx+c)) \sinh(dx+c)^{12} - 556920(78ab^2 + 85b^3) \cosh(dx+c)^{11} + 153153(5720b^3 \cosh(dx+c)^7 - 17017b^3 \cosh(dx+c)^5 + 880(6ab^2 + 17b^3) \cosh(dx+c)^3 - 40(78ab^2 + 85b^3) \cosh(dx+c)) \sinh(dx+c)^{10} + 340340(192a^2b + 936ab^2 + 595b^3) \cosh(dx+c)^9 + 765765(1430b^3 \cosh(dx+c)^9 - 7293b^3 \cosh(dx+c)^7 + 792(6ab^2 + 17b^3) \cosh(dx+c)^5 - 120(78ab^2 + 85b^3) \cosh(dx+c)^3 + 4(192a^2b + 936ab^2 + 595b^3) \cosh(dx+c)) \sinh(dx+c)^8 - 437580(1728a^2b + 3432ab^2 + 1547b^3) \cosh(dx+c)^7 + 255255(2184b^3 \cosh(dx+c)^{11} - 17017b^3 \cosh(dx+c)^9 + 3168(6ab^2 + 17b^3) \cosh(dx+c)^7 - 1008(78ab^2 + 85b^3) \cosh(dx+c)^5 + 112(192a^2b + 936ab^2 + 595b^3) \cosh(dx+c)^3 - 12(1728a^2b + 3432ab^2 + 1547b^3) \cosh(dx+c)) \sinh(dx+c)^6 + 1225224(512a^3 + 3456a^2b + 4290ab^2 + 1547b^3) \cosh(dx+c)^5 + 765765(140b^3 \cosh(dx+c)^{13} - 1547b^3 \cosh(dx+c)^{11} + 440(6ab^2 + 17b^3) \cosh(dx+c)^9 - 240(78ab^2 + 85b^3) \cosh(dx+c)^7 + 56(192a^2b + 936ab^2 + 595b^3) \cosh(dx+c)^5 - 20(1728a^2b + 3432ab^2 + 1547b^3) \cosh(dx+c)^3 + 8(512a^3 + 3456a^2b + 4290ab^2 + 1547b^3) \cosh(dx+c)) \sinh(dx+c)^4 - 2042040(2560a^3 + 8064a^2b + 7722ab^2 + 2431b^3) \cosh(dx+c)^3 + 765765(8b^3 \cosh(dx+c)^{15} - 119b^3 \cosh(dx+c)^{13} + 48(6ab^2 + 17b^3) \cosh(dx+c)^{11} - 40(78ab^2 + 85b^3) \cosh(dx+c)^9 + 16(192a^2b + 936ab^2 + 595b^3) \cosh(dx+c)^7 - 12(1728a^2b + 3432ab^2 + 1547b^3) \cosh(dx+c)^5 + 16(512a^3 + 3456a^2b + 4290ab^2 + 1547b^3) \cosh(dx+c)^3 - 8(2560a^3 + 8064a^2b + 7722ab^2 + 2431b^3) \cosh(dx+c)) \sinh(dx+c)^2 + 1531530(20480a^3 + 48384a^2b + 41184ab^2 + 12155b^3) \cosh(dx+c)) / d$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.7945, size = 934, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/100370350080*(45045*b^3*e^{(17*d*x + 17*c)} - 867867*b^3*e^{(15*d*x + 15*c)} \\ & + 2827440*a*b^2*e^{(13*d*x + 13*c)} + 8011080*b^3*e^{(13*d*x + 13*c)} - 4343976 \\ & 0*a*b^2*e^{(11*d*x + 11*c)} - 47338200*b^3*e^{(11*d*x + 11*c)} + 65345280*a^2*b \\ & *e^{(9*d*x + 9*c)} + 318558240*a*b^2*e^{(9*d*x + 9*c)} + 202502300*b^3*e^{(9*d*x \\ & + 9*c)} - 756138240*a^2*b*e^{(7*d*x + 7*c)} - 1501774560*a*b^2*e^{(7*d*x + 7*c)} \\ &) - 676936260*b^3*e^{(7*d*x + 7*c)} + 627314688*a^3*e^{(5*d*x + 5*c)} + 4234374 \\ & 144*a^2*b*e^{(5*d*x + 5*c)} + 5256210960*a*b^2*e^{(5*d*x + 5*c)} + 1895421528*b \\ & ^3*e^{(5*d*x + 5*c)} - 5227622400*a^3*e^{(3*d*x + 3*c)} - 16467010560*a^2*b*e^{(\\ & 3*d*x + 3*c)} - 15768632880*a*b^2*e^{(3*d*x + 3*c)} - 4964199240*b^3*e^{(3*d*x \\ & + 3*c)} + 31365734400*a^3*e^{(d*x + c)} + 74101547520*a^2*b*e^{(d*x + c)} + 6307 \\ & 4531520*a*b^2*e^{(d*x + c)} + 18615747150*b^3*e^{(d*x + c)} + (31365734400*a^3* \\ & e^{(16*d*x + 16*c)} + 74101547520*a^2*b*e^{(16*d*x + 16*c)} + 63074531520*a*b^2 \\ & *e^{(16*d*x + 16*c)} + 18615747150*b^3*e^{(16*d*x + 16*c)} - 5227622400*a^3*e^{(\\ & 14*d*x + 14*c)} - 16467010560*a^2*b*e^{(14*d*x + 14*c)} - 15768632880*a*b^2*e^{ \\ & (14*d*x + 14*c)} - 4964199240*b^3*e^{(14*d*x + 14*c)} + 627314688*a^3*e^{(12*d* \\ & x + 12*c)} + 4234374144*a^2*b*e^{(12*d*x + 12*c)} + 5256210960*a*b^2*e^{(12*d*x \\ & + 12*c)} + 1895421528*b^3*e^{(12*d*x + 12*c)} - 756138240*a^2*b*e^{(10*d*x + 1 \\ & 0*c)} - 1501774560*a*b^2*e^{(10*d*x + 10*c)} - 676936260*b^3*e^{(10*d*x + 10*c)} \\ & + 65345280*a^2*b*e^{(8*d*x + 8*c)} + 318558240*a*b^2*e^{(8*d*x + 8*c)} + 20250 \\ & 2300*b^3*e^{(8*d*x + 8*c)} - 43439760*a*b^2*e^{(6*d*x + 6*c)} - 47338200*b^3*e^{ \\ & (6*d*x + 6*c)} + 2827440*a*b^2*e^{(4*d*x + 4*c)} + 8011080*b^3*e^{(4*d*x + 4*c)} \\ & - 867867*b^3*e^{(2*d*x + 2*c)} + 45045*b^3)*e^{(-17*d*x - 17*c)}/d \end{aligned}$$

3.208 $\int \sinh^3(c + dx) \left(a + b \sinh^4(c + dx)\right)^3 dx$

Optimal. Leaf size=183

$$\frac{b(3a^2 + 30ab + 35b^2) \cosh^7(c + dx)}{7d} + \frac{3b^2(a + 7b) \cosh^{11}(c + dx)}{11d} - \frac{5b^2(3a + 7b) \cosh^9(c + dx)}{9d} - \frac{3b(a + b)(3a + 7b) \cosh^5(c + dx)}{5d}$$

[Out] -(((a + b)^3*Cosh[c + d*x])/d) + ((a + b)^2*(a + 7*b)*Cosh[c + d*x]^3)/(3*d) - (3*b*(a + b)*(3*a + 7*b)*Cosh[c + d*x]^5)/(5*d) + (b*(3*a^2 + 30*a*b + 35*b^2)*Cosh[c + d*x]^7)/(7*d) - (5*b^2*(3*a + 7*b)*Cosh[c + d*x]^9)/(9*d) + (3*b^2*(a + 7*b)*Cosh[c + d*x]^11)/(11*d) - (7*b^3*Cosh[c + d*x]^13)/(13*d) + (b^3*Cosh[c + d*x]^15)/(15*d)

Rubi [A] time = 0.175103, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3215, 1153}

$$\frac{b(3a^2 + 30ab + 35b^2) \cosh^7(c + dx)}{7d} + \frac{3b^2(a + 7b) \cosh^{11}(c + dx)}{11d} - \frac{5b^2(3a + 7b) \cosh^9(c + dx)}{9d} - \frac{3b(a + b)(3a + 7b) \cosh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] -(((a + b)^3*Cosh[c + d*x])/d) + ((a + b)^2*(a + 7*b)*Cosh[c + d*x]^3)/(3*d) - (3*b*(a + b)*(3*a + 7*b)*Cosh[c + d*x]^5)/(5*d) + (b*(3*a^2 + 30*a*b + 35*b^2)*Cosh[c + d*x]^7)/(7*d) - (5*b^2*(3*a + 7*b)*Cosh[c + d*x]^9)/(9*d) + (3*b^2*(a + 7*b)*Cosh[c + d*x]^11)/(11*d) - (7*b^3*Cosh[c + d*x]^13)/(13*d) + (b^3*Cosh[c + d*x]^15)/(15*d)

Rule 3215

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 1153

Int[((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],

x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rubi steps

$$\begin{aligned} \int \sinh^3(c + dx) (a + b \sinh^4(c + dx))^3 dx &= -\frac{\text{Subst}\left(\int (1 - x^2) (a + b - 2bx^2 + bx^4)^3 dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{\text{Subst}\left(\int ((a + b)^3 - (a + b)^2(a + 7b)x^2 + 3b(a + b)(3a + 7b)x^4 - b(3a^2 + 6ab + 3b^2)x^6) dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{(a + b)^3 \cosh(c + dx)}{d} + \frac{(a + b)^2(a + 7b) \cosh^3(c + dx)}{3d} - \frac{3b(a + b)(3a + 6ab + 3b^2) \cosh^5(c + dx)}{15d} + \frac{b(3a^2 + 6ab + 3b^2) \cosh^7(c + dx)}{105d} \end{aligned}$$

Mathematica [A] time = 2.47, size = 185, normalized size = 1.01

$$\frac{-135135(8960a^2b + 4096a^3 + 7392ab^2 + 2145b^3) \cosh(c + dx) + 15015(16128a^2b + 4096a^3 + 15840ab^2 + 5005b^3) \cosh^3(c + dx) - 19305(256a^2 + 880ab + 455b^2) \cosh^5(c + dx) + 7425b \cosh^7(c + dx) - 429b \cosh^{15}(c + dx)}{(738017280d)}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (-135135*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*Cosh[c + d*x] + 15015*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*Cosh[3*(c + d*x)] + b*(-27027*(1792*a^2 + 2640*a*b + 1001*b^2)*Cosh[5*(c + d*x)] + 19305*(256*a^2 + 880*a*b + 455*b^2)*Cosh[7*(c + d*x)] - 7*b*(715*(528*a + 455*b)*Cosh[9*(c + d*x)] - 1755*(16*a + 35*b)*Cosh[11*(c + d*x)] + 7425*b*Cosh[13*(c + d*x)] - 429*b*Cosh[15*(c + d*x)])))/(738017280*d)

Maple [A] time = 0.059, size = 218, normalized size = 1.2

$$\frac{1}{d} \left(b^3 \left(-\frac{2048}{6435} + \frac{(\sinh(dx + c))^{14}}{15} - \frac{14(\sinh(dx + c))^{12}}{195} + \frac{56(\sinh(dx + c))^{10}}{715} - \frac{112(\sinh(dx + c))^8}{1287} + \frac{128(\sinh(dx + c))^6}{1287} - \frac{64(\sinh(dx + c))^4}{1287} + \frac{32(\sinh(dx + c))^2}{1287} - \frac{1}{1287} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x)

```
[Out] 1/d*(b^3*(-2048/6435+1/15*sinh(d*x+c)^14-14/195*sinh(d*x+c)^12+56/715*sinh(d*x+c)^10-112/1287*sinh(d*x+c)^8+128/1287*sinh(d*x+c)^6-256/2145*sinh(d*x+c)^4+1024/6435*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(-256/693+1/11*sinh(d*x+c)^10-10/99*sinh(d*x+c)^8+80/693*sinh(d*x+c)^6-32/231*sinh(d*x+c)^4+128/693*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.07978, size = 676, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/210862080*b^3*((7425*e^(-2*d*x - 2*c) - 61425*e^(-4*d*x - 4*c) + 325325*e^(-6*d*x - 6*c) - 1254825*e^(-8*d*x - 8*c) + 3864861*e^(-10*d*x - 10*c) - 10735725*e^(-12*d*x - 12*c) + 41409225*e^(-14*d*x - 14*c) - 429)*e^(15*d*x + 15*c)/d + (41409225*e^(-d*x - c) - 10735725*e^(-3*d*x - 3*c) + 3864861*e^(-5*d*x - 5*c) - 1254825*e^(-7*d*x - 7*c) + 325325*e^(-9*d*x - 9*c) - 61425*e^(-11*d*x - 11*c) + 7425*e^(-13*d*x - 13*c) - 429*e^(-15*d*x - 15*c))/d) - 1/473088*a*b^2*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(11*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*x - 11*c))/d) - 3/4480*a^2*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.73061, size = 2207, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/738017280*(3003*b^3*cosh(d*x + c)^15 + 45045*b^3*cosh(d*x + c)*sinh(d*x +
c)^14 - 51975*b^3*cosh(d*x + c)^13 + 15015*(91*b^3*cosh(d*x + c)^3 - 45*b^
3*cosh(d*x + c))*sinh(d*x + c)^12 + 12285*(16*a*b^2 + 35*b^3)*cosh(d*x + c)
^11 + 9009*(1001*b^3*cosh(d*x + c)^5 - 1650*b^3*cosh(d*x + c)^3 + 15*(16*a*
b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 - 5005*(528*a*b^2 + 455*b^3)*
cosh(d*x + c)^9 + 45045*(429*b^3*cosh(d*x + c)^7 - 1485*b^3*cosh(d*x + c)^5
+ 45*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^3 - (528*a*b^2 + 455*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^8 + 19305*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x +
c)^7 + 15015*(1001*b^3*cosh(d*x + c)^9 - 5940*b^3*cosh(d*x + c)^7 + 378*(16
*a*b^2 + 35*b^3)*cosh(d*x + c)^5 - 28*(528*a*b^2 + 455*b^3)*cosh(d*x + c)^3
+ 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 270
27*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 + 45045*(91*b^3*cos
h(d*x + c)^11 - 825*b^3*cosh(d*x + c)^9 + 90*(16*a*b^2 + 35*b^3)*cosh(d*x +
c)^7 - 14*(528*a*b^2 + 455*b^3)*cosh(d*x + c)^5 + 15*(256*a^2*b + 880*a*b^
2 + 455*b^3)*cosh(d*x + c)^3 - 3*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(
d*x + c))*sinh(d*x + c)^4 + 15015*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5
005*b^3)*cosh(d*x + c)^3 + 45045*(7*b^3*cosh(d*x + c)^13 - 90*b^3*cosh(d*x
+ c)^11 + 15*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^9 - 4*(528*a*b^2 + 455*b^3)*
cosh(d*x + c)^7 + 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c)^5 - 6*(
1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 + (4096*a^3 + 16128*a^2
*b + 15840*a*b^2 + 5005*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 135135*(4096*
a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*cosh(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.72125, size = 779, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/1476034560*(3003*b^3*e^(15*d*x + 15*c) - 51975*b^3*e^(13*d*x + 13*c) + 19
6560*a*b^2*e^(11*d*x + 11*c) + 429975*b^3*e^(11*d*x + 11*c) - 2642640*a*b^2
*e^(9*d*x + 9*c) - 2277275*b^3*e^(9*d*x + 9*c) + 4942080*a^2*b*e^(7*d*x + 7
*c) + 16988400*a*b^2*e^(7*d*x + 7*c) + 8783775*b^3*e^(7*d*x + 7*c) - 484323
84*a^2*b*e^(5*d*x + 5*c) - 71351280*a*b^2*e^(5*d*x + 5*c) - 27054027*b^3*e^
(5*d*x + 5*c) + 61501440*a^3*e^(3*d*x + 3*c) + 242161920*a^2*b*e^(3*d*x + 3
*c) + 237837600*a*b^2*e^(3*d*x + 3*c) + 75150075*b^3*e^(3*d*x + 3*c) - 5535
12960*a^3*e^(d*x + c) - 1210809600*a^2*b*e^(d*x + c) - 998917920*a*b^2*e^(d
*x + c) - 289864575*b^3*e^(d*x + c) - (553512960*a^3*e^(14*d*x + 14*c) + 12
10809600*a^2*b*e^(14*d*x + 14*c) + 998917920*a*b^2*e^(14*d*x + 14*c) + 2898
64575*b^3*e^(14*d*x + 14*c) - 61501440*a^3*e^(12*d*x + 12*c) - 242161920*a^
2*b*e^(12*d*x + 12*c) - 237837600*a*b^2*e^(12*d*x + 12*c) - 75150075*b^3*e^
(12*d*x + 12*c) + 48432384*a^2*b*e^(10*d*x + 10*c) + 71351280*a*b^2*e^(10*d
*x + 10*c) + 27054027*b^3*e^(10*d*x + 10*c) - 4942080*a^2*b*e^(8*d*x + 8*c)
- 16988400*a*b^2*e^(8*d*x + 8*c) - 8783775*b^3*e^(8*d*x + 8*c) + 2642640*a
*b^2*e^(6*d*x + 6*c) + 2277275*b^3*e^(6*d*x + 6*c) - 196560*a*b^2*e^(4*d*x
+ 4*c) - 429975*b^3*e^(4*d*x + 4*c) + 51975*b^3*e^(2*d*x + 2*c) - 3003*b^3)
*e^(-15*d*x - 15*c))/d
```

3.209 $\int \sinh(c + dx) \left(a + b \sinh^4(c + dx)\right)^3 dx$

Optimal. Leaf size=143

$$\frac{b^2(a+5b)\cosh^9(c+dx)}{3d} - \frac{4b^2(3a+5b)\cosh^7(c+dx)}{7d} + \frac{3b(a+b)(a+5b)\cosh^5(c+dx)}{5d} - \frac{2b(a+b)^2\cosh^3(c+dx)}{d} + \dots$$

[Out] $((a + b)^3 \text{Cosh}[c + d*x])/d - (2*b*(a + b)^2*\text{Cosh}[c + d*x]^3)/d + (3*b*(a + b)*(a + 5*b)*\text{Cosh}[c + d*x]^5)/(5*d) - (4*b^2*(3*a + 5*b)*\text{Cosh}[c + d*x]^7)/(7*d) + (b^2*(a + 5*b)*\text{Cosh}[c + d*x]^9)/(3*d) - (6*b^3*\text{Cosh}[c + d*x]^11)/(11*d) + (b^3*\text{Cosh}[c + d*x]^13)/(13*d)$

Rubi [A] time = 0.15744, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3215, 1090}

$$\frac{b^2(a+5b)\cosh^9(c+dx)}{3d} - \frac{4b^2(3a+5b)\cosh^7(c+dx)}{7d} + \frac{3b(a+b)(a+5b)\cosh^5(c+dx)}{5d} - \frac{2b(a+b)^2\cosh^3(c+dx)}{d} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]*(a + b*\text{Sinh}[c + d*x]^4)^3, x]$

[Out] $((a + b)^3 \text{Cosh}[c + d*x])/d - (2*b*(a + b)^2*\text{Cosh}[c + d*x]^3)/d + (3*b*(a + b)*(a + 5*b)*\text{Cosh}[c + d*x]^5)/(5*d) - (4*b^2*(3*a + 5*b)*\text{Cosh}[c + d*x]^7)/(7*d) + (b^2*(a + 5*b)*\text{Cosh}[c + d*x]^9)/(3*d) - (6*b^3*\text{Cosh}[c + d*x]^11)/(11*d) + (b^3*\text{Cosh}[c + d*x]^13)/(13*d)$

Rule 3215

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \text{Cos}[e + f*x]/ff], x] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[(m-1)/2]$

Rule 1090

$\text{Int}[(a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4]^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x^2 + c*x^4)^p, x], x] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{IGtQ}[p, 0]$

Rubi steps

$$\int \sinh(c + dx) (a + b \sinh^4(c + dx))^3 dx = \frac{\text{Subst}\left(\int (a + b - 2bx^2 + bx^4)^3 dx, x, \cosh(c + dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(a^3 \left(1 + \frac{b(3a^2 + 3ab + b^2)}{a^3}\right) - 6b(a + b)^2 x^2 + 12b^2(a + b) \left(1 + \frac{a+b}{4b}\right) x^4 - \dots\right) dx, x, \cosh(c + dx)\right)}{d}$$

$$= \frac{(a + b)^3 \cosh(c + dx)}{d} - \frac{2b(a + b)^2 \cosh^3(c + dx)}{d} + \frac{3b(a + b)(a + 5b) \cosh^5(c + dx)}{5d}$$

Mathematica [A] time = 0.90112, size = 157, normalized size = 1.1

$$\frac{-15015b(1280a^2 + 1344ab + 429b^2) \cosh(3(c + dx)) + 3003b(768a^2 + 1728ab + 715b^2) \cosh(5(c + dx)) + 60060(1920a^3 + 1512a^2b + 429b^3) \cosh(7(c + dx)) - 15015b^2(1280a^2 + 1344ab + 429b^2) \cosh(9(c + dx)) + 10010b^2(8a + 13b) \cosh(11(c + dx)) - 17745b^3 \cosh(13(c + dx))}{(61501440d)}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*Cosh[c + d*x] - 15015*b*(1280*a^2 + 1344*a*b + 429*b^2)*Cosh[3*(c + d*x)] + 3003*b*(768*a^2 + 1728*a*b + 715*b^2)*Cosh[5*(c + d*x)] - 4290*b^2*(216*a + 143*b)*Cosh[7*(c + d*x)] + 10010*b^2*(8*a + 13*b)*Cosh[9*(c + d*x)] - 17745*b^3*Cosh[11*(c + d*x)] + 1155*b^3*Cosh[13*(c + d*x)])/(61501440*d)

Maple [A] time = 0.02, size = 176, normalized size = 1.2

$$\frac{1}{d} \left(b^3 \left(\frac{1024}{3003} + \frac{(\sinh(dx + c))^{12}}{13} - \frac{12(\sinh(dx + c))^{10}}{143} + \frac{40(\sinh(dx + c))^8}{429} - \frac{320(\sinh(dx + c))^6}{3003} + \frac{128(\sinh(dx + c))^4}{1001} - \frac{512(\sinh(dx + c))^2}{105} + \frac{64}{315} \right) \cosh(dx + c) + 3ab^2 \left(\frac{128}{315} + \frac{1}{9} \sinh^2(dx + c) - \frac{8}{63} \sinh^4(dx + c) + \frac{16}{105} \sinh^6(dx + c) - \frac{64}{315} \sinh^8(dx + c) \right) \cosh(dx + c) + 3a^2b \left(\frac{8}{15} + \frac{1}{5} \sinh^2(dx + c) - \frac{8}{15} \sinh^4(dx + c) + \frac{16}{105} \sinh^6(dx + c) - \frac{64}{315} \sinh^8(dx + c) \right) \cosh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x)

[Out] 1/d*(b^3*(1024/3003+1/13*sinh(d*x+c)^12-12/143*sinh(d*x+c)^10+40/429*sinh(d*x+c)^8-320/3003*sinh(d*x+c)^6+128/1001*sinh(d*x+c)^4-512/3003*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(128/315+1/9*sinh(d*x+c)^2-8/63*sinh(d*x+c)^4+16/105*sinh(d*x+c)^6-64/315*sinh(d*x+c)^8)*cosh(d*x+c)+3*a^2*b*(8/15+1/5*sinh(d*x+c)^2-8/15*sinh(d*x+c)^4+16/105*sinh(d*x+c)^6-64/315*sinh(d*x+c)^8)*cosh(d*x+c)

$$+c)^4 - 4/15 * \sinh(d*x+c)^2 * \cosh(d*x+c) + a^3 * \cosh(d*x+c)$$

Maxima [B] time = 1.04483, size = 539, normalized size = 3.77

$$-\frac{1}{24600576} b^3 \left(\frac{(3549 e^{(-2dx-2c)} - 26026 e^{(-4dx-4c)} + 122694 e^{(-6dx-6c)} - 429429 e^{(-8dx-8c)} + 1288287 e^{(-10dx-10c)} - 5153148 e^{(-12dx-12c)} - 231) e^{(13dx+13c)}}{d} - (5153148 e^{(-dx-c)} - 1288287 e^{(-3dx-3c)} + 429429 e^{(-5dx-5c)} - 122694 e^{(-7dx-7c)} + 26026 e^{(-9dx-9c)} - 3549 e^{(-11dx-11c)} + 231) e^{(-13dx-13c)}}{d} - \frac{1}{53760} a * b^2 * ((405 e^{(-2dx-2c)} - 2268 e^{(-4dx-4c)} + 8820 e^{(-6dx-6c)} - 39690 e^{(-8dx-8c)} - 35) e^{(9dx+9c)}}{d} - (39690 e^{(-dx-c)} - 8820 e^{(-3dx-3c)} + 2268 e^{(-5dx-5c)} - 405 e^{(-7dx-7c)} + 35) e^{(-9dx-9c)}}{d} + \frac{1}{160} a^2 * b * (3 e^{(5dx+5c)}}{d} - 25 e^{(3dx+3c)}}{d} + 150 e^{(dx+c)}}{d} + 150 e^{(-dx-c)}}{d} - 25 e^{(-3dx-3c)}}{d} + 3 e^{(-5dx-5c)}}{d} + a^3 * \cosh(dx+c) / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] -1/24600576*b^3*((3549*e^(-2*d*x - 2*c) - 26026*e^(-4*d*x - 4*c) + 122694*e^(-6*d*x - 6*c) - 429429*e^(-8*d*x - 8*c) + 1288287*e^(-10*d*x - 10*c) - 5153148*e^(-12*d*x - 12*c) - 231)*e^(13*d*x + 13*c)/d - (5153148*e^(-d*x - c) - 1288287*e^(-3*d*x - 3*c) + 429429*e^(-5*d*x - 5*c) - 122694*e^(-7*d*x - 7*c) + 26026*e^(-9*d*x - 9*c) - 3549*e^(-11*d*x - 11*c) + 231)*e^(-13*d*x - 13*c))/d) - 1/53760*a*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c) + 35)*e^(-9*d*x - 9*c))/d) + 1/160*a^2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^3*cosh(d*x + c)/d

Fricas [B] time = 1.73622, size = 1620, normalized size = 11.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] 1/61501440*(1155*b^3*cosh(d*x + c)^13 + 15015*b^3*cosh(d*x + c)*sinh(d*x + c)^12 - 17745*b^3*cosh(d*x + c)^11 + 15015*(22*b^3*cosh(d*x + c)^3 - 13*b^3*cosh(d*x + c))*sinh(d*x + c)^10 + 10010*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^9 + 45045*(33*b^3*cosh(d*x + c)^5 - 65*b^3*cosh(d*x + c)^3 + 2*(8*a*b^2 + 13*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 - 4290*(216*a*b^2 + 143*b^3)*cosh(d*x + c)^7 + 30030*(66*b^3*cosh(d*x + c)^7 - 273*b^3*cosh(d*x + c)^5 + 28*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^3 - (216*a*b^2 + 143*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 3003*(768*a^2*b + 1728*a*b^2 + 715*b^3)*cosh(d*x + c)^5 + 15015

$$\begin{aligned} &*(55*b^3*\cosh(d*x + c)^9 - 390*b^3*\cosh(d*x + c)^7 + 84*(8*a*b^2 + 13*b^3)* \\ &\cosh(d*x + c)^5 - 10*(216*a*b^2 + 143*b^3)*\cosh(d*x + c)^3 + (768*a^2*b + 1 \\ &728*a*b^2 + 715*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 15015*(1280*a^2*b + 1 \\ &344*a*b^2 + 429*b^3)*\cosh(d*x + c)^3 + 15015*(6*b^3*\cosh(d*x + c)^11 - 65*b \\ &^3*\cosh(d*x + c)^9 + 24*(8*a*b^2 + 13*b^3)*\cosh(d*x + c)^7 - 6*(216*a*b^2 + \\ &143*b^3)*\cosh(d*x + c)^5 + 2*(768*a^2*b + 1728*a*b^2 + 715*b^3)*\cosh(d*x + \\ &c)^3 - 3*(1280*a^2*b + 1344*a*b^2 + 429*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^ \\ &2 + 60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*\cosh(d*x + c))/d \end{aligned}$$

Sympy [A] time = 169.693, size = 377, normalized size = 2.64

$$\left\{ \begin{array}{l} \frac{a^3 \cosh(c+dx)}{d} + \frac{3a^2b \sinh^4(c+dx) \cosh(c+dx)}{d} - \frac{4a^2b \sinh^2(c+dx) \cosh^3(c+dx)}{d} + \frac{8a^2b \cosh^5(c+dx)}{5d} + \frac{3ab^2 \sinh^8(c+dx) \cosh(c+dx)}{d} - \frac{8ab^2 \sinh^6(c+dx)}{d} \\ x(a + b \sinh^4(c))^3 \sinh(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/(5*d) + 3*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*a*b**2*sinh(c + d*x)**6*cosh(c + d*x)**3/d + 48*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 192*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*a*b**2*cosh(c + d*x)**9/(105*d) + b**3*sinh(c + d*x)**12*cosh(c + d*x)/d - 4*b**3*sinh(c + d*x)**10*cosh(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**8*cosh(c + d*x)**5/d - 64*b**3*sinh(c + d*x)**6*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c + d*x)**4*cosh(c + d*x)**9/(21*d) - 512*b**3*sinh(c + d*x)**2*cosh(c + d*x)**11/(231*d) + 1024*b**3*cosh(c + d*x)**13/(3003*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**3*sinh(c), True))

Giac [B] time = 1.67781, size = 621, normalized size = 4.34

$$1155 b^3 e^{(13 dx+13 c)} - 17745 b^3 e^{(11 dx+11 c)} + 80080 a b^2 e^{(9 dx+9 c)} + 130130 b^3 e^{(9 dx+9 c)} - 926640 a b^2 e^{(7 dx+7 c)} - 613470 b^3 e^{(7 dx+7 c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

```
[Out] 1/123002880*(1155*b^3*e^(13*d*x + 13*c) - 17745*b^3*e^(11*d*x + 11*c) + 800
80*a*b^2*e^(9*d*x + 9*c) + 130130*b^3*e^(9*d*x + 9*c) - 926640*a*b^2*e^(7*d
*x + 7*c) - 613470*b^3*e^(7*d*x + 7*c) + 2306304*a^2*b*e^(5*d*x + 5*c) + 51
89184*a*b^2*e^(5*d*x + 5*c) + 2147145*b^3*e^(5*d*x + 5*c) - 19219200*a^2*b*
e^(3*d*x + 3*c) - 20180160*a*b^2*e^(3*d*x + 3*c) - 6441435*b^3*e^(3*d*x + 3
*c) + 61501440*a^3*e^(d*x + c) + 115315200*a^2*b*e^(d*x + c) + 90810720*a*b
^2*e^(d*x + c) + 25765740*b^3*e^(d*x + c) + (61501440*a^3*e^(12*d*x + 12*c)
+ 115315200*a^2*b*e^(12*d*x + 12*c) + 90810720*a*b^2*e^(12*d*x + 12*c) + 2
5765740*b^3*e^(12*d*x + 12*c) - 19219200*a^2*b*e^(10*d*x + 10*c) - 20180160
*a*b^2*e^(10*d*x + 10*c) - 6441435*b^3*e^(10*d*x + 10*c) + 2306304*a^2*b*e^
(8*d*x + 8*c) + 5189184*a*b^2*e^(8*d*x + 8*c) + 2147145*b^3*e^(8*d*x + 8*c)
- 926640*a*b^2*e^(6*d*x + 6*c) - 613470*b^3*e^(6*d*x + 6*c) + 80080*a*b^2*
e^(4*d*x + 4*c) + 130130*b^3*e^(4*d*x + 4*c) - 17745*b^3*e^(2*d*x + 2*c) +
1155*b^3)*e^(-13*d*x - 13*c))/d
```

3.210 $\int \operatorname{csch}(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=158

$$\frac{b(3a^2 + 9ab + 5b^2) \cosh^3(c + dx)}{3d} - \frac{b(3a^2 + 3ab + b^2) \cosh(c + dx)}{d} - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2(3a + 10b) \cosh^7(c + dx)}{7d}$$

[Out] $-\left(\frac{a^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) - \frac{b(3a^2 + 3ab + b^2) \operatorname{Cosh}[c + d*x]}{d} + \frac{b(3a^2 + 9ab + 5b^2) \operatorname{Cosh}[c + d*x]^3}{3d} - \frac{b^2(9a + 10b) \operatorname{Cosh}[c + d*x]^5}{5d} + \frac{b^2(3a + 10b) \operatorname{Cosh}[c + d*x]^7}{7d} - \frac{5b^3 \operatorname{Cosh}[c + d*x]^9}{9d} + \frac{b^3 \operatorname{Cosh}[c + d*x]^{11}}{11d}$

Rubi [A] time = 0.135437, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3215, 1153, 206}

$$\frac{b(3a^2 + 9ab + 5b^2) \cosh^3(c + dx)}{3d} - \frac{b(3a^2 + 3ab + b^2) \cosh(c + dx)}{d} - \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{d} + \frac{b^2(3a + 10b) \cosh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x] * (a + b \operatorname{Sinh}[c + d*x]^4)^3, x]$

[Out] $-\left(\frac{a^3 \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]}{d}\right) - \frac{b(3a^2 + 3ab + b^2) \operatorname{Cosh}[c + d*x]}{d} + \frac{b(3a^2 + 9ab + 5b^2) \operatorname{Cosh}[c + d*x]^3}{3d} - \frac{b^2(9a + 10b) \operatorname{Cosh}[c + d*x]^5}{5d} + \frac{b^2(3a + 10b) \operatorname{Cosh}[c + d*x]^7}{7d} - \frac{5b^3 \operatorname{Cosh}[c + d*x]^9}{9d} + \frac{b^3 \operatorname{Cosh}[c + d*x]^{11}}{11d}$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)} * ((a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2} * (a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x] \}; \operatorname{FreeQ}\{a, b, e, f, p\}, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 1153

$\operatorname{Int}[(d_.) + (e_.)(x_.)^2]^{(q_.)} * ((a_.) + (b_.)(x_.)^2 + (c_.)(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(d + e*x^2)^q * (a + b*x^2 + c*x^4)^p, x], x] \}; \operatorname{FreeQ}\{a, b, c, d, e\}, x\} \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0] \&\& \operatorname{NeQ}[c*d^2 - b*d*e, 0]$

+ a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}(c+dx) (a+b \sinh^4(c+dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^3}{1-x^2} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\operatorname{Subst}\left(\int \left(b(3a^2+3ab+b^2) - b(3a^2+9ab+5b^2)x^2 + b^2(9a+10b)x^4 - b^3x^6\right) dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{b(3a^2+3ab+b^2) \cosh(c+dx)}{d} + \frac{b(3a^2+9ab+5b^2) \cosh^3(c+dx)}{3d} - \frac{b^3 \cosh^5(c+dx)}{5d} \\ &= -\frac{a^3 \tanh^{-1}(\cosh(c+dx))}{d} - \frac{b(3a^2+3ab+b^2) \cosh(c+dx)}{d} + \frac{b(3a^2+9ab+5b^2) \cosh^3(c+dx)}{3d} - \frac{b^3 \cosh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.380613, size = 139, normalized size = 0.88

$$\frac{-20790b(384a^2 + 280ab + 77b^2) \cosh(c+dx) + 3548160a^3 \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 2079b^2(112a + 55b) \cosh(5(c+dx))}{(3548160d)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (-20790*b*(384*a^2 + 280*a*b + 77*b^2)*Cosh[c + d*x] + 6930*b*(8*a + 5*b)*(16*a + 11*b)*Cosh[3*(c + d*x)] - 2079*b^2*(112*a + 55*b)*Cosh[5*(c + d*x)] + 495*b^2*(48*a + 55*b)*Cosh[7*(c + d*x)] - 4235*b^3*Cosh[9*(c + d*x)] + 315*b^3*Cosh[11*(c + d*x)] + 3548160*a^3*Log[Tanh[(c + d*x)/2]])/(3548160*d)

Maple [A] time = 0.039, size = 148, normalized size = 0.9

$$\frac{1}{d} \left(-2a^3 \operatorname{Artanh}(e^{dx+c}) + 3a^2b \left(-\frac{2}{3} + \frac{1}{3} (\sinh(dx+c))^2 \right) \cosh(dx+c) + 3ab^2 \left(-\frac{16}{35} + \frac{1}{7} (\sinh(dx+c))^6 - \frac{6}{35} (\sinh(dx+c))^4 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] $\frac{1}{d}(-2a^3\operatorname{arctanh}(\exp(d*x+c))+3a^2b(-\frac{2}{3}+\frac{1}{3}\sinh(d*x+c)^2)\cosh(d*x+c)+3ab^2(-\frac{16}{35}+\frac{1}{7}\sinh(d*x+c)^6-\frac{6}{35}\sinh(d*x+c)^4+\frac{8}{35}\sinh(d*x+c)^2)\cosh(d*x+c)+b^3(-\frac{256}{693}+\frac{1}{11}\sinh(d*x+c)^{10}-\frac{10}{99}\sinh(d*x+c)^8+\frac{80}{693}\sinh(d*x+c)^6-\frac{32}{231}\sinh(d*x+c)^4+\frac{128}{693}\sinh(d*x+c)^2)\cosh(d*x+c))$

Maxima [B] time = 1.06644, size = 441, normalized size = 2.79

$$-\frac{1}{1419264}b^3\left(\frac{(847e^{(-2dx-2c)} - 5445e^{(-4dx-4c)} + 22869e^{(-6dx-6c)} - 76230e^{(-8dx-8c)} + 320166e^{(-10dx-10c)} - 63)e^{(11dx+11c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] $-\frac{1}{1419264}b^3\left(\frac{(847e^{(-2d*x - 2*c)} - 5445e^{(-4d*x - 4*c)} + 22869e^{(-6d*x - 6*c)} - 76230e^{(-8d*x - 8*c)} + 320166e^{(-10d*x - 10*c)} - 63)e^{(11d*x + 11*c)}}{d} + \frac{(320166e^{(-d*x - c)} - 76230e^{(-3d*x - 3*c)} + 22869e^{(-5d*x - 5*c)} - 5445e^{(-7d*x - 7*c)} + 847e^{(-9d*x - 9*c)} - 63e^{(-11d*x - 11*c)})}{d} - \frac{3}{4480}ab^2\left(\frac{(49e^{(-2d*x - 2*c)} - 245e^{(-4d*x - 4*c)} + 1225e^{(-6d*x - 6*c)} - 5)e^{(7d*x + 7*c)}}{d} + \frac{(1225e^{(-d*x - c)} - 245e^{(-3d*x - 3*c)} + 49e^{(-5d*x - 5*c)} - 5e^{(-7d*x - 7*c)})}{d} + \frac{1}{8}a^2b\left(\frac{e^{(3d*x + 3*c)}}{d} - \frac{9e^{(d*x + c)}}{d} - \frac{9e^{(-d*x - c)}}{d} + \frac{e^{(-3d*x - 3*c)}}{d}\right) + a^3\log(\tanh(\frac{1}{2}d*x + \frac{1}{2}c))/d\right)$

Fricas [B] time = 2.04014, size = 10637, normalized size = 67.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")`

[Out] $\frac{1}{7096320}(315b^3\cosh(d*x + c)^{22} + 6930b^3\cosh(d*x + c)\sinh(d*x + c)^{21} + 315b^3\sinh(d*x + c)^{22} - 4235b^3\cosh(d*x + c)^{20} + 385(189b^3\co$

$$\begin{aligned}
& \text{sh}(d*x + c)^2 - 11*b^3*\sinh(d*x + c)^{20} + 7700*(63*b^3*\cosh(d*x + c)^3 - 1 \\
& 1*b^3*\cosh(d*x + c))*\sinh(d*x + c)^{19} + 495*(48*a*b^2 + 55*b^3)*\cosh(d*x + \\
& c)^{18} + 55*(41895*b^3*\cosh(d*x + c)^4 - 14630*b^3*\cosh(d*x + c)^2 + 432*a*b \\
& ^2 + 495*b^3)*\sinh(d*x + c)^{18} + 330*(25137*b^3*\cosh(d*x + c)^5 - 14630*b^3 \\
& *\cosh(d*x + c)^3 + 27*(48*a*b^2 + 55*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{17} - \\
& 2079*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^{16} + 33*(712215*b^3*\cosh(d*x + c)^ \\
& 6 - 621775*b^3*\cosh(d*x + c)^4 - 7056*a*b^2 - 3465*b^3 + 2295*(48*a*b^2 + 5 \\
& 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{16} + 528*(101745*b^3*\cosh(d*x + c)^7 \\
& - 124355*b^3*\cosh(d*x + c)^5 + 765*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^3 - 63 \\
& *(112*a*b^2 + 55*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{15} + 6930*(128*a^2*b + 1 \\
& 68*a*b^2 + 55*b^3)*\cosh(d*x + c)^{14} + 330*(305235*b^3*\cosh(d*x + c)^8 - 497 \\
& 420*b^3*\cosh(d*x + c)^6 + 4590*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^4 + 2688*a \\
& ^2*b + 3528*a*b^2 + 1155*b^3 - 756*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^2)*\si \\
& nh(d*x + c)^{14} + 4620*(33915*b^3*\cosh(d*x + c)^9 - 71060*b^3*\cosh(d*x + c)^ \\
& 7 + 918*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^5 - 252*(112*a*b^2 + 55*b^3)*\cosh \\
& (d*x + c)^3 + 21*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^{13} - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^{12} + 2310*(88 \\
& 179*b^3*\cosh(d*x + c)^{10} - 230945*b^3*\cosh(d*x + c)^8 + 3978*(48*a*b^2 + 55 \\
& *b^3)*\cosh(d*x + c)^6 - 1638*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^4 - 3456*a^ \\
& 2*b - 2520*a*b^2 - 693*b^3 + 273*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x \\
& + c)^2)*\sinh(d*x + c)^{12} + 8*(27776385*b^3*\cosh(d*x + c)^{11} - 88913825*b^3* \\
& \cosh(d*x + c)^9 + 1969110*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^7 - 1135134*(11 \\
& 2*a*b^2 + 55*b^3)*\cosh(d*x + c)^5 + 315315*(128*a^2*b + 168*a*b^2 + 55*b^3) \\
& *\cosh(d*x + c)^3 - 31185*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c))*\si \\
& nh(d*x + c)^{11} - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^{10} + \\
& 22*(9258795*b^3*\cosh(d*x + c)^{12} - 35565530*b^3*\cosh(d*x + c)^{10} + 984555*(\\
& 48*a*b^2 + 55*b^3)*\cosh(d*x + c)^8 - 756756*(112*a*b^2 + 55*b^3)*\cosh(d*x + \\
& c)^6 + 315315*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^4 - 362880*a^ \\
& 2*b - 264600*a*b^2 - 72765*b^3 - 62370*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cos \\
& h(d*x + c)^2)*\sinh(d*x + c)^{10} + 220*(712215*b^3*\cosh(d*x + c)^{13} - 3233230 \\
& *b^3*\cosh(d*x + c)^{11} + 109395*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^9 - 108108 \\
& *(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^7 + 63063*(128*a^2*b + 168*a*b^2 + 55*b \\
& ^3)*\cosh(d*x + c)^5 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^ \\
& 3 - 945*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 6 \\
& 930*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^8 + 330*(305235*b^3*\cosh \\
& (d*x + c)^{14} - 1616615*b^3*\cosh(d*x + c)^{12} + 65637*(48*a*b^2 + 55*b^3)*\cos \\
& h(d*x + c)^{10} - 81081*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^8 + 63063*(128*a^2 \\
& *b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^6 - 31185*(384*a^2*b + 280*a*b^2 + 7 \\
& 7*b^3)*\cosh(d*x + c)^4 + 2688*a^2*b + 3528*a*b^2 + 1155*b^3 - 2835*(384*a^2 \\
& *b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 2640*(20349*b^3 \\
& *\cosh(d*x + c)^{15} - 124355*b^3*\cosh(d*x + c)^{13} + 5967*(48*a*b^2 + 55*b^3)* \\
& \cosh(d*x + c)^{11} - 9009*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^9 + 9009*(128*a^ \\
& 2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^7 - 6237*(384*a^2*b + 280*a*b^2 + 7 \\
& 7*b^3)*\cosh(d*x + c)^5 - 945*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c) \\
& ^3 + 21*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 2
\end{aligned}$$

$$\begin{aligned}
& 079*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^6 + 231*(101745*b^3*\cosh(d*x + c)^16 \\
& - 710600*b^3*\cosh(d*x + c)^14 + 39780*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^12 \\
& - 72072*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^10 + 90090*(128*a^2*b + 168*a*b \\
& ^2 + 55*b^3)*\cosh(d*x + c)^8 - 83160*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^6 \\
& - 18900*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^4 - 1008*a*b^2 - 495*b^3 + 840*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 \\
& + 462*(17955*b^3*\cosh(d*x + c)^17 - 142120*b^3*\cosh(d*x + c)^15 + 9180*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^13 - 19656*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^11 + 30030*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^9 - 35640*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^7 - 11340*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^5 + 840*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^3 - 27*(112*a*b^2 + 55*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 4235*b^3*\cosh(d*x + c)^2 + 495*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^4 + 165*(13965*b^3*\cosh(d*x + c)^18 - 124355*b^3*\cosh(d*x + c)^16 + 9180*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^14 - 22932*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^12 + 42042*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^10 - 62370*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^8 - 26460*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^6 + 2940*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^4 + 144*a*b^2 + 165*b^3 - 189*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 660*(735*b^3*\cosh(d*x + c)^19 - 7315*b^3*\cosh(d*x + c)^17 + 612*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^15 - 1764*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^13 + 3822*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^11 - 6930*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^9 - 3780*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^7 + 588*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^5 - 63*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^3 + 3*(48*a*b^2 + 55*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 315*b^3 + 55*(1323*b^3*\cosh(d*x + c)^20 - 14630*b^3*\cosh(d*x + c)^18 + 1377*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^16 - 4536*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^14 + 11466*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^12 - 24948*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^10 - 17010*(384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^8 + 3528*(128*a^2*b + 168*a*b^2 + 55*b^3)*\cosh(d*x + c)^6 - 567*(112*a*b^2 + 55*b^3)*\cosh(d*x + c)^4 - 77*b^3 + 54*(48*a*b^2 + 55*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 7096320*(a^3*\cosh(d*x + c)^11 + 11*a^3*\cosh(d*x + c)^10*\sinh(d*x + c) + 55*a^3*\cosh(d*x + c)^9*\sinh(d*x + c)^2 + 165*a^3*\cosh(d*x + c)^8*\sinh(d*x + c)^3 + 330*a^3*\cosh(d*x + c)^7*\sinh(d*x + c)^4 + 462*a^3*\cosh(d*x + c)^6*\sinh(d*x + c)^5 + 462*a^3*\cosh(d*x + c)^5*\sinh(d*x + c)^6 + 330*a^3*\cosh(d*x + c)^4*\sinh(d*x + c)^7 + 165*a^3*\cosh(d*x + c)^3*\sinh(d*x + c)^8 + 55*a^3*\cosh(d*x + c)^2*\sinh(d*x + c)^9 + 11*a^3*\cosh(d*x + c)*\sinh(d*x + c)^10 + a^3*\sinh(d*x + c)^11)*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) + 7096320*(a^3*\cosh(d*x + c)^11 + 11*a^3*\cosh(d*x + c)^10*\sinh(d*x + c) + 55*a^3*\cosh(d*x + c)^9*\sinh(d*x + c)^2 + 165*a^3*\cosh(d*x + c)^8*\sinh(d*x + c)^3 + 330*a^3*\cosh(d*x + c)^7*\sinh(d*x + c)^4 + 462*a^3*\cosh(d*x + c)^6*\sinh(d*x + c)^5 + 462*a^3*\cosh(d*x + c)^5*\sinh(d*x + c)^6 + 330*a^3*\cosh(d*x + c)^4*\sinh(d*x + c)^7 + 165*a^3*\cosh(d*x + c)^3*\sinh(d*x + c)^8 + 55*a^3*\cosh(d*x + c)^2*\sinh(d*x + c)^9 + 11*a^3*\cosh(d*x + c)*\sinh(d*x + c)^10 + a^3*\sinh(d*x + c)^11)
\end{aligned}$$

$$\begin{aligned} & ^{11}) \cdot \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 22 \cdot (315 \cdot b^3 \cdot \cosh(dx + c)^{21} \\ & - 3850 \cdot b^3 \cdot \cosh(dx + c)^{19} + 405 \cdot (48 \cdot a \cdot b^2 + 55 \cdot b^3) \cdot \cosh(dx + c)^{17} - 15 \\ & 12 \cdot (112 \cdot a \cdot b^2 + 55 \cdot b^3) \cdot \cosh(dx + c)^{15} + 4410 \cdot (128 \cdot a^2 \cdot b + 168 \cdot a \cdot b^2 + 55 \\ & \cdot b^3) \cdot \cosh(dx + c)^{13} - 11340 \cdot (384 \cdot a^2 \cdot b + 280 \cdot a \cdot b^2 + 77 \cdot b^3) \cdot \cosh(dx + \\ & c)^{11} - 9450 \cdot (384 \cdot a^2 \cdot b + 280 \cdot a \cdot b^2 + 77 \cdot b^3) \cdot \cosh(dx + c)^9 + 2520 \cdot (128 \cdot a \\ & ^2 \cdot b + 168 \cdot a \cdot b^2 + 55 \cdot b^3) \cdot \cosh(dx + c)^7 - 567 \cdot (112 \cdot a \cdot b^2 + 55 \cdot b^3) \cdot \cosh(\\ & dx + c)^5 - 385 \cdot b^3 \cdot \cosh(dx + c) + 90 \cdot (48 \cdot a \cdot b^2 + 55 \cdot b^3) \cdot \cosh(dx + c)^3 \\ &) \cdot \sinh(dx + c) / (d \cdot \cosh(dx + c)^{11} + 11 \cdot d \cdot \cosh(dx + c)^{10} \cdot \sinh(dx + c) \\ & + 55 \cdot d \cdot \cosh(dx + c)^9 \cdot \sinh(dx + c)^2 + 165 \cdot d \cdot \cosh(dx + c)^8 \cdot \sinh(dx + c) \\ &)^3 + 330 \cdot d \cdot \cosh(dx + c)^7 \cdot \sinh(dx + c)^4 + 462 \cdot d \cdot \cosh(dx + c)^6 \cdot \sinh(dx + \\ & c)^5 + 462 \cdot d \cdot \cosh(dx + c)^5 \cdot \sinh(dx + c)^6 + 330 \cdot d \cdot \cosh(dx + c)^4 \cdot \sinh(dx + \\ & c)^7 + 165 \cdot d \cdot \cosh(dx + c)^3 \cdot \sinh(dx + c)^8 + 55 \cdot d \cdot \cosh(dx + c)^2 \cdot \sinh(dx + c)^9 \\ & + 11 \cdot d \cdot \cosh(dx + c) \cdot \sinh(dx + c)^{10} + d \cdot \sinh(dx + c)^{11} \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)*(a+b*sinh(dx+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.65024, size = 570, normalized size = 3.61

$$-\frac{a^3 \log(e^{dx+c} + 1)}{d} + \frac{a^3 \log(|e^{dx+c} - 1|)}{d} - \frac{(7983360 a^2 b e^{10 dx+10 c} + 5821200 a b^2 e^{10 dx+10 c} + 1600830 b^3 e^{10 dx+10 c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)*(a+b*sinh(dx+c)^4)^3,x, algorithm="giac")

[Out] $-a^3 \log(e^{dx+c} + 1)/d + a^3 \log(\text{abs}(e^{dx+c} - 1))/d - 1/7096320 \cdot (7983360 \cdot a^2 \cdot b \cdot e^{10 \cdot dx + 10 \cdot c} + 5821200 \cdot a \cdot b^2 \cdot e^{10 \cdot dx + 10 \cdot c} + 1600830 \cdot b^3 \cdot e^{10 \cdot dx + 10 \cdot c} - 887040 \cdot a^2 \cdot b \cdot e^{8 \cdot dx + 8 \cdot c} - 1164240 \cdot a \cdot b^2 \cdot e^{8 \cdot dx + 8 \cdot c} - 381150 \cdot b^3 \cdot e^{8 \cdot dx + 8 \cdot c} + 232848 \cdot a \cdot b^2 \cdot e^{6 \cdot dx + 6 \cdot c} + 114345 \cdot b^3 \cdot e^{6 \cdot dx + 6 \cdot c} - 23760 \cdot a \cdot b^2 \cdot e^{4 \cdot dx + 4 \cdot c} - 27225 \cdot b^3 \cdot e^{4 \cdot dx + 4 \cdot c})$

$$\begin{aligned}
& + 4*c) + 4235*b^3*e^{(2*d*x + 2*c)} - 315*b^3)*e^{(-11*d*x - 11*c)}/d + 1/7096 \\
& 320*(315*b^3*d^{10}*e^{(11*d*x + 11*c)} - 4235*b^3*d^{10}*e^{(9*d*x + 9*c)} + 23760 \\
& *a*b^2*d^{10}*e^{(7*d*x + 7*c)} + 27225*b^3*d^{10}*e^{(7*d*x + 7*c)} - 232848*a*b^2 \\
& *d^{10}*e^{(5*d*x + 5*c)} - 114345*b^3*d^{10}*e^{(5*d*x + 5*c)} + 887040*a^2*b*d^{10} \\
& *e^{(3*d*x + 3*c)} + 1164240*a*b^2*d^{10}*e^{(3*d*x + 3*c)} + 381150*b^3*d^{10}*e^{(\\
& 3*d*x + 3*c)} - 7983360*a^2*b*d^{10}*e^{(d*x + c)} - 5821200*a*b^2*d^{10}*e^{(d*x + \\
& c)} - 1600830*b^3*d^{10}*e^{(d*x + c)})/d^{11}
\end{aligned}$$

3.211 $\int \operatorname{csch}^3(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=148

$$\frac{b(3a^2 + 3ab + b^2) \cosh(c + dx)}{d} + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{3b^2(a + 2b) \cosh^5(c + dx)}{5d}$$

```
[Out] (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]
)/d - (2*b^2*(3*a + 2*b)*Cosh[c + d*x]^3)/(3*d) + (3*b^2*(a + 2*b)*Cosh[c +
d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d
) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rubi [A] time = 0.209365, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3215, 1157, 1810, 206}

$$\frac{b(3a^2 + 3ab + b^2) \cosh(c + dx)}{d} + \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} - \frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} + \frac{3b^2(a + 2b) \cosh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]
)/d - (2*b^2*(3*a + 2*b)*Cosh[c + d*x]^3)/(3*d) + (3*b^2*(a + 2*b)*Cosh[c +
d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d
) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^3(c + dx) (a + b \sinh^4(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^3}{(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-a^3 - 6a^2b - 6ab^2 - 2b^3 + 2b(3a^2 + 9ab + 5b^2)x^2}{(1-x^2)^2} dx, x, \cosh(c + dx)\right)}{d} \\ &= -\frac{a^3 \coth(c + dx) \operatorname{csch}(c + dx)}{2d} - \frac{\operatorname{Subst}\left(\int \left(-2b(3a^2 + 3ab + b^2) + 4b^2(3a + 2b)x\right) dx, x, \cosh(c + dx)\right)}{d} \\ &= \frac{b(3a^2 + 3ab + b^2) \cosh(c + dx)}{d} - \frac{2b^2(3a + 2b) \cosh^3(c + dx)}{3d} + \frac{3b^2(a + 2b) \cosh^2(c + dx)}{2d} \\ &= \frac{a^3 \tanh^{-1}(\cosh(c + dx))}{2d} + \frac{b(3a^2 + 3ab + b^2) \cosh(c + dx)}{d} - \frac{2b^2(3a + 2b) \cosh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.340198, size = 155, normalized size = 1.05

$$\frac{1890b(128a^2 + 80ab + 21b^2) \cosh(c + dx) - 10080a^3 \operatorname{csch}^2\left(\frac{1}{2}(c + dx)\right) - 10080a^3 \operatorname{sech}^2\left(\frac{1}{2}(c + dx)\right) - 40320a^3 \log\left(\tanh\left(\frac{1}{2}(c + dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

[Out] $(1890*b*(128*a^2 + 80*a*b + 21*b^2)*\text{Cosh}[c + d*x] - 1260*b^2*(20*a + 7*b)*\text{Cosh}[3*(c + d*x)] + 3024*a*b^2*\text{Cosh}[5*(c + d*x)] + 2268*b^3*\text{Cosh}[5*(c + d*x)] - 405*b^3*\text{Cosh}[7*(c + d*x)] + 35*b^3*\text{Cosh}[9*(c + d*x)] - 10080*a^3*\text{Csch}[(c + d*x)/2]^2 - 40320*a^3*\text{Log}[\text{Tanh}[(c + d*x)/2]] - 10080*a^3*\text{Sech}[(c + d*x)/2]^2)/(80640*d)$

Maple [A] time = 0.049, size = 130, normalized size = 0.9

$\frac{1}{d} \left(a^3 \left(-\frac{\text{csch}(dx+c) \coth(dx+c)}{2} + \text{Artanh}(e^{dx+c}) \right) + 3a^2b \cosh(dx+c) + 3ab^2 \left(\frac{8}{15} + \frac{1}{5} (\sinh(dx+c))^4 - \frac{4}{15} (\sinh(dx+c))^2 \right) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] $1/d*(a^3*(-1/2*\text{csch}(d*x+c)*\text{coth}(d*x+c)+\text{arctanh}(\exp(d*x+c)))+3*a^2*b*\text{cosh}(d*x+c)+3*a*b^2*(8/15+1/5*\sinh(d*x+c)^4-4/15*\sinh(d*x+c)^2)*\text{cosh}(d*x+c)+b^3*(128/315+1/9*\sinh(d*x+c)^8-8/63*\sinh(d*x+c)^6+16/105*\sinh(d*x+c)^4-64/315*\sinh(d*x+c)^2)*\text{cosh}(d*x+c)$

Maxima [B] time = 1.07909, size = 451, normalized size = 3.05

$-\frac{1}{161280} b^3 \left(\frac{(405 e^{(-2dx-2c)} - 2268 e^{(-4dx-4c)} + 8820 e^{(-6dx-6c)} - 39690 e^{(-8dx-8c)} - 35) e^{(9dx+9c)}}{d} - \frac{39690 e^{(-dx-c)} - 8820 e^{(-3dx-3c)} + 2268 e^{(-5dx-5c)} - 405 e^{(-7dx-7c)} + 35 e^{(-9dx-9c)}}{d} + \frac{1}{160} a^2 b^2 (3 e^{(5dx+5c)})/d - 25 e^{(3dx+x+3c)}/d + 150 e^{(dx+c)}/d + 150 e^{(-dx-c)}/d - 25 e^{(-3dx-3c)}/d + 3 e^{(-5dx-5c)}/d + \frac{3}{2} a^2 b (e^{(dx+c)}/d + e^{(-dx-c)}/d) + \frac{1}{2} a^3 (\log(e^{(-dx-c)} + 1)/d - \log(e^{(-dx-c)} - 1)/d + 2(e^{(-dx-c)} + e^{(-3dx-3c)})/(d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1))) \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] $-1/161280*b^3*((405*e^{(-2*d*x - 2*c)} - 2268*e^{(-4*d*x - 4*c)} + 8820*e^{(-6*d*x - 6*c)} - 39690*e^{(-8*d*x - 8*c)} - 35)*e^{(9*d*x + 9*c)}/d - (39690*e^{(-d*x - c)} - 8820*e^{(-3*d*x - 3*c)} + 2268*e^{(-5*d*x - 5*c)} - 405*e^{(-7*d*x - 7*c)} + 35*e^{(-9*d*x - 9*c)})/d + 1/160*a^2*b^2*(3*e^{(5*d*x + 5*c)})/d - 25*e^{(3*d*x + 3*c)}/d + 150*e^{(d*x + c)}/d + 150*e^{(-d*x - c)}/d - 25*e^{(-3*d*x - 3*c)}/d + 3*e^{(-5*d*x - 5*c)}/d + 3/2*a^2*b*(e^{(d*x + c)}/d + e^{(-d*x - c)}/d) + 1/2*a^3*(log(e^{(-d*x - c)} + 1)/d - log(e^{(-d*x - c)} - 1)/d + 2*(e^{(-d*x - c)} + e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1)))$

Fricas [B] time = 2.12155, size = 13697, normalized size = 92.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] $\frac{1}{161280} \cdot (35b^3 \cosh(dx+c)^{22} + 770b^3 \cosh(dx+c) \sinh(dx+c)^{21} + 35b^3 \sinh(dx+c)^{22} - 475b^3 \cosh(dx+c)^{20} + 5(1617b^3 \cosh(dx+c)^2 - 95b^3) \sinh(dx+c)^{20} + 100(539b^3 \cosh(dx+c)^3 - 95b^3 \cosh(dx+c)) \sinh(dx+c)^{19} + (3024ab^2 + 3113b^3) \cosh(dx+c)^{18} + (256025b^3 \cosh(dx+c)^4 - 90250b^3 \cosh(dx+c)^2 + 3024ab^2 + 3113b^3) \sinh(dx+c)^{18} + 6(153615b^3 \cosh(dx+c)^5 - 90250b^3 \cosh(dx+c)^3 + 3(3024ab^2 + 3113b^3) \cosh(dx+c)) \sinh(dx+c)^{17} - 9(3472ab^2 + 1529b^3) \cosh(dx+c)^{16} + 3(870485b^3 \cosh(dx+c)^6 - 767125b^3 \cosh(dx+c)^4 - 10416ab^2 - 4587b^3 + 51(3024ab^2 + 3113b^3) \cosh(dx+c)^2) \sinh(dx+c)^{16} + 48(124355b^3 \cosh(dx+c)^7 - 153425b^3 \cosh(dx+c)^5 + 17(3024ab^2 + 3113b^3) \cosh(dx+c)^3 - 3(3472ab^2 + 1529b^3) \cosh(dx+c)) \sinh(dx+c)^{15} + 126(1920a^2b + 1624ab^2 + 473b^3) \cosh(dx+c)^{14} + 6(1865325b^3 \cosh(dx+c)^8 - 3068500b^3 \cosh(dx+c)^6 + 510(3024ab^2 + 3113b^3) \cosh(dx+c)^4 + 40320a^2b + 34104ab^2 + 9933b^3 - 180(3472ab^2 + 1529b^3) \cosh(dx+c)^2) \sinh(dx+c)^{14} + 4(4352425b^3 \cosh(dx+c)^9 - 9205500b^3 \cosh(dx+c)^7 + 2142(3024ab^2 + 3113b^3) \cosh(dx+c)^5 - 1260(3472ab^2 + 1529b^3) \cosh(dx+c)^3 + 441(1920a^2b + 1624ab^2 + 473b^3) \cosh(dx+c)) \sinh(dx+c)^{13} - 630(256a^3 + 384a^2b + 280ab^2 + 77b^3) \cosh(dx+c)^{12} + 2(11316305b^3 \cosh(dx+c)^{10} - 29917875b^3 \cosh(dx+c)^8 + 9282(3024ab^2 + 3113b^3) \cosh(dx+c)^6 - 8190(3472ab^2 + 1529b^3) \cosh(dx+c)^4 - 80640a^3 - 120960a^2b - 88200ab^2 - 24255b^3 + 5733(1920a^2b + 1624ab^2 + 473b^3) \cosh(dx+c)^2) \sinh(dx+c)^{12} + 8(3086265b^3 \cosh(dx+c)^{11} - 9972625b^3 \cosh(dx+c)^9 + 3978(3024ab^2 + 3113b^3) \cosh(dx+c)^7 - 4914(3472ab^2 + 1529b^3) \cosh(dx+c)^5 + 5733(1920a^2b + 1624ab^2 + 473b^3) \cosh(dx+c)^3 - 945(256a^3 + 384a^2b + 280ab^2 + 77b^3) \cosh(dx+c)) \sinh(dx+c)^{11} - 630(256a^3 + 384a^2b + 280ab^2 + 77b^3) \cosh(dx+c)^{10} + 2(11316305b^3 \cosh(dx+c)^{12} - 43879550b^3 \cosh(dx+c)^{10} + 21879(3024ab^2 + 3113b^3) \cosh(dx+c)^8 - 36036(3472ab^2 + 1529b^3) \cosh(dx+c)^6 + 63063(1920a^2b + 1624ab^2 + 473b^3) \cosh(dx+c)^4 - 80640a^3 - 120960a^2b - 88200ab^2 - 24255b^3 - 20790(256a^3 + 384a^2b + 280ab^2 + 77b^3) \cosh(dx+c)^2) \sinh(dx+c)^{10} + 4(4352425b^3 \cosh(dx+c)^{13} - 19945250b^3 \cosh(dx+c)^{11} + 12155(3024ab^2 + 3113b^3) \cosh(dx+c)^9 - 25740(3472ab^2 + 1529b^3) \cosh(dx+c)^7 + 63063(1920a^2b + 1624ab^2 + 473b^3) \cosh(dx+c)^5 - 34650(256a$

$$\begin{aligned}
&^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^3 - 1575*(256*a^3 + 384* \\
&a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 126*(1920*a^2* \\
&b + 1624*a*b^2 + 473*b^3)*\cosh(d*x + c)^8 + 6*(1865325*b^3*\cosh(d*x + c)^14 \\
&- 9972625*b^3*\cosh(d*x + c)^12 + 7293*(3024*a*b^2 + 3113*b^3)*\cosh(d*x + c \\
&)^10 - 19305*(3472*a*b^2 + 1529*b^3)*\cosh(d*x + c)^8 + 63063*(1920*a^2*b + \\
&1624*a*b^2 + 473*b^3)*\cosh(d*x + c)^6 - 51975*(256*a^3 + 384*a^2*b + 280*a* \\
&b^2 + 77*b^3)*\cosh(d*x + c)^4 + 40320*a^2*b + 34104*a*b^2 + 9933*b^3 - 4725 \\
&*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^ \\
&8 + 48*(124355*b^3*\cosh(d*x + c)^15 - 767125*b^3*\cosh(d*x + c)^13 + 663*(30 \\
&24*a*b^2 + 3113*b^3)*\cosh(d*x + c)^11 - 2145*(3472*a*b^2 + 1529*b^3)*\cosh(d \\
&*x + c)^9 + 9009*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*\cosh(d*x + c)^7 - 1039 \\
&5*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^5 - 1575*(256*a^ \\
&3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^3 + 21*(1920*a^2*b + 1624 \\
&*a*b^2 + 473*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 9*(3472*a*b^2 + 1529*b^3 \\
&)*\cosh(d*x + c)^6 + 3*(870485*b^3*\cosh(d*x + c)^16 - 6137000*b^3*\cosh(d*x + \\
&c)^14 + 6188*(3024*a*b^2 + 3113*b^3)*\cosh(d*x + c)^12 - 24024*(3472*a*b^2 \\
&+ 1529*b^3)*\cosh(d*x + c)^10 + 126126*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*c \\
&osh(d*x + c)^8 - 194040*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x \\
&+ c)^6 - 44100*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^4 \\
&- 10416*a*b^2 - 4587*b^3 + 1176*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*\cosh(d* \\
&x + c)^2)*\sinh(d*x + c)^6 + 6*(153615*b^3*\cosh(d*x + c)^17 - 1227400*b^3*co \\
&sh(d*x + c)^15 + 1428*(3024*a*b^2 + 3113*b^3)*\cosh(d*x + c)^13 - 6552*(3472 \\
&*a*b^2 + 1529*b^3)*\cosh(d*x + c)^11 + 42042*(1920*a^2*b + 1624*a*b^2 + 473* \\
&b^3)*\cosh(d*x + c)^9 - 83160*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cos \\
&h(d*x + c)^7 - 26460*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + \\
&c)^5 + 1176*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*\cosh(d*x + c)^3 - 9*(3472*a \\
&*b^2 + 1529*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 475*b^3*\cosh(d*x + c)^2 + \\
&(3024*a*b^2 + 3113*b^3)*\cosh(d*x + c)^4 + (256025*b^3*\cosh(d*x + c)^18 - 2 \\
&301375*b^3*\cosh(d*x + c)^16 + 3060*(3024*a*b^2 + 3113*b^3)*\cosh(d*x + c)^14 \\
&- 16380*(3472*a*b^2 + 1529*b^3)*\cosh(d*x + c)^12 + 126126*(1920*a^2*b + 16 \\
&24*a*b^2 + 473*b^3)*\cosh(d*x + c)^10 - 311850*(256*a^3 + 384*a^2*b + 280*a* \\
&b^2 + 77*b^3)*\cosh(d*x + c)^8 - 132300*(256*a^3 + 384*a^2*b + 280*a*b^2 + 7 \\
&7*b^3)*\cosh(d*x + c)^6 + 8820*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*\cosh(d*x \\
&+ c)^4 + 3024*a*b^2 + 3113*b^3 - 135*(3472*a*b^2 + 1529*b^3)*\cosh(d*x + c)^ \\
&2)*\sinh(d*x + c)^4 + 4*(13475*b^3*\cosh(d*x + c)^19 - 135375*b^3*\cosh(d*x + \\
&c)^17 + 204*(3024*a*b^2 + 3113*b^3)*\cosh(d*x + c)^15 - 1260*(3472*a*b^2 + 1 \\
&529*b^3)*\cosh(d*x + c)^13 + 11466*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*\cosh(\\
&d*x + c)^11 - 34650*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c \\
&)^9 - 18900*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^7 + 17 \\
&64*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*\cosh(d*x + c)^5 - 45*(3472*a*b^2 + 1 \\
&529*b^3)*\cosh(d*x + c)^3 + (3024*a*b^2 + 3113*b^3)*\cosh(d*x + c))*\sinh(d*x \\
&+ c)^3 + 35*b^3 + (8085*b^3*\cosh(d*x + c)^20 - 90250*b^3*\cosh(d*x + c)^18 + \\
&153*(3024*a*b^2 + 3113*b^3)*\cosh(d*x + c)^16 - 1080*(3472*a*b^2 + 1529*b^3 \\
&)*\cosh(d*x + c)^14 + 11466*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*\cosh(d*x + c \\
&)^12 - 41580*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^10 -
\end{aligned}$$

$$\begin{aligned}
& 28350*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh(d*x + c)^8 + 3528*(19 \\
& 20*a^2*b + 1624*a*b^2 + 473*b^3)*\cosh(d*x + c)^6 - 135*(3472*a*b^2 + 1529*b \\
& ^3)*\cosh(d*x + c)^4 - 475*b^3 + 6*(3024*a*b^2 + 3113*b^3)*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^2 + 80640*(a^3*\cosh(d*x + c)^13 + 13*a^3*\cosh(d*x + c)*\sinh(d \\
& *x + c)^12 + a^3*\sinh(d*x + c)^13 - 2*a^3*\cosh(d*x + c)^11 + a^3*\cosh(d*x + \\
& c)^9 + 2*(39*a^3*\cosh(d*x + c)^2 - a^3)*\sinh(d*x + c)^11 + 22*(13*a^3*\cosh \\
& (d*x + c)^3 - a^3*\cosh(d*x + c))*\sinh(d*x + c)^10 + (715*a^3*\cosh(d*x + c)^ \\
& 4 - 110*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x + c)^9 + 3*(429*a^3*\cosh(d*x + \\
& c)^5 - 110*a^3*\cosh(d*x + c)^3 + 3*a^3*\cosh(d*x + c))*\sinh(d*x + c)^8 + 12* \\
& (143*a^3*\cosh(d*x + c)^6 - 55*a^3*\cosh(d*x + c)^4 + 3*a^3*\cosh(d*x + c)^2)* \\
& \sinh(d*x + c)^7 + 12*(143*a^3*\cosh(d*x + c)^7 - 77*a^3*\cosh(d*x + c)^5 + 7* \\
& a^3*\cosh(d*x + c)^3)*\sinh(d*x + c)^6 + 3*(429*a^3*\cosh(d*x + c)^8 - 308*a^3 \\
& *\cosh(d*x + c)^6 + 42*a^3*\cosh(d*x + c)^4)*\sinh(d*x + c)^5 + (715*a^3*\cosh(\\
& d*x + c)^9 - 660*a^3*\cosh(d*x + c)^7 + 126*a^3*\cosh(d*x + c)^5)*\sinh(d*x + \\
& c)^4 + 2*(143*a^3*\cosh(d*x + c)^10 - 165*a^3*\cosh(d*x + c)^8 + 42*a^3*\cosh(\\
& d*x + c)^6)*\sinh(d*x + c)^3 + 2*(39*a^3*\cosh(d*x + c)^11 - 55*a^3*\cosh(d*x \\
& + c)^9 + 18*a^3*\cosh(d*x + c)^7)*\sinh(d*x + c)^2 + (13*a^3*\cosh(d*x + c)^12 \\
& - 22*a^3*\cosh(d*x + c)^10 + 9*a^3*\cosh(d*x + c)^8)*\sinh(d*x + c))*\log(\cosh \\
& (d*x + c) + \sinh(d*x + c) + 1) - 80640*(a^3*\cosh(d*x + c)^13 + 13*a^3*\cosh(\\
& d*x + c)*\sinh(d*x + c)^12 + a^3*\sinh(d*x + c)^13 - 2*a^3*\cosh(d*x + c)^11 + \\
& a^3*\cosh(d*x + c)^9 + 2*(39*a^3*\cosh(d*x + c)^2 - a^3)*\sinh(d*x + c)^11 + \\
& 22*(13*a^3*\cosh(d*x + c)^3 - a^3*\cosh(d*x + c))*\sinh(d*x + c)^10 + (715*a^3 \\
& *\cosh(d*x + c)^4 - 110*a^3*\cosh(d*x + c)^2 + a^3)*\sinh(d*x + c)^9 + 3*(429* \\
& a^3*\cosh(d*x + c)^5 - 110*a^3*\cosh(d*x + c)^3 + 3*a^3*\cosh(d*x + c))*\sinh(d \\
& *x + c)^8 + 12*(143*a^3*\cosh(d*x + c)^6 - 55*a^3*\cosh(d*x + c)^4 + 3*a^3*\co \\
& sh(d*x + c)^2)*\sinh(d*x + c)^7 + 12*(143*a^3*\cosh(d*x + c)^7 - 77*a^3*\cosh(\\
& d*x + c)^5 + 7*a^3*\cosh(d*x + c)^3)*\sinh(d*x + c)^6 + 3*(429*a^3*\cosh(d*x + \\
& c)^8 - 308*a^3*\cosh(d*x + c)^6 + 42*a^3*\cosh(d*x + c)^4)*\sinh(d*x + c)^5 + \\
& (715*a^3*\cosh(d*x + c)^9 - 660*a^3*\cosh(d*x + c)^7 + 126*a^3*\cosh(d*x + c) \\
& ^5)*\sinh(d*x + c)^4 + 2*(143*a^3*\cosh(d*x + c)^10 - 165*a^3*\cosh(d*x + c)^8 \\
& + 42*a^3*\cosh(d*x + c)^6)*\sinh(d*x + c)^3 + 2*(39*a^3*\cosh(d*x + c)^11 - 5 \\
& 5*a^3*\cosh(d*x + c)^9 + 18*a^3*\cosh(d*x + c)^7)*\sinh(d*x + c)^2 + (13*a^3*\c \\
& osh(d*x + c)^12 - 22*a^3*\cosh(d*x + c)^10 + 9*a^3*\cosh(d*x + c)^8)*\sinh(d*x \\
& + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 2*(385*b^3*\cosh(d*x + c)^21 \\
& - 4750*b^3*\cosh(d*x + c)^19 + 9*(3024*a*b^2 + 3113*b^3)*\cosh(d*x + c)^17 - \\
& 72*(3472*a*b^2 + 1529*b^3)*\cosh(d*x + c)^15 + 882*(1920*a^2*b + 1624*a*b^2 \\
& + 473*b^3)*\cosh(d*x + c)^13 - 3780*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b \\
& ^3)*\cosh(d*x + c)^11 - 3150*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*\cosh \\
& (d*x + c)^9 + 504*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*\cosh(d*x + c)^7 - 27* \\
& (3472*a*b^2 + 1529*b^3)*\cosh(d*x + c)^5 - 475*b^3*\cosh(d*x + c) + 2*(3024*a \\
& *b^2 + 3113*b^3)*\cosh(d*x + c)^3)*\sinh(d*x + c))/(d*\cosh(d*x + c)^13 + 13*d \\
& *\cosh(d*x + c)*\sinh(d*x + c)^12 + d*\sinh(d*x + c)^13 - 2*d*\cosh(d*x + c)^11 \\
& + 2*(39*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^11 + 22*(13*d*\cosh(d*x + c)^3 \\
& - d*\cosh(d*x + c))*\sinh(d*x + c)^10 + d*\cosh(d*x + c)^9 + (715*d*\cosh(d*x \\
& + c)^4 - 110*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^9 + 3*(429*d*\cosh(d*x + c
\end{aligned}$$

)⁵ - 110*d*cosh(d*x + c)³ + 3*d*cosh(d*x + c))*sinh(d*x + c)⁸ + 12*(143*d*cosh(d*x + c)⁶ - 55*d*cosh(d*x + c)⁴ + 3*d*cosh(d*x + c)²)*sinh(d*x + c)⁷ + 12*(143*d*cosh(d*x + c)⁷ - 77*d*cosh(d*x + c)⁵ + 7*d*cosh(d*x + c)³)*sinh(d*x + c)⁶ + 3*(429*d*cosh(d*x + c)⁸ - 308*d*cosh(d*x + c)⁶ + 42*d*cosh(d*x + c)⁴)*sinh(d*x + c)⁵ + (715*d*cosh(d*x + c)⁹ - 660*d*cosh(d*x + c)⁷ + 126*d*cosh(d*x + c)⁵)*sinh(d*x + c)⁴ + 2*(143*d*cosh(d*x + c)¹⁰ - 165*d*cosh(d*x + c)⁸ + 42*d*cosh(d*x + c)⁶)*sinh(d*x + c)³ + 2*(39*d*cosh(d*x + c)¹¹ - 55*d*cosh(d*x + c)⁹ + 18*d*cosh(d*x + c)⁷)*sinh(d*x + c)² + (13*d*cosh(d*x + c)¹² - 22*d*cosh(d*x + c)¹⁰ + 9*d*cosh(d*x + c)⁸)*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.66912, size = 455, normalized size = 3.07

$$\frac{a^3 \log(e^{(dx+c)} + e^{(-dx-c)} + 2)}{4d} - \frac{a^3 \log(e^{(dx+c)} + e^{(-dx-c)} - 2)}{4d} - \frac{a^3(e^{(dx+c)} + e^{(-dx-c)})}{\left((e^{(dx+c)} + e^{(-dx-c)})^2 - 4\right)d} + \frac{35b^3d^8(e^{(dx+c)} + e^{(-dx-c)})^9}{\left((e^{(dx+c)} + e^{(-dx-c)})^2 - 4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] 1/4*a^3*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/4*a^3*log(e^(d*x + c) + e^(-d*x - c) - 2)/d - a^3*(e^(d*x + c) + e^(-d*x - c))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)*d) + 1/161280*(35*b^3*d^8*(e^(d*x + c) + e^(-d*x - c))^9 - 720*b^3*d^8*(e^(d*x + c) + e^(-d*x - c))^7 + 3024*a*b^2*d^8*(e^(d*x + c) + e^(-d*x - c))^5 + 6048*b^3*d^8*(e^(d*x + c) + e^(-d*x - c))^5 - 40320*a*b^2*d^8*(e^(d*x + c) + e^(-d*x - c))^3 - 26880*b^3*d^8*(e^(d*x + c) + e^(-d*x - c))^3 + 241920*a^2*b*d^8*(e^(d*x + c) + e^(-d*x - c)) + 241920*a*b^2*d^8*(e^(d*x + c) + e^(-d*x - c)) + 80640*b^3*d^8*(e^(d*x + c) + e^(-d*x - c))) /d^9

3.212 $\int \operatorname{csch}^5(c + dx) (a + b \sinh^4(c + dx))^3 dx$

Optimal. Leaf size=142

$$-\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d} - \frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d} + \frac{b^2(a+b)\cosh^3(c+dx)}{d}$$

[Out] $(-3a^2(a+8b)\operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]])/(8d) - (b^2(3a+b)\operatorname{Cosh}[c+dx])/d + (b^2(a+b)\operatorname{Cosh}[c+dx]^3)/d - (3b^3\operatorname{Cosh}[c+dx]^5)/(5d) + (b^3\operatorname{Cosh}[c+dx]^7)/(7d) + (3a^3\operatorname{Coth}[c+dx]*\operatorname{Csch}[c+dx])/(8d) - (a^3\operatorname{Coth}[c+dx]*\operatorname{Csch}[c+dx]^3)/(4d)$

Rubi [A] time = 0.272746, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 1810, 206}

$$-\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d} - \frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d} + \frac{b^2(a+b)\cosh^3(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c+dx]^5(a+b\operatorname{Sinh}[c+dx]^4)^3, x]$

[Out] $(-3a^2(a+8b)\operatorname{ArcTanh}[\operatorname{Cosh}[c+dx]])/(8d) - (b^2(3a+b)\operatorname{Cosh}[c+dx])/d + (b^2(a+b)\operatorname{Cosh}[c+dx]^3)/d - (3b^3\operatorname{Cosh}[c+dx]^5)/(5d) + (b^3\operatorname{Cosh}[c+dx]^7)/(7d) + (3a^3\operatorname{Coth}[c+dx]*\operatorname{Csch}[c+dx])/(8d) - (a^3\operatorname{Coth}[c+dx]*\operatorname{Csch}[c+dx]^3)/(4d)$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{((m-1)/2)}*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 1157

$\operatorname{Int}[(d_.) + (e_.)*(x_)^2)^{(q_.)}*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{Qx = \operatorname{PolynomialQuotient}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x$

```
, 0]], -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^5(c+dx) (a+b \sinh^4(c+dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^3}{(1-x^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^3 \coth(c+dx) \operatorname{csch}^3(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{-3a^3-12a^2b-12ab^2-4b^3+4b(3a^2+9ab+5b^2)}{(1-x^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{3a^3 \coth(c+dx) \operatorname{csch}(c+dx)}{8d} - \frac{a^3 \coth(c+dx) \operatorname{csch}^3(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{3a^3-12a^2b-12ab^2-4b^3+4b(3a^2+9ab+5b^2)}{(1-x^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{3a^3 \coth(c+dx) \operatorname{csch}(c+dx)}{8d} - \frac{a^3 \coth(c+dx) \operatorname{csch}^3(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{3a^3-12a^2b-12ab^2-4b^3+4b(3a^2+9ab+5b^2)}{(1-x^2)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{b^2(3a+b) \cosh(c+dx)}{d} + \frac{b^2(a+b) \cosh^3(c+dx)}{d} - \frac{3b^3 \cosh^5(c+dx)}{5d} + \frac{b^2(a+b) \cosh^7(c+dx)}{7d} \\
&= -\frac{3a^2(a+8b) \tanh^{-1}(\cosh(c+dx))}{8d} - \frac{b^2(3a+b) \cosh(c+dx)}{d} + \frac{b^2(a+b) \cosh^3(c+dx)}{3d} - \frac{b^2(a+b) \cosh^5(c+dx)}{5d} + \frac{b^2(a+b) \cosh^7(c+dx)}{7d}
\end{aligned}$$

Mathematica [A] time = 0.382854, size = 173, normalized size = 1.22

$$\frac{6720a^2b \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 35a^3 \operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right) + 210a^3 \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) + 35a^3 \operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right) + 210a^3 \operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (-35*b^2*(144*a + 35*b)*Cosh[c + d*x] + 35*b^2*(16*a + 7*b)*Cosh[3*(c + d*x)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*x)] + 210*a^3*Csch[(c + d*x)/2]^2 - 35*a^3*Csch[(c + d*x)/2]^4 + 840*a^3*Log[Tanh[(c + d*x)/2]] + 6720*a^2*b*Log[Tanh[(c + d*x)/2]] + 210*a^3*Sech[(c + d*x)/2]^2 + 35*a^3*Sech[(c + d*x)/2]^4)/(2240*d)

Maple [A] time = 0.046, size = 125, normalized size = 0.9

$$\frac{1}{d} \left(a^3 \left(\left(-\frac{\operatorname{csch}(dx+c)^3}{4} + \frac{3 \operatorname{csch}(dx+c)}{8} \right) \coth(dx+c) - \frac{3 \operatorname{Artanh}(e^{dx+c})}{4} \right) - 6a^2b \operatorname{Artanh}(e^{dx+c}) + 3ab^2 \left(-\frac{2}{3} + \frac{1}{3} \operatorname{csch}^2(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] $\frac{1}{d} \cdot (a^3 \cdot ((-1/4 \cdot \operatorname{csch}(d \cdot x + c)^3 + 3/8 \cdot \operatorname{csch}(d \cdot x + c)) \cdot \operatorname{coth}(d \cdot x + c) - 3/4 \cdot \operatorname{arctanh}(\exp(d \cdot x + c))) - 6 \cdot a^2 \cdot b \cdot \operatorname{arctanh}(\exp(d \cdot x + c)) + 3 \cdot a \cdot b^2 \cdot (-2/3 + 1/3 \cdot \sinh(d \cdot x + c)^2) \cdot \cosh(d \cdot x + c) + b^3 \cdot (-16/35 + 1/7 \cdot \sinh(d \cdot x + c)^6 - 6/35 \cdot \sinh(d \cdot x + c)^4 + 8/35 \cdot \sinh(d \cdot x + c)^2) \cdot \cosh(d \cdot x + c))$

Maxima [B] time = 1.05803, size = 459, normalized size = 3.23

$$-\frac{1}{4480} b^3 \left(\frac{(49 e^{(-2dx-2c)} - 245 e^{(-4dx-4c)} + 1225 e^{(-6dx-6c)} - 5) e^{(7dx+7c)}}{d} + \frac{1225 e^{(-dx-c)} - 245 e^{(-3dx-3c)} + 49 e^{(-5dx-5c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] $-1/4480 \cdot b^3 \cdot ((49 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} - 245 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 1225 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} - 5) \cdot e^{(7 \cdot d \cdot x + 7 \cdot c)} / d + (1225 \cdot e^{(-d \cdot x - c)} - 245 \cdot e^{(-3 \cdot d \cdot x - 3 \cdot c)} + 49 \cdot e^{(-5 \cdot d \cdot x - 5 \cdot c)} - 5 \cdot e^{(-7 \cdot d \cdot x - 7 \cdot c)}) / d) + 1/8 \cdot a \cdot b^2 \cdot (e^{(3 \cdot d \cdot x + 3 \cdot c)} / d - 9 \cdot e^{(d \cdot x + c)} / d - 9 \cdot e^{(-d \cdot x - c)} / d + e^{(-3 \cdot d \cdot x - 3 \cdot c)} / d) - 1/8 \cdot a^3 \cdot (3 \cdot \log(e^{(-d \cdot x - c)} + 1) / d - 3 \cdot \log(e^{(-d \cdot x - c)} - 1) / d + 2 \cdot (3 \cdot e^{(-d \cdot x - c)} - 11 \cdot e^{(-3 \cdot d \cdot x - 3 \cdot c)} - 11 \cdot e^{(-5 \cdot d \cdot x - 5 \cdot c)} + 3 \cdot e^{(-7 \cdot d \cdot x - 7 \cdot c)}) / (d \cdot (4 \cdot e^{(-2 \cdot d \cdot x - 2 \cdot c)} - 6 \cdot e^{(-4 \cdot d \cdot x - 4 \cdot c)} + 4 \cdot e^{(-6 \cdot d \cdot x - 6 \cdot c)} - e^{(-8 \cdot d \cdot x - 8 \cdot c)} - 1))) - 3 \cdot a^2 \cdot b \cdot (\log(e^{(-d \cdot x - c)} + 1) / d - \log(e^{(-d \cdot x - c)} - 1) / d)$

Fricas [B] time = 2.4282, size = 17403, normalized size = 122.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")`

[Out] $1/4480 \cdot (5 \cdot b^3 \cdot \cosh(d \cdot x + c)^{22} + 110 \cdot b^3 \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^{21} + 5 \cdot b^3 \cdot \sinh(d \cdot x + c)^{22} - 69 \cdot b^3 \cdot \cosh(d \cdot x + c)^{20} + 3 \cdot (385 \cdot b^3 \cdot \cosh(d \cdot x + c)^2 - 23 \cdot b^3) \cdot \sinh(d \cdot x + c)^{20} + 20 \cdot (385 \cdot b^3 \cdot \cosh(d \cdot x + c)^3 - 69 \cdot b^3 \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^{19} + (560 \cdot a \cdot b^2 + 471 \cdot b^3) \cdot \cosh(d \cdot x + c)^{18} + (36575 \cdot b^3 \cdot \cosh(d \cdot x + c)^4 - 13110 \cdot b^3 \cdot \cosh(d \cdot x + c)^2 + 560 \cdot a \cdot b^2 + 471 \cdot b^3) \cdot \sinh(d \cdot x + c)^{18} + 18 \cdot (7315 \cdot b^3 \cdot \cosh(d \cdot x + c)^5 - 4370 \cdot b^3 \cdot \cosh(d \cdot x + c)^3 + (5$

$$\begin{aligned}
& 60*a*b^2 + 471*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{17} - (7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^{16} + (373065*b^3*\cosh(d*x + c)^6 - 334305*b^3*\cosh(d*x + c)^4 - 7280*a*b^2 - 2519*b^3 + 153*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{16} + 16*(53295*b^3*\cosh(d*x + c)^7 - 66861*b^3*\cosh(d*x + c)^5 + 51*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^3 - (7280*a*b^2 + 2519*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{15} + 6*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^{14} + 6*(266475*b^3*\cosh(d*x + c)^8 - 445740*b^3*\cosh(d*x + c)^6 + 510*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^4 + 560*a^3 + 3080*a*b^2 + 891*b^3 - 20*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{14} + 4*(621775*b^3*\cosh(d*x + c)^9 - 1337220*b^3*\cosh(d*x + c)^7 + 2142*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^5 - 140*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^3 + 21*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{13} - 14*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^{12} + 2*(1616615*b^3*\cosh(d*x + c)^{10} - 4345965*b^3*\cosh(d*x + c)^8 + 9282*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^6 - 910*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^4 - 6160*a^3 - 5880*a*b^2 - 1617*b^3 + 273*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{12} + 24*(146965*b^3*\cosh(d*x + c)^{11} - 482885*b^3*\cosh(d*x + c)^9 + 1326*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^7 - 182*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^5 + 91*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^3 - 7*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^{11} - 14*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^{10} + 2*(1616615*b^3*\cosh(d*x + c)^{12} - 6374082*b^3*\cosh(d*x + c)^{10} + 21879*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^8 - 4004*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^6 + 3003*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^4 - 6160*a^3 - 5880*a*b^2 - 1617*b^3 - 462*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{10} + 4*(621775*b^3*\cosh(d*x + c)^{13} - 2897310*b^3*\cosh(d*x + c)^{11} + 12155*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^9 - 2860*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^7 + 3003*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^5 - 770*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^3 - 35*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 6*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^8 + 6*(266475*b^3*\cosh(d*x + c)^{14} - 1448655*b^3*\cosh(d*x + c)^{12} + 7293*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^{10} - 2145*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^8 + 3003*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^6 - 1155*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^4 + 560*a^3 + 3080*a*b^2 + 891*b^3 - 105*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 16*(53295*b^3*\cosh(d*x + c)^{15} - 334305*b^3*\cosh(d*x + c)^{13} + 1989*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^{11} - 715*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^9 + 1287*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^7 - 693*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^5 - 105*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^3 + 3*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 - (7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^6 + (373065*b^3*\cosh(d*x + c)^{16} - 2674440*b^3*\cosh(d*x + c)^{14} + 18564*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^{12} - 8008*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^{10} + 18018*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^8 - 12936*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^6 - 2940*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^4 - 7280*a*b^2 -
\end{aligned}$$

$$\begin{aligned}
& 2519*b^3 + 168*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^6 + 6*(21945*b^3*\cosh(d*x + c)^{17} - 178296*b^3*\cosh(d*x + c)^{15} + 1428 \\
& *(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^{13} - 728*(7280*a*b^2 + 2519*b^3)*\cosh(\\
& d*x + c)^{11} + 2002*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^9 - 1848* \\
& (880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^7 - 588*(880*a^3 + 840*a*b^2 \\
& + 231*b^3)*\cosh(d*x + c)^5 + 56*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + \\
& c)^3 - (7280*a*b^2 + 2519*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 69*b^3*\cos \\
& h(d*x + c)^2 + (560*a*b^2 + 471*b^3)*\cosh(d*x + c)^4 + (36575*b^3*\cosh(d*x \\
& + c)^{18} - 334305*b^3*\cosh(d*x + c)^{16} + 3060*(560*a*b^2 + 471*b^3)*\cosh(d*x \\
& + c)^{14} - 1820*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^{12} + 6006*(560*a^3 + \\
& 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^{10} - 6930*(880*a^3 + 840*a*b^2 + 231*b^ \\
& 3)*\cosh(d*x + c)^8 - 2940*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^6 + \\
& 420*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^4 + 560*a*b^2 + 471*b^3 \\
& - 15*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(1925*b^ \\
& 3*\cosh(d*x + c)^{19} - 19665*b^3*\cosh(d*x + c)^{17} + 204*(560*a*b^2 + 471*b^3) \\
& *\cosh(d*x + c)^{15} - 140*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^{13} + 546*(560 \\
& *a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^{11} - 770*(880*a^3 + 840*a*b^2 + \\
& 231*b^3)*\cosh(d*x + c)^9 - 420*(880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c \\
&)^7 + 84*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^5 - 5*(7280*a*b^2 + \\
& 2519*b^3)*\cosh(d*x + c)^3 + (560*a*b^2 + 471*b^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^3 + 5*b^3 + 3*(385*b^3*\cosh(d*x + c)^{20} - 4370*b^3*\cosh(d*x + c)^{18} + \\
& 51*(560*a*b^2 + 471*b^3)*\cosh(d*x + c)^{16} - 40*(7280*a*b^2 + 2519*b^3)*\cosh \\
& (d*x + c)^{14} + 182*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x + c)^{12} - 308* \\
& (880*a^3 + 840*a*b^2 + 231*b^3)*\cosh(d*x + c)^{10} - 210*(880*a^3 + 840*a*b^2 \\
& + 231*b^3)*\cosh(d*x + c)^8 + 56*(560*a^3 + 3080*a*b^2 + 891*b^3)*\cosh(d*x \\
& + c)^6 - 5*(7280*a*b^2 + 2519*b^3)*\cosh(d*x + c)^4 - 23*b^3 + 2*(560*a*b^2 \\
& + 471*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 1680*((a^3 + 8*a^2*b)*\cosh(d* \\
& x + c)^{15} + 15*(a^3 + 8*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^{14} + (a^3 + 8*a^ \\
& 2*b)*\sinh(d*x + c)^{15} - 4*(a^3 + 8*a^2*b)*\cosh(d*x + c)^{13} - (4*a^3 + 32*a^ \\
& 2*b - 105*(a^3 + 8*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{13} + 13*(35*(a^3 + \\
& 8*a^2*b)*\cosh(d*x + c)^3 - 4*(a^3 + 8*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^ \\
& 12 + 6*(a^3 + 8*a^2*b)*\cosh(d*x + c)^{11} + 3*(455*(a^3 + 8*a^2*b)*\cosh(d*x + \\
& c)^4 + 2*a^3 + 16*a^2*b - 104*(a^3 + 8*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^{11} + 11*(273*(a^3 + 8*a^2*b)*\cosh(d*x + c)^5 - 104*(a^3 + 8*a^2*b)*\cosh(\\
& d*x + c)^3 + 6*(a^3 + 8*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^{10} - 4*(a^3 + 8 \\
& *a^2*b)*\cosh(d*x + c)^9 + (5005*(a^3 + 8*a^2*b)*\cosh(d*x + c)^6 - 2860*(a^3 \\
& + 8*a^2*b)*\cosh(d*x + c)^4 - 4*a^3 - 32*a^2*b + 330*(a^3 + 8*a^2*b)*\cosh(d \\
& *x + c)^2)*\sinh(d*x + c)^9 + 9*(715*(a^3 + 8*a^2*b)*\cosh(d*x + c)^7 - 572*(\\
& a^3 + 8*a^2*b)*\cosh(d*x + c)^5 + 110*(a^3 + 8*a^2*b)*\cosh(d*x + c)^3 - 4*(a \\
& ^3 + 8*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^8 + (a^3 + 8*a^2*b)*\cosh(d*x + c \\
&)^7 + (6435*(a^3 + 8*a^2*b)*\cosh(d*x + c)^8 - 6864*(a^3 + 8*a^2*b)*\cosh(d*x \\
& + c)^6 + 1980*(a^3 + 8*a^2*b)*\cosh(d*x + c)^4 + a^3 + 8*a^2*b - 144*(a^3 + \\
& 8*a^2*b)*\cosh(d*x + c)^2)*\sinh(d*x + c)^7 + (5005*(a^3 + 8*a^2*b)*\cosh(d*x \\
& + c)^9 - 6864*(a^3 + 8*a^2*b)*\cosh(d*x + c)^7 + 2772*(a^3 + 8*a^2*b)*\cosh(\\
& d*x + c)^5 - 336*(a^3 + 8*a^2*b)*\cosh(d*x + c)^3 + 7*(a^3 + 8*a^2*b)*\cosh(d
\end{aligned}$$

$$\begin{aligned}
& *x + c)) * \sinh(dx + c)^6 + 3 * (1001 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{10} - 1716 * \\
& (a^3 + 8 * a^2 * b) * \cosh(dx + c)^8 + 924 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^6 - 168 \\
& * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^4 + 7 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^2) * \sinh(\\
& dx + c)^5 + (1365 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{11} - 2860 * (a^3 + 8 * a^2 * b) * \\
& \cosh(dx + c)^9 + 1980 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^7 - 504 * (a^3 + 8 * a^2 * b) \\
&) * \cosh(dx + c)^5 + 35 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^3) * \sinh(dx + c)^4 + (\\
& 455 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{12} - 1144 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{11} \\
& 0 + 990 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^8 - 336 * (a^3 + 8 * a^2 * b) * \cosh(dx + c) \\
& ^6 + 35 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^4) * \sinh(dx + c)^3 + 3 * (35 * (a^3 + 8 * a \\
& ^2 * b) * \cosh(dx + c)^{13} - 104 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{11} + 110 * (a^3 + \\
& 8 * a^2 * b) * \cosh(dx + c)^9 - 48 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^7 + 7 * (a^3 + 8 * \\
& a^2 * b) * \cosh(dx + c)^5) * \sinh(dx + c)^2 + (15 * (a^3 + 8 * a^2 * b) * \cosh(dx + c) \\
& ^{14} - 52 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{12} + 66 * (a^3 + 8 * a^2 * b) * \cosh(dx + c) \\
&)^{10} - 36 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^8 + 7 * (a^3 + 8 * a^2 * b) * \cosh(dx + c) \\
& ^6) * \sinh(dx + c)) * \log(\cosh(dx + c) + \sinh(dx + c) + 1) + 1680 * ((a^3 + 8 * \\
& a^2 * b) * \cosh(dx + c)^{15} + 15 * (a^3 + 8 * a^2 * b) * \cosh(dx + c) * \sinh(dx + c)^{14} \\
& + (a^3 + 8 * a^2 * b) * \sinh(dx + c)^{15} - 4 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{13} - \\
& (4 * a^3 + 32 * a^2 * b - 105 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^2) * \sinh(dx + c)^{13} + \\
& 13 * (35 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^3 - 4 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)) * \\
& \sinh(dx + c)^{12} + 6 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{11} + 3 * (455 * (a^3 + 8 * a^2 \\
& * b) * \cosh(dx + c)^4 + 2 * a^3 + 16 * a^2 * b - 104 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^2) * \sinh(dx + c)^{11} + 11 * (273 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^5 - 104 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^3 + 6 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)) * \sinh(dx + c)^{10} - 4 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^9 + (5005 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^6 - 2860 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^4 - 4 * a^3 - 32 * a^2 * b + 330 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^2) * \sinh(dx + c)^9 + 9 * (715 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^7 - 572 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^5 + 110 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^3 - 4 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)) * \sinh(dx + c)^8 + (a^3 + 8 * a^2 * b) * \cosh(dx + c)^7 + (6435 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^8 - 6864 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^6 + 1980 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^4 + a^3 + 8 * a^2 * b - 144 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^2) * \sinh(dx + c)^7 + (5005 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^9 - 6864 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^7 + 2772 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^5 - 336 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^3 + 7 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)) * \sinh(dx + c)^6 + 3 * (1001 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{10} - 1716 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^8 + 924 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^6 - 168 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^4 + 7 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^2) * \sinh(dx + c)^5 + (1365 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{11} - 2860 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^9 + 1980 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^7 - 504 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^5 + 35 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^3) * \sinh(dx + c)^4 + (455 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{12} - 1144 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{10} + 990 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^8 - 336 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^6 + 35 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^4) * \sinh(dx + c)^3 + 3 * (35 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{13} - 104 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^{11} + 110 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^9 - 48 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^7 + 7 * (a^3 + 8 * a^2 * b) * \cosh(dx + c)^5) * \sinh(dx + c)^2 + (15 * (a^3 + 8 * a^2 * b)
\end{aligned}$$


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*cosh(d*x + c)^14 - 52*(a^3 + 8*a^2*b)*cosh(d*x + c)^12 + 66*(a^3 + 8*a^2*b
)*cosh(d*x + c)^10 - 36*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 + 7*(a^3 + 8*a^2*b)
*cosh(d*x + c)^6)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2
*(55*b^3*cosh(d*x + c)^21 - 690*b^3*cosh(d*x + c)^19 + 9*(560*a*b^2 + 471*b
^3)*cosh(d*x + c)^17 - 8*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^15 + 42*(560
*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^13 - 84*(880*a^3 + 840*a*b^2 + 2
31*b^3)*cosh(d*x + c)^11 - 70*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)
^9 + 24*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^7 - 3*(7280*a*b^2 +
2519*b^3)*cosh(d*x + c)^5 - 69*b^3*cosh(d*x + c) + 2*(560*a*b^2 + 471*b^3)*
cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c)^15 + 15*d*cosh(d*x + c)*si
nh(d*x + c)^14 + d*sinh(d*x + c)^15 - 4*d*cosh(d*x + c)^13 + (105*d*cosh(d*
x + c)^2 - 4*d)*sinh(d*x + c)^13 + 13*(35*d*cosh(d*x + c)^3 - 4*d*cosh(d*x
+ c))*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^11 + 3*(455*d*cosh(d*x + c)^4 -
104*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^11 + 11*(273*d*cosh(d*x + c)^5 -
104*d*cosh(d*x + c)^3 + 6*d*cosh(d*x + c))*sinh(d*x + c)^10 - 4*d*cosh(d*x
+ c)^9 + (5005*d*cosh(d*x + c)^6 - 2860*d*cosh(d*x + c)^4 + 330*d*cosh(d*x
+ c)^2 - 4*d)*sinh(d*x + c)^9 + 9*(715*d*cosh(d*x + c)^7 - 572*d*cosh(d*x
+ c)^5 + 110*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^8 + d*cos
h(d*x + c)^7 + (6435*d*cosh(d*x + c)^8 - 6864*d*cosh(d*x + c)^6 + 1980*d*co
sh(d*x + c)^4 - 144*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + (5005*d*cosh(d
*x + c)^9 - 6864*d*cosh(d*x + c)^7 + 2772*d*cosh(d*x + c)^5 - 336*d*cosh(d*
x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 3*(1001*d*cosh(d*x + c)^10
- 1716*d*cosh(d*x + c)^8 + 924*d*cosh(d*x + c)^6 - 168*d*cosh(d*x + c)^4 +
7*d*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (1365*d*cosh(d*x + c)^11 - 2860*d*co
sh(d*x + c)^9 + 1980*d*cosh(d*x + c)^7 - 504*d*cosh(d*x + c)^5 + 35*d*cosh(
d*x + c)^3)*sinh(d*x + c)^4 + (455*d*cosh(d*x + c)^12 - 1144*d*cosh(d*x + c
)^10 + 990*d*cosh(d*x + c)^8 - 336*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4
)*sinh(d*x + c)^3 + 3*(35*d*cosh(d*x + c)^13 - 104*d*cosh(d*x + c)^11 + 110
*d*cosh(d*x + c)^9 - 48*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5)*sinh(d*x +
c)^2 + (15*d*cosh(d*x + c)^14 - 52*d*cosh(d*x + c)^12 + 66*d*cosh(d*x + c)
^10 - 36*d*cosh(d*x + c)^8 + 7*d*cosh(d*x + c)^6)*sinh(d*x + c))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.74344, size = 404, normalized size = 2.85

$$\frac{3(a^3 + 8a^2b)\log(e^{(dx+c)} + e^{(-dx-c)} + 2)}{16d} + \frac{3(a^3 + 8a^2b)\log(e^{(dx+c)} + e^{(-dx-c)} - 2)}{16d} + \frac{3a^3(e^{(dx+c)} + e^{(-dx-c)})^3 - 20a^3(e^{(dx+c)} + e^{(-dx-c)})^2 - 20a^3(e^{(dx+c)} + e^{(-dx-c)})}{4((e^{(dx+c)} + e^{(-dx-c)})^2 - 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -3/16*(a^3 + 8*a^2*b)*\log(e^{(d*x + c)} + e^{(-d*x - c)} + 2)/d + 3/16*(a^3 + 8 \\ & *a^2*b)*\log(e^{(d*x + c)} + e^{(-d*x - c)} - 2)/d + 1/4*(3*a^3*(e^{(d*x + c)} + e \\ & ^{(-d*x - c)})^3 - 20*a^3*(e^{(d*x + c)} + e^{(-d*x - c)}))/(((e^{(d*x + c)} + e^{(- \\ & d*x - c)})^2 - 4)^2*d) + 1/4480*(5*b^3*d^6*(e^{(d*x + c)} + e^{(-d*x - c)})^7 - \\ & 84*b^3*d^6*(e^{(d*x + c)} + e^{(-d*x - c)})^5 + 560*a*b^2*d^6*(e^{(d*x + c)} + e^{ \\ & (-d*x - c)})^3 + 560*b^3*d^6*(e^{(d*x + c)} + e^{(-d*x - c)})^3 - 6720*a*b^2*d^6 \\ & *(e^{(d*x + c)} + e^{(-d*x - c)}) - 2240*b^3*d^6*(e^{(d*x + c)} + e^{(-d*x - c)}))/ \\ & d^7 \end{aligned}$$

3.213 $\int \operatorname{csch}^7(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=156

$$\frac{a^2(5a + 24b) \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{a^2(5a + 24b) \coth(c + dx) \operatorname{csch}(c + dx)}{16d} - \frac{a^3 \coth(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a^3 \coth(c + dx) \operatorname{csch}^3(c + dx)}{6d}$$

[Out] $(a^2(5a + 24b) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(16*d) + (b^2(3a + b) \operatorname{Cosh}[c + d*x])/d - (2*b^3 \operatorname{Cosh}[c + d*x]^3)/(3*d) + (b^3 \operatorname{Cosh}[c + d*x]^5)/(5*d) - (a^2(5a + 24b) \operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x])/(16*d) + (5*a^3 \operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x]^3)/(24*d) - (a^3 \operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x]^5)/(6*d)$

Rubi [A] time = 0.304455, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 1810, 206}

$$\frac{a^2(5a + 24b) \tanh^{-1}(\cosh(c + dx))}{16d} - \frac{a^2(5a + 24b) \coth(c + dx) \operatorname{csch}(c + dx)}{16d} - \frac{a^3 \coth(c + dx) \operatorname{csch}^5(c + dx)}{6d} + \frac{5a^3 \coth(c + dx) \operatorname{csch}^3(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^7*(a + b*\operatorname{Sinh}[c + d*x]^4)^3, x]$

[Out] $(a^2(5a + 24b) \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]])/(16*d) + (b^2(3a + b) \operatorname{Cosh}[c + d*x])/d - (2*b^3 \operatorname{Cosh}[c + d*x]^3)/(3*d) + (b^3 \operatorname{Cosh}[c + d*x]^5)/(5*d) - (a^2(5a + 24b) \operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x])/(16*d) + (5*a^3 \operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x]^3)/(24*d) - (a^3 \operatorname{Coth}[c + d*x] \operatorname{Csch}[c + d*x]^5)/(6*d)$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] :> \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}[\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 1157

$\operatorname{Int}[(d_.) + (e_.)*(x_.)^2]^{(q_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] :> \operatorname{With}[\{Qx = \operatorname{PolynomialQuotient}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = \operatorname{Coeff}[\operatorname{PolynomialRemainder}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x$

```
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^7(c+dx) (a+b \sinh^4(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^3}{(1-x^2)^4} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^3 \coth(c+dx) \operatorname{csch}^5(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-5a^3-18a^2b-18ab^2-6b^3+6b(3a^2+9ab)}{\dots} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{5a^3 \coth(c+dx) \operatorname{csch}^3(c+dx)}{24d} - \frac{a^3 \coth(c+dx) \operatorname{csch}^5(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{b^2(3a+b) \cosh(c+dx)}{\dots} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^2(5a+24b) \coth(c+dx) \operatorname{csch}(c+dx)}{16d} + \frac{5a^3 \coth(c+dx) \operatorname{csch}^3(c+dx)}{24d} \\
&= -\frac{a^2(5a+24b) \coth(c+dx) \operatorname{csch}(c+dx)}{16d} + \frac{5a^3 \coth(c+dx) \operatorname{csch}^3(c+dx)}{24d} \\
&= \frac{b^2(3a+b) \cosh(c+dx)}{d} - \frac{2b^3 \cosh^3(c+dx)}{3d} + \frac{b^3 \cosh^5(c+dx)}{5d} - \frac{a^2(5a+24b) \tanh^{-1}(\cosh(c+dx))}{16d} \\
&= \frac{a^2(5a+24b) \tanh^{-1}(\cosh(c+dx))}{16d} + \frac{b^2(3a+b) \cosh(c+dx)}{d} - \frac{2b^3 \cosh^3(c+dx)}{3d} + \frac{b^3 \cosh^5(c+dx)}{5d} - \frac{a^2(5a+24b) \tanh^{-1}(\cosh(c+dx))}{16d}
\end{aligned}$$

Mathematica [A] time = 0.702756, size = 223, normalized size = 1.43

$$\frac{720a^2b \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) + 720a^2b \operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right) + 2880a^2b \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 5a^3 \operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right) - 30a^3 \operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{1920d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^3,x]

[Out]
$$\begin{aligned}
& -(-240*b^2*(24*a + 5*b)*\operatorname{Cosh}[c + d*x] + 200*b^3*\operatorname{Cosh}[3*(c + d*x)] - 24*b^3* \\
& \operatorname{Cosh}[5*(c + d*x)] + 150*a^3*\operatorname{Csch}[(c + d*x)/2]^2 + 720*a^2*b*\operatorname{Csch}[(c + d*x)/ \\
& 2]^2 - 30*a^3*\operatorname{Csch}[(c + d*x)/2]^4 + 5*a^3*\operatorname{Csch}[(c + d*x)/2]^6 + 600*a^3*\operatorname{Log} \\
& [\operatorname{Tanh}[(c + d*x)/2]] + 2880*a^2*b*\operatorname{Log}[\operatorname{Tanh}[(c + d*x)/2]] + 150*a^3*\operatorname{Sech}[(c + \\
& d*x)/2]^2 + 720*a^2*b*\operatorname{Sech}[(c + d*x)/2]^2 + 30*a^3*\operatorname{Sech}[(c + d*x)/2]^4 + 5 \\
& *a^3*\operatorname{Sech}[(c + d*x)/2]^6)/(1920*d)
\end{aligned}$$

Maple [A] time = 0.047, size = 128, normalized size = 0.8

$$\frac{1}{d} \left(a^3 \left(\left(-\frac{(\operatorname{csch}(dx+c))^5}{6} + \frac{5(\operatorname{csch}(dx+c))^3}{24} - \frac{5\operatorname{csch}(dx+c)}{16} \right) \coth(dx+c) + \frac{5 \operatorname{Artanh}(e^{dx+c})}{8} \right) + 3a^2b \left(-\frac{1}{2} \operatorname{csch}(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] `1/d*(a^3*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x+c)+5/8*arctanh(exp(d*x+c)))+3*a^2*b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a*b^2*cosh(d*x+c)+b^3*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))`

Maxima [B] time = 1.11351, size = 527, normalized size = 3.38

$$\frac{1}{480} b^3 \left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d} \right) + \frac{3}{2} ab^2 \left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] `1/480*b^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^3*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))`

Fricas [B] time = 2.39735, size = 22631, normalized size = 145.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] $\frac{1}{480} \cdot (3b^3 \cosh(dx+c)^{22} + 66b^3 \cosh(dx+c) \sinh(dx+c)^{21} + 3b^3 \sinh(dx+c)^{22} - 43b^3 \cosh(dx+c)^{20} + (693b^3 \cosh(dx+c)^2 - 43b^3) \sinh(dx+c)^{20} + 20(231b^3 \cosh(dx+c)^3 - 43b^3 \cosh(dx+c)) \sinh(dx+c)^{19} + 15(48ab^2 + 23b^3) \cosh(dx+c)^{18} + 5(4389b^3 \cosh(dx+c)^4 - 1634b^3 \cosh(dx+c)^2 + 144ab^2 + 69b^3) \sinh(dx+c)^{18} + 6(13167b^3 \cosh(dx+c)^5 - 8170b^3 \cosh(dx+c)^3 + 45(48ab^2 + 23b^3) \cosh(dx+c)) \sinh(dx+c)^{17} - 15(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx+c)^{16} + 3(74613b^3 \cosh(dx+c)^6 - 69445b^3 \cosh(dx+c)^4 - 100a^3 - 480a^2b - 1200ab^2 - 395b^3 + 765(48ab^2 + 23b^3) \cosh(dx+c)^2) \sinh(dx+c)^{16} + 48(10659b^3 \cosh(dx+c)^7 - 13889b^3 \cosh(dx+c)^5 + 255(48ab^2 + 23b^3) \cosh(dx+c)^3 - 5(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx+c)) \sinh(dx+c)^{15} + 10(170a^3 + 432a^2b + 648ab^2 + 187b^3) \cosh(dx+c)^{14} + 10(95931b^3 \cosh(dx+c)^8 - 166668b^3 \cosh(dx+c)^6 + 4590(48ab^2 + 23b^3) \cosh(dx+c)^4 + 170a^3 + 432a^2b + 648ab^2 + 187b^3 - 180(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx+c)^2) \sinh(dx+c)^{14} + 20(74613b^3 \cosh(dx+c)^9 - 166668b^3 \cosh(dx+c)^7 + 6426(48ab^2 + 23b^3) \cosh(dx+c)^5 - 420(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx+c)^3 + 7(170a^3 + 432a^2b + 648ab^2 + 187b^3) \cosh(dx+c)) \sinh(dx+c)^{13} - 90(44a^3 + 32a^2b + 40ab^2 + 11b^3) \cosh(dx+c)^{12} + 2(969969b^3 \cosh(dx+c)^{10} - 2708355b^3 \cosh(dx+c)^8 + 139230(48ab^2 + 23b^3) \cosh(dx+c)^6 - 13650(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx+c)^4 - 1980a^3 - 1440a^2b - 1800ab^2 - 495b^3 + 455(170a^3 + 432a^2b + 648ab^2 + 187b^3) \cosh(dx+c)^2) \sinh(dx+c)^{12} + 8(264537b^3 \cosh(dx+c)^{11} - 902785b^3 \cosh(dx+c)^9 + 59670(48ab^2 + 23b^3) \cosh(dx+c)^7 - 8190(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx+c)^5 + 455(170a^3 + 432a^2b + 648ab^2 + 187b^3) \cosh(dx+c)^3 - 135(44a^3 + 32a^2b + 40ab^2 + 11b^3) \cosh(dx+c)) \sinh(dx+c)^{11} - 90(44a^3 + 32a^2b + 40ab^2 + 11b^3) \cosh(dx+c)^{10} + 2(969969b^3 \cosh(dx+c)^{12} - 3972254b^3 \cosh(dx+c)^{10} + 328185(48ab^2 + 23b^3) \cosh(dx+c)^8 - 60060(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx+c)^6 + 5005(170a^3 + 432a^2b + 648ab^2 + 187b^3) \cosh(dx+c)^4 - 1980a^3 - 1440a^2b - 1800ab^2 - 495b^3 - 2970(44a^3 + 32a^2b + 40ab^2 + 11b^3) \cosh(dx+c)^2) \sinh(dx+c)^{10} + 20(74613b^3 \cosh(dx+c)^{13} - 361114b^3 \cosh(dx+c)^{11} + 36465(48ab^2 + 23b^3) \cosh(dx+c)^9 - 8580(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx+c)^7 + 1001(170a^3 + 432a^2b + 648ab^2 + 187b^3) \cosh(dx+c)^5 - 990(44a^3 + 32a^2b + 40ab^2 + 11b^3) \cosh(dx+c)^3 - 45(44a^3 + 32a^2b + 40ab^2 + 11b^3) \cosh(dx+c)) \sinh(dx+c)^9 + 10(170a^3 + 432a^2b + 648ab^2 + 187b^3) \cosh(dx+c)^8 + 10(95931b^3 \cosh(dx+c)^{14} - 541671b^3 \cosh(dx+c)^{12} + 65637(48ab^2 + 23b^3) \cosh(dx+c)^{10} - 19305(20a^3 + 96a^2b + 240ab^2 + 79$

$$\begin{aligned}
& *b^3) * \cosh(dx + c)^8 + 3003 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^6 - 4455 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^4 + 170a^3 + 432a^2b + 648ab^2 + 187b^3 - 405 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^2 * \sinh(dx + c)^8 + 16 * (31977b^3 * \cosh(dx + c)^15 - 208335b^3 * \cosh(dx + c)^13 + 29835 * (48ab^2 + 23b^3) * \cosh(dx + c)^11 - 10725 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c)^9 + 2145 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^7 - 4455 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^5 - 675 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^3 + 5 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c) * \sinh(dx + c)^7 - 15 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c)^6 + (223839b^3 * \cosh(dx + c)^16 - 1666680b^3 * \cosh(dx + c)^14 + 278460 * (48ab^2 + 23b^3) * \cosh(dx + c)^12 - 120120 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c)^10 + 30030 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^8 - 83160 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^6 - 18900 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^4 - 300a^3 - 1440a^2b - 3600ab^2 - 1185b^3 + 280 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 2 * (39501b^3 * \cosh(dx + c)^17 - 333336b^3 * \cosh(dx + c)^15 + 64260 * (48ab^2 + 23b^3) * \cosh(dx + c)^13 - 32760 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c)^11 + 10010 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^9 - 35640 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^7 - 11340 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^5 + 280 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^3 - 45 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c) * \sinh(dx + c)^5 - 43b^3 * \cosh(dx + c)^2 + 15 * (48ab^2 + 23b^3) * \cosh(dx + c)^4 + 5 * (4389b^3 * \cosh(dx + c)^18 - 41667b^3 * \cosh(dx + c)^16 + 9180 * (48ab^2 + 23b^3) * \cosh(dx + c)^14 - 5460 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c)^12 + 2002 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^10 - 8910 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^8 - 3780 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^6 + 140 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^4 + 144ab^2 + 69b^3 - 45 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 20 * (231b^3 * \cosh(dx + c)^19 - 2451b^3 * \cosh(dx + c)^17 + 612 * (48ab^2 + 23b^3) * \cosh(dx + c)^15 - 420 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c)^13 + 182 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^11 - 990 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^9 - 540 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^7 + 28 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^5 - 15 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c)^3 + 3 * (48ab^2 + 23b^3) * \cosh(dx + c) * \sinh(dx + c)^3 + 3b^3 + (693b^3 * \cosh(dx + c)^20 - 8170b^3 * \cosh(dx + c)^18 + 2295 * (48ab^2 + 23b^3) * \cosh(dx + c)^16 - 1800 * (20a^3 + 96a^2b + 240ab^2 + 79b^3) * \cosh(dx + c)^14 + 910 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^12 - 5940 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^10 - 4050 * (44a^3 + 32a^2b + 40ab^2 + 11b^3) * \cosh(dx + c)^8 + 280 * (170a^3 + 432a^2b + 648ab^2 + 187b^3) * \cosh(dx + c)^6 - 225 * (20a^3
\end{aligned}$$

$$\begin{aligned}
& + 96a^2b + 240ab^2 + 79b^3) \cosh(dx + c)^4 - 43b^3 + 90(48ab^2 + \\
& 23b^3) \cosh(dx + c)^2 \sinh(dx + c)^2 + 30((5a^3 + 24a^2b) \cosh(dx \\
& + c)^{17} + 17(5a^3 + 24a^2b) \cosh(dx + c) \sinh(dx + c)^{16} + (5a^3 + 2 \\
& 4a^2b) \sinh(dx + c)^{17} - 6(5a^3 + 24a^2b) \cosh(dx + c)^{15} - 2(15a \\
& ^3 + 72a^2b - 68(5a^3 + 24a^2b) \cosh(dx + c)^2) \sinh(dx + c)^{15} + 1 \\
& 0(68(5a^3 + 24a^2b) \cosh(dx + c)^3 - 9(5a^3 + 24a^2b) \cosh(dx + \\
& c)) \sinh(dx + c)^{14} + 15(5a^3 + 24a^2b) \cosh(dx + c)^{13} + 5(476(5a \\
& ^3 + 24a^2b) \cosh(dx + c)^4 + 15a^3 + 72a^2b - 126(5a^3 + 24a^2b) \\
& * \cosh(dx + c)^2) \sinh(dx + c)^{13} + 13(476(5a^3 + 24a^2b) \cosh(dx + \\
& c)^5 - 210(5a^3 + 24a^2b) \cosh(dx + c)^3 + 15(5a^3 + 24a^2b) \cosh(\\
& dx + c)) \sinh(dx + c)^{12} - 20(5a^3 + 24a^2b) \cosh(dx + c)^{11} + 2(61 \\
& 88(5a^3 + 24a^2b) \cosh(dx + c)^6 - 4095(5a^3 + 24a^2b) \cosh(dx + \\
& c)^4 - 50a^3 - 240a^2b + 585(5a^3 + 24a^2b) \cosh(dx + c)^2) \sinh(dx \\
& + c)^{11} + 22(884(5a^3 + 24a^2b) \cosh(dx + c)^7 - 819(5a^3 + 24a^ \\
& 2b) \cosh(dx + c)^5 + 195(5a^3 + 24a^2b) \cosh(dx + c)^3 - 10(5a^3 + \\
& 24a^2b) \cosh(dx + c)) \sinh(dx + c)^{10} + 15(5a^3 + 24a^2b) \cosh(dx \\
& + c)^9 + 5(4862(5a^3 + 24a^2b) \cosh(dx + c)^8 - 6006(5a^3 + 24a^2 \\
& b) \cosh(dx + c)^6 + 2145(5a^3 + 24a^2b) \cosh(dx + c)^4 + 15a^3 + 72 \\
& a^2b - 220(5a^3 + 24a^2b) \cosh(dx + c)^2) \sinh(dx + c)^9 + 5(4862 \\
& (5a^3 + 24a^2b) \cosh(dx + c)^9 - 7722(5a^3 + 24a^2b) \cosh(dx + c)^ \\
& 7 + 3861(5a^3 + 24a^2b) \cosh(dx + c)^5 - 660(5a^3 + 24a^2b) \cosh(dx \\
& + c)^3 + 27(5a^3 + 24a^2b) \cosh(dx + c)) \sinh(dx + c)^8 - 6(5a^3 \\
& + 24a^2b) \cosh(dx + c)^7 + 2(9724(5a^3 + 24a^2b) \cosh(dx + c)^{10} \\
& - 19305(5a^3 + 24a^2b) \cosh(dx + c)^8 + 12870(5a^3 + 24a^2b) \cosh(\\
& dx + c)^6 - 3300(5a^3 + 24a^2b) \cosh(dx + c)^4 - 15a^3 - 72a^2b + \\
& 270(5a^3 + 24a^2b) \cosh(dx + c)^2) \sinh(dx + c)^7 + 2(6188(5a^3 + \\
& 24a^2b) \cosh(dx + c)^{11} - 15015(5a^3 + 24a^2b) \cosh(dx + c)^9 + 128 \\
& 70(5a^3 + 24a^2b) \cosh(dx + c)^7 - 4620(5a^3 + 24a^2b) \cosh(dx + \\
& c)^5 + 630(5a^3 + 24a^2b) \cosh(dx + c)^3 - 21(5a^3 + 24a^2b) \cosh(\\
& dx + c)) \sinh(dx + c)^6 + (5a^3 + 24a^2b) \cosh(dx + c)^5 + (6188(5a \\
& ^3 + 24a^2b) \cosh(dx + c)^{12} - 18018(5a^3 + 24a^2b) \cosh(dx + c)^{10} \\
& + 19305(5a^3 + 24a^2b) \cosh(dx + c)^8 - 9240(5a^3 + 24a^2b) \cosh(\\
& dx + c)^6 + 1890(5a^3 + 24a^2b) \cosh(dx + c)^4 + 5a^3 + 24a^2b - 1 \\
& 26(5a^3 + 24a^2b) \cosh(dx + c)^2) \sinh(dx + c)^5 + 5(476(5a^3 + 24 \\
& a^2b) \cosh(dx + c)^{13} - 1638(5a^3 + 24a^2b) \cosh(dx + c)^{11} + 2145 \\
& (5a^3 + 24a^2b) \cosh(dx + c)^9 - 1320(5a^3 + 24a^2b) \cosh(dx + c)^ \\
& 7 + 378(5a^3 + 24a^2b) \cosh(dx + c)^5 - 42(5a^3 + 24a^2b) \cosh(dx \\
& + c)^3 + (5a^3 + 24a^2b) \cosh(dx + c)) \sinh(dx + c)^4 + 10(68(5a^3 \\
& + 24a^2b) \cosh(dx + c)^{14} - 273(5a^3 + 24a^2b) \cosh(dx + c)^{12} + 4 \\
& 29(5a^3 + 24a^2b) \cosh(dx + c)^{10} - 330(5a^3 + 24a^2b) \cosh(dx + \\
& c)^8 + 126(5a^3 + 24a^2b) \cosh(dx + c)^6 - 21(5a^3 + 24a^2b) \cosh(\\
& dx + c)^4 + (5a^3 + 24a^2b) \cosh(dx + c)^2) \sinh(dx + c)^3 + 2(68(5 \\
& a^3 + 24a^2b) \cosh(dx + c)^{15} - 315(5a^3 + 24a^2b) \cosh(dx + c)^{13} \\
& + 585(5a^3 + 24a^2b) \cosh(dx + c)^{11} - 550(5a^3 + 24a^2b) \cosh(dx \\
& + c)^9 + 270(5a^3 + 24a^2b) \cosh(dx + c)^7 - 63(5a^3 + 24a^2b) \c
\end{aligned}$$

$$\begin{aligned}
& \text{osh}(d*x + c)^5 + 5*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^3*\sinh(d*x + c)^2 + (1 \\
& 7*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{16} - 90*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c) \\
& ^{14} + 195*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{12} - 220*(5*a^3 + 24*a^2*b)*\text{cosh} \\
& (d*x + c)^{10} + 135*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^8 - 42*(5*a^3 + 24*a^2* \\
& b)*\text{cosh}(d*x + c)^6 + 5*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^4*\sinh(d*x + c))* \\
& \log(\text{cosh}(d*x + c) + \sinh(d*x + c) + 1) - 30*((5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c) \\
&)^{17} + 17*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)*\sinh(d*x + c)^{16} + (5*a^3 + 24*a \\
& ^2*b)*\sinh(d*x + c)^{17} - 6*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{15} - 2*(15*a^3 \\
& + 72*a^2*b - 68*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^2)*\sinh(d*x + c)^{15} + 10*(\\
& 68*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^3 - 9*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)) \\
& *\sinh(d*x + c)^{14} + 15*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{13} + 5*(476*(5*a^3 \\
& + 24*a^2*b)*\text{cosh}(d*x + c)^4 + 15*a^3 + 72*a^2*b - 126*(5*a^3 + 24*a^2*b)*\text{co} \\
& sh(d*x + c)^2)*\sinh(d*x + c)^{13} + 13*(476*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^ \\
& 5 - 210*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^3 + 15*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x \\
& + c))*\sinh(d*x + c)^{12} - 20*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{11} + 2*(6188* \\
& (5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^6 - 4095*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^ \\
& 4 - 50*a^3 - 240*a^2*b + 585*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^2)*\sinh(d*x + \\
& c)^{11} + 22*(884*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^7 - 819*(5*a^3 + 24*a^2*b) \\
&)*\text{cosh}(d*x + c)^5 + 195*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^3 - 10*(5*a^3 + 24 \\
& *a^2*b)*\text{cosh}(d*x + c))*\sinh(d*x + c)^{10} + 15*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + \\
& c)^9 + 5*(4862*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^8 - 6006*(5*a^3 + 24*a^2*b) \\
& *\text{cosh}(d*x + c)^6 + 2145*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^4 + 15*a^3 + 72*a^ \\
& 2*b - 220*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^2)*\sinh(d*x + c)^9 + 5*(4862*(5* \\
& a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^9 - 7722*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^7 + \\
& 3861*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^5 - 660*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x \\
& + c)^3 + 27*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c))*\sinh(d*x + c)^8 - 6*(5*a^3 + \\
& 24*a^2*b)*\text{cosh}(d*x + c)^7 + 2*(9724*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{10} - 1 \\
& 9305*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^8 + 12870*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x \\
& + c)^6 - 3300*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^4 - 15*a^3 - 72*a^2*b + 270 \\
& *(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^2)*\sinh(d*x + c)^7 + 2*(6188*(5*a^3 + 24* \\
& a^2*b)*\text{cosh}(d*x + c)^{11} - 15015*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^9 + 12870* \\
& (5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^7 - 4620*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^ \\
& 5 + 630*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^3 - 21*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x \\
& + c))*\sinh(d*x + c)^6 + (5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^5 + (6188*(5*a^3 \\
& + 24*a^2*b)*\text{cosh}(d*x + c)^{12} - 18018*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{10} + \\
& 19305*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^8 - 9240*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x \\
& + c)^6 + 1890*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^4 + 5*a^3 + 24*a^2*b - 126* \\
& (5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^2)*\sinh(d*x + c)^5 + 5*(476*(5*a^3 + 24*a^ \\
& 2*b)*\text{cosh}(d*x + c)^{13} - 1638*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{11} + 2145*(5* \\
& a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^9 - 1320*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^7 + \\
& 378*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^5 - 42*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + \\
& c)^3 + (5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c))*\sinh(d*x + c)^4 + 10*(68*(5*a^3 + \\
& 24*a^2*b)*\text{cosh}(d*x + c)^{14} - 273*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{12} + 429* \\
& (5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^{10} - 330*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^ \\
& 8 + 126*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x + c)^6 - 21*(5*a^3 + 24*a^2*b)*\text{cosh}(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^4 + (5a^3 + 24a^2b) \cosh(dx + c)^2 \sinh(dx + c)^3 + 2(68(5a^3 + 24a^2b) \cosh(dx + c)^{15} - 315(5a^3 + 24a^2b) \cosh(dx + c)^{13} + 585(5a^3 + 24a^2b) \cosh(dx + c)^{11} - 550(5a^3 + 24a^2b) \cosh(dx + c)^9 + 270(5a^3 + 24a^2b) \cosh(dx + c)^7 - 63(5a^3 + 24a^2b) \cosh(dx + c)^5 + 5(5a^3 + 24a^2b) \cosh(dx + c)^3) \sinh(dx + c)^2 + (17(5a^3 + 24a^2b) \cosh(dx + c)^{16} - 90(5a^3 + 24a^2b) \cosh(dx + c)^{14} + 195(5a^3 + 24a^2b) \cosh(dx + c)^{12} - 220(5a^3 + 24a^2b) \cosh(dx + c)^{10} + 135(5a^3 + 24a^2b) \cosh(dx + c)^8 - 42(5a^3 + 24a^2b) \cosh(dx + c)^6 + 5(5a^3 + 24a^2b) \cosh(dx + c)^4) \sinh(dx + c) \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 2(33b^3 \cosh(dx + c)^{21} - 430b^3 \cosh(dx + c)^{19} + 135(48ab^2 + 23b^3) \cosh(dx + c)^{17} - 120(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx + c)^{15} + 70(170a^3 + 432a^2b + 648ab^2 + 187b^3) \cosh(dx + c)^{13} - 540(44a^3 + 32a^2b + 40ab^2 + 11b^3) \cosh(dx + c)^{11} - 450(44a^3 + 32a^2b + 40ab^2 + 11b^3) \cosh(dx + c)^9 + 40(170a^3 + 432a^2b + 648ab^2 + 187b^3) \cosh(dx + c)^7 - 45(20a^3 + 96a^2b + 240ab^2 + 79b^3) \cosh(dx + c)^5 - 43b^3 \cosh(dx + c) + 30(48ab^2 + 23b^3) \cosh(dx + c)^3) \sinh(dx + c)) / (d \cosh(dx + c)^{17} + 17d \cosh(dx + c) \sinh(dx + c)^{16} + d \sinh(dx + c)^{17} - 6d \cosh(dx + c)^{15} + 2(68d \cosh(dx + c)^2 - 3d) \sinh(dx + c)^{15} + 10(68d \cosh(dx + c)^3 - 9d \cosh(dx + c)) \sinh(dx + c)^{14} + 15d \cosh(dx + c)^{13} + 5(476d \cosh(dx + c)^4 - 126d \cosh(dx + c)^2 + 3d) \sinh(dx + c)^{13} + 13(476d \cosh(dx + c)^5 - 210d \cosh(dx + c)^3 + 15d \cosh(dx + c)) \sinh(dx + c)^{12} - 20d \cosh(dx + c)^{11} + 2(6188d \cosh(dx + c)^6 - 4095d \cosh(dx + c)^4 + 585d \cosh(dx + c)^2 - 10d) \sinh(dx + c)^{11} + 22(884d \cosh(dx + c)^7 - 819d \cosh(dx + c)^5 + 195d \cosh(dx + c)^3 - 10d \cosh(dx + c)) \sinh(dx + c)^{10} + 15d \cosh(dx + c)^9 + 5(4862d \cosh(dx + c)^8 - 6006d \cosh(dx + c)^6 + 2145d \cosh(dx + c)^4 - 220d \cosh(dx + c)^2 + 3d) \sinh(dx + c)^9 + 5(4862d \cosh(dx + c)^9 - 7722d \cosh(dx + c)^7 + 3861d \cosh(dx + c)^5 - 660d \cosh(dx + c)^3 + 27d \cosh(dx + c)) \sinh(dx + c)^8 - 6d \cosh(dx + c)^7 + 2(9724d \cosh(dx + c)^7 + c)^{10} - 19305d \cosh(dx + c)^8 + 12870d \cosh(dx + c)^6 - 3300d \cosh(dx + c)^4 + 270d \cosh(dx + c)^2 - 3d) \sinh(dx + c)^7 + 2(6188d \cosh(dx + c)^{11} - 15015d \cosh(dx + c)^9 + 12870d \cosh(dx + c)^7 - 4620d \cosh(dx + c)^5 + 630d \cosh(dx + c)^3 - 21d \cosh(dx + c)) \sinh(dx + c)^6 + d \cosh(dx + c)^5 + (6188d \cosh(dx + c)^{12} - 18018d \cosh(dx + c)^{10} + 19305d \cosh(dx + c)^8 - 9240d \cosh(dx + c)^6 + 1890d \cosh(dx + c)^4 - 126d \cosh(dx + c)^2 + d) \sinh(dx + c)^5 + 5(476d \cosh(dx + c)^{13} - 1638d \cosh(dx + c)^{11} + 2145d \cosh(dx + c)^9 - 1320d \cosh(dx + c)^7 + 378d \cosh(dx + c)^5 - 42d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c)^4 + 10(68d \cosh(dx + c)^{14} - 273d \cosh(dx + c)^{12} + 429d \cosh(dx + c)^{10} - 330d \cosh(dx + c)^8 + 126d \cosh(dx + c)^6 - 21d \cosh(dx + c)^4 + d \cosh(dx + c)^2) \sinh(dx + c)^3 + 2(68d \cosh(dx + c)^{15} - 315d \cosh(dx + c)^{13} + 585d \cosh(dx + c)^{11} - 550d \cosh(dx + c)^9 + 270d \cosh(dx + c)^7 - 63d \cosh(dx + c)^5 + 5d \cosh(dx + c)^3) \sinh(dx + c)^2 + (17d \cosh(dx + c)^{16} - 90d \cosh(dx + c)^{14} + 195d \cosh(dx + c)^{12}
\end{aligned}$$

$12 - 220*d*\cosh(d*x + c)^{10} + 135*d*\cosh(d*x + c)^8 - 42*d*\cosh(d*x + c)^6 + 5*d*\cosh(d*x + c)^4*\sinh(d*x + c)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.68381, size = 463, normalized size = 2.97

$$\frac{(5a^3 + 24a^2b) \log(e^{dx+c} + e^{-dx-c} + 2)}{32d} - \frac{(5a^3 + 24a^2b) \log(e^{dx+c} + e^{-dx-c} - 2)}{32d} - \frac{15a^3(e^{dx+c} + e^{-dx-c})^5 + 72a^2b(e^{dx+c} + e^{-dx-c})^5}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] $\frac{1}{32}*(5*a^3 + 24*a^2*b)*\log(e^{d*x + c} + e^{-d*x - c} + 2)/d - \frac{1}{32}*(5*a^3 + 24*a^2*b)*\log(e^{d*x + c} + e^{-d*x - c} - 2)/d - \frac{1}{24}*(15*a^3*(e^{d*x + c} + e^{-d*x - c})^5 + 72*a^2*b*(e^{d*x + c} + e^{-d*x - c})^5 - 160*a^3*(e^{d*x + c} + e^{-d*x - c})^3 - 576*a^2*b*(e^{d*x + c} + e^{-d*x - c})^3 + 528*a^3*(e^{d*x + c} + e^{-d*x - c}) + 1152*a^2*b*(e^{d*x + c} + e^{-d*x - c})))/(((e^{d*x + c} + e^{-d*x - c})^2 - 4)^3*d) + \frac{1}{480}*(3*b^3*d^4*(e^{d*x + c} + e^{-d*x - c})^5 - 40*b^3*d^4*(e^{d*x + c} + e^{-d*x - c})^3 + 720*a*b^2*d^4*(e^{d*x + c} + e^{-d*x - c}) + 240*b^3*d^4*(e^{d*x + c} + e^{-d*x - c}))/d^5$

3.214 $\int \operatorname{csch}^9(c + dx) \left(a + b \sinh^4(c + dx)\right)^3 dx$

Optimal. Leaf size=171

$$\frac{a(35a^2 + 144ab + 384b^2) \tanh^{-1}(\cosh(c + dx))}{128d} - \frac{a^2(35a + 144b) \coth(c + dx) \operatorname{csch}^3(c + dx)}{192d} + \frac{a^2(35a + 144b) \coth(c + dx)}{128d}$$

```
[Out] -(a*(35*a^2 + 144*a*b + 384*b^2)*ArcTanh[Cosh[c + d*x]])/(128*d) - (b^3*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x]^3)/(3*d) + (a^2*(35*a + 144*b)*Coth[c + d*x]*Csch[c + d*x])/(128*d) - (a^2*(35*a + 144*b)*Coth[c + d*x]*Csch[c + d*x]^3)/(192*d) + (7*a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(48*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^7)/(8*d)
```

Rubi [A] time = 0.331856, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 1153, 206}

$$\frac{a(35a^2 + 144ab + 384b^2) \tanh^{-1}(\cosh(c + dx))}{128d} - \frac{a^2(35a + 144b) \coth(c + dx) \operatorname{csch}^3(c + dx)}{192d} + \frac{a^2(35a + 144b) \coth(c + dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(a*(35*a^2 + 144*a*b + 384*b^2)*ArcTanh[Cosh[c + d*x]])/(128*d) - (b^3*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x]^3)/(3*d) + (a^2*(35*a + 144*b)*Coth[c + d*x]*Csch[c + d*x])/(128*d) - (a^2*(35*a + 144*b)*Coth[c + d*x]*Csch[c + d*x]^3)/(192*d) + (7*a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(48*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^7)/(8*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1157

```
Int[((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^9(c+dx) (a+b \sinh^4(c+dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^3}{(1-x^2)^5} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^3 \operatorname{coth}(c+dx) \operatorname{csch}^7(c+dx)}{8d} + \frac{\operatorname{Subst}\left(\int \frac{-(a+2b)(7a^2+10ab+4b^2)+8b(3a^2+9ab)}{(1-x^2)^5} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{7a^3 \operatorname{coth}(c+dx) \operatorname{csch}^5(c+dx)}{48d} - \frac{a^3 \operatorname{coth}(c+dx) \operatorname{csch}^7(c+dx)}{8d} - \frac{\operatorname{Subst}\left(\int \frac{-(a+2b)(7a^2+10ab+4b^2)+8b(3a^2+9ab)}{(1-x^2)^5} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^2(35a+144b) \operatorname{coth}(c+dx) \operatorname{csch}^3(c+dx)}{192d} + \frac{7a^3 \operatorname{coth}(c+dx) \operatorname{csch}^5(c+dx)}{48d} \\
&= \frac{a^2(35a+144b) \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{128d} - \frac{a^2(35a+144b) \operatorname{coth}(c+dx)}{192d} \\
&= \frac{a^2(35a+144b) \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{128d} - \frac{a^2(35a+144b) \operatorname{coth}(c+dx)}{192d} \\
&= -\frac{b^3 \cosh(c+dx)}{d} + \frac{b^3 \cosh^3(c+dx)}{3d} + \frac{a^2(35a+144b) \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{128d} \\
&= -\frac{a(35a^2+144ab+384b^2) \tanh^{-1}(\cosh(c+dx))}{128d} - \frac{b^3 \cosh(c+dx)}{d} + \frac{b^3 \cosh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 1.69414, size = 219, normalized size = 1.28

$$a \left(48 (35a^2 + 144ab + 384b^2) \log \left(\tanh \left(\frac{1}{2}(c+dx) \right) \right) - 3a^2 \operatorname{csch}^8 \left(\frac{1}{2}(c+dx) \right) + 20a^2 \operatorname{csch}^6 \left(\frac{1}{2}(c+dx) \right) + 3a^2 \operatorname{sech}^8 \left(\frac{1}{2}(c+dx) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] $(-4608*b^3*\operatorname{Cosh}[c+d*x] + 512*b^3*\operatorname{Cosh}[3*(c+d*x)] + a*(12*a*(35*a + 144*b)*\operatorname{Csch}[(c+d*x)/2]^2 - 18*a*(5*a + 16*b)*\operatorname{Csch}[(c+d*x)/2]^4 + 20*a^2*\operatorname{Csch}[(c+d*x)/2]^6 - 3*a^2*\operatorname{Csch}[(c+d*x)/2]^8 + 48*(35*a^2 + 144*a*b + 384*b^2)*\operatorname{Log}[\operatorname{Tanh}[(c+d*x)/2]] + 12*a*(35*a + 144*b)*\operatorname{Sech}[(c+d*x)/2]^2 + 18*a*(5*a + 16*b)*\operatorname{Sech}[(c+d*x)/2]^4 + 20*a^2*\operatorname{Sech}[(c+d*x)/2]^6 + 3*a^2*\operatorname{Sech}[(c+d*x)/2]^8)/(6144*d)$

Maple [A] time = 0.083, size = 143, normalized size = 0.8

$$\frac{1}{d} \left(a^3 \left(-\frac{(\operatorname{csch}(dx+c))^7}{8} + \frac{7(\operatorname{csch}(dx+c))^5}{48} - \frac{35(\operatorname{csch}(dx+c))^3}{192} + \frac{35\operatorname{csch}(dx+c)}{128} \right) \coth(dx+c) - \frac{35 \operatorname{Arctanh}(e^{dx+c})}{64} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] `1/d*(a^3*((-1/8*csch(d*x+c)^7+7/48*csch(d*x+c)^5-35/192*csch(d*x+c)^3+35/128*csch(d*x+c))*coth(d*x+c)-35/64*arctanh(exp(d*x+c)))+3*a^2*b*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d*x+c)))-6*a*b^2*arctanh(exp(d*x+c))+b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))`

Maxima [B] time = 1.13245, size = 625, normalized size = 3.65

$$\frac{1}{24} b^3 \left(\frac{e^{3dx+3c}}{d} - \frac{9e^{dx+c}}{d} - \frac{9e^{-dx-c}}{d} + \frac{e^{-3dx-3c}}{d} \right) - \frac{1}{384} a^3 \left(\frac{105 \log(e^{-dx-c} + 1)}{d} - \frac{105 \log(e^{-dx-c} - 1)}{d} + \frac{2(105 e^{-dx-c} - 1)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] `1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) - 1/384*a^3*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*x - c) - 1)/d + 2*(105*e^(-d*x - c) - 805*e^(-3*d*x - 3*c) + 2681*e^(-5*d*x - 5*c) - 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) + 2681*e^(-11*d*x - 11*c) - 805*e^(-13*d*x - 13*c) + 105*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x - 2*c) - 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) - 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) - 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) - e^(-16*d*x - 16*c) - 1))) - 3/8*a^2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)`

Fricas [B] time = 2.76965, size = 30027, normalized size = 175.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] $\frac{1}{384} \cdot (16 \cdot b^3 \cdot \cosh(d \cdot x + c)^{22} + 352 \cdot b^3 \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^{21} + 16 \cdot b^3 \cdot \sinh(d \cdot x + c)^{22} - 272 \cdot b^3 \cdot \cosh(d \cdot x + c)^{20} + 16 \cdot (231 \cdot b^3 \cdot \cosh(d \cdot x + c)^2 - 17 \cdot b^3) \cdot \sinh(d \cdot x + c)^{20} + 320 \cdot (77 \cdot b^3 \cdot \cosh(d \cdot x + c)^3 - 17 \cdot b^3 \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^{19} + 2 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)^{18} + 2 \cdot (58520 \cdot b^3 \cdot \cosh(d \cdot x + c)^4 - 25840 \cdot b^3 \cdot \cosh(d \cdot x + c)^2 + 105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \sinh(d \cdot x + c)^{18} + 12 \cdot (35112 \cdot b^3 \cdot \cosh(d \cdot x + c)^5 - 2 \cdot 5840 \cdot b^3 \cdot \cosh(d \cdot x + c)^3 + 3 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^{17} - 2 \cdot (805 \cdot a^3 + 3312 \cdot a^2 \cdot b + 1880 \cdot b^3) \cdot \cosh(d \cdot x + c)^{16} + 2 \cdot (596904 \cdot b^3 \cdot \cosh(d \cdot x + c)^6 - 658920 \cdot b^3 \cdot \cosh(d \cdot x + c)^4 - 805 \cdot a^3 - 3312 \cdot a^2 \cdot b - 1880 \cdot b^3 + 153 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)^2) \cdot \sinh(d \cdot x + c)^{16} + 32 \cdot (85272 \cdot b^3 \cdot \cosh(d \cdot x + c)^7 - 131784 \cdot b^3 \cdot \cosh(d \cdot x + c)^5 + 51 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)^3 - (805 \cdot a^3 + 3312 \cdot a^2 \cdot b + 1880 \cdot b^3) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^{15} + 2 \cdot (2681 \cdot a^3 + 7344 \cdot a^2 \cdot b + 2512 \cdot b^3) \cdot \cosh(d \cdot x + c)^{14} + 2 \cdot (2558160 \cdot b^3 \cdot \cosh(d \cdot x + c)^8 - 5271360 \cdot b^3 \cdot \cosh(d \cdot x + c)^6 + 3060 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)^4 + 2681 \cdot a^3 + 7344 \cdot a^2 \cdot b + 2512 \cdot b^3 - 120 \cdot (805 \cdot a^3 + 3312 \cdot a^2 \cdot b + 1880 \cdot b^3) \cdot \cosh(d \cdot x + c)^2) \cdot \sinh(d \cdot x + c)^{14} + 4 \cdot (1989680 \cdot b^3 \cdot \cosh(d \cdot x + c)^9 - 5271360 \cdot b^3 \cdot \cosh(d \cdot x + c)^7 + 4284 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)^5 - 280 \cdot (805 \cdot a^3 + 3312 \cdot a^2 \cdot b + 1880 \cdot b^3) \cdot \cosh(d \cdot x + c)^3 + 7 \cdot (2681 \cdot a^3 + 7344 \cdot a^2 \cdot b + 2512 \cdot b^3) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^{13} - 2 \cdot (5053 \cdot a^3 + 4464 \cdot a^2 \cdot b + 1232 \cdot b^3) \cdot \cosh(d \cdot x + c)^{12} + 2 \cdot (5173168 \cdot b^3 \cdot \cosh(d \cdot x + c)^{10} - 17131920 \cdot b^3 \cdot \cosh(d \cdot x + c)^8 + 18564 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)^6 - 1820 \cdot (805 \cdot a^3 + 3312 \cdot a^2 \cdot b + 1880 \cdot b^3) \cdot \cosh(d \cdot x + c)^4 - 5053 \cdot a^3 - 4464 \cdot a^2 \cdot b - 1232 \cdot b^3 + 91 \cdot (2681 \cdot a^3 + 7344 \cdot a^2 \cdot b + 2512 \cdot b^3) \cdot \cosh(d \cdot x + c)^2) \cdot \sinh(d \cdot x + c)^{12} + 8 \cdot (1410864 \cdot b^3 \cdot \cosh(d \cdot x + c)^{11} - 5710640 \cdot b^3 \cdot \cosh(d \cdot x + c)^9 + 7956 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)^7 - 1092 \cdot (805 \cdot a^3 + 3312 \cdot a^2 \cdot b + 1880 \cdot b^3) \cdot \cosh(d \cdot x + c)^5 + 91 \cdot (2681 \cdot a^3 + 7344 \cdot a^2 \cdot b + 2512 \cdot b^3) \cdot \cosh(d \cdot x + c)^3 - 3 \cdot (5053 \cdot a^3 + 4464 \cdot a^2 \cdot b + 1232 \cdot b^3) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^{11} - 2 \cdot (5053 \cdot a^3 + 4464 \cdot a^2 \cdot b + 1232 \cdot b^3) \cdot \cosh(d \cdot x + c)^{10} + 2 \cdot (5173168 \cdot b^3 \cdot \cosh(d \cdot x + c)^{12} - 25126816 \cdot b^3 \cdot \cosh(d \cdot x + c)^{10} + 43758 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)^8 - 8008 \cdot (805 \cdot a^3 + 3312 \cdot a^2 \cdot b + 1880 \cdot b^3) \cdot \cosh(d \cdot x + c)^6 + 1001 \cdot (2681 \cdot a^3 + 7344 \cdot a^2 \cdot b + 2512 \cdot b^3) \cdot \cosh(d \cdot x + c)^4 - 5053 \cdot a^3 - 4464 \cdot a^2 \cdot b - 1232 \cdot b^3 - 66 \cdot (5053 \cdot a^3 + 4464 \cdot a^2 \cdot b + 1232 \cdot b^3) \cdot \cosh(d \cdot x + c)^2) \cdot \sinh(d \cdot x + c)^{10} + 4 \cdot (1989680 \cdot b^3 \cdot \cosh(d \cdot x + c)^{13} - 11421280 \cdot b^3 \cdot \cosh(d \cdot x + c)^{11} + 24310 \cdot (105 \cdot a^3 + 432 \cdot a^2 \cdot b + 728 \cdot b^3) \cdot \cosh(d \cdot x + c)^9 - 5720 \cdot (805 \cdot a^3 + 3312 \cdot a^2 \cdot b + 1880 \cdot b^3) \cdot \cosh(d \cdot x + c)^7$

$$\begin{aligned}
& 7 + 1001*(2681*a^3 + 7344*a^2*b + 2512*b^3)*\cosh(d*x + c)^5 - 110*(5053*a^3 \\
& + 4464*a^2*b + 1232*b^3)*\cosh(d*x + c)^3 - 5*(5053*a^3 + 4464*a^2*b + 1232 \\
& *b^3)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 2*(2681*a^3 + 7344*a^2*b + 2512*b^3) \\
& *\cosh(d*x + c)^8 + 2*(2558160*b^3*\cosh(d*x + c)^14 - 17131920*b^3*\cosh(d*x \\
& + c)^12 + 43758*(105*a^3 + 432*a^2*b + 728*b^3)*\cosh(d*x + c)^10 - 12870*(8 \\
& 05*a^3 + 3312*a^2*b + 1880*b^3)*\cosh(d*x + c)^8 + 3003*(2681*a^3 + 7344*a^2 \\
& *b + 2512*b^3)*\cosh(d*x + c)^6 - 495*(5053*a^3 + 4464*a^2*b + 1232*b^3)*\cos \\
& h(d*x + c)^4 + 2681*a^3 + 7344*a^2*b + 2512*b^3 - 45*(5053*a^3 + 4464*a^2*b \\
& + 1232*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^8 + 16*(170544*b^3*\cosh(d*x + c \\
&)^15 - 1317840*b^3*\cosh(d*x + c)^13 + 3978*(105*a^3 + 432*a^2*b + 728*b^3)* \\
& \cosh(d*x + c)^11 - 1430*(805*a^3 + 3312*a^2*b + 1880*b^3)*\cosh(d*x + c)^9 + \\
& 429*(2681*a^3 + 7344*a^2*b + 2512*b^3)*\cosh(d*x + c)^7 - 99*(5053*a^3 + 44 \\
& 64*a^2*b + 1232*b^3)*\cosh(d*x + c)^5 - 15*(5053*a^3 + 4464*a^2*b + 1232*b^3 \\
&)*\cosh(d*x + c)^3 + (2681*a^3 + 7344*a^2*b + 2512*b^3)*\cosh(d*x + c))*\sinh(\\
& d*x + c)^7 - 2*(805*a^3 + 3312*a^2*b + 1880*b^3)*\cosh(d*x + c)^6 + 2*(59690 \\
& 4*b^3*\cosh(d*x + c)^16 - 5271360*b^3*\cosh(d*x + c)^14 + 18564*(105*a^3 + 43 \\
& 2*a^2*b + 728*b^3)*\cosh(d*x + c)^12 - 8008*(805*a^3 + 3312*a^2*b + 1880*b^3 \\
&)*\cosh(d*x + c)^10 + 3003*(2681*a^3 + 7344*a^2*b + 2512*b^3)*\cosh(d*x + c)^ \\
& 8 - 924*(5053*a^3 + 4464*a^2*b + 1232*b^3)*\cosh(d*x + c)^6 - 210*(5053*a^3 \\
& + 4464*a^2*b + 1232*b^3)*\cosh(d*x + c)^4 - 805*a^3 - 3312*a^2*b - 1880*b^3 \\
& + 28*(2681*a^3 + 7344*a^2*b + 2512*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + \\
& 4*(105336*b^3*\cosh(d*x + c)^17 - 1054272*b^3*\cosh(d*x + c)^15 + 4284*(105*a \\
& ^3 + 432*a^2*b + 728*b^3)*\cosh(d*x + c)^13 - 2184*(805*a^3 + 3312*a^2*b + 1 \\
& 880*b^3)*\cosh(d*x + c)^11 + 1001*(2681*a^3 + 7344*a^2*b + 2512*b^3)*\cosh(d* \\
& x + c)^9 - 396*(5053*a^3 + 4464*a^2*b + 1232*b^3)*\cosh(d*x + c)^7 - 126*(50 \\
& 53*a^3 + 4464*a^2*b + 1232*b^3)*\cosh(d*x + c)^5 + 28*(2681*a^3 + 7344*a^2*b \\
& + 2512*b^3)*\cosh(d*x + c)^3 - 3*(805*a^3 + 3312*a^2*b + 1880*b^3)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^5 - 272*b^3*\cosh(d*x + c)^2 + 2*(105*a^3 + 432*a^2*b + \\
& 728*b^3)*\cosh(d*x + c)^4 + 2*(58520*b^3*\cosh(d*x + c)^18 - 658920*b^3*\cosh \\
& (d*x + c)^16 + 3060*(105*a^3 + 432*a^2*b + 728*b^3)*\cosh(d*x + c)^14 - 1820 \\
& *(805*a^3 + 3312*a^2*b + 1880*b^3)*\cosh(d*x + c)^12 + 1001*(2681*a^3 + 7344 \\
& *a^2*b + 2512*b^3)*\cosh(d*x + c)^10 - 495*(5053*a^3 + 4464*a^2*b + 1232*b^3 \\
&)*\cosh(d*x + c)^8 - 210*(5053*a^3 + 4464*a^2*b + 1232*b^3)*\cosh(d*x + c)^6 \\
& + 70*(2681*a^3 + 7344*a^2*b + 2512*b^3)*\cosh(d*x + c)^4 + 105*a^3 + 432*a^2 \\
& *b + 728*b^3 - 15*(805*a^3 + 3312*a^2*b + 1880*b^3)*\cosh(d*x + c)^2*\sinh(d \\
& *x + c)^4 + 8*(3080*b^3*\cosh(d*x + c)^19 - 38760*b^3*\cosh(d*x + c)^17 + 204 \\
& *(105*a^3 + 432*a^2*b + 728*b^3)*\cosh(d*x + c)^15 - 140*(805*a^3 + 3312*a^2 \\
& *b + 1880*b^3)*\cosh(d*x + c)^13 + 91*(2681*a^3 + 7344*a^2*b + 2512*b^3)*\cos \\
& h(d*x + c)^11 - 55*(5053*a^3 + 4464*a^2*b + 1232*b^3)*\cosh(d*x + c)^9 - 30* \\
& (5053*a^3 + 4464*a^2*b + 1232*b^3)*\cosh(d*x + c)^7 + 14*(2681*a^3 + 7344*a^ \\
& 2*b + 2512*b^3)*\cosh(d*x + c)^5 - 5*(805*a^3 + 3312*a^2*b + 1880*b^3)*\cosh(\\
& d*x + c)^3 + (105*a^3 + 432*a^2*b + 728*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& + 16*b^3 + 2*(1848*b^3*\cosh(d*x + c)^20 - 25840*b^3*\cosh(d*x + c)^18 + 153 \\
& *(105*a^3 + 432*a^2*b + 728*b^3)*\cosh(d*x + c)^16 - 120*(805*a^3 + 3312*a^2 \\
& *b + 1880*b^3)*\cosh(d*x + c)^14 + 91*(2681*a^3 + 7344*a^2*b + 2512*b^3)*\cos
\end{aligned}$$

$$\begin{aligned}
& h(dx + c)^{12} - 66*(5053*a^3 + 4464*a^2*b + 1232*b^3)*\cosh(dx + c)^{10} - 45 \\
& *(5053*a^3 + 4464*a^2*b + 1232*b^3)*\cosh(dx + c)^8 + 28*(2681*a^3 + 7344*a \\
& ^2*b + 2512*b^3)*\cosh(dx + c)^6 - 15*(805*a^3 + 3312*a^2*b + 1880*b^3)*\cos \\
& h(dx + c)^4 - 136*b^3 + 6*(105*a^3 + 432*a^2*b + 728*b^3)*\cosh(dx + c)^2) \\
& *sinh(dx + c)^2 - 3*((35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^{19} + 1 \\
& 9*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)*sinh(dx + c)^{18} + (35*a^3 \\
& + 144*a^2*b + 384*a*b^2)*sinh(dx + c)^{19} - 8*(35*a^3 + 144*a^2*b + 384*a* \\
& b^2)*\cosh(dx + c)^{17} - (280*a^3 + 1152*a^2*b + 3072*a*b^2 - 171*(35*a^3 + \\
& 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^2)*sinh(dx + c)^{17} + 17*(57*(35*a^3 + \\
& 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^3 - 8*(35*a^3 + 144*a^2*b + 384*a*b^2 \\
&)*\cosh(dx + c))*sinh(dx + c)^{16} + 28*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cos \\
& h(dx + c)^{15} + 4*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^4 + 2 \\
& 45*a^3 + 1008*a^2*b + 2688*a*b^2 - 272*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cos \\
& h(dx + c)^2)*sinh(dx + c)^{15} + 4*(2907*(35*a^3 + 144*a^2*b + 384*a*b^2)*\c \\
& osh(dx + c)^5 - 1360*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^3 + 10 \\
& 5*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c))*sinh(dx + c)^{14} - 56*(35 \\
& *a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^{13} + 28*(969*(35*a^3 + 144*a^2* \\
& b + 384*a*b^2)*\cosh(dx + c)^6 - 680*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(\\
& dx + c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 105*(35*a^3 + 144*a^2*b + 384 \\
& *a*b^2)*\cosh(dx + c)^2)*sinh(dx + c)^{13} + 52*(969*(35*a^3 + 144*a^2*b + 3 \\
& 84*a*b^2)*\cosh(dx + c)^7 - 952*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + \\
& c)^5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^3 - 14*(35*a^3 + \\
& 144*a^2*b + 384*a*b^2)*\cosh(dx + c))*sinh(dx + c)^{12} + 70*(35*a^3 + 144* \\
& a^2*b + 384*a*b^2)*\cosh(dx + c)^{11} + 2*(37791*(35*a^3 + 144*a^2*b + 384*a* \\
& b^2)*\cosh(dx + c)^8 - 49504*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c) \\
& ^6 + 19110*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^4 + 1225*a^3 + 50 \\
& 40*a^2*b + 13440*a*b^2 - 2184*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c \\
&)^2)*sinh(dx + c)^{11} + 22*(4199*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx \\
& + c)^9 - 7072*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^7 + 3822*(35*a \\
& ^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^5 - 728*(35*a^3 + 144*a^2*b + 384 \\
& *a*b^2)*\cosh(dx + c)^3 + 35*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c) \\
&)*sinh(dx + c)^{10} - 56*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^9 + \\
& 2*(46189*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^{10} - 97240*(35*a^3 \\
& + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^8 + 70070*(35*a^3 + 144*a^2*b + 384* \\
& a*b^2)*\cosh(dx + c)^6 - 20020*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + \\
& c)^4 - 980*a^3 - 4032*a^2*b - 10752*a*b^2 + 1925*(35*a^3 + 144*a^2*b + 384* \\
& a*b^2)*\cosh(dx + c)^2)*sinh(dx + c)^9 + 2*(37791*(35*a^3 + 144*a^2*b + 38 \\
& 4*a*b^2)*\cosh(dx + c)^{11} - 97240*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx \\
& + c)^9 + 90090*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^7 - 36036*(3 \\
& 5*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^5 + 5775*(35*a^3 + 144*a^2*b + \\
& 384*a*b^2)*\cosh(dx + c)^3 - 252*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx \\
& + c))*sinh(dx + c)^8 + 28*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^ \\
& 7 + 4*(12597*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^{12} - 38896*(35* \\
& a^3 + 144*a^2*b + 384*a*b^2)*\cosh(dx + c)^{10} + 45045*(35*a^3 + 144*a^2*b + \\
& 384*a*b^2)*\cosh(dx + c)^8 - 24024*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d
\end{aligned}$$

$$\begin{aligned}
& *x + c)^6 + 5775*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^4 + 245*a^3 \\
& + 1008*a^2*b + 2688*a*b^2 - 504*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x \\
& + c)^2)*\sinh(d*x + c)^7 + 28*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x \\
& + c)^13 - 3536*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^11 + 5005*(3 \\
& 5*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^9 - 3432*(35*a^3 + 144*a^2*b + \\
& 384*a*b^2)*\cosh(d*x + c)^7 + 1155*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d* \\
& x + c)^5 - 168*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^3 + 7*(35*a^3 \\
& + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^6 - 8*(35*a^3 + 144* \\
& a^2*b + 384*a*b^2)*\cosh(d*x + c)^5 + 4*(2907*(35*a^3 + 144*a^2*b + 384*a*b^ \\
& 2)*\cosh(d*x + c)^14 - 12376*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^ \\
& 12 + 21021*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^10 - 18018*(35*a^ \\
& 3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^8 + 8085*(35*a^3 + 144*a^2*b + 384 \\
& *a*b^2)*\cosh(d*x + c)^6 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + \\
& c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 147*(35*a^3 + 144*a^2*b + 384*a*b^2 \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 4*(969*(35*a^3 + 144*a^2*b + 384*a*b^2 \\
&)*\cosh(d*x + c)^15 - 4760*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^13 \\
& + 9555*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^11 - 10010*(35*a^3 + \\
& 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^9 + 5775*(35*a^3 + 144*a^2*b + 384*a* \\
& b^2)*\cosh(d*x + c)^7 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^ \\
& 5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^3 - 10*(35*a^3 + 144 \\
& *a^2*b + 384*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^4 + (35*a^3 + 144*a^2*b + \\
& 384*a*b^2)*\cosh(d*x + c)^3 + (969*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x \\
& + c)^16 - 5440*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^14 + 12740*(\\
& 35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^12 - 16016*(35*a^3 + 144*a^2* \\
& b + 384*a*b^2)*\cosh(d*x + c)^10 + 11550*(35*a^3 + 144*a^2*b + 384*a*b^2)*\co \\
& sh(d*x + c)^8 - 4704*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^6 + 980 \\
& *(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^4 + 35*a^3 + 144*a^2*b + 38 \\
& 4*a*b^2 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c \\
&)^3 + (171*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^17 - 1088*(35*a^3 \\
& + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^15 + 2940*(35*a^3 + 144*a^2*b + 384 \\
& *a*b^2)*\cosh(d*x + c)^13 - 4368*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + \\
& c)^11 + 3850*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^9 - 2016*(35*a \\
& ^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^7 + 588*(35*a^3 + 144*a^2*b + 384 \\
& *a*b^2)*\cosh(d*x + c)^5 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c) \\
& ^3 + 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (1 \\
& 9*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^18 - 136*(35*a^3 + 144*a^2 \\
& *b + 384*a*b^2)*\cosh(d*x + c)^16 + 420*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cos \\
& h(d*x + c)^14 - 728*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^12 + 770 \\
& *(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^10 - 504*(35*a^3 + 144*a^2* \\
& b + 384*a*b^2)*\cosh(d*x + c)^8 + 196*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(\\
& d*x + c)^6 - 40*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^4 + 3*(35*a^ \\
& 3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c))*\log(\cosh(d*x + c \\
&) + \sinh(d*x + c) + 1) + 3*((35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c)^ \\
& 19 + 19*(35*a^3 + 144*a^2*b + 384*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^18 + (\\
& 35*a^3 + 144*a^2*b + 384*a*b^2)*\sinh(d*x + c)^19 - 8*(35*a^3 + 144*a^2*b +
\end{aligned}$$

$$\begin{aligned}
& 384*a*b^2)*\cosh(d*x + c)^{17} - (280*a^3 + 1152*a^2*b + 3072*a*b^2 - 171*(35* \\
& a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^2)*\sinh(d*x + c)^{17} + 17*(57*(35 \\
& *a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^3 - 8*(35*a^3 + 144*a^2*b + 384 \\
& *a*b^2))*\cosh(d*x + c))*\sinh(d*x + c)^{16} + 28*(35*a^3 + 144*a^2*b + 384*a*b^ \\
& 2))*\cosh(d*x + c)^{15} + 4*(969*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c) \\
& ^4 + 245*a^3 + 1008*a^2*b + 2688*a*b^2 - 272*(35*a^3 + 144*a^2*b + 384*a*b^ \\
& 2))*\cosh(d*x + c)^2)*\sinh(d*x + c)^{15} + 4*(2907*(35*a^3 + 144*a^2*b + 384*a* \\
& b^2))*\cosh(d*x + c)^5 - 1360*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^ \\
& 3 + 105*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c))*\sinh(d*x + c)^{14} - \\
& 56*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^{13} + 28*(969*(35*a^3 + 14 \\
& 4*a^2*b + 384*a*b^2))*\cosh(d*x + c)^6 - 680*(35*a^3 + 144*a^2*b + 384*a*b^2) \\
& *\cosh(d*x + c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 105*(35*a^3 + 144*a^2*b \\
& + 384*a*b^2))*\cosh(d*x + c)^2)*\sinh(d*x + c)^{13} + 52*(969*(35*a^3 + 144*a^2 \\
& *b + 384*a*b^2))*\cosh(d*x + c)^7 - 952*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh \\
& (d*x + c)^5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^3 - 14*(35 \\
& *a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c))*\sinh(d*x + c)^{12} + 70*(35*a^3 \\
& + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^{11} + 2*(37791*(35*a^3 + 144*a^2*b + \\
& 384*a*b^2))*\cosh(d*x + c)^8 - 49504*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d* \\
& x + c)^6 + 19110*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^4 + 1225*a^ \\
& 3 + 5040*a^2*b + 13440*a*b^2 - 2184*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d \\
& *x + c)^2)*\sinh(d*x + c)^{11} + 22*(4199*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cos \\
& h(d*x + c)^9 - 7072*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^7 + 3822 \\
& *(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^5 - 728*(35*a^3 + 144*a^2*b \\
& + 384*a*b^2))*\cosh(d*x + c)^3 + 35*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d* \\
& x + c))*\sinh(d*x + c)^{10} - 56*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c \\
&)^9 + 2*(46189*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^{10} - 97240*(3 \\
& 5*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^8 + 70070*(35*a^3 + 144*a^2*b \\
& + 384*a*b^2))*\cosh(d*x + c)^6 - 20020*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(\\
& d*x + c)^4 - 980*a^3 - 4032*a^2*b - 10752*a*b^2 + 1925*(35*a^3 + 144*a^2*b \\
& + 384*a*b^2))*\cosh(d*x + c)^2)*\sinh(d*x + c)^9 + 2*(37791*(35*a^3 + 144*a^2* \\
& b + 384*a*b^2))*\cosh(d*x + c)^{11} - 97240*(35*a^3 + 144*a^2*b + 384*a*b^2))*\co \\
& sh(d*x + c)^9 + 90090*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^7 - 36 \\
& 036*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^5 + 5775*(35*a^3 + 144*a \\
& ^2*b + 384*a*b^2))*\cosh(d*x + c)^3 - 252*(35*a^3 + 144*a^2*b + 384*a*b^2))*\co \\
& sh(d*x + c))*\sinh(d*x + c)^8 + 28*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x \\
& + c)^7 + 4*(12597*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^{12} - 3889 \\
& 6*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^{10} + 45045*(35*a^3 + 144*a \\
& ^2*b + 384*a*b^2))*\cosh(d*x + c)^8 - 24024*(35*a^3 + 144*a^2*b + 384*a*b^2))* \\
& \cosh(d*x + c)^6 + 5775*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^4 + 2 \\
& 45*a^3 + 1008*a^2*b + 2688*a*b^2 - 504*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cos \\
& h(d*x + c)^2)*\sinh(d*x + c)^7 + 28*(969*(35*a^3 + 144*a^2*b + 384*a*b^2))*\co \\
& sh(d*x + c)^{13} - 3536*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^{11} + 5 \\
& 005*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^9 - 3432*(35*a^3 + 144*a \\
& ^2*b + 384*a*b^2))*\cosh(d*x + c)^7 + 1155*(35*a^3 + 144*a^2*b + 384*a*b^2))*\c \\
& osh(d*x + c)^5 - 168*(35*a^3 + 144*a^2*b + 384*a*b^2))*\cosh(d*x + c)^3 + 7*(
\end{aligned}$$

$$\begin{aligned}
& 35a^3 + 144a^2b + 384ab^2) \cosh(dx + c) \sinh(dx + c)^6 - 8(35a^3 \\
& + 144a^2b + 384ab^2) \cosh(dx + c)^5 + 4(2907(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{14} - 12376(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{12} + 21021(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{10} - 18018(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^8 + 8085(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^6 - 1764(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^4 - 70a^3 - 288a^2b - 768ab^2 + 147(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^2) \sinh(dx + c)^5 + 4(969(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{15} - 4760(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{13} + 9555(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{11} - 10010(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^9 + 5775(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^7 - 1764(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^5 + 245(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^3 - 10(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c) \sinh(dx + c)^4 + (35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^3 + (969(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{16} - 5440(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{14} + 12740(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{12} - 16016(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{10} + 11550(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^8 - 4704(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^6 + 980(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^4 + 35a^3 + 144a^2b + 384ab^2 - 80(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^2) \sinh(dx + c)^3 + (171(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{17} - 1088(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{15} + 2940(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{13} - 4368(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{11} + 3850(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^9 - 2016(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^7 + 588(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^5 - 80(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^3 + 3(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c) \sinh(dx + c)^2 + (19(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{18} - 136(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{16} + 420(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{14} - 728(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{12} + 770(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^{10} - 504(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^8 + 196(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^6 - 40(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^4 + 3(35a^3 + 144a^2b + 384ab^2) \cosh(dx + c)^2) \sinh(dx + c) \log(\cosh(dx + c) + \sinh(dx + c) - 1) + 4(88b^3 \cosh(dx + c)^{21} - 1360b^3 \cosh(dx + c)^{19} + 9(105a^3 + 432a^2b + 728b^3) \cosh(dx + c)^{17} - 8(805a^3 + 3312a^2b + 1880b^3) \cosh(dx + c)^{15} + 7(2681a^3 + 7344a^2b + 2512b^3) \cosh(dx + c)^{13} - 6(5053a^3 + 4464a^2b + 1232b^3) \cosh(dx + c)^{11} - 5(5053a^3 + 4464a^2b + 1232b^3) \cosh(dx + c)^9 + 4(2681a^3 + 7344a^2b + 2512b^3) \cosh(dx + c)^7 - 3(805a^3 + 3312a^2b + 1880b^3) \cosh(dx + c)^5 - 136b^3 \cosh(dx + c) + 2(105a^3 + 432a^2b + 728b^3) \cosh(dx + c)^3) \sinh(dx + c) / (d \cosh(dx + c)^{19} + 19d \cosh(dx + c) \sinh(dx + c)^{18} + d \sinh(dx + c)^{19} - 8d \cosh(dx + c)^{17} + (171d \cosh(dx + c)^2 - 8d) \sinh(dx + c)^{17} + 17(57d \cosh(dx + c)^3 - 8d \cosh
\end{aligned}$$

$$\begin{aligned}
& (d*x + c))*\sinh(d*x + c)^{16} + 28*d*\cosh(d*x + c)^{15} + 4*(969*d*\cosh(d*x + c) \\
&)^4 - 272*d*\cosh(d*x + c)^2 + 7*d)*\sinh(d*x + c)^{15} + 4*(2907*d*\cosh(d*x + \\
& c)^5 - 1360*d*\cosh(d*x + c)^3 + 105*d*\cosh(d*x + c))*\sinh(d*x + c)^{14} - 56* \\
& d*\cosh(d*x + c)^{13} + 28*(969*d*\cosh(d*x + c)^6 - 680*d*\cosh(d*x + c)^4 + 10 \\
& 5*d*\cosh(d*x + c)^2 - 2*d)*\sinh(d*x + c)^{13} + 52*(969*d*\cosh(d*x + c)^7 - 9 \\
& 52*d*\cosh(d*x + c)^5 + 245*d*\cosh(d*x + c)^3 - 14*d*\cosh(d*x + c))*\sinh(d*x \\
& + c)^{12} + 70*d*\cosh(d*x + c)^{11} + 2*(37791*d*\cosh(d*x + c)^8 - 49504*d*\cos \\
& h(d*x + c)^6 + 19110*d*\cosh(d*x + c)^4 - 2184*d*\cosh(d*x + c)^2 + 35*d)*\sin \\
& h(d*x + c)^{11} + 22*(4199*d*\cosh(d*x + c)^9 - 7072*d*\cosh(d*x + c)^7 + 3822* \\
& d*\cosh(d*x + c)^5 - 728*d*\cosh(d*x + c)^3 + 35*d*\cosh(d*x + c))*\sinh(d*x + \\
& c)^{10} - 56*d*\cosh(d*x + c)^9 + 2*(46189*d*\cosh(d*x + c)^{10} - 97240*d*\cosh(d \\
& *x + c)^8 + 70070*d*\cosh(d*x + c)^6 - 20020*d*\cosh(d*x + c)^4 + 1925*d*\cosh \\
& (d*x + c)^2 - 28*d)*\sinh(d*x + c)^9 + 2*(37791*d*\cosh(d*x + c)^{11} - 97240*d \\
& *\cosh(d*x + c)^9 + 90090*d*\cosh(d*x + c)^7 - 36036*d*\cosh(d*x + c)^5 + 5775 \\
& *d*\cosh(d*x + c)^3 - 252*d*\cosh(d*x + c))*\sinh(d*x + c)^8 + 28*d*\cosh(d*x + \\
& c)^7 + 4*(12597*d*\cosh(d*x + c)^{12} - 38896*d*\cosh(d*x + c)^{10} + 45045*d*\co \\
& sh(d*x + c)^8 - 24024*d*\cosh(d*x + c)^6 + 5775*d*\cosh(d*x + c)^4 - 504*d*\co \\
& sh(d*x + c)^2 + 7*d)*\sinh(d*x + c)^7 + 28*(969*d*\cosh(d*x + c)^{13} - 3536*d* \\
& cosh(d*x + c)^{11} + 5005*d*\cosh(d*x + c)^9 - 3432*d*\cosh(d*x + c)^7 + 1155*d \\
& *\cosh(d*x + c)^5 - 168*d*\cosh(d*x + c)^3 + 7*d*\cosh(d*x + c))*\sinh(d*x + c) \\
& ^6 - 8*d*\cosh(d*x + c)^5 + 4*(2907*d*\cosh(d*x + c)^{14} - 12376*d*\cosh(d*x + \\
& c)^{12} + 21021*d*\cosh(d*x + c)^{10} - 18018*d*\cosh(d*x + c)^8 + 8085*d*\cosh(d* \\
& x + c)^6 - 1764*d*\cosh(d*x + c)^4 + 147*d*\cosh(d*x + c)^2 - 2*d)*\sinh(d*x + \\
& c)^5 + 4*(969*d*\cosh(d*x + c)^{15} - 4760*d*\cosh(d*x + c)^{13} + 9555*d*\cosh(d \\
& *x + c)^{11} - 10010*d*\cosh(d*x + c)^9 + 5775*d*\cosh(d*x + c)^7 - 1764*d*\cosh \\
& (d*x + c)^5 + 245*d*\cosh(d*x + c)^3 - 10*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + \\
& d*\cosh(d*x + c)^3 + (969*d*\cosh(d*x + c)^{16} - 5440*d*\cosh(d*x + c)^{14} + 12 \\
& 740*d*\cosh(d*x + c)^{12} - 16016*d*\cosh(d*x + c)^{10} + 11550*d*\cosh(d*x + c)^8 \\
& - 4704*d*\cosh(d*x + c)^6 + 980*d*\cosh(d*x + c)^4 - 80*d*\cosh(d*x + c)^2 + \\
& d)*\sinh(d*x + c)^3 + (171*d*\cosh(d*x + c)^{17} - 1088*d*\cosh(d*x + c)^{15} + 29 \\
& 40*d*\cosh(d*x + c)^{13} - 4368*d*\cosh(d*x + c)^{11} + 3850*d*\cosh(d*x + c)^9 - \\
& 2016*d*\cosh(d*x + c)^7 + 588*d*\cosh(d*x + c)^5 - 80*d*\cosh(d*x + c)^3 + 3*d \\
& *\cosh(d*x + c))*\sinh(d*x + c)^2 + (19*d*\cosh(d*x + c)^{18} - 136*d*\cosh(d*x + \\
& c)^{16} + 420*d*\cosh(d*x + c)^{14} - 728*d*\cosh(d*x + c)^{12} + 770*d*\cosh(d*x + \\
& c)^{10} - 504*d*\cosh(d*x + c)^8 + 196*d*\cosh(d*x + c)^6 - 40*d*\cosh(d*x + c) \\
& ^4 + 3*d*\cosh(d*x + c)^2)*\sinh(d*x + c)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**9*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70297, size = 473, normalized size = 2.77

$$-\frac{(35a^3 + 144a^2b + 384ab^2)\log(e^{(dx+c)} + e^{(-dx-c)} + 2)}{256d} + \frac{(35a^3 + 144a^2b + 384ab^2)\log(e^{(dx+c)} + e^{(-dx-c)} - 2)}{256d} + \frac{b^3d^2}{256d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/256*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2)
/d + 1/256*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x - c)
- 2)/d + 1/24*(b^3*d^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*b^3*d^2*(e^(d*x
+ c) + e^(-d*x - c)))/d^3 + 1/192*(105*a^3*(e^(d*x + c) + e^(-d*x - c))^7 +
432*a^2*b*(e^(d*x + c) + e^(-d*x - c))^7 - 1540*a^3*(e^(d*x + c) + e^(-d*x
- c))^5 - 6336*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5 + 8176*a^3*(e^(d*x + c
) + e^(-d*x - c))^3 + 29952*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 - 17856*a^
3*(e^(d*x + c) + e^(-d*x - c)) - 46080*a^2*b*(e^(d*x + c) + e^(-d*x - c)))/
(((e^(d*x + c) + e^(-d*x - c))^2 - 4)^4*d)
```


3.215 $\int \operatorname{csch}^{11}(c + dx) \left(a + b \sinh^4(c + dx)\right)^3 dx$

Optimal. Leaf size=189

$$\frac{3a(21a^2 + 80ab + 128b^2) \tanh^{-1}(\cosh(c + dx))}{256d} - \frac{3a(21a^2 + 80ab + 128b^2) \coth(c + dx) \operatorname{csch}(c + dx)}{256d} - \frac{a^2(21a + 80b)}{256d}$$

```
[Out] (3*a*(21*a^2 + 80*a*b + 128*b^2)*ArcTanh[Cosh[c + d*x]])/(256*d) + (b^3*Cos
h[c + d*x])/d - (3*a*(21*a^2 + 80*a*b + 128*b^2)*Coth[c + d*x]*Csch[c + d*x
])/(256*d) + (a^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^3)/(128*d) - (a
^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(160*d) + (9*a^3*Coth[c + d
*x]*Csch[c + d*x]^7)/(80*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^9)/(10*d)
```

Rubi [A] time = 0.369044, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 388, 206}

$$\frac{3a(21a^2 + 80ab + 128b^2) \tanh^{-1}(\cosh(c + dx))}{256d} - \frac{3a(21a^2 + 80ab + 128b^2) \coth(c + dx) \operatorname{csch}(c + dx)}{256d} - \frac{a^2(21a + 80b)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*a*(21*a^2 + 80*a*b + 128*b^2)*ArcTanh[Cosh[c + d*x]])/(256*d) + (b^3*Cos
h[c + d*x])/d - (3*a*(21*a^2 + 80*a*b + 128*b^2)*Coth[c + d*x]*Csch[c + d*x
])/(256*d) + (a^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^3)/(128*d) - (a
^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(160*d) + (9*a^3*Coth[c + d
*x]*Csch[c + d*x]^7)/(80*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^9)/(10*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^{11}(c+dx) (a+b \sinh^4(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^3}{(1-x^2)^6} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^3 \operatorname{coth}(c+dx) \operatorname{csch}^9(c+dx)}{10d} - \frac{\operatorname{Subst}\left(\int \frac{-9a^3-30a^2b-30ab^2-10b^3+10b(3a^2+5ab+3b^2)x^2}{(1-x^2)^6} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{9a^3 \operatorname{coth}(c+dx) \operatorname{csch}^7(c+dx)}{80d} - \frac{a^3 \operatorname{coth}(c+dx) \operatorname{csch}^9(c+dx)}{10d} + \frac{\operatorname{Subst}\left(\int \frac{9a^3+30a^2b+30ab^2+10b^3-10b(3a^2+5ab+3b^2)x^2}{(1-x^2)^6} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^2(21a+80b) \operatorname{coth}(c+dx) \operatorname{csch}^5(c+dx)}{160d} + \frac{9a^3 \operatorname{coth}(c+dx) \operatorname{csch}^7(c+dx)}{80d} \\
&= \frac{a^2(21a+80b) \operatorname{coth}(c+dx) \operatorname{csch}^3(c+dx)}{128d} - \frac{a^2(21a+80b) \operatorname{coth}(c+dx) \operatorname{csch}^5(c+dx)}{160d} \\
&= -\frac{3a(21a^2+80ab+128b^2) \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{256d} + \frac{a^2(21a+80b) \operatorname{coth}(c+dx) \operatorname{csch}^3(c+dx)}{128d} \\
&= \frac{b^3 \cosh(c+dx)}{d} - \frac{3a(21a^2+80ab+128b^2) \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{256d} + \frac{a^2(21a+80b) \operatorname{coth}(c+dx) \operatorname{csch}^3(c+dx)}{128d} \\
&= \frac{3a(21a^2+80ab+128b^2) \tanh^{-1}(\cosh(c+dx))}{256d} + \frac{b^3 \cosh(c+dx)}{d} - \frac{3a(21a^2+80ab+128b^2) \operatorname{coth}(c+dx) \operatorname{csch}(c+dx)}{256d}
\end{aligned}$$

Mathematica [A] time = 2.47304, size = 265, normalized size = 1.4

$$\frac{b^3 \cosh(c+dx)}{d} - \frac{a \left(60(21a^2+80ab+128b^2) \operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) + 60(21a^2+80ab+128b^2) \operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right) + 240 \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (b^3*Cosh[c + d*x])/d - (a*(60*(21*a^2 + 80*a*b + 128*b^2)*Csch[(c + d*x)/2]^2 - 40*a*(7*a + 24*b)*Csch[(c + d*x)/2]^4 + 10*a*(7*a + 16*b)*Csch[(c + d*x)/2]^6 - 15*a^2*Csch[(c + d*x)/2]^8 + 2*a^2*Csch[(c + d*x)/2]^10 + 240*(21*a^2 + 80*a*b + 128*b^2)*Log[Tanh[(c + d*x)/2]] + 60*(21*a^2 + 80*a*b + 128*b^2)*Sech[(c + d*x)/2]^2 + 40*a*(7*a + 24*b)*Sech[(c + d*x)/2]^4 + 10*a*(7*a + 16*b)*Sech[(c + d*x)/2]^6 + 15*a^2*Sech[(c + d*x)/2]^8 + 2*a^2*Sech[(c + d*x)/2]^10)/d

$c + d*x)/2]^{10})/(20480*d)$

Maple [A] time = 0.081, size = 166, normalized size = 0.9

$$\frac{1}{d} \left(a^3 \left(\left(-\frac{(\operatorname{csch}(dx+c))^9}{10} + \frac{9(\operatorname{csch}(dx+c))^7}{80} - \frac{21(\operatorname{csch}(dx+c))^5}{160} + \frac{21(\operatorname{csch}(dx+c))^3}{128} - \frac{63\operatorname{csch}(dx+c)}{256} \right) \operatorname{coth}(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] `1/d*(a^3*((-1/10*csch(d*x+c)^9+9/80*csch(d*x+c)^7-21/160*csch(d*x+c)^5+21/128*csch(d*x+c)^3-63/256*csch(d*x+c))*coth(d*x+c)+63/128*arctanh(exp(d*x+c))+3*a^2*b*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x+c)+5/8*arctanh(exp(d*x+c)))+3*a*b^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b^3*cosh(d*x+c))`

Maxima [B] time = 1.14825, size = 774, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] `1/2*b^3*(e^(d*x+c)/d + e^(-d*x-c)/d) + 1/1280*a^3*(315*log(e^(-d*x-c)+1)/d - 315*log(e^(-d*x-c)-1)/d + 2*(315*e^(-d*x-c) - 3045*e^(-3*d*x-3*c) + 13188*e^(-5*d*x-5*c) - 33660*e^(-7*d*x-7*c) + 55970*e^(-9*d*x-9*c) + 55970*e^(-11*d*x-11*c) - 33660*e^(-13*d*x-13*c) + 13188*e^(-15*d*x-15*c) - 3045*e^(-17*d*x-17*c) + 315*e^(-19*d*x-19*c))/(d*(10*e^(-2*d*x-2*c) - 45*e^(-4*d*x-4*c) + 120*e^(-6*d*x-6*c) - 210*e^(-8*d*x-8*c) + 252*e^(-10*d*x-10*c) - 210*e^(-12*d*x-12*c) + 120*e^(-14*d*x-14*c) - 45*e^(-16*d*x-16*c) + 10*e^(-18*d*x-18*c) - e^(-20*d*x-20*c) - 1))) + 1/16*a^2*b*(15*log(e^(-d*x-c)+1)/d - 15*log(e^(-d*x-c)-1)/d + 2*(15*e^(-d*x-c) - 85*e^(-3*d*x-3*c) + 198*e^(-5*d*x-5*c) + 198*e^(-7*d*x-7*c) - 85*e^(-9*d*x-9*c) + 15*e^(-11*d*x-11*c))/(d*(6*e^(-2*d*x-2*c) - 15*e^(-4*d*x-4*c) + 20*e^(-6*d*x-6*c) - 15*e^(-8*d*x-8*c) + 6*e^(-10*d*x-10*c) - e^(-12*d*x-12*c) - 1))) + 3/2*a*b^2*(log(e^(-d*x-c)+1)/d - log(e^(-d*x-c)-1)/d + 2*(e^(-d*x-c) + e^(-3*d*x-c) - 1)))`

$$3*c))/ (d*(2*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} - 1)))$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**11*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.70407, size = 653, normalized size = 3.46

$$\frac{b^3(e^{(dx+c)} + e^{(-dx-c)})}{2d} + \frac{3(21a^3 + 80a^2b + 128ab^2) \log(e^{(dx+c)} + e^{(-dx-c)} + 2)}{512d} - \frac{3(21a^3 + 80a^2b + 128ab^2) \log(e^{(dx+c)} + e^{(-dx-c)} - 2)}{512d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] $\frac{1}{2}b^3(e^{(d*x + c)} + e^{(-d*x - c)})/d + \frac{3}{512}(21*a^3 + 80*a^2*b + 128*a*b^2)*\log(e^{(d*x + c)} + e^{(-d*x - c)} + 2)/d - \frac{3}{512}(21*a^3 + 80*a^2*b + 128*a*b^2)*\log(e^{(d*x + c)} + e^{(-d*x - c)} - 2)/d - \frac{1}{640}(315*a^3*(e^{(d*x + c)} + e^{(-d*x - c)})^9 + 1200*a^2*b*(e^{(d*x + c)} + e^{(-d*x - c)})^9 + 1920*a*b^2*(e^{(d*x + c)} + e^{(-d*x - c)})^9 - 5880*a^3*(e^{(d*x + c)} + e^{(-d*x - c)})^7 -$

$$\begin{aligned}
& 22400*a^2*b*(e^{(d*x + c)} + e^{(-d*x - c)})^7 - 30720*a*b^2*(e^{(d*x + c)} + e^{(-d*x - c)})^7 + 43008*a^3*(e^{(d*x + c)} + e^{(-d*x - c)})^5 + 163840*a^2*b*(e^{(d*x + c)} + e^{(-d*x - c)})^5 + 184320*a*b^2*(e^{(d*x + c)} + e^{(-d*x - c)})^5 - \\
& 151680*a^3*(e^{(d*x + c)} + e^{(-d*x - c)})^3 - 542720*a^2*b*(e^{(d*x + c)} + e^{(-d*x - c)})^3 - 491520*a*b^2*(e^{(d*x + c)} + e^{(-d*x - c)})^3 + 247040*a^3*(e^{(d*x + c)} + e^{(-d*x - c)}) + 675840*a^2*b*(e^{(d*x + c)} + e^{(-d*x - c)}) + 491 \\
& 520*a*b^2*(e^{(d*x + c)} + e^{(-d*x - c)})/(((e^{(d*x + c)} + e^{(-d*x - c)})^2 - 4)^5*d)
\end{aligned}$$

3.216 $\int \operatorname{csch}^{13}(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=220

$$\frac{(840a^2b + 231a^3 + 1152ab^2 + 1024b^3) \tanh^{-1}(\cosh(c + dx))}{1024d} - \frac{a(77a^2 + 280ab + 384b^2) \coth(c + dx) \operatorname{csch}^3(c + dx)}{512d}$$

[Out] $-\left(\left(231a^3 + 840a^2b + 1152ab^2 + 1024b^3\right) \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]]\right) / (1024d) + (3a(77a^2 + 280ab + 384b^2) \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]) / (1024d) - (a(77a^2 + 280ab + 384b^2) \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^3) / (512d) + (7a^2(11a + 40b) \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^5) / (640d) - (3a^2(11a + 40b) \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^7) / (320d) + (11a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^9) / (120d) - (a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^{11}) / (12d)$

Rubi [A] time = 0.397589, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1157, 1814, 385, 206}

$$\frac{(840a^2b + 231a^3 + 1152ab^2 + 1024b^3) \tanh^{-1}(\cosh(c + dx))}{1024d} - \frac{a(77a^2 + 280ab + 384b^2) \coth(c + dx) \operatorname{csch}^3(c + dx)}{512d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + dx]^{13} (a + b \operatorname{Sinh}[c + dx]^4)^3, x]$

[Out] $-\left(\left(231a^3 + 840a^2b + 1152ab^2 + 1024b^3\right) \operatorname{ArcTanh}[\operatorname{Cosh}[c + dx]]\right) / (1024d) + (3a(77a^2 + 280ab + 384b^2) \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]) / (1024d) - (a(77a^2 + 280ab + 384b^2) \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^3) / (512d) + (7a^2(11a + 40b) \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^5) / (640d) - (3a^2(11a + 40b) \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^7) / (320d) + (11a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^9) / (120d) - (a^3 \operatorname{Coth}[c + dx] \operatorname{Csch}[c + dx]^{11}) / (12d)$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)} ((a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + fx], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2x^2)^{(m-1)/2} (a + b - 2bff^2x^2 + bff^4x^4)^p, x], x, \operatorname{Cos}[e + fx]/ff], x]] /; \operatorname{FreeQ}\{a, b, e, f, p, x\} \ \&\amp; \operatorname{IntegerQ}[(m-1)/2]$

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2,
x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x,
0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^{13}(c+dx) (a+b \sinh^4(c+dx))^3 dx &= -\frac{\operatorname{Subst}\left(\int \frac{(a+b-2bx^2+bx^4)^3}{(1-x^2)^7} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a^3 \coth(c+dx) \operatorname{csch}^{11}(c+dx)}{12d} + \frac{\operatorname{Subst}\left(\int \frac{-11a^3-36a^2b-36ab^2-12b^3+12b(3a^2}{\dots} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{11a^3 \coth(c+dx) \operatorname{csch}^9(c+dx)}{120d} - \frac{a^3 \coth(c+dx) \operatorname{csch}^{11}(c+dx)}{12d} - \frac{\dots}{d} \\
&= -\frac{3a^2(11a+40b) \coth(c+dx) \operatorname{csch}^7(c+dx)}{320d} + \frac{11a^3 \coth(c+dx) \operatorname{csch}^9(c+dx)}{120d} \\
&= \frac{7a^2(11a+40b) \coth(c+dx) \operatorname{csch}^5(c+dx)}{640d} - \frac{3a^2(11a+40b) \coth(c+dx) \operatorname{csch}^7(c+dx)}{320d} \\
&= -\frac{a(77a^2+280ab+384b^2) \coth(c+dx) \operatorname{csch}^3(c+dx)}{512d} + \frac{7a^2(11a+40b) \coth(c+dx) \operatorname{csch}^5(c+dx)}{320d} \\
&= \frac{3a(77a^2+280ab+384b^2) \coth(c+dx) \operatorname{csch}(c+dx)}{1024d} - \frac{a(77a^2+280ab+384b^2) \coth(c+dx) \operatorname{csch}^3(c+dx)}{512d} \\
&= -\frac{(231a^3+840a^2b+1152ab^2+1024b^3) \tanh^{-1}(\cosh(c+dx))}{1024d} + \frac{3a(77a^2+280ab+384b^2) \coth(c+dx) \operatorname{csch}(c+dx)}{1024d}
\end{aligned}$$

Mathematica [A] time = 2.06945, size = 246, normalized size = 1.12

$$15360(840a^2b + 231a^3 + 1152ab^2 + 1024b^3) \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + 2a(750629a^2 + 2074200ab + 1422720b^2) \cosh(c+dx)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^13*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (-30*a*(76555*a^2 + 75816*a*b + 45696*b^2)*Coth[c + d*x]*Csch[c + d*x]^11 + 2*a*(750629*a^2 + 2074200*a*b + 1422720*b^2)*Cosh[3*(c + d*x)]*Csch[c + d*x]^12 - 9*a*(77099*a^2 + 280360*a*b + 246400*b^2)*Cosh[5*(c + d*x)]*Csch[c + d*x]^12 + 63*a*(3421*a^2 + 12440*a*b + 14720*b^2)*Cosh[7*(c + d*x)]*Csch[c + d*x]^12 - 525*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[9*(c + d*x)]*Csch[c + d*x]^12 + 45*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[11*(c + d*x)]*Csch[c + d*x]^12

$x]^{12} + 15360*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*\text{Log}[\text{Tanh}[(c + d*x)/2]]/(15728640*d)$

Maple [A] time = 0.085, size = 202, normalized size = 0.9

$$\frac{1}{d} \left(a^3 \left(-\frac{(\text{csch}(dx+c))^{11}}{12} + \frac{11(\text{csch}(dx+c))^9}{120} - \frac{33(\text{csch}(dx+c))^7}{320} + \frac{77(\text{csch}(dx+c))^5}{640} - \frac{77(\text{csch}(dx+c))^3}{512} + \frac{231}{1024} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] $\frac{1}{d} * (a^3 * ((-1/12 * \text{csch}(d*x+c)^{11} + 11/120 * \text{csch}(d*x+c)^9 - 33/320 * \text{csch}(d*x+c)^7 + 77/640 * \text{csch}(d*x+c)^5 - 77/512 * \text{csch}(d*x+c)^3 + 231/1024 * \text{csch}(d*x+c)) * \text{coth}(d*x+c) - 231/512 * \text{arctanh}(\exp(d*x+c))) + 3*a^2*b * ((-1/8 * \text{csch}(d*x+c)^7 + 7/48 * \text{csch}(d*x+c)^5 - 35/192 * \text{csch}(d*x+c)^3 + 35/128 * \text{csch}(d*x+c)) * \text{coth}(d*x+c) - 35/64 * \text{arctanh}(\exp(d*x+c))) + 3*a*b^2 * ((-1/4 * \text{csch}(d*x+c)^3 + 3/8 * \text{csch}(d*x+c)) * \text{coth}(d*x+c) - 3/4 * \text{arctanh}(\exp(d*x+c))) - 2*b^3 * \text{arctanh}(\exp(d*x+c)))$

Maxima [B] time = 1.15229, size = 972, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] $-1/15360*a^3*(3465*\log(e^{-d*x-c}) + 1)/d - 3465*\log(e^{-d*x-c} - 1)/d + 2*(3465*e^{-d*x-c} - 40425*e^{-3*d*x-3*c} + 215523*e^{-5*d*x-5*c} - 693891*e^{-7*d*x-7*c} + 1501258*e^{-9*d*x-9*c} - 2296650*e^{-11*d*x-11*c} - 2296650*e^{-13*d*x-13*c} + 1501258*e^{-15*d*x-15*c} - 693891*e^{-17*d*x-17*c} + 215523*e^{-19*d*x-19*c} - 40425*e^{-21*d*x-21*c} + 3465*e^{-23*d*x-23*c})/(d*(12*e^{-2*d*x-2*c} - 66*e^{-4*d*x-4*c} + 220*e^{-6*d*x-6*c} - 495*e^{-8*d*x-8*c} + 792*e^{-10*d*x-10*c} - 924*e^{-12*d*x-12*c} + 792*e^{-14*d*x-14*c} - 495*e^{-16*d*x-16*c} + 220*e^{-18*d*x-18*c} - 66*e^{-20*d*x-20*c} + 12*e^{-22*d*x-22*c} - e^{-24*d*x-24*c} - 1))) - 1/128*a^2*b*(105*\log(e^{-d*x-c}) + 1)/d - 105*\log(e^{-d*x-c} - 1)/d + 2*(105*e^{-d*x-c} - 805*e^{-3*d*x-3*c} + 2681*e^{-5*d*x-5*c} - 5053*e^{-7*d*x-7*c} - 5053*e^{-9*d*x-9*c} + 2681*e^{-11*d*x-11*c} - 2681*e^{-13*d*x-13*c} + 1053*e^{-15*d*x-15*c} - 1053*e^{-17*d*x-17*c} + 2681*e^{-19*d*x-19*c} - 1053*e^{-21*d*x-21*c} + 1053*e^{-23*d*x-23*c})/(d*(12*e^{-2*d*x-2*c} - 66*e^{-4*d*x-4*c} + 220*e^{-6*d*x-6*c} - 495*e^{-8*d*x-8*c} + 792*e^{-10*d*x-10*c} - 924*e^{-12*d*x-12*c} + 792*e^{-14*d*x-14*c} - 495*e^{-16*d*x-16*c} + 220*e^{-18*d*x-18*c} - 66*e^{-20*d*x-20*c} + 12*e^{-22*d*x-22*c} - e^{-24*d*x-24*c} - 1))) - 2*b^2*(105*\log(e^{-d*x-c}) + 1)/d - 105*\log(e^{-d*x-c} - 1)/d + 2*(105*e^{-d*x-c} - 805*e^{-3*d*x-3*c} + 2681*e^{-5*d*x-5*c} - 5053*e^{-7*d*x-7*c} - 5053*e^{-9*d*x-9*c} + 2681*e^{-11*d*x-11*c} - 2681*e^{-13*d*x-13*c} + 1053*e^{-15*d*x-15*c} - 1053*e^{-17*d*x-17*c} + 2681*e^{-19*d*x-19*c} - 1053*e^{-21*d*x-21*c} + 1053*e^{-23*d*x-23*c})/(d*(12*e^{-2*d*x-2*c} - 66*e^{-4*d*x-4*c} + 220*e^{-6*d*x-6*c} - 495*e^{-8*d*x-8*c} + 792*e^{-10*d*x-10*c} - 924*e^{-12*d*x-12*c} + 792*e^{-14*d*x-14*c} - 495*e^{-16*d*x-16*c} + 220*e^{-18*d*x-18*c} - 66*e^{-20*d*x-20*c} + 12*e^{-22*d*x-22*c} - e^{-24*d*x-24*c} - 1))) - 2*b^3*(105*\log(e^{-d*x-c}) + 1)/d - 105*\log(e^{-d*x-c} - 1)/d + 2*(105*e^{-d*x-c} - 805*e^{-3*d*x-3*c} + 2681*e^{-5*d*x-5*c} - 5053*e^{-7*d*x-7*c} - 5053*e^{-9*d*x-9*c} + 2681*e^{-11*d*x-11*c} - 2681*e^{-13*d*x-13*c} + 1053*e^{-15*d*x-15*c} - 1053*e^{-17*d*x-17*c} + 2681*e^{-19*d*x-19*c} - 1053*e^{-21*d*x-21*c} + 1053*e^{-23*d*x-23*c})/(d*(12*e^{-2*d*x-2*c} - 66*e^{-4*d*x-4*c} + 220*e^{-6*d*x-6*c} - 495*e^{-8*d*x-8*c} + 792*e^{-10*d*x-10*c} - 924*e^{-12*d*x-12*c} + 792*e^{-14*d*x-14*c} - 495*e^{-16*d*x-16*c} + 220*e^{-18*d*x-18*c} - 66*e^{-20*d*x-20*c} + 12*e^{-22*d*x-22*c} - e^{-24*d*x-24*c} - 1)))$

$$\begin{aligned}
& 11*c) - 805*e^{(-13*d*x - 13*c)} + 105*e^{(-15*d*x - 15*c)})/(d*(8*e^{(-2*d*x - 2*c)} - 28*e^{(-4*d*x - 4*c)} + 56*e^{(-6*d*x - 6*c)} - 70*e^{(-8*d*x - 8*c)} + 56*e^{(-10*d*x - 10*c)} - 28*e^{(-12*d*x - 12*c)} + 8*e^{(-14*d*x - 14*c)} - e^{(-16*d*x - 16*c)} - 1))) - 3/8*a*b^2*(3*\log(e^{(-d*x - c)} + 1)/d - 3*\log(e^{(-d*x - c)} - 1)/d + 2*(3*e^{(-d*x - c)} - 11*e^{(-3*d*x - 3*c)} - 11*e^{(-5*d*x - 5*c)} + 3*e^{(-7*d*x - 7*c)}))/(d*(4*e^{(-2*d*x - 2*c)} - 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} - e^{(-8*d*x - 8*c)} - 1))) - b^3*(\log(e^{(-d*x - c)} + 1)/d - \log(e^{(-d*x - c)} - 1)/d)
\end{aligned}$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**13*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.65125, size = 730, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

```
[Out] -1/2048*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d + 1/2048*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d + 1/7680*(3465*a^3*(e^(d*x + c) + e^(-d*x - c))^11 + 12600*a^2*b*(e^(d*x + c) + e^(-d*x - c))^11 + 17280*a*b^2*(e^(d*x + c) + e^(-d*x - c))^11 - 78540*a^3*(e^(d*x + c) + e^(-d*x - c))^9 - 285600*a^2*b*(e^(d*x + c) + e^(-d*x - c))^9 - 391680*a*b^2*(e^(d*x + c) + e^(-d*x - c))^9 + 731808*a^3*(e^(d*x + c) + e^(-d*x - c))^7 + 2661120*a^2*b*(e^(d*x + c) + e^(-d*x - c))^7 + 3502080*a*b^2*(e^(d*x + c) + e^(-d*x - c))^7 - 3560832*a^3*(e^(d*x + c) + e^(-d*x - c))^5 - 12948480*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5 - 15482880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 9391360*a^3*(e^(d*x + c) + e^(-d*x - c))^3 + 32839680*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 + 33914880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12180480*a^3*(e^(d*x + c) + e^(-d*x - c)) - 34283520*a^2*b*(e^(d*x + c) + e^(-d*x - c)) - 29491200*a*b^2*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)^6*d)
```

3.217 $\int \sinh^2(c + dx) (a + b \sinh^4(c + dx))^3 dx$

Optimal. Leaf size=255

$$\frac{b(1920a^2 + 12312ab + 10579b^2) \sinh(c + dx) \cosh^5(c + dx)}{3840d} - \frac{b(4992a^2 + 10728ab + 5549b^2) \sinh(c + dx) \cosh^3(c + dx)}{3072d}$$

```
[Out] -((1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*x)/2048 + ((1024*a^3 + 422
4*a^2*b + 4632*a*b^2 + 1619*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(2048*d) - (b
*(4992*a^2 + 10728*a*b + 5549*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(3072*d)
+ (b*(1920*a^2 + 12312*a*b + 10579*b^2)*Cosh[c + d*x]^5*Sinh[c + d*x])/(384
0*d) - (b^2*(6888*a + 11821*b)*Cosh[c + d*x]^7*Sinh[c + d*x])/(4480*d) + (b
^2*(504*a + 2593*b)*Cosh[c + d*x]^9*Sinh[c + d*x])/(1680*d) - (85*b^3*Cosh[
c + d*x]^11*Sinh[c + d*x])/(168*d) + (b^3*Cosh[c + d*x]^13*Sinh[c + d*x])/(
14*d)
```

Rubi [A] time = 0.557087, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3217, 1257, 1814, 1157, 385, 206}

$$\frac{b(1920a^2 + 12312ab + 10579b^2) \sinh(c + dx) \cosh^5(c + dx)}{3840d} - \frac{b(4992a^2 + 10728ab + 5549b^2) \sinh(c + dx) \cosh^3(c + dx)}{3072d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*x)/2048 + ((1024*a^3 + 422
4*a^2*b + 4632*a*b^2 + 1619*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(2048*d) - (b
*(4992*a^2 + 10728*a*b + 5549*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(3072*d)
+ (b*(1920*a^2 + 12312*a*b + 10579*b^2)*Cosh[c + d*x]^5*Sinh[c + d*x])/(384
0*d) - (b^2*(6888*a + 11821*b)*Cosh[c + d*x]^7*Sinh[c + d*x])/(4480*d) + (b
^2*(504*a + 2593*b)*Cosh[c + d*x]^9*Sinh[c + d*x])/(1680*d) - (85*b^3*Cosh[
c + d*x]^11*Sinh[c + d*x])/(168*d) + (b^3*Cosh[c + d*x]^13*Sinh[c + d*x])/(
14*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)
```

$^{(m/2 + 2*p + 1)}, x], x, \text{Tan}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, e, f\}, x] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[p]$

Rule 1257

$\text{Int}[(x_)^{(m_)}*((d_)+(e_)*(x_)^2)^{(q_)}*((a_)+(b_)*(x_)^2+(c_)*(x_)^4)^{(p_)}, x_Symbol] :> \text{Simp}[((-d)^{(m/2-1)}*(c*d^2-b*d*e+a*e^2)^p*x*(d+e*x^2)^{(q+1)})/(2*e^{(2*p+m/2)}*(q+1)), x] + \text{Dist}[1/(2*e^{(2*p+m/2)}*(q+1)), \text{Int}[(d+e*x^2)^{(q+1)}*\text{ExpandToSum}[\text{Together}[(1*(2*e^{(2*p+m/2)}*(q+1)*x^m*(a+b*x^2+c*x^4))^p - (-d)^{(m/2-1)}*(c*d^2-b*d*e+a*e^2)^p*(d+e*(2*q+3)*x^2))]/(d+e*x^2)], x], x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2-4*a*c, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{ILtQ}[q, -1] \&\& \text{IGtQ}[m/2, 0]$

Rule 1814

$\text{Int}[(Pq_)*((a_)+(b_)*(x_)^2)^{(p_)}, x_Symbol] :> \text{With}[\{Q = \text{PolynomialQuotient}[Pq, a + b*x^2, x], f = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + b*x^2, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[Pq, a + b*x^2, x], x, 1]\}, \text{Simp}[(a*g - b*f*x)*(a + b*x^2)^{(p+1)})/(2*a*b*(p+1)), x] + \text{Dist}[1/(2*a*(p+1)), \text{Int}[(a + b*x^2)^{(p+1)}*\text{ExpandToSum}[2*a*(p+1)*Q + f*(2*p+3), x], x], x]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{PolyQ}[Pq, x] \&\& \text{LtQ}[p, -1]$

Rule 1157

$\text{Int}[(d_)+(e_)*(x_)^2)^{(q_)}*((a_)+(b_)*(x_)^2+(c_)*(x_)^4)^{(p_)}, x_Symbol] :> \text{With}[\{Qx = \text{PolynomialQuotient}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = \text{Coeff}[\text{PolynomialRemainder}[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]\}, -\text{Simp}[(R*x*(d + e*x^2)^{(q+1)})/(2*d*(q+1)), x] + \text{Dist}[1/(2*d*(q+1)), \text{Int}[(d + e*x^2)^{(q+1)}*\text{ExpandToSum}[2*d*(q+1)*Qx + R*(2*q+3), x], x], x]] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b^2-4*a*c, 0] \&\& \text{NeQ}[c*d^2-b*d*e+a*e^2, 0] \&\& \text{IGtQ}[p, 0] \&\& \text{LtQ}[q, -1]$

Rule 385

$\text{Int}[(a_)+(b_)*(x_)^{(n_)})^{(p_)}*((c_)+(d_)*(x_)^{(n_)}), x_Symbol] :> -\text{Simp}[(b*c - a*d)*x*(a + b*x^n)^{(p+1)})/(a*b*n*(p+1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(a*b*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}, x], x] /; \text{FreeQ}[\{a, b, c, d, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{LtQ}[p, -1] || \text{ILtQ}[1/n + p, 0])$

Rule 206

$\text{Int}[(a_)+(b_)*(x_)^2)^{(-1)}, x_Symbol] :> \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{Gt}$

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \sinh^2(c + dx) (a + b \sinh^4(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{x^2(a-2ax^2+(a+b)x^4)^3}{(1-x^2)^8} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{b^3 \cosh^{13}(c + dx) \sinh(c + dx)}{14d} + \frac{\text{Subst}\left(\int \frac{-b^3+14(a^3-b^3)x^2-14(5a^3+b^3)x^4+14(1}{\dots} dx, x, \tanh(c + dx)\right)}{d} \\
 &= -\frac{85b^3 \cosh^{11}(c + dx) \sinh(c + dx)}{168d} + \frac{b^3 \cosh^{13}(c + dx) \sinh(c + dx)}{14d} - \dots \\
 &= \frac{b^2(504a + 2593b) \cosh^9(c + dx) \sinh(c + dx)}{1680d} - \frac{85b^3 \cosh^{11}(c + dx) \sinh(c + dx)}{168d} \\
 &= -\frac{b^2(6888a + 11821b) \cosh^7(c + dx) \sinh(c + dx)}{4480d} + \frac{b^2(504a + 2593b) \cosh^9(c + dx) \sinh(c + dx)}{1680d} \\
 &= \frac{b(1920a^2 + 12312ab + 10579b^2) \cosh^5(c + dx) \sinh(c + dx)}{3840d} - \frac{b^2(6888a + 11821b) \cosh^7(c + dx) \sinh(c + dx)}{4480d} \\
 &= -\frac{b(4992a^2 + 10728ab + 5549b^2) \cosh^3(c + dx) \sinh(c + dx)}{3072d} + \frac{b(1920a^2 + 12312ab + 10579b^2) \cosh^5(c + dx) \sinh(c + dx)}{3840d} \\
 &= \frac{(1024a^3 + 4224a^2b + 4632ab^2 + 1619b^3) \cosh(c + dx) \sinh(c + dx)}{2048d} - \frac{b(4992a^2 + 10728ab + 5549b^2) \cosh^3(c + dx) \sinh(c + dx)}{3072d} \\
 &= -\frac{(1024a^3 + 1920a^2b + 1512ab^2 + 429b^3)x}{2048} + \frac{(1024a^3 + 4224a^2b + 4632ab^2 + 1619b^3) \cosh(c + dx) \sinh(c + dx)}{2048d}
 \end{aligned}$$

Mathematica [A] time = 0.680134, size = 189, normalized size = 0.74

$$\frac{-840(1920a^2b + 1024a^3 + 1512ab^2 + 429b^3)(c + dx) - 105b(2304a^2 + 2880ab + 1001b^2) \sinh(4(c + dx)) + 35b(768a^3 + 4224a^2b + 4632ab^2 + 1619b^3) \cosh(4(c + dx)) \sinh(4(c + dx))}{2048d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] $(-840*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*(c + d*x) + 105*(4096*a^3 + 11520*a^2*b + 10080*a*b^2 + 3003*b^3)*\text{Sinh}[2*(c + d*x)] - 105*b*(2304*a^2 + 2880*a*b + 1001*b^2)*\text{Sinh}[4*(c + d*x)] + 35*b*(768*a^2 + 2160*a*b + 1001*b^2)*\text{Sinh}[6*(c + d*x)] - 105*b^2*(120*a + 91*b)*\text{Sinh}[8*(c + d*x)] + 21*b^2*(48*a + 91*b)*\text{Sinh}[10*(c + d*x)] - 245*b^3*\text{Sinh}[12*(c + d*x)] + 15*b^3*\text{Sinh}[14*(c + d*x)])/(1720320*d)$

Maple [A] time = 0.054, size = 240, normalized size = 0.9

$$\frac{1}{d} \left(b^3 \left(\frac{(\sinh(dx+c))^{13}}{14} - \frac{13(\sinh(dx+c))^{11}}{168} + \frac{143(\sinh(dx+c))^9}{1680} - \frac{429(\sinh(dx+c))^7}{4480} + \frac{143(\sinh(dx+c))^5}{1280} - \frac{143(\sinh(dx+c))^3}{1024} + \frac{429}{2048} \right) \cosh(dx+c) - \frac{429}{2048} dx - \frac{429}{2048} c + 3ab^2 \left(\frac{(\sinh(dx+c))^9}{10} - \frac{9}{80} \sinh(dx+c)^7 + \frac{21}{160} \sinh(dx+c)^5 - \frac{21}{128} \sinh(dx+c)^3 + \frac{63}{256} \sinh(dx+c) \right) \cosh(dx+c) - \frac{63}{256} dx - \frac{63}{256} c + 3a^2b \left(\frac{(\sinh(dx+c))^5}{6} - \frac{5}{24} \sinh(dx+c)^3 + \frac{5}{16} \sinh(dx+c) \right) \cosh(dx+c) - \frac{5}{16} dx - \frac{5}{16} c + a^3 \left(\frac{1}{2} \cosh(dx+c) \sinh(dx+c) - \frac{1}{2} dx - \frac{1}{2} c \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] $1/d*(b^3*((1/14*\sinh(d*x+c)^13-13/168*\sinh(d*x+c)^11+143/1680*\sinh(d*x+c)^9-429/4480*\sinh(d*x+c)^7+143/1280*\sinh(d*x+c)^5-143/1024*\sinh(d*x+c)^3+429/2048*\sinh(d*x+c))*\cosh(d*x+c)-429/2048*d*x-429/2048*c)+3*a*b^2*((1/10*\sinh(d*x+c)^9-9/80*\sinh(d*x+c)^7+21/160*\sinh(d*x+c)^5-21/128*\sinh(d*x+c)^3+63/256*\sinh(d*x+c))*\cosh(d*x+c)-63/256*d*x-63/256*c)+3*a^2*b*((1/6*\sinh(d*x+c)^5-5/24*\sinh(d*x+c)^3+5/16*\sinh(d*x+c))*\cosh(d*x+c)-5/16*d*x-5/16*c)+a^3*(1/2*\cosh(d*x+c)*\sinh(d*x+c)-1/2*d*x-1/2*c))$

Maxima [A] time = 1.21768, size = 597, normalized size = 2.34

$$-\frac{1}{8} a^3 \left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d} \right) - \frac{1}{3440640} b^3 \left(\frac{(245 e^{(-2dx-2c)} - 1911 e^{(-4dx-4c)} + 9555 e^{(-6dx-6c)} - 35035 e^{(-8dx-8c)} + 105105 e^{(-10dx-10c)} - 315315 e^{(-12dx-12c)} - 15) e^{(14dx+14c)}}{d} + \frac{720720(dx+c)}{d} + (315315 e^{(-2dx-2c)} - 105105 e^{(-4dx-4c)} + 35035 e^{(-6dx-6c)} - 9555 e^{(-8dx-8c)} + 1911 e^{(-10dx-10c)} - 245 e^{(-12dx-12c)} + 15) e^{(-14dx-14c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] $-1/8*a^3*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/3440640*b^3*((245*e^{(-2*d*x - 2*c)} - 1911*e^{(-4*d*x - 4*c)} + 9555*e^{(-6*d*x - 6*c)} - 35035*e^{(-8*d*x - 8*c)} + 105105*e^{(-10*d*x - 10*c)} - 315315*e^{(-12*d*x - 12*c)} - 15)*e^{(14*d*x + 14*c)}/d + 720720*(d*x + c)/d + (315315*e^{(-2*d*x - 2*c)} - 105105*e^{(-4*d*x - 4*c)} + 35035*e^{(-6*d*x - 6*c)} - 9555*e^{(-8*d*x - 8*c)} + 1911*e^{(-10*d*x - 10*c)} - 245*e^{(-12*d*x - 12*c)} + 15)*e^{(-14*d*x - 14*c)}/d)$

$$\begin{aligned} & - 3/20480*a*b^2*((25*e^{(-2*d*x - 2*c)} - 150*e^{(-4*d*x - 4*c)} + 600*e^{(-6*d*x - 6*c)} - 2100*e^{(-8*d*x - 8*c)} - 2)*e^{(10*d*x + 10*c)}/d + 5040*(d*x + c)/d + (2100*e^{(-2*d*x - 2*c)} - 600*e^{(-4*d*x - 4*c)} + 150*e^{(-6*d*x - 6*c)} - 25*e^{(-8*d*x - 8*c)} + 2*e^{(-10*d*x - 10*c)})/d) - 1/128*a^2*b*((9*e^{(-2*d*x - 2*c)} - 45*e^{(-4*d*x - 4*c)} - 1)*e^{(6*d*x + 6*c)}/d + 120*(d*x + c)/d + (45*e^{(-2*d*x - 2*c)} - 9*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})/d) \end{aligned}$$

Fricas [B] time = 1.40475, size = 1688, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/860160*(105*b^3*cosh(d*x + c)*sinh(d*x + c)^{13} + 210*(13*b^3*cosh(d*x + c)^3 - 7*b^3*cosh(d*x + c))*sinh(d*x + c)^{11} + 35*(429*b^3*cosh(d*x + c)^5 - 770*b^3*cosh(d*x + c)^3 + 3*(48*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 60*(429*b^3*cosh(d*x + c)^7 - 1617*b^3*cosh(d*x + c)^5 + 21*(48*a*b^2 + 91*b^3)*cosh(d*x + c)^3 - 7*(120*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 21*(715*b^3*cosh(d*x + c)^9 - 4620*b^3*cosh(d*x + c)^7 + 126*(48*a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 140*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^3 + 5*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 70*(39*b^3*cosh(d*x + c)^{11} - 385*b^3*cosh(d*x + c)^9 + 18*(48*a*b^2 + 91*b^3)*cosh(d*x + c)^7 - 42*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^5 + 5*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 - 3*(2304*a^2*b + 2880*a*b^2 + 1001*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 420*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*d*x + 105*(b^3*cosh(d*x + c)^{13} - 14*b^3*cosh(d*x + c)^{11} + (48*a*b^2 + 91*b^3)*cosh(d*x + c)^9 - 4*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^7 + (768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 - 2*(2304*a^2*b + 2880*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 + (4096*a^3 + 11520*a^2*b + 10080*a*b^2 + 3003*b^3)*cosh(d*x + c))*sinh(d*x + c))/d \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.70269, size = 756, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/3440640*(15*b^3*e^{(14*d*x + 14*c)} - 245*b^3*e^{(12*d*x + 12*c)} + 1008*a*b^2*e^{(10*d*x + 10*c)} + 1911*b^3*e^{(10*d*x + 10*c)} - 12600*a*b^2*e^{(8*d*x + 8*c)} - 9555*b^3*e^{(8*d*x + 8*c)} + 26880*a^2*b*e^{(6*d*x + 6*c)} + 75600*a*b^2*e^{(6*d*x + 6*c)} + 35035*b^3*e^{(6*d*x + 6*c)} - 241920*a^2*b*e^{(4*d*x + 4*c)} - 302400*a*b^2*e^{(4*d*x + 4*c)} - 105105*b^3*e^{(4*d*x + 4*c)} + 430080*a^3*e^{(2*d*x + 2*c)} + 1209600*a^2*b*e^{(2*d*x + 2*c)} + 1058400*a*b^2*e^{(2*d*x + 2*c)} + 315315*b^3*e^{(2*d*x + 2*c)} - 1680*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*(d*x + c) + (2230272*a^3*e^{(14*d*x + 14*c)} + 4181760*a^2*b*e^{(14*d*x + 14*c)} + 3293136*a*b^2*e^{(14*d*x + 14*c)} + 934362*b^3*e^{(14*d*x + 14*c)} - 430080*a^3*e^{(12*d*x + 12*c)} - 1209600*a^2*b*e^{(12*d*x + 12*c)} - 1058400*a*b^2*e^{(12*d*x + 12*c)} - 315315*b^3*e^{(12*d*x + 12*c)} + 241920*a^2*b*e^{(10*d*x + 10*c)} + 302400*a*b^2*e^{(10*d*x + 10*c)} + 105105*b^3*e^{(10*d*x + 10*c)} - 26880*a^2*b*e^{(8*d*x + 8*c)} - 75600*a*b^2*e^{(8*d*x + 8*c)} - 35035*b^3*e^{(8*d*x + 8*c)} + 12600*a*b^2*e^{(6*d*x + 6*c)} + 9555*b^3*e^{(6*d*x + 6*c)} - 1008*a*b^2*e^{(4*d*x + 4*c)} - 1911*b^3*e^{(4*d*x + 4*c)} + 245*b^3*e^{(2*d*x + 2*c)} - 15*b^3)*e^{(-14*d*x - 14*c)})/d \end{aligned}$$

$$3.218 \quad \int (a + b \sinh^4(c + dx))^3 dx$$

Optimal. Leaf size=211

$$\frac{b(1152a^2 + 3912ab + 2279b^2) \sinh(c + dx) \cosh^3(c + dx)}{1536d} - \frac{b(1920a^2 + 2232ab + 793b^2) \sinh(c + dx) \cosh(c + dx)}{1024d} +$$

```
[Out] ((1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*x)/1024 - (b*(1920*a^2 + 223
2*a*b + 793*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(1024*d) + (b*(1152*a^2 + 391
2*a*b + 2279*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(1536*d) - (b^2*(3000*a +
3481*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(1920*d) + (3*b^2*(40*a + 139*b)*Cos
h[c + d*x]^7*Sinh[c + d*x])/(320*d) - (61*b^3*Cosh[c + d*x]^9*Sinh[c + d*x]
)/(120*d) + (b^3*Cosh[c + d*x]^11*Sinh[c + d*x])/(12*d)
```

Rubi [A] time = 0.386844, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3209, 1157, 1814, 385, 206}

$$\frac{b(1152a^2 + 3912ab + 2279b^2) \sinh(c + dx) \cosh^3(c + dx)}{1536d} - \frac{b(1920a^2 + 2232ab + 793b^2) \sinh(c + dx) \cosh(c + dx)}{1024d} +$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*x)/1024 - (b*(1920*a^2 + 223
2*a*b + 793*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(1024*d) + (b*(1152*a^2 + 391
2*a*b + 2279*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(1536*d) - (b^2*(3000*a +
3481*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(1920*d) + (3*b^2*(40*a + 139*b)*Cos
h[c + d*x]^7*Sinh[c + d*x])/(320*d) - (61*b^3*Cosh[c + d*x]^9*Sinh[c + d*x]
)/(120*d) + (b^3*Cosh[c + d*x]^11*Sinh[c + d*x])/(12*d)
```

Rule 3209

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^4)^(p_), x_Symbol] := With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
+ b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x] /;
FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2,
x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x,
0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^4(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^3}{(1-x^2)^7} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b^3 \cosh^{11}(c + dx) \sinh(c + dx)}{12d} - \frac{\text{Subst}\left(\int \frac{-12a^3+b^3+12(5a^3+b^3)x^2-12(10a^3+3a^2b-b^3)x^4+12(10a^3+3a^2b-b^3)x^6}{(1-x^2)^6} dx, x, \tanh(c + dx)\right)}{d} \\
&= -\frac{61b^3 \cosh^9(c + dx) \sinh(c + dx)}{120d} + \frac{b^3 \cosh^{11}(c + dx) \sinh(c + dx)}{12d} + \frac{\text{Subst}\left(\int \frac{3(40a^3+12a^2b-b^3)x^8-12(10a^3+3a^2b-b^3)x^{10}+12(10a^3+3a^2b-b^3)x^{12}}{(1-x^2)^5} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{3b^2(40a + 139b) \cosh^7(c + dx) \sinh(c + dx)}{320d} - \frac{61b^3 \cosh^9(c + dx) \sinh(c + dx)}{120d} + \frac{b^3 \cosh^{11}(c + dx) \sinh(c + dx)}{12d} \\
&= -\frac{b^2(3000a + 3481b) \cosh^5(c + dx) \sinh(c + dx)}{1920d} + \frac{3b^2(40a + 139b) \cosh^7(c + dx) \sinh(c + dx)}{320d} \\
&= \frac{b(1152a^2 + 3912ab + 2279b^2) \cosh^3(c + dx) \sinh(c + dx)}{1536d} - \frac{b^2(3000a + 3481b) \cosh^5(c + dx) \sinh(c + dx)}{1920d} \\
&= -\frac{b(1920a^2 + 2232ab + 793b^2) \cosh(c + dx) \sinh(c + dx)}{1024d} + \frac{b(1152a^2 + 3912ab + 2279b^2) \cosh^3(c + dx) \sinh(c + dx)}{1536d} \\
&= \frac{(1024a^3 + 1152a^2b + 840ab^2 + 231b^3)x}{1024} - \frac{b(1920a^2 + 2232ab + 793b^2) \cosh(c + dx) \sinh(c + dx)}{1024d}
\end{aligned}$$

Mathematica [A] time = 0.389919, size = 156, normalized size = 0.74

$$\frac{120(1152a^2b + 1024a^3 + 840ab^2 + 231b^3)(c + dx) - 720b(128a^2 + 112ab + 33b^2)\sinh(2(c + dx)) + 45b(256a^2 + 448ab + 165b^2)\sinh(4(c + dx)) - 40b^2(96a + 55b)\sinh(6(c + dx)) + 45b^2(8a + 11b)\sinh(8(c + dx)) - 72b^3\sinh(10(c + dx)) + 5b^3\sinh(12(c + dx))}{122880d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (120*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*(c + d*x) - 720*b*(128*a^2 + 112*a*b + 33*b^2)*Sinh[2*(c + d*x)] + 45*b*(256*a^2 + 448*a*b + 165*b^2)*Sinh[4*(c + d*x)] - 40*b^2*(96*a + 55*b)*Sinh[6*(c + d*x)] + 45*b^2*(8*a + 11*b)*Sinh[8*(c + d*x)] - 72*b^3*Sinh[10*(c + d*x)] + 5*b^3*Sinh[12*(c + d*x)])/(122880*d)

Maple [A] time = 0.056, size = 193, normalized size = 0.9

$$\frac{1}{d} \left(b^3 \left(\left(\frac{(\sinh(dx+c))^{11}}{12} - \frac{11(\sinh(dx+c))^9}{120} + \frac{33(\sinh(dx+c))^7}{320} - \frac{77(\sinh(dx+c))^5}{640} + \frac{77(\sinh(dx+c))^3}{512} - \frac{231}{1024} \right) \cosh(dx+c) + \frac{31}{1024} d x + \frac{231}{1024} c \right) + 3 a b^2 \left(\frac{(\sinh(dx+c))^7}{8} - \frac{7(\sinh(dx+c))^5}{48} + \frac{35(\sinh(dx+c))^3}{92} - \frac{35(\sinh(dx+c))}{128} \right) \cosh(dx+c) + \frac{35}{128} d x + \frac{35}{128} c \right) + 3 a^2 b \left(\frac{(\sinh(dx+c))^3}{4} - \frac{3(\sinh(dx+c))}{8} \right) \cosh(dx+c) + \frac{3}{8} d x + \frac{3}{8} c \right) + a^3 (dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(d*x+c)^4)^3,x)

[Out] 1/d*(b^3*((1/12*sinh(d*x+c)^11-11/120*sinh(d*x+c)^9+33/320*sinh(d*x+c)^7-77/640*sinh(d*x+c)^5+77/512*sinh(d*x+c)^3-231/1024)*cosh(d*x+c)+231/1024*d*x+231/1024*c)+3*a*b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/92*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+3*a^2*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^3*(d*x+c))

Maxima [A] time = 1.1122, size = 464, normalized size = 2.2

$$\frac{3}{64} a^2 b \left(24 x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + a^3 x - \frac{1}{245760} b^3 \left(\frac{(72e^{(-2dx-2c)} - 495e^{(-4dx-4c)} + 2200e^{(-6dx-6c)} - 7425e^{(-8dx-8c)} + 23760e^{(-10dx-10c)} - 5)e^{(12dx+12c)}}{d} - 55440(dx+c) - (23760e^{(-2dx-2c)} - 7425e^{(-4dx-4c)} + 2200e^{(-6dx-6c)} - 495e^{(-8dx-8c)} + 72e^{(-10dx-10c)} - 5e^{(-12dx-12c)})}{d} - \frac{1}{2048} a b^2 \left((32e^{(-2dx-2c)} - 168e^{(-4dx-4c)} + 672e^{(-6dx-6c)} - 3)e^{(8dx+8c)} \right) / d - 1680(dx+c) - (672e^{(-2dx-2c)} - 168e^{(-4dx-4c)} + 32e^{(-6dx-6c)} - 3e^{(-8dx-8c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + a^3*x - 1/245760*b^3*((72*e^(-2*d*x - 2*c) - 495*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 7425*e^(-8*d*x - 8*c) + 23760*e^(-10*d*x - 10*c) - 5)*e^(12*d*x + 12*c)/d - 55440*(d*x + c) - (23760*e^(-2*d*x - 2*c) - 7425*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 495*e^(-8*d*x - 8*c) + 72*e^(-10*d*x - 10*c) - 5*e^(-12*d*x - 12*c))/d) - 1/2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c) - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d)

Fricas [B] time = 1.48421, size = 1200, normalized size = 5.69

$$15b^3 \cosh(dx+c) \sinh(dx+c)^{11} + 5(55b^3 \cosh(dx+c)^3 - 36b^3 \cosh(dx+c)) \sinh(dx+c)^9 + 90(11b^3 \cosh(dx+c) \sinh(dx+c)^7 - 11b^3 \cosh(dx+c) \sinh(dx+c)^5 + 11b^3 \cosh(dx+c) \sinh(dx+c)^3 - 11b^3 \cosh(dx+c) \sinh(dx+c)) \sinh(dx+c)^5 + 11b^3 \cosh(dx+c) \sinh(dx+c)^3 - 11b^3 \cosh(dx+c) \sinh(dx+c) + 11b^3 \cosh(dx+c) \sinh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] $\frac{1}{30720} \cdot (15b^3 \cosh(dx+c) \sinh(dx+c)^{11} + 5(55b^3 \cosh(dx+c)^3 - 36b^3 \cosh(dx+c)) \sinh(dx+c)^9 + 90(11b^3 \cosh(dx+c)^5 - 24b^3 \cosh(dx+c)^3 + (8ab^2 + 11b^3) \cosh(dx+c)) \sinh(dx+c)^7 + 6(165b^3 \cosh(dx+c)^7 - 756b^3 \cosh(dx+c)^5 + 105(8ab^2 + 11b^3) \cosh(dx+c)^3 - 10(96ab^2 + 55b^3) \cosh(dx+c)) \sinh(dx+c)^5 + 5(55b^3 \cosh(dx+c)^9 - 432b^3 \cosh(dx+c)^7 + 126(8ab^2 + 11b^3) \cosh(dx+c)^5 - 40(96ab^2 + 55b^3) \cosh(dx+c)^3 + 9(256a^2b + 448ab^2 + 165b^3) \cosh(dx+c)) \sinh(dx+c)^3 + 30(1024a^3 + 1152a^2b + 840ab^2 + 231b^3) dx + 15(b^3 \cosh(dx+c)^{11} - 12b^3 \cosh(dx+c)^9 + 6(8ab^2 + 11b^3) \cosh(dx+c)^7 - 4(96ab^2 + 55b^3) \cosh(dx+c)^5 + 3(256a^2b + 448ab^2 + 165b^3) \cosh(dx+c)^3 - 24(128a^2b + 112ab^2 + 33b^3) \cosh(dx+c)) \sinh(dx+c)) / d$

Sympy [A] time = 103.123, size = 666, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)**4)**3,x)

[Out] $\text{Piecewise}((a^3x + 9a^2bx \sinh(c+dx))^4/8 - 9a^2bx \sinh(c+dx)^2 \cosh(c+dx)^2/4 + 9a^2bx \cosh(c+dx)^4/8 + 15a^2b \sinh(c+dx)^3 \cosh(c+dx)/(8d) - 9a^2b \sinh(c+dx) \cosh(c+dx)^3/(8d) + 105ab^2x \sinh(c+dx)^8/128 - 105ab^2x \sinh(c+dx)^6 \cosh(c+dx)^2/32 + 315ab^2x \sinh(c+dx)^4 \cosh(c+dx)^4/64 - 105ab^2x \sinh(c+dx)^2 \cosh(c+dx)^6/32 + 105ab^2x \cosh(c+dx)^8/128 + 279ab^2 \sinh(c+dx)^7 \cosh(c+dx)/(128d) - 511ab^2 \sinh(c+dx)^5 \cosh(c+dx)^3/(128d) + 385ab^2 \sinh(c+dx)^3 \cosh(c+dx)^5/(128d) - 105ab^2 \sinh(c+dx) \cosh(c+dx)^7/(128d) + 231b^3x \sinh(c+dx)^{12}/1024 - 693b^3x \sinh(c+dx)^{10} \cosh(c+dx)^2/512 + 3465b^3x \sinh(c+dx)^8 \cosh(c+dx)^4/1024 - 1155b^3x \sinh(c+dx)^6 \cosh(c+dx)^6/256 + 3465b^3x \sinh(c+dx)^4 \cosh(c+dx)^8/1024 - 693b^3x \sinh(c+dx)^2 \cosh(c+dx)^{10}/512 + 231b^3x \cosh(c+dx)^{12}/1024 + 793b^3 \sinh(c+dx)^{11} \cosh(c+dx)/(1024d) - 7337b^3 \sinh(c+dx)^9 \cosh(c+dx)^3/(3072d) + 9273b^3 \sinh(c+dx)^7 \cosh(c+dx)^5/(2560d) - 7623b^3 \sinh(c+dx)^5 \cosh(c+dx)^7/(2560d) + 1309b^3 \sinh(c+dx)^3 \cosh(c+dx)^9/(1024d)$

- 231*b**3*sinh(c + d*x)*cosh(c + d*x)**11/(1024*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**3, True))

Giac [B] time = 1.14116, size = 601, normalized size = 2.85

$$5b^3e^{(12dx+12c)} - 72b^3e^{(10dx+10c)} + 360ab^2e^{(8dx+8c)} + 495b^3e^{(8dx+8c)} - 3840ab^2e^{(6dx+6c)} - 2200b^3e^{(6dx+6c)} + 11520a^2be^{(4dx+4c)} - 7425b^3e^{(4dx+4c)} + 92160a^2be^{(2dx+2c)} - 80640ab^2e^{(2dx+2c)} - 23760b^3e^{(2dx+2c)} + 240(1024a^3 + 1152a^2b + 840ab^2 + 231b^3)(dx + c) - (301056a^3e^{(12dx+12c)} + 338688a^2be^{(12dx+12c)} + 246960ab^2e^{(12dx+12c)} + 67914b^3e^{(12dx+12c)} - 92160a^2be^{(10dx+10c)} - 80640ab^2e^{(10dx+10c)} - 23760b^3e^{(10dx+10c)} + 11520a^2be^{(8dx+8c)} + 20160ab^2e^{(8dx+8c)} + 7425b^3e^{(8dx+8c)} - 3840ab^2e^{(6dx+6c)} - 2200b^3e^{(6dx+6c)} + 360ab^2e^{(4dx+4c)} + 495b^3e^{(4dx+4c)} - 72b^3e^{(2dx+2c)} + 5b^3)e^{(-12dx-12c)})/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] 1/245760*(5*b^3*e^(12*d*x + 12*c) - 72*b^3*e^(10*d*x + 10*c) + 360*a*b^2*e^(8*d*x + 8*c) + 495*b^3*e^(8*d*x + 8*c) - 3840*a*b^2*e^(6*d*x + 6*c) - 2200*b^3*e^(6*d*x + 6*c) + 11520*a^2*b*e^(4*d*x + 4*c) + 20160*a*b^2*e^(4*d*x + 4*c) + 7425*b^3*e^(4*d*x + 4*c) - 92160*a^2*b*e^(2*d*x + 2*c) - 80640*a*b^2*e^(2*d*x + 2*c) - 23760*b^3*e^(2*d*x + 2*c) + 240*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*(d*x + c) - (301056*a^3*e^(12*d*x + 12*c) + 338688*a^2*b*e^(12*d*x + 12*c) + 246960*a*b^2*e^(12*d*x + 12*c) + 67914*b^3*e^(12*d*x + 12*c) - 92160*a^2*b*e^(10*d*x + 10*c) - 80640*a*b^2*e^(10*d*x + 10*c) - 23760*b^3*e^(10*d*x + 10*c) + 11520*a^2*b*e^(8*d*x + 8*c) + 20160*a*b^2*e^(8*d*x + 8*c) + 7425*b^3*e^(8*d*x + 8*c) - 3840*a*b^2*e^(6*d*x + 6*c) - 2200*b^3*e^(6*d*x + 6*c) + 360*a*b^2*e^(4*d*x + 4*c) + 495*b^3*e^(4*d*x + 4*c) - 72*b^3*e^(2*d*x + 2*c) + 5*b^3)*e^(-12*d*x - 12*c))/d

3.219 $\int \operatorname{csch}^2(c + dx) \left(a + b \sinh^4(c + dx)\right)^3 dx$

Optimal. Leaf size=181

$$\frac{b(384a^2 + 528ab + 193b^2) \sinh(c + dx) \cosh(c + dx)}{256d} - \frac{3}{256}bx(128a^2 + 80ab + 21b^2) - \frac{a^3 \coth(c + dx)}{d} + \frac{b^2(80a + 171b)}{d}$$

[Out] $(-3*b*(128*a^2 + 80*a*b + 21*b^2)*x)/256 - (a^3*\operatorname{Coth}[c + d*x])/d + (b*(384*a^2 + 528*a*b + 193*b^2)*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(256*d) - (b^2*(208*a + 149*b)*\operatorname{Cosh}[c + d*x]^3*\operatorname{Sinh}[c + d*x])/(128*d) + (b^2*(80*a + 171*b)*\operatorname{Cosh}[c + d*x]^5*\operatorname{Sinh}[c + d*x])/(160*d) - (41*b^3*\operatorname{Cosh}[c + d*x]^7*\operatorname{Sinh}[c + d*x])/(80*d) + (b^3*\operatorname{Cosh}[c + d*x]^9*\operatorname{Sinh}[c + d*x])/(10*d)$

Rubi [A] time = 0.438191, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3217, 1259, 1805, 453, 206}

$$\frac{b(384a^2 + 528ab + 193b^2) \sinh(c + dx) \cosh(c + dx)}{256d} - \frac{3}{256}bx(128a^2 + 80ab + 21b^2) - \frac{a^3 \coth(c + dx)}{d} + \frac{b^2(80a + 171b)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^2*(a + b*\operatorname{Sinh}[c + d*x]^4)^3, x]$

[Out] $(-3*b*(128*a^2 + 80*a*b + 21*b^2)*x)/256 - (a^3*\operatorname{Coth}[c + d*x])/d + (b*(384*a^2 + 528*a*b + 193*b^2)*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(256*d) - (b^2*(208*a + 149*b)*\operatorname{Cosh}[c + d*x]^3*\operatorname{Sinh}[c + d*x])/(128*d) + (b^2*(80*a + 171*b)*\operatorname{Cosh}[c + d*x]^5*\operatorname{Sinh}[c + d*x])/(160*d) - (41*b^3*\operatorname{Cosh}[c + d*x]^7*\operatorname{Sinh}[c + d*x])/(80*d) + (b^3*\operatorname{Cosh}[c + d*x]^9*\operatorname{Sinh}[c + d*x])/(10*d)$

Rule 3217

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]] /; \operatorname{FreeQ}\{a, b, e, f\}, x\} \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 1259

$\operatorname{Int}[(x_.)^{(m_.)}*((d_.) + (e_.)*(x_.)^2)^{(q_.)}*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[((-d)^{(m/2 - 1)}*(c*d^2 - b*d*e + a*e^2)^p*x*(d$

```
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 453

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^n)^(p_)*((c_) + (d_)*(x_)^n), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^2(c+dx) (a+b \sinh^4(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^3}{x^2(1-x^2)^6} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^3 \cosh^9(c+dx) \sinh(c+dx)}{10d} - \frac{\operatorname{Subst}\left(\int \frac{-10a^3+(50a^3+b^3)x^2-10(10a^3+3a^2b-b^3)}{\dots} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{41b^3 \cosh^7(c+dx) \sinh(c+dx)}{80d} + \frac{b^3 \cosh^9(c+dx) \sinh(c+dx)}{10d} + \dots \\
&= \frac{b^2(80a+171b) \cosh^5(c+dx) \sinh(c+dx)}{160d} - \frac{41b^3 \cosh^7(c+dx) \sinh(c+dx)}{80d} \\
&= -\frac{b^2(208a+149b) \cosh^3(c+dx) \sinh(c+dx)}{128d} + \frac{b^2(80a+171b) \cosh^5(c+dx) \sinh(c+dx)}{160d} \\
&= \frac{b(384a^2+528ab+193b^2) \cosh(c+dx) \sinh(c+dx)}{256d} - \frac{b^2(208a+149b)}{256d} \\
&= -\frac{a^3 \coth(c+dx)}{d} + \frac{b(384a^2+528ab+193b^2) \cosh(c+dx) \sinh(c+dx)}{256d} \\
&= -\frac{3}{256}b(128a^2+80ab+21b^2)x - \frac{a^3 \coth(c+dx)}{d} + \frac{b(384a^2+528ab+193b^2) \cosh(c+dx) \sinh(c+dx)}{256d}
\end{aligned}$$

Mathematica [A] time = 0.770606, size = 134, normalized size = 0.74

$$\frac{-120b(128a^2+80ab+21b^2)(c+dx) + 60b(128a^2+120ab+35b^2)\sinh(2(c+dx)) - 10240a^3 \coth(c+dx) - 120b^2(128a^2+80ab+21b^2)}{10240d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (-120*b*(128*a^2 + 80*a*b + 21*b^2)*(c + d*x) - 10240*a^3*Coth[c + d*x] + 60*b*(128*a^2 + 120*a*b + 35*b^2)*Sinh[2*(c + d*x)] - 120*b^2*(12*a + 5*b)*Sinh[4*(c + d*x)] + 10*b^2*(16*a + 15*b)*Sinh[6*(c + d*x)] - 25*b^3*Sinh[8*(c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)

Maple [A] time = 0.039, size = 163, normalized size = 0.9

$$\frac{1}{d} \left(-a^3 \coth(dx+c) + 3a^2b \left(\frac{1}{2} \cosh(dx+c) \sinh(dx+c) - \frac{1}{2} dx - \frac{c}{2} \right) + 3ab^2 \left(\left(\frac{1}{6} (\sinh(dx+c))^5 - \frac{5 (\sinh(dx+c))}{24} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x)

[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a*b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/128*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c))

Maxima [A] time = 1.09203, size = 383, normalized size = 2.12

$$-\frac{3}{8} a^2 b \left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{-2dx-2c}}{d} \right) - \frac{1}{20480} b^3 \left(\frac{(25e^{-2dx-2c} - 150e^{-4dx-4c} + 600e^{-6dx-6c} - 2100e^{-8dx-8c} - 2)e^{10dx+10c}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] -3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/20480*b^3*((2*5*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1)))

Fricas [B] time = 1.44548, size = 1247, normalized size = 6.89

$$2b^3 \cosh(dx+c)^{11} + 22b^3 \cosh(dx+c) \sinh(dx+c)^{10} - 27b^3 \cosh(dx+c)^9 + 3(110b^3 \cosh(dx+c)^3 - 81b^3 \cosh(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] $\frac{1}{20480} \cdot (2b^3 \cosh(dx+c)^{11} + 22b^3 \cosh(dx+c) \sinh(dx+c)^{10} - 27b^3 \cosh(dx+c)^9 + 3(110b^3 \cosh(dx+c)^3 - 81b^3 \cosh(dx+c)) \sinh(dx+c)^8 + 5(32ab^2 + 35b^3) \cosh(dx+c)^7 + 7(132b^3 \cosh(dx+c)^5 - 324b^3 \cosh(dx+c)^3 + 5(32ab^2 + 35b^3) \cosh(dx+c)) \sinh(dx+c)^6 - 50(32ab^2 + 15b^3) \cosh(dx+c)^5 + (660b^3 \cosh(dx+c)^7 - 3402b^3 \cosh(dx+c)^5 + 175(32ab^2 + 35b^3) \cosh(dx+c)^3 - 250(32ab^2 + 15b^3) \cosh(dx+c)) \sinh(dx+c)^4 + 60(128a^2b + 144ab^2 + 45b^3) \cosh(dx+c)^3 + (110b^3 \cosh(dx+c)^9 - 972b^3 \cosh(dx+c)^7 + 105(32ab^2 + 35b^3) \cosh(dx+c)^5 - 500(32ab^2 + 15b^3) \cosh(dx+c)^3 + 180(128a^2b + 144ab^2 + 45b^3) \cosh(dx+c)) \sinh(dx+c)^2 - 20(1024a^3 + 384a^2b + 360ab^2 + 105b^3) \cosh(dx+c) + 80(256a^3 - 3(128a^2b + 80ab^2 + 21b^3)dx) \sinh(dx+c)) / (d \sinh(dx+c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.70399, size = 531, normalized size = 2.93

$$-\frac{3(128a^2b + 80ab^2 + 21b^3)(dx+c)}{256d} - \frac{2a^3}{d(e^{(2dx+2c)} - 1)} + \frac{(35072a^2be^{(10dx+10c)} + 21920ab^2e^{(10dx+10c)} + 5754b^3e^{(10dx+10c)})}{d(e^{(2dx+2c)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] $-3/256 \cdot (128a^2b + 80ab^2 + 21b^3) \cdot (dx+c) / d - 2a^3 / (d \cdot (e^{(2dx+2c)} - 1)) + 1/20480 \cdot (35072a^2b \cdot e^{(10dx+10c)} + 21920ab^2 \cdot e^{(10dx+10c)} + 5754b^3 \cdot e^{(10dx+10c)} - 7680a^2b \cdot e^{(8dx+8c)} - 7200ab^2 \cdot e^{(8dx+8c)} - 2100b^3 \cdot e^{(8dx+8c)} + 1440ab^2 \cdot e^{(6dx+6c)}) / (d \cdot (e^{(2dx+2c)} - 1))$

$$\begin{aligned} &+ 600*b^3*e^{(6*d*x + 6*c)} - 160*a*b^2*e^{(4*d*x + 4*c)} - 150*b^3*e^{(4*d*x + 4*c)} \\ &+ 25*b^3*e^{(2*d*x + 2*c)} - 2*b^3*e^{(-10*d*x - 10*c)}/d + 1/20480*(2*b^3*d^4*e^{(10*d*x + 10*c)} \\ &- 25*b^3*d^4*e^{(8*d*x + 8*c)} + 160*a*b^2*d^4*e^{(6*d*x + 6*c)} + 150*b^3*d^4*e^{(6*d*x + 6*c)} \\ &- 1440*a*b^2*d^4*e^{(4*d*x + 4*c)} - 600*b^3*d^4*e^{(4*d*x + 4*c)} + 7680*a^2*b*d^4*e^{(2*d*x + 2*c)} \\ &+ 7200*a*b^2*d^4*e^{(2*d*x + 2*c)} + 2100*b^3*d^4*e^{(2*d*x + 2*c)})/d^5 \end{aligned}$$

3.220 $\int \operatorname{csch}^4(c + dx) \left(a + b \sinh^4(c + dx)\right)^3 dx$

Optimal. Leaf size=161

$$\frac{1}{128}bx(384a^2 + 144ab + 35b^2) - \frac{a^3 \coth^3(c + dx)}{3d} + \frac{a^3 \coth(c + dx)}{d} + \frac{b^2(144a + 163b) \sinh(c + dx) \cosh^3(c + dx)}{192d}$$

[Out] (b*(384*a^2 + 144*a*b + 35*b^2)*x)/128 + (a^3*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) - (3*b^2*(80*a + 31*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (b^2*(144*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b^3*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^3*Cosh[c + d*x]^7*Sinh[c + d*x])/(8*d)

Rubi [A] time = 0.384284, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3217, 1259, 1805, 1261, 207}

$$\frac{1}{128}bx(384a^2 + 144ab + 35b^2) - \frac{a^3 \coth^3(c + dx)}{3d} + \frac{a^3 \coth(c + dx)}{d} + \frac{b^2(144a + 163b) \sinh(c + dx) \cosh^3(c + dx)}{192d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (b*(384*a^2 + 144*a*b + 35*b^2)*x)/128 + (a^3*Coth[c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) - (3*b^2*(80*a + 31*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (b^2*(144*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b^3*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^3*Cosh[c + d*x]^7*Sinh[c + d*x])/(8*d)

Rule 3217

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1259

Int[(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d

```

+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]

```

Rule 1805

```

Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]

```

Rule 1261

```

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (
c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

```

Rule 207

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(c+dx) (a+b \sinh^4(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^3}{x^4(1-x^2)^5} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^3 \cosh^7(c+dx) \sinh(c+dx)}{8d} + \frac{\operatorname{Subst}\left(\int \frac{8a^3-40a^3x^2+(80a^3+24a^2b-b^3)x^4-8(10a^3+24a^2b-b^3)x^6}{x^4} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{25b^3 \cosh^5(c+dx) \sinh(c+dx)}{48d} + \frac{b^3 \cosh^7(c+dx) \sinh(c+dx)}{8d} - \frac{3b^2(80a+31b) \cosh(c+dx) \sinh(c+dx)}{128d} \\
&= \frac{b^2(144a+163b) \cosh^3(c+dx) \sinh(c+dx)}{192d} - \frac{25b^3 \cosh^5(c+dx) \sinh(c+dx)}{48d} \\
&= -\frac{3b^2(80a+31b) \cosh(c+dx) \sinh(c+dx)}{128d} + \frac{b^2(144a+163b) \cosh^3(c+dx) \sinh(c+dx)}{192d} \\
&= -\frac{3b^2(80a+31b) \cosh(c+dx) \sinh(c+dx)}{128d} + \frac{b^2(144a+163b) \cosh^3(c+dx) \sinh(c+dx)}{192d} \\
&= \frac{a^3 \coth(c+dx)}{d} - \frac{a^3 \coth^3(c+dx)}{3d} - \frac{3b^2(80a+31b) \cosh(c+dx) \sinh(c+dx)}{128d} \\
&= \frac{1}{128} b (384a^2 + 144ab + 35b^2) x + \frac{a^3 \coth(c+dx)}{d} - \frac{a^3 \coth^3(c+dx)}{3d} - \frac{3b^2(80a+31b) \cosh(c+dx) \sinh(c+dx)}{128d}
\end{aligned}$$

Mathematica [A] time = 0.782114, size = 131, normalized size = 0.81

$$\frac{b(9216a^2c + 9216a^2dx - 96b(24a + 7b) \sinh(2(c+dx)) + 24b(12a + 7b) \sinh(4(c+dx)) + 3456abc + 3456abdx - 32b^2c^2 - 32b^2dx^2 + 32b^2c^2 + 32b^2dx^2)}{3072d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (-1024*a^3*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + b*(9216*a^2*c + 3456*a*b*c + 840*b^2*c + 9216*a^2*d*x + 3456*a*b*d*x + 840*b^2*d*x - 96*b*(24*a + 7*b)*Sinh[2*(c + d*x)] + 24*b*(12*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh[6*(c + d*x)] + 3*b^2*Sinh[8*(c + d*x)]))/(3072*d)

Maple [A] time = 0.048, size = 137, normalized size = 0.9

$$\frac{1}{d} \left(a^3 \left(\frac{2}{3} - \frac{(\operatorname{csch}(dx+c))^2}{3} \right) \coth(dx+c) + 3a^2b(dx+c) + 3ab^2 \left(\left(\frac{1}{4} (\sinh(dx+c))^3 - \frac{3}{8} \sinh(dx+c) \right) \cosh(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] `1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(d*x+c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b^3*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c))`

Maxima [A] time = 1.10793, size = 381, normalized size = 2.37

$$\frac{3}{64} ab^2 \left(24x + \frac{e^{4dx+4c}}{d} - \frac{8e^{2dx+2c}}{d} + \frac{8e^{-2dx-2c}}{d} - \frac{e^{-4dx-4c}}{d} \right) + 3a^2bx - \frac{1}{6144} b^3 \left(\frac{(32e^{-2dx-2c} - 168e^{-4dx-4c} + 672e^{-6dx-6c} - 32e^{-8dx-8c})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] `3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 3*a^2*b*x - 1/6144*b^3*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 32*e^(-8*d*x - 8*c))/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))`

Fricas [B] time = 1.36759, size = 1476, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")`

```
[Out] 1/6144*(3*b^3*cosh(d*x + c)^11 + 33*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 41
*b^3*cosh(d*x + c)^9 + 9*(55*b^3*cosh(d*x + c)^3 - 41*b^3*cosh(d*x + c))*si
nh(d*x + c)^8 + 3*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^7 + 21*(66*b^3*cosh(d*x
+ c)^5 - 164*b^3*cosh(d*x + c)^3 + (96*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh
(d*x + c)^6 - 3*(1056*a*b^2 + 425*b^3)*cosh(d*x + c)^5 + 3*(330*b^3*cosh(d*
x + c)^7 - 1722*b^3*cosh(d*x + c)^5 + 35*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^
3 - 5*(1056*a*b^2 + 425*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(512*a^3 +
972*a*b^2 + 319*b^3)*cosh(d*x + c)^3 - 16*(256*a^3 - 3*(384*a^2*b + 144*a*b
^2 + 35*b^3)*d*x)*sinh(d*x + c)^3 + 3*(55*b^3*cosh(d*x + c)^9 - 492*b^3*cos
h(d*x + c)^7 + 21*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 10*(1056*a*b^2 + 42
5*b^3)*cosh(d*x + c)^3 + 8*(512*a^3 + 972*a*b^2 + 319*b^3)*cosh(d*x + c))*s
inh(d*x + c)^2 - 24*(512*a^3 + 204*a*b^2 + 63*b^3)*cosh(d*x + c) + 48*(256*
a^3 - 3*(384*a^2*b + 144*a*b^2 + 35*b^3)*d*x - (256*a^3 - 3*(384*a^2*b + 14
4*a*b^2 + 35*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3 +
3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70406, size = 423, normalized size = 2.63

$$\frac{(384a^2b + 144ab^2 + 35b^3)(dx + c)}{128d} - \frac{(19200a^2be^{(8dx+8c)} + 7200ab^2e^{(8dx+8c)} + 1750b^3e^{(8dx+8c)} - 2304ab^2e^{(6dx+6c)} - 672b^3e^{(6dx+6c)} + 288a*b^2e^{(4dx+4c)} + 168b^3e^{(4dx+4c)} - 32b^3e^{(2dx+2c)} + 3b^3)e^{(-8dx - 8c)}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/128*(384*a^2*b + 144*a*b^2 + 35*b^3)*(d*x + c)/d - 1/6144*(19200*a^2*b*e^
(8*d*x + 8*c) + 7200*a*b^2*e^(8*d*x + 8*c) + 1750*b^3*e^(8*d*x + 8*c) - 230
4*a*b^2*e^(6*d*x + 6*c) - 672*b^3*e^(6*d*x + 6*c) + 288*a*b^2*e^(4*d*x + 4*
c) + 168*b^3*e^(4*d*x + 4*c) - 32*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-8*d*x -
```

$$\begin{aligned} & 8*c)/d + 1/6144*(3*b^3*d^3*e^(8*d*x + 8*c) - 32*b^3*d^3*e^(6*d*x + 6*c) + 2 \\ & 88*a*b^2*d^3*e^(4*d*x + 4*c) + 168*b^3*d^3*e^(4*d*x + 4*c) - 2304*a*b^2*d^3 \\ & *e^(2*d*x + 2*c) - 672*b^3*d^3*e^(2*d*x + 2*c))/d^4 - 4/3*(3*a^3*e^(2*d*x + \\ & 2*c) - a^3)/(d*(e^(2*d*x + 2*c) - 1)^3) \end{aligned}$$

3.221 $\int \operatorname{csch}^6(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=148

$$-\frac{a^2(a+3b)\operatorname{coth}(c+dx)}{d} - \frac{a^3\operatorname{coth}^5(c+dx)}{5d} + \frac{2a^3\operatorname{coth}^3(c+dx)}{3d} + \frac{b^2(24a+11b)\sinh(c+dx)\cosh(c+dx)}{16d} - \frac{1}{16}b^2x$$

[Out] $-(b^2(24a+5b)x)/16 - (a^2(a+3b)\operatorname{Coth}[c+dx])/d + (2a^3\operatorname{Coth}[c+dx]^3)/(3d) - (a^3\operatorname{Coth}[c+dx]^5)/(5d) + (b^2(24a+11b)\operatorname{Cosh}[c+dx]*\operatorname{Sinh}[c+dx])/(16d) - (13b^3\operatorname{Cosh}[c+dx]^3*\operatorname{Sinh}[c+dx])/(24d) + (b^3\operatorname{Cosh}[c+dx]^5*\operatorname{Sinh}[c+dx])/(6d)$

Rubi [A] time = 0.355256, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3217, 1259, 1805, 1802, 207}

$$-\frac{a^2(a+3b)\operatorname{coth}(c+dx)}{d} - \frac{a^3\operatorname{coth}^5(c+dx)}{5d} + \frac{2a^3\operatorname{coth}^3(c+dx)}{3d} + \frac{b^2(24a+11b)\sinh(c+dx)\cosh(c+dx)}{16d} - \frac{1}{16}b^2x$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c+dx]^6*(a+b*\operatorname{Sinh}[c+dx]^4)^3,x]$

[Out] $-(b^2(24a+5b)x)/16 - (a^2(a+3b)\operatorname{Coth}[c+dx])/d + (2a^3\operatorname{Coth}[c+dx]^3)/(3d) - (a^3\operatorname{Coth}[c+dx]^5)/(5d) + (b^2(24a+11b)\operatorname{Cosh}[c+dx]*\operatorname{Sinh}[c+dx])/(16d) - (13b^3\operatorname{Cosh}[c+dx]^3*\operatorname{Sinh}[c+dx])/(24d) + (b^3\operatorname{Cosh}[c+dx]^5*\operatorname{Sinh}[c+dx])/(6d)$

Rule 3217

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_)]^{(m_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m+1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + 2*a*ff^2*x^2 + (a+b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]\} /; \operatorname{FreeQ}\{a, b, e, f\}, x\} \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 1259

$\operatorname{Int}[(x_)^{(m_)}*((d_.) + (e_.)*(x_)^2)^{(q_)}*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[((-d)^{(m/2 - 1)}*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^{(q+1)})/(2*e^{(2*p + m/2)}*(q+1)), x] + \operatorname{Dist}[(-d)^{(m/2 - 1)}/(2*e^{(2*p + m/2)}*(q+1)), x]$

```
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^6(c+dx) (a+b \sinh^4(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^3}{x^6(1-x^2)^4} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^3 \cosh^5(c+dx) \sinh(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-6a^3+30a^3x^2-6a^2(10a+3b)x^4+(60a^3+5b^2)x^6}{x^6(1-x^2)^4} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{13b^3 \cosh^3(c+dx) \sinh(c+dx)}{24d} + \frac{b^3 \cosh^5(c+dx) \sinh(c+dx)}{6d} + \frac{b^2(24a+11b) \cosh(c+dx) \sinh(c+dx)}{16d} - \frac{13b^3 \cosh^3(c+dx) \sinh(c+dx)}{24d} \\
&= \frac{b^2(24a+11b) \cosh(c+dx) \sinh(c+dx)}{16d} - \frac{13b^3 \cosh^3(c+dx) \sinh(c+dx)}{24d} \\
&= -\frac{a^2(a+3b) \operatorname{coth}(c+dx)}{d} + \frac{2a^3 \operatorname{coth}^3(c+dx)}{3d} - \frac{a^3 \operatorname{coth}^5(c+dx)}{5d} + \frac{b^2(24a+11b) \cosh(c+dx) \sinh(c+dx)}{16d} - \frac{13b^3 \cosh^3(c+dx) \sinh(c+dx)}{24d} \\
&= -\frac{1}{16}b^2(24a+5b)x - \frac{a^2(a+3b) \operatorname{coth}(c+dx)}{d} + \frac{2a^3 \operatorname{coth}^3(c+dx)}{3d} - \frac{a^3 \operatorname{coth}^5(c+dx)}{5d} + \frac{b^2(24a+11b) \cosh(c+dx) \sinh(c+dx)}{16d} - \frac{13b^3 \cosh^3(c+dx) \sinh(c+dx)}{24d}
\end{aligned}$$

Mathematica [A] time = 1.15998, size = 110, normalized size = 0.74

$$\frac{5b^2(9(16a+5b) \sinh(2(c+dx)) - 288ac - 288adx - 9b \sinh(4(c+dx)) + b \sinh(6(c+dx)) - 60bc - 60bdx) - 64a^2 \operatorname{coth}^5(c+dx)}{960d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (-64*a^2*Coth[c + d*x]*(8*a + 45*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*x]^4) + 5*b^2*(-288*a*c - 60*b*c - 288*a*d*x - 60*b*d*x + 9*(16*a + 5*b)*Sinh[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)]))/(960*d)

Maple [A] time = 0.046, size = 126, normalized size = 0.9

$$\frac{1}{d} \left(a^3 \left(-\frac{8}{15} - \frac{(\operatorname{csch}(dx+c))^4}{5} + \frac{4(\operatorname{csch}(dx+c))^2}{15} \right) \operatorname{coth}(dx+c) - 3a^2b \operatorname{coth}(dx+c) + 3ab^2 \left(\frac{1}{2} \cosh(dx+c) \sinh(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] $1/d*(a^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-3*a^2*b*c$
 $oth(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^3*((1/6*si$
 $nh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*$
 $c))$

Maxima [B] time = 1.06571, size = 485, normalized size = 3.28

$$-\frac{3}{8}ab^2\left(4x - \frac{e^{2dx+2c}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{384}b^3\left(\frac{(9e^{(-2dx-2c)} - 45e^{(-4dx-4c)} - 1)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] $-3/8*a*b^2*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 1/384*b^3*((9*e$
 $^{(-2*d*x - 2*c)} - 45*e^{(-4*d*x - 4*c)} - 1)*e^{(6*d*x + 6*c)}/d + 120*(d*x + c$
 $)/d + (45*e^{(-2*d*x - 2*c)} - 9*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)})/d - 16$
 $/15*a^3*(5*e^{(-2*d*x - 2*c)}/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} +$
 $10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) - 10*e^{$
 $(-4*d*x - 4*c)}/(d*(5*e^{(-2*d*x - 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x$
 $- 6*c)} - 5*e^{(-8*d*x - 8*c)} + e^{(-10*d*x - 10*c)} - 1)) - 1/(d*(5*e^{(-2*d*x$
 $- 2*c)} - 10*e^{(-4*d*x - 4*c)} + 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + e$
 $^{(-10*d*x - 10*c)} - 1))) + 6*a^2*b/(d*(e^{(-2*d*x - 2*c)} - 1))$

Fricas [B] time = 1.5227, size = 2006, normalized size = 13.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")`

[Out] $1/1920*(5*b^3*cosh(d*x + c)^11 + 55*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 70$
 $*b^3*cosh(d*x + c)^9 + 15*(55*b^3*cosh(d*x + c)^3 - 42*b^3*cosh(d*x + c))*s$
 $inh(d*x + c)^8 + 20*(36*a*b^2 + 25*b^3)*cosh(d*x + c)^7 + 70*(33*b^3*cosh(d$

$$\begin{aligned} & *x + c)^5 - 84*b^3*\cosh(d*x + c)^3 + 2*(36*a*b^2 + 25*b^3)*\cosh(d*x + c))*s \\ & \sinh(d*x + c)^6 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 1625*b^3)*\cosh(d*x + \\ & c)^5 + 8*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*\sinh(d*x + c)^5 \\ & + 5*(330*b^3*\cosh(d*x + c)^7 - 1764*b^3*\cosh(d*x + c)^5 + 140*(36*a*b^2 + \\ & 25*b^3)*\cosh(d*x + c)^3 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 1625*b^3)*c \\ & \cosh(d*x + c))*\sinh(d*x + c)^4 + 20*(256*a^3 + 864*a^2*b + 324*a*b^2 + 125*b \\ & ^3)*\cosh(d*x + c)^3 - 40*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x - \\ & 2*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(\\ & d*x + c)^3 + 5*(55*b^3*\cosh(d*x + c)^9 - 504*b^3*\cosh(d*x + c)^7 + 84*(36*a \\ & *b^2 + 25*b^3)*\cosh(d*x + c)^5 - 2*(1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 16 \\ & 25*b^3)*\cosh(d*x + c)^3 + 12*(256*a^3 + 864*a^2*b + 324*a*b^2 + 125*b^3)*co \\ & sh(d*x + c))*\sinh(d*x + c)^2 - 10*(1024*a^3 + 1152*a^2*b + 360*a*b^2 + 131* \\ & b^3)*\cosh(d*x + c) + 40*((128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)* \\ & \cosh(d*x + c)^4 + 256*a^3 + 1440*a^2*b - 30*(24*a*b^2 + 5*b^3)*d*x - 3*(128 \\ & *a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c \\ &))/(d*\sinh(d*x + c)^5 + 5*(2*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^3 + 5*(d* \\ & \cosh(d*x + c)^4 - 3*d*\cosh(d*x + c)^2 + 2*d)*\sinh(d*x + c)) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.70671, size = 414, normalized size = 2.8

$$-\frac{(24ab^2 + 5b^3)(dx + c)}{16d} + \frac{(528ab^2e^{(6dx+6c)} + 110b^3e^{(6dx+6c)} - 144ab^2e^{(4dx+4c)} - 45b^3e^{(4dx+4c)} + 9b^3e^{(2dx+2c)} - b^3)e^{(6dx+6c)}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] $-1/16*(24*a*b^2 + 5*b^3)*(d*x + c)/d + 1/384*(528*a*b^2*e^{(6*d*x + 6*c)} + 110*b^3*e^{(6*d*x + 6*c)} - 144*a*b^2*e^{(4*d*x + 4*c)} - 45*b^3*e^{(4*d*x + 4*c)})$

$$\begin{aligned}
& + 9*b^3*e^{(2*d*x + 2*c)} - b^3*e^{(-6*d*x - 6*c)}/d + 1/384*(b^3*d^2*e^{(6*d*x + 6*c)} - 9*b^3*d^2*e^{(4*d*x + 4*c)} + 144*a*b^2*d^2*e^{(2*d*x + 2*c)} + 45*b^3*d^2*e^{(2*d*x + 2*c)})/d^3 - 2/15*(45*a^2*b*e^{(8*d*x + 8*c)} - 180*a^2*b*e^{(6*d*x + 6*c)} + 80*a^3*e^{(4*d*x + 4*c)} + 270*a^2*b*e^{(4*d*x + 4*c)} - 40*a^3*e^{(2*d*x + 2*c)} - 180*a^2*b*e^{(2*d*x + 2*c)} + 8*a^3 + 45*a^2*b)/(d*(e^{(2*d*x + 2*c)} - 1)^5)
\end{aligned}$$

3.222 $\int \operatorname{csch}^8(c + dx) \left(a + b \sinh^4(c + dx)\right)^3 dx$

Optimal. Leaf size=133

$$-\frac{a^2(a+b)\operatorname{coth}^3(c+dx)}{d} + \frac{a^2(a+3b)\operatorname{coth}(c+dx)}{d} - \frac{a^3\operatorname{coth}^7(c+dx)}{7d} + \frac{3a^3\operatorname{coth}^5(c+dx)}{5d} + \frac{3}{8}b^2x(8a+b) + \frac{b^3\sinh}{8}$$

[Out] (3*b^2*(8*a + b)*x)/8 + (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^2*(a + b)*Coth[c + d*x]^3)/d + (3*a^3*Coth[c + d*x]^5)/(5*d) - (a^3*Coth[c + d*x]^7)/(7*d) - (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d)

Rubi [A] time = 0.288891, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3217, 1259, 1805, 1802, 207}

$$-\frac{a^2(a+b)\operatorname{coth}^3(c+dx)}{d} + \frac{a^2(a+3b)\operatorname{coth}(c+dx)}{d} - \frac{a^3\operatorname{coth}^7(c+dx)}{7d} + \frac{3a^3\operatorname{coth}^5(c+dx)}{5d} + \frac{3}{8}b^2x(8a+b) + \frac{b^3\sinh}{8}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (3*b^2*(8*a + b)*x)/8 + (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^2*(a + b)*Coth[c + d*x]^3)/d + (3*a^3*Coth[c + d*x]^5)/(5*d) - (a^3*Coth[c + d*x]^7)/(7*d) - (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d)

Rule 3217

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1259

Int[(x_.)^(m_.)*((d_.) + (e_.)*(x_.)^2)^(q_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^

```
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^8(c+dx) (a+b \sinh^4(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^3}{x^8(1-x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^3 \cosh^3(c+dx) \sinh(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{4a^3-20a^3x^2+4a^2(10a+3b)x^4-4a^2(10a+9)}{x^8} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{5b^3 \cosh(c+dx) \sinh(c+dx)}{8d} + \frac{b^3 \cosh^3(c+dx) \sinh(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{4a^3-20a^3x^2+4a^2(10a+3b)x^4-4a^2(10a+9)}{x^8} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{5b^3 \cosh(c+dx) \sinh(c+dx)}{8d} + \frac{b^3 \cosh^3(c+dx) \sinh(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{4a^3-20a^3x^2+4a^2(10a+3b)x^4-4a^2(10a+9)}{x^8} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{a^2(a+3b) \coth(c+dx)}{d} - \frac{a^2(a+b) \coth^3(c+dx)}{d} + \frac{3a^3 \coth^5(c+dx)}{5d} - \frac{\operatorname{Subst}\left(\int \frac{4a^3-20a^3x^2+4a^2(10a+3b)x^4-4a^2(10a+9)}{x^8} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{3}{8}b^2(8a+b)x + \frac{a^2(a+3b) \coth(c+dx)}{d} - \frac{a^2(a+b) \coth^3(c+dx)}{d} + \frac{3a^3}{d} - \frac{\operatorname{Subst}\left(\int \frac{4a^3-20a^3x^2+4a^2(10a+3b)x^4-4a^2(10a+9)}{x^8} dx, x, \tanh(c+dx)\right)}{d}
\end{aligned}$$

Mathematica [A] time = 0.691789, size = 106, normalized size = 0.8

$$\frac{35b^2(12(8a+b)(c+dx) - 8b \sinh(2(c+dx)) + b \sinh(4(c+dx))) - 32a^2 \coth(c+dx) ((8a+35b) \operatorname{csch}^2(c+dx) - 2(8a+b) \operatorname{csch}(c+dx) \sinh(c+dx) + b \sinh(4(c+dx)))}{1120d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (-32*a^2*Coth[c + d*x]*(-2*(8*a + 35*b) + (8*a + 35*b)*Csch[c + d*x]^2 - 6*a*Csch[c + d*x]^4 + 5*a*Csch[c + d*x]^6) + 35*b^2*(12*(8*a + b)*(c + d*x) - 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(1120*d)

Maple [A] time = 0.089, size = 121, normalized size = 0.9

$$\frac{1}{d} \left(a^3 \left(\frac{16}{35} - \frac{(\operatorname{csch}(dx+c))^6}{7} + \frac{6(\operatorname{csch}(dx+c))^4}{35} - \frac{8(\operatorname{csch}(dx+c))^2}{35} \right) \coth(dx+c) + 3a^2b \left(\frac{2}{3} - \frac{1}{3} \operatorname{csch}(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x)

[Out] $1/d*(a^3*(16/35-1/7*\operatorname{csch}(d*x+c)^6+6/35*\operatorname{csch}(d*x+c)^4-8/35*\operatorname{csch}(d*x+c)^2)*\operatorname{coth}(d*x+c)+3*a^2*b*(2/3-1/3*\operatorname{csch}(d*x+c)^2)*\operatorname{coth}(d*x+c)+3*a*b^2*(d*x+c)+b^3*(1/4*\sinh(d*x+c)^3-3/8*\sinh(d*x+c))*\operatorname{cosh}(d*x+c)+3/8*d*x+3/8*c)$

Maxima [B] time = 1.14781, size = 725, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] $1/64*b^3*(24*x + e^{(4*d*x + 4*c)}/d - 8*e^{(2*d*x + 2*c)}/d + 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d) + 3*a*b^2*x + 32/35*a^3*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1)) - 21*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1)) + 35*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1)) - 1/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1))) + 4*a^2*b*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)))$

Fricas [B] time = 1.49259, size = 2464, normalized size = 18.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")`

[Out] $1/2240*(35*b^3*\cosh(d*x + c)^{11} + 385*b^3*\cosh(d*x + c)*\sinh(d*x + c)^{10} - 525*b^3*\cosh(d*x + c)^9 + 525*(11*b^3*\cosh(d*x + c)^3 - 9*b^3*\cosh(d*x + c))*\sinh(d*x + c)^8 + (1024*a^3 + 4480*a^2*b + 2695*b^3)*\cosh(d*x + c)^7 - 8*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*\sinh(d*x + c)^7 + 7*(2310*b^3*\cosh(d*x + c)^5 - 6300*b^3*\cosh(d*x + c)^3 + (1024*a^3 + 4480*a^2*b + 26$

```

95*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 7*(1024*a^3 + 4480*a^2*b + 975*b^3
)*cosh(d*x + c)^5 + 56*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x - 3*(
128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x +
c)^5 + 35*(330*b^3*cosh(d*x + c)^7 - 1890*b^3*cosh(d*x + c)^5 + (1024*a^3 +
4480*a^2*b + 2695*b^3)*cosh(d*x + c)^3 - (1024*a^3 + 4480*a^2*b + 975*b^3)
*cosh(d*x + c))*sinh(d*x + c)^4 + 42*(512*a^3 + 1600*a^2*b + 215*b^3)*cosh(
d*x + c)^3 - 56*(5*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^4 + 384*a^3 + 1680*a^2*b - 315*(8*a*b^2 + b^3)*d*x - 10*(128*a^3 + 56
0*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(27
5*b^3*cosh(d*x + c)^9 - 2700*b^3*cosh(d*x + c)^7 + 3*(1024*a^3 + 4480*a^2*b
+ 2695*b^3)*cosh(d*x + c)^5 - 10*(1024*a^3 + 4480*a^2*b + 975*b^3)*cosh(d*
x + c)^3 + 18*(512*a^3 + 1600*a^2*b + 215*b^3)*cosh(d*x + c))*sinh(d*x + c)
^2 - 70*(512*a^3 + 576*a^2*b + 63*b^3)*cosh(d*x + c) - 56*((128*a^3 + 560*a
^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 5*(128*a^3 + 560*a^2*b -
105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 640*a^3 - 2800*a^2*b + 525*(8*a*
b^2 + b^3)*d*x + 9*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x
+ c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^7 + 7*(3*d*cosh(d*x + c)^2 - d)*si
nh(d*x + c)^5 + 7*(5*d*cosh(d*x + c)^4 - 10*d*cosh(d*x + c)^2 + 3*d)*sinh(d
*x + c)^3 + 7*(d*cosh(d*x + c)^6 - 5*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^
2 - 5*d)*sinh(d*x + c))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**8*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.70678, size = 356, normalized size = 2.68

$$\frac{3(8ab^2 + b^3)(dx + c)}{8d} - \frac{(144ab^2e^{(4dx+4c)} + 18b^3e^{(4dx+4c)} - 8b^3e^{(2dx+2c)} + b^3)e^{(-4dx-4c)}}{64d} + \frac{b^3de^{(4dx+4c)} - 8b^3de^{(2dx+2c)}}{64d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

```
[Out] 3/8*(8*a*b^2 + b^3)*(d*x + c)/d - 1/64*(144*a*b^2*e^(4*d*x + 4*c) + 18*b^3*
e^(4*d*x + 4*c) - 8*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c)/d + 1/64*(b
^3*d*e^(4*d*x + 4*c) - 8*b^3*d*e^(2*d*x + 2*c))/d^2 - 4/35*(105*a^2*b*e^(10
*d*x + 10*c) - 455*a^2*b*e^(8*d*x + 8*c) + 280*a^3*e^(6*d*x + 6*c) + 770*a^
2*b*e^(6*d*x + 6*c) - 168*a^3*e^(4*d*x + 4*c) - 630*a^2*b*e^(4*d*x + 4*c) +
56*a^3*e^(2*d*x + 2*c) + 245*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 35*a^2*b)/(d*
(e^(2*d*x + 2*c) - 1)^7)
```


3.223 $\int \operatorname{csch}^{10}(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=140

$$\frac{a(a^2 + 3ab + 3b^2) \operatorname{coth}(c + dx)}{d} - \frac{3a^2(2a + b) \operatorname{coth}^5(c + dx)}{5d} + \frac{2a^2(2a + 3b) \operatorname{coth}^3(c + dx)}{3d} - \frac{a^3 \operatorname{coth}^9(c + dx)}{9d} + \frac{4a^3}{9d}$$

[Out] $-(b^3x)/2 - (a(a^2 + 3ab + 3b^2)\operatorname{Coth}[c + dx])/d + (2a^2(2a + 3b)\operatorname{Coth}[c + dx]^3)/(3d) - (3a^2(2a + b)\operatorname{Coth}[c + dx]^5)/(5d) + (4a^3\operatorname{Coth}[c + dx]^7)/(7d) - (a^3\operatorname{Coth}[c + dx]^9)/(9d) + (b^3\operatorname{Cosh}[c + dx]*\operatorname{Sinh}[c + dx])/(2d)$

Rubi [A] time = 0.222652, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3217, 1259, 1802, 207}

$$\frac{a(a^2 + 3ab + 3b^2) \operatorname{coth}(c + dx)}{d} - \frac{3a^2(2a + b) \operatorname{coth}^5(c + dx)}{5d} + \frac{2a^2(2a + 3b) \operatorname{coth}^3(c + dx)}{3d} - \frac{a^3 \operatorname{coth}^9(c + dx)}{9d} + \frac{4a^3}{9d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + dx]^{10}(a + b\operatorname{Sinh}[c + dx]^4)^3, x]$

[Out] $-(b^3x)/2 - (a(a^2 + 3ab + 3b^2)\operatorname{Coth}[c + dx])/d + (2a^2(2a + 3b)\operatorname{Coth}[c + dx]^3)/(3d) - (3a^2(2a + b)\operatorname{Coth}[c + dx]^5)/(5d) + (4a^3\operatorname{Coth}[c + dx]^7)/(7d) - (a^3\operatorname{Coth}[c + dx]^9)/(9d) + (b^3\operatorname{Cosh}[c + dx]*\operatorname{Sinh}[c + dx])/(2d)$

Rule 3217

$\operatorname{Int}[\sin[(e_.) + (f_.)x]^{(m_)}((a_.) + (b_.)\sin[(e_.) + (f_.)x]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + fx], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m(a + 2a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \operatorname{Tan}[e + fx]/ff], x]\} /; \operatorname{FreeQ}\{a, b, e, f, x\} \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 1259

$\operatorname{Int}[(x_.)^{(m_)}((d_.) + (e_.)x_.)^{(q_)}((a_.) + (b_.)x_.)^2 + (c_.)x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Simp}[((-d)^{(m/2 - 1)}(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^{(q + 1)})/(2*e^{(2*p + m/2)}(q + 1)), x] + \operatorname{Dist}[(-d)^{(m/2 - 1)}/(2*e^{(2*p + m/2)}(q + 1)), x]$

```
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^{10}(c + dx) (a + b \sinh^4(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a - 2ax^2 + (a+b)x^4)^3}{x^{10}(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b^3 \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-2a^3 + 10a^3x^2 - 2a^2(10a+3b)x^4 + 2a^2(10a+9b)x^6 - 2a^2(10a+3b)x^8 + 2a^2(10a+9b)x^{10}}{x^{10}(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b^3 \cosh(c + dx) \sinh(c + dx)}{2d} - \frac{\operatorname{Subst}\left(\int \left(-\frac{2a^3}{x^{10}} + \frac{8a^3}{x^8} - \frac{6a^2(2a+b)}{x^6} + \frac{4a^2(2a+3b)}{x^4} - \frac{2a^2(10a+3b)}{x^2} + 2a^2\right) dx, x, \tanh(c + dx)\right)}{d} \\
&= -\frac{a(a^2 + 3ab + 3b^2) \operatorname{coth}(c + dx)}{d} + \frac{2a^2(2a + 3b) \operatorname{coth}^3(c + dx)}{3d} - \frac{3a^2(2a + 3b)}{3d} \\
&= -\frac{b^3x}{2} - \frac{a(a^2 + 3ab + 3b^2) \operatorname{coth}(c + dx)}{d} + \frac{2a^2(2a + 3b) \operatorname{coth}^3(c + dx)}{3d} - \frac{3a^2(2a + 3b)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.579417, size = 115, normalized size = 0.82

$$\frac{315b^3(\sinh(2(c + dx)) - 2(c + dx)) - 4a \operatorname{coth}(c + dx) (35a^2 \operatorname{csch}^8(c + dx) - 40a^2 \operatorname{csch}^6(c + dx) + 128a^2 + 3a(16a + 63b) \operatorname{csch}^4(c + dx))}{1260d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^10*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] $(-4*a*\text{Coth}[c + d*x]*(128*a^2 + 504*a*b + 945*b^2 - 4*a*(16*a + 63*b)*\text{Csch}[c + d*x]^2 + 3*a*(16*a + 63*b)*\text{Csch}[c + d*x]^4 - 40*a^2*\text{Csch}[c + d*x]^6 + 35*a^2*\text{Csch}[c + d*x]^8) + 315*b^3*(-2*(c + d*x) + \text{Sinh}[2*(c + d*x)]))/ (1260*d)$

Maple [A] time = 0.084, size = 130, normalized size = 0.9

$$\frac{1}{d} \left(a^3 \left(-\frac{128}{315} - \frac{(\text{csch}(dx+c))^8}{9} + \frac{8(\text{csch}(dx+c))^6}{63} - \frac{16(\text{csch}(dx+c))^4}{105} + \frac{64(\text{csch}(dx+c))^2}{315} \right) \text{coth}(dx+c) + 3a^2b \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x)

[Out] $1/d*(a^3*(-128/315-1/9*\text{csch}(d*x+c)^8+8/63*\text{csch}(d*x+c)^6-16/105*\text{csch}(d*x+c)^4+64/315*\text{csch}(d*x+c)^2)*\text{coth}(d*x+c)+3*a^2*b*(-8/15-1/5*\text{csch}(d*x+c)^4+4/15*\text{csch}(d*x+c)^2)*\text{coth}(d*x+c)-3*a*b^2*\text{coth}(d*x+c)+b^3*(1/2*\text{cosh}(d*x+c)*\text{sinh}(d*x+c)-1/2*d*x-1/2*c))$

Maxima [B] time = 1.21684, size = 1137, normalized size = 8.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] $-1/8*b^3*(4*x - e^{(2*d*x + 2*c)}/d + e^{(-2*d*x - 2*c)}/d) - 256/315*a^3*(9*e^{(-2*d*x - 2*c)}/(d*(9*e^{(-2*d*x - 2*c)} - 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} - 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} - 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} - 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} - 1)) - 36*e^{(-4*d*x - 4*c)}/(d*(9*e^{(-2*d*x - 2*c)} - 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} - 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} - 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} - 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} - 1)) + 84*e^{(-6*d*x - 6*c)}/(d*(9*e^{(-2*d*x - 2*c)} - 36*e^{(-4*d*x - 4*c)} + 84*e^{(-6*d*x - 6*c)} - 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} - 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} - 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} - 1)) + 84*e^{(-6*d*x - 6*c)} - 126*e^{(-8*d*x - 8*c)} + 126*e^{(-10*d*x - 10*c)} - 84*e^{(-12*d*x - 12*c)} + 36*e^{(-14*d*x - 14*c)} - 9*e^{(-16*d*x - 16*c)} + e^{(-18*d*x - 18*c)} - 1))$

$$\begin{aligned}
& (-18dx - 18c) - 1) - 126e^{(-8dx - 8c)} / (d(9e^{(-2dx - 2c)} - 36e^{(-4dx - 4c)} + 84e^{(-6dx - 6c)} - 126e^{(-8dx - 8c)} + 126e^{(-10dx - 10c)} - 84e^{(-12dx - 12c)} + 36e^{(-14dx - 14c)} - 9e^{(-16dx - 16c)} + e^{(-18dx - 18c)} - 1)) - 1 / (d(9e^{(-2dx - 2c)} - 36e^{(-4dx - 4c)} + 84e^{(-6dx - 6c)} - 126e^{(-8dx - 8c)} + 126e^{(-10dx - 10c)} - 84e^{(-12dx - 12c)} + 36e^{(-14dx - 14c)} - 9e^{(-16dx - 16c)} + e^{(-18dx - 18c)} - 1))) - 16/5a^2b(5e^{(-2dx - 2c)} / (d(5e^{(-2dx - 2c)} - 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} - 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} - 1)) - 10e^{(-4dx - 4c)} / (d(5e^{(-2dx - 2c)} - 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} - 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} - 1)) - 1 / (d(5e^{(-2dx - 2c)} - 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} - 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} - 1))) + 6ab^2 / (d(e^{(-2dx - 2c)} - 1))
\end{aligned}$$

Fricas [B] time = 1.56726, size = 3646, normalized size = 26.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^10*(a+b*sinh(dx+c)^4)^3,x, algorithm="fricas")

[Out] $1/2520*(315b^3\cosh(dx + c)^{11} + 3465b^3\cosh(dx + c)\sinh(dx + c)^{10} - (1024a^3 + 4032a^2b + 7560a*b^2 + 2835b^3)*\cosh(dx + c)^9 - 4*(315b^3dx - 256a^3 - 1008a^2b - 1890a*b^2)*\sinh(dx + c)^9 + 9*(5775b^3\cosh(dx + c)^3 - (1024a^3 + 4032a^2b + 7560a*b^2 + 2835b^3)*\cosh(dx + c))*\sinh(dx + c)^8 + 9*(1024a^3 + 4032a^2b + 5880a*b^2 + 1225b^3)*\cosh(dx + c)^7 + 36*(315b^3dx - 256a^3 - 1008a^2b - 1890a*b^2 - 4*(315b^3dx - 256a^3 - 1008a^2b - 1890a*b^2))*\cosh(dx + c)^2*\sinh(dx + c)^7 + 21*(6930b^3\cosh(dx + c)^5 - 4*(1024a^3 + 4032a^2b + 7560a*b^2 + 2835b^3)*\cosh(dx + c))^3 + 3*(1024a^3 + 4032a^2b + 5880a*b^2 + 1225b^3)*\cosh(dx + c))*\sinh(dx + c)^6 - 9*(4096a^3 + 16128a^2b + 16800a*b^2 + 2625b^3)*\cosh(dx + c)^5 - 36*(1260b^3dx + 14*(315b^3dx - 256a^3 - 1008a^2b - 1890a*b^2))*\cosh(dx + c)^4 - 1024a^3 - 4032a^2b - 7560a*b^2 - 21*(315b^3dx - 256a^3 - 1008a^2b - 1890a*b^2))*\cosh(dx + c)^2*\sinh(dx + c)^5 + 9*(11550b^3\cosh(dx + c)^7 - 14*(1024a^3 + 4032a^2b + 7560a*b^2 + 2835b^3)*\cosh(dx + c))^5 + 35*(1024a^3 + 4032a^2b + 5880a*b^2 + 1225b^3)*\cosh(dx + c))^3 - 5*(4096a^3 + 16128a^2b + 16800a*b^2 + 2625b^3)*\cosh(dx + c))*\sinh(dx + c)^4 + 42*(2048a^3 + 6144a^2b + 5040a*b^2 + 675b^3)*\cosh(dx + c)^3 - 12*(28*(315b^3dx - 256a^3 - 1008a^2b - 1890a*b^2))*\cosh(dx + c)^6 - 8820b^3dx - 105*(315b^3dx - 256a^3 - 1008a^2b - 1890a*b^2))*\cosh(dx + c)^4 + 7168a^3 + 28224$

```

*a^2*b + 52920*a*b^2 + 120*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(1925*b^3*cosh(d*x + c)^9 - 4*(1024*
a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^7 + 21*(1024*a^3 +
4032*a^2*b + 5880*a*b^2 + 1225*b^3)*cosh(d*x + c)^5 - 10*(4096*a^3 + 16128*
a^2*b + 16800*a*b^2 + 2625*b^3)*cosh(d*x + c)^3 + 14*(2048*a^3 + 6144*a^2*b
+ 5040*a*b^2 + 675*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 126*(1024*a^3 + 1
152*a^2*b + 840*a*b^2 + 105*b^3)*cosh(d*x + c) - 36*((315*b^3*d*x - 256*a^3
- 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^8 - 7*(315*b^3*d*x - 256*a^3 - 10
08*a^2*b - 1890*a*b^2)*cosh(d*x + c)^6 + 4410*b^3*d*x + 20*(315*b^3*d*x - 2
56*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 - 3584*a^3 - 14112*a^2*b
- 26460*a*b^2 - 28*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^9 + 9*(4*d*cosh(d*x + c)^2 - d)*
sinh(d*x + c)^7 + 9*(14*d*cosh(d*x + c)^4 - 21*d*cosh(d*x + c)^2 + 4*d)*sin
h(d*x + c)^5 + 3*(28*d*cosh(d*x + c)^6 - 105*d*cosh(d*x + c)^4 + 120*d*cosh
(d*x + c)^2 - 28*d)*sinh(d*x + c)^3 + 9*(d*cosh(d*x + c)^8 - 7*d*cosh(d*x +
c)^6 + 20*d*cosh(d*x + c)^4 - 28*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**10*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.71514, size = 495, normalized size = 3.54

$$-\frac{(dx+c)b^3}{2d} + \frac{b^3e^{2dx+2c}}{8d} + \frac{(2b^3e^{2dx+2c} - b^3)e^{(-2dx-2c)}}{8d} - \frac{2(945ab^2e^{16dx+16c} - 7560ab^2e^{14dx+14c} + 5040a^2be^{12dx+12c})}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] -1/2*(d*x + c)*b^3/d + 1/8*b^3*e^(2*d*x + 2*c)/d + 1/8*(2*b^3*e^(2*d*x + 2*c) - b^3)*e^(-2*d*x - 2*c)/d - 2/315*(945*a*b^2*e^(16*d*x + 16*c) - 7560*a*b^2*e^(14*d*x + 14*c) + 5040*a^2*b*e^(12*d*x + 12*c) + 26460*a*b^2*e^(12*d*x + 12*c))

$$\begin{aligned} & x + 12c) - 22680a^2b e^{(10dx + 10c)} - 52920a^2b^2 e^{(10dx + 10c)} + \\ & 16128a^3 e^{(8dx + 8c)} + 40824a^2b e^{(8dx + 8c)} + 66150a^2b^2 e^{(8 \\ & dx + 8c)} - 10752a^3 e^{(6dx + 6c)} - 37296a^2b e^{(6dx + 6c)} - 529 \\ & 20a^2b^2 e^{(6dx + 6c)} + 4608a^3 e^{(4dx + 4c)} + 18144a^2b e^{(4dx \\ & + 4c)} + 26460a^2b^2 e^{(4dx + 4c)} - 1152a^3 e^{(2dx + 2c)} - 4536a^2b \\ & b e^{(2dx + 2c)} - 7560a^2b^2 e^{(2dx + 2c)} + 128a^3 + 504a^2b + 945a \\ & ab^2)/(d(e^{(2dx + 2c)} - 1)^9) \end{aligned}$$

3.224 $\int \operatorname{csch}^{12}(c + dx) \left(a + b \sinh^4(c + dx)\right)^3 dx$

Optimal. Leaf size=147

$$-\frac{a(5a^2 + 9ab + 3b^2) \operatorname{coth}^3(c + dx)}{3d} + \frac{a(a^2 + 3ab + 3b^2) \operatorname{coth}(c + dx)}{d} - \frac{a^2(10a + 3b) \operatorname{coth}^7(c + dx)}{7d} + \frac{a^2(10a + 9b) \operatorname{coth}^{11}(c + dx)}{5d}$$

```
[Out] b^3*x + (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a*(5*a^2 + 9*a*b + 3*b^2)*Coth[c + d*x]^3)/(3*d) + (a^2*(10*a + 9*b)*Coth[c + d*x]^5)/(5*d) - (a^2*(10*a + 3*b)*Coth[c + d*x]^7)/(7*d) + (5*a^3*Coth[c + d*x]^9)/(9*d) - (a^3*Coth[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.149097, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3217, 1261, 207}

$$-\frac{a(5a^2 + 9ab + 3b^2) \operatorname{coth}^3(c + dx)}{3d} + \frac{a(a^2 + 3ab + 3b^2) \operatorname{coth}(c + dx)}{d} - \frac{a^2(10a + 3b) \operatorname{coth}^7(c + dx)}{7d} + \frac{a^2(10a + 9b) \operatorname{coth}^{11}(c + dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] b^3*x + (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a*(5*a^2 + 9*a*b + 3*b^2)*Coth[c + d*x]^3)/(3*d) + (a^2*(10*a + 9*b)*Coth[c + d*x]^5)/(5*d) - (a^2*(10*a + 3*b)*Coth[c + d*x]^7)/(7*d) + (5*a^3*Coth[c + d*x]^9)/(9*d) - (a^3*Coth[c + d*x]^11)/(11*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)]/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 1261

```
Int[((f_.)*(x_.))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
```

$b^2 - 4ac, 0$ && IGtQ[p, 0] && IGtQ[q, -2]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{csch}^{12}(c+dx) (a+b \sinh^4(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^3}{x^{12}(1-x^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{12}} - \frac{5a^3}{x^{10}} + \frac{a^2(10a+3b)}{x^8} - \frac{a^2(10a+9b)}{x^6} + \frac{a(5a^2+9ab+3b^2)}{x^4} - \frac{a(a^2+3ab+3b^2)}{x^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{a(a^2+3ab+3b^2) \operatorname{coth}(c+dx)}{d} - \frac{a(5a^2+9ab+3b^2) \operatorname{coth}^3(c+dx)}{3d} + \frac{a^2}{3d} \\ &= b^3x + \frac{a(a^2+3ab+3b^2) \operatorname{coth}(c+dx)}{d} - \frac{a(5a^2+9ab+3b^2) \operatorname{coth}^3(c+dx)}{3d} \end{aligned}$$

Mathematica [A] time = 6.10498, size = 239, normalized size = 1.63

$$\frac{\operatorname{csch}^3(c+dx) (-2376a^2b \cosh(c+dx) - 640a^3 \cosh(c+dx) - 3465ab^2 \cosh(c+dx))}{3465d} + \frac{2 \operatorname{csch}(c+dx) (2376a^2b \cosh(c+dx) - 640a^3 \cosh(c+dx) - 3465ab^2 \cosh(c+dx))}{3465d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] (b^3*(c + d*x))/d + (2*(640*a^3*Cosh[c + d*x] + 2376*a^2*b*Cosh[c + d*x] + 3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x])/(3465*d) + ((-640*a^3*Cosh[c + d*x] - 2376*a^2*b*Cosh[c + d*x] - 3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x]^3)/(3465*d) + (2*(80*a^3*Cosh[c + d*x] + 297*a^2*b*Cosh[c + d*x])*Csch[c + d*x]^5)/(1155*d) + ((-80*a^3*Cosh[c + d*x] - 297*a^2*b*Cosh[c + d*x])*Csch[c + d*x]^7)/(693*d) + (10*a^3*Coth[c + d*x]*Csch[c + d*x]^8)/(99*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^10)/(11*d)

Maple [A] time = 0.086, size = 145, normalized size = 1.

$$\frac{1}{d} \left(a^3 \left(\frac{256}{693} - \frac{(\operatorname{csch}(dx+c))^{10}}{11} + \frac{10(\operatorname{csch}(dx+c))^8}{99} - \frac{80(\operatorname{csch}(dx+c))^6}{693} + \frac{32(\operatorname{csch}(dx+c))^4}{231} - \frac{128(\operatorname{csch}(dx+c))}{693} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x)`

[Out] $\frac{1}{d} \left(a^3 \left(\frac{256}{693} - \frac{(\operatorname{csch}(dx+c))^{10}}{11} + \frac{10(\operatorname{csch}(dx+c))^8}{99} - \frac{80(\operatorname{csch}(dx+c))^6}{693} + \frac{32(\operatorname{csch}(dx+c))^4}{231} - \frac{128(\operatorname{csch}(dx+c))}{693} \right) + 3a^2b \left(\frac{16}{35} - \frac{1}{7} \operatorname{csch}(dx+c)^6 + \frac{6}{35} \operatorname{csch}(dx+c)^4 - \frac{8}{35} \operatorname{csch}(dx+c)^2 \right) \operatorname{coth}(dx+c) + 3ab^2 \left(\frac{2}{3} - \frac{1}{3} \operatorname{csch}(dx+c)^2 \right) \operatorname{coth}(dx+c) + b^3(dx+c) \right)$

Maxima [B] time = 1.24004, size = 1743, normalized size = 11.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")`

[Out] $b^3x + \frac{512}{693}a^3 \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} - 55e^{-4dx-4c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} + 165e^{-6dx-6c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} - 330e^{-8dx-8c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} + 462e^{-10dx-10c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} - 462e^{-12dx-12c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} + 330e^{-14dx-14c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} - 165e^{-16dx-16c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} + 55e^{-18dx-18c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} - 11e^{-20dx-20c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} + e^{-22dx-22c} \frac{(11e^{-2dx-2c})}{(d(11e^{-2dx-2c}) - 55e^{-4dx-4c} + 165e^{-6dx-6c} - 330e^{-8dx-8c} + 462e^{-10dx-10c} - 462e^{-12dx-12c} + 330e^{-14dx-14c} - 165e^{-16dx-16c} + 55e^{-18dx-18c} - 11e^{-20dx-20c} + e^{-22dx-22c} - 1)} - 1)$

$$\begin{aligned}
& (-20*d*x - 20*c) + e^{(-22*d*x - 22*c) - 1}) - 1/(d*(11*e^{(-2*d*x - 2*c)} - 5* \\
& 5*e^{(-4*d*x - 4*c)} + 165*e^{(-6*d*x - 6*c)} - 330*e^{(-8*d*x - 8*c)} + 462*e^{(- \\
& 10*d*x - 10*c)} - 462*e^{(-12*d*x - 12*c)} + 330*e^{(-14*d*x - 14*c)} - 165*e^{(- \\
& 16*d*x - 16*c)} + 55*e^{(-18*d*x - 18*c)} - 11*e^{(-20*d*x - 20*c)} + e^{(-22*d*x \\
& - 22*c) - 1})) + 96/35*a^2*b*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} - \\
& 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10 \\
& *d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1)) - 21*e^{(-4*d \\
& *x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c} \\
&) - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{ \\
& (-14*d*x - 14*c)} - 1)) + 35*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{ \\
& (-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x \\
& - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1)) - 1/(d*(7*e^{(-2*d \\
& *x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} \\
& + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1))) \\
& + 4*a*b^2*(3*e^{(-2*d*x - 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} \\
& + e^{(-6*d*x - 6*c)} - 1)) - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + \\
& e^{(-6*d*x - 6*c)} - 1)))
\end{aligned}$$

Fricas [B] time = 1.72119, size = 4510, normalized size = 30.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] 1/3465*(2*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^11 + 22*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + (3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*sinh(d*x + c)^11 - 22*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^9 - 11*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2) * cosh(d*x + c)^2)*sinh(d*x + c)^9 + 66*(5*(640*a^3 + 2376*a^2*b + 3465*a*b^2) * cosh(d*x + c)^3 - 3*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 110*(640*a^3 + 2376*a^2*b + 3087*a*b^2)*cosh(d*x + c)^7 + 11*(17325*b^3*d*x + 30*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^4 - 6400*a^3 - 23760*a^2*b - 34650*a*b^2 - 36*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 154*(6*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^5 - 12*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^3 + 5*(640*a^3 + 2376*a^2*b + 3087*a*b^2) * cosh(d*x + c))*sinh(d*x + c)^6 - 330*(640*a^3 + 2376*a^2*b + 2415*a*b^2) * cosh(d*x + c)^5 + 33*(14*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2) * cosh(d*x + c)^6 - 17325*b^3*d*x - 42*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b

$$\begin{aligned}
& b - 6930*a*b^2)*\cosh(d*x + c)^4 + 6400*a^3 + 23760*a^2*b + 34650*a*b^2 + 35 \\
& *(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*\cosh(d*x + c)^2)*\sinh(\\
& d*x + c)^5 + 22*(30*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*\cosh(d*x + c)^7 - 1 \\
& 26*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*\cosh(d*x + c)^5 + 175*(640*a^3 + 237 \\
& 6*a^2*b + 3087*a*b^2)*\cosh(d*x + c)^3 - 75*(640*a^3 + 2376*a^2*b + 2415*a*b \\
& ^2)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 660*(640*a^3 + 1872*a^2*b + 1533*a*b^2 \\
&)*\cosh(d*x + c)^3 + 11*(15*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b \\
& ^2)*\cosh(d*x + c)^8 - 84*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2 \\
&)*\cosh(d*x + c)^6 + 103950*b^3*d*x + 175*(3465*b^3*d*x - 1280*a^3 - 4752*a^ \\
& 2*b - 6930*a*b^2)*\cosh(d*x + c)^4 - 38400*a^3 - 142560*a^2*b - 207900*a*b^2 \\
& - 150*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*\cosh(d*x + c)^2) \\
& *\sinh(d*x + c)^3 + 22*(5*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*\cosh(d*x + c)^ \\
& 9 - 36*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*\cosh(d*x + c)^7 + 105*(640*a^3 + \\
& 2376*a^2*b + 3087*a*b^2)*\cosh(d*x + c)^5 - 150*(640*a^3 + 2376*a^2*b + 241 \\
& 5*a*b^2)*\cosh(d*x + c)^3 + 90*(640*a^3 + 1872*a^2*b + 1533*a*b^2)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^2 - 4620*(128*a^3 + 144*a^2*b + 105*a*b^2)*\cosh(d*x + c \\
&) + 11*((3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*\cosh(d*x + c)^1 \\
& 0 - 9*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*\cosh(d*x + c)^8 + \\
& 35*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*\cosh(d*x + c)^6 - 1 \\
& 45530*b^3*d*x - 75*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*\cosh \\
& (d*x + c)^4 + 53760*a^3 + 199584*a^2*b + 291060*a*b^2 + 90*(3465*b^3*d*x - \\
& 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(d*\sinh \\
& (d*x + c)^11 + 11*(5*d*\cosh(d*x + c)^2 - d)*\sinh(d*x + c)^9 + 11*(30*d*\cosh \\
& (d*x + c)^4 - 36*d*\cosh(d*x + c)^2 + 5*d)*\sinh(d*x + c)^7 + 33*(14*d*\cosh(d \\
& *x + c)^6 - 42*d*\cosh(d*x + c)^4 + 35*d*\cosh(d*x + c)^2 - 5*d)*\sinh(d*x + c \\
&)^5 + 11*(15*d*\cosh(d*x + c)^8 - 84*d*\cosh(d*x + c)^6 + 175*d*\cosh(d*x + c) \\
& ^4 - 150*d*\cosh(d*x + c)^2 + 30*d)*\sinh(d*x + c)^3 + 11*(d*\cosh(d*x + c)^10 \\
& - 9*d*\cosh(d*x + c)^8 + 35*d*\cosh(d*x + c)^6 - 75*d*\cosh(d*x + c)^4 + 90*d \\
& *\cosh(d*x + c)^2 - 42*d)*\sinh(d*x + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**12*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.6987, size = 485, normalized size = 3.3

$$\frac{(dx + c)b^3}{d} - \frac{4(10395 ab^2 e^{(18dx+18c)} - 86625 ab^2 e^{(16dx+16c)} + 83160 a^2 b e^{(14dx+14c)} + 318780 ab^2 e^{(14dx+14c)} - 382536 a^2 b e^{(12dx+12c)} + 295680 a^3 e^{(10dx+10c)} + 715176 a^2 b e^{(10dx+10c)} + 921690 a b^2 e^{(10dx+10c)} - 211200 a^3 e^{(8dx+8c)} - 700920 a^2 b e^{(8dx+8c)} - 824670 a b^2 e^{(8dx+8c)} + 105600 a^3 e^{(6dx+6c)} + 392040 a^2 b e^{(6dx+6c)} + 485100 a b^2 e^{(6dx+6c)} - 35200 a^3 e^{(4dx+4c)} - 130680 a^2 b e^{(4dx+4c)} - 180180 a b^2 e^{(4dx+4c)} + 7040 a^3 e^{(2dx+2c)} + 26136 a^2 b e^{(2dx+2c)} + 38115 a b^2 e^{(2dx+2c)} - 640 a^3 - 2376 a^2 b - 3465 a b^2)}{(d*(e^{(2dx+2c)} - 1)^{11})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] (d*x + c)*b^3/d - 4/3465*(10395*a*b^2*e^(18*d*x + 18*c) - 86625*a*b^2*e^(16*d*x + 16*c) + 83160*a^2*b*e^(14*d*x + 14*c) + 318780*a*b^2*e^(14*d*x + 14*c) - 382536*a^2*b*e^(12*d*x + 12*c) - 679140*a*b^2*e^(12*d*x + 12*c) + 295680*a^3*e^(10*d*x + 10*c) + 715176*a^2*b*e^(10*d*x + 10*c) + 921690*a*b^2*e^(10*d*x + 10*c) - 211200*a^3*e^(8*d*x + 8*c) - 700920*a^2*b*e^(8*d*x + 8*c) - 824670*a*b^2*e^(8*d*x + 8*c) + 105600*a^3*e^(6*d*x + 6*c) + 392040*a^2*b*e^(6*d*x + 6*c) + 485100*a*b^2*e^(6*d*x + 6*c) - 35200*a^3*e^(4*d*x + 4*c) - 130680*a^2*b*e^(4*d*x + 4*c) - 180180*a*b^2*e^(4*d*x + 4*c) + 7040*a^3*e^(2*d*x + 2*c) + 26136*a^2*b*e^(2*d*x + 2*c) + 38115*a*b^2*e^(2*d*x + 2*c) - 640*a^3 - 2376*a^2*b - 3465*a*b^2)/(d*(e^(2*d*x + 2*c) - 1)^11)

3.225 $\int \operatorname{csch}^{14}(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=144

$$-\frac{a^2(5a+b)\operatorname{coth}^9(c+dx)}{3d} + \frac{4a^2(5a+3b)\operatorname{coth}^7(c+dx)}{7d} - \frac{a^3\operatorname{coth}^{13}(c+dx)}{13d} + \frac{6a^3\operatorname{coth}^{11}(c+dx)}{11d} - \frac{3a(a+b)(5a+b)}{5d}$$

[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*a*(a + b)^2*Coth[c + d*x]^3)/d - (3*a*(a + b)*(5*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^2*(5*a + 3*b)*Coth[c + d*x]^7)/(7*d) - (a^2*(5*a + b)*Coth[c + d*x]^9)/(3*d) + (6*a^3*Coth[c + d*x]^11)/(11*d) - (a^3*Coth[c + d*x]^13)/(13*d)

Rubi [A] time = 0.130776, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3217, 1108}

$$-\frac{a^2(5a+b)\operatorname{coth}^9(c+dx)}{3d} + \frac{4a^2(5a+3b)\operatorname{coth}^7(c+dx)}{7d} - \frac{a^3\operatorname{coth}^{13}(c+dx)}{13d} + \frac{6a^3\operatorname{coth}^{11}(c+dx)}{11d} - \frac{3a(a+b)(5a+b)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*a*(a + b)^2*Coth[c + d*x]^3)/d - (3*a*(a + b)*(5*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^2*(5*a + 3*b)*Coth[c + d*x]^7)/(7*d) - (a^2*(5*a + b)*Coth[c + d*x]^9)/(3*d) + (6*a^3*Coth[c + d*x]^11)/(11*d) - (a^3*Coth[c + d*x]^13)/(13*d)

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol]
:> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 1108

```
Int[((d_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[p, 0] && !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \operatorname{csch}^{14}(c+dx) (a+b \sinh^4(c+dx))^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(a-2ax^2+(a+b)x^4)^3}{x^{14}} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{14}} - \frac{6a^3}{x^{12}} + \frac{3a^2(5a+b)}{x^{10}} - \frac{4a^2(5a+3b)}{x^8} + \frac{3a(a+b)(5a+b)}{x^6} - \frac{6a(a+b)^2}{x^4} + \frac{(a+b)^3}{x^2}\right) dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a+b)^3 \operatorname{coth}(c+dx)}{d} + \frac{2a(a+b)^2 \operatorname{coth}^3(c+dx)}{d} - \frac{3a(a+b)(5a+b) \operatorname{coth}(c+dx)}{5d}$$

Mathematica [B] time = 3.20677, size = 350, normalized size = 2.43

$$\frac{\operatorname{csch}^{13}(c+dx) (8580 (1152a^2b + 1024a^3 + 840ab^2 + 231b^3) \cosh(c+dx) - 6435 (2944a^2b + 1024a^3 + 2408ab^2 + 693b^3))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] -((8580*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c + d*x] - 6435*(1024*a^3 + 2944*a^2*b + 2408*a*b^2 + 693*b^3)*Cosh[3*(c + d*x)] + 3660800*a^3*Cosh[5*(c + d*x)] + 13087360*a^2*b*Cosh[5*(c + d*x)] + 13093080*a*b^2*Cosh[5*(c + d*x)] + 4129125*b^3*Cosh[5*(c + d*x)] - 1464320*a^3*Cosh[7*(c + d*x)] - 5234944*a^2*b*Cosh[7*(c + d*x)] - 6390384*a*b^2*Cosh[7*(c + d*x)] - 2312310*b^3*Cosh[7*(c + d*x)] + 399360*a^3*Cosh[9*(c + d*x)] + 1427712*a^2*b*Cosh[9*(c + d*x)] + 1873872*a*b^2*Cosh[9*(c + d*x)] + 810810*b^3*Cosh[9*(c + d*x)] - 66560*a^3*Cosh[11*(c + d*x)] - 237952*a^2*b*Cosh[11*(c + d*x)] - 312312*a*b^2*Cosh[11*(c + d*x)] - 165165*b^3*Cosh[11*(c + d*x)] + 5120*a^3*Cosh[13*(c + d*x)] + 18304*a^2*b*Cosh[13*(c + d*x)] + 24024*a*b^2*Cosh[13*(c + d*x)] + 15015*b^3*Cosh[13*(c + d*x)])*Csch[c + d*x]^13)/(61501440*d)

Maple [A] time = 0.098, size = 177, normalized size = 1.2

$$\frac{1}{d} \left(a^3 \left(-\frac{1024}{3003} - \frac{(\operatorname{csch}(dx+c))^{12}}{13} + \frac{12(\operatorname{csch}(dx+c))^{10}}{143} - \frac{40(\operatorname{csch}(dx+c))^8}{429} + \frac{320(\operatorname{csch}(dx+c))^6}{3003} - \frac{128(\operatorname{csch}(dx+c))^4}{1001} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{csch}(d*x+c)^{14}*(a+b*\sinh(d*x+c)^4)^3,x)$

[Out] $\frac{1}{d}*(a^3*(-1024/3003-1/13*\text{csch}(d*x+c)^{12}+12/143*\text{csch}(d*x+c)^{10}-40/429*\text{csch}(d*x+c)^8+320/3003*\text{csch}(d*x+c)^6-128/1001*\text{csch}(d*x+c)^4+512/3003*\text{csch}(d*x+c)^2)*\text{coth}(d*x+c)+3*a^2*b*(-128/315-1/9*\text{csch}(d*x+c)^8+8/63*\text{csch}(d*x+c)^6-16/105*\text{csch}(d*x+c)^4+64/315*\text{csch}(d*x+c)^2)*\text{coth}(d*x+c)+3*a*b^2*(-8/15-1/5*\text{csch}(d*x+c)^4+4/15*\text{csch}(d*x+c)^2)*\text{coth}(d*x+c)-b^3*\text{coth}(d*x+c))$

Maxima [B] time = 1.30355, size = 2587, normalized size = 17.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\text{csch}(d*x+c)^{14}*(a+b*\sinh(d*x+c)^4)^3,x, \text{algorithm}=\text{"maxima"})$

[Out]
$$\frac{-2048/3003*a^3*(13*e^{(-2*d*x - 2*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) - 78*e^{(-4*d*x - 4*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) + 286*e^{(-6*d*x - 6*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) - 715*e^{(-8*d*x - 8*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) + 1287*e^{(-10*d*x - 10*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) - 1716*e^{(-12*d*x - 12*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) - 1716*e^{(-14*d*x - 14*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) - 1287*e^{(-16*d*x - 16*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) + 715*e^{(-18*d*x - 18*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) - 286*e^{(-20*d*x - 20*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) + 78*e^{(-22*d*x - 22*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) - 13*e^{(-24*d*x - 24*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1)) + e^{(-26*d*x - 26*c)} / (d*(13*e^{(-2*d*x - 2*c)} - 78*e^{(-4*d*x - 4*c)} + 286*e^{(-6*d*x - 6*c)} - 715*e^{(-8*d*x - 8*c)} + 1287*e^{(-10*d*x - 10*c)} - 1716*e^{(-12*d*x - 12*c)} + 1716*e^{(-14*d*x - 14*c)} - 1287*e^{(-16*d*x - 16*c)} + 715*e^{(-18*d*x - 18*c)} - 286*e^{(-20*d*x - 20*c)} + 78*e^{(-22*d*x - 22*c)} - 13*e^{(-24*d*x - 24*c)} + e^{(-26*d*x - 26*c)} - 1))$$

```
16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x -
22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 1/(d*(13*e^(-2*d
*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*
c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x -
14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x -
20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c)
- 1))) - 256/105*a^2*b*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-
4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x
- 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 1
6*c) + e^(-18*d*x - 18*c) - 1)) - 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c
) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*
e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2
*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8
*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c
) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 126*e^(-8*d*x - 8*c)/
(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^
(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14
*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 1/(d*(9*e^
(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x
- 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 1
4*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1))) - 16/5*a*b^2*(5*e^(-
2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)
/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1))) + 2*b^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.80262, size = 6778, normalized size = 47.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -4/15015*((2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^1
2 - 48*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 +
(2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*sinh(d*x + c)^12 - 52*(6
40*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^10 - 2*(16640*a^
3 + 59488*a^2*b + 78078*a*b^2 + 90090*b^3 - 33*(2560*a^3 + 9152*a^2*b + 120
```


$$\begin{aligned}
& 12*a*b^2 + 15015*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^{10} - 40*(22*(640*a^3 + \\
& 2288*a^2*b + 3003*a*b^2)*\cosh(d*x + c)^3 - 13*(640*a^3 + 2288*a^2*b + 3003 \\
& *a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^9 + 78*(2560*a^3 + 9152*a^2*b + 13552* \\
& a*b^2 + 12705*b^3)*\cosh(d*x + c)^8 + 3*(165*(2560*a^3 + 9152*a^2*b + 12012* \\
& a*b^2 + 15015*b^3)*\cosh(d*x + c)^4 + 66560*a^3 + 237952*a^2*b + 352352*a*b^ \\
& 2 + 330330*b^3 - 780*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*\cosh(d* \\
& x + c)^2)*\sinh(d*x + c)^8 - 96*(33*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*\cosh \\
& (d*x + c)^5 - 65*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*\cosh(d*x + c)^3 + 52*(\\
& 320*a^3 + 1144*a^2*b + 1309*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^7 - 572*(12 \\
& 80*a^3 + 4576*a^2*b + 7581*a*b^2 + 5775*b^3)*\cosh(d*x + c)^6 + 4*(231*(2560 \\
& *a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*\cosh(d*x + c)^6 - 2730*(640*a^ \\
& 3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*\cosh(d*x + c)^4 - 183040*a^3 - 6543 \\
& 68*a^2*b - 1084083*a*b^2 - 825825*b^3 + 546*(2560*a^3 + 9152*a^2*b + 13552* \\
& a*b^2 + 12705*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 - 24*(132*(640*a^3 + 22 \\
& 88*a^2*b + 3003*a*b^2)*\cosh(d*x + c)^7 - 546*(640*a^3 + 2288*a^2*b + 3003*a \\
& *b^2)*\cosh(d*x + c)^5 + 1456*(320*a^3 + 1144*a^2*b + 1309*a*b^2)*\cosh(d*x + \\
& c)^3 - 143*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^5 + 143*(12800*a^3 + 53824*a^2*b + 79884*a*b^2 + 51975*b^3)*\cosh(d*x + c \\
&)^4 + (495*(2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*\cosh(d*x + c)^ \\
& 8 - 10920*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*\cosh(d*x + c)^6 + \\
& 5460*(2560*a^3 + 9152*a^2*b + 13552*a*b^2 + 12705*b^3)*\cosh(d*x + c)^4 + 18 \\
& 30400*a^3 + 7696832*a^2*b + 11423412*a*b^2 + 7432425*b^3 - 8580*(1280*a^3 + \\
& 4576*a^2*b + 7581*a*b^2 + 5775*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 - 16* \\
& (55*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*\cosh(d*x + c)^9 - 390*(640*a^3 + 22 \\
& 88*a^2*b + 3003*a*b^2)*\cosh(d*x + c)^7 + 2184*(320*a^3 + 1144*a^2*b + 1309* \\
& a*b^2)*\cosh(d*x + c)^5 - 715*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*\cosh(d*x \\
& + c)^3 + 143*(3200*a^3 + 9424*a^2*b + 6489*a*b^2)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^3 + 4392960*a^3 + 10323456*a^2*b + 12108096*a*b^2 + 6936930*b^3 - 3432* \\
& (960*a^3 + 4664*a^2*b + 5859*a*b^2 + 3465*b^3)*\cosh(d*x + c)^2 + 6*(11*(256 \\
& 0*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*\cosh(d*x + c)^10 - 390*(640*a \\
& ^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*\cosh(d*x + c)^8 + 364*(2560*a^3 + \\
& 9152*a^2*b + 13552*a*b^2 + 12705*b^3)*\cosh(d*x + c)^6 - 1430*(1280*a^3 + 45 \\
& 76*a^2*b + 7581*a*b^2 + 5775*b^3)*\cosh(d*x + c)^4 - 549120*a^3 - 2667808*a^ \\
& 2*b - 3351348*a*b^2 - 1981980*b^3 + 143*(12800*a^3 + 53824*a^2*b + 79884*a* \\
& b^2 + 51975*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 8*(6*(640*a^3 + 2288*a^ \\
& 2*b + 3003*a*b^2)*\cosh(d*x + c)^11 - 65*(640*a^3 + 2288*a^2*b + 3003*a*b^2) \\
& *\cosh(d*x + c)^9 + 624*(320*a^3 + 1144*a^2*b + 1309*a*b^2)*\cosh(d*x + c)^7 \\
& - 429*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*\cosh(d*x + c)^5 + 286*(3200*a^3 \\
& + 9424*a^2*b + 6489*a*b^2)*\cosh(d*x + c)^3 - 858*(960*a^3 + 1528*a^2*b + 90 \\
& 3*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^14 + 14*d*\cosh(d*x \\
& + c)*\sinh(d*x + c)^13 + d*\sinh(d*x + c)^14 - 14*d*\cosh(d*x + c)^12 + 7*(13* \\
& d*\cosh(d*x + c)^2 - 2*d)*\sinh(d*x + c)^12 + 4*(91*d*\cosh(d*x + c)^3 - 36*d* \\
& \cosh(d*x + c))*\sinh(d*x + c)^11 + 91*d*\cosh(d*x + c)^10 + 7*(143*d*\cosh(d*x \\
& + c)^4 - 132*d*\cosh(d*x + c)^2 + 13*d)*\sinh(d*x + c)^10 + 2*(1001*d*\cosh(d \\
& *x + c)^5 - 1320*d*\cosh(d*x + c)^3 + 325*d*\cosh(d*x + c))*\sinh(d*x + c)^9 -
\end{aligned}$$

$$\begin{aligned}
& 364*d*\cosh(d*x + c)^8 + 7*(429*d*\cosh(d*x + c)^6 - 990*d*\cosh(d*x + c)^4 + \\
& 585*d*\cosh(d*x + c)^2 - 52*d)*\sinh(d*x + c)^8 + 8*(429*d*\cosh(d*x + c)^7 - \\
& 1188*d*\cosh(d*x + c)^5 + 975*d*\cosh(d*x + c)^3 - 208*d*\cosh(d*x + c))*\sinh \\
& (d*x + c)^7 + 1001*d*\cosh(d*x + c)^6 + 7*(429*d*\cosh(d*x + c)^8 - 1848*d*\cosh \\
& (d*x + c)^6 + 2730*d*\cosh(d*x + c)^4 - 1456*d*\cosh(d*x + c)^2 + 143*d)*\sinh \\
& (d*x + c)^6 + 2*(1001*d*\cosh(d*x + c)^9 - 4752*d*\cosh(d*x + c)^7 + 8190*d \\
& *\cosh(d*x + c)^5 - 5824*d*\cosh(d*x + c)^3 + 1287*d*\cosh(d*x + c))*\sinh(d*x \\
& + c)^5 - 2002*d*\cosh(d*x + c)^4 + 7*(143*d*\cosh(d*x + c)^10 - 990*d*\cosh(d*x \\
& + c)^8 + 2730*d*\cosh(d*x + c)^6 - 3640*d*\cosh(d*x + c)^4 + 2145*d*\cosh(d*x \\
& + c)^2 - 286*d)*\sinh(d*x + c)^4 + 4*(91*d*\cosh(d*x + c)^11 - 660*d*\cosh(d \\
& *x + c)^9 + 1950*d*\cosh(d*x + c)^7 - 2912*d*\cosh(d*x + c)^5 + 2145*d*\cosh(d \\
& *x + c)^3 - 572*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3003*d*\cosh(d*x + c)^2 + \\
& 7*(13*d*\cosh(d*x + c)^12 - 132*d*\cosh(d*x + c)^10 + 585*d*\cosh(d*x + c)^8 \\
& - 1456*d*\cosh(d*x + c)^6 + 2145*d*\cosh(d*x + c)^4 - 1716*d*\cosh(d*x + c)^2 \\
& + 429*d)*\sinh(d*x + c)^2 + 2*(7*d*\cosh(d*x + c)^13 - 72*d*\cosh(d*x + c)^11 \\
& + 325*d*\cosh(d*x + c)^9 - 832*d*\cosh(d*x + c)^7 + 1287*d*\cosh(d*x + c)^5 - \\
& 1144*d*\cosh(d*x + c)^3 + 429*d*\cosh(d*x + c))*\sinh(d*x + c) - 1716*d)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**14*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.66735, size = 760, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] $-2/15015*(15015*b^3*e^{(24*d*x + 24*c)} - 180180*b^3*e^{(22*d*x + 22*c)} + 240240*a*b^2*e^{(20*d*x + 20*c)} + 990990*b^3*e^{(20*d*x + 20*c)} - 2042040*a*b^2*e^{(18*d*x + 18*c)} - 3303300*b^3*e^{(18*d*x + 18*c)} + 2306304*a^2*b*e^{(16*d*x$

$$\begin{aligned}
& + 16*c) + 7711704*a*b^2*e^{(16*d*x + 16*c)} + 7432425*b^3*e^{(16*d*x + 16*c)} - \\
& 10762752*a^2*b*e^{(14*d*x + 14*c)} - 17008992*a*b^2*e^{(14*d*x + 14*c)} - 1189 \\
& 1880*b^3*e^{(14*d*x + 14*c)} + 8785920*a^3*e^{(12*d*x + 12*c)} + 20646912*a^2*b \\
& *e^{(12*d*x + 12*c)} + 24216192*a*b^2*e^{(12*d*x + 12*c)} + 13873860*b^3*e^{(12* \\
& d*x + 12*c)} - 6589440*a^3*e^{(10*d*x + 10*c)} - 21250944*a^2*b*e^{(10*d*x + 10 \\
& *c)} - 23207184*a*b^2*e^{(10*d*x + 10*c)} - 11891880*b^3*e^{(10*d*x + 10*c)} + 3 \\
& 660800*a^3*e^{(8*d*x + 8*c)} + 13087360*a^2*b*e^{(8*d*x + 8*c)} + 15135120*a*b^ \\
& 2*e^{(8*d*x + 8*c)} + 7432425*b^3*e^{(8*d*x + 8*c)} - 1464320*a^3*e^{(6*d*x + 6* \\
& c)} - 5234944*a^2*b*e^{(6*d*x + 6*c)} - 6630624*a*b^2*e^{(6*d*x + 6*c)} - 330330 \\
& 0*b^3*e^{(6*d*x + 6*c)} + 399360*a^3*e^{(4*d*x + 4*c)} + 1427712*a^2*b*e^{(4*d*x \\
& + 4*c)} + 1873872*a*b^2*e^{(4*d*x + 4*c)} + 990990*b^3*e^{(4*d*x + 4*c)} - 6656 \\
& 0*a^3*e^{(2*d*x + 2*c)} - 237952*a^2*b*e^{(2*d*x + 2*c)} - 312312*a*b^2*e^{(2*d* \\
& x + 2*c)} - 180180*b^3*e^{(2*d*x + 2*c)} + 5120*a^3 + 18304*a^2*b + 24024*a*b^ \\
& 2 + 15015*b^3)/(d*(e^{(2*d*x + 2*c)} - 1)^13)
\end{aligned}$$

3.226 $\int \operatorname{csch}^{16}(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=182

$$\frac{a(35a^2 + 30ab + 3b^2) \operatorname{coth}^7(c + dx)}{7d} - \frac{3a^2(7a + b) \operatorname{coth}^{11}(c + dx)}{11d} + \frac{5a^2(7a + 3b) \operatorname{coth}^9(c + dx)}{9d} - \frac{a^3 \operatorname{coth}^{15}(c + dx)}{15d} +$$

[Out] $((a + b)^3 \operatorname{Coth}[c + d*x])/d - ((a + b)^2 (7*a + b) \operatorname{Coth}[c + d*x]^3)/(3*d) + (3*a*(a + b)*(7*a + 3*b) \operatorname{Coth}[c + d*x]^5)/(5*d) - (a*(35*a^2 + 30*a*b + 3*b^2) \operatorname{Coth}[c + d*x]^7)/(7*d) + (5*a^2*(7*a + 3*b) \operatorname{Coth}[c + d*x]^9)/(9*d) - (3*a^2*(7*a + b) \operatorname{Coth}[c + d*x]^11)/(11*d) + (7*a^3 \operatorname{Coth}[c + d*x]^13)/(13*d) - (a^3 \operatorname{Coth}[c + d*x]^15)/(15*d)$

Rubi [A] time = 0.158821, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3217, 1261}

$$\frac{a(35a^2 + 30ab + 3b^2) \operatorname{coth}^7(c + dx)}{7d} - \frac{3a^2(7a + b) \operatorname{coth}^{11}(c + dx)}{11d} + \frac{5a^2(7a + 3b) \operatorname{coth}^9(c + dx)}{9d} - \frac{a^3 \operatorname{coth}^{15}(c + dx)}{15d} +$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^{16} (a + b \operatorname{Sinh}[c + d*x]^4)^3, x]$

[Out] $((a + b)^3 \operatorname{Coth}[c + d*x])/d - ((a + b)^2 (7*a + b) \operatorname{Coth}[c + d*x]^3)/(3*d) + (3*a*(a + b)*(7*a + 3*b) \operatorname{Coth}[c + d*x]^5)/(5*d) - (a*(35*a^2 + 30*a*b + 3*b^2) \operatorname{Coth}[c + d*x]^7)/(7*d) + (5*a^2*(7*a + 3*b) \operatorname{Coth}[c + d*x]^9)/(9*d) - (3*a^2*(7*a + b) \operatorname{Coth}[c + d*x]^11)/(11*d) + (7*a^3 \operatorname{Coth}[c + d*x]^13)/(13*d) - (a^3 \operatorname{Coth}[c + d*x]^15)/(15*d)$

Rule 3217

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)} ((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m (a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]] /; \operatorname{FreeQ}\{a, b, e, f\}, x] \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 1261

$\operatorname{Int}[(f_.)*(x_.)]^{(m_.)} ((d_.) + (e_.)*(x_.)^2)^{(q_.)} ((a_.) + (b_.)*(x_.)^2 + (c_.)*(x_.)^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ExpandIntegrand}[(f*x)^m (d + e*x^2)^q (a + b*x^2 + c*x^4)^p, x], x]$

$(a + b*x^2 + c*x^4)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, m, q\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{IGtQ}[p, 0] \ \&\& \ \text{IGtQ}[q, -2]$

Rubi steps

$$\int \text{csch}^{16}(c + dx) (a + b \sinh^4(c + dx))^3 dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)(a-2ax^2+(a+b)x^4)^3}{x^{16}} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^{16}} - \frac{7a^3}{x^{14}} + \frac{3a^2(7a+b)}{x^{12}} - \frac{5a^2(7a+3b)}{x^{10}} + \frac{a(35a^2+30ab+3b^2)}{x^8} + \frac{3a(-7a-3b)}{x^6}\right) dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{(a + b)^3 \coth(c + dx)}{d} - \frac{(a + b)^2(7a + b) \coth^3(c + dx)}{3d} + \frac{3a(a + b)(7a + b) \coth^5(c + dx)}{5d} - \frac{3a^2(7a + b) \coth^7(c + dx)}{7d} + \frac{7a^3 \coth^9(c + dx)}{9d} - \frac{7a^3 \coth^{11}(c + dx)}{11d} + \frac{7a^3 \coth^{13}(c + dx)}{13d} - \frac{7a^3 \coth^{15}(c + dx)}{15d}$$

Mathematica [B] time = 4.50951, size = 404, normalized size = 2.22

$$\frac{\text{csch}^{15}(c + dx) (45045 (1152a^2b + 1024a^3 + 840ab^2 + 231b^3) \cosh(c + dx) - 5005 (20352a^2b + 7168a^3 + 16632ab^2 + 4785b^3) \cosh^3(c + dx) + 21525504a^3 \cosh^5(c + dx) + 74954880a^2b \cosh^5(c + dx) + 74162088a^2b^2 \cosh^5(c + dx) + 23288265b^3 \cosh^5(c + dx) - 9784320a^3 \cosh^7(c + dx) - 34070400a^2b \cosh^7(c + dx) - 39999960a^2b^2 \cosh^7(c + dx) - 14189175b^3 \cosh^7(c + dx) + 3261440a^3 \cosh^9(c + dx) + 11356800a^2b \cosh^9(c + dx) + 14054040a^2b^2 \cosh^9(c + dx) + 5720715b^3 \cosh^9(c + dx) - 752640a^3 \cosh^{11}(c + dx) - 2620800a^2b \cosh^{11}(c + dx) - 3243240a^2b^2 \cosh^{11}(c + dx) - 1486485b^3 \cosh^{11}(c + dx) + 107520a^3 \cosh^{13}(c + dx) + 374400a^2b \cosh^{13}(c + dx) + 463320a^2b^2 \cosh^{13}(c + dx) + 225225b^3 \cosh^{13}(c + dx) - 7168a^3 \cosh^{15}(c + dx) - 24960a^2b \cosh^{15}(c + dx) - 30888a^2b^2 \cosh^{15}(c + dx) - 15015b^3 \cosh^{15}(c + dx)) \text{Csch}[c + dx]^{15}}{(369008640*d)}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^16*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] $-\left((45045(1024a^3 + 1152a^2b + 840ab^2 + 231b^3) \cosh[c + d*x] - 5005(7168a^3 + 20352a^2b + 16632a^2b^2 + 4785b^3) \cosh^3[c + d*x] + 21525504a^3 \cosh^5[c + d*x] + 74954880a^2b \cosh^5[c + d*x] + 74162088a^2b^2 \cosh^5[c + d*x] + 23288265b^3 \cosh^5[c + d*x] - 9784320a^3 \cosh^7[c + d*x] - 34070400a^2b \cosh^7[c + d*x] - 39999960a^2b^2 \cosh^7[c + d*x] - 14189175b^3 \cosh^7[c + d*x] + 3261440a^3 \cosh^9[c + d*x] + 11356800a^2b \cosh^9[c + d*x] + 14054040a^2b^2 \cosh^9[c + d*x] + 5720715b^3 \cosh^9[c + d*x] - 752640a^3 \cosh^{11}[c + d*x] - 2620800a^2b \cosh^{11}[c + d*x] - 3243240a^2b^2 \cosh^{11}[c + d*x] - 1486485b^3 \cosh^{11}[c + d*x] + 107520a^3 \cosh^{13}[c + d*x] + 374400a^2b \cosh^{13}[c + d*x] + 463320a^2b^2 \cosh^{13}[c + d*x] + 225225b^3 \cosh^{13}[c + d*x] - 7168a^3 \cosh^{15}[c + d*x] - 24960a^2b \cosh^{15}[c + d*x] - 30888a^2b^2 \cosh^{15}[c + d*x] - 15015b^3 \cosh^{15}[c + d*x]) \text{Csch}[c + d*x]^{15}\right) / (369008640*d)$

Maple [A] time = 0.085, size = 218, normalized size = 1.2

$$\frac{1}{d} \left(a^3 \left(\frac{2048}{6435} - \frac{(\operatorname{csch}(dx+c))^{14}}{15} + \frac{14(\operatorname{csch}(dx+c))^{12}}{195} - \frac{56(\operatorname{csch}(dx+c))^{10}}{715} + \frac{112(\operatorname{csch}(dx+c))^8}{1287} - \frac{128(\operatorname{csch}(dx+c))^6}{1287} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x)

[Out] 1/d*(a^3*(2048/6435-1/15*csch(d*x+c)^14+14/195*csch(d*x+c)^12-56/715*csch(d*x+c)^10+112/1287*csch(d*x+c)^8-128/1287*csch(d*x+c)^6+256/2145*csch(d*x+c)^4-1024/6435*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(256/693-1/11*csch(d*x+c)^10+10/99*csch(d*x+c)^8-80/693*csch(d*x+c)^6+32/231*csch(d*x+c)^4-128/693*csch(d*x+c)^2)*coth(d*x+c)+3*a*b^2*(16/35-1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*coth(d*x+c)+b^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))

Maxima [B] time = 1.38143, size = 3687, normalized size = 20.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] 4096/6435*a^3*(15*e^(-2*d*x - 2*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 105*e^(-4*d*x - 4*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 455*e^(-6*d*x - 6*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 1365*e^(-8*d*x - 8*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 3003*e^(-10*d*x - 10*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 5005*e^(-12*d*x - 12*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 6435*e^(-14*d*x - 14*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 6435*e^(-16*d*x - 16*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 5005*e^(-18*d*x - 18*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 3003*e^(-20*d*x - 20*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 1365*e^(-22*d*x - 22*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 455*e^(-24*d*x - 24*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 15*e^(-28*d*x - 28*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - e^(-30*d*x - 30*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1))

$$\begin{aligned}
& - 4*c) + 455*e^{(-6*d*x - 6*c)} - 1365*e^{(-8*d*x - 8*c)} + 3003*e^{(-10*d*x - 10*c)} \\
& - 5005*e^{(-12*d*x - 12*c)} + 6435*e^{(-14*d*x - 14*c)} - 6435*e^{(-16*d*x - 16*c)} \\
& + 5005*e^{(-18*d*x - 18*c)} - 3003*e^{(-20*d*x - 20*c)} + 1365*e^{(-22*d*x - 22*c)} \\
& - 455*e^{(-24*d*x - 24*c)} + 105*e^{(-26*d*x - 26*c)} - 15*e^{(-28*d*x - 28*c)} \\
& + e^{(-30*d*x - 30*c)} - 1) + 3003*e^{(-10*d*x - 10*c)}/(d*(15*e^{(-2*d*x - 2*c)} \\
& - 105*e^{(-4*d*x - 4*c)} + 455*e^{(-6*d*x - 6*c)} - 1365*e^{(-8*d*x - 8*c)} \\
& + 3003*e^{(-10*d*x - 10*c)} - 5005*e^{(-12*d*x - 12*c)} + 6435*e^{(-14*d*x - 14*c)} \\
& - 6435*e^{(-16*d*x - 16*c)} + 5005*e^{(-18*d*x - 18*c)} - 3003*e^{(-20*d*x - 20*c)} \\
& + 1365*e^{(-22*d*x - 22*c)} - 455*e^{(-24*d*x - 24*c)} + 105*e^{(-26*d*x - 26*c)} \\
& - 15*e^{(-28*d*x - 28*c)} + e^{(-30*d*x - 30*c)} - 1) - 5005*e^{(-12*d*x - 12*c)}/ \\
& (d*(15*e^{(-2*d*x - 2*c)} - 105*e^{(-4*d*x - 4*c)} + 455*e^{(-6*d*x - 6*c)} - 1365*e^{(-8*d*x - 8*c)} \\
& + 3003*e^{(-10*d*x - 10*c)} - 5005*e^{(-12*d*x - 12*c)} + 6435*e^{(-14*d*x - 14*c)} \\
& - 6435*e^{(-16*d*x - 16*c)} + 5005*e^{(-18*d*x - 18*c)} - 3003*e^{(-20*d*x - 20*c)} \\
& + 1365*e^{(-22*d*x - 22*c)} - 455*e^{(-24*d*x - 24*c)} + 105*e^{(-26*d*x - 26*c)} \\
& - 15*e^{(-28*d*x - 28*c)} + e^{(-30*d*x - 30*c)} - 1) + 6435*e^{(-14*d*x - 14*c)}/ \\
& (d*(15*e^{(-2*d*x - 2*c)} - 105*e^{(-4*d*x - 4*c)} + 455*e^{(-6*d*x - 6*c)} - 1365*e^{(-8*d*x - 8*c)} \\
& + 3003*e^{(-10*d*x - 10*c)} - 5005*e^{(-12*d*x - 12*c)} + 6435*e^{(-14*d*x - 14*c)} \\
& - 6435*e^{(-16*d*x - 16*c)} + 5005*e^{(-18*d*x - 18*c)} - 3003*e^{(-20*d*x - 20*c)} \\
& + 1365*e^{(-22*d*x - 22*c)} - 455*e^{(-24*d*x - 24*c)} + 105*e^{(-26*d*x - 26*c)} - 15 \\
& *e^{(-28*d*x - 28*c)} + e^{(-30*d*x - 30*c)} - 1) - 1/(d*(15*e^{(-2*d*x - 2*c)} \\
& - 105*e^{(-4*d*x - 4*c)} + 455*e^{(-6*d*x - 6*c)} - 1365*e^{(-8*d*x - 8*c)} + 3003 \\
& *e^{(-10*d*x - 10*c)} - 5005*e^{(-12*d*x - 12*c)} + 6435*e^{(-14*d*x - 14*c)} - 6435 \\
& *e^{(-16*d*x - 16*c)} + 5005*e^{(-18*d*x - 18*c)} - 3003*e^{(-20*d*x - 20*c)} \\
& + 1365*e^{(-22*d*x - 22*c)} - 455*e^{(-24*d*x - 24*c)} + 105*e^{(-26*d*x - 26*c)} \\
& - 15*e^{(-28*d*x - 28*c)} + e^{(-30*d*x - 30*c)} - 1)) + 512/231*a^2*b*(11*e^{(-2*d*x - 2*c)}/ \\
& (d*(11*e^{(-2*d*x - 2*c)} - 55*e^{(-4*d*x - 4*c)} + 165*e^{(-6*d*x - 6*c)} - 330 \\
& *e^{(-8*d*x - 8*c)} + 462*e^{(-10*d*x - 10*c)} - 462*e^{(-12*d*x - 12*c)} + 330 \\
& *e^{(-14*d*x - 14*c)} - 165*e^{(-16*d*x - 16*c)} + 55*e^{(-18*d*x - 18*c)} - 11 \\
& *e^{(-20*d*x - 20*c)} + e^{(-22*d*x - 22*c)} - 1) - 55*e^{(-4*d*x - 4*c)}/ \\
& (d*(11*e^{(-2*d*x - 2*c)} - 55*e^{(-4*d*x - 4*c)} + 165*e^{(-6*d*x - 6*c)} - 330 \\
& *e^{(-8*d*x - 8*c)} + 462*e^{(-10*d*x - 10*c)} - 462*e^{(-12*d*x - 12*c)} + 3 \\
& 30*e^{(-14*d*x - 14*c)} - 165*e^{(-16*d*x - 16*c)} + 55*e^{(-18*d*x - 18*c)} - 11 \\
& *e^{(-20*d*x - 20*c)} + e^{(-22*d*x - 22*c)} - 1) + 165*e^{(-6*d*x - 6*c)}/(d*(11 \\
& *e^{(-2*d*x - 2*c)} - 55*e^{(-4*d*x - 4*c)} + 165*e^{(-6*d*x - 6*c)} - 330*e^{(-8 \\
& *d*x - 8*c)} + 462*e^{(-10*d*x - 10*c)} - 462*e^{(-12*d*x - 12*c)} + 330*e^{(-14 \\
& *d*x - 14*c)} - 165*e^{(-16*d*x - 16*c)} + 55*e^{(-18*d*x - 18*c)} - 11*e^{(-20*d \\
& *x - 20*c)} + e^{(-22*d*x - 22*c)} - 1) - 330*e^{(-8*d*x - 8*c)}/(d*(11*e^{(-2*d \\
& *x - 2*c)} - 55*e^{(-4*d*x - 4*c)} + 165*e^{(-6*d*x - 6*c)} - 330*e^{(-8*d*x - 8*c)} \\
&) + 462*e^{(-10*d*x - 10*c)} - 462*e^{(-12*d*x - 12*c)} + 330*e^{(-14*d*x - 14*c)} \\
&) - 165*e^{(-16*d*x - 16*c)} + 55*e^{(-18*d*x - 18*c)} - 11*e^{(-20*d*x - 20*c)} \\
& + e^{(-22*d*x - 22*c)} - 1) + 462*e^{(-10*d*x - 10*c)}/(d*(11*e^{(-2*d*x - 2*c)} \\
& - 55*e^{(-4*d*x - 4*c)} + 165*e^{(-6*d*x - 6*c)} - 330*e^{(-8*d*x - 8*c)} + 462 \\
& *e^{(-10*d*x - 10*c)} - 462*e^{(-12*d*x - 12*c)} + 330*e^{(-14*d*x - 14*c)} - 165 \\
& *e^{(-16*d*x - 16*c)} + 55*e^{(-18*d*x - 18*c)} - 11*e^{(-20*d*x - 20*c)} + e^{(-22
\end{aligned}$$

$$\begin{aligned}
& *d*x - 22*c) - 1)) - 1/(d*(11*e^{(-2*d*x - 2*c)} - 55*e^{(-4*d*x - 4*c)} + 165* \\
& e^{(-6*d*x - 6*c)} - 330*e^{(-8*d*x - 8*c)} + 462*e^{(-10*d*x - 10*c)} - 462*e^{(- \\
& 12*d*x - 12*c)} + 330*e^{(-14*d*x - 14*c)} - 165*e^{(-16*d*x - 16*c)} + 55*e^{(-1 \\
& 8*d*x - 18*c)} - 11*e^{(-20*d*x - 20*c)} + e^{(-22*d*x - 22*c)} - 1))) + 96/35*a \\
& *b^2*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35* \\
& e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d \\
& *x - 12*c)} + e^{(-14*d*x - 14*c)} - 1)) - 21*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x \\
& - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + \\
& 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1)) + \\
& 35*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6 \\
& *d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - \\
& 12*c)} + e^{(-14*d*x - 14*c)} - 1)) - 1/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x \\
& - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} \\
& - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1))) + 4/3*b^3*(3*e^{(-2*d*x - \\
& 2*c)}/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)) \\
& - 1/(d*(3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + e^{(-6*d*x - 6*c)} - 1)))
\end{aligned}$$

Fricas [B] time = 1.88967, size = 8951, normalized size = 49.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] $8/45045*((3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*\cosh(d*x + c)^13 + 13*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^12 - 2*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*\sinh(d*x + c)^13 - 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*\cosh(d*x + c)^11 + 6*(8960*a^3 + 31200*a^2*b + 38610*a*b^2 + 65065*b^3 - 26*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^11 + 11*(26*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*\cosh(d*x + c)^3 - 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^10 + 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*\cosh(d*x + c)^9 - 10*(143*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*\cosh(d*x + c)^4 + 37632*a^3 + 131040*a^2*b + 216216*a*b^2 + 234234*b^3 - 165*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^9 + 9*(143*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*\cosh(d*x + c)^5 - 275*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*\cosh(d*x + c)^3 + 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^8 - 182*(8960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*\cosh(d*x + c)^7 - 4*(858*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*\cosh(d*x$

$$\begin{aligned}
& + c)^6 - 2475*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*\cosh(d*x + c) \\
&)^4 - 407680*a^3 - 1419600*a^2*b - 2918916*a*b^2 - 2147145*b^3 + 3780*(896* \\
& a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^7 \\
& + 2*(858*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*\cosh(d*x + c)^7 \\
& - 3465*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*\cosh(d*x + c)^5 \\
& + 8820*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*\cosh(d*x + c)^3 - 63 \\
& 7*(8960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*\cosh(d*x + c))*\sinh(d* \\
& x + c)^6 + 1365*(3584*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*\cosh(d*x + \\
& c)^5 - 6*(429*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*\cosh(d*x + c) \\
&)^8 - 2310*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*\cosh(d*x + c)^6 \\
& + 8820*(896*a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*\cosh(d*x + c)^4 + 81 \\
& 5360*a^3 + 3800160*a^2*b + 6396390*a*b^2 + 3578575*b^3 - 1274*(4480*a^3 + 1 \\
& 5600*a^2*b + 32076*a*b^2 + 23595*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^5 + 5* \\
& (143*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*\cosh(d*x + c)^9 - 9 \\
& 90*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*\cosh(d*x + c)^7 + 529 \\
& 2*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*\cosh(d*x + c)^5 - 1274*(8 \\
& 960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*\cosh(d*x + c)^3 + 1365*(35 \\
& 84*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 \\
& - 429*(25088*a^3 + 24000*a^2*b + 3492*a*b^2 - 10395*b^3)*\cosh(d*x + c)^3 - \\
& 2*(286*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*\cosh(d*x + c)^10 - \\
& 2475*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*\cosh(d*x + c)^8 + 17 \\
& 640*(896*a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*\cosh(d*x + c)^6 - 6370*(\\
& 4480*a^3 + 15600*a^2*b + 32076*a*b^2 + 23595*b^3)*\cosh(d*x + c)^4 - 5381376 \\
& *a^3 - 32329440*a^2*b - 40980654*a*b^2 - 19324305*b^3 + 13650*(1792*a^3 + 8 \\
& 352*a^2*b + 14058*a*b^2 + 7865*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 3*(2 \\
& 6*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*\cosh(d*x + c)^11 - 275 \\
& *(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*\cosh(d*x + c)^9 + 2520* \\
& (1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*\cosh(d*x + c)^7 - 1274*(896 \\
& 0*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*\cosh(d*x + c)^5 + 4550*(3584 \\
& *a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*\cosh(d*x + c)^3 - 429*(25088*a^3 \\
& + 24000*a^2*b + 3492*a*b^2 - 10395*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 2 \\
& 860*(1792*a^3 - 1248*a^2*b - 108*a*b^2 + 693*b^3)*\cosh(d*x + c) - 2*(13*(17 \\
& 92*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*\cosh(d*x + c)^12 - 165*(1792* \\
& a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*\cosh(d*x + c)^10 + 1890*(896*a^3 \\
& + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*\cosh(d*x + c)^8 - 1274*(4480*a^3 + 1 \\
& 5600*a^2*b + 32076*a*b^2 + 23595*b^3)*\cosh(d*x + c)^6 + 6825*(1792*a^3 + 83 \\
& 52*a^2*b + 14058*a*b^2 + 7865*b^3)*\cosh(d*x + c)^4 + 20500480*a^3 + 5491200 \\
& 0*a^2*b + 59304960*a*b^2 + 25765740*b^3 - 1287*(12544*a^3 + 75360*a^2*b + 9 \\
& 5526*a*b^2 + 45045*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c))/(d*\cosh(d*x + c)^17 \\
& + 17*d*\cosh(d*x + c)*\sinh(d*x + c)^16 + d*\sinh(d*x + c)^17 - 15*d*\cosh(d*x \\
& + c)^15 + (136*d*\cosh(d*x + c)^2 - 15*d)*\sinh(d*x + c)^15 + 5*(136*d*\cosh(\\
& d*x + c)^3 - 45*d*\cosh(d*x + c))*\sinh(d*x + c)^14 + 104*d*\cosh(d*x + c)^13 \\
& + (2380*d*\cosh(d*x + c)^4 - 1575*d*\cosh(d*x + c)^2 + 106*d)*\sinh(d*x + c)^1 \\
& 3 + 13*(476*d*\cosh(d*x + c)^5 - 525*d*\cosh(d*x + c)^3 + 104*d*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^12 - 440*d*\cosh(d*x + c)^11 + (12376*d*\cosh(d*x + c)^6 - 20
\end{aligned}$$

$$\begin{aligned}
& 475*d*\cosh(d*x + c)^4 + 8268*d*\cosh(d*x + c)^2 - 470*d)*\sinh(d*x + c)^{11} + \\
& 11*(1768*d*\cosh(d*x + c)^7 - 4095*d*\cosh(d*x + c)^5 + 2704*d*\cosh(d*x + c)^3 - \\
& 440*d*\cosh(d*x + c))*\sinh(d*x + c)^{10} + 1260*d*\cosh(d*x + c)^9 + 5*(486 \\
& 2*d*\cosh(d*x + c)^8 - 15015*d*\cosh(d*x + c)^6 + 15158*d*\cosh(d*x + c)^4 - 5 \\
& 170*d*\cosh(d*x + c)^2 + 294*d)*\sinh(d*x + c)^9 + (24310*d*\cosh(d*x + c)^9 - \\
& 96525*d*\cosh(d*x + c)^7 + 133848*d*\cosh(d*x + c)^5 - 72600*d*\cosh(d*x + c) \\
& ^3 + 11340*d*\cosh(d*x + c))*\sinh(d*x + c)^8 - 2548*d*\cosh(d*x + c)^7 + (194 \\
& 48*d*\cosh(d*x + c)^{10} - 96525*d*\cosh(d*x + c)^8 + 181896*d*\cosh(d*x + c)^6 \\
& - 155100*d*\cosh(d*x + c)^4 + 52920*d*\cosh(d*x + c)^2 - 3458*d)*\sinh(d*x + c) \\
& ^7 + (12376*d*\cosh(d*x + c)^{11} - 75075*d*\cosh(d*x + c)^9 + 178464*d*\cosh(d \\
& *x + c)^7 - 203280*d*\cosh(d*x + c)^5 + 105840*d*\cosh(d*x + c)^3 - 17836*d*c \\
& osh(d*x + c))*\sinh(d*x + c)^6 + 3640*d*\cosh(d*x + c)^5 + (6188*d*\cosh(d*x + \\
& c)^{12} - 45045*d*\cosh(d*x + c)^{10} + 136422*d*\cosh(d*x + c)^8 - 217140*d*cos \\
& h(d*x + c)^6 + 185220*d*\cosh(d*x + c)^4 - 72618*d*\cosh(d*x + c)^2 + 6370*d) \\
& *\sinh(d*x + c)^5 + 5*(476*d*\cosh(d*x + c)^{13} - 4095*d*\cosh(d*x + c)^{11} + 14 \\
& 872*d*\cosh(d*x + c)^9 - 29040*d*\cosh(d*x + c)^7 + 31752*d*\cosh(d*x + c)^5 - \\
& 17836*d*\cosh(d*x + c)^3 + 3640*d*\cosh(d*x + c))*\sinh(d*x + c)^4 - 3432*d*c \\
& osh(d*x + c)^3 + (680*d*\cosh(d*x + c)^{14} - 6825*d*\cosh(d*x + c)^{12} + 30316* \\
& d*\cosh(d*x + c)^{10} - 77550*d*\cosh(d*x + c)^8 + 123480*d*\cosh(d*x + c)^6 - 1 \\
& 21030*d*\cosh(d*x + c)^4 + 63700*d*\cosh(d*x + c)^2 - 9438*d)*\sinh(d*x + c)^3 \\
& + (136*d*\cosh(d*x + c)^{15} - 1575*d*\cosh(d*x + c)^{13} + 8112*d*\cosh(d*x + c) \\
& ^{11} - 24200*d*\cosh(d*x + c)^9 + 45360*d*\cosh(d*x + c)^7 - 53508*d*\cosh(d*x \\
& + c)^5 + 36400*d*\cosh(d*x + c)^3 - 10296*d*\cosh(d*x + c))*\sinh(d*x + c)^2 + \\
& 1430*d*\cosh(d*x + c) + (17*d*\cosh(d*x + c)^{16} - 225*d*\cosh(d*x + c)^{14} + 1 \\
& 378*d*\cosh(d*x + c)^{12} - 5170*d*\cosh(d*x + c)^{10} + 13230*d*\cosh(d*x + c)^8 \\
& - 24206*d*\cosh(d*x + c)^6 + 31850*d*\cosh(d*x + c)^4 - 28314*d*\cosh(d*x + c) \\
& ^2 + 11440*d)*\sinh(d*x + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**16*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.70699, size = 838, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")`

[Out]
$$\begin{aligned} & -4/45045*(45045*b^3*e^{(26*d*x + 26*c)} - 55555*b^3*e^{(24*d*x + 24*c)} + 1081 \\ & 080*a*b^2*e^{(22*d*x + 22*c)} + 3153150*b^3*e^{(22*d*x + 22*c)} - 9297288*a*b^2 \\ & *e^{(20*d*x + 20*c)} - 10900890*b^3*e^{(20*d*x + 20*c)} + 11531520*a^2*b*e^{(18* \\ & d*x + 18*c)} + 35675640*a*b^2*e^{(18*d*x + 18*c)} + 25600575*b^3*e^{(18*d*x + 1 \\ & 8*c)} - 54362880*a^2*b*e^{(16*d*x + 16*c)} - 80463240*a*b^2*e^{(16*d*x + 16*c)} \\ & - 43108065*b^3*e^{(16*d*x + 16*c)} + 46126080*a^3*e^{(14*d*x + 14*c)} + 1062547 \\ & 20*a^2*b*e^{(14*d*x + 14*c)} + 118301040*a*b^2*e^{(14*d*x + 14*c)} + 53513460*b \\ & ^3*e^{(14*d*x + 14*c)} - 35875840*a^3*e^{(12*d*x + 12*c)} - 113393280*a^2*b*e^{(\\ & 12*d*x + 12*c)} - 118918800*a*b^2*e^{(12*d*x + 12*c)} - 49549500*b^3*e^{(12*d*x \\ & + 12*c)} + 21525504*a^3*e^{(10*d*x + 10*c)} + 74954880*a^2*b*e^{(10*d*x + 10*c \\ &)} + 83459376*a*b^2*e^{(10*d*x + 10*c)} + 34189155*b^3*e^{(10*d*x + 10*c)} - 978 \\ & 4320*a^3*e^{(8*d*x + 8*c)} - 34070400*a^2*b*e^{(8*d*x + 8*c)} - 41081040*a*b^2* \\ & e^{(8*d*x + 8*c)} - 17342325*b^3*e^{(8*d*x + 8*c)} + 3261440*a^3*e^{(6*d*x + 6*c \\ &)} + 11356800*a^2*b*e^{(6*d*x + 6*c)} + 14054040*a*b^2*e^{(6*d*x + 6*c)} + 62762 \\ & 70*b^3*e^{(6*d*x + 6*c)} - 752640*a^3*e^{(4*d*x + 4*c)} - 2620800*a^2*b*e^{(4*d* \\ & x + 4*c)} - 3243240*a*b^2*e^{(4*d*x + 4*c)} - 1531530*b^3*e^{(4*d*x + 4*c)} + 10 \\ & 7520*a^3*e^{(2*d*x + 2*c)} + 374400*a^2*b*e^{(2*d*x + 2*c)} + 463320*a*b^2*e^{(2 \\ & *d*x + 2*c)} + 225225*b^3*e^{(2*d*x + 2*c)} - 7168*a^3 - 24960*a^2*b - 30888*a \\ & *b^2 - 15015*b^3)/(d*(e^{(2*d*x + 2*c)} - 1)^{15}) \end{aligned}$$

3.227 $\int \operatorname{csch}^{18}(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=221

$$-\frac{a(70a^2 + 45ab + 3b^2) \operatorname{coth}^9(c + dx)}{9d} + \frac{4a(14a^2 + 15ab + 3b^2) \operatorname{coth}^7(c + dx)}{7d} - \frac{(a + b)(28a^2 + 17ab + b^2) \operatorname{coth}^5(c + dx)}{5d}$$

[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*(a + b)^2*(4*a + b)*Coth[c + d*x]^3)/(3*d) - ((a + b)*(28*a^2 + 17*a*b + b^2)*Coth[c + d*x]^5)/(5*d) + (4*a*(14*a^2 + 15*a*b + 3*b^2)*Coth[c + d*x]^7)/(7*d) - (a*(70*a^2 + 45*a*b + 3*b^2)*Coth[c + d*x]^9)/(9*d) + (2*a^2*(28*a + 9*b)*Coth[c + d*x]^11)/(11*d) - (a^2*(28*a + 3*b)*Coth[c + d*x]^13)/(13*d) + (8*a^3*Coth[c + d*x]^15)/(15*d) - (a^3*Coth[c + d*x]^17)/(17*d)

Rubi [A] time = 0.191817, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3217, 1261}

$$-\frac{a(70a^2 + 45ab + 3b^2) \operatorname{coth}^9(c + dx)}{9d} + \frac{4a(14a^2 + 15ab + 3b^2) \operatorname{coth}^7(c + dx)}{7d} - \frac{(a + b)(28a^2 + 17ab + b^2) \operatorname{coth}^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*(a + b)^2*(4*a + b)*Coth[c + d*x]^3)/(3*d) - ((a + b)*(28*a^2 + 17*a*b + b^2)*Coth[c + d*x]^5)/(5*d) + (4*a*(14*a^2 + 15*a*b + 3*b^2)*Coth[c + d*x]^7)/(7*d) - (a*(70*a^2 + 45*a*b + 3*b^2)*Coth[c + d*x]^9)/(9*d) + (2*a^2*(28*a + 9*b)*Coth[c + d*x]^11)/(11*d) - (a^2*(28*a + 3*b)*Coth[c + d*x]^13)/(13*d) + (8*a^3*Coth[c + d*x]^15)/(15*d) - (a^3*Coth[c + d*x]^17)/(17*d)

Rule 3217

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \operatorname{csch}^{18}(c + dx) (a + b \sinh^4(c + dx))^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2 (a-2ax^2+(a+b)x^4)^3}{x^{18}} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{18}} - \frac{8a^3}{x^{16}} + \frac{a^2(28a+3b)}{x^{14}} - \frac{2a^2(28a+9b)}{x^{12}} + \frac{a(70a^2+45ab+3b^2)}{x^{10}} - \frac{4a(14a^2+3b^2)}{x^8} + \frac{3b^2}{x^6}\right) dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{(a+b)^3 \operatorname{coth}(c + dx)}{d} + \frac{2(a+b)^2(4a+b) \operatorname{coth}^3(c + dx)}{3d} - \frac{(a+b)(28a^2+3b^2)}{3d} \operatorname{coth}(c + dx)$$

Mathematica [B] time = 6.18111, size = 494, normalized size = 2.24

$$\operatorname{csch}^{17}(c + dx) \left(-784143360a^2b \operatorname{cosh}(c + dx) + 1568286720a^2b \operatorname{cosh}(3(c + dx)) - 1211857920a^2b \operatorname{cosh}(5(c + dx)) + 605928960a^2b \operatorname{cosh}(7(c + dx)) + 692659968a^2b \operatorname{cosh}(9(c + dx)) - 233049600a^2b \operatorname{cosh}(11(c + dx)) - 277717440a^2b \operatorname{cosh}(13(c + dx)) - 108738630b^3 \operatorname{cosh}(9(c + dx)) + 19496960a^3 \operatorname{cosh}(11(c + dx)) + 66585600a^2b \operatorname{cosh}(11(c + dx)) + 79347840a^2b \operatorname{cosh}(11(c + dx)) + 33693660b^3 \operatorname{cosh}(11(c + dx)) - 3899392a^3 \operatorname{cosh}(13(c + dx)) - 13317120a^2b \operatorname{cosh}(13(c + dx)) - 15869568a^2b \operatorname{cosh}(13(c + dx)) - 6942936b^3 \operatorname{cosh}(13(c + dx)) + 487424a^3 \operatorname{cosh}(15(c + dx)) + 1664640a^2b \operatorname{cosh}(15(c + dx)) + 1983696a^2b \operatorname{cosh}(15(c + dx)) + 867867b^3 \operatorname{cosh}(15(c + dx)) - 28672a^3 \operatorname{cosh}(17(c + dx)) - 97920a^2b \operatorname{cosh}(17(c + dx)) - 116688a^2b \operatorname{cosh}(17(c + dx)) - \dots\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]

[Out] ((-697016320*a^3*Cosh[c + d*x] - 784143360*a^2*b*Cosh[c + d*x] - 571771200*a*b^2*Cosh[c + d*x] - 157237080*b^3*Cosh[c + d*x] + 557613056*a^3*Cosh[3*(c + d*x)] + 1568286720*a^2*b*Cosh[3*(c + d*x)] + 1280767488*a*b^2*Cosh[3*(c + d*x)] + 368384016*b^3*Cosh[3*(c + d*x)] - 354844672*a^3*Cosh[5*(c + d*x)] - 1211857920*a^2*b*Cosh[5*(c + d*x)] - 1189284096*a*b^2*Cosh[5*(c + d*x)] - 372263892*b^3*Cosh[5*(c + d*x)] + 177422336*a^3*Cosh[7*(c + d*x)] + 605928960*a^2*b*Cosh[7*(c + d*x)] + 692659968*a*b^2*Cosh[7*(c + d*x)] + 242288046*b^3*Cosh[7*(c + d*x)] - 68239360*a^3*Cosh[9*(c + d*x)] - 233049600*a^2*b*Cosh[9*(c + d*x)] - 277717440*a*b^2*Cosh[9*(c + d*x)] - 108738630*b^3*Cosh[9*(c + d*x)] + 19496960*a^3*Cosh[11*(c + d*x)] + 66585600*a^2*b*Cosh[11*(c + d*x)] + 79347840*a*b^2*Cosh[11*(c + d*x)] + 33693660*b^3*Cosh[11*(c + d*x)] - 3899392*a^3*Cosh[13*(c + d*x)] - 13317120*a^2*b*Cosh[13*(c + d*x)] - 15869568*a^2*b*Cosh[13*(c + d*x)] - 6942936*b^3*Cosh[13*(c + d*x)] + 487424*a^3*Cosh[15*(c + d*x)] + 1664640*a^2*b*Cosh[15*(c + d*x)] + 1983696*a^2*b*Cosh[15*(c + d*x)] + 867867*b^3*Cosh[15*(c + d*x)] - 28672*a^3*Cosh[17*(c + d*x)] - 97920*a^2*b*Cosh[17*(c + d*x)] - 116688*a^2*b*Cosh[17*(c + d*x)] -

51051*b^3*Cosh[17*(c + d*x)]*Csch[c + d*x]^17)/(6273146880*d)

Maple [A] time = 0.085, size = 258, normalized size = 1.2

$$\frac{1}{d} \left(a^3 \left(-\frac{32768}{109395} - \frac{(\operatorname{csch}(dx+c))^{16}}{17} + \frac{16(\operatorname{csch}(dx+c))^{14}}{255} - \frac{224(\operatorname{csch}(dx+c))^{12}}{3315} + \frac{896(\operatorname{csch}(dx+c))^{10}}{12155} - \frac{1792(\operatorname{csch}(dx+c))^{8}}{21879} + \frac{2048(\operatorname{csch}(dx+c))^{6}}{21879} - \frac{4096(\operatorname{csch}(dx+c))^{4}}{36465} + \frac{16384(\operatorname{csch}(dx+c))^{2}}{109395} \right) \operatorname{coth}(dx+c) + 3a^2b(-1024/3003 - 1/13 \operatorname{csch}(dx+c)^{12} + 12/143 \operatorname{csch}(dx+c)^{10} - 40/429 \operatorname{csch}(dx+c)^8 + 320/3003 \operatorname{csch}(dx+c)^6 - 128/1001 \operatorname{csch}(dx+c)^4 + 512/3003 \operatorname{csch}(dx+c)^2) \operatorname{coth}(dx+c) + 3ab^2(-128/315 - 1/9 \operatorname{csch}(dx+c)^8 + 8/63 \operatorname{csch}(dx+c)^6 - 16/105 \operatorname{csch}(dx+c)^4 + 64/315 \operatorname{csch}(dx+c)^2) \operatorname{coth}(dx+c) + b^3(-8/15 - 1/5 \operatorname{csch}(dx+c)^4 + 4/15 \operatorname{csch}(dx+c)^2) \operatorname{coth}(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x)

[Out] 1/d*(a^3*(-32768/109395-1/17*csch(d*x+c)^16+16/255*csch(d*x+c)^14-224/3315*csch(d*x+c)^12+896/12155*csch(d*x+c)^10-1792/21879*csch(d*x+c)^8+2048/21879*csch(d*x+c)^6-4096/36465*csch(d*x+c)^4+16384/109395*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-1024/3003-1/13*csch(d*x+c)^12+12/143*csch(d*x+c)^10-40/429*csch(d*x+c)^8+320/3003*csch(d*x+c)^6-128/1001*csch(d*x+c)^4+512/3003*csch(d*x+c)^2)*coth(d*x+c)+3*a*b^2*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x+c)^6-16/105*csch(d*x+c)^4+64/315*csch(d*x+c)^2)*coth(d*x+c)+b^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)

Maxima [B] time = 1.29654, size = 5021, normalized size = 22.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] -65536/109395*a^3*(17*e^(-2*d*x - 2*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1) - 136*e^(-4*d*x - 4*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x -

$$126e^{(-8dx - 8c)} + 126e^{(-10dx - 10c)} - 84e^{(-12dx - 12c)} + 36e^{(-14dx - 14c)} - 9e^{(-16dx - 16c)} + e^{(-18dx - 18c)} - 1))) - 16/15b^3(5e^{(-2dx - 2c)}/(d(5e^{(-2dx - 2c)} - 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} - 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} - 1)) - 10e^{(-4dx - 4c)}/(d(5e^{(-2dx - 2c)} - 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} - 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} - 1)) - 1/(d(5e^{(-2dx - 2c)} - 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} - 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} - 1)))$$

Fricas [B] time = 1.90939, size = 11491, normalized size = 52.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)^18*(a+b*sinh(dx+c)^4)^3,x, algorithm="fricas")

[Out] $-16/765765*((28672a^3 + 97920a^2b + 116688ab^2 + 561561b^3)*\cosh(dx + c)^{14} - 14*(28672a^3 + 97920a^2b + 116688ab^2 - 459459b^3)*\cosh(dx + c)*\sinh(dx + c)^{13} + (28672a^3 + 97920a^2b + 116688ab^2 + 561561b^3)*\sinh(dx + c)^{14} - 34*(14336a^3 + 48960a^2b + 58344ab^2 + 213213b^3)*\cosh(dx + c)^{12} - (487424a^3 + 1664640a^2b + 1983696ab^2 + 7249242b^3 - 91*(28672a^3 + 97920a^2b + 116688ab^2 + 561561b^3)*\cosh(dx + c)^2)*\sinh(dx + c)^{12} - 4*(91*(28672a^3 + 97920a^2b + 116688ab^2 - 459459b^3)*\cosh(dx + c)^3 - 204*(7168a^3 + 24480a^2b + 29172ab^2 - 81081b^3)*\cosh(dx + c))*\sinh(dx + c)^{11} + 17*(229376a^3 + 783360a^2b + 1798368ab^2 + 2573571b^3)*\cosh(dx + c)^{10} + (1001*(28672a^3 + 97920a^2b + 116688ab^2 + 561561b^3)*\cosh(dx + c)^4 + 3899392a^3 + 13317120a^2b + 30572256ab^2 + 43750707b^3 - 2244*(14336a^3 + 48960a^2b + 58344ab^2 + 213213b^3)*\cosh(dx + c)^2)*\sinh(dx + c)^{10} - 2*(1001*(28672a^3 + 97920a^2b + 116688ab^2 - 459459b^3)*\cosh(dx + c)^5 - 7480*(7168a^3 + 24480a^2b + 29172ab^2 - 81081b^3)*\cosh(dx + c)^3 + 85*(229376a^3 + 783360a^2b + 68640ab^2 - 1756755b^3)*\cosh(dx + c))*\sinh(dx + c)^9 - 68*(286720a^3 + 979200a^2b + 3040752ab^2 + 2411409b^3)*\cosh(dx + c)^8 + (3003*(28672a^3 + 97920a^2b + 116688ab^2 + 561561b^3)*\cosh(dx + c)^6 - 16830*(14336a^3 + 48960a^2b + 58344ab^2 + 213213b^3)*\cosh(dx + c)^4 - 19496960a^3 - 66585600a^2b - 206771136ab^2 - 163975812b^3 + 765*(229376a^3 + 783360a^2b + 1798368ab^2 + 2573571b^3)*\cosh(dx + c)^2)*\sinh(dx + c)^8 - 8*(429*(28672a^3 + 97920a^2b + 116688ab^2 - 459459b^3)*\cosh(dx + c)^7 - 6732*(7168a^3 + 24480a^2b + 29172ab^2 - 81081b^3)*\cosh(dx + c)^5 + 255*(229376a^3 + 783360a^2b + 68640ab^2 - 1756755b^3)*\cosh(dx + c)^3 - 272*(71680a^3 + 244800a^2b - 176748ab^2$

$$\begin{aligned}
& 2 - 351351*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 17*(4014080*a^3 + 23592960 \\
& *a^2*b + 45412224*a*b^2 + 25138113*b^3)*\cosh(d*x + c)^6 + (3003*(28672*a^3 \\
& + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*\cosh(d*x + c)^8 - 31416*(14336*a \\
& ^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b^3)*\cosh(d*x + c)^6 + 3570*(229376 \\
& *a^3 + 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*\cosh(d*x + c)^4 + 682393 \\
& 60*a^3 + 401080320*a^2*b + 772007808*a*b^2 + 427347921*b^3 - 1904*(286720*a \\
& ^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^6 - 2*(1001*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*\cosh \\
& (d*x + c)^9 - 26928*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*\cosh \\
& (d*x + c)^7 + 2142*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^3)* \\
& \cosh(d*x + c)^5 - 7616*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 - 351351*b^ \\
& 3)*\cosh(d*x + c)^3 + 51*(4014080*a^3 + 3824640*a^2*b - 12739584*a*b^2 - 115 \\
& 94583*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 442*(401408*a^3 + 3176640*a^2*b \\
& + 4162488*a*b^2 + 1857471*b^3)*\cosh(d*x + c)^4 + (1001*(28672*a^3 + 97920* \\
& a^2*b + 116688*a*b^2 + 561561*b^3)*\cosh(d*x + c)^10 - 16830*(14336*a^3 + 48 \\
& 960*a^2*b + 58344*a*b^2 + 213213*b^3)*\cosh(d*x + c)^8 + 3570*(229376*a^3 + \\
& 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*\cosh(d*x + c)^6 - 4760*(286720* \\
& a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*\cosh(d*x + c)^4 - 1774223 \\
& 36*a^3 - 1404074880*a^2*b - 1839819696*a*b^2 - 821002182*b^3 + 255*(4014080 \\
& *a^3 + 23592960*a^2*b + 45412224*a*b^2 + 25138113*b^3)*\cosh(d*x + c)^2)*\sin \\
& h(d*x + c)^4 - 4*(91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)* \\
& \cosh(d*x + c)^11 - 3740*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)* \\
& \cosh(d*x + c)^9 + 510*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^ \\
& 3)*\cosh(d*x + c)^7 - 3808*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 - 351351 \\
& *b^3)*\cosh(d*x + c)^5 + 85*(4014080*a^3 + 3824640*a^2*b - 12739584*a*b^2 - \\
& 11594583*b^3)*\cosh(d*x + c)^3 - 884*(200704*a^3 - 217440*a^2*b - 480876*a*b \\
& ^2 - 297297*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 557613056*a^3 - 173631744 \\
& 0*a^2*b - 1775057856*a*b^2 - 680611932*b^3 + 221*(4759552*a^3 + 12643200*a^ \\
& 2*b + 13669392*a*b^2 + 5435199*b^3)*\cosh(d*x + c)^2 + (91*(28672*a^3 + 9792 \\
& 0*a^2*b + 116688*a*b^2 + 561561*b^3)*\cosh(d*x + c)^12 - 2244*(14336*a^3 + 4 \\
& 8960*a^2*b + 58344*a*b^2 + 213213*b^3)*\cosh(d*x + c)^10 + 765*(229376*a^3 + \\
& 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*\cosh(d*x + c)^8 - 1904*(286720 \\
& *a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*\cosh(d*x + c)^6 + 255*(4 \\
& 014080*a^3 + 23592960*a^2*b + 45412224*a*b^2 + 25138113*b^3)*\cosh(d*x + c)^ \\
& 4 + 1051860992*a^3 + 2794147200*a^2*b + 3020935632*a*b^2 + 1201178979*b^3 - \\
& 2652*(401408*a^3 + 3176640*a^2*b + 4162488*a*b^2 + 1857471*b^3)*\cosh(d*x + \\
& c)^2)*\sinh(d*x + c)^2 - 2*(7*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459 \\
& 459*b^3)*\cosh(d*x + c)^13 - 408*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 810 \\
& 81*b^3)*\cosh(d*x + c)^11 + 85*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 17 \\
& 56755*b^3)*\cosh(d*x + c)^9 - 1088*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 \\
& - 351351*b^3)*\cosh(d*x + c)^7 + 51*(4014080*a^3 + 3824640*a^2*b - 12739584* \\
& a*b^2 - 11594583*b^3)*\cosh(d*x + c)^5 - 1768*(200704*a^3 - 217440*a^2*b - 4 \\
& 80876*a*b^2 - 297297*b^3)*\cosh(d*x + c)^3 - 5967*(57344*a^3 + 62080*a^2*b + \\
& 64944*a*b^2 + 33033*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^20 \\
& + 20*d*\cosh(d*x + c)*\sinh(d*x + c)^19 + d*\sinh(d*x + c)^20 - 17*d*\cosh(d*x
\end{aligned}$$

$$\begin{aligned}
& + c)^{18} + (190*d*\cosh(d*x + c)^2 - 17*d)*\sinh(d*x + c)^{18} + 6*(190*d*\cosh(\\
& d*x + c)^3 - 51*d*\cosh(d*x + c))*\sinh(d*x + c)^{17} + 136*d*\cosh(d*x + c)^{16} \\
& + 17*(285*d*\cosh(d*x + c)^4 - 153*d*\cosh(d*x + c)^2 + 8*d)*\sinh(d*x + c)^{16} \\
& + 272*(57*d*\cosh(d*x + c)^5 - 51*d*\cosh(d*x + c)^3 + 8*d*\cosh(d*x + c))*\si \\
& nh(d*x + c)^{15} - 681*d*\cosh(d*x + c)^{14} + 3*(12920*d*\cosh(d*x + c)^6 - 1734 \\
& 0*d*\cosh(d*x + c)^4 + 5440*d*\cosh(d*x + c)^2 - 227*d)*\sinh(d*x + c)^{14} + 2* \\
& (38760*d*\cosh(d*x + c)^7 - 72828*d*\cosh(d*x + c)^5 + 38080*d*\cosh(d*x + c)^ \\
& 3 - 4753*d*\cosh(d*x + c))*\sinh(d*x + c)^{13} + 2397*d*\cosh(d*x + c)^{12} + (125 \\
& 970*d*\cosh(d*x + c)^8 - 315588*d*\cosh(d*x + c)^6 + 247520*d*\cosh(d*x + c)^4 \\
& - 61971*d*\cosh(d*x + c)^2 + 2397*d)*\sinh(d*x + c)^{12} + 4*(41990*d*\cosh(d*x \\
& + c)^9 - 135252*d*\cosh(d*x + c)^7 + 148512*d*\cosh(d*x + c)^5 - 61789*d*\cos \\
& h(d*x + c)^3 + 7089*d*\cosh(d*x + c))*\sinh(d*x + c)^{11} - 6324*d*\cosh(d*x + c \\
&)^{10} + (184756*d*\cosh(d*x + c)^{10} - 743886*d*\cosh(d*x + c)^8 + 1089088*d*\co \\
& sh(d*x + c)^6 - 681681*d*\cosh(d*x + c)^4 + 158202*d*\cosh(d*x + c)^2 - 6324* \\
& d)*\sinh(d*x + c)^{10} + 2*(83980*d*\cosh(d*x + c)^{11} - 413270*d*\cosh(d*x + c)^ \\
& 9 + 777920*d*\cosh(d*x + c)^7 - 679679*d*\cosh(d*x + c)^5 + 259930*d*\cosh(d*x \\
& + c)^3 - 30260*d*\cosh(d*x + c))*\sinh(d*x + c)^9 + 13056*d*\cosh(d*x + c)^8 \\
& + 3*(41990*d*\cosh(d*x + c)^{12} - 247962*d*\cosh(d*x + c)^{10} + 583440*d*\cosh(d \\
& *x + c)^8 - 681681*d*\cosh(d*x + c)^6 + 395505*d*\cosh(d*x + c)^4 - 94860*d*\c \\
& osh(d*x + c)^2 + 4352*d)*\sinh(d*x + c)^8 + 8*(9690*d*\cosh(d*x + c)^{13} - 676 \\
& 26*d*\cosh(d*x + c)^{11} + 194480*d*\cosh(d*x + c)^9 - 291291*d*\cosh(d*x + c)^7 \\
& + 233937*d*\cosh(d*x + c)^5 - 90780*d*\cosh(d*x + c)^3 + 11696*d*\cosh(d*x + \\
& c))*\sinh(d*x + c)^7 - 21828*d*\cosh(d*x + c)^6 + (38760*d*\cosh(d*x + c)^{14} - \\
& 315588*d*\cosh(d*x + c)^{12} + 1089088*d*\cosh(d*x + c)^{10} - 2045043*d*\cosh(d* \\
& x + c)^8 + 2214828*d*\cosh(d*x + c)^6 - 1328040*d*\cosh(d*x + c)^4 + 365568*d \\
& *\cosh(d*x + c)^2 - 21828*d)*\sinh(d*x + c)^6 + 2*(7752*d*\cosh(d*x + c)^{15} - \\
& 72828*d*\cosh(d*x + c)^{13} + 297024*d*\cosh(d*x + c)^{11} - 679679*d*\cosh(d*x + \\
& c)^9 + 935748*d*\cosh(d*x + c)^7 - 762552*d*\cosh(d*x + c)^5 + 327488*d*\cosh(\\
& d*x + c)^3 - 51204*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 30498*d*\cosh(d*x + c) \\
& ^4 + (4845*d*\cosh(d*x + c)^{16} - 52020*d*\cosh(d*x + c)^{14} + 247520*d*\cosh(d* \\
& x + c)^{12} - 681681*d*\cosh(d*x + c)^{10} + 1186515*d*\cosh(d*x + c)^8 - 1328040 \\
& *d*\cosh(d*x + c)^6 + 913920*d*\cosh(d*x + c)^4 - 327420*d*\cosh(d*x + c)^2 + \\
& 30498*d)*\sinh(d*x + c)^4 + 4*(285*d*\cosh(d*x + c)^{17} - 3468*d*\cosh(d*x + c) \\
& ^{15} + 19040*d*\cosh(d*x + c)^{13} - 61789*d*\cosh(d*x + c)^{11} + 129965*d*\cosh(d \\
& *x + c)^9 - 181560*d*\cosh(d*x + c)^7 + 163744*d*\cosh(d*x + c)^5 - 85340*d*\c \\
& osh(d*x + c)^3 + 18122*d*\cosh(d*x + c))*\sinh(d*x + c)^3 - 36686*d*\cosh(d*x \\
& + c)^2 + (190*d*\cosh(d*x + c)^{18} - 2601*d*\cosh(d*x + c)^{16} + 16320*d*\cosh(d \\
& *x + c)^{14} - 61971*d*\cosh(d*x + c)^{12} + 158202*d*\cosh(d*x + c)^{10} - 284580* \\
& d*\cosh(d*x + c)^8 + 365568*d*\cosh(d*x + c)^6 - 327420*d*\cosh(d*x + c)^4 + 1 \\
& 82988*d*\cosh(d*x + c)^2 - 36686*d)*\sinh(d*x + c)^2 + 2*(10*d*\cosh(d*x + c)^ \\
& 19 - 153*d*\cosh(d*x + c)^{17} + 1088*d*\cosh(d*x + c)^{15} - 4753*d*\cosh(d*x + c \\
&)^{13} + 14178*d*\cosh(d*x + c)^{11} - 30260*d*\cosh(d*x + c)^9 + 46784*d*\cosh(d* \\
& x + c)^7 - 51204*d*\cosh(d*x + c)^5 + 36244*d*\cosh(d*x + c)^3 - 11934*d*\cosh \\
& (d*x + c))*\sinh(d*x + c) + 19448*d)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**18*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.6542, size = 917, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -16/765765*(510510*b^3*e^{(28*d*x + 28*c)} - 6381375*b^3*e^{(26*d*x + 26*c)} + \\ & 14702688*a*b^2*e^{(24*d*x + 24*c)} + 36807771*b^3*e^{(24*d*x + 24*c)} - 1274232 \\ & 96*a*b^2*e^{(22*d*x + 22*c)} - 129771642*b^3*e^{(22*d*x + 22*c)} + 168030720*a^2*b*e^{(20*d*x + 20*c)} + \\ & 494290368*a*b^2*e^{(20*d*x + 20*c)} + 312227916*b^3*e^{(20*d*x + 20*c)} - 798145920*a^2*b*e^{(18*d*x + 18*c)} - \\ & 1132457040*a*b^2*e^{(18*d*x + 18*c)} - 541906365*b^3*e^{(18*d*x + 18*c)} + 697016320*a^3*e^{(16*d*x + 16*c)} + \\ & 1582289280*a^2*b*e^{(16*d*x + 16*c)} + 1704228240*a*b^2*e^{(16*d*x + 16*c)} + 699143445*b^3*e^{(16*d*x + 16*c)} - \\ & 557613056*a^3*e^{(14*d*x + 14*c)} - 1736317440*a^2*b*e^{(14*d*x + 14*c)} - 1775057856*a*b^2*e^{(14*d*x + 14*c)} - \\ & 680611932*b^3*e^{(14*d*x + 14*c)} + 354844672*a^3*e^{(12*d*x + 12*c)} + 1211857920*a^2*b*e^{(12*d*x + 12*c)} + \\ & 1316707392*a*b^2*e^{(12*d*x + 12*c)} + 502035534*b^3*e^{(12*d*x + 12*c)} - 177422336*a^3*e^{(10*d*x + 10*c)} - 605928960*a^2*b*e^{(10*d*x + 10*c)} - \\ & 707362656*a*b^2*e^{(10*d*x + 10*c)} - 279095817*b^3*e^{(10*d*x + 10*c)} + 68239360*a^3*e^{(8*d*x + 8*c)} + \\ & 233049600*a^2*b*e^{(8*d*x + 8*c)} + 277717440*a*b^2*e^{(8*d*x + 8*c)} + 115120005*b^3*e^{(8*d*x + 8*c)} - 19496960*a^3*e^{(6*d*x + 6*c)} - \\ & 66585600*a^2*b*e^{(6*d*x + 6*c)} - 79347840*a*b^2*e^{(6*d*x + 6*c)} - 34204170*b^3*e^{(6*d*x + 6*c)} + 3899392*a^3*e^{(4*d*x + 4*c)} + \\ & 13317120*a^2*b*e^{(4*d*x + 4*c)} + 15869568*a*b^2*e^{(4*d*x + 4*c)} + 6942936*b^3*e^{(4*d*x + 4*c)} - 487424*a^3*e^{(2*d*x + 2*c)} - 1664640*a^2*b*e^{(2*d*x + 2*c)} - \\ & 1983696*a*b^2*e^{(2*d*x + 2*c)} - 867867*b^3*e^{(2*d*x + 2*c)} + 28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 51051*b^3)/(d*(e^{(2*d*x + 2*c)} - 1)^{17}) \end{aligned}$$

3.228 $\int \operatorname{csch}^{20}(c + dx) \left(a + b \sinh^4(c + dx) \right)^3 dx$

Optimal. Leaf size=248

$$\frac{3a(42a^2 + 21ab + b^2) \operatorname{coth}^{11}(c + dx)}{11d} + \frac{a(42a^2 + 35ab + 5b^2) \operatorname{coth}^9(c + dx)}{3d} - \frac{(105a^2b + 84a^3 + 30ab^2 + b^3) \operatorname{coth}^7(c + dx)}{7d}$$

[Out] $((a + b)^3 \operatorname{Coth}[c + d*x])/d - ((a + b)^2 (3*a + b) \operatorname{Coth}[c + d*x]^3)/d + (3*(a + b) * (12*a^2 + 9*a*b + b^2) \operatorname{Coth}[c + d*x]^5)/(5*d) - ((84*a^3 + 105*a^2*b + 30*a*b^2 + b^3) \operatorname{Coth}[c + d*x]^7)/(7*d) + (a*(42*a^2 + 35*a*b + 5*b^2) \operatorname{Coth}[c + d*x]^9)/(3*d) - (3*a*(42*a^2 + 21*a*b + b^2) \operatorname{Coth}[c + d*x]^11)/(11*d) + (21*a^2*(4*a + b) \operatorname{Coth}[c + d*x]^13)/(13*d) - (a^2*(12*a + b) \operatorname{Coth}[c + d*x]^15)/(5*d) + (9*a^3 \operatorname{Coth}[c + d*x]^17)/(17*d) - (a^3 \operatorname{Coth}[c + d*x]^19)/(19*d)$

Rubi [A] time = 0.222522, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3217, 1261}

$$\frac{3a(42a^2 + 21ab + b^2) \operatorname{coth}^{11}(c + dx)}{11d} + \frac{a(42a^2 + 35ab + 5b^2) \operatorname{coth}^9(c + dx)}{3d} - \frac{(105a^2b + 84a^3 + 30ab^2 + b^3) \operatorname{coth}^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^{20} * (a + b * \operatorname{Sinh}[c + d*x]^4)^3, x]$

[Out] $((a + b)^3 \operatorname{Coth}[c + d*x])/d - ((a + b)^2 (3*a + b) \operatorname{Coth}[c + d*x]^3)/d + (3*(a + b) * (12*a^2 + 9*a*b + b^2) \operatorname{Coth}[c + d*x]^5)/(5*d) - ((84*a^3 + 105*a^2*b + 30*a*b^2 + b^3) \operatorname{Coth}[c + d*x]^7)/(7*d) + (a*(42*a^2 + 35*a*b + 5*b^2) \operatorname{Coth}[c + d*x]^9)/(3*d) - (3*a*(42*a^2 + 21*a*b + b^2) \operatorname{Coth}[c + d*x]^11)/(11*d) + (21*a^2*(4*a + b) \operatorname{Coth}[c + d*x]^13)/(13*d) - (a^2*(12*a + b) \operatorname{Coth}[c + d*x]^15)/(5*d) + (9*a^3 \operatorname{Coth}[c + d*x]^17)/(17*d) - (a^3 \operatorname{Coth}[c + d*x]^19)/(19*d)$

Rule 3217

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)} * ((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m * (a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]\} /; \operatorname{FreeQ}\{a, b, e, f\}, x\} \&\& \operatorname{IntegerQ}[m/2] \&\& \operatorname{IntegerQ}[p]$

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \operatorname{csch}^{20}(c+dx) (a+b \sinh^4(c+dx))^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3 (a-2ax^2+(a+b)x^4)^3}{x^{20}} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{20}} - \frac{9a^3}{x^{18}} + \frac{3a^2(12a+b)}{x^{16}} - \frac{21a^2(4a+b)}{x^{14}} + \frac{3a(42a^2+21ab+b^2)}{x^{12}} - \frac{3a(42a^2+35ab+b^2)}{x^{10}}\right) dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a+b)^3 \operatorname{coth}(c+dx)}{d} - \frac{(a+b)^2(3a+b) \operatorname{coth}^3(c+dx)}{d} + \frac{3(a+b)(12a^2+35ab+b^2)}{d}$$

Mathematica [B] time = 6.16765, size = 548, normalized size = 2.21

$$\operatorname{csch}^{19}(c+dx) \left(-8939234304a^2b \cosh(c+dx) + 18149354496a^2b \cosh(3(c+dx)) - 14582690304a^2b \cosh(5(c+dx)) + \dots\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^20*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-7945986048*a^3*Cosh[c + d*x] - 8939234304*a^2*b*Cosh[c + d*x] - 6518191680*a*b^2*Cosh[c + d*x] - 1792502712*b^3*Cosh[c + d*x] + 6501261312*a^3*Cosh[3*(c + d*x)] + 18149354496*a^2*b*Cosh[3*(c + d*x)] + 14814072000*a*b^2*Cosh[3*(c + d*x)] + 4260103848*b^3*Cosh[3*(c + d*x)] - 4334174208*a^3*Cosh[5*(c + d*x)] - 14582690304*a^2*b*Cosh[5*(c + d*x)] - 14221509120*a*b^2*Cosh[5*(c + d*x)] - 4440518082*b^3*Cosh[5*(c + d*x)] + 2333786112*a^3*Cosh[7*(c + d*x)] + 7852217856*a^2*b*Cosh[7*(c + d*x)] + 8803791360*a*b^2*Cosh[7*(c + d*x)] + 3047642598*b^3*Cosh[7*(c + d*x)] - 1000194048*a^3*Cosh[9*(c + d*x)] - 3365236224*a^2*b*Cosh[9*(c + d*x)] - 3906077760*a*b^2*Cosh[9*(c + d*x)] - 1489040982*b^3*Cosh[9*(c + d*x)] + 333398016*a^3*Cosh[11*(c + d*x)] + 1121745408*a^2*b*Cosh[11*(c + d*x)] + 1302025920*a*b^2*Cosh[11*(c + d*x)] + 527386002*b^3*Cosh[11*(c + d*x)] - 83349504*a^3*Cosh[13*(c + d*x)] - 280436352*a^2*b*Cosh[13*(c + d*x)] - 325506480*a*b^2*Cosh[13*(c + d*x)] - 134271423*b^3*Cosh[13*(c + d*x)] + 14708736*a^3*Cosh[15*(c + d*x)] + 49488768*a^2*b*C
```

osh[15*(c + d*x)] + 57442320*a*b^2*Cosh[15*(c + d*x)] + 23694957*b^3*Cosh[15*(c + d*x)] - 1634304*a^3*Cosh[17*(c + d*x)] - 5498752*a^2*b*Cosh[17*(c + d*x)] - 6382480*a*b^2*Cosh[17*(c + d*x)] - 2632773*b^3*Cosh[17*(c + d*x)] + 86016*a^3*Cosh[19*(c + d*x)] + 289408*a^2*b*Cosh[19*(c + d*x)] + 335920*a*b^2*Cosh[19*(c + d*x)] + 138567*b^3*Cosh[19*(c + d*x)]*Csch[c + d*x]^19)/(79459860480*d)

Maple [A] time = 0.09, size = 298, normalized size = 1.2

$$\frac{1}{d} \left(a^3 \left(\frac{65536}{230945} - \frac{(\operatorname{csch}(dx+c))^{18}}{19} + \frac{18(\operatorname{csch}(dx+c))^{16}}{323} - \frac{96(\operatorname{csch}(dx+c))^{14}}{1615} + \frac{1344(\operatorname{csch}(dx+c))^{12}}{20995} - \frac{16128(\operatorname{csch}(dx+c))^{10}}{230945} + \frac{3584(\operatorname{csch}(dx+c))^8}{46189} - \frac{4096(\operatorname{csch}(dx+c))^6}{46189} + \frac{24576(\operatorname{csch}(dx+c))^4}{230945} - \frac{32768(\operatorname{csch}(dx+c))^2}{230945} \right) \operatorname{coth}(dx+c) + 3a^2b(2048/6435 - 1/15 \operatorname{csch}(dx+c)^{14} + 14/195 \operatorname{csch}(dx+c)^{12} - 56/715 \operatorname{csch}(dx+c)^{10} + 112/1287 \operatorname{csch}(dx+c)^8 - 128/1287 \operatorname{csch}(dx+c)^6 + 256/2145 \operatorname{csch}(dx+c)^4 - 1024/6435 \operatorname{csch}(dx+c)^2) \operatorname{coth}(dx+c) + 3ab^2(256/693 - 1/11 \operatorname{csch}(dx+c)^{10} + 10/99 \operatorname{csch}(dx+c)^8 - 80/693 \operatorname{csch}(dx+c)^6 + 32/231 \operatorname{csch}(dx+c)^4 - 128/693 \operatorname{csch}(dx+c)^2) \operatorname{coth}(dx+c) + b^3(16/35 - 1/7 \operatorname{csch}(dx+c)^6 + 6/35 \operatorname{csch}(dx+c)^4 - 8/35 \operatorname{csch}(dx+c)^2) \operatorname{coth}(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x)

[Out] 1/d*(a^3*(65536/230945-1/19*csch(d*x+c)^18+18/323*csch(d*x+c)^16-96/1615*csch(d*x+c)^14+1344/20995*csch(d*x+c)^12-16128/230945*csch(d*x+c)^10+3584/46189*csch(d*x+c)^8-4096/46189*csch(d*x+c)^6+24576/230945*csch(d*x+c)^4-32768/230945*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(2048/6435-1/15*csch(d*x+c)^14+14/195*csch(d*x+c)^12-56/715*csch(d*x+c)^10+112/1287*csch(d*x+c)^8-128/1287*csch(d*x+c)^6+256/2145*csch(d*x+c)^4-1024/6435*csch(d*x+c)^2)*coth(d*x+c)+3*a*b^2*(256/693-1/11*csch(d*x+c)^10+10/99*csch(d*x+c)^8-80/693*csch(d*x+c)^6+32/231*csch(d*x+c)^4-128/693*csch(d*x+c)^2)*coth(d*x+c)+b^3*(16/35-1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*coth(d*x+c))

Maxima [B] time = 1.29003, size = 6592, normalized size = 26.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] 131072/230945*a^3*(19*e^(-2*d*x - 2*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c)

$$\begin{aligned}
& c) - 15e^{(-28dx - 28c)} + e^{(-30dx - 30c)} - 1)) - 5005e^{(-12dx - 12c)} \\
& 2c)/(d*(15e^{(-2dx - 2c)} - 105e^{(-4dx - 4c)} + 455e^{(-6dx - 6c)} \\
& - 1365e^{(-8dx - 8c)} + 3003e^{(-10dx - 10c)} - 5005e^{(-12dx - 12c)} \\
& + 6435e^{(-14dx - 14c)} - 6435e^{(-16dx - 16c)} + 5005e^{(-18dx - 18c)} \\
& - 3003e^{(-20dx - 20c)} + 1365e^{(-22dx - 22c)} - 455e^{(-24dx - 24c)} \\
& + 105e^{(-26dx - 26c)} - 15e^{(-28dx - 28c)} + e^{(-30dx - 30c)} \\
& - 1)) + 6435e^{(-14dx - 14c)}/(d*(15e^{(-2dx - 2c)} - 105e^{(-4dx - 4c)} \\
& + 455e^{(-6dx - 6c)} - 1365e^{(-8dx - 8c)} + 3003e^{(-10dx - 10c)} \\
& - 5005e^{(-12dx - 12c)} + 6435e^{(-14dx - 14c)} - 6435e^{(-16dx - 16c)} \\
& + 5005e^{(-18dx - 18c)} - 3003e^{(-20dx - 20c)} + 1365e^{(-22dx - 22c)} \\
& - 455e^{(-24dx - 24c)} + 105e^{(-26dx - 26c)} - 15e^{(-28dx - 28c)} \\
& + e^{(-30dx - 30c)} - 1)) - 1/(d*(15e^{(-2dx - 2c)} - 105e^{(-4dx - 4c)} \\
& + 455e^{(-6dx - 6c)} - 1365e^{(-8dx - 8c)} + 3003e^{(-10dx - 10c)} \\
& - 5005e^{(-12dx - 12c)} + 6435e^{(-14dx - 14c)} - 6435e^{(-16dx - 16c)} \\
& + 5005e^{(-18dx - 18c)} - 3003e^{(-20dx - 20c)} + 1365e^{(-22dx - 22c)} \\
& - 455e^{(-24dx - 24c)} + 105e^{(-26dx - 26c)} - 15e^{(-28dx - 28c)} \\
& + e^{(-30dx - 30c)} - 1))) + 512/231*a*b^2*(11e^{(-2dx - 2c)} \\
& 2c)/(d*(11e^{(-2dx - 2c)} - 55e^{(-4dx - 4c)} + 165e^{(-6dx - 6c)} - 330e^{(-8dx - 8c)} \\
& + 462e^{(-10dx - 10c)} - 462e^{(-12dx - 12c)} + 330e^{(-14dx - 14c)} \\
& - 165e^{(-16dx - 16c)} + 55e^{(-18dx - 18c)} - 11e^{(-20dx - 20c)} + e^{(-22dx - 22c)} \\
& - 1)) - 55e^{(-4dx - 4c)}/(d*(11e^{(-2dx - 2c)} - 55e^{(-4dx - 4c)} \\
& + 165e^{(-6dx - 6c)} - 330e^{(-8dx - 8c)} + 462e^{(-10dx - 10c)} - 462e^{(-12dx - 12c)} \\
& + 330e^{(-14dx - 14c)} - 165e^{(-16dx - 16c)} + 55e^{(-18dx - 18c)} - 11e^{(-20dx - 20c)} \\
& + e^{(-22dx - 22c)} - 1)) + 165e^{(-6dx - 6c)}/(d*(11e^{(-2dx - 2c)} \\
& - 55e^{(-4dx - 4c)} + 165e^{(-6dx - 6c)} - 330e^{(-8dx - 8c)} \\
& + 462e^{(-10dx - 10c)} - 462e^{(-12dx - 12c)} + 330e^{(-14dx - 14c)} \\
& - 165e^{(-16dx - 16c)} + 55e^{(-18dx - 18c)} - 11e^{(-20dx - 20c)} + e^{(-22dx - 22c)} \\
& - 1)) - 330e^{(-8dx - 8c)}/(d*(11e^{(-2dx - 2c)} - 55e^{(-4dx - 4c)} \\
& + 165e^{(-6dx - 6c)} - 330e^{(-8dx - 8c)} + 462e^{(-10dx - 10c)} \\
& - 462e^{(-12dx - 12c)} + 330e^{(-14dx - 14c)} - 165e^{(-16dx - 16c)} \\
& + 55e^{(-18dx - 18c)} - 11e^{(-20dx - 20c)} + e^{(-22dx - 22c)} \\
& - 1)) + 462e^{(-10dx - 10c)}/(d*(11e^{(-2dx - 2c)} - 55e^{(-4dx - 4c)} \\
& + 165e^{(-6dx - 6c)} - 330e^{(-8dx - 8c)} + 462e^{(-10dx - 10c)} \\
& - 462e^{(-12dx - 12c)} + 330e^{(-14dx - 14c)} - 165e^{(-16dx - 16c)} \\
& + 55e^{(-18dx - 18c)} - 11e^{(-20dx - 20c)} + e^{(-22dx - 22c)} \\
& - 1)) - 1/(d*(11e^{(-2dx - 2c)} - 55e^{(-4dx - 4c)} + 165e^{(-6dx - 6c)} \\
& - 330e^{(-8dx - 8c)} + 462e^{(-10dx - 10c)} - 462e^{(-12dx - 12c)} \\
& + 330e^{(-14dx - 14c)} - 165e^{(-16dx - 16c)} + 55e^{(-18dx - 18c)} \\
& - 11e^{(-20dx - 20c)} + e^{(-22dx - 22c)} - 1))) + 32/35*b^3*(7e^{(-2dx - 2c)} \\
& 2c)/(d*(7e^{(-2dx - 2c)} - 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} - 35e^{(-8dx - 8c)} \\
& + 21e^{(-10dx - 10c)} - 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} - 1)) - 21e^{(-4dx - 4c)}/(d*(7e^{(-2dx - 2c)} \\
& - 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} - 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} \\
& - 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} - 1)) + 35e^{(-6dx - 6c)} -
\end{aligned}$$

$$\frac{6*c}{(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1)) - 1/(d*(7*e^{(-2*d*x - 2*c)} - 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} - 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} - 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} - 1))}$$

Fricas [B] time = 2.26097, size = 13925, normalized size = 56.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] 64/4849845*((43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^15 + 15*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)*sinh(d*x + c)^14 - 2*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*sinh(d*x + c)^15 - 19*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^13 + 2*(408576*a^3 + 1374688*a^2*b + 1595620*a*b^2 + 15935205*b^3 - 105*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(35*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^3 - 19*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 + 57*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^11 - 6*(455*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^4 + 1225728*a^3 + 4124064*a^2*b + 17719780*a*b^2 + 31639465*b^3 - 494*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 + 11*(273*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^5 - 494*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^3 + 57*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 - 969*(43008*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c)^9 - 2*(5005*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^6 - 13585*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^4 - 20837376*a^3 - 70109088*a^2*b - 419480100*a*b^2 - 351267345*b^3 + 3135*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(715*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^7 - 2717*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^5 + 1045*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^3 - 969*(43008*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 6783*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c)^7 - 6*(2145*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cos

$$\begin{aligned}
& h(dx + c)^8 - 10868*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*c \\
& \cosh(dx + c)^6 + 6270*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3) \\
& * \cosh(dx + c)^4 + 27783168*a^3 + 248673824*a^2*b + 549145220*a*b^2 + 303 \\
& 230785*b^3 - 11628*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^3)*c \\
& \sinh(dx + c)^2 * \sinh(dx + c)^7 + (5005*(43008*a^3 + 144704*a^2*b + 167960*a \\
& *b^2 - 2355639*b^3)* \cosh(dx + c)^9 - 32604*(43008*a^3 + 144704*a^2*b + 167 \\
& 960*a*b^2 - 1538823*b^3)* \cosh(dx + c)^7 + 26334*(129024*a^3 + 434112*a^2*b \\
& - 857480*a*b^2 - 2914769*b^3)* \cosh(dx + c)^5 - 81396*(43008*a^3 + 144704*a \\
& a^2*b - 529880*a*b^2 - 586443*b^3)* \cosh(dx + c)^3 + 47481*(24576*a^3 - 545 \\
& 92*a^2*b - 293800*a*b^2 - 189761*b^3)* \cosh(dx + c)) * \sinh(dx + c)^6 - 323* \\
& (1548288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)* \cosh(dx + c)^ \\
& 5 - 2*(3003*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)* \cosh(dx \\
& + c)^10 - 24453*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)* \cosh(dx \\
& *x + c)^8 + 26334*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3)*c \\
& \cosh(dx + c)^6 - 122094*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^3) \\
& * \cosh(dx + c)^4 - 250048512*a^3 - 3065771296*a^2*b - 4040697700*a*b^2 - \\
& 1763542209*b^3 + 20349*(86016*a^3 + 769888*a^2*b + 1700140*a*b^2 + 938795*b \\
& ^3)* \cosh(dx + c)^2 * \sinh(dx + c)^5 + (1365*(43008*a^3 + 144704*a^2*b + 16 \\
& 7960*a*b^2 - 2355639*b^3)* \cosh(dx + c)^11 - 13585*(43008*a^3 + 144704*a^2* \\
& b + 167960*a*b^2 - 1538823*b^3)* \cosh(dx + c)^9 + 18810*(129024*a^3 + 43411 \\
& 2*a^2*b - 857480*a*b^2 - 2914769*b^3)* \cosh(dx + c)^7 - 122094*(43008*a^3 + \\
& 144704*a^2*b - 529880*a*b^2 - 586443*b^3)* \cosh(dx + c)^5 + 237405*(24576* \\
& a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)* \cosh(dx + c)^3 - 1615*(1548 \\
& 288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)* \cosh(dx + c)) * \sinh \\
& (dx + c)^4 - 323*(8687616*a^3 + 15456448*a^2*b + 15194920*a*b^2 + 6026163* \\
& b^3)* \cosh(dx + c)^3 - 2*(455*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247 \\
& 103*b^3)* \cosh(dx + c)^12 - 5434*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 8 \\
& 38695*b^3)* \cosh(dx + c)^10 + 9405*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 \\
& + 1665235*b^3)* \cosh(dx + c)^8 - 81396*(21504*a^3 + 72352*a^2*b + 432900*a \\
& *b^2 + 362505*b^3)* \cosh(dx + c)^6 + 33915*(86016*a^3 + 769888*a^2*b + 1700 \\
& 140*a*b^2 + 938795*b^3)* \cosh(dx + c)^4 + 2569943040*a^3 + 6422325280*a^2*b \\
& + 6933472780*a*b^2 + 2675035935*b^3 - 3230*(774144*a^3 + 9491552*a^2*b + 1 \\
& 2509900*a*b^2 + 5459883*b^3)* \cosh(dx + c)^2 * \sinh(dx + c)^3 + (105*(43008 \\
& *a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)* \cosh(dx + c)^13 - 1482*(\\
& 43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)* \cosh(dx + c)^11 + 3 \\
& 135*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)* \cosh(dx + c)^ \\
& 9 - 34884*(43008*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)* \cosh(dx + \\
& c)^7 + 142443*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)* \cosh(dx \\
& *x + c)^5 - 3230*(1548288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3) \\
& * \cosh(dx + c)^3 - 969*(8687616*a^3 + 15456448*a^2*b + 15194920*a*b^2 + 6 \\
& 026163*b^3)* \cosh(dx + c)) * \sinh(dx + c)^2 + 12597*(86016*a^3 + 215488*a^2* \\
& b + 179720*a*b^2 + 65703*b^3)* \cosh(dx + c) - 2*(15*(21504*a^3 + 72352*a^2* \\
& b + 83980*a*b^2 + 1247103*b^3)* \cosh(dx + c)^14 - 247*(21504*a^3 + 72352*a^ \\
& 2*b + 83980*a*b^2 + 838695*b^3)* \cosh(dx + c)^12 + 627*(64512*a^3 + 217056* \\
& a^2*b + 932620*a*b^2 + 1665235*b^3)* \cosh(dx + c)^10 - 8721*(21504*a^3 + 72
\end{aligned}$$

$$\begin{aligned}
& 352*a^2*b + 432900*a*b^2 + 362505*b^3)*\cosh(d*x + c)^8 + 6783*(86016*a^3 + \\
& 769888*a^2*b + 1700140*a*b^2 + 938795*b^3)*\cosh(d*x + c)^6 - 1615*(774144*a \\
& ^3 + 9491552*a^2*b + 12509900*a*b^2 + 5459883*b^3)*\cosh(d*x + c)^4 - 270885 \\
& 8880*a^3 - 8648596320*a^2*b - 8918927940*a*b^2 - 3269488365*b^3 + 4845*(159 \\
& 1296*a^3 + 3976672*a^2*b + 4293172*a*b^2 + 1656369*b^3)*\cosh(d*x + c)^2)*\si \\
& nh(d*x + c))/(d*\cosh(d*x + c)^23 + 23*d*\cosh(d*x + c)*\sinh(d*x + c)^22 + d* \\
& \sinh(d*x + c)^23 - 19*d*\cosh(d*x + c)^21 + (253*d*\cosh(d*x + c)^2 - 19*d)*s \\
& inh(d*x + c)^21 + 7*(253*d*\cosh(d*x + c)^3 - 57*d*\cosh(d*x + c))*\sinh(d*x + \\
& c)^20 + 171*d*\cosh(d*x + c)^19 + (8855*d*\cosh(d*x + c)^4 - 3990*d*\cosh(d*x \\
& + c)^2 + 171*d)*\sinh(d*x + c)^19 + 19*(1771*d*\cosh(d*x + c)^5 - 1330*d*\cos \\
& h(d*x + c)^3 + 171*d*\cosh(d*x + c))*\sinh(d*x + c)^18 - 969*d*\cosh(d*x + c)^ \\
& 17 + 57*(1771*d*\cosh(d*x + c)^6 - 1995*d*\cosh(d*x + c)^4 + 513*d*\cosh(d*x + \\
& c)^2 - 17*d)*\sinh(d*x + c)^17 + 969*(253*d*\cosh(d*x + c)^7 - 399*d*\cosh(d* \\
& x + c)^5 + 171*d*\cosh(d*x + c)^3 - 17*d*\cosh(d*x + c))*\sinh(d*x + c)^16 + 3 \\
& 875*d*\cosh(d*x + c)^15 + (490314*d*\cosh(d*x + c)^8 - 1031016*d*\cosh(d*x + c \\
&)^6 + 662796*d*\cosh(d*x + c)^4 - 131784*d*\cosh(d*x + c)^2 + 3877*d)*\sinh(d* \\
& x + c)^15 + (817190*d*\cosh(d*x + c)^9 - 2209320*d*\cosh(d*x + c)^7 + 1988388 \\
& *d*\cosh(d*x + c)^5 - 658920*d*\cosh(d*x + c)^3 + 58125*d*\cosh(d*x + c))*\sinh \\
& (d*x + c)^14 - 11609*d*\cosh(d*x + c)^13 + (1144066*d*\cosh(d*x + c)^10 - 386 \\
& 6310*d*\cosh(d*x + c)^8 + 4639572*d*\cosh(d*x + c)^6 - 2306220*d*\cosh(d*x + c \\
&)^4 + 407085*d*\cosh(d*x + c)^2 - 11647*d)*\sinh(d*x + c)^13 + 13*(104006*d*c \\
& osh(d*x + c)^11 - 429590*d*\cosh(d*x + c)^9 + 662796*d*\cosh(d*x + c)^7 - 461 \\
& 244*d*\cosh(d*x + c)^5 + 135625*d*\cosh(d*x + c)^3 - 11609*d*\cosh(d*x + c))*s \\
& inh(d*x + c)^12 + 26961*d*\cosh(d*x + c)^11 + (1352078*d*\cosh(d*x + c)^12 - \\
& 6701604*d*\cosh(d*x + c)^10 + 12924522*d*\cosh(d*x + c)^8 - 11992344*d*\cosh(d \\
& *x + c)^6 + 5292105*d*\cosh(d*x + c)^4 - 908466*d*\cosh(d*x + c)^2 + 27303*d) \\
& *\sinh(d*x + c)^11 + (1144066*d*\cosh(d*x + c)^13 - 6701604*d*\cosh(d*x + c)^1 \\
& 1 + 15796638*d*\cosh(d*x + c)^9 - 18845112*d*\cosh(d*x + c)^7 + 11636625*d*co \\
& sh(d*x + c)^5 - 3320174*d*\cosh(d*x + c)^3 + 296571*d*\cosh(d*x + c))*\sinh(d* \\
& x + c)^10 - 49419*d*\cosh(d*x + c)^9 + (817190*d*\cosh(d*x + c)^14 - 5584670* \\
& d*\cosh(d*x + c)^12 + 15796638*d*\cosh(d*x + c)^10 - 23556390*d*\cosh(d*x + c) \\
& ^8 + 19404385*d*\cosh(d*x + c)^6 - 8327605*d*\cosh(d*x + c)^4 + 1501665*d*\cos \\
& h(d*x + c)^2 - 51357*d)*\sinh(d*x + c)^9 + 3*(163438*d*\cosh(d*x + c)^15 - 12 \\
& 88770*d*\cosh(d*x + c)^13 + 4308174*d*\cosh(d*x + c)^11 - 7852130*d*\cosh(d*x \\
& + c)^9 + 8311875*d*\cosh(d*x + c)^7 - 4980261*d*\cosh(d*x + c)^5 + 1482855*d* \\
& cosh(d*x + c)^3 - 148257*d*\cosh(d*x + c))*\sinh(d*x + c)^8 + 71706*d*\cosh(d* \\
& x + c)^7 + 3*(81719*d*\cosh(d*x + c)^16 - 736440*d*\cosh(d*x + c)^14 + 287211 \\
& 6*d*\cosh(d*x + c)^12 - 6281704*d*\cosh(d*x + c)^10 + 8316165*d*\cosh(d*x + c) \\
& ^8 - 6662084*d*\cosh(d*x + c)^6 + 3003330*d*\cosh(d*x + c)^4 - 616284*d*\cosh(\\
& d*x + c)^2 + 26486*d)*\sinh(d*x + c)^7 + (100947*d*\cosh(d*x + c)^17 - 103101 \\
& 6*d*\cosh(d*x + c)^15 + 4639572*d*\cosh(d*x + c)^13 - 11992344*d*\cosh(d*x + c \\
&)^11 + 19394375*d*\cosh(d*x + c)^9 - 19921044*d*\cosh(d*x + c)^7 + 12455982*d \\
& *\cosh(d*x + c)^5 - 4151196*d*\cosh(d*x + c)^3 + 501942*d*\cosh(d*x + c))*\sinh \\
& (d*x + c)^6 - 80750*d*\cosh(d*x + c)^5 + (33649*d*\cosh(d*x + c)^18 - 386631* \\
& d*\cosh(d*x + c)^16 + 1988388*d*\cosh(d*x + c)^14 - 5996172*d*\cosh(d*x + c)^1
\end{aligned}$$

$$\begin{aligned}
& 2 + 11642631*d*\cosh(d*x + c)^{10} - 14989689*d*\cosh(d*x + c)^8 + 12613986*d*\cosh(d*x + c)^6 - 6470982*d*\cosh(d*x + c)^4 + 1668618*d*\cosh(d*x + c)^2 - 104006*d)*\sinh(d*x + c)^5 + (8855*d*\cosh(d*x + c)^{19} - 113715*d*\cosh(d*x + c)^{17} + 662796*d*\cosh(d*x + c)^{15} - 2306220*d*\cosh(d*x + c)^{13} + 5289375*d*\cosh(d*x + c)^{11} - 8300435*d*\cosh(d*x + c)^9 + 8897130*d*\cosh(d*x + c)^7 - 6226794*d*\cosh(d*x + c)^5 + 2509710*d*\cosh(d*x + c)^3 - 403750*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 65246*d*\cosh(d*x + c)^3 + (1771*d*\cosh(d*x + c)^{20} - 25270*d*\cosh(d*x + c)^{18} + 165699*d*\cosh(d*x + c)^{16} - 658920*d*\cosh(d*x + c)^{14} + 1764035*d*\cosh(d*x + c)^{12} - 3331042*d*\cosh(d*x + c)^{10} + 4504995*d*\cosh(d*x + c)^8 - 4313988*d*\cosh(d*x + c)^6 + 2781030*d*\cosh(d*x + c)^4 - 1040060*d*\cosh(d*x + c)^2 + 119510*d)*\sinh(d*x + c)^3 + (253*d*\cosh(d*x + c)^{21} - 3990*d*\cosh(d*x + c)^{19} + 29241*d*\cosh(d*x + c)^{17} - 131784*d*\cosh(d*x + c)^{15} + 406875*d*\cosh(d*x + c)^{13} - 905502*d*\cosh(d*x + c)^{11} + 1482855*d*\cosh(d*x + c)^9 - 1779084*d*\cosh(d*x + c)^7 + 1505826*d*\cosh(d*x + c)^5 - 807500*d*\cosh(d*x + c)^3 + 195738*d*\cosh(d*x + c))*\sinh(d*x + c)^2 - 25194*d*\cosh(d*x + c) + (23*d*\cosh(d*x + c)^{22} - 399*d*\cosh(d*x + c)^{20} + 3249*d*\cosh(d*x + c)^{18} - 16473*d*\cosh(d*x + c)^{16} + 58155*d*\cosh(d*x + c)^{14} - 151411*d*\cosh(d*x + c)^{12} + 300333*d*\cosh(d*x + c)^{10} - 462213*d*\cosh(d*x + c)^8 + 556206*d*\cosh(d*x + c)^6 - 520030*d*\cosh(d*x + c)^4 + 358530*d*\cosh(d*x + c)^2 - 125970*d)*\sinh(d*x + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**20*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [B] time = 1.66426, size = 995, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

```
[Out] -32/4849845*(4849845*b^3*e^(30*d*x + 30*c) - 61108047*b^3*e^(28*d*x + 28*c)
+ 155195040*a*b^2*e^(26*d*x + 26*c) + 355978623*b^3*e^(26*d*x + 26*c) - 13
52413920*a*b^2*e^(24*d*x + 24*c) - 1270797957*b^3*e^(24*d*x + 24*c) + 18623
40480*a^2*b*e^(22*d*x + 22*c) + 5287716720*a*b^2*e^(22*d*x + 22*c) + 310653
3573*b^3*e^(22*d*x + 22*c) - 8897848960*a^2*b*e^(20*d*x + 20*c) - 122567130
40*a*b^2*e^(20*d*x + 20*c) - 5504019807*b^3*e^(20*d*x + 20*c) + 7945986048*
a^3*e^(18*d*x + 18*c) + 17837083264*a^2*b*e^(18*d*x + 18*c) + 18774904720*a
*b^2*e^(18*d*x + 18*c) + 7296522519*b^3*e^(18*d*x + 18*c) - 6501261312*a^3*
e^(16*d*x + 16*c) - 20011694976*a^2*b*e^(16*d*x + 16*c) - 20101788720*a*b^2
*e^(16*d*x + 16*c) - 7366637421*b^3*e^(16*d*x + 16*c) + 4334174208*a^3*e^(1
4*d*x + 14*c) + 14582690304*a^2*b*e^(14*d*x + 14*c) + 15573923040*a*b^2*e^(
14*d*x + 14*c) + 5711316039*b^3*e^(14*d*x + 14*c) - 2333786112*a^3*e^(12*d*
x + 12*c) - 7852217856*a^2*b*e^(12*d*x + 12*c) - 8958986400*a*b^2*e^(12*d*x
+ 12*c) - 3403621221*b^3*e^(12*d*x + 12*c) + 1000194048*a^3*e^(10*d*x + 10
*c) + 3365236224*a^2*b*e^(10*d*x + 10*c) + 3906077760*a*b^2*e^(10*d*x + 10*
c) + 1550149029*b^3*e^(10*d*x + 10*c) - 333398016*a^3*e^(8*d*x + 8*c) - 112
1745408*a^2*b*e^(8*d*x + 8*c) - 1302025920*a*b^2*e^(8*d*x + 8*c) - 53223584
7*b^3*e^(8*d*x + 8*c) + 83349504*a^3*e^(6*d*x + 6*c) + 280436352*a^2*b*e^(6
*d*x + 6*c) + 325506480*a*b^2*e^(6*d*x + 6*c) + 134271423*b^3*e^(6*d*x + 6*
c) - 14708736*a^3*e^(4*d*x + 4*c) - 49488768*a^2*b*e^(4*d*x + 4*c) - 574423
20*a*b^2*e^(4*d*x + 4*c) - 23694957*b^3*e^(4*d*x + 4*c) + 1634304*a^3*e^(2*
d*x + 2*c) + 5498752*a^2*b*e^(2*d*x + 2*c) + 6382480*a*b^2*e^(2*d*x + 2*c)
+ 2632773*b^3*e^(2*d*x + 2*c) - 86016*a^3 - 289408*a^2*b - 335920*a*b^2 - 1
38567*b^3)/(d*(e^(2*d*x + 2*c) - 1)^19)
```

$$3.229 \quad \int \frac{\sinh^7(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=148

$$-\frac{a \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{a \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\cosh^3(c+dx)}{3bd} + \frac{\cosh(c+dx)}{bd}$$

[Out] $-(a \operatorname{ArcTan}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]])/(2 \operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]] * b^{7/4} * d) + (a \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]])/(2 \operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]] * b^{7/4} * d) + \operatorname{Cosh}[c + d*x]/(b*d) - \operatorname{Cosh}[c + d*x]^3/(3*b*d)$

Rubi [A] time = 0.254065, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3215, 1170, 1166, 205, 208}

$$-\frac{a \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{a \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\cosh^3(c+dx)}{3bd} + \frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sinh}[c + d*x]^7/(a - b*\operatorname{Sinh}[c + d*x]^4), x]$

[Out] $-(a \operatorname{ArcTan}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]])/(2 \operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]] * b^{7/4} * d) + (a \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]])/(2 \operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]] * b^{7/4} * d) + \operatorname{Cosh}[c + d*x]/(b*d) - \operatorname{Cosh}[c + d*x]^3/(3*b*d)$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p}, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x \&\& \operatorname{IntegerQ}[(m - 1)/2]$

Rule 1170

Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{b} + \frac{x^2}{b} + \frac{a-ax^2}{b(a-b+2bx^2-bx^4)}\right) dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)}{bd} - \frac{\cosh^3(c+dx)}{3bd} - \frac{\text{Subst}\left(\int \frac{a-ax^2}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{bd} \\
&= \frac{\cosh(c+dx)}{bd} - \frac{\cosh^3(c+dx)}{3bd} + \frac{a \text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b+b-bx^2}} dx, x, \cosh(c+dx)\right)}{2bd} + \frac{a \text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b+b-bx^2}} dx, x, \cosh(c+dx)\right)}{2bd} \\
&= -\frac{a \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{7/4}d} + \frac{a \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{7/4}d} + \frac{\cosh(c+dx)}{bd} - \frac{\cosh^3(c+dx)}{3bd}
\end{aligned}$$

Mathematica [C] time = 0.336444, size = 390, normalized size = 2.64

$$-3a\text{RootSum}\left[-16\#1^4a + \#1^8b - 4\#1^6b + 6\#1^4b - 4\#1^2b + b\&, \frac{2\#1^6 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right)}{\dots}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4), x]

[Out] (18*Cosh[c + d*x] - 2*Cosh[3*(c + d*x)] - 3*a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 &, (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-b*#1 - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) &])/(24*b*d)

Maple [B] time = 0.082, size = 270, normalized size = 1.8

$$-\frac{1}{3bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-3} + \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} + \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} - \frac{a}{2db^2} \sqrt{ab} \arctan\left(\frac{1}{4} \left(- \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4), x)`

[Out]
$$-1/3/d/b/(\tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)-1/2/d*a/b^2*(a*b)^{(1/2)/(-a*b-(a*b)^{(1/2)*a})^{(1/2)}}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a)/(-a*b-(a*b)^{(1/2)*a})^{(1/2)}))-1/2/d*a/b^2*(a*b)^{(1/2)/(-a*b+(a*b)^{(1/2)*a})^{(1/2)}}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a)/(-a*b+(a*b)^{(1/2)*a})^{(1/2)}))+1/3/d/b/(\tanh(1/2*d*x+1/2*c)-1)^3+1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(e^{(6dx+6c)} - 9e^{(4dx+4c)} - 9e^{(2dx+2c)} + 1)e^{(-3dx-3c)}}{24bd} - \frac{1}{128} \int \frac{256(ae^{(7dx+7c)} - 3ae^{(5dx+5c)} + 3ae^{(3dx+3c)} - ae^{(dx+c)})}{b^2e^{(8dx+8c)} - 4b^2e^{(6dx+6c)} - 4b^2e^{(2dx+2c)} + b^2 - 2(8abe^{(4dx+4c)} - 3b^2e^{(4c)})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4), x, algorithm="maxima")`

[Out]
$$-1/24*(e^{(6*d*x + 6*c)} - 9*e^{(4*d*x + 4*c)} - 9*e^{(2*d*x + 2*c)} + 1)*e^{(-3*d*x - 3*c)/(b*d)} - 1/128*\int(256*(a*e^{(7*d*x + 7*c)} - 3*a*e^{(5*d*x + 5*c)} + 3*a*e^{(3*d*x + 3*c)} - a*e^{(d*x + c)})/(b^2*e^{(8*d*x + 8*c)} - 4*b^2*e^{(6*d*x + 6*c)} - 4*b^2*e^{(2*d*x + 2*c)} + b^2 - 2*(8*a*b*e^{(4*c)} - 3*b^2*e^{(4*c)}))*e^{(4*d*x)}, x)$$

Fricas [B] time = 2.63877, size = 3729, normalized size = 25.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out]
$$-1/24*(\cosh(d*x + c)^6 + 6*\cosh(d*x + c)*\sinh(d*x + c)^5 + \sinh(d*x + c)^6 + 3*(5*\cosh(d*x + c)^2 - 3)*\sinh(d*x + c)^4 - 9*\cosh(d*x + c)^4 + 4*(5*\cosh(d*x + c)^3 - 9*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(5*\cosh(d*x + c)^4 - 18*\cosh(d*x + c)^2 - 3)*\sinh(d*x + c)^2 - 6*(b*d*\cosh(d*x + c)^3 + 3*b*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*b*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*d*\sinh(d*x + c)^3)*\sqrt{-((a*b^3 - b^4)*d^2*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)}) + a^2}/((a*b^3 - b^4)*d^2))*\log(a^3*\cosh(d*x + c)^2 + 2*a^3*\cosh(d*x + c)*\sinh(d*x + c) + a^3*\sinh(d*x + c)^2 + a^3 + 2*(a^2*b^2*d*\cosh(d*x + c) + a^2*b^2*d*\sinh(d*x + c) - ((a*b^5 - b^6)*d^3*\cosh(d*x + c) + (a*b^5 - b^6)*d^3*\sinh(d*x + c)))*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)})*\sqrt{-((a*b^3 - b^4)*d^2*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)}) + a^2}/((a*b^3 - b^4)*d^2))) + 6*(b*d*\cosh(d*x + c)^3 + 3*b*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*b*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*d*\sinh(d*x + c)^3)*\sqrt{-((a*b^3 - b^4)*d^2*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)}) + a^2}/((a*b^3 - b^4)*d^2))*\log(a^3*\cosh(d*x + c)^2 + 2*a^3*\cosh(d*x + c)*\sinh(d*x + c) + a^3*\sinh(d*x + c)^2 + a^3 - 2*(a^2*b^2*d*\cosh(d*x + c) + a^2*b^2*d*\sinh(d*x + c) - ((a*b^5 - b^6)*d^3*\cosh(d*x + c) + (a*b^5 - b^6)*d^3*\sinh(d*x + c)))*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)})*\sqrt{-((a*b^3 - b^4)*d^2*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)}) + a^2}/((a*b^3 - b^4)*d^2))) - 6*(b*d*\cosh(d*x + c)^3 + 3*b*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*b*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*d*\sinh(d*x + c)^3)*\sqrt{((a*b^3 - b^4)*d^2*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)}) - a^2}/((a*b^3 - b^4)*d^2))*\log(a^3*\cosh(d*x + c)^2 + 2*a^3*\cosh(d*x + c)*\sinh(d*x + c) + a^3*\sinh(d*x + c)^2 + a^3 + 2*(a^2*b^2*d*\cosh(d*x + c) + a^2*b^2*d*\sinh(d*x + c) + ((a*b^5 - b^6)*d^3*\cosh(d*x + c) + (a*b^5 - b^6)*d^3*\sinh(d*x + c)))*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)})*\sqrt{((a*b^3 - b^4)*d^2*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)}) - a^2}/((a*b^3 - b^4)*d^2))) + 6*(b*d*\cosh(d*x + c)^3 + 3*b*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*b*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*d*\sinh(d*x + c)^3)*\sqrt{((a*b^3 - b^4)*d^2*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)}) - a^2}/((a*b^3 - b^4)*d^2))*\log(a^3*\cosh(d*x + c)^2 + 2*a^3*\cosh(d*x + c)*\sinh(d*x + c) + a^3*\sinh(d*x + c)^2 + a^3 - 2*(a^2*b^2*d*\cosh(d*x + c) + a^2*b^2*d*\sinh(d*x + c) + ((a*b^5 - b^6)*d^3*\cosh(d*x + c) + (a*b^5 - b^6)*d^3*\sinh(d*x + c)))*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)})*\sqrt{((a*b^3 - b^4)*d^2*\sqrt{a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)}) - a^2}/((a*b^3 - b^4)*d^2))) - 9*\cosh(d*x + c)^2 + 6*(\cosh(d*x + c)^5 - 6*\cosh(d*x + c)^3 - 3*\cosh(d*x + c))*\sinh(d*x + c) + 1)/(b*d*\cosh(d*x + c)^3 + 3*b*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*b*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*d*\sinh(d*x + c)^3)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.230 \quad \int \frac{\sinh^5(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=139

$$\frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\cosh(c+dx)}{bd}$$

[Out] (Sqrt[a]*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(5/4)*d) + (Sqrt[a]*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) - Cosh[c + d*x]/(b*d)

Rubi [A] time = 0.197599, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3215, 1170, 1093, 205, 208}

$$\frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]

[Out] (Sqrt[a]*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(5/4)*d) + (Sqrt[a]*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) - Cosh[c + d*x]/(b*d)

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1170

Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]

Rule 1093

Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{d} \\
 &= \frac{\text{Subst}\left(\int \left(-\frac{1}{b} + \frac{a}{b(a-b+2bx^2-bx^4)}\right) dx, x, \cosh(c+dx)\right)}{d} \\
 &= -\frac{\cosh(c+dx)}{bd} + \frac{a \text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{bd} \\
 &= -\frac{\cosh(c+dx)}{bd} - \frac{\sqrt{a} \text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b+b-bx^2}} dx, x, \cosh(c+dx)\right)}{2\sqrt{bd}} + \frac{\sqrt{a} \text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b+b-bx^2}} dx, x, \cosh(c+dx)\right)}{2\sqrt{bd}} \\
 &= \frac{\sqrt{a} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{a-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{bb^{5/4}d}}} + \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{a+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{bb^{5/4}d}}} - \frac{\cosh(c+dx)}{bd}
 \end{aligned}$$

Mathematica [C] time = 0.259954, size = 235, normalized size = 1.69

$$a \operatorname{RootSum} \left[-16 \#1^4 a + \#1^8 b - 4 \#1^6 b + 6 \#1^4 b - 4 \#1^2 b + b \&, \frac{2 \#1^3 \log \left(-\#1 \sinh \left(\frac{1}{2} (c+dx) \right) + \#1 \cosh \left(\frac{1}{2} (c+dx) \right) - \sinh \left(\frac{1}{2} (c+dx) \right) - \cosh \left(\frac{1}{2} (c+dx) \right) \right)}{2bd} \right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]

[Out] $-(2 \operatorname{Cosh}[c + d*x] + a \operatorname{RootSum}[b - 4*b*\#1^2 - 16*a*\#1^4 + 6*b*\#1^4 - 4*b*\#1^6 + b*\#1^8 \&, (-c*\#1) - d*x*\#1 - 2*\operatorname{Log}[-\operatorname{Cosh}[(c + d*x)/2] - \operatorname{Sinh}[(c + d*x)/2] + \operatorname{Cosh}[(c + d*x)/2]*\#1 - \operatorname{Sinh}[(c + d*x)/2]*\#1*\#1 + c*\#1^3 + d*x*\#1^3 + 2*\operatorname{Log}[-\operatorname{Cosh}[(c + d*x)/2] - \operatorname{Sinh}[(c + d*x)/2] + \operatorname{Cosh}[(c + d*x)/2]*\#1 - \operatorname{Sinh}[(c + d*x)/2]*\#1*\#1^3]/(-b - 8*a*\#1^2 + 3*b*\#1^2 - 3*b*\#1^4 + b*\#1^6) \&])/(2*b*d)$

Maple [A] time = 0.048, size = 175, normalized size = 1.3

$$-\frac{1}{bd} \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) + 1 \right)^{-1} - \frac{a}{2bd} \arctan \left(\frac{1}{4} \left(-2 \left(\tanh \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 a + 4 \sqrt{ab} + 2a \right) \frac{1}{\sqrt{-ab - \sqrt{aba}}} \right) \frac{1}{\sqrt{-ab - \sqrt{aba}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x)

[Out] $-1/d/b/(\tanh(1/2*d*x+1/2*c)+1)-1/2/d*a/b/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a}/(-a*b-(a*b)^{(1/2)*a})^{(1/2)})+1/2/d*a/b/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a}/(-a*b+(a*b)^{(1/2)*a})^{(1/2)})+1/d/b/(\tanh(1/2*d*x+1/2*c)-1)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(e^{(2dx+2c)} + 1)e^{(-dx-c)}}{2bd} - \frac{1}{32} \int \frac{256 (ae^{(5dx+5c)} - ae^{(3dx+3c)})}{b^2 e^{(8dx+8c)} - 4b^2 e^{(6dx+6c)} - 4b^2 e^{(2dx+2c)} + b^2 - 2(8abe^{(4c)} - 3b^2 e^{(4c)})e^{(4dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] $-1/2*(e^{(2*d*x + 2*c)} + 1)*e^{(-d*x - c)}/(b*d) - 1/32*\text{integrate}(256*(a*e^{(5*d*x + 5*c)} - a*e^{(3*d*x + 3*c)})/(b^2*e^{(8*d*x + 8*c)} - 4*b^2*e^{(6*d*x + 6*c)} - 4*b^2*e^{(2*d*x + 2*c)} + b^2 - 2*(8*a*b*e^{(4*c)} - 3*b^2*e^{(4*c)})*e^{(4*d*x)}), x)$

Fricas [B] time = 2.11141, size = 2755, normalized size = 19.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out] $1/4*((b*d*\cosh(d*x + c) + b*d*\sinh(d*x + c))*\sqrt{-((a*b^2 - b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a}/((a*b^2 - b^3)*d^2))*\log(a^2*\cosh(d*x + c)^2 + 2*a^2*\cosh(d*x + c)*\sinh(d*x + c) + a^2*\sinh(d*x + c)^2 + a^2 + 2*(a^2*b*d*\cosh(d*x + c) + a^2*b*d*\sinh(d*x + c) - ((a*b^4 - b^5)*d^3*\cosh(d*x + c) + (a*b^4 - b^5)*d^3*\sinh(d*x + c))*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)}))\sqrt{-((a*b^2 - b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a}/((a*b^2 - b^3)*d^2)) - (b*d*\cosh(d*x + c) + b*d*\sinh(d*x + c))*\sqrt{-((a*b^2 - b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a}/((a*b^2 - b^3)*d^2))*\log(a^2*\cosh(d*x + c)^2 + 2*a^2*\cosh(d*x + c)*\sinh(d*x + c) + a^2*\sinh(d*x + c)^2 + a^2 - 2*(a^2*b*d*\cosh(d*x + c) + a^2*b*d*\sinh(d*x + c) - ((a*b^4 - b^5)*d^3*\cosh(d*x + c) + (a*b^4 - b^5)*d^3*\sinh(d*x + c))*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)}))\sqrt{-((a*b^2 - b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a}/((a*b^2 - b^3)*d^2)) + (b*d*\cosh(d*x + c) + b*d*\sinh(d*x + c))*\sqrt{((a*b^2 - b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a}/((a*b^2 - b^3)*d^2))*\log(a^2*\cosh(d*x + c)^2 + 2*a^2*\cosh(d*x + c)*\sinh(d*x + c) + a^2*\sinh(d*x + c)^2 + a^2 + 2*(a^2*b*d*\cosh(d*x + c) + a^2*b*d*\sinh(d*x + c) + ((a*b^4 - b^5)*d^3*\cosh(d*x + c) + (a*b^4 - b^5)*d^3*\sinh(d*x + c))*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)}))\sqrt{((a*b^2 - b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a}/((a*b^2 - b^3)*d^2)) - (b*d*\cosh(d*x + c) + b*d*\sinh(d*x + c))*\sqrt{((a*b^2 - b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a}/((a*b^2 - b^3)*d^2))*\log(a^2*\cosh(d*x + c)^2 + 2*a^2*\cosh(d*x + c)*\sinh(d*x + c) + a^2*\sinh(d*x + c)^2 + a^2 - 2*(a^2*b*d*\cosh(d*x + c) + a^2*b*d*\sinh(d*x + c) + ((a*b^4 - b^5)*d^3*\cosh(d*x + c) + (a*b^4 - b^5)*d^3*\sinh(d*x + c))*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)}))\sqrt{((a*b^2 - b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a}/((a*b^2 - b^3)*d^2)) - 2*\cosh(d*x + c)^2 - 4*\cosh(d*x + c)*\sinh(d*x + c) - 2*\sinh(d*x + c)^2 - 2)/(b*d*\cosh(d*x + c) + b*d*$

$\sinh(dx + c)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(dx+c)**5/(a-b*sinh(dx+c)**4),x)`

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(dx+c)^5/(a-b*sinh(dx+c)^4),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.231 \quad \int \frac{\sinh^3(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

[Out] $-\text{ArcTan}[(b^{1/4} \cdot \text{Cosh}[c + d \cdot x]) / \text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]]] / (2 \cdot \text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]] \cdot b^{3/4} \cdot d) + \text{ArcTanh}[(b^{1/4} \cdot \text{Cosh}[c + d \cdot x]) / \text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]]] / (2 \cdot \text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]] \cdot b^{3/4} \cdot d)$

Rubi [A] time = 0.114616, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3215, 1166, 205, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d \cdot x]^3 / (a - b \cdot \text{Sinh}[c + d \cdot x]^4), x]$

[Out] $-\text{ArcTan}[(b^{1/4} \cdot \text{Cosh}[c + d \cdot x]) / \text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]]] / (2 \cdot \text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]] \cdot b^{3/4} \cdot d) + \text{ArcTanh}[(b^{1/4} \cdot \text{Cosh}[c + d \cdot x]) / \text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]]] / (2 \cdot \text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]] \cdot b^{3/4} \cdot d)$

Rule 3215

$\text{Int}[\sin[(e_.) + (f_.)(x_.)]^{(m_.)} \cdot ((a_.) + (b_.) \cdot \sin[(e_.) + (f_.)(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e + f \cdot x], x]\}, -\text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2 \cdot x^2)^{(m-1)/2} \cdot (a + b - 2 \cdot b \cdot ff^2 \cdot x^2 + b \cdot ff^4 \cdot x^4)^p, x], x, \text{Cos}[e + f \cdot x]/ff], x] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \ \&\amp; \ \text{IntegerQ}[(m-1)/2]$

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)} dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b+b-bx^2}} dx, x, \cosh(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b+b-bx^2}} dx, x, \cosh(c+dx)\right)}{2d}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{a-\sqrt{b}}}\right)}{2\sqrt{a-\sqrt{b}}b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{a+\sqrt{b}}}\right)}{2\sqrt{a+\sqrt{b}}b^{3/4}d}$$

Mathematica [C] time = 0.177202, size = 365, normalized size = 3.17

$$\text{RootSum}\left[-16\#1^4 a + \#1^8 b - 4\#1^6 b + 6\#1^4 b - 4\#1^2 b + b \& \& \frac{2\#1^6 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right)}{\dots}\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4), x]
```

```
[Out] -RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c -
d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
```

$$\frac{\text{Sinh}[(c + d*x)/2]^{*1} + 3*c^{*1^2} + 3*d*x^{*1^2} + 6*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]^{*1} - \text{Sinh}[(c + d*x)/2]^{*1^2} - 3*c^{*1^4} - 3*d*x^{*1^4} - 6*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]^{*1} - \text{Sinh}[(c + d*x)/2]^{*1^4} + c^{*1^6} + d*x^{*1^6} + 2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]^{*1} - \text{Sinh}[(c + d*x)/2]^{*1^6}]/(-b^{*1} - 8*a^{*1^3} + 3*b^{*1^3} - 3*b^{*1^5} + b^{*1^7}) \&]/(8*d}$$

Maple [A] time = 0.029, size = 142, normalized size = 1.2

$$-\frac{1}{2bd}\sqrt{ab}\arctan\left(\frac{1}{4}\left(-2(\tanh(1/2dx + c/2))^2a + 4\sqrt{ab} + 2a\right)\frac{1}{\sqrt{-ab - \sqrt{aba}}}\right)\frac{1}{\sqrt{-ab - \sqrt{aba}}} - \frac{1}{2bd}\sqrt{ab}\arctan\left(\frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x)

[Out]
$$-1/2/d*(a*b)^{(1/2)}/b/(-a*b-(a*b)^{(1/2)*a}^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a)/(-a*b-(a*b)^{(1/2)*a}^{(1/2)})-1/2/d*(a*b)^{(1/2)}/b/(-a*b+(a*b)^{(1/2)*a}^{(1/2)*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a)/(-a*b+(a*b)^{(1/2)*a}^{(1/2)})})}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh(dx + c)^3}{b \sinh(dx + c)^4 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] -integrate(sinh(d*x + c)^3/(b*sinh(d*x + c)^4 - a), x)

Fricas [B] time = 1.90177, size = 2202, normalized size = 19.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a
*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) - ((a*b^2 - b^3)*d^3*
cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4
+ b^5)*d^4)))*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4
)) + 1)/((a*b - b^2)*d^2)) + 1) - 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c) + b*d*s
inh(d*x + c) - ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*
x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)))*sqrt(-((a*b - b^2)*d^2*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2)) + 1) + 1/4*sq
rt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2
)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) + ((a*b^2 - b^3)*d^3*cosh(d*x
+ c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*
d^4)))*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/
((a*b - b^2)*d^2)) + 1) - 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b
^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x +
c) + ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)))*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2)) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.232 \quad \int \frac{\sinh(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{bd}\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{bd}\sqrt{\sqrt{a}+\sqrt{b}}}$$

[Out] ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)

Rubi [A] time = 0.103533, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3215, 1093, 205, 208}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{bd}\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{bd}\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4),x]

[Out] ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] - Sqrt[b]]*b^(1/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1093


```
Int[((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{d} \\ &= -\frac{\sqrt{b} \text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b+b-bx^2}} dx, x, \cosh(c+dx)\right)}{2\sqrt{ad}} + \frac{\sqrt{b} \text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b+b-bx^2}} dx, x, \cosh(c+dx)\right)}{2\sqrt{ad}} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{a-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a-\sqrt{b}}\sqrt[4]{bd}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{a+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{a+\sqrt{b}}\sqrt[4]{bd}} \end{aligned}$$

Mathematica [C] time = 0.167964, size = 221, normalized size = 1.77

$$\frac{\text{RootSum}\left[-16\#1^4 a + \#1^8 b - 4\#1^6 b + 6\#1^4 b - 4\#1^2 b + b \&, \frac{2\#1^3 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}\right]}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4), x]
```

```
[Out] -RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 &, (-c*#1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3 + 2*Log[-Cosh[(c + d*
```

$x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2] * \#1 - \text{Sinh}[(c + d*x)/2] * \#1 * \#1^3) / (-b - 8*a*\#1^2 + 3*b*\#1^2 - 3*b*\#1^4 + b*\#1^6) \&] / (2*d)$

Maple [A] time = 0.04, size = 126, normalized size = 1.

$$-\frac{1}{2d} \arctan\left(\frac{1}{4}\left(-2(\tanh(1/2 dx + c/2))^2 a + 4\sqrt{ab} + 2a\right)\frac{1}{\sqrt{-ab - \sqrt{aba}}}\right)\frac{1}{\sqrt{-ab - \sqrt{aba}}} + \frac{1}{2d} \arctan\left(\frac{1}{4}\left(2(\tanh(1/2 dx + c/2))^2 a - 4\sqrt{ab} + 2a\right)\frac{1}{\sqrt{-ab - \sqrt{aba}}}\right)\frac{1}{\sqrt{-ab - \sqrt{aba}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x)

[Out] $-1/2/d/(-a*b-(a*b)^{(1/2)*a}^{(1/2)*a})*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)+2*a}}/(-a*b-(a*b)^{(1/2)*a}^{(1/2)})+1/2/d/(-a*b+(a*b)^{(1/2)*a}^{(1/2)})*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)-2*a}}/(-a*b+(a*b)^{(1/2)*a}^{(1/2)}))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh(dx+c)}{b \sinh(dx+c)^4 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] -integrate(sinh(d*x + c)/(b*sinh(d*x + c)^4 - a), x)

Fricas [B] time = 1.98088, size = 2256, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")

```
[Out] 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + 1) - 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + 1) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) + ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + 1) - 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) + ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.233 \quad \int \frac{\operatorname{csch}(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=136

$$-\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

[Out] $-(b^{1/4} \operatorname{ArcTan}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]])/(2*a*\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]*d) - \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/(a*d) + (b^{1/4} \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]])/(2*a*\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]*d)$

Rubi [A] time = 0.163084, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3215, 1170, 207, 1166, 205, 208}

$$-\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]/(a - b*\operatorname{Sinh}[c + d*x]^4), x]$

[Out] $-(b^{1/4} \operatorname{ArcTan}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]])/(2*a*\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]*d) - \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/(a*d) + (b^{1/4} \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]])/(2*a*\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]*d)$

Rule 3215

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] :> \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Cos}[e + f*x], x]\}, -\operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, \operatorname{Cos}[e + f*x]/ff], x] /; \operatorname{FreeQ}[\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)} dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(-\frac{1}{a(-1+x^2)} + \frac{b-bx^2}{a(a-b+2bx^2-bx^4)}\right) dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \cosh(c+dx)\right)}{ad} - \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{ad} \\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{b \operatorname{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2} dx, x, \cosh(c+dx)\right)}{2ad} + \frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-b-bx^2} dx, x, \cosh(c+dx)\right)}{2ad} \\
&= -\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}d}
\end{aligned}$$

Mathematica [C] time = 0.248951, size = 385, normalized size = 2.83

$$8 \log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - b \operatorname{RootSum}\left[-16\#1^4 a + \#1^8 b - 4\#1^6 b + 6\#1^4 b - 4\#1^2 b + b\sqrt{b}, \frac{2\#1^6 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{b}}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4), x]

[Out] (8*Log[Tanh[(c + d*x)/2]] - b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(- (b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) &])/(8*a*d)

Maple [A] time = 0.053, size = 159, normalized size = 1.2

$$-\frac{1}{2da}\sqrt{ab}\arctan\left(\frac{1}{4}\left(-2(\tanh(1/2dx+c/2))^2a+4\sqrt{ab}+2a\right)\frac{1}{\sqrt{-ab-\sqrt{aba}}}\right)\frac{1}{\sqrt{-ab-\sqrt{aba}}}-\frac{1}{2da}\sqrt{ab}\arctan\left(\frac{1}{4}\left(2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x)

[Out]
$$-1/2/d*(a*b)^{(1/2)}/a/(-a*b-(a*b)^{(1/2)*a}^{(1/2)*}\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a}/(-a*b-(a*b)^{(1/2)*a}^{(1/2)})-1/2/d*(a*b)^{(1/2)}/a/(-a*b+(a*b)^{(1/2)*a}^{(1/2)*}\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a}/(-a*b+(a*b)^{(1/2)*a}^{(1/2)})+1/d/a*\ln(\tanh(1/2*d*x+1/2*c)))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{-c}\right)}{ad}+\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{-c}\right)}{ad}-2\int\frac{be^{(7dx+7c)}-3be^{(5dx+5c)}+3be^{(3dx+3c)}-be^{(dx+c)}}{abe^{(8dx+8c)}-4abe^{(6dx+6c)}-4abe^{(2dx+2c)}+ab-2\left(8a^2e^{(4c)}-3\right)}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out]
$$-\log\left(\left(e^{(d*x+c)}+1\right)*e^{-c}\right)/(a*d)+\log\left(\left(e^{(d*x+c)}-1\right)*e^{-c}\right)/(a*d)-2*\int\left(\left(b*e^{(7*d*x+7*c)}-3*b*e^{(5*d*x+5*c)}+3*b*e^{(3*d*x+3*c)}-b*e^{(d*x+c)}\right)/\left(a*b*e^{(8*d*x+8*c)}-4*a*b*e^{(6*d*x+6*c)}-4*a*b*e^{(2*d*x+2*c)}+a*b-2*(8*a^2*e^{(4*c)}-3*a*b*e^{(4*c)})*e^{(4*d*x)}\right),x$$

Fricas [B] time = 2.26058, size = 2431, normalized size = 17.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out]
$$1/4*(a*d*\sqrt{-((a^3-a^2*b)*d^2*\sqrt{b/((a^5-2*a^4*b+a^3*b^2)*d^4)})+b}/((a^3-a^2*b)*d^2))*\log(b*\cosh(d*x+c)^2+2*b*\cosh(d*x+c)*\sinh(d*x+c))$$

$$\begin{aligned}
& + c) + b \sinh(dx + c)^2 + 2*(a*b*d*\cosh(dx + c) + a*b*d*\sinh(dx + c) - \\
& ((a^4 - a^3*b)*d^3*\cosh(dx + c) + (a^4 - a^3*b)*d^3*\sinh(dx + c))*\sqrt{b/} \\
& ((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*\sqrt{-((a^3 - a^2*b)*d^2*\sqrt{b/((a^5 - 2} \\
& *a^4*b + a^3*b^2)*d^4)) + b}/((a^3 - a^2*b)*d^2)) + b) - a*d*\sqrt{-((a^3 - \\
& a^2*b)*d^2*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b}/((a^3 - a^2*b)*d^2)} \\
&)*\log(b*\cosh(dx + c)^2 + 2*b*\cosh(dx + c)*\sinh(dx + c) + b*\sinh(dx + c) \\
& ^2 - 2*(a*b*d*\cosh(dx + c) + a*b*d*\sinh(dx + c) - ((a^4 - a^3*b)*d^3*\cosh \\
& (dx + c) + (a^4 - a^3*b)*d^3*\sinh(dx + c))*\sqrt{b/((a^5 - 2*a^4*b + a^3*b \\
& ^2)*d^4)))*\sqrt{-((a^3 - a^2*b)*d^2*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4))} \\
& + b)/((a^3 - a^2*b)*d^2)) + b) + a*d*\sqrt{((a^3 - a^2*b)*d^2*\sqrt{b/((a^5 \\
& - 2*a^4*b + a^3*b^2)*d^4)) - b}/((a^3 - a^2*b)*d^2))*\log(b*\cosh(dx + c)^2 \\
& + 2*b*\cosh(dx + c)*\sinh(dx + c) + b*\sinh(dx + c)^2 + 2*(a*b*d*\cosh(dx + \\
& c) + a*b*d*\sinh(dx + c) + ((a^4 - a^3*b)*d^3*\cosh(dx + c) + (a^4 - a^3*b) \\
&)*d^3*\sinh(dx + c))*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*\sqrt{((a^3 - \\
& a^2*b)*d^2*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b}/((a^3 - a^2*b)*d^2)} \\
&) + b) - a*d*\sqrt{((a^3 - a^2*b)*d^2*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)} \\
&) - b)/((a^3 - a^2*b)*d^2))*\log(b*\cosh(dx + c)^2 + 2*b*\cosh(dx + c)*\sinh \\
& (dx + c) + b*\sinh(dx + c)^2 - 2*(a*b*d*\cosh(dx + c) + a*b*d*\sinh(dx + c) \\
& + ((a^4 - a^3*b)*d^3*\cosh(dx + c) + (a^4 - a^3*b)*d^3*\sinh(dx + c))*\sqrt{ \\
& b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*\sqrt{((a^3 - a^2*b)*d^2*\sqrt{b/((a^5 - \\
& 2*a^4*b + a^3*b^2)*d^4)) - b}/((a^3 - a^2*b)*d^2)) + b) - 4*\log(\cosh(dx + \\
& c) + \sinh(dx + c) + 1) + 4*\log(\cosh(dx + c) + \sinh(dx + c) - 1))/(a*d)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)/(a-b*sinh(dx+c)**4),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)/(a-b*sinh(dx+c)^4),x, algorithm="giac")

```
[Out] Exception raised: NotImplementedError
```

$$3.234 \quad \int \frac{\operatorname{csch}^3(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=184

$$\frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{b^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}} + \frac{1}{4ad(1-\cosh(c+dx))} - \frac{1}{4ad(\cosh(c+dx)+1)} + \frac{\tanh^{-1}(\cos}{2a}$$

[Out] (b^(3/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + ArcTanh[Cosh[c + d*x]]/(2*a*d) + (b^(3/4)*ArcTanH[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqrt[a] + Sqrt[b]]*d) + 1/(4*a*d*(1 - Cosh[c + d*x])) - 1/(4*a*d*(1 + Cosh[c + d*x]))

Rubi [A] time = 0.200735, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3215, 1170, 207, 1093, 205, 208}

$$\frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{b^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}} + \frac{1}{4ad(1-\cosh(c+dx))} - \frac{1}{4ad(\cosh(c+dx)+1)} + \frac{\tanh^{-1}(\cos}{2a}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4), x]

[Out] (b^(3/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + ArcTanh[Cosh[c + d*x]]/(2*a*d) + (b^(3/4)*ArcTanH[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqrt[a] + Sqrt[b]]*d) + 1/(4*a*d*(1 - Cosh[c + d*x])) - 1/(4*a*d*(1 + Cosh[c + d*x]))

Rule 3215

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -

1)/2]

Rule 1170

Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1093

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+2bx^2-bx^4)} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2} + \frac{1}{4a(1+x)^2} - \frac{1}{2a(-1+x^2)} + \frac{b}{a(a-b+2bx^2-bx^4)}\right) dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{1}{4ad(1-\cosh(c+dx))} - \frac{1}{4ad(1+\cosh(c+dx))} - \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \cosh(c+dx)\right)}{2ad} + \frac{b}{4ad} \\
&= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{1}{4ad(1-\cosh(c+dx))} - \frac{1}{4ad(1+\cosh(c+dx))} - \frac{b^{3/2} \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \cosh(c+dx)\right)}{2ad} \\
&= \frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2} \sqrt{\sqrt{a}-\sqrt{b}d}} + \frac{\tanh^{-1}(\cosh(c+dx))}{2ad} + \frac{b^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2} \sqrt{\sqrt{a}+\sqrt{b}d}} + \frac{b}{4ad(1-\cosh(c+dx))}
\end{aligned}$$

Mathematica [C] time = 0.378545, size = 265, normalized size = 1.44

$$4b\operatorname{RootSum}\left[-16\#1^4a + \#1^8b - 4\#1^6b + 6\#1^4b - 4\#1^2b + b\&, \frac{2\#1^3 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right)}{\dots}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4), x]

[Out] $-(\operatorname{Csch}[(c+d*x)/2]^2 + 4*\operatorname{Log}[\operatorname{Tanh}[(c+d*x)/2]] + 4*b*\operatorname{RootSum}[b - 4*b*\#1^2 - 16*a*\#1^4 + 6*b*\#1^4 - 4*b*\#1^6 + b*\#1^8 \&, (-c*\#1) - d*x*\#1 - 2*\operatorname{Log}[-\operatorname{Cosh}[(c+d*x)/2] - \operatorname{Sinh}[(c+d*x)/2] + \operatorname{Cosh}[(c+d*x)/2]*\#1 - \operatorname{Sinh}[(c+d*x)/2]*\#1]*\#1 + c*\#1^3 + d*x*\#1^3 + 2*\operatorname{Log}[-\operatorname{Cosh}[(c+d*x)/2] - \operatorname{Sinh}[(c+d*x)/2]] + \operatorname{Cosh}[(c+d*x)/2]*\#1 - \operatorname{Sinh}[(c+d*x)/2]*\#1]*\#1^3)/(-b - 8*a*\#1^2 + 3*b*\#1^2 - 3*b*\#1^4 + b*\#1^6) \&] + \operatorname{Sech}[(c+d*x)/2]^2/(8*a*d)$

Maple [A] time = 0.066, size = 190, normalized size = 1.

$$\frac{1}{8da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 - \frac{b}{2da} \arctan\left(\frac{1}{4} \left(-2 \left(\tanh\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 a + 4\sqrt{ab} + 2a \right) \frac{1}{\sqrt{-ab - \sqrt{aba}}} \right) \frac{1}{\sqrt{-ab - \sqrt{aba}}} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x)`

[Out] $\frac{1}{8} \frac{d}{a} \tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 - \frac{1}{2} \frac{d*b}{a} \frac{1}{(-a*b - (a*b)^{1/2}*a)^{1/2}} \arctan\left(\frac{1}{4} \frac{(-2*\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2*a + 4*(a*b)^{1/2} + 2*a)}{(-a*b - (a*b)^{1/2}*a)^{1/2}}\right) + \frac{1}{2} \frac{d*b}{a} \frac{1}{(-a*b + (a*b)^{1/2}*a)^{1/2}} \arctan\left(\frac{1}{4} \frac{(2*\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2*a + 4*(a*b)^{1/2} - 2*a)}{(-a*b + (a*b)^{1/2}*a)^{1/2}}\right) - \frac{1}{8} \frac{d}{a} \frac{1}{\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)^2 - 1} \frac{d}{a} \ln\left(\tanh\left(\frac{1}{2}d*x + \frac{1}{2}c\right)\right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{3dx+3c} + e^{(dx+c)}}{ade^{4dx+4c} - 2ade^{2dx+2c} + ad} + \frac{\log\left(\left(e^{(dx+c)} + 1\right)e^{-c}\right)}{2ad} - \frac{\log\left(\left(e^{(dx+c)} - 1\right)e^{-c}\right)}{2ad} - 8 \int \frac{be^{8dx+8c} - 4abe^{6dx+6c} - 4abe^{4dx+4c} + a^2d}{abe^{8dx+8c} - 4abe^{6dx+6c} - 4abe^{4dx+4c} + a^2d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")`

[Out] $-\frac{(e^{3d*x + 3c} + e^{(d*x + c)})}{(a*d*e^{4d*x + 4c} - 2*a*d*e^{2d*x + 2c} + a*d)} + \frac{1}{2} \frac{\log((e^{(d*x + c)} + 1)*e^{-c})}{(a*d)} - \frac{1}{2} \frac{\log((e^{(d*x + c)} - 1)*e^{-c})}{(a*d)} - 8 \int \frac{(b*e^{5d*x + 5c} - b*e^{3d*x + 3c})}{(a*b*e^{8d*x + 8c} - 4*a*b*e^{6d*x + 6c} - 4*a*b*e^{2d*x + 2c} + a*b - 2*(8*a^2*e^{4c} - 3*a*b*e^{4c})*e^{4d*x})} dx$

Fricas [B] time = 2.58645, size = 4716, normalized size = 25.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")`

[Out] $-\frac{1}{4} \frac{(4*\cosh(d*x + c)^3 + 12*\cosh(d*x + c)*\sinh(d*x + c)^2 + 4*\sinh(d*x + c)^3 - (a*d*\cosh(d*x + c)^4 + 4*a*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + a*d*\sinh(d*x + c)^4 - 2*a*d*\cosh(d*x + c)^2 + 2*(3*a*d*\cosh(d*x + c)^2 - a*d)*\sinh(d*x + c)^2 + a*d + 4*(a*d*\cosh(d*x + c)^3 - a*d*\cosh(d*x + c))*\sinh(d*x + c))\sqrt{-((a^4 - a^3*b)*d^2*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)} + b^2)}}{((a^4 - a^3*b)*d^2)} \log(b^2*\cosh(d*x + c)^2 + 2*b^2*\cosh(d*x + c)*\sinh(d*x + c))$

$$\begin{aligned}
& *x + c) + b^2 \sinh(dx + c)^2 + b^2 + 2*(a^2*b*d*\cosh(dx + c) + a^2*b*d*\sinh(dx + c) - ((a^5 - a^4*b)*d^3*\cosh(dx + c) + (a^5 - a^4*b)*d^3*\sinh(dx + c))*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)})*\sqrt{-((a^4 - a^3*b)*d^2*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)} + b^2)/((a^4 - a^3*b)*d^2)}) + (a*d*\cosh(dx + c)^4 + 4*a*d*\cosh(dx + c)*\sinh(dx + c)^3 + a*d*\sinh(dx + c)^4 - 2*a*d*\cosh(dx + c)^2 + 2*(3*a*d*\cosh(dx + c)^2 - a*d)*\sinh(dx + c)^2 + a*d + 4*(a*d*\cosh(dx + c)^3 - a*d*\cosh(dx + c))*\sinh(dx + c))*\sqrt{-((a^4 - a^3*b)*d^2*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)} + b^2)/((a^4 - a^3*b)*d^2)})*\log(b^2*\cosh(dx + c)^2 + 2*b^2*\cosh(dx + c)*\sinh(dx + c) + b^2*\sinh(dx + c)^2 + b^2 - 2*(a^2*b*d*\cosh(dx + c) + a^2*b*d*\sinh(dx + c) - ((a^5 - a^4*b)*d^3*\cosh(dx + c) + (a^5 - a^4*b)*d^3*\sinh(dx + c))*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)})*\sqrt{-((a^4 - a^3*b)*d^2*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)} + b^2)/((a^4 - a^3*b)*d^2)}) - (a*d*\cosh(dx + c)^4 + 4*a*d*\cosh(dx + c)*\sinh(dx + c)^3 + a*d*\sinh(dx + c)^4 - 2*a*d*\cosh(dx + c)^2 + 2*(3*a*d*\cosh(dx + c)^2 - a*d)*\sinh(dx + c)^2 + a*d + 4*(a*d*\cosh(dx + c)^3 - a*d*\cosh(dx + c))*\sinh(dx + c))*\sqrt{((a^4 - a^3*b)*d^2*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)} - b^2)/((a^4 - a^3*b)*d^2)})*\log(b^2*\cosh(dx + c)^2 + 2*b^2*\cosh(dx + c)*\sinh(dx + c) + b^2*\sinh(dx + c)^2 + b^2 + 2*(a^2*b*d*\cosh(dx + c) + a^2*b*d*\sinh(dx + c) + ((a^5 - a^4*b)*d^3*\cosh(dx + c) + (a^5 - a^4*b)*d^3*\sinh(dx + c))*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)})*\sqrt{((a^4 - a^3*b)*d^2*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)} - b^2)/((a^4 - a^3*b)*d^2)}) + (a*d*\cosh(dx + c)^4 + 4*a*d*\cosh(dx + c)*\sinh(dx + c)^3 + a*d*\sinh(dx + c)^4 - 2*a*d*\cosh(dx + c)^2 + 2*(3*a*d*\cosh(dx + c)^2 - a*d)*\sinh(dx + c)^2 + a*d + 4*(a*d*\cosh(dx + c)^3 - a*d*\cosh(dx + c))*\sinh(dx + c))*\sqrt{((a^4 - a^3*b)*d^2*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)} - b^2)/((a^4 - a^3*b)*d^2)})*\log(b^2*\cosh(dx + c)^2 + 2*b^2*\cosh(dx + c)*\sinh(dx + c) + b^2*\sinh(dx + c)^2 + b^2 - 2*(a^2*b*d*\cosh(dx + c) + a^2*b*d*\sinh(dx + c) + ((a^5 - a^4*b)*d^3*\cosh(dx + c) + (a^5 - a^4*b)*d^3*\sinh(dx + c))*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)})*\sqrt{((a^4 - a^3*b)*d^2*\sqrt{b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)} - b^2)/((a^4 - a^3*b)*d^2)}) - 2*(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 - 1)*\sinh(dx + c)^2 - 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 - \cosh(dx + c))*\sinh(dx + c) + 1)*\log(\cosh(dx + c) + \sinh(dx + c) + 1) + 2*(\cosh(dx + c)^4 + 4*\cosh(dx + c)*\sinh(dx + c)^3 + \sinh(dx + c)^4 + 2*(3*\cosh(dx + c)^2 - 1)*\sinh(dx + c)^2 - 2*\cosh(dx + c)^2 + 4*(\cosh(dx + c)^3 - \cosh(dx + c))*\sinh(dx + c) + 1)*\log(\cosh(dx + c) + \sinh(dx + c) - 1) + 4*(3*\cosh(dx + c)^2 + 1)*\sinh(dx + c) + 4*\cosh(dx + c))/(a*d*\cosh(dx + c)^4 + 4*a*d*\cosh(dx + c)*\sinh(dx + c)^3 + a*d*\sinh(dx + c)^4 - 2*a*d*\cosh(dx + c)^2 + 2*(3*a*d*\cosh(dx + c)^2 - a*d)*\sinh(dx + c)^2 + a*d + 4*(a*d*\cosh(dx + c)^3 - a*d*\cosh(dx + c))*\sinh(dx + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```


$$3.235 \quad \int \frac{\sinh^6(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=175

$$\frac{a^{3/4} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{a^{3/4} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{1}{4bd(1-\tanh(c+dx))} + \frac{1}{4bd(\tanh(c+dx)+1)}$$

[Out] x/(2*b) - (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) + (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) - 1/(4*b*d*(1 - Tanh[c + d*x])) + 1/(4*b*d*(1 + Tanh[c + d*x]))

Rubi [A] time = 0.258026, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3217, 1287, 207, 1130, 208}

$$\frac{a^{3/4} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{a^{3/4} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{1}{4bd(1-\tanh(c+dx))} + \frac{1}{4bd(\tanh(c+dx)+1)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]

[Out] x/(2*b) - (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) + (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) - 1/(4*b*d*(1 - Tanh[c + d*x])) + 1/(4*b*d*(1 + Tanh[c + d*x]))

Rule 3217

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
+ b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
+ q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(a-2ax^2+(a-b)x^4)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{1}{4b(-1+x)^2} - \frac{1}{4b(1+x)^2} - \frac{1}{2b(-1+x^2)} + \frac{ax^2}{b(a-2ax^2+(a-b)x^4)}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{1}{4bd(1-\tanh(c+dx))} + \frac{1}{4bd(1+\tanh(c+dx))} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \tanh(c+dx)\right)}{2bd} + \\
&= \frac{x}{2b} - \frac{1}{4bd(1-\tanh(c+dx))} + \frac{1}{4bd(1+\tanh(c+dx))} + \frac{(a(\sqrt{a}+\sqrt{b})) \text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+x} dx, x, \tanh(c+dx)\right)}{2b^{3/2}d} \\
&= \frac{x}{2b} - \frac{a^{3/4} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{3/2}d} + \frac{a^{3/4} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{3/2}d} - \frac{1}{4bd(1-\tanh(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.867199, size = 158, normalized size = 0.9

$$\frac{2a \tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b+a}}}\right)}{\sqrt{\sqrt{a}\sqrt{b+a}}} + \frac{2a \tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b-a}}}\right)}{\sqrt{\sqrt{a}\sqrt{b-a}}} + 2\sqrt{b}(c+dx) - \sqrt{b} \sinh(2(c+dx))}{4b^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]

[Out] (2*Sqrt[b]*(c + d*x) + (2*a*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])]/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (2*a*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]])]/Sqrt[a + Sqrt[a]*Sqrt[b]] - Sqrt[b]*Sinh[2*(c + d*x)])/(4*b^(3/2)*d)

Maple [C] time = 0.051, size = 223, normalized size = 1.3

$$\frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-2} - \frac{1}{2bd} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)^{-1} + \frac{1}{2bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{a}{bd} \sum_{R=\text{RootOf}(a_Z^8-4a_Z^6+...)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x)`

[Out] $\frac{1}{2} \frac{d}{d} \frac{b}{b} (\tanh(\frac{1}{2}d*x + \frac{1}{2}c) + 1)^{-2} - \frac{1}{2} \frac{d}{d} \frac{b}{b} (\tanh(\frac{1}{2}d*x + \frac{1}{2}c) + 1) + \frac{1}{2} \frac{d}{d} \frac{b}{b} \ln(\tanh(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \frac{1}{d} \frac{a}{a} \frac{b}{b} \sum((\frac{R^4 - R^2}{R^7 a - 3 R^5 a + 3 R^3 a - 8 R^3 b - R a}) * \ln(\tanh(\frac{1}{2}d*x + \frac{1}{2}c) - R), R = \text{RootOf}(a Z^8 - 4 a Z^6 + (6 a - 16 b) Z^4 - 4 a Z^2 + a)) - \frac{1}{2} \frac{d}{d} \frac{b}{b} (\tanh(\frac{1}{2}d*x + \frac{1}{2}c) - 1)^{-2} - \frac{1}{2} \frac{d}{d} \frac{b}{b} (\tanh(\frac{1}{2}d*x + \frac{1}{2}c) - 1) - \frac{1}{2} \frac{d}{d} \frac{b}{b} \ln(\tanh(\frac{1}{2}d*x + \frac{1}{2}c) - 1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(4 dx e^{(2dx+2c)} - e^{(4dx+4c)} + 1)e^{(-2dx-2c)}}{8bd} - \frac{1}{64} \int \frac{256 (ae^{(6dx+6c)} - 2ae^{(4dx+4c)} + ae^{(2dx+2c)})}{b^2 e^{(8dx+8c)} - 4b^2 e^{(6dx+6c)} - 4b^2 e^{(2dx+2c)} + b^2 - 2(8abe^{(4c)} - 3b^2 e^{(4c)})e^{(4dx+4c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")`

[Out] $\frac{1}{8} * (4 * d * x * e^{(2 * d * x + 2 * c)} - e^{(4 * d * x + 4 * c)} + 1) * e^{(-2 * d * x - 2 * c)} / (b * d) - \frac{1}{64} * \text{integrate}(256 * (a * e^{(6 * d * x + 6 * c)} - 2 * a * e^{(4 * d * x + 4 * c)} + a * e^{(2 * d * x + 2 * c)}) / (b^2 * e^{(8 * d * x + 8 * c)} - 4 * b^2 * e^{(6 * d * x + 6 * c)} - 4 * b^2 * e^{(2 * d * x + 2 * c)} + b^2 - 2 * (8 * a * b * e^{(4 * c)} - 3 * b^2 * e^{(4 * c)}) * e^{(4 * d * x)}), x)$

Fricas [B] time = 2.39824, size = 3123, normalized size = 17.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")`

[Out] $\frac{1}{8} * (4 * d * x * \cosh(d * x + c)^2 - \cosh(d * x + c)^4 - 4 * \cosh(d * x + c) * \sinh(d * x + c)^3 - \sinh(d * x + c)^4 + 2 * (2 * d * x - 3 * \cosh(d * x + c)^2) * \sinh(d * x + c)^2 - 2 * (b * d * \cosh(d * x + c)^2 + 2 * b * d * \cosh(d * x + c) * \sinh(d * x + c) + b * d * \sinh(d * x + c)^2) * \sqrt{((a * b^3 - b^4) * d^2 * \sqrt{a^3 / ((a^2 * b^5 - 2 * a * b^6 + b^7) * d^4)}) + a^2} / ((a * b^3 - b^4) * d^2) * \log(a^2 * \cosh(d * x + c)^2 + 2 * a^2 * \cosh(d * x + c) * \sinh(d * x + c) + a^2 * \sinh(d * x + c)^2 + 2 * (a^2 * b^2 - a * b^3) * d^2 * \sqrt{a^3 / ((a^2 * b^5 - 2 * a * b^6 + b^7) * d^4)}) - a^2 + 2 * ((a * b^4 - b^5) * d^3 * \sqrt{a^3 / ((a^2 * b^5 - 2 * a * b^6 + b^7) * d^4)})$

$$\begin{aligned}
& a*b^6 + b^7)*d^4)) - a^2*b*d)*\sqrt{((a*b^3 - b^4)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))} + 2*(b*d*\cosh(d*x + c)^2 \\
& + 2*b*d*\cosh(d*x + c)*\sinh(d*x + c) + b*d*\sinh(d*x + c)^2)*\sqrt{((a*b^3 - b^4)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2)} \\
&))*\log(a^2*\cosh(d*x + c)^2 + 2*a^2*\cosh(d*x + c)*\sinh(d*x + c) + a^2*\sinh(d*x + c)^2 + 2*(a^2*b^2 - a*b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)} \\
&)) - a^2 - 2*((a*b^4 - b^5)*d^3*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2*b*d)*\sqrt{((a*b^3 - b^4)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))} \\
& + 2*(b*d*\cosh(d*x + c)^2 + 2*b*d*\cosh(d*x + c)*\sinh(d*x + c) + b*d*\sinh(d*x + c)^2)*\sqrt{-((a*b^3 - b^4)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)} \\
& - a^2)/((a*b^3 - b^4)*d^2))*\log(a^2*\cosh(d*x + c)^2 + 2*a^2*\cosh(d*x + c)*\sinh(d*x + c) + a^2*\sinh(d*x + c)^2 - 2*(a^2*b^2 - a*b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)} \\
& - a^2 + 2*((a*b^4 - b^5)*d^3*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)} + a^2*b*d)*\sqrt{-((a*b^3 - b^4)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)} - a^2)/((a*b^3 - b^4)*d^2))} \\
& - 2*(b*d*\cosh(d*x + c)^2 + 2*b*d*\cosh(d*x + c)*\sinh(d*x + c) + b*d*\sinh(d*x + c)^2)*\sqrt{-((a*b^3 - b^4)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)} - a^2)/((a*b^3 - b^4)*d^2))*\log(a^2*\cosh(d*x + c)^2 + 2*a^2*\cosh(d*x + c)*\sinh(d*x + c) + a^2*\sinh(d*x + c)^2 - 2*(a^2*b^2 - a*b^3)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)} - a^2 - 2*((a*b^4 - b^5)*d^3*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)} + a^2*b*d)*\sqrt{-((a*b^3 - b^4)*d^2*\sqrt{a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)} - a^2)/((a*b^3 - b^4)*d^2))} + 4*(2*d*x*\cosh(d*x + c) - \cosh(d*x + c)^3)*\sinh(d*x + c) + 1)/(b*d*\cosh(d*x + c)^2 + 2*b*d*\cosh(d*x + c)*\sinh(d*x + c) + b*d*\sinh(d*x + c)^2)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4),x)

[Out] Timed out

Giac [A] time = 1.30782, size = 82, normalized size = 0.47

$$-\frac{(2e^{2dx+2c} - 1)e^{(-2dx-2c)}}{8bd} + \frac{dx + c}{2bd} - \frac{e^{(2dx+2c)}}{8bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/8*(2*e^(2*d*x + 2*c) - 1)*e^(-2*d*x - 2*c)/(b*d) + 1/2*(d*x + c)/(b*d) -  
1/8*e^(2*d*x + 2*c)/(b*d)
```

$$3.236 \quad \int \frac{\sinh^4(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=127

$$\frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{x}{b}$$

[Out] $-(x/b) + (a^{1/4} \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]] \text{Tanh}[c + d*x])/a^{1/4}]) / (2 * \text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]] * b * d) + (a^{1/4} \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]] \text{Tanh}[c + d*x])/a^{1/4}]) / (2 * \text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]] * b * d)$

Rubi [A] time = 0.206337, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3217, 1287, 207, 1166, 208}

$$\frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{x}{b}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]^4/(a - b*\text{Sinh}[c + d*x]^4), x]$

[Out] $-(x/b) + (a^{1/4} \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]] \text{Tanh}[c + d*x])/a^{1/4}]) / (2 * \text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]] * b * d) + (a^{1/4} \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]] \text{Tanh}[c + d*x])/a^{1/4}]) / (2 * \text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]] * b * d)$

Rule 3217

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff^{(m + 1)}/f, \text{Subst}[\text{Int}[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \text{Tan}[e + f*x]/ff], x]] /; \text{FreeQ}[\{a, b, e, f\}, x] \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[p]$

Rule 1287

Int[(((f_)*(x_)^(m_))*((d_) + (e_)*(x_)^2)^(q_))/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[q] && IntegerQ[m]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^4(c + dx)}{a - b \sinh^4(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a-2ax^2+(a-b)x^4)} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{1}{b(-1+x^2)} + \frac{a(1-x^2)}{b(a-2ax^2+(a-b)x^4)}\right) dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \tanh(c + dx)\right)}{bd} + \frac{a \text{Subst}\left(\int \frac{1-x^2}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c + dx)\right)}{bd} \\
 &= -\frac{x}{b} - \frac{(\sqrt{a}(\sqrt{a} + \sqrt{b})) \text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2} dx, x, \tanh(c + dx)\right)}{2bd} - \frac{\left(a\left(1 - \frac{\sqrt{b}}{\sqrt{a}}\right)\right) \text{Subst}\left(\int \frac{1}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c + dx)\right)}{2bd} \\
 &= -\frac{x}{b} + \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}bd}} + \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}bd}}
 \end{aligned}$$

Mathematica [A] time = 0.449812, size = 143, normalized size = 1.13

$$\frac{\frac{\sqrt{a} \tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b+a}}}\right)}{\sqrt{\sqrt{a}\sqrt{b+a}}} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b-a}}}\right)}{\sqrt{\sqrt{a}\sqrt{b-a}}}}{2bd} - 2(c+dx)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4), x]

[Out] $(-2*(c + d*x) - (\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a] - \text{Sqrt}[b])*\text{Tanh}[c + d*x])/(\text{Sqrt}[-a + \text{Sqrt}[a]*\text{Sqrt}[b]])/\text{Sqrt}[-a + \text{Sqrt}[a]*\text{Sqrt}[b]] + (\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a] + \text{Sqrt}[b])*\text{Tanh}[c + d*x])/(\text{Sqrt}[a + \text{Sqrt}[a]*\text{Sqrt}[b]])/\text{Sqrt}[a + \text{Sqrt}[a]*\text{Sqrt}[b]])/(2*b*d)$

Maple [C] time = 0.033, size = 144, normalized size = 1.1

$$-\frac{1}{bd} \ln\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right) - \frac{a}{4bd} \sum_{\substack{_R=\text{RootOf}(a_Z^8-4a_Z^6+(6a-16b)_Z^4-4a_Z^2+a) \\ _R^7a-3_R^5a+3_R^3a-8_R^3b-_R}} \frac{_R^6-3_R^4+3_R^2-1}{_R^7a-3_R^5a+3_R^3a-8_R^3b-_R}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4), x)

[Out] $-1/d/b*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/4/d*a/b*\text{sum}((_R^6-3*_R^4+3*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R), _R=\text{RootOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))+1/d/b*\ln(\tanh(1/2*d*x+1/2*c)-1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-16a \int \frac{e^{(4dx+4c)}}{b^2e^{(8dx+8c)} - 4b^2e^{(6dx+6c)} - 4b^2e^{(2dx+2c)} + b^2 - 2(8abe^{(4c)} - 3b^2e^{(4c)})e^{(4dx)}} dx - \frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4), x, algorithm="maxima")

```
[Out] -16*a*integrate(e^(4*d*x + 4*c)/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c)
) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*
x)), x) - x/b
```

Fricas [B] time = 2.16358, size = 2128, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a
)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b
^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - b*d)*sq
rt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a)/((a*b^2
- b^3)*d^2)) - 1) - b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 +
b^5)*d^4)) + a)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3
- 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d
^4)) - b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
+ a)/((a*b^2 - b^3)*d^2)) - 1) - b*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-2*(a*b - b^2)*d^2*
sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 -
2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a
*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2)) - 1) + b*sqrt(-((a*b^2 - b^3)*d
^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-2
*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*((a*b^2 - b^3)*d^3*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(
a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2)) - 1) + 4*x)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32317, size = 18, normalized size = 0.14

$$-\frac{dx + c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -(d*x + c)/(b*d)
```

$$3.237 \quad \int \frac{\sinh^2(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=125

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{bd}\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{bd}\sqrt{\sqrt{a}-\sqrt{b}}}$$

[Out] -ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[b]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)

Rubi [A] time = 0.120626, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3217, 1130, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{bd}\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{bd}\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]

[Out] -ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] - Sqrt[b]]*Sqrt[b]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)

Rule 3217

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\left(1 - \frac{\sqrt{a}}{\sqrt{b}}\right) \text{Subst}\left(\int \frac{1}{-a+\sqrt{a}\sqrt{b}+(a-b)x^2} dx, x, \tanh(c+dx)\right)}{2d} + \frac{\left(1 + \frac{\sqrt{a}}{\sqrt{b}}\right) \text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2} dx, x, \tanh(c+dx)\right)}{2d} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{bd}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{bd}} \end{aligned}$$

Mathematica [A] time = 0.349201, size = 127, normalized size = 1.02

$$\frac{\frac{\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{2\sqrt{bd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4), x]
```

```
[Out] (ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/Sqrt[-a + Sqrt[a]*Sqrt[b]] + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[b]*d)
```

Maple [C] time = 0.031, size = 94, normalized size = 0.8

$$-\frac{1}{d} \sum_{_R=\text{RootOf}(a_Z^8-4a_Z^6+(6a-16b)_Z^4-4a_Z^2+a)} \frac{_R^4 - _R^2}{_R^7 a - 3 _R^5 a + 3 _R^3 a - 8 _R^3 b - _R a} \ln \left(\tanh \left(\frac{dx}{2} + \frac{c}{2} \right) - _R \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x)

[Out] -1/d*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh(dx+c)^2}{b \sinh(dx+c)^4 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] -integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^4 - a), x)

Fricas [B] time = 2.21265, size = 2130, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out] -1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - a*d)*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a*b - b^2)*d^2)) - 1) + 1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b -

$$\begin{aligned}
& 2*a^2*b^2 + a*b^3)*d^4)) + \cosh(d*x + c)^2 + 2*\cosh(d*x + c)*\sinh(d*x + c) \\
& + \sinh(d*x + c)^2 - 2*((a^2*b - a*b^2)*d^3*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} \\
& - a*d)*\sqrt{((a*b - b^2)*d^2*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} + 1)/((a*b - b^2)*d^2)} \\
& - 1) + 1/4*\sqrt{-((a*b - b^2)*d^2*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} - 1)/((a*b - b^2)*d^2)} \\
& * \log(-2*(a^2 - a*b)*d^2*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} + \cosh(d*x + c)^2 + 2*\cosh(d*x + c) \\
& * \sinh(d*x + c) + \sinh(d*x + c)^2 + 2*((a^2*b - a*b^2)*d^3*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} \\
& + a*d)*\sqrt{-((a*b - b^2)*d^2*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} - 1)/((a*b - b^2)*d^2)} \\
& - 1) - 1/4*\sqrt{-((a*b - b^2)*d^2*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} - 1)/((a*b - b^2)*d^2)} \\
& * \log(-2*(a^2 - a*b)*d^2*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} + \cosh(d*x + c)^2 + 2*\cosh(d*x + c) \\
& * \sinh(d*x + c) + \sinh(d*x + c)^2 - 2*((a^2*b - a*b^2)*d^3*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} \\
& + a*d)*\sqrt{-((a*b - b^2)*d^2*\sqrt{1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)} - 1)/((a*b - b^2)*d^2)} - 1) \\
&)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)

[Out] Timed out

Giac [A] time = 1.616, size = 1, normalized size = 0.01

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")

[Out] 0

$$3.238 \quad \int \frac{1}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)

Rubi [A] time = 0.0976941, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3209, 1166, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

[In] Int[(a - b*Sinh[c + d*x]^4)^(-1), x]

[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)

Rule 3209

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[p]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2

- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{a - b \sinh^4(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\left(1 - \frac{\sqrt{b}}{\sqrt{a}}\right) \text{Subst}\left(\int \frac{1}{-a+\sqrt{a}\sqrt{b}+(a-b)x^2} dx, x, \tanh(c + dx)\right)}{2d} - \frac{\left(1 + \frac{\sqrt{b}}{\sqrt{a}}\right) \text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2} dx, x, \tanh(c + dx)\right)}{2d} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}d}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}d}} \end{aligned}$$

Mathematica [A] time = 0.2393, size = 128, normalized size = 1.11

$$\frac{\frac{\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{2\sqrt{ad}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-1), x]

[Out] (-(ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]]) + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]]/(2*Sqrt[a]*d)

Maple [C] time = 0.033, size = 102, normalized size = 0.9

$$\frac{1}{4d} \sum_{_R=\text{RootOf}(a_Z^8-4a_Z^6+(6a-16b)_Z^4-4a_Z^2+a)} \frac{-_R^6+3_R^4-3_R^2+1}{_R^7a-3_R^5a+3_R^3a-8_R^3b-_Ra} \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a-b*sinh(d*x+c)^4),x)

[Out] 1/4/d*sum((-_R^6+3*_R^4-3*_R^2+1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{b \sinh(dx+c)^4 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")

[Out] -integrate(1/(b*sinh(d*x + c)^4 - a), x)

Fricas [B] time = 2.16271, size = 2130, normalized size = 18.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")

[Out] -1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a^2 - a*b)*d^2)) - b) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a^2 - a*b)*d^2))

$$\begin{aligned} &^2*d^4)) + 1)/((a^2 - a*b)*d^2))*\log(2*(a^3 - a^2*b)*d^2*\sqrt{b/((a^5 - 2* \\ &a^4*b + a^3*b^2)*d^4)) + b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c \\ &) + b*\sinh(d*x + c)^2 - 2*((a^4 - a^3*b)*d^3*\sqrt{b/((a^5 - 2*a^4*b + a^3*b \\ &^2)*d^4)) - a*b*d)*\sqrt{((a^2 - a*b)*d^2*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)* \\ &d^4)) + 1)/((a^2 - a*b)*d^2)) - b) + 1/4*\sqrt{-((a^2 - a*b)*d^2*\sqrt{b/((a^ \\ &5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*\log(-2*(a^3 - a^2*b)*d \\ &^2*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*\cosh(d*x + c)^2 + 2*b*\cosh(d \\ &*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*((a^4 - a^3*b)*d^3*\sqrt{b/((a \\ &^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*\sqrt{-((a^2 - a*b)*d^2*\sqrt{b/((a^5 \\ &- 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2)) - b) - 1/4*\sqrt{-((a^2 - \\ &a*b)*d^2*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*\log(-2*(a^3 - a^2*b)*d^2*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*\cosh(d* \\ &x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 - 2*((a^4 - \\ &a^3*b)*d^3*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*\sqrt{-((a^2 - a \\ &*b)*d^2*\sqrt{b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2)) - b \\ &)} \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b*sinh(d*x+c)**4),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{b \sinh(dx + c)^4 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="giac")

[Out] integrate(-1/(b*sinh(d*x + c)^4 - a), x)

$$3.239 \quad \int \frac{\operatorname{csch}^2(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=139

$$-\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\operatorname{coth}(c+dx)}{ad}$$

[Out] $-(\operatorname{Sqrt}[b] \operatorname{ArcTanh}[(\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]] \operatorname{Tanh}[c + d*x])/a^{(1/4)}])/(2*a^{(5/4)} \operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]*d) + (\operatorname{Sqrt}[b] \operatorname{ArcTanh}[(\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]] \operatorname{Tanh}[c + d*x])/a^{(1/4)}])/(2*a^{(5/4)} \operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]*d) - \operatorname{Coth}[c + d*x]/(a*d)$

Rubi [A] time = 0.181358, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3217, 1287, 1130, 208}

$$-\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\operatorname{coth}(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]^2/(a - b*\operatorname{Sinh}[c + d*x]^4), x]$

[Out] $-(\operatorname{Sqrt}[b] \operatorname{ArcTanh}[(\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]] \operatorname{Tanh}[c + d*x])/a^{(1/4)}])/(2*a^{(5/4)} \operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]*d) + (\operatorname{Sqrt}[b] \operatorname{ArcTanh}[(\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]] \operatorname{Tanh}[c + d*x])/a^{(1/4)}])/(2*a^{(5/4)} \operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]*d) - \operatorname{Coth}[c + d*x]/(a*d)$

Rule 3217

$\operatorname{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff^{(m + 1)}/f, \operatorname{Subst}[\operatorname{Int}[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]\} /; \operatorname{FreeQ}\{a, b, e, f\}, x \ \&\& \operatorname{IntegerQ}[m/2] \ \&\& \operatorname{IntegerQ}[p]$

Rule 1287

Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[q] && IntegerQ[m]

Rule 1130

Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 - q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GeQ[m, 2]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{csch}^2(c+dx)}{a-b \sinh^4(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^2(a-2ax^2+(a-b)x^4)} dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^2} + \frac{bx^2}{a(a-2ax^2+(a-b)x^4)}\right) dx, x, \tanh(c+dx)\right)}{d} \\
 &= -\frac{\operatorname{coth}(c+dx)}{ad} + \frac{b \operatorname{Subst}\left(\int \frac{x^2}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{ad} \\
 &= -\frac{\operatorname{coth}(c+dx)}{ad} + \frac{((\sqrt{a} + \sqrt{b})\sqrt{b}) \operatorname{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2} dx, x, \tanh(c+dx)\right)}{2ad} + \frac{\left((1 - \frac{\sqrt{b}}{\sqrt{a}})\sqrt{b}\right) \operatorname{Subst}\left(\int \frac{1}{-a+\sqrt{a}\sqrt{b}+(a-b)x^2} dx, x, \tanh(c+dx)\right)}{2ad} \\
 &= -\frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{bd}}} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{bd}}} - \frac{\operatorname{coth}(c+dx)}{ad}
 \end{aligned}$$

Mathematica [A] time = 0.813646, size = 143, normalized size = 1.03

$$\frac{\frac{\sqrt{b} \tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b+a}}}\right)}{\sqrt{\sqrt{a}\sqrt{b+a}}} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b-a}}}\right)}{\sqrt{\sqrt{a}\sqrt{b-a}}} - 2 \coth(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4), x]

[Out] ((Sqrt[b]*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - 2*Cot h[c + d*x])/(2*a*d)

Maple [C] time = 0.059, size = 135, normalized size = 1.

$$-\frac{1}{2da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{1}{2da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^{-1} - \frac{b}{da} \sum_{R=\text{RootOf}(aZ^8-4aZ^6+(6a-16b)Z^4-4aZ^2+a)} \frac{R^4 - \dots}{R^7a - 3R^5a + 3R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4), x)

[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a*b*sum((R^4-R^2)/(R^7*a-3*R^5*a+3*R^3*a-8*R^3*b-R*a)*ln(tanh(1/2*d*x+1/2*c)-R), R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{ade^{(2dx+2c)} - ad} - 4 \int \frac{be^{(6dx+6c)} - 2be^{(4dx+4c)} + be^{(2dx+2c)}}{abe^{(8dx+8c)} - 4abe^{(6dx+6c)} - 4abe^{(2dx+2c)} + ab - 2(8a^2e^{(4c)} - 3abe^{(4c)})e^{(4dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4), x, algorithm="maxima")

```
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(6*d*x + 6*c) - 2*b*e^(4*d*x + 4*c) + b*e^(2*d*x + 2*c))/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*x)), x)
```

Fricas [B] time = 2.28628, size = 2840, normalized size = 20.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 + 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 - 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 + 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))) + (a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 - 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))) + 8)/(a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)

[Out] Timed out

Giac [A] time = 2.00349, size = 28, normalized size = 0.2

$$-\frac{2}{ad(e^{2dx+2c}-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")

[Out] -2/(a*d*(e^(2*d*x + 2*c) - 1))

$$3.240 \quad \int \frac{\operatorname{csch}^4(c+dx)}{a-b \sinh^4(c+dx)} dx$$

Optimal. Leaf size=148

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\operatorname{coth}^3(c+dx)}{3ad} + \frac{\operatorname{coth}(c+dx)}{ad}$$

[Out] (b*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + (b*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) + Coth[c + d*x]/(a*d) - Coth[c + d*x]^3/(3*a*d)

Rubi [A] time = 0.211268, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3217, 1287, 1166, 208}

$$\frac{b \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}} - \frac{\operatorname{coth}^3(c+dx)}{3ad} + \frac{\operatorname{coth}(c+dx)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4), x]

[Out] (b*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + (b*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) + Coth[c + d*x]/(a*d) - Coth[c + d*x]^3/(3*a*d)

Rule 3217

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1287

Int[(((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_))/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[q] && IntegerQ[m]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{x^4(a-2ax^2+(a-b)x^4)} dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^4} - \frac{1}{ax^2} + \frac{b-bx^2}{a(a-2ax^2+(a-b)x^4)}\right) dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} + \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{ad} \\
 &= \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad} - \frac{((\sqrt{a} + \sqrt{b})b) \operatorname{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2} dx, x, \tanh(c+dx)\right)}{2a^{3/2}d} \\
 &= \frac{b \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}d}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}d}} + \frac{\operatorname{coth}(c+dx)}{ad} - \frac{\operatorname{coth}^3(c+dx)}{3ad}
 \end{aligned}$$

Mathematica [A] time = 2.19296, size = 165, normalized size = 1.11

$$\frac{3b \tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b+a}}}\right) - 3b \tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b-a}}}\right)}{\sqrt{\sqrt{a}\sqrt{b+a}} - \sqrt{\sqrt{a}\sqrt{b-a}}} + 4\sqrt{a} \coth(c+dx) - 2\sqrt{a} \coth(c+dx) \operatorname{csch}^2(c+dx)$$

$$6a^{3/2}d$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4), x]

[Out] ((-3*b*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (3*b*ArcTanH[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + 4*Sqrt[a]*Coth[c + d*x] - 2*Sqrt[a]*Coth[c + d*x]*Csch[c + d*x]^2)/(6*a^(3/2)*d)

Maple [C] time = 0.075, size = 179, normalized size = 1.2

$$-\frac{1}{24da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^3 + \frac{3}{8da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) - \frac{b}{4da} \sum_{_R=\text{RootOf}(a_Z^8-4a_Z^6+(6a-16b)_Z^4-4a_Z^2+a)} \frac{_R^6 - 3_R}{_R^7 a - 3_R^5 a + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4), x)

[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)-1/4/d/a*b*sum((_R^6-3*_R^4+3*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R), _R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/24/d/a/tanh(1/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-16b \int \frac{e^{(4dx+4c)}}{abe^{(8dx+8c)} - 4abe^{(6dx+6c)} - 4abe^{(2dx+2c)} + ab - 2(8a^2e^{(4c)} - 3abe^{(4c)})e^{(4dx)}} dx - \frac{4(3e^{(2dx+2c)}}{3(ade^{(6dx+6c)} - 3ade^{(4dx+4c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4), x, algorithm="maxima")

```
[Out] -16*b*integrate(e^(4*d*x + 4*c)/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c)
) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*
x)), x) - 4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*d*x
+ 4*c) + 3*a*d*e^(2*d*x + 2*c) - a*d)
```

Fricas [B] time = 2.21904, size = 5195, normalized size = 35.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*d*si
nh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sin
h(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cos
h(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c
)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x
+ c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5
/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(b^4*cosh(
d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b^4*sinh(d*x + c)^2 - b^4
+ 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 2*(a^
2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt
(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4
- a^3*b)*d^2))) - 3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c
)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^
2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3
- 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*
cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a
*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*
d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*l
og(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b^4*sinh(d*x +
c)^2 - b^4 + 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d
^4)) - 2*(a^2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)
*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) +
b^2)/((a^4 - a^3*b)*d^2))) + 3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*
sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*co
sh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cos
h(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c
)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x
+ c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((
a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^2)/((a^4 - a
```

$$\begin{aligned} &^3*b*d^2))*\log(b^4*\cosh(d*x + c)^2 + 2*b^4*\cosh(d*x + c)*\sinh(d*x + c) + b \\ &^4*\sinh(d*x + c)^2 - b^4 - 2*(a^5*b - a^4*b^2)*d^2*\sqrt{b^5/((a^9 - 2*a^8*b \\ &+ a^7*b^2)*d^4)) + 2*(a^2*b^3*d + (a^7 - a^6*b)*d^3*\sqrt{b^5/((a^9 - 2*a^8 \\ &*b + a^7*b^2)*d^4)))*\sqrt{-((a^4 - a^3*b)*d^2*\sqrt{b^5/((a^9 - 2*a^8*b + a^ \\ &7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))} - 3*(a*d*\cosh(d*x + c)^6 + 6*a*d* \\ &\cosh(d*x + c)*\sinh(d*x + c)^5 + a*d*\sinh(d*x + c)^6 - 3*a*d*\cosh(d*x + c)^4 \\ &+ 3*(5*a*d*\cosh(d*x + c)^2 - a*d)*\sinh(d*x + c)^4 + 3*a*d*\cosh(d*x + c)^2 \\ &+ 4*(5*a*d*\cosh(d*x + c)^3 - 3*a*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 3*(5*a* \\ &d*\cosh(d*x + c)^4 - 6*a*d*\cosh(d*x + c)^2 + a*d)*\sinh(d*x + c)^2 - a*d + 6* \\ &(a*d*\cosh(d*x + c)^5 - 2*a*d*\cosh(d*x + c)^3 + a*d*\cosh(d*x + c))*\sinh(d*x \\ &+ c))*\sqrt{-((a^4 - a^3*b)*d^2*\sqrt{b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - \\ &b^2)/((a^4 - a^3*b)*d^2))*\log(b^4*\cosh(d*x + c)^2 + 2*b^4*\cosh(d*x + c)*\sin \\ &h(d*x + c) + b^4*\sinh(d*x + c)^2 - b^4 - 2*(a^5*b - a^4*b^2)*d^2*\sqrt{b^5/(\\ &(a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 2*(a^2*b^3*d + (a^7 - a^6*b)*d^3*\sqrt{b^5 \\ &/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*\sqrt{-((a^4 - a^3*b)*d^2*\sqrt{b^5/((a^9 \\ &- 2*a^8*b + a^7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))} - 48*\cosh(d*x + c)^ \\ &2 - 96*\cosh(d*x + c)*\sinh(d*x + c) - 48*\sinh(d*x + c)^2 + 16)/(a*d*\cosh(d*x \\ &+ c)^6 + 6*a*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + a*d*\sinh(d*x + c)^6 - 3*a*d \\ &*cosh(d*x + c)^4 + 3*(5*a*d*\cosh(d*x + c)^2 - a*d)*\sinh(d*x + c)^4 + 3*a*d* \\ &\cosh(d*x + c)^2 + 4*(5*a*d*\cosh(d*x + c)^3 - 3*a*d*\cosh(d*x + c))*\sinh(d*x \\ &+ c)^3 + 3*(5*a*d*\cosh(d*x + c)^4 - 6*a*d*\cosh(d*x + c)^2 + a*d)*\sinh(d*x + \\ &c)^2 - a*d + 6*(a*d*\cosh(d*x + c)^5 - 2*a*d*\cosh(d*x + c)^3 + a*d*\cosh(d*x \\ &+ c))*\sinh(d*x + c)) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)

[Out] Timed out

Giac [A] time = 2.85349, size = 46, normalized size = 0.31

$$-\frac{4(3e^{(2dx+2c)} - 1)}{3ad(e^{(2dx+2c)} - 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*(e^(2*d*x + 2*c) - 1)^3)
```

$$3.241 \quad \int \frac{\sinh^9(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^2} dx$$

Optimal. Leaf size=235

$$\frac{a \cosh(c+dx) (a-b \cosh^2(c+dx)+b)}{4b^2d(a-b) (a-b \cosh^4(c+dx)+2b \cosh^2(c+dx)-b)} - \frac{\sqrt{a} (5\sqrt{a}-6\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d (\sqrt{a}-\sqrt{b})^{3/2}} - \frac{\sqrt{a} (5\sqrt{a}+6\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{9/4}d (\sqrt{a}+\sqrt{b})^{3/2}}$$

[Out] $-(\text{Sqrt}[a]*(5*\text{Sqrt}[a]-6*\text{Sqrt}[b])*ArcTan[(b^{(1/4)}*Cosh[c+d*x])/Sqrt[Sqrt[a]-Sqrt[b]])/(8*(\text{Sqrt}[a]-\text{Sqrt}[b])^{(3/2)}*b^{(9/4)}*d)-(\text{Sqrt}[a]*(5*\text{Sqrt}[a]+6*\text{Sqrt}[b])*ArcTanh[(b^{(1/4)}*Cosh[c+d*x])/Sqrt[Sqrt[a]+Sqrt[b]])/(8*(\text{Sqrt}[a]+\text{Sqrt}[b])^{(3/2)}*b^{(9/4)}*d)+Cosh[c+d*x]/(b^2*d)+(a*Cosh[c+d*x]*(a+b-b*Cosh[c+d*x]^2))/(4*(a-b)*b^2*d*(a-b+2*b*Cosh[c+d*x]^2-b*Cosh[c+d*x]^4))$

Rubi [A] time = 0.485868, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3215, 1205, 1676, 1166, 205, 208}

$$\frac{a \cosh(c+dx) (a-b \cosh^2(c+dx)+b)}{4b^2d(a-b) (a-b \cosh^4(c+dx)+2b \cosh^2(c+dx)-b)} - \frac{\sqrt{a} (5\sqrt{a}-6\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d (\sqrt{a}-\sqrt{b})^{3/2}} - \frac{\sqrt{a} (5\sqrt{a}+6\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{9/4}d (\sqrt{a}+\sqrt{b})^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c+d*x]^9/(a-b*\text{Sinh}[c+d*x]^4)^2,x]$

[Out] $-(\text{Sqrt}[a]*(5*\text{Sqrt}[a]-6*\text{Sqrt}[b])*ArcTan[(b^{(1/4)}*Cosh[c+d*x])/Sqrt[Sqrt[a]-Sqrt[b]])/(8*(\text{Sqrt}[a]-\text{Sqrt}[b])^{(3/2)}*b^{(9/4)}*d)-(\text{Sqrt}[a]*(5*\text{Sqrt}[a]+6*\text{Sqrt}[b])*ArcTanh[(b^{(1/4)}*Cosh[c+d*x])/Sqrt[Sqrt[a]+Sqrt[b]])/(8*(\text{Sqrt}[a]+\text{Sqrt}[b])^{(3/2)}*b^{(9/4)}*d)+Cosh[c+d*x]/(b^2*d)+(a*Cosh[c+d*x]*(a+b-b*Cosh[c+d*x]^2))/(4*(a-b)*b^2*d*(a-b+2*b*Cosh[c+d*x]^2-b*Cosh[c+d*x]^4))$

Rule 3215

$\text{Int}[\sin[(e_.)+(f_.)*(x_.)]^{(m_.)}*((a_.)+(b_.)*\sin[(e_.)+(f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] :> \text{With}[\{ff = \text{FreeFactors}[\text{Cos}[e+f*x], x]\}, -\text{Dist}[ff/f, S$

ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 1205

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]

Rule 1676

Int[(Pq)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{a \cosh(c+dx) (a+b-b\cosh^2(c+dx))}{4(a-b)b^2d (a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{2a\left(a+\frac{a^2}{b}-4b\right)-2a(7a-8)}{a-b+2bx^2-} dx, x, \cosh(c+dx)\right)}{8a(b-a)} \\
&= \frac{a \cosh(c+dx) (a+b-b\cosh^2(c+dx))}{4(a-b)b^2d (a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} - \frac{\text{Subst}\left(\int \left(-\frac{8a(a-b)}{b} + \frac{2(a^2(5a-b)}{b(a-b)}\right) dx, x, \cosh(c+dx)\right)}{8a(b-a)} \\
&= \frac{\cosh(c+dx)}{b^2d} + \frac{a \cosh(c+dx) (a+b-b\cosh^2(c+dx))}{4(a-b)b^2d (a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{a^2}{a} dx, x, \cosh(c+dx)\right)}{8a(b-a)} \\
&= \frac{\cosh(c+dx)}{b^2d} + \frac{a \cosh(c+dx) (a+b-b\cosh^2(c+dx))}{4(a-b)b^2d (a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} + \frac{(\sqrt{a}(5\sqrt{a}-6\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right) - \sqrt{a}(5\sqrt{a}+6\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right))}{8(\sqrt{a}-\sqrt{b})^{3/2} b^{9/4}d} + \frac{\sqrt{a}(5\sqrt{a}+6\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right) - \sqrt{a}(5\sqrt{a}-6\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8(\sqrt{a}+\sqrt{b})^{3/2} b^{9/4}d} + \frac{\text{Subst}\left(\int \frac{a^2}{a} dx, x, \cosh(c+dx)\right)}{8a(b-a)}
\end{aligned}$$

Mathematica [C] time = 0.948725, size = 615, normalized size = 2.62

$$\frac{a\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{40\#1^4a \log(-\#1 \sinh(\frac{1}{2}(c+dx))+\#1 \cosh(\frac{1}{2}(c+dx))-\sinh(\frac{1}{2}(c+dx))-\cosh(\frac{1}{2}(c+dx))) - 40\#1^2a \log(-\#1 \sinh(\frac{1}{2}(c+dx)))}{b^2d}\right]}{b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] (32*Cosh[c + d*x] + (32*a*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)])))/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (- (b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 20*a*c*#1^2 + 27*b*c*#1^2 - 20*a*d*x*#1^2 + 27*b*d*x*#1^2 - 40*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 54*b*Log[-Cosh[(c + d*x)/2] - Sinh[

```
(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1)*#1^2 + 20*a*c*#1^4 - 27*b*c*#1^4 + 20*a*d*x*#1^4 - 27*b*d*x*#1^4 + 40*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1)*#1^4 - 54*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1)*#1^4 + b*c*#1^6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1)*#1^6)/(- (b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32*b^2*d)
```

Maple [B] time = 0.082, size = 1191, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d/b^2/(tanh(1/2*d*x+1/2*c)+1)-1/2/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^6+1/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^6+3/2/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-4/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-3/2/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)+1/8/d*a/b^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+5/8/d*a^2/b^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-3/4/d*a/b/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/8/d*a/b^2/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-5/8/d*a^2/b^2/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
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$$(a*b)^{(1/2)}*(a)^{(1/2)}+3/4/d*a/b/(a-b)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)})-1/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out]
$$\begin{aligned} & 1/2*(a*b - b^2 + (a*b*e^{(10*c)} - b^2*e^{(10*c)})*e^{(10*d*x)} - (2*a*b*e^{(8*c)} \\ & - 3*b^2*e^{(8*c)})*e^{(8*d*x)} - (20*a^2*e^{(6*c)} - 17*a*b*e^{(6*c)} + 2*b^2*e^{(6*c)} \\ &)*e^{(6*d*x)} - (20*a^2*e^{(4*c)} - 17*a*b*e^{(4*c)} + 2*b^2*e^{(4*c)})*e^{(4*d*x)} \\ & - (2*a*b*e^{(2*c)} - 3*b^2*e^{(2*c)})*e^{(2*d*x)})/((a*b^3*d*e^{(9*c)} - b^4*d*e^{(9*c)}) \\ &)*e^{(9*d*x)} - 4*(a*b^3*d*e^{(7*c)} - b^4*d*e^{(7*c)})*e^{(7*d*x)} - 2*(8*a^2*b^2*d*e^{(5*c)} \\ & - 11*a*b^3*d*e^{(5*c)} + 3*b^4*d*e^{(5*c)})*e^{(5*d*x)} - 4*(a*b^3*d*e^{(3*c)} - b^4*d*e^{(3*c)}) \\ &)*e^{(3*d*x)} + (a*b^3*d*e^c - b^4*d*e^c)*e^{(d*x)} + 1/512*\int(256*(a*b*e^{(7*d*x + 7*c)} - a*b*e^{(d*x + c)} \\ & + (20*a^2*e^{(5*c)} - 27*a*b*e^{(5*c)})*e^{(5*d*x)} - (20*a^2*e^{(3*c)} - 27*a*b*e^{(3*c)})*e^{(3*d*x)}) \\ &)/(a*b^3 - b^4 + (a*b^3*e^{(8*c)} - b^4*e^{(8*c)})*e^{(8*d*x)} - 4*(a*b^3*e^{(6*c)} - b^4*e^{(6*c)}) \\ &)*e^{(6*d*x)} - 2*(8*a^2*b^2*e^{(4*c)} - 11*a*b^3*e^{(4*c)} + 3*b^4*e^{(4*c)})*e^{(4*d*x)} - 4*(a*b^3*e^{(2*c)} - b^4*e^{(2*c)}) \\ &)*e^{(2*d*x)}, x \end{aligned}$$

Fricas [B] time = 3.04473, size = 17667, normalized size = 75.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/16*(8*(a*b - b^2)*\cosh(d*x + c)^{10} + 80*(a*b - b^2)*\cosh(d*x + c)*\sinh(d*x \\ & + c)^9 + 8*(a*b - b^2)*\sinh(d*x + c)^{10} - 8*(2*a*b - 3*b^2)*\cosh(d*x + c)^8 \\ & + 8*(45*(a*b - b^2)*\cosh(d*x + c)^2 - 2*a*b + 3*b^2)*\sinh(d*x + c)^8 + 6 \\ & 4*(15*(a*b - b^2)*\cosh(d*x + c)^3 - (2*a*b - 3*b^2)*\cosh(d*x + c))*\sinh(d*x \\ & + c)^7 - 8*(20*a^2 - 17*a*b + 2*b^2)*\cosh(d*x + c)^6 + 8*(210*(a*b - b^2)* \\ & \cosh(d*x + c)^4 - 28*(2*a*b - 3*b^2)*\cosh(d*x + c)^2 - 20*a^2 + 17*a*b - 2*b^2)*\sinh(d*x + c)^6 \\ & + 16*(126*(a*b - b^2)*\cosh(d*x + c)^5 - 28*(2*a*b - 3*b^2)*\sinh(d*x + c)^6 \end{aligned}$$

$$\begin{aligned}
& b^2) \cosh(dx + c)^3 - 3(20a^2 - 17ab + 2b^2) \cosh(dx + c) \sinh(dx + c)^5 - 8(20a^2 - 17ab + 2b^2) \cosh(dx + c)^4 + 8(210(ab - b^2) \cosh(dx + c)^6 - 70(2ab - 3b^2) \cosh(dx + c)^4 - 15(20a^2 - 17ab + 2b^2) \cosh(dx + c)^2 - 20a^2 + 17ab - 2b^2) \sinh(dx + c)^4 + 32(30(ab - b^2) \cosh(dx + c)^7 - 14(2ab - 3b^2) \cosh(dx + c)^5 - 5(20a^2 - 17ab + 2b^2) \cosh(dx + c)^3 - (20a^2 - 17ab + 2b^2) \cosh(dx + c)) \sinh(dx + c)^3 - 8(2ab - 3b^2) \cosh(dx + c)^2 + 8(45(ab - b^2) \cosh(dx + c)^8 - 28(2ab - 3b^2) \cosh(dx + c)^6 - 15(20a^2 - 17ab + 2b^2) \cosh(dx + c)^4 - 6(20a^2 - 17ab + 2b^2) \cosh(dx + c)^2 - 2ab + 3b^2) \sinh(dx + c)^2 + ((ab^3 - b^4)d \cosh(dx + c)^9 + 9(ab^3 - b^4)d \cosh(dx + c) \sinh(dx + c)^8 + (ab^3 - b^4)d \sinh(dx + c)^9 - 4(ab^3 - b^4)d \cosh(dx + c)^7 + 4(9(ab^3 - b^4)d \cosh(dx + c)^2 - (ab^3 - b^4)d) \sinh(dx + c)^7 - 2(8a^2b^2 - 11ab^3 + 3b^4)d \cosh(dx + c)^5 + 28(3(ab^3 - b^4)d \cosh(dx + c)^3 - (ab^3 - b^4)d \cosh(dx + c)) \sinh(dx + c)^6 + 2(63(ab^3 - b^4)d \cosh(dx + c)^4 - 42(ab^3 - b^4)d \cosh(dx + c)^2 - (8a^2b^2 - 11ab^3 + 3b^4)d) \sinh(dx + c)^5 - 4(ab^3 - b^4)d \cosh(dx + c)^3 + 2(63(ab^3 - b^4)d \cosh(dx + c)^5 - 70(ab^3 - b^4)d \cosh(dx + c)^3 - 5(8a^2b^2 - 11ab^3 + 3b^4)d \cosh(dx + c)) \sinh(dx + c)^4 + 4(21(ab^3 - b^4)d \cosh(dx + c)^6 - 35(ab^3 - b^4)d \cosh(dx + c)^4 - 5(8a^2b^2 - 11ab^3 + 3b^4)d \cosh(dx + c)^2 - (ab^3 - b^4)d) \sinh(dx + c)^3 + (ab^3 - b^4)d \cosh(dx + c) + 4(9(ab^3 - b^4)d \cosh(dx + c)^7 - 21(ab^3 - b^4)d \cosh(dx + c)^5 - 5(8a^2b^2 - 11ab^3 + 3b^4)d \cosh(dx + c)^3 - 3(ab^3 - b^4)d \cosh(dx + c)) \sinh(dx + c)^2 + (9(ab^3 - b^4)d \cosh(dx + c)^8 - 28(ab^3 - b^4)d \cosh(dx + c)^6 - 10(8a^2b^2 - 11ab^3 + 3b^4)d \cosh(dx + c)^4 - 12(ab^3 - b^4)d \cosh(dx + c)^2 + (ab^3 - b^4)d) \sinh(dx + c)) \sqrt{-((a^3b^4 - 3a^2b^5 + 3ab^6 - b^7)d^2 \sqrt{(625a^7 - 3450a^6b + 7161a^5b^2 - 6624a^4b^3 + 2304a^3b^4)/((a^6b^9 - 6a^5b^10 + 15a^4b^11 - 20a^3b^12 + 15a^2b^13 - 6ab^14 + b^15)d^4))} + 15a^3 - 47a^2b + 36ab^2)/((a^3b^4 - 3a^2b^5 + 3ab^6 - b^7)d^2) \log(-625a^5 + 2625a^4b - 3684a^3b^2 + 1728a^2b^3 - (625a^5 - 2625a^4b + 3684a^3b^2 - 1728a^2b^3) \cosh(dx + c)^2 - 2(625a^5 - 2625a^4b + 3684a^3b^2 - 1728a^2b^3) \cosh(dx + c) \sinh(dx + c) - (625a^5 - 2625a^4b + 3684a^3b^2 - 1728a^2b^3) \sinh(dx + c)^2 + 2((125a^5b^2 - 520a^4b^3 + 723a^3b^4 - 336a^2b^5) d \cosh(dx + c) + (125a^5b^2 - 520a^4b^3 + 723a^3b^4 - 336a^2b^5) d \sinh(dx + c) - 2((2a^4b^7 - 9a^3b^8 + 15a^2b^9 - 11ab^10 + 3b^11) d^3 \cosh(dx + c) + (2a^4b^7 - 9a^3b^8 + 15a^2b^9 - 11ab^10 + 3b^11) d^3 \sinh(dx + c))) \sqrt{(625a^7 - 3450a^6b + 7161a^5b^2 - 6624a^4b^3 + 2304a^3b^4)/((a^6b^9 - 6a^5b^10 + 15a^4b^11 - 20a^3b^12 + 15a^2b^13 - 6ab^14 + b^15)d^4))} \sqrt{-((a^3b^4 - 3a^2b^5 + 3ab^6 - b^7)d^2 \sqrt{(625a^7 - 3450a^6b + 7161a^5b^2 - 6624a^4b^3 + 2304a^3b^4)/((a^6b^9 - 6a^5b^10 + 15a^4b^11 - 20a^3b^12 + 15a^2b^13 - 6ab^14 + b^15)d^4))} + 15a^3 - 47a^2b + 36ab^2)/((a^3b^4 - 3a^2b^5 + 3ab^6 - b^7)d^2)) - ((ab^3 - b^4)d \cosh(dx + c)^9 + 9(ab^3 - b^4)d \cosh(dx + c) \sinh(dx + c) \sinh(dx + c)
\end{aligned}$$

$$\begin{aligned}
& *x + c)^8 + (a*b^3 - b^4)*d*\sinh(d*x + c)^9 - 4*(a*b^3 - b^4)*d*\cosh(d*x + \\
& c)^7 + 4*(9*(a*b^3 - b^4)*d*\cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*\sinh(d*x + c \\
&)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*\cosh(d*x + c)^5 + 28*(3*(a*b^3 - b \\
& ^4)*d*\cosh(d*x + c)^3 - (a*b^3 - b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 2* \\
& (63*(a*b^3 - b^4)*d*\cosh(d*x + c)^4 - 42*(a*b^3 - b^4)*d*\cosh(d*x + c)^2 - \\
& (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*\sinh(d*x + c)^5 - 4*(a*b^3 - b^4)*d*\cosh(\\
& d*x + c)^3 + 2*(63*(a*b^3 - b^4)*d*\cosh(d*x + c)^5 - 70*(a*b^3 - b^4)*d*\cos \\
& h(d*x + c)^3 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + \\
& c)^4 + 4*(21*(a*b^3 - b^4)*d*\cosh(d*x + c)^6 - 35*(a*b^3 - b^4)*d*\cosh(d*x \\
& + c)^4 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*\cosh(d*x + c)^2 - (a*b^3 - b^4 \\
&)*d)*\sinh(d*x + c)^3 + (a*b^3 - b^4)*d*\cosh(d*x + c) + 4*(9*(a*b^3 - b^4)*d \\
& *\cosh(d*x + c)^7 - 21*(a*b^3 - b^4)*d*\cosh(d*x + c)^5 - 5*(8*a^2*b^2 - 11*a \\
& *b^3 + 3*b^4)*d*\cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*\cosh(d*x + c))*\sinh(d*x \\
& + c)^2 + (9*(a*b^3 - b^4)*d*\cosh(d*x + c)^8 - 28*(a*b^3 - b^4)*d*\cosh(d*x \\
& + c)^6 - 10*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*\cosh(d*x + c)^4 - 12*(a*b^3 - \\
& b^4)*d*\cosh(d*x + c)^2 + (a*b^3 - b^4)*d)*\sinh(d*x + c))*\sqrt{-((a^3*b^4 - \\
& 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*\sqrt{((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - \\
& 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3* \\
& b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4))} + 15*a^3 - 47*a^2*b + 36*a*b^2) \\
& /((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*\log(-625*a^5 + 2625*a^4*b - 3 \\
& 684*a^3*b^2 + 1728*a^2*b^3 - (625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^ \\
& 2*b^3)*\cosh(d*x + c)^2 - 2*(625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2* \\
& b^3)*\cosh(d*x + c)*\sinh(d*x + c) - (625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1 \\
& 728*a^2*b^3)*\sinh(d*x + c)^2 - 2*((125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 \\
& - 336*a^2*b^5)*d*\cosh(d*x + c) + (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - \\
& 336*a^2*b^5)*d*\sinh(d*x + c) - 2*((2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11 \\
& *a*b^10 + 3*b^11)*d^3*\cosh(d*x + c) + (2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - \\
& 11*a*b^10 + 3*b^11)*d^3*\sinh(d*x + c))*\sqrt{((625*a^7 - 3450*a^6*b + 7161*a \\
& ^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 \\
& - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)))*\sqrt{-((a^3*b^4 - 3*a \\
& ^2*b^5 + 3*a*b^6 - b^7)*d^2*\sqrt{((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 662 \\
& 4*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^1 \\
& 2 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4))} + 15*a^3 - 47*a^2*b + 36*a*b^2)/((\\
& a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + ((a*b^3 - b^4)*d*\cosh(d*x + c \\
&)^9 + 9*(a*b^3 - b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^8 + (a*b^3 - b^4)*d*\sin \\
& h(d*x + c)^9 - 4*(a*b^3 - b^4)*d*\cosh(d*x + c)^7 + 4*(9*(a*b^3 - b^4)*d*\cos \\
& h(d*x + c)^2 - (a*b^3 - b^4)*d)*\sinh(d*x + c)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + \\
& 3*b^4)*d*\cosh(d*x + c)^5 + 28*(3*(a*b^3 - b^4)*d*\cosh(d*x + c)^3 - (a*b^3 \\
& - b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 2*(63*(a*b^3 - b^4)*d*\cosh(d*x + \\
& c)^4 - 42*(a*b^3 - b^4)*d*\cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)* \\
& d)*\sinh(d*x + c)^5 - 4*(a*b^3 - b^4)*d*\cosh(d*x + c)^3 + 2*(63*(a*b^3 - b^4 \\
&)*d*\cosh(d*x + c)^5 - 70*(a*b^3 - b^4)*d*\cosh(d*x + c)^3 - 5*(8*a^2*b^2 - 1 \\
& 1*a*b^3 + 3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(21*(a*b^3 - b^4)*d*c \\
& osh(d*x + c)^6 - 35*(a*b^3 - b^4)*d*\cosh(d*x + c)^4 - 5*(8*a^2*b^2 - 11*a*b \\
& ^3 + 3*b^4)*d*\cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*\sinh(d*x + c)^3 + (a*b^3 -
\end{aligned}$$

$$\begin{aligned}
& b^4) * d * \cosh(d * x + c) + 4 * (9 * (a * b^3 - b^4) * d * \cosh(d * x + c)^7 - 21 * (a * b^3 - \\
& b^4) * d * \cosh(d * x + c)^5 - 5 * (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^3 \\
& - 3 * (a * b^3 - b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)^2 + (9 * (a * b^3 - b^4) * d * \cosh(d * x + c)^8 - 28 * (a * b^3 - b^4) * d * \cosh(d * x + c)^6 - 10 * (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^4 - 12 * (a * b^3 - b^4) * d * \cosh(d * x + c)^2 + (a * b^3 - b^4) * d) * \sinh(d * x + c)) * \sqrt{((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2 * \sqrt{((625 * a^7 - 3450 * a^6 * b + 7161 * a^5 * b^2 - 6624 * a^4 * b^3 + 2304 * a^3 * b^4) / ((a^6 * b^9 - 6 * a^5 * b^{10} + 15 * a^4 * b^{11} - 20 * a^3 * b^{12} + 15 * a^2 * b^{13} - 6 * a * b^{14} + b^{15}) * d^4)) - 15 * a^3 + 47 * a^2 * b - 36 * a * b^2) / ((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2))} * \log(-625 * a^5 + 2625 * a^4 * b - 3684 * a^3 * b^2 + 1728 * a^2 * b^3 - (625 * a^5 - 2625 * a^4 * b + 3684 * a^3 * b^2 - 1728 * a^2 * b^3) * \cosh(d * x + c)^2 - 2 * (625 * a^5 - 2625 * a^4 * b + 3684 * a^3 * b^2 - 1728 * a^2 * b^3) * \sinh(d * x + c)^2 + 2 * ((125 * a^5 * b^2 - 520 * a^4 * b^3 + 723 * a^3 * b^4 - 336 * a^2 * b^5) * d * \cosh(d * x + c) + (125 * a^5 * b^2 - 520 * a^4 * b^3 + 723 * a^3 * b^4 - 336 * a^2 * b^5) * d * \sinh(d * x + c) + 2 * ((2 * a^4 * b^7 - 9 * a^3 * b^8 + 15 * a^2 * b^9 - 11 * a * b^{10} + 3 * b^{11}) * d^3 * \cosh(d * x + c) + (2 * a^4 * b^7 - 9 * a^3 * b^8 + 15 * a^2 * b^9 - 11 * a * b^{10} + 3 * b^{11}) * d^3 * \sinh(d * x + c))) * \sqrt{((625 * a^7 - 3450 * a^6 * b + 7161 * a^5 * b^2 - 6624 * a^4 * b^3 + 2304 * a^3 * b^4) / ((a^6 * b^9 - 6 * a^5 * b^{10} + 15 * a^4 * b^{11} - 20 * a^3 * b^{12} + 15 * a^2 * b^{13} - 6 * a * b^{14} + b^{15}) * d^4))} * \sqrt{((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2 * \sqrt{((625 * a^7 - 3450 * a^6 * b + 7161 * a^5 * b^2 - 6624 * a^4 * b^3 + 2304 * a^3 * b^4) / ((a^6 * b^9 - 6 * a^5 * b^{10} + 15 * a^4 * b^{11} - 20 * a^3 * b^{12} + 15 * a^2 * b^{13} - 6 * a * b^{14} + b^{15}) * d^4)) - 15 * a^3 + 47 * a^2 * b - 36 * a * b^2) / ((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2))} - ((a * b^3 - b^4) * d * \cosh(d * x + c)^9 + 9 * (a * b^3 - b^4) * d * \cosh(d * x + c) * \sinh(d * x + c)^8 + (a * b^3 - b^4) * d * \sinh(d * x + c)^9 - 4 * (a * b^3 - b^4) * d * \cosh(d * x + c)^7 + 4 * (9 * (a * b^3 - b^4) * d * \cosh(d * x + c)^2 - (a * b^3 - b^4) * d) * \sinh(d * x + c)^7 - 2 * (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^5 + 28 * (3 * (a * b^3 - b^4) * d * \cosh(d * x + c)^3 - (a * b^3 - b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)^6 + 2 * (63 * (a * b^3 - b^4) * d * \cosh(d * x + c)^4 - 42 * (a * b^3 - b^4) * d * \cosh(d * x + c)^2 - (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d) * \sinh(d * x + c)^5 - 4 * (a * b^3 - b^4) * d * \cosh(d * x + c)^3 + 2 * (63 * (a * b^3 - b^4) * d * \cosh(d * x + c)^5 - 70 * (a * b^3 - b^4) * d * \cosh(d * x + c)^3 - 5 * (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)^4 + 4 * (21 * (a * b^3 - b^4) * d * \cosh(d * x + c)^6 - 35 * (a * b^3 - b^4) * d * \cosh(d * x + c)^4 - 5 * (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^2 - (a * b^3 - b^4) * d) * \sinh(d * x + c)^3 + (a * b^3 - b^4) * d * \cosh(d * x + c) + 4 * (9 * (a * b^3 - b^4) * d * \cosh(d * x + c)^7 - 21 * (a * b^3 - b^4) * d * \cosh(d * x + c)^5 - 5 * (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^3 - 3 * (a * b^3 - b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)^2 + (9 * (a * b^3 - b^4) * d * \cosh(d * x + c)^8 - 28 * (a * b^3 - b^4) * d * \cosh(d * x + c)^6 - 10 * (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^4 - 12 * (a * b^3 - b^4) * d * \cosh(d * x + c)^2 + (a * b^3 - b^4) * d) * \sinh(d * x + c)) * \sqrt{((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2 * \sqrt{((625 * a^7 - 3450 * a^6 * b + 7161 * a^5 * b^2 - 6624 * a^4 * b^3 + 2304 * a^3 * b^4) / ((a^6 * b^9 - 6 * a^5 * b^{10} + 15 * a^4 * b^{11} - 20 * a^3 * b^{12} + 15 * a^2 * b^{13} - 6 * a * b^{14} + b^{15}) * d^4)) - 15 * a^3 + 47 * a^2 * b - 36 * a * b^2) / ((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2))} * \log(-625 * a^5 + 2625 * a^4 * b - 3684 * a^3 * b^2 + 1728 * a^2 * b^3 - (625 * a^5 - 2625 * a^4 * b + 3684 * a^3 * b^2
\end{aligned}$$

$$\begin{aligned}
& - 1728a^2b^3 \cosh(dx + c)^2 - 2(625a^5 - 2625a^4b + 3684a^3b^2 - \\
& 1728a^2b^3) \cosh(dx + c) \sinh(dx + c) - (625a^5 - 2625a^4b + 3684a^3b^2 - \\
& 1728a^2b^3) \sinh(dx + c)^2 - 2((125a^5b^2 - 520a^4b^3 + 72 \\
& 3a^3b^4 - 336a^2b^5) d \cosh(dx + c) + (125a^5b^2 - 520a^4b^3 + 723 \\
& a^3b^4 - 336a^2b^5) d \sinh(dx + c) + 2((2a^4b^7 - 9a^3b^8 + 15a^2b^9 - \\
& 11ab^{10} + 3b^{11}) d^3 \cosh(dx + c) + (2a^4b^7 - 9a^3b^8 + 15a^2b^9 - \\
& 11ab^{10} + 3b^{11}) d^3 \sinh(dx + c)) \sqrt{(625a^7 - 3450a^6b + 7161a^5b^2 - \\
& 6624a^4b^3 + 2304a^3b^4) / ((a^6b^9 - 6a^5b^{10} + 15a^4b^{11} - 20a^3b^{12} + \\
& 15a^2b^{13} - 6ab^{14} + b^{15}) d^4)} \sqrt{((a^3b^4 - 3a^2b^5 + 3ab^6 - b^7) d^2 \sqrt{(625a^7 - 3450a^6b + 7161a^5b^2 - \\
& 6624a^4b^3 + 2304a^3b^4) / ((a^6b^9 - 6a^5b^{10} + 15a^4b^{11} - 20a^3b^{12} + \\
& 15a^2b^{13} - 6ab^{14} + b^{15}) d^4)} - 15a^3 + 47a^2b - 36ab^2) / ((a^3b^4 - 3a^2b^5 + 3ab^6 - b^7) d^2)} + 8ab - 8b^2 + 16(\\
& 5(ab - b^2) \cosh(dx + c)^9 - 4(2ab - 3b^2) \cosh(dx + c)^7 - 3(20a^2 - 17ab + 2b^2) \cosh(dx + c)^5 - 2(20a^2 - 17ab + 2b^2) \cosh(dx + c)^3 - (2ab - 3b^2) \cosh(dx + c) \sinh(dx + c)) / ((ab^3 - b^4) d \cosh(dx + c)^9 + 9(ab^3 - b^4) d \cosh(dx + c) \sinh(dx + c)^8 + (ab^3 - b^4) d \sinh(dx + c)^9 - 4(ab^3 - b^4) d \cosh(dx + c)^7 + 4(9(ab^3 - b^4) d \cosh(dx + c)^2 - (ab^3 - b^4) d) \sinh(dx + c)^7 - 2(8a^2b^2 - 11ab^3 + 3b^4) d \cosh(dx + c)^5 + 28(3(ab^3 - b^4) d \cosh(dx + c)^3 - (ab^3 - b^4) d \cosh(dx + c) \sinh(dx + c)^6 + 2(63(ab^3 - b^4) d \cosh(dx + c)^4 - 42(ab^3 - b^4) d \cosh(dx + c)^2 - (8a^2b^2 - 11ab^3 + 3b^4) d) \sinh(dx + c)^5 - 4(ab^3 - b^4) d \cosh(dx + c)^3 + 2(63(ab^3 - b^4) d \cosh(dx + c)^5 - 70(ab^3 - b^4) d \cosh(dx + c)^3 - 5(8a^2b^2 - 11ab^3 + 3b^4) d \cosh(dx + c)) \sinh(dx + c)^4 + 4(21(ab^3 - b^4) d \cosh(dx + c)^6 - 35(ab^3 - b^4) d \cosh(dx + c)^4 - 5(8a^2b^2 - 11ab^3 + 3b^4) d \cosh(dx + c)^2 - (ab^3 - b^4) d) \sinh(dx + c)^3 + (ab^3 - b^4) d \cosh(dx + c) + 4(9(ab^3 - b^4) d \cosh(dx + c)^7 - 21(ab^3 - b^4) d \cosh(dx + c)^5 - 5(8a^2b^2 - 11ab^3 + 3b^4) d \cosh(dx + c)^3 - 3(ab^3 - b^4) d \cosh(dx + c) \sinh(dx + c)^2 + (9(ab^3 - b^4) d \cosh(dx + c)^8 - 28(ab^3 - b^4) d \cosh(dx + c)^6 - 10(8a^2b^2 - 11ab^3 + 3b^4) d \cosh(dx + c)^4 - 12(ab^3 - b^4) d \cosh(dx + c)^2 + (ab^3 - b^4) d) \sinh(dx + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)**9/(a-b*sinh(dx+c)**4)**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.242 \quad \int \frac{\sinh^7(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$$

Optimal. Leaf size=210

$$\frac{(3\sqrt{a}-4\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d(\sqrt{a}-\sqrt{b})^{3/2}} - \frac{(3\sqrt{a}+4\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d(\sqrt{a}+\sqrt{b})^{3/2}} - \frac{a \cosh(c+dx)(2-\cosh^2(c+dx))}{4bd(a-b)(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx))}$$

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*ArcTanH[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.370462, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3215, 1205, 1166, 205, 208}

$$\frac{(3\sqrt{a}-4\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d(\sqrt{a}-\sqrt{b})^{3/2}} - \frac{(3\sqrt{a}+4\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d(\sqrt{a}+\sqrt{b})^{3/2}} - \frac{a \cosh(c+dx)(2-\cosh^2(c+dx))}{4bd(a-b)(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*ArcTanH[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
```

1)/2]

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{a \cosh(c+dx)(2-\cosh^2(c+dx))}{4(a-b)bd(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} + \frac{\text{Subst}\left(\int \frac{4a(a-2b)-2a(3a-4b)x^2}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{8a(a-b)} \\
&= -\frac{a \cosh(c+dx)(2-\cosh^2(c+dx))}{4(a-b)bd(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} - \frac{(3a-\sqrt{a}\sqrt{b}-4b)\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{8(a-b)b} \\
&= \frac{(3\sqrt{a}-4\sqrt{b})\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8(\sqrt{a}-\sqrt{b})^{3/2}b^{7/4}d} - \frac{(3\sqrt{a}+4\sqrt{b})\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8(\sqrt{a}+\sqrt{b})^{3/2}b^{7/4}d} - \frac{a}{4(a-b)bd}
\end{aligned}$$

Mathematica [C] time = 0.663144, size = 737, normalized size = 3.51

$$\text{RootSum}\left[-16\#1^4a + \#1^8b - 4\#1^6b + 6\#1^4b - 4\#1^2b + b\sqrt{\frac{-6\#1^6a \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right)}{8(\sqrt{a}-\sqrt{b})^{3/2}b^{7/4}d} - \frac{(3\sqrt{a}+4\sqrt{b})\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8(\sqrt{a}+\sqrt{b})^{3/2}b^{7/4}d} - \frac{a}{4(a-b)bd}}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] -((-16*a*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (3*a*c - 4*b*c + 3*a*d*x - 4*b*d*x + 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 5*a*c*#1^2 + 12*b*c*#1^2 - 5*a*d*x*#1^2 + 12*b*d*x*#1^2 - 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 5*a*c*#1^4 - 12*b*c*#1^4 + 5*a*d*x*#1^4 - 12*b*d*x*#1^4 + 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 3*a*c*#1^6 + 4*b*c*#1^6 - 3*a*d*x*#1^6 + 4*b*d*x*#1^6 - 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 + 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c +

$$\frac{d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x)/2]*\#1*\#1^6)/(-(b*\#1) - 8*a*\#1^3 + 3*b*\#1^3 - 3*b*\#1^5 + b*\#1^7) \&])/(32*(a - b)*b*d)$$

Maple [B] time = 0.184, size = 1200, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x)`

[Out] $\frac{1}{4} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 (a*b)^{\frac{1}{2}} - \frac{1}{2} \frac{d}{a} \frac{b}{b}} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 (a*b)^{\frac{1}{2}} - \frac{1}{4} \frac{d}{b}} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 - \frac{1}{4} \frac{d}{b^2}} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) (a*b)^{\frac{1}{2}} - \frac{1}{4} \frac{d}{b}} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) + \frac{3}{8} \frac{d}{a} \frac{b}{b^2}} \frac{1}{(a-b)} \frac{1}{(-a*b + (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}} \arctan\left(\frac{1}{4} \frac{2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 a + 4(a*b)^{\frac{1}{2}} - 2a}{(-a*b + (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}}\right) (a*b)^{\frac{1}{2}} - \frac{1}{2} \frac{d}{b} \frac{1}{(a-b)} \frac{1}{(-a*b + (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}} \arctan\left(\frac{1}{4} \frac{2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 a + 4(a*b)^{\frac{1}{2}} - 2a}{(-a*b + (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}}\right) (a*b)^{\frac{1}{2}} - \frac{1}{8} \frac{d}{a} \frac{b}{(a-b)} \frac{1}{(-a*b + (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}} \arctan\left(\frac{1}{4} \frac{2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 a + 4(a*b)^{\frac{1}{2}} - 2a}{(-a*b + (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}}\right) - \frac{1}{4} \frac{d}{b^2} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 (a*b)^{\frac{1}{2}} + \frac{1}{2} \frac{d}{a} \frac{b}{b}} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 (a*b)^{\frac{1}{2}} - \frac{1}{4} \frac{d}{b}} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 - \frac{1}{4} \frac{d}{b}} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 - \frac{1}{4} \frac{d}{b}} \frac{1}{(\tanh(\frac{1}{2}d*x + \frac{1}{2}c)^4 - 4(a*b)^{\frac{1}{2}})} \frac{1}{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 - 2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 + 1} \frac{1}{(a-b) (a*b)^{\frac{1}{2}} + \frac{3}{8} \frac{d}{a} \frac{b}{b^2}} \frac{1}{(a-b)} \frac{1}{(-a*b - (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}} \arctan\left(\frac{1}{4} \frac{-2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 a + 4(a*b)^{\frac{1}{2}} + 2a}{(-a*b - (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}}\right) (a*b)^{\frac{1}{2}} - \frac{1}{2} \frac{d}{b} \frac{1}{(a-b)} \frac{1}{(-a*b - (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}} \arctan\left(\frac{1}{4} \frac{-2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 a + 4(a*b)^{\frac{1}{2}} + 2a}{(-a*b - (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}}\right) (a*b)^{\frac{1}{2}} + \frac{1}{8} \frac{d}{a} \frac{b}{(a-b)} \frac{1}{(-a*b - (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}} \arctan\left(\frac{1}{4} \frac{-2 \tanh(\frac{1}{2}d*x + \frac{1}{2}c)^2 a + 4(a*b)^{\frac{1}{2}} + 2a}{(-a*b - (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}}\right) \frac{1}{(-a*b - (a*b)^{\frac{1}{2}}) a^{\frac{1}{2}}}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{ae^{7dx+7c} - 5ae^{5dx+5c} - 5ae^{3dx+3c} + ae^{dx+c}}{2(ab^2d - b^3d + (ab^2de^{8c} - b^3de^{8c}))e^{8dx} - 4(ab^2de^{6c} - b^3de^{6c})e^{6dx} - 2(8a^2bde^{4c} - 11ab^2de^{4c} + 3b^3de^{4c})e^{4dx} - 2(8a^2bde^{2c} - 11ab^2de^{2c} + 3b^3de^{2c})e^{2dx} - 2(ab^2de^{0c} - b^3de^{0c})e^{0dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out]
$$-1/2*(a*e^{(7*d*x + 7*c)} - 5*a*e^{(5*d*x + 5*c)} - 5*a*e^{(3*d*x + 3*c)} + a*e^{(d*x + c)})/(a*b^2*d - b^3*d + (a*b^2*d*e^{(8*c)} - b^3*d*e^{(8*c)})*e^{(8*d*x)} - 4*(a*b^2*d*e^{(6*c)} - b^3*d*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^2*b*d*e^{(4*c)} - 11*a*b^2*d*e^{(4*c)} + 3*b^3*d*e^{(4*c)})*e^{(4*d*x)} - 4*(a*b^2*d*e^{(2*c)} - b^3*d*e^{(2*c)})*e^{(2*d*x)}) + 1/128*integrate(64*((3*a*e^{(7*c)} - 4*b*e^{(7*c)})*e^{(7*d*x)} - (5*a*e^{(5*c)} - 12*b*e^{(5*c)})*e^{(5*d*x)} + (5*a*e^{(3*c)} - 12*b*e^{(3*c)})*e^{(3*d*x)} - (3*a*e^{(c)} - 4*b*e^{(c)})*e^{(d*x)}))/(a*b^2 - b^3 + (a*b^2*e^{(8*c)} - b^3*e^{(8*c)})*e^{(8*d*x)} - 4*(a*b^2*e^{(6*c)} - b^3*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^2*b*e^{(4*c)} - 11*a*b^2*e^{(4*c)} + 3*b^3*e^{(4*c)})*e^{(4*d*x)} - 4*(a*b^2*e^{(2*c)} - b^3*e^{(2*c)})*e^{(2*d*x)}), x)$$

Fricas [B] time = 2.53825, size = 14307, normalized size = 68.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out]
$$-1/16*(8*a*\cosh(d*x + c)^7 + 56*a*\cosh(d*x + c)*\sinh(d*x + c)^6 + 8*a*\sinh(d*x + c)^7 - 40*a*\cosh(d*x + c)^5 + 8*(21*a*\cosh(d*x + c)^2 - 5*a)*\sinh(d*x + c)^5 + 40*(7*a*\cosh(d*x + c)^3 - 5*a*\cosh(d*x + c))*\sinh(d*x + c)^4 - 40*a*\cosh(d*x + c)^3 + 40*(7*a*\cosh(d*x + c)^4 - 10*a*\cosh(d*x + c)^2 - a)*\sinh(d*x + c)^3 + 8*(21*a*\cosh(d*x + c)^5 - 50*a*\cosh(d*x + c)^3 - 15*a*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 -$$

$$\begin{aligned}
& 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)} + 3*a^2 - 15*a*b + 16*b^2)/(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*\log(-81*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*\cosh(d*x + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*\cosh(d*x + c)*\sinh(d*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*\sinh(d*x + c)^2 + 2*(2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*\cosh(d*x + c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*\sinh(d*x + c) - ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*\cosh(d*x + c) + (3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*\sinh(d*x + c)))*\sqrt{(81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)})*\sqrt{-(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)} + 3*a^2 - 15*a*b + 16*b^2)/(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)} + 3*a^2 - 15*a*b + 16*b^2)/(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*\log(-81*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*\cosh(d*x + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*\cosh(d*x + c)*\sinh(d*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*\sinh(d*x + c)^2 - 2*(2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*\cosh(d*x + c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*\sinh(d*x + c) - ((3*
\end{aligned}$$

$$\begin{aligned}
& a^4 b^5 - 14 a^3 b^6 + 24 a^2 b^7 - 18 a b^8 + 5 b^9) d^3 \cosh(dx + c) + (\\
& 3 a^4 b^5 - 14 a^3 b^6 + 24 a^2 b^7 - 18 a b^8 + 5 b^9) d^3 \sinh(dx + c)) * \\
& \sqrt{((81 a^5 - 522 a^4 b + 1273 a^3 b^2 - 1392 a^2 b^3 + 576 a b^4) / ((a^6 b^7 - 6 a^5 b^8 + 15 a^4 b^9 - 20 a^3 b^{10} + 15 a^2 b^{11} - 6 a b^{12} + b^{13}) * \\
& d^4))) * \sqrt{-((a^3 b^3 - 3 a^2 b^4 + 3 a b^5 - b^6) d^2 * \sqrt{((81 a^5 - 522 a^4 b + 1273 a^3 b^2 - 1392 a^2 b^3 + 576 a b^4) / ((a^6 b^7 - 6 a^5 b^8 + 15 \\
& a^4 b^9 - 20 a^3 b^{10} + 15 a^2 b^{11} - 6 a b^{12} + b^{13}) d^4)) + 3 a^2 - 15 \\
& a b + 16 b^2) / ((a^3 b^3 - 3 a^2 b^4 + 3 a b^5 - b^6) d^2))} - ((a b^2 - b^3) \\
&) * d * \cosh(dx + c)^8 + 8 (a b^2 - b^3) * d * \cosh(dx + c) * \sinh(dx + c)^7 + (a \\
& b^2 - b^3) * d * \sinh(dx + c)^8 - 4 (a b^2 - b^3) * d * \cosh(dx + c)^6 + 4 (7 (a \\
& b^2 - b^3) * d * \cosh(dx + c)^2 - (a b^2 - b^3) * d) * \sinh(dx + c)^6 - 2 (8 a^2 b \\
& b - 11 a b^2 + 3 b^3) * d * \cosh(dx + c)^4 + 8 (7 (a b^2 - b^3) * d * \cosh(dx + c \\
&)^3 - 3 (a b^2 - b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 (35 (a b^2 - b^3) \\
&) * d * \cosh(dx + c)^4 - 30 (a b^2 - b^3) * d * \cosh(dx + c)^2 - (8 a^2 b - 11 a \\
& b^2 + 3 b^3) * d) * \sinh(dx + c)^4 - 4 (a b^2 - b^3) * d * \cosh(dx + c)^2 + 8 (7 \\
& (a b^2 - b^3) * d * \cosh(dx + c)^5 - 10 (a b^2 - b^3) * d * \cosh(dx + c)^3 - (8 a \\
& ^2 b - 11 a b^2 + 3 b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4 (7 (a b^2 - b \\
& ^3) * d * \cosh(dx + c)^6 - 15 (a b^2 - b^3) * d * \cosh(dx + c)^4 - 3 (8 a^2 b - 1 \\
& 1 a b^2 + 3 b^3) * d * \cosh(dx + c)^2 - (a b^2 - b^3) * d) * \sinh(dx + c)^2 + (a \\
& b^2 - b^3) * d + 8 ((a b^2 - b^3) * d * \cosh(dx + c)^7 - 3 (a b^2 - b^3) * d * \cosh(\\
& dx + c)^5 - (8 a^2 b - 11 a b^2 + 3 b^3) * d * \cosh(dx + c)^3 - (a b^2 - b^3) \\
&) * d * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{((a^3 b^3 - 3 a^2 b^4 + 3 a b^5 - b^6) \\
&) d^2 * \sqrt{((81 a^5 - 522 a^4 b + 1273 a^3 b^2 - 1392 a^2 b^3 + 576 a b^4) / (\\
& a^6 b^7 - 6 a^5 b^8 + 15 a^4 b^9 - 20 a^3 b^{10} + 15 a^2 b^{11} - 6 a b^{12} + \\
& b^{13}) d^4)) - 3 a^2 + 15 a b - 16 b^2) / ((a^3 b^3 - 3 a^2 b^4 + 3 a b^5 - b^6) \\
&) d^2)} * \log(-81 a^3 + 405 a^2 b - 680 a b^2 + 384 b^3 - (81 a^3 - 405 a^2 b \\
& + 680 a b^2 - 384 b^3) * \cosh(dx + c)^2 - 2 (81 a^3 - 405 a^2 b + 680 a b^2 \\
& - 384 b^3) * \cosh(dx + c) * \sinh(dx + c) - (81 a^3 - 405 a^2 b + 680 a b^2 \\
& - 384 b^3) * \sinh(dx + c)^2 + 2 (2 (9 a^3 b^2 - 47 a^2 b^3 + 82 a b^4 - 48 b \\
& ^5) * d * \cosh(dx + c) + 2 (9 a^3 b^2 - 47 a^2 b^3 + 82 a b^4 - 48 b^5) * d * \sinh \\
& (dx + c) + ((3 a^4 b^5 - 14 a^3 b^6 + 24 a^2 b^7 - 18 a b^8 + 5 b^9) d^3 * \c \\
& osh(dx + c) + (3 a^4 b^5 - 14 a^3 b^6 + 24 a^2 b^7 - 18 a b^8 + 5 b^9) d^3 \\
& * \sinh(dx + c)) * \sqrt{((81 a^5 - 522 a^4 b + 1273 a^3 b^2 - 1392 a^2 b^3 + 57 \\
& 6 a b^4) / ((a^6 b^7 - 6 a^5 b^8 + 15 a^4 b^9 - 20 a^3 b^{10} + 15 a^2 b^{11} - 6 \\
& a b^{12} + b^{13}) d^4))) * \sqrt{((a^3 b^3 - 3 a^2 b^4 + 3 a b^5 - b^6) d^2 * \sqrt{ \\
& ((81 a^5 - 522 a^4 b + 1273 a^3 b^2 - 1392 a^2 b^3 + 576 a b^4) / ((a^6 b^7 - \\
& 6 a^5 b^8 + 15 a^4 b^9 - 20 a^3 b^{10} + 15 a^2 b^{11} - 6 a b^{12} + b^{13}) d^4) \\
&) - 3 a^2 + 15 a b - 16 b^2) / ((a^3 b^3 - 3 a^2 b^4 + 3 a b^5 - b^6) d^2))} \\
& + ((a b^2 - b^3) * d * \cosh(dx + c)^8 + 8 (a b^2 - b^3) * d * \cosh(dx + c) * \sinh(d \\
& x + c)^7 + (a b^2 - b^3) * d * \sinh(dx + c)^8 - 4 (a b^2 - b^3) * d * \cosh(dx + \\
& c)^6 + 4 (7 (a b^2 - b^3) * d * \cosh(dx + c)^2 - (a b^2 - b^3) * d) * \sinh(dx + c \\
&)^6 - 2 (8 a^2 b - 11 a b^2 + 3 b^3) * d * \cosh(dx + c)^4 + 8 (7 (a b^2 - b^3) \\
&) * d * \cosh(dx + c)^3 - 3 (a b^2 - b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 (\\
& 35 (a b^2 - b^3) * d * \cosh(dx + c)^4 - 30 (a b^2 - b^3) * d * \cosh(dx + c)^2 - (\\
& 8 a^2 b - 11 a b^2 + 3 b^3) * d) * \sinh(dx + c)^4 - 4 (a b^2 - b^3) * d * \cosh(dx
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + \\
& 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)}*\log(-81*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*\cosh(d*x + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*\cosh(d*x + c)*\sinh(d*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*\sinh(d*x + c)^2 - 2*(2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*\cosh(d*x + c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*\sinh(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*\cosh(d*x + c) + (3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*\sinh(d*x + c)))*\sqrt{(81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)))*\sqrt{((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)} + 8*a*\cosh(d*x + c) + 8*(7*a*\cosh(d*x + c)^6 - 25*a*\cosh(d*x + c)^4 - 15*a*\cosh(d*x + c)^2 + a)*\sinh(d*x + c))/((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.243 \quad \int \frac{\sinh^5(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$$

Optimal. Leaf size=217

$$\frac{(\sqrt{a}-2\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{ab}^{5/4}d(\sqrt{a}-\sqrt{b})^{3/2}} - \frac{(\sqrt{a}+2\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{ab}^{5/4}d(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{\cosh(c+dx)(a-b \cosh^2(c+dx)+b)}{4bd(a-b)(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx))}$$

[Out] -((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) - ((Sqrt[a] + 2*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(5/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(4*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rubi [A] time = 0.294387, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3215, 1205, 1166, 205, 208}

$$\frac{(\sqrt{a}-2\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{ab}^{5/4}d(\sqrt{a}-\sqrt{b})^{3/2}} - \frac{(\sqrt{a}+2\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{ab}^{5/4}d(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{\cosh(c+dx)(a-b \cosh^2(c+dx)+b)}{4bd(a-b)(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] -((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) - ((Sqrt[a] + 2*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(5/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(4*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rule 3215

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -

1)/2]

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{4(a-b)bd(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{2a(a-3b)+2abx^2}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{8a(a-b)bd} \\
&= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{4(a-b)bd(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} + \frac{(\sqrt{a}-2\sqrt{b})\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b+b^2x^2}} dx, x, \cosh(c+dx)\right)}{8\sqrt{a}(\sqrt{a}-\sqrt{b})} \\
&= -\frac{(\sqrt{a}-2\sqrt{b})\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}(\sqrt{a}-\sqrt{b})^{3/2}b^{5/4}d} - \frac{(\sqrt{a}+2\sqrt{b})\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}(\sqrt{a}+\sqrt{b})^{3/2}b^{5/4}d} + \frac{\cosh(c+dx)}{4(a-b)bd}
\end{aligned}$$

Mathematica [C] time = 0.551238, size = 597, normalized size = 2.75

$$\text{RootSum}\left[-16\#1^4a + \#1^8b - 4\#1^6b + 6\#1^4b - 4\#1^2b + b\&\mathfrak{x}, \frac{8\#1^4a \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right)}{\dots}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] ((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (- (b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 4*a*c*#1^2 + 11*b*c*#1^2 - 4*a*d*x*#1^2 + 11*b*d*x*#1^2 - 8*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 22*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 4*a*c*#1^4 - 11*b*c*#1^4 + 4*a*d*x*#1^4 - 11*b*d*x*#1^4 + 8*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 22*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(- (b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) &])/(32*(a - b)*b*d)

Maple [B] time = 0.077, size = 1116, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\sinh(dx+c))^5 / (a-b*\sinh(dx+c))^4 dx, x$

[Out]
$$\begin{aligned} & -1/2/d*a/b/(\tanh(1/2*d*x+1/2*c))^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a}/(a-b)* \\ & \tanh(1/2*d*x+1/2*c)^{6+1/d}/(\tanh(1/2*d*x+1/2*c))^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a}/(a-b)* \\ & \tanh(1/2*d*x+1/2*c)^{6+3/2/d}*/b/(\tanh(1/2*d*x+1/2*c))^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a}/(a-b)* \\ & \tanh(1/2*d*x+1/2*c)^{4-4/d}/(\tanh(1/2*d*x+1/2*c))^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a}/(a-b)* \\ & \tanh(1/2*d*x+1/2*c)^{4-3/2/d}*/b/(\tanh(1/2*d*x+1/2*c))^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a}/(a-b)* \\ & \tanh(1/2*d*x+1/2*c)^{2-1/d}/(\tanh(1/2*d*x+1/2*c))^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a}/(a-b)* \\ & \tanh(1/2*d*x+1/2*c)^{2+1/2/d}/(\tanh(1/2*d*x+1/2*c))^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a} \\ & *a/b/(a-b)+1/8/d/b/(a-b)/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*a}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c))^{2*a+4*(a*b)^{(1/2)+2*a}}/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*a})^{(1/2)}) \\ & *(a*b)^{(1/2)+1/8/d}*/b/(a-b)/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*a}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c))^{2*a+4*(a*b)^{(1/2)+2*a}}/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*a})^{(1/2)}) \\ & -1/4/d/(a-b)/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*a}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c))^{2*a+4*(a*b)^{(1/2)+2*a}}/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*a})^{(1/2)})+1/8/d/b/(a-b)/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*a} \\ & *\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c))^{2*a+4*(a*b)^{(1/2)-2*a}}/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*a})^{(1/2)}) \\ & *(a*b)^{(1/2)-1/8/d}*/b/(a-b)/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*a}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c))^{2*a+4*(a*b)^{(1/2)-2*a}}/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*a})^{(1/2)}) \\ & +1/4/d/(a-b)/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*a}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c))^{2*a+4*(a*b)^{(1/2)-2*a}}/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*a})^{(1/2)}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(4ae^{(5c)} + be^{(5c)})e^{(5dx)} + (4ae^{(3c)} + be^{(3c)})e^{(3dx)} - be^{(7dx+7c)} - be^{(dx+c)}}{2(ab^2d - b^3d + (ab^2de^{(8c)} - b^3de^{(8c)})e^{(8dx)} - 4(ab^2de^{(6c)} - b^3de^{(6c)})e^{(6dx)} - 2(8a^2bde^{(4c)} - 11ab^2de^{(4c)} + 3b^3de^{(4c)})e^{(4dx)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out]
$$-1/2*((4*a*e^{(5*c)} + b*e^{(5*c)})e^{(5*d*x)} + (4*a*e^{(3*c)} + b*e^{(3*c)})e^{(3*d*x)} - b*e^{(7*d*x + 7*c)} - b*e^{(d*x + c)})/(a*b^2*d - b^3*d + (a*b^2*d*e^{(8*c)} - b^3*d*e^{(8*c)})e^{(8*d*x)} - 4*(a*b^2*d*e^{(6*c)} - b^3*d*e^{(6*c)})e^{(6*d*x)} - 2*(8*a^2*b*d*e^{(4*c)} - 11*a*b^2*d*e^{(4*c)} + 3*b^3*d*e^{(4*c)})e^{(4*d*x)} - 4*(a*b^2*d*e^{(2*c)} - b^3*d*e^{(2*c)})e^{(2*d*x)}) + 1/32*integrate(16*((4*a*e^{(5*c)} - 11*b*e^{(5*c)})e^{(5*d*x)} - (4*a*e^{(3*c)} - 11*b*e^{(3*c)})e^{(3*d*x)} + b*e^{(7*d*x + 7*c)} - b*e^{(d*x + c)})/(a*b^2 - b^3 + (a*b^2*e^{(8*c)} - b^3*e^{(8*c)})e^{(8*d*x)} - 4*(a*b^2*e^{(6*c)} - b^3*e^{(6*c)})e^{(6*d*x)} - 2*(8*a^2*b*e^{(4*c)} - 11*a*b^2*e^{(4*c)} + 3*b^3*e^{(4*c)})e^{(4*d*x)} - 4*(a*b^2*e^{(2*c)} - b^3*e^{(2*c)})e^{(2*d*x)}), x)$$

Fricas [B] time = 2.66042, size = 13905, normalized size = 64.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out]
$$1/16*(8*b*\cosh(d*x + c)^7 + 56*b*\cosh(d*x + c)*\sinh(d*x + c)^6 + 8*b*\sinh(d*x + c)^7 - 8*(4*a + b)*\cosh(d*x + c)^5 + 8*(21*b*\cosh(d*x + c)^2 - 4*a - b)*\sinh(d*x + c)^5 + 40*(7*b*\cosh(d*x + c)^3 - (4*a + b)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 8*(4*a + b)*\cosh(d*x + c)^3 + 8*(35*b*\cosh(d*x + c)^4 - 10*(4*a + b)*\cosh(d*x + c)^2 - 4*a - b)*\sinh(d*x + c)^3 + 8*(21*b*\cosh(d*x + c)^5 - 10*(4*a + b)*\cosh(d*x + c)^3 - 3*(4*a + b)*\cosh(d*x + c))*\sinh(d*x + c)^2 + ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 +$$

$$\begin{aligned}
& 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^{10} + a*b^{11})*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*\log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*\cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*\cosh(d*x + c)*\sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*\sinh(d*x + c)^2 + 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*\cosh(d*x + c) + (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*\sinh(d*x + c) - 2*((a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*\cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*\sinh(d*x + c)))*\sqrt{((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^{10} + a*b^{11})*d^4)))*\sqrt{((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*\sqrt{(a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^{10} + a*b^{11})*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))} \\
&) - ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*\sqrt{(a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^{10} + a*b^{11})*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*\log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*\cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*\cosh(d*x + c)*\sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*\sinh(d*x + c)^2 - 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*\cosh(d*x + c) + (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*\sinh(d*x + c) - 2*((a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*\cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*\sinh(d*x + c)))*\sqrt{((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^{10} + a*b^{11})*d^4)))*\sqrt{((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*\sqrt{(a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^{10} + a*b^{11})*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))} \\
& + ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x +
\end{aligned}$$

$$\begin{aligned}
& c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + \\
& c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3 \\
&)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2* \\
& (35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - \\
& (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d* \\
& x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d \\
& *x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + \\
& 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 \\
& - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(\\
& d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^ \\
& 2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 \\
& - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-((a^4*b^2 - 3*a^3*b^ \\
& 3 + 3*a^2*b^4 - a*b^5)*d^2*\sqrt{(a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 6 \\
& 4*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2 \\
& *b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^ \\
& 4 - a*b^5)*d^2))*\log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b + \\
& 28*a*b^2 - 32*b^3)*\cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)* \\
& \cosh(d*x + c)*\sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*\sinh(d*x \\
& + c)^2 + 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*\cosh(d*x + c) + (\\
& a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*\sinh(d*x + c) + 2*((a^4*b^5 - \\
& 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*\cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 + 3 \\
& *a^2*b^7 - a*b^8)*d^3*\sinh(d*x + c))*\sqrt{(a^4 - 10*a^3*b + 41*a^2*b^2 - 80 \\
& *a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 \\
& - 6*a^2*b^10 + a*b^11)*d^4)))*\sqrt{-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - \\
& a*b^5)*d^2*\sqrt{(a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 \\
& - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)* \\
& d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2)) \\
&) - ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh \\
& (d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x \\
& + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + \\
& c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^ \\
& 3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2 \\
& *(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - \\
& (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d \\
& *x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(\\
& d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 \\
& + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 \\
& - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh \\
& (d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b \\
& ^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^ \\
& 3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-((a^4*b^2 - 3*a^3*b \\
& ^3 + 3*a^2*b^4 - a*b^5)*d^2*\sqrt{(a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + \\
& 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^ \\
& 2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b \\
& ^4 - a*b^5)*d^2))*\log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b +
\end{aligned}$$

$$\begin{aligned}
& 28*a*b^2 - 32*b^3)*\cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3) \\
& * \cosh(d*x + c)*\sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*\sinh(d*x \\
& + c)^2 - 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*\cosh(d*x + c) + \\
& (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*\sinh(d*x + c) + 2*((a^4*b^5 - \\
& 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*\cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 + \\
& 3*a^2*b^7 - a*b^8)*d^3*\sinh(d*x + c))*\sqrt{(a^4 - 10*a^3*b + 41*a^2*b^2 - 8 \\
& 0*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3* \\
& b^9 - 6*a^2*b^{10} + a*b^{11})*d^4)}*\sqrt{-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - \\
& a*b^5)*d^2*\sqrt{(a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 \\
& - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^{10} + a*b^{11}) \\
& *d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2) \\
&)) + 8*b*\cosh(d*x + c) + 8*(7*b*\cosh(d*x + c)^6 - 5*(4*a + b)*\cosh(d*x + c) \\
& ^4 - 3*(4*a + b)*\cosh(d*x + c)^2 + b)*\sinh(d*x + c))/((a*b^2 - b^3)*d*\cosh(\\
& d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3 \\
&)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3 \\
&)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a* \\
& b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(\\
& a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(\\
& d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b \\
& ^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - \\
& b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11 \\
& *a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cos \\
& h(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + \\
& 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3 \\
&)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^ \\
& 5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d \\
& *x + c))*\sinh(d*x + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.244 \quad \int \frac{\sinh^3(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^2} dx$$

Optimal. Leaf size=186

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{ab}^{3/4}d(\sqrt{a}-\sqrt{b})^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{ab}^{3/4}d(\sqrt{a}+\sqrt{b})^{3/2}} - \frac{\cosh(c+dx)(2-\cosh^2(c+dx))}{4d(a-b)(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx)-b)}$$

[Out] -ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rubi [A] time = 0.205211, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3215, 1178, 1166, 205, 208}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{ab}^{3/4}d(\sqrt{a}-\sqrt{b})^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{ab}^{3/4}d(\sqrt{a}+\sqrt{b})^{3/2}} - \frac{\cosh(c+dx)(2-\cosh^2(c+dx))}{4d(a-b)(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx)-b)}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] -ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rule 3215

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -

1)/2]

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
- 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\cosh(c+dx)(2-\cosh^2(c+dx))}{4(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} + \frac{\text{Subst}\left(\int \frac{-4ab+2abx^2}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{8a(a-b)bd} \\
&= -\frac{\cosh(c+dx)(2-\cosh^2(c+dx))}{4(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b+b-bx^2}} dx, x, \cosh(c+dx)\right)}{8\sqrt{a}(\sqrt{a}-\sqrt{b})} \\
&= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}(\sqrt{a}-\sqrt{b})^{3/2}b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}(\sqrt{a}+\sqrt{b})^{3/2}b^{3/4}d} - \frac{\cosh(c+dx)(2-\cosh^2(c+dx))}{4(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))}
\end{aligned}$$

Mathematica [C] time = 0.485186, size = 422, normalized size = 2.27

$$\text{RootSum}\left[-16\#1^4a + \#1^8b - 4\#1^6b + 6\#1^4b - 4\#1^2b + b\sqrt{\frac{2\#1^6 \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right)}{a-b+2bx^2-bx^4}}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] -((16*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 7*c*#1^2 + 7*d*x*#1^2 + 14*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 7*c*#1^4 - 7*d*x*#1^4 - 14*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(- (b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) &])/(32*(a - b)*d)

Maple [B] time = 0.058, size = 767, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(dx+c)^3/(a-b*\sinh(dx+c)^4)^2,x)$

[Out]
$$-1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}/(a-b)*\tanh(1/2*d*x+1/2*c)^{6-3/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}/(a-b)*\tanh(1/2*d*x+1/2*c)^{4+4/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}/a/(a-b)*\tanh(1/2*d*x+1/2*c)^{4*b+5/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}/(a-b)*\tanh(1/2*d*x+1/2*c)^{2-1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}/(a-b)+1/8/d/(a-b)/(-a*b-(a*b)^{(1/2)*a}^{(1/2)*a}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)+2*a})/(-a*b-(a*b)^{(1/2)*a}^{(1/2)*a})^{(1/2)}-1/8/d/b/(a-b)/(-a*b-(a*b)^{(1/2)*a}^{(1/2)*a}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)+2*a})/(-a*b-(a*b)^{(1/2)*a}^{(1/2)*a})^{(1/2)})*(a*b)^{(1/2)}-1/8/d/b/(a-b)/(-a*b+(a*b)^{(1/2)*a}^{(1/2)*a}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)-2*a})/(-a*b+(a*b)^{(1/2)*a}^{(1/2)*a})^{(1/2)})*(a*b)^{(1/2)}-1/8/d/(a-b)/(-a*b+(a*b)^{(1/2)*a}^{(1/2)*a}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)-2*a})/(-a*b+(a*b)^{(1/2)*a}^{(1/2)*a})^{(1/2)})$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{(7dx+7c)} - 5e^{(5dx+5c)} - 5e^{(3dx+3c)} + e^{(dx+c)}}{2(abd - b^2d + (abde^{(8c)} - b^2de^{(8c)})e^{(8dx)} - 4(abde^{(6c)} - b^2de^{(6c)})e^{(6dx)} - 2(8a^2de^{(4c)} - 11abde^{(4c)} + 3b^2de^{(4c)})e^{(4dx)} - \dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(dx+c)^3/(a-b*\sinh(dx+c)^4)^2,x, \text{algorithm}=\text{"maxima"})$

[Out]
$$-1/2*(e^{(7*d*x + 7*c)} - 5*e^{(5*d*x + 5*c)} - 5*e^{(3*d*x + 3*c)} + e^{(d*x + c)})/(a*b*d - b^2*d + (a*b*d*e^{(8*c)} - b^2*d*e^{(8*c)})*e^{(8*d*x)} - 4*(a*b*d*e^{(6*c)} - b^2*d*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^2*d*e^{(4*c)} - 11*a*b*d*e^{(4*c)} + 3*b^2*d*e^{(4*c)})*e^{(4*d*x)} - 4*(a*b*d*e^{(2*c)} - b^2*d*e^{(2*c)})*e^{(2*d*x)}) - 1/8*\text{integrate}(4*(e^{(7*d*x + 7*c)} - 7*e^{(5*d*x + 5*c)} + 7*e^{(3*d*x + 3*c)} - e^{(d*x + c)})/(a*b - b^2 + (a*b*e^{(8*c)} - b^2*e^{(8*c)})*e^{(8*d*x)} - 4*(a*b*e^{(6*c)} - b^2*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^2*e^{(4*c)} - 11*a*b*e^{(4*c)} + 3*b^2*e^{(4*c)})*e^{(4*d*x)} - 4*(a*b*e^{(2*c)} - b^2*e^{(2*c)})*e^{(2*d*x)}), x)$$

Fricas [B] time = 2.36398, size = 11888, normalized size = 63.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

```
[Out] -1/16*(8*cosh(d*x + c)^7 + 56*cosh(d*x + c)*sinh(d*x + c)^6 + 8*sinh(d*x + c)^7 + 8*(21*cosh(d*x + c)^2 - 5)*sinh(d*x + c)^5 - 40*cosh(d*x + c)^5 + 40*(7*cosh(d*x + c)^3 - 5*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(7*cosh(d*x + c)^4 - 10*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^3 - 40*cosh(d*x + c)^3 + 8*(21*cosh(d*x + c)^5 - 50*cosh(d*x + c)^3 - 15*cosh(d*x + c))*sinh(d*x + c)^2 - ((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cosh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d*x + c)^2 + 2*(2*(a^2*b + 3*a*b^2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*sinh(d*x + c) - ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) + a + 3*b) + ((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*co
```

$$\begin{aligned}
& \operatorname{sh}(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d*\operatorname{sinh}(d*x + c)^4 - 4*(a*b - b^2) \\
& *d*\operatorname{cosh}(d*x + c)^2 + 8*(7*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)^5 - 10*(a*b - b^2)*d* \\
& \operatorname{cosh}(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*\operatorname{cosh}(d*x + c))*\operatorname{sinh}(d*x + c)^3 \\
& + 4*(7*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)^6 - 15*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)^4 - \\
& 3*(8*a^2 - 11*a*b + 3*b^2)*d*\operatorname{cosh}(d*x + c)^2 - (a*b - b^2)*d*\operatorname{sinh}(d*x + c) \\
& ^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*\operatorname{cosh}(d*x + c)^7 - 3*(a*b - b^2)*d*\operatorname{cos} \\
& \operatorname{h}(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*\operatorname{cosh}(d*x + c)^3 - (a*b - b^2)*d*\operatorname{c} \\
& \operatorname{osh}(d*x + c))*\operatorname{sinh}(d*x + c))*\operatorname{sqrt}(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4) \\
& *d^2*\operatorname{sqrt}((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4 \\
& *b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2 \\
& + 3*a^2*b^3 - a*b^4)*d^2))*\log((a + 3*b)*\operatorname{cosh}(d*x + c)^2 + 2*(a + 3*b)*\operatorname{cos} \\
& \operatorname{h}(d*x + c)*\operatorname{sinh}(d*x + c) + (a + 3*b)*\operatorname{sinh}(d*x + c)^2 - 2*(2*(a^2*b + 3*a*b^ \\
& 2)*d*\operatorname{cosh}(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*\operatorname{sinh}(d*x + c) - ((a^5*b^2 - 2*a^ \\
& 4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*\operatorname{cosh}(d*x + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2 \\
& *b^5 - a*b^6)*d^3*\operatorname{sinh}(d*x + c))*\operatorname{sqrt}((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a \\
& ^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))*\operatorname{s} \\
& \operatorname{qrt}(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*\operatorname{sqrt}((a^2 + 6*a*b + 9*b^2) \\
&)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 \\
& + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) + \\
& a + 3*b) - ((a*b - b^2)*d*\operatorname{cosh}(d*x + c)^8 + 8*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)*\operatorname{s} \\
& \operatorname{inh}(d*x + c)^7 + (a*b - b^2)*d*\operatorname{sinh}(d*x + c)^8 - 4*(a*b - b^2)*d*\operatorname{cosh}(d*x + \\
& c)^6 + 4*(7*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)^2 - (a*b - b^2)*d)*\operatorname{sinh}(d*x + c)^6 \\
& - 2*(8*a^2 - 11*a*b + 3*b^2)*d*\operatorname{cosh}(d*x + c)^4 + 8*(7*(a*b - b^2)*d*\operatorname{cosh}(d \\
& *x + c)^3 - 3*(a*b - b^2)*d*\operatorname{cosh}(d*x + c))*\operatorname{sinh}(d*x + c)^5 + 2*(35*(a*b - b \\
& ^2)*d*\operatorname{cosh}(d*x + c)^4 - 30*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)^2 - (8*a^2 - 11*a*b \\
& + 3*b^2)*d)*\operatorname{sinh}(d*x + c)^4 - 4*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)^2 + 8*(7*(a*b - \\
& b^2)*d*\operatorname{cosh}(d*x + c)^5 - 10*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)^3 - (8*a^2 - 11*a*b \\
& b + 3*b^2)*d*\operatorname{cosh}(d*x + c))*\operatorname{sinh}(d*x + c)^3 + 4*(7*(a*b - b^2)*d*\operatorname{cosh}(d*x + \\
& c)^6 - 15*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*\operatorname{cos} \\
& \operatorname{h}(d*x + c)^2 - (a*b - b^2)*d)*\operatorname{sinh}(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b \\
& ^2)*d*\operatorname{cosh}(d*x + c)^7 - 3*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)^5 - (8*a^2 - 11*a*b + \\
& 3*b^2)*d*\operatorname{cosh}(d*x + c)^3 - (a*b - b^2)*d*\operatorname{cosh}(d*x + c))*\operatorname{sinh}(d*x + c))*\operatorname{sqr} \\
& \operatorname{t}(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*\operatorname{sqrt}((a^2 + 6*a*b + 9*b^2)/((\\
& a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a \\
& *b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*\log((\\
& a + 3*b)*\operatorname{cosh}(d*x + c)^2 + 2*(a + 3*b)*\operatorname{cosh}(d*x + c)*\operatorname{sinh}(d*x + c) + (a + 3 \\
& *b)*\operatorname{sinh}(d*x + c)^2 + 2*(2*(a^2*b + 3*a*b^2)*d*\operatorname{cosh}(d*x + c) + 2*(a^2*b + 3 \\
& *a*b^2)*d*\operatorname{sinh}(d*x + c) + ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*\operatorname{co} \\
& \operatorname{sh}(d*x + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*\operatorname{sinh}(d*x + c))* \\
& \operatorname{sqrt}((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 \\
& + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))*\operatorname{sqrt}(((a^4*b - 3*a^3*b^2 + 3*a^2*b \\
& ^3 - a*b^4)*d^2*\operatorname{sqrt}((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b \\
& ^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b \\
& - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) + a + 3*b) + ((a*b - b^2)*d*\operatorname{cosh}(d*x \\
& + c)^8 + 8*(a*b - b^2)*d*\operatorname{cosh}(d*x + c)*\operatorname{sinh}(d*x + c)^7 + (a*b - b^2)*d*\operatorname{sin}
\end{aligned}$$

$$\begin{aligned}
& h(dx + c)^8 - 4*(a*b - b^2)*d*\cosh(dx + c)^6 + 4*(7*(a*b - b^2)*d*\cosh(dx + c)^2 - (a*b - b^2)*d)*\sinh(dx + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*\cosh(dx + c)^4 + 8*(7*(a*b - b^2)*d*\cosh(dx + c)^3 - 3*(a*b - b^2)*d*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(35*(a*b - b^2)*d*\cosh(dx + c)^4 - 30*(a*b - b^2)*d*\cosh(dx + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*\sinh(dx + c)^4 - 4*(a*b - b^2)*d*\cosh(dx + c)^2 + 8*(7*(a*b - b^2)*d*\cosh(dx + c)^5 - 10*(a*b - b^2)*d*\cosh(dx + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(7*(a*b - b^2)*d*\cosh(dx + c)^6 - 15*(a*b - b^2)*d*\cosh(dx + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*\cosh(dx + c)^2 - (a*b - b^2)*d)*\sinh(dx + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*\cosh(dx + c)^7 - 3*(a*b - b^2)*d*\cosh(dx + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*\cosh(dx + c)^3 - (a*b - b^2)*d*\cosh(dx + c))*\sinh(dx + c))*\sqrt{((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*\sqrt{(a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)}*\log((a + 3*b)*\cosh(dx + c)^2 + 2*(a + 3*b)*\cosh(dx + c)*\sinh(dx + c) + (a + 3*b)*\sinh(dx + c)^2 - 2*(2*(a^2*b + 3*a*b^2)*d*\cosh(dx + c) + 2*(a^2*b + 3*a*b^2)*d*\sinh(dx + c) + ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*\cosh(dx + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*\sinh(dx + c))*\sqrt{(a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)))*\sqrt{((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*\sqrt{(a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)} + a + 3*b) + 8*(7*\cosh(dx + c)^6 - 25*\cosh(dx + c)^4 - 15*\cosh(dx + c)^2 + 1)*\sinh(dx + c) + 8*\cosh(dx + c))/((a*b - b^2)*d*\cosh(dx + c)^8 + 8*(a*b - b^2)*d*\cosh(dx + c)*\sinh(dx + c)^7 + (a*b - b^2)*d*\sinh(dx + c)^8 - 4*(a*b - b^2)*d*\cosh(dx + c)^6 + 4*(7*(a*b - b^2)*d*\cosh(dx + c)^2 - (a*b - b^2)*d)*\sinh(dx + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*\cosh(dx + c)^4 + 8*(7*(a*b - b^2)*d*\cosh(dx + c)^3 - 3*(a*b - b^2)*d*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(35*(a*b - b^2)*d*\cosh(dx + c)^4 - 30*(a*b - b^2)*d*\cosh(dx + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*\sinh(dx + c)^4 - 4*(a*b - b^2)*d*\cosh(dx + c)^2 + 8*(7*(a*b - b^2)*d*\cosh(dx + c)^5 - 10*(a*b - b^2)*d*\cosh(dx + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(7*(a*b - b^2)*d*\cosh(dx + c)^6 - 15*(a*b - b^2)*d*\cosh(dx + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*\cosh(dx + c)^2 - (a*b - b^2)*d)*\sinh(dx + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*\cosh(dx + c)^7 - 3*(a*b - b^2)*d*\cosh(dx + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*\cosh(dx + c)^3 - (a*b - b^2)*d*\cosh(dx + c))*\sinh(dx + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.245 \quad \int \frac{\sinh(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^2} dx$$

Optimal. Leaf size=221

$$\frac{(3\sqrt{a}-2\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{bd}(\sqrt{a}-\sqrt{b})^{3/2}} + \frac{(3\sqrt{a}+2\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{bd}(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{\cosh(c+dx)(a-b \cosh^2(c+dx))}{4ad(a-b)(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx))}$$

[Out] $((3*\text{Sqrt}[a] - 2*\text{Sqrt}[b])*\text{ArcTan}[(b^{(1/4)}*\text{Cosh}[c + d*x])/ \text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]]]) / (8*a^{(3/2)}*(\text{Sqrt}[a] - \text{Sqrt}[b])^{(3/2)}*b^{(1/4)}*d) + ((3*\text{Sqrt}[a] + 2*\text{Sqrt}[b])*\text{ArcTanh}[(b^{(1/4)}*\text{Cosh}[c + d*x])/ \text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]]]) / (8*a^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b])^{(3/2)}*b^{(1/4)}*d) + (\text{Cosh}[c + d*x]*(a + b - b*\text{Cosh}[c + d*x]^2)) / (4*a*(a - b)*d*(a - b + 2*b*\text{Cosh}[c + d*x]^2 - b*\text{Cosh}[c + d*x]^4))$

Rubi [A] time = 0.298505, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {3215, 1092, 1166, 205, 208}

$$\frac{(3\sqrt{a}-2\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{bd}(\sqrt{a}-\sqrt{b})^{3/2}} + \frac{(3\sqrt{a}+2\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{bd}(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{\cosh(c+dx)(a-b \cosh^2(c+dx))}{4ad(a-b)(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sinh}[c + d*x]/(a - b*\text{Sinh}[c + d*x]^4)^2, x]$

[Out] $((3*\text{Sqrt}[a] - 2*\text{Sqrt}[b])*\text{ArcTan}[(b^{(1/4)}*\text{Cosh}[c + d*x])/ \text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]]]) / (8*a^{(3/2)}*(\text{Sqrt}[a] - \text{Sqrt}[b])^{(3/2)}*b^{(1/4)}*d) + ((3*\text{Sqrt}[a] + 2*\text{Sqrt}[b])*\text{ArcTanh}[(b^{(1/4)}*\text{Cosh}[c + d*x])/ \text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]]]) / (8*a^{(3/2)}*(\text{Sqrt}[a] + \text{Sqrt}[b])^{(3/2)}*b^{(1/4)}*d) + (\text{Cosh}[c + d*x]*(a + b - b*\text{Cosh}[c + d*x]^2)) / (4*a*(a - b)*d*(a - b + 2*b*\text{Cosh}[c + d*x]^2 - b*\text{Cosh}[c + d*x]^4))$

Rule 3215

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{\text{ff} = \text{FreeFactors}[\text{Cos}[e + f*x], x]\}, -\text{Dist}[\text{ff}/f, \text{Subst}[\text{Int}[(1 - \text{ff}^2*x^2)^{(m-1)/2}*(a + b - 2*b*\text{ff}^2*x^2 + b*\text{ff}^4*x^4)^p, x], x, \text{Cos}[e + f*x]/\text{ff}], x] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[(m -$

1)/2]

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 -
2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
- 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{4a(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2-2(4(a-b)b+4b^2)}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{8a(a-b)d} \\
&= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{4a(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} - \frac{((3\sqrt{a}-2\sqrt{b})\sqrt{b})\text{Subst}\left(\int \frac{1}{\sqrt{a-bx^2-bx^4}} dx, x, \cosh(c+dx)\right)}{8a^{3/2}(\sqrt{a}-\sqrt{b})} \\
&= \frac{(3\sqrt{a}-2\sqrt{b})\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}(\sqrt{a}-\sqrt{b})^{3/2}\sqrt[4]{bd}} + \frac{(3\sqrt{a}+2\sqrt{b})\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}(\sqrt{a}+\sqrt{b})^{3/2}\sqrt[4]{bd}} + \frac{\cosh(c+dx)}{4a(a-b)d}
\end{aligned}$$

Mathematica [C] time = 0.35422, size = 597, normalized size = 2.7

$$\text{RootSum}\left[-16\#1^4 a + \#1^8 b - 4\#1^6 b + 6\#1^4 b - 4\#1^2 b + b\&, \frac{-24\#1^4 a \log(-\#1 \sinh(\frac{1}{2}(c+dx)) + \#1 \cosh(\frac{1}{2}(c+dx)) - \sinh(\frac{1}{2}(c+dx)) - \cosh(\frac{1}{2}(c+dx)))}{(32\#1^4 a^2 - 16\#1^4 a b + 6\#1^4 b^2 - 4\#1^2 b^2 + b^2) \sqrt{b} \sqrt{a-b} \sqrt{a+b}}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] ((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 &, (-b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 12*a*c*#1^2 - 5*b*c*#1^2 + 12*a*d*x*#1^2 - 5*b*d*x*#1^2 + 24*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 12*a*c*#1^4 + 5*b*c*#1^4 - 12*a*d*x*#1^4 + 5*b*d*x*#1^4 - 24*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) &]/(32*a*(a - b)*d)

Maple [B] time = 0.072, size = 1112, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(dx+c)/(a-b*\sinh(dx+c)^4)^2, x)$

[Out]
$$-1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*\tanh(1/2*d*x+1/2*c)^6+1/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)/a*\tanh(1/2*d*x+1/2*c)^6*b+3/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*\tanh(1/2*d*x+1/2*c)^4-4/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*\tanh(1/2*d*x+1/2*c)^4*b-3/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*\tanh(1/2*d*x+1/2*c)^2-1/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*\tanh(1/2*d*x+1/2*c)^2*b+1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)+1/8/d/(a-b)/a/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a})/(-a*b-(a*b)^{(1/2)*a})^{(1/2)})*(a*b)^{(1/2)-3/8}/d/(a-b)/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a})/(-a*b-(a*b)^{(1/2)*a})^{(1/2)})+1/4/d/(a-b)/a/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a})/(-a*b-(a*b)^{(1/2)*a})^{(1/2)})*b+1/8/d/(a-b)/a/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a})/(-a*b+(a*b)^{(1/2)*a})^{(1/2)})*(a*b)^{(1/2)+3/8}/d/(a-b)/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a})/(-a*b+(a*b)^{(1/2)*a})^{(1/2)})-1/4/d/(a-b)/a/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a})/(-a*b+(a*b)^{(1/2)*a})^{(1/2)})*b$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(4ae^{5c} + be^{5c})e^{5dx} + (4ae^{3c} + be^{3c})e^{3dx} - be^{7dx+7c} - be^{dx+c}}{2(a^2bd - ab^2d + (a^2bde^{8c} - ab^2de^{8c})e^{8dx} - 4(a^2bde^{6c} - ab^2de^{6c})e^{6dx} - 2(8a^3de^{4c} - 11a^2bde^{4c} + 3ab^2de^{4c}))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out]
$$-1/2*((4*a*e^{(5*c)} + b*e^{(5*c)})e^{(5*d*x)} + (4*a*e^{(3*c)} + b*e^{(3*c)})e^{(3*d*x)} - b*e^{(7*d*x + 7*c)} - b*e^{(d*x + c)})/(a^2*b*d - a*b^2*d + (a^2*b*d*e^{(8*c)} - a*b^2*d*e^{(8*c)})e^{(8*d*x)} - 4*(a^2*b*d*e^{(6*c)} - a*b^2*d*e^{(6*c)})e^{(6*d*x)} - 2*(8*a^3*d*e^{(4*c)} - 11*a^2*b*d*e^{(4*c)} + 3*a*b^2*d*e^{(4*c)})e^{(4*d*x)} - 4*(a^2*b*d*e^{(2*c)} - a*b^2*d*e^{(2*c)})e^{(2*d*x)}) + 1/2*integrate(-((12*a*e^{(5*c)} - 5*b*e^{(5*c)})e^{(5*d*x)} - (12*a*e^{(3*c)} - 5*b*e^{(3*c)})e^{(3*d*x)} - b*e^{(7*d*x + 7*c)} + b*e^{(d*x + c)})/(a^2*b - a*b^2 + (a^2*b*e^{(8*c)} - a*b^2*e^{(8*c)})e^{(8*d*x)} - 4*(a^2*b*e^{(6*c)} - a*b^2*e^{(6*c)})e^{(6*d*x)} - 2*(8*a^3*e^{(4*c)} - 11*a^2*b*e^{(4*c)} + 3*a*b^2*e^{(4*c)})e^{(4*d*x)} - 4*(a^2*b*e^{(2*c)} - a*b^2*e^{(2*c)})e^{(2*d*x)}), x)$$

Fricas [B] time = 2.73134, size = 13570, normalized size = 61.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out]
$$1/16*(8*b*\cosh(d*x + c)^7 + 56*b*\cosh(d*x + c)*\sinh(d*x + c)^6 + 8*b*\sinh(d*x + c)^7 - 8*(4*a + b)*\cosh(d*x + c)^5 + 8*(21*b*\cosh(d*x + c)^2 - 4*a - b)*\sinh(d*x + c)^5 + 40*(7*b*\cosh(d*x + c)^3 - (4*a + b)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 8*(4*a + b)*\cosh(d*x + c)^3 + 8*(35*b*\cosh(d*x + c)^4 - 10*(4*a + b)*\cosh(d*x + c)^2 - 4*a - b)*\sinh(d*x + c)^3 + 8*(21*b*\cosh(d*x + c)^5 - 10*(4*a + b)*\cosh(d*x + c)^3 - 3*(4*a + b)*\cosh(d*x + c))*\sinh(d*x + c)^2 + ((a^2*b - a*b^2)*d*\cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*\sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a$$

$$\begin{aligned}
&^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) * \log((81*a^2 - 81*a*b + 20*b^2)*\cosh(d*x + c))^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (81*a^2 - 81*a*b + 20*b^2)*\sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 + 2*((27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*\sinh(d*x + c) - 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d^3*\cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d^3*\sinh(d*x + c)))*\sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*\sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) - ((a^2*b - a*b^2)*d*\cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*\sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) * \log((81*a^2 - 81*a*b + 20*b^2)*\cosh(d*x + c))^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (81*a^2 - 81*a*b + 20*b^2)*\sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 - 2*((27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*\sinh(d*x + c) - 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d^3*\cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d^3*\sinh(d*x + c)))*\sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*\sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) + ((a^2*b - a*b^2)*d*\cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*\sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)
\end{aligned}$$

$$\begin{aligned}
&^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(d*x \\
&+ c))*\sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 30*(a^2*b \\
&- a*b^2)*d*\cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + c) \\
&^4 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x \\
&+ c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2 \\
&)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 \\
&- 15*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d* \\
&\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + \\
&8*((a^2*b - a*b^2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 \\
&- (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh(d \\
&*x + c))*\sinh(d*x + c))*\sqrt{((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt{ \\
&t((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 \\
&+ 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 \\
&- 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*\log((81*a^2 - 81*a*b + 20*b^2)*\cosh(\\
&d*x + c)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (81 \\
&*a^2 - 81*a*b + 20*b^2)*\sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 + 2*((27 \\
&*a^4 - 24*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b \\
&^2)*d*\sinh(d*x + c) + 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3 \\
&*b^5)*d^3*\cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^ \\
&3*b^5)*d^3*\sinh(d*x + c))*\sqrt{((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b \\
&^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*\sqrt{ \\
&t(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt{((81*a^2 - 90*a*b + 25*b^2 \\
&)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + \\
&a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3 \\
&*b^3)*d^2))} - ((a^2*b - a*b^2)*d*\cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*\cos \\
&h(d*x + c)*\sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*\sinh(d*x + c)^8 - 4*(a^2*b - \\
&a*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b \\
&- a*b^2)*d)*\sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + \\
&c)^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(d* \\
&x + c))*\sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 30*(a^2 \\
&*b - a*b^2)*d*\cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + \\
&c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d* \\
&x + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b \\
&^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c) \\
&^6 - 15*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)* \\
&d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d \\
&+ 8*((a^2*b - a*b^2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^ \\
&5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh \\
&(d*x + c))*\sinh(d*x + c))*\sqrt{((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt{ \\
&t((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^ \\
&4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^ \\
&6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*\log((81*a^2 - 81*a*b + 20*b^2)*\cos \\
&h(d*x + c)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (\\
&81*a^2 - 81*a*b + 20*b^2)*\sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 - 2*((\\
&27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2
\end{aligned}$$

$$\begin{aligned}
& *b^2)*d*\sinh(d*x + c) + 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a \\
& ^3*b^5)*d^3*\cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + \\
& a^3*b^5)*d^3*\sinh(d*x + c))*\sqrt{(81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8 \\
& *b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*s \\
& \sqrt{((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt{(81*a^2 - 90*a*b + 25*b \\
& ^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 \\
& + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a \\
& ^3*b^3)*d^2))} + 8*b*\cosh(d*x + c) + 8*(7*b*\cosh(d*x + c)^6 - 5*(4*a + b)*c \\
& \cosh(d*x + c)^4 - 3*(4*a + b)*\cosh(d*x + c)^2 + b)*\sinh(d*x + c))/((a^2*b - \\
& a*b^2)*d*\cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^ \\
& 7 + (a^2*b - a*b^2)*d*\sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 \\
& + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c) \\
&)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^ \\
& 2)*d*\cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + \\
& 2*(35*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh(d*x + \\
& c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)* \\
& d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - 10*(a^2*b - a \\
& b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c))*\sinh \\
& (d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d \\
& *\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b \\
& b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*co \\
& sh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3 \\
& *a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.246 \quad \int \frac{\operatorname{csch}(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^2} dx$$

Optimal. Leaf size=325

$$\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2 d \sqrt{\sqrt{a}-\sqrt{b}}} - \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2} d (\sqrt{a}-\sqrt{b})^{3/2}} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2 d \sqrt{\sqrt{a}+\sqrt{b}}} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2} d (\sqrt{a}+\sqrt{b})^{3/2}} - \frac{\tanh^{-1}}{\dots}$$

[Out] $-(b^{1/4} \operatorname{ArcTan}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]])/(8*a^{3/2}*(\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b])^{3/2}*d) - (b^{1/4} \operatorname{ArcTan}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]])/(2*a^2*\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]*d) - \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/(a^2*d) + (b^{1/4} \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]])/(8*a^{3/2}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b])^{3/2}*d) + (b^{1/4} \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]])/(2*a^2*\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]*d) - (b*\operatorname{Cosh}[c + d*x]*(2 - \operatorname{Cosh}[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*\operatorname{Cosh}[c + d*x]^2 - b*\operatorname{Cosh}[c + d*x]^4))$

Rubi [A] time = 0.361405, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$\frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2 d \sqrt{\sqrt{a}-\sqrt{b}}} - \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2} d (\sqrt{a}-\sqrt{b})^{3/2}} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2 d \sqrt{\sqrt{a}+\sqrt{b}}} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2} d (\sqrt{a}+\sqrt{b})^{3/2}} - \frac{\tanh^{-1}}{\dots}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Csch}[c + d*x]/(a - b*\operatorname{Sinh}[c + d*x]^4)^2, x]$

[Out] $-(b^{1/4} \operatorname{ArcTan}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]])/(8*a^{3/2}*(\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b])^{3/2}*d) - (b^{1/4} \operatorname{ArcTan}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]])/(2*a^2*\operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]]*d) - \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]]/(a^2*d) + (b^{1/4} \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]])/(8*a^{3/2}*(\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b])^{3/2}*d) + (b^{1/4} \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x])/\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]])/(2*a^2*\operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]]*d) - (b*\operatorname{Cosh}[c + d*x]*(2 - \operatorname{Cosh}[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*\operatorname{Cosh}[c + d*x]^2 - b*\operatorname{Cosh}[c + d*x]^4))$

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{csch}(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left(-\frac{1}{a^2(-1+x^2)} + \frac{b-bx^2}{a(a-b+2bx^2-bx^4)^2} + \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)}\right) dx, x, \cosh(c+dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \cosh(c+dx)\right)}{a^2 d} - \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{a^2 d} - \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)} dx, x, \cosh(c+dx)\right)}{a^2 d} \\
 &= \frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{b \cosh(c+dx) (2 - \cosh^2(c+dx))}{4a(a-b)d (a-b+2b \cosh^2(c+dx) - b \cosh^4(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)} dx, x, \cosh(c+dx)\right)}{a^2 d} \\
 &= \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2 \sqrt{\sqrt{a}-\sqrt{b}d}} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2 \sqrt{\sqrt{a}+\sqrt{b}d}} - \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)} dx, x, \cosh(c+dx)\right)}{a^2 d} \\
 &= \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2} (\sqrt{a}-\sqrt{b})^{3/2} d} - \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2 \sqrt{\sqrt{a}-\sqrt{b}d}} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\sqrt[4]{b} \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2} (\sqrt{a}+\sqrt{b})^{3/2} d}
 \end{aligned}$$

Mathematica [C] time = 0.837766, size = 761, normalized size = 2.34

$$\frac{b \operatorname{RootSum}\left[-16\#1^4 a + \#1^8 b - 4\#1^6 b + 6\#1^4 b - 4\#1^2 b + b \&, \frac{10\#1^6 a \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right) + \#1 \cosh\left(\frac{1}{2}(c+dx)\right) - \sinh\left(\frac{1}{2}(c+dx)\right) - \cosh\left(\frac{1}{2}(c+dx)\right)\right) - 38\#1^4 a \log\left(-\#1 \sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d}\right]}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^2, x]

[Out] ((16*a*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)])))/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + 32*Log[Tanh[(c + d*x)/2]] - (b*

```

RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-5*a*c
+ 4*b*c - 5*a*d*x + 4*b*d*x - 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 8*b*Log[-Cosh[(c + d*x)
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 1
9*a*c*#1^2 - 12*b*c*#1^2 + 19*a*d*x*#1^2 - 12*b*d*x*#1^2 + 38*a*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*
#1]*#1^2 - 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 19*a*c*#1^4 + 12*b*c*#1^4 - 19*a*d*x*
#1^4 + 12*b*d*x*#1^4 - 38*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Co
sh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 24*b*Log[-Cosh[(c + d*x)/
2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4
+ 5*a*c*#1^6 - 4*b*c*#1^6 + 5*a*d*x*#1^6 - 4*b*d*x*#1^6 + 10*a*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#
1]*#1^6 - 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^
5 + b*#1^7) & ])/(a - b)/(32*a^2*d)

```

Maple [B] time = 0.096, size = 966, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int \operatorname{csch}(d*x+c)/(a-b*\sinh(d*x+c)^4)^2, x$

[Out]
$$\begin{aligned}
& -1/2/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)/a*\tanh(1/2*d*x+1/2*c)^6*b-3/2/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*\tanh(1/2*d*x+1/2*c)^4*b+4/d*b^2/a^2/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*\tanh(1/2*d*x+1/2*c)^4+5/2/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*\tanh(1/2*d*x+1/2*c)^2*b-1/2/d*b/a/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)-5/8/d/(a-b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+1/2/d*b/a^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+1/8/d/(a-b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*b-5/8/d/(a-b)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)
\end{aligned}$$

$$\begin{aligned} & /2) * a)^{(1/2)} * \arctan(1/4 * (2 * \tanh(1/2 * d * x + 1/2 * c))^2 * a + 4 * (a * b)^{(1/2)} - 2 * a) / (-a * b \\ & + (a * b)^{(1/2)} * a)^{(1/2)} * (a * b)^{(1/2)} + 1/2 / d * b / a^2 / (a - b) / (-a * b + (a * b)^{(1/2)} * a)^{(1/2)} \\ & /2) * \arctan(1/4 * (2 * \tanh(1/2 * d * x + 1/2 * c))^2 * a + 4 * (a * b)^{(1/2)} - 2 * a) / (-a * b + (a * b)^{(1/2)} * a)^{(1/2)} \\ & /2) * a)^{(1/2)} * (a * b)^{(1/2)} - 1/8 / d / (a - b) / a / (-a * b + (a * b)^{(1/2)} * a)^{(1/2)} * \arctan(\\ & 1/4 * (2 * \tanh(1/2 * d * x + 1/2 * c))^2 * a + 4 * (a * b)^{(1/2)} - 2 * a) / (-a * b + (a * b)^{(1/2)} * a)^{(1/2)} \\ &) * b + 1/d / a^2 * \ln(\tanh(1/2 * d * x + 1/2 * c)) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{be^{(7dx+7c)} - 5be^{(5dx+5c)} - 5be^{(3dx+3c)} + be^{(dx+c)}}{2(a^2bd - ab^2d + (a^2bde^{(8c)} - ab^2de^{(8c)})e^{(8dx)} - 4(a^2bde^{(6c)} - ab^2de^{(6c)})e^{(6dx)} - 2(8a^3de^{(4c)} - 11a^2bde^{(4c)} + 3ab^2de^{(4c)})e^{(4dx)} - 2(a^2bde^{(2c)} - ab^2de^{(2c)})e^{(2dx)} - \log((e^{(dx+c)} + 1)e^{(-c)}) / (a^2*d) + \log((e^{(dx+c)} - 1)e^{(-c)}) / (a^2*d) - 2 \int \frac{1}{4} ((5a * b * e^{(7c)} - 4b^2 * e^{(7c)}) * e^{(7dx)} - (19a * b * e^{(5c)} - 12b^2 * e^{(5c)}) * e^{(5dx)} + (19a * b * e^{(3c)} - 12b^2 * e^{(3c)}) * e^{(3dx)} - (5a * b * e^c - 4b^2 * e^c) * e^{(dx)}) / (a^3 * b - a^2 * b^2 + (a^3 * b * e^{(8c)} - a^2 * b^2 * e^{(8c)}) * e^{(8dx)} - 4(a^3 * b * e^{(6c)} - a^2 * b^2 * e^{(6c)}) * e^{(6dx)} - 2(8a^4 * e^{(4c)} - 11a^3 * b * e^{(4c)} + 3a^2 * b^2 * e^{(4c)}) * e^{(4dx)} - 4(a^3 * b * e^{(2c)} - a^2 * b^2 * e^{(2c)}) * e^{(2dx)}) , x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out] -1/2*(b*e^(7*d*x + 7*c) - 5*b*e^(5*d*x + 5*c) - 5*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/4*((5*a*b*e^(7*c) - 4*b^2*e^(7*c))*e^(7*d*x) - (19*a*b*e^(5*c) - 12*b^2*e^(5*c))*e^(5*d*x) + (19*a*b*e^(3*c) - 12*b^2*e^(3*c))*e^(3*d*x) - (5*a*b*e^c - 4*b^2*e^c)*e^(d*x))/(a^3*b - a^2*b^2 + (a^3*b*e^(8*c) - a^2*b^2*e^(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c) - a^2*b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^4*e^(4*c) - 11*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x), x)

Fricas [B] time = 4.22233, size = 17785, normalized size = 54.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out] -1/16*(8*a*b*cosh(d*x + c)^7 + 56*a*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*a*b*sinh(d*x + c)^7 - 40*a*b*cosh(d*x + c)^5 + 8*(21*a*b*cosh(d*x + c)^2 - 5*a

$$\begin{aligned}
& *b) * \sinh(dx + c)^5 - 40*a*b * \cosh(dx + c)^3 + 40*(7*a*b * \cosh(dx + c)^3 - \\
& 5*a*b * \cosh(dx + c)) * \sinh(dx + c)^4 + 40*(7*a*b * \cosh(dx + c)^4 - 10*a*b * \cosh(dx + c)^2 - a*b) * \sinh(dx + c)^3 + 8*a*b * \cosh(dx + c) + 8*(21*a*b * \cosh(dx + c)^5 - 50*a*b * \cosh(dx + c)^3 - 15*a*b * \cosh(dx + c)) * \sinh(dx + c)^2 + ((a^3*b - a^2*b^2) * d * \cosh(dx + c)^8 + 8*(a^3*b - a^2*b^2) * d * \cosh(dx + c) * \sinh(dx + c)^7 + (a^3*b - a^2*b^2) * d * \sinh(dx + c)^8 - 4*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^6 + 4*(7*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^2 - (a^3*b - a^2*b^2) * d) * \sinh(dx + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2) * d * \cosh(dx + c)^4 + 8*(7*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^3 - 3*(a^3*b - a^2*b^2) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(35*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^4 - 30*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*b^2) * d) * \sinh(dx + c)^4 - 4*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^2 + 8*(7*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^5 - 10*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^3 - (8*a^4 - 11*a^3*b + 3*a^2*b^2) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(7*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^6 - 15*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^4 - 3*(8*a^4 - 11*a^3*b + 3*a^2*b^2) * d * \cosh(dx + c)^2 - (a^3*b - a^2*b^2) * d) * \sinh(dx + c)^2 + (a^3*b - a^2*b^2) * d + 8*((a^3*b - a^2*b^2) * d * \cosh(dx + c)^7 - 3*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2) * d * \cosh(dx + c)^3 - (a^3*b - a^2*b^2) * d * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) * d^2 * \sqrt{(625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5) / ((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6) * d^4))} + 35*a^2*b - 47*a*b^2 + 16*b^3) / ((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) * d^2) * \log(-625*a^3*b + 1125*a^2*b^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4) * \cosh(dx + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4) * \cosh(dx + c) * \sinh(dx + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4) * \sinh(dx + c)^2 + 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4) * d * \cosh(dx + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4) * d * \sinh(dx + c) - ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4) * d^3 * \cosh(dx + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4) * d^3 * \sinh(dx + c)) * \sqrt{(625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5) / ((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6) * d^4))} * \sqrt{-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) * d^2 * \sqrt{(625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5) / ((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6) * d^4))} + 35*a^2*b - 47*a*b^2 + 16*b^3) / ((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3) * d^2))) - ((a^3*b - a^2*b^2) * d * \cosh(dx + c)^8 + 8*(a^3*b - a^2*b^2) * d * \cosh(dx + c) * \sinh(dx + c)^7 + (a^3*b - a^2*b^2) * d * \sinh(dx + c)^8 - 4*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^6 + 4*(7*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^2 - (a^3*b - a^2*b^2) * d) * \sinh(dx + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2) * d * \cosh(dx + c)^4 + 8*(7*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^3 - 3*(a^3*b - a^2*b^2) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(35*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^4 - 30*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*b^2) * d) * \sinh(dx + c)^4 - 4*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^2 + 8*(7*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^5 - 10*(a^3*b - a^2*b^2) * d * \cosh(dx + c)^3 -
\end{aligned}$$

$$\begin{aligned}
& (8a^4 - 11a^3b + 3a^2b^2) * d * \cosh(dx + c) * \sinh(dx + c)^3 + 4 * (7 * (a^3b - a^2b^2) * d * \cosh(dx + c)^6 - 15 * (a^3b - a^2b^2) * d * \cosh(dx + c)^4 - \\
& 3 * (8a^4 - 11a^3b + 3a^2b^2) * d * \cosh(dx + c)^2 - (a^3b - a^2b^2) * d) * \sinh(dx + c)^2 + (a^3b - a^2b^2) * d + 8 * ((a^3b - a^2b^2) * d * \cosh(dx + c) \\
& ^7 - 3 * (a^3b - a^2b^2) * d * \cosh(dx + c)^5 - (8a^4 - 11a^3b + 3a^2b^2) * d * \cosh(dx + c)^3 - (a^3b - a^2b^2) * d * \cosh(dx + c)) * \sinh(dx + c) * \sqrt{ \\
& - ((a^7 - 3a^6b + 3a^5b^2 - a^4b^3) * d^2 * \sqrt{(625a^4b - 1450a^3b^2 + 1241a^2b^3 - 464ab^4 + 64b^5)} / ((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6) * d^4)) + 35a^2b - 47ab^2 + \\
& 16b^3} / ((a^7 - 3a^6b + 3a^5b^2 - a^4b^3) * d^2)) * \log(-625a^3b + 1125a^2b^2 - 664ab^3 + 128b^4 - (625a^3b - 1125a^2b^2 + 664ab^3 - 128b^4) * \cosh(dx + c)^2 - 2 * (625a^3b - 1125a^2b^2 + 664ab^3 - 128b^4) * \\
& \cosh(dx + c) * \sinh(dx + c) - (625a^3b - 1125a^2b^2 + 664ab^3 - 128b^4) * \sinh(dx + c)^2 - 2 * (2 * (75a^5b - 137a^4b^2 + 82a^3b^3 - 16a^2b^4) * d * \cosh(dx + c) + 2 * (75a^5b - 137a^4b^2 + 82a^3b^3 - 16a^2b^4) * d * \\
& \sinh(dx + c) - ((5a^{10} - 18a^9b + 24a^8b^2 - 14a^7b^3 + 3a^6b^4) * d^3 * \cosh(dx + c) + (5a^{10} - 18a^9b + 24a^8b^2 - 14a^7b^3 + 3a^6b^4) * d^3 * \sinh(dx + c))) * \sqrt{(625a^4b - 1450a^3b^2 + 1241a^2b^3 - 464a \\
& ab^4 + 64b^5)} / ((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6) * d^4)) * \sqrt{-((a^7 - 3a^6b + 3a^5b^2 - a^4b^3) * \\
& d^2 * \sqrt{(625a^4b - 1450a^3b^2 + 1241a^2b^3 - 464ab^4 + 64b^5)} / ((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6) * d^4)) + 35a^2b - 47ab^2 + 16b^3} / ((a^7 - 3a^6b + 3a^5b^2 - a^4 \\
& * b^3) * d^2))) + ((a^3b - a^2b^2) * d * \cosh(dx + c)^8 + 8 * (a^3b - a^2b^2) * d * \cosh(dx + c) * \sinh(dx + c)^7 + (a^3b - a^2b^2) * d * \sinh(dx + c)^8 - 4 * (a \\
& ^3b - a^2b^2) * d * \cosh(dx + c)^6 + 4 * (7 * (a^3b - a^2b^2) * d * \cosh(dx + c)^2 - (a^3b - a^2b^2) * d) * \sinh(dx + c)^6 - 2 * (8a^4 - 11a^3b + 3a^2b^2) \\
& * d * \cosh(dx + c)^4 + 8 * (7 * (a^3b - a^2b^2) * d * \cosh(dx + c)^3 - 3 * (a^3b - a^2b^2) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 * (35 * (a^3b - a^2b^2) * d * \cosh(dx + c) \\
& ^4 - 30 * (a^3b - a^2b^2) * d * \cosh(dx + c)^2 - (8a^4 - 11a^3b + 3a^2b^2) * d) * \sinh(dx + c)^4 - 4 * (a^3b - a^2b^2) * d * \cosh(dx + c)^2 + 8 * (7 \\
& * (a^3b - a^2b^2) * d * \cosh(dx + c)^5 - 10 * (a^3b - a^2b^2) * d * \cosh(dx + c)^3 - (8a^4 - 11a^3b + 3a^2b^2) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4 * (7 \\
& * (a^3b - a^2b^2) * d * \cosh(dx + c)^6 - 15 * (a^3b - a^2b^2) * d * \cosh(dx + c)^4 - 3 * (8a^4 - 11a^3b + 3a^2b^2) * d * \cosh(dx + c)^2 - (a^3b - a^2b^2) \\
& * d) * \sinh(dx + c)^2 + (a^3b - a^2b^2) * d + 8 * ((a^3b - a^2b^2) * d * \cosh(dx + c)^7 - 3 * (a^3b - a^2b^2) * d * \cosh(dx + c)^5 - (8a^4 - 11a^3b + 3a^2 \\
& * b^2) * d * \cosh(dx + c)^3 - (a^3b - a^2b^2) * d * \cosh(dx + c)) * \sinh(dx + c) * \sqrt{((a^7 - 3a^6b + 3a^5b^2 - a^4b^3) * d^2 * \sqrt{(625a^4b - 1450a^3 \\
& * b^2 + 1241a^2b^3 - 464ab^4 + 64b^5)} / ((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6) * d^4)) - 35a^2b + 47ab^2 - 16b^3} / ((a^7 - 3a^6b + 3a^5b^2 - a^4b^3) * d^2)) * \log(-625a^3b + 1 \\
& 125a^2b^2 - 664ab^3 + 128b^4 - (625a^3b - 1125a^2b^2 + 664ab^3 - 128b^4) * \cosh(dx + c)^2 - 2 * (625a^3b - 1125a^2b^2 + 664ab^3 - 128b^4) * \cosh(dx + c) * \sinh(dx + c) - (625a^3b - 1125a^2b^2 + 664ab^3 - 1
\end{aligned}$$

$$\begin{aligned}
& 28*b^4)*\sinh(d*x + c)^2 + 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*\cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*\sinh(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*\cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*\sinh(d*x + c))*\sqrt{((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)))*\sqrt{((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*\sqrt{((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))} - ((a^3*b - a^2*b^2)*d*\cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*\sinh(d*x + c)^8 - 4*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*\sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^3 - 3*(a^3*b - a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^4 - 30*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^3 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*\cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*\sinh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*\cosh(d*x + c)^7 - 3*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*\cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*\sqrt{((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*\log(-625*a^3*b + 1125*a^2*b^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*\cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*\sinh(d*x + c)^2 - 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*\cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*\sinh(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*\cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*\sinh(d*x + c))*\sqrt{((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)))*\sqrt{((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*\sqrt{((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))} + 16*((a*b - b^2)*\cosh(d*x + c)^8 + 8*(a*b - b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b - b^2)*\sinh(d*x + c)^8 - 4*(a*b - b^2)*\cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*\cosh(d*x + c)^2 - a*b + b^2)*\sinh(d*x + c)^6
\end{aligned}$$

$$\begin{aligned}
& + 8*(7*(a*b - b^2)*\cosh(d*x + c)^3 - 3*(a*b - b^2)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^5 - 2*(8*a^2 - 11*a*b + 3*b^2)*\cosh(d*x + c)^4 + 2*(35*(a*b - b^2)*\cos \\
& h(d*x + c)^4 - 30*(a*b - b^2)*\cosh(d*x + c)^2 - 8*a^2 + 11*a*b - 3*b^2)*\sin \\
& h(d*x + c)^4 + 8*(7*(a*b - b^2)*\cosh(d*x + c)^5 - 10*(a*b - b^2)*\cosh(d*x + \\
& c)^3 - (8*a^2 - 11*a*b + 3*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(a*b - \\
& b^2)*\cosh(d*x + c)^2 + 4*(7*(a*b - b^2)*\cosh(d*x + c)^6 - 15*(a*b - b^2)*\co \\
& sh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*\cosh(d*x + c)^2 - a*b + b^2)*\sin \\
& h(d*x + c)^2 + a*b - b^2 + 8*((a*b - b^2)*\cosh(d*x + c)^7 - 3*(a*b - b^2)*\c \\
& osh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*\cosh(d*x + c)^3 - (a*b - b^2)*\cos \\
& h(d*x + c))*\sinh(d*x + c))*\log(\cosh(d*x + c) + \sinh(d*x + c) + 1) - 16*((a* \\
& b - b^2)*\cosh(d*x + c)^8 + 8*(a*b - b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a \\
& *b - b^2)*\sinh(d*x + c)^8 - 4*(a*b - b^2)*\cosh(d*x + c)^6 + 4*(7*(a*b - b^2 \\
&)*\cosh(d*x + c)^2 - a*b + b^2)*\sinh(d*x + c)^6 + 8*(7*(a*b - b^2)*\cosh(d*x \\
& + c)^3 - 3*(a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(8*a^2 - 11*a*b + \\
& 3*b^2)*\cosh(d*x + c)^4 + 2*(35*(a*b - b^2)*\cosh(d*x + c)^4 - 30*(a*b - b^2 \\
&)*\cosh(d*x + c)^2 - 8*a^2 + 11*a*b - 3*b^2)*\sinh(d*x + c)^4 + 8*(7*(a*b - b \\
& ^2)*\cosh(d*x + c)^5 - 10*(a*b - b^2)*\cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3* \\
& b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 4*(a*b - b^2)*\cosh(d*x + c)^2 + 4*(7* \\
& (a*b - b^2)*\cosh(d*x + c)^6 - 15*(a*b - b^2)*\cosh(d*x + c)^4 - 3*(8*a^2 - 1 \\
& 1*a*b + 3*b^2)*\cosh(d*x + c)^2 - a*b + b^2)*\sinh(d*x + c)^2 + a*b - b^2 + 8 \\
& *((a*b - b^2)*\cosh(d*x + c)^7 - 3*(a*b - b^2)*\cosh(d*x + c)^5 - (8*a^2 - 11 \\
& *a*b + 3*b^2)*\cosh(d*x + c)^3 - (a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\l \\
& og(\cosh(d*x + c) + \sinh(d*x + c) - 1) + 8*(7*a*b*\cosh(d*x + c)^6 - 25*a*b*\c \\
& osh(d*x + c)^4 - 15*a*b*\cosh(d*x + c)^2 + a*b)*\sinh(d*x + c))/((a^3*b - a^2 \\
& *b^2)*d*\cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c) \\
& ^7 + (a^3*b - a^2*b^2)*d*\sinh(d*x + c)^8 - 4*(a^3*b - a^2*b^2)*d*\cosh(d*x + \\
& c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*\sin \\
& h(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a \\
& ^3*b - a^2*b^2)*d*\cosh(d*x + c)^3 - 3*(a^3*b - a^2*b^2)*d*\cosh(d*x + c))*\si \\
& nh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^4 - 30*(a^3*b - a^2 \\
& *b^2)*d*\cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d)*\sinh(d*x + c)^4 \\
& - 4*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^3*b - a^2*b^2)*d*\cosh(d* \\
& x + c)^5 - 10*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^3 - (8*a^4 - 11*a^3*b + 3*a \\
& ^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^3*b - a^2*b^2)*d*\cosh(d* \\
& x + c)^6 - 15*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^4 - 11*a^3*b + 3 \\
& *a^2*b^2)*d*\cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*\sinh(d*x + c)^2 + (a^3*b \\
& - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*\cosh(d*x + c)^7 - 3*(a^3*b - a^2*b^2 \\
&)*d*\cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*\cosh(d*x + c)^3 - (a \\
& ^3*b - a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.247 \quad \int \frac{\sinh^8(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$$

Optimal. Leaf size=320

$$\frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d(\sqrt{a}-\sqrt{b})^{3/2}} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d(\sqrt{a}+\sqrt{b})^{3/2}} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}$$

[Out] x/b^2 - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^2*d) - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tanh[c + d*x]/(4*(a - b)*b*d) + Tanh[c + d*x]^5/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rubi [A] time = 0.461176, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3217, 1313, 1275, 12, 1122, 1166, 208, 1287, 207}

$$\frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d(\sqrt{a}-\sqrt{b})^{3/2}} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d(\sqrt{a}+\sqrt{b})^{3/2}} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] x/b^2 - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^2*d) - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tanh[c + d*x]/(4*(a - b)*b*d) + Tanh[c + d*x]^5/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] &&
IntegerQ[m/2] && IntegerQ[p]
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] := -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1
))/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:= Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1287

Int[(((f_.)*(x_)^(m_.))*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[q] && IntegerQ[m]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^8}{(1-x^2)(a-2ax^2+(a-b)x^4)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^4(a-ax^2)}{(a-2ax^2+(a-b)x^4)^2} dx, x, \tanh(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a-2ax^2+(a-b)x^4)} dx, x, \tanh(c+dx)\right)}{bd} \\
&= \frac{\tanh^5(c+dx)}{4bd(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} + \frac{\text{Subst}\left(\int -\frac{2abx^4}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{8ab^2d} \\
&= \frac{\tanh^5(c+dx)}{4bd(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \tanh(c+dx)\right)}{b^2d} \\
&= \frac{x}{b^2} - \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\tanh^5(c+dx)}{4bd(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} + \frac{(\sqrt{a}(\sqrt{a}+\sqrt{b}))}{b^2} \\
&= \frac{x}{b^2} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^2d} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^2d} - \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{(\sqrt{a}(\sqrt{a}+\sqrt{b}))}{b^2} \\
&= \frac{x}{b^2} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^2d} + \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8(\sqrt{a}-\sqrt{b})^{3/2}b^{3/2}d} - \frac{\sqrt[4]{a} \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^2d} + \frac{(\sqrt{a}(\sqrt{a}+\sqrt{b}))}{b^2}
\end{aligned}$$

Mathematica [A] time = 4.6367, size = 262, normalized size = 0.82

$$\frac{\frac{\sqrt{a}(4\sqrt{a}+5\sqrt{b}) \tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{(\sqrt{a}+\sqrt{b})\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{a}(4\sqrt{a}-5\sqrt{b}) \tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{(\sqrt{a}-\sqrt{b})\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{2ab(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{(a-b)(8a+4b \cosh(2(c+dx))-b \cosh(4(c+dx))-3b)} + 8(c+dx)}{8b^2d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] (8*(c + d*x) + (Sqrt[a]*(4*Sqrt[a] - 5*Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b]) * Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])]/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b])) - (Sqrt[a]*(4*Sqrt[a] + 5*Sqrt[b])*ArcTanh[((Sqrt[a] + S

$$\frac{\text{qrt}[b]) * \text{Tanh}[c + d*x] / \text{Sqrt}[a + \text{Sqrt}[a] * \text{Sqrt}[b]]}{((\text{Sqrt}[a] + \text{Sqrt}[b]) * \text{Sqrt}[a + \text{Sqrt}[a] * \text{Sqrt}[b]]) + (2*a*b*(-6*\text{Sinh}[2*(c + d*x)] + \text{Sinh}[4*(c + d*x)])} / ((a - b)*(8*a - 3*b + 4*b*\text{Cosh}[2*(c + d*x)] - b*\text{Cosh}[4*(c + d*x)])) / (8*b^2*d)$$

Maple [C] time = 0.063, size = 574, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x)`

[Out] $\frac{1}{d*b^2} \ln(\tanh(1/2*d*x+1/2*c)+1) - 1/2/d*a/b / (\tanh(1/2*d*x+1/2*c)^{8*a-4} \tanh(1/2*d*x+1/2*c)^{6*a+6} \tanh(1/2*d*x+1/2*c)^{4*a-16} b \tanh(1/2*d*x+1/2*c)^{4-4} \tanh(1/2*d*x+1/2*c)^{2*a+a} / (a-b) \tanh(1/2*d*x+1/2*c)^{7+5/2} / d*a/b / (\tanh(1/2*d*x+1/2*c)^{8*a-4} \tanh(1/2*d*x+1/2*c)^{6*a+6} \tanh(1/2*d*x+1/2*c)^{4*a-16} b \tanh(1/2*d*x+1/2*c)^{4-4} \tanh(1/2*d*x+1/2*c)^{2*a+a} / (a-b) \tanh(1/2*d*x+1/2*c)^5 + 5/2/d*a/b / (\tanh(1/2*d*x+1/2*c)^{8*a-4} \tanh(1/2*d*x+1/2*c)^{6*a+6} \tanh(1/2*d*x+1/2*c)^{4*a-16} b \tanh(1/2*d*x+1/2*c)^{4-4} \tanh(1/2*d*x+1/2*c)^{2*a+a} / (a-b) \tanh(1/2*d*x+1/2*c)^3 - 1/2/d*a/b / (\tanh(1/2*d*x+1/2*c)^{8*a-4} \tanh(1/2*d*x+1/2*c)^{6*a+6} \tanh(1/2*d*x+1/2*c)^{4*a-16} b \tanh(1/2*d*x+1/2*c)^{4-4} \tanh(1/2*d*x+1/2*c)^{2*a+a} / (a-b) \tanh(1/2*d*x+1/2*c) + 1/16/d*a/b^2 / (a-b) \sum((4*a-5*b) * _R^6 + (-12*a+19*b) * _R^4 + (12*a-19*b) * _R^2 - 4*a+5*b) / (_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a) * \ln(\tanh(1/2*d*x+1/2*c)-_R), _R = \text{RootOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a)) - 1/d/b^2 * \ln(\tanh(1/2*d*x+1/2*c)-1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")`

[Out] $\frac{1}{2} * (2*(a*b*d*e^{(8*c)} - b^2*d*e^{(8*c)}) * x * e^{(8*d*x)} + a*b + 2*(a*b*d - b^2*d) * x + (a*b*e^{(6*c)} - 8*(a*b*d*e^{(6*c)} - b^2*d*e^{(6*c)}) * x) * e^{(6*d*x)} - (8*a^2 * e^{(4*c)} - 3*a*b * e^{(4*c)} + 4*(8*a^2*d * e^{(4*c)} - 11*a*b*d * e^{(4*c)} + 3*b^2*d * e^{(4*c)}) * x) * e^{(4*d*x)} - (5*a*b * e^{(2*c)} + 8*(a*b*d * e^{(2*c)} - b^2*d * e^{(2*c)}))$

```

*x)*e^(2*d*x))/(a*b^3*d - b^4*d + (a*b^3*d*e^(8*c) - b^4*d*e^(8*c))*e^(8*d*
x) - 4*(a*b^3*d*e^(6*c) - b^4*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*d*e^(4*c)
- 11*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) - 4*(a*b^3*d*e^(2*c) - b
^4*d*e^(2*c))*e^(2*d*x)) + 1/256*integrate(256*(a*b*e^(6*d*x + 6*c) + a*b*e
^(2*d*x + 2*c) + 2*(8*a^2*e^(4*c) - 11*a*b*e^(4*c))*e^(4*d*x))/(a*b^3 - b^4
+ (a*b^3*e^(8*c) - b^4*e^(8*c))*e^(8*d*x) - 4*(a*b^3*e^(6*c) - b^4*e^(6*c)
)*e^(6*d*x) - 2*(8*a^2*b^2*e^(4*c) - 11*a*b^3*e^(4*c) + 3*b^4*e^(4*c))*e^(4
*d*x) - 4*(a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)

```

Fricas [B] time = 3.8798, size = 15757, normalized size = 49.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*(a*b - b^2)*d*x*cosh(d*x + c)^8 + 128*(a*b - b^2)*d*x*cosh(d*x + c
)*sinh(d*x + c)^7 + 16*(a*b - b^2)*d*x*sinh(d*x + c)^8 - 8*(8*(a*b - b^2)*d
*x - a*b)*cosh(d*x + c)^6 + 8*(56*(a*b - b^2)*d*x*cosh(d*x + c)^2 - 8*(a*b
- b^2)*d*x + a*b)*sinh(d*x + c)^6 + 16*(56*(a*b - b^2)*d*x*cosh(d*x + c)^3
- 3*(8*(a*b - b^2)*d*x - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(4*(8*a^2
- 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x + c)^4 + 8*(140*(a*b - b^2)
*d*x*cosh(d*x + c)^4 - 4*(8*a^2 - 11*a*b + 3*b^2)*d*x - 15*(8*(a*b - b^2)*d
*x - a*b)*cosh(d*x + c)^2 - 8*a^2 + 3*a*b)*sinh(d*x + c)^4 + 32*(28*(a*b -
b^2)*d*x*cosh(d*x + c)^5 - 5*(8*(a*b - b^2)*d*x - a*b)*cosh(d*x + c)^3 - (4
*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x + c))*sinh(d*x + c)
^3 + 16*(a*b - b^2)*d*x - 8*(8*(a*b - b^2)*d*x + 5*a*b)*cosh(d*x + c)^2 + 8
*(56*(a*b - b^2)*d*x*cosh(d*x + c)^6 - 15*(8*(a*b - b^2)*d*x - a*b)*cosh(d*
x + c)^4 - 8*(a*b - b^2)*d*x - 6*(4*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 -
3*a*b)*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^2 + ((a*b^3 - b^4)*d*cosh(d*x
+ c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^3 - b^4)*d
*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d
*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 3*(a
*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*cosh(d
*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*
b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + 8*(7*(a*b^3 -
b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (8*a^2*b^2 -
11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*
cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*
b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a*b^3

```

$$\begin{aligned}
& - b^4) * d + 8 * ((a * b^3 - b^4) * d * \cosh(d * x + c)^7 - 3 * (a * b^3 - b^4) * d * \cosh(d * x \\
& + c)^5 - (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^3 - (a * b^3 - b^4) * d \\
& * \cosh(d * x + c)) * \sinh(d * x + c)) * \sqrt{-((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) \\
& * d^2 * \sqrt{(64 * a^5 - 464 * a^4 * b + 1241 * a^3 * b^2 - 1450 * a^2 * b^3 + 625 * a * b^4) / ((\\
& a^6 * b^7 - 6 * a^5 * b^8 + 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 * a^2 * b^{11} - 6 * a * b^{12} + b \\
& ^{13}) * d^4)) - 16 * a^3 + 47 * a^2 * b - 35 * a * b^2) / ((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - \\
& - b^7) * d^2)) * \log(2 * (16 * a^4 * b^3 - 73 * a^3 * b^4 + 123 * a^2 * b^5 - 91 * a * b^6 + 25 * b^7) \\
& * d^2 * \sqrt{(64 * a^5 - 464 * a^4 * b + 1241 * a^3 * b^2 - 1450 * a^2 * b^3 + 625 * a * b^4) / ((\\
& a^6 * b^7 - 6 * a^5 * b^8 + 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 * a^2 * b^{11} - 6 * a * b^{12} \\
& + b^{13}) * d^4)) + 128 * a^3 - 664 * a^2 * b + 1125 * a * b^2 - 625 * b^3 - (128 * a^3 - 664 \\
& * a^2 * b + 1125 * a * b^2 - 625 * b^3) * \cosh(d * x + c)^2 - 2 * (128 * a^3 - 664 * a^2 * b + 1 \\
& 125 * a * b^2 - 625 * b^3) * \cosh(d * x + c) * \sinh(d * x + c) - (128 * a^3 - 664 * a^2 * b + 1 \\
& 125 * a * b^2 - 625 * b^3) * \sinh(d * x + c)^2 + 2 * (2 * (2 * a^4 * b^5 - 9 * a^3 * b^6 + 15 * a^2 \\
& * b^7 - 11 * a * b^8 + 3 * b^9) * d^3 * \sqrt{(64 * a^5 - 464 * a^4 * b + 1241 * a^3 * b^2 - 1450 \\
& * a^2 * b^3 + 625 * a * b^4) / ((a^6 * b^7 - 6 * a^5 * b^8 + 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 \\
& * a^2 * b^{11} - 6 * a * b^{12} + b^{13}) * d^4)) + (24 * a^3 * b^2 - 127 * a^2 * b^3 + 220 * a * b^4 \\
& - 125 * b^5) * d) * \sqrt{-((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2 * \sqrt{(64 * a^5 \\
& - 464 * a^4 * b + 1241 * a^3 * b^2 - 1450 * a^2 * b^3 + 625 * a * b^4) / ((a^6 * b^7 - 6 * a^5 * b^8 \\
& + 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 * a^2 * b^{11} - 6 * a * b^{12} + b^{13}) * d^4)) - 16 * a \\
& ^3 + 47 * a^2 * b - 35 * a * b^2) / ((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2)) - (\\
& (a * b^3 - b^4) * d * \cosh(d * x + c)^8 + 8 * (a * b^3 - b^4) * d * \cosh(d * x + c) * \sinh(d * x \\
& + c)^7 + (a * b^3 - b^4) * d * \sinh(d * x + c)^8 - 4 * (a * b^3 - b^4) * d * \cosh(d * x + c)^6 \\
& + 4 * (7 * (a * b^3 - b^4) * d * \cosh(d * x + c)^2 - (a * b^3 - b^4) * d) * \sinh(d * x + c)^6 \\
& - 2 * (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^4 + 8 * (7 * (a * b^3 - b^4) * \\
& d * \cosh(d * x + c)^3 - 3 * (a * b^3 - b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)^5 + 2 * (3 \\
& 5 * (a * b^3 - b^4) * d * \cosh(d * x + c)^4 - 30 * (a * b^3 - b^4) * d * \cosh(d * x + c)^2 - (8 \\
& * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d) * \sinh(d * x + c)^4 - 4 * (a * b^3 - b^4) * d * \cosh(d * \\
& x + c)^2 + 8 * (7 * (a * b^3 - b^4) * d * \cosh(d * x + c)^5 - 10 * (a * b^3 - b^4) * d * \cosh(d \\
& * x + c)^3 - (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)^3 \\
& + 4 * (7 * (a * b^3 - b^4) * d * \cosh(d * x + c)^6 - 15 * (a * b^3 - b^4) * d * \cosh(d * x + c)^ \\
& 4 - 3 * (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^2 - (a * b^3 - b^4) * d) * \sinh \\
& (d * x + c)^2 + (a * b^3 - b^4) * d + 8 * ((a * b^3 - b^4) * d * \cosh(d * x + c)^7 - 3 * (\\
& a * b^3 - b^4) * d * \cosh(d * x + c)^5 - (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x \\
& + c)^3 - (a * b^3 - b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)) * \sqrt{-((a^3 * b^4 - 3 * \\
& a^2 * b^5 + 3 * a * b^6 - b^7) * d^2 * \sqrt{(64 * a^5 - 464 * a^4 * b + 1241 * a^3 * b^2 - 1450 \\
& * a^2 * b^3 + 625 * a * b^4) / ((a^6 * b^7 - 6 * a^5 * b^8 + 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 \\
& * a^2 * b^{11} - 6 * a * b^{12} + b^{13}) * d^4)) - 16 * a^3 + 47 * a^2 * b - 35 * a * b^2) / ((a^3 * b^4 \\
& - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2)) * \log(2 * (16 * a^4 * b^3 - 73 * a^3 * b^4 + 123 * a^2 * b^5 - 91 * a * b^6 + 25 * b^7) \\
& * d^2 * \sqrt{(64 * a^5 - 464 * a^4 * b + 1241 * a^3 * b^2 - 1450 * a^2 * b^3 + 625 * a * b^4) / ((\\
& a^6 * b^7 - 6 * a^5 * b^8 + 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 * a^2 * b^{11} - 6 * a * b^{12} + b^{13}) * d^4)) + 128 * a^3 - 664 * a^2 * b + 1125 * a * b^2 - \\
& 625 * b^3 - (128 * a^3 - 664 * a^2 * b + 1125 * a * b^2 - 625 * b^3) * \cosh(d * x + c)^2 - 2 * \\
& (128 * a^3 - 664 * a^2 * b + 1125 * a * b^2 - 625 * b^3) * \cosh(d * x + c) * \sinh(d * x + c) - \\
& (128 * a^3 - 664 * a^2 * b + 1125 * a * b^2 - 625 * b^3) * \sinh(d * x + c)^2 - 2 * (2 * (2 * a^4 * \\
& b^5 - 9 * a^3 * b^6 + 15 * a^2 * b^7 - 11 * a * b^8 + 3 * b^9) * d^3 * \sqrt{(64 * a^5 - 464 * a^4
\end{aligned}$$

$$\begin{aligned}
& *b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^{10} + 15*a^2*b^{11} - 6*a*b^{12} + b^{13})*d^4)) + (24*a^3*b^2 - \\
& 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d)*\sqrt{-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*\sqrt{(64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a* \\
& b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^{10} + 15*a^2*b^{11} - 6*a*b^{12} + b^{13})*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3 \\
& *a*b^6 - b^7)*d^2))) - ((a*b^3 - b^4)*d*\cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d \\
& *\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^3 - b^4)*d*\sinh(d*x + c)^8 - 4*(a*b^3 \\
& - b^4)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d*\cosh(d*x + c)^2 - (a*b^3 - \\
& b^4)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*\cosh(d*x + c) \\
& ^4 + 8*(7*(a*b^3 - b^4)*d*\cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*\cosh(d*x + c)^4 - 30*(a*b^3 - b^4 \\
&)*d*\cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*\sinh(d*x + c)^4 - 4 \\
& *(a*b^3 - b^4)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^3 - b^4)*d*\cosh(d*x + c)^5 - 1 \\
& 0*(a*b^3 - b^4)*d*\cosh(d*x + c)^3 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*\cosh(d \\
& *x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*\cosh(d*x + c)^6 - 15*(a*b^3 \\
& - b^4)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*\cosh(d*x + c \\
&)^2 - (a*b^3 - b^4)*d)*\sinh(d*x + c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4) \\
& *d*\cosh(d*x + c)^7 - 3*(a*b^3 - b^4)*d*\cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a* \\
& b^3 + 3*b^4)*d*\cosh(d*x + c)^3 - (a*b^3 - b^4)*d*\cosh(d*x + c))*\sinh(d*x + \\
& c))*\sqrt{((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*\sqrt{(64*a^5 - 464*a^4* \\
& b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4 \\
& *b^9 - 20*a^3*b^{10} + 15*a^2*b^{11} - 6*a*b^{12} + b^{13})*d^4)) + 16*a^3 - 47*a^2 \\
& *b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*\log(-2*(16*a^4* \\
& b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*\sqrt{(64*a^5 - 464* \\
& a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15 \\
& *a^4*b^9 - 20*a^3*b^{10} + 15*a^2*b^{11} - 6*a*b^{12} + b^{13})*d^4)) + 128*a^3 - 6 \\
& 64*a^2*b + 1125*a*b^2 - 625*b^3 - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b \\
& ^3)*\cosh(d*x + c)^2 - 2*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*\cosh(d \\
& *x + c)*\sinh(d*x + c) - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*\sinh(d \\
& *x + c)^2 + 2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^ \\
& 3*\sqrt{(64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6 \\
& *b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^{10} + 15*a^2*b^{11} - 6*a*b^{12} + b^{13} \\
&)*d^4)) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d)*\sqrt{((a^3*b^ \\
& 4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*\sqrt{(64*a^5 - 464*a^4*b + 1241*a^3*b^2 \\
& - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^{1 \\
& 0 + 15*a^2*b^{11} - 6*a*b^{12} + b^{13})*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((\\
& a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + ((a*b^3 - b^4)*d*\cosh(d*x + c \\
&)^8 + 8*(a*b^3 - b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^3 - b^4)*d*\sin \\
& h(d*x + c)^8 - 4*(a*b^3 - b^4)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d*cos \\
& h(d*x + c)^2 - (a*b^3 - b^4)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b^3 + \\
& 3*b^4)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*\cosh(d*x + c)^3 - 3*(a*b^3 \\
& - b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*\cosh(d*x + \\
& c)^4 - 30*(a*b^3 - b^4)*d*\cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4) \\
& *d)*\sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^3 - b^4)
\end{aligned}$$

$$\begin{aligned}
&) * d * \cosh(d * x + c)^5 - 10 * (a * b^3 - b^4) * d * \cosh(d * x + c)^3 - (8 * a^2 * b^2 - 11 * \\
& a * b^3 + 3 * b^4) * d * \cosh(d * x + c) * \sinh(d * x + c)^3 + 4 * (7 * (a * b^3 - b^4) * d * \cosh \\
& (d * x + c)^6 - 15 * (a * b^3 - b^4) * d * \cosh(d * x + c)^4 - 3 * (8 * a^2 * b^2 - 11 * a * b^3 \\
& + 3 * b^4) * d * \cosh(d * x + c)^2 - (a * b^3 - b^4) * d) * \sinh(d * x + c)^2 + (a * b^3 - b^4) \\
& * d + 8 * ((a * b^3 - b^4) * d * \cosh(d * x + c)^7 - 3 * (a * b^3 - b^4) * d * \cosh(d * x + c) \\
& ^5 - (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^3 - (a * b^3 - b^4) * d * \cosh \\
& (d * x + c)) * \sinh(d * x + c)) * \sqrt{((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2 * \\
& \sqrt{(64 * a^5 - 464 * a^4 * b + 1241 * a^3 * b^2 - 1450 * a^2 * b^3 + 625 * a * b^4) / ((a^6 * b \\
& ^7 - 6 * a^5 * b^8 + 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 * a^2 * b^{11} - 6 * a * b^{12} + b^{13}) * \\
& d^4)) + 16 * a^3 - 47 * a^2 * b + 35 * a * b^2) / ((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) \\
& * d^2)) * \log(-2 * (16 * a^4 * b^3 - 73 * a^3 * b^4 + 123 * a^2 * b^5 - 91 * a * b^6 + 25 * b^7) * \\
& d^2 * \sqrt{(64 * a^5 - 464 * a^4 * b + 1241 * a^3 * b^2 - 1450 * a^2 * b^3 + 625 * a * b^4) / ((a \\
& ^6 * b^7 - 6 * a^5 * b^8 + 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 * a^2 * b^{11} - 6 * a * b^{12} + b^{13}) * \\
& d^4)) + 128 * a^3 - 664 * a^2 * b + 1125 * a * b^2 - 625 * b^3 - (128 * a^3 - 664 * a^2 \\
& * b + 1125 * a * b^2 - 625 * b^3) * \cosh(d * x + c)^2 - 2 * (128 * a^3 - 664 * a^2 * b + 1125 * \\
& a * b^2 - 625 * b^3) * \cosh(d * x + c) * \sinh(d * x + c) - (128 * a^3 - 664 * a^2 * b + 1125 * \\
& a * b^2 - 625 * b^3) * \sinh(d * x + c)^2 - 2 * (2 * (2 * a^4 * b^5 - 9 * a^3 * b^6 + 15 * a^2 * b^7 \\
& - 11 * a * b^8 + 3 * b^9) * d^3 * \sqrt{(64 * a^5 - 464 * a^4 * b + 1241 * a^3 * b^2 - 1450 * a^2 \\
& * b^3 + 625 * a * b^4) / ((a^6 * b^7 - 6 * a^5 * b^8 + 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 * a^2 \\
& * b^{11} - 6 * a * b^{12} + b^{13}) * d^4)) - (24 * a^3 * b^2 - 127 * a^2 * b^3 + 220 * a * b^4 - 12 \\
& 5 * b^5) * d) * \sqrt{((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2 * \sqrt{(64 * a^5 - 46 \\
& 4 * a^4 * b + 1241 * a^3 * b^2 - 1450 * a^2 * b^3 + 625 * a * b^4) / ((a^6 * b^7 - 6 * a^5 * b^8 + \\
& 15 * a^4 * b^9 - 20 * a^3 * b^{10} + 15 * a^2 * b^{11} - 6 * a * b^{12} + b^{13}) * d^4)) + 16 * a^3 - \\
& 47 * a^2 * b + 35 * a * b^2) / ((a^3 * b^4 - 3 * a^2 * b^5 + 3 * a * b^6 - b^7) * d^2)) + 8 * a * b \\
& + 16 * (8 * (a * b - b^2) * d * x * \cosh(d * x + c)^7 - 3 * (8 * (a * b - b^2) * d * x - a * b) * \cosh(\\
& d * x + c)^5 - 2 * (4 * (8 * a^2 - 11 * a * b + 3 * b^2) * d * x + 8 * a^2 - 3 * a * b) * \cosh(d * x + \\
& c)^3 - (8 * (a * b - b^2) * d * x + 5 * a * b) * \cosh(d * x + c)) * \sinh(d * x + c)) / ((a * b^3 - \\
& b^4) * d * \cosh(d * x + c)^8 + 8 * (a * b^3 - b^4) * d * \cosh(d * x + c) * \sinh(d * x + c)^7 + \\
& (a * b^3 - b^4) * d * \sinh(d * x + c)^8 - 4 * (a * b^3 - b^4) * d * \cosh(d * x + c)^6 + 4 * (7 * \\
& (a * b^3 - b^4) * d * \cosh(d * x + c)^2 - (a * b^3 - b^4) * d) * \sinh(d * x + c)^6 - 2 * (8 * a \\
& ^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^4 + 8 * (7 * (a * b^3 - b^4) * d * \cosh(d * \\
& x + c)^3 - 3 * (a * b^3 - b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)^5 + 2 * (35 * (a * b^3 \\
& - b^4) * d * \cosh(d * x + c)^4 - 30 * (a * b^3 - b^4) * d * \cosh(d * x + c)^2 - (8 * a^2 * b^2 \\
& - 11 * a * b^3 + 3 * b^4) * d) * \sinh(d * x + c)^4 - 4 * (a * b^3 - b^4) * d * \cosh(d * x + c)^2 \\
& + 8 * (7 * (a * b^3 - b^4) * d * \cosh(d * x + c)^5 - 10 * (a * b^3 - b^4) * d * \cosh(d * x + c)^3 \\
& - (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c)^3 + 4 * (7 * (\\
& a * b^3 - b^4) * d * \cosh(d * x + c)^6 - 15 * (a * b^3 - b^4) * d * \cosh(d * x + c)^4 - 3 * (8 * \\
& a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^2 - (a * b^3 - b^4) * d) * \sinh(d * x + \\
& c)^2 + (a * b^3 - b^4) * d + 8 * ((a * b^3 - b^4) * d * \cosh(d * x + c)^7 - 3 * (a * b^3 - b \\
& ^4) * d * \cosh(d * x + c)^5 - (8 * a^2 * b^2 - 11 * a * b^3 + 3 * b^4) * d * \cosh(d * x + c)^3 - \\
& (a * b^3 - b^4) * d * \cosh(d * x + c)) * \sinh(d * x + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [A] time = 1.19348, size = 201, normalized size = 0.63

$$\frac{abe^{(6dx+6c)} - 8a^2e^{(4dx+4c)} + 3abe^{(4dx+4c)} - 5abe^{(2dx+2c)} + ab}{2(ab^2d - b^3d)(be^{(8dx+8c)} - 4be^{(6dx+6c)} - 16ae^{(4dx+4c)} + 6be^{(4dx+4c)} - 4be^{(2dx+2c)} + b)} + \frac{dx+c}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] 1/2*(a*b*e^(6*d*x + 6*c) - 8*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) - 5*a*b*e^(2*d*x + 2*c) + a*b)/((a*b^2*d - b^3*d)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)) + (d*x + c)/(b^2*d)

$$3.248 \quad \int \frac{\sinh^6(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$$

Optimal. Leaf size=233

$$\frac{(2\sqrt{a} - 3\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{ab^3}d(\sqrt{a} - \sqrt{b})^{3/2}} - \frac{(2\sqrt{a} + 3\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{ab^3}d(\sqrt{a} + \sqrt{b})^{3/2}} + \frac{\tanh(c+dx)}{4bd(a-b)} + \frac{\tanh(c+dx)}{4bd((a-b)\tanh(c+dx))}$$

[Out] ((2*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - ((2*Sqrt[a] + 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(1/4)*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) + Tanh[c + d*x]/(4*(a - b)*b*d) + (Sinh[c + d*x]^2*Tanh[c + d*x]^3)/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rubi [A] time = 0.337798, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3217, 1120, 1279, 1166, 208}

$$\frac{(2\sqrt{a} - 3\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{ab^3}d(\sqrt{a} - \sqrt{b})^{3/2}} - \frac{(2\sqrt{a} + 3\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{ab^3}d(\sqrt{a} + \sqrt{b})^{3/2}} + \frac{\tanh(c+dx)}{4bd(a-b)} + \frac{\tanh(c+dx)}{4bd((a-b)\tanh(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] ((2*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - ((2*Sqrt[a] + 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(1/4)*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) + Tanh[c + d*x]/(4*(a - b)*b*d) + (Sinh[c + d*x]^2*Tanh[c + d*x]^3)/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rule 3217

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1

)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1120

Int[((d_)*(x_))^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1279

Int[((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p + 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^6}{(a-2ax^2+(a-b)x^4)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\text{sech}^2(c+dx)\tanh^3(c+dx)}{4bd(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{x^2(6a-2ax^2)}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{8abd} \\
&= \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\text{sech}^2(c+dx)\tanh^3(c+dx)}{4bd(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} + \frac{\text{Subst}\left(\int \frac{-2a^2-2a(a-b)x^2}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{8abd} \\
&= \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\text{sech}^2(c+dx)\tanh^3(c+dx)}{4bd(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} - \frac{\left(a - \frac{2\sqrt{a}(a-2b)}{\sqrt{b}} - 3b\right)}{8abd} \\
&= \frac{(2\sqrt{a}-3\sqrt{b})\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}(\sqrt{a}-\sqrt{b})^{3/2}b^{3/2}d} - \frac{(2\sqrt{a}+3\sqrt{b})\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}(\sqrt{a}+\sqrt{b})^{3/2}b^{3/2}d} + \frac{3b}{8abd}
\end{aligned}$$

Mathematica [A] time = 2.58959, size = 238, normalized size = 1.02

$$\frac{\sqrt{b}(-\sqrt{a}\sqrt{b}-2a+3b)\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{b}(\sqrt{a}\sqrt{b}-2a+3b)\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} - \frac{4b\sinh(2(c+dx))(-2a+b\cosh(2(c+dx))-b)}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}$$

$$8b^2d(a-b)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] ((Sqrt[b]*(-2*a + Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])]/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*(-2*a - Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]])]/Sqrt[a + Sqrt[a]*Sqrt[b]] - (4*b*(-2*a - b + b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)]/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))/(8*(a - b)*b^2*d)

Maple [C] time = 0.054, size = 716, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x)`

[Out] $\frac{1}{2} \frac{d a}{b} \frac{1}{\tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{8 a-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6 a+6} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4 a-16} b \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2 a+a}}{(a-b) \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{7-1} \frac{d a}{b} \frac{1}{\tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{8 a-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6 a+6} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4 a-16} b \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2 a+a}}{(a-b) \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5-2} \frac{d}{\tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{8 a-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6 a+6} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4 a-16} b \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2 a+a}}{(a-b) \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{5-1} \frac{d a}{b} \frac{1}{\tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{8 a-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6 a+6} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4 a-16} b \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2 a+a}}{(a-b) \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3-2} \frac{d}{\tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{8 a-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6 a+6} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4 a-16} b \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2 a+a}}{(a-b) \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{3+1} \frac{d a}{b} \frac{1}{\tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{8 a-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{6 a+6} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4 a-16} b \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{4-4} \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{2 a+a}}{(a-b) \tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)^{-1} \frac{16}{d} \frac{b}{(a-b)} \sum\left(-_R^{6 a+(-5 a+12 b)} *_R^{4+(5 a-12 b)} *_R^{2+a}\right) / \left(_R^{7 a-3} *_R^{5 a+3} *_R^{3 a-8} *_R^{3 b-}_R^a\right) * \ln\left(\tanh\left(\frac{1}{2} d x+\frac{1}{2} c\right)-_R\right), _R=\text{RootOf}\left(a *_Z^{8-4} a *_Z^{6+(6 a-16 b)} *_Z^{4-4} a *_Z^{2+a}\right)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2 a e^{(6 c)}-b e^{(6 c)}\right) e^{(6 d x)}-\left(8 a e^{(4 c)}-3 b e^{(4 c)}\right) e^{(4 d x)}-\left(2 a e^{(2 c)}+3 b e^{(2 c)}\right) e^{(2 d x)}+b}{2\left(a b^2 d-b^3 d+\left(a b^2 d e^{(8 c)}-b^3 d e^{(8 c)}\right) e^{(8 d x)}-4\left(a b^2 d e^{(6 c)}-b^3 d e^{(6 c)}\right) e^{(6 d x)}-2\left(8 a^2 b d e^{(4 c)}-11 a b^2 d e^{(4 c)}+3 b^3 d e^{(4 c)}\right) e^{(4 d x)}-4\left(a b^2 d e^{(2 c)}-b^3 d e^{(2 c)}\right) e^{(2 d x)}+b) e^{(2 d x)}+6 b e^{(4 d x+4 c)}}{\left(a b^2- b^3+\left(a b^2 e^{(8 c)}-b^3 e^{(8 c)}\right) e^{(8 d x)}-4\left(a b^2 e^{(6 c)}-b^3 e^{(6 c)}\right) e^{(6 d x)}-2\left(8 a^2 b d e^{(4 c)}-11 a b^2 d e^{(4 c)}+3 b^3 d e^{(4 c)}\right) e^{(4 d x)}-4\left(a b^2 e^{(2 c)}-b^3 e^{(2 c)}\right) e^{(2 d x)}+b\right) e^{(2 d x)}+6 b e^{(4 d x+4 c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")`

[Out] $-1/2 * \left(\left(2 a e^{(6 c)} - b e^{(6 c)} \right) e^{(6 d x)} - \left(8 a e^{(4 c)} - 3 b e^{(4 c)} \right) e^{(4 d x)} - \left(2 a e^{(2 c)} + 3 b e^{(2 c)} \right) e^{(2 d x)} + b \right) / \left(a b^2 d - b^3 d + \left(a b^2 d e^{(8 c)} - b^3 d e^{(8 c)} \right) e^{(8 d x)} - 4 \left(a b^2 d e^{(6 c)} - b^3 d e^{(6 c)} \right) e^{(6 d x)} - 2 \left(8 a^2 b d e^{(4 c)} - 11 a b^2 d e^{(4 c)} + 3 b^3 d e^{(4 c)} \right) e^{(4 d x)} - 4 \left(a b^2 d e^{(2 c)} - b^3 d e^{(2 c)} \right) e^{(2 d x)} + b \right) e^{(2 d x)} + 6 b e^{(4 d x + 4 c)} / \left(a b^2 - b^3 + \left(a b^2 e^{(8 c)} - b^3 e^{(8 c)} \right) e^{(8 d x)} - 4 \left(a b^2 e^{(6 c)} - b^3 e^{(6 c)} \right) e^{(6 d x)} - 2 \left(8 a^2 b e^{(4 c)} - 11 a b^2 e^{(4 c)} + 3 b^3 e^{(4 c)} \right) e^{(4 d x)} - 4 \left(a b^2 e^{(2 c)} - b^3 e^{(2 c)} \right) e^{(2 d x)} + b \right) e^{(2 d x)} + 6 b e^{(4 d x + 4 c)}, x$

Fricas [B] time = 3.5546, size = 13423, normalized size = 57.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/16*(8*(2*a - b)*\cosh(d*x + c)^6 + 48*(2*a - b)*\cosh(d*x + c)*\sinh(d*x + c)^5 + 8*(2*a - b)*\sinh(d*x + c)^6 - 8*(8*a - 3*b)*\cosh(d*x + c)^4 + 8*(15*(2*a - b)*\cosh(d*x + c)^2 - 8*a + 3*b)*\sinh(d*x + c)^4 + 32*(5*(2*a - b)*\cosh(d*x + c)^3 - (8*a - 3*b)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 8*(2*a + 3*b)*\cosh(d*x + c)^2 + 8*(15*(2*a - b)*\cosh(d*x + c)^4 - 6*(8*a - 3*b)*\cosh(d*x + c)^2 - 2*a - 3*b)*\sinh(d*x + c)^2 - ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))\sqrt{-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)}*\log(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*\sqrt{(25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 - 81*a*b + 81*b^2)*\cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*\sinh(d*x + c)^2 - 20*a^2 + 81*a*b - 81*b^2 + 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*\sqrt{(25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 2*(5*a^3*b - 19*a^2*b^2 + 18*a*b^3)*d)*\sqrt{-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)})) + ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c$$

$$\begin{aligned}
&)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c) \\
&)^2 - (a*b^2 - b^3)*d*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*c \\
& \cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*c \\
& \cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30* \\
& (a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\sinh(d*x \\
& + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x \\
& + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)* \\
& d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 1 \\
& 5*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d \\
& *x + c)^2 - (a*b^2 - b^3)*d*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 \\
& - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 1 \\
& 1*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d* \\
& x + c))*\sqrt{-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(25*a^2 - 90* \\
& a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 \\
& - 6*a^2*b^8 + a*b^9)*d^4))} - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 \\
& + 3*a*b^5 - b^6)*d^2))*\log(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b \\
& ^4 + 9*a*b^5)*d^2*\sqrt{(25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1 \\
& 5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} + (20*a^2 - \\
& 81*a*b + 81*b^2)*\cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*\cosh(d*x + \\
& c)*\sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*\sinh(d*x + c)^2 - 20*a^2 + 81 \\
& *a*b - 81*b^2 - 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7 \\
&)*d^3*\sqrt{(25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - \\
& 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} - 2*(5*a^3*b - 19*a^2*b^ \\
& 2 + 18*a*b^3)*d*\sqrt{-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(25* \\
& a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15 \\
& *a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - \\
& 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a \\
& *b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c) \\
& ^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c) \\
& ^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*co \\
& sh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*co \\
& sh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(\\
& a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\sinh(d*x + \\
& c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + \\
& c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d \\
& *\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15 \\
& *(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d* \\
& x + c)^2 - (a*b^2 - b^3)*d*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - \\
& b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11 \\
& *a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x \\
& + c))*\sqrt{((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{(25*a^2 - 90*a* \\
& b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - \\
& 6*a^2*b^8 + a*b^9)*d^4))} + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + \\
& 3*a*b^5 - b^6)*d^2))*\log(-2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^ \\
& 4 + 9*a*b^5)*d^2*\sqrt{(25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15
\end{aligned}$$

$$\begin{aligned}
& *a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 - 81*a*b + 81*b^2)*\cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*\cosh(d*x + c) \\
&)*\sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*\sinh(d*x + c)^2 - 20*a^2 + 81*a*b - 81*b^2 + 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7) \\
& *d^3*\sqrt{((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} \\
& + 2*(5*a^3*b - 19*a^2*b^2 + 18*a*b^3)*d)*\sqrt{((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} \\
& + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)) - ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 \\
& - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh \\
& (d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*\log(-2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*\sqrt{((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} + (20*a^2 - 81*a*b + 81*b^2)*\cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*\cosh(d*x + c)*\sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*\sinh(d*x + c)^2 - 20*a^2 + 81*a*b - 81*b^2 - 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*\sqrt{((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} + 2*(5*a^3*b - 19*a^2*b^2 + 18*a*b^3)*d)*\sqrt{((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)) + 16*(3*(2*a - b)*\cosh(d*x + c)^5 - 2*(8*a - 3*b)*\cosh(d*x + c)^3 - (2*a + 3*b)*\cosh(d*x + c))*\sinh(d*x + c) + 8*b)/((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*\sqrt{((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))} + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))}
\end{aligned}$$

$$b - 11ab^2 + 3b^3)d \sinh(dx + c)^4 - 4(a^2b - b^3)d \cosh(dx + c)^2 + 8(7(a^2b - b^3)d \cosh(dx + c)^5 - 10(a^2b - b^3)d \cosh(dx + c)^3 - (8a^2b - 11ab^2 + 3b^3)d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^2b - b^3)d \cosh(dx + c)^6 - 15(a^2b - b^3)d \cosh(dx + c)^4 - 3(8a^2b - 11ab^2 + 3b^3)d \cosh(dx + c)^2 - (a^2b - b^3)d) \sinh(dx + c)^2 + (a^2b - b^3)d + 8((a^2b - b^3)d \cosh(dx + c)^7 - 3(a^2b - b^3)d \cosh(dx + c)^5 - (8a^2b - 11ab^2 + 3b^3)d \cosh(dx + c)^3 - (a^2b - b^3)d \cosh(dx + c)) \sinh(dx + c)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)**6/(a-b*sinh(dx+c)**4)**2,x)

[Out] Timed out

Giac [A] time = 4.43949, size = 205, normalized size = 0.88

$$\frac{2ae^{(6dx+6c)} - be^{(6dx+6c)} - 8ae^{(4dx+4c)} + 3be^{(4dx+4c)} - 2ae^{(2dx+2c)} - 3be^{(2dx+2c)} + b}{2(abd - b^2d)(be^{(8dx+8c)} - 4be^{(6dx+6c)} - 16ae^{(4dx+4c)} + 6be^{(4dx+4c)} - 4be^{(2dx+2c)} + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^6/(a-b*sinh(dx+c)^4)^2,x, algorithm="giac")

[Out]
$$-1/2*(2*a*e^{(6*d*x + 6*c)} - b*e^{(6*d*x + 6*c)} - 8*a*e^{(4*d*x + 4*c)} + 3*b*e^{(4*d*x + 4*c)} - 2*a*e^{(2*d*x + 2*c)} - 3*b*e^{(2*d*x + 2*c)} + b)/((a*b*d - b^2*d)*(b*e^{(8*d*x + 8*c)} - 4*b*e^{(6*d*x + 6*c)} - 16*a*e^{(4*d*x + 4*c)} + 6*b*e^{(4*d*x + 4*c)} - 4*b*e^{(2*d*x + 2*c)} + b))$$

$$3.249 \quad \int \frac{\sinh^4(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$$

Optimal. Leaf size=195

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{bd}(\sqrt{a}-\sqrt{b})^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{bd}(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{\tanh^5(c+dx)}{4ad((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a)} - \frac{\tanh(c+dx)}{4ad}$$

[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tanh[c + d*x]/(4*a*(a - b)*d) + Tanh[c + d*x]^5/(4*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rubi [A] time = 0.248514, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3217, 1275, 12, 1122, 1166, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{bd}(\sqrt{a}-\sqrt{b})^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{bd}(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{\tanh^5(c+dx)}{4ad((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a)} - \frac{\tanh(c+dx)}{4ad}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tanh[c + d*x]/(4*a*(a - b)*d) + Tanh[c + d*x]^5/(4*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rule 3217

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f}, x] &&

IntegerQ[m/2] && IntegerQ[p]

Rule 1275

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 1122

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)), x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m + 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^4(1-x^2)}{(a-2ax^2+(a-b)x^4)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh^5(c+dx)}{4ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} + \frac{\text{Subst}\left(\int -\frac{2bx^4}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{8abd} \\
&= \frac{\tanh^5(c+dx)}{4ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{x^4}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{4ad} \\
&= -\frac{\tanh(c+dx)}{4a(a-b)d} + \frac{\tanh^5(c+dx)}{4ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} + \frac{\text{Subst}\left(\int \frac{a-2ax^2}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{4ad} \\
&= -\frac{\tanh(c+dx)}{4a(a-b)d} + \frac{\tanh^5(c+dx)}{4ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} - \frac{(\sqrt{a}+\sqrt{b})^2 \text{Subst}\left(\int \frac{1}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c+dx)\right)}{4ad} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}(\sqrt{a}-\sqrt{b})^{3/2}\sqrt{bd}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}(\sqrt{a}+\sqrt{b})^{3/2}\sqrt{bd}} - \frac{\tanh(c+dx)}{4a(a-b)d} + \frac{\tanh^5(c+dx)}{4ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))}
\end{aligned}$$

Mathematica [A] time = 4.13686, size = 225, normalized size = 1.15

$$\frac{(\sqrt{a}-\sqrt{b})\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{(\sqrt{a}+\sqrt{b})\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}} - \frac{2(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}$$

$$8d(a-b)$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2, x]

[Out] -(((Sqrt[a] + Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((Sqrt[a] - Sqrt[b])*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (2*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))/(8*(a - b)*d)

Maple [C] time = 0.046, size = 490, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\sinh(dx+c)^4/(a-b*\sinh(dx+c)^4)^2, x)$

[Out]
$$-1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/(a-b)*\tanh(1/2*d*x+1/2*c)^{7+5/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/(a-b)*\tanh(1/2*d*x+1/2*c)^{5+5/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/(a-b)*\tanh(1/2*d*x+1/2*c)^{3-1/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/(a-b)*\tanh(1/2*d*x+1/2*c)-1/16/d/(a-b)*\sum((_R^6-7*_R^4+7*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R), _R=\text{RootOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(8ae^{4c} - 3be^{4c})e^{4dx} - be^{(6dx+6c)} + 5be^{(2dx+2c)} - b}{2(ab^2d - b^3d + (ab^2de^{8c} - b^3de^{8c})e^{8dx}) - 4(ab^2de^{6c} - b^3de^{6c})e^{6dx} - 2(8a^2bde^{4c} - 11ab^2de^{4c} + 3b^3de^{4c})e^{4dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\sinh(dx+c)^4/(a-b*\sinh(dx+c)^4)^2, x, \text{algorithm}=\text{"maxima"})$

[Out]
$$-1/2*((8*a*e^{4*c} - 3*b*e^{4*c})*e^{4*d*x} - b*e^{(6*d*x + 6*c)} + 5*b*e^{(2*d*x + 2*c)} - b)/(a*b^2*d - b^3*d + (a*b^2*d*e^{(8*c)} - b^3*d*e^{(8*c)})*e^{(8*d*x)} - 4*(a*b^2*d*e^{(6*c)} - b^3*d*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^2*b*d*e^{(4*c)} - 11*a*b^2*d*e^{(4*c)} + 3*b^3*d*e^{(4*c)})*e^{(4*d*x)} - 4*(a*b^2*d*e^{(2*c)} - b^3*d*e^{(2*c)})*e^{(2*d*x)}) + 1/16*\text{integrate}(16*(e^{(6*d*x + 6*c)} - 6*e^{(4*d*x + 4*c)} + e^{(2*d*x + 2*c)})/(a*b - b^2 + (a*b*e^{(8*c)} - b^2*e^{(8*c)})*e^{(8*d*x)} - 4*(a*b*e^{(6*c)} - b^2*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^2*e^{(4*c)} - 11*a*b*e^{(4*c)} + 3*b^2*e^{(4*c)})*e^{(4*d*x)} - 4*(a*b*e^{(2*c)} - b^2*e^{(2*c)})*e^{(2*d*x)}), x)$$

Fricas [B] time = 2.72877, size = 12467, normalized size = 63.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out]
$$\frac{1}{16} \cdot (8 \cdot b \cdot \cosh(d \cdot x + c)^6 + 48 \cdot b \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^5 + 8 \cdot b \cdot \sinh(d \cdot x + c)^6 - 8 \cdot (8 \cdot a - 3 \cdot b) \cdot \cosh(d \cdot x + c)^4 + 8 \cdot (15 \cdot b \cdot \cosh(d \cdot x + c)^2 - 8 \cdot a + 3 \cdot b) \cdot \sinh(d \cdot x + c)^4 + 32 \cdot (5 \cdot b \cdot \cosh(d \cdot x + c)^3 - (8 \cdot a - 3 \cdot b) \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^3 - 40 \cdot b \cdot \cosh(d \cdot x + c)^2 + 8 \cdot (15 \cdot b \cdot \cosh(d \cdot x + c)^4 - 6 \cdot (8 \cdot a - 3 \cdot b) \cdot \cosh(d \cdot x + c)^2 - 5 \cdot b) \cdot \sinh(d \cdot x + c)^2 + ((a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^8 + 8 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^7 + (a \cdot b^2 - b^3) \cdot d \cdot \sinh(d \cdot x + c)^8 - 4 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^6 + 4 \cdot (7 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^2 - (a \cdot b^2 - b^3) \cdot d) \cdot \sinh(d \cdot x + c)^6 - 2 \cdot (8 \cdot a^2 \cdot b - 11 \cdot a \cdot b^2 + 3 \cdot b^3) \cdot d \cdot \cosh(d \cdot x + c)^4 + 8 \cdot (7 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^3 - 3 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^5 + 2 \cdot (35 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^4 - 30 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^2 - (8 \cdot a^2 \cdot b - 11 \cdot a \cdot b^2 + 3 \cdot b^3) \cdot d) \cdot \sinh(d \cdot x + c)^4 - 4 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^2 + 8 \cdot (7 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^5 - 10 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^3 - (8 \cdot a^2 \cdot b - 11 \cdot a \cdot b^2 + 3 \cdot b^3) \cdot d \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^3 + 4 \cdot (7 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^6 - 15 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^4 - 3 \cdot (8 \cdot a^2 \cdot b - 11 \cdot a \cdot b^2 + 3 \cdot b^3) \cdot d \cdot \cosh(d \cdot x + c)^2 - (a \cdot b^2 - b^3) \cdot d) \cdot \sinh(d \cdot x + c)^2 + (a \cdot b^2 - b^3) \cdot d + 8 \cdot ((a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^7 - 3 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^5 - (8 \cdot a^2 \cdot b - 11 \cdot a \cdot b^2 + 3 \cdot b^3) \cdot d \cdot \cosh(d \cdot x + c)^3 - (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)) \cdot \sqrt{((a^4 \cdot b - 3 \cdot a^3 \cdot b^2 + 3 \cdot a^2 \cdot b^3 - a \cdot b^4) \cdot d^2 \cdot \sqrt{(9 \cdot a^2 + 6 \cdot a \cdot b + b^2) / ((a^9 \cdot b - 6 \cdot a^8 \cdot b^2 + 15 \cdot a^7 \cdot b^3 - 20 \cdot a^6 \cdot b^4 + 15 \cdot a^5 \cdot b^5 - 6 \cdot a^4 \cdot b^6 + a^3 \cdot b^7) \cdot d^4)) + a + 3 \cdot b) / ((a^4 \cdot b - 3 \cdot a^3 \cdot b^2 + 3 \cdot a^2 \cdot b^3 - a \cdot b^4) \cdot d^2)} \cdot \log(2 \cdot (a^5 - 3 \cdot a^4 \cdot b + 3 \cdot a^3 \cdot b^2 - a^2 \cdot b^3) \cdot d^2 \cdot \sqrt{(9 \cdot a^2 + 6 \cdot a \cdot b + b^2) / ((a^9 \cdot b - 6 \cdot a^8 \cdot b^2 + 15 \cdot a^7 \cdot b^3 - 20 \cdot a^6 \cdot b^4 + 15 \cdot a^5 \cdot b^5 - 6 \cdot a^4 \cdot b^6 + a^3 \cdot b^7) \cdot d^4)) + (3 \cdot a + b) \cdot \cosh(d \cdot x + c)^2 + 2 \cdot (3 \cdot a + b) \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c) + (3 \cdot a + b) \cdot \sinh(d \cdot x + c)^2 + 2 \cdot (2 \cdot (a^6 \cdot b - 3 \cdot a^5 \cdot b^2 + 3 \cdot a^4 \cdot b^3 - a^3 \cdot b^4) \cdot d^3 \cdot \sqrt{(9 \cdot a^2 + 6 \cdot a \cdot b + b^2) / ((a^9 \cdot b - 6 \cdot a^8 \cdot b^2 + 15 \cdot a^7 \cdot b^3 - 20 \cdot a^6 \cdot b^4 + 15 \cdot a^5 \cdot b^5 - 6 \cdot a^4 \cdot b^6 + a^3 \cdot b^7) \cdot d^4)) - (3 \cdot a^3 + 4 \cdot a^2 \cdot b + a \cdot b^2) \cdot d) \cdot \sqrt{((a^4 \cdot b - 3 \cdot a^3 \cdot b^2 + 3 \cdot a^2 \cdot b^3 - a \cdot b^4) \cdot d^2 \cdot \sqrt{(9 \cdot a^2 + 6 \cdot a \cdot b + b^2) / ((a^9 \cdot b - 6 \cdot a^8 \cdot b^2 + 15 \cdot a^7 \cdot b^3 - 20 \cdot a^6 \cdot b^4 + 15 \cdot a^5 \cdot b^5 - 6 \cdot a^4 \cdot b^6 + a^3 \cdot b^7) \cdot d^4)) + a + 3 \cdot b) / ((a^4 \cdot b - 3 \cdot a^3 \cdot b^2 + 3 \cdot a^2 \cdot b^3 - a \cdot b^4) \cdot d^2)} - 3 \cdot a - b) - ((a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^8 + 8 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c) \cdot \sinh(d \cdot x + c)^7 + (a \cdot b^2 - b^3) \cdot d \cdot \sinh(d \cdot x + c)^8 - 4 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^6 + 4 \cdot (7 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^2 - (a \cdot b^2 - b^3) \cdot d) \cdot \sinh(d \cdot x + c)^6 - 2 \cdot (8 \cdot a^2 \cdot b - 11 \cdot a \cdot b^2 + 3 \cdot b^3) \cdot d \cdot \cosh(d \cdot x + c)^4 + 8 \cdot (7 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^3 - 3 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)) \cdot \sinh(d \cdot x + c)^5 + 2 \cdot (35 \cdot (a \cdot b^2 - b^3) \cdot d \cdot \cosh(d \cdot x + c)^4 - 3$$

$$\begin{aligned}
& 0*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*\sqrt{(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*\log(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*\sqrt{(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a + b)*\cosh(d*x + c)^2 + 2*(3*a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (3*a + b)*\sinh(d*x + c)^2 - 2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*\sqrt{(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (3*a^3 + 4*a^2*b + a*b^2)*d*\sqrt{((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*\sqrt{(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 3*a - b) - ((a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 - b^3)*d*\sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*\sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*\cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*\cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*\cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*\sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*\cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*\cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*\cosh(d*x + c)^3 - (a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*\sqrt{(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*\log(-2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*\sqrt{(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a + b)*\cosh(d*x + c)^2 + 2*(3*a + b)*\cosh(d*x + c)*\sinh(d*x + c) + (3*a + b)*\sinh(d*x + c)^2 + 2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*\sqrt{(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a^3 + 4*a^2*b + a*b^2)*d*\sqrt{((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*\sqrt{(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6
\end{aligned}$$

$$\begin{aligned}
& *a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 3*a - b) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d* \\
& cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - \\
& b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - \\
& b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 \\
& + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*s \\
& inh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d \\
& *cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b \\
& ^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a* \\
& b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c) \\
&)*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3) \\
& *d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a* \\
& b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d \\
& *x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3 \\
&)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(- \\
& (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-2*(\\
& a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*cosh(d*x + c)*sinh(d*x + c) \\
& + (3*a + b)*sinh(d*x + c)^2 - 2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a^3 + 4*a^2*b + a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 3*a - b) + 16*(3*b*cosh(d*x + c)^5 - 2*(8*a - 3*b)*cosh(d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c) + 8*b)/((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [A] time = 4.33748, size = 171, normalized size = 0.88

$$\frac{be^{(6dx+6c)} - 8ae^{(4dx+4c)} + 3be^{(4dx+4c)} - 5be^{(2dx+2c)} + b}{2(abd - b^2d)(be^{(8dx+8c)} - 4be^{(6dx+6c)} - 16ae^{(4dx+4c)} + 6be^{(4dx+4c)} - 4be^{(2dx+2c)} + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] 1/2*(b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 5*b*e^(2*d*x + 2*c) + b)/((a*b*d - b^2*d)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b))

$$3.250 \quad \int \frac{\sinh^2(c+dx)}{(a-b \sinh^4(c+dx))^2} dx$$

Optimal. Leaf size=220

$$-\frac{(2\sqrt{a}-\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{bd}(\sqrt{a}-\sqrt{b})^{3/2}} + \frac{(2\sqrt{a}+\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{bd}(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{\tanh(c+dx)(a-(a+b))}{4ad(a-b)((a-b)\tanh^4(c+dx))}$$

[Out] -((2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) + ((2*Sqrt[a] + Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(5/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) + (Tanh[c + d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a*(a - b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rubi [A] time = 0.297621, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3217, 1333, 1166, 208}

$$-\frac{(2\sqrt{a}-\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{bd}(\sqrt{a}-\sqrt{b})^{3/2}} + \frac{(2\sqrt{a}+\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{bd}(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{\tanh(c+dx)(a-(a+b))}{4ad(a-b)((a-b)\tanh^4(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] -((2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) + ((2*Sqrt[a] + Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(5/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) + (Tanh[c + d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a*(a - b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rule 3217

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1

)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1333

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{x^2(1-x^2)^2}{(a-2ax^2+(a-b)x^4)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh(c+dx)(a-(a+b)\tanh^2(c+dx))}{4a(a-b)d(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} - \frac{\text{Subst}\left(\int \frac{\frac{2a^2b-2a(3a-b)bx^2}{a-b} - \frac{a-b}{a-2ax^2+(a-b)x^4}}{a-2ax^2+(a-b)x^4} dx, x\right)}{8a^2bd} \\
&= \frac{\tanh(c+dx)(a-(a+b)\tanh^2(c+dx))}{4a(a-b)d(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} - \frac{(2a-\sqrt{a}\sqrt{b}-b)\text{Subst}\left(\int \frac{1}{a-2ax^2+(a-b)x^4} dx, x\right)}{8a(\sqrt{a}\sqrt{b}-b)} \\
&= -\frac{(2\sqrt{a}-\sqrt{b})\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}(\sqrt{a}-\sqrt{b})^{3/2}\sqrt{bd}} + \frac{(2\sqrt{a}+\sqrt{b})\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}(\sqrt{a}+\sqrt{b})^{3/2}\sqrt{bd}} + \frac{1}{4a}
\end{aligned}$$

Mathematica [A] time = 2.06416, size = 253, normalized size = 1.15

$$\frac{\frac{\sqrt{a}(-\sqrt{a}\sqrt{b}+2a-b)\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{a}(\sqrt{a}\sqrt{b}+2a-b)\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{4\sqrt{a}\sinh(2(c+dx))(2a-b\cosh(2(c+dx))+b)}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}}{8a^{3/2}d(a-b)}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] ((Sqrt[a]*(2*a + Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])]/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (Sqrt[a]*(2*a - Sqrt[a]*Sqrt[b] - b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]])]/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (4*Sqrt[a]*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)]/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)

Maple [C] time = 0.069, size = 708, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x)

[Out] $\frac{1}{2}d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}\tanh(1/2*d*x+1/2*c)^{6*a+6}\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*\tanh(1/2*d*x+1/2*c)^7-1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}\tanh(1/2*d*x+1/2*c)^{6*a+6}\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*\tanh(1/2*d*x+1/2*c)^5-2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}\tanh(1/2*d*x+1/2*c)^{6*a+6}\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*\tanh(1/2*d*x+1/2*c)^5*b-1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}\tanh(1/2*d*x+1/2*c)^{6*a+6}\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*\tanh(1/2*d*x+1/2*c)^3-2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}\tanh(1/2*d*x+1/2*c)^{6*a+6}\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*\tanh(1/2*d*x+1/2*c)^3*b+1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}\tanh(1/2*d*x+1/2*c)^{6*a+6}\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*\tanh(1/2*d*x+1/2*c)-1/16/d/a/(a-b)*\sum((-_R^6*a+(11*a-4*b)*_R^4+(-11*a+4*b)*_R^2+a)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R),_R=\text{RootOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(2ae^{6c} - be^{6c})e^{6dx} - (8ae^{4c} - 3be^{4c})e^{4dx} - (2ae^{2c} + 3be^{2c})e^{2dx} + 2(a^2bd - ab^2d + (a^2bde^{8c} - ab^2de^{8c})e^{8dx}) - 4(a^2bde^{6c} - ab^2de^{6c})e^{6dx} - 2(8a^3de^{4c} - 11a^2bde^{4c} + 3ab^2de^{4c})e^{4dx} + (2a^2bde^{2c} - ab^2de^{2c})e^{2dx}}{2(a^2bd - ab^2d + (a^2bde^{8c} - ab^2de^{8c})e^{8dx}) - 4(a^2bde^{6c} - ab^2de^{6c})e^{6dx} - 2(8a^3de^{4c} - 11a^2bde^{4c} + 3ab^2de^{4c})e^{4dx} + (2a^2bde^{2c} - ab^2de^{2c})e^{2dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out] $-1/2*((2*a*e^{6*c} - b*e^{6*c})*e^{6*d*x} - (8*a*e^{4*c} - 3*b*e^{4*c})*e^{4*d*x} - (2*a*e^{2*c} + 3*b*e^{2*c})*e^{2*d*x} + b)/(a^2*b*d - a*b^2*d + (a^2*b*d*e^{8*c} - a*b^2*d*e^{8*c})*e^{8*d*x} - 4*(a^2*b*d*e^{6*c} - a*b^2*d*e^{6*c})*e^{6*d*x} - 2*(8*a^3*d*e^{4*c} - 11*a^2*b*d*e^{4*c} + 3*a*b^2*d*e^{4*c})*e^{4*d*x} - 4*(a^2*b*d*e^{2*c} - a*b^2*d*e^{2*c})*e^{2*d*x}) - 1/4*\text{integrate}(4*((2*a*e^{6*c} - b*e^{6*c})*e^{6*d*x} - 2*(4*a*e^{4*c} - b*e^{4*c}))*e^{4*d*x} + (2*a*e^{2*c} - b*e^{2*c})*e^{2*d*x})/(a^2*b - a*b^2 + (a^2*b*e^{8*c} - a*b^2*e^{8*c})*e^{8*d*x} - 4*(a^2*b*e^{6*c} - a*b^2*e^{6*c})*e^{6*d*x} - 2*(8*a^3*e^{4*c} - 11*a^2*b*e^{4*c} + 3*a*b^2*e^{4*c})*e^{4*d*x} - 4*(a^2*b*e^{2*c} - a*b^2*e^{2*c})*e^{2*d*x}), x)$

Fricas [B] time = 3.51966, size = 14390, normalized size = 65.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/16*(8*(2*a - b)*\cosh(d*x + c)^6 + 48*(2*a - b)*\cosh(d*x + c)*\sinh(d*x + \\ & c)^5 + 8*(2*a - b)*\sinh(d*x + c)^6 - 8*(8*a - 3*b)*\cosh(d*x + c)^4 + 8*(15* \\ & (2*a - b)*\cosh(d*x + c)^2 - 8*a + 3*b)*\sinh(d*x + c)^4 + 32*(5*(2*a - b)*\co \\ & sh(d*x + c)^3 - (8*a - 3*b)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 8*(2*a + 3*b)* \\ & \cosh(d*x + c)^2 + 8*(15*(2*a - b)*\cosh(d*x + c)^4 - 6*(8*a - 3*b)*\cosh(d*x \\ & + c)^2 - 2*a - 3*b)*\sinh(d*x + c)^2 + ((a^2*b - a*b^2)*d*\cosh(d*x + c)^8 + \\ & 8*(a^2*b - a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*\sinh(\\ & d*x + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*c \\ & osh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + \\ & 3*a*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - 3*(a \\ & ^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\co \\ & sh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + \\ & 3*a*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a \\ & ^2*b - a*b^2)*d*\cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - (8 \\ & *a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - \\ & a*b^2)*d*\cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 \\ & - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c) \\ & ^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - \\ & a*b^2)*d*\cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - \\ & (a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-((a^5*b - 3*a^4*b^2 \\ & + 3*a^3*b^3 - a^2*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 \\ & + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6 \\ & *b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 \\ & - a^2*b^4)*d^2))*\log(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4 \\ &)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a \\ & ^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) \\ & + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)* \\ & \cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\cosh(d*x + c)*\sinh(\\ & d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\sinh(d*x + c)^2 + 2*((3*a^8* \\ & b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*\sqrt{(64*a^4 - 80*a^ \\ & 3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20* \\ & a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 2*(8*a^5 - 5*a^4*b + a^ \\ & 3*b^2)*d)*\sqrt{-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*\sqrt{(64*a^4 \\ & - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b \\ & ^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b \\ & ^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))} - ((a^2*b - a*b^2)*d* \end{aligned}$$

$$\begin{aligned}
& \cosh(dx + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(dx + c)*\sinh(dx + c)^7 + (a^2*b - a*b^2)*d*\sinh(dx + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(dx + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(dx + c)^2 - (a^2*b - a*b^2)*d)*\sinh(dx + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(dx + c)^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(dx + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(dx + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh(dx + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(dx + c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(dx + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(dx + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(dx + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(dx + c)^6 - 15*(a^2*b - a*b^2)*d*\cosh(dx + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(dx + c)^2 - (a^2*b - a*b^2)*d)*\sinh(dx + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*\cosh(dx + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(dx + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(dx + c)^3 - (a^2*b - a*b^2)*d*\cosh(dx + c))*\sinh(dx + c))*\sqrt{-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*\log(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\cosh(dx + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\cosh(dx + c)*\sinh(dx + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\sinh(dx + c)^2 - 2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d)*\sqrt{-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))} - ((a^2*b - a*b^2)*d*\cosh(dx + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(dx + c)*\sinh(dx + c)^7 + (a^2*b - a*b^2)*d*\sinh(dx + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(dx + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(dx + c)^2 - (a^2*b - a*b^2)*d)*\sinh(dx + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(dx + c)^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(dx + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(dx + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh(dx + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(dx + c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(dx + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(dx + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(dx + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(dx + c)^6 - 15*(a^2*b - a*b^2)*d*\cosh(dx + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(dx + c)^2 - (a^2*b - a*b^2)*d)*\sinh(dx + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*\cosh(dx + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(dx + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(dx + c)^3 - (a^2*b - a*b^2)*d*\cosh(dx + c))*\sinh(dx + c))*\sqrt{((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2)}
\end{aligned}$$

$$\begin{aligned}
& - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^{11}*b - 6*a^{10}*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*\log(-2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^{11}*b - 6*a^{10}*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\cosh(d*x + c)*\sinh(d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\sinh(d*x + c)^2 + 2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^{11}*b - 6*a^{10}*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d)*\sqrt{((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^{11}*b - 6*a^{10}*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))) + ((a^2*b - a*b^2)*d*\cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*\sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^{11}*b - 6*a^{10}*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*\log(-2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^{11}*b - 6*a^{10}*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\cosh(d*x + c)*\sinh(d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*\sinh(d*x + c)^2 - 2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^{11}*b - 6*a^{10}*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d)*\sqrt{((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*\sqrt{(64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^{11}*b - 6*a^{10}*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))
\end{aligned}$$

$$\begin{aligned}
& + 3a^3b^3 - a^2b^4)d^2)) + 16*(3*(2*a - b)*\cosh(d*x + c)^5 - 2*(8*a - \\
& 3*b)*\cosh(d*x + c)^3 - (2*a + 3*b)*\cosh(d*x + c))*\sinh(d*x + c) + 8*b)/((a \\
& ^2*b - a*b^2)*d*\cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(d*x + c)*\sinh(d* \\
& x + c)^7 + (a^2*b - a*b^2)*d*\sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(d*x \\
& + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(\\
& d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^2*b \\
& - a*b^2)*d*\cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + \\
& c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh \\
& (d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^2*b - \\
& a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - 10*(a^2 \\
& *b - a*b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c \\
&))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 - 15*(a^2*b - a \\
& *b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^2 \\
& - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^ \\
& 2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - (8*a^3 - 11*a^ \\
& 2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d* \\
& x + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [A] time = 4.2299, size = 205, normalized size = 0.93

$$\frac{2ae^{(6dx+6c)} - be^{(6dx+6c)} - 8ae^{(4dx+4c)} + 3be^{(4dx+4c)} - 2ae^{(2dx+2c)} - 3be^{(2dx+2c)} + b}{2(a^2d - abd)(be^{(8dx+8c)} - 4be^{(6dx+6c)} - 16ae^{(4dx+4c)} + 6be^{(4dx+4c)} - 4be^{(2dx+2c)} + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] -1/2*(2*a*e^(6*d*x + 6*c) - b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + b)/((a^2*d - a

$$*b*d)*(b*e^{(8*d*x + 8*c)} - 4*b*e^{(6*d*x + 6*c)} - 16*a*e^{(4*d*x + 4*c)} + 6*b*e^{(4*d*x + 4*c)} - 4*b*e^{(2*d*x + 2*c)} + b))$$

$$3.251 \quad \int \frac{1}{(a - b \sinh^4(c + dx))^2} dx$$

Optimal. Leaf size=210

$$\frac{(4\sqrt{a} - 3\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d(\sqrt{a} - \sqrt{b})^{3/2}} + \frac{(4\sqrt{a} + 3\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d(\sqrt{a} + \sqrt{b})^{3/2}} - \frac{b \tanh(c + dx)(1 - 2 \tanh^2(c + dx))}{4ad(a - b)((a - b) \tanh^4(c + dx) + a)}$$

[Out] $((4*\text{Sqrt}[a] - 3*\text{Sqrt}[b])* \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]]*\text{Tanh}[c + d*x])/a^{(1/4)}])/(8*a^{(7/4)}*(\text{Sqrt}[a] - \text{Sqrt}[b])^{(3/2)}*d) + ((4*\text{Sqrt}[a] + 3*\text{Sqrt}[b])* \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]]*\text{Tanh}[c + d*x])/a^{(1/4)}])/(8*a^{(7/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b])^{(3/2)}*d) - (b*\text{Tanh}[c + d*x]*(1 - 2*\text{Tanh}[c + d*x]^2))/(4*a*(a - b)*d*(a - 2*a*\text{Tanh}[c + d*x]^2 + (a - b)*\text{Tanh}[c + d*x]^4))$

Rubi [A] time = 0.246026, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3209, 1205, 1166, 208}

$$\frac{(4\sqrt{a} - 3\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d(\sqrt{a} - \sqrt{b})^{3/2}} + \frac{(4\sqrt{a} + 3\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d(\sqrt{a} + \sqrt{b})^{3/2}} - \frac{b \tanh(c + dx)(1 - 2 \tanh^2(c + dx))}{4ad(a - b)((a - b) \tanh^4(c + dx) + a)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a - b*\text{Sinh}[c + d*x]^4)^{-2}, x]$

[Out] $((4*\text{Sqrt}[a] - 3*\text{Sqrt}[b])* \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[a] - \text{Sqrt}[b]]*\text{Tanh}[c + d*x])/a^{(1/4)}])/(8*a^{(7/4)}*(\text{Sqrt}[a] - \text{Sqrt}[b])^{(3/2)}*d) + ((4*\text{Sqrt}[a] + 3*\text{Sqrt}[b])* \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[a] + \text{Sqrt}[b]]*\text{Tanh}[c + d*x])/a^{(1/4)}])/(8*a^{(7/4)}*(\text{Sqrt}[a] + \text{Sqrt}[b])^{(3/2)}*d) - (b*\text{Tanh}[c + d*x]*(1 - 2*\text{Tanh}[c + d*x]^2))/(4*a*(a - b)*d*(a - 2*a*\text{Tanh}[c + d*x]^2 + (a - b)*\text{Tanh}[c + d*x]^4))$

Rule 3209

$\text{Int}[(a + (b_*)*\sin[(e_*) + (f_*)*(x_*)]^4)^{(p_*)}, x_Symbol] \rightarrow \text{With}[\{\text{ff} = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[\text{ff}/f, \text{Subst}[\text{Int}[(a + 2*a*\text{ff}^2*x^2 + (a + b)*\text{ff}^4*x^4)^p/(1 + \text{ff}^2*x^2)^{(2*p + 1)}, x], x, \text{Tan}[e + f*x]/\text{ff}], x] \text{ ;}$
 $\text{FreeQ}\{\{a, b, e, f\}, x\} \ \&\amp; \ \text{IntegerQ}[p]$

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a - b \sinh^4(c + dx))^2} dx &= \frac{\text{Subst} \left(\int \frac{(1-x^2)^3}{(a-2ax^2+(a-b)x^4)^2} dx, x, \tanh(c + dx) \right)}{d} \\
&= -\frac{b \tanh(c + dx) (1 - 2 \tanh^2(c + dx))}{4a(a-b)d (a - 2a \tanh^2(c + dx) + (a-b) \tanh^4(c + dx))} - \frac{\text{Subst} \left(\int \frac{-\frac{2a(4a-3b)b}{a-b} + \frac{4a(2a-b)}{a-b}}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c + dx) \right)}{8a^2d} \\
&= -\frac{b \tanh(c + dx) (1 - 2 \tanh^2(c + dx))}{4a(a-b)d (a - 2a \tanh^2(c + dx) + (a-b) \tanh^4(c + dx))} - \frac{(4a - \sqrt{a}\sqrt{b} - 3b) \text{Subst} \left(\int \frac{1}{a-2ax^2+(a-b)x^4} dx, x, \tanh(c + dx) \right)}{8a^2d} \\
&= \frac{(4\sqrt{a} - 3\sqrt{b}) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{8a^{7/4} (\sqrt{a} - \sqrt{b})^{3/2} d} + \frac{(4\sqrt{a} + 3\sqrt{b}) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{8a^{7/4} (\sqrt{a} + \sqrt{b})^{3/2} d}
\end{aligned}$$

Mathematica [A] time = 2.93151, size = 230, normalized size = 1.1

$$\frac{(-\sqrt{a}\sqrt{b}+4a-3b) \tanh^{-1} \left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}} \right) - (\sqrt{a}\sqrt{b}+4a-3b) \tanh^{-1} \left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}} \right)}{\sqrt{\sqrt{a}\sqrt{b}+a} \sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{2\sqrt{ab}(\sinh(4(c+dx))-6 \sinh(2(c+dx)))}{8a+4b \cosh(2(c+dx))-b \cosh(4(c+dx))-3b}$$

$$8a^{3/2}d(a-b)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-2), x]

[Out] (-(((4*a + Sqrt[a]*Sqrt[b] - 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])/Sqrt[-a + Sqrt[a]*Sqrt[b]]) + ((4*a - Sqrt[a]*Sqrt[b] - 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (2*Sqrt[a]*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)

Maple [C] time = 0.062, size = 534, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a-b*sinh(d*x+c)^4)^2,x)

[Out]
$$-1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})*b/a/(a-b)*\tanh(1/2*d*x+1/2*c)^{7+5/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/a/(a-b)*\tanh(1/2*d*x+1/2*c)^{5*b+5/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/a/(a-b)*\tanh(1/2*d*x+1/2*c)^{3*b-1/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})*b/a/(a-b)*\tanh(1/2*d*x+1/2*c)-1/16/d/a/(a-b)*\text{sum}(((4*a-3*b)*_R^6+(-12*a+5*b)*_R^4+(-5*b+12*a)*_R^2-4*a+3*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R),_R=\text{RootOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(8ae^{4c} - 3be^{4c})e^{4dx} - be^{6dx+6c} + 5be^{2dx+2c} - b}{2(a^2bd - ab^2d + (a^2bde^{8c} - ab^2de^{8c}))e^{8dx} - 4(a^2bde^{6c} - ab^2de^{6c})e^{6dx} - 2(8a^3de^{4c} - 11a^2bde^{4c} + 3ab^2de^{4c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")

[Out]
$$-1/2*((8*a*e^{4*c} - 3*b*e^{4*c})*e^{4*d*x} - b*e^{6*d*x + 6*c} + 5*b*e^{2*d*x + 2*c} - b)/(a^2*b*d - a*b^2*d + (a^2*b*d*e^{8*c} - a*b^2*d*e^{8*c})*e^{8*d*x} - 4*(a^2*b*d*e^{6*c} - a*b^2*d*e^{6*c})*e^{6*d*x} - 2*(8*a^3*d*e^{4*c} - 11*a^2*b*d*e^{4*c} + 3*a*b^2*d*e^{4*c})*e^{4*d*x} - 4*(a^2*b*d*e^{2*c} - a*b^2*d*e^{2*c})*e^{2*d*x}) + \text{integrate}(-2*(8*a*e^{4*c} - 5*b*e^{4*c})*e^{4*d*x} - b*e^{6*d*x + 6*c} - b*e^{2*d*x + 2*c})/(a^2*b - a*b^2 + (a^2*b*e^{8*c} - a*b^2*e^{8*c})*e^{8*d*x} - 4*(a^2*b*e^{6*c} - a*b^2*e^{6*c})*e^{6*d*x} - 2*(8*a^3*e^{4*c} - 11*a^2*b*e^{4*c} + 3*a*b^2*e^{4*c})*e^{4*d*x} - 4*(a^2*b*e^{2*c} - a*b^2*e^{2*c})*e^{2*d*x}), x)$$

Fricas [B] time = 3.53477, size = 14862, normalized size = 70.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")

[Out]
$$\frac{1}{16} \cdot (8b \cosh(dx+c)^6 + 48b \cosh(dx+c) \sinh(dx+c)^5 + 8b \sinh(dx+c)^6 - 8(8a-3b) \cosh(dx+c)^4 + 8(15b \cosh(dx+c)^2 - 8a+3b) \sinh(dx+c)^4 + 32(5b \cosh(dx+c)^3 - (8a-3b) \cosh(dx+c) \sinh(dx+c)^3 - 40b \cosh(dx+c)^2 + 8(15b \cosh(dx+c)^4 - 6(8a-3b) \cosh(dx+c)^2 - 5b) \sinh(dx+c)^2 - ((a^2b - ab^2)d \cosh(dx+c)^8 + 8(a^2b - ab^2)d \cosh(dx+c) \sinh(dx+c)^7 + (a^2b - ab^2)d \sinh(dx+c)^8 - 4(a^2b - ab^2)d \cosh(dx+c)^6 + 4(7(a^2b - ab^2)d \cosh(dx+c)^2 - (a^2b - ab^2)d) \sinh(dx+c)^6 - 2(8a^3 - 11a^2b + 3ab^2)d \cosh(dx+c)^4 + 8(7(a^2b - ab^2)d \cosh(dx+c)^3 - 3(a^2b - ab^2)d \cosh(dx+c)) \sinh(dx+c)^5 + 2(35(a^2b - ab^2)d \cosh(dx+c)^4 - 30(a^2b - ab^2)d \cosh(dx+c)^2 - (8a^3 - 11a^2b + 3ab^2)d) \sinh(dx+c)^4 - 4(a^2b - ab^2)d \cosh(dx+c)^2 + 8(7(a^2b - ab^2)d \cosh(dx+c)^5 - 10(a^2b - ab^2)d \cosh(dx+c)^3 - (8a^3 - 11a^2b + 3ab^2)d \cosh(dx+c)) \sinh(dx+c)^3 + 4(7(a^2b - ab^2)d \cosh(dx+c)^6 - 15(a^2b - ab^2)d \cosh(dx+c)^4 - 3(8a^3 - 11a^2b + 3ab^2)d \cosh(dx+c)^2 - (a^2b - ab^2)d) \sinh(dx+c)^2 + (a^2b - ab^2)d + 8((a^2b - ab^2)d \cosh(dx+c)^7 - 3(a^2b - ab^2)d \cosh(dx+c)^5 - (8a^3 - 11a^2b + 3ab^2)d \cosh(dx+c)^3 - (a^2b - ab^2)d \cosh(dx+c)) \sinh(dx+c)) \sqrt{-((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2 \sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} - 16a^2 + 15ab - 3b^2)/((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2)} \cdot \log(384a^3b - 680a^2b^2 + 405ab^3 - 81b^4 + 2(16a^8 - 57a^7b + 75a^6b^2 - 43a^5b^3 + 9a^4b^4)d^2 \sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} - (384a^3b - 680a^2b^2 + 405ab^3 - 81b^4) \cosh(dx+c)^2 - 2(384a^3b - 680a^2b^2 + 405ab^3 - 81b^4) \cosh(dx+c) \sinh(dx+c) - (384a^3b - 680a^2b^2 + 405ab^3 - 81b^4) \sinh(dx+c)^2 + 2(2(2a^{10} - 7a^9b + 9a^8b^2 - 5a^7b^3 + a^6b^4)d^3 \sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} + (120a^5b - 217a^4b^2 + 132a^3b^3 - 27a^2b^4)d) \sqrt{-((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2 \sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} - 16a^2 + 15ab - 3b^2)/((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2)})) + ((a^2b - ab^2)d \cosh(dx+c)^8 + 8(a^2b - ab^2)d \cosh(dx+c) \sinh(dx+c)^7 + (a^2b - ab^2)d \sinh(dx+c)^8 - 4(a^2b - ab^2)d \cosh(dx+c)^6 + 4(7(a^2b - ab^2)d \cosh(dx+c)^2 - (a^2b - ab^2)d) \sinh(dx+c)^6 - 2(8a^3 - 11a^2b + 3ab^2)d \cosh(dx+c)^4 + 8(7(a^2b - ab^2)d \cosh(dx+c)^3 - 3(a^2b - ab^2)d \cosh(dx+c)) \sinh(dx+c)^5 + 2(35(a^2b - ab^2)d \cosh(dx+c)^4$$

$$\begin{aligned}
& - 30*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2) \\
& *d*\cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt{(576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*\log(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4 + 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*\sqrt{(576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*\cosh(d*x + c)^2 - 2*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*\cosh(d*x + c)*\sinh(d*x + c) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*\sinh(d*x + c)^2 - 2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*\sqrt{(576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + (120*a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d)*\sqrt{-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt{(576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + ((a^2*b - a*b^2)*d*\cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*\sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*\sqrt{(576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*\log(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4 - 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 -
\end{aligned}$$

$$\begin{aligned}
& 43a^5b^3 + 9a^4b^4)d^2\sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} - (384a^3b - 680a^2b^2 + 405ab^3 - 81b^4)*\cosh(dx + c)^2 - 2*(384a^3b - 680a^2b^2 + 405ab^3 - 81b^4)*\cosh(dx + c)*\sinh(dx + c) - (384a^3b - 680a^2b^2 + 405ab^3 - 81b^4)*\sinh(dx + c)^2 + 2*(2*(2a^{10} - 7a^9b + 9a^8b^2 - 5a^7b^3 + a^6b^4)d^3\sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} - (120a^5b - 217a^4b^2 + 132a^3b^3 - 27a^2b^4)d)*\sqrt{((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2\sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} + 16a^2 - 15ab + 3b^2)/((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2))} - ((a^2b - ab^2)d*\cosh(dx + c)^8 + 8*(a^2b - ab^2)d*\cosh(dx + c)*\sinh(dx + c)^7 + (a^2b - ab^2)d*\sinh(dx + c)^8 - 4*(a^2b - ab^2)d*\cosh(dx + c)^6 + 4*(7*(a^2b - ab^2)d*\cosh(dx + c)^2 - (a^2b - ab^2)d)*\sinh(dx + c)^6 - 2*(8a^3 - 11a^2b + 3ab^2)d*\cosh(dx + c)^4 + 8*(7*(a^2b - ab^2)d*\cosh(dx + c)^3 - 3*(a^2b - ab^2)d*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(35*(a^2b - ab^2)d*\cosh(dx + c)^4 - 30*(a^2b - ab^2)d*\cosh(dx + c)^2 - (8a^3 - 11a^2b + 3ab^2)d)*\sinh(dx + c)^4 - 4*(a^2b - ab^2)d*\cosh(dx + c)^2 + 8*(7*(a^2b - ab^2)d*\cosh(dx + c)^5 - 10*(a^2b - ab^2)d*\cosh(dx + c)^3 - (8a^3 - 11a^2b + 3ab^2)d*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(7*(a^2b - ab^2)d*\cosh(dx + c)^6 - 15*(a^2b - ab^2)d*\cosh(dx + c)^4 - 3*(8a^3 - 11a^2b + 3ab^2)d*\cosh(dx + c)^2 - (a^2b - ab^2)d)*\sinh(dx + c)^2 + (a^2b - ab^2)d + 8*((a^2b - ab^2)d*\cosh(dx + c)^7 - 3*(a^2b - ab^2)d*\cosh(dx + c)^5 - (8a^3 - 11a^2b + 3ab^2)d*\cosh(dx + c)^3 - (a^2b - ab^2)d*\cosh(dx + c))*\sinh(dx + c))*\sqrt{((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2\sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} + 16a^2 - 15ab + 3b^2)/((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2)}*\log(384a^3b - 680a^2b^2 + 405ab^3 - 81b^4 - 2*(16a^8 - 57a^7b + 75a^6b^2 - 43a^5b^3 + 9a^4b^4)d^2\sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} - (384a^3b - 680a^2b^2 + 405ab^3 - 81b^4)*\cosh(dx + c)^2 - 2*(384a^3b - 680a^2b^2 + 405ab^3 - 81b^4)*\cosh(dx + c)*\sinh(dx + c) - (384a^3b - 680a^2b^2 + 405ab^3 - 81b^4)*\sinh(dx + c)^2 - 2*(2*(2a^{10} - 7a^9b + 9a^8b^2 - 5a^7b^3 + a^6b^4)d^3\sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} - (120a^5b - 217a^4b^2 + 132a^3b^3 - 27a^2b^4)d)*\sqrt{((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2\sqrt{(576a^4b - 1392a^3b^2 + 1273a^2b^3 - 522ab^4 + 81b^5)/((a^{13} - 6a^{12}b + 15a^{11}b^2 - 20a^{10}b^3 + 15a^9b^4 - 6a^8b^5 + a^7b^6)d^4)} + 16a^2 - 15ab + 3b^2)/((a^6 - 3a^5b + 3a^4b^2 - a^3b^3)d^2))} + 16*(3b*\cosh(dx + c)^5 - 2*(8a - 3b)*\cosh(dx +
\end{aligned}$$

$$c)^3 - 5*b*\cosh(d*x + c))*\sinh(d*x + c) + 8*b)/((a^2*b - a*b^2)*d*\cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*\sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*\sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*\cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*\cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*\cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*\cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*\sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*\cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*\cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*\cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [A] time = 4.28869, size = 171, normalized size = 0.81

$$\frac{be^{6dx+6c} - 8ae^{4dx+4c} + 3be^{4dx+4c} - 5be^{2dx+2c} + b}{2(a^2d - abd)(be^{8dx+8c} - 4be^{6dx+6c} - 16ae^{4dx+4c} + 6be^{4dx+4c} - 4be^{2dx+2c} + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out] $\frac{1}{2} * (b * e^{(6 * d * x + 6 * c)} - 8 * a * e^{(4 * d * x + 4 * c)} + 3 * b * e^{(4 * d * x + 4 * c)} - 5 * b * e^{(2 * d * x + 2 * c)} + b) / ((a^2 * d - a * b * d) * (b * e^{(8 * d * x + 8 * c)} - 4 * b * e^{(6 * d * x + 6 * c)} - 16 * a * e^{(4 * d * x + 4 * c)} + 6 * b * e^{(4 * d * x + 4 * c)} - 4 * b * e^{(2 * d * x + 2 * c)} + b))$

$$3.252 \quad \int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^2} dx$$

Optimal. Leaf size=237

$$\frac{\sqrt{b}(6\sqrt{a}-5\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d(\sqrt{a}-\sqrt{b})^{3/2}} + \frac{\sqrt{b}(6\sqrt{a}+5\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{b \tanh(c+dx)}{4a^2d(a-b)((a-b) \tanh(c+dx))}$$

```
[Out] -((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
d*x])/a^(1/4)])/(8*a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((6*Sqrt[a] + 5*
Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/
(8*a^(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Coth[c + d*x]/(a^2*d) + (b*Tanh[c
+ d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - 2*a*Tanh[c + d
*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 0.533692, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3217, 1334, 1664, 1166, 208}

$$\frac{\sqrt{b}(6\sqrt{a}-5\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d(\sqrt{a}-\sqrt{b})^{3/2}} + \frac{\sqrt{b}(6\sqrt{a}+5\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d(\sqrt{a}+\sqrt{b})^{3/2}} + \frac{b \tanh(c+dx)}{4a^2d(a-b)((a-b) \tanh(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2, x]
```

```
[Out] -((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
d*x])/a^(1/4)])/(8*a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((6*Sqrt[a] + 5*
Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/
(8*a^(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Coth[c + d*x]/(a^2*d) + (b*Tanh[c
+ d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - 2*a*Tanh[c + d
*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
```

) / f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff, x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1334

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :=> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && ILtQ[m/2, 0]

Rule 1664

Int[(Pq_)*((d_)*(x_)^(m_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_)), x_Symbol] :=> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :=> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^4}{x^2(a-2ax^2+(a-b)x^4)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b \tanh(c+dx) (a - (a+b) \tanh^2(c+dx))}{4a^2(a-b)d (a - 2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))} - \operatorname{Subst}\left(\int \frac{-8ab + \frac{2a(8a-7b)bx^2}{a-b}}{x^2(a-2ax^2+(a-b)x^4)} dx, x, \tanh(c+dx)\right) \\
&= \frac{b \tanh(c+dx) (a - (a+b) \tanh^2(c+dx))}{4a^2(a-b)d (a - 2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))} - \operatorname{Subst}\left(\int \left(-\frac{8b}{x^2} + \frac{2b^2(a-c)}{(a-b)(a-2ax^2+(a-b)x^4)}\right) dx, x, \tanh(c+dx)\right) \\
&= -\frac{\operatorname{coth}(c+dx)}{a^2d} + \frac{b \tanh(c+dx) (a - (a+b) \tanh^2(c+dx))}{4a^2(a-b)d (a - 2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))} - \frac{b \operatorname{Subst}\left(\int \frac{1}{x} dx, x, \tanh(c+dx)\right)}{a^2d} \\
&= -\frac{\operatorname{coth}(c+dx)}{a^2d} + \frac{b \tanh(c+dx) (a - (a+b) \tanh^2(c+dx))}{4a^2(a-b)d (a - 2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))} + \frac{\left((7a + 2b) \operatorname{arctanh}(\tanh(c+dx))\right)}{a^2d} \\
&= -\frac{(6\sqrt{a} - 5\sqrt{b}) \sqrt{b} \operatorname{tanh}^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4} (\sqrt{a} - \sqrt{b})^{3/2} d} + \frac{(6\sqrt{a} + 5\sqrt{b}) \sqrt{b} \operatorname{tanh}^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4} (\sqrt{a} + \sqrt{b})^{3/2} d}
\end{aligned}$$

Mathematica [A] time = 1.87298, size = 272, normalized size = 1.15

$$\frac{(6a\sqrt{b}+5\sqrt{ab}) \operatorname{tanh}^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \operatorname{tanh}(c+dx)}{\sqrt{\sqrt{a}\sqrt{b+a}}}\right)}{(\sqrt{a}+\sqrt{b})\sqrt{\sqrt{a}\sqrt{b+a}}} + \frac{(6a\sqrt{b}-5\sqrt{ab}) \operatorname{tanh}^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \operatorname{tanh}(c+dx)}{\sqrt{\sqrt{a}\sqrt{b-a}}}\right)}{(\sqrt{a}-\sqrt{b})\sqrt{\sqrt{a}\sqrt{b-a}}} + \frac{4\sqrt{ab} \sinh(2(c+dx))(2a-b \cosh(2(c+dx))+b)}{(a-b)(8a+4b \cosh(2(c+dx))-b \cosh(4(c+dx))-3b)} - 8\sqrt{a} \operatorname{coth}(c+dx)$$

$$8a^{5/2}d$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]

[Out] (((6*a*Sqrt[b] - 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + ((6*a*Sqrt[b] + 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt[b]]) - 8*Sqrt[a]*Coth[c + d*x] + (4*Sqrt[a]*b*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)]/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))

$c + d*x]])))/(8*a^{(5/2)*d}$

Maple [C] time = 0.09, size = 765, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{csch}(d*x+c)^2/(a-b*\sinh(d*x+c)^4)^2, x)$

[Out]
$$-1/2/d/a^2*\tanh(1/2*d*x+1/2*c)+1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})*b/a/(a-b)*\tanh(1/2*d*x+1/2*c)^{7-1/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/a/(a-b)*\tanh(1/2*d*x+1/2*c)^{5*b-2}/d*b^2/a^2/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/(a-b)*\tanh(1/2*d*x+1/2*c)^{5-1/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/a/(a-b)*\tanh(1/2*d*x+1/2*c)^{3*b-2}/d*b^2/a^2/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})/(a-b)*\tanh(1/2*d*x+1/2*c)^{3+1/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4}*\tanh(1/2*d*x+1/2*c)^{6*a+6}*\tanh(1/2*d*x+1/2*c)^{4*a-16}*b*\tanh(1/2*d*x+1/2*c)^{4-4}*\tanh(1/2*d*x+1/2*c)^{2*a+a})*b/a/(a-b)*\tanh(1/2*d*x+1/2*c)-1/16/d*b/a^2/(a-b)*\text{sum}((-R^6*a+(27*a-20*b)*R^4+(-27*a+20*b)*R^2+a)/(-R^7*a-3*R^5*a+3*R^3*a-8*R^3*b-R*a)*\ln(\tanh(1/2*d*x+1/2*c)-R), R=\text{RootOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/2/d/a^2/\tanh(1/2*d*x+1/2*c)$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\text{csch}(d*x+c)^2/(a-b*\sinh(d*x+c)^4)^2, x, \text{algorithm}=\text{"maxima"})$

[Out]
$$1/2*(4*a*b - 5*b^2 + (6*a*b*e^{(8*c)} - 5*b^2*e^{(8*c)})*e^{(8*d*x)} - 2*(13*a*b*e^{(6*c)} - 10*b^2*e^{(6*c)})*e^{(6*d*x)} - 2*(32*a^2*e^{(4*c)} - 47*a*b*e^{(4*c)} +$$

$$15*b^2*e^{(4*c)}*e^{(4*d*x)} - 2*(7*a*b*e^{(2*c)} - 10*b^2*e^{(2*c)})*e^{(2*d*x)}/(a^3*b*d - a^2*b^2*d - (a^3*b*d*e^{(10*c)} - a^2*b^2*d*e^{(10*c)})*e^{(10*d*x)} + 5*(a^3*b*d*e^{(8*c)} - a^2*b^2*d*e^{(8*c)})*e^{(8*d*x)} + 2*(8*a^4*d*e^{(6*c)} - 13*a^3*b*d*e^{(6*c)} + 5*a^2*b^2*d*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^4*d*e^{(4*c)} - 13*a^3*b*d*e^{(4*c)} + 5*a^2*b^2*d*e^{(4*c)})*e^{(4*d*x)} - 5*(a^3*b*d*e^{(2*c)} - a^2*b^2*d*e^{(2*c)})*e^{(2*d*x)}) - 4*\integrate(1/4*((6*a*b*e^{(6*c)} - 5*b^2*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a*b*e^{(4*c)} - 5*b^2*e^{(4*c)})*e^{(4*d*x)} + (6*a*b*e^{(2*c)} - 5*b^2*e^{(2*c)})*e^{(2*d*x)})/(a^3*b - a^2*b^2 + (a^3*b*e^{(8*c)} - a^2*b^2*e^{(8*c)})*e^{(8*d*x)} - 4*(a^3*b*e^{(6*c)} - a^2*b^2*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^4*e^{(4*c)} - 11*a^3*b*e^{(4*c)} + 3*a^2*b^2*e^{(4*c)})*e^{(4*d*x)} - 4*(a^3*b*e^{(2*c)} - a^2*b^2*e^{(2*c)})*e^{(2*d*x)}), x)$$

Fricas [B] time = 4.28504, size = 20254, normalized size = 85.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")`

[Out]
$$\begin{aligned} & -1/16*(8*(6*a*b - 5*b^2)*\cosh(d*x + c)^8 + 64*(6*a*b - 5*b^2)*\cosh(d*x + c) \\ & * \sinh(d*x + c)^7 + 8*(6*a*b - 5*b^2)*\sinh(d*x + c)^8 - 16*(13*a*b - 10*b^2) \\ & * \cosh(d*x + c)^6 + 16*(14*(6*a*b - 5*b^2)*\cosh(d*x + c)^2 - 13*a*b + 10*b^2) \\ & * \sinh(d*x + c)^6 + 32*(14*(6*a*b - 5*b^2)*\cosh(d*x + c)^3 - 3*(13*a*b - 10 \\ & * b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 16*(32*a^2 - 47*a*b + 15*b^2)*\cosh(d \\ & * x + c)^4 + 16*(35*(6*a*b - 5*b^2)*\cosh(d*x + c)^4 - 15*(13*a*b - 10*b^2)*\c \\ & osh(d*x + c)^2 - 32*a^2 + 47*a*b - 15*b^2)*\sinh(d*x + c)^4 + 64*(7*(6*a*b - \\ & 5*b^2)*\cosh(d*x + c)^5 - 5*(13*a*b - 10*b^2)*\cosh(d*x + c)^3 - (32*a^2 - 4 \\ & 7*a*b + 15*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 16*(7*a*b - 10*b^2)*\cosh(d \\ & * x + c)^2 + 16*(14*(6*a*b - 5*b^2)*\cosh(d*x + c)^6 - 15*(13*a*b - 10*b^2)*\c \\ & osh(d*x + c)^4 - 6*(32*a^2 - 47*a*b + 15*b^2)*\cosh(d*x + c)^2 - 7*a*b + 10* \\ & b^2)*\sinh(d*x + c)^2 + ((a^3*b - a^2*b^2)*d*\cosh(d*x + c)^10 + 10*(a^3*b - \\ & a^2*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*\sinh(d*x + c \\ &)^10 - 5*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*\cos \\ & h(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*\sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b + \\ & 5*a^2*b^2)*d*\cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^3 - \\ & (a^3*b - a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b \\ & ^2)*d*\cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 - (8*a^4 - 1 \\ & 3*a^3*b + 5*a^2*b^2)*d)*\sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)* \\ & d*\cosh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^5 - 70*(a^3*b - \\ & a^2*b^2)*d*\cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + \\ & c))*\sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^6 - 175*(a^ \end{aligned}$$

$$\begin{aligned}
& 3*b - a^2*b^2)*d*\cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*\cosh \\
& (d*x + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*\sinh(d*x + c)^4 + 5*(a^3*b \\
& - a^2*b^2)*d*\cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^7 - \\
& 35*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d \\
& *\cosh(d*x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x \\
& + c)^3 + (45*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d \\
& *\cosh(d*x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + c)^4 + 12 \\
& *(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)* \\
& \sinh(d*x + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*\cosh(d*x + \\
& c)^9 - 20*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^ \\
& 2*b^2)*d*\cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + c) \\
& ^3 + 5*(a^3*b - a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-((a^7 - 3*a^ \\
& 6*b + 3*a^5*b^2 - a^4*b^3)*d^2*\sqrt{((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2 \\
& *b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 \\
& + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)} \\
& /((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*\log(1728*a^3*b^2 - 3684*a^2*b \\
& ^3 + 2625*a*b^4 - 625*b^5 + 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b \\
& ^3 + 25*a^5*b^4)*d^2*\sqrt{((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 345 \\
& 0*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11* \\
& b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b \\
& ^4 - 625*b^5)*\cosh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 \\
& - 625*b^5)*\cosh(d*x + c)*\sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 26 \\
& 25*a*b^4 - 625*b^5)*\sinh(d*x + c)^2 + 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - \\
& 22*a^8*b^3 + 5*a^7*b^4)*d^3*\sqrt{((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 \\
& ^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + \\
& 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 2*(144*a^6*b - 303*a^5*b^2 + 21 \\
& 3*a^4*b^3 - 50*a^3*b^4)*d)*\sqrt{-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2 \\
& *\sqrt{((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/ \\
& (a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a \\
& ^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - \\
& a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*\cosh(d*x + c)^10 + 10*(a^3*b - a^2* \\
& b^2)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*\sinh(d*x + c)^10 \\
& - 5*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*\cosh(d* \\
& x + c)^2 - (a^3*b - a^2*b^2)*d)*\sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b + 5*a \\
& ^2*b^2)*d*\cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^3 - (a^ \\
& 3*b - a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b^2)* \\
& d*\cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 - (8*a^4 - 13*a^ \\
& 3*b + 5*a^2*b^2)*d)*\sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*co \\
& sh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^5 - 70*(a^3*b - a^2 \\
& *b^2)*d*\cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*\cosh(d*x + c) \\
& *\sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^6 - 175*(a^3*b \\
& - a^2*b^2)*d*\cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*\cosh(d*x \\
& + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*\sinh(d*x + c)^4 + 5*(a^3*b - a^ \\
& 2*b^2)*d*\cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*\cosh(d*x + c)^7 - 35*(\\
& a^3*b - a^2*b^2)*d*\cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cos
\end{aligned}$$

$$\begin{aligned}
& h(dx + c)^3 + (8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c) \sinh(dx + c) \\
&)^3 + (45(a^3b - a^2b^2)d \cosh(dx + c)^8 - 140(a^3b - a^2b^2)d \cosh(dx + c)^6 - 30(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^4 + 12(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^2 + 5(a^3b - a^2b^2)d) \sinh(dx + c)^2 - (a^3b - a^2b^2)d + 2(5(a^3b - a^2b^2)d \cosh(dx + c)^9 - 20(a^3b - a^2b^2)d \cosh(dx + c)^7 - 6(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^5 + 4(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^3 + 5(a^3b - a^2b^2)d \cosh(dx + c) \sinh(dx + c)) \sqrt{-(a^7 - 3a^6b + 3a^5b^2 - a^4b^3)d^2 \sqrt{((2304a^4b^3 - 6624a^3b^4 + 7161a^2b^5 - 3450ab^6 + 625b^7)/(a^{15} - 6a^{14}b + 15a^{13}b^2 - 20a^{12}b^3 + 15a^{11}b^4 - 6a^{10}b^5 + a^9b^6)d^4)} - 36a^2b + 47ab^2 - 15b^3)/(a^7 - 3a^6b + 3a^5b^2 - a^4b^3)d^2)} \log(1728a^3b^2 - 3684a^2b^3 + 2625ab^4 - 625b^5 + 2(36a^9 - 133a^8b + 183a^7b^2 - 111a^6b^3 + 25a^5b^4)d^2 \sqrt{((2304a^4b^3 - 6624a^3b^4 + 7161a^2b^5 - 3450ab^6 + 625b^7)/(a^{15} - 6a^{14}b + 15a^{13}b^2 - 20a^{12}b^3 + 15a^{11}b^4 - 6a^{10}b^5 + a^9b^6)d^4)} - (1728a^3b^2 - 3684a^2b^3 + 2625ab^4 - 625b^5) \cosh(dx + c)^2 - 2(1728a^3b^2 - 3684a^2b^3 + 2625ab^4 - 625b^5) \cosh(dx + c) \sinh(dx + c) - (1728a^3b^2 - 3684a^2b^3 + 2625ab^4 - 625b^5) \sinh(dx + c)^2 - 2((7a^{11} - 26a^{10}b + 36a^9b^2 - 22a^8b^3 + 5a^7b^4)d^3 \sqrt{((2304a^4b^3 - 6624a^3b^4 + 7161a^2b^5 - 3450ab^6 + 625b^7)/(a^{15} - 6a^{14}b + 15a^{13}b^2 - 20a^{12}b^3 + 15a^{11}b^4 - 6a^{10}b^5 + a^9b^6)d^4)} + 2(144a^6b - 303a^5b^2 + 213a^4b^3 - 50a^3b^4)d) \sqrt{-(a^7 - 3a^6b + 3a^5b^2 - a^4b^3)d^2 \sqrt{((2304a^4b^3 - 6624a^3b^4 + 7161a^2b^5 - 3450ab^6 + 625b^7)/(a^{15} - 6a^{14}b + 15a^{13}b^2 - 20a^{12}b^3 + 15a^{11}b^4 - 6a^{10}b^5 + a^9b^6)d^4)} - 36a^2b + 47ab^2 - 15b^3)/(a^7 - 3a^6b + 3a^5b^2 - a^4b^3)d^2)) - ((a^3b - a^2b^2)d \cosh(dx + c)^{10} + 10(a^3b - a^2b^2)d \cosh(dx + c) \sinh(dx + c)^9 + (a^3b - a^2b^2)d \sinh(dx + c)^{10} - 5(a^3b - a^2b^2)d \cosh(dx + c)^8 + 5(9(a^3b - a^2b^2)d \cosh(dx + c)^2 - (a^3b - a^2b^2)d) \sinh(dx + c)^8 - 2(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^6 + 40(3(a^3b - a^2b^2)d \cosh(dx + c)^3 - (a^3b - a^2b^2)d \cosh(dx + c) \sinh(dx + c))^7 + 2(105(a^3b - a^2b^2)d \cosh(dx + c)^4 - 70(a^3b - a^2b^2)d \cosh(dx + c)^2 - (8a^4 - 13a^3b + 5a^2b^2)d) \sinh(dx + c)^6 + 2(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^4 + 4(63(a^3b - a^2b^2)d \cosh(dx + c)^5 - 70(a^3b - a^2b^2)d \cosh(dx + c)^3 - 3(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c) \sinh(dx + c))^5 + 2(105(a^3b - a^2b^2)d \cosh(dx + c)^6 - 175(a^3b - a^2b^2)d \cosh(dx + c)^4 - 15(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^2 + (8a^4 - 13a^3b + 5a^2b^2)d) \sinh(dx + c)^4 + 5(a^3b - a^2b^2)d \cosh(dx + c)^2 + 8(15(a^3b - a^2b^2)d \cosh(dx + c)^7 - 35(a^3b - a^2b^2)d \cosh(dx + c)^5 - 5(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^3 + (8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c) \sinh(dx + c))^3 + (45(a^3b - a^2b^2)d \cosh(dx + c)^8 - 140(a^3b - a^2b^2)d \cosh(dx + c)^6 - 30(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^4 + 12(8a^4 - 13a^3b + 5a^2b^2)d \cosh(dx + c)^2 + 5(a^3b - a^2b^2)d) \sinh(dx + c)^2
\end{aligned}$$

$$\begin{aligned}
& + c)^2 - (a^3b - a^2b^2)d + 2*(5*(a^3b - a^2b^2)*d*\cosh(d*x + c)^9 - \\
& 20*(a^3b - a^2b^2)*d*\cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3b + 5*a^2b^2)*d \\
& *\cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3b + 5*a^2b^2)*d*\cosh(d*x + c)^3 + 5*(\\
& a^3b - a^2b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^7 - 3*a^6b + 3*a \\
& ^5b^2 - a^4b^3)*d^2*\sqrt{((2304*a^4b^3 - 6624*a^3b^4 + 7161*a^2b^5 - 34 \\
& 50*a*b^6 + 625*b^7)/((a^15 - 6*a^14b + 15*a^13b^2 - 20*a^12b^3 + 15*a^11 \\
& *b^4 - 6*a^10b^5 + a^9b^6)*d^4)) + 36*a^2b - 47*a*b^2 + 15*b^3)/((a^7 - \\
& 3*a^6b + 3*a^5b^2 - a^4b^3)*d^2))*\log(1728*a^3b^2 - 3684*a^2b^3 + 2625 \\
& *a*b^4 - 625*b^5 - 2*(36*a^9 - 133*a^8b + 183*a^7b^2 - 111*a^6b^3 + 25*a \\
& ^5b^4)*d^2*\sqrt{((2304*a^4b^3 - 6624*a^3b^4 + 7161*a^2b^5 - 3450*a*b^6 + \\
& 625*b^7)/((a^15 - 6*a^14b + 15*a^13b^2 - 20*a^12b^3 + 15*a^11b^4 - 6*a \\
& ^10b^5 + a^9b^6)*d^4)) - (1728*a^3b^2 - 3684*a^2b^3 + 2625*a*b^4 - 625*b \\
& ^5)*\cosh(d*x + c)^2 - 2*(1728*a^3b^2 - 3684*a^2b^3 + 2625*a*b^4 - 625*b \\
& ^5)*\cosh(d*x + c)*\sinh(d*x + c) - (1728*a^3b^2 - 3684*a^2b^3 + 2625*a*b^4 \\
& - 625*b^5)*\sinh(d*x + c)^2 + 2*((7*a^11 - 26*a^10b + 36*a^9b^2 - 22*a^8b \\
& ^3 + 5*a^7b^4)*d^3*\sqrt{((2304*a^4b^3 - 6624*a^3b^4 + 7161*a^2b^5 - 3450 \\
& *a*b^6 + 625*b^7)/((a^15 - 6*a^14b + 15*a^13b^2 - 20*a^12b^3 + 15*a^11b \\
& ^4 - 6*a^10b^5 + a^9b^6)*d^4)) - 2*(144*a^6b - 303*a^5b^2 + 213*a^4b^3 \\
& - 50*a^3b^4)*d)*\sqrt{((a^7 - 3*a^6b + 3*a^5b^2 - a^4b^3)*d^2*\sqrt{((230 \\
& 4*a^4b^3 - 6624*a^3b^4 + 7161*a^2b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6* \\
& a^14b + 15*a^13b^2 - 20*a^12b^3 + 15*a^11b^4 - 6*a^10b^5 + a^9b^6)*d^ \\
& 4)) + 36*a^2b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6b + 3*a^5b^2 - a^4b^3)* \\
& d^2)) + ((a^3b - a^2b^2)*d*\cosh(d*x + c)^10 + 10*(a^3b - a^2b^2)*d*cos \\
& h(d*x + c)*\sinh(d*x + c)^9 + (a^3b - a^2b^2)*d*\sinh(d*x + c)^10 - 5*(a^3* \\
& b - a^2b^2)*d*\cosh(d*x + c)^8 + 5*(9*(a^3b - a^2b^2)*d*\cosh(d*x + c)^2 - \\
& (a^3b - a^2b^2)*d)*\sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3b + 5*a^2b^2)*d* \\
& \cosh(d*x + c)^6 + 40*(3*(a^3b - a^2b^2)*d*\cosh(d*x + c)^3 - (a^3b - a^2* \\
& b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^3b - a^2b^2)*d*\cosh(d*x \\
& + c)^4 - 70*(a^3b - a^2b^2)*d*\cosh(d*x + c)^2 - (8*a^4 - 13*a^3b + 5*a^ \\
& 2b^2)*d)*\sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3b + 5*a^2b^2)*d*\cosh(d*x + c \\
&)^4 + 4*(63*(a^3b - a^2b^2)*d*\cosh(d*x + c)^5 - 70*(a^3b - a^2b^2)*d*co \\
& sh(d*x + c)^3 - 3*(8*a^4 - 13*a^3b + 5*a^2b^2)*d*\cosh(d*x + c))*\sinh(d*x \\
& + c)^5 + 2*(105*(a^3b - a^2b^2)*d*\cosh(d*x + c)^6 - 175*(a^3b - a^2b^2) \\
& *d*\cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3b + 5*a^2b^2)*d*\cosh(d*x + c)^2 + \\
& (8*a^4 - 13*a^3b + 5*a^2b^2)*d)*\sinh(d*x + c)^4 + 5*(a^3b - a^2b^2)*d*c \\
& osh(d*x + c)^2 + 8*(15*(a^3b - a^2b^2)*d*\cosh(d*x + c)^7 - 35*(a^3b - a^ \\
& 2b^2)*d*\cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3b + 5*a^2b^2)*d*\cosh(d*x + c) \\
& ^3 + (8*a^4 - 13*a^3b + 5*a^2b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (45* \\
& (a^3b - a^2b^2)*d*\cosh(d*x + c)^8 - 140*(a^3b - a^2b^2)*d*\cosh(d*x + c) \\
& ^6 - 30*(8*a^4 - 13*a^3b + 5*a^2b^2)*d*\cosh(d*x + c)^4 + 12*(8*a^4 - 13*a \\
& ^3b + 5*a^2b^2)*d*\cosh(d*x + c)^2 + 5*(a^3b - a^2b^2)*d)*\sinh(d*x + c)^ \\
& 2 - (a^3b - a^2b^2)*d + 2*(5*(a^3b - a^2b^2)*d*\cosh(d*x + c)^9 - 20*(a^ \\
& 3b - a^2b^2)*d*\cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3b + 5*a^2b^2)*d*\cosh(\\
& d*x + c)^5 + 4*(8*a^4 - 13*a^3b + 5*a^2b^2)*d*\cosh(d*x + c)^3 + 5*(a^3b \\
& - a^2b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{((a^7 - 3*a^6b + 3*a^5b^2
\end{aligned}$$

$$\begin{aligned}
& - a^4 b^3) d^2 \sqrt{(2304 a^4 b^3 - 6624 a^3 b^4 + 7161 a^2 b^5 - 3450 a b^6 + 625 b^7) / ((a^{15} - 6 a^{14} b + 15 a^{13} b^2 - 20 a^{12} b^3 + 15 a^{11} b^4 - 6 a^{10} b^5 + a^9 b^6) d^4)} + 36 a^2 b - 47 a b^2 + 15 b^3) / ((a^7 - 3 a^6 b + 3 a^5 b^2 - a^4 b^3) d^2) * \log(1728 a^3 b^2 - 3684 a^2 b^3 + 2625 a b^4 - 625 b^5 - 2 * (36 a^9 - 133 a^8 b + 183 a^7 b^2 - 111 a^6 b^3 + 25 a^5 b^4) * d^2 \sqrt{(2304 a^4 b^3 - 6624 a^3 b^4 + 7161 a^2 b^5 - 3450 a b^6 + 625 b^7) / ((a^{15} - 6 a^{14} b + 15 a^{13} b^2 - 20 a^{12} b^3 + 15 a^{11} b^4 - 6 a^{10} b^5 + a^9 b^6) d^4)} - (1728 a^3 b^2 - 3684 a^2 b^3 + 2625 a b^4 - 625 b^5) * \cosh(d x + c)^2 - 2 * (1728 a^3 b^2 - 3684 a^2 b^3 + 2625 a b^4 - 625 b^5) * \cosh(d x + c) * \sinh(d x + c) - (1728 a^3 b^2 - 3684 a^2 b^3 + 2625 a b^4 - 625 b^5) * \sinh(d x + c)^2 - 2 * ((7 a^{11} - 26 a^{10} b + 36 a^9 b^2 - 22 a^8 b^3 + 5 a^7 b^4) * d^3 \sqrt{(2304 a^4 b^3 - 6624 a^3 b^4 + 7161 a^2 b^5 - 3450 a b^6 + 625 b^7) / ((a^{15} - 6 a^{14} b + 15 a^{13} b^2 - 20 a^{12} b^3 + 15 a^{11} b^4 - 6 a^{10} b^5 + a^9 b^6) d^4)} - 2 * (144 a^6 b - 303 a^5 b^2 + 213 a^4 b^3 - 50 a^3 b^4) * d) * \sqrt{((a^7 - 3 a^6 b + 3 a^5 b^2 - a^4 b^3) d^2 \sqrt{(2304 a^4 b^3 - 6624 a^3 b^4 + 7161 a^2 b^5 - 3450 a b^6 + 625 b^7) / ((a^{15} - 6 a^{14} b + 15 a^{13} b^2 - 20 a^{12} b^3 + 15 a^{11} b^4 - 6 a^{10} b^5 + a^9 b^6) d^4)} + 36 a^2 b - 47 a b^2 + 15 b^3) / ((a^7 - 3 a^6 b + 3 a^5 b^2 - a^4 b^3) d^2))} + 32 a b - 40 b^2 + 32 * (2 * (6 a b - 5 b^2) * \cosh(d x + c)^7 - 3 * (13 a b - 10 b^2) * \cosh(d x + c)^5 - 2 * (32 a^2 - 47 a b + 15 b^2) * \cosh(d x + c)^3 - (7 a b - 10 b^2) * \cosh(d x + c) * \sinh(d x + c)) / ((a^3 b - a^2 b^2) * d * \cosh(d x + c)^{10} + 10 * (a^3 b - a^2 b^2) * d * \cosh(d x + c) * \sinh(d x + c)^9 + (a^3 b - a^2 b^2) * d * \sinh(d x + c)^{10} - 5 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^8 + 5 * (9 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^2 - (a^3 b - a^2 b^2) * d) * \sinh(d x + c)^8 - 2 * (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c)^6 + 40 * (3 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^3 - (a^3 b - a^2 b^2) * d * \cosh(d x + c) * \sinh(d x + c)^7 + 2 * (105 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^4 - 70 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^2 - (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d) * \sinh(d x + c)^6 + 2 * (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c)^4 + 4 * (63 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^5 - 70 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^3 - 3 * (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c) * \sinh(d x + c)^5 + 2 * (105 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^6 - 175 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^4 - 15 * (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c)^2 + (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d) * \sinh(d x + c)^4 + 5 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^2 + 8 * (15 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^7 - 35 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^5 - 5 * (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c)^3 + (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c) * \sinh(d x + c)^3 + (45 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^8 - 140 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^6 - 30 * (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c)^4 + 12 * (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c)^2 + 5 * (a^3 b - a^2 b^2) * d) * \sinh(d x + c)^2 - (a^3 b - a^2 b^2) * d + 2 * (5 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^9 - 20 * (a^3 b - a^2 b^2) * d * \cosh(d x + c)^7 - 6 * (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c)^5 + 4 * (8 a^4 - 13 a^3 b + 5 a^2 b^2) * d * \cosh(d x + c)^3 + 5 * (a^3 b - a^2 b^2) * d * \cosh(d x + c) * \sinh(d x + c)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)

[Out] Timed out

Giac [A] time = 5.79865, size = 321, normalized size = 1.35

$$\frac{6abe^{(8dx+8c)} - 5b^2e^{(8dx+8c)} - 26abe^{(6dx+6c)} + 20b^2e^{(6dx+6c)} - 64a^2e^{(4dx+4c)} + 94abe^{(4dx+4c)} - 30b^2e^{(4dx+4c)} - 14abe^{(2dx+2c)} + 4a^2b - 5b^2}{2(a^3d - a^2bd)(be^{(10dx+10c)} - 5be^{(8dx+8c)} - 16ae^{(6dx+6c)} + 10be^{(6dx+6c)} + 16ae^{(4dx+4c)} - 10be^{(4dx+4c)} - 5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")

[Out]
$$\begin{aligned} & -1/2*(6*a*b*e^{(8*d*x + 8*c)} - 5*b^2*e^{(8*d*x + 8*c)} - 26*a*b*e^{(6*d*x + 6*c)} \\ & + 20*b^2*e^{(6*d*x + 6*c)} - 64*a^2*e^{(4*d*x + 4*c)} + 94*a*b*e^{(4*d*x + 4*c)} \\ & - 30*b^2*e^{(4*d*x + 4*c)} - 14*a*b*e^{(2*d*x + 2*c)} + 20*b^2*e^{(2*d*x + 2*c)} \\ & + 4*a*b - 5*b^2)/((a^3*d - a^2*b*d)*(b*e^{(10*d*x + 10*c)} - 5*b*e^{(8*d*x + 8*c)} \\ & - 16*a*e^{(6*d*x + 6*c)} + 10*b*e^{(6*d*x + 6*c)} + 16*a*e^{(4*d*x + 4*c)} \\ & - 10*b*e^{(4*d*x + 4*c)} + 5*b*e^{(2*d*x + 2*c)} - b)) \end{aligned}$$

$$3.253 \quad \int \frac{\sinh^9(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^3} dx$$

Optimal. Leaf size=315

$$\frac{\cosh(c+dx) \left(9a^2 - 2b(2a-5b) \cosh^2(c+dx) - 11ab - 10b^2\right)}{32b^2d(a-b)^2 \left(a-b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b\right)} + \frac{a \cosh(c+dx) \left(a-b \cosh^2(c+dx) + b\right)}{8b^2d(a-b) \left(a-b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b\right)}$$

```
[Out] ((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) + ((5*a + 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) + (a*Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(2*a - 5*b)*b*Cosh[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.577358, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3215, 1205, 1678, 1166, 205, 208}

$$\frac{\cosh(c+dx) \left(9a^2 - 2b(2a-5b) \cosh^2(c+dx) - 11ab - 10b^2\right)}{32b^2d(a-b)^2 \left(a-b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b\right)} + \frac{a \cosh(c+dx) \left(a-b \cosh^2(c+dx) + b\right)}{8b^2d(a-b) \left(a-b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) + ((5*a + 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) + (a*Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(2*a - 5*b)*b*Cosh[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

$\text{Int}[(a_ + (b_ .)*(x_)^2)^{-1}, x_Symbol] \text{ :> Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \text{ /; FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\sinh^9(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^4}{(a-b+2bx^2-bx^4)^3} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{a \cosh(c+dx)(a+b-b\cosh^2(c+dx))}{8(a-b)b^2d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a(a^2+ab-8b^2)}{b}-2a(11a-b)}{(a-b+2bx^2-bx^4)^3} dx, x, \cosh(c+dx)\right)}{16b^2d} \\ &= \frac{a \cosh(c+dx)(a+b-b\cosh^2(c+dx))}{8(a-b)b^2d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} - \frac{\cosh(c+dx)(9a^2-11ab-b^2)}{32(a-b)^2b^2d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} \\ &= \frac{a \cosh(c+dx)(a+b-b\cosh^2(c+dx))}{8(a-b)b^2d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} - \frac{\cosh(c+dx)(9a^2-11ab-b^2)}{32(a-b)^2b^2d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} \\ &= \frac{(5a-14\sqrt{a}\sqrt{b}+12b) \tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}(\sqrt{a}-\sqrt{b})^{5/2}b^{9/4}d} + \frac{(5a+14\sqrt{a}\sqrt{b}+12b) \tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}(\sqrt{a}+\sqrt{b})^{5/2}b^{9/4}d} \end{aligned}$$

Mathematica [C] time = 1.68906, size = 1021, normalized size = 3.24

result too large to display

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] ((32*Cosh[c + d*x]*(-9*a^2 + 13*a*b + 5*b^2 + (2*a - 5*b)*b*Cosh[2*(c + d*x)])))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a - b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 - RootSum[b - 4*b*#1^2 - 16*a*#1^4 +

$$\begin{aligned}
& 6*b^{1^4} - 4*b^{1^6} + b^{1^8} \& , (-2*a*b*c + 5*b^2*c - 2*a*b*d*x + 5*b^2*d*x \\
& x - 4*a*b*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 \\
& - \text{Sinh}[(c + d*x)/2]*\#1] + 10*b^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] \\
&] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x)/2]*\#1] - 10*a^2*c*\#1^2 + 28*a*b*c \\
& *\#1^2 - 39*b^2*c*\#1^2 - 10*a^2*d*x*\#1^2 + 28*a*b*d*x*\#1^2 - 39*b^2*d*x*\#1^2 \\
& - 20*a^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 \\
& - \text{Sinh}[(c + d*x)/2]*\#1]*\#1^2 + 56*a*b*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d \\
& *x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x)/2]*\#1]*\#1^2 - 78*b^2*\text{Log}[-\text{Co} \\
& sh[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x) \\
& /2]*\#1]*\#1^2 + 10*a^2*c*\#1^4 - 28*a*b*c*\#1^4 + 39*b^2*c*\#1^4 + 10*a^2*d*x*\# \\
& 1^4 - 28*a*b*d*x*\#1^4 + 39*b^2*d*x*\#1^4 + 20*a^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{S} \\
& inh[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x)/2]*\#1]*\#1^4 - 56*a \\
& *b*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh} \\
& [(c + d*x)/2]*\#1]*\#1^4 + 78*b^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] \\
& + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x)/2]*\#1]*\#1^4 + 2*a*b*c*\#1^6 - 5*b^2*c \\
& *\#1^6 + 2*a*b*d*x*\#1^6 - 5*b^2*d*x*\#1^6 + 4*a*b*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{S} \\
& inh[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x)/2]*\#1]*\#1^6 - 10*b \\
& ^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh} \\
& [(c + d*x)/2]*\#1]*\#1^6)/(- (b*\#1) - 8*a*\#1^3 + 3*b*\#1^3 - 3*b*\#1^5 + b*\#1^7) \\
& \&])/(128*(a - b)^2*b^2*d)
\end{aligned}$$

Maple [B] time = 0.099, size = 3542, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(d*x+c)^9/(a-b*\sinh(d*x+c)^4)^3,x)$

[Out] $\begin{aligned}
& -1/16/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(-2*\tan \\
& h(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}+2*a)/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)})*(a*b)^{(\\
& 1/2)}-77/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2* \\
& d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^ \\
& 2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^2-11/16/d/(\tanh(1/2*d*x+1/2*c)^8*a- \\
& 4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c \\
&)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c \\
&)^14-35/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2* \\
& d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^ \\
& 3/b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^12+85/16/d/(\tanh(1/2*d*x+1/2*c)^8 \\
& *a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/ \\
& 2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/ \\
& 2*c)^12+105/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(
\end{aligned}$

$$\begin{aligned}
& \frac{1}{2}d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{10}*a^3-11/64/d/b/(a^2-2*a*b+b^2)* \\
& a/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)})-5/64/d/b^2/(a^2-2*a*b+b^2)*a^2/(-a*b \\
& -(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}+2*a)/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)})+11/64/d/b/(a^2-2*a*b+b^2)*a/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}+2*a)/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)})+35/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8*a^3+5/32/d/b/(a^2-2*a*b+b^2)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)})*(a*b)^{(1/2)}+5/32/d/b/(a^2-2*a*b+b^2)/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}+2*a)/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)})*(a*b)^{(1/2)}+5/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{14}+865/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8*a^2+175/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^6*a^3-849/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^6*a^2-105/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4+383/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4-407/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{10}*a^2+5/64/d/b^2/(a^2-2*a*b+b^2)*a^2/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)})-1/16/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)})*(a*b)^{(1/2)}+88/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8+20/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^6-5/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)+11/1
\end{aligned}$$

$$\begin{aligned}
& 6/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)}} \\
&)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}^{2*a^2/b/(a^2-2*a*b+b^2)} \\
& -106/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)}} \\
&)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}^{2/(a^2-2*a*b+b^2)} \\
& * \tanh(1/2*d*x+1/2*c)^{8*a+189/4}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)}} \\
&)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}^{2/(a^2-2*a*b+b^2)} * \tanh(1/2*d*x+1/2*c)^{6*a-31/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)}} \\
&)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}^{2*a/(a^2-2*a*b+b^2)} \\
& * \tanh(1/2*d*x+1/2*c)^{4-3/4}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)}} \\
&)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}^{2*a/(a^2-2*a*b+b^2)} * \tanh(1/2*d*x+1/2*c)^{2+3/4}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)}} \\
&)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}^{2*a/(a^2-2*a*b+b^2)} * \tanh(1/2*d*x+1/2*c)^{14-13/2}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)}} \\
&)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}^{2*a/(a^2-2*a*b+b^2)} * \tanh(1/2*d*x+1/2*c)^{12+163/4}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)}} \\
&)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}^{2/(a^2-2*a*b+b^2)} * \tanh(1/2*d*x+1/2*c)^{10*a-20}/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^{6*a+6*\tanh(1/2*d*x+1/2*c)}} \\
&)^{4*a-16*b*\tanh(1/2*d*x+1/2*c)^{4-4*\tanh(1/2*d*x+1/2*c)^{2*a+a}}^{2*b/(a^2-2*a*b+b^2)} * \tanh(1/2*d*x+1/2*c)^{10-3/16}/d/(a^2-2*a*b+b^2)/(-a*b-(a*b)^{(1/2)*a}^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)+2*a})/(-a*b-(a*b)^{(1/2)*a}^{(1/2)})+3/16}/d/(a^2-2*a*b+b^2)/(-a*b+(a*b)^{(1/2)*a}^{(1/2)*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)-2*a})/(-a*b+(a*b)^{(1/2)*a}^{(1/2)})}
\end{aligned}$$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] Timed out

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**9/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.254 \quad \int \frac{\sinh^7(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$$

Optimal. Leaf size=290

$$\frac{3(\sqrt{a}-2\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{ab}^{7/4}d(\sqrt{a}-\sqrt{b})^{5/2}} - \frac{3(\sqrt{a}+2\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{ab}^{7/4}d(\sqrt{a}+\sqrt{b})^{5/2}} + \frac{\cosh(c+dx)(-3(a-3b)\cosh^2(c+dx)-32bd(a-b)^2(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)))}{32bd(a-b)^2(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx))}$$

[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) + (Cosh[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cosh[c + d*x]^2))/(32*(a - b)^2*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rubi [A] time = 0.467115, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$\frac{3(\sqrt{a}-2\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{ab}^{7/4}d(\sqrt{a}-\sqrt{b})^{5/2}} - \frac{3(\sqrt{a}+2\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{ab}^{7/4}d(\sqrt{a}+\sqrt{b})^{5/2}} + \frac{\cosh(c+dx)(-3(a-3b)\cosh^2(c+dx)-32bd(a-b)^2(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)))}{32bd(a-b)^2(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) + (Cosh[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cosh[c + d*x]^2))/(32*(a - b)^2*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
```

Rt[-(a/b), 2]]/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^7(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a-b+2bx^2-bx^4)^3} dx, x, \cosh(c+dx)\right)}{d} \\
 &= -\frac{a \cosh(c+dx) (2 - \cosh^2(c+dx))}{8(a-b)bd (a-b+2b \cosh^2(c+dx) - b \cosh^4(c+dx))^2} + \frac{\text{Subst}\left(\int \frac{4a(a-4b)-2a(3a-8b)x^2}{(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{16a(a-b)} \\
 &= -\frac{a \cosh(c+dx) (2 - \cosh^2(c+dx))}{8(a-b)bd (a-b+2b \cosh^2(c+dx) - b \cosh^4(c+dx))^2} + \frac{\cosh(c+dx) (5a - 17b)}{32(a-b)^2bd (a-b+2b \cosh^2(c+dx) - b \cosh^4(c+dx))} \\
 &= -\frac{a \cosh(c+dx) (2 - \cosh^2(c+dx))}{8(a-b)bd (a-b+2b \cosh^2(c+dx) - b \cosh^4(c+dx))^2} + \frac{\cosh(c+dx) (5a - 17b)}{32(a-b)^2bd (a-b+2b \cosh^2(c+dx) - b \cosh^4(c+dx))} \\
 &= \frac{3(\sqrt{a}-2\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}(\sqrt{a}-\sqrt{b})^{5/2} b^{7/4} d} - \frac{3(\sqrt{a}+2\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}(\sqrt{a}+\sqrt{b})^{5/2} b^{7/4} d} - \frac{a}{8(a-b)bd}
 \end{aligned}$$

Mathematica [C] time = 1.29495, size = 802, normalized size = 2.77

$$\frac{32 \cosh(c+dx)(-7a+25b+3(a-3b) \cosh(2(c+dx)))}{8a-3b+4b \cosh(2(c+dx))-b \cosh(4(c+dx))} - 3\text{RootSum}\left[b\#1^8 - 4b\#1^6 - 16a\#1^4 + 6b\#1^4 - 4b\#1^2 + b\&, \frac{-ac\#1^6+3bc\#1^6-adx\#1^6}{\dots}\right]$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3, x]

[Out] ((-32*Cosh[c + d*x]*(-7*a + 25*b + 3*(a - 3*b)*Cosh[2*(c + d*x)])))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a - b)*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)])))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 - 3*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (a*c - 3*b*c + a*d*x - 3*b*d*x + 2*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 6*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +

$$\begin{aligned} & d*x)/2]*\#1] - 3*a*c*\#1^2 + 17*b*c*\#1^2 - 3*a*d*x*\#1^2 + 17*b*d*x*\#1^2 - 6* \\ & a*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh} \\ & (c + d*x)/2]*\#1]*\#1^2 + 34*b*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{C} \\ & \text{osh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x)/2]*\#1]*\#1^2 + 3*a*c*\#1^4 - 17*b*c*\#1^4 \\ & + 3*a*d*x*\#1^4 - 17*b*d*x*\#1^4 + 6*a*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d* \\ & x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x)/2]*\#1]*\#1^4 - 34*b*\text{Log}[-\text{Cosh} \\ & (c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x)/2] \\ & *\#1]*\#1^4 - a*c*\#1^6 + 3*b*c*\#1^6 - a*d*x*\#1^6 + 3*b*d*x*\#1^6 - 2*a*\text{Log}[-\text{Co} \\ & \text{sh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]*\#1 - \text{Sinh}[(c + d*x) \\ & /2]*\#1]*\#1^6 + 6*b*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d \\ & *x)/2]*\#1 - \text{Sinh}[(c + d*x)/2]*\#1]*\#1^6)/(- (b*\#1) - 8*a*\#1^3 + 3*b*\#1^3 - 3* \\ & b*\#1^5 + b*\#1^7) \&])/(256*(a - b)^2*b*d) \end{aligned}$$

Maple [B] time = 0.088, size = 2563, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(d*x+c)^7/(a-b*\sinh(d*x+c)^4)^3, x)$

[Out]
$$\begin{aligned} & 3/64/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*a}\arctan(1/4*(-2*\tanh \\ & (1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a)/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*a})^{(1/2)*a} \\ & +1/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1 \\ & /2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(\\ & a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^2-32/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1 \\ & /2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*ta \\ & nh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8-3/8/ \\ & d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^ \\ & 4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2* \\ & a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{12}-9/64/d/b/(a^2-2*a*b+b^2)/(-a*b+(a*b)^{(1/2)* \\ & a})^{(1/2)*a}\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a)/(-a*b+(a* \\ & b)^{(1/2)*a})^{(1/2)*a})^{(1/2)*a})^{(1/2)*a}-9/64/d/b/(a^2-2*a*b+b^2)/(-a*b-(a*b)^{(1/2)*a} \\ &)^{(1/2)*a}\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a)/(-a*b-(a* \\ & b)^{(1/2)*a})^{(1/2)*a})^{(1/2)*a})^{(1/2)*a}-35/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d \\ & *x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1 \\ & /2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8*a^2+5/d/(\tan \\ & h(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16 \\ & *b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*t \\ & anh(1/2*d*x+1/2*c)^6*a^2-25/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2 \\ & *c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x \\ & +1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4+2/d/(\tanh(1/2* \end{aligned}$$

$$\begin{aligned}
& d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^10*a^2+3/64/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)} \\
& *\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)} \\
&)*(a*b)^{(1/2)}+8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6 \\
& *a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8-42/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^6-1/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)+13/4/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8*a+95/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^6*a-27/4/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4+19/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^2-3/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^14+15/4/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^12-111/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^10*a+10/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^10-1/4/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)+8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4+3/32/d/(a^2-2*a*b+b^2)/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}+2*a)/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)})-3/32/d/(a^2-2*a*b+b^2)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)})
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out]
$$\frac{1}{16} \left(3(a b e^{15c} - 3b^2 e^{15c}) e^{15dx} - (23 a b e^{13c} - 77 b^2 e^{13c}) e^{13dx} + (16 a^2 e^{11c} + 131 a b e^{11c} - 177 b^2 e^{11c}) e^{11dx} - (144 a^2 e^{9c} + 367 a b e^{9c} - 109 b^2 e^{9c}) e^{9dx} - (144 a^2 e^{7c} + 367 a b e^{7c} - 109 b^2 e^{7c}) e^{7dx} + (16 a^2 e^{5c} + 131 a b e^{5c} - 177 b^2 e^{5c}) e^{5dx} - (23 a b e^{3c} - 77 b^2 e^{3c}) e^{3dx} + 3(a b e^c - 3b^2 e^c) e^{dx} \right) / (a^2 b^3 d - 2 a b^4 d + b^5 d + (a^2 b^3 d e^{16c} - 2 a b^4 d e^{16c} + b^5 d e^{16c}) e^{16dx} - 8(a^2 b^3 d e^{14c} - 2 a b^4 d e^{14c} + b^5 d e^{14c}) e^{14dx} - 4(8 a^3 b^2 d e^{12c} - 23 a^2 b^3 d e^{12c} + 22 a b^4 d e^{12c} - 7 b^5 d e^{12c}) e^{12dx} + 8(16 a^3 b^2 d e^{10c} - 39 a^2 b^3 d e^{10c} + 30 a b^4 d e^{10c} - 7 b^5 d e^{10c}) e^{10dx} + 2(128 a^4 b d e^{8c} - 352 a^3 b^2 d e^{8c} + 355 a^2 b^3 d e^{8c} - 166 a b^4 d e^{8c} + 35 b^5 d e^{8c}) e^{8dx} + 8(16 a^3 b^2 d e^{6c} - 39 a^2 b^3 d e^{6c} + 30 a b^4 d e^{6c} - 7 b^5 d e^{6c}) e^{6dx} - 4(8 a^3 b^2 d e^{4c} - 23 a^2 b^3 d e^{4c} + 22 a b^4 d e^{4c} - 7 b^5 d e^{4c}) e^{4dx} - 8(a^2 b^3 d e^{2c} - 2 a b^4 d e^{2c} + b^5 d e^{2c}) e^{2dx} \Big) + \frac{1}{128} \int (24(a e^{7c} - 3 b e^{7c}) e^{7dx} - (3 a e^{5c} - 17 b e^{5c}) e^{5dx} + (3 a e^{3c} - 17 b e^{3c}) e^{3dx} - (a e^c - 3 b e^c) e^{dx}) / (a^2 b^2 - 2 a b^3 + b^4 + (a^2 b^2 e^{8c} - 2 a b^3 e^{8c} + b^4 e^{8c}) e^{8dx} - 4(a^2 b^2 e^{6c} - 2 a b^3 e^{6c} + b^4 e^{6c}) e^{6dx} - 2(8 a^3 b e^{4c} - 19 a^2 b^2 e^{4c} + 14 a b^3 e^{4c} - 3 b^4 e^{4c}) e^{4dx} - 4(a^2 b^2 e^{2c} - 2 a b^3 e^{2c} + b^4 e^{2c}) e^{2dx}) dx$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```


$$3.255 \quad \int \frac{\sinh^5(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$$

Optimal. Leaf size=313

$$\frac{\cosh(c+dx)(a^2+2b(2a+b)\cosh^2(c+dx)-11ab-2b^2)}{32abd(a-b)^2(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b)} \frac{(-10\sqrt{a}\sqrt{b}+3a+4b)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d(\sqrt{a}-\sqrt{b})^{5/2}} (10\sqrt{a}\sqrt{b})$$

[Out] -((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) - ((3*a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a + b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rubi [A] time = 0.498432, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$\frac{\cosh(c+dx)(a^2+2b(2a+b)\cosh^2(c+dx)-11ab-2b^2)}{32abd(a-b)^2(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b)} \frac{(-10\sqrt{a}\sqrt{b}+3a+4b)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d(\sqrt{a}-\sqrt{b})^{5/2}} (10\sqrt{a}\sqrt{b})$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] -((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) - ((3*a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a + b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\sinh^5(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{(a-b+2bx^2-bx^4)^3} dx, x, \cosh(c+dx)\right)}{d} \\ &= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{8(a-b)bd(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{2a(a-7b)+10abx^2}{(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{16a(a-b)bd} \\ &= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{8(a-b)bd(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} - \frac{\cosh(c+dx)(a^2-11ab-2b^2)}{32a(a-b)^2bd(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} \\ &= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{8(a-b)bd(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} - \frac{\cosh(c+dx)(a^2-11ab-2b^2)}{32a(a-b)^2bd(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} \\ &= -\frac{(3a-10\sqrt{a}\sqrt{b}+4b)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}(\sqrt{a}-\sqrt{b})^{5/2}b^{5/4}d} - \frac{(3a+10\sqrt{a}\sqrt{b}+4b)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}(\sqrt{a}+\sqrt{b})^{5/2}b^{5/4}d} \end{aligned}$$

Mathematica [C] time = 1.90983, size = 1019, normalized size = 3.26

result too large to display

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] -((32*Cosh[c + d*x]*(a^2 - 9*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x)]))/(a*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) - (512*(a - b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (2*a*b*c + b^2*c + 2*a*b*d*x + b^2*d*x + 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a^2*c*#1^2 - 32*a*b*c*#1^2 + 5*b^2*c*#1^2))

$$\begin{aligned}
& 2*c\#1^2 + 6*a^2*d*x\#1^2 - 32*a*b*d*x\#1^2 + 5*b^2*d*x\#1^2 + 12*a^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]\#1 - \text{Sinh}[(c + d*x)/2]\#1]\#1^2 - 64*a*b*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]\#1 - \text{Sinh}[(c + d*x)/2]\#1]\#1^2 + 10*b^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]\#1 - \text{Sinh}[(c + d*x)/2]\#1]\#1^2 - 6*a^2*c\#1^4 + 32*a*b*c\#1^4 - 5*b^2*c\#1^4 - 6*a^2*d*x\#1^4 + 32*a*b*d*x\#1^4 - 5*b^2*d*x\#1^4 - 12*a^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]\#1 - \text{Sinh}[(c + d*x)/2]\#1]\#1^4 + 64*a*b*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]\#1 - \text{Sinh}[(c + d*x)/2]\#1]\#1^4 - 10*b^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]\#1 - \text{Sinh}[(c + d*x)/2]\#1]\#1^4 - 2*a*b*c\#1^6 - b^2*c\#1^6 - 2*a*b*d*x\#1^6 - b^2*d*x\#1^6 - 4*a*b*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]\#1 - \text{Sinh}[(c + d*x)/2]\#1]\#1^6 - 2*b^2*\text{Log}[-\text{Cosh}[(c + d*x)/2] - \text{Sinh}[(c + d*x)/2] + \text{Cosh}[(c + d*x)/2]\#1 - \text{Sinh}[(c + d*x)/2]\#1]\#1^6)/(- (b\#1) - 8*a\#1^3 + 3*b\#1^3 - 3*b\#1^5 + b\#1^7) &]/a)/(128*(a - b)^2*b*d)
\end{aligned}$$

Maple [B] time = 0.096, size = 3511, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x)

[Out] $\frac{1}{32} \frac{d}{(a^2 - 2ab + b^2)} \frac{1}{a} \frac{(-ab - (ab)^{1/2}a)^{1/2} \arctan(1/4(-2 \tanh(1/2 dx + 1/2 c)^2 a + 4(ab)^{1/2} + 2a)/(-ab - (ab)^{1/2}a)^{1/2}) (ab)^{1/2} + 4/d (\tanh(1/2 dx + 1/2 c)^8 a - 4 \tanh(1/2 dx + 1/2 c)^6 a + 6 \tanh(1/2 dx + 1/2 c)^4 a - 16 b \tanh(1/2 dx + 1/2 c)^4 - 4 \tanh(1/2 dx + 1/2 c)^2 a + a)^2 / a b^2}{(a^2 - 2ab + b^2) \tanh(1/2 dx + 1/2 c)^6 + 1/16 d b / (a^2 - 2ab + b^2) \frac{1}{a} \frac{(-ab - (ab)^{1/2}a)^{1/2} \arctan(1/4(-2 \tanh(1/2 dx + 1/2 c)^2 a + 4(ab)^{1/2} + 2a)/(-ab - (ab)^{1/2}a)^{1/2}) - 1/16 d b / (a^2 - 2ab + b^2) \frac{1}{a} \frac{(-ab + (ab)^{1/2}a)^{1/2} \arctan(1/4(2 \tanh(1/2 dx + 1/2 c)^2 a + 4(ab)^{1/2} - 2a)/(-ab + (ab)^{1/2}a)^{1/2}) (ab)^{1/2} + 1/32 d / (a^2 - 2ab + b^2) \frac{1}{a} \frac{(-ab + (ab)^{1/2}a)^{1/2} \arctan(1/4(2 \tanh(1/2 dx + 1/2 c)^2 a + 4(ab)^{1/2} - 2a)/(-ab + (ab)^{1/2}a)^{1/2}) (ab)^{1/2} - 4/d (\tanh(1/2 dx + 1/2 c)^8 a - 4 \tanh(1/2 dx + 1/2 c)^6 a + 6 \tanh(1/2 dx + 1/2 c)^4 a - 16 b \tanh(1/2 dx + 1/2 c)^4 - 4 \tanh(1/2 dx + 1/2 c)^2 a + a)^2}{a \tanh(1/2 dx + 1/2 c)^{10} - 21/16 d / (\tanh(1/2 dx + 1/2 c)^8 a - 4 \tanh(1/2 dx + 1/2 c)^6 a + 6 \tanh(1/2 dx + 1/2 c)^4 a - 16 b \tanh(1/2 dx + 1/2 c)^4 - 4 \tanh(1/2 dx + 1/2 c)^2 a + a)^2} \frac{1}{a b^2} \frac{1}{(a^2 - 2ab + b^2) \tanh(1/2 dx + 1/2 c)^{10} - 21/16 d / (\tanh(1/2 dx + 1/2 c)^8 a - 4 \tanh(1/2 dx + 1/2 c)^6 a + 6 \tanh(1/2 dx + 1/2 c)^4 a - 16 b \tanh(1/2 dx + 1/2 c)^4 - 4 \tanh(1/2 dx + 1/2 c)^2 a + a)^2} \frac{1}{a b^2}$

$$\begin{aligned}
& ^2/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^8-3/16/d/(\tanh(1/2dx+1/2c)^8a-4* \\
& \tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2a^2/b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^{14}+21/16/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2a^2/b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^{12}-3/64/d/b/(a^2-2ab+b^2) a/(-ab+(ab)^{1/2}) a^{1/2} \arctan(1/4*(2*\tanh(1/2dx+1/2c)^2a+4*(ab)^{1/2}-2a)/(-ab+(ab)^{1/2}) a^{1/2})+3/64/d/b/(a^2-2ab+b^2) a/(-ab-(ab)^{1/2}) a^{1/2} \arctan(1/4*(-2*\tanh(1/2dx+1/2c)^2a+4*(ab)^{1/2}+2a)/(-ab-(ab)^{1/2}) a^{1/2})+1/16/d/b/(a^2-2ab+b^2)/(-ab+(ab)^{1/2}) a^{1/2} \arctan(1/4*(2*\tanh(1/2dx+1/2c)^2a+4*(ab)^{1/2}-2a)/(-ab+(ab)^{1/2}) a^{1/2})*(ab)^{1/2}+1/16/d/b/(a^2-2ab+b^2)/(-ab-(ab)^{1/2}) a^{1/2} \arctan(1/4*(-2*\tanh(1/2dx+1/2c)^2a+4*(ab)^{1/2}+2a)/(-ab-(ab)^{1/2}) a^{1/2})*(ab)^{1/2}+105/16/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2/b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^8a^2-105/16/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2/b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^6a^2+63/16/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2a^2/b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^4-63/16/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^8+113/4/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^6+3/16/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2a^2/b/(a^2-2ab+b^2)-183/16/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^8a-9/16/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^6a+87/16/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2a/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^4+1/4/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^2-1/4/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^{14}+3/2/d/(\tanh(1/2dx+1/2c)^8a-4*\tanh(1/2dx+1/2c)^6a+6*\tanh(1/2dx+1/2c)^4a-16*b*\tanh(1/2dx+1/2c)^4-4*\tanh(1/2dx+1/2c)^2a+a)^2b/(a^2-2ab+b^2) \tanh(1/2dx+1/2c)^{12}-37/16/d/(t
\end{aligned}$$

$$\begin{aligned} & \operatorname{anh}(1/2*d*x+1/2*c)^{8*a-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{6*a+6*\operatorname{tanh}(1/2*d*x+1/2*c)^{4*a-16*b*\operatorname{tanh}(1/2*d*x+1/2*c)^{4-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{2*a+a}}^2*a/(a^2-2*a*b+b^2)} \\ & * \operatorname{tanh}(1/2*d*x+1/2*c)^{2+13/16/d/(\operatorname{tanh}(1/2*d*x+1/2*c)^{8*a-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{6*a+6*\operatorname{tanh}(1/2*d*x+1/2*c)^{4-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{2*a+a}}^2*a/(a^2-2*a*b+b^2)} \\ &)^{12+225/16/d/(\operatorname{tanh}(1/2*d*x+1/2*c)^{8*a-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{6*a+6*\operatorname{tanh}(1/2*d*x+1/2*c)^{4-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{2*a+a}}^2/(a^2-2*a*b+b^2)} \\ &)^{10*a-17/4/d/(\operatorname{tanh}(1/2*d*x+1/2*c)^{8*a-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{6*a+6*\operatorname{tanh}(1/2*d*x+1/2*c)^{4-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{2*a+a}}^2*b/(a^2-2*a*b+b^2)} \\ &)^{10+3/16/d/(\operatorname{tanh}(1/2*d*x+1/2*c)^{8*a-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{6*a+6*\operatorname{tanh}(1/2*d*x+1/2*c)^{4-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{2*a+a}}^2*a/(a^2-2*a*b+b^2)} \\ &)^{-15/2/d/(\operatorname{tanh}(1/2*d*x+1/2*c)^{8*a-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{6*a+6*\operatorname{tanh}(1/2*d*x+1/2*c)^{4-4*\operatorname{tanh}(1/2*d*x+1/2*c)^{2*a+a}}^2*b/(a^2-2*a*b+b^2)} \\ &)^{-4-13/64/d/(a^2-2*a*b+b^2)/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(-2*\operatorname{tanh}(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)+2*a}/(-a*b-(a*b)^{(1/2)*a})^{(1/2)})+13/64/d/(a^2-2*a*b+b^2)/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(2*\operatorname{tanh}(1/2*d*x+1/2*c)^{2*a+4*(a*b)^{(1/2)-2*a}/(-a*b+(a*b)^{(1/2)*a})^{(1/2)})} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] $\frac{1}{8} * ((2*a*b^2*e^{(15*c)} + b^3*e^{(15*c)}) * e^{(15*d*x)} + (2*a^2*b*e^{(13*c)} - 24*a*b^2*e^{(13*c)} - 5*b^3*e^{(13*c)}) * e^{(13*d*x)} - (70*a^2*b*e^{(11*c)} - 76*a*b^2*e^{(11*c)} - 9*b^3*e^{(11*c)}) * e^{(11*d*x)} + (96*a^3*e^{(9*c)} + 164*a^2*b*e^{(9*c)} - 54*a*b^2*e^{(9*c)} - 5*b^3*e^{(9*c)}) * e^{(9*d*x)} + (96*a^3*e^{(7*c)} + 164*a^2*b*e^{(7*c)} - 54*a*b^2*e^{(7*c)} - 5*b^3*e^{(7*c)}) * e^{(7*d*x)} - (70*a^2*b*e^{(5*c)} - 76*a*b^2*e^{(5*c)} - 9*b^3*e^{(5*c)}) * e^{(5*d*x)} + (2*a^2*b*e^{(3*c)} - 24*a*b^2*e^{(3*c)} - 5*b^3*e^{(3*c)}) * e^{(3*d*x)} + (2*a*b^2*e^c + b^3*e^c) * e^{(d*x)}) / (a^3*b^3*d - 2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^{(16*c)} - 2*a^2*b^4*d*e^{(16*c)} + a*b^5*d*e^{(16*c)}) * e^{(16*d*x)} - 8*(a^3*b^3*d*e^{(14*c)} - 2*a^2*b^4*d*e^{(14*c)} + a*b^5*d*e^{(14*c)}) * e^{(14*d*x)} - 4*(8*a^4*b^2*d*e^{(12*c)} - 23*a^3*b^3*d*e^{(12*c)} + 22*a^2*b^4*d*e^{(12*c)} - 7*a*b^5*d*e^{(12*c)}) * e^{(12*d*x)} + 8*(16*a^4*b^2*d*e^{(10*c)} - 39*a^3*b^3*d*e^{(10*c)} + 30*a^2*b^4*d*e^{(10*c)} - 7*a*$

$$\begin{aligned}
& b^5 d e^{(10c)} e^{(10dx)} + 2(128 a^5 b d e^{(8c)} - 352 a^4 b^2 d e^{(8c)} \\
& + 355 a^3 b^3 d e^{(8c)} - 166 a^2 b^4 d e^{(8c)} + 35 a b^5 d e^{(8c)}) e^{(8 \\
& dx)} + 8(16 a^4 b^2 d e^{(6c)} - 39 a^3 b^3 d e^{(6c)} + 30 a^2 b^4 d e^{(6c)} \\
& - 7 a b^5 d e^{(6c)}) e^{(6dx)} - 4(8 a^4 b^2 d e^{(4c)} - 23 a^3 b^3 d e^{(4c)} \\
& + 22 a^2 b^4 d e^{(4c)} - 7 a b^5 d e^{(4c)}) e^{(4dx)} - 8(a^3 b^3 d e^{(2c)} \\
& - 2 a^2 b^4 d e^{(2c)} + a b^5 d e^{(2c)}) e^{(2dx)} + 1/32 \text{integrate}(4((2 a b e^{(7c)} + b^2 e^{(7c)}) e^{(7dx)} + (6 a^2 e^{(5c)} - 32 a b e^{(5c)} \\
& + 5 b^2 e^{(5c)}) e^{(5dx)} - (6 a^2 e^{(3c)} - 32 a b e^{(3c)} + 5 b^2 e^{(3c)}) e^{(3dx)} - (2 a b e^c + b^2 e^c) e^{(dx)}) / (a^3 b^2 - 2 a^2 b^3 + a \\
& b^4 + (a^3 b^2 e^{(8c)} - 2 a^2 b^3 e^{(8c)} + a b^4 e^{(8c)}) e^{(8dx)} - 4(a^3 b^2 e^{(6c)} - 2 a^2 b^3 e^{(6c)} + a b^4 e^{(6c)}) e^{(6dx)} - 2(8 a^4 b e^{(4c)} - 19 a^3 b^2 e^{(4c)} + 14 a^2 b^3 e^{(4c)} - 3 a b^4 e^{(4c)}) e^{(4 \\
& dx)} - 4(a^3 b^2 e^{(2c)} - 2 a^2 b^3 e^{(2c)} + a b^4 e^{(2c)}) e^{(2dx)}), \\
& x)
\end{aligned}$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)^5/(a-b*sinh(dx+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)**5/(a-b*sinh(dx+c)**4)**3,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```


$$3.256 \quad \int \frac{\sinh^3(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$$

Optimal. Leaf size=288

$$-\frac{(5\sqrt{a}-2\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d(\sqrt{a}-\sqrt{b})^{5/2}} + \frac{(5\sqrt{a}+2\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d(\sqrt{a}+\sqrt{b})^{5/2}} - \frac{\cosh(c+dx)(-5a+b) \cosh^2(c+dx)}{32ad(a-b)^2(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx))}$$

[Out] -((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] + 2*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(8*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(11*a + b - (5*a + b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rubi [A] time = 0.518835, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3215, 1178, 1166, 205, 208}

$$-\frac{(5\sqrt{a}-2\sqrt{b}) \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d(\sqrt{a}-\sqrt{b})^{5/2}} + \frac{(5\sqrt{a}+2\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d(\sqrt{a}+\sqrt{b})^{5/2}} - \frac{\cosh(c+dx)(-5a+b) \cosh^2(c+dx)}{32ad(a-b)^2(a-b \cosh^4(c+dx)+2b \cosh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] -((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] + 2*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(8*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(11*a + b - (5*a + b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^3(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1-x^2}{(a-b+2bx^2-bx^4)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= -\frac{\cosh(c+dx)(2-\cosh^2(c+dx))}{8(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} + \frac{\text{Subst}\left(\int \frac{-12ab+10abx^2}{(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{16a(a-b)d} \\
&= -\frac{\cosh(c+dx)(2-\cosh^2(c+dx))}{8(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} - \frac{\cosh(c+dx)(11a+b)}{32a(a-b)^2d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} \\
&= -\frac{\cosh(c+dx)(2-\cosh^2(c+dx))}{8(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} - \frac{\cosh(c+dx)(11a+b)}{32a(a-b)^2d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} \\
&= -\frac{(5\sqrt{a}-2\sqrt{b})\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}(\sqrt{a}-\sqrt{b})^{5/2}b^{3/4}d} + \frac{(5\sqrt{a}+2\sqrt{b})\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}(\sqrt{a}+\sqrt{b})^{5/2}b^{3/4}d} - \frac{\cosh(c+dx)}{8(a-b)d}
\end{aligned}$$

Mathematica [C] time = 1.30044, size = 802, normalized size = 2.78

$$\frac{32\cosh(c+dx)(-17a-b+(5a+b)\cosh(2(c+dx)))}{a(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))} + \frac{\text{RootSum}\left[b^{\#1^8}-4b^{\#1^6}-16a^{\#1^4}+6b^{\#1^4}-4b^{\#1^2}+b\&, \frac{-5ac^{\#1^6}-bc^{\#1^6}-5adx^{\#1^6}-bdx^{\#1^6}-10a\log(\#1\cosh(\frac{1}{2}(c+dx)))}{b^{\#1^8}-4b^{\#1^6}-16a^{\#1^4}+6b^{\#1^4}-4b^{\#1^2}+b\&}\right]}{a(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))}$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] ((32*Cosh[c + d*x]*(-17*a - b + (5*a + b)*Cosh[2*(c + d*x)])))/(a*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (512*(a - b)*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)])))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)]^2 + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 &, (5*a*c + b*c + 5*a*d*x + b*d*x + 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 47*a*c*#1^2 + 5*b*c*#1^2 - 47*a*d*x*#1^2 + 5*b*d*x*#1^2 - 94*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh

$$\frac{[(c + dx)/2]^{\#1} - \text{Sinh}[(c + dx)/2]^{\#1} \cdot \#1^2 + 47a \cdot c^{\#1^4} - 5b \cdot c^{\#1^4} + 47a \cdot d^{\#1^4} - 5b \cdot d^{\#1^4} + 94a \cdot \text{Log}[-\text{Cosh}[(c + dx)/2] - \text{Sinh}[(c + dx)/2] + \text{Cosh}[(c + dx)/2]^{\#1} - \text{Sinh}[(c + dx)/2]^{\#1} \cdot \#1^4 - 10b \cdot \text{Log}[-\text{Cosh}[(c + dx)/2] - \text{Sinh}[(c + dx)/2] + \text{Cosh}[(c + dx)/2]^{\#1} - \text{Sinh}[(c + dx)/2]^{\#1} \cdot \#1^4 - 5a \cdot c^{\#1^6} - b \cdot c^{\#1^6} - 5a \cdot d^{\#1^6} - b \cdot d^{\#1^6} - 10a \cdot \text{Log}[-\text{Cosh}[(c + dx)/2] - \text{Sinh}[(c + dx)/2] + \text{Cosh}[(c + dx)/2]^{\#1} - \text{Sinh}[(c + dx)/2]^{\#1} \cdot \#1^6 - 2b \cdot \text{Log}[-\text{Cosh}[(c + dx)/2] - \text{Sinh}[(c + dx)/2] + \text{Cosh}[(c + dx)/2]^{\#1} - \text{Sinh}[(c + dx)/2]^{\#1} \cdot \#1^6] / (-b^{\#1} - 8a^{\#1^3} + 3b^{\#1^3} - 3b^{\#1^5} + b^{\#1^7}) \&] / a / (256 \cdot (a - b)^2 \cdot d}$$

Maple [B] time = 0.082, size = 2916, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(dx+c)^3/(a-b \cdot \sinh(dx+c)^4)^3, x)$

[Out]
$$\begin{aligned} & -1/64/d/(a^2-2ab+b^2)/a/(-ab-(ab)^{1/2}a)^{1/2} \cdot \arctan(1/4 \cdot (-2 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^2 \cdot a+4 \cdot (ab)^{1/2}+2a)/(-ab-(ab)^{1/2}a)^{1/2}) \cdot (ab)^{1/2} \\ & +14/d/(\tanh(1/2 \cdot dx+1/2 \cdot c)^8 \cdot a-4 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^6 \cdot a+6 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^4 \cdot a-16 \cdot b \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^4-4 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^2 \cdot a+a)^2/a \cdot b^2/(a^2-2ab+b^2) \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^6-1/32/d \cdot b/(a^2-2ab+b^2)/a/(-ab-(ab)^{1/2}a)^{1/2} \cdot \arctan(1/4 \cdot (-2 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^2 \cdot a+4 \cdot (ab)^{1/2}+2a)/(-ab-(ab)^{1/2}a)^{1/2})+1/32/d \cdot b/(a^2-2ab+b^2)/a/(-ab+(ab)^{1/2}a)^{1/2} \cdot \arctan(1/4 \cdot (2 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^2 \cdot a+4 \cdot (ab)^{1/2}-2a)/(-ab+(ab)^{1/2}a)^{1/2}) \cdot (ab)^{1/2}+18/d/(\tanh(1/2 \cdot dx+1/2 \cdot c)^8 \cdot a-4 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^6 \cdot a+6 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^4 \cdot a-16 \cdot b \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^4-4 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^2 \cdot a+a)^2 \cdot b^2/(a^2-2ab+b^2)/a \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^{10}-4/d/(\tanh(1/2 \cdot dx+1/2 \cdot c)^8 \cdot a-4 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^6 \cdot a+6 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^4 \cdot a-16 \cdot b \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^4-4 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^2 \cdot a+a)^2/a/(a^2-2ab+b^2) \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^4 \cdot b^2-4/d/(\tanh(1/2 \cdot dx+1/2 \cdot c)^8 \cdot a-4 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^6 \cdot a+6 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^4 \cdot a-16 \cdot b \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^4-4 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^2 \cdot a+a)^2/a \cdot b^2/(a^2-2ab+b^2) \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^8-5/64/d/b/(a^2-2ab+b^2)/(-ab+(ab)^{1/2}a)^{1/2} \cdot \arctan(1/4 \cdot (2 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^2 \cdot a+4 \cdot (ab)^{1/2}-2a)/(-ab+(ab)^{1/2}a)^{1/2}) \cdot (ab)^{1/2}-5/64/d/b/(a^2-2ab+b^2)/(-ab-(ab)^{1/2}a)^{1/2} \cdot \arctan(1/4 \cdot (-2 \cdot \tanh(1/2 \cdot dx+1/2 \cdot c)^2 \cdot a+4 \cdot (ab)^{1/2}+2a)/(-ab-(ab)^{1/2}a)^{1/2}) \cdot (ab)^{1/2}+3 \end{aligned}$$

$$\begin{aligned}
& 2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)} \\
&)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2* \\
& a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8*b^3+275/4/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(\\
& 1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*t \\
& \tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8-533/8/d \\
& /(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4 \\
& *a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b \\
& ^2)*\tanh(1/2*d*x+1/2*c)^6-175/8/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1 \\
& /2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d \\
& *x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8*a+55/2/d/(\tanh(1/2 \\
& *d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*ta \\
& nh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2 \\
& *d*x+1/2*c)^6*a-141/8/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+ \\
& 6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^ \\
& 2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4-17/8/d/(\tanh(1/2*d*x+1/2*c) \\
&)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x \\
& +1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2 \\
& *c)^2+1/8/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d \\
& *x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(\\
& a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^14+29/4/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tan \\
& h(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4 \\
& *\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^12+11/2 \\
& /d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c) \\
& ^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b \\
& +b^2)*\tanh(1/2*d*x+1/2*c)^2-1/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1 \\
& /2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d \\
& *x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^14+1/8/d/(\tanh(1/2 \\
& *d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*ta \\
& nh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1 \\
& /2*d*x+1/2*c)^12+15/2/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+ \\
& 6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^ \\
& 2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^10*a-219/8/d/(\tanh(1/2*d*x+1/2 \\
& *c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d \\
& *x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1 \\
& /2*c)^10+1/4/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/ \\
& 2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/ \\
& (a^2-2*a*b+b^2)*b-5/8/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+ \\
& 6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^ \\
& 2*a+a)^2*a/(a^2-2*a*b+b^2)+79/4/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1 \\
& /2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d \\
& *x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4+1/8/d/(a^2-2*a*b \\
& +b^2)/(-a*b-(a*b)^{(1/2)*a}^{(1/2)*a}\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(\\
& a*b)^{(1/2)+2*a}/(-a*b-(a*b)^{(1/2)*a}^{(1/2)})-1/8/d/(a^2-2*a*b+b^2)/(-a*b+(a* \\
& b)^{(1/2)*a}^{(1/2)*a}\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a}/ \\
& (-a*b+(a*b)^{(1/2)*a}^{(1/2)}))
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/16*((5*a*b*e^{(15*c)} + b^2*e^{(15*c)})*e^{(15*d*x)} - (49*a*b*e^{(13*c)} + 5*b^2*e^{(13*c)})*e^{(13*d*x)} - 3*(48*a^2*e^{(11*c)} - 55*a*b*e^{(11*c)} - 3*b^2*e^{(11*c)})*e^{(11*d*x)} + (784*a^2*e^{(9*c)} - 377*a*b*e^{(9*c)} - 5*b^2*e^{(9*c)})*e^{(9*d*x)} + (784*a^2*e^{(7*c)} - 377*a*b*e^{(7*c)} - 5*b^2*e^{(7*c)})*e^{(7*d*x)} - 3*(48*a^2*e^{(5*c)} - 55*a*b*e^{(5*c)} - 3*b^2*e^{(5*c)})*e^{(5*d*x)} - (49*a*b*e^{(3*c)} + 5*b^2*e^{(3*c)})*e^{(3*d*x)} + (5*a*b*e^c + b^2*e^c)*e^{(d*x)})/(a^3*b^2*d - 2*a^2*b^3*d + a*b^4*d + (a^3*b^2*d*e^{(16*c)} - 2*a^2*b^3*d*e^{(16*c)} + a*b^4*d*e^{(16*c)})*e^{(16*d*x)} - 8*(a^3*b^2*d*e^{(14*c)} - 2*a^2*b^3*d*e^{(14*c)} + a*b^4*d*e^{(14*c)})*e^{(14*d*x)} - 4*(8*a^4*b*d*e^{(12*c)} - 23*a^3*b^2*d*e^{(12*c)} + 22*a^2*b^3*d*e^{(12*c)} - 7*a*b^4*d*e^{(12*c)})*e^{(12*d*x)} + 8*(16*a^4*b*d*e^{(10*c)} - 39*a^3*b^2*d*e^{(10*c)} + 30*a^2*b^3*d*e^{(10*c)} - 7*a*b^4*d*e^{(10*c)})*e^{(10*d*x)} + 2*(128*a^5*d*e^{(8*c)} - 352*a^4*b*d*e^{(8*c)} + 355*a^3*b^2*d*e^{(8*c)} - 166*a^2*b^3*d*e^{(8*c)} + 35*a*b^4*d*e^{(8*c)})*e^{(8*d*x)} + 8*(16*a^4*b*d*e^{(6*c)} - 39*a^3*b^2*d*e^{(6*c)} + 30*a^2*b^3*d*e^{(6*c)} - 7*a*b^4*d*e^{(6*c)})*e^{(6*d*x)} - 4*(8*a^4*b*d*e^{(4*c)} - 23*a^3*b^2*d*e^{(4*c)} + 22*a^2*b^3*d*e^{(4*c)} - 7*a*b^4*d*e^{(4*c)})*e^{(4*d*x)} - 8*(a^3*b^2*d*e^{(2*c)} - 2*a^2*b^3*d*e^{(2*c)} + a*b^4*d*e^{(2*c)})*e^{(2*d*x)}) - 1/8*integrate(1/2*((5*a*e^{(7*c)} + b*e^{(7*c)})*e^{(7*d*x)} - (47*a*e^{(5*c)} - 5*b*e^{(5*c)})*e^{(5*d*x)} + (47*a*e^{(3*c)} - 5*b*e^{(3*c)})*e^{(3*d*x)} - (5*a*e^c + b*e^c)*e^{(d*x)})/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^{(8*c)} - 2*a^2*b^2*e^{(8*c)} + a*b^3*e^{(8*c)})*e^{(8*d*x)} - 4*(a^3*b*e^{(6*c)} - 2*a^2*b^2*e^{(6*c)} + a*b^3*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^4*e^{(4*c)} - 19*a^3*b*e^{(4*c)} + 14*a^2*b^2*e^{(4*c)} - 3*a*b^3*e^{(4*c)})*e^{(4*d*x)} - 4*(a^3*b*e^{(2*c)} - 2*a^2*b^2*e^{(2*c)} + a*b^3*e^{(2*c)})*e^{(2*d*x)}), x) \end{aligned}$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**3,x)`

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.257 \quad \int \frac{\sinh(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^3} dx$$

Optimal. Leaf size=313

$$\frac{\cosh(c+dx) \left((7a-3b)(a+2b) - 6b(2a-b) \cosh^2(c+dx) \right)}{32a^2d(a-b)^2 \left(a-b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b \right)} + \frac{3 \left(-10\sqrt{a}\sqrt{b} + 7a + 4b \right) \tan^{-1} \left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}} \right)}{64a^{5/2} \sqrt[4]{bd} \left(\sqrt{a} - \sqrt{b} \right)^{5/2}} + \frac{3 \left(10\sqrt{a}\sqrt{b} + 7a + 4b \right) \tan^{-1} \left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}} \right)}{64a^{5/2} \sqrt[4]{bd} \left(\sqrt{a} + \sqrt{b} \right)^{5/2}}$$

[Out] (3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) + (3*(7*a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) + (Cosh[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a - b)*b*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rubi [A] time = 0.463176, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3215, 1092, 1178, 1166, 205, 208}

$$\frac{\cosh(c+dx) \left((7a-3b)(a+2b) - 6b(2a-b) \cosh^2(c+dx) \right)}{32a^2d(a-b)^2 \left(a-b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b \right)} + \frac{3 \left(-10\sqrt{a}\sqrt{b} + 7a + 4b \right) \tan^{-1} \left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}} \right)}{64a^{5/2} \sqrt[4]{bd} \left(\sqrt{a} - \sqrt{b} \right)^{5/2}} + \frac{3 \left(10\sqrt{a}\sqrt{b} + 7a + 4b \right) \tan^{-1} \left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}} \right)}{64a^{5/2} \sqrt[4]{bd} \left(\sqrt{a} + \sqrt{b} \right)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] (3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) + (3*(7*a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) + (Cosh[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a - b)*b*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1092

```
Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := -Simp[(x*(b^2 - 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_.) + (e_.)*(x_)^2)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_.) + (e_.)*(x_)^2)/((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a-b+2bx^2-bx^4)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{8a(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2-4(a-b)b+}{(a-b+2bx^2-bx^4)} dx, x, \cosh(c+dx)\right)}{16a(a-b)d} \\
&= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{8a(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} + \frac{\cosh(c+dx)((7a-3b)(a+2b))}{32a^2(a-b)^2d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} \\
&= \frac{\cosh(c+dx)(a+b-b\cosh^2(c+dx))}{8a(a-b)d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))^2} + \frac{\cosh(c+dx)((7a-3b)(a+2b))}{32a^2(a-b)^2d(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx))} \\
&= \frac{3(7a-10\sqrt{a}\sqrt{b}+4b)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}(\sqrt{a}-\sqrt{b})^{5/2}\sqrt[4]{bd}} + \frac{3(7a+10\sqrt{a}\sqrt{b}+4b)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}(\sqrt{a}+\sqrt{b})^{5/2}\sqrt[4]{bd}}
\end{aligned}$$

Mathematica [C] time = 1.36658, size = 1018, normalized size = 3.25

result too large to display

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3, x]

[Out] ((32*Cosh[c + d*x]*(7*a^2 + 5*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2*(c + d*x)])))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a - b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 + 3*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*a*b*c + b^2*c - 2*a*b*d*x + b^2*d*x - 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 14*a^2*c*#1^2 - 12*a*b*c*#1^2 + 5*b^2*c*#1^2 + 14*a^2*d*x*#1^2 - 12*a*b*d*x*#1^2 + 5*b^2*d*x*#1^2 + 28*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 24*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]

$$\begin{aligned}
& + \operatorname{Cosh}[(c + d*x)/2]*\#1 - \operatorname{Sinh}[(c + d*x)/2]*\#1*\#1^2 + 10*b^2*\operatorname{Log}[-\operatorname{Cosh}[(c \\
& + d*x)/2] - \operatorname{Sinh}[(c + d*x)/2] + \operatorname{Cosh}[(c + d*x)/2]*\#1 - \operatorname{Sinh}[(c + d*x)/2]*\#1 \\
&]*\#1^2 - 14*a^2*c*\#1^4 + 12*a*b*c*\#1^4 - 5*b^2*c*\#1^4 - 14*a^2*d*x*\#1^4 + 1 \\
& 2*a*b*d*x*\#1^4 - 5*b^2*d*x*\#1^4 - 28*a^2*\operatorname{Log}[-\operatorname{Cosh}[(c + d*x)/2] - \operatorname{Sinh}[(c + \\
& d*x)/2] + \operatorname{Cosh}[(c + d*x)/2]*\#1 - \operatorname{Sinh}[(c + d*x)/2]*\#1*\#1^4 + 24*a*b*\operatorname{Log}[- \\
& \operatorname{Cosh}[(c + d*x)/2] - \operatorname{Sinh}[(c + d*x)/2] + \operatorname{Cosh}[(c + d*x)/2]*\#1 - \operatorname{Sinh}[(c + d* \\
& x)/2]*\#1*\#1^4 - 10*b^2*\operatorname{Log}[-\operatorname{Cosh}[(c + d*x)/2] - \operatorname{Sinh}[(c + d*x)/2] + \operatorname{Cosh}[(\\
& c + d*x)/2]*\#1 - \operatorname{Sinh}[(c + d*x)/2]*\#1*\#1^4 + 2*a*b*c*\#1^6 - b^2*c*\#1^6 + 2 \\
& *a*b*d*x*\#1^6 - b^2*d*x*\#1^6 + 4*a*b*\operatorname{Log}[-\operatorname{Cosh}[(c + d*x)/2] - \operatorname{Sinh}[(c + d*x \\
&)/2] + \operatorname{Cosh}[(c + d*x)/2]*\#1 - \operatorname{Sinh}[(c + d*x)/2]*\#1*\#1^6 - 2*b^2*\operatorname{Log}[-\operatorname{Cosh}[\\
& (c + d*x)/2] - \operatorname{Sinh}[(c + d*x)/2] + \operatorname{Cosh}[(c + d*x)/2]*\#1 - \operatorname{Sinh}[(c + d*x)/2] \\
& *\#1*\#1^6)/(-b*\#1) - 8*a*\#1^3 + 3*b*\#1^3 - 3*b*\#1^5 + b*\#1^7) &])/(128*a^ \\
& 2*(a - b)^2*d)
\end{aligned}$$

Maple [B] time = 0.106, size = 3512, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}(\sinh(d*x+c)/(a-b*\sinh(d*x+c)^4)^3, x)$

[Out] $\begin{aligned}
& 12/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2* \\
& c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2 \\
& *a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{10}*b^3-12/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\tanh(1 \\
& /2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*ta \\
& nh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^6*b^3+5/ \\
& 4/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c \\
&)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a* \\
& b+b^2)*\tanh(1/2*d*x+1/2*c)^2*b^2-5/4/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\tanh(1/2* \\
& d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(\\
& 1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{14}*b^2-3/16/d \\
& /a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^{(1/2)*a)^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x \\
& +1/2*c)^2*a+4*(a*b)^{(1/2)+2*a)/(-a*b-(a*b)^{(1/2)*a)^{(1/2)})}*b^2+3/16/d/a^2/(\\
& a^2-2*a*b+b^2)/(-a*b+(a*b)^{(1/2)*a)^{(1/2)*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c) \\
& ^2*a+4*(a*b)^{(1/2)-2*a)/(-a*b+(a*b)^{(1/2)*a)^{(1/2)})}*b^2+3/16/d/(a^2-2*a*b+b \\
& ^2)/a/(-a*b-(a*b)^{(1/2)*a)^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(\\
& a*b)^{(1/2)+2*a)/(-a*b-(a*b)^{(1/2)*a)^{(1/2)})*(a*b)^{(1/2)+37/4/d/(\tanh(1/2*d* \\
& x+1/2*c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(\\
& 1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*\tanh(\\
& 1/2*d*x+1/2*c)^6+27/64/d*b/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^{(1/2)*a)^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c) \\
& ^2*a+4*(a*b)^{(1/2)+2*a)/(-a*b-(a*b)^{(1/2)*a)^{(1/2)})}
\end{aligned}$

$$\begin{aligned}
& ^{(1/2)}) - 27/64/d*b/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)})+3/ \\
& 16/d/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)})*(a*b)^{(1/2)}-197 \\
& /4/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/(a^2-2 \\
& *a*b+b^2)/a*\tanh(1/2*d*x+1/2*c)^10+1/2/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tan \\
& h(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4*b^2+19/2/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^ \\
& 4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*\tanh(1/2*d*x+1/2*c)^12*b^2+118/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d \\
& *x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8-3/32/d/a \\
& ^2/(a^2-2*a*b+b^2)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}-2*a)/(-a*b+(a*b)^{(1/2)}*a)^{(1/2)})*(a*b)^{(1/2)}*b-3/32/d/a \\
& ^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)}*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)}+2*a)/(-a*b-(a*b)^{(1/2)}*a)^{(1/2)})*(a*b)^{(1/2)}*b-4 \\
& 0/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2* \\
& a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8*b^3-1231/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tan \\
& h(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4 \\
& *\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8+831/1 \\
& 6/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a* \\
& b+b^2)*\tanh(1/2*d*x+1/2*c)^6+385/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d \\
& *x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^8*a-385/16/d/(ta \\
& nh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-1 \\
& 6*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tan \\
& h(1/2*d*x+1/2*c)^6*a+231/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2* \\
& c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+ \\
& 1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4+3/16/d/(\tanh(1/2*d* \\
& x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(\\
& 1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2* \\
& d*x+1/2*c)^2+37/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\t \\
& anh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a \\
& +a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^14-283/16/d/(\tanh(1/2*d*x+1/2*c \\
&)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x \\
& +1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2 \\
& *c)^12-77/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/ \\
& 2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2* \\
& a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^2-11/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4* \\
& \tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^ \\
& 4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^14+7
\end{aligned}$$

$$\begin{aligned} & 7/16/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{12}-231/16/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{10*a}+857/16/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{10}-5/16/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*b+11/16/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)-209/16/d/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4-21/64/d/(a^2-2*a*b+b^2)/(-a*b-(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(-2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)+2*a}/(-a*b-(a*b)^{(1/2)*a})^{(1/2)})+21/64/d/(a^2-2*a*b+b^2)/(-a*b+(a*b)^{(1/2)*a})^{(1/2)*\arctan(1/4*(2*\tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^{(1/2)-2*a}/(-a*b+(a*b)^{(1/2)*a})^{(1/2)})} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] $\frac{1}{8} * (3 * (2 * a * b^2 * e^{(15*c)} - b^3 * e^{(15*c)}) * e^{(15*d*x)} - (14 * a^2 * b * e^{(13*c)} + 28 * a * b^2 * e^{(13*c)} - 15 * b^3 * e^{(13*c)}) * e^{(13*d*x)} - (86 * a^2 * b * e^{(11*c)} - 128 * a * b^2 * e^{(11*c)} + 27 * b^3 * e^{(11*c)}) * e^{(11*d*x)} + (352 * a^3 * e^{(9*c)} - 60 * a^2 * b * e^{(9*c)} - 106 * a * b^2 * e^{(9*c)} + 15 * b^3 * e^{(9*c)}) * e^{(9*d*x)} + (352 * a^3 * e^{(7*c)} - 60 * a^2 * b * e^{(7*c)} - 106 * a * b^2 * e^{(7*c)} + 15 * b^3 * e^{(7*c)}) * e^{(7*d*x)} - (86 * a^2 * b * e^{(5*c)} - 128 * a * b^2 * e^{(5*c)} + 27 * b^3 * e^{(5*c)}) * e^{(5*d*x)} - (14 * a^2 * b * e^{(3*c)} + 28 * a * b^2 * e^{(3*c)} - 15 * b^3 * e^{(3*c)}) * e^{(3*d*x)} + 3 * (2 * a * b^2 * e^c - b^3 * e^c) * e^{(d*x)}) / (a^4 * b^2 * d - 2 * a^3 * b^3 * d + a^2 * b^4 * d + (a^4 * b^2 * d * e^{(16*c)} - 2 * a^3 * b^3 * d * e^{(16*c)} + a^2 * b^4 * d * e^{(16*c)}) * e^{(16*d*x)} - 8 * (a^4 * b^2 * d * e^{(14*c)} - 2 * a^3 * b^3 * d * e^{(14*c)} + a^2 * b^4 * d * e^{(14*c)}) * e^{(14*d*x)} - 4 * (8 * a^5 * b * d * e^{(12*c)} - 23 * a^4 * b^2 * d * e^{(12*c)} + 22 * a^3 * b^3 * d * e^{(12*c)} - 7 * a^2 * b^4 * d * e^{(12*c)}) * e^{(12*d*x)} + 8 * (16 * a^5 * b * d * e^{(10*c)} - 39 * a^4 * b^2 * d * e^{(10*c)} + 30 * a^3 * b^3 * d * e^{(10*c)} - 7 * a^2 * b^4 * d * e^{(10*c)}) * e^{(10*d*x)} + 2 * (128 * a^6 * d * e^{(8*c)} - 352 * a^5 * b * d * e^{(8*c)} + 355 * a^4 * b^2 * d * e^{(8*c)} - 166 * a^3 * b^3 * d * e^{(8*c)} + 35 * a^2 * b^4 * d * e^{(8*c)}) * e^{(8*d*x)} + 8 * (16 * a^5 * b * d * e^{(6*c)} - 39 * a^4 * b^2 * d * e^{(6*c)} +$

$$30a^3b^3d^6e^{6c} - 7a^2b^4d^6e^{6c})e^{6dx} - 4(8a^5bd^4e^{4c} - 23a^4b^2d^4e^{4c} + 22a^3b^3d^4e^{4c} - 7a^2b^4d^4e^{4c})e^{4dx} - 8(a^4b^2d^2e^{2c} - 2a^3b^3d^2e^{2c} + a^2b^4d^2e^{2c})e^{2dx} + \frac{1}{2} \int \frac{3}{4} ((2ab^7e^{7c} - b^2e^{7c})e^{7dx} - (14a^2e^{5c} - 12ab^5e^{5c} + 5b^2e^{5c})e^{5dx} + (14a^2e^{3c} - 12ab^3e^{3c} + 5b^2e^{3c})e^{3dx} - (2ab^c - b^2e^c)e^{dx}) / (a^4b - 2a^3b^2 + a^2b^3 + (a^4be^{8c} - 2a^3b^2e^{8c} + a^2b^3e^{8c})e^{8dx} - 4(a^4be^{6c} - 2a^3b^2e^{6c} + a^2b^3e^{6c})e^{6dx} - 2(8a^5e^{4c} - 19a^4be^{4c} + 14a^3b^2e^{4c} - 3a^2b^3e^{4c})e^{4dx} - 4(a^4be^{2c} - 2a^3b^2e^{2c} + a^2b^3e^{2c})e^{2dx}), x$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)/(a-b*sinh(dx+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(dx+c)/(a-b*sinh(dx+c)**4)**3,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.258 \quad \int \frac{\operatorname{csch}(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^3} dx$$

Optimal. Leaf size=617

$$\frac{b \cosh(c+dx) \left(2 - \cosh^2(c+dx)\right)}{4a^2d(a-b) \left(a - b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b\right)} - \frac{b \cosh(c+dx) \left(-5a + b\right) \cosh^2(c+dx) + 11a + b}{32a^2d(a-b)^2 \left(a - b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b\right)}$$

[Out] $-\left(\left(5\sqrt{a} - 2\sqrt{b}\right)b^{1/4}\operatorname{ArcTan}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} - \sqrt{b}\right)}\right]\right)/\left(64a^{5/2}\left(\sqrt{a} - \sqrt{b}\right)^{5/2}d\right) - \left(b^{1/4}\operatorname{ArcTan}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} - \sqrt{b}\right)}\right]\right)/\left(8a^{5/2}\left(\sqrt{a} - \sqrt{b}\right)^{3/2}d\right) - \left(b^{1/4}\operatorname{ArcTan}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} - \sqrt{b}\right)}\right]\right)/\left(2a^3\sqrt{\left(\sqrt{a} - \sqrt{b}\right)}d\right) - \operatorname{ArcTanh}\left[\operatorname{Cosh}[c + d*x]\right]/\left(a^3d\right) + \left(b^{1/4}\operatorname{ArcTanh}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} + \sqrt{b}\right)}\right]\right)/\left(8a^{5/2}\left(\sqrt{a} + \sqrt{b}\right)^{3/2}d\right) + \left(b^{1/4}\operatorname{ArcTanh}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} + \sqrt{b}\right)}\right]\right)/\left(2a^3\sqrt{\left(\sqrt{a} + \sqrt{b}\right)}d\right) + \left(\left(5\sqrt{a} + 2\sqrt{b}\right)b^{1/4}\operatorname{ArcTanh}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} + \sqrt{b}\right)}\right]\right)/\left(64a^{5/2}\left(\sqrt{a} + \sqrt{b}\right)^{5/2}d\right) - \left(b\operatorname{Cosh}[c + d*x]\left(2 - \operatorname{Cosh}[c + d*x]^2\right)\right)/\left(8a\left(a - b\right)d\left(a - b + 2b\operatorname{Cosh}[c + d*x]^2 - b\operatorname{Cosh}[c + d*x]^4\right)^2\right) - \left(b\operatorname{Cosh}[c + d*x]\left(2 - \operatorname{Cosh}[c + d*x]^2\right)\right)/\left(4a^2\left(a - b\right)d\left(a - b + 2b\operatorname{Cosh}[c + d*x]^2 - b\operatorname{Cosh}[c + d*x]^4\right)\right) - \left(b\operatorname{Cosh}[c + d*x]\left(11a + b - \left(5a + b\right)\operatorname{Cosh}[c + d*x]^2\right)\right)/\left(32a^2\left(a - b\right)^2d\left(a - b + 2b\operatorname{Cosh}[c + d*x]^2 - b\operatorname{Cosh}[c + d*x]^4\right)\right)$

Rubi [A] time = 0.81855, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$\frac{b \cosh(c+dx) \left(2 - \cosh^2(c+dx)\right)}{4a^2d(a-b) \left(a - b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b\right)} - \frac{b \cosh(c+dx) \left(-5a + b\right) \cosh^2(c+dx) + 11a + b}{32a^2d(a-b)^2 \left(a - b \cosh^4(c+dx) + 2b \cosh^2(c+dx) - b\right)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}\left[\operatorname{Csch}[c + d*x]/\left(a - b\operatorname{Sinh}[c + d*x]^4\right)^3, x\right]$

[Out] $-\left(\left(5\sqrt{a} - 2\sqrt{b}\right)b^{1/4}\operatorname{ArcTan}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} - \sqrt{b}\right)}\right]\right)/\left(64a^{5/2}\left(\sqrt{a} - \sqrt{b}\right)^{5/2}d\right) - \left(b^{1/4}\operatorname{ArcTan}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} - \sqrt{b}\right)}\right]\right)/\left(8a^{5/2}\left(\sqrt{a} - \sqrt{b}\right)^{3/2}d\right) - \left(b^{1/4}\operatorname{ArcTan}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} - \sqrt{b}\right)}\right]\right)/\left(2a^3\sqrt{\left(\sqrt{a} - \sqrt{b}\right)}d\right) - \operatorname{ArcTanh}\left[\operatorname{Cosh}[c + d*x]\right]/\left(a^3d\right) + \left(b^{1/4}\operatorname{ArcTanh}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} + \sqrt{b}\right)}\right]\right)/\left(8a^{5/2}\left(\sqrt{a} + \sqrt{b}\right)^{3/2}d\right) + \left(b^{1/4}\operatorname{ArcTanh}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} + \sqrt{b}\right)}\right]\right)/\left(2a^3\sqrt{\left(\sqrt{a} + \sqrt{b}\right)}d\right) + \left(\left(5\sqrt{a} + 2\sqrt{b}\right)b^{1/4}\operatorname{ArcTanh}\left[\left(b^{1/4}\operatorname{Cosh}[c + d*x]\right)/\sqrt{\left(\sqrt{a} + \sqrt{b}\right)}\right]\right)/\left(64a^{5/2}\left(\sqrt{a} + \sqrt{b}\right)^{5/2}d\right) - \left(b\operatorname{Cosh}[c + d*x]\left(2 - \operatorname{Cosh}[c + d*x]^2\right)\right)/\left(8a\left(a - b\right)d\left(a - b + 2b\operatorname{Cosh}[c + d*x]^2 - b\operatorname{Cosh}[c + d*x]^4\right)^2\right) - \left(b\operatorname{Cosh}[c + d*x]\left(2 - \operatorname{Cosh}[c + d*x]^2\right)\right)/\left(4a^2\left(a - b\right)d\left(a - b + 2b\operatorname{Cosh}[c + d*x]^2 - b\operatorname{Cosh}[c + d*x]^4\right)\right) - \left(b\operatorname{Cosh}[c + d*x]\left(11a + b - \left(5a + b\right)\operatorname{Cosh}[c + d*x]^2\right)\right)/\left(32a^2\left(a - b\right)^2d\left(a - b + 2b\operatorname{Cosh}[c + d*x]^2 - b\operatorname{Cosh}[c + d*x]^4\right)\right)$

$$\begin{aligned} & (b^{1/4} \operatorname{Cosh}[c + d*x]) / \operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]] / (8*a^{5/2} * (\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b])^{3/2} * d) - (b^{1/4} \operatorname{ArcTan}[(b^{1/4} \operatorname{Cosh}[c + d*x]) / \operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]])] / (2*a^3 * \operatorname{Sqrt}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[b]] * d) - \operatorname{ArcTanh}[\operatorname{Cosh}[c + d*x]] / (a^3 * d) \\ & + (b^{1/4} \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x]) / \operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]])] / (8*a^{5/2} * (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b])^{3/2} * d) + (b^{1/4} \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x]) / \operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]])] / (2*a^3 * \operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]] * d) + ((5 * \operatorname{Sqrt}[a] + 2 * \operatorname{Sqrt}[b]) * b^{1/4} \operatorname{ArcTanh}[(b^{1/4} \operatorname{Cosh}[c + d*x]) / \operatorname{Sqrt}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b]])] / (64*a^{5/2} * (\operatorname{Sqrt}[a] + \operatorname{Sqrt}[b])^{5/2} * d) - (b * \operatorname{Cosh}[c + d*x] * (2 - \operatorname{Cosh}[c + d*x]^2)) / (8*a*(a - b)*d*(a - b + 2*b*\operatorname{Cosh}[c + d*x]^2 - b*\operatorname{Cosh}[c + d*x]^4)^2) - (b * \operatorname{Cosh}[c + d*x] * (2 - \operatorname{Cosh}[c + d*x]^2)) / (4*a^2*(a - b)*d*(a - b + 2*b*\operatorname{Cosh}[c + d*x]^2 - b*\operatorname{Cosh}[c + d*x]^4)) - (b * \operatorname{Cosh}[c + d*x] * (11*a + b - (5*a + b) * \operatorname{Cosh}[c + d*x]^2)) / (32*a^2*(a - b)^2*d*(a - b + 2*b*\operatorname{Cosh}[c + d*x]^2 - b*\operatorname{Cosh}[c + d*x]^4)) \end{aligned}$$

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^(m - 1)/2*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p, x], x, Cos[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 1238

```
Int[((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 207

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 1178

```
Int[((d_.) + (e_.)*(x_)^2)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)^3} dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(-\frac{1}{a^3(-1+x^2)} + \frac{b-bx^2}{a(a-b+2bx^2-bx^4)^3} + \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)^2} + \frac{b-bx^2}{a^3(a-b+2bx^2-bx^4)}\right) dx, x, \cosh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \cosh(c+dx)\right)}{a^3 d} - \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4} dx, x, \cosh(c+dx)\right)}{a^3 d} - \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a^2(a-b+2bx^2-bx^4)^2} dx, x, \cosh(c+dx)\right)}{a^3 d} \\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} - \frac{b \cosh(c+dx) (2 - \cosh^2(c+dx))}{8a(a-b)d (a-b+2b \cosh^2(c+dx) - b \cosh^4(c+dx))^2} - \frac{4\sqrt{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3 \sqrt{\sqrt{a}-\sqrt{b}d}} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{4\sqrt{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3 \sqrt{\sqrt{a}+\sqrt{b}d}} - \frac{4\sqrt{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^5} \\
&= -\frac{4\sqrt{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2} (\sqrt{a}-\sqrt{b})^{3/2} d} - \frac{4\sqrt{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3 \sqrt{\sqrt{a}-\sqrt{b}d}} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{4\sqrt{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{5/2} (\sqrt{a}+\sqrt{b})^{3/2} d} \\
&= -\frac{(5\sqrt{a}-2\sqrt{b}) \sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2} (\sqrt{a}-\sqrt{b})^{5/2} d} - \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2} (\sqrt{a}-\sqrt{b})^{3/2} d} - \frac{\sqrt[4]{b} \tan^{-1}\left(\frac{\sqrt[4]{b} \cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3 \sqrt{\sqrt{a}+\sqrt{b}d}}
\end{aligned}$$

Mathematica [C] time = 5.75688, size = 1189, normalized size = 1.93

result too large to display

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] ((32*a*b*Cosh[c + d*x]*(-41*a + 23*b + (13*a - 7*b)*Cosh[2*(c + d*x)])))/((a - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a^2*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)])))/((a - b)*(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2) + 256*Log[Tanh[(c + d*x)/2]] - (b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-45*a^

```

2*c + 71*a*b*c - 32*b^2*c - 45*a^2*d*x + 71*a*b*d*x - 32*b^2*d*x - 90*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 142*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 64*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 199*a^2*c*#1^2 - 253*a*b*c*#1^2 + 96*b^2*c*#1^2 + 199*a^2*d*x*#1^2 - 253*a*b*d*x*#1^2 + 96*b^2*d*x*#1^2 + 398*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 506*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 192*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 199*a^2*c*#1^4 + 253*a*b*c*#1^4 - 96*b^2*c*#1^4 - 199*a^2*d*x*#1^4 + 253*a*b*d*x*#1^4 - 96*b^2*d*x*#1^4 - 398*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 506*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 192*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 45*a^2*c*#1^6 - 71*a*b*c*#1^6 + 32*b^2*c*#1^6 + 45*a^2*d*x*#1^6 - 71*a*b*d*x*#1^6 + 32*b^2*d*x*#1^6 + 90*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 142*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 + 64*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b)^2)/(256*a^3*d)

```

Maple [B] time = 0.138, size = 3159, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x)
```

```

[Out] -1/2/d*b^2/a^3/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/2/d*b^2/a^3/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+26/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*b^3+70/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*b^3-53/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/

```

$$\begin{aligned}
& 2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/ \\
& a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*b^2+5/8/d/(tanh(1/2*d*x+1/2*c)^8*a- \\
& 4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c \\
&)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14 \\
& *b^2-5/32/d/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*t \\
& anh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*b^2+5 \\
& /32/d/a^2/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2 \\
& *d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*b^2-45/64/d/ \\
& (a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/ \\
& 2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1161/8/ \\
& d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^ \\
& 4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2* \\
& a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+1/4/d*b/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)* \\
& a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a \\
& *b)^(1/2)*a)^(1/2))-1/4/d*b/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*ar \\
& ctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a) \\
& ^{(1/2)})-45/64/d/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2* \\
& tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b \\
&)^(1/2)-327/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1 \\
& /2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2 \\
& *b^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^10+185/4/d/(tanh(1/2*d*x+1/2*c)^ \\
& 8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1 \\
& /2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c \\
&)^4*b^2+43/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/ \\
& 2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/ \\
& (a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^12*b^2+537/4/d/(tanh(1/2*d*x+1/2*c)^8 \\
& *a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/ \\
& 2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/ \\
& 2*c)^8+71/64/d/a^2/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2 \\
& *tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a* \\
& b)^(1/2)*b+71/64/d/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/ \\
& 4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2) \\
&)*(a*b)^(1/2)*b-216/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6* \\
& tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2* \\
& a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*b^3-315/8/d/(tanh(1/2*d*x+ \\
& 1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/ \\
& 2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d* \\
& x+1/2*c)^8+50/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1 \\
& /2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2 \\
& *b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+10/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta \\
& nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4- \\
& 4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/d/ \\
& (tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4* \\
& a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^ \\
& 2)*tanh(1/2*d*x+1/2*c)^14+5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
\end{aligned}$$

$$\begin{aligned}
& *c)^{6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x \\
& +1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{12+3/4/d*b^2/a}/(\tanh \\
& (1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16* \\
& b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)+13/d \\
& /(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4 \\
& *a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b \\
& ^2)*\tanh(1/2*d*x+1/2*c)^{10-16/d*b^3/a^2}/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2 \\
& *d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh \\
& (1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4+96/d*b^4/a^3 \\
& /(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4 \\
& *a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2 \\
&)*\tanh(1/2*d*x+1/2*c)^8-8/d*b^3/a^2}/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x \\
& +1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2 \\
& *d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^{12-9/8/d}/(\tanh(1/2 \\
& *d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*ta \\
& nh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*b+1/d/a^ \\
& 3*\ln(\tanh(1/2*d*x+1/2*c))-257/8/d}/(\tanh(1/2*d*x+1/2*c)^{8*a-4*\tanh(1/2*d*x+1 \\
& /2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d \\
& *x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^4
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned}
& -1/16*((13*a*b^2*e^{(15*c)} - 7*b^3*e^{(15*c)})e^{(15*d*x)} - (121*a*b^2*e^{(13*c)} \\
&) - 67*b^3*e^{(13*c)})e^{(13*d*x)} - (272*a^2*b*e^{(11*c)} - 461*a*b^2*e^{(11*c)} \\
& + 159*b^3*e^{(11*c)})e^{(11*d*x)} + (1424*a^2*b*e^{(9*c)} - 1121*a*b^2*e^{(9*c)} + \\
& 99*b^3*e^{(9*c)})e^{(9*d*x)} + (1424*a^2*b*e^{(7*c)} - 1121*a*b^2*e^{(7*c)} + 99* \\
& b^3*e^{(7*c)})e^{(7*d*x)} - (272*a^2*b*e^{(5*c)} - 461*a*b^2*e^{(5*c)} + 159*b^3*e \\
& ^{(5*c)})e^{(5*d*x)} - (121*a*b^2*e^{(3*c)} - 67*b^3*e^{(3*c)})e^{(3*d*x)} + (13*a* \\
& b^2*e^c - 7*b^3*e^c)e^{(d*x)}/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b \\
& ^2*d*e^{(16*c)} - 2*a^3*b^3*d*e^{(16*c)} + a^2*b^4*d*e^{(16*c)})e^{(16*d*x)} - 8*(\\
& a^4*b^2*d*e^{(14*c)} - 2*a^3*b^3*d*e^{(14*c)} + a^2*b^4*d*e^{(14*c)})e^{(14*d*x)} \\
& - 4*(8*a^5*b*d*e^{(12*c)} - 23*a^4*b^2*d*e^{(12*c)} + 22*a^3*b^3*d*e^{(12*c)} - 7 \\
& *a^2*b^4*d*e^{(12*c)})e^{(12*d*x)} + 8*(16*a^5*b*d*e^{(10*c)} - 39*a^4*b^2*d*e^{(\\
& 10*c)} + 30*a^3*b^3*d*e^{(10*c)} - 7*a^2*b^4*d*e^{(10*c)})e^{(10*d*x)} + 2*(128*a \\
& ^6*d*e^{(8*c)} - 352*a^5*b*d*e^{(8*c)} + 355*a^4*b^2*d*e^{(8*c)} - 166*a^3*b^3*d* \\
& e^{(8*c)} + 35*a^2*b^4*d*e^{(8*c)})e^{(8*d*x)} + 8*(16*a^5*b*d*e^{(6*c)} - 39*a^4*
\end{aligned}$$

$$b^2 d e^{(6c)} + 30 a^3 b^3 d e^{(6c)} - 7 a^2 b^4 d e^{(6c)}) e^{(6dx)} - 4(8 a^5 b d e^{(4c)} - 23 a^4 b^2 d e^{(4c)} + 22 a^3 b^3 d e^{(4c)} - 7 a^2 b^4 d e^{(4c)}) e^{(4dx)} - 8(a^4 b^2 d e^{(2c)} - 2 a^3 b^3 d e^{(2c)} + a^2 b^4 d e^{(2c)}) e^{(2dx)} - \log((e^{(dx+c)} + 1) e^{(-c)}) / (a^3 d) + \log((e^{(dx+c)} - 1) e^{(-c)}) / (a^3 d) - 2 \int (1/32((45 a^2 b e^{(7c)} - 71 a b^2 e^{(7c)} + 32 b^3 e^{(7c)}) e^{(7dx)} - (199 a^2 b e^{(5c)} - 253 a b^2 e^{(5c)} + 96 b^3 e^{(5c)}) e^{(5dx)} + (199 a^2 b e^{(3c)} - 253 a b^2 e^{(3c)} + 96 b^3 e^{(3c)}) e^{(3dx)} - (45 a^2 b e^c - 71 a b^2 e^c + 32 b^3 e^c) e^{(dx)}) / (a^5 b - 2 a^4 b^2 + a^3 b^3 + (a^5 b e^{(8c)} - 2 a^4 b^2 e^{(8c)} + a^3 b^3 e^{(8c)}) e^{(8dx)} - 4(a^5 b e^{(6c)} - 2 a^4 b^2 e^{(6c)} + a^3 b^3 e^{(6c)}) e^{(6dx)} - 2(8 a^6 e^{(4c)} - 19 a^5 b e^{(4c)} + 14 a^4 b^2 e^{(4c)} - 3 a^3 b^3 e^{(4c)}) e^{(4dx)} - 4(a^5 b e^{(2c)} - 2 a^4 b^2 e^{(2c)} + a^3 b^3 e^{(2c)}) e^{(2dx)}), x)$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)/(a-b*sinh(dx+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(dx+c)/(a-b*sinh(dx+c)**4)**3,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```


$$3.259 \quad \int \frac{\sinh^8(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^3} dx$$

Optimal. Leaf size=319

$$\frac{(2\sqrt{a}-5\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d(\sqrt{a}-\sqrt{b})^{5/2}} + \frac{(2\sqrt{a}+5\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d(\sqrt{a}+\sqrt{b})^{5/2}} + \frac{\tanh^9(c+dx)}{8ad((a-b) \tanh^4(c+dx)-2)}$$

```
[Out] -((2*Sqrt[a] - 5*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a
^(1/4)])/(64*a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] + 5
*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(
3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tanh[c + d*x])/(32*a
*(a - b)^2*b*d) - Tanh[c + d*x]^3/(32*a*(a - b)*b*d) + Tanh[c + d*x]^9/(8*a
*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (Sech[c + d*x]^
2*Tanh[c + d*x]^5)/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*
x]^4))
```

Rubi [A] time = 0.48526, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {3217, 1275, 12, 1120, 1279, 1166, 208}

$$\frac{(2\sqrt{a}-5\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d(\sqrt{a}-\sqrt{b})^{5/2}} + \frac{(2\sqrt{a}+5\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d(\sqrt{a}+\sqrt{b})^{5/2}} + \frac{\tanh^9(c+dx)}{8ad((a-b) \tanh^4(c+dx)-2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((2*Sqrt[a] - 5*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a
^(1/4)])/(64*a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] + 5
*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(
3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tanh[c + d*x])/(32*a
*(a - b)^2*b*d) - Tanh[c + d*x]^3/(32*a*(a - b)*b*d) + Tanh[c + d*x]^9/(8*a
*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (Sech[c + d*x]^
2*Tanh[c + d*x]^5)/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*
x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2
)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
IntegerQ[m/2] && IntegerQ[p]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] := Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1
)*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:= -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^(
m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
```

```
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^8(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^8(1-x^2)}{(a-2ax^2+(a-b)x^4)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh^9(c+dx)}{8ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))^2} + \frac{\text{Subst}\left(\int -\frac{2bx^8}{(a-2ax^2+(a-b)x^4)^2} dx, x, \tanh(c+dx)\right)}{16abd} \\
&= \frac{\tanh^9(c+dx)}{8ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{x^8}{(a-2ax^2+(a-b)x^4)^2} dx, x, \tanh(c+dx)\right)}{8ad} \\
&= \frac{\tanh^9(c+dx)}{8ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))^2} - \frac{\text{sech}^2(c+dx)\tanh^5(c+dx)}{32abd(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} \\
&= -\frac{\tanh^3(c+dx)}{32a(a-b)bd} + \frac{\tanh^9(c+dx)}{8ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))^2} - \frac{\text{sech}^2(c+dx)\tanh^5(c+dx)}{32abd(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))} \\
&= -\frac{(a+5b)\tanh(c+dx)}{32a(a-b)^2bd} - \frac{\tanh^3(c+dx)}{32a(a-b)bd} + \frac{\tanh^9(c+dx)}{8ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))^2} \\
&= -\frac{(a+5b)\tanh(c+dx)}{32a(a-b)^2bd} - \frac{\tanh^3(c+dx)}{32a(a-b)bd} + \frac{\tanh^9(c+dx)}{8ad(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx))^2} \\
&= -\frac{(2\sqrt{a}-5\sqrt{b})\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}(\sqrt{a}-\sqrt{b})^{5/2}b^{3/2}d} + \frac{(2\sqrt{a}+5\sqrt{b})\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}(\sqrt{a}+\sqrt{b})^{5/2}b^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 3.95342, size = 331, normalized size = 1.04

$$\frac{(2a^{3/2}\sqrt{b}-8\sqrt{a}b^{3/2}+ab+5b^2)\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{b}(2\sqrt{a}-5\sqrt{b})(\sqrt{a}+\sqrt{b})^2\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{8b\sinh(2(c+dx))(5b-2a)\cosh(2(c+dx))}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))}$$

$$64b^2d(a-b)^2$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3, x]

```
[Out] (((2*sqrt[a] - 5*sqrt[b])*(sqrt[a] + sqrt[b])^2*sqrt[b]*ArcTan[((sqrt[a] -
sqrt[b])*Tanh[c + d*x])/sqrt[-a + sqrt[a]*sqrt[b]]])/(sqrt[a]*sqrt[-a + sqrt
[a]*sqrt[b])) + ((2*a^(3/2)*sqrt[b] + a*b - 8*sqrt[a]*b^(3/2) + 5*b^2)*Arc
TanH[((sqrt[a] + sqrt[b])*Tanh[c + d*x])/sqrt[a + sqrt[a]*sqrt[b]]])/(sqrt[
a]*sqrt[a + sqrt[a]*sqrt[b])) + (8*b*(5*a - 14*b + (-2*a + 5*b)*Cosh[2*(c +
d*x)])*Sinh[2*(c + d*x)]/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c
+ d*x)]) + (64*a*(a - b)*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)])))/(-8
*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2/(64*(a - b)^2*b^
2*d)
```

Maple [C] time = 0.092, size = 2236, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -1/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+5/16/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^3+49/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-9/16/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^5*a^2-165/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1
/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a+9/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^5+5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/
b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a^2+377/16/d/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7
*a-49/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-
```

$$\begin{aligned}
& 2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7+5/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2* \\
& *d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh \\
& (1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9*a^2+377/16 \\
& /d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c) \\
& ^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b \\
& ^2)*\tanh(1/2*d*x+1/2*c)^9*a-49/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/ \\
& 2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d* \\
& x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9-9/16/d/(\tanh(1/2* \\
& d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh \\
& (1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*\tanh(1/ \\
& 2*d*x+1/2*c)^11*a^2-165/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c) \\
& ^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/ \\
& 2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^11*a+9/d/(\tanh(1/2*d*x+1/ \\
& 2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2* \\
& d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+ \\
& 1/2*c)^11+5/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(\\
& 1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^ \\
& 2*a^2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^13+49/16/d/(\tanh(1/2*d*x+1/2*c) \\
& ^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+ \\
& 1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2* \\
& c)^13-1/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2* \\
& d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^ \\
& 2/b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^15-5/16/d/(\tanh(1/2*d*x+1/2*c)^8*a- \\
& 4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c) \\
&)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^15 \\
& -1/128/d/b/(a^2-2*a*b+b^2)*\text{sum}(((a+5*b)*_R^6+(5*a-47*b)*_R^4+(-5*a+47*b)*_R \\
& ^2-a-5*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_ \\
& R),_R=\text{RootOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] $-1/8*(2*a*b^2 - 5*b^3 + (a*b^2*e^{(14*c)} - 4*b^3*e^{(14*c)})*e^{(14*d*x)} - (32*a^2*b*e^{(12*c)} - 58*a*b^2*e^{(12*c)} - b^3*e^{(12*c)})*e^{(12*d*x)} + 3*(48*a^2*b*e^{(10*c)} - 73*a*b^2*e^{(10*c)} + 20*b^3*e^{(10*c)})*e^{(10*d*x)} + (256*a^3*e^{(8*c)} - 832*a^2*b*e^{(8*c)} + 550*a*b^2*e^{(8*c)} - 175*b^3*e^{(8*c)})*e^{(8*d*x)} + (112*a^2*b*e^{(6*c)} - 533*a*b^2*e^{(6*c)} + 220*b^3*e^{(6*c)})*e^{(6*d*x)} - (32*a$

$$\begin{aligned} & ^2*b*e^{(4*c)} - 158*a*b^2*e^{(4*c)} + 141*b^3*e^{(4*c)})*e^{(4*d*x)} - (17*a*b^2*e^{(2*c)} - 44*b^3*e^{(2*c)})*e^{(2*d*x)})/(a^2*b^4*d - 2*a*b^5*d + b^6*d + (a^2*b^4*d*e^{(16*c)} - 2*a*b^5*d*e^{(16*c)} + b^6*d*e^{(16*c)})*e^{(16*d*x)} - 8*(a^2*b^4*d*e^{(14*c)} - 2*a*b^5*d*e^{(14*c)} + b^6*d*e^{(14*c)})*e^{(14*d*x)} - 4*(8*a^3*b^3*d*e^{(12*c)} - 23*a^2*b^4*d*e^{(12*c)} + 22*a*b^5*d*e^{(12*c)} - 7*b^6*d*e^{(12*c)})*e^{(12*d*x)} + 8*(16*a^3*b^3*d*e^{(10*c)} - 39*a^2*b^4*d*e^{(10*c)} + 30*a*b^5*d*e^{(10*c)} - 7*b^6*d*e^{(10*c)})*e^{(10*d*x)} + 2*(128*a^4*b^2*d*e^{(8*c)} - 352*a^3*b^3*d*e^{(8*c)} + 355*a^2*b^4*d*e^{(8*c)} - 166*a*b^5*d*e^{(8*c)} + 35*b^6*d*e^{(8*c)})*e^{(8*d*x)} + 8*(16*a^3*b^3*d*e^{(6*c)} - 39*a^2*b^4*d*e^{(6*c)} + 30*a*b^5*d*e^{(6*c)} - 7*b^6*d*e^{(6*c)})*e^{(6*d*x)} - 4*(8*a^3*b^3*d*e^{(4*c)} - 23*a^2*b^4*d*e^{(4*c)} + 22*a*b^5*d*e^{(4*c)} - 7*b^6*d*e^{(4*c)})*e^{(4*d*x)} - 8*(a^2*b^4*d*e^{(2*c)} - 2*a*b^5*d*e^{(2*c)} + b^6*d*e^{(2*c)})*e^{(2*d*x)}) - 1/256*\text{integrate}(64*((a*e^{(6*c)} - 4*b*e^{(6*c)})*e^{(6*d*x)} + (a*e^{(2*c)} - 4*b*e^{(2*c)})*e^{(2*d*x)} + 18*b*e^{(4*d*x + 4*c)})/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2*e^{(8*c)} - 2*a*b^3*e^{(8*c)} + b^4*e^{(8*c)})*e^{(8*d*x)} - 4*(a^2*b^2*e^{(6*c)} - 2*a*b^3*e^{(6*c)} + b^4*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^3*b*e^{(4*c)} - 19*a^2*b^2*e^{(4*c)} + 14*a*b^3*e^{(4*c)} - 3*b^4*e^{(4*c)})*e^{(4*d*x)} - 4*(a^2*b^2*e^{(2*c)} - 2*a*b^3*e^{(2*c)} + b^4*e^{(2*c)})*e^{(2*d*x)}), x) \end{aligned}$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [A] time = 17.4165, size = 527, normalized size = 1.65

$$ab^2e^{(14dx+14c)} - 4b^3e^{(14dx+14c)} - 32a^2be^{(12dx+12c)} + 58ab^2e^{(12dx+12c)} + b^3e^{(12dx+12c)} + 144a^2be^{(10dx+10c)} - 219ab^2e^{(10dx+10c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out]
$$\frac{-1/8*(a*b^2*e^{(14*d*x + 14*c)} - 4*b^3*e^{(14*d*x + 14*c)} - 32*a^2*b*e^{(12*d*x + 12*c)} + 58*a*b^2*e^{(12*d*x + 12*c)} + b^3*e^{(12*d*x + 12*c)} + 144*a^2*b*e^{(10*d*x + 10*c)} - 219*a*b^2*e^{(10*d*x + 10*c)} + 60*b^3*e^{(10*d*x + 10*c)} + 256*a^3*e^{(8*d*x + 8*c)} - 832*a^2*b*e^{(8*d*x + 8*c)} + 550*a*b^2*e^{(8*d*x + 8*c)} - 175*b^3*e^{(8*d*x + 8*c)} + 112*a^2*b*e^{(6*d*x + 6*c)} - 533*a*b^2*e^{(6*d*x + 6*c)} + 220*b^3*e^{(6*d*x + 6*c)} - 32*a^2*b*e^{(4*d*x + 4*c)} + 158*a*b^2*e^{(4*d*x + 4*c)} - 141*b^3*e^{(4*d*x + 4*c)} - 17*a*b^2*e^{(2*d*x + 2*c)} + 44*b^3*e^{(2*d*x + 2*c)} + 2*a*b^2 - 5*b^3)/(a^2*b^2*d - 2*a*b^3*d + b^4*d)*(b*e^{(8*d*x + 8*c)} - 4*b*e^{(6*d*x + 6*c)} - 16*a*e^{(4*d*x + 4*c)} + 6*b*e^{(4*d*x + 4*c)} - 4*b*e^{(2*d*x + 2*c)} + b)^2}$$

$$3.260 \quad \int \frac{\sinh^6(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^3} dx$$

Optimal. Leaf size=345

$$\frac{(-10\sqrt{a}\sqrt{b} + 4a + 3b) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d(\sqrt{a}-\sqrt{b})^{5/2}} - \frac{(10\sqrt{a}\sqrt{b} + 4a + 3b) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d(\sqrt{a}+\sqrt{b})^{5/2}} + \frac{\tanh(c+dx)}{32abd((a-b))}$$

```
[Out] ((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
d*x])/a^(1/4)]/(64*a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) - ((4*a +
10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/
a^(1/4)]/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) + (Tanh[c + d*x]
*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2
*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((2*a*(a^
2 - a*b - 8*b^2))/(a - b)^3 - ((2*a^2 + 15*a*b + 3*b^2)*Tanh[c + d*x]^2)/(a
- b)^2))/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 0.732552, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3217, 1333, 1678, 1166, 208}

$$\frac{(-10\sqrt{a}\sqrt{b} + 4a + 3b) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d(\sqrt{a}-\sqrt{b})^{5/2}} - \frac{(10\sqrt{a}\sqrt{b} + 4a + 3b) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d(\sqrt{a}+\sqrt{b})^{5/2}} + \frac{\tanh(c+dx)}{32abd((a-b))}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
d*x])/a^(1/4)]/(64*a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) - ((4*a +
10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/
a^(1/4)]/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) + (Tanh[c + d*x]
*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2
*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((2*a*(a^
2 - a*b - 8*b^2))/(a - b)^3 - ((2*a^2 + 15*a*b + 3*b^2)*Tanh[c + d*x]^2)/(a
- b)^2))/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2
)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] := With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
+ 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^6(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^6(1-x^2)^2}{(a-2ax^2+(a-b)x^4)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh(c+dx)(a(a+3b) - (a^2+6ab+b^2)\tanh^2(c+dx))}{8(a-b)^3d(a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^3b(a+3b)}{(a-b)^3} + \frac{2a^2b(5a^2+)}{(a-2)}}{(a-2)}\right)}{8(a-b)^3d(a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} \\
&= \frac{\tanh(c+dx)(a(a+3b) - (a^2+6ab+b^2)\tanh^2(c+dx))}{8(a-b)^3d(a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} + \frac{\tanh(c+dx)\left(\frac{2a(a^2-ab-8b^2)}{(a-b)^3}\right)}{32abd(a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} \\
&= \frac{\tanh(c+dx)(a(a+3b) - (a^2+6ab+b^2)\tanh^2(c+dx))}{8(a-b)^3d(a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} + \frac{\tanh(c+dx)\left(\frac{2a(a^2-ab-8b^2)}{(a-b)^3}\right)}{32abd(a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} \\
&= \frac{(4a-10\sqrt{a}\sqrt{b}+3b)\tanh^{-1}\left(\frac{\sqrt{a-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}(\sqrt{a}-\sqrt{b})^{5/2}b^{3/2}d} - \frac{(4a+10\sqrt{a}\sqrt{b}+3b)\tanh^{-1}\left(\frac{\sqrt{a+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}(\sqrt{a}+\sqrt{b})^{5/2}b^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 3.30722, size = 351, normalized size = 1.02

$$\frac{4\sinh(2(c+dx))(4a^2+3b(a+b)\cosh(2(c+dx))-19ab-3b^2)}{ab(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)} + \frac{(10\sqrt{a}\sqrt{b}+4a+3b)(\sqrt{a}-\sqrt{b})^2\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{ab^{3/2}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{(\sqrt{a}+\sqrt{b})^2(-10\sqrt{a}\sqrt{b}+4a+3b)\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{ab^{3/2}\sqrt{\sqrt{a}\sqrt{b}+a}}$$

$$64d(a-b)^2$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] -(((Sqrt[a] + Sqrt[b])^2*(4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])]/(a*Sqrt[-a + Sqrt[a]*Sqrt[b]])*b^(3/2)) + ((Sqrt[a] - Sqrt[b])^2*(4*a + 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])]/(a*Sqrt[-a + Sqrt[a]*Sqrt[b]])*b^(3/2))

$$\frac{\text{ArcTanh}\left(\frac{(\sqrt{a} + \sqrt{b})\tanh(c + dx)}{\sqrt{a + \sqrt{a}\sqrt{b}}}\right)}{a\sqrt{a + \sqrt{a}\sqrt{b}}b^{3/2}} + \frac{4(4a^2 - 19ab - 3b^2 + 3b(a + b)\cosh[2(c + dx)]\sinh[2(c + dx)])}{a^2b(8a - 3b + 4b\cosh[2(c + dx)] - b\cosh[4(c + dx)])} - \frac{128(a - b)(2a + b - b\cosh[2(c + dx)])\sinh[2(c + dx)]}{(b(-8a + 3b - 4b\cosh[2(c + dx)] + b\cosh[4(c + dx)])^2)} \frac{1}{(64(a - b)^2d)}$$

Maple [C] time = 0.088, size = 2681, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\sinh(dx+c))^6 / (a-b\sinh(dx+c))^4 dx$

[Out]
$$\frac{12}{d} \frac{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2}{a^2 b^2 (a^2 - 2ab + b^2)} + \frac{12}{d} \frac{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2}{a^2 b^2 (a^2 - 2ab + b^2)} + \frac{5}{8} \frac{d}{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2} + \frac{5}{8} \frac{d}{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2} + \frac{9}{8} \frac{d}{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2} + \frac{9}{8} \frac{d}{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2} + \frac{5}{8} \frac{d}{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2} + \frac{1}{4} \frac{d}{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2} + \frac{1}{4} \frac{d}{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2} + \frac{1}{4} \frac{d}{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2} + \frac{1}{4} \frac{d}{\tanh(1/2 dx + 1/2 c)^{8a-4} \tanh(1/2 dx + 1/2 c)^{6a+6} \tanh(1/2 dx + 1/2 c)^{4a-16} b \tanh(1/2 dx + 1/2 c)^{4-4} \tanh(1/2 dx + 1/2 c)^{2a+a}^2}$$

```

c)^13+1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d
*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+19/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh
(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*
tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11*a-35/4/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)^11-3/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-3/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3+19/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tan
h(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a
)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a-35/4/d/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5
-27/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a+41/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-27/4/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^9*a+41/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+1/4/d/(tanh(1/2*d*x+1
/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2
*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x
+1/2*c)^15+1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/64/d/a/b/(a^2-2*a*b+b^2)*sum((a*(
-a-2*b)*_R^6+(-5*a^2+32*a*b-6*b^2)*_R^4+(5*a^2-32*a*b+6*b^2)*_R^2+a^2+2*a*b
)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Ro
tOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))

```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*a*b^2 + 3*b^3 - (4*a^2*b*e^(14*c) - 13*a*b^2*e^(14*c) + 3*b^3*e^(14*c))
)*e^(14*d*x) + 3*(8*a^2*b*e^(12*c) - 33*a*b^2*e^(12*c) + 7*b^3*e^(12*c))
)*e^(12*d*x) - (64*a^3*e^(10*c) + 68*a^2*b*e^(10*c) - 225*a*b^2*e^(10*c) +
63*b^3*e^(10*c))*e^(10*d*x) + 3*(128*a^3*e^(8*c) + 32*a^2*b*e^(8*c) - 61*a*
b^2*e^(8*c) + 35*b^3*e^(8*c))*e^(8*d*x) + (64*a^3*e^(6*c) + 452*a^2*b*e^(6*
c) - 9*a*b^2*e^(6*c) - 105*b^3*e^(6*c))*e^(6*d*x) - 3*(40*a^2*b*e^(4*c) - 2
9*a*b^2*e^(4*c) - 21*b^3*e^(4*c))*e^(4*d*x) + (4*a^2*b*e^(2*c) - 37*a*b^2*e
^(2*c) - 21*b^3*e^(2*c))*e^(2*d*x))/(a^3*b^3*d - 2*a^2*b^4*d + a*b^5*d + (a
^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*c) + a*b^5*d*e^(16*c))*e^(16*d*x) - 8
*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(14*c) + a*b^5*d*e^(14*c))*e^(14*d*x)
- 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3*d*e^(12*c) + 22*a^2*b^4*d*e^(12*c) -
7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b^2*d*e^(10*c) - 39*a^3*b^3*d*e
^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*b^5*d*e^(10*c))*e^(10*d*x) + 2*(128*a
^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c) + 355*a^3*b^3*d*e^(8*c) - 166*a^2*b^
4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8*d*x) + 8*(16*a^4*b^2*d*e^(6*c) - 39*
a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*c) - 7*a*b^5*d*e^(6*c))*e^(6*d*x) - 4
*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e^(4*c) + 22*a^2*b^4*d*e^(4*c) - 7*a*b
^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d*e^(2*c) - 2*a^2*b^4*d*e^(2*c) + a*b^
5*d*e^(2*c))*e^(2*d*x)) + 1/64*integrate(8*((4*a^2*e^(6*c) - 13*a*b*e^(6*c)
+ 3*b^2*e^(6*c))*e^(6*d*x) + 6*(7*a*b*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + (
4*a^2*e^(2*c) - 13*a*b*e^(2*c) + 3*b^2*e^(2*c))*e^(2*d*x))/(a^3*b^2 - 2*a^2
*b^3 + a*b^4 + (a^3*b^2*e^(8*c) - 2*a^2*b^3*e^(8*c) + a*b^4*e^(8*c))*e^(8*d
*x) - 4*(a^3*b^2*e^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c))*e^(6*d*x) - 2
*(8*a^4*b*e^(4*c) - 19*a^3*b^2*e^(4*c) + 14*a^2*b^3*e^(4*c) - 3*a*b^4*e^(4*
c))*e^(4*d*x) - 4*(a^3*b^2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c))*e^(
2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [A] time = 18.2037, size = 609, normalized size = 1.77

$4 a^2 b e^{14 d x+14 c} - 13 a b^2 e^{14 d x+14 c} + 3 b^3 e^{14 d x+14 c} - 24 a^2 b e^{12 d x+12 c} + 99 a b^2 e^{12 d x+12 c} - 21 b^3 e^{12 d x+12 c} + 64 a^3 e^{10 d x+10 c} - 225 a^2 b e^{10 d x+10 c} + 63 b^3 e^{10 d x+10 c} - 384 a^3 e^{8 d x+8 c} - 96 a^2 b e^{8 d x+8 c} + 183 a b^2 e^{8 d x+8 c} - 105 b^3 e^{8 d x+8 c} - 64 a^3 e^{6 d x+6 c} - 452 a^2 b e^{6 d x+6 c} + 9 a b^2 e^{6 d x+6 c} + 105 b^3 e^{6 d x+6 c} + 120 a^2 b e^{4 d x+4 c} - 87 a b^2 e^{4 d x+4 c} - 63 b^3 e^{4 d x+4 c} - 4 a^2 b e^{2 d x+2 c} + 37 a b^2 e^{2 d x+2 c} + 21 b^3 e^{2 d x+2 c} - 3 a b^2 - 3 b^3) / ((a^3 b d - 2 a^2 b^2 d + a b^3 d) * (b e^{8 d x+8 c} - 4 b e^{6 d x+6 c} - 16 a e^{4 d x+4 c} + 6 b e^{4 d x+4 c} - 4 b e^{2 d x+2 c} + b)^2)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] $1/16*(4*a^2*b*e^{(14*d*x + 14*c)} - 13*a*b^2*e^{(14*d*x + 14*c)} + 3*b^3*e^{(14*d*x + 14*c)} - 24*a^2*b*e^{(12*d*x + 12*c)} + 99*a*b^2*e^{(12*d*x + 12*c)} - 21*b^3*e^{(12*d*x + 12*c)} + 64*a^3*e^{(10*d*x + 10*c)} + 68*a^2*b*e^{(10*d*x + 10*c)} - 225*a*b^2*e^{(10*d*x + 10*c)} + 63*b^3*e^{(10*d*x + 10*c)} - 384*a^3*e^{(8*d*x + 8*c)} - 96*a^2*b*e^{(8*d*x + 8*c)} + 183*a*b^2*e^{(8*d*x + 8*c)} - 105*b^3*e^{(8*d*x + 8*c)} - 64*a^3*e^{(6*d*x + 6*c)} - 452*a^2*b*e^{(6*d*x + 6*c)} + 9*a*b^2*e^{(6*d*x + 6*c)} + 105*b^3*e^{(6*d*x + 6*c)} + 120*a^2*b*e^{(4*d*x + 4*c)} - 87*a*b^2*e^{(4*d*x + 4*c)} - 63*b^3*e^{(4*d*x + 4*c)} - 4*a^2*b*e^{(2*d*x + 2*c)} + 37*a*b^2*e^{(2*d*x + 2*c)} + 21*b^3*e^{(2*d*x + 2*c)} - 3*a*b^2 - 3*b^3) / ((a^3*b*d - 2*a^2*b^2*d + a*b^3*d)*(b*e^{(8*d*x + 8*c)} - 4*b*e^{(6*d*x + 6*c)} - 16*a*e^{(4*d*x + 4*c)} + 6*b*e^{(4*d*x + 4*c)} - 4*b*e^{(2*d*x + 2*c)} + b)^2)$

$$3.261 \quad \int \frac{\sinh^4(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$$

Optimal. Leaf size=314

$$\frac{\tanh(c+dx) \left(\frac{9a^2-24ab-b^2}{(a-b)^3} - \frac{(17a+3b) \tanh^2(c+dx)}{(a-b)^2} \right)}{32ad \left((a-b) \tanh^4(c+dx) - 2a \tanh^2(c+dx) + a \right)} + \frac{3(2\sqrt{a}-\sqrt{b}) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{64a^{7/4} \sqrt{bd} (\sqrt{a}-\sqrt{b})^{5/2}} - \frac{3(2\sqrt{a}+\sqrt{b}) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{64a^{7/4} \sqrt{bd} (\sqrt{a}+\sqrt{b})^{5/2}}$$

[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] + Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(7/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tanh[c + d*x]*(3*a + b - 4*(a + b)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (Tanh[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 - ((17*a + 3*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rubi [A] time = 0.64847, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3217, 1333, 1678, 1166, 208}

$$\frac{\tanh(c+dx) \left(\frac{9a^2-24ab-b^2}{(a-b)^3} - \frac{(17a+3b) \tanh^2(c+dx)}{(a-b)^2} \right)}{32ad \left((a-b) \tanh^4(c+dx) - 2a \tanh^2(c+dx) + a \right)} + \frac{3(2\sqrt{a}-\sqrt{b}) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{64a^{7/4} \sqrt{bd} (\sqrt{a}-\sqrt{b})^{5/2}} - \frac{3(2\sqrt{a}+\sqrt{b}) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{64a^{7/4} \sqrt{bd} (\sqrt{a}+\sqrt{b})^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] + Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(7/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tanh[c + d*x]*(3*a + b - 4*(a + b)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (Tanh[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 - ((17*a + 3*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^4)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] := With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d =
Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
+ 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sinh^4(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^4(1-x^2)^3}{(a-2ax^2+(a-b)x^4)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{b \tanh(c+dx) (3a+b-4(a+b)\tanh^2(c+dx))}{8(a-b)^3 d (a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^2b^2(3a+b)}{(a-b)^3} - \frac{8a^2(3a-b)}{(a-b)^3}}{(a-2ax^2+(a-b)x^4)^3} dx, x, \tanh(c+dx)\right)}{(a-b)^3 d} \\
&= -\frac{b \tanh(c+dx) (3a+b-4(a+b)\tanh^2(c+dx))}{8(a-b)^3 d (a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} - \frac{\tanh(c+dx) \left(\frac{9a^2-24ab-8a^2}{(a-b)^3}\right)}{32ad (a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} \\
&= -\frac{b \tanh(c+dx) (3a+b-4(a+b)\tanh^2(c+dx))}{8(a-b)^3 d (a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} - \frac{\tanh(c+dx) \left(\frac{9a^2-24ab-8a^2}{(a-b)^3}\right)}{32ad (a-2a\tanh^2(c+dx) + (a-b)\tanh^4(c+dx))^2} \\
&= \frac{3(2\sqrt{a}-\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4} (\sqrt{a}-\sqrt{b})^{5/2} \sqrt{bd}} - \frac{3(2\sqrt{a}+\sqrt{b}) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4} (\sqrt{a}+\sqrt{b})^{5/2} \sqrt{bd}}
\end{aligned}$$

Mathematica [A] time = 4.7506, size = 316, normalized size = 1.01

$$\frac{3(2a^{3/2}-3a\sqrt{b}+b^{3/2}) \tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{3(2a^{3/2}+3a\sqrt{b}-b^{3/2}) \tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{8 \sinh(2(c+dx))(2a+b) \cosh(2(c+dx))-7a-2b}{a(8a+4b \cosh(2(c+dx))-b \cosh(4(c+dx))-3b)} + \dots$$

$$64d(a-b)^2$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3, x]

[Out] ((-3*(2*a^(3/2) + 3*a*Sqrt[b] - b^(3/2))*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])]/(a^(3/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]])*Sqrt[b] - (3*(2*a^(3/2) - 3*a*Sqrt[b] + b^(3/2))*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]])]/(a^(3/2)*Sqrt[a + Sqrt[a]*Sqrt[b]])*Sqrt[b] + (8*(-7*a - 2*b + (2*a + b)*Cosh[2*(c + d*x)])*Sinh[2*(c +

$$\begin{aligned}
& a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c) \\
&)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9-177/16/d/(\tanh(1/2*d \\
& *x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh \\
& (1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d \\
& *x+1/2*c)^11*a+131/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+ \\
& 6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^ \\
& 2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^11+1/d/(\tanh(1/2*d*x+1/2*c)^ \\
& 8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1 \\
& /2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*\tanh(1/2*d*x+1/2*c) \\
&)^11*b^2+77/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(\\
& 1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^ \\
& 2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^13-23/16/d/(\tanh(1/2*d*x+1/2*c)^8*a \\
& -4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2* \\
& c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^1 \\
& 3-9/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+ \\
& 1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2 \\
& -2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^15+3/16/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1 \\
& /2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*ta \\
& nh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^15*b-3/128/d \\
& /(a^2-2*a*b+b^2)/a*\text{sum}(((3*a-b)*_R^6+(-17*a+3*b)*_R^4+(17*a-3*b)*_R^2-3*a+b) \\
&)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R),_R=\text{Ro} \\
& \text{otOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] $\begin{aligned}
& 1/8*(3*a*b^2*e^{(14*d*x + 14*c)} + 2*a*b^2 + b^3 - 3*(10*a*b^2*e^{(12*c)} - b^3 \\
& *e^{(12*c)})*e^{(12*d*x)} - (80*a^2*b*e^{(10*c)} - 111*a*b^2*e^{(10*c)} + 16*b^3*e^{ \\
& (10*c)})*e^{(10*d*x)} + (256*a^3*e^{(8*c)} - 64*a^2*b*e^{(8*c)} - 26*a*b^2*e^{(8*c)} \\
& + 35*b^3*e^{(8*c)})*e^{(8*d*x)} + (336*a^2*b*e^{(6*c)} - 95*a*b^2*e^{(6*c)} - 40*b \\
& ^3*e^{(6*c)})*e^{(6*d*x)} - (64*a^2*b*e^{(4*c)} - 54*a*b^2*e^{(4*c)} - 25*b^3*e^{(4* \\
& c)})*e^{(4*d*x)} - (19*a*b^2*e^{(2*c)} + 8*b^3*e^{(2*c)})*e^{(2*d*x)})/(a^3*b^3*d - \\
& 2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^{(16*c)} - 2*a^2*b^4*d*e^{(16*c)} + a*b^5* \\
& d*e^{(16*c)})*e^{(16*d*x)} - 8*(a^3*b^3*d*e^{(14*c)} - 2*a^2*b^4*d*e^{(14*c)} + a*b \\
& ^5*d*e^{(14*c)})*e^{(14*d*x)} - 4*(8*a^4*b^2*d*e^{(12*c)} - 23*a^3*b^3*d*e^{(12*c)} \\
& + 22*a^2*b^4*d*e^{(12*c)} - 7*a*b^5*d*e^{(12*c)})*e^{(12*d*x)} + 8*(16*a^4*b^2*d \\
& *e^{(10*c)} - 39*a^3*b^3*d*e^{(10*c)} + 30*a^2*b^4*d*e^{(10*c)} - 7*a*b^5*d*e^{(10
\end{aligned}$

```

*c))e^(10*d*x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c) + 355*a^3*
b^3*d*e^(8*c) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))e^(8*d*x) + 8*(
16*a^4*b^2*d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*c) - 7*a*b^
5*d*e^(6*c))e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e^(4*c) + 22
*a^2*b^4*d*e^(4*c) - 7*a*b^5*d*e^(4*c))e^(4*d*x) - 8*(a^3*b^3*d*e^(2*c) -
2*a^2*b^4*d*e^(2*c) + a*b^5*d*e^(2*c))e^(2*d*x)) + 1/16*integrate(-12*(2*(
4*a*e^(4*c) - b*e^(4*c))e^(4*d*x) - a*e^(6*d*x + 6*c) - a*e^(2*d*x + 2*c))
/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(8*c) - 2*a^2*b^2*e^(8*c) + a*b^3*e^
(8*c))e^(8*d*x) - 4*(a^3*b*e^(6*c) - 2*a^2*b^2*e^(6*c) + a*b^3*e^(6*c))e^
(6*d*x) - 2*(8*a^4*e^(4*c) - 19*a^3*b*e^(4*c) + 14*a^2*b^2*e^(4*c) - 3*a*b^
3*e^(4*c))e^(4*d*x) - 4*(a^3*b*e^(2*c) - 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c)
)*e^(2*d*x)), x)

```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 17.54, size = 489, normalized size = 1.56

$$3ab^2e^{(14dx+14c)} - 30ab^2e^{(12dx+12c)} + 3b^3e^{(12dx+12c)} - 80a^2be^{(10dx+10c)} + 111ab^2e^{(10dx+10c)} - 16b^3e^{(10dx+10c)} + 256a^3e^{(10dx+10c)}$$

$$8(a^3b^3e^{(10dx+10c)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")`

[Out]
$$\frac{1}{8} \cdot (3ab^2e^{14dx+14c} - 30a^2be^{12dx+12c} + 3b^3e^{12dx+12c} - 80a^2be^{10dx+10c} + 111a^2b^2e^{10dx+10c} - 16b^3e^{10dx+10c} + 256a^3e^{8dx+8c} - 64a^2be^{8dx+8c} - 26a^2b^2e^{8dx+8c} + 35b^3e^{8dx+8c} + 336a^2be^{6dx+6c} - 95a^2b^2e^{6dx+6c} - 40b^3e^{6dx+6c} - 64a^2be^{4dx+4c} + 54a^2b^2e^{4dx+4c} + 25b^3e^{4dx+4c} - 19a^2b^2e^{2dx+2c} - 8b^3e^{2dx+2c} + 2a^2b^2 + b^3) / ((a^3bd - 2a^2b^2d + ab^3d)(be^{8dx+8c} - 4be^{6dx+6c} - 16ae^{4dx+4c} + 6be^{4dx+4c} - 4be^{2dx+2c} + b)^2)$$

$$3.262 \quad \int \frac{\sinh^2(c+dx)}{\left(a-b \sinh^4(c+dx)\right)^3} dx$$

Optimal. Leaf size=348

$$\frac{\tanh(c+dx) \left(\frac{2a(5a^2-9ab-4b^2)}{(a-b)^3} - \frac{5(2a^2+3ab-b^2)\tanh^2(c+dx)}{(a-b)^2} \right)}{32a^2d \left((a-b)\tanh^4(c+dx) - 2a\tanh^2(c+dx) + a \right)} + \frac{b \tanh(c+dx) \left(a(a+3b) - (a^2+6ab+b^2)\tanh^2(c+dx) \right)}{8ad(a-b)^3 \left((a-b)\tanh^4(c+dx) - 2a\tanh^2(c+dx) + a \right)^2}$$

```
[Out] -((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) + ((12*a + 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) + (b*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((2*a*(5*a^2 - 9*a*b - 4*b^2))/(a - b)^3 - (5*(2*a^2 + 3*a*b - b^2)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 0.664354, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3217, 1333, 1678, 1166, 208}

$$\frac{\tanh(c+dx) \left(\frac{2a(5a^2-9ab-4b^2)}{(a-b)^3} - \frac{5(2a^2+3ab-b^2)\tanh^2(c+dx)}{(a-b)^2} \right)}{32a^2d \left((a-b)\tanh^4(c+dx) - 2a\tanh^2(c+dx) + a \right)} + \frac{b \tanh(c+dx) \left(a(a+3b) - (a^2+6ab+b^2)\tanh^2(c+dx) \right)}{8ad(a-b)^3 \left((a-b)\tanh^4(c+dx) - 2a\tanh^2(c+dx) + a \right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) + ((12*a + 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) + (b*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((2*a*(5*a^2 - 9*a*b - 4*b^2))/(a - b)^3 - (5*(2*a^2 + 3*a*b - b^2)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

*x]^4))

Rule 3217

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 1333

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*Simp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]

Rule 1678

Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{d = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\sinh^2(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{x^2(1-x^2)^4}{(a-2ax^2+(a-b)x^4)^3} dx, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{b \tanh(c+dx) (a(a+3b) - (a^2 + 6ab + b^2) \tanh^2(c+dx))}{8a(a-b)^3 d (a - 2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^2b^2(a+3b)}{(a-b)^3} - \frac{2ab(8a^3)}{(a-b)^3}}{dx}, x, \tanh(c+dx)\right)}{d} \\
 &= \frac{b \tanh(c+dx) (a(a+3b) - (a^2 + 6ab + b^2) \tanh^2(c+dx))}{8a(a-b)^3 d (a - 2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} + \frac{\tanh(c+dx) \left(\frac{2a(5a^2-9ab-b^2)}{(a-b)^3}\right)}{32a^2 d (a - 2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} \\
 &= \frac{b \tanh(c+dx) (a(a+3b) - (a^2 + 6ab + b^2) \tanh^2(c+dx))}{8a(a-b)^3 d (a - 2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} + \frac{\tanh(c+dx) \left(\frac{2a(5a^2-9ab-b^2)}{(a-b)^3}\right)}{32a^2 d (a - 2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} \\
 &= -\frac{(12a - 14\sqrt{a}\sqrt{b} + 5b) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4} (\sqrt{a} - \sqrt{b})^{5/2} \sqrt{bd}} + \frac{(12a + 14\sqrt{a}\sqrt{b} + 5b) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4} (\sqrt{a} + \sqrt{b})^{5/2} \sqrt{bd}}
 \end{aligned}$$

Mathematica [A] time = 4.87274, size = 343, normalized size = 0.99

$$\frac{4 \sinh(2(c+dx))(12a^2+b(5b-11a) \cosh(2(c+dx))+11ab-5b^2)}{8a+4b \cosh(2(c+dx))-b \cosh(4(c+dx))-3b} + \frac{(14\sqrt{a}\sqrt{b}+12a+5b)(\sqrt{a}-\sqrt{b})^2 \tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{(\sqrt{a}+\sqrt{b})^2(-14\sqrt{a}\sqrt{b}+12a+5b) \tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}$$

$$64a^2d(a-b)^2$$

Antiderivative was successfully verified.

[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] (((Sqrt[a] + Sqrt[b])^2*(12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]])]/(Sqrt[-a + Sqrt[a]*S

$$\begin{aligned} & \text{qrt}[b]]*\text{Sqrt}[b)) + ((\text{Sqrt}[a] - \text{Sqrt}[b])^2*(12*a + 14*\text{Sqrt}[a]*\text{Sqrt}[b] + 5*b) \\ & * \text{ArcTanh}(((\text{Sqrt}[a] + \text{Sqrt}[b])*\text{Tanh}[c + d*x])/ \text{Sqrt}[a + \text{Sqrt}[a]*\text{Sqrt}[b]]))/(\text{S} \\ & \text{qrt}[a + \text{Sqrt}[a]*\text{Sqrt}[b]]*\text{Sqrt}[b]) + (4*(12*a^2 + 11*a*b - 5*b^2 + b*(-11*a \\ & + 5*b)*\text{Cosh}[2*(c + d*x)])*\text{Sinh}[2*(c + d*x)]/(8*a - 3*b + 4*b*\text{Cosh}[2*(c + d \\ & *x)] - b*\text{Cosh}[4*(c + d*x)]) + (128*a*(a - b)*(2*a + b - b*\text{Cosh}[2*(c + d*x)] \\ &)*\text{Sinh}[2*(c + d*x)]/(-8*a + 3*b - 4*b*\text{Cosh}[2*(c + d*x)] + b*\text{Cosh}[4*(c + d* \\ & x)]))^2)/(64*a^2*(a - b)^2*d) \end{aligned}$$

Maple [C] time = 0.095, size = 2670, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\sinh(d*x+c)^2/(a-b*\sinh(d*x+c)^4)^3, x)$

[Out]
$$\begin{aligned} & -1/64/d/a^2/(a^2-2*a*b+b^2)*\text{sum}((a*(-5*a+2*b)*_R^6+(39*a^2-28*a*b+10*b^2)* \\ & _R^4+(-39*a^2+28*a*b-10*b^2)*_R^2+5*a^2-2*a*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_ \\ & R^3*b-_R*a)*\ln(\tanh(1/2*d*x+1/2*c)-_R), _R=\text{RootOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b) \\ & *_Z^4-4*a*_Z^2+a))+97/2/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6* \\ & a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c \\ &)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9+97/2/d/(\tanh(1/2*d*x \\ & +1/2*c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1 \\ & /2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*\tanh(1 \\ & /2*d*x+1/2*c)^7-20/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*t \\ & \text{anh}(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a \\ & +a)^2/a^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7*b^3+9/4/d/(\tanh(1/2*d*x+1/2 \\ & *c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d \\ & *x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1 \\ & /2*c)^3*b^2+9/4/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh \\ & (1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a) \\ & ^2/a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^13*b^2-20/d/(\tanh(1/2*d*x+1/2*c))^8 \\ & *a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/ \\ & 2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2* \\ & c)^9*b^3-27/4/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1 \\ & /2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2 \\ & /(a^2-2*a*b+b^2)/a*\tanh(1/2*d*x+1/2*c)^5*b^2-27/4/d/(\tanh(1/2*d*x+1/2*c))^8* \\ & a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2 \\ & *c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*\tanh(1/2*d*x+1/2*c)^ \\ & 11*b^2-5/2/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2* \\ & d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/ \\ & (a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^13-5/2/d/(\tanh(1/2*d*x+1/2*c))^8*a-4*\text{tan} \end{aligned}$$

$$\begin{aligned}
& h(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4 \\
& * \tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3-1/4/d \\
& /(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4 \\
& *a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2 \\
&)*\tanh(1/2*d*x+1/2*c)^15*b-1/4/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/ \\
& 2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d* \\
& x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)*b+45/8/d/(\tanh(1/2*d* \\
& x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(\\
& 1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d* \\
& x+1/2*c)^11*a+3/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh \\
& (1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a) \\
& ^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^11-25/8/d/(\tanh(1/2*d*x+1/2*c)^8*a \\
& -4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2* \\
& c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^1 \\
& 3-25/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+ \\
& 1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2 \\
& -2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3+45/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/ \\
& 2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tan \\
& h(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^5*a+3/d/(\tanh \\
& (1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16* \\
& b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tan \\
& h(1/2*d*x+1/2*c)^5-25/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6 \\
& *a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2* \\
& c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7*a-1/4/d/(\tanh(1/2*d*x+1/2 \\
& *c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d \\
& *x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1 \\
& /2*c)^7-25/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/ \\
& 2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/ \\
& (a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9*a-1/4/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tan \\
& h(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4- \\
& 4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9+5/8/ \\
& d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^ \\
& 4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+ \\
& b^2)*\tanh(1/2*d*x+1/2*c)^15+5/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1 \\
& /2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d \\
& *x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/16*(11*a*b^2 - 5*b^3 + (12*a^2*b*e^{(14*c)} - 11*a*b^2*e^{(14*c)} + 5*b^3*e^{(14*c)}) \\ & *e^{(14*d*x)} - (104*a^2*b*e^{(12*c)} - 85*a*b^2*e^{(12*c)} + 35*b^3*e^{(12*c)}) \\ & *e^{(12*d*x)} - (320*a^3*e^{(10*c)} - 652*a^2*b*e^{(10*c)} + 407*a*b^2*e^{(10*c)} \\ & - 105*b^3*e^{(10*c)}) \\ & *e^{(10*d*x)} + (1408*a^3*e^{(8*c)} - 1696*a^2*b*e^{(8*c)} + 865*a*b^2*e^{(8*c)} - 175*b^3*e^{(8*c)}) \\ & *e^{(8*d*x)} + (320*a^3*e^{(6*c)} + 756*a^2*b*e^{(6*c)} - 849*a*b^2*e^{(6*c)} + 175*b^3*e^{(6*c)}) \\ & *e^{(6*d*x)} - (248*a^2*b*e^{(4*c)} - 383*a*b^2*e^{(4*c)} + 105*b^3*e^{(4*c)}) \\ & *e^{(4*d*x)} - (12*a^2*b*e^{(2*c)} + 77*a*b^2*e^{(2*c)} - 35*b^3*e^{(2*c)}) \\ & *e^{(2*d*x)}) / (a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^{(16*c)} - 2*a^3*b^3*d*e^{(16*c)} + a^2*b^4*d*e^{(16*c)}) \\ & *e^{(16*d*x)} - 8*(a^4*b^2*d*e^{(14*c)} - 2*a^3*b^3*d*e^{(14*c)} + a^2*b^4*d*e^{(14*c)}) \\ & *e^{(14*d*x)} - 4*(8*a^5*b*d*e^{(12*c)} - 23*a^4*b^2*d*e^{(12*c)} + 22*a^3*b^3*d*e^{(12*c)} - 7*a^2*b^4*d*e^{(12*c)}) \\ & *e^{(12*d*x)} + 8*(16*a^5*b*d*e^{(10*c)} - 39*a^4*b^2*d*e^{(10*c)} + 30*a^3*b^3*d*e^{(10*c)} - 7*a^2*b^4*d*e^{(10*c)}) \\ & *e^{(10*d*x)} + 2*(128*a^6*d*e^{(8*c)} - 352*a^5*b*d*e^{(8*c)} + 355*a^4*b^2*d*e^{(8*c)} - 166*a^3*b^3*d*e^{(8*c)} + 35*a^2*b^4*d*e^{(8*c)}) \\ & *e^{(8*d*x)} + 8*(16*a^5*b*d*e^{(6*c)} - 39*a^4*b^2*d*e^{(6*c)} + 30*a^3*b^3*d*e^{(6*c)} - 7*a^2*b^4*d*e^{(6*c)}) \\ & *e^{(6*d*x)} - 4*(8*a^5*b*d*e^{(4*c)} - 23*a^4*b^2*d*e^{(4*c)} + 22*a^3*b^3*d*e^{(4*c)} - 7*a^2*b^4*d*e^{(4*c)}) \\ & *e^{(4*d*x)} - 8*(a^4*b^2*d*e^{(2*c)} - 2*a^3*b^3*d*e^{(2*c)} + a^2*b^4*d*e^{(2*c)}) \\ & *e^{(2*d*x)}) - 1/4*integrate(1/2*((12*a^2*e^{(6*c)} - 11*a*b*e^{(6*c)} + 5*b^2*e^{(6*c)}) \\ & *e^{(6*d*x)} - 2*(32*a^2*e^{(4*c)} - 19*a*b*e^{(4*c)} + 5*b^2*e^{(4*c)}) \\ & *e^{(4*d*x)} + (12*a^2*e^{(2*c)} - 11*a*b*e^{(2*c)} + 5*b^2*e^{(2*c)}) \\ & *e^{(2*d*x)}) / (a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^4*b*e^{(8*c)} - 2*a^3*b^2*e^{(8*c)} + a^2*b^3*e^{(8*c)}) \\ & *e^{(8*d*x)} - 4*(a^4*b*e^{(6*c)} - 2*a^3*b^2*e^{(6*c)} + a^2*b^3*e^{(6*c)}) \\ & *e^{(6*d*x)} - 2*(8*a^5*e^{(4*c)} - 19*a^4*b*e^{(4*c)} + 14*a^3*b^2*e^{(4*c)} - 3*a^2*b^3*e^{(4*c)}) \\ & *e^{(4*d*x)} - 4*(a^4*b*e^{(2*c)} - 2*a^3*b^2*e^{(2*c)} + a^2*b^3*e^{(2*c)}) \\ & *e^{(2*d*x)}), x \end{aligned}$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [A] time = 17.2049, size = 608, normalized size = 1.75

$$\frac{12 a^2 b e^{(14 d x+14 c)} - 11 a b^2 e^{(14 d x+14 c)} + 5 b^3 e^{(14 d x+14 c)} - 104 a^2 b e^{(12 d x+12 c)} + 85 a b^2 e^{(12 d x+12 c)} - 35 b^3 e^{(12 d x+12 c)} - 320 a^3 e^{(10 d x+10 c)} + 652 a^2 b e^{(10 d x+10 c)} - 407 a b^2 e^{(10 d x+10 c)} + 105 b^3 e^{(10 d x+10 c)} + 1408 a^3 e^{(8 d x+8 c)} - 1696 a^2 b e^{(8 d x+8 c)} + 865 a b^2 e^{(8 d x+8 c)} - 175 b^3 e^{(8 d x+8 c)} + 320 a^3 e^{(6 d x+6 c)} + 756 a^2 b e^{(6 d x+6 c)} - 849 a b^2 e^{(6 d x+6 c)} + 175 b^3 e^{(6 d x+6 c)} - 248 a^2 b e^{(4 d x+4 c)} + 383 a b^2 e^{(4 d x+4 c)} - 105 b^3 e^{(4 d x+4 c)} - 12 a^2 b e^{(2 d x+2 c)} - 77 a b^2 e^{(2 d x+2 c)} + 35 b^3 e^{(2 d x+2 c)} + 11 a b^2 - 5 b^3}{((a^4 d - 2 a^3 b d + a^2 b^2 d) * (b e^{(8 d x+8 c)} - 4 b e^{(6 d x+6 c)} - 16 a e^{(4 d x+4 c)} + 6 b e^{(4 d x+4 c)} - 4 b e^{(2 d x+2 c)} + b)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out]
$$\frac{-1/16*(12*a^2*b*e^{(14*d*x + 14*c)} - 11*a*b^2*e^{(14*d*x + 14*c)} + 5*b^3*e^{(14*d*x + 14*c)} - 104*a^2*b*e^{(12*d*x + 12*c)} + 85*a*b^2*e^{(12*d*x + 12*c)} - 35*b^3*e^{(12*d*x + 12*c)} - 320*a^3*e^{(10*d*x + 10*c)} + 652*a^2*b*e^{(10*d*x + 10*c)} - 407*a*b^2*e^{(10*d*x + 10*c)} + 105*b^3*e^{(10*d*x + 10*c)} + 1408*a^3*e^{(8*d*x + 8*c)} - 1696*a^2*b*e^{(8*d*x + 8*c)} + 865*a*b^2*e^{(8*d*x + 8*c)} - 175*b^3*e^{(8*d*x + 8*c)} + 320*a^3*e^{(6*d*x + 6*c)} + 756*a^2*b*e^{(6*d*x + 6*c)} - 849*a*b^2*e^{(6*d*x + 6*c)} + 175*b^3*e^{(6*d*x + 6*c)} - 248*a^2*b*e^{(4*d*x + 4*c)} + 383*a*b^2*e^{(4*d*x + 4*c)} - 105*b^3*e^{(4*d*x + 4*c)} - 12*a^2*b*e^{(2*d*x + 2*c)} - 77*a*b^2*e^{(2*d*x + 2*c)} + 35*b^3*e^{(2*d*x + 2*c)} + 11*a*b^2 - 5*b^3)/((a^4*d - 2*a^3*b*d + a^2*b^2*d)*(b*e^{(8*d*x + 8*c)} - 4*b*e^{(6*d*x + 6*c)} - 16*a*e^{(4*d*x + 4*c)} + 6*b*e^{(4*d*x + 4*c)} - 4*b*e^{(2*d*x + 2*c)} + b)^2)}$$

$$3.263 \quad \int \frac{1}{(a-b \sinh^4(c+dx))^3} dx$$

Optimal. Leaf size=320

$$\frac{b \tanh(c+dx) \left(\frac{17a^2-40ab+7b^2}{(a-b)^3} - \frac{(33a-13b) \tanh^2(c+dx)}{(a-b)^2} \right)}{32a^2d \left((a-b) \tanh^4(c+dx) - 2a \tanh^2(c+dx) + a \right)} + \frac{(-50\sqrt{a}\sqrt{b} + 32a + 21b) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{64a^{11/4}d (\sqrt{a} - \sqrt{b})^{5/2}} + \frac{(50\sqrt{a}}{64a^{11/4}d (\sqrt{a} - \sqrt{b})^{5/2}}$$

[Out] ((32*a - 50*sqrt[a]*sqrt[b] + 21*b)*ArcTanh[(sqrt[sqrt[a] - sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(64*a^(11/4)*(sqrt[a] - sqrt[b])^(5/2)*d) + ((32*a + 50*sqrt[a]*sqrt[b] + 21*b)*ArcTanh[(sqrt[sqrt[a] + sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(64*a^(11/4)*(sqrt[a] + sqrt[b])^(5/2)*d) - (b^2*Tanh[c + d*x]*(3*a + b - 4*(a + b)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (b*Tanh[c + d*x]*((17*a^2 - 40*a*b + 7*b^2)/(a - b)^3 - ((33*a - 13*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rubi [A] time = 0.60877, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3209, 1205, 1678, 1166, 208}

$$\frac{b \tanh(c+dx) \left(\frac{17a^2-40ab+7b^2}{(a-b)^3} - \frac{(33a-13b) \tanh^2(c+dx)}{(a-b)^2} \right)}{32a^2d \left((a-b) \tanh^4(c+dx) - 2a \tanh^2(c+dx) + a \right)} + \frac{(-50\sqrt{a}\sqrt{b} + 32a + 21b) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{64a^{11/4}d (\sqrt{a} - \sqrt{b})^{5/2}} + \frac{(50\sqrt{a}}{64a^{11/4}d (\sqrt{a} - \sqrt{b})^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a - b*Sinh[c + d*x]^4)^(-3), x]

[Out] ((32*a - 50*sqrt[a]*sqrt[b] + 21*b)*ArcTanh[(sqrt[sqrt[a] - sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(64*a^(11/4)*(sqrt[a] - sqrt[b])^(5/2)*d) + ((32*a + 50*sqrt[a]*sqrt[b] + 21*b)*ArcTanh[(sqrt[sqrt[a] + sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(64*a^(11/4)*(sqrt[a] + sqrt[b])^(5/2)*d) - (b^2*Tanh[c + d*x]*(3*a + b - 4*(a + b)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (b*Tanh[c + d*x]*((17*a^2 - 40*a*b + 7*b^2)/(a - b)^3 - ((33*a - 13*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))

Rule 3209

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^4)^(p_), x_Symbol] := With[{ff =
  FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
  + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x] /;
  FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1205

```
Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x
_Symbol] := With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := With[{d =
  Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
+ b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
+ 7)*(b*d - 2*a*e)*x^2, x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a - b \sinh^4(c + dx))^3} dx &= \frac{\text{Subst} \left(\int \frac{(1-x^2)^5}{(a-2ax^2+(a-b)x^4)^3} dx, x, \tanh(c + dx) \right)}{d} \\
&= -\frac{b^2 \tanh(c + dx) (3a + b - 4(a + b) \tanh^2(c + dx))}{8a(a - b)^3 d (a - 2a \tanh^2(c + dx) + (a - b) \tanh^4(c + dx))^2} - \frac{\text{Subst} \left(\int \frac{-\frac{2ab(8a^3 - 24a^2b + 27ab^2 - 8b^3)}{(a-b)^3}}{dx} \right)}{d} \\
&= -\frac{b^2 \tanh(c + dx) (3a + b - 4(a + b) \tanh^2(c + dx))}{8a(a - b)^3 d (a - 2a \tanh^2(c + dx) + (a - b) \tanh^4(c + dx))^2} - \frac{b \tanh(c + dx) \left(\frac{17a^2 - 40ab + 27b^2}{(a-b)^3} \right)}{32a^2 d (a - 2a \tanh^2(c + dx) + (a - b) \tanh^4(c + dx))^2} \\
&= -\frac{b^2 \tanh(c + dx) (3a + b - 4(a + b) \tanh^2(c + dx))}{8a(a - b)^3 d (a - 2a \tanh^2(c + dx) + (a - b) \tanh^4(c + dx))^2} - \frac{b \tanh(c + dx) \left(\frac{17a^2 - 40ab + 27b^2}{(a-b)^3} \right)}{32a^2 d (a - 2a \tanh^2(c + dx) + (a - b) \tanh^4(c + dx))^2} \\
&= \frac{(32a - 50\sqrt{a}\sqrt{b} + 21b) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{64a^{11/4} (\sqrt{a} - \sqrt{b})^{5/2} d} + \frac{(32a + 50\sqrt{a}\sqrt{b} + 21b) \tanh^{-1} \left(\frac{\sqrt{\sqrt{a}+\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}} \right)}{64a^{11/4} (\sqrt{a} + \sqrt{b})^{5/2} d}
\end{aligned}$$

Mathematica [A] time = 2.98798, size = 333, normalized size = 1.04

$$\frac{\frac{64a^{3/2}b(a-b)(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{(-8a-4b\cosh(2(c+dx))+b\cosh(4(c+dx))+3b)^2} + \frac{(50\sqrt{a}\sqrt{b}+32a+21b)(\sqrt{a}-\sqrt{b})^2 \tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b+a}}}\right)}{\sqrt{\sqrt{a}\sqrt{b+a}}}}{64a^{5/2}d(a-b)^2} - \frac{(\sqrt{a}+\sqrt{b})^2(-50\sqrt{a}\sqrt{b}+32a+21b)\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b-a}}}\right)}{\sqrt{\sqrt{a}\sqrt{b-a}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-3), x]

[Out] (-(((Sqrt[a] + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]]) + ((Sqrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (8*Sqrt[a]*b*(-19*a + 10*b + (6*a - 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])

$$\begin{aligned}
& *d*x) - 4*(8*a^5*b*d*e^{(12*c)} - 23*a^4*b^2*d*e^{(12*c)} + 22*a^3*b^3*d*e^{(12*c)} \\
& c) - 7*a^2*b^4*d*e^{(12*c)})*e^{(12*d*x)} + 8*(16*a^5*b*d*e^{(10*c)} - 39*a^4*b^2 \\
& *d*e^{(10*c)} + 30*a^3*b^3*d*e^{(10*c)} - 7*a^2*b^4*d*e^{(10*c)})*e^{(10*d*x)} + 2* \\
& (128*a^6*d*e^{(8*c)} - 352*a^5*b*d*e^{(8*c)} + 355*a^4*b^2*d*e^{(8*c)} - 166*a^3* \\
& b^3*d*e^{(8*c)} + 35*a^2*b^4*d*e^{(8*c)})*e^{(8*d*x)} + 8*(16*a^5*b*d*e^{(6*c)} - 3 \\
& 9*a^4*b^2*d*e^{(6*c)} + 30*a^3*b^3*d*e^{(6*c)} - 7*a^2*b^4*d*e^{(6*c)})*e^{(6*d*x)} \\
& - 4*(8*a^5*b*d*e^{(4*c)} - 23*a^4*b^2*d*e^{(4*c)} + 22*a^3*b^3*d*e^{(4*c)} - 7*a \\
& ^2*b^4*d*e^{(4*c)})*e^{(4*d*x)} - 8*(a^4*b^2*d*e^{(2*c)} - 2*a^3*b^3*d*e^{(2*c)} + \\
& a^2*b^4*d*e^{(2*c)})*e^{(2*d*x)} + \text{integrate}(1/4*((7*a*b*e^{(6*c)} - 4*b^2*e^{(6* \\
& c)))*e^{(6*d*x)} - 2*(32*a^2*e^{(4*c)} - 40*a*b*e^{(4*c)} + 17*b^2*e^{(4*c)})*e^{(4*d \\
& *x)} + (7*a*b*e^{(2*c)} - 4*b^2*e^{(2*c)})*e^{(2*d*x)})/(a^4*b - 2*a^3*b^2 + a^2*b \\
& ^3 + (a^4*b*e^{(8*c)} - 2*a^3*b^2*e^{(8*c)} + a^2*b^3*e^{(8*c)})*e^{(8*d*x)} - 4*(a \\
& ^4*b*e^{(6*c)} - 2*a^3*b^2*e^{(6*c)} + a^2*b^3*e^{(6*c)})*e^{(6*d*x)} - 2*(8*a^5*e^{ \\
& (4*c)} - 19*a^4*b*e^{(4*c)} + 14*a^3*b^2*e^{(4*c)} - 3*a^2*b^3*e^{(4*c)})*e^{(4*d*x)} \\
&) - 4*(a^4*b*e^{(2*c)} - 2*a^3*b^2*e^{(2*c)} + a^2*b^3*e^{(2*c)})*e^{(2*d*x)}, x)
\end{aligned}$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a-b*sinh(d*x+c))^4)^3,x, algorithm="fricas")`

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a-b*sinh(d*x+c)**4)**3,x)`

[Out] Timed out

Giac [A] time = 17.8292, size = 529, normalized size = 1.65

$$\frac{7ab^2e^{(14dx+14c)} - 4b^3e^{(14dx+14c)} - 32a^2be^{(12dx+12c)} - 2ab^2e^{(12dx+12c)} + 7b^3e^{(12dx+12c)} - 16a^2be^{(10dx+10c)} + 3ab^2e^{(10dx+10c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-b*sinh(d*x+c))^4,x, algorithm="giac")

[Out] $\frac{1}{8} \cdot (7ab^2e^{(14dx+14c)} - 4b^3e^{(14dx+14c)} - 32a^2be^{(12dx+12c)} - 2ab^2e^{(12dx+12c)} + 7b^3e^{(12dx+12c)} - 16a^2be^{(10dx+10c)} + 3ab^2e^{(10dx+10c)} + 28b^3e^{(10dx+10c)} + 768a^3e^{(8dx+8c)} - 960a^2be^{(8dx+8c)} + 498ab^2e^{(8dx+8c)} - 105b^3e^{(8dx+8c)} + 784a^2be^{(6dx+6c)} - 723ab^2e^{(6dx+6c)} + 140b^3e^{(6dx+6c)} - 160a^2be^{(4dx+4c)} + 266ab^2e^{(4dx+4c)} - 91b^3e^{(4dx+4c)} - 55ab^2e^{(2dx+2c)} + 28b^3e^{(2dx+2c)} + 6ab^2 - 3b^3) / ((a^4d - 2a^3bd + a^2b^2d) \cdot (be^{(8dx+8c)} - 4be^{(6dx+6c)} - 16ae^{(4dx+4c)} + 6be^{(4dx+4c)} - 4be^{(2dx+2c)} + b)^2)$

$$3.264 \quad \int \frac{\operatorname{csch}^2(c+dx)}{(a-b \sinh^4(c+dx))^3} dx$$

Optimal. Leaf size=359

$$\frac{b^2 \tanh(c+dx) (a(a+3b) - (a^2 + 6ab + b^2) \tanh^2(c+dx))}{8a^2d(a-b)^3 ((a-b) \tanh^4(c+dx) - 2a \tanh^2(c+dx) + a)^2} + \frac{b \tanh(c+dx) \left(\frac{2a^2(9a-17b)}{(a-b)^3} - \frac{(18a^2+15ab-13b^2) \tanh^2(c+dx)}{(a-b)^2} \right)}{32a^3d ((a-b) \tanh^4(c+dx) - 2a \tanh^2(c+dx) + a)}$$

```
[Out] (-3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + (3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot h[c + d*x]/(a^3*d) + (b^2*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*a^2*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (b*Tanh[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 - ((18*a^2 + 15*a*b - 13*b^2)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 1.1577, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3217, 1334, 1669, 1664, 1166, 208}

$$\frac{b^2 \tanh(c+dx) (a(a+3b) - (a^2 + 6ab + b^2) \tanh^2(c+dx))}{8a^2d(a-b)^3 ((a-b) \tanh^4(c+dx) - 2a \tanh^2(c+dx) + a)^2} + \frac{b \tanh(c+dx) \left(\frac{2a^2(9a-17b)}{(a-b)^3} - \frac{(18a^2+15ab-13b^2) \tanh^2(c+dx)}{(a-b)^2} \right)}{32a^3d ((a-b) \tanh^4(c+dx) - 2a \tanh^2(c+dx) + a)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + (3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot h[c + d*x]/(a^3*d) + (b^2*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*a^2*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (b*Tanh[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 - ((18*a^2 + 15*a*b - 13*b^2)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a - 2*a*Tanh
```

$[c + d*x]^2 + (a - b)*\text{Tanh}[c + d*x]^4)$

Rule 3217

$\text{Int}[\sin[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^4)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff^{(m + 1)}/f, \text{Subst}[\text{Int}[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p]/(1 + ff^2*x^2)^{(m/2 + 2*p + 1)}, x], x, \text{Tan}[e + f*x]/ff], x]] /; \text{FreeQ}\{a, b, e, f\}, x\} \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[p]$

Rule 1334

$\text{Int}[(x_)^{(m_.)}*((d_.) + (e_.)*(x_)^2)^{(q_.)}*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{f = \text{Coeff}[\text{PolynomialRemainder}[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 0], g = \text{Coeff}[\text{PolynomialRemainder}[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x], x, 2]\}, \text{Simp}[(x*(a + b*x^2 + c*x^4)^{(p + 1)}*(a*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p + 1)*(b^2 - 4*a*c)), \text{Int}[x^m*(a + b*x^2 + c*x^4)^{(p + 1)}*\text{Simp}[\text{ExpandToSum}[(2*a*(p + 1)*(b^2 - 4*a*c)*\text{PolynomialQuotient}[x^m*(d + e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^{(2 - m)}, x], x], x]] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1] \&\& \text{IGtQ}[q, 1] \&\& \text{ILtQ}[m/2, 0]$

Rule 1669

$\text{Int}[(Pq_)*(x_)^{(m_.)}*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{d = \text{Coeff}[\text{PolynomialRemainder}[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0], e = \text{Coeff}[\text{PolynomialRemainder}[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]\}, \text{Simp}[(x*(a + b*x^2 + c*x^4)^{(p + 1)}*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + \text{Dist}[1/(2*a*(p + 1)*(b^2 - 4*a*c)), \text{Int}[x^m*(a + b*x^2 + c*x^4)^{(p + 1)}*\text{ExpandToSum}[(2*a*(p + 1)*(b^2 - 4*a*c)*\text{PolynomialQuotient}[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^{(2 - m)}, x], x]] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{PolyQ}[Pq, x^2] \&\& \text{GtQ}[\text{Expon}[Pq, x^2], 1] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{LtQ}[p, -1] \&\& \text{ILtQ}[m/2, 0]$

Rule 1664

$\text{Int}[(Pq_)*((d_.)*(x_.))^{(m_.)}*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /; \text{FreeQ}\{a, b, c, d, m\}, x\} \&\& \text{PolyQ}[Pq, x^2] \&\& \text{IGtQ}[p, -2]$

Rule 1166

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{csch}^2(c+dx)}{(a-b\sinh^4(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^6}{x^2(a-2ax^2+(a-b)x^4)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{b^2 \tanh(c+dx) (a(a+3b) - (a^2+6ab+b^2) \tanh^2(c+dx))}{8a^2(a-b)^3 d (a-2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} - \operatorname{Subst}\left(\int \frac{-16ab + \frac{2ab(32a^3-9)}{(a-b)^3}}{\dots} dx\right) \\
&= \frac{b^2 \tanh(c+dx) (a(a+3b) - (a^2+6ab+b^2) \tanh^2(c+dx))}{8a^2(a-b)^3 d (a-2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} + \frac{b \tanh(c+dx) \left(\frac{2a^2(9a-1)}{(a-b)^3}\right)}{32a^3 d (a-2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} \\
&= \frac{b^2 \tanh(c+dx) (a(a+3b) - (a^2+6ab+b^2) \tanh^2(c+dx))}{8a^2(a-b)^3 d (a-2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} + \frac{b \tanh(c+dx) \left(\frac{2a^2(9a-1)}{(a-b)^3}\right)}{32a^3 d (a-2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} \\
&= -\frac{\operatorname{coth}(c+dx)}{a^3 d} + \frac{b^2 \tanh(c+dx) (a(a+3b) - (a^2+6ab+b^2) \tanh^2(c+dx))}{8a^2(a-b)^3 d (a-2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} + \frac{b \tanh(c+dx)}{32a^3 d} \\
&= -\frac{\operatorname{coth}(c+dx)}{a^3 d} + \frac{b^2 \tanh(c+dx) (a(a+3b) - (a^2+6ab+b^2) \tanh^2(c+dx))}{8a^2(a-b)^3 d (a-2a \tanh^2(c+dx) + (a-b) \tanh^4(c+dx))^2} + \frac{b \tanh(c+dx)}{32a^3 d} \\
&= -\frac{3\sqrt{b} (20a - 34\sqrt{a}\sqrt{b} + 15b) \tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}} \tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4} (\sqrt{a}-\sqrt{b})^{5/2} d} + \frac{3\sqrt{b} (20a + 34\sqrt{a}\sqrt{b} + 15b)}{64a^{13/4} (\sqrt{a}+\sqrt{b})^{5/2} d}
\end{aligned}$$

Mathematica [A] time = 3.45587, size = 357, normalized size = 0.99

$$\frac{4b \sinh(2(c+dx))(28a^2+b(13b-19a) \cosh(2(c+dx))+3ab-13b^2)}{(a-b)^2(8a+4b \cosh(2(c+dx))-b \cosh(4(c+dx))-3b)} + \frac{3\sqrt{b}(34\sqrt{a}\sqrt{b}+20a+15b) \tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b+a}}}\right)}{(\sqrt{a}+\sqrt{b})^2 \sqrt{\sqrt{a}\sqrt{b+a}}} + \frac{3\sqrt{b}(-34\sqrt{a}\sqrt{b}+20a+15b) \tan^{-1}\left(\frac{(\sqrt{a}-\sqrt{b}) \tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b+a}}}\right)}{(\sqrt{a}-\sqrt{b})^2 \sqrt{\sqrt{a}\sqrt{b+a}}}$$

$$64a^3d$$

Antiderivative was successfully verified.

[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]

[Out] ((3*sqrt[b]*(20*a - 34*sqrt[a]*sqrt[b] + 15*b)*ArcTan[((sqrt[a] - sqrt[b])*Tanh[c + d*x])/sqrt[-a + sqrt[a]*sqrt[b]]])/((sqrt[a] - sqrt[b])^2*sqrt[-a + sqrt[a]*sqrt[b]]) + (3*sqrt[b]*(20*a + 34*sqrt[a]*sqrt[b] + 15*b)*ArcTanh(((sqrt[a] + sqrt[b])*Tanh[c + d*x])/sqrt[a + sqrt[a]*sqrt[b]]))/((sqrt[a] + sqrt[b])^2*sqrt[a + sqrt[a]*sqrt[b]]) - 64*Coth[c + d*x] + (4*b*(28*a^2 + 3*a*b - 13*b^2 + b*(-19*a + 13*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)]/((a - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (128*a*b*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)]/((a - b)*(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)]^2)))/(64*a^3*d)

Maple [C] time = 0.158, size = 2747, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x)

[Out] -3/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-19/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-19/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-3/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15+25/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-

$$\begin{aligned}
& 2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9+25/4/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2 \\
& *d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh \\
& (1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^7+153/2/ \\
& d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^ \\
& 4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a* \\
& b+b^2)*\tanh(1/2*d*x+1/2*c)^7*b^3-2/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d* \\
& x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/ \\
& 2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3*b^2-2/d/(\tanh \\
& (1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16* \\
& b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*\tanh \\
& (1/2*d*x+1/2*c)^13*b^2+153/2/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/ \\
& 2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d* \\
& x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9*b^3-7/2/d/(\tanh \\
& (1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16* \\
& b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*\tanh \\
& (1/2*d*x+1/2*c)^5*b^2-7/2/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c) \\
&)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1 \\
& /2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*\tanh(1/2*d*x+1/2*c)^11*b^2-3/64/d*b/a^3/(a \\
& ^2-2*a*b+b^2)*\text{sum}((a*(-3*a+2*b))*_R^6+(49*a^2-72*a*b+30*b^2))*_R^4+(-49*a^2+7 \\
& 2*a*b-30*b^2))*_R^2+3*a^2-2*a*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*\ln \\
& (\tanh(1/2*d*x+1/2*c)-_R),_R=\text{RootOf}(a*_Z^8-4*a*_Z^6+(6*a-16*b))*_Z^4-4*a*_Z^2 \\
& +a))+17/4/d*b^3/a^2/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh \\
& (1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+ \\
& a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^13-52/d*b^4/a^3/(\tanh(1/2*d*x+1/2* \\
& c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d* \\
& x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2* \\
& c)^7-52/d*b^4/a^3/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh \\
& (1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a) \\
& ^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^9+17/4/d*b^3/a^2/(\tanh(1/2*d*x+1/2*c) \\
&)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x \\
& +1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c \\
&)^3-45/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d* \\
& x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a \\
& ^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^13-45/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh \\
& (1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4* \\
& \tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^3+9/8/d/ \\
& (\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4* \\
& a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2) \\
& *\tanh(1/2*d*x+1/2*c)^15*b+9/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2 \\
& *c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4-4*\tanh(1/2*d*x \\
& +1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)*b-1/2/d/a^3*\tanh(1/2*d \\
& *x+1/2*c)-1/2/d/a^3/\tanh(1/2*d*x+1/2*c)+81/8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh \\
& (1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*c)^4*a-16*b*\tanh(1/2*d*x+1/2*c)^4 \\
& -4*\tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*\tanh(1/2*d*x+1/2*c)^11+81 \\
& /8/d/(\tanh(1/2*d*x+1/2*c)^8*a-4*\tanh(1/2*d*x+1/2*c)^6*a+6*\tanh(1/2*d*x+1/2*
\end{aligned}$$

$$c)^4 a - 16 b \tanh(1/2 d x + 1/2 c)^4 - 4 \tanh(1/2 d x + 1/2 c)^2 a + a)^2 b / (a^2 - 2 a b + b^2) \tanh(1/2 d x + 1/2 c)^5 - 45/8 d / (\tanh(1/2 d x + 1/2 c)^8 a - 4 \tanh(1/2 d x + 1/2 c)^6 a + 6 \tanh(1/2 d x + 1/2 c)^4 a - 16 b \tanh(1/2 d x + 1/2 c)^4 - 4 \tanh(1/2 d x + 1/2 c)^2 a + a)^2 b / (a^2 - 2 a b + b^2) \tanh(1/2 d x + 1/2 c)^7 - 45/8 d / (\tanh(1/2 d x + 1/2 c)^8 a - 4 \tanh(1/2 d x + 1/2 c)^6 a + 6 \tanh(1/2 d x + 1/2 c)^4 a - 16 b \tanh(1/2 d x + 1/2 c)^4 - 4 \tanh(1/2 d x + 1/2 c)^2 a + a)^2 b / (a^2 - 2 a b + b^2) \tanh(1/2 d x + 1/2 c)^9$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out]
$$\frac{1}{16} (32 a^2 b^2 - 83 a b^3 + 45 b^4 + 3(20 a^2 b^2 e^{16c} - 33 a b^3 e^{16c} + 15 b^4 e^{16c})) e^{16 d x} - 12(43 a^2 b^2 e^{14c} - 68 a b^3 e^{14c} + 30 b^4 e^{14c}) e^{14 d x} - 4(400 a^3 b e^{12c} - 1137 a^2 b^2 e^{12c} + 1031 a b^3 e^{12c} - 315 b^4 e^{12c}) e^{12 d x} + 12(592 a^3 b e^{10c} - 1237 a^2 b^2 e^{10c} + 886 a b^3 e^{10c} - 210 b^4 e^{10c}) e^{10 d x} + 2(4096 a^4 e^{8c} - 12192 a^3 b e^{8c} + 13634 a^2 b^2 e^{8c} - 7113 a b^3 e^{8c} + 1575 b^4 e^{8c}) e^{8 d x} + 4(880 a^3 b e^{6c} - 2855 a^2 b^2 e^{6c} + 2512 a b^3 e^{6c} - 630 b^4 e^{6c}) e^{6 d x} - 4(256 a^3 b e^{4c} - 823 a^2 b^2 e^{4c} + 903 a b^3 e^{4c} - 315 b^4 e^{4c}) e^{4 d x} - 12(19 a^2 b^2 e^{2c} - 54 a b^3 e^{2c} + 30 b^4 e^{2c}) e^{2 d x} / (a^5 b^2 d - 2 a^4 b^3 d + a^3 b^4 d - (a^5 b^2 d e^{18c} - 2 a^4 b^3 d e^{18c} + a^3 b^4 d e^{18c})) e^{18 d x} + 9(a^5 b^2 d e^{16c} - 2 a^4 b^3 d e^{16c} + a^3 b^4 d e^{16c}) e^{16 d x} + 4(8 a^6 b d e^{14c} - 25 a^5 b^2 d e^{14c} + 26 a^4 b^3 d e^{14c} - 9 a^3 b^4 d e^{14c}) e^{14 d x} - 4(40 a^6 b d e^{12c} - 101 a^5 b^2 d e^{12c} + 82 a^4 b^3 d e^{12c} - 21 a^3 b^4 d e^{12c}) e^{12 d x} - 2(128 a^7 d e^{10c} - 416 a^6 b d e^{10c} + 511 a^5 b^2 d e^{10c} - 286 a^4 b^3 d e^{10c} + 63 a^3 b^4 d e^{10c}) e^{10 d x} + 2(128 a^7 d e^{8c} - 416 a^6 b d e^{8c} + 511 a^5 b^2 d e^{8c} - 286 a^4 b^3 d e^{8c} + 63 a^3 b^4 d e^{8c}) e^{8 d x} + 4(40 a^6 b d e^{6c} - 101 a^5 b^2 d e^{6c} + 82 a^4 b^3 d e^{6c} - 21 a^3 b^4 d e^{6c}) e^{6 d x} - 4(8 a^6 b d e^{4c} - 25 a^5 b^2 d e^{4c} + 26 a^4 b^3 d e^{4c} - 9 a^3 b^4 d e^{4c}) e^{4 d x} - 9(a^5 b^2 d e^{2c} - 2 a^4 b^3 d e^{2c} + a^3 b^4 d e^{2c}) e^{2 d x} - 4 \int \frac{3}{32} ((20 a^2 b e^{6c} - 33 a b^2 e^{6c} + 15 b^3 e^{6c}) e^{6 d x} - 2(32 a^2 b e^{4c} - 41 a b^2 e^{4c} + 15 b^3 e^{4c}) e^{4 d x} + (20 a^2 b e^{2c} - 33 a b^2 e^{2c} + 15 b^3 e^{2c}) e^{2 d x}) /$$

$$(a^5b - 2a^4b^2 + a^3b^3 + (a^5b e^{(8c)} - 2a^4b^2 e^{(8c)} + a^3b^3 e^{(8c)}) e^{(8dx)} - 4(a^5b e^{(6c)} - 2a^4b^2 e^{(6c)} + a^3b^3 e^{(6c)}) e^{(6dx)} - 2(8a^6 e^{(4c)} - 19a^5b e^{(4c)} + 14a^4b^2 e^{(4c)} - 3a^3b^3 e^{(4c)}) e^{(4dx)} - 4(a^5b e^{(2c)} - 2a^4b^2 e^{(2c)} + a^3b^3 e^{(2c)}) e^{(2dx)}), x)$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [A] time = 23.5837, size = 660, normalized size = 1.84

$$28 a^2 b^2 e^{(14 dx + 14 c)} - 35 a b^3 e^{(14 dx + 14 c)} + 13 b^4 e^{(14 dx + 14 c)} - 232 a^2 b^2 e^{(12 dx + 12 c)} + 269 a b^3 e^{(12 dx + 12 c)} - 91 b^4 e^{(12 dx + 12 c)} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] $-1/16*(28*a^2*b^2*e^{(14*d*x + 14*c)} - 35*a*b^3*e^{(14*d*x + 14*c)} + 13*b^4*e^{(14*d*x + 14*c)} - 232*a^2*b^2*e^{(12*d*x + 12*c)} + 269*a*b^3*e^{(12*d*x + 12*c)} - 91*b^4*e^{(12*d*x + 12*c)})$

$$\begin{aligned}
& *c) - 91*b^4*e^{(12*d*x + 12*c)} - 576*a^3*b*e^{(10*d*x + 10*c)} + 1372*a^2*b^2 \\
& *e^{(10*d*x + 10*c)} - 1039*a*b^3*e^{(10*d*x + 10*c)} + 273*b^4*e^{(10*d*x + 10* \\
& c)} + 2432*a^3*b*e^{(8*d*x + 8*c)} - 3488*a^2*b^2*e^{(8*d*x + 8*c)} + 1913*a*b^3 \\
& *e^{(8*d*x + 8*c)} - 455*b^4*e^{(8*d*x + 8*c)} + 576*a^3*b*e^{(6*d*x + 6*c)} + 10 \\
& 60*a^2*b^2*e^{(6*d*x + 6*c)} - 1689*a*b^3*e^{(6*d*x + 6*c)} + 455*b^4*e^{(6*d*x \\
& + 6*c)} - 376*a^2*b^2*e^{(4*d*x + 4*c)} + 679*a*b^3*e^{(4*d*x + 4*c)} - 273*b^4* \\
& e^{(4*d*x + 4*c)} - 28*a^2*b^2*e^{(2*d*x + 2*c)} - 117*a*b^3*e^{(2*d*x + 2*c)} + \\
& 91*b^4*e^{(2*d*x + 2*c)} + 19*a*b^3 - 13*b^4)/((a^5*d - 2*a^4*b*d + a^3*b^2*d \\
&)*(b*e^{(8*d*x + 8*c)} - 4*b*e^{(6*d*x + 6*c)} - 16*a*e^{(4*d*x + 4*c)} + 6*b*e^{(\\
& 4*d*x + 4*c)} - 4*b*e^{(2*d*x + 2*c)} + b)^2) - 2/(a^3*d*(e^{(2*d*x + 2*c)} - 1) \\
&)
\end{aligned}$$

$$3.265 \quad \int \frac{1}{1-\sinh^4(x)} dx$$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}(\sqrt{2}\tanh(x))}{2\sqrt{2}} + \frac{\tanh(x)}{2}$$

[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2

Rubi [A] time = 0.018092, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {3209, 388, 206}

$$\frac{\tanh^{-1}(\sqrt{2}\tanh(x))}{2\sqrt{2}} + \frac{\tanh(x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^4)^(-1),x]

[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2

Rule 3209

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^4]^(p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[p]

Rule 388

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{1 - \sinh^4(x)} dx &= \text{Subst} \left(\int \frac{1 - x^2}{1 - 2x^2} dx, x, \tanh(x) \right) \\ &= \frac{\tanh(x)}{2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \tanh(x) \right) \\ &= \frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{2\sqrt{2}} + \frac{\tanh(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.0995156, size = 24, normalized size = 0.96

$$\frac{1}{4} \left(\sqrt{2} \tanh^{-1}(\sqrt{2} \tanh(x)) + 2 \tanh(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sinh[x]^4)^(-1), x]

[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4

Maple [B] time = 0.02, size = 55, normalized size = 2.2

$$\tanh\left(\frac{x}{2}\right) \left(\left(\tanh\left(\frac{x}{2}\right) \right)^2 + 1 \right)^{-1} + \frac{\sqrt{2}}{4} \text{Arctanh}\left(\frac{\sqrt{2}}{4} (2 \tanh(x/2) - 2)\right) + \frac{\sqrt{2}}{4} \text{Arctanh}\left(\frac{\sqrt{2}}{4} (2 \tanh(x/2) + 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-sinh(x)^4), x)

[Out] tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))

Maxima [B] time = 1.54143, size = 93, normalized size = 3.72

$$\frac{1}{8} \sqrt{2} \log\left(-\frac{\sqrt{2} - e^{(-x)} + 1}{\sqrt{2} + e^{(-x)} - 1}\right) - \frac{1}{8} \sqrt{2} \log\left(-\frac{\sqrt{2} - e^{(-x)} - 1}{\sqrt{2} + e^{(-x)} + 1}\right) + \frac{1}{e^{(-2x)} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^4),x, algorithm="maxima")

[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + 1/(e^(-2*x) + 1)

Fricas [B] time = 2.04111, size = 382, normalized size = 15.28

$$\frac{(\sqrt{2} \cosh(x)^2 + 2\sqrt{2} \cosh(x) \sinh(x) + \sqrt{2} \sinh(x)^2 + \sqrt{2}) \log\left(-\frac{3(2\sqrt{2}-3) \cosh(x)^2 - 4(3\sqrt{2}-4) \cosh(x) \sinh(x) + 3(2\sqrt{2}-3) \sinh(x)^2}{\cosh(x)^2 + \sinh(x)^2 - 3}\right)}{8(\cosh(x)^2 + 2 \cosh(x) \sinh(x) + \sinh(x)^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^4),x, algorithm="fricas")

[Out] 1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) - 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)**4),x)

[Out] Timed out

Giac [B] time = 1.14865, size = 65, normalized size = 2.6

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{|-4\sqrt{2}+2e^{(2x)}-6|}{|4\sqrt{2}+2e^{(2x)}-6|}\right)-\frac{1}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^4),x, algorithm="giac")

[Out] -1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x) - 6)) - 1/(e^(2*x) + 1)

$$3.266 \quad \int \frac{1}{1+\sinh^4(x)} dx$$

Optimal. Leaf size=176

$$-\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(2\tanh^2(x)-2\sqrt{1+\sqrt{2}}\tanh(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(\sqrt{2}\tanh^2(x)+\sqrt{2(1+\sqrt{2})}\tanh(x)+1\right)$$

[Out] -ArcTan[(Sqrt[1 + Sqrt[2]] - 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[1 + Sqrt[2]]) + ArcTan[(Sqrt[1 + Sqrt[2]] + 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[1 + Sqrt[2]]) - (Sqrt[1 + Sqrt[2]]*Log[Sqrt[2] - 2*Sqrt[1 + Sqrt[2]]*Tanh[x] + 2*Tanh[x]^2])/8 + (Sqrt[1 + Sqrt[2]]*Log[1 + Sqrt[2*(1 + Sqrt[2])]]*Tanh[x] + Sqrt[2]*Tanh[x]^2)/8

Rubi [A] time = 0.159089, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {3209, 1169, 634, 618, 204, 628}

$$-\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(2\tanh^2(x)-2\sqrt{1+\sqrt{2}}\tanh(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(\sqrt{2}\tanh^2(x)+\sqrt{2(1+\sqrt{2})}\tanh(x)+1\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^4)^(-1), x]

[Out] -ArcTan[(Sqrt[1 + Sqrt[2]] - 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[1 + Sqrt[2]]) + ArcTan[(Sqrt[1 + Sqrt[2]] + 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[1 + Sqrt[2]]) - (Sqrt[1 + Sqrt[2]]*Log[Sqrt[2] - 2*Sqrt[1 + Sqrt[2]]*Tanh[x] + 2*Tanh[x]^2])/8 + (Sqrt[1 + Sqrt[2]]*Log[1 + Sqrt[2*(1 + Sqrt[2])]]*Tanh[x] + Sqrt[2]*Tanh[x]^2)/8

Rule 3209

Int[((a_) + (b_)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[p]

Rule 1169

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
(d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :=> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :=> Dist[-2, Subst[In
t[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :=> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :=> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1 + \sinh^4(x)} dx &= \text{Subst} \left(\int \frac{1 - x^2}{1 - 2x^2 + 2x^4} dx, x, \tanh(x) \right) \\
&= \frac{\text{Subst} \left(\int \frac{\sqrt{1+\sqrt{2}} - (1+\frac{1}{\sqrt{2}})x}{\frac{1}{\sqrt{2}} - \sqrt{1+\sqrt{2}}x + x^2} dx, x, \tanh(x) \right)}{2\sqrt{2}(1 + \sqrt{2})} + \frac{\text{Subst} \left(\int \frac{\sqrt{1+\sqrt{2}} + (1+\frac{1}{\sqrt{2}})x}{\frac{1}{\sqrt{2}} + \sqrt{1+\sqrt{2}}x + x^2} dx, x, \tanh(x) \right)}{2\sqrt{2}(1 + \sqrt{2})} \\
&= \frac{1}{8}\sqrt{3 - 2\sqrt{2}} \text{Subst} \left(\int \frac{1}{\frac{1}{\sqrt{2}} - \sqrt{1 + \sqrt{2}}x + x^2} dx, x, \tanh(x) \right) + \frac{1}{8}\sqrt{3 - 2\sqrt{2}} \text{Subst} \left(\int \frac{1}{\frac{1}{\sqrt{2}} + \sqrt{1 + \sqrt{2}}x + x^2} dx, x, \tanh(x) \right) \\
&= -\frac{1}{8}\sqrt{1 + \sqrt{2}} \log \left(\sqrt{2} - 2\sqrt{1 + \sqrt{2}} \tanh(x) + 2 \tanh^2(x) \right) + \frac{1}{8}\sqrt{1 + \sqrt{2}} \log \left(1 + \sqrt{2} \left(1 + \sqrt{2} \right) \tanh(x) \right) \\
&= -\frac{1}{4}\sqrt{-1 + \sqrt{2}} \tan^{-1} \left(\frac{\sqrt{1 + \sqrt{2}} - 2 \tanh(x)}{\sqrt{-1 + \sqrt{2}}} \right) + \frac{1}{4}\sqrt{-1 + \sqrt{2}} \tan^{-1} \left(\frac{\sqrt{1 + \sqrt{2}} + 2 \tanh(x)}{\sqrt{-1 + \sqrt{2}}} \right) - \frac{1}{8}\sqrt{3 - 2\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.072179, size = 45, normalized size = 0.26

$$\frac{\tanh^{-1}(\sqrt{1-i} \tanh(x))}{2\sqrt{1-i}} + \frac{\tanh^{-1}(\sqrt{1+i} \tanh(x))}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^4)^(-1), x]

[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(2*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]/(2*Sqrt[1 + I])

Maple [C] time = 0.024, size = 44, normalized size = 0.3

$$\frac{1}{4} \sum_{_R=\text{RootOf}(2_Z^4-2_Z^2+1)} \text{ln} \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 + (-4_R^3 + 4_R) \tanh \left(\frac{x}{2} \right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+sinh(x)^4), x)

[Out] $\frac{1}{4} \sum (_R \ln(\tanh(1/2*x)^2 + (-4*_R^3 + 4*_R) \tanh(1/2*x) + 1), _R = \text{RootOf}(2*_Z^4 - 2*_Z^2 + 1))$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(x)^4 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^4),x, algorithm="maxima")`

[Out] `integrate(1/(sinh(x)^4 + 1), x)`

Fricas [B] time = 2.257, size = 1874, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^4),x, algorithm="fricas")`

[Out] $\frac{1}{16} 2^{1/4} (\sqrt{2} + 1) \sqrt{-2\sqrt{2} + 4} \log((2^{3/4} e^{2x} - 2^{1/4} (3\sqrt{2} + 4)) \sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + e^{4x} - 2e^{2x} + 5) - \frac{1}{16} 2^{1/4} (\sqrt{2} + 1) \sqrt{-2\sqrt{2} + 4} \log(-(2^{3/4} e^{2x} - 2^{1/4} (3\sqrt{2} + 4)) \sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + e^{4x} - 2e^{2x} + 5) - \frac{1}{4} 2^{1/4} \sqrt{-2\sqrt{2} + 4} \arctan(1/14 (\sqrt{2} (5\sqrt{2} + 6) + 8\sqrt{2} + 4) e^{2x} - 1/28 (2\sqrt{2} (5\sqrt{2} + 6) - (2^{3/4} (8\sqrt{2} + 11) + 2 \cdot 2^{1/4} (5\sqrt{2} + 6)) \sqrt{-2\sqrt{2} + 4} + 16\sqrt{2} + 8) \sqrt{-(2^{3/4} e^{2x} - 2^{1/4} (3\sqrt{2} + 4)) \sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + e^{4x} - 2e^{2x} + 5} - 1/14 \sqrt{2} (3\sqrt{2} (2) - 2) - 1/28 ((2^{3/4} (8\sqrt{2} + 11) + 2 \cdot 2^{1/4} (5\sqrt{2} + 6)) e^{2x} - 2^{3/4} (2\sqrt{2} + 1) - 2 \cdot 2^{1/4} (3\sqrt{2} - 2)) \sqrt{-2\sqrt{2} (2) + 4} - 1/7 \sqrt{2} + 3/7) - 1/4 2^{1/4} \sqrt{-2\sqrt{2} + 4} \arctan(-1/14 (\sqrt{2} (5\sqrt{2} + 6) + 8\sqrt{2} + 4) e^{2x} + 1/28 (2\sqrt{2} (5\sqrt{2} (2) + 6) + (2^{3/4} (8\sqrt{2} + 11) + 2 \cdot 2^{1/4} (5\sqrt{2} + 6)) \sqrt{-2\sqrt{2} (2) + 4} + 16\sqrt{2} + 8) \sqrt{(2^{3/4} e^{2x} - 2^{1/4} (3\sqrt{2} + 4)) \sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + e^{4x} - 2e^{2x} + 5} + 1/14 \sqrt{2} (2) (3\sqrt{2} - 2) - 1/28 ((2^{3/4} (8\sqrt{2} + 11) + 2 \cdot 2^{1/4} (5\sqrt{2} + 6)) e^{2x} - 2^{3/4} (2\sqrt{2} + 1) - 2 \cdot 2^{1/4} (3\sqrt{2} - 2)) \sqrt{(-2\sqrt{2} + 4) + 4\sqrt{2} + e^{4x} - 2e^{2x} + 5})$

$-2\sqrt{2} + 4) + 1/7\sqrt{2} - 3/7)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)**4),x)

[Out] Timed out

Giac [C] time = 1.24228, size = 293, normalized size = 1.66

$$\left(\frac{1}{16}i + \frac{1}{16}\right) \sqrt{2\sqrt{2}-2} \left(-\frac{i}{\sqrt{2}-1} + 1\right) \log\left(2\sqrt{10\sqrt{2}+14}\left(-\frac{i}{5\sqrt{2}+7} + 1\right) + (4i+2)e^{2x} - 10\right) - \left(\frac{1}{16}i + \frac{1}{16}\right) \sqrt{2\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^4),x, algorithm="giac")

[Out] $(1/16*I + 1/16)*\sqrt{2*\sqrt{2} - 2}*(-I/(\sqrt{2} - 1) + 1)*\log(2*\sqrt{10*\sqrt{2} + 14}*(-I/(5*\sqrt{2} + 7) + 1) + (4*I + 2)*e^{2*x} - 10) - (1/16*I + 1/16)*\sqrt{2*\sqrt{2} - 2}*(-I/(\sqrt{2} - 1) + 1)*\log(-2*\sqrt{10*\sqrt{2} + 14}*(-I/(5*\sqrt{2} + 7) + 1) + (4*I + 2)*e^{2*x} - 10) + (1/16*I + 1/16)*\sqrt{2*\sqrt{2} + 2}*(-I/(\sqrt{2} + 1) + 1)*\log(2*\sqrt{2*\sqrt{2} - 2}*(I/(\sqrt{2} - 1) + 1) + 2*e^{2*x} - 4*I - 2) - (1/16*I + 1/16)*\sqrt{2*\sqrt{2} + 2}*(-I/(\sqrt{2} + 1) + 1)*\log(-2*\sqrt{2*\sqrt{2} - 2}*(I/(\sqrt{2} - 1) + 1) + 2*e^{2*x} - 4*I - 2)$

$$3.267 \quad \int \frac{1}{a+b \sinh^5(x)} dx$$

Optimal. Leaf size=435

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}} + \frac{2(-1)^{9/10} \tanh^{-1}\left(\frac{(-1)^{9/10}\left(\sqrt[5]{a} \tanh\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}\right)}{\sqrt{\sqrt[5]{-1}b^{2/5}-(-1)^{4/5}a^{2/5}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{-1}b^{2/5}-(-1)^{4/5}a^{2/5}}} + \frac{2\sqrt[5]{-1} \tanh^{-1}\left(\frac{\sqrt[5]{-1}\sqrt[5]{a} \tanh\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}} + \frac{2(-1)^{9/10}}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}$$

[Out] $(-2*\text{ArcTanh}[(b^{(1/5)} - a^{(1/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[a^{(2/5)} + b^{(2/5)}]])/(5*a^{(4/5)}*\text{Sqrt}[a^{(2/5)} + b^{(2/5)}]) + (2*(-1)^{(9/10)}*\text{ArcTanh}[((-1)^{(9/10)}*((-1)^{(1/5)}*b^{(1/5)} + a^{(1/5)}*\text{Tanh}[x/2])]/ \text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} + (-1)^{(1/5)}*b^{(2/5)})])/(5*a^{(4/5)}*\text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} + (-1)^{(1/5)}*b^{(2/5)})]) + (2*(-1)^{(1/5)}*\text{ArcTanh}[(b^{(1/5)} + (-1)^{(1/5)}*a^{(1/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[(-1)^{(2/5)}*a^{(2/5)} + b^{(2/5)}]])/(5*a^{(4/5)}*\text{Sqrt}[(-1)^{(2/5)}*a^{(2/5)} + b^{(2/5)}]) + (2*(-1)^{(9/10)}*\text{ArcTanh}[((-1)^{(3/10)}*(b^{(1/5)} + (-1)^{(3/5)}*a^{(1/5)}*\text{Tanh}[x/2])]/ \text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} + (-1)^{(3/5)}*b^{(2/5)})])/(5*a^{(4/5)}*\text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} + (-1)^{(3/5)}*b^{(2/5)})]) - (2*(-1)^{(9/10)}*\text{ArcTanh}[(I*b^{(1/5)} - (-1)^{(9/10)}*a^{(1/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} - b^{(2/5)})])/(5*a^{(4/5)}*\text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} - b^{(2/5)})])$

Rubi [A] time = 0.974696, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$, Rules used = {3213, 2660, 618, 206, 204}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}} + \frac{2(-1)^{9/10} \tanh^{-1}\left(\frac{(-1)^{9/10}\left(\sqrt[5]{a} \tanh\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}\right)}{\sqrt{\sqrt[5]{-1}b^{2/5}-(-1)^{4/5}a^{2/5}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{-1}b^{2/5}-(-1)^{4/5}a^{2/5}}} + \frac{2\sqrt[5]{-1} \tanh^{-1}\left(\frac{\sqrt[5]{-1}\sqrt[5]{a} \tanh\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}} + \frac{2(-1)^{9/10}}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[x]^5)^(-1), x]

[Out] $(-2*\text{ArcTanh}[(b^{(1/5)} - a^{(1/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[a^{(2/5)} + b^{(2/5)}]])/(5*a^{(4/5)}*\text{Sqrt}[a^{(2/5)} + b^{(2/5)}]) + (2*(-1)^{(9/10)}*\text{ArcTanh}[((-1)^{(9/10)}*((-1)^{(1/5)}*b^{(1/5)} + a^{(1/5)}*\text{Tanh}[x/2])]/ \text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} + (-1)^{(1/5)}*b^{(2/5)})])/(5*a^{(4/5)}*\text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} + (-1)^{(1/5)}*b^{(2/5)})]) + (2*(-1)^{(1/5)}*\text{ArcTanh}[(b^{(1/5)} + (-1)^{(1/5)}*a^{(1/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[(-1)^{(2/5)}*a^{(2/5)} + b^{(2/5)}]])/(5*a^{(4/5)}*\text{Sqrt}[(-1)^{(2/5)}*a^{(2/5)} + b^{(2/5)}]) + (2*(-1)^{(9/10)}*\text{ArcTanh}[((-1)^{(3/10)}*(b^{(1/5)} + (-1)^{(3/5)}*a^{(1/5)}*\text{Tanh}[x/2])]/ \text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} + (-1)^{(3/5)}*b^{(2/5)})])/(5*a^{(4/5)}*\text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} + (-1)^{(3/5)}*b^{(2/5)})]) - (2*(-1)^{(9/10)}*\text{ArcTanh}[(I*b^{(1/5)} - (-1)^{(9/10)}*a^{(1/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} - b^{(2/5)})])/(5*a^{(4/5)}*\text{Sqrt}[-((-1)^{(4/5)}*a^{(2/5)} - b^{(2/5)})])$

$$\frac{\sqrt{-((-1)^{4/5}a^{2/5}) + (-1)^{3/5}b^{2/5}}}{(5a^{4/5}\sqrt{-((-1)^{4/5}a^{2/5}) + (-1)^{3/5}b^{2/5}}) - (2(-1)^{9/10}\text{ArcTanh}[(Ib^{1/5} - (-1)^{9/10}a^{1/5}\text{Tanh}[x/2])]/\sqrt{-((-1)^{4/5}a^{2/5}) - b^{2/5}})]/(5a^{4/5}\sqrt{-((-1)^{4/5}a^{2/5}) - b^{2/5}})}$$

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{a + b \sinh^5(x)} dx &= \int \left(\frac{(-1)^{9/10}}{5a^{4/5} \left(-(-1)^{9/10} \sqrt[5]{a} - i \sqrt[5]{b} \sinh(x) \right)} - \frac{(-1)^{9/10}}{5a^{4/5} \left(-(-1)^{9/10} \sqrt[5]{a} - \sqrt[10]{-1} \sqrt[5]{b} \sinh(x) \right)} - \frac{(-1)^{9/10}}{5a^{4/5} \left(-(-1)^{9/10} \sqrt[5]{a} + i \sqrt[5]{b} \sinh(x) \right)} \right) dx \\
&= \frac{(-1)^{9/10} \int \frac{1}{-(-1)^{9/10} \sqrt[5]{a} - i \sqrt[5]{b} \sinh(x)} dx}{5a^{4/5}} - \frac{(-1)^{9/10} \int \frac{1}{-(-1)^{9/10} \sqrt[5]{a} - \sqrt[10]{-1} \sqrt[5]{b} \sinh(x)} dx}{5a^{4/5}} - \frac{(-1)^{9/10} \int \frac{1}{-(-1)^{9/10} \sqrt[5]{a} + i \sqrt[5]{b} \sinh(x)} dx}{5a^{4/5}} \\
&= \frac{(2(-1)^{9/10}) \operatorname{Subst} \left(\int \frac{1}{-(-1)^{9/10} \sqrt[5]{a} - 2i \sqrt[5]{b} x + (-1)^{9/10} \sqrt[5]{a} x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right)}{5a^{4/5}} - \frac{(2(-1)^{9/10}) \operatorname{Subst} \left(\int \frac{1}{-(-1)^{9/10} \sqrt[5]{a} - \sqrt[10]{-1} \sqrt[5]{b} x + (-1)^{9/10} \sqrt[5]{a} x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right)}{5a^{4/5}} - \frac{(2(-1)^{9/10}) \operatorname{Subst} \left(\int \frac{1}{-(-1)^{9/10} \sqrt[5]{a} + 2i \sqrt[5]{b} x + (-1)^{9/10} \sqrt[5]{a} x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right)}{5a^{4/5}} \\
&= \frac{(4(-1)^{9/10}) \operatorname{Subst} \left(\int \frac{1}{-4(-1)^{4/5} (a^{2/5} + b^{2/5}) - x^2} dx, x, -2(-1)^{9/10} \sqrt[5]{b} + 2(-1)^{9/10} \sqrt[5]{a} \tanh\left(\frac{x}{2}\right) \right)}{5a^{4/5}} + \frac{(4(-1)^{9/10}) \operatorname{Subst} \left(\int \frac{1}{-4(-1)^{4/5} (a^{2/5} + b^{2/5}) - x^2} dx, x, -2(-1)^{9/10} \sqrt[5]{b} + 2(-1)^{9/10} \sqrt[5]{a} \tanh\left(\frac{x}{2}\right) \right)}{5a^{4/5}} \\
&= \frac{2(-1)^{7/10} \tan^{-1} \left(\frac{i \sqrt[5]{b} + (-1)^{7/10} \sqrt[5]{a} \tanh\left(\frac{x}{2}\right)}{\sqrt{(-1)^{2/5} a^{2/5} + b^{2/5}}} \right)}{5a^{4/5} \sqrt{(-1)^{2/5} a^{2/5} + b^{2/5}}} - \frac{2 \tanh^{-1} \left(\frac{\sqrt[5]{b} - \sqrt[5]{a} \tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5} + b^{2/5}}} \right)}{5a^{4/5} \sqrt{a^{2/5} + b^{2/5}}} - \frac{2(-1)^{9/10} \tanh^{-1} \left(\frac{i \sqrt[5]{b} - (-1)^{9/10} \sqrt[5]{a} \tanh\left(\frac{x}{2}\right)}{\sqrt{(-1)^{4/5} a^{2/5} + b^{2/5}}} \right)}{5a^{4/5} \sqrt{(-1)^{4/5} a^{2/5} + b^{2/5}}}
\end{aligned}$$

Mathematica [C] time = 0.325722, size = 141, normalized size = 0.32

$$\frac{8}{5} \operatorname{RootSum} \left[32\#1^5 a + \#1^{10} b - 5\#1^8 b + 10\#1^6 b - 10\#1^4 b + 5\#1^2 b - b \&, \frac{\#1^3 x + 2\#1^3 \log\left(-\#1 \sinh\left(\frac{x}{2}\right) + \#1 \cosh\left(\frac{x}{2}\right) - \sqrt{(-1)^{2/5} a^{2/5} + b^{2/5}}\right)}{16\#1^3 a + \#1^8 b - 4\#1^6 b + 6\#1^4 b - 4\#1^2 b - b} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[x]^5)^(-1), x]

[Out] (8*RootSum[-b + 5*b*#1^2 - 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 - 5*b*#1^8 + b*#1^10 &, (x*#1^3 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^3)/(b - 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8) &])/5

Maple [C] time = 0.032, size = 113, normalized size = 0.3

$$\frac{1}{5} \sum_{_R=\operatorname{RootOf}(a_Z^{10}-5a_Z^8+10a_Z^6-32b_Z^5-10a_Z^4+5a_Z^2-a)} \frac{-_R^8 + 4_R^6 - 6_R^4 + 4_R^2 - 1}{-_R^9 a - 4_R^7 a + 6_R^5 a - 16_R^4 b - 4_R^3 a + _R a} \ln \left(\tanh \left(\frac{x}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b*sinh(x)^5),x)`

[Out] $\frac{1}{5} \sum \left(\frac{-R^8 + 4R^6 - 6R^4 + 4R^2 - 1}{(R^9 a - 4R^7 a + 6R^5 a - 16R^4 b - 4R^3 a + R a) \ln(\tanh(1/2 x) - R)} \right), R = \text{RootOf}(Z^{10} a - 5Z^8 a + 10Z^6 a - 32Z^5 b - 10Z^4 a + 5Z^2 a - a)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \sinh(x)^5 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sinh(x)^5),x, algorithm="maxima")`

[Out] `integrate(1/(b*sinh(x)^5 + a), x)`

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sinh(x)^5),x, algorithm="fricas")`

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \sinh^5(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sinh(x)**5),x)`

[Out] Integral(1/(a + b*sinh(x)**5), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \sinh(x)^5 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="giac")

[Out] integrate(1/(b*sinh(x)^5 + a), x)

$$3.268 \quad \int \frac{1}{a+b \sinh^6(x)} dx$$

Optimal. Leaf size=175

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

[Out] ArcTanh[(Sqrt[a^(1/3) - b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)])

Rubi [A] time = 0.279326, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[x]^6)^(-1), x]

[Out] ArcTanh[(Sqrt[a^(1/3) - b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)])

Rule 3211

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(n_))^(1), x_Symbol] :> Module[{k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b), n/2])), x], {k, 1, n/2}], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a + b \sinh^6(x)} dx = \frac{\int \frac{1}{1 + \frac{\sqrt[3]{b} \sinh^2(x)}{\sqrt[3]{a}}} dx}{3a} + \frac{\int \frac{1}{1 - \frac{\sqrt[3]{-1} \sqrt[3]{b} \sinh^2(x)}{\sqrt[3]{a}}} dx}{3a} + \frac{\int \frac{1}{1 + \frac{(-1)^{2/3} \sqrt[3]{b} \sinh^2(x)}{\sqrt[3]{a}}} dx}{3a}$$

$$= \frac{\text{Subst} \left(\int \frac{1}{1 - \left(1 - \frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right) x^2} dx, x, \tanh(x) \right)}{3a} + \frac{\text{Subst} \left(\int \frac{1}{1 - \left(1 + \frac{\sqrt[3]{-1} \sqrt[3]{b}}{\sqrt[3]{a}}\right) x^2} dx, x, \tanh(x) \right)}{3a} + \frac{\text{Subst} \left(\int \frac{1}{1 - \left(1 + \frac{(-1)^{2/3} \sqrt[3]{b}}{\sqrt[3]{a}}\right) x^2} dx, x, \tanh(x) \right)}{3a}$$

$$= \frac{\tanh^{-1} \left(\frac{\sqrt{\sqrt[3]{a} - \sqrt[3]{b}} \tanh(x)}{\sqrt[6]{a}} \right)}{3a^{5/6} \sqrt{\sqrt[3]{a} - \sqrt[3]{b}}} + \frac{\tanh^{-1} \left(\frac{\sqrt{\sqrt[3]{a} + \sqrt[3]{-1} \sqrt[3]{b}} \tanh(x)}{\sqrt[6]{a}} \right)}{3a^{5/6} \sqrt{\sqrt[3]{a} + \sqrt[3]{-1} \sqrt[3]{b}}} + \frac{\tanh^{-1} \left(\frac{\sqrt{\sqrt[3]{a} - (-1)^{2/3} \sqrt[3]{b}} \tanh(x)}{\sqrt[6]{a}} \right)}{3a^{5/6} \sqrt{\sqrt[3]{a} - (-1)^{2/3} \sqrt[3]{b}}}$$

Mathematica [C] time = 0.16913, size = 134, normalized size = 0.77

$$\frac{16}{3} \text{RootSum} \left[64\#1^3 a + \#1^6 b - 6\#1^5 b + 15\#1^4 b - 20\#1^3 b + 15\#1^2 b - 6\#1 b + b\&, \frac{\#1^2 x + \#1^2 \log(-\#1 \sinh(x) + \#1 \cosh(x))}{32\#1^2 a + \#1^5 b - 5\#1^4 b + 10\#1^3 b} \right]$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^6)^(-1), x]
```

```
[Out] (16*RootSum[b - 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*
b*#1^5 + b*#1^6 & , (x*#1^2 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]
*#1]*#1^2)/(-b + 5*b*#1 + 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3 - 5*b*#1^4 + b*
#1^5) & ])/3
```

Maple [C] time = 0.03, size = 128, normalized size = 0.7

$$\frac{1}{6} \sum_{_R=\text{RootOf}(a_Z^{12}-6a_Z^{10}+15a_Z^8+(-20a+64b)_Z^6+15a_Z^4-6a_Z^2+a)} \frac{-_R^{10}+5_R^8-10_R^6+10_R^4-5_R^2+1}{_R^{11}a-5_R^9a+10_R^7a-10_R^5a+32_R^5b+5_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(x)^6),x)

[Out] 1/6*sum((-_R^10+5*_R^8-10*_R^6+10*_R^4-5*_R^2+1)/(_R^11*a-5*_R^9*a+10*_R^7*a-10*_R^5*a+32*_R^5*b+5*_R^3*a-_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^12-6*a*_Z^10+15*a*_Z^8+(-20*a+64*b)*_Z^6+15*a*_Z^4-6*a*_Z^2+a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \sinh(x)^6 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="maxima")

[Out] integrate(1/(b*sinh(x)^6 + a), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \sinh^6(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**6),x)
```

```
[Out] Integral(1/(a + b*sinh(x)**6), x)
```

Giac [A] time = 1.37914, size = 1, normalized size = 0.01

0

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="giac")
```

```
[Out] 0
```

$$3.269 \quad \int \frac{1}{a+b \sinh^8(x)} dx$$

Optimal. Leaf size=245

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

[Out] $-\text{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}-b^{1/4}} \tanh(x)}{(-a)^{1/8}}\right]/(4*(-a)^{7/8} \sqrt{(-a)^{1/4}-b^{1/4}}) - \text{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}-I*b^{1/4}} \tanh(x)}{(-a)^{1/8}}\right]/(4*(-a)^{7/8} \sqrt{(-a)^{1/4}-I*b^{1/4}}) - \text{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}+I*b^{1/4}} \tanh(x)}{(-a)^{1/8}}\right]/(4*(-a)^{7/8} \sqrt{(-a)^{1/4}+I*b^{1/4}}) - \text{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}+b^{1/4}} \tanh(x)}{(-a)^{1/8}}\right]/(4*(-a)^{7/8} \sqrt{(-a)^{1/4}+b^{1/4}})$

Rubi [A] time = 0.516823, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}} \tanh(x)}{(-a)^{5/8}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Sinh}[x]^8)^{-1}, x]$

[Out] $-\text{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}-I*b^{1/4}} \tanh(x)}{(-a)^{1/8}}\right]/(4*(-a)^{7/8} \sqrt{(-a)^{1/4}-I*b^{1/4}}) - \text{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}+I*b^{1/4}} \tanh(x)}{(-a)^{1/8}}\right]/(4*(-a)^{7/8} \sqrt{(-a)^{1/4}+I*b^{1/4}}) - \text{ArcTanh}\left[\frac{\sqrt{(-a)^{1/4}+b^{1/4}} \tanh(x)}{(-a)^{1/8}}\right]/(4*(-a)^{7/8} \sqrt{(-a)^{1/4}+b^{1/4}}) - \text{ArcTanh}\left[\frac{\sqrt{(-a)^{5/4}+a*b^{1/4}} \tanh(x)}{(-a)^{5/8}}\right]/(4*(-a)^{3/8} \sqrt{(-a)^{5/4}+a*b^{1/4}})$

Rule 3211

$\text{Int}[(a_+ + (b_+)*\sin[e_+] + (f_+)*(x_+))^{(n_+)}^{-1}, x_Symbol] \rightarrow \text{Module}[\{k\}, \text{Dist}[2/(a*n), \text{Sum}[\text{Int}[1/(1 - \text{Sin}[e + f*x]^2/((-1)^{(4*k)/n})*\text{Rt}[-(a/b),$

n/2))), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\int \frac{1}{a + b \sinh^8(x)} dx = \frac{\int \frac{1}{1 - \frac{4\sqrt{b} \sinh^2(x)}{\sqrt[4]{-a}}} dx}{4a} + \frac{\int \frac{1}{1 - i \frac{4\sqrt{b} \sinh^2(x)}{\sqrt[4]{-a}}} dx}{4a} + \frac{\int \frac{1}{1 + i \frac{4\sqrt{b} \sinh^2(x)}{\sqrt[4]{-a}}} dx}{4a} + \frac{\int \frac{1}{1 + \frac{4\sqrt{b} \sinh^2(x)}{\sqrt[4]{-a}}} dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1 - \left(1 - \frac{4\sqrt{b}}{\sqrt[4]{-a}}\right)x^2} dx, x, \tanh(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1 - \left(1 - i \frac{4\sqrt{b}}{\sqrt[4]{-a}}\right)x^2} dx, x, \tanh(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1 - \left(1 + i \frac{4\sqrt{b}}{\sqrt[4]{-a}}\right)x^2} dx, x, \tanh(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1 + \left(1 + \frac{4\sqrt{b}}{\sqrt[4]{-a}}\right)x^2} dx, x, \tanh(x)\right)}{4a}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a} - i\sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8} \sqrt{\sqrt[4]{-a} - i\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a} + i\sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8} \sqrt{\sqrt[4]{-a} + i\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a} + \sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8} \sqrt{\sqrt[4]{-a} + \sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a} - \sqrt[4]{b}} \tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8} \sqrt{\sqrt[4]{-a} - \sqrt[4]{b}}}$$

Mathematica [C] time = 0.245581, size = 160, normalized size = 0.65

$$16\text{RootSum}\left[256\#1^4 a + \#1^8 b - 8\#1^7 b + 28\#1^6 b - 56\#1^5 b + 70\#1^4 b - 56\#1^3 b + 28\#1^2 b - 8\#1 b + b \&, \frac{\#1^3 x + \#1^3}{128\#1^3 a + \#1^7 b - \dots}\right]$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[x]^8)^(-1), x]

[Out] 16*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 - 56*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 &, (x*#1^3 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]*#1]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2 + 128*a*#1^3 +

$$35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) \&]$$

Maple [C] time = 0.033, size = 162, normalized size = 0.7

$$\frac{1}{8} \sum_{_R=\text{RootOf}(a_Z^{16}-8a_Z^{14}+28a_Z^{12}-56a_Z^{10}+(70a+256b)_Z^8-56a_Z^6+28a_Z^4-8a_Z^2+a)} \frac{-_R^{14} + 7_R^{12} - 21_R^{10} + 35_R^8 - 35_R^6 + 21_R^4 - 7_R^2 + 1}{-_R^{15}a - 7_R^{13}a + 21_R^{11}a - 35_R^9a + 35_R^7a - 128_R^7b - 21_R^5a + 7_R^3a - a} \ln(\tanh(1/2*x) - _R)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(x)^8),x)

[Out] 1/8*sum((-_R^14+7*_R^12-21*_R^10+35*_R^8-35*_R^6+21*_R^4-7*_R^2+1)/(_R^15*a-7*_R^13*a+21*_R^11*a-35*_R^9*a+35*_R^7*a+128*_R^7*b-21*_R^5*a+7*_R^3*a-_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^16-8*a*_Z^14+28*a*_Z^12-56*a*_Z^10+(70*a+256*b)*_Z^8-56*a*_Z^6+28*a*_Z^4-8*a*_Z^2+a))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b \sinh(x)^8 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="maxima")

[Out] integrate(1/(b*sinh(x)^8 + a), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sinh(x)**8),x)`

[Out] Timed out

Giac [A] time = 1.90444, size = 1, normalized size = 0.

0

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*sinh(x)^8),x, algorithm="giac")`

[Out] 0

$$3.270 \quad \int \frac{1}{1+\sinh^5(x)} dx$$

Optimal. Leaf size=242

$$-\frac{1}{5}\sqrt{2} \tanh^{-1}\left(\frac{1 - \tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + \frac{2(-1)^{9/10} \tanh^{-1}\left(\frac{(-1)^{7/10}(\sqrt[5]{-1} \tanh\left(\frac{x}{2}\right) + 1)}{\sqrt{-(-1)^{2/5}(1+(-1)^{2/5})}}\right)}{5\sqrt{-(-1)^{2/5}(1+(-1)^{2/5})}} - \frac{2(-1)^{4/5} \tanh^{-1}\left(\frac{1-(-1)^{4/5} \tanh\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}} - \frac{2(-1)^3}{5}$$

[Out] $(-2*(-1)^{(3/5)}*\text{ArcTan}[(1 + (-1)^{(3/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[-1 + (-1)^{(1/5)}]])/(5 * \text{Sqrt}[-1 + (-1)^{(1/5)}]) + (2*(-1)^{(9/10)}*\text{ArcTan}[(1 - (-1)^{(9/10)}*\text{Tanh}[x/2])/ \text{Sqrt}[1 + (-1)^{(4/5)}]])/(5*\text{Sqrt}[1 + (-1)^{(4/5)}]) - (\text{Sqrt}[2]*\text{ArcTanh}[(1 - \text{Tanh}[x/2])/ \text{Sqrt}[2]])/5 + (2*(-1)^{(9/10)}*\text{ArcTanh}[((-1)^{(7/10)}*(1 + (-1)^{(1/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[-((-1)^{(2/5)}*(1 + (-1)^{(2/5))})]])/(5*\text{Sqrt}[-((-1)^{(2/5)}*(1 + (-1)^{(2/5))})]) - (2*(-1)^{(4/5)}*\text{ArcTanh}[(1 - (-1)^{(4/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(3/5)}]])/(5*\text{Sqrt}[1 - (-1)^{(3/5)}])$

Rubi [A] time = 0.5185, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {3213, 2660, 618, 204, 206, 617}

$$-\frac{1}{5}\sqrt{2} \tanh^{-1}\left(\frac{1 - \tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right) + \frac{2(-1)^{9/10} \tanh^{-1}\left(\frac{(-1)^{7/10}(\sqrt[5]{-1} \tanh\left(\frac{x}{2}\right) + 1)}{\sqrt{-(-1)^{2/5}(1+(-1)^{2/5})}}\right)}{5\sqrt{-(-1)^{2/5}(1+(-1)^{2/5})}} - \frac{2(-1)^{4/5} \tanh^{-1}\left(\frac{1-(-1)^{4/5} \tanh\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}} - \frac{2(-1)^3}{5}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^5)^(-1), x]

[Out] $(-2*(-1)^{(3/5)}*\text{ArcTan}[(1 + (-1)^{(3/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[-1 + (-1)^{(1/5)}]])/(5 * \text{Sqrt}[-1 + (-1)^{(1/5)}]) + (2*(-1)^{(9/10)}*\text{ArcTan}[(1 - (-1)^{(9/10)}*\text{Tanh}[x/2])/ \text{Sqrt}[1 + (-1)^{(4/5)}]])/(5*\text{Sqrt}[1 + (-1)^{(4/5)}]) - (\text{Sqrt}[2]*\text{ArcTanh}[(1 - \text{Tanh}[x/2])/ \text{Sqrt}[2]])/5 + (2*(-1)^{(9/10)}*\text{ArcTanh}[((-1)^{(7/10)}*(1 + (-1)^{(1/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[-((-1)^{(2/5)}*(1 + (-1)^{(2/5))})]])/(5*\text{Sqrt}[-((-1)^{(2/5)}*(1 + (-1)^{(2/5))})]) - (2*(-1)^{(4/5)}*\text{ArcTanh}[(1 - (-1)^{(4/5)}*\text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(3/5)}]])/(5*\text{Sqrt}[1 - (-1)^{(3/5)}])$

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1 + \sinh^5(x)} dx &= \int \left(-\frac{(-1)^{9/10}}{5(-(-1)^{9/10} - i \sinh(x))} - \frac{(-1)^{9/10}}{5(-(-1)^{9/10} - \sqrt[10]{-1} \sinh(x))} - \frac{(-1)^{9/10}}{5(-(-1)^{9/10} + (-1)^{3/10} \sinh(x))} \right) dx \\
&= -\left(\frac{1}{5}(-1)^{9/10} \int \frac{1}{-(-1)^{9/10} - i \sinh(x)} dx\right) - \frac{1}{5}(-1)^{9/10} \int \frac{1}{-(-1)^{9/10} - \sqrt[10]{-1} \sinh(x)} dx - \frac{1}{5}(-1)^{9/10} \int \frac{1}{-(-1)^{9/10} + (-1)^{3/10} \sinh(x)} dx \\
&= -\left(\frac{1}{5} (2(-1)^{9/10}) \text{Subst} \left(\int \frac{1}{-(-1)^{9/10} - 2ix + (-1)^{9/10}x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right)\right) - \frac{1}{5} (2(-1)^{9/10}) \text{Subst} \left(\int \frac{1}{-(-1)^{9/10} - 2ix + (-1)^{9/10}x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right) \\
&= -\left(\frac{2}{5} \text{Subst} \left(\int \frac{1}{2-x^2} dx, x, 1 - \tanh\left(\frac{x}{2}\right) \right)\right) + \frac{1}{5} (4(-1)^{9/10}) \text{Subst} \left(\int \frac{1}{4(-1)^{3/5} (1 - \sqrt[5]{-1}) - x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right) \\
&= \frac{2(-1)^{9/10} \tan^{-1} \left(\frac{i(-1)^{9/10} \tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{4/5}}} \right)}{5\sqrt{1+(-1)^{4/5}}} - \frac{1}{5} \sqrt{2} \tanh^{-1} \left(\frac{1 - \tanh\left(\frac{x}{2}\right)}{\sqrt{2}} \right) - \frac{2 \sqrt[10]{-1} \tanh^{-1} \left(\frac{i - \sqrt[10]{-1} \tanh\left(\frac{x}{2}\right)}{\sqrt{-1 + \sqrt[5]{-1}}} \right)}{5\sqrt{-1 + \sqrt[5]{-1}}}
\end{aligned}$$

Mathematica [C] time = 0.915842, size = 439, normalized size = 1.81

$$\frac{1}{10} \left(2\sqrt{2} \tanh^{-1} \left(\frac{\tanh\left(\frac{x}{2}\right) - 1}{\sqrt{2}} \right) - \text{RootSum} \left[\#1^8 - 2\#1^7 - 2\#1^5 + 14\#1^4 + 2\#1^3 + 2\#1 + 1 \&, \frac{\#1^6 x - 4\#1^5 x + 9\#1^4 x - \dots}{\dots} \right] \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^5)^(-1), x]

[Out] (2*sqrt(2)*ArcTanh[(-1 + Tanh[x/2])/sqrt(2)] - RootSum[1 + 2*#1 + 2*#1^3 + 14*#1^4 - 2*#1^5 - 2*#1^7 + #1^8 &, (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1] - 4*x*#1 - 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1 - 9*x*#1^2 - 18*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^2 - 24*x*#1^3 - 48*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^3 + 9*x*#1^4 + 18*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 - 4*x*#1^5 - 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(1 + 3*#1^2 + 28*#1^3 - 5*#1^4 - 7*#1^6 + 4*#1^7) &])/10

Maple [C] time = 0.033, size = 124, normalized size = 0.5

$$\frac{2}{5} \sum_{_R=\text{RootOf}(_Z^8+2_Z^7+2_Z^5+14_Z^4-2_Z^3-2_Z+1)} \frac{-2_R^6 - 3_R^5 + 2_R^4 + 2_R^3 - 2_R^2 - 3_R + 2}{4_R^7 + 7_R^6 + 5_R^4 + 28_R^3 - 3_R^2 - 1} \ln \left(\tanh\left(\frac{x}{2}\right) - _R \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+sinh(x)^5),x)`

[Out] $2/5 \sum((-2*_R^6-3*_R^5+2*_R^4+2*_R^3-2*_R^2-3*_R+2)/(4*_R^7+7*_R^6+5*_R^4+28*_R^3-3*_R^2-1)*\ln(\tanh(1/2*x)-_R),_R=\text{RootOf}(_Z^8+2*_Z^7+2*_Z^5+14*_Z^4-2*_Z^3-2*_Z+1))+1/5*2^{(1/2)}*\text{arctanh}(1/4*(2*\tanh(1/2*x)-2)*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{10} \sqrt{2} \log\left(-\frac{\sqrt{2}-e^x-1}{\sqrt{2}+e^x+1}\right) - \int \frac{2(e^{7x}-4e^{6x}+9e^{5x}-24e^{4x}-9e^{3x}-4e^{2x}-e^x)}{5(e^{8x}-2e^{7x}-2e^{5x}+14e^{4x}+2e^{3x}+2e^x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^5),x, algorithm="maxima")`

[Out] `1/10*sqrt(2)*log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - integrate(2/5*(e^(7*x) - 4*e^(6*x) + 9*e^(5*x) - 24*e^(4*x) - 9*e^(3*x) - 4*e^(2*x) - e^x)/(e^(8*x) - 2*e^(7*x) - 2*e^(5*x) + 14*e^(4*x) + 2*e^(3*x) + 2*e^x + 1), x)`

Fricas [B] time = 3.48642, size = 11367, normalized size = 46.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^5),x, algorithm="fricas")`

[Out] $-1/200*\sqrt{2}*\sqrt{2*\sqrt{2}*(2*\sqrt{5}-5)*\sqrt{\sqrt{5}+3}}-4*\sqrt{5}+20*(8*\sqrt{5}+24)^{(1/4)}*(3*\sqrt{5}-5)*\sqrt{2*\sqrt{5}+5}*\sqrt{\sqrt{5}+3}*\arctan(1/40*\sqrt{2}*((11*\sqrt{5}-25)*e^x-7*\sqrt{5}+15)*\sqrt{2*\sqrt{5}+5}*\sqrt{\sqrt{5}+3})+1/80*\sqrt{2}*(\sqrt{2}*((11*\sqrt{5}-25)*e^x+4*\sqrt{5}-10)*\sqrt{2*\sqrt{5}+5}*\sqrt{\sqrt{5}+3})+2*((3*\sqrt{5}-5)*e^x+7*\sqrt{5}-15)*\sqrt{2*\sqrt{5}+5})*\sqrt{\sqrt{5}+3}+1/12800*(80*\sqrt{2}*(5*\sqrt{5}-11)*\sqrt{2*\sqrt{5}+5}*\sqrt{\sqrt{5}+3})+40*\sqrt{2}*(\sqrt{2}*(5*\sqrt{5}-11)*\sqrt{2*\sqrt{5}+5}*\sqrt{\sqrt{5}+3})+2*\sqrt{2*\sqrt{5}+5}*(\sqrt{5}-3))*\sqrt{\sqrt{5}+3}+\sqrt{2*\sqrt{2}*(2*s$

$$\begin{aligned}
& \sqrt{5} - 5) \sqrt{\sqrt{5} + 3} - 4 \sqrt{5} + 20) * ((\sqrt{2} * (7 \sqrt{5} - 15) * \\
& \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 * (11 \sqrt{5} - 25) \sqrt{2 \sqrt{5} + 5}) * (8 \sqrt{5} + 24)^{3/4} + 4 * (\sqrt{2} * (17 \sqrt{5} - 35) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 * (11 \sqrt{5} - 25) \sqrt{2 \sqrt{5} + 5}) * (8 \sqrt{5} + 24)^{1/4}) + 320 \sqrt{2 \sqrt{5} + 5} * (\sqrt{5} - 4) \sqrt{-20 \sqrt{2} \sqrt{\sqrt{5} + 3} * (\sqrt{5} - 3) + 40 * (\sqrt{5} - 1) e^x - 2 * (\sqrt{2} * ((2 \sqrt{5} - 5) e^x - 3 \sqrt{5} + 5) \sqrt{\sqrt{5} + 3} + 2 * (\sqrt{5} - 5) e^x - 3 \sqrt{5} + 5) \sqrt{2 \sqrt{2} * (2 \sqrt{5} - 5) \sqrt{\sqrt{5} + 3} - 4 \sqrt{5} + 20) * (8 \sqrt{5} + 24)^{1/4} + 80 e^{2x} + 80) + 1/640 \sqrt{2 \sqrt{2} * (2 \sqrt{5} - 5) \sqrt{\sqrt{5} + 3} - 4 \sqrt{5} + 20) * ((\sqrt{2} * ((3 \sqrt{5} - 7) e^x + 8 \sqrt{5} - 18) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 * ((5 \sqrt{5} - 11) e^x + 2 \sqrt{5} - 4) \sqrt{2 \sqrt{5} + 5}) * (8 \sqrt{5} + 24)^{3/4} + 4 * (\sqrt{2} * ((7 \sqrt{5} - 17) e^x + 8 \sqrt{5} - 18) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 * ((5 \sqrt{5} - 11) e^x + 5 \sqrt{5} - 9) \sqrt{2 \sqrt{5} + 5}) * (8 \sqrt{5} + 24)^{1/4}) + 1/20 * (2 * (4 \sqrt{5} - 5) e^x - \sqrt{5} + 5) \sqrt{2 \sqrt{5} + 5}) - 1/200 \sqrt{2} \sqrt{2 \sqrt{2} * (2 \sqrt{5} - 5) \sqrt{\sqrt{5} + 3} - 4 \sqrt{5} + 20) * (8 \sqrt{5} + 24)^{1/4} * (3 \sqrt{5} - 5) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} * \arctan(-1/40 \sqrt{2} * ((11 \sqrt{5} - 25) e^x - 7 \sqrt{5} + 15) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} - 1/80 \sqrt{2} * (\sqrt{2} * ((11 \sqrt{5} - 25) e^x + 4 \sqrt{5} - 10) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 * ((3 \sqrt{5} - 5) e^x + 7 \sqrt{5} - 15) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} - 1/12800 * (80 \sqrt{2} * (5 \sqrt{5} - 11) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 40 \sqrt{2} * (\sqrt{2} * (5 \sqrt{5} - 11) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 \sqrt{2 \sqrt{5} + 5} * (\sqrt{5} - 3)) \sqrt{\sqrt{5} + 3} - \sqrt{2 \sqrt{2} * (2 \sqrt{5} - 5) \sqrt{\sqrt{5} + 3} - 4 \sqrt{5} + 20) * ((\sqrt{2} * (7 \sqrt{5} - 15) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 * (11 \sqrt{5} - 25) \sqrt{2 \sqrt{5} + 5}) * (8 \sqrt{5} + 24)^{3/4} + 4 * (\sqrt{2} * (17 \sqrt{5} - 35) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 * (11 \sqrt{5} - 25) \sqrt{2 \sqrt{5} + 5}) * (8 \sqrt{5} + 24)^{1/4}) + 320 \sqrt{2 \sqrt{5} + 5} * (\sqrt{5} - 4) \sqrt{-20 \sqrt{2} \sqrt{\sqrt{5} + 3} * (\sqrt{5} - 3) + 40 * (\sqrt{5} - 1) e^x + 2 * (\sqrt{2} * ((2 \sqrt{5} - 5) e^x - 3 \sqrt{5} + 5) \sqrt{\sqrt{5} + 3} + 2 * (\sqrt{5} - 5) e^x - 3 \sqrt{5} + 5) \sqrt{2 \sqrt{2} * (2 \sqrt{5} - 5) \sqrt{\sqrt{5} + 3} - 4 \sqrt{5} + 20) * (8 \sqrt{5} + 24)^{1/4} + 80 e^{2x} + 80) + 1/640 \sqrt{2 \sqrt{2} * (2 \sqrt{5} - 5) \sqrt{\sqrt{5} + 3} - 4 \sqrt{5} + 20) * ((\sqrt{2} * ((3 \sqrt{5} - 7) e^x + 8 \sqrt{5} - 18) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 * ((5 \sqrt{5} - 11) e^x + 2 \sqrt{5} - 4) \sqrt{2 \sqrt{5} + 5}) * (8 \sqrt{5} + 24)^{3/4} + 4 * (\sqrt{2} * ((7 \sqrt{5} - 17) e^x + 8 \sqrt{5} - 18) \sqrt{2 \sqrt{5} + 5}) \sqrt{\sqrt{5} + 3} + 2 * ((5 \sqrt{5} - 11) e^x + 5 \sqrt{5} - 9) \sqrt{2 \sqrt{5} + 5}) * (8 \sqrt{5} + 24)^{1/4}) - 1/20 * (2 * (4 \sqrt{5} - 5) e^x - \sqrt{5} + 5) \sqrt{2 \sqrt{5} + 5}) + 1/400 \sqrt{-(2 \sqrt{5} + 5) \sqrt{-8 \sqrt{5} + 24} + 4 \sqrt{5} + 20) * (3 \sqrt{5} + 5) \sqrt{-2 \sqrt{5} + 5} * (-8 \sqrt{5} + 24)^{3/4} * \arctan(-1/1280 * ((4 * (5 \sqrt{5} + 11) e^x + ((3 \sqrt{5} + 7) e^x + 8 \sqrt{5} + 18) \sqrt{-8 \sqrt{5} + 24} + 8 \sqrt{5} + 16) * (-8 \sqrt{5} + 24)^{3/4} + 4 * (4 * (5 \sqrt{5} + 11) e^x + ((7 \sqrt{5} + 17) e^x + 8 \sqrt{5} + 18) \sqrt{-8 \sqrt{5} + 24} + 20 \sqrt{5} + 36) * (-8 \sqrt{5} + 24)^{1/4}) \sqrt{-(2}
\end{aligned}$$

$$\begin{aligned}
& * \sqrt{5} + 5) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} + 20) \sqrt{-2\sqrt{5} + 5} \\
& + 1/25600 * (((7\sqrt{5} + 15) \sqrt{-8\sqrt{5} + 24} + 44\sqrt{5} + 100) * (-8 \\
& * \sqrt{5} + 24)^{3/4} + 4 * ((17\sqrt{5} + 35) \sqrt{-8\sqrt{5} + 24} + 44\sqrt{5} \\
& (5) + 100) * (-8\sqrt{5} + 24)^{1/4}) * \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + \\
& 24} + 4\sqrt{5} + 20) \sqrt{-2\sqrt{5} + 5} + 20 * (((5\sqrt{5} + 11) \sqrt{-8 \\
& * \sqrt{5} + 24} + 4\sqrt{5} + 12) \sqrt{-8\sqrt{5} + 24} + 4 * (5\sqrt{5} + 11) \\
& * \sqrt{-8\sqrt{5} + 24} + 32\sqrt{5} + 128) \sqrt{-2\sqrt{5} + 5}) * \sqrt{-40 * (\\
& \sqrt{5} + 1) e^x + (4 * (\sqrt{5} + 5) e^x + ((2\sqrt{5} + 5) e^x - 3\sqrt{5} - 5) \\
& * \sqrt{-8\sqrt{5} + 24} - 6\sqrt{5} - 10) \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + \\
& 24} + 4\sqrt{5} + 20) * (-8\sqrt{5} + 24)^{1/4} + 10 * (\sqrt{5} + 3) * \\
& \sqrt{-8\sqrt{5} + 24} + 80 e^{2x} + 80) - 1/320 * (32 * (4\sqrt{5} + 5) e^x + \\
& 4 * ((11\sqrt{5} + 25) e^x - 7\sqrt{5} - 15) \sqrt{-8\sqrt{5} + 24} + (4 * (3\sqrt{5} \\
& + 5) e^x + ((11\sqrt{5} + 25) e^x + 4\sqrt{5} + 10) \sqrt{-8\sqrt{5} + 24} + 28\sqrt{5} \\
& + 60) \sqrt{-8\sqrt{5} + 24} - 16\sqrt{5} - 80) \sqrt{-2\sqrt{5} + 5}) + 1/400 * \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} \\
& + 20) * (3\sqrt{5} + 5) \sqrt{-2\sqrt{5} + 5} * (-8\sqrt{5} + 24)^{3/4} * \arctan(\\
& -1/1280 * ((4 * (5\sqrt{5} + 11) e^x + ((3\sqrt{5} + 7) e^x + 8\sqrt{5} + 18) * \\
& \sqrt{-8\sqrt{5} + 24} + 8\sqrt{5} + 16) * (-8\sqrt{5} + 24)^{3/4} + 4 * (4 * (5\sqrt{5} \\
& + 11) e^x + ((7\sqrt{5} + 17) e^x + 8\sqrt{5} + 18) \sqrt{-8\sqrt{5} + 24} + 20\sqrt{5} \\
& + 36) * (-8\sqrt{5} + 24)^{1/4}) * \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} \\
& + 20) \sqrt{-2\sqrt{5} + 5} + 1/25600 * (((7\sqrt{5} + 15) \sqrt{-8\sqrt{5} + 24} + 44\sqrt{5} \\
& + 100) * (-8\sqrt{5} + 24)^{3/4} + 4 * ((17\sqrt{5} + 35) \sqrt{-8\sqrt{5} + 24} + 44\sqrt{5} \\
& + 100) * (-8\sqrt{5} + 24)^{1/4}) * \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} \\
& + 20) \sqrt{-2\sqrt{5} + 5} - 20 * (((5\sqrt{5} + 11) \sqrt{-8\sqrt{5} + 24} + 4 \\
& * \sqrt{5} + 12) \sqrt{-8\sqrt{5} + 24} + 4 * (5\sqrt{5} + 11) \sqrt{-8\sqrt{5} + 24} + 32\sqrt{5} \\
& + 128) \sqrt{-2\sqrt{5} + 5}) * \sqrt{-40 * (\sqrt{5} + 1) e^x - \\
& (4 * (\sqrt{5} + 5) e^x + ((2\sqrt{5} + 5) e^x - 3\sqrt{5} - 5) \sqrt{-8\sqrt{5} (\\
& 5) + 24} - 6\sqrt{5} - 10) \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} \\
& + 20) * (-8\sqrt{5} + 24)^{1/4} + 10 * (\sqrt{5} + 3) \sqrt{-8\sqrt{5} + 24} + 80 e^{2x} \\
& + 80) + 1/320 * (32 * (4\sqrt{5} + 5) e^x + 4 * ((11\sqrt{5} + 25) e^x - 7\sqrt{5} - 15) \\
& * \sqrt{-8\sqrt{5} + 24} + (4 * (3\sqrt{5} + 5) e^x + (\\
& (11\sqrt{5} + 25) e^x + 4\sqrt{5} + 10) \sqrt{-8\sqrt{5} + 24} + 28\sqrt{5} \\
& + 60) \sqrt{-8\sqrt{5} + 24} - 16\sqrt{5} - 80) \sqrt{-2\sqrt{5} + 5}) + 1/80 \\
& 0 * (\sqrt{2} * (3\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) + 8\sqrt{5}}) * \sqrt{2\sqrt{2} * (2 \\
& * \sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) - 4\sqrt{5} + 20) * (8\sqrt{5} + 24)^{1/4}} * \log(-4\sqrt{2} \\
& * \sqrt{2} * \sqrt{(\sqrt{5} + 3) * (\sqrt{5} - 3) + 8 * (\sqrt{5} - 1) e^x} + 2/5 * (\sqrt{2} \\
& * ((2\sqrt{5} - 5) e^x - 3\sqrt{5} + 5) \sqrt{(\sqrt{5} + 3) + 2 * (\sqrt{5} \\
& - 5) e^x - 3\sqrt{5} + 5) \sqrt{2\sqrt{2} * (2\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) \\
& - 4\sqrt{5} + 20) * (8\sqrt{5} + 24)^{1/4}} + 16 e^{2x} + 16) - 1/800 * (\sqrt{2} \\
& * (3\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) + 8\sqrt{5}}) * \sqrt{2\sqrt{2} * (2\sqrt{5} \\
& - 5) \sqrt{(\sqrt{5} + 3) - 4\sqrt{5} + 20) * (8\sqrt{5} + 24)^{1/4}} * \log(-4\sqrt{2} \\
& * \sqrt{2} * \sqrt{(\sqrt{5} + 3) * (\sqrt{5} - 3) + 8 * (\sqrt{5} - 1) e^x} - 2/5 * (\sqrt{2} * (\\
& (2\sqrt{5} - 5) e^x - 3\sqrt{5} + 5) \sqrt{(\sqrt{5} + 3) + 2 * (\sqrt{5} - 5) e^x \\
& - 3\sqrt{5} + 5) \sqrt{2\sqrt{2} * (2\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) - 4\sqrt{5}
\end{aligned}$$


```
t(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) + 1/1600*((3*sqrt(5) +
5)*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(
5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4)*log(-8*(sqrt(5) + 1)*e^x
+ 1/5*(4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5) - 5)*sqrt(-8
*sqrt(5) + 24) - 6*sqrt(5) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24
) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(
5) + 24) + 16*e^(2*x) + 16) - 1/1600*((3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24)
+ 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20
)*(-8*sqrt(5) + 24)^(1/4)*log(-8*(sqrt(5) + 1)*e^x - 1/5*(4*(sqrt(5) + 5)*e
^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5) - 5)*sqrt(-8*sqrt(5) + 24) - 6*sqrt(5
) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*s
qrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(5) + 24) + 16*e^(2*x) + 1
6) + 1/10*sqrt(2)*log(-(2*(sqrt(2) - 1)*e^x + 2*sqrt(2) - e^(2*x) - 3)/(e^(
2*x) + 2*e^x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**5),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(x)^5 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(sinh(x)^5 + 1), x)
```

$$3.271 \quad \int \frac{1}{1+\sinh^6(x)} dx$$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}} + \frac{\tanh(x)}{3}$$

[Out] ArcTanh[Sqrt[1 + (-1)^(1/3)]*Tanh[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTanh[Sqrt[1 - (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tanh[x]/3

Rubi [A] time = 0.137859, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$, Rules used = {3211, 3181, 206, 3175, 3767, 8}

$$\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}} + \frac{\tanh(x)}{3}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^6)^(-1), x]

[Out] ArcTanh[Sqrt[1 + (-1)^(1/3)]*Tanh[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTanh[Sqrt[1 - (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tanh[x]/3

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(m_), x_Symbol] := Module[{k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b), n/2])), x], {k, 1, n/2}], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 3175

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{1 + \sinh^6(x)} dx &= \frac{1}{3} \int \frac{1}{1 + \sinh^2(x)} dx + \frac{1}{3} \int \frac{1}{1 - \sqrt[3]{-1} \sinh^2(x)} dx + \frac{1}{3} \int \frac{1}{1 + (-1)^{2/3} \sinh^2(x)} dx \\
 &= \frac{1}{3} \int \operatorname{sech}^2(x) dx + \frac{1}{3} \operatorname{Subst} \left(\int \frac{1}{1 - (1 + \sqrt[3]{-1}) x^2} dx, x, \tanh(x) \right) + \frac{1}{3} \operatorname{Subst} \left(\int \frac{1}{1 - (1 - (-1)^{2/3}) x^2} dx, x, \tanh(x) \right) \\
 &= \frac{\tanh^{-1} \left(\sqrt{1 + \sqrt[3]{-1}} \tanh(x) \right)}{3\sqrt{1 + \sqrt[3]{-1}}} + \frac{\tanh^{-1} \left(\sqrt{1 - (-1)^{2/3}} \tanh(x) \right)}{3\sqrt{1 - (-1)^{2/3}}} + \frac{1}{3} i \operatorname{Subst} \left(\int 1 dx, x, -i \tanh(x) \right) \\
 &= \frac{\tanh^{-1} \left(\sqrt{1 + \sqrt[3]{-1}} \tanh(x) \right)}{3\sqrt{1 + \sqrt[3]{-1}}} + \frac{\tanh^{-1} \left(\sqrt{1 - (-1)^{2/3}} \tanh(x) \right)}{3\sqrt{1 - (-1)^{2/3}}} + \frac{\tanh(x)}{3}
 \end{aligned}$$

Mathematica [C] time = 0.201319, size = 87, normalized size = 1.23

$$\frac{1}{18} \left(6 \tanh(x) + \sqrt[4]{-3} \left((-3 - i\sqrt{3}) \tan^{-1} \left(\frac{1}{2} \sqrt[4]{-3} (1 + i\sqrt{3}) \tanh(x) \right) - (\sqrt{3} + 3i) \tan^{-1} \left(\frac{1}{2} \sqrt[4]{-3} (3 + i\sqrt{3}) \tanh(x) \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^6)^(-1),x]

[Out] $((-3)^{1/4} * ((-3 - I * \text{Sqrt}[3]) * \text{ArcTan}[((-3)^{1/4} * (1 + I * \text{Sqrt}[3]) * \text{Tanh}[x]) / 2] - (3 * I + \text{Sqrt}[3]) * \text{ArcTan}[((-1/3)^{1/4} * (3 + I * \text{Sqrt}[3]) * \text{Tanh}[x]) / 2]) + 6 * \text{Tanh}[x]) / 18$

Maple [C] time = 0.033, size = 61, normalized size = 0.9

$$\frac{1}{6} \sum_{_R=\text{RootOf}(3_Z^4-3_Z^2+1)} _R \ln \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 + (-6_R^3 + 6_R) \tanh \left(\frac{x}{2} \right) + 1 \right) + \frac{2}{3} \tanh \left(\frac{x}{2} \right) \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 + 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+sinh(x)^6),x)

[Out] $1/6 * \text{sum}(_R * \ln(\tanh(1/2 * x)^2 + (-6 * _R^3 + 6 * _R) * \tanh(1/2 * x) + 1), _R = \text{RootOf}(3 * _Z^4 - 3 * _Z^2 + 1)) + 2/3 * \tanh(1/2 * x) / (\tanh(1/2 * x)^2 + 1)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{3(e^{2x} + 1)} - \int \frac{4(e^{6x} - 10e^{4x} + e^{2x})}{3(e^{8x} - 8e^{6x} + 30e^{4x} - 8e^{2x} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^6),x, algorithm="maxima")

[Out] $-2/3 / (e^{2x} + 1) - \text{integrate}(4/3 * (e^{6x} - 10 * e^{4x} + e^{2x}) / (e^{8x} - 8 * e^{6x} + 30 * e^{4x} - 8 * e^{2x} + 1), x)$

Fricas [B] time = 1.84705, size = 2303, normalized size = 32.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^6),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/144*(4*(12^{1/4}*\sqrt{6}*e^{2*x} + 12^{1/4}*\sqrt{6})*\sqrt{-4*\sqrt{3} + 8} \\ &)*\arctan((\sqrt{3} + 2)*e^{2*x} + 1/216*\sqrt{-6*(12^{1/4}*\sqrt{6}*(\sqrt{3} + 3) \\ &)*e^{2*x} - 12^{1/4}*\sqrt{6}*(5*\sqrt{3} + 9))*\sqrt{-4*\sqrt{3} + 8} + 144* \\ & \sqrt{3} + 36*e^{4*x} - 144*e^{2*x} + 252)*((12^{3/4}*\sqrt{6}*(\sqrt{3} + 3) \\ & + 3*12^{1/4}*\sqrt{6}*(\sqrt{3} + 3))*\sqrt{-4*\sqrt{3} + 8} - 36*\sqrt{3} - 72) \\ & - 2/3*\sqrt{3}*(2*\sqrt{3} - 3) - 1/36*(12^{3/4}*\sqrt{6}*(\sqrt{3} - 3) + (12 \\ & ^{3/4}*\sqrt{6}*(\sqrt{3} + 3) + 3*12^{1/4}*\sqrt{6}*(\sqrt{3} + 3))*e^{2*x} + \\ & 3*12^{1/4}*\sqrt{6}*(\sqrt{3} - 3))*\sqrt{-4*\sqrt{3} + 8} - 2*\sqrt{3} + 4) + 4 \\ & *(12^{1/4}*\sqrt{6}*e^{2*x} + 12^{1/4}*\sqrt{6})*\sqrt{-4*\sqrt{3} + 8}*\arctan(\\ & -(\sqrt{3} + 2)*e^{2*x} + 1/216*\sqrt{6*(12^{1/4}*\sqrt{6}*(\sqrt{3} + 3)*e^{2*x} \\ &) - 12^{1/4}*\sqrt{6}*(5*\sqrt{3} + 9))*\sqrt{-4*\sqrt{3} + 8} + 144*\sqrt{3} + \\ & 36*e^{4*x} - 144*e^{2*x} + 252)*((12^{3/4}*\sqrt{6}*(\sqrt{3} + 3) + 3*12^{1/4} \\ &)*\sqrt{6}*(\sqrt{3} + 3))*\sqrt{-4*\sqrt{3} + 8} + 36*\sqrt{3} + 72) + 2/3*\sqrt{3} \\ & *(2*\sqrt{3} - 3) - 1/36*(12^{3/4}*\sqrt{6}*(\sqrt{3} - 3) + (12^{3/4}*\sqrt{6} \\ &)*(\sqrt{3} + 3) + 3*12^{1/4}*\sqrt{6}*(\sqrt{3} + 3))*e^{2*x} + 3*12^{1/4} \\ &)*\sqrt{6}*(\sqrt{3} - 3))*\sqrt{-4*\sqrt{3} + 8} + 2*\sqrt{3} - 4) - (12^{1/4} \\ &)*\sqrt{6}*(\sqrt{3} + 2)*e^{2*x} + 12^{1/4}*\sqrt{6}*(\sqrt{3} + 2))*\sqrt{-4*\sqrt{3} \\ & + 8}*\log(6*(12^{1/4}*\sqrt{6}*(\sqrt{3} + 3)*e^{2*x} - 12^{1/4}*\sqrt{6}*(5*\sqrt{3} \\ & + 9))*\sqrt{-4*\sqrt{3} + 8} + 144*\sqrt{3} + 36*e^{4*x} - 144*e^{2*x} + 252) \\ & + (12^{1/4}*\sqrt{6}*(\sqrt{3} + 2)*e^{2*x} + 12^{1/4}*\sqrt{6}*(\sqrt{3} + 2))*\sqrt{-4*\sqrt{3} \\ & + 8}*\log(-6*(12^{1/4}*\sqrt{6}*(\sqrt{3} + 3)*e^{2*x} - 12^{1/4}*\sqrt{6}*(5*\sqrt{3} \\ & + 9))*\sqrt{-4*\sqrt{3} + 8} + 144*\sqrt{3} + 36*e^{4*x} - 144*e^{2*x} + 252) \\ & + 96)/(e^{2*x} + 1) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)**6),x)

[Out] Timed out

Giac [A] time = 1.27431, size = 14, normalized size = 0.2

$$\frac{2}{3(e^{2x} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="giac")
```

```
[Out] -2/3/(e^(2*x) + 1)
```

$$3.272 \quad \int \frac{1}{1 + \sinh^8(x)} dx$$

Optimal. Leaf size=129

$$\frac{\tanh^{-1}\left(\sqrt{1 - \sqrt[4]{-1}} \tanh(x)\right)}{4\sqrt{1 - \sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1 + \sqrt[4]{-1}} \tanh(x)\right)}{4\sqrt{1 + \sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1 - (-1)^{3/4}} \tanh(x)\right)}{4\sqrt{1 - (-1)^{3/4}}} + \frac{\tanh^{-1}\left(\sqrt{1 + (-1)^{3/4}} \tanh(x)\right)}{4\sqrt{1 + (-1)^{3/4}}}$$

[Out] ArcTanh[Sqrt[1 - (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTanh[Sqrt[1 + (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTanh[Sqrt[1 - (-1)^(3/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTanh[Sqrt[1 + (-1)^(3/4)]*Tanh[x]]/(4*Sqrt[1 + (-1)^(3/4)])

Rubi [A] time = 0.197778, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\sqrt{1 - \sqrt[4]{-1}} \tanh(x)\right)}{4\sqrt{1 - \sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1 + \sqrt[4]{-1}} \tanh(x)\right)}{4\sqrt{1 + \sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1 - (-1)^{3/4}} \tanh(x)\right)}{4\sqrt{1 - (-1)^{3/4}}} + \frac{\tanh^{-1}\left(\sqrt{1 + (-1)^{3/4}} \tanh(x)\right)}{4\sqrt{1 + (-1)^{3/4}}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^8)^(-1), x]

[Out] ArcTanh[Sqrt[1 - (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTanh[Sqrt[1 + (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTanh[Sqrt[1 - (-1)^(3/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTanh[Sqrt[1 + (-1)^(3/4)]*Tanh[x]]/(4*Sqrt[1 + (-1)^(3/4)])

Rule 3211

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^(n_))^(m_), x_Symbol] :> Module[{k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b), n/2])], x], {k, 1, n/2}], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]

Rule 3181

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2

), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{1 + \sinh^8(x)} dx &= \frac{1}{4} \int \frac{1}{1 - \sqrt[4]{-1} \sinh^2(x)} dx + \frac{1}{4} \int \frac{1}{1 + \sqrt[4]{-1} \sinh^2(x)} dx + \frac{1}{4} \int \frac{1}{1 - (-1)^{3/4} \sinh^2(x)} dx + \frac{1}{4} \int \frac{1}{1 + (-1)^{3/4} \sinh^2(x)} dx \\ &= \frac{1}{4} \text{Subst} \left(\int \frac{1}{1 - (1 - \sqrt[4]{-1}) x^2} dx, x, \tanh(x) \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1 - (1 + \sqrt[4]{-1}) x^2} dx, x, \tanh(x) \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1 - (-1)^{3/4} x^2} dx, x, \tanh(x) \right) + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1 + (-1)^{3/4} x^2} dx, x, \tanh(x) \right) \\ &= \frac{\tanh^{-1} \left(\sqrt{1 - \sqrt[4]{-1}} \tanh(x) \right)}{4\sqrt{1 - \sqrt[4]{-1}}} + \frac{\tanh^{-1} \left(\sqrt{1 + \sqrt[4]{-1}} \tanh(x) \right)}{4\sqrt{1 + \sqrt[4]{-1}}} + \frac{\tanh^{-1} \left(\sqrt{1 - (-1)^{3/4}} \tanh(x) \right)}{4\sqrt{1 - (-1)^{3/4}}} + \frac{\tanh^{-1} \left(\sqrt{1 + (-1)^{3/4}} \tanh(x) \right)}{4\sqrt{1 + (-1)^{3/4}}} \end{aligned}$$

Mathematica [C] time = 0.123296, size = 127, normalized size = 0.98

$$16\text{RootSum} \left[\#1^8 - 8\#1^7 + 28\#1^6 - 56\#1^5 + 326\#1^4 - 56\#1^3 + 28\#1^2 - 8\#1 + 1 \&, \frac{\#1^3 x + \#1^3 \log(-\#1 \sinh(x) + \#1 \cosh(x))}{\#1^7 - 7\#1^6 + 21\#1^5 - 35\#1^4 + 16\#1^3} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^8)^(-1), x]

[Out] 16*RootSum[1 - 8*#1 + 28*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8*#1^7 + #1^8 &, (x*#1^3 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]*#1^3])/(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^5 - 7*#1^6 + #1^7) &]

Maple [C] time = 0.036, size = 64, normalized size = 0.5

$$\frac{1}{8} \sum_{_R=\text{RootOf}(2_Z^8-4_Z^6+6_Z^4-4_Z^2+1)} _R \ln \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 + (-4_R^7 + 8_R^5 - 12_R^3 + 8_R) \tanh \left(\frac{x}{2} \right) + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+sinh(x)^8),x)`

[Out] `1/8*sum(_R*ln(tanh(1/2*x)^2+(-4*_R^7+8*_R^5-12*_R^3+8*_R)*tanh(1/2*x)+1),_R
=RootOf(2*_Z^8-4*_Z^6+6*_Z^4-4*_Z^2+1))`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(x)^8 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^8),x, algorithm="maxima")`

[Out] `integrate(1/(sinh(x)^8 + 1), x)`

Fricas [B] time = 2.8748, size = 11979, normalized size = 92.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+sinh(x)^8),x, algorithm="fricas")`

[Out] `1/16*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2)
+ 4)^(3/4)*sqrt(2*sqrt(2) + 3)*(sqrt(2) - 1)*arctan(1/31*(2*(13*sqrt(2) -
20)*e^(2*x) + 23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 1/
496*(32*(10*sqrt(2) - 13)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + ((355*
sqrt(2) - 508)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 6*(59*sqrt(2) - 86
)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 4*((82*sqrt(2) - 119)*sqrt(2
*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + (85*sqrt(2) - 126)*sqrt(2*sqrt(2) + 3))
*(2*sqrt(2) + 4)^(1/4)*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt
(2) + 8) + 4*((76*sqrt(2) - 105)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) +
2*(53*sqrt(2) - 72)*sqrt(2*sqrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 16*(23*sqrt(
2) - 33)*sqrt(2*sqrt(2) + 3))*sqrt(4*(sqrt(2) - 1)*e^(2*x) - (2*(sqrt(2) -
1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) - 5*sqrt(2) + 6)*sqrt(2*sqrt(2) + 4) -
6*sqrt(2) + 6)*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*`

$$\begin{aligned}
& (2\sqrt{2} + 4)^{1/4} - 4\sqrt{2}\sqrt{2\sqrt{2} + 4}(\sqrt{2} - 2) - 4\sqrt{2} + 2 \\
& *e^{4x} + 10 + 1/248 * (((254\sqrt{2} - 355)e^{2x} - 102\sqrt{2} + 145) * \\
& \sqrt{2\sqrt{2} + 4} * \sqrt{2\sqrt{2} + 3} + 2 * (3 * (43\sqrt{2} - 59)e^{2x} - \\
& 23\sqrt{2} + 33) * \sqrt{2\sqrt{2} + 3}) * (2\sqrt{2} + 4)^{3/4} + 2 * (((119\sqrt{2} \\
& (2) - 164)e^{2x} - 39\sqrt{2} + 60) * \sqrt{2\sqrt{2} + 4} * \sqrt{2\sqrt{2} + \\
& 3} + 2 * ((63\sqrt{2} - 85)e^{2x} - 17\sqrt{2} + 19) * \sqrt{2\sqrt{2} + 3}) * (\\
& 2\sqrt{2} + 4)^{1/4}) * \sqrt{2\sqrt{2}\sqrt{2\sqrt{2} + 4}} * (2\sqrt{2} - 3) - 4\sqrt{2} \\
&) + 8 + 1/124 * (((105\sqrt{2} - 152)e^{2x} + 13\sqrt{2} - 20) * \sqrt{2\sqrt{2} \\
& (2) + 4} * \sqrt{2\sqrt{2} + 3} + 4 * ((36\sqrt{2} - 53)e^{2x} - 23\sqrt{2} + \\
& 33) * \sqrt{2\sqrt{2} + 3}) * \sqrt{2\sqrt{2} + 4} + 1/31 * ((33\sqrt{2} - 46)e^{2 \\
& *x} - 3\sqrt{2} + 7) * \sqrt{2\sqrt{2} + 3}) + 1/16 * \sqrt{2\sqrt{2}\sqrt{2\sqrt{2} + 4} \\
& * (2\sqrt{2} - 3) - 4\sqrt{2} + 8) * (2\sqrt{2} + 4)^{3/4} * \sqrt{2\sqrt{2} + 3} \\
& * (\sqrt{2} - 1) * \arctan(-1/31 * (2 * (13\sqrt{2} - 20)e^{2x} + 23\sqrt{2} - 33) \\
& * \sqrt{2\sqrt{2} + 4} * \sqrt{2\sqrt{2} + 3} - 1/496 * (32 * (10\sqrt{2} - 13) * \sqrt{2} \\
& \sqrt{2\sqrt{2} + 4} * \sqrt{2\sqrt{2} + 3} - ((355\sqrt{2} - 508) * \sqrt{2\sqrt{2} \\
& + 4} * \sqrt{2\sqrt{2} + 3} + 6 * (59\sqrt{2} - 86) * \sqrt{2\sqrt{2} + 3})) * (2\sqrt{2} \\
& (2) + 4)^{3/4} + 4 * ((82\sqrt{2} - 119) * \sqrt{2\sqrt{2} + 4} * \sqrt{2\sqrt{2} + \\
& 3} + (85\sqrt{2} - 126) * \sqrt{2\sqrt{2} + 3})) * (2\sqrt{2} + 4)^{1/4}) * \sqrt{2} \\
& * \sqrt{2\sqrt{2} + 4} * (2\sqrt{2} - 3) - 4\sqrt{2} + 8 + 4 * ((76\sqrt{2} - 10 \\
& 5) * \sqrt{2\sqrt{2} + 4} * \sqrt{2\sqrt{2} + 3} + 2 * (53\sqrt{2} - 72) * \sqrt{2\sqrt{2} \\
& t(2) + 3}) * \sqrt{2\sqrt{2} + 4} + 16 * (23\sqrt{2} - 33) * \sqrt{2\sqrt{2} + 3}) * \\
& \sqrt{4 * (\sqrt{2} - 1) * e^{2x} + (2 * (\sqrt{2} - 1) * e^{2x} + ((\sqrt{2} - 2) * e^{ \\
& (2x) - 5\sqrt{2} + 6) * \sqrt{2\sqrt{2} + 4} - 6\sqrt{2} + 6) * \sqrt{2\sqrt{2} * \sqrt{2} \\
& \sqrt{2} + 4} * (2\sqrt{2} - 3) - 4\sqrt{2} + 8) * (2\sqrt{2} + 4)^{1/4} - 4\sqrt{2} \\
& (2\sqrt{2} + 4) * (\sqrt{2} - 2) - 4\sqrt{2} + 2 * e^{4x} + 10 + 1/248 * (((254 \\
& * \sqrt{2} - 355) * e^{2x} - 102\sqrt{2} + 145) * \sqrt{2\sqrt{2} + 4} * \sqrt{2\sqrt{2} \\
& t(2) + 3} + 2 * (3 * (43\sqrt{2} - 59) * e^{2x} - 23\sqrt{2} + 33) * \sqrt{2\sqrt{2} \\
&) + 3}) * (2\sqrt{2} + 4)^{3/4} + 2 * (((119\sqrt{2} - 164) * e^{2x} - 39\sqrt{2} \\
&) + 60) * \sqrt{2\sqrt{2} + 4} * \sqrt{2\sqrt{2} + 3} + 2 * ((63\sqrt{2} - 85) * e^{2 \\
& *x} - 17\sqrt{2} + 19) * \sqrt{2\sqrt{2} + 3}) * (2\sqrt{2} + 4)^{1/4}) * \sqrt{2\sqrt{2} \\
& \sqrt{2\sqrt{2} + 4} * (2\sqrt{2} - 3) - 4\sqrt{2} + 8) - 1/124 * (((105\sqrt{2} \\
& - 152) * e^{2x} + 13\sqrt{2} - 20) * \sqrt{2\sqrt{2} + 4} * \sqrt{2\sqrt{2} + 3} + \\
& 4 * ((36\sqrt{2} - 53) * e^{2x} - 23\sqrt{2} + 33) * \sqrt{2\sqrt{2} + 3}) * \sqrt{2} \\
& \sqrt{2\sqrt{2} + 4} - 1/31 * ((33\sqrt{2} - 46) * e^{2x} - 3\sqrt{2} + 7) * \sqrt{2\sqrt{2} \\
& rt(2) + 3}) - 1/16 * \sqrt{-2 * (2\sqrt{2} + 3) * \sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} \\
& + 8) * (\sqrt{2} + 1) * (-2\sqrt{2} + 4)^{3/4} * \sqrt{-2\sqrt{2} + 3} * \arctan(1/31 \\
& * (2 * (13\sqrt{2} + 20) * e^{2x} + 23\sqrt{2} + 33) * \sqrt{-2\sqrt{2} + 4} * \sqrt{ \\
& -2\sqrt{2} + 3} - 1/496 * (32 * (10\sqrt{2} + 13) * \sqrt{-2\sqrt{2} + 4} * \sqrt{-2\sqrt{2} \\
& \sqrt{2} + 3} - (((355\sqrt{2} + 508) * \sqrt{-2\sqrt{2} + 4} * \sqrt{-2\sqrt{2} + \\
& 3} + 6 * (59\sqrt{2} + 86) * \sqrt{-2\sqrt{2} + 3})) * (-2\sqrt{2} + 4)^{3/4} + 4 * \\
& ((82\sqrt{2} + 119) * \sqrt{-2\sqrt{2} + 4} * \sqrt{-2\sqrt{2} + 3} + (85\sqrt{2} \\
& + 126) * \sqrt{-2\sqrt{2} + 3})) * (-2\sqrt{2} + 4)^{1/4}) * \sqrt{-2 * (2\sqrt{2} + \\
& 3) * \sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8) + 4 * ((76\sqrt{2} + 105) * \sqrt{-2\sqrt{2} \\
& rt(2) + 4} * \sqrt{-2\sqrt{2} + 3} + 2 * (53\sqrt{2} + 72) * \sqrt{-2\sqrt{2} + 3}) \\
& * \sqrt{-2\sqrt{2} + 4} + 16 * (23\sqrt{2} + 33) * \sqrt{-2\sqrt{2} + 3}) * \sqrt{-4 *
\end{aligned}$$

$$\begin{aligned}
& (\sqrt{2} + 1)e^{2x} - (2(\sqrt{2} + 1)e^{2x} + ((\sqrt{2} + 2)e^{2x} - 5\sqrt{2} - 6)\sqrt{-2\sqrt{2} + 4} - 6\sqrt{2} - 6)\sqrt{-2(2\sqrt{2} + 3)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8})^{1/4} + 4(\sqrt{2} + 2)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 2e^{4x} + 10) - 1/248 * (((254\sqrt{2} + 355)e^{2x} - 102\sqrt{2} - 145)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + 2(3(43\sqrt{2} + 59)e^{2x} - 23\sqrt{2} - 33)\sqrt{-2\sqrt{2} + 3})^{3/4} + 2(((119\sqrt{2} + 164)e^{2x} - 39\sqrt{2} - 60)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + 2((63\sqrt{2} + 85)e^{2x} - 17\sqrt{2} - 19)\sqrt{-2\sqrt{2} + 3}))^{1/4} * \sqrt{-2(2\sqrt{2} + 3)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8} + 1/124 * (((105\sqrt{2} + 152)e^{2x} + 13\sqrt{2} + 20)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + 4((36\sqrt{2} + 53)e^{2x} - 23\sqrt{2} - 33)\sqrt{-2\sqrt{2} + 3})\sqrt{-2\sqrt{2} + 4} + 1/31 * ((33\sqrt{2} + 46)e^{2x} - 3\sqrt{2} - 7)\sqrt{-2\sqrt{2} + 3}) - 1/16 * \sqrt{-2(2\sqrt{2} + 3)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8} * (\sqrt{2} + 1) * (-2\sqrt{2} + 4)^{3/4} * \sqrt{-2\sqrt{2} + 3} * \arctan(-1/31 * (2(13\sqrt{2} + 20)e^{2x} + 23\sqrt{2} + 33)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + 1/496 * (32(10\sqrt{2} + 13)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + ((355\sqrt{2} + 508)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + 6(59\sqrt{2} + 86)\sqrt{-2\sqrt{2} + 3}))^{1/4} * (-2\sqrt{2} + 4)^{3/4} + 4((82\sqrt{2} + 119)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + (85\sqrt{2} + 126)\sqrt{-2\sqrt{2} + 3}) * (-2\sqrt{2} + 4)^{1/4} * \sqrt{-2(2\sqrt{2} + 3)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8} + 4((76\sqrt{2} + 105)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + 2(53\sqrt{2} + 72)\sqrt{-2\sqrt{2} + 3})\sqrt{-2\sqrt{2} + 4} + 16(23\sqrt{2} + 33)\sqrt{-2\sqrt{2} + 3})\sqrt{-4(\sqrt{2} + 1)e^{2x} + (2(\sqrt{2} + 1)e^{2x} + ((\sqrt{2} + 2)e^{2x} - 5\sqrt{2} - 6)\sqrt{-2\sqrt{2} + 4} - 6\sqrt{2} - 6)\sqrt{-2(2\sqrt{2} + 3)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8})^{1/4} + 4(\sqrt{2} + 2)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 2e^{4x} + 10) - 1/248 * (((254\sqrt{2} + 355)e^{2x} - 102\sqrt{2} - 145)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + 2(3(43\sqrt{2} + 59)e^{2x} - 23\sqrt{2} - 33)\sqrt{-2\sqrt{2} + 3})^{3/4} + 2(((119\sqrt{2} + 164)e^{2x} - 39\sqrt{2} - 60)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + 2((63\sqrt{2} + 85)e^{2x} - 17\sqrt{2} - 19)\sqrt{-2\sqrt{2} + 3}))^{1/4} * \sqrt{-2(2\sqrt{2} + 3)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8} - 1/124 * (((105\sqrt{2} + 152)e^{2x} + 13\sqrt{2} + 20)\sqrt{-2\sqrt{2} + 4}\sqrt{-2\sqrt{2} + 3} + 4((36\sqrt{2} + 53)e^{2x} - 23\sqrt{2} - 33)\sqrt{-2\sqrt{2} + 3})\sqrt{-2\sqrt{2} + 4} - 1/31 * ((33\sqrt{2} + 46)e^{2x} - 3\sqrt{2} - 7)\sqrt{-2\sqrt{2} + 3}) + 1/64 * \sqrt{-2(2\sqrt{2} + 3)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8} * ((\sqrt{2} + 1)\sqrt{-2\sqrt{2} + 4} + 2\sqrt{2}) * (-2\sqrt{2} + 4)^{1/4} * \log(-2(\sqrt{2} + 1)e^{2x} + 1/2 * (2(\sqrt{2} + 1)e^{2x} + ((\sqrt{2} + 2)e^{2x} - 5\sqrt{2} - 6)\sqrt{-2\sqrt{2} + 4} - 6\sqrt{2} - 6)\sqrt{-2(2\sqrt{2} + 3)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8})^{1/4} + 2(\sqrt{2} + 2)\sqrt{-2\sqrt{2} + 4} + 2\sqrt{2} + e^{4x} + 5) - 1/64 * \sqrt{-2(2\sqrt{2} + 3)\sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8} * ((\sqrt{2} + 1)\sqrt{-2\sqrt{2} + 4} + 2\sqrt{2}) * (-2\sqrt{2} + 4)
\end{aligned}$$

$$\begin{aligned}
& + 4)^{(1/4)} \cdot \log(-2 \cdot (\sqrt{2} + 1) \cdot e^{2x} - 1/2 \cdot (2 \cdot (\sqrt{2} + 1) \cdot e^{2x} + (\sqrt{2} + 2) \cdot e^{2x} - 5\sqrt{2} - 6) \cdot \sqrt{-2\sqrt{2} + 4} - 6\sqrt{2} - 6) \cdot \sqrt{-2 \cdot (2\sqrt{2} + 3) \cdot \sqrt{-2\sqrt{2} + 4} + 4\sqrt{2} + 8} \cdot (-2\sqrt{2} + 4)^{(1/4)} + 2 \cdot (\sqrt{2} + 2) \cdot \sqrt{-2\sqrt{2} + 4} + 2\sqrt{2} + e^{4x} + 5) + 1/64 \cdot \sqrt{2\sqrt{2} \cdot \sqrt{2\sqrt{2} + 4}} \cdot (2\sqrt{2} - 3) - 4\sqrt{2} + 8) \cdot (\sqrt{2\sqrt{2} + 4} \cdot (\sqrt{2} - 1) + 2\sqrt{2}) \cdot (2\sqrt{2} + 4)^{(1/4)} \cdot \log(2 \cdot (\sqrt{2} - 1) \cdot e^{2x} + 1/2 \cdot (2 \cdot (\sqrt{2} - 1) \cdot e^{2x} + ((\sqrt{2} - 2) \cdot e^{2x} - 5\sqrt{2} + 6) \cdot \sqrt{2\sqrt{2} + 4} - 6\sqrt{2} + 6) \cdot \sqrt{2\sqrt{2} \cdot \sqrt{2\sqrt{2} + 4}} \cdot (2\sqrt{2} - 3) - 4\sqrt{2} + 8) \cdot (2\sqrt{2} + 4)^{(1/4)} - 2\sqrt{2\sqrt{2} + 4} \cdot (\sqrt{2} - 2) - 2\sqrt{2} + e^{4x} + 5) - 1/64 \cdot \sqrt{2\sqrt{2} \cdot \sqrt{2\sqrt{2} + 4}} \cdot (2\sqrt{2} - 3) - 4\sqrt{2} + 8) \cdot (\sqrt{2\sqrt{2} + 4} \cdot (\sqrt{2} - 1) + 2\sqrt{2}) \cdot (2\sqrt{2} + 4)^{(1/4)} \cdot \log(2 \cdot (\sqrt{2} - 1) \cdot e^{2x} - 1/2 \cdot (2 \cdot (\sqrt{2} - 1) \cdot e^{2x} + ((\sqrt{2} - 2) \cdot e^{2x} - 5\sqrt{2} + 6) \cdot \sqrt{2\sqrt{2} + 4} - 6\sqrt{2} + 6) \cdot \sqrt{2\sqrt{2} \cdot \sqrt{2\sqrt{2} + 4}} \cdot (2\sqrt{2} - 3) - 4\sqrt{2} + 8) \cdot (2\sqrt{2} + 4)^{(1/4)} - 2\sqrt{2\sqrt{2} + 4} \cdot (\sqrt{2} - 2) - 2\sqrt{2} + e^{4x} + 5)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)**8),x)

[Out] Timed out

Giac [A] time = 1.27901, size = 1, normalized size = 0.01

0

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+sinh(x)^8),x, algorithm="giac")

[Out] 0

$$3.273 \quad \int \frac{1}{1 - \sinh^5(x)} dx$$

Optimal. Leaf size=228

$$-\frac{2 \tanh^{-1}\left(\frac{(-1)^{3/5} - \tanh\left(\frac{x}{2}\right)}{\sqrt{1 - \sqrt[5]{-1}}}\right)}{5\sqrt{1 - \sqrt[5]{-1}}} + \frac{1}{5}\sqrt{2} \tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right) + 1}{\sqrt{2}}\right) + \frac{2 \tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right) + (-1)^{4/5}}{\sqrt{1 - (-1)^{3/5}}}\right)}{5\sqrt{1 - (-1)^{3/5}}} - \frac{2 \sqrt[10]{-1} \tanh^{-1}\left(\frac{(-1)^{3/10}((-1)^{4/5} + \tanh\left(\frac{x}{2}\right))}{\sqrt{\sqrt[5]{-1} + (-1)^{3/5}}}\right)}{5\sqrt{\sqrt[5]{-1} + (-1)^{3/5}}}$$

[Out] $(-2*(-1)^{(1/10)}*\text{ArcTan}[(I + (-1)^{(1/10)}*\text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(1/5)}]])/(5*\text{Sqrt}[1 - (-1)^{(1/5)}]) - (2*\text{ArcTanh}[((-1)^{(3/5)} - \text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(1/5)}]])/(5*\text{Sqrt}[1 - (-1)^{(1/5)}]) + (\text{Sqrt}[2]*\text{ArcTanh}[(1 + \text{Tanh}[x/2])/ \text{Sqrt}[2]])/5 + (2*\text{ArcTanh}[((-1)^{(4/5)} + \text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(3/5)}]])/(5*\text{Sqrt}[1 - (-1)^{(3/5)}]) - (2*(-1)^{(1/10)}*\text{ArcTanh}[((-1)^{(3/10)}*(1 + (-1)^{(4/5)}*\text{Tanh}[x/2]))/ \text{Sqrt}[(-1)^{(1/5)} + (-1)^{(3/5)}]])/(5*\text{Sqrt}[(-1)^{(1/5)} + (-1)^{(3/5)}])$

Rubi [A] time = 0.417044, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {3213, 2660, 618, 204, 617, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{(-1)^{3/5} - \tanh\left(\frac{x}{2}\right)}{\sqrt{1 - \sqrt[5]{-1}}}\right)}{5\sqrt{1 - \sqrt[5]{-1}}} + \frac{1}{5}\sqrt{2} \tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right) + 1}{\sqrt{2}}\right) + \frac{2 \tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right) + (-1)^{4/5}}{\sqrt{1 - (-1)^{3/5}}}\right)}{5\sqrt{1 - (-1)^{3/5}}} - \frac{2 \sqrt[10]{-1} \tanh^{-1}\left(\frac{(-1)^{3/10}((-1)^{4/5} + \tanh\left(\frac{x}{2}\right))}{\sqrt{\sqrt[5]{-1} + (-1)^{3/5}}}\right)}{5\sqrt{\sqrt[5]{-1} + (-1)^{3/5}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^5)^(-1), x]

[Out] $(-2*(-1)^{(1/10)}*\text{ArcTan}[(I + (-1)^{(1/10)}*\text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(1/5)}]])/(5*\text{Sqrt}[1 - (-1)^{(1/5)}]) - (2*\text{ArcTanh}[((-1)^{(3/5)} - \text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(1/5)}]])/(5*\text{Sqrt}[1 - (-1)^{(1/5)}]) + (\text{Sqrt}[2]*\text{ArcTanh}[(1 + \text{Tanh}[x/2])/ \text{Sqrt}[2]])/5 + (2*\text{ArcTanh}[((-1)^{(4/5)} + \text{Tanh}[x/2])/ \text{Sqrt}[1 - (-1)^{(3/5)}]])/(5*\text{Sqrt}[1 - (-1)^{(3/5)}]) - (2*(-1)^{(1/10)}*\text{ArcTanh}[((-1)^{(3/10)}*(1 + (-1)^{(4/5)}*\text{Tanh}[x/2]))/ \text{Sqrt}[(-1)^{(1/5)} + (-1)^{(3/5)}]])/(5*\text{Sqrt}[(-1)^{(1/5)} + (-1)^{(3/5)}])$

Rule 3213

Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :=>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f

, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{1}{1 - \sinh^5(x)} dx &= \int \left(\frac{\sqrt[10]{-1}}{5(\sqrt[10]{-1} - i \sinh(x))} + \frac{\sqrt[10]{-1}}{5(\sqrt[10]{-1} - \sqrt[10]{-1} \sinh(x))} + \frac{\sqrt[10]{-1}}{5(\sqrt[10]{-1} + (-1)^{3/10} \sinh(x))} + \frac{\sqrt[10]{-1}}{5(\sqrt[10]{-1} + (-1)^{3/10} \sinh(x))} \right) dx \\
&= \frac{1}{5} \sqrt[10]{-1} \int \frac{1}{\sqrt[10]{-1} - i \sinh(x)} dx + \frac{1}{5} \sqrt[10]{-1} \int \frac{1}{\sqrt[10]{-1} - \sqrt[10]{-1} \sinh(x)} dx + \frac{1}{5} \sqrt[10]{-1} \int \frac{1}{\sqrt[10]{-1} + (-1)^{3/10} \sinh(x)} dx \\
&= \frac{1}{5} \left(2 \sqrt[10]{-1} \right) \text{Subst} \left(\int \frac{1}{\sqrt[10]{-1} - 2ix - \sqrt[10]{-1}x^2} dx, x, \tanh\left(\frac{x}{2}\right) \right) + \frac{1}{5} \left(2 \sqrt[10]{-1} \right) \text{Subst} \left(\int \frac{1}{\sqrt[10]{-1} - 2 \sqrt[10]{-1}x} dx, x, \tanh\left(\frac{x}{2}\right) \right) \\
&= \frac{2}{5} \text{Subst} \left(\int \frac{1}{2 - x^2} dx, x, 1 + \tanh\left(\frac{x}{2}\right) \right) - \frac{1}{5} \left(4 \sqrt[10]{-1} \right) \text{Subst} \left(\int \frac{1}{-4(1 - \sqrt[5]{-1}) - x^2} dx, x, -2i - 2 \sqrt[10]{-1} \right) \\
&= -\frac{2 \sqrt[10]{-1} \tan^{-1} \left(\frac{i + \sqrt[10]{-1} \tanh\left(\frac{x}{2}\right)}{\sqrt{1 - \sqrt[5]{-1}}} \right)}{5 \sqrt{1 - \sqrt[5]{-1}}} - \frac{2 \tanh^{-1} \left(\frac{(-1)^{3/5} - \tanh\left(\frac{x}{2}\right)}{\sqrt{1 - \sqrt[5]{-1}}} \right)}{5 \sqrt{1 - \sqrt[5]{-1}}} + \frac{1}{5} \sqrt{2} \tanh^{-1} \left(\frac{1 + \tanh\left(\frac{x}{2}\right)}{\sqrt{2}} \right) + \frac{2 \tanh^{-1} \left(\frac{-2i - 2 \sqrt[10]{-1}}{1 - \sqrt[5]{-1}} \right)}{5 \sqrt{1 - \sqrt[5]{-1}}}
\end{aligned}$$

Mathematica [C] time = 0.868295, size = 437, normalized size = 1.92

$$\frac{1}{10} \left(\text{RootSum} \left[\#1^8 + 2\#1^7 + 2\#1^5 + 14\#1^4 - 2\#1^3 - 2\#1 + 1 \&, \frac{\#1^6 x + 4\#1^5 x + 9\#1^4 x + 24\#1^3 x - 9\#1^2 x + 2\#1^6 \log(-x - 2 \log[-\cosh(x/2) - \sinh(x/2) + \cosh(x/2)] \#1 - \sinh(x/2) \#1 + 4*x*\#1 + 8*\log[-\cosh(x/2) - \sinh(x/2) + \cosh(x/2)] \#1 - \sinh(x/2) \#1 \#1 - 9*x*\#1^2 - 18*\log[-\cosh(x/2) - \sinh(x/2) + \cosh(x/2)] \#1 - \sinh(x/2) \#1 \#1^2 + 24*x*\#1^3 + 48*\log[-\cosh(x/2) - \sinh(x/2) + \cosh(x/2)] \#1 - \sinh(x/2) \#1 \#1^3 + 9*x*\#1^4 + 18*\log[-\cosh(x/2) - \sinh(x/2) + \cosh(x/2)] \#1 - \sinh(x/2) \#1 \#1^4 + 4*x*\#1^5 + 8*\log[-\cosh(x/2) - \sinh(x/2) + \cosh(x/2)] \#1 - \sinh(x/2) \#1 \#1^5 + x*\#1^6 + 2*\log[-\cosh(x/2) - \sinh(x/2) + \cosh(x/2)] \#1 - \sinh(x/2) \#1 \#1^6)/(-1 - 3*\#1^2 + 28*\#1^3 + 5*\#1^4 + 7*\#1^6 + 4*\#1^7) \&] \right) / 10$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sinh[x]^5)^(-1), x]

[Out] (2*sqrt(2)*ArcTanh[(1 + Tanh[x/2])/sqrt(2)] + RootSum[1 - 2*#1 - 2*#1^3 + 14*#1^4 + 2*#1^5 + 2*#1^7 + #1^8 &, (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]]*#1 - Sinh[x/2]*#1 + 4*x*#1 + 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]]*#1 - Sinh[x/2]*#1*#1 - 9*x*#1^2 - 18*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]]*#1 - Sinh[x/2]*#1*#1^2 + 24*x*#1^3 + 48*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]]*#1 - Sinh[x/2]*#1*#1^3 + 9*x*#1^4 + 18*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]]*#1 - Sinh[x/2]*#1*#1^4 + 4*x*#1^5 + 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]]*#1 - Sinh[x/2]*#1*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]]*#1 - Sinh[x/2]*#1*#1^6)/(-1 - 3*#1^2 + 28*#1^3 + 5*#1^4 + 7*#1^6 + 4*#1^7) &])/10

Maple [C] time = 0.036, size = 124, normalized size = 0.5

$$\frac{2}{5} \sum_{_R=\text{RootOf}(_Z^8-2_Z^7-2_Z^5+14_Z^4+2_Z^3+2_Z+1)} \frac{-2_R^6+3_R^5+2_R^4-2_R^3-2_R^2+3_R+2}{4_R^7-7_R^6-5_R^4+28_R^3+3_R^2+1} \ln \left(\tanh\left(\frac{x}{2}\right) - \frac{2_R}{1 - 2_R - 2_R^3 + 14_R^4 + 2_R^5 + 2_R^7 + R^8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1-sinh(x)^5),x)`

[Out] $2/5 \sum((-2*_R^6 + 3*_R^5 + 2*_R^4 - 2*_R^3 - 2*_R^2 + 3*_R + 2) / (4*_R^7 - 7*_R^6 - 5*_R^4 + 28*_R^3 + 3*_R^2 + 1) * \ln(\tanh(1/2*x) - _R), _R = \text{RootOf}(_Z^8 - 2*_Z^7 - 2*_Z^5 + 14*_Z^4 + 2*_Z^3 + 2*_Z + 1)) + 1/5 * 2^{(1/2)} * \operatorname{arctanh}(1/4 * (2 * \tanh(1/2*x) + 2) * 2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{10} \sqrt{2} \log\left(-\frac{\sqrt{2} - e^x + 1}{\sqrt{2} + e^x - 1}\right) + \int \frac{2(e^{7x} + 4e^{6x} + 9e^{5x} + 24e^{4x} - 9e^{3x} + 4e^{2x} - e^x)}{5(e^{8x} + 2e^{7x} + 2e^{5x} + 14e^{4x} - 2e^{3x} - 2e^x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^5),x, algorithm="maxima")`

[Out] $-1/10 * \sqrt{2} * \log(-(\sqrt{2} - e^x + 1) / (\sqrt{2} + e^x - 1)) + \operatorname{integrate}(2/5 * (e^{(7*x)} + 4*e^{(6*x)} + 9*e^{(5*x)} + 24*e^{(4*x)} - 9*e^{(3*x)} + 4*e^{(2*x)} - e^x) / (e^{(8*x)} + 2*e^{(7*x)} + 2*e^{(5*x)} + 14*e^{(4*x)} - 2*e^{(3*x)} - 2*e^x + 1), x)$

Fricas [B] time = 4.08645, size = 11359, normalized size = 49.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^5),x, algorithm="fricas")`

[Out] $1/200 * \sqrt{2} * \sqrt{2 * \sqrt{2} * (2 * \sqrt{5} - 5) * \sqrt{\sqrt{5} + 3}} - 4 * \sqrt{5} + 20 * (8 * \sqrt{5} + 24)^{(1/4)} * (3 * \sqrt{5} - 5) * \sqrt{2 * \sqrt{5} + 5} * \sqrt{\sqrt{5} + 3} * \operatorname{arctan}(1/40 * \sqrt{2} * ((11 * \sqrt{5} - 25) * e^x + 7 * \sqrt{5} - 15) * \sqrt{2 * \sqrt{5} + 5} * \sqrt{\sqrt{5} + 3}) + 1/80 * \sqrt{2} * (\sqrt{2} * ((11 * \sqrt{5} - 25) * e^x - 4 * \sqrt{5} + 10) * \sqrt{2 * \sqrt{5} + 5} * \sqrt{\sqrt{5} + 3}) + 2 * ((3 * \sqrt{5} - 5) * e^x - 7 * \sqrt{5} + 15) * \sqrt{2 * \sqrt{5} + 5} * \sqrt{\sqrt{5} + 3} + 1/1280 * (80 * \sqrt{2} * (5 * \sqrt{5} - 11) * \sqrt{2 * \sqrt{5} + 5} * \sqrt{\sqrt{5} + 3}) + 40 * \sqrt{2} * (\sqrt{2} * (5 * \sqrt{5} - 11) * \sqrt{2 * \sqrt{5} + 5} * \sqrt{\sqrt{5} + 3}) + 2 * \sqrt{2 * \sqrt{5} + 5} * (\sqrt{5} - 3) * \sqrt{\sqrt{5} + 3} + \sqrt{2 * \sqrt{2} * (2 * \sqrt{5} - 5) * \sqrt{\sqrt{5} + 3}}$

$$\begin{aligned}
& \sqrt{5} + 5) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} + 20) \sqrt{-2\sqrt{5} + 5} + \\
& 1/25600 * (((7\sqrt{5} + 15) \sqrt{-8\sqrt{5} + 24} + 44\sqrt{5} + 100) * (-8\sqrt{5} \\
& \sqrt{5} + 24)^{3/4} + 4 * ((17\sqrt{5} + 35) \sqrt{-8\sqrt{5} + 24} + 44\sqrt{5} (\\
& 5) + 100) * (-8\sqrt{5} + 24)^{1/4}) * \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + \\
& 24} + 4\sqrt{5} + 20) \sqrt{-2\sqrt{5} + 5} - 20 * (((5\sqrt{5} + 11) \sqrt{-8\sqrt{5} \\
& \sqrt{5} + 24} + 4\sqrt{5} + 12) \sqrt{-8\sqrt{5} + 24} + 4 * (5\sqrt{5} + 11) * \\
& \sqrt{-8\sqrt{5} + 24} + 32\sqrt{5} + 128) \sqrt{-2\sqrt{5} + 5}) * \sqrt{40 * (\sqrt{5} + 1) e^x + (4 * (\sqrt{5} + 5) e^x + ((2\sqrt{5} + 5) e^x + 3\sqrt{5} + \\
& 5) \sqrt{-8\sqrt{5} + 24} + 6\sqrt{5} + 10) \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} \\
& \sqrt{5} + 24} + 4\sqrt{5} + 20) * (-8\sqrt{5} + 24)^{1/4} + 10 * (\sqrt{5} + 3) \sqrt{ \\
& \sqrt{-8\sqrt{5} + 24} + 80 e^{(2x)} + 80) + 1/320 * (32 * (4\sqrt{5} + 5) e^x + 4 * \\
& ((11\sqrt{5} + 25) e^x + 7\sqrt{5} + 15) \sqrt{-8\sqrt{5} + 24} + (4 * (3\sqrt{5} \\
& \sqrt{5} + 5) e^x + ((11\sqrt{5} + 25) e^x - 4\sqrt{5} - 10) \sqrt{-8\sqrt{5} + 2 \\
& 4} - 28\sqrt{5} - 60) \sqrt{-8\sqrt{5} + 24} + 16\sqrt{5} + 80) \sqrt{-2\sqrt{5} \\
& \sqrt{5} + 5)) - 1/400 * \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} + \\
& 20) * (3\sqrt{5} + 5) \sqrt{-2\sqrt{5} + 5} * (-8\sqrt{5} + 24)^{3/4} * \arctan(-1 \\
& /1280 * ((4 * (5\sqrt{5} + 11) e^x + ((3\sqrt{5} + 7) e^x - 8\sqrt{5} - 18) \sqrt{ \\
& \sqrt{-8\sqrt{5} + 24} - 8\sqrt{5} - 16) * (-8\sqrt{5} + 24)^{3/4} + 4 * (4 * (5\sqrt{5} \\
& \sqrt{5} + 11) e^x + ((7\sqrt{5} + 17) e^x - 8\sqrt{5} - 18) \sqrt{-8\sqrt{5} + 2 \\
& 4} - 20\sqrt{5} - 36) * (-8\sqrt{5} + 24)^{1/4}) * \sqrt{-(2\sqrt{5} + 5) \sqrt{ \\
& \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} + 20) \sqrt{-2\sqrt{5} + 5} + 1/25600 * (((7\sqrt{5} \\
& \sqrt{5} + 15) \sqrt{-8\sqrt{5} + 24} + 44\sqrt{5} + 100) * (-8\sqrt{5} + 24)^{3/4} \\
& + 4 * ((17\sqrt{5} + 35) \sqrt{-8\sqrt{5} + 24} + 44\sqrt{5} + 100) * (-8\sqrt{5} (\\
& 5) + 24)^{1/4}) * \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} + 2 \\
& 0) \sqrt{-2\sqrt{5} + 5} + 20 * (((5\sqrt{5} + 11) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{5} \\
& \sqrt{5} + 12) \sqrt{-8\sqrt{5} + 24} + 4 * (5\sqrt{5} + 11) \sqrt{-8\sqrt{5} + 2 \\
& 4} + 32\sqrt{5} + 128) \sqrt{-2\sqrt{5} + 5}) * \sqrt{40 * (\sqrt{5} + 1) e^x - (4 \\
& * (\sqrt{5} + 5) e^x + ((2\sqrt{5} + 5) e^x + 3\sqrt{5} + 5) \sqrt{-8\sqrt{5} (\\
& + 24} + 6\sqrt{5} + 10) \sqrt{-(2\sqrt{5} + 5) \sqrt{-8\sqrt{5} + 24} + 4\sqrt{ \\
& \sqrt{5} + 20) * (-8\sqrt{5} + 24)^{1/4} + 10 * (\sqrt{5} + 3) \sqrt{-8\sqrt{5} + 24} \\
& + 80 e^{(2x)} + 80) - 1/320 * (32 * (4\sqrt{5} + 5) e^x + 4 * ((11\sqrt{5} + 25) * \\
& e^x + 7\sqrt{5} + 15) \sqrt{-8\sqrt{5} + 24} + (4 * (3\sqrt{5} + 5) e^x + ((11 \\
& \sqrt{5} + 25) e^x - 4\sqrt{5} - 10) \sqrt{-8\sqrt{5} + 24} - 28\sqrt{5} - 6 \\
& 0) \sqrt{-8\sqrt{5} + 24} + 16\sqrt{5} + 80) \sqrt{-2\sqrt{5} + 5)) - 1/800 * (\\
& \sqrt{2} * (3\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) + 8\sqrt{5}}) \sqrt{2\sqrt{2} * (2\sqrt{ \\
& \sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) - 4\sqrt{5} + 20) * (8\sqrt{5} + 24)^{1/4} * \log(- \\
& 4\sqrt{2} * \sqrt{(\sqrt{5} + 3) * (\sqrt{5} - 3) - 8 * (\sqrt{5} - 1) e^x + 2/5 * (\sqrt{ \\
& 2} * ((2\sqrt{5} - 5) e^x + 3\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) + 2 * (\sqrt{5} - \\
& 5) e^x + 3\sqrt{5} - 5) \sqrt{2\sqrt{2} * (2\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) - \\
& 4\sqrt{5} + 20) * (8\sqrt{5} + 24)^{1/4} + 16 e^{(2x)} + 16) + 1/800 * (\sqrt{2} * \\
& (3\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) + 8\sqrt{5}}) \sqrt{2\sqrt{2} * (2\sqrt{5} - \\
& 5) \sqrt{(\sqrt{5} + 3) - 4\sqrt{5} + 20) * (8\sqrt{5} + 24)^{1/4} * \log(-4\sqrt{2} \\
&) * \sqrt{(\sqrt{5} + 3) * (\sqrt{5} - 3) - 8 * (\sqrt{5} - 1) e^x - 2/5 * (\sqrt{2} * ((2\sqrt{ \\
& \sqrt{5} - 5) e^x + 3\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) + 2 * (\sqrt{5} - 5) e^x + \\
& 3\sqrt{5} - 5) \sqrt{2\sqrt{2} * (2\sqrt{5} - 5) \sqrt{(\sqrt{5} + 3) - 4\sqrt{5} (5
\end{aligned}$$

) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) - 1/1600*((3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4)*log(8*(sqrt(5) + 1)*e^x + 1/5*(4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x + 3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 6*sqrt(5) + 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(5) + 24) + 16*e^(2*x) + 16) + 1/1600*((3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4)*log(8*(sqrt(5) + 1)*e^x - 1/5*(4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x + 3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 6*sqrt(5) + 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(5) + 24) + 16*e^(2*x) + 16) + 1/10*sqrt(2)*log((2*(sqrt(2) - 1)*e^x - 2*sqrt(2) + e^(2*x) + 3)/(e^(2*x) - 2*e^x - 1))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)**5),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sinh(x)^5 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^5),x, algorithm="giac")

[Out] integrate(-1/(sinh(x)^5 - 1), x)

$$3.274 \quad \int \frac{1}{1 - \sinh^6(x)} dx$$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\sqrt{1 - \sqrt[3]{-1}} \tanh(x)\right)}{3\sqrt{1 - \sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1 + (-1)^{2/3}} \tanh(x)\right)}{3\sqrt{1 + (-1)^{2/3}}}$$

[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(3*Sqrt[2]) + ArcTanh[Sqrt[1 - (-1)^(1/3)]*Tanh[x]]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTanh[Sqrt[1 + (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[1 + (-1)^(2/3)])

Rubi [A] time = 0.0998656, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\sqrt{1 - \sqrt[3]{-1}} \tanh(x)\right)}{3\sqrt{1 - \sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1 + (-1)^{2/3}} \tanh(x)\right)}{3\sqrt{1 + (-1)^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^6)^(-1), x]

[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(3*Sqrt[2]) + ArcTanh[Sqrt[1 - (-1)^(1/3)]*Tanh[x]]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTanh[Sqrt[1 + (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[1 + (-1)^(2/3)])

Rule 3211

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(m_), x_Symbol] := Module[{k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x])^2/((-1)^((4*k)/n)*Rt[-(a/b), n/2]), x], {k, 1, n/2}], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{1 - \sinh^6(x)} dx &= \frac{1}{3} \int \frac{1}{1 - \sinh^2(x)} dx + \frac{1}{3} \int \frac{1}{1 + \sqrt[3]{-1} \sinh^2(x)} dx + \frac{1}{3} \int \frac{1}{1 - (-1)^{2/3} \sinh^2(x)} dx \\ &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \tanh(x) \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{1 - (1 - \sqrt[3]{-1}) x^2} dx, x, \tanh(x) \right) + \frac{1}{3} \text{Subst} \left(\int \frac{1}{1 - (-1)^{2/3} x^2} dx, x, \tanh(x) \right) \\ &= \frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\sqrt{1 - \sqrt[3]{-1}} \tanh(x)\right)}{3\sqrt{1 - \sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1 + (-1)^{2/3}} \tanh(x)\right)}{3\sqrt{1 + (-1)^{2/3}}} \end{aligned}$$

Mathematica [C] time = 0.431108, size = 70, normalized size = 0.84

$$\frac{1}{6} \left(\sqrt{2} \tanh^{-1}(\sqrt{2} \tanh(x)) + i\sqrt{3} \left(\tan^{-1}\left(\frac{1 - 2i \tanh(x)}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1 + 2i \tanh(x)}{\sqrt{3}}\right) \right) - \tan^{-1}(\text{csch}(x)\text{sech}(x)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sinh[x]^6)^(-1), x]

[Out] (-ArcTan[Csch[x]*Sech[x]] + I*Sqrt[3]*(ArcTan[(1 - (2*I)*Tanh[x])/Sqrt[3]] - ArcTan[(1 + (2*I)*Tanh[x])/Sqrt[3]])) + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]/6

Maple [C] time = 0.032, size = 160, normalized size = 1.9

$$\frac{1}{3} \sum_{_R=\text{RootOf}(_Z^4-2_Z^3+2_Z^2+2_Z+1)} \frac{-_R^2 + _R + 1}{2_R^3 - 3_R^2 + 2_R + 1} \ln\left(\tanh\left(\frac{x}{2}\right) - _R\right) + \frac{1}{3} \sum_{_R=\text{RootOf}(_Z^4+2_Z^3+2_Z^2-2_Z+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-sinh(x)^6), x)

[Out] $\frac{1}{3} \sum \left(\frac{-R^2 + R + 1}{2R^3 - 3R^2 + 2R + 1} \ln(\tanh(1/2x) - R), R = \text{RootOf}(Z^4 - 2Z^3 + 2Z^2 + 2Z + 1) \right) + \frac{1}{3} \sum \left(\frac{-R^2 - R + 1}{2R^3 + 3R^2 + 2R - 1} \ln(\tanh(1/2x) - R), R = \text{RootOf}(Z^4 + 2Z^3 + 2Z^2 - 2Z + 1) \right) + \frac{1}{6} 2^{1/2} \operatorname{arctanh}\left(\frac{1}{4} (2 \tanh(1/2x) - 2) 2^{1/2}\right) + \frac{1}{6} 2^{1/2} \operatorname{arctanh}\left(\frac{1}{4} (2 \tanh(1/2x) + 2) 2^{1/2}\right)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12} \sqrt{2} \log\left(-\frac{\sqrt{2} - e^x + 1}{\sqrt{2} + e^x - 1}\right) + \frac{1}{12} \sqrt{2} \log\left(-\frac{\sqrt{2} - e^x - 1}{\sqrt{2} + e^x + 1}\right) + \int \frac{e^{(3x)} + 4e^{(2x)} - e^x}{3(e^{(4x)} + 2e^{(3x)} + 2e^{(2x)} - 2e^x + 1)} dx - \int \frac{e^{(3x)}}{3(e^{(4x)} - 2e^{(3x)} + 2e^{(2x)} - 2e^x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^6), x, algorithm="maxima")

[Out] $-\frac{1}{12} \sqrt{2} \log\left(\frac{-\sqrt{2} - e^x + 1}{\sqrt{2} + e^x - 1}\right) + \frac{1}{12} \sqrt{2} \log\left(\frac{-\sqrt{2} - e^x - 1}{\sqrt{2} + e^x + 1}\right) + \int \frac{1}{3} (e^{(3x)} + 4e^{(2x)} - e^x) / (e^{(4x)} + 2e^{(3x)} + 2e^{(2x)} - 2e^x + 1), x - \int \frac{1}{3} (e^{(3x)} - 4e^{(2x)} - e^x) / (e^{(4x)} - 2e^{(3x)} + 2e^{(2x)} + 2e^x + 1), x$

Fricas [B] time = 1.74409, size = 505, normalized size = 6.08

$$-\frac{1}{12} \sqrt{3} \log(16\sqrt{3} + 4e^{(4x)} + 28) + \frac{1}{12} \sqrt{3} \log(-16\sqrt{3} + 4e^{(4x)} + 28) + \frac{1}{12} \sqrt{2} \log\left(\frac{2(2\sqrt{2} - 3)e^{(2x)} - 12\sqrt{2} + e^{(4x)}}{e^{(4x)} - 6e^{(2x)} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^6), x, algorithm="fricas")

[Out] $-\frac{1}{12} \sqrt{3} \log(16\sqrt{3} + 4e^{(4x)} + 28) + \frac{1}{12} \sqrt{3} \log(-16\sqrt{3} + 4e^{(4x)} + 28) + \frac{1}{12} \sqrt{2} \log\left(\frac{2(2\sqrt{2} - 3)e^{(2x)} - 12\sqrt{2} + e^{(4x)}}{e^{(4x)} - 6e^{(2x)} + 1}\right) - \frac{1}{3} \arctan\left(\frac{\sqrt{3} + 2}{e^{(2x)} + 1}\right) + \frac{1}{2} (\sqrt{3} + 2) \sqrt{-16\sqrt{3} + 4e^{(4x)} + 28} + \frac{1}{3} \arctan\left(\frac{-\sqrt{3} - 2}{e^{(2x)} + 1}\right) + \frac{1}{3} \arctan\left(\frac{\sqrt{3} - 2}{e^{(2x)} + 1}\right)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)**6),x)

[Out] Timed out

Giac [B] time = 1.17511, size = 193, normalized size = 2.33

$$-\frac{1}{36} \left((2\sqrt{3}-3)e^{4x} + 2\sqrt{3}-3 \right) \arctan\left(\frac{e^{2x}}{\sqrt{3}+2}\right) + \frac{1}{36} \left((2\sqrt{3}+3)e^{4x} + 2\sqrt{3}+3 \right) \arctan\left(-\frac{e^{2x}}{\sqrt{3}-2}\right) - \frac{1}{12} \sqrt{3} \log$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-sinh(x)^6),x, algorithm="giac")

[Out]
$$-1/36*((2*\text{sqrt}(3) - 3)*e^{4*x} + 2*\text{sqrt}(3) - 3)*\arctan(e^{2*x}/(\text{sqrt}(3) + 2)) + 1/36*((2*\text{sqrt}(3) + 3)*e^{4*x} + 2*\text{sqrt}(3) + 3)*\arctan(-e^{2*x}/(\text{sqrt}(3) - 2)) - 1/12*\text{sqrt}(3)*\log((\text{sqrt}(3) + 2)^2 + e^{4*x}) + 1/12*\text{sqrt}(3)*\log((\text{sqrt}(3) - 2)^2 + e^{4*x}) - 1/12*\text{sqrt}(2)*\log(\text{abs}(-4*\text{sqrt}(2) + 2*e^{2*x}) - 6) / \text{abs}(4*\text{sqrt}(2) + 2*e^{2*x}) - 6))$$

$$3.275 \quad \int \frac{1}{1-\sinh^8(x)} dx$$

Optimal. Leaf size=69

$$\frac{\tanh^{-1}(\sqrt{1-i}\tanh(x))}{4\sqrt{1-i}} + \frac{\tanh^{-1}(\sqrt{1+i}\tanh(x))}{4\sqrt{1+i}} + \frac{\tanh^{-1}(\sqrt{2}\tanh(x))}{4\sqrt{2}} + \frac{\tanh(x)}{4}$$

[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(4*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]/(4*Sqrt[1 + I]) + ArcTanh[Sqrt[2]*Tanh[x]]/(4*Sqrt[2]) + Tanh[x]/4

Rubi [A] time = 0.0772041, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.6$, Rules used = {3211, 3181, 206, 3175, 3767, 8}

$$\frac{\tanh^{-1}(\sqrt{1-i}\tanh(x))}{4\sqrt{1-i}} + \frac{\tanh^{-1}(\sqrt{1+i}\tanh(x))}{4\sqrt{1+i}} + \frac{\tanh^{-1}(\sqrt{2}\tanh(x))}{4\sqrt{2}} + \frac{\tanh(x)}{4}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^8)^(-1), x]

[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(4*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]/(4*Sqrt[1 + I]) + ArcTanh[Sqrt[2]*Tanh[x]]/(4*Sqrt[2]) + Tanh[x]/4

Rule 3211

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(m_), x_Symbol] := Module[{k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b), n/2])), x], {k, 1, n/2}], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]

Rule 3181

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(m_), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(m_), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

Q[a, 0] || LtQ[b, 0])

Rule 3175

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^p, x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^n, x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{1 - \sinh^8(x)} dx &= \frac{1}{4} \int \frac{1}{1 - \sinh^2(x)} dx + \frac{1}{4} \int \frac{1}{1 - i \sinh^2(x)} dx + \frac{1}{4} \int \frac{1}{1 + i \sinh^2(x)} dx + \frac{1}{4} \int \frac{1}{1 + \sinh^2(x)} dx \\ &= \frac{1}{4} \int \operatorname{sech}^2(x) dx + \frac{1}{4} \operatorname{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \tanh(x) \right) + \frac{1}{4} \operatorname{Subst} \left(\int \frac{1}{1 - (1 + i)x^2} dx, x, \tanh(x) \right) \\ &= \frac{\tanh^{-1}(\sqrt{1 - i} \tanh(x))}{4\sqrt{1 - i}} + \frac{\tanh^{-1}(\sqrt{1 + i} \tanh(x))}{4\sqrt{1 + i}} + \frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{4\sqrt{2}} + \frac{1}{4} i \operatorname{Subst} \left(\int 1 dx \right) \\ &= \frac{\tanh^{-1}(\sqrt{1 - i} \tanh(x))}{4\sqrt{1 - i}} + \frac{\tanh^{-1}(\sqrt{1 + i} \tanh(x))}{4\sqrt{1 + i}} + \frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{4\sqrt{2}} + \frac{\tanh(x)}{4} \end{aligned}$$

Mathematica [A] time = 0.482848, size = 64, normalized size = 0.93

$$\frac{1}{8} \left(\frac{2 \tanh^{-1}(\sqrt{1 - i} \tanh(x))}{\sqrt{1 - i}} + \frac{2 \tanh^{-1}(\sqrt{1 + i} \tanh(x))}{\sqrt{1 + i}} + \sqrt{2} \tanh^{-1}(\sqrt{2} \tanh(x)) + 2 \tanh(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sinh[x]^8)^(-1), x]

[Out] $((2*\text{ArcTanh}[\text{Sqrt}[1 - I]*\text{Tanh}[x]])/\text{Sqrt}[1 - I] + (2*\text{ArcTanh}[\text{Sqrt}[1 + I]*\text{Tanh}[x]])/\text{Sqrt}[1 + I] + \text{Sqrt}[2]*\text{ArcTanh}[\text{Sqrt}[2]*\text{Tanh}[x]] + 2*\text{Tanh}[x])/8$

Maple [C] time = 0.033, size = 99, normalized size = 1.4

$$\frac{1}{8} \sum_{_R=\text{RootOf}(2_Z^4-2_Z^2+1)} _R \ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + (-4_R^3 + 4_R) \tanh\left(\frac{x}{2}\right) + 1\right) + \frac{1}{2} \tanh\left(\frac{x}{2}\right) \left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right)^{-1} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1-sinh(x)^8),x)`

[Out] $1/8*\text{sum}(_R*\ln(\tanh(1/2*x)^2+(-4*_R^3+4*_R)*\tanh(1/2*x)+1), _R=\text{RootOf}(2*_Z^4-2*_Z^2+1))+1/2*\tanh(1/2*x)/(\tanh(1/2*x)^2+1)+1/8*2^{(1/2)}*\text{arctanh}(1/4*(2*\tanh(1/2*x)-2)*2^{(1/2)})+1/8*2^{(1/2)}*\text{arctanh}(1/4*(2*\tanh(1/2*x)+2)*2^{(1/2)})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{16} \sqrt{2} \log\left(-\frac{\sqrt{2}-e^x+1}{\sqrt{2}+e^x-1}\right) + \frac{1}{16} \sqrt{2} \log\left(-\frac{\sqrt{2}-e^x-1}{\sqrt{2}+e^x+1}\right) - \frac{1}{2(e^{2x}+1)} + 8 \int \frac{e^{4x}}{e^{8x}-4e^{6x}+22e^{4x}-4e^{2x}+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^8),x, algorithm="maxima")`

[Out] $-1/16*\text{sqrt}(2)*\log(-(\text{sqrt}(2) - e^x + 1)/(\text{sqrt}(2) + e^x - 1)) + 1/16*\text{sqrt}(2)*\log(-(\text{sqrt}(2) - e^x - 1)/(\text{sqrt}(2) + e^x + 1)) - 1/2/(e^{2x} + 1) + 8*\text{integrate}(e^{4x}/(e^{8x} - 4e^{6x} + 22e^{4x} - 4e^{2x} + 1), x)$

Fricas [B] time = 1.85785, size = 2198, normalized size = 31.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1-sinh(x)^8),x, algorithm="fricas")`

```
[Out] -1/32*(4*(2^(1/4)*e^(2*x) + 2^(1/4))*sqrt(-2*sqrt(2) + 4)*arctan(1/14*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) - 1/28*(2*sqrt(2)*(5*sqrt(2) + 6) - (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4) + 16*sqrt(2) + 8)*sqrt(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 1/14*sqrt(2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2) + 4) - 1/7*sqrt(2) + 3/7) + 4*(2^(1/4)*e^(2*x) + 2^(1/4))*sqrt(-2*sqrt(2) + 4)*arctan(-1/14*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) + 1/28*(2*sqrt(2)*(5*sqrt(2) + 6) + (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4) + 16*sqrt(2) + 8)*sqrt((2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) + 1/14*sqrt(2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2) + 4) + 1/7*sqrt(2) - 3/7 - (2^(1/4)*(sqrt(2) + 1)*e^(2*x) + 2^(1/4)*(sqrt(2) + 1))*sqrt(-2*sqrt(2) + 4)*log((2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) + (2^(1/4)*(sqrt(2) + 1)*e^(2*x) + 2^(1/4)*(sqrt(2) + 1))*sqrt(-2*sqrt(2) + 4)*log(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 2*(sqrt(2)*e^(2*x) + sqrt(2))*log((2*(2*sqrt(2) - 3)*e^(2*x) - 12*sqrt(2) + e^(4*x) + 17)/(e^(4*x) - 6*e^(2*x) + 1)) + 16)/(e^(2*x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**8),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38856, size = 65, normalized size = 0.94

$$-\frac{1}{16} \sqrt{2} \log \left(\frac{|-4\sqrt{2} + 2e^{(2x)} - 6|}{|4\sqrt{2} + 2e^{(2x)} - 6|} \right) - \frac{1}{2(e^{(2x)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x) - 6)) - 1/2/(e^(2*x) + 1)
```

$$3.276 \quad \int \frac{\cosh^5(x)}{a+a \sinh^2(x)} dx$$

Optimal. Leaf size=18

$$\frac{\sinh^3(x)}{3a} + \frac{\sinh(x)}{a}$$

[Out] Sinh[x]/a + Sinh[x]^3/(3*a)

Rubi [A] time = 0.0516122, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3175, 2633}

$$\frac{\sinh^3(x)}{3a} + \frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^5/(a + a*Sinh[x]^2),x]

[Out] Sinh[x]/a + Sinh[x]^3/(3*a)

Rule 3175

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^5(x)}{a + a \sinh^2(x)} dx &= \frac{\int \cosh^3(x) dx}{a} \\ &= \frac{i \operatorname{Subst}\left(\int (1-x^2) dx, x, -i \sinh(x)\right)}{a} \\ &= \frac{\sinh(x)}{a} + \frac{\sinh^3(x)}{3a} \end{aligned}$$

Mathematica [A] time = 0.0034921, size = 19, normalized size = 1.06

$$\frac{\frac{3 \sinh(x)}{4} + \frac{1}{12} \sinh(3x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^5/(a + a*Sinh[x]^2),x]

[Out] ((3*Sinh[x])/4 + Sinh[3*x]/12)/a

Maple [B] time = 0.025, size = 67, normalized size = 3.7

$$2 \frac{1}{a} \left(-1/6 (\tanh(x/2) + 1)^{-3} + 1/4 (\tanh(x/2) + 1)^{-2} - 1/2 (\tanh(x/2) + 1)^{-1} - 1/6 (\tanh(x/2) - 1)^{-3} - 1/4 (\tanh(x/2) - 1)^{-2} + 1/2 (\tanh(x/2) - 1)^{-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^5/(a+a*sinh(x)^2),x)

[Out] 2/a*(-1/6/(tanh(1/2*x)+1)^3+1/4/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2*x)+1)-1/6/(tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2-1/2/(tanh(1/2*x)-1))

Maxima [B] time = 1.03111, size = 46, normalized size = 2.56

$$\frac{(9e^{(-2x)} + 1)e^{(3x)}}{24a} - \frac{9e^{(-x)} + e^{(-3x)}}{24a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="maxima")

[Out] 1/24*(9*e^(-2*x) + 1)*e^(3*x)/a - 1/24*(9*e^(-x) + e^(-3*x))/a

Fricas [A] time = 1.47582, size = 65, normalized size = 3.61

$$\frac{\sinh(x)^3 + 3(\cosh(x)^2 + 3)\sinh(x)}{12a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="fricas")

[Out] 1/12*(sinh(x)^3 + 3*(cosh(x)^2 + 3)*sinh(x))/a

Sympy [B] time = 10.0482, size = 124, normalized size = 6.89

$$\frac{6 \tanh^5\left(\frac{x}{2}\right)}{3a \tanh^6\left(\frac{x}{2}\right) - 9a \tanh^4\left(\frac{x}{2}\right) + 9a \tanh^2\left(\frac{x}{2}\right) - 3a} + \frac{4 \tanh^3\left(\frac{x}{2}\right)}{3a \tanh^6\left(\frac{x}{2}\right) - 9a \tanh^4\left(\frac{x}{2}\right) + 9a \tanh^2\left(\frac{x}{2}\right) - 3a} - \frac{3a \tanh^6\left(\frac{x}{2}\right)}{3a \tanh^6\left(\frac{x}{2}\right) - 9a \tanh^4\left(\frac{x}{2}\right) + 9a \tanh^2\left(\frac{x}{2}\right) - 3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**5/(a+a*sinh(x)**2),x)

[Out] -6*tanh(x/2)**5/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**2 - 3*a) + 4*tanh(x/2)**3/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**2 - 3*a) - 6*tanh(x/2)/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**2 - 3*a)

Giac [A] time = 1.14123, size = 39, normalized size = 2.17

$$\frac{(9e^{2x} + 1)e^{-3x} - e^{3x} - 9e^x}{24a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a
```


$$3.277 \quad \int \frac{\cosh^4(x)}{a+a \sinh^2(x)} dx$$

Optimal. Leaf size=20

$$\frac{x}{2a} + \frac{\sinh(x) \cosh(x)}{2a}$$

[Out] x/(2*a) + (Cosh[x]*Sinh[x])/(2*a)

Rubi [A] time = 0.0501591, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3175, 2635, 8}

$$\frac{x}{2a} + \frac{\sinh(x) \cosh(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^4/(a + a*Sinh[x]^2),x]

[Out] x/(2*a) + (Cosh[x]*Sinh[x])/(2*a)

Rule 3175

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p, x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^n, x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{a + a \sinh^2(x)} dx &= \frac{\int \cosh^2(x) dx}{a} \\ &= \frac{\cosh(x) \sinh(x)}{2a} + \frac{\int 1 dx}{2a} \\ &= \frac{x}{2a} + \frac{\cosh(x) \sinh(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0024033, size = 18, normalized size = 0.9

$$\frac{\frac{x}{2} + \frac{1}{4} \sinh(2x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^4/(a + a*Sinh[x]^2),x]

[Out] (x/2 + Sinh[2*x]/4)/a

Maple [B] time = 0.022, size = 78, normalized size = 3.9

$$-\frac{1}{2a} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-2} + \frac{1}{2a} \left(\tanh\left(\frac{x}{2}\right) + 1 \right)^{-1} + \frac{1}{2a} \ln\left(\tanh\left(\frac{x}{2}\right) + 1\right) + \frac{1}{2a} \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^{-2} + \frac{1}{2a} \left(\tanh\left(\frac{x}{2}\right) - 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^4/(a+a*sinh(x)^2),x)

[Out] -1/2/a/(tanh(1/2*x)+1)^2+1/2/a/(tanh(1/2*x)+1)+1/2/a*ln(tanh(1/2*x)+1)+1/2/a/(tanh(1/2*x)-1)^2+1/2/a/(tanh(1/2*x)-1)-1/2/a*ln(tanh(1/2*x)-1)

Maxima [A] time = 1.05272, size = 34, normalized size = 1.7

$$\frac{x}{2a} + \frac{e^{(2x)}}{8a} - \frac{e^{(-2x)}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="maxima")

[Out] 1/2*x/a + 1/8*e^(2*x)/a - 1/8*e^(-2*x)/a

Fricas [A] time = 1.46211, size = 39, normalized size = 1.95

$$\frac{\cosh(x) \sinh(x) + x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="fricas")

[Out] 1/2*(cosh(x)*sinh(x) + x)/a

Sympy [B] time = 6.11654, size = 153, normalized size = 7.65

$$\frac{x \tanh^4\left(\frac{x}{2}\right)}{2a \tanh^4\left(\frac{x}{2}\right) - 4a \tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{2x \tanh^2\left(\frac{x}{2}\right)}{2a \tanh^4\left(\frac{x}{2}\right) - 4a \tanh^2\left(\frac{x}{2}\right) + 2a} + \frac{x}{2a \tanh^4\left(\frac{x}{2}\right) - 4a \tanh^2\left(\frac{x}{2}\right) + 2a} + \frac{1}{2a \tanh^4\left(\frac{x}{2}\right) - 4a \tanh^2\left(\frac{x}{2}\right) + 2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**4/(a+a*sinh(x)**2),x)

[Out] x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 2*x*tanh(x/2)**2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + x/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + 2*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + 2*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)

Giac [A] time = 1.13112, size = 38, normalized size = 1.9

$$\frac{(2e^{2x} + 1)e^{-2x} - 4x - e^{2x}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="giac")

[Out] $-1/8*((2*e^{(2*x)} + 1)*e^{(-2*x)} - 4*x - e^{(2*x)})/a$

$$3.278 \quad \int \frac{\cosh^3(x)}{a+a \sinh^2(x)} dx$$

Optimal. Leaf size=6

$$\frac{\sinh(x)}{a}$$

[Out] Sinh[x]/a

Rubi [A] time = 0.0439533, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3175, 2637}

$$\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^3/(a + a*Sinh[x]^2),x]

[Out] Sinh[x]/a

Rule 3175

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{\cosh^3(x)}{a+a \sinh^2(x)} dx = \frac{\int \cosh(x) dx}{a} = \frac{\sinh(x)}{a}$$

Mathematica [A] time = 0.0016884, size = 6, normalized size = 1.

$$\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^3/(a + a*Sinh[x]^2),x]

[Out] Sinh[x]/a

Maple [A] time = 0.012, size = 7, normalized size = 1.2

$$\frac{\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^3/(a+a*sinh(x)^2),x)

[Out] sinh(x)/a

Maxima [B] time = 1.17094, size = 23, normalized size = 3.83

$$-\frac{e^{(-x)}}{2a} + \frac{e^x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")

[Out] -1/2*e^(-x)/a + 1/2*e^x/a

Fricas [A] time = 1.44102, size = 15, normalized size = 2.5

$$\frac{\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] sinh(x)/a
```

Sympy [B] time = 3.29542, size = 17, normalized size = 2.83

$$-\frac{2 \tanh\left(\frac{x}{2}\right)}{a \tanh^2\left(\frac{x}{2}\right) - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*sinh(x)**2),x)
```

```
[Out] -2*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

Giac [B] time = 1.10998, size = 19, normalized size = 3.17

$$-\frac{e^{(-x)} - e^x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/a
```

$$3.279 \quad \int \frac{\cosh^2(x)}{a + a \sinh^2(x)} dx$$

Optimal. Leaf size=5

$$\frac{x}{a}$$

[Out] x/a

Rubi [A] time = 0.0415225, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {3175, 8}

$$\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^2/(a + a*Sinh[x]^2),x]

[Out] x/a

Rule 3175

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p, x_Symbol] :> Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{\cosh^2(x)}{a + a \sinh^2(x)} dx = \frac{\int 1 dx}{a} = \frac{x}{a}$$

Mathematica [A] time = 0.0003036, size = 5, normalized size = 1.

$$\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^2/(a + a*Sinh[x]^2),x]

[Out] x/a

Maple [C] time = 0.012, size = 11, normalized size = 2.2

$$2 \frac{\text{Artanh}(\tanh(x/2))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^2/(a+a*sinh(x)^2),x)

[Out] 2/a*arctanh(tanh(1/2*x))

Maxima [A] time = 1.11412, size = 7, normalized size = 1.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="maxima")

[Out] x/a

Fricas [A] time = 1.42275, size = 7, normalized size = 1.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] x/a
```

Sympy [A] time = 1.77234, size = 2, normalized size = 0.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*sinh(x)**2),x)
```

```
[Out] x/a
```

Giac [A] time = 1.15493, size = 7, normalized size = 1.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] x/a
```

$$3.280 \quad \int \frac{\cosh(x)}{a+a \sinh^2(x)} dx$$

Optimal. Leaf size=7

$$\frac{\tan^{-1}(\sinh(x))}{a}$$

[Out] ArcTan[Sinh[x]]/a

Rubi [A] time = 0.0276392, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3175, 3770}

$$\frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]/(a + a*Sinh[x]^2),x]

[Out] ArcTan[Sinh[x]]/a

Rule 3175

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^p, x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cosh(x)}{a+a \sinh^2(x)} dx &= \int \frac{\operatorname{sech}(x) dx}{a} \\ &= \frac{\tan^{-1}(\sinh(x))}{a} \end{aligned}$$

Mathematica [A] time = 0.0027435, size = 12, normalized size = 1.71

$$\frac{2 \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]/(a + a*Sinh[x]^2),x]

[Out] (2*ArcTan[Tanh[x/2]])/a

Maple [A] time = 0.008, size = 8, normalized size = 1.1

$$\frac{\arctan(\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)/(a+a*sinh(x)^2),x)

[Out] arctan(sinh(x))/a

Maxima [A] time = 1.57621, size = 14, normalized size = 2.

$$-\frac{2 \arctan\left(e^{-x}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="maxima")

[Out] -2*arctan(e^(-x))/a

Fricas [A] time = 1.44555, size = 42, normalized size = 6.

$$\frac{2 \arctan(\cosh(x) + \sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="fricas")`

[Out] `2*arctan(cosh(x) + sinh(x))/a`

Sympy [A] time = 0.250555, size = 5, normalized size = 0.71

$$\frac{\operatorname{atan}(\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)/(a+a*sinh(x)**2),x)`

[Out] `atan(sinh(x))/a`

Giac [A] time = 1.10845, size = 11, normalized size = 1.57

$$\frac{2 \operatorname{arctan}(e^x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="giac")`

[Out] `2*arctan(e^x)/a`

$$3.281 \quad \int \frac{\operatorname{sech}(x)}{a+a \sinh^2(x)} dx$$

Optimal. Leaf size=22

$$\frac{\tan^{-1}(\sinh(x))}{2a} + \frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

[Out] ArcTan[Sinh[x]]/(2*a) + (Sech[x]*Tanh[x])/(2*a)

Rubi [A] time = 0.0454984, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3175, 3768, 3770}

$$\frac{\tan^{-1}(\sinh(x))}{2a} + \frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]/(a + a*Sinh[x]^2),x]

[Out] ArcTan[Sinh[x]]/(2*a) + (Sech[x]*Tanh[x])/(2*a)

Rule 3175

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] *(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}(x)}{a + a \sinh^2(x)} dx &= \frac{\int \operatorname{sech}^3(x) dx}{a} \\ &= \frac{\operatorname{sech}(x) \tanh(x)}{2a} + \frac{\int \operatorname{sech}(x) dx}{2a} \\ &= \frac{\tan^{-1}(\sinh(x))}{2a} + \frac{\operatorname{sech}(x) \tanh(x)}{2a} \end{aligned}$$

Mathematica [A] time = 0.0037607, size = 20, normalized size = 0.91

$$\frac{\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{2} \tanh(x) \operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]/(a + a*Sinh[x]^2),x]

[Out] (ArcTan[Tanh[x/2]] + (Sech[x]*Tanh[x])/2)/a

Maple [B] time = 0.021, size = 50, normalized size = 2.3

$$-\frac{1}{a} \left(\tanh\left(\frac{x}{2}\right)\right)^3 \left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right)^{-2} + \frac{1}{a} \tanh\left(\frac{x}{2}\right) \left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right)^{-2} + \frac{1}{a} \arctan\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)/(a+a*sinh(x)^2),x)

[Out] -1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3+1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)+1/a*arctan(tanh(1/2*x))

Maxima [B] time = 1.52255, size = 54, normalized size = 2.45

$$\frac{e^{(-x)} - e^{(-3x)}}{2ae^{(-2x)} + ae^{(-4x)} + a} - \frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="maxima")

[Out] (e^{-x} - e^{-3x})/(2*a*e^{-2x} + a*e^{-4x} + a) - arctan(e^{-x})/a

Fricas [B] time = 1.48705, size = 522, normalized size = 23.73

$$\frac{\cosh(x)^3 + 3 \cosh(x) \sinh(x)^2 + \sinh(x)^3 + (\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 2(3 \cosh(x)^2 + 1) \sinh(x))}{a \cosh(x)^4 + 4 a \cosh(x) \sinh(x)^3 + a \sinh(x)^4 + 2 a \cosh(x)^2 + 2 a \sinh(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="fricas")

[Out] (cosh(x)³ + 3*cosh(x)*sinh(x)² + sinh(x)³ + (cosh(x)⁴ + 4*cosh(x)*sinh(x)³ + sinh(x)⁴ + 2*(3*cosh(x)² + 1)*sinh(x)² + 2*cosh(x)² + 4*(cosh(x)³ + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)² - 1)*sinh(x) - cosh(x))/(a*cosh(x)⁴ + 4*a*cosh(x)*sinh(x)³ + a*sinh(x)⁴ + 2*a*cosh(x)² + 2*(3*a*cosh(x)² + a)*sinh(x)² + 4*(a*cosh(x)³ + a*cosh(x))*sinh(x) + a)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}(x)}{\sinh^2(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)/(a+a*sinh(x)**2),x)

[Out] Integral(sech(x)/(sinh(x)**2 + 1), x)/a

Giac [B] time = 1.13744, size = 70, normalized size = 3.18

$$\frac{\pi + 2 \arctan\left(\frac{1}{2}(e^{2x} - 1)e^{-x}\right)}{4a} - \frac{e^{-x} - e^x}{\left((e^{-x} - e^x)^2 + 4\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - (e^(-x) - e^x)/(((e^(-x) - e^x)^2 + 4)*a)
```

$$3.282 \quad \int \frac{\operatorname{sech}^3(x)}{a+a \sinh^2(x)} dx$$

Optimal. Leaf size=35

$$\frac{3 \tan^{-1}(\sinh(x))}{8a} + \frac{\tanh(x)\operatorname{sech}^3(x)}{4a} + \frac{3 \tanh(x)\operatorname{sech}(x)}{8a}$$

[Out] (3*ArcTan[Sinh[x]])/(8*a) + (3*Sech[x]*Tanh[x])/(8*a) + (Sech[x]^3*Tanh[x])/(4*a)

Rubi [A] time = 0.0598158, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3175, 3768, 3770}

$$\frac{3 \tan^{-1}(\sinh(x))}{8a} + \frac{\tanh(x)\operatorname{sech}^3(x)}{4a} + \frac{3 \tanh(x)\operatorname{sech}(x)}{8a}$$

Antiderivative was successfully verified.

[In] Int[Sech[x]^3/(a + a*Sinh[x]^2),x]

[Out] (3*ArcTan[Sinh[x]])/(8*a) + (3*Sech[x]*Tanh[x])/(8*a) + (Sech[x]^3*Tanh[x])/(4*a)

Rule 3175

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p, x_Symbol] := Dist[a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0] && IntegerQ[p]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^n, x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(x)}{a + a \sinh^2(x)} dx &= \frac{\int \operatorname{sech}^5(x) dx}{a} \\
&= \frac{\operatorname{sech}^3(x) \tanh(x)}{4a} + \frac{3 \int \operatorname{sech}^3(x) dx}{4a} \\
&= \frac{3 \operatorname{sech}(x) \tanh(x)}{8a} + \frac{\operatorname{sech}^3(x) \tanh(x)}{4a} + \frac{3 \int \operatorname{sech}(x) dx}{8a} \\
&= \frac{3 \tan^{-1}(\sinh(x))}{8a} + \frac{3 \operatorname{sech}(x) \tanh(x)}{8a} + \frac{\operatorname{sech}^3(x) \tanh(x)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0044584, size = 34, normalized size = 0.97

$$\frac{\frac{3}{4} \tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{4} \tanh(x) \operatorname{sech}^3(x) + \frac{3}{8} \tanh(x) \operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[x]^3/(a + a*Sinh[x]^2),x]

[Out] ((3*ArcTan[Tanh[x/2]])/4 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4)/a

Maple [B] time = 0.026, size = 94, normalized size = 2.7

$$-\frac{5}{4a} \left(\tanh\left(\frac{x}{2}\right)\right)^7 \left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right)^{-4} + \frac{3}{4a} \left(\tanh\left(\frac{x}{2}\right)\right)^5 \left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right)^{-4} - \frac{3}{4a} \left(\tanh\left(\frac{x}{2}\right)\right)^3 \left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + 1\right)^{-4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(x)^3/(a+a*sinh(x)^2),x)

[Out] -5/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7+3/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^5-3/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3+5/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)+3/4/a*arctan(tanh(1/2*x))

Maxima [B] time = 1.51705, size = 93, normalized size = 2.66

$$\frac{3e^{-x} + 11e^{-3x} - 11e^{-5x} - 3e^{-7x}}{4(4ae^{-2x} + 6ae^{-4x} + 4ae^{-6x} + ae^{-8x} + a)} - \frac{3 \arctan(e^{-x})}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")

[Out] 1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) - 3/4*arctan(e^(-x))/a

Fricas [B] time = 1.49142, size = 1607, normalized size = 45.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")

[Out] 1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11)*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (105*cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5 + 110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x) - 3*cosh(x))/(a*cosh(x)^8 + 8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*cosh(x)^6 + 4*(7*a*cosh(x)^2 + a)*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^5 + 6*a*cosh(x)^4 + 2*(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*sinh(x)^4 + 8*(7*a*cosh(x)^5 + 10*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 4*a*cosh(x)^2 + 4*(7*a*cosh(x)^6 + 15*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*sinh(x)^2 + 8*(a*cosh(x)^7 + 3*a*cosh(x)^5 + 3*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(x)}{\sinh^2(x)+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)**3/(a+a*sinh(x)**2),x)

[Out] Integral(sech(x)**3/(sinh(x)**2 + 1), x)/a

Giac [B] time = 1.15806, size = 90, normalized size = 2.57

$$\frac{3 \left(\pi + 2 \arctan \left(\frac{1}{2} (e^{2x} - 1) e^{-x} \right) \right)}{16 a} - \frac{3 (e^{-x} - e^x)^3 + 20 e^{-x} - 20 e^x}{4 \left((e^{-x} - e^x)^2 + 4 \right)^2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")

[Out] 3/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - 1/4*(3*(e^(-x) - e^x)^3 + 20*e^(-x) - 20*e^x)/(((e^(-x) - e^x)^2 + 4)^2*a)

3.283 $\int \cosh^4(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=89

$$\frac{(6a - b) \sinh(c + dx) \cosh^3(c + dx)}{24d} + \frac{(6a - b) \sinh(c + dx) \cosh(c + dx)}{16d} + \frac{1}{16}x(6a - b) + \frac{b \sinh(c + dx) \cosh^5(c + dx)}{6d}$$

[Out] $((6*a - b)*x)/16 + ((6*a - b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(16*d) + ((6*a - b)*\text{Cosh}[c + d*x]^3*\text{Sinh}[c + d*x])/(24*d) + (b*\text{Cosh}[c + d*x]^5*\text{Sinh}[c + d*x])/ (6*d)$

Rubi [A] time = 0.0571499, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3191, 385, 199, 206}

$$\frac{(6a - b) \sinh(c + dx) \cosh^3(c + dx)}{24d} + \frac{(6a - b) \sinh(c + dx) \cosh(c + dx)}{16d} + \frac{1}{16}x(6a - b) + \frac{b \sinh(c + dx) \cosh^5(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cosh}[c + d*x]^4*(a + b*\text{Sinh}[c + d*x]^2), x]$

[Out] $((6*a - b)*x)/16 + ((6*a - b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(16*d) + ((6*a - b)*\text{Cosh}[c + d*x]^3*\text{Sinh}[c + d*x])/(24*d) + (b*\text{Cosh}[c + d*x]^5*\text{Sinh}[c + d*x])/ (6*d)$

Rule 3191

$\text{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^{(m/2 + p + 1)}, x], x, \text{Tan}[e + f*x]/ff], x]] /;$ $\text{FreeQ}\{a, b, e, f\}, x\} \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ \text{IntegerQ}[p]$

Rule 385

$\text{Int}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow -\text{Simp}[(b*c - a*d)*x*(a + b*x^n)^{(p + 1)}/(a*b*n*(p + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), \text{Int}[(a + b*x^n)^{(p + 1)}, x], x] /;$ $\text{FreeQ}\{a, b, c, d, n, p\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ (\text{LtQ}[p, -1] \ || \ \text{ILtQ}[1/n + p, 0])$

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \cosh^4(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int \frac{a - (a-b)x^2}{(1-x^2)^4} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{b \cosh^5(c + dx) \sinh(c + dx)}{6d} + \frac{(6a - b) \text{Subst}\left(\int \frac{1}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{6d} \\
 &= \frac{(6a - b) \cosh^3(c + dx) \sinh(c + dx)}{24d} + \frac{b \cosh^5(c + dx) \sinh(c + dx)}{6d} + \frac{(6a - b) \cosh^3(c + dx) \sinh(c + dx)}{24d} \\
 &= \frac{(6a - b) \cosh(c + dx) \sinh(c + dx)}{16d} + \frac{(6a - b) \cosh^3(c + dx) \sinh(c + dx)}{24d} \\
 &= \frac{1}{16}(6a - b)x + \frac{(6a - b) \cosh(c + dx) \sinh(c + dx)}{16d} + \frac{(6a - b) \cosh^3(c + dx) \sinh(c + dx)}{24d}
 \end{aligned}$$

Mathematica [A] time = 0.168358, size = 63, normalized size = 0.71

$$\frac{(48a - 3b) \sinh(2(c + dx)) + 3(2a + b) \sinh(4(c + dx)) + 72ac + 72adx + b \sinh(6(c + dx)) - 12bdx}{192d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2), x]

[Out] (72*a*c + 72*a*d*x - 12*b*d*x + (48*a - 3*b)*Sinh[2*(c + d*x)] + 3*(2*a + b)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)

Maple [A] time = 0.059, size = 95, normalized size = 1.1

$$\frac{1}{d} \left(b \left(\frac{\sinh(dx+c) (\cosh(dx+c))^5}{6} - \frac{\sinh(dx+c) \left(\frac{(\cosh(dx+c))^3}{4} + \frac{3 \cosh(dx+c)}{8} \right)}{6} - \frac{dx}{16} - \frac{c}{16} \right) + a \left(\frac{(\cosh(dx+c))^3}{4} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)`

[Out] `1/d*(b*(1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)-1/16*d*x-1/16*c)+a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)`

Maxima [A] time = 1.01327, size = 205, normalized size = 2.3

$$\frac{1}{64} a \left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) + \frac{1}{384} b \left(\frac{(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + 1)e^{(6dx+6c)}}{d} - \frac{24(dx+c)}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")`

[Out] `1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/384*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d)`

Fricas [A] time = 1.48793, size = 306, normalized size = 3.44

$$\frac{3b \cosh(dx+c) \sinh(dx+c)^5 + 2(5b \cosh(dx+c)^3 + 3(2a+b) \cosh(dx+c)) \sinh(dx+c)^3 + 6(6a-b)dx + 3(b \cosh(dx+c)^3 + 3(2a+b) \cosh(dx+c)) \sinh(dx+c)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")`

[Out] `1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - b)*d*x + 3*(b*cosh(d*x + c)^3 + 3*(2*a + b)*cosh(d*x + c))*sinh(d*x + c)`

$$+ 2*(2*a + b)*\cosh(d*x + c)^3 + (16*a - b)*\cosh(d*x + c))*\sinh(d*x + c))/d$$

Sympy [A] time = 4.28559, size = 250, normalized size = 2.81

$$\left\{ \begin{array}{l} \frac{3ax \sinh^4(c+dx)}{8} - \frac{3ax \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3ax \cosh^4(c+dx)}{8} - \frac{3a \sinh^3(c+dx) \cosh(c+dx)}{8d} + \frac{5a \sinh(c+dx) \cosh^3(c+dx)}{8d} + \frac{bx \sinh^6(c+dx)}{16} \\ x(a + b \sinh^2(c)) \cosh^4(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2), x)

[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*a*x*cosh(c + d*x)**4/8 - 3*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 5*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*x*sinh(c + d*x)**6/16 - 3*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 3*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - b*x*cosh(c + d*x)**6/16 - b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**4, True))

Giac [B] time = 1.20001, size = 221, normalized size = 2.48

$$\frac{24(dx + c)(6a - b) + be^{(6dx+6c)} + 6ae^{(4dx+4c)} + 3be^{(4dx+4c)} + 48ae^{(2dx+2c)} - 3be^{(2dx+2c)} - (132ae^{(6dx+6c)} - 22be^{(6dx+6c)})}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2), x, algorithm="giac")

[Out] 1/384*(24*(d*x + c)*(6*a - b) + b*e^(6*d*x + 6*c) + 6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) + 48*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) - (132*a*e^(6*d*x + 6*c) - 22*b*e^(6*d*x + 6*c) + 48*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + b)*e^(-6*d*x - 6*c))/d

3.284 $\int \cosh^3(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=46

$$\frac{(a + b) \sinh^3(c + dx)}{3d} + \frac{a \sinh(c + dx)}{d} + \frac{b \sinh^5(c + dx)}{5d}$$

[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^5)/(5*d)

Rubi [A] time = 0.0416845, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3190, 373}

$$\frac{(a + b) \sinh^3(c + dx)}{3d} + \frac{a \sinh(c + dx)}{d} + \frac{b \sinh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]

[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^5)/(5*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 373

Int[((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cosh^3(c+dx) (a+b \sinh^2(c+dx)) dx &= \frac{\text{Subst}\left(\int (1+x^2)(a+bx^2) dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int (a+(a+b)x^2+bx^4) dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{a \sinh(c+dx)}{d} + \frac{(a+b) \sinh^3(c+dx)}{3d} + \frac{b \sinh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.139109, size = 48, normalized size = 1.04

$$\frac{\sinh(c+dx)(4(5a+2b) \cosh(2(c+dx)) + 100a + 3b \cosh(4(c+dx)) - 11b)}{120d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2), x]

[Out] ((100*a - 11*b + 4*(5*a + 2*b)*Cosh[2*(c + d*x)] + 3*b*Cosh[4*(c + d*x)])*Sinh[c + d*x])/(120*d)

Maple [A] time = 0.053, size = 65, normalized size = 1.4

$$\frac{1}{d} \left(b \left(\frac{\sinh(dx+c) (\cosh(dx+c))^4}{5} - \frac{\sinh(dx+c) \left(\frac{2}{3} + \frac{(\cosh(dx+c))^2}{3} \right)}{5} \right) + a \left(\frac{2}{3} + \frac{(\cosh(dx+c))^2}{3} \right) \sinh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(b*(1/5*sinh(d*x+c)*cosh(d*x+c)^4-1/5*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))

Maxima [B] time = 1.01288, size = 184, normalized size = 4.

$$\frac{1}{480} b \left(\frac{(5e^{-2dx-2c} - 30e^{-4dx-4c} + 3)e^{5dx+5c}}{d} + \frac{30e^{-dx-c} - 5e^{-3dx-3c} - 3e^{-5dx-5c}}{d} \right) + \frac{1}{24} a \left(\frac{e^{3dx+3c}}{d} + \frac{9e^{dx+c}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{1}{480}b\left(\frac{5e^{-2dx-2c}-30e^{-4dx-4c}+3}{d}e^{5dx+5c}+\frac{30e^{-dx-c}-5e^{-3dx-3c}-3e^{-5dx-5c}}{d}\right)+\frac{1}{24}a\left(\frac{e^{3dx+3c}}{d}+9\frac{e^{dx+c}}{d}-9\frac{e^{-dx-c}}{d}-\frac{e^{-3dx-3c}}{d}\right)$

Fricas [A] time = 1.48221, size = 220, normalized size = 4.78

$$\frac{3b \sinh(dx+c)^5 + 5(6b \cosh(dx+c)^2 + 4a+b) \sinh(dx+c)^3 + 15(b \cosh(dx+c)^4 + (4a+b) \cosh(dx+c)^2 + 12a)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $\frac{1}{240}\left(3b \sinh(dx+c)^5 + 5(6b \cosh(dx+c)^2 + 4a+b) \sinh(dx+c)^3 + 15(b \cosh(dx+c)^4 + (4a+b) \cosh(dx+c)^2 + 12a - 2b) \sinh(dx+c)\right)/d$

Sympy [A] time = 2.04711, size = 85, normalized size = 1.85

$$\begin{cases} -\frac{2a \sinh^3(c+dx)}{3d} + \frac{a \sinh(c+dx) \cosh^2(c+dx)}{d} - \frac{2b \sinh^5(c+dx)}{15d} + \frac{b \sinh^3(c+dx) \cosh^2(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a + b \sinh^2(c)) \cosh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)

[Out] Piecewise((-2*a*sinh(c + d*x)**3/(3*d) + a*sinh(c + d*x)*cosh(c + d*x)**2/d - 2*b*sinh(c + d*x)**5/(15*d) + b*sinh(c + d*x)**3*cosh(c + d*x)**2/(3*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**3, True))

Giac [B] time = 1.17355, size = 166, normalized size = 3.61

$$\frac{3be^{5dx+5c} + 20ae^{3dx+3c} + 5be^{3dx+3c} + 180ae^{dx+c} - 30be^{dx+c} - (180ae^{4dx+4c} - 30be^{4dx+4c} + 20ae^{2dx+2c} + 5b)}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*b*e^(5*d*x + 5*c) + 20*a*e^(3*d*x + 3*c) + 5*b*e^(3*d*x + 3*c) + 180*a*e^(d*x + c) - 30*b*e^(d*x + c) - (180*a*e^(4*d*x + 4*c) - 30*b*e^(4*d*x + 4*c) + 20*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 3*b)*e^(-5*d*x - 5*c))/d
```

3.285 $\int \cosh^2(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=61

$$\frac{(4a - b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{1}{8}x(4a - b) + \frac{b \sinh(c + dx) \cosh^3(c + dx)}{4d}$$

[Out] $((4*a - b)*x)/8 + ((4*a - b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(8*d) + (b*\text{Cosh}[c + d*x]^3*\text{Sinh}[c + d*x])/(4*d)$

Rubi [A] time = 0.0488858, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3191, 385, 199, 206}

$$\frac{(4a - b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{1}{8}x(4a - b) + \frac{b \sinh(c + dx) \cosh^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cosh}[c + d*x]^2*(a + b*\text{Sinh}[c + d*x]^2), x]$

[Out] $((4*a - b)*x)/8 + ((4*a - b)*\text{Cosh}[c + d*x]*\text{Sinh}[c + d*x])/(8*d) + (b*\text{Cosh}[c + d*x]^3*\text{Sinh}[c + d*x])/(4*d)$

Rule 3191

$\text{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \text{ :> With}\{ff = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^{(m/2 + p + 1)}, x], x, \text{Tan}[e + f*x]/ff], x]] \text{ /; FreeQ}\{a, b, e, f\}, x\} \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[p]$

Rule 385

$\text{Int}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \text{ :> -Simp}[(b*c - a*d)*x*(a + b*x^n)^{(p + 1)}/(a*b*n*(p + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), \text{Int}[(a + b*x^n)^{(p + 1)}, x], x] \text{ /; FreeQ}\{a, b, c, d, n, p\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& (\text{LtQ}[p, -1] \text{ || ILtQ}[1/n + p, 0])$

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)
)/ (a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \cosh^2(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{\text{Subst} \left(\int \frac{a - (a-b)x^2}{(1-x^2)^3} dx, x, \tanh(c + dx) \right)}{d} \\ &= \frac{b \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{(4a - b) \text{Subst} \left(\int \frac{1}{(1-x^2)^2} dx, x, \tanh(c + dx) \right)}{4d} \\ &= \frac{(4a - b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{(4a - b)}{8d} \\ &= \frac{1}{8}(4a - b)x + \frac{(4a - b) \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b \cosh^3(c + dx) \sinh(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.0695163, size = 43, normalized size = 0.7

$$\frac{8a \sinh(2(c + dx)) + 16ac + 16adx + b \sinh(4(c + dx)) - 4bdx}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2), x]
```

```
[Out] (16*a*c + 16*a*d*x - 4*b*d*x + 8*a*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)])
/(32*d)
```

Maple [A] time = 0.026, size = 70, normalized size = 1.2

$$\frac{1}{d} \left(b \left(\frac{\sinh(dx+c) \cosh(dx+c)^3}{4} - \frac{\cosh(dx+c) \sinh(dx+c)}{8} - \frac{dx}{8} - \frac{c}{8} \right) + a \left(\frac{\cosh(dx+c) \sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)

[Out] 1/d*(b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+a*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c))

Maxima [A] time = 1.00693, size = 103, normalized size = 1.69

$$\frac{1}{8} a \left(4x + \frac{e^{2dx+2c}}{d} - \frac{e^{-2dx-2c}}{d} \right) - \frac{1}{64} b \left(\frac{8(dx+c)}{d} - \frac{e^{4dx+4c}}{d} + \frac{e^{-4dx-4c}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/64*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)

Fricas [A] time = 1.45926, size = 153, normalized size = 2.51

$$\frac{b \cosh(dx+c) \sinh(dx+c)^3 + (4a-b)dx + (b \cosh(dx+c)^3 + 4a \cosh(dx+c)) \sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a - b)*d*x + (b*cosh(d*x + c)^3 + 4*a*cosh(d*x + c))*sinh(d*x + c))/d

Sympy [A] time = 1.14033, size = 150, normalized size = 2.46

$$\left\{ \begin{array}{l} -\frac{ax \sinh^2(c+dx)}{2} + \frac{ax \cosh^2(c+dx)}{2} + \frac{a \sinh(c+dx) \cosh(c+dx)}{2d} - \frac{bx \sinh^4(c+dx)}{8} + \frac{bx \sinh^2(c+dx) \cosh^2(c+dx)}{4} - \frac{bx \cosh^4(c+dx)}{8} + \frac{b \sinh^3(c+dx)}{8} \\ x(a + b \sinh^2(c)) \cosh^2(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)

[Out] Piecewise((-a*x*sinh(c + d*x)**2/2 + a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x)*cosh(c + d*x)/(2*d) - b*x*sinh(c + d*x)**4/8 + b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - b*x*cosh(c + d*x)**4/8 + b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**2, True))

Giac [A] time = 1.12573, size = 124, normalized size = 2.03

$$\frac{8(dx+c)(4a-b) + be^{4dx+4c} + 8ae^{2dx+2c} - (24ae^{4dx+4c} - 6be^{4dx+4c} + 8ae^{2dx+2c} + b)e^{-4dx-4c}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] 1/64*(8*(d*x + c)*(4*a - b) + b*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) - (24*a*e^(4*d*x + 4*c) - 6*b*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) + b)*e^(-4*d*x - 4*c))/d

3.286 $\int \cosh(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=28

$$\frac{a \sinh(c + dx)}{d} + \frac{b \sinh^3(c + dx)}{3d}$$

[Out] (a*Sinh[c + d*x])/d + (b*Sinh[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0216874, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3190}

$$\frac{a \sinh(c + dx)}{d} + \frac{b \sinh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]

[Out] (a*Sinh[c + d*x])/d + (b*Sinh[c + d*x]^3)/(3*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rubi steps

$$\begin{aligned} \int \cosh(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{\text{Subst}\left(\int (a + bx^2) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{a \sinh(c + dx)}{d} + \frac{b \sinh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0117645, size = 39, normalized size = 1.39

$$\frac{a \sinh(c) \cosh(dx)}{d} + \frac{a \cosh(c) \sinh(dx)}{d} + \frac{b \sinh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]

[Out] (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d + (b*Sinh[c + d*x]^3)/(3*d)

Maple [A] time = 0.01, size = 25, normalized size = 0.9

$$\frac{1}{d} \left(\frac{b (\sinh(dx + c))^3}{3} + a \sinh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x)

[Out] 1/d*(1/3*b*sinh(d*x+c)^3+a*sinh(d*x+c))

Maxima [A] time = 1.0335, size = 35, normalized size = 1.25

$$\frac{b \sinh(dx + c)^3}{3d} + \frac{a \sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/3*b*sinh(d*x + c)^3/d + a*sinh(d*x + c)/d

Fricas [A] time = 1.4489, size = 103, normalized size = 3.68

$$\frac{b \sinh(dx + c)^3 + 3(b \cosh(dx + c)^2 + 4a - b) \sinh(dx + c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $1/12*(b*\sinh(d*x + c)^3 + 3*(b*\cosh(d*x + c)^2 + 4*a - b)*\sinh(d*x + c))/d$

Sympy [A] time = 0.487066, size = 36, normalized size = 1.29

$$\begin{cases} \frac{a \sinh(c+dx)}{d} + \frac{b \sinh^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a + b \sinh^2(c)) \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2),x)`

[Out] `Piecewise((a*sinh(c + d*x)/d + b*sinh(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c), True))`

Giac [B] time = 1.13263, size = 97, normalized size = 3.46

$$\frac{be^{(3dx+3c)} + 12ae^{(dx+c)} - 3be^{(dx+c)} - (12ae^{(2dx+2c)} - 3be^{(2dx+2c)} + b)e^{(-3dx-3c)}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")`

[Out] $1/24*(b*e^{(3*d*x + 3*c)} + 12*a*e^{(d*x + c)} - 3*b*e^{(d*x + c)} - (12*a*e^{(2*d*x + 2*c)} - 3*b*e^{(2*d*x + 2*c)} + b)*e^{(-3*d*x - 3*c)})/d$

3.287 $\int \operatorname{sech}(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=28

$$\frac{(a - b) \tan^{-1}(\sinh(c + dx))}{d} + \frac{b \sinh(c + dx)}{d}$$

[Out] ((a - b)*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x])/d

Rubi [A] time = 0.0319274, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3190, 388, 203}

$$\frac{(a - b) \tan^{-1}(\sinh(c + dx))}{d} + \frac{b \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2),x]

[Out] ((a - b)*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x])/d

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 388

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}(c+dx)(a+b\sinh^2(c+dx))dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{1+x^2}dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{b\sinh(c+dx)}{d} + \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{1+x^2}dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{(a-b)\tan^{-1}(\sinh(c+dx))}{d} + \frac{b\sinh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.013777, size = 37, normalized size = 1.32

$$\frac{a \tan^{-1}(\sinh(c+dx))}{d} + \frac{b \sinh(c+dx)}{d} - \frac{b \tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2), x]

[Out] (a*ArcTan[Sinh[c + d*x]])/d - (b*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x])/d

Maple [A] time = 0.028, size = 39, normalized size = 1.4

$$\frac{b \sinh(dx+c)}{d} + 2 \frac{a \arctan(e^{dx+c})}{d} - 2 \frac{b \arctan(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2), x)

[Out] b*sinh(d*x+c)/d+2/d*a*arctan(exp(d*x+c))-2/d*b*arctan(exp(d*x+c))

Maxima [A] time = 1.51922, size = 76, normalized size = 2.71

$$\frac{1}{2}b\left(\frac{4 \arctan(e^{-dx-c})}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) + \frac{a \arctan(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{1}{2}b(4\arctan(e^{-dx-c})/d + e^{dx+c}/d - e^{-dx-c}/d) + a\arctan(\sinh(dx+c))/d$

Fricas [B] time = 1.51464, size = 282, normalized size = 10.07

$$\frac{b \cosh(dx+c)^2 + 2b \cosh(dx+c) \sinh(dx+c) + b \sinh(dx+c)^2 + 4((a-b) \cosh(dx+c) + (a-b) \sinh(dx+c)) \arctan(\cosh(dx+c) + \sinh(dx+c))}{2(d \cosh(dx+c) + d \sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $\frac{1}{2}(b \cosh(dx+c)^2 + 2b \cosh(dx+c) \sinh(dx+c) + b \sinh(dx+c)^2 + 4((a-b) \cosh(dx+c) + (a-b) \sinh(dx+c)) \arctan(\cosh(dx+c) + \sinh(dx+c)) - b)/(d \cosh(dx+c) + d \sinh(dx+c))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sinh^2(c + dx)) \operatorname{sech}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2),x)

[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x), x)

Giac [A] time = 1.12814, size = 61, normalized size = 2.18

$$\frac{2(a-b) \arctan(e^{dx+c})}{d} + \frac{be^{dx+c}}{2d} - \frac{be^{-dx-c}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2*(a - b)*arctan(e^(d*x + c))/d + 1/2*b*e^(d*x + c)/d - 1/2*b*e^(-d*x - c)/  
d
```


3.288 $\int \operatorname{sech}^2(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=19

$$\frac{(a - b) \tanh(c + dx)}{d} + bx$$

[Out] b*x + ((a - b)*Tanh[c + d*x])/d

Rubi [A] time = 0.0340843, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3191, 388, 206}

$$\frac{(a - b) \tanh(c + dx)}{d} + bx$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2), x]

[Out] b*x + ((a - b)*Tanh[c + d*x])/d

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 388

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 206

Int[((a_.) + (b_.)*(x_.)^(2))^(p_.), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^2(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a - (a-b)x^2}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{(a-b) \tanh(c + dx)}{d} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\
&= bx + \frac{(a-b) \tanh(c + dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0138384, size = 36, normalized size = 1.89

$$\frac{a \tanh(c + dx)}{d} + \frac{b \tanh^{-1}(\tanh(c + dx))}{d} - \frac{b \tanh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2), x]

[Out] (b*ArcTanh[Tanh[c + d*x]])/d + (a*Tanh[c + d*x])/d - (b*Tanh[c + d*x])/d

Maple [A] time = 0.056, size = 29, normalized size = 1.5

$$\frac{\tanh(dx + c) a + b(dx + c - \tanh(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(tanh(d*x+c)*a+b*(d*x+c-tanh(d*x+c)))

Maxima [B] time = 1.04978, size = 63, normalized size = 3.32

$$b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)}\right) + \frac{2a}{d(e^{(-2dx-2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")`

[Out] `b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 2*a/(d*(e^(-2*d*x - 2*c) + 1))`

Fricas [B] time = 1.51005, size = 101, normalized size = 5.32

$$\frac{(bdx - a + b) \cosh(dx + c) + (a - b) \sinh(dx + c)}{d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")`

[Out] `((b*d*x - a + b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))/(d*cosh(d*x + c))`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sinh^2(c + dx)) \operatorname{sech}^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)`

[Out] `Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**2, x)`

Giac [A] time = 1.15137, size = 46, normalized size = 2.42

$$\frac{(dx + c)b}{d} - \frac{2(a - b)}{d(e^{2dx+2c} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")`

[Out] `(d*x + c)*b/d - 2*(a - b)/(d*(e^(2*d*x + 2*c) + 1))`

3.289 $\int \operatorname{sech}^3(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=42

$$\frac{(a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(a - b) \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

[Out] ((a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - b)*Sech[c + d*x]*Tanh[c + d*x])/ (2*d)

Rubi [A] time = 0.0395048, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3190, 385, 203}

$$\frac{(a + b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(a - b) \tanh(c + dx) \operatorname{sech}(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2), x]

[Out] ((a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - b)*Sech[c + d*x]*Tanh[c + d*x])/ (2*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 385

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^3(c+dx)(a+b\sinh^2(c+dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx)\right)}{2d} \\ &= \frac{(a+b)\tan^{-1}(\sinh(c+dx))}{2d} + \frac{(a-b)\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.0229631, size = 71, normalized size = 1.69

$$\frac{a \tan^{-1}(\sinh(c+dx))}{2d} + \frac{a \tanh(c+dx)\operatorname{sech}(c+dx)}{2d} + \frac{b \tan^{-1}(\sinh(c+dx))}{2d} - \frac{b \tanh(c+dx)\operatorname{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2), x]

[Out] (a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Sech[c + d*x]*Tanh[c + d*x])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)

Maple [B] time = 0.069, size = 82, normalized size = 2.

$$\frac{a \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{a \arctan(e^{dx+c})}{d} - \frac{b \sinh(dx+c)}{d (\cosh(dx+c))^2} + \frac{b \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{b \arctan(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2), x)

[Out] 1/2/d*a*sech(d*x+c)*tanh(d*x+c)+1/d*a*arctan(exp(d*x+c))-1/d*b*sinh(d*x+c)/cosh(d*x+c)^2+1/2/d*b*sech(d*x+c)*tanh(d*x+c)+1/d*b*arctan(exp(d*x+c))

Maxima [B] time = 1.52533, size = 184, normalized size = 4.38

$$-b \left(\frac{\arctan(e^{(-dx-c)})}{d} + \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right) - a \left(\frac{\arctan(e^{(-dx-c)})}{d} - \frac{e^{(-dx-c)} - e^{(-3dx-3c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))

Fricas [B] time = 1.52328, size = 903, normalized size = 21.5

$$(a - b) \cosh(dx + c)^3 + 3(a - b) \cosh(dx + c) \sinh(dx + c)^2 + (a - b) \sinh(dx + c)^3 + ((a + b) \cosh(dx + c)^4 + 4(a + b) \cosh(dx + c) \sinh(dx + c)^3 + 6(a + b) \sinh(dx + c)^4 + 4(a + b) \cosh(dx + c)^2 \sinh(dx + c)^2 + 2(a + b) \sinh(dx + c)^4 + 2(3(a + b) \cosh(dx + c)^2 + a + b) \sinh(dx + c)^2 + 4((a + b) \cosh(dx + c)^3 + (a + b) \cosh(dx + c)) \sinh(dx + c) + a + b) \arctan(\cosh(dx + c) + \sinh(dx + c)) - (a - b) \cosh(dx + c) + (3(a - b) \cosh(dx + c)^2 - a + b) \sinh(dx + c) / (d \cosh(dx + c)^4 + 4d \cosh(dx + c) \sinh(dx + c)^3 + d \sinh(dx + c)^4 + 2d \cosh(dx + c)^2 + 2(3d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 4(d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c) + d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] ((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [B] time = 1.18395, size = 143, normalized size = 3.4

$$\frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{2dx+2c} - 1\right)e^{-dx-c}\right)\right)(a+b)}{4d} + \frac{a\left(e^{dx+c} - e^{-dx-c}\right) - b\left(e^{dx+c} - e^{-dx-c}\right)}{\left(\left(e^{dx+c} - e^{-dx-c}\right)^2 + 4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a + b)/d + (a*(e^(d*x + c) - e^(-d*x - c)) - b*(e^(d*x + c) - e^(-d*x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)*d)

3.290 $\int \operatorname{sech}^4(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=32

$$\frac{a \tanh(c + dx)}{d} - \frac{(a - b) \tanh^3(c + dx)}{3d}$$

[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0337702, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {3191}

$$\frac{a \tanh(c + dx)}{d} - \frac{(a - b) \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]

[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^4(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int (a - (a - b)x^2) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{a \tanh(c + dx)}{d} - \frac{(a - b) \tanh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0074434, size = 44, normalized size = 1.38

$$-\frac{a \tanh^3(c + dx)}{3d} + \frac{a \tanh(c + dx)}{d} + \frac{b \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2), x]

[Out] (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^3)/(3*d)

Maple [B] time = 0.062, size = 65, normalized size = 2.

$$\frac{1}{d} \left(a \left(\frac{2}{3} + \frac{(\operatorname{sech}(dx+c))^2}{3} \right) \tanh(dx+c) + b \left(-\frac{\sinh(dx+c)}{2(\cosh(dx+c))^3} + \frac{\tanh(dx+c)}{2} \left(\frac{2}{3} + \frac{(\operatorname{sech}(dx+c))^2}{3} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(a*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))

Maxima [B] time = 1.0453, size = 250, normalized size = 7.81

$$\frac{4}{3} a \left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} + \frac{1}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right) + \frac{2}{3} b \left(\frac{1}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2), x, algorithm="maxima")

[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2/3*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))

Fricas [B] time = 1.38691, size = 427, normalized size = 13.34

$$\frac{4 \left((a+2b) \cosh(dx+c)^2 - 2(a-b) \cosh(dx+c) \sinh(dx+c) \right) + 3 \left(d \cosh(dx+c)^4 + 4d \cosh(dx+c) \sinh(dx+c)^3 + d \sinh(dx+c)^4 + 4d \cosh(dx+c)^2 + 2 \left(3d \cosh(dx+c)^2 + \dots \right) \right)}{3 \left(d \cosh(dx+c)^4 + 4d \cosh(dx+c) \sinh(dx+c)^3 + d \sinh(dx+c)^4 + 4d \cosh(dx+c)^2 + 2 \left(3d \cosh(dx+c)^2 + \dots \right) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$-4/3*((a + 2*b)*\cosh(d*x + c)^2 - 2*(a - b)*\cosh(d*x + c)*\sinh(d*x + c) + (a + 2*b)*\sinh(d*x + c)^2 + 3*a)/(d*\cosh(d*x + c)^4 + 4*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + d*\sinh(d*x + c)^4 + 4*d*\cosh(d*x + c)^2 + 2*(3*d*\cosh(d*x + c)^2 + 2*d)*\sinh(d*x + c)^2 + 4*(d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c) + 3*d)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [A] time = 1.18553, size = 63, normalized size = 1.97

$$\frac{2(3be^{4dx+4c} + 6ae^{2dx+2c} + 2a + b)}{3d(e^{2dx+2c} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out]
$$-2/3*(3*b*e^{(4*d*x + 4*c)} + 6*a*e^{(2*d*x + 2*c)} + 2*a + b)/(d*(e^{(2*d*x + 2*c)} + 1)^3)$$

3.291 $\int \operatorname{sech}^5(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=70

$$\frac{(3a + b) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{(a - b) \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d} + \frac{(3a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d}$$

[Out] $((3*a + b)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*d) + ((3*a + b)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*d) + ((a - b)*\operatorname{Sech}[c + d*x]^3*\operatorname{Tanh}[c + d*x])/(4*d)$

Rubi [A] time = 0.0470149, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3190, 385, 199, 203}

$$\frac{(3a + b) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{(a - b) \tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d} + \frac{(3a + b) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[c + d*x]^5*(a + b*\operatorname{Sinh}[c + d*x]^2), x]$

[Out] $((3*a + b)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*d) + ((3*a + b)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*d) + ((a - b)*\operatorname{Sech}[c + d*x]^3*\operatorname{Tanh}[c + d*x])/(4*d)$

Rule 3190

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*ff^2*x^2)^p, x], x, \operatorname{Sin}[e + f*x]/ff], x]] /;$ $\operatorname{FreeQ}\{a, b, e, f, p\}, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 385

$\operatorname{Int}[(a_. + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow -\operatorname{Simp}[(b*c - a*d)*x*(a + b*x^n)^{(p+1)}/(a*b*n*(p+1)), x] - \operatorname{Dist}[(a*d - b*c*(n*(p+1) + 1))/(a*b*n*(p+1)), \operatorname{Int}[(a + b*x^n)^{(p+1)}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d, n, p\}, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& (\operatorname{LtQ}[p, -1] \mid\mid \operatorname{ILtQ}[1/n + p, 0])$

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)) / (a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1) / (a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]) / (Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^5(c + dx) (a + b \sinh^2(c + dx)) dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(a-b)\operatorname{sech}^3(c + dx) \tanh(c + dx)}{4d} + \frac{(3a+b)\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{4d} \\ &= \frac{(3a+b)\operatorname{sech}(c + dx) \tanh(c + dx)}{8d} + \frac{(a-b)\operatorname{sech}^3(c + dx) \tanh(c + dx)}{4d} + \frac{(a-b)\operatorname{sech}^5(c + dx)}{4d} \\ &= \frac{(3a+b)\tan^{-1}(\sinh(c + dx))}{8d} + \frac{(3a+b)\operatorname{sech}(c + dx) \tanh(c + dx)}{8d} + \frac{(a-b)\operatorname{sech}^5(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.149494, size = 60, normalized size = 0.86

$$\frac{(3a + b)\tan^{-1}(\sinh(c + dx)) + 2(a - b)\tanh(c + dx)\operatorname{sech}^3(c + dx) + (3a + b)\tanh(c + dx)\operatorname{sech}(c + dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2), x]
```

```
[Out] ((3*a + b)*ArcTan[Sinh[c + d*x]] + (3*a + b)*Sech[c + d*x]*Tanh[c + d*x] + 2*(a - b)*Sech[c + d*x]^3*Tanh[c + d*x]) / (8*d)
```

Maple [A] time = 0.072, size = 124, normalized size = 1.8

$$\frac{\tanh(dx+c) a (\operatorname{sech}(dx+c))^3}{4d} + \frac{3 a \operatorname{sech}(dx+c) \tanh(dx+c)}{8d} + \frac{3 a \arctan(e^{dx+c})}{4d} - \frac{b \sinh(dx+c)}{3d (\cosh(dx+c))^4} + \frac{b \tanh(dx+c)}{3d (\cosh(dx+c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x)`

[Out] $\frac{1}{4} \frac{a \tanh(dx+c) \operatorname{sech}(dx+c)^3 + 3/8 d a \operatorname{sech}(dx+c) \tanh(dx+c) + 3/4 d a a \arctan(\exp(dx+c)) - 1/3 d b \sinh(dx+c) / \cosh(dx+c)^4 + 1/12 d b \tanh(dx+c) \operatorname{sech}(dx+c)^3 + 1/8 d b \operatorname{sech}(dx+c) \tanh(dx+c) + 1/4 d b \arctan(\exp(dx+c))}{d}$

Maxima [B] time = 1.53705, size = 308, normalized size = 4.4

$$-\frac{1}{4} a \left(\frac{3 \arctan(e^{-dx-c})}{d} - \frac{3 e^{-dx-c} + 11 e^{-3dx-3c} - 11 e^{-5dx-5c} - 3 e^{-7dx-7c}}{d(4 e^{-2dx-2c} + 6 e^{-4dx-4c} + 4 e^{-6dx-6c} + e^{-8dx-8c} + 1)} \right) - \frac{1}{4} b \left(\frac{\arctan(e^{-dx-c})}{d} - \frac{e^{-dx-c} - 7 e^{-3dx-3c} + 7 e^{-5dx-5c} - e^{-7dx-7c}}{d(4 e^{-2dx-2c} + 6 e^{-4dx-4c} + 4 e^{-6dx-6c} + e^{-8dx-8c} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")`

[Out] $-\frac{1}{4} a \left(\frac{3 \arctan(e^{-dx-c})}{d} - \frac{3 e^{-dx-c} + 11 e^{-3dx-3c} - 11 e^{-5dx-5c} - 3 e^{-7dx-7c}}{d(4 e^{-2dx-2c} + 6 e^{-4dx-4c} + 4 e^{-6dx-6c} + e^{-8dx-8c} + 1)} \right) - \frac{1}{4} b \left(\frac{\arctan(e^{-dx-c})}{d} - \frac{e^{-dx-c} - 7 e^{-3dx-3c} + 7 e^{-5dx-5c} - e^{-7dx-7c}}{d(4 e^{-2dx-2c} + 6 e^{-4dx-4c} + 4 e^{-6dx-6c} + e^{-8dx-8c} + 1)} \right)$

Fricas [B] time = 1.60049, size = 2838, normalized size = 40.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")`

[Out] $\frac{1}{4} ((3a + b) \cosh(dx+c)^7 + 7(3a + b) \cosh(dx+c) \sinh(dx+c)^6 + (3a + b) \sinh(dx+c)^7 + (11a - 7b) \cosh(dx+c)^5 + (21(3a + b) \cosh(dx+c)^4 + 7(3a + b) \sinh(dx+c)^4 + 7(3a + b) \cosh(dx+c) \sinh(dx+c)^3 + (3a + b) \sinh(dx+c)^3 + 7(3a + b) \cosh(dx+c) \sinh(dx+c)^2 + (3a + b) \sinh(dx+c)^2 + 7(3a + b) \cosh(dx+c) \sinh(dx+c) + (3a + b) \sinh(dx+c)) / d$

$$\begin{aligned} & \cosh(dx + c)^2 + 11a - 7b) \sinh(dx + c)^5 + 5(7(3a + b) \cosh(dx + c) \\ &)^3 + (11a - 7b) \cosh(dx + c) \sinh(dx + c)^4 - (11a - 7b) \cosh(dx + c) \\ &)^3 + (35(3a + b) \cosh(dx + c)^4 + 10(11a - 7b) \cosh(dx + c)^2 - 1 \\ & 1a + 7b) \sinh(dx + c)^3 + (21(3a + b) \cosh(dx + c)^5 + 10(11a - 7b) \\ &) \cosh(dx + c)^3 - 3(11a - 7b) \cosh(dx + c) \sinh(dx + c)^2 + ((3a + b) \\ &) \cosh(dx + c)^8 + 8(3a + b) \cosh(dx + c) \sinh(dx + c)^7 + (3a + b) \\ &) \sinh(dx + c)^8 + 4(3a + b) \cosh(dx + c)^6 + 4(7(3a + b) \cosh(dx + c) \\ &)^2 + 3a + b) \sinh(dx + c)^6 + 8(7(3a + b) \cosh(dx + c)^3 + 3(3a + b) \\ &) \cosh(dx + c) \sinh(dx + c)^5 + 6(3a + b) \cosh(dx + c)^4 + 2(35(3 \\ & a + b) \cosh(dx + c)^4 + 30(3a + b) \cosh(dx + c)^2 + 9a + 3b) \sinh(dx \\ & x + c)^4 + 8(7(3a + b) \cosh(dx + c)^5 + 10(3a + b) \cosh(dx + c)^3 + \\ & 3(3a + b) \cosh(dx + c)) \sinh(dx + c)^3 + 4(3a + b) \cosh(dx + c)^2 + \\ & 4(7(3a + b) \cosh(dx + c)^6 + 15(3a + b) \cosh(dx + c)^4 + 9(3a + b) \\ &) \cosh(dx + c)^2 + 3a + b) \sinh(dx + c)^2 + 8((3a + b) \cosh(dx + c)^7 \\ & + 3(3a + b) \cosh(dx + c)^5 + 3(3a + b) \cosh(dx + c)^3 + (3a + b) \cos \\ & h(dx + c)) \sinh(dx + c) + 3a + b) \arctan(\cosh(dx + c) + \sinh(dx + c)) \\ & - (3a + b) \cosh(dx + c) + (7(3a + b) \cosh(dx + c)^6 + 5(11a - 7b) \cosh(dx + c) \\ &)^4 - 3(11a - 7b) \cosh(dx + c)^2 - 3a - b) \sinh(dx + c)) / (\\ & d \cosh(dx + c)^8 + 8d \cosh(dx + c) \sinh(dx + c)^7 + d \sinh(dx + c)^8 + \\ & 4d \cosh(dx + c)^6 + 4(7d \cosh(dx + c)^2 + d) \sinh(dx + c)^6 + 8(7d \\ &) \cosh(dx + c)^3 + 3d \cosh(dx + c) \sinh(dx + c)^5 + 6d \cosh(dx + c)^4 \\ & + 2(35d \cosh(dx + c)^4 + 30d \cosh(dx + c)^2 + 3d) \sinh(dx + c)^4 + \\ & 8(7d \cosh(dx + c)^5 + 10d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx \\ & + c)^3 + 4d \cosh(dx + c)^2 + 4(7d \cosh(dx + c)^6 + 15d \cosh(dx + c) \\ &)^4 + 9d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 8(d \cosh(dx + c)^7 + 3d \\ &) \cosh(dx + c)^5 + 3d \cosh(dx + c)^3 + d \cosh(dx + c) \sinh(dx + c) + d \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)**5*(a+b*sinh(dx+c)**2),x)

[Out] Timed out

Giac [B] time = 1.15173, size = 209, normalized size = 2.99

$$\frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{2dx+2c} - 1\right)e^{-dx-c}\right)\right)(3a + b)}{16d} + \frac{3a\left(e^{dx+c} - e^{-dx-c}\right)^3 + b\left(e^{dx+c} - e^{-dx-c}\right)^3 + 20a\left(e^{dx+c} - e^{-dx-c}\right)}{4\left(\left(e^{dx+c} - e^{-dx-c}\right)^2 + 4\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] 1/16*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a + b)/d + 1/4*(3*a*(e^(d*x + c) - e^(-d*x - c))^3 + b*(e^(d*x + c) - e^(-d*x - c))^3 + 20*a*(e^(d*x + c) - e^(-d*x - c)) - 4*b*(e^(d*x + c) - e^(-d*x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2*d)

3.292 $\int \operatorname{sech}^6(c + dx) (a + b \sinh^2(c + dx)) dx$

Optimal. Leaf size=54

$$\frac{(a-b) \tanh^5(c+dx)}{5d} - \frac{(2a-b) \tanh^3(c+dx)}{3d} + \frac{a \tanh(c+dx)}{d}$$

[Out] (a*Tanh[c + d*x])/d - ((2*a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)*Tanh[c + d*x]^5)/(5*d)

Rubi [A] time = 0.0474164, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3191, 373}

$$\frac{(a-b) \tanh^5(c+dx)}{5d} - \frac{(2a-b) \tanh^3(c+dx)}{3d} + \frac{a \tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2),x]

[Out] (a*Tanh[c + d*x])/d - ((2*a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)*Tanh[c + d*x]^5)/(5*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^6(c+dx)(a+b\sinh^2(c+dx))dx &= \frac{\operatorname{Subst}\left(\int(1-x^2)(a-(a-b)x^2)dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int(a-(2a-b)x^2+(a-b)x^4)dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{a \tanh(c+dx)}{d} - \frac{(2a-b) \tanh^3(c+dx)}{3d} + \frac{(a-b) \tanh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.050378, size = 102, normalized size = 1.89

$$\frac{a \tanh^5(c+dx)}{5d} - \frac{2a \tanh^3(c+dx)}{3d} + \frac{a \tanh(c+dx)}{d} + \frac{2b \tanh(c+dx)}{15d} - \frac{b \tanh(c+dx) \operatorname{sech}^4(c+dx)}{5d} + \frac{b \tanh(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2), x]

[Out] (a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh[c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (2*a*Tanh[c + d*x]^3)/(3*d) + (a*Tanh[c + d*x]^5)/(5*d)

Maple [A] time = 0.066, size = 85, normalized size = 1.6

$$\frac{1}{d} \left(a \left(\frac{8}{15} + \frac{(\operatorname{sech}(dx+c))^4}{5} + \frac{4(\operatorname{sech}(dx+c))^2}{15} \right) \tanh(dx+c) + b \left(-\frac{\sinh(dx+c)}{4(\cosh(dx+c))^5} + \frac{\tanh(dx+c)}{4} \right) \left(\frac{8}{15} + \frac{(\operatorname{sech}(dx+c))^4}{5} + \frac{4(\operatorname{sech}(dx+c))^2}{15} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*(a*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))

Maxima [B] time = 1.07042, size = 656, normalized size = 12.15

$$\frac{16}{15} a \left(\frac{5e^{(-2dx-2c)}}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} + \frac{10}{d(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} + 10e^{(-6dx-6c)} + 5e^{(-8dx-8c)} + e^{(-10dx-10c)} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] $\frac{16}{15}a \frac{(5e^{-2dx-2c})}{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + 10e^{-4dx-4c} \frac{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))}{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + \frac{1}{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + \frac{4}{15}b \frac{(5e^{-2dx-2c})}{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} - 5e^{-4dx-4c} \frac{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))}{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + 15e^{-6dx-6c} \frac{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))}{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + \frac{1}{(d(5e^{-2dx-2c} + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))}$

Fricas [B] time = 1.45769, size = 930, normalized size = 17.22

$$\frac{15(d \cosh(dx+c))^7 + 7d \cosh(dx+c) \sinh(dx+c)^6 + d \sinh(dx+c)^7 + 5d \cosh(dx+c)^5 + (21d \cosh(dx+c)^2 + 5d \sinh(dx+c)^2)}{(d \cosh(dx+c))^7 + 7d \cosh(dx+c) \sinh(dx+c)^6 + d \sinh(dx+c)^7 + 5d \cosh(dx+c)^5 + (21d \cosh(dx+c)^2 + 5d \sinh(dx+c)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $-\frac{8}{15}(2(a+4b)\cosh(dx+c)^3 + 6(a+4b)\cosh(dx+c)\sinh(dx+c)^2 - (2a-7b)\sinh(dx+c)^3 + 30a\cosh(dx+c) - (3(2a-7b)\cosh(dx+c)^2 - 10a+5b)\sinh(dx+c)) / (d\cosh(dx+c)^7 + 7d\cosh(dx+c)\sinh(dx+c)^6 + d\sinh(dx+c)^7 + 5d\cosh(dx+c)^5 + (21d\cosh(dx+c)^2 + 5d)\sinh(dx+c)^5 + 5(7d\cosh(dx+c)^3 + 5d\cosh(dx+c))\sinh(dx+c)^4 + 11d\cosh(dx+c)^3 + (35d\cosh(dx+c)^4 + 50d\cosh(dx+c)^2 + 9d)\sinh(dx+c)^3 + (21d\cosh(dx+c)^5 + 50d\cosh(dx+c)^3 + 33d\cosh(dx+c))\sinh(dx+c)^2 + 15d\cosh(dx+c) + (7d\cosh(dx+c)^6 + 25d\cosh(dx+c)^4 + 27d\cosh(dx+c)^2 + 5d)\sinh(dx+c)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2),x)`

[Out] Timed out

Giac [A] time = 1.15458, size = 112, normalized size = 2.07

$$\frac{4(15be^{(6dx+6c)} + 40ae^{(4dx+4c)} - 5be^{(4dx+4c)} + 20ae^{(2dx+2c)} + 5be^{(2dx+2c)} + 4a + b)}{15d(e^{(2dx+2c)} + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="giac")`

[Out]
$$-4/15*(15*b*e^{(6*d*x + 6*c)} + 40*a*e^{(4*d*x + 4*c)} - 5*b*e^{(4*d*x + 4*c)} + 20*a*e^{(2*d*x + 2*c)} + 5*b*e^{(2*d*x + 2*c)} + 4*a + b)/(d*(e^{(2*d*x + 2*c)} + 1)^5)$$

3.293 $\int \cosh^4(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=159

$$\frac{(48a^2 - 16ab + 3b^2) \sinh(c + dx) \cosh^3(c + dx)}{192d} + \frac{(48a^2 - 16ab + 3b^2) \sinh(c + dx) \cosh(c + dx)}{128d} + \frac{1}{128}x(48a^2 - 16ab + 3b^2)$$

[Out] ((48*a^2 - 16*a*b + 3*b^2)*x)/128 + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + ((10*a - 3*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(8*d)

Rubi [A] time = 0.174047, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3191, 413, 385, 199, 206}

$$\frac{(48a^2 - 16ab + 3b^2) \sinh(c + dx) \cosh^3(c + dx)}{192d} + \frac{(48a^2 - 16ab + 3b^2) \sinh(c + dx) \cosh(c + dx)}{128d} + \frac{1}{128}x(48a^2 - 16ab + 3b^2)$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((48*a^2 - 16*a*b + 3*b^2)*x)/128 + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + ((10*a - 3*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(8*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p

+ q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \cosh^4(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\text{Subst} \left(\int \frac{(a - (a-b)x^2)^2}{(1-x^2)^5} dx, x, \tanh(c + dx) \right)}{d} \\
&= \frac{b \cosh^7(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))}{8d} - \frac{\text{Subst} \left(\int \frac{-a(8a - (a-b)x^2)^2}{(1-x^2)^5} dx, x, \tanh(c + dx) \right)}{8d} \\
&= \frac{(10a - 3b)b \cosh^5(c + dx) \sinh(c + dx)}{48d} + \frac{b \cosh^7(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))}{8d} \\
&= \frac{(48a^2 - 16ab + 3b^2) \cosh^3(c + dx) \sinh(c + dx)}{192d} + \frac{(10a - 3b)b \cosh^5(c + dx) \sinh(c + dx)}{48d} \\
&= \frac{(48a^2 - 16ab + 3b^2) \cosh(c + dx) \sinh(c + dx)}{128d} + \frac{(48a^2 - 16ab + 3b^2) \cosh^3(c + dx) \sinh(c + dx)}{192d} \\
&= \frac{1}{128} (48a^2 - 16ab + 3b^2) x + \frac{(48a^2 - 16ab + 3b^2) \cosh(c + dx) \sinh(c + dx)}{128d}
\end{aligned}$$

Mathematica [A] time = 0.309086, size = 98, normalized size = 0.62

$$\frac{24(48a^2 - 16ab + 3b^2)(c + dx) + 24(4a^2 + 4ab - b^2) \sinh(4(c + dx)) + 32ab \sinh(6(c + dx)) + 96a(8a - b) \sinh(2(c + dx))}{3072d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (24*(48*a^2 - 16*a*b + 3*b^2)*(c + d*x) + 96*a*(8*a - b)*Sinh[2*(c + d*x)] + 24*(4*a^2 + 4*a*b - b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] + 3*b^2*Sinh[8*(c + d*x)])/(3072*d)

Maple [A] time = 0.031, size = 172, normalized size = 1.1

$$\frac{1}{d} \left(b^2 \left(\frac{(\sinh(dx + c))^3 (\cosh(dx + c))^5}{8} - \frac{\sinh(dx + c) (\cosh(dx + c))^5}{16} + \frac{\sinh(dx + c)}{16} \left(\frac{(\cosh(dx + c))^3}{4} + \frac{3 \cosh(dx + c)}{8} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)`

[Out] $\frac{1}{d} \cdot (b^2 \cdot (\frac{1}{8} \sinh(d*x+c)^3 \cosh(d*x+c)^5 - \frac{1}{16} \sinh(d*x+c) \cosh(d*x+c)^5 + \frac{1}{16} (\frac{1}{4} \cosh(d*x+c)^3 + \frac{3}{8} \cosh(d*x+c)) \sinh(d*x+c) + \frac{3}{128} d*x + \frac{3}{128} c) + 2 \cdot a \cdot b \cdot (\frac{1}{6} \sinh(d*x+c) \cosh(d*x+c)^5 - \frac{1}{6} (\frac{1}{4} \cosh(d*x+c)^3 + \frac{3}{8} \cosh(d*x+c)) \sinh(d*x+c) - \frac{1}{16} d*x - \frac{1}{16} c) + a^2 \cdot ((\frac{1}{4} \cosh(d*x+c)^3 + \frac{3}{8} \cosh(d*x+c)) \sinh(d*x+c) + \frac{3}{8} d*x + \frac{3}{8} c))$

Maxima [A] time = 1.11009, size = 304, normalized size = 1.91

$\frac{1}{64} a^2 \left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{2048} b^2 \left(\frac{(8e^{(-4dx-4c)} - 1)e^{(8dx+8c)}}{d} - \frac{48(dx+c)}{d} - \frac{8e^{(-4dx-4c)}}{d} \right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $\frac{1}{64} a^2 \cdot (24x + \frac{e^{(4d*x + 4*c)}}{d} + \frac{8 \cdot e^{(2*d*x + 2*c)}}{d} - \frac{8 \cdot e^{(-2*d*x - 2*c)}}{d} - \frac{e^{(-4*d*x - 4*c)}}{d} - \frac{1}{2048} b^2 \cdot ((8 \cdot e^{(-4*d*x - 4*c)} - 1) \cdot e^{(8*d*x + 8*c)} / d - 48 \cdot (d*x + c) / d - (8 \cdot e^{(-4*d*x - 4*c)} - e^{(-8*d*x - 8*c)}) / d) + \frac{1}{192} a \cdot b \cdot ((3 \cdot e^{(-2*d*x - 2*c)} - 3 \cdot e^{(-4*d*x - 4*c)} + 1) \cdot e^{(6*d*x + 6*c)} / d - 24 \cdot (d*x + c) / d + (3 \cdot e^{(-2*d*x - 2*c)} - 3 \cdot e^{(-4*d*x - 4*c)} - e^{(-6*d*x - 6*c)}) / d)$

Fricas [A] time = 1.48994, size = 532, normalized size = 3.35

$3b^2 \cosh(dx+c) \sinh(dx+c)^7 + 3(7b^2 \cosh(dx+c)^3 + 8ab \cosh(dx+c)) \sinh(dx+c)^5 + (21b^2 \cosh(dx+c)^5 + 8a^2 \cosh(dx+c)^3 + 12(4a^2 + 4ab - b^2) \cosh(dx+c)) \sinh(dx+c)^3 + 3(48a^2 - 16ab + 3b^2) d*x + 3(b^2 \cosh(dx+c)^7 + 8ab \cosh(dx+c)^5 + 4(4a^2 + 4ab - b^2) \cosh(dx+c)^3 + 8(8a^2 - ab) \cosh(dx+c)) \sinh(dx+c)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $\frac{1}{384} \cdot (3 \cdot b^2 \cdot \cosh(d*x + c) \cdot \sinh(d*x + c)^7 + 3 \cdot (7 \cdot b^2 \cdot \cosh(d*x + c)^3 + 8 \cdot a \cdot b \cdot \cosh(d*x + c)) \cdot \sinh(d*x + c)^5 + (21 \cdot b^2 \cdot \cosh(d*x + c)^5 + 80 \cdot a \cdot b \cdot \cosh(d*x + c)^3 + 12 \cdot (4 \cdot a^2 + 4 \cdot a \cdot b - b^2) \cdot \cosh(d*x + c)) \cdot \sinh(d*x + c)^3 + 3 \cdot (48 \cdot a^2 - 16 \cdot a \cdot b + 3 \cdot b^2) \cdot d \cdot x + 3 \cdot (b^2 \cdot \cosh(d*x + c)^7 + 8 \cdot a \cdot b \cdot \cosh(d*x + c)^5 + 4 \cdot (4 \cdot a^2 + 4 \cdot a \cdot b - b^2) \cdot \cosh(d*x + c)^3 + 8 \cdot (8 \cdot a^2 - a \cdot b) \cdot \cosh(d*x + c)) \cdot \sinh(d*x + c)$

*sinh(d*x + c))/d

Sympy [A] time = 12.8862, size = 481, normalized size = 3.03

$$\left\{ \frac{3a^2x \sinh^4(c+dx)}{8} - \frac{3a^2x \sinh^2(c+dx) \cosh^2(c+dx)}{4} + \frac{3a^2x \cosh^4(c+dx)}{8} - \frac{3a^2 \sinh^3(c+dx) \cosh(c+dx)}{8d} + \frac{5a^2 \sinh(c+dx) \cosh^3(c+dx)}{8d} + \frac{abx \sinh^6(c)}{8} \right. \\ \left. x (a + b \sinh^2(c))^2 \cosh^4(c) \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 - 3*a**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 5*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + a*b*x*sinh(c + d*x)**6/8 - 3*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 3*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/8 - a*b*x*cosh(c + d*x)**6/8 - a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8*d) + a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + 3*b**2*x*sinh(c + d*x)**8/128 - 3*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 9*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 3*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 3*b**2*x*cosh(c + d*x)**8/128 - 3*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 11*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(128*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**4, True))

Giac [A] time = 1.23409, size = 354, normalized size = 2.23

$$3b^2e^{(8dx+8c)} + 32abe^{(6dx+6c)} + 96a^2e^{(4dx+4c)} + 96abe^{(4dx+4c)} - 24b^2e^{(4dx+4c)} + 768a^2e^{(2dx+2c)} - 96abe^{(2dx+2c)} + 48(48$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/6144*(3*b^2*e^(8*d*x + 8*c) + 32*a*b*e^(6*d*x + 6*c) + 96*a^2*e^(4*d*x + 4*c) + 96*a*b*e^(4*d*x + 4*c) - 24*b^2*e^(4*d*x + 4*c) + 768*a^2*e^(2*d*x + 2*c) - 96*a*b*e^(2*d*x + 2*c) + 48*(48*a^2 - 16*a*b + 3*b^2)*(d*x + c) - (2400*a^2*e^(8*d*x + 8*c) - 800*a*b*e^(8*d*x + 8*c) + 150*b^2*e^(8*d*x + 8*c) + 768*a^2*e^(6*d*x + 6*c) - 96*a*b*e^(6*d*x + 6*c) + 96*a^2*e^(4*d*x + 4*c

$$\frac{c) + 96*a*b*e^{(4*d*x + 4*c)} - 24*b^2*e^{(4*d*x + 4*c)} + 32*a*b*e^{(2*d*x + 2*c)} + 3*b^2*e^{(-8*d*x - 8*c)}}{d}$$

$$3.294 \quad \int \cosh^3(c + dx) \left(a + b \sinh^2(c + dx) \right)^2 dx$$

Optimal. Leaf size=74

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{b(2a + b) \sinh^5(c + dx)}{5d} + \frac{a(a + 2b) \sinh^3(c + dx)}{3d} + \frac{b^2 \sinh^7(c + dx)}{7d}$$

[Out] (a^2*Sinh[c + d*x])/d + (a*(a + 2*b)*Sinh[c + d*x]^3)/(3*d) + (b*(2*a + b)*Sinh[c + d*x]^5)/(5*d) + (b^2*Sinh[c + d*x]^7)/(7*d)

Rubi [A] time = 0.0723943, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3190, 373}

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{b(2a + b) \sinh^5(c + dx)}{5d} + \frac{a(a + 2b) \sinh^3(c + dx)}{3d} + \frac{b^2 \sinh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (a^2*Sinh[c + d*x])/d + (a*(a + 2*b)*Sinh[c + d*x]^3)/(3*d) + (b*(2*a + b)*Sinh[c + d*x]^5)/(5*d) + (b^2*Sinh[c + d*x]^7)/(7*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cosh^3(c+dx) (a+b\sinh^2(c+dx))^2 dx &= \frac{\text{Subst}\left(\int (1+x^2)(a+bx^2)^2 dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int (a^2+a(a+2b)x^2+b(2a+b)x^4+b^2x^6) dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{a^2 \sinh(c+dx)}{d} + \frac{a(a+2b) \sinh^3(c+dx)}{3d} + \frac{b(2a+b) \sinh^5(c+dx)}{5d} + \frac{b^2 \sinh^7(c+dx)}{7d} \end{aligned}$$

Mathematica [A] time = 0.114626, size = 64, normalized size = 0.86

$$\frac{105a^2 \sinh(c+dx) + 21b(2a+b) \sinh^5(c+dx) + 35a(a+2b) \sinh^3(c+dx) + 15b^2 \sinh^7(c+dx)}{105d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (105*a^2*Sinh[c + d*x] + 35*a*(a + 2*b)*Sinh[c + d*x]^3 + 21*b*(2*a + b)*Sinh[c + d*x]^5 + 15*b^2*Sinh[c + d*x]^7)/(105*d)

Maple [A] time = 0.031, size = 128, normalized size = 1.7

$$\frac{1}{d} \left(b^2 \left(\frac{(\sinh(dx+c))^3 (\cosh(dx+c))^4}{7} - \frac{3 \sinh(dx+c) (\cosh(dx+c))^4}{35} + \frac{3 \sinh(dx+c)}{35} \left(\frac{2}{3} + \frac{(\cosh(dx+c))^2}{3} \right) \right) \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*(b^2*(1/7*sinh(d*x+c)^3*cosh(d*x+c)^4-3/35*sinh(d*x+c)*cosh(d*x+c)^4+3/35*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+2*a*b*(1/5*sinh(d*x+c)*cosh(d*x+c)^4-1/5*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+a^2*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))

Maxima [B] time = 1.08167, size = 327, normalized size = 4.42

$$-\frac{1}{4480} b^2 \left(\frac{(7e^{(-2dx-2c)} + 35e^{(-4dx-4c)} - 105e^{(-6dx-6c)} - 5)e^{(7dx+7c)}}{d} + \frac{105e^{(-dx-c)} - 35e^{(-3dx-3c)} - 7e^{(-5dx-5c)} + 5e^{(-7dx-7c)}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $-1/4480*b^2*((7*e^{(-2*d*x - 2*c)} + 35*e^{(-4*d*x - 4*c)} - 105*e^{(-6*d*x - 6*c)} - 5)*e^{(7*d*x + 7*c)}/d + (105*e^{(-d*x - c)} - 35*e^{(-3*d*x - 3*c)} - 7*e^{(-5*d*x - 5*c)} + 5*e^{(-7*d*x - 7*c)})/d) + 1/240*a*b*((5*e^{(-2*d*x - 2*c)} - 30*e^{(-4*d*x - 4*c)} + 3)*e^{(5*d*x + 5*c)}/d + (30*e^{(-d*x - c)} - 5*e^{(-3*d*x - 3*c)} - 3*e^{(-5*d*x - 5*c)})/d) + 1/24*a^2*(e^{(3*d*x + 3*c)}/d + 9*e^{(d*x + c)}/d - 9*e^{(-d*x - c)}/d - e^{(-3*d*x - 3*c)}/d)$

Fricas [B] time = 1.46437, size = 463, normalized size = 6.26

$15 b^2 \sinh(dx + c)^7 + 21 (15 b^2 \cosh(dx + c)^2 + 8 ab - b^2) \sinh(dx + c)^5 + 35 (15 b^2 \cosh(dx + c)^4 + 6 (8 ab - b^2) \cosh(dx + c)^2 + 16 a^2 + 8 ab - 3 b^2) \sinh(dx + c)^3 + 105 (b^2 \cosh(dx + c)^6 + (8 ab - b^2) \cosh(dx + c)^4 + (16 a^2 + 8 ab - 3 b^2) \cosh(dx + c)^2 + 48 a^2 - 16 ab + 3 b^2) \sinh(dx + c) / d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $1/6720*(15*b^2*\sinh(d*x + c)^7 + 21*(15*b^2*\cosh(d*x + c)^2 + 8*a*b - b^2)*\sinh(d*x + c)^5 + 35*(15*b^2*\cosh(d*x + c)^4 + 6*(8*a*b - b^2)*\cosh(d*x + c)^2 + 16*a^2 + 8*a*b - 3*b^2)*\sinh(d*x + c)^3 + 105*(b^2*\cosh(d*x + c)^6 + (8*a*b - b^2)*\cosh(d*x + c)^4 + (16*a^2 + 8*a*b - 3*b^2)*\cosh(d*x + c)^2 + 48*a^2 - 16*a*b + 3*b^2)*\sinh(d*x + c))/d$

Sympy [A] time = 6.89745, size = 136, normalized size = 1.84

$$\left\{ \begin{array}{l} -\frac{2a^2 \sinh^3(c+dx)}{3d} + \frac{a^2 \sinh(c+dx) \cosh^2(c+dx)}{d} - \frac{4ab \sinh^5(c+dx)}{15d} + \frac{2ab \sinh^3(c+dx) \cosh^2(c+dx)}{3d} - \frac{2b^2 \sinh^7(c+dx)}{35d} + \frac{b^2 \sinh^5(c+dx) \cosh^2(c+dx)}{5d} \\ x (a + b \sinh^2(c))^2 \cosh^3(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Piecewise((-2*a**2*sinh(c + d*x)**3/(3*d) + a**2*sinh(c + d*x)*cosh(c + d*x)**2/d - 4*a*b*sinh(c + d*x)**5/(15*d) + 2*a*b*sinh(c + d*x)**3*cosh(c + d*x)**2/(3*d) - 2*b**2*sinh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**5*cosh(c + d*x)**2/(3*d), True)

```
+ d*x)**2/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**3, True))
```

Giac [B] time = 1.22079, size = 332, normalized size = 4.49

$$15 b^2 e^{(7 dx+7 c)} + 168 a b e^{(5 dx+5 c)} - 21 b^2 e^{(5 dx+5 c)} + 560 a^2 e^{(3 dx+3 c)} + 280 a b e^{(3 dx+3 c)} - 105 b^2 e^{(3 dx+3 c)} + 5040 a^2 e^{(dx+c)} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*b^2*e^(7*d*x + 7*c) + 168*a*b*e^(5*d*x + 5*c) - 21*b^2*e^(5*d*x
+ 5*c) + 560*a^2*e^(3*d*x + 3*c) + 280*a*b*e^(3*d*x + 3*c) - 105*b^2*e^(3*
d*x + 3*c) + 5040*a^2*e^(d*x + c) - 1680*a*b*e^(d*x + c) + 315*b^2*e^(d*x +
c) - (5040*a^2*e^(6*d*x + 6*c) - 1680*a*b*e^(6*d*x + 6*c) + 315*b^2*e^(6*d
*x + 6*c) + 560*a^2*e^(4*d*x + 4*c) + 280*a*b*e^(4*d*x + 4*c) - 105*b^2*e^(
4*d*x + 4*c) + 168*a*b*e^(2*d*x + 2*c) - 21*b^2*e^(2*d*x + 2*c) + 15*b^2)*e
^(-7*d*x - 7*c))/d
```

3.295 $\int \cosh^2(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=119

$$\frac{(8a^2 - 4ab + b^2) \sinh(c + dx) \cosh(c + dx)}{16d} + \frac{1}{16}x(8a^2 - 4ab + b^2) + \frac{b(8a - 3b) \sinh(c + dx) \cosh^3(c + dx)}{24d} + \frac{b \sinh(c + dx) \cosh^5(c + dx)}{24d}$$

[Out] $((8a^2 - 4ab + b^2)x)/16 + ((8a^2 - 4ab + b^2) \cosh[c + dx] \sinh[c + dx])/(16d) + ((8a - 3b)b \cosh[c + dx]^3 \sinh[c + dx])/(24d) + (b \cosh[c + dx]^5 \sinh[c + dx] (a - (a - b) \tanh[c + dx]^2))/(6d)$

Rubi [A] time = 0.14271, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3191, 413, 385, 199, 206}

$$\frac{(8a^2 - 4ab + b^2) \sinh(c + dx) \cosh(c + dx)}{16d} + \frac{1}{16}x(8a^2 - 4ab + b^2) + \frac{b(8a - 3b) \sinh(c + dx) \cosh^3(c + dx)}{24d} + \frac{b \sinh(c + dx) \cosh^5(c + dx)}{24d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\cosh[c + dx]^2 (a + b \sinh[c + dx]^2)^2, x]$

[Out] $((8a^2 - 4ab + b^2)x)/16 + ((8a^2 - 4ab + b^2) \cosh[c + dx] \sinh[c + dx])/(16d) + ((8a - 3b)b \cosh[c + dx]^3 \sinh[c + dx])/(24d) + (b \cosh[c + dx]^5 \sinh[c + dx] (a - (a - b) \tanh[c + dx]^2))/(6d)$

Rule 3191

$\text{Int}[\cos[(e.) + (f.)x]^{(m)} ((a.) + (b.) \sin[(e.) + (f.)x]^2)^{(p.)}], x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Tan}[e + fx], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(a + (a + b)ff^2x^2)^p / (1 + ff^2x^2)^{(m/2 + p + 1)}], x], \text{Tan}[e + fx]/ff], x] /; \text{FreeQ}\{a, b, e, f\}, x \ \&\& \ \text{IntegerQ}[m/2] \ \&\& \ \text{IntegerQ}[p]$

Rule 413

$\text{Int}[(a.) + (b.)x^{(n)}]^{(p)} ((c.) + (d.)x^{(n)})^{(q)}, x_Symbol] \rightarrow \text{Simp}[(a*d - c*b)x^{(n)} (a + b*x^n)^{(p+1)} (c + d*x^n)^{(q-1)} / (a*b*n*(p+1)), x] - \text{Dist}[1/(a*b*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)} (c + d*x^n)^{(q-2)} \text{Simp}[c*(a*d - c*b*(n*(p+1) + 1)) + d*(a*d*(n*(q-1) + 1) - b*c*(n*(p+q) + 1)]*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[q, 1] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[(b*c - a*d)*x*(a + b*x^n)^(p + 1)/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \cosh^2(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int \frac{(a - (a-b)x^2)^2}{(1-x^2)^4} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{b \cosh^5(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))}{6d} - \frac{\text{Subst}\left(\int \frac{-a(6x^2 - 1)}{(1-x^2)^4} dx, x, \tanh(c + dx)\right)}{6d} \\
 &= \frac{(8a - 3b)b \cosh^3(c + dx) \sinh(c + dx)}{24d} + \frac{b \cosh^5(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))}{6d} \\
 &= \frac{(8a^2 - 4ab + b^2) \cosh(c + dx) \sinh(c + dx)}{16d} + \frac{(8a - 3b)b \cosh^3(c + dx) \sinh(c + dx)}{24d} \\
 &= \frac{1}{16} (8a^2 - 4ab + b^2) x + \frac{(8a^2 - 4ab + b^2) \cosh(c + dx) \sinh(c + dx)}{16d} + \frac{(8a - 3b)b \cosh^3(c + dx) \sinh(c + dx)}{24d}
 \end{aligned}$$

Mathematica [A] time = 0.294553, size = 79, normalized size = 0.66

$$\frac{12(8a^2 - 4ab + b^2)(c + dx) + 3(16a^2 - b^2)\sinh(2(c + dx)) + 3b(4a - b)\sinh(4(c + dx)) + b^2\sinh(6(c + dx))}{192d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (12*(8*a^2 - 4*a*b + b^2)*(c + d*x) + 3*(16*a^2 - b^2)*Sinh[2*(c + d*x)] + 3*(4*a - b)*b*Sinh[4*(c + d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)

Maple [A] time = 0.032, size = 134, normalized size = 1.1

$$\frac{1}{d} \left(b^2 \left(\frac{(\sinh(dx+c))^3 (\cosh(dx+c))^3}{6} - \frac{\sinh(dx+c) (\cosh(dx+c))^3}{8} + \frac{\cosh(dx+c) \sinh(dx+c)}{16} + \frac{dx}{16} + \frac{c}{16} \right) + 2a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*(b^2*(1/6*sinh(d*x+c)^3*cosh(d*x+c)^3-1/8*sinh(d*x+c)*cosh(d*x+c)^3+1/16*cosh(d*x+c)*sinh(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c))

Maxima [A] time = 1.15212, size = 231, normalized size = 1.94

$$\frac{1}{8} a^2 \left(4x + \frac{e^{2dx+2c}}{d} - \frac{e^{-2dx-2c}}{d} \right) - \frac{1}{384} b^2 \left(\frac{(3e^{-2dx-2c} + 3e^{-4dx-4c} - 1)e^{6dx+6c}}{d} - \frac{24(dx+c)}{d} - \frac{3e^{-2dx-2c} + 3e^{-4dx-4c}}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/384*b^2*((3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d - (3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d) - 1/32*a*b

$$*(8*(d*x + c)/d - e^{(4*d*x + 4*c)/d} + e^{(-4*d*x - 4*c)/d})$$

Fricas [A] time = 1.4468, size = 347, normalized size = 2.92

$$\frac{3b^2 \cosh(dx + c) \sinh(dx + c)^5 + 2(5b^2 \cosh(dx + c)^3 + 3(4ab - b^2) \cosh(dx + c)) \sinh(dx + c)^3 + 6(8a^2 - 4ab + b^2) \cosh(dx + c) \sinh(dx + c)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c))^2,x, algorithm="fricas")

[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^2*cosh(d*x + c)^3 + 3*(4*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 - 4*a*b + b^2)*d*x + 3*(b^2*cosh(d*x + c)^5 + 2*(4*a*b - b^2)*cosh(d*x + c)^3 + (16*a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))/d

Sympy [A] time = 4.5846, size = 314, normalized size = 2.64

$$\left\{ \begin{array}{l} \frac{a^2 x \sinh^2(c+dx)}{2} + \frac{a^2 x \cosh^2(c+dx)}{2} + \frac{a^2 \sinh(c+dx) \cosh(c+dx)}{2d} - \frac{abx \sinh^4(c+dx)}{4} + \frac{abx \sinh^2(c+dx) \cosh^2(c+dx)}{2} - \frac{abx \cosh^4(c+dx)}{4} + \frac{abx \sinh^2(c+dx) \cosh^2(c+dx)}{2} \\ x(a + b \sinh^2(c))^2 \cosh^2(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Piecewise((-a**2*x*sinh(c + d*x)**2/2 + a**2*x*cosh(c + d*x)**2/2 + a**2*sinh(c + d*x)*cosh(c + d*x)/(2*d) - a*b*x*sinh(c + d*x)**4/4 + a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**2/2 - a*b*x*cosh(c + d*x)**4/4 + a*b*sinh(c + d*x)*3*cosh(c + d*x)/(4*d) + a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) - b**2*x*sinh(c + d*x)**6/16 + 3*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 - 3*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 + b**2*x*cosh(c + d*x)**6/16 + b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) - b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**2, True))

Giac [A] time = 1.18555, size = 277, normalized size = 2.33

$$\frac{b^2 e^{(6dx+6c)} + 12abe^{(4dx+4c)} - 3b^2 e^{(4dx+4c)} + 48a^2 e^{(2dx+2c)} - 3b^2 e^{(2dx+2c)} + 24(8a^2 - 4ab + b^2)(dx + c) - (176a^2 e^{(6dx+6c)} + 12ab^2 e^{(4dx+4c)} - 3b^2 e^{(4dx+4c)} + 48a^2 e^{(2dx+2c)} - 3b^2 e^{(2dx+2c)} + 24(8a^2 - 4ab + b^2)(dx + c) - (176a^2 e^{(6dx+6c)} + 12ab^2 e^{(4dx+4c)} - 3b^2 e^{(4dx+4c)} + 48a^2 e^{(2dx+2c)} - 3b^2 e^{(2dx+2c)} + 24(8a^2 - 4ab + b^2)(dx + c))}{384ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")`

[Out] $\frac{1}{384}(b^2e^{(6dx + 6c)} + 12ab e^{(4dx + 4c)} - 3b^2e^{(4dx + 4c)} + 48a^2e^{(2dx + 2c)} - 3b^2e^{(2dx + 2c)} + 24(8a^2 - 4ab + b^2)(dx + c) - (176a^2e^{(6dx + 6c)} - 88ab e^{(6dx + 6c)} + 22b^2e^{(6dx + 6c)} + 48a^2e^{(4dx + 4c)} - 3b^2e^{(4dx + 4c)} + 12ab e^{(2dx + 2c)} - 3b^2e^{(2dx + 2c)} + b^2)e^{(-6dx - 6c)})/d$

$$3.296 \quad \int \cosh(c + dx) \left(a + b \sinh^2(c + dx) \right)^2 dx$$

Optimal. Leaf size=49

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{2ab \sinh^3(c + dx)}{3d} + \frac{b^2 \sinh^5(c + dx)}{5d}$$

[Out] (a^2*Sinh[c + d*x])/d + (2*a*b*Sinh[c + d*x]^3)/(3*d) + (b^2*Sinh[c + d*x]^5)/(5*d)

Rubi [A] time = 0.0370079, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3190, 194}

$$\frac{a^2 \sinh(c + dx)}{d} + \frac{2ab \sinh^3(c + dx)}{3d} + \frac{b^2 \sinh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (a^2*Sinh[c + d*x])/d + (2*a*b*Sinh[c + d*x]^3)/(3*d) + (b^2*Sinh[c + d*x]^5)/(5*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 194

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \cosh(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\text{Subst}\left(\int (a + bx^2)^2 dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int (a^2 + 2abx^2 + b^2x^4) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{a^2 \sinh(c + dx)}{d} + \frac{2ab \sinh^3(c + dx)}{3d} + \frac{b^2 \sinh^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.0560869, size = 44, normalized size = 0.9

$$\frac{a^2 \sinh(c + dx) + \frac{2}{3} ab \sinh^3(c + dx) + \frac{1}{5} b^2 \sinh^5(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (a^2*Sinh[c + d*x] + (2*a*b*Sinh[c + d*x]^3)/3 + (b^2*Sinh[c + d*x]^5)/5)/d

Maple [A] time = 0.011, size = 41, normalized size = 0.8

$$\frac{1}{d} \left(\frac{b^2 (\sinh(dx + c))^5}{5} + \frac{2a (\sinh(dx + c))^3 b}{3} + a^2 \sinh(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*(1/5*b^2*sinh(d*x+c)^5+2/3*a*sinh(d*x+c)^3*b+a^2*sinh(d*x+c))

Maxima [A] time = 1.16507, size = 61, normalized size = 1.24

$$\frac{b^2 \sinh(dx + c)^5}{5d} + \frac{2ab \sinh(dx + c)^3}{3d} + \frac{a^2 \sinh(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{5}b^2\sinh(dx+c)^5/d + \frac{2}{3}a*b*\sinh(dx+c)^3/d + a^2*\sinh(dx+c)/d$

Fricas [B] time = 1.47832, size = 261, normalized size = 5.33

$$\frac{3b^2\sinh(dx+c)^5 + 5(6b^2\cosh(dx+c)^2 + 8ab - 3b^2)\sinh(dx+c)^3 + 15(b^2\cosh(dx+c)^4 + (8ab - 3b^2)\cosh(dx+c)^2 + 16a^2 - 8ab + 2b^2)\sinh(dx+c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $\frac{1}{240}(3b^2\sinh(dx+c)^5 + 5(6b^2\cosh(dx+c)^2 + 8ab - 3b^2)\sinh(dx+c)^3 + 15(b^2\cosh(dx+c)^4 + (8ab - 3b^2)\cosh(dx+c)^2 + 16a^2 - 8ab + 2b^2)\sinh(dx+c))/d$

Sympy [A] time = 2.01711, size = 58, normalized size = 1.18

$$\begin{cases} \frac{a^2\sinh(c+dx)}{d} + \frac{2ab\sinh^3(c+dx)}{3d} + \frac{b^2\sinh^5(c+dx)}{5d} & \text{for } d \neq 0 \\ x(a + b\sinh^2(c))^2 \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*sinh(c + d*x)**3/(3*d) + b**2*sinh(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c), True))

Giac [B] time = 1.17326, size = 221, normalized size = 4.51

$$\frac{3b^2e^{(5dx+5c)} + 40abe^{(3dx+3c)} - 15b^2e^{(3dx+3c)} + 240a^2e^{(dx+c)} - 120abe^{(dx+c)} + 30b^2e^{(dx+c)} - (240a^2e^{(4dx+4c)} - 120abe^{(4dx+4c)})}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^2*e^(5*d*x + 5*c) + 40*a*b*e^(3*d*x + 3*c) - 15*b^2*e^(3*d*x + 3*c) + 240*a^2*e^(d*x + c) - 120*a*b*e^(d*x + c) + 30*b^2*e^(d*x + c) - (240*a^2*e^(4*d*x + 4*c) - 120*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 40*a*b*e^(2*d*x + 2*c) - 15*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-5*d*x - 5*c))/d
```

3.297 $\int \operatorname{sech}(c + dx) \left(a + b \sinh^2(c + dx) \right)^2 dx$

Optimal. Leaf size=55

$$\frac{b(2a - b) \sinh(c + dx)}{d} + \frac{(a - b)^2 \tan^{-1}(\sinh(c + dx))}{d} + \frac{b^2 \sinh^3(c + dx)}{3d}$$

[Out] $((a - b)^2 \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/d + ((2*a - b)*b*\operatorname{Sinh}[c + d*x])/d + (b^2*\operatorname{Sinh}[c + d*x]^3)/(3*d)$

Rubi [A] time = 0.0577477, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3190, 390, 203}

$$\frac{b(2a - b) \sinh(c + dx)}{d} + \frac{(a - b)^2 \tan^{-1}(\sinh(c + dx))}{d} + \frac{b^2 \sinh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[c + d*x]*(a + b*\operatorname{Sinh}[c + d*x]^2)^2, x]$

[Out] $((a - b)^2 \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/d + ((2*a - b)*b*\operatorname{Sinh}[c + d*x])/d + (b^2*\operatorname{Sinh}[c + d*x]^3)/(3*d)$

Rule 3190

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*ff^2*x^2)^p, x], x, \operatorname{Sin}[e + f*x]/ff], x]] /;$ $\operatorname{FreeQ}\{a, b, e, f, p\}, x] \ \&\& \ \operatorname{IntegerQ}[(m - 1)/2]$

Rule 390

$\operatorname{Int}[((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(a + b*x^n)^p, (c + d*x^n)^{-q}, x], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{IGtQ}[p, 0] \ \&\& \ \operatorname{ILtQ}[q, 0] \ \&\& \ \operatorname{GeQ}[p, -q]$

Rule 203

$\operatorname{Int}[((a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTan}[(\operatorname{Rt}[b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x] \ \&\& \ \operatorname{PosQ}[a/b] \ \&\& \ (\operatorname{GtQ}[a$

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \operatorname{sech}(c+dx) (a+b \sinh^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^2}{1+x^2} dx, x, \sinh(c+dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left((2a-b)b + b^2x^2 + \frac{(a-b)^2}{1+x^2}\right) dx, x, \sinh(c+dx)\right)}{d} \\
 &= \frac{(2a-b)b \sinh(c+dx)}{d} + \frac{b^2 \sinh^3(c+dx)}{3d} + \frac{(a-b)^2 \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx)\right)}{d} \\
 &= \frac{(a-b)^2 \tan^{-1}(\sinh(c+dx))}{d} + \frac{(2a-b)b \sinh(c+dx)}{d} + \frac{b^2 \sinh^3(c+dx)}{3d}
 \end{aligned}$$

Mathematica [A] time = 0.227902, size = 70, normalized size = 1.27

$$\frac{\sinh(c+dx) \left(b(6a + b(\sinh^2(c+dx) - 3)) + \frac{3(a-b)^2 \tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (Sinh[c + d*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c + d*x]^2] + b*(6*a + b*(-3 + Sinh[c + d*x]^2))))/(3*d)

Maple [A] time = 0.038, size = 89, normalized size = 1.6

$$2 \frac{a^2 \arctan(e^{dx+c})}{d} + 2 \frac{ab \sinh(dx+c)}{d} - 4 \frac{ab \arctan(e^{dx+c})}{d} + \frac{b^2 (\sinh(dx+c))^3}{3d} - \frac{b^2 \sinh(dx+c)}{d} + 2 \frac{b^2 \arctan(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)

[Out] $2/d*a^2*\arctan(\exp(d*x+c))+2/d*a*b*\sinh(d*x+c)-4/d*a*b*\arctan(\exp(d*x+c))+1/3*b^2*\sinh(d*x+c)^3/d-b^2*\sinh(d*x+c)/d+2/d*b^2*\arctan(\exp(d*x+c))$

Maxima [B] time = 1.72436, size = 180, normalized size = 3.27

$$-\frac{1}{24}b^2\left(\frac{(15e^{(-2dx-2c)}-1)e^{(3dx+3c)}}{d}-\frac{15e^{(-dx-c)}-e^{(-3dx-3c)}}{d}+\frac{48\arctan(e^{(-dx-c)})}{d}\right)+ab\left(\frac{4\arctan(e^{(-dx-c)})}{d}+\frac{e^{(dx+c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $-1/24*b^2*((15*e^{(-2*d*x - 2*c)} - 1)*e^{(3*d*x + 3*c)}/d - (15*e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/d + 48*\arctan(e^{(-d*x - c)})/d) + a*b*(4*\arctan(e^{(-d*x - c)})/d + e^{(d*x + c)}/d - e^{(-d*x - c)}/d) + a^2*\arctan(\sinh(d*x + c))/d$

Fricas [B] time = 1.60384, size = 1143, normalized size = 20.78

$$b^2 \cosh(dx + c)^6 + 6b^2 \cosh(dx + c) \sinh(dx + c)^5 + b^2 \sinh(dx + c)^6 + 3(8ab - 5b^2) \cosh(dx + c)^4 + 3(5b^2 \cosh(dx + c)^2 + 8ab - 5b^2) \sinh(dx + c)^4 + 4(5b^2 \cosh(dx + c)^3 + 3(8ab - 5b^2) \cosh(dx + c)) \sinh(dx + c)^3 - 3(8ab - 5b^2) \cosh(dx + c)^2 + 3(5b^2 \cosh(dx + c)^4 + 6(8ab - 5b^2) \cosh(dx + c)^2 - 8ab + 5b^2) \sinh(dx + c)^2 - b^2 + 48((a^2 - 2ab + b^2) \cosh(dx + c)^3 + 3(a^2 - 2ab + b^2) \cosh(dx + c)^2 \sinh(dx + c) + 3(a^2 - 2ab + b^2) \cosh(dx + c) \sinh(dx + c)^2 + (a^2 - 2ab + b^2) \sinh(dx + c)^3) \arctan(\cosh(dx + c) + \sinh(dx + c)) + 6(b^2 \cosh(dx + c)^5 + 2(8ab - 5b^2) \cosh(dx + c)^3 - (8ab - 5b^2) \cosh(dx + c)) \sinh(dx + c) / (d \cosh(dx + c)^3 + 3d \cosh(dx + c)^2 \sinh(dx + c) + 3d \cosh(dx + c) \sinh(dx + c)^2 + d \sinh(dx + c)^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $1/24*(b^2*\cosh(d*x + c)^6 + 6*b^2*\cosh(d*x + c)*\sinh(d*x + c)^5 + b^2*\sinh(d*x + c)^6 + 3*(8*a*b - 5*b^2)*\cosh(d*x + c)^4 + 3*(5*b^2*\cosh(d*x + c)^2 + 8*a*b - 5*b^2)*\sinh(d*x + c)^4 + 4*(5*b^2*\cosh(d*x + c)^3 + 3*(8*a*b - 5*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 3*(8*a*b - 5*b^2)*\cosh(d*x + c)^2 + 3*(5*b^2*\cosh(d*x + c)^4 + 6*(8*a*b - 5*b^2)*\cosh(d*x + c)^2 - 8*a*b + 5*b^2)*\sinh(d*x + c)^2 - b^2 + 48*((a^2 - 2*a*b + b^2)*\cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*\sinh(d*x + c)^3)*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) + 6*(b^2*\cosh(d*x + c)^5 + 2*(8*a*b - 5*b^2)*\cosh(d*x + c)^3 - (8*a*b - 5*b^2)*\cosh(d*x + c))*\sinh(d*x + c)/(d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + d*\sinh(d*x + c)^3)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.18227, size = 159, normalized size = 2.89

$$\frac{2(a^2 - 2ab + b^2) \arctan(e^{(dx+c)})}{d} - \frac{(24abe^{(2dx+2c)} - 15b^2e^{(2dx+2c)} + b^2)e^{(-3dx-3c)}}{24d} + \frac{b^2d^2e^{(3dx+3c)} + 24abd^2e^{(dx+c)} - 15b^2d^2e^{(3dx+3c)}}{24d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 2*(a^2 - 2*a*b + b^2)*arctan(e^(d*x + c))/d - 1/24*(24*a*b*e^(2*d*x + 2*c) - 15*b^2*e^(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c)/d + 1/24*(b^2*d^2*e^(3*d*x + 3*c) + 24*a*b*d^2*e^(d*x + c) - 15*b^2*d^2*e^(d*x + c))/d^3

3.298 $\int \operatorname{sech}^2(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=53

$$\frac{(a-b)^2 \tanh(c+dx)}{d} + \frac{1}{2}bx(4a-3b) + \frac{b^2 \sinh(c+dx) \cosh(c+dx)}{2d}$$

[Out] $((4*a - 3*b)*b*x)/2 + (b^2*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(2*d) + ((a - b)^2*\operatorname{Tanh}[c + d*x])/d$

Rubi [A] time = 0.0949116, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3191, 390, 385, 206}

$$\frac{(a-b)^2 \tanh(c+dx)}{d} + \frac{1}{2}bx(4a-3b) + \frac{b^2 \sinh(c+dx) \cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[c + d*x]^2*(a + b*\operatorname{Sinh}[c + d*x]^2)^2, x]$

[Out] $((4*a - 3*b)*b*x)/2 + (b^2*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(2*d) + ((a - b)^2*\operatorname{Tanh}[c + d*x])/d$

Rule 3191

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^{(m/2 + p + 1)}], x], x, \operatorname{Tan}[e + f*x]/ff], x]] \;/; \operatorname{FreeQ}\{a, b, e, f\}, x] \ \&\& \operatorname{IntegerQ}[m/2] \ \&\& \operatorname{IntegerQ}[p]$

Rule 390

$\operatorname{Int}[((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(a + b*x^n)^p, (c + d*x^n)^{-q}], x] \;/; \operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{ILtQ}[q, 0] \ \&\& \operatorname{GeQ}[p, -q]$

Rule 385

$\operatorname{Int}[((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow -\operatorname{Simp}[(b*c - a*d)*x*(a + b*x^n)^{(p + 1)}/(a*b*n*(p + 1)), x] - \operatorname{Dist}[(a*d - b$

c(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \operatorname{sech}^2(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a - (a-b)x^2)^2}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left((a-b)^2 + \frac{(2a-b)b - 2(a-b)bx^2}{(1-x^2)^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{(a-b)^2 \tanh(c + dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{(2a-b)b - 2(a-b)bx^2}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{b^2 \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{(a-b)^2 \tanh(c + dx)}{d} + \frac{((4a-3b)b) \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{1}{2}(4a-3b)bx + \frac{b^2 \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{(a-b)^2 \tanh(c + dx)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.291894, size = 50, normalized size = 0.94

$$\frac{2b(4a-3b)(c+dx) + 4(a-b)^2 \tanh(c+dx) + b^2 \sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2, x]

[Out] (2*(4*a - 3*b)*b*(c + d*x) + b^2*Sinh[2*(c + d*x)] + 4*(a - b)^2*Tanh[c + d*x])/(4*d)

Maple [A] time = 0.037, size = 71, normalized size = 1.3

$$\frac{1}{d} \left(a^2 \tanh(dx + c) + 2ab(dx + c - \tanh(dx + c)) + b^2 \left(\frac{(\sinh(dx + c))^3}{2 \cosh(dx + c)} - \frac{3dx}{2} - \frac{3c}{2} + \frac{3 \tanh(dx + c)}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*tanh(d*x+c)+2*a*b*(d*x+c-tanh(d*x+c))+b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))

Maxima [B] time = 1.21566, size = 161, normalized size = 3.04

$$2ab \left(x + \frac{c}{d} - \frac{2}{d(e^{(-2dx-2c)} + 1)} \right) - \frac{1}{8} b^2 \left(\frac{12(dx + c)}{d} + \frac{e^{(-2dx-2c)}}{d} - \frac{17e^{(-2dx-2c)} + 1}{d(e^{(-2dx-2c)} + e^{(-4dx-4c)})} \right) + \frac{2a^2}{d(e^{(-2dx-2c)} + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 2*a*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) - 1/8*b^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)))) + 2*a^2/(d*(e^(-2*d*x - 2*c) + 1))

Fricas [A] time = 1.53539, size = 235, normalized size = 4.43

$$\frac{b^2 \sinh(dx + c)^3 + 4 \left((4ab - 3b^2)dx - 2a^2 + 4ab - 2b^2 \right) \cosh(dx + c) + (3b^2 \cosh(dx + c)^2 + 8a^2 - 16ab + 9b^2) \sinh(dx + c)}{8d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/8*(b^2*sinh(d*x + c)^3 + 4*((4*a*b - 3*b^2)*d*x - 2*a^2 + 4*a*b - 2*b^2)*cosh(d*x + c) + (3*b^2*cosh(d*x + c)^2 + 8*a^2 - 16*a*b + 9*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.18936, size = 184, normalized size = 3.47

$$\frac{b^2 e^{(2dx+2c)}}{8d} + \frac{(4ab - 3b^2)(dx + c)}{2d} - \frac{4abe^{(4dx+4c)} - 3b^2e^{(4dx+4c)} + 16a^2e^{(2dx+2c)} - 28abe^{(2dx+2c)} + 14b^2e^{(2dx+2c)} + b^2}{8d(e^{(4dx+4c)} + e^{(2dx+2c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/8*b^2*e^(2*d*x + 2*c)/d + 1/2*(4*a*b - 3*b^2)*(d*x + c)/d - 1/8*(4*a*b*e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 28*a*b*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x + 2*c) + b^2)/(d*(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))

3.299 $\int \operatorname{sech}^3(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=64

$$\frac{(a + 3b)(a - b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(a - b)^2 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d} + \frac{b^2 \sinh(c + dx)}{d}$$

[Out] $((a - b)(a + 3b) \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*d) + (b^2 \operatorname{Sinh}[c + d*x])/d + (a - b)^2 \operatorname{Sech}[c + d*x] \operatorname{Tanh}[c + d*x]/(2*d)$

Rubi [A] time = 0.079862, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 390, 385, 203}

$$\frac{(a + 3b)(a - b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(a - b)^2 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d} + \frac{b^2 \sinh(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[c + d*x]^3 (a + b \operatorname{Sinh}[c + d*x]^2)^2, x]$

[Out] $((a - b)(a + 3b) \operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(2*d) + (b^2 \operatorname{Sinh}[c + d*x])/d + (a - b)^2 \operatorname{Sech}[c + d*x] \operatorname{Tanh}[c + d*x]/(2*d)$

Rule 3190

$\operatorname{Int}[\cos[(e_.) + (f_.)(x_.)]^{(m_.)} ((a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2} (a + b*ff^2*x^2)^p, x], x, \operatorname{Sin}[e + f*x]/ff], x]] /;$ $\operatorname{FreeQ}\{a, b, e, f, p\}, x] \ \&\& \ \operatorname{IntegerQ}[(m-1)/2]$

Rule 390

$\operatorname{Int}[((a_.) + (b_.)(x_.)^{(n_.)})^{(p_.)} ((c_.) + (d_.)(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(a + b*x^n)^p, (c + d*x^n)^{-q}], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{IGtQ}[p, 0] \ \&\& \ \operatorname{ILtQ}[q, 0] \ \&\& \ \operatorname{GeQ}[p, -q]$

Rule 385

$\operatorname{Int}[((a_.) + (b_.)(x_.)^{(n_.)})^{(p_.)} ((c_.) + (d_.)(x_.)^{(n_.)}), x_Symbol] \rightarrow -\operatorname{Simp}[(b*c - a*d)*x*(a + b*x^n)^{(p+1)} / (a*b*n*(p+1)), x] - \operatorname{Dist}[(a*d - b$

c(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \operatorname{sech}^3(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left(b^2 + \frac{a^2 - b^2 + 2(a-b)bx^2}{(1+x^2)^2}\right) dx, x, \sinh(c + dx)\right)}{d} \\
 &= \frac{b^2 \sinh(c + dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{a^2 - b^2 + 2(a-b)bx^2}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\
 &= \frac{b^2 \sinh(c + dx)}{d} + \frac{(a - b)^2 \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} + \frac{((a - b)(a + 3b)) \operatorname{Sech}(c + dx)}{2a} \\
 &= \frac{(a - b)(a + 3b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{b^2 \sinh(c + dx)}{d} + \frac{(a - b)^2 \operatorname{sech}(c + dx)}{2a}
 \end{aligned}$$

Mathematica [C] time = 7.36673, size = 253, normalized size = 3.95

$$\operatorname{csch}^3(c + dx) \left(-64 \sinh^6(c + dx) (a + b \sinh^2(c + dx))^2 \operatorname{HypergeometricPFQ}\left(\left\{\frac{3}{2}, 2, 2, 2\right\}, \left\{1, 1, \frac{9}{2}\right\}, -\sinh^2(c + dx)\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (Csch[c + d*x]^3*(-64*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 - 35*(a^2*(375 + 37*Sinh[c + d*x]^2) + b^2*Sinh[c + d*x]^4*(303 + 61*Sinh[c + d*x]^2) + 2*a*b*Si

nh[c + d*x]^2*(375 + 61*Sinh[c + d*x]^2)) + (105*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^2*Sinh[c + d*x]^4*(101 + 54*Sinh[c + d*x]^2 + Sinh[c + d*x]^4) + 2*a*b*Sinh[c + d*x]^2*(125 + 62*Sinh[c + d*x]^2 + Sinh[c + d*x]^4) + a^2*(125 + 54*Sinh[c + d*x]^2 + 9*Sinh[c + d*x]^4)))/Sqrt[-Sinh[c + d*x]^2]))/(1680*d)

Maple [B] time = 0.047, size = 169, normalized size = 2.6

$$\frac{a^2 \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{a^2 \arctan(e^{dx+c})}{d} - 2 \frac{ab \sinh(dx+c)}{d (\cosh(dx+c))^2} + \frac{ab \operatorname{sech}(dx+c) \tanh(dx+c)}{d} + 2 \frac{ab \arctan(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/2/d*a^2*sech(d*x+c)*tanh(d*x+c)+1/d*a^2*arctan(exp(d*x+c))-2/d*a*b*sinh(d*x+c)/cosh(d*x+c)^2+1/d*a*b*sech(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(exp(d*x+c))+1/d*b^2*sinh(d*x+c)^3/cosh(d*x+c)^2+3/d*b^2*sinh(d*x+c)/cosh(d*x+c)^2-3/2/d*b^2*sech(d*x+c)*tanh(d*x+c)-3/d*b^2*arctan(exp(d*x+c))

Maxima [B] time = 1.89297, size = 316, normalized size = 4.94

$$\frac{1}{2} b^2 \left(\frac{6 \arctan(e^{-dx-c})}{d} - \frac{e^{-dx-c}}{d} + \frac{4e^{-2dx-2c} - e^{-4dx-4c} + 1}{d(e^{-dx-c} + 2e^{-3dx-3c} + e^{-5dx-5c})} \right) - 2ab \left(\frac{\arctan(e^{-dx-c})}{d} + \frac{e^{-dx-c} - e^{-3dx-3c}}{d(2e^{-2dx-2c} + e^{-4dx-4c} + 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) - 2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a^2*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))

Fricas [B] time = 1.62602, size = 1916, normalized size = 29.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $\frac{1}{2}(b^2 \cosh(dx+c)^6 + 6b^2 \cosh(dx+c) \sinh(dx+c)^5 + b^2 \sinh(dx+c)^6 + (2a^2 - 4ab + 3b^2) \cosh(dx+c)^4 + (15b^2 \cosh(dx+c)^2 + 2a^2 - 4ab + 3b^2) \sinh(dx+c)^4 + 4(5b^2 \cosh(dx+c)^3 + (2a^2 - 4ab + 3b^2) \cosh(dx+c) \sinh(dx+c)^3 - (2a^2 - 4ab + 3b^2) \cosh(dx+c)^2 + (15b^2 \cosh(dx+c)^4 + 6(2a^2 - 4ab + 3b^2) \cosh(dx+c)^2 - 2a^2 + 4ab - 3b^2) \sinh(dx+c)^2 - b^2 + 2((a^2 + 2ab - 3b^2) \cosh(dx+c)^5 + 5(a^2 + 2ab - 3b^2) \cosh(dx+c) \sinh(dx+c)^4 + (a^2 + 2ab - 3b^2) \sinh(dx+c)^5 + 2(a^2 + 2ab - 3b^2) \cosh(dx+c)^3 + 2(5(a^2 + 2ab - 3b^2) \cosh(dx+c)^2 + a^2 + 2ab - 3b^2) \sinh(dx+c)^3 + 2(5(a^2 + 2ab - 3b^2) \cosh(dx+c)^3 + 3(a^2 + 2ab - 3b^2) \cosh(dx+c)) \sinh(dx+c)^2 + (a^2 + 2ab - 3b^2) \cosh(dx+c) + (5(a^2 + 2ab - 3b^2) \cosh(dx+c)^4 + 6(a^2 + 2ab - 3b^2) \cosh(dx+c)^2 + a^2 + 2ab - 3b^2) \sinh(dx+c)) \arctan(\cosh(dx+c) + \sinh(dx+c)) + 2(3b^2 \cosh(dx+c)^5 + 2(2a^2 - 4ab + 3b^2) \cosh(dx+c)^3 - (2a^2 - 4ab + 3b^2) \cosh(dx+c)) \sinh(dx+c)) / (d \cosh(dx+c)^5 + 5d \cosh(dx+c) \sinh(dx+c)^4 + d \sinh(dx+c)^5 + 2d \cosh(dx+c)^3 + 2(5d \cosh(dx+c)^2 + d) \sinh(dx+c)^3 + 2(5d \cosh(dx+c)^3 + 3d \cosh(dx+c)) \sinh(dx+c)^2 + d \cosh(dx+c) + (5d \cosh(dx+c)^4 + 6d \cosh(dx+c)^2 + d) \sinh(dx+c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.2036, size = 225, normalized size = 3.52

$$\frac{b^2(e^{(dx+c)} - e^{(-dx-c)})}{2d} + \frac{\left(\pi + 2 \arctan\left(\frac{1}{2}(e^{(2dx+2c)} - 1)e^{(-dx-c)}\right)\right)(a^2 + 2ab - 3b^2)}{4d} + \frac{a^2(e^{(dx+c)} - e^{(-dx-c)}) - 2ab(e^{(dx+c)} - e^{(-dx-c)})}{(e^{(dx+c)} - e^{(-dx-c)})^2 + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*b^2*(e^(d*x + c) - e^(-d*x - c))/d + 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^2 + 2*a*b - 3*b^2)/d + (a^2*(e^(d*x + c) - e^(-d*x - c)) - 2*a*b*(e^(d*x + c) - e^(-d*x - c)) + b^2*(e^(d*x + c) - e^(-d*x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)*d)

3.300 $\int \operatorname{sech}^4(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=47

$$\frac{(a^2 - b^2) \tanh(c + dx)}{d} - \frac{(a - b)^2 \tanh^3(c + dx)}{3d} + b^2 x$$

[Out] $b^2 x + ((a^2 - b^2) \operatorname{Tanh}[c + d x]) / d - ((a - b)^2 \operatorname{Tanh}[c + d x]^3) / (3 d)$

Rubi [A] time = 0.062746, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3191, 390, 206}

$$\frac{(a^2 - b^2) \tanh(c + dx)}{d} - \frac{(a - b)^2 \tanh^3(c + dx)}{3d} + b^2 x$$

Antiderivative was successfully verified.

[In] `Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]`

[Out] $b^2 x + ((a^2 - b^2) \operatorname{Tanh}[c + d x]) / d - ((a - b)^2 \operatorname{Tanh}[c + d x]^3) / (3 d)$

Rule 3191

`Int[cos[(e_.) + (f_.)*(x_.)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]`

Rule 390

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^4(c+dx) (a+b \sinh^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-(a-b)x^2)^2}{1-x^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(a^2 - b^2 - (a-b)^2 x^2 + \frac{b^2}{1-x^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{(a^2 - b^2) \tanh(c+dx)}{d} - \frac{(a-b)^2 \tanh^3(c+dx)}{3d} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{1-x^2} dx, x\right)}{d} \\
&= b^2 x + \frac{(a^2 - b^2) \tanh(c+dx)}{d} - \frac{(a-b)^2 \tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.331525, size = 57, normalized size = 1.21

$$\frac{(a-b) \tanh(c+dx) \operatorname{sech}^2(c+dx) ((a+2b) \cosh(2(c+dx)) + 2a+b) + 3b^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (3*b^2*(c + d*x) + (a - b)*(2*a + b + (a + 2*b)*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Tanh[c + d*x])/(3*d)

Maple [B] time = 0.046, size = 96, normalized size = 2.

$$\frac{1}{d} \left(a^2 \left(\frac{2}{3} + \frac{(\operatorname{sech}(dx+c))^2}{3} \right) \tanh(dx+c) + 2ab \left(-\frac{1}{2} \frac{\sinh(dx+c)}{(\cosh(dx+c))^3} + \frac{1}{2} \left(\frac{2}{3} + \frac{1}{3} (\operatorname{sech}(dx+c))^2 \right) \tanh(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)

Maxima [B] time = 1.11575, size = 360, normalized size = 7.66

$$\frac{1}{3}b^2\left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)}\right) + \frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} + d\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))

Fricas [B] time = 1.48203, size = 495, normalized size = 10.53

$$\frac{(3b^2dx - 2a^2 - 2ab + 4b^2) \cosh(dx + c)^3 + 3(3b^2dx - 2a^2 - 2ab + 4b^2) \cosh(dx + c) \sinh(dx + c)^2 + 2(a^2 + ab - 2b^2) \sinh(dx + c)^3}{3(d \cosh(dx + c))^3 + 3d \cosh(dx + c) \sinh(dx + c)^2 + 3d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] 1/3*((3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b - 2*b^2)*sinh(d*x + c)^3 + 3*(3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c) + 6*((a^2 + a*b - 2*b^2)*cosh(d*x + c)^2 + a^2 - a*b)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.1663, size = 132, normalized size = 2.81

$$\frac{(dx+c)b^2}{d} - \frac{4(3abe^{4dx+4c} - 3b^2e^{4dx+4c} + 3a^2e^{2dx+2c} - 3b^2e^{2dx+2c} + a^2 + ab - 2b^2)}{3d(e^{2dx+2c} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] (d*x + c)*b^2/d - 4/3*(3*a*b*e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c) + a^2 + a*b - 2*b^2)/(d*(e^(2*d*x + 2*c) + 1)^3)

3.301 $\int \operatorname{sech}^5(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=96

$$\frac{(3a^2 + 2ab + 3b^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{3(a^2 - b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{(a - b) \tanh(c + dx) \operatorname{sech}^3(c + dx) (a + b \sinh^2(c + dx))^2}{4d}$$

[Out] $((3*a^2 + 2*a*b + 3*b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*d) + (3*(a^2 - b^2)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*d) + ((a - b)*\operatorname{Sech}[c + d*x]^3*(a + b*\operatorname{Sinh}[c + d*x]^2)*\operatorname{Tanh}[c + d*x])/(4*d)$

Rubi [A] time = 0.0927501, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 413, 385, 203}

$$\frac{(3a^2 + 2ab + 3b^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{3(a^2 - b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{8d} + \frac{(a - b) \tanh(c + dx) \operatorname{sech}^3(c + dx) (a + b \sinh^2(c + dx))^2}{4d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[c + d*x]^5*(a + b*\operatorname{Sinh}[c + d*x]^2)^2, x]$

[Out] $((3*a^2 + 2*a*b + 3*b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*d) + (3*(a^2 - b^2)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*d) + ((a - b)*\operatorname{Sech}[c + d*x]^3*(a + b*\operatorname{Sinh}[c + d*x]^2)*\operatorname{Tanh}[c + d*x])/(4*d)$

Rule 3190

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*ff^2*x^2)^p, x], x, \operatorname{Sin}[e + f*x]/ff], x]] /; \operatorname{FreeQ}[\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 413

$\operatorname{Int}(((a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Simp}(((a*d - c*b)*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)})/(a*b*n*(p+1)), x] - \operatorname{Dist}[1/(a*b*n*(p+1)), \operatorname{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-2)}*\operatorname{Simp}[c*(a*d - c*b*(n*(p+1) + 1)) + d*(a*d*(n*(q-1) + 1) - b*c*(n*(p+q) + 1)]*x^n, x], x] /; \operatorname{FreeQ}[\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[p, -1] \&\& \operatorname{GtQ}[q, 1] \&\& \operatorname{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^5(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(a - b)\operatorname{sech}^3(c + dx) (a + b \sinh^2(c + dx)) \tanh(c + dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{a(3a+b^2x^2)}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{4d} \\ &= \frac{3(a^2 - b^2)\operatorname{sech}(c + dx) \tanh(c + dx)}{8d} + \frac{(a - b)\operatorname{sech}^3(c + dx) (a + b \sinh^2(c + dx)) \tanh(c + dx)}{4d} \\ &= \frac{(3a^2 + 2ab + 3b^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{3(a^2 - b^2)\operatorname{sech}(c + dx) \tanh(c + dx)}{8d} \end{aligned}$$

Mathematica [C] time = 5.94609, size = 303, normalized size = 3.16

$$\operatorname{csch}^3(c + dx) \left(128 \sinh^6(c + dx) (7a^2 + 12ab \sinh^2(c + dx) + 5b^2 \sinh^4(c + dx)) \operatorname{HypergeometricPFQ}\left(\left\{\frac{3}{2}, 2, 2, 2\right\}, \left\{1, 1, 1, 9/2\right\}, \sinh^2(c + dx)\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] -(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 + 128*Hypergeo

metricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(7*a^2 + 12*a*b*Sinh[c + d*x]^2 + 5*b^2*Sinh[c + d*x]^4) + 35*(3375*a^2 + a*(657*a + 4643*b + 607*b*Cosh[2*(c + d*x)])*Sinh[c + d*x]^2 + 1947*b^2*Sinh[c + d*x]^4 + 485*b^2*Sinh[c + d*x]^6) - (105*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(1125*a^2 + 2*a*(297*a + 875*b)*Sinh[c + d*x]^2 + (37*a^2 + 988*a*b + 649*b^2)*Sinh[c + d*x]^4 + 2*b*(11*a + 189*b)*Sinh[c + d*x]^6 + 9*b^2*Sinh[c + d*x]^8))/Sqrt[-Sinh[c + d*x]^2]))/(6720*d)

Maple [B] time = 0.051, size = 237, normalized size = 2.5

$$\frac{a^2 \tanh(dx + c) (\operatorname{sech}(dx + c))^3}{4d} + \frac{3a^2 \operatorname{sech}(dx + c) \tanh(dx + c)}{8d} + \frac{3a^2 \arctan(e^{dx+c})}{4d} - \frac{2ab \sinh(dx + c)}{3d (\cosh(dx + c))^4} + \frac{ab \tanh(dx + c)}{3d (\cosh(dx + c))^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/4/d*a^2*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a^2*sech(d*x+c)*tanh(d*x+c)+3/4/d*a^2*arctan(exp(d*x+c))-2/3/d*a*b*sinh(d*x+c)/cosh(d*x+c)^4+1/6/d*a*b*tanh(d*x+c)*sech(d*x+c)^3+1/4/d*a*b*sech(d*x+c)*tanh(d*x+c)+1/2/d*a*b*arctan(exp(d*x+c))-1/d*b^2*sinh(d*x+c)^3/cosh(d*x+c)^4-1/d*b^2*sinh(d*x+c)/cosh(d*x+c)^4+1/4/d*b^2*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*b^2*sech(d*x+c)*tanh(d*x+c)+3/4/d*b^2*arctan(exp(d*x+c))

Maxima [B] time = 1.72498, size = 468, normalized size = 4.88

$$-\frac{1}{4}b^2\left(\frac{3\arctan(e^{-dx-c})}{d} + \frac{5e^{-dx-c} - 3e^{-3dx-3c} + 3e^{-5dx-5c} - 5e^{-7dx-7c}}{d(4e^{-2dx-2c} + 6e^{-4dx-4c} + 4e^{-6dx-6c} + e^{-8dx-8c} + 1)}\right) - \frac{1}{4}a^2\left(\frac{3\arctan(e^{-dx-c})}{d} - \frac{3e^{-5dx-5c} - 5e^{-7dx-7c}}{d(4e^{-2dx-2c} + 6e^{-4dx-4c} + 4e^{-6dx-6c} + e^{-8dx-8c} + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/4*a^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/2*a*b*(arctan(e^(-d*x - c))

$$\frac{1}{d} - \frac{(e^{-(d*x - c)} - 7*e^{(-3*d*x - 3*c)} + 7*e^{(-5*d*x - 5*c)} - e^{(-7*d*x - 7*c)})}{(d*(4*e^{(-2*d*x - 2*c)} + 6*e^{(-4*d*x - 4*c)} + 4*e^{(-6*d*x - 6*c)} + e^{(-8*d*x - 8*c)} + 1))}$$

Fricas [B] time = 1.5918, size = 3672, normalized size = 38.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $\frac{1}{4} * ((3*a^2 + 2*a*b - 5*b^2) * \cosh(d*x + c)^7 + 7*(3*a^2 + 2*a*b - 5*b^2) * \cosh(d*x + c) * \sinh(d*x + c)^6 + (3*a^2 + 2*a*b - 5*b^2) * \sinh(d*x + c)^7 + (11*a^2 - 14*a*b + 3*b^2) * \cosh(d*x + c)^5 + (21*(3*a^2 + 2*a*b - 5*b^2) * \cosh(d*x + c)^2 + 11*a^2 - 14*a*b + 3*b^2) * \sinh(d*x + c)^5 + 5*(7*(3*a^2 + 2*a*b - 5*b^2) * \cosh(d*x + c)^3 + (11*a^2 - 14*a*b + 3*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^4 - (11*a^2 - 14*a*b + 3*b^2) * \cosh(d*x + c)^3 + (35*(3*a^2 + 2*a*b - 5*b^2) * \cosh(d*x + c)^4 + 10*(11*a^2 - 14*a*b + 3*b^2) * \cosh(d*x + c)^2 - 11*a^2 + 14*a*b - 3*b^2) * \sinh(d*x + c)^3 + (21*(3*a^2 + 2*a*b - 5*b^2) * \cosh(d*x + c)^5 + 10*(11*a^2 - 14*a*b + 3*b^2) * \cosh(d*x + c)^3 - 3*(11*a^2 - 14*a*b + 3*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^2 + ((3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^8 + 8*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c) * \sinh(d*x + c)^7 + (3*a^2 + 2*a*b + 3*b^2) * \sinh(d*x + c)^8 + 4*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^6 + 4*(7*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^2 + 3*a^2 + 2*a*b + 3*b^2) * \sinh(d*x + c)^6 + 8*(7*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^3 + 3*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^5 + 6*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^4 + 2*(35*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^4 + 30*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^2 + 9*a^2 + 6*a*b + 9*b^2) * \sinh(d*x + c)^4 + 8*(7*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^5 + 10*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^3 + 3*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 4*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^2 + 4*(7*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^6 + 15*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^4 + 9*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^2 + 3*a^2 + 2*a*b + 3*b^2) * \sinh(d*x + c)^2 + 3*a^2 + 2*a*b + 3*b^2 + 8*((3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^7 + 3*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^5 + 3*(3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)^3 + (3*a^2 + 2*a*b + 3*b^2) * \cosh(d*x + c)) * \sinh(d*x + c)) * \arctan(\cosh(d*x + c) + \sinh(d*x + c)) - (3*a^2 + 2*a*b - 5*b^2) * \cosh(d*x + c) + (7*(3*a^2 + 2*a*b - 5*b^2) * \cosh(d*x + c)^6 + 5*(11*a^2 - 14*a*b + 3*b^2) * \cosh(d*x + c)^4 - 3*(11*a^2 - 14*a*b + 3*b^2) * \cosh(d*x + c)^2 - 3*a^2 - 2*a*b + 5*b^2) * \sinh(d*x + c)) / (d * \cosh(d*x + c)^8 + 8*d * \cosh(d*x + c) * \sinh(d*x + c)^7 + d * \sinh(d*x + c)^8 + 4*d * \cosh(d*x + c)^6 + 4*(7*d * \cosh(d*x + c)^2 + d) * \sinh(d$

$x + c)^6 + 8(7d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^5 + 6$
 $d \cosh(dx + c)^4 + 2(35d \cosh(dx + c)^4 + 30d \cosh(dx + c)^2 + 3d)$
 $\sinh(dx + c)^4 + 8(7d \cosh(dx + c)^5 + 10d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^3 + 4d \cosh(dx + c)^2 + 4(7d \cosh(dx + c)^6 +$
 $15d \cosh(dx + c)^4 + 9d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 8(d \cosh(dx + c)^7 + 3d \cosh(dx + c)^5 + 3d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c) + d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)**5*(a+b*sinh(dx+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.16042, size = 297, normalized size = 3.09

$$\frac{\left(\pi + 2 \arctan\left(\frac{1}{2} \left(e^{2dx+2c} - 1\right) e^{-dx-c}\right)\right) \left(3a^2 + 2ab + 3b^2\right)}{16d} + \frac{3a^2 \left(e^{dx+c} - e^{-dx-c}\right)^3 + 2ab \left(e^{dx+c} - e^{-dx-c}\right)^3 - 5b^2 \left(e^{dx+c} - e^{-dx-c}\right)^3}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^5*(a+b*sinh(dx+c)^2)^2,x, algorithm="giac")

[Out] 1/16*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a^2 + 2*a*b + 3*b^2)/d + 1/4*(3*a^2*(e^(d*x + c) - e^(-d*x - c))^3 + 2*a*b*(e^(d*x + c) - e^(-d*x - c))^3 - 5*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 20*a^2*(e^(d*x + c) - e^(-d*x - c)) - 8*a*b*(e^(d*x + c) - e^(-d*x - c)) - 12*b^2*(e^(d*x + c) - e^(-d*x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2*d)

3.302 $\int \operatorname{sech}^6(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=57

$$\frac{a^2 \tanh(c + dx)}{d} + \frac{(a - b)^2 \tanh^5(c + dx)}{5d} - \frac{2a(a - b) \tanh^3(c + dx)}{3d}$$

[Out] (a^2*Tanh[c + d*x])/d - (2*a*(a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)^2*Tanh[c + d*x]^5)/(5*d)

Rubi [A] time = 0.0601492, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3191, 194}

$$\frac{a^2 \tanh(c + dx)}{d} + \frac{(a - b)^2 \tanh^5(c + dx)}{5d} - \frac{2a(a - b) \tanh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (a^2*Tanh[c + d*x])/d - (2*a*(a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)^2*Tanh[c + d*x]^5)/(5*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 194

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^6(c+dx) (a+b \sinh^2(c+dx))^2 dx &= \frac{\operatorname{Subst}\left(\int (a-(a-b)x^2)^2 dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int (a^2-2a(a-b)x^2+(a-b)^2x^4) dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{a^2 \tanh(c+dx)}{d} - \frac{2a(a-b) \tanh^3(c+dx)}{3d} + \frac{(a-b)^2 \tanh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.373003, size = 69, normalized size = 1.21

$$\frac{\tanh(c+dx) \left(2(2a^2+ab-3b^2) \operatorname{sech}^2(c+dx) + 8a^2 + 3(a-b)^2 \operatorname{sech}^4(c+dx) + 4ab + 3b^2\right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((8*a^2 + 4*a*b + 3*b^2 + 2*(2*a^2 + a*b - 3*b^2)*Sech[c + d*x]^2 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(15*d)

Maple [B] time = 0.048, size = 158, normalized size = 2.8

$$\frac{1}{d} \left(a^2 \left(\frac{8}{15} + \frac{(\operatorname{sech}(dx+c))^4}{5} + \frac{4(\operatorname{sech}(dx+c))^2}{15} \right) \tanh(dx+c) + 2ab \left(-\frac{1}{4} \frac{\sinh(dx+c)}{(\cosh(dx+c))^5} + \frac{1}{4} \left(\frac{8}{15} + \frac{1}{5} (\operatorname{sech}(dx+c))^4 \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*(a^2*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))

Maxima [B] time = 1.12718, size = 942, normalized size = 16.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out]
$$\frac{16}{15}a^2 \frac{(5e^{-2dx-2c})}{(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + 10e^{-4dx-4c} \frac{(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + 1/(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)) + 8/15ab \frac{(5e^{-2dx-2c})}{(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} - 5e^{-4dx-4c} \frac{(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + 15e^{-6dx-6c} \frac{(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + 1/(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1)) + 2/5b^2 \frac{(10e^{-4dx-4c})}{(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + 5e^{-8dx-8c} \frac{(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))} + 1/(d(5e^{-2dx-2c}) + 10e^{-4dx-4c} + 10e^{-6dx-6c} + 5e^{-8dx-8c} + e^{-10dx-10c} + 1))$$

Fricas [B] time = 1.45971, size = 1040, normalized size = 18.25

$$\frac{4 \left((4a^2 + 2ab + 9b^2) \cosh(dx+c)^4 - 8(2a^2 + ab - 3b^2) \cosh(dx+c) \sinh(dx+c)^3 + (4a^2 + 2ab - 3b^2) \sinh(dx+c)^4 \right)}{15 \left(d \cosh(dx+c)^6 + 6d \cosh(dx+c) \sinh(dx+c)^5 + d \sinh(dx+c)^6 + 6d \cosh(dx+c)^4 + 3(5d \cosh(dx+c)^2 + 5d) \sinh(dx+c)^3 + 3(5d \cosh(dx+c)^2 + 5d) \sinh(dx+c) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\frac{-4/15 \left((4a^2 + 2ab + 9b^2) \cosh(dx+c)^4 - 8(2a^2 + ab - 3b^2) \cosh(dx+c) \sinh(dx+c)^3 + (4a^2 + 2ab - 3b^2) \sinh(dx+c)^4 + 20(a^2 + 2ab) \cosh(dx+c)^2 + 2(3(4a^2 + 2ab + 9b^2) \cosh(dx+c)^2 + 10a^2 + 20ab) \sinh(dx+c)^2 + 40a^2 - 10ab + 15b^2 - 8((2a^2 + ab - 3b^2) \cosh(dx+c)^3 + 5(a^2 - ab) \cosh(dx+c)) \sinh(dx+c) \right)}{(d \cosh(dx+c)^6 + 6d \cosh(dx+c) \sinh(dx+c)^5 + d \sinh(dx+c)^6 + 6d \cosh(dx+c)^4 + 3(5d \cosh(dx+c)^2 + 2d) \sinh(dx+c)^4 + 4(5d \cosh(dx+c)^3 + 4d \cosh(dx+c)) \sinh(dx+c)^3 + 15d \cosh(dx+c)^2 + 3(5d \cosh(dx+c)^4 + 12d \cosh(dx+c)^2 + 5d) \sinh(dx+c)}$$

)² + 2*(3*d*cosh(d*x + c)⁵ + 8*d*cosh(d*x + c)³ + 5*d*cosh(d*x + c))*sinh(d*x + c) + 10*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.22235, size = 173, normalized size = 3.04

$$\frac{2 \left(15 b^2 e^{(8dx+8c)} + 60 a b e^{(6dx+6c)} + 80 a^2 e^{(4dx+4c)} - 20 a b e^{(4dx+4c)} + 30 b^2 e^{(4dx+4c)} + 40 a^2 e^{(2dx+2c)} + 20 a b e^{(2dx+2c)} + 8 a^2 \right)}{15 d \left(e^{(2dx+2c)} + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] -2/15*(15*b²*e^(8*d*x + 8*c) + 60*a*b*e^(6*d*x + 6*c) + 80*a²*e^(4*d*x + 4*c) - 20*a*b*e^(4*d*x + 4*c) + 30*b²*e^(4*d*x + 4*c) + 40*a²*e^(2*d*x + 2*c) + 20*a*b*e^(2*d*x + 2*c) + 8*a² + 4*a*b + 3*b²)/(d*(e^(2*d*x + 2*c) + 1)⁵)

3.303 $\int \operatorname{sech}^7(c + dx) (a + b \sinh^2(c + dx))^2 dx$

Optimal. Leaf size=131

$$\frac{(5a^2 + 2ab + b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{(5a^2 + 2ab + b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{16d} + \frac{(a - b)(5a + 3b) \tanh(c + dx) \operatorname{sech}^3(c + dx)}{24d}$$

```
[Out] ((5*a^2 + 2*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((5*a^2 + 2*a*b + b^2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Sech[c + d*x]^3*Tanh[c + d*x])/(24*d) + ((a - b)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)*Tanh[c + d*x])/(6*d)
```

Rubi [A] time = 0.132593, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3190, 413, 385, 199, 203}

$$\frac{(5a^2 + 2ab + b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{(5a^2 + 2ab + b^2) \tanh(c + dx) \operatorname{sech}(c + dx)}{16d} + \frac{(a - b)(5a + 3b) \tanh(c + dx) \operatorname{sech}^3(c + dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 2*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((5*a^2 + 2*a*b + b^2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Sech[c + d*x]^3*Tanh[c + d*x])/(24*d) + ((a - b)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)*Tanh[c + d*x])/(6*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
```

+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \operatorname{sech}^7(c + dx) (a + b \sinh^2(c + dx))^2 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^2}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{d} \\
 &= \frac{(a - b)\operatorname{sech}^5(c + dx) (a + b \sinh^2(c + dx)) \tanh(c + dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{a(5a+b)}{(1+x^2)^4} dx, x, \sinh(c + dx)\right)}{6d} \\
 &= \frac{(a - b)(5a + 3b)\operatorname{sech}^3(c + dx) \tanh(c + dx)}{24d} + \frac{(a - b)\operatorname{sech}^5(c + dx) (a + b \sinh^2(c + dx))}{6d} \\
 &= \frac{(5a^2 + 2ab + b^2) \operatorname{sech}(c + dx) \tanh(c + dx)}{16d} + \frac{(a - b)(5a + 3b)\operatorname{sech}^3(c + dx)}{24d} \\
 &= \frac{(5a^2 + 2ab + b^2) \tan^{-1}(\sinh(c + dx))}{16d} + \frac{(5a^2 + 2ab + b^2) \operatorname{sech}(c + dx) \tanh(c + dx)}{16d}
 \end{aligned}$$

Mathematica [C] time = 10.1684, size = 715, normalized size = 5.46

$$\operatorname{csch}^3(c+dx) \left(32 \left(-\sinh^2(c+dx) \right)^{3/2} \sinh^4(c+dx) \left(5a^2 + 9ab \sinh^2(c+dx) + 4b^2 \sinh^4(c+dx) \right) \operatorname{HypergeometricP}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (Csch[c + d*x]^3*(65625*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] + 36855*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 + 91875*a*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 + 1680*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 + 54180*a*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 + 32970*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 + 1365*a*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 + 19845*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 + 525*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8 - 65625*a^2*Sqrt[-Sinh[c + d*x]^2] + 14980*a^2*(-Sinh[c + d*x]^2)^(3/2) + 91875*a*b*(-Sinh[c + d*x]^2)^(3/2) + 8855*b^2*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2) + 16*a^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2) + 32*a*b*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2) + 16*b^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(-Sinh[c + d*x]^2)^(3/2) - 23555*a*b*(-Sinh[c + d*x]^2)^(5/2) - 32970*b^2*(-Sinh[c + d*x]^2)^(5/2) + 32*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2)*(5*a^2 + 9*a*b*Sinh[c + d*x]^2 + 4*b^2*Sinh[c + d*x]^4) + 4*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2)*(155*a^2 + 242*a*b*Sinh[c + d*x]^2 + 95*b^2*Sinh[c + d*x]^4))/(2520*d*Sqrt[-Sinh[c + d*x]^2])

Maple [B] time = 0.082, size = 302, normalized size = 2.3

$$\frac{a^2 \tanh(dx+c) (\operatorname{sech}(dx+c))^5}{6d} + \frac{5a^2 \tanh(dx+c) (\operatorname{sech}(dx+c))^3}{24d} + \frac{5a^2 \operatorname{sech}(dx+c) \tanh(dx+c)}{16d} + \frac{5a^2 \arctan}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x)

[Out] $1/6/d*a^2*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^5+5/24/d*a^2*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^3+5/16/d*a^2*\operatorname{sech}(d*x+c)*\tanh(d*x+c)+5/8/d*a^2*\arctan(\exp(d*x+c))-2/5/d*a*b*\sinh(d*x+c)/\cosh(d*x+c)^6+1/15/d*a*b*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^5+1/12/d*a*b*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^3+1/8/d*a*b*\operatorname{sech}(d*x+c)*\tanh(d*x+c)+1/4/d*a*b*\arctan(\exp(d*x+c))-1/3/d*b^2*\sinh(d*x+c)^3/\cosh(d*x+c)^6-1/5/d*b^2*\sinh(d*x+c)/\cosh(d*x+c)^6+1/30/d*b^2*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^5+1/24/d*b^2*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^3+1/16/d*b^2*\operatorname{sech}(d*x+c)*\tanh(d*x+c)+1/8/d*b^2*\arctan(\exp(d*x+c))$

Maxima [B] time = 1.79108, size = 652, normalized size = 4.98

$$-\frac{1}{24}a^2\left(\frac{15\arctan(e^{-dx-c})}{d}-\frac{15e^{-dx-c}+85e^{-3dx-3c}+198e^{-5dx-5c}-198e^{-7dx-7c}-85e^{-9dx-9c}-15e^{-11dx-11c}}{d(6e^{-2dx-2c}+15e^{-4dx-4c}+20e^{-6dx-6c}+15e^{-8dx-8c}+6e^{-10dx-10c}+e^{-12dx-12c})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $-1/24*a^2*(15*\arctan(e^{-d*x-c})/d - (15*e^{-d*x-c} + 85*e^{-3*d*x-3*c} + 198*e^{-5*d*x-5*c} - 198*e^{-7*d*x-7*c} - 85*e^{-9*d*x-9*c} - 15*e^{-11*d*x-11*c})/(d*(6*e^{-2*d*x-2*c} + 15*e^{-4*d*x-4*c} + 20*e^{-6*d*x-6*c} + 15*e^{-8*d*x-8*c} + 6*e^{-10*d*x-10*c} + e^{-12*d*x-12*c} + 1))) - 1/12*a*b*(3*\arctan(e^{-d*x-c})/d - (3*e^{-d*x-c} + 17*e^{-3*d*x-3*c} - 114*e^{-5*d*x-5*c} + 114*e^{-7*d*x-7*c} - 17*e^{-9*d*x-9*c} - 3*e^{-11*d*x-11*c})/(d*(6*e^{-2*d*x-2*c} + 15*e^{-4*d*x-4*c} + 20*e^{-6*d*x-6*c} + 15*e^{-8*d*x-8*c} + 6*e^{-10*d*x-10*c} + e^{-12*d*x-12*c} + 1))) - 1/24*b^2*(3*\arctan(e^{-d*x-c})/d - (3*e^{-d*x-c} - 47*e^{-3*d*x-3*c} + 78*e^{-5*d*x-5*c} - 78*e^{-7*d*x-7*c} + 47*e^{-9*d*x-9*c} - 3*e^{-11*d*x-11*c})/(d*(6*e^{-2*d*x-2*c} + 15*e^{-4*d*x-4*c} + 20*e^{-6*d*x-6*c} + 15*e^{-8*d*x-8*c} + 6*e^{-10*d*x-10*c} + e^{-12*d*x-12*c} + 1)))$

Fricas [B] time = 1.71361, size = 7356, normalized size = 56.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

```
[Out] 1/24*(3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 + 33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^11 + (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 85*a^2 + 34*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 + 6*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c))^2 + 33*a^2 - 38*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^3 + (33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^5 + 6*(231*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 21*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^2 - 33*a^2 + 38*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(165*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 21*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^5 + 35*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^3 + (495*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^6 + 210*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^4 - 60*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^2 - 85*a^2 - 34*a*b + 47*b^2)*sinh(d*x + c)^3 + 3*(55*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 12*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^7 + 42*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^5 - 20*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^3 - (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 12*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 6*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^10 + 6*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^10 + 20*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 15*(33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 18*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 24*(33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 30*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 315*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(5*a^2 + 2*a*b + b^2))*cosh(d*x + c)^2 + 25*a^2 + 10*a*b + 5*b^2)*sinh(d*x + c)^6 + 24*(33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 63*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 35*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 70*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 20*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 36*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 42*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 20*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 6*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
```

$$\begin{aligned}
& c)^{10} + 45(5a^2 + 2ab + b^2)\cosh(dx + c)^8 + 70(5a^2 + 2ab + b^2) \\
& * \cosh(dx + c)^6 + 50(5a^2 + 2ab + b^2)\cosh(dx + c)^4 + 15(5a^2 + 2 \\
& * ab + b^2)\cosh(dx + c)^2 + 5a^2 + 2ab + b^2)\sinh(dx + c)^2 + 5a^2 \\
& + 2ab + b^2 + 12((5a^2 + 2ab + b^2)\cosh(dx + c)^{11} + 5(5a^2 + 2a \\
& * b + b^2)\cosh(dx + c)^9 + 10(5a^2 + 2ab + b^2)\cosh(dx + c)^7 + 10(\\
& 5a^2 + 2ab + b^2)\cosh(dx + c)^5 + 5(5a^2 + 2ab + b^2)\cosh(dx + c \\
&)^3 + (5a^2 + 2ab + b^2)\cosh(dx + c))\sinh(dx + c))\arctan(\cosh(dx + \\
& c) + \sinh(dx + c)) - 3(5a^2 + 2ab + b^2)\cosh(dx + c) + 3(11(5a^2 \\
& + 2ab + b^2)\cosh(dx + c)^{10} + 3(85a^2 + 34ab - 47b^2)\cosh(dx + \\
& c)^8 + 14(33a^2 - 38ab + 13b^2)\cosh(dx + c)^6 - 10(33a^2 - 38ab \\
& + 13b^2)\cosh(dx + c)^4 - (85a^2 + 34ab - 47b^2)\cosh(dx + c)^2 - 5a \\
& a^2 - 2ab - b^2)\sinh(dx + c))/(d\cosh(dx + c)^{12} + 12d\cosh(dx + c) \\
& \sinh(dx + c)^{11} + d\sinh(dx + c)^{12} + 6d\cosh(dx + c)^{10} + 6(11d\cosh \\
& (dx + c)^2 + d)\sinh(dx + c)^{10} + 20(11d\cosh(dx + c)^3 + 3d\cosh(dx \\
& + c))\sinh(dx + c)^9 + 15d\cosh(dx + c)^8 + 15(33d\cosh(dx + c)^4 + \\
& 18d\cosh(dx + c)^2 + d)\sinh(dx + c)^8 + 24(33d\cosh(dx + c)^5 + 30d \\
& * \cosh(dx + c)^3 + 5d\cosh(dx + c))\sinh(dx + c)^7 + 20d\cosh(dx + c)^ \\
& 6 + 4(231d\cosh(dx + c)^6 + 315d\cosh(dx + c)^4 + 105d\cosh(dx + c)^ \\
& 2 + 5d)\sinh(dx + c)^6 + 24(33d\cosh(dx + c)^7 + 63d\cosh(dx + c)^5 \\
& + 35d\cosh(dx + c)^3 + 5d\cosh(dx + c))\sinh(dx + c)^5 + 15d\cosh(dx \\
& + c)^4 + 15(33d\cosh(dx + c)^8 + 84d\cosh(dx + c)^6 + 70d\cosh(dx + \\
& c)^4 + 20d\cosh(dx + c)^2 + d)\sinh(dx + c)^4 + 20(11d\cosh(dx + c)^ \\
& 9 + 36d\cosh(dx + c)^7 + 42d\cosh(dx + c)^5 + 20d\cosh(dx + c)^3 + 3 \\
& d\cosh(dx + c))\sinh(dx + c)^3 + 6d\cosh(dx + c)^2 + 6(11d\cosh(dx + \\
& c)^{10} + 45d\cosh(dx + c)^8 + 70d\cosh(dx + c)^6 + 50d\cosh(dx + c)^4 \\
& + 15d\cosh(dx + c)^2 + d)\sinh(dx + c)^2 + 12(d\cosh(dx + c)^{11} + 5d \\
& * \cosh(dx + c)^9 + 10d\cosh(dx + c)^7 + 10d\cosh(dx + c)^5 + 5d\cosh(dx \\
& * x + c)^3 + d\cosh(dx + c))\sinh(dx + c) + d)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)**7*(a+b*sinh(dx+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.1794, size = 394, normalized size = 3.01

$$\frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{2dx+2c} - 1\right)e^{-dx-c}\right)\right)\left(5a^2 + 2ab + b^2\right)}{32d} + \frac{15a^2\left(e^{dx+c} - e^{-dx-c}\right)^5 + 6ab\left(e^{dx+c} - e^{-dx-c}\right)^5 + 3b^2\left(e^{dx+c} - e^{-dx-c}\right)^5}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/32*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(5*a^2 + 2*a*b + b^2)/d + 1/24*(15*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 6*a*b*(e^(d*x + c) - e^(-d*x - c))^5 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 160*a^2*(e^(d*x + c) - e^(-d*x - c))^3 + 64*a*b*(e^(d*x + c) - e^(-d*x - c))^3 - 32*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) - e^(-d*x - c)) - 96*a*b*(e^(d*x + c) - e^(-d*x - c)) - 48*b^2*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3*d

3.304 $\int \cosh^4(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=238

$$\frac{b(44a^2 - 28ab + 5b^2) \sinh(c + dx) \cosh^5(c + dx)}{160d} + \frac{(4a - b)(8a^2 - 2ab + b^2) \sinh(c + dx) \cosh^3(c + dx)}{128d} + \frac{3(4a - b)(8a^2 - 2ab + b^2) \sinh(c + dx) \cosh(c + dx)}{128d}$$

[Out] (3*(4*a - b)*(8*a^2 - 2*a*b + b^2)*x)/256 + (3*(4*a - b)*(8*a^2 - 2*a*b + b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + ((4*a - b)*(8*a^2 - 2*a*b + b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b*(44*a^2 - 28*a*b + 5*b^2)*Cosh[c + d*x]^5*Sinh[c + d*x])/(160*d) + (b*Cosh[c + d*x]^9*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(10*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a*(10*a - b) - 5*(a - b)*(2*a - b)*Tanh[c + d*x]^2))/(80*d)

Rubi [A] time = 0.331757, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3191, 413, 526, 385, 199, 206}

$$\frac{b(44a^2 - 28ab + 5b^2) \sinh(c + dx) \cosh^5(c + dx)}{160d} + \frac{(4a - b)(8a^2 - 2ab + b^2) \sinh(c + dx) \cosh^3(c + dx)}{128d} + \frac{3(4a - b)(8a^2 - 2ab + b^2) \sinh(c + dx) \cosh(c + dx)}{128d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (3*(4*a - b)*(8*a^2 - 2*a*b + b^2)*x)/256 + (3*(4*a - b)*(8*a^2 - 2*a*b + b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + ((4*a - b)*(8*a^2 - 2*a*b + b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b*(44*a^2 - 28*a*b + 5*b^2)*Cosh[c + d*x]^5*Sinh[c + d*x])/(160*d) + (b*Cosh[c + d*x]^9*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(10*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a*(10*a - b) - 5*(a - b)*(2*a - b)*Tanh[c + d*x]^2))/(80*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 413


```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \cosh^4(c + dx) (a + b \sinh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a-(a-b)x^2)^3}{(1-x^2)^6} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b \cosh^9(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))^2}{10d} - \frac{\text{Subst}\left(\int \frac{(-a(1-x^2)^3)}{(1-x^2)^6} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b \cosh^9(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))^2}{10d} + \frac{b \cosh^7(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))^2}{10d} \\
&= \frac{b(44a^2 - 28ab + 5b^2) \cosh^5(c + dx) \sinh(c + dx)}{160d} + \frac{b \cosh^9(c + dx) \sinh(c + dx)}{160d} \\
&= \frac{(4a - b)(8a^2 - 2ab + b^2) \cosh^3(c + dx) \sinh(c + dx)}{128d} + \frac{b(44a^2 - 28ab + 5b^2) \cosh^9(c + dx) \sinh(c + dx)}{160d} \\
&= \frac{3(4a - b)(8a^2 - 2ab + b^2) \cosh(c + dx) \sinh(c + dx)}{256d} + \frac{(4a - b)(8a^2 - 2ab + b^2) \cosh^9(c + dx) \sinh(c + dx)}{256d} \\
&= \frac{3}{256}(4a - b)(8a^2 - 2ab + b^2)x + \frac{3(4a - b)(8a^2 - 2ab + b^2) \cosh(c + dx) \sinh(c + dx)}{256d}
\end{aligned}$$

Mathematica [A] time = 0.520767, size = 144, normalized size = 0.61

$$\frac{120(4a - b)(8a^2 - 2ab + b^2)(c + dx) - 10b(b^2 - 16a^2) \sinh(6(c + dx)) + 20(-24a^2b + 128a^3 + b^3) \sinh(2(c + dx)) + 40b^3 \cosh(10(c + dx))}{10240d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (120*(4*a - b)*(8*a^2 - 2*a*b + b^2)*(c + d*x) + 20*(128*a^3 - 24*a^2*b + b^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*Sinh[4*(c + d*x)] - 10*b*(-16*a^2 + b^2)*Sinh[6*(c + d*x)] + 5*(6*a - b)*b^2*Sinh[8*(c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)

Maple [A] time = 0.032, size = 267, normalized size = 1.1

$$\frac{1}{d} \left(b^3 \left(\frac{(\sinh(dx + c))^5 (\cosh(dx + c))^5}{10} - \frac{(\sinh(dx + c))^3 (\cosh(dx + c))^5}{16} + \frac{\sinh(dx + c) (\cosh(dx + c))^5}{32} - \frac{\sinh(dx + c)}{32} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cosh(dx+c)^4*(a+b*\sinh(dx+c)^2)^3,x)$

[Out] $\frac{1}{d}*(b^3*(\frac{1}{10}*\sinh(dx+c)^5*\cosh(dx+c)^5-1/16*\sinh(dx+c)^3*\cosh(dx+c)^5+1/32*\sinh(dx+c)*\cosh(dx+c)^5-1/32*(1/4*\cosh(dx+c)^3+3/8*\cosh(dx+c))*\sinh(dx+c)-3/256*d*x-3/256*c)+3*a*b^2*(1/8*\sinh(dx+c)^3*\cosh(dx+c)^5-1/16*\sinh(dx+c)*\cosh(dx+c)^5+1/16*(1/4*\cosh(dx+c)^3+3/8*\cosh(dx+c))*\sinh(dx+c)+3/128*d*x+3/128*c)+3*a^2*b*(1/6*\sinh(dx+c)*\cosh(dx+c)^5-1/6*(1/4*\cosh(dx+c)^3+3/8*\cosh(dx+c))*\sinh(dx+c)-1/16*d*x-1/16*c)+a^3*((1/4*\cosh(dx+c)^3+3/8*\cosh(dx+c))*\sinh(dx+c)+3/8*d*x+3/8*c))$

Maxima [A] time = 1.10372, size = 490, normalized size = 2.06

$$\frac{1}{64} a^3 \left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d} \right) - \frac{1}{20480} b^3 \left(\frac{(5e^{(-2dx-2c)} + 10e^{(-4dx-4c)} - 40e^{(-6dx-6c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cosh(dx+c)^4*(a+b*\sinh(dx+c)^2)^3,x, \text{algorithm}="maxima")$

[Out] $\frac{1}{64}a^3*(24*x + e^{(4*d*x + 4*c)}/d + 8*e^{(2*d*x + 2*c)}/d - 8*e^{(-2*d*x - 2*c)}/d - e^{(-4*d*x - 4*c)}/d - 1/20480*b^3*((5*e^{(-2*d*x - 2*c)} + 10*e^{(-4*d*x - 4*c)} - 40*e^{(-6*d*x - 6*c)} - 20*e^{(-8*d*x - 8*c)} - 2)*e^{(10*d*x + 10*c)}/d + 240*(d*x + c)/d + (20*e^{(-2*d*x - 2*c)} + 40*e^{(-4*d*x - 4*c)} - 10*e^{(-6*d*x - 6*c)} - 5*e^{(-8*d*x - 8*c)} + 2*e^{(-10*d*x - 10*c)})/d) - 3/2048*a*b^2*((8*e^{(-4*d*x - 4*c)} - 1)*e^{(8*d*x + 8*c)}/d - 48*(d*x + c)/d - (8*e^{(-4*d*x - 4*c)} - e^{(-8*d*x - 8*c)})/d) + 1/128*a^2*b*((3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} + 1)*e^{(6*d*x + 6*c)}/d - 24*(d*x + c)/d + (3*e^{(-2*d*x - 2*c)} - 3*e^{(-4*d*x - 4*c)} - e^{(-6*d*x - 6*c)})/d)$

Fricas [A] time = 1.49742, size = 906, normalized size = 3.81

$$5b^3 \cosh(dx+c) \sinh(dx+c)^9 + 10(6b^3 \cosh(dx+c)^3 + (6ab^2 - b^3) \cosh(dx+c)) \sinh(dx+c)^7 + (126b^3 \cosh(dx+c) \sinh(dx+c)^5 + (12ab^2 - 3b^3) \cosh(dx+c)) \sinh(dx+c)^5 + (12ab^2 - 3b^3) \cosh(dx+c) \sinh(dx+c)^3 + 3b^3 \cosh(dx+c) \sinh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cosh(dx+c)^4*(a+b*\sinh(dx+c)^2)^3,x, \text{algorithm}="fricas")$

```
[Out] 1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 + (
6*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^5 +
70*(6*a*b^2 - b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - b^3)*cosh(d*x + c))*sin
h(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2 - b^3)*cosh(d*x + c)^
5 + 5*(16*a^2*b - b^3)*cosh(d*x + c)^3 + 4*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(32*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*
d*x + 5*(b^3*cosh(d*x + c)^9 + 2*(6*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(16*a^
2*b - b^3)*cosh(d*x + c)^5 + 8*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*cosh(d*x
+ c)^3 + 2*(128*a^3 - 24*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 34.7673, size = 774, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise(((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c +
d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 - 3*a**3*sinh(c + d*x)**3*cosh(c +
d*x)/(8*d) + 5*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 3*a**2*b*x*sinh
(c + d*x)**6/16 - 9*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 9*a**2*
b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 3*a**2*b*x*cosh(c + d*x)**6/16 -
3*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + a**2*b*sinh(c + d*x)**3*c
osh(c + d*x)**3/(2*d) + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 9*
a*b**2*x*sinh(c + d*x)**8/128 - 9*a*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**
2/32 + 27*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 9*a*b**2*x*sinh(c
+ d*x)**2*cosh(c + d*x)**6/32 + 9*a*b**2*x*cosh(c + d*x)**8/128 - 9*a*b**2
*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 33*a*b**2*sinh(c + d*x)**5*cosh(c
+ d*x)**3/(128*d) + 33*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(128*d) -
9*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 3*b**3*x*sinh(c + d*x)**1
0/256 - 15*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 15*b**3*x*sinh(c
+ d*x)**6*cosh(c + d*x)**4/128 - 15*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)**
6/128 + 15*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 3*b**3*x*cosh(c +
d*x)**10/256 - 3*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) + 7*b**3*sinh
(c + d*x)**7*cosh(c + d*x)**3/(128*d) + b**3*sinh(c + d*x)**5*cosh(c + d*x)
**5/(10*d) - 7*b**3*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 3*b**3*sinh
(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cos
h(c)**4, True))
```

Giac [A] time = 1.26231, size = 598, normalized size = 2.51

$$2b^3e^{(10dx+10c)} + 30ab^2e^{(8dx+8c)} - 5b^3e^{(8dx+8c)} + 160a^2be^{(6dx+6c)} - 10b^3e^{(6dx+6c)} + 320a^3e^{(4dx+4c)} + 480a^2be^{(4dx+4c)} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{20480} \cdot (2b^3e^{(10dx+10c)} + 30a^2b^2e^{(8dx+8c)} - 5b^3e^{(8dx+8c)} + 160a^2be^{(6dx+6c)} - 10b^3e^{(6dx+6c)} + 320a^3e^{(4dx+4c)} + 480a^2be^{(4dx+4c)} - 240a^2b^2e^{(4dx+4c)} + 40b^3e^{(4dx+4c)} + 2560a^3e^{(2dx+2c)} - 480a^2be^{(2dx+2c)} + 20b^3e^{(2dx+2c)} + 240 \cdot (32a^3 - 16a^2b + 6ab^2 - b^3) \cdot (dx + c) - (8768a^3e^{(10dx+10c)} - 4384a^2be^{(10dx+10c)} + 1644a^2b^2e^{(10dx+10c)} - 274b^3e^{(10dx+10c)} + 2560a^3e^{(8dx+8c)} - 480a^2be^{(8dx+8c)} + 20b^3e^{(8dx+8c)} + 320a^3e^{(6dx+6c)} + 480a^2be^{(6dx+6c)} - 240a^2b^2e^{(6dx+6c)} + 40b^3e^{(6dx+6c)} + 160a^2be^{(4dx+4c)} - 10b^3e^{(4dx+4c)} + 30a^2b^2e^{(2dx+2c)} - 5b^3e^{(2dx+2c)} + 2b^3) \cdot e^{(-10dx-10c)}) / d$

3.305 $\int \cosh^3(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=98

$$\frac{a^2(a + 3b) \sinh^3(c + dx)}{3d} + \frac{a^3 \sinh(c + dx)}{d} + \frac{b^2(3a + b) \sinh^7(c + dx)}{7d} + \frac{3ab(a + b) \sinh^5(c + dx)}{5d} + \frac{b^3 \sinh^9(c + dx)}{9d}$$

[Out] (a^3*Sinh[c + d*x])/d + (a^2*(a + 3*b)*Sinh[c + d*x]^3)/(3*d) + (3*a*b*(a + b)*Sinh[c + d*x]^5)/(5*d) + (b^2*(3*a + b)*Sinh[c + d*x]^7)/(7*d) + (b^3*Sinh[c + d*x]^9)/(9*d)

Rubi [A] time = 0.0902005, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3190, 373}

$$\frac{a^2(a + 3b) \sinh^3(c + dx)}{3d} + \frac{a^3 \sinh(c + dx)}{d} + \frac{b^2(3a + b) \sinh^7(c + dx)}{7d} + \frac{3ab(a + b) \sinh^5(c + dx)}{5d} + \frac{b^3 \sinh^9(c + dx)}{9d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (a^3*Sinh[c + d*x])/d + (a^2*(a + 3*b)*Sinh[c + d*x]^3)/(3*d) + (3*a*b*(a + b)*Sinh[c + d*x]^5)/(5*d) + (b^2*(3*a + b)*Sinh[c + d*x]^7)/(7*d) + (b^3*Sinh[c + d*x]^9)/(9*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 373

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\int \cosh^3(c + dx) (a + b \sinh^2(c + dx))^3 dx = \frac{\text{Subst}\left(\int (1 + x^2) (a + bx^2)^3 dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int (a^3 + a^2(a + 3b)x^2 + 3ab(a + b)x^4 + b^2(3a + b)x^6 + b^3x^8) dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{a^3 \sinh(c + dx)}{d} + \frac{a^2(a + 3b) \sinh^3(c + dx)}{3d} + \frac{3ab(a + b) \sinh^5(c + dx)}{5d} + \dots$$

Mathematica [A] time = 0.233608, size = 83, normalized size = 0.85

$$\frac{105a^2(a + 3b) \sinh^3(c + dx) + 315a^3 \sinh(c + dx) + 45b^2(3a + b) \sinh^7(c + dx) + 189ab(a + b) \sinh^5(c + dx) + 35b^3 \sinh^9(c + dx)}{315d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (315*a^3*Sinh[c + d*x] + 105*a^2*(a + 3*b)*Sinh[c + d*x]^3 + 189*a*b*(a + b)*Sinh[c + d*x]^5 + 45*b^2*(3*a + b)*Sinh[c + d*x]^7 + 35*b^3*Sinh[c + d*x]^9)/(315*d)

Maple [B] time = 0.035, size = 209, normalized size = 2.1

$$\frac{1}{d} \left(b^3 \left(\frac{(\sinh(dx + c))^5 (\cosh(dx + c))^4}{9} - \frac{5 (\sinh(dx + c))^3 (\cosh(dx + c))^4}{63} + \frac{\sinh(dx + c) (\cosh(dx + c))^4}{21} - \frac{\sinh(dx + c)}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/d*(b^3*(1/9*sinh(d*x+c)^5*cosh(d*x+c)^4-5/63*sinh(d*x+c)^3*cosh(d*x+c)^4+1/21*sinh(d*x+c)*cosh(d*x+c)^4-1/21*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+3*a*b^2*(1/7*sinh(d*x+c)^3*cosh(d*x+c)^4-3/35*sinh(d*x+c)*cosh(d*x+c)^4+3/35*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+3*a^2*b*(1/5*sinh(d*x+c)*cosh(d*x+c)^4-1/5*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+a^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))

Maxima [B] time = 1.09882, size = 471, normalized size = 4.81

$$-\frac{1}{32256} b^3 \left(\frac{(27 e^{(-2dx-2c)} - 168 e^{(-6dx-6c)} + 378 e^{(-8dx-8c)} - 7) e^{(9dx+9c)}}{d} - \frac{378 e^{(-dx-c)} - 168 e^{(-3dx-3c)} + 27 e^{(-7dx-7c)} - 7}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] -1/32256*b^3*((27*e^(-2*d*x - 2*c) - 168*e^(-6*d*x - 6*c) + 378*e^(-8*d*x - 8*c) - 7)*e^(9*d*x + 9*c)/d - (378*e^(-d*x - c) - 168*e^(-3*d*x - 3*c) + 27*e^(-7*d*x - 7*c) - 7)*e^(-9*d*x - 9*c)/d) - 3/4480*a*b^2*((7*e^(-2*d*x - 2*c) + 35*e^(-4*d*x - 4*c) - 105*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (105*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) - 7*e^(-5*d*x - 5*c) + 5*e^(-7*d*x - 7*c))/d) + 1/160*a^2*b*((5*e^(-2*d*x - 2*c) - 30*e^(-4*d*x - 4*c) + 3)*e^(5*d*x + 5*c)/d + (30*e^(-d*x - c) - 5*e^(-3*d*x - 3*c) - 3*e^(-5*d*x - 5*c))/d) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)

Fricas [B] time = 1.52027, size = 787, normalized size = 8.03

$$35b^3 \sinh(dx + c)^9 + 45(28b^3 \cosh(dx + c)^2 + 12ab^2 - 3b^3) \sinh(dx + c)^7 + 63(70b^3 \cosh(dx + c)^4 + 48a^2b - 12ab^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] 1/80640*(35*b^3*sinh(d*x + c)^9 + 45*(28*b^3*cosh(d*x + c)^2 + 12*a*b^2 - 3*b^3)*sinh(d*x + c)^7 + 63*(70*b^3*cosh(d*x + c)^4 + 48*a^2*b - 12*a*b^2 + 45*(4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 105*(28*b^3*cosh(d*x + c)^6 + 45*(4*a*b^2 - b^3)*cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b - 36*a*b^2 + 8*b^3 + 72*(4*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 315*(b^3*cosh(d*x + c)^8 + 3*(4*a*b^2 - b^3)*cosh(d*x + c)^6 + 12*(4*a^2*b - a*b^2)*cosh(d*x + c)^4 + 192*a^3 - 96*a^2*b + 36*a*b^2 - 6*b^3 + 4*(16*a^3 + 12*a^2*b - 9*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/d

Sympy [A] time = 19.4941, size = 182, normalized size = 1.86

$$\left\{ \begin{array}{l} -\frac{2a^3 \sinh^3(c+dx)}{3d} + \frac{a^3 \sinh(c+dx) \cosh^2(c+dx)}{d} - \frac{2a^2 b \sinh^5(c+dx)}{5d} + \frac{a^2 b \sinh^3(c+dx) \cosh^2(c+dx)}{d} - \frac{6ab^2 \sinh^7(c+dx)}{35d} + \frac{3ab^2 \sinh^5(c+dx) \cosh^2(c+dx)}{5d} \\ x \left(a + b \sinh^2(c) \right)^3 \cosh^3(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Piecewise((-2*a**3*sinh(c + d*x)**3/(3*d) + a**3*sinh(c + d*x)*cosh(c + d*x)**2/d - 2*a**2*b*sinh(c + d*x)**5/(5*d) + a**2*b*sinh(c + d*x)**3*cosh(c + d*x)**2/d - 6*a*b**2*sinh(c + d*x)**7/(35*d) + 3*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)**2/(5*d) - 2*b**3*sinh(c + d*x)**9/(63*d) + b**3*sinh(c + d*x)**7*cosh(c + d*x)**2/(7*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cosh(c)**3, True))

Giac [B] time = 1.26016, size = 506, normalized size = 5.16

$$35 b^3 e^{(9dx+9c)} + 540 ab^2 e^{(7dx+7c)} - 135 b^3 e^{(7dx+7c)} + 3024 a^2 b e^{(5dx+5c)} - 756 ab^2 e^{(5dx+5c)} + 6720 a^3 e^{(3dx+3c)} + 5040 a^2 b e^{(3dx+3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/161280*(35*b^3*e^(9*d*x + 9*c) + 540*a*b^2*e^(7*d*x + 7*c) - 135*b^3*e^(7*d*x + 7*c) + 3024*a^2*b*e^(5*d*x + 5*c) - 756*a*b^2*e^(5*d*x + 5*c) + 6720*a^3*e^(3*d*x + 3*c) + 5040*a^2*b*e^(3*d*x + 3*c) - 3780*a*b^2*e^(3*d*x + 3*c) + 840*b^3*e^(3*d*x + 3*c) + 60480*a^3*e^(d*x + c) - 30240*a^2*b*e^(d*x + c) + 11340*a*b^2*e^(d*x + c) - 1890*b^3*e^(d*x + c) - (60480*a^3*e^(8*d*x + 8*c) - 30240*a^2*b*e^(8*d*x + 8*c) + 11340*a*b^2*e^(8*d*x + 8*c) - 1890*b^3*e^(8*d*x + 8*c) + 6720*a^3*e^(6*d*x + 6*c) + 5040*a^2*b*e^(6*d*x + 6*c) - 3780*a*b^2*e^(6*d*x + 6*c) + 840*b^3*e^(6*d*x + 6*c) + 3024*a^2*b*e^(4*d*x + 4*c) - 756*a*b^2*e^(4*d*x + 4*c) + 540*a*b^2*e^(2*d*x + 2*c) - 135*b^3*e^(2*d*x + 2*c) + 35*b^3)*e^(-9*d*x - 9*c))/d

3.306 $\int \cosh^2(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=203

$$\frac{b(88a^2 - 68ab + 15b^2) \sinh(c + dx) \cosh^3(c + dx)}{192d} + \frac{(-48a^2b + 64a^3 + 24ab^2 - 5b^3) \sinh(c + dx) \cosh(c + dx)}{128d} + \frac{1}{128}x$$

```
[Out] ((64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*x)/128 + ((64*a^3 - 48*a^2*b + 24*a
*b^2 - 5*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (b*(88*a^2 - 68*a*b +
15*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + (b*Cosh[c + d*x]^7*Sinh[c
+ d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(8*d) + (b*Cosh[c + d*x]^5*Sinh[c +
d*x]*(a*(8*a - b) - (8*a - 5*b)*(a - b)*Tanh[c + d*x]^2))/(48*d)
```

Rubi [A] time = 0.268032, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3191, 413, 526, 385, 199, 206}

$$\frac{b(88a^2 - 68ab + 15b^2) \sinh(c + dx) \cosh^3(c + dx)}{192d} + \frac{(-48a^2b + 64a^3 + 24ab^2 - 5b^3) \sinh(c + dx) \cosh(c + dx)}{128d} + \frac{1}{128}x$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*x)/128 + ((64*a^3 - 48*a^2*b + 24*a
*b^2 - 5*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (b*(88*a^2 - 68*a*b +
15*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + (b*Cosh[c + d*x]^7*Sinh[c
+ d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(8*d) + (b*Cosh[c + d*x]^5*Sinh[c +
d*x]*(a*(8*a - b) - (8*a - 5*b)*(a - b)*Tanh[c + d*x]^2))/(48*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
```

```

1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

```

Rule 526

```

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f
_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
1) + (b*e - a*f)*(n*q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n},
x] && LtQ[p, -1] && GtQ[q, 0]

```

Rule 385

```

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])

```

Rule 199

```

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])

```

Rule 206

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \cosh^2(c + dx) (a + b \sinh^2(c + dx))^3 dx &= \frac{\text{Subst}\left(\int \frac{(a-(a-b)x^2)^3}{(1-x^2)^5} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b \cosh^7(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))^2}{8d} - \frac{\text{Subst}\left(\int \frac{(-a(8b^2 - 3a^2 + 2abx^2 - b^2x^4))}{(1-x^2)^5} dx, x, \tanh(c + dx)\right)}{d} \\
&= \frac{b \cosh^7(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))^2}{8d} + \frac{b \cosh^5(c + dx) \sinh(c + dx) (a - (a-b) \tanh^2(c + dx))}{8d} \\
&= \frac{b(88a^2 - 68ab + 15b^2) \cosh^3(c + dx) \sinh(c + dx)}{192d} + \frac{b \cosh^7(c + dx) \sinh(c + dx)}{192d} \\
&= \frac{(64a^3 - 48a^2b + 24ab^2 - 5b^3) \cosh(c + dx) \sinh(c + dx)}{128d} + \frac{b(88a^2 - 68ab + 15b^2) \cosh^3(c + dx) \sinh(c + dx)}{192d} \\
&= \frac{1}{128} (64a^3 - 48a^2b + 24ab^2 - 5b^3) x + \frac{(64a^3 - 48a^2b + 24ab^2 - 5b^3) \cosh(c + dx) \sinh(c + dx)}{128d}
\end{aligned}$$

Mathematica [A] time = 0.302241, size = 120, normalized size = 0.59

$$\frac{24(-48a^2b + 64a^3 + 24ab^2 - 5b^3)(c + dx) + 24b(12a^2 - 6ab + b^2) \sinh(4(c + dx)) + 48(16a^3 - 3ab^2 + b^3) \sinh(2(c + dx))}{3072d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (24*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*(c + d*x) + 48*(16*a^3 - 3*a*b^2 + b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 6*a*b + b^2)*Sinh[4*(c + d*x)] + 16*(3*a - b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh[8*(c + d*x)])/(3072*d)

Maple [A] time = 0.032, size = 216, normalized size = 1.1

$$\frac{1}{d} \left(b^3 \left(\frac{(\sinh(dx + c))^5 (\cosh(dx + c))^3}{8} - \frac{5 (\sinh(dx + c))^3 (\cosh(dx + c))^3}{48} + \frac{5 \sinh(dx + c) (\cosh(dx + c))^3}{64} - \frac{5 \cosh(dx + c)}{64} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)`

[Out] $\frac{1}{d}*(b^3*(\frac{1}{8}*\sinh(d*x+c)^5*\cosh(d*x+c)^3-5/48*\sinh(d*x+c)^3*\cosh(d*x+c)^3+5/64*\sinh(d*x+c)*\cosh(d*x+c)^3-5/128*\cosh(d*x+c)*\sinh(d*x+c)-5/128*d*x-5/128*c)+3*a*b^2*(\frac{1}{6}*\sinh(d*x+c)^3*\cosh(d*x+c)^3-1/8*\sinh(d*x+c)*\cosh(d*x+c)^3+1/16*\cosh(d*x+c)*\sinh(d*x+c)+1/16*d*x+1/16*c)+3*a^2*b*(\frac{1}{4}*\sinh(d*x+c)*\cosh(d*x+c)^3-1/8*\cosh(d*x+c)*\sinh(d*x+c)-1/8*d*x-1/8*c)+a^3*(\frac{1}{2}*\cosh(d*x+c)*\sinh(d*x+c)+1/2*d*x+1/2*c))$

Maxima [A] time = 1.11971, size = 387, normalized size = 1.91

$$\frac{1}{8}a^3\left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{6144}b^3\left(\frac{(16e^{(-2dx-2c)} - 24e^{(-4dx-4c)} - 48e^{(-6dx-6c)} - 3)e^{(8dx+8c)}}{d} + \frac{240(dx+c)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $\frac{1}{8}a^3*(4*x + e^{(2*d*x + 2*c)}/d - e^{(-2*d*x - 2*c)}/d) - \frac{1}{6144}b^3*((16*e^{(-2*d*x - 2*c)} - 24*e^{(-4*d*x - 4*c)} - 48*e^{(-6*d*x - 6*c)} - 3)*e^{(8*d*x + 8*c)}/d + 240*(d*x + c)/d + (48*e^{(-2*d*x - 2*c)} + 24*e^{(-4*d*x - 4*c)} - 16*e^{(-6*d*x - 6*c)} + 3*e^{(-8*d*x - 8*c)})/d) - \frac{1}{128}a*b^2*((3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} - 1)*e^{(6*d*x + 6*c)}/d - 24*(d*x + c)/d - (3*e^{(-2*d*x - 2*c)} + 3*e^{(-4*d*x - 4*c)} - e^{(-6*d*x - 6*c)})/d) - \frac{3}{64}a^2*b*(8*(d*x + c)/d - e^{(4*d*x + 4*c)}/d + e^{(-4*d*x - 4*c)}/d)$

Fricas [A] time = 1.48108, size = 626, normalized size = 3.08

$$3b^3 \cosh(dx+c) \sinh(dx+c)^7 + 3(7b^3 \cosh(dx+c)^3 + 4(3ab^2 - b^3) \cosh(dx+c)) \sinh(dx+c)^5 + (21b^3 \cosh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")`

[Out] $\frac{1}{384}*(3*b^3*\cosh(d*x+c)*\sinh(d*x+c)^7 + 3*(7*b^3*\cosh(d*x+c)^3 + 4*(3*a*b^2 - b^3)*\cosh(d*x+c))*\sinh(d*x+c)^5 + (21*b^3*\cosh(d*x+c)^5 + 40*(3*a*b^2 - b^3)*\cosh(d*x+c)^3 + 12*(12*a^2*b - 6*a*b^2 + b^3)*\cosh(d*x+c))*\sinh(d*x+c)^3 + 3*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*d*x + 3*(b$

$$\frac{3 \cosh(dx + c)^7 + 4(3ab^2 - b^3) \cosh(dx + c)^5 + 4(12a^2b - 6ab^2 + b^3) \cosh(dx + c)^3 + 4(16a^3 - 3ab^2 + b^3) \cosh(dx + c) \sinh(dx + c)}{d}$$

Sympy [A] time = 13.7037, size = 559, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)**2*(a+b*sinh(dx+c)**2)**3,x)

[Out] Piecewise((-a**3*x*sinh(c + dx)**2/2 + a**3*x*cosh(c + dx)**2/2 + a**3*sinh(c + dx)*cosh(c + dx)/(2*d) - 3*a**2*b*x*sinh(c + dx)**4/8 + 3*a**2*b*x*sinh(c + dx)**2*cosh(c + dx)**2/4 - 3*a**2*b*x*cosh(c + dx)**4/8 + 3*a**2*b*sinh(c + dx)**3*cosh(c + dx)/(8*d) + 3*a**2*b*sinh(c + dx)*cosh(c + dx)**3/(8*d) - 3*a*b**2*x*sinh(c + dx)**6/16 + 9*a*b**2*x*sinh(c + dx)**4*cosh(c + dx)**2/16 - 9*a*b**2*x*sinh(c + dx)**2*cosh(c + dx)**4/16 + 3*a*b**2*x*cosh(c + dx)**6/16 + 3*a*b**2*sinh(c + dx)**5*cosh(c + dx)/(16*d) + a*b**2*sinh(c + dx)**3*cosh(c + dx)**3/(2*d) - 3*a*b**2*sinh(c + dx)*cosh(c + dx)**5/(16*d) - 5*b**3*x*sinh(c + dx)**8/128 + 5*b**3*x*sinh(c + dx)**6*cosh(c + dx)**2/32 - 15*b**3*x*sinh(c + dx)**4*cosh(c + dx)**4/64 + 5*b**3*x*sinh(c + dx)**2*cosh(c + dx)**6/32 - 5*b**3*x*cosh(c + dx)**8/128 + 5*b**3*sinh(c + dx)**7*cosh(c + dx)/(128*d) + 73*b**3*sinh(c + dx)**5*cosh(c + dx)**3/(384*d) - 55*b**3*sinh(c + dx)**3*cosh(c + dx)**5/(384*d) + 5*b**3*sinh(c + dx)*cosh(c + dx)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cosh(c)**2, True))

Giac [A] time = 1.27872, size = 482, normalized size = 2.37

$$3b^3e^{(8dx+8c)} + 48ab^2e^{(6dx+6c)} - 16b^3e^{(6dx+6c)} + 288a^2be^{(4dx+4c)} - 144ab^2e^{(4dx+4c)} + 24b^3e^{(4dx+4c)} + 768a^3e^{(2dx+2c)} - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^2*(a+b*sinh(dx+c)^2)^3,x, algorithm="giac")

[Out] 1/6144*(3*b^3*e^(8*d*x + 8*c) + 48*a*b^2*e^(6*d*x + 6*c) - 16*b^3*e^(6*d*x + 6*c) + 288*a^2*b*e^(4*d*x + 4*c) - 144*a*b^2*e^(4*d*x + 4*c) + 24*b^3*e^(

$$\begin{aligned} &4*d*x + 4*c) + 768*a^3*e^(2*d*x + 2*c) - 144*a*b^2*e^(2*d*x + 2*c) + 48*b^3 \\ &*e^(2*d*x + 2*c) + 48*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*(d*x + c) - (3 \\ &200*a^3*e^(8*d*x + 8*c) - 2400*a^2*b*e^(8*d*x + 8*c) + 1200*a*b^2*e^(8*d*x \\ &+ 8*c) - 250*b^3*e^(8*d*x + 8*c) + 768*a^3*e^(6*d*x + 6*c) - 144*a*b^2*e^(6 \\ &*d*x + 6*c) + 48*b^3*e^(6*d*x + 6*c) + 288*a^2*b*e^(4*d*x + 4*c) - 144*a*b^2 \\ &e^(4*d*x + 4*c) + 24*b^3*e^(4*d*x + 4*c) + 48*a*b^2*e^(2*d*x + 2*c) - 16* \\ &b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-8*d*x - 8*c))/d \end{aligned}$$

3.307 $\int \cosh(c + dx) \left(a + b \sinh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=67

$$\frac{a^2 b \sinh^3(c + dx)}{d} + \frac{a^3 \sinh(c + dx)}{d} + \frac{3ab^2 \sinh^5(c + dx)}{5d} + \frac{b^3 \sinh^7(c + dx)}{7d}$$

[Out] (a^3*Sinh[c + d*x])/d + (a^2*b*Sinh[c + d*x]^3)/d + (3*a*b^2*Sinh[c + d*x]^5)/(5*d) + (b^3*Sinh[c + d*x]^7)/(7*d)

Rubi [A] time = 0.0454635, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3190, 194}

$$\frac{a^2 b \sinh^3(c + dx)}{d} + \frac{a^3 \sinh(c + dx)}{d} + \frac{3ab^2 \sinh^5(c + dx)}{5d} + \frac{b^3 \sinh^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (a^3*Sinh[c + d*x])/d + (a^2*b*Sinh[c + d*x]^3)/d + (3*a*b^2*Sinh[c + d*x]^5)/(5*d) + (b^3*Sinh[c + d*x]^7)/(7*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \cosh(c+dx) (a+b \sinh^2(c+dx))^3 dx &= \frac{\text{Subst}\left(\int (a+bx^2)^3 dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int (a^3+3a^2bx^2+3ab^2x^4+b^3x^6) dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{a^3 \sinh(c+dx)}{d} + \frac{a^2b \sinh^3(c+dx)}{d} + \frac{3ab^2 \sinh^5(c+dx)}{5d} + \frac{b^3 \sinh^7(c+dx)}{7d} \end{aligned}$$

Mathematica [A] time = 0.11578, size = 59, normalized size = 0.88

$$\frac{a^2b \sinh^3(c+dx) + a^3 \sinh(c+dx) + \frac{3}{5}ab^2 \sinh^5(c+dx) + \frac{1}{7}b^3 \sinh^7(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (a^3*Sinh[c + d*x] + a^2*b*Sinh[c + d*x]^3 + (3*a*b^2*Sinh[c + d*x]^5)/5 + (b^3*Sinh[c + d*x]^7)/7)/d

Maple [A] time = 0.011, size = 56, normalized size = 0.8

$$\frac{1}{d} \left(\frac{b^3 (\sinh(dx+c))^7}{7} + \frac{3ab^2 (\sinh(dx+c))^5}{5} + a^2b (\sinh(dx+c))^3 + a^3 \sinh(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/d*(1/7*b^3*sinh(d*x+c)^7+3/5*a*b^2*sinh(d*x+c)^5+a^2*b*sinh(d*x+c)^3+a^3*sinh(d*x+c))

Maxima [A] time = 1.05374, size = 85, normalized size = 1.27

$$\frac{b^3 \sinh(dx+c)^7}{7d} + \frac{3ab^2 \sinh(dx+c)^5}{5d} + \frac{a^2b \sinh(dx+c)^3}{d} + \frac{a^3 \sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*sinh(d*x + c)^7/d + 3/5*a*b^2*sinh(d*x + c)^5/d + a^2*b*sinh(d*x + c)^3/d + a^3*sinh(d*x + c)/d
```

Fricas [B] time = 1.54785, size = 512, normalized size = 7.64

$$5b^3 \sinh(dx + c)^7 + 7(15b^3 \cosh(dx + c)^2 + 12ab^2 - 5b^3) \sinh(dx + c)^5 + 35(5b^3 \cosh(dx + c)^4 + 16a^2b - 12ab^2 + 3b^3) \sinh(dx + c)^3 + 35a^2b \sinh(dx + c) + 3a^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^3*sinh(d*x + c)^7 + 7*(15*b^3*cosh(d*x + c)^2 + 12*a*b^2 - 5*b^3)*sinh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^4 + 16*a^2*b - 12*a*b^2 + 3*b^3 + 2*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 35*(b^3*cosh(d*x + c)^6 + (12*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3 + 3*(16*a^2*b - 12*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/d
```

Sympy [A] time = 6.6694, size = 75, normalized size = 1.12

$$\begin{cases} \frac{a^3 \sinh(c+dx)}{d} + \frac{a^2 b \sinh^3(c+dx)}{d} + \frac{3ab^2 \sinh^5(c+dx)}{5d} + \frac{b^3 \sinh^7(c+dx)}{7d} & \text{for } d \neq 0 \\ x(a + b \sinh^2(c))^3 \cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)/d + a**2*b*sinh(c + d*x)**3/d + 3*a*b**2*sinh(c + d*x)**5/(5*d) + b**3*sinh(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cosh(c), True))
```

Giac [B] time = 1.2839, size = 387, normalized size = 5.78

$$5b^3e^{(7dx+7c)} + 84ab^2e^{(5dx+5c)} - 35b^3e^{(5dx+5c)} + 560a^2be^{(3dx+3c)} - 420ab^2e^{(3dx+3c)} + 105b^3e^{(3dx+3c)} + 2240a^3e^{(dx+c)} -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/4480*(5*b^3*e^(7*d*x + 7*c) + 84*a*b^2*e^(5*d*x + 5*c) - 35*b^3*e^(5*d*x + 5*c) + 560*a^2*b*e^(3*d*x + 3*c) - 420*a*b^2*e^(3*d*x + 3*c) + 105*b^3*e^(3*d*x + 3*c) + 2240*a^3*e^(d*x + c) - 1680*a^2*b*e^(d*x + c) + 840*a*b^2*e^(d*x + c) - 175*b^3*e^(d*x + c) - (2240*a^3*e^(6*d*x + 6*c) - 1680*a^2*b*e^(6*d*x + 6*c) + 840*a*b^2*e^(6*d*x + 6*c) - 175*b^3*e^(6*d*x + 6*c) + 560*a^2*b*e^(4*d*x + 4*c) - 420*a*b^2*e^(4*d*x + 4*c) + 105*b^3*e^(4*d*x + 4*c) + 84*a*b^2*e^(2*d*x + 2*c) - 35*b^3*e^(2*d*x + 2*c) + 5*b^3)*e^(-7*d*x - 7*c))/d

3.308 $\int \operatorname{sech}(c + dx) \left(a + b \sinh^2(c + dx) \right)^3 dx$

Optimal. Leaf size=86

$$\frac{b(3a^2 - 3ab + b^2) \sinh(c + dx)}{d} + \frac{b^2(3a - b) \sinh^3(c + dx)}{3d} + \frac{(a - b)^3 \tan^{-1}(\sinh(c + dx))}{d} + \frac{b^3 \sinh^5(c + dx)}{5d}$$

[Out] ((a - b)^3*ArcTan[Sinh[c + d*x]])/d + (b*(3*a^2 - 3*a*b + b^2)*Sinh[c + d*x])/d + ((3*a - b)*b^2*Sinh[c + d*x]^3)/(3*d) + (b^3*Sinh[c + d*x]^5)/(5*d)

Rubi [A] time = 0.0755837, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3190, 390, 203}

$$\frac{b(3a^2 - 3ab + b^2) \sinh(c + dx)}{d} + \frac{b^2(3a - b) \sinh^3(c + dx)}{3d} + \frac{(a - b)^3 \tan^{-1}(\sinh(c + dx))}{d} + \frac{b^3 \sinh^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((a - b)^3*ArcTan[Sinh[c + d*x]])/d + (b*(3*a^2 - 3*a*b + b^2)*Sinh[c + d*x])/d + ((3*a - b)*b^2*Sinh[c + d*x]^3)/(3*d) + (b^3*Sinh[c + d*x]^5)/(5*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 203

Int[((a_.) + (b_.)*(x_.)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}(c+dx) (a+b \sinh^2(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^3}{1+x^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(b(3a^2-3ab+b^2) + (3a-b)b^2x^2 + b^3x^4 + \frac{(a-b)^3}{1+x^2}\right) dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{b(3a^2-3ab+b^2) \sinh(c+dx)}{d} + \frac{(3a-b)b^2 \sinh^3(c+dx)}{3d} + \frac{b^3 \sinh^5(c+dx)}{5d} \\ &+ \frac{(a-b)^3 \tan^{-1}(\sinh(c+dx))}{d} + \frac{b(3a^2-3ab+b^2) \sinh(c+dx)}{d} + \frac{(3a-b)b^3}{d} \end{aligned}$$

Mathematica [A] time = 0.530513, size = 100, normalized size = 1.16

$$\frac{\sinh(c+dx) \left(b(45a^2 + 15ab(\sinh^2(c+dx) - 3) + b^2(3\sinh^4(c+dx) - 5\sinh^2(c+dx) + 15)) + \frac{15(a-b)^3 \tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}} \right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (Sinh[c + d*x]*((15*(a - b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c + d*x]^2] + b*(45*a^2 + 15*a*b*(-3 + Sinh[c + d*x]^2) + b^2*(15 - 5*Sinh[c + d*x]^2 + 3*Sinh[c + d*x]^4))))/(15*d)

Maple [A] time = 0.056, size = 155, normalized size = 1.8

$$2 \frac{a^3 \arctan(e^{dx+c})}{d} + 3 \frac{a^2 b \sinh(dx+c)}{d} - 6 \frac{a^2 b \arctan(e^{dx+c})}{d} + \frac{ab^2 (\sinh(dx+c))^3}{d} - 3 \frac{ab^2 \sinh(dx+c)}{d} + 6 \frac{ab^2 \arctan(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)

[Out] $2/d*a^3*\arctan(\exp(d*x+c))+3/d*a^2*b*\sinh(d*x+c)-6/d*a^2*b*\arctan(\exp(d*x+c))$
 $+1/d*a*b^2*\sinh(d*x+c)^3-3/d*a*b^2*\sinh(d*x+c)+6/d*a*b^2*\arctan(\exp(d*x+c))$
 $+1/5*b^3*\sinh(d*x+c)^5/d-1/3*b^3*\sinh(d*x+c)^3/d+b^3*\sinh(d*x+c)/d-2/d*b^3*\arctan(\exp(d*x+c))$

Maxima [B] time = 1.69462, size = 315, normalized size = 3.66

$$-\frac{1}{480} b^3 \left(\frac{(35 e^{(-2dx-2c)} - 330 e^{(-4dx-4c)} - 3) e^{(5dx+5c)}}{d} + \frac{330 e^{(-dx-c)} - 35 e^{(-3dx-3c)} + 3 e^{(-5dx-5c)}}{d} - \frac{960 \arctan(e^{(-dx-c)})}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")`

[Out] $-1/480*b^3*((35*e^{(-2*d*x - 2*c)} - 330*e^{(-4*d*x - 4*c)} - 3)*e^{(5*d*x + 5*c)}/d$
 $+ (330*e^{(-d*x - c)} - 35*e^{(-3*d*x - 3*c)} + 3*e^{(-5*d*x - 5*c)})/d - 960$
 $*\arctan(e^{(-d*x - c)})/d - 1/8*a*b^2*((15*e^{(-2*d*x - 2*c)} - 1)*e^{(3*d*x + 3*c)}/d$
 $- (15*e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/d + 48*\arctan(e^{(-d*x - c)})/d$
 $+ 3/2*a^2*b*(4*\arctan(e^{(-d*x - c)})/d + e^{(d*x + c)}/d - e^{(-d*x - c)}/d) +$
 $a^3*\arctan(\sinh(d*x + c))/d$

Fricas [B] time = 1.62245, size = 2795, normalized size = 32.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")`

[Out] $1/480*(3*b^3*\cosh(d*x + c)^{10} + 30*b^3*\cosh(d*x + c)*\sinh(d*x + c)^9 + 3*b^3*$
 $\sinh(d*x + c)^{10} + 5*(12*a*b^2 - 7*b^3)*\cosh(d*x + c)^8 + 5*(27*b^3*\cosh(d*x + c)^2$
 $+ 12*a*b^2 - 7*b^3)*\sinh(d*x + c)^8 + 40*(9*b^3*\cosh(d*x + c)^3$
 $+ (12*a*b^2 - 7*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 30*(24*a^2*b - 30*a*b^2$
 $+ 11*b^3)*\cosh(d*x + c)^6 + 10*(63*b^3*\cosh(d*x + c)^4 + 72*a^2*b - 90*a$
 $*b^2 + 33*b^3 + 14*(12*a*b^2 - 7*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*$
 $(189*b^3*\cosh(d*x + c)^5 + 70*(12*a*b^2 - 7*b^3)*\cosh(d*x + c)^3 + 45*(24*a^2*b$
 $- 30*a*b^2 + 11*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 30*(24*a^2*b - 3$
 $0*a*b^2 + 11*b^3)*\cosh(d*x + c)^4 + 10*(63*b^3*\cosh(d*x + c)^6 + 35*(12*a*b^2$
 $- 7*b^3)*\cosh(d*x + c)^4 - 72*a^2*b + 90*a*b^2 - 33*b^3 + 45*(24*a^2*b -$

$$\begin{aligned}
& 30*a*b^2 + 11*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^4 + 40*(9*b^3*\cosh(d*x + \\
& c)^7 + 7*(12*a*b^2 - 7*b^3)*\cosh(d*x + c)^5 + 15*(24*a^2*b - 30*a*b^2 + 11 \\
& *b^3)*\cosh(d*x + c)^3 - 3*(24*a^2*b - 30*a*b^2 + 11*b^3)*\cosh(d*x + c))*\sin \\
& h(d*x + c)^3 - 3*b^3 - 5*(12*a*b^2 - 7*b^3)*\cosh(d*x + c)^2 + 5*(27*b^3*\cos \\
& h(d*x + c)^8 + 28*(12*a*b^2 - 7*b^3)*\cosh(d*x + c)^6 + 90*(24*a^2*b - 30*a* \\
& b^2 + 11*b^3)*\cosh(d*x + c)^4 - 12*a*b^2 + 7*b^3 - 36*(24*a^2*b - 30*a*b^2 \\
& + 11*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 960*((a^3 - 3*a^2*b + 3*a*b^2 \\
& - b^3)*\cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)^4* \\
& \sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)^3*\sinh(d*x \\
& + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^ \\
& 3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)*\sinh(d*x + c)^4 + (a^3 \\
& - 3*a^2*b + 3*a*b^2 - b^3)*\sinh(d*x + c)^5)*\arctan(\cosh(d*x + c) + \sinh(d*x \\
& + c)) + 10*(3*b^3*\cosh(d*x + c)^9 + 4*(12*a*b^2 - 7*b^3)*\cosh(d*x + c)^7 + \\
& 18*(24*a^2*b - 30*a*b^2 + 11*b^3)*\cosh(d*x + c)^5 - 12*(24*a^2*b - 30*a*b^ \\
& 2 + 11*b^3)*\cosh(d*x + c)^3 - (12*a*b^2 - 7*b^3)*\cosh(d*x + c))*\sinh(d*x + \\
& c))/(d*\cosh(d*x + c)^5 + 5*d*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*d*\cosh(d*x \\
& + c)^3*\sinh(d*x + c)^2 + 10*d*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + 5*d*\cosh(d* \\
& x + c)*\sinh(d*x + c)^4 + d*\sinh(d*x + c)^5)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.279, size = 309, normalized size = 3.59

$$\frac{2(a^3 - 3a^2b + 3ab^2 - b^3) \arctan(e^{(dx+c)})}{d} - \frac{(720a^2be^{(4dx+4c)} - 900ab^2e^{(4dx+4c)} + 330b^3e^{(4dx+4c)} + 60ab^2e^{(2dx+2c)} - \dots)}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\arctan(e^{(d*x + c)})/d - 1/480*(720*a^2*b* \\ e^{(4*d*x + 4*c)} - 900*a*b^2*e^{(4*d*x + 4*c)} + 330*b^3*e^{(4*d*x + 4*c)} + 60*$

$$\begin{aligned} & a*b^2*e^{(2*d*x + 2*c)} - 35*b^3*e^{(2*d*x + 2*c)} + 3*b^3)*e^{(-5*d*x - 5*c)}/d \\ & + 1/480*(3*b^3*d^4*e^{(5*d*x + 5*c)} + 60*a*b^2*d^4*e^{(3*d*x + 3*c)} - 35*b^3* \\ & d^4*e^{(3*d*x + 3*c)} + 720*a^2*b*d^4*e^{(d*x + c)} - 900*a*b^2*d^4*e^{(d*x + c)} \\ & + 330*b^3*d^4*e^{(d*x + c)})/d^5 \end{aligned}$$

3.309 $\int \operatorname{sech}^2(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=92

$$\frac{3}{8}bx(8a^2 - 12ab + 5b^2) + \frac{3b^2(4a - 3b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{(a - b)^3 \tanh(c + dx)}{d} + \frac{b^3 \sinh(c + dx) \cosh^3(c + dx)}{4d}$$

[Out] (3*b*(8*a^2 - 12*a*b + 5*b^2)*x)/8 + (3*(4*a - 3*b)*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + ((a - b)^3*Tanh[c + d*x])/d

Rubi [A] time = 0.1291, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3191, 390, 1157, 385, 206}

$$\frac{3}{8}bx(8a^2 - 12ab + 5b^2) + \frac{3b^2(4a - 3b) \sinh(c + dx) \cosh(c + dx)}{8d} + \frac{(a - b)^3 \tanh(c + dx)}{d} + \frac{b^3 \sinh(c + dx) \cosh^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (3*b*(8*a^2 - 12*a*b + 5*b^2)*x)/8 + (3*(4*a - 3*b)*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + ((a - b)^3*Tanh[c + d*x])/d

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_))^(p_)*((c_.) + (d_.)*(x_.)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2,
x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x],
0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^2(c + dx) (a + b \sinh^2(c + dx))^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(a - (a-b)x^2)^3}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left((a-b)^3 + \frac{b(3a^2 - 3ab + b^2) - 3(a-b)(2a-b)bx^2 + 3(a-b)^2bx^4}{(1-x^2)^3}\right) dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{(a-b)^3 \tanh(c + dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{b(3a^2 - 3ab + b^2) - 3(a-b)(2a-b)bx^2 + 3(a-b)^2bx^4}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{b^3 \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{(a-b)^3 \tanh(c + dx)}{d} - \frac{\operatorname{Subst}\left(\int \frac{-3(2a-b)^2}{(1-x^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= \frac{3(4a - 3b)b^2 \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b^3 \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{(a-b)^3 \tanh(c + dx)}{d}$$

$$= \frac{3}{8}b(8a^2 - 12ab + 5b^2)x + \frac{3(4a - 3b)b^2 \cosh(c + dx) \sinh(c + dx)}{8d} + \frac{b^3 \cosh^3(c + dx) \sinh(c + dx)}{4d} + \frac{(a-b)^3 \tanh(c + dx)}{d}$$

Mathematica [A] time = 0.510685, size = 78, normalized size = 0.85

$$\frac{12b(8a^2 - 12ab + 5b^2)(c + dx) + 8b^2(3a - 2b)\sinh(2(c + dx)) + 32(a - b)^3 \tanh(c + dx) + b^3 \sinh(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (12*b*(8*a^2 - 12*a*b + 5*b^2)*(c + d*x) + 8*(3*a - 2*b)*b^2*Sinh[2*(c + d*x)] + b^3*Sinh[4*(c + d*x)] + 32*(a - b)^3*Tanh[c + d*x])/(32*d)

Maple [A] time = 0.039, size = 131, normalized size = 1.4

$$\frac{1}{d} \left(a^3 \tanh(dx + c) + 3a^2b(dx + c - \tanh(dx + c)) + 3ab^2 \left(\frac{1}{2} \frac{(\sinh(dx + c))^3}{\cosh(dx + c)} - \frac{3}{2} dx - \frac{3}{2} c + \frac{3}{2} \tanh(dx + c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*tanh(d*x+c)+3*a^2*b*(d*x+c-tanh(d*x+c))+3*a*b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^3*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+15/8*c-15/8*tanh(d*x+c)))

Maxima [B] time = 1.09097, size = 290, normalized size = 3.15

$$3a^2b \left(x + \frac{c}{d} - \frac{2}{d(e^{-2dx-2c} + 1)} \right) + \frac{1}{64} b^3 \left(\frac{120(dx + c)}{d} + \frac{16e^{(-2dx-2c)} - e^{(-4dx-4c)}}{d} - \frac{15e^{(-2dx-2c)} + 144e^{(-4dx-4c)} - 1}{d(e^{(-4dx-4c)} + e^{(-6dx-6c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 1/64*b^3*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)))) - 3/8*a*b^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))

$$-2*d*x - 2*c) + e^{(-4*d*x - 4*c)})) + 2*a^3/(d*(e^{(-2*d*x - 2*c)} + 1))$$

Fricas [B] time = 1.55093, size = 435, normalized size = 4.73

$$\frac{b^3 \sinh(dx + c)^5 + (10b^3 \cosh(dx + c)^2 + 24ab^2 - 15b^3) \sinh(dx + c)^3 - 8(8a^3 - 24a^2b + 24ab^2 - 8b^3 - 3(8a^2b - 12ab^2 + 5b^3)) \cosh(dx + c) + (5b^3 \cosh(dx + c)^4 + 64a^3 - 192a^2b + 216ab^2 - 80b^3 + 9(8a^2b^2 - 5b^3)) \sinh(dx + c)}{64d \cosh(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c))^2)^3,x, algorithm="fricas")

[Out] 1/64*(b^3*sinh(d*x + c)^5 + (10*b^3*cosh(d*x + c)^2 + 24*a*b^2 - 15*b^3)*sinh(d*x + c)^3 - 8*(8*a^3 - 24*a^2*b + 24*a*b^2 - 8*b^3 - 3*(8*a^2*b - 12*a*b^2 + 5*b^3))*cosh(d*x + c) + (5*b^3*cosh(d*x + c)^4 + 64*a^3 - 192*a^2*b + 216*a*b^2 - 80*b^3 + 9*(8*a^2*b^2 - 5*b^3))*sinh(d*x + c))/(d*cosh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.30828, size = 284, normalized size = 3.09

$$\frac{3(8a^2b - 12ab^2 + 5b^3)(dx + c)}{8d} - \frac{(144a^2be^{4dx+4c} - 216ab^2e^{4dx+4c} + 90b^3e^{4dx+4c} + 24ab^2e^{2dx+2c} - 16b^3e^{2dx+2c})}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c))^2)^3,x, algorithm="giac")

```
[Out] 3/8*(8*a^2*b - 12*a*b^2 + 5*b^3)*(d*x + c)/d - 1/64*(144*a^2*b*e^(4*d*x + 4*c) - 216*a*b^2*e^(4*d*x + 4*c) + 90*b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 16*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c)/d + 1/64*(b^3*d*e^(4*d*x + 4*c) + 24*a*b^2*d*e^(2*d*x + 2*c) - 16*b^3*d*e^(2*d*x + 2*c))/d^2 - 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)/(d*(e^(2*d*x + 2*c) + 1))
```

3.310 $\int \operatorname{sech}^3(c + dx) \left(a + b \sinh^2(c + dx)\right)^3 dx$

Optimal. Leaf size=91

$$\frac{b^2(3a - 2b) \sinh(c + dx)}{d} + \frac{(a + 5b)(a - b)^2 \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(a - b)^3 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d} + \frac{b^3 \sinh^3(c + dx)}{3d}$$

[Out] ((a - b)^2*(a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((3*a - 2*b)*b^2*Sinh[c + d*x])/d + (b^3*Sinh[c + d*x]^3)/(3*d) + ((a - b)^3*Sech[c + d*x]*Tanh[c + d*x])/(2*d)

Rubi [A] time = 0.0957924, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 390, 385, 203}

$$\frac{b^2(3a - 2b) \sinh(c + dx)}{d} + \frac{(a + 5b)(a - b)^2 \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(a - b)^3 \tanh(c + dx) \operatorname{sech}(c + dx)}{2d} + \frac{b^3 \sinh^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((a - b)^2*(a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((3*a - 2*b)*b^2*Sinh[c + d*x])/d + (b^3*Sinh[c + d*x]^3)/(3*d) + ((a - b)^3*Sech[c + d*x]*Tanh[c + d*x])/(2*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[
((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[
{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^3(c + dx) (a + b \sinh^2(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^3}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left((3a - 2b)b^2 + b^3x^2 + \frac{(a-b)^2(a+2b)+3(a-b)^2bx^2}{(1+x^2)^2}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(3a - 2b)b^2 \sinh(c + dx)}{d} + \frac{b^3 \sinh^3(c + dx)}{3d} + \frac{\operatorname{Subst}\left(\int \frac{(a-b)^2(a+2b)+3(a-b)^2bx^2}{(1+x^2)^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(3a - 2b)b^2 \sinh(c + dx)}{d} + \frac{b^3 \sinh^3(c + dx)}{3d} + \frac{(a - b)^3 \operatorname{sech}(c + dx) \tanh(c + dx)}{2d} \\ &= \frac{(a - b)^2(a + 5b) \tan^{-1}(\sinh(c + dx))}{2d} + \frac{(3a - 2b)b^2 \sinh(c + dx)}{d} + \frac{b^3 \sinh^3(c + dx)}{3d} \end{aligned}$$

Mathematica [C] time = 7.32553, size = 347, normalized size = 3.81

$$\operatorname{csch}^5(c + dx) \left(-256 \sinh^8(c + dx) (a + b \sinh^2(c + dx))^3 \operatorname{HypergeometricPFQ}\left(\left\{\frac{3}{2}, 2, 2, 2, 2\right\}, \left\{1, 1, 1, \frac{11}{2}\right\}, -\sinh^2(c + dx)\right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3 + 21*(36015*a^3 + 5*a^2*(3224*a + 21609*b)*Sinh[c + d*x]^2 + 3*a*(491*a^2 + 16120*a*b + 36015*b^2)*Sinh[c + d*x]^4 + 3*b*(753*a^2 + 18280*a*b + 10805*b^2)*Sinh[c + d*x]^6 + b^2*(2259*a + 17320*b)*Sinh[c + d*x]^8 + 753*b^3*Sinh[c + d*x]^10) - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(2401*a^3 + 3*a^2*(625*a + 2401*b)*Sinh[c + d*x]^2 + 3*a*(81*a^2 + 1875*a*b + 2401*b^2)*Sinh[c + d*x]^4 + (-47*a^3 + 585*a^2*b + 6057*a*b^2 + 2161*b^3)*Sinh[c + d*x]^6 + 3*b*(a^2 + 243*a*b + 625*b^2)*Sinh[c + d*x]^8 + 3*b^2*(a + 81*b)*Sinh[c + d*x]^10 + b^3*Sinh[c + d*x]^12))/Sqrt[-Sinh[c + d*x]^2]))/(30240*d)
```

Maple [B] time = 0.055, size = 286, normalized size = 3.1

$$\frac{a^3 \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + \frac{a^3 \arctan(e^{dx+c})}{d} - 3 \frac{a^2 b \sinh(dx+c)}{d (\cosh(dx+c))^2} + \frac{3 a^2 b \operatorname{sech}(dx+c) \tanh(dx+c)}{2d} + 3 \frac{a^2 b \operatorname{arctan}(e^{dx+c})}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d*a^3*sech(d*x+c)*tanh(d*x+c)+1/d*a^3*arctan(exp(d*x+c))-3/d*a^2*b*sinh(d*x+c)/cosh(d*x+c)^2+3/2/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/d*a^2*b*arctan(exp(d*x+c))+3/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^2+9/d*a*b^2*sinh(d*x+c)/cosh(d*x+c)^2-9/2/d*a*b^2*sech(d*x+c)*tanh(d*x+c)-9/d*a*b^2*arctan(exp(d*x+c))+1/3/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^2-5/3/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^2-5/d*b^3*sinh(d*x+c)/cosh(d*x+c)^2+5/2/d*b^3*sech(d*x+c)*tanh(d*x+c)+5/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.59216, size = 482, normalized size = 5.3

$$\frac{1}{24} b^3 \left(\frac{27 e^{(-dx-c)} - e^{(-3dx-3c)}}{d} - \frac{120 \arctan(e^{(-dx-c)})}{d} - \frac{25 e^{(-2dx-2c)} + 77 e^{(-4dx-4c)} + 3 e^{(-6dx-6c)} - 1}{d(e^{(-3dx-3c)} + 2 e^{(-5dx-5c)} + e^{(-7dx-7c)})} \right) + \frac{3}{2} ab^2 \left(\frac{6 \arctan(e^{(-dx-c)})}{d} - \frac{25 e^{(-2dx-2c)} + 77 e^{(-4dx-4c)} + 3 e^{(-6dx-6c)} - 1}{d(e^{(-3dx-3c)} + 2 e^{(-5dx-5c)} + e^{(-7dx-7c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(
```


$$d*(e^{(-3*d*x - 3*c)} + 2*e^{(-5*d*x - 5*c)} + e^{(-7*d*x - 7*c)})) + 3/2*a*b^2*(6*\arctan(e^{(-d*x - c)})/d - e^{(-d*x - c)}/d + (4*e^{(-2*d*x - 2*c)} - e^{(-4*d*x - 4*c)} + 1)/(d*(e^{(-d*x - c)} + 2*e^{(-3*d*x - 3*c)} + e^{(-5*d*x - 5*c)}))) - 3*a^2*b*(\arctan(e^{(-d*x - c)})/d + (e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1))) - a^3*(\arctan(e^{(-d*x - c)})/d - (e^{(-d*x - c)} - e^{(-3*d*x - 3*c)})/(d*(2*e^{(-2*d*x - 2*c)} + e^{(-4*d*x - 4*c)} + 1)))$$

Fricas [B] time = 1.81228, size = 4188, normalized size = 46.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $1/24*(b^3*\cosh(d*x + c)^{10} + 10*b^3*\cosh(d*x + c)*\sinh(d*x + c)^9 + b^3*\sinh(d*x + c)^{10} + (36*a*b^2 - 25*b^3)*\cosh(d*x + c)^8 + (45*b^3*\cosh(d*x + c)^2 + 36*a*b^2 - 25*b^3)*\sinh(d*x + c)^8 + 8*(15*b^3*\cosh(d*x + c)^3 + (36*a*b^2 - 25*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*\cosh(d*x + c)^6 + 2*(105*b^3*\cosh(d*x + c)^4 + 12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3 + 14*(36*a*b^2 - 25*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*b^3*\cosh(d*x + c)^5 + 14*(36*a*b^2 - 25*b^3)*\cosh(d*x + c)^3 + 3*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 2*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*\cosh(d*x + c)^4 + 2*(105*b^3*\cosh(d*x + c)^6 + 35*(36*a*b^2 - 25*b^3)*\cosh(d*x + c)^4 - 12*a^3 + 36*a^2*b - 54*a*b^2 + 25*b^3 + 15*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(15*b^3*\cosh(d*x + c)^7 + 7*(36*a*b^2 - 25*b^3)*\cosh(d*x + c)^5 + 5*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*\cosh(d*x + c)^3 - (12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 - b^3 - (36*a*b^2 - 25*b^3)*\cosh(d*x + c)^2 + (45*b^3*\cosh(d*x + c)^8 + 28*(36*a*b^2 - 25*b^3)*\cosh(d*x + c)^6 + 30*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*\cosh(d*x + c)^4 - 36*a*b^2 + 25*b^3 - 12*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 24*((a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^6 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\sinh(d*x + c)^7 + 2*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + (2*a^3 + 6*a^2*b - 18*a*b^2 + 10*b^3 + 21*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 5*(7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + 2*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + (35*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3 + 20*(a^3 + 3*a^2*b$

```

- 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(a^3 + 3*a^2*b -
9*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cos
h(d*x + c)^3 + 3*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^2 + (7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(a^3 + 3
*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 3*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c))
+ 2*(5*b^3*cosh(d*x + c)^9 + 4*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^7 + 6*(12*
a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^5 - 4*(12*a^3 - 36*a^2*b
+ 54*a*b^2 - 25*b^3)*cosh(d*x + c)^3 - (36*a*b^2 - 25*b^3)*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*si
nh(d*x + c)^7 + 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x
+ c)^5 + 5*(7*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*c
osh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^3 + (21*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))
*sinh(d*x + c)^2 + (7*d*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d
*x + c)^2)*sinh(d*x + c))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43009, size = 354, normalized size = 3.89

$$\frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{2dx+2c} - 1\right)e^{-dx-c}\right)\right)\left(a^3 + 3a^2b - 9ab^2 + 5b^3\right)}{4d} + \frac{a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 3a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 3b^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{\left(e^{(dx+c)} - e^{(-dx-c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^3 + 3*a^2*b
- 9*a*b^2 + 5*b^3)/d + (a^3*(e^(d*x + c) - e^(-d*x - c)) - 3*a^2*b*(e^(d*x
```

$$\begin{aligned} &+ c) - e^{(-d*x - c)} + 3*a*b^2*(e^{(d*x + c)} - e^{(-d*x - c)}) - b^3*(e^{(d*x + c)} \\ &- e^{(-d*x - c)})/(((e^{(d*x + c)} - e^{(-d*x - c)})^2 + 4)*d) + 1/24*(b^3*d \\ &^2*(e^{(d*x + c)} - e^{(-d*x - c)})^3 + 36*a*b^2*d^2*(e^{(d*x + c)} - e^{(-d*x - c)} \\ &)) - 24*b^3*d^2*(e^{(d*x + c)} - e^{(-d*x - c)})/d^3 \end{aligned}$$

3.311 $\int \operatorname{sech}^4(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=82

$$\frac{1}{2}b^2x(6a - 5b) - \frac{(a - b)^3 \tanh^3(c + dx)}{3d} + \frac{(a - b)^2(a + 2b) \tanh(c + dx)}{d} + \frac{b^3 \sinh(c + dx) \cosh(c + dx)}{2d}$$

[Out] $((6*a - 5*b)*b^2*x)/2 + (b^3*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(2*d) + ((a - b)^2*(a + 2*b)*\operatorname{Tanh}[c + d*x])/d - ((a - b)^3*\operatorname{Tanh}[c + d*x]^3)/(3*d)$

Rubi [A] time = 0.108489, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3191, 390, 385, 206}

$$\frac{1}{2}b^2x(6a - 5b) - \frac{(a - b)^3 \tanh^3(c + dx)}{3d} + \frac{(a - b)^2(a + 2b) \tanh(c + dx)}{d} + \frac{b^3 \sinh(c + dx) \cosh(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[c + d*x]^4*(a + b*\operatorname{Sinh}[c + d*x]^2)^3, x]$

[Out] $((6*a - 5*b)*b^2*x)/2 + (b^3*\operatorname{Cosh}[c + d*x]*\operatorname{Sinh}[c + d*x])/(2*d) + ((a - b)^2*(a + 2*b)*\operatorname{Tanh}[c + d*x])/d - ((a - b)^3*\operatorname{Tanh}[c + d*x]^3)/(3*d)$

Rule 3191

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^{(m/2 + p + 1)}], x], \operatorname{Tan}[e + f*x]/ff], x]\} /;$ $\operatorname{FreeQ}\{a, b, e, f\}, x\} \ \&\& \ \operatorname{IntegerQ}[m/2] \ \&\& \ \operatorname{IntegerQ}[p]$

Rule 390

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(a + b*x^n)^p, (c + d*x^n)^{-q}], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x\} \ \&\& \ \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \ \operatorname{IGtQ}[n, 0] \ \&\& \ \operatorname{IGtQ}[p, 0] \ \&\& \ \operatorname{ILtQ}[q, 0] \ \&\& \ \operatorname{GeQ}[p, -q]$

Rule 385

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(n_.)}]^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow -\operatorname{Simp}[(b*c - a*d)*x*(a + b*x^n)^{(p + 1)}/(a*b*n*(p + 1)), x] - \operatorname{Dist}[(a*d - b$

c(n*(p + 1) + 1)/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^4(c + dx) (a + b \sinh^2(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a - (a-b)x^2)^3}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left((a-b)^2(a+2b) - (a-b)^3x^2 + \frac{(3a-2b)b^2 - 3(a-b)b^2x^2}{(1-x^2)^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{(a-b)^2(a+2b) \tanh(c + dx)}{d} - \frac{(a-b)^3 \tanh^3(c + dx)}{3d} + \frac{\operatorname{Subst}\left(\int \frac{(3a-2b)b^2}{(1-x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{b^3 \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{(a-b)^2(a+2b) \tanh(c + dx)}{d} - \frac{(a-b)^3 \tanh^3(c + dx)}{3d} \\ &= \frac{1}{2}(6a - 5b)b^2x + \frac{b^3 \cosh(c + dx) \sinh(c + dx)}{2d} + \frac{(a-b)^2(a+2b) \tanh(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.738259, size = 84, normalized size = 1.02

$$\frac{6b^2(6a - 5b)(c + dx) + 2(a - b)^2 \tanh(c + dx) \operatorname{sech}^2(c + dx) ((2a + 7b) \cosh(2(c + dx)) + 4a + 5b) + 3b^3 \sinh(2(c + dx))}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3, x]

[Out] (6*(6*a - 5*b)*b^2*(c + d*x) + 3*b^3*Sinh[2*(c + d*x)] + 2*(a - b)^2*(4*a + 5*b + (2*a + 7*b)*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Tanh[c + d*x]/(12*d)

Maple [A] time = 0.054, size = 148, normalized size = 1.8

$$\frac{1}{d} \left(a^3 \left(\frac{2}{3} + \frac{(\operatorname{sech}(dx+c))^2}{3} \right) \tanh(dx+c) + 3a^2b \left(-\frac{1}{2} \frac{\sinh(dx+c)}{(\cosh(dx+c))^3} + \frac{1}{2} \left(\frac{2}{3} + \frac{1}{3} (\operatorname{sech}(dx+c))^2 \right) \tanh(dx+c) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3)+b^3*(1/2*sinh(d*x+c)^5/cosh(d*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))

Maxima [B] time = 1.14396, size = 516, normalized size = 6.29

$$ab^2 \left(3x + \frac{3c}{d} - \frac{4(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + 2)}{d(3e^{(-2dx-2c)} + 3e^{(-4dx-4c)} + e^{(-6dx-6c)} + 1)} \right) - \frac{1}{24} b^3 \left(\frac{60(dx+c)}{d} + \frac{3e^{(-2dx-2c)}}{d} - \frac{121e^{(-2dx-2c)} + 201e^{(-4dx-4c)}}{d(e^{(-2dx-2c)} + 3e^{(-4dx-4c)})} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 1/24*b^3*(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a^2*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))

Fricas [B] time = 1.5977, size = 788, normalized size = 9.61

$$3b^3 \sinh(dx+c)^5 - 4(4a^3 + 6a^2b - 24ab^2 + 14b^3 - 3(6ab^2 - 5b^3)dx) \cosh(dx+c)^3 - 12(4a^3 + 6a^2b - 24ab^2 + 14b^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $\frac{1}{24}*(3*b^3*\sinh(d*x + c)^5 - 4*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*\cosh(d*x + c)^3 - 12*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (30*b^3*\cosh(d*x + c)^2 + 16*a^3 + 24*a^2*b - 96*a*b^2 + 65*b^3)*\sinh(d*x + c)^3 - 12*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*\cosh(d*x + c) + 3*(5*b^3*\cosh(d*x + c)^4 + 16*a^3 - 24*a^2*b + 10*b^3 + (16*a^3 + 24*a^2*b - 96*a*b^2 + 65*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(d*\cosh(d*x + c)^3 + 3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + 3*d*\cosh(d*x + c))$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.37152, size = 290, normalized size = 3.54

$$\frac{b^3 e^{(2dx+2c)}}{8d} + \frac{(6ab^2 - 5b^3)(dx+c)}{2d} - \frac{(12ab^2 e^{(2dx+2c)} - 10b^3 e^{(2dx+2c)} + b^3) e^{(-2dx-2c)}}{8d} - \frac{2(9a^2 b e^{(4dx+4c)} - 18ab^2 e^{(4dx+4c)})}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{8}*b^3*e^{(2*d*x + 2*c)}/d + \frac{1}{2}*(6*a*b^2 - 5*b^3)*(d*x + c)/d - \frac{1}{8}*(12*a*b^2*e^{(2*d*x + 2*c)} - 10*b^3*e^{(2*d*x + 2*c)} + b^3)*e^{(-2*d*x - 2*c)}/d - \frac{2}{3}*(9*a^2*b*e^{(4*d*x + 4*c)} - 18*a*b^2*e^{(4*d*x + 4*c)} + 9*b^3*e^{(4*d*x + 4*c)} + 6*a^3*e^{(2*d*x + 2*c)} - 18*a*b^2*e^{(2*d*x + 2*c)} + 12*b^3*e^{(2*d*x + 2*c)} + 2*a^3 + 3*a^2*b - 12*a*b^2 + 7*b^3)/(d*(e^{(2*d*x + 2*c)} + 1)^3)$

3.312 $\int \operatorname{sech}^5(c + dx) \left(a + b \sinh^2(c + dx)\right)^3 dx$

Optimal. Leaf size=103

$$\frac{3(a-b)\left((a+b)^2 + 4b^2\right) \tan^{-1}(\sinh(c+dx))}{8d} + \frac{(a-b)^3 \tanh(c+dx) \operatorname{sech}^3(c+dx)}{4d} + \frac{3(a-b)^2(a+3b) \tanh(c+dx) \operatorname{sech}(c+dx)}{8d}$$

[Out] (3*(a - b)*(4*b^2 + (a + b)^2)*ArcTan[Sinh[c + d*x]]/(8*d) + (b^3*Sinh[c + d*x])/d + (3*(a - b)^2*(a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + ((a - b)^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)

Rubi [A] time = 0.132009, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3190, 390, 1157, 385, 203}

$$\frac{3(a-b)\left((a+b)^2 + 4b^2\right) \tan^{-1}(\sinh(c+dx))}{8d} + \frac{(a-b)^3 \tanh(c+dx) \operatorname{sech}^3(c+dx)}{4d} + \frac{3(a-b)^2(a+3b) \tanh(c+dx) \operatorname{sech}(c+dx)}{8d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (3*(a - b)*(4*b^2 + (a + b)^2)*ArcTan[Sinh[c + d*x]]/(8*d) + (b^3*Sinh[c + d*x])/d + (3*(a - b)^2*(a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + ((a - b)^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 1157


```

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

```

Rule 385

```

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])

```

Rule 203

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^5(c + dx) (a + b \sinh^2(c + dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^3}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(b^3 + \frac{a^3 - b^3 + 3b(a^2 - b^2)x^2 + 3(a-b)b^2x^4}{(1+x^2)^3}\right) dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{b^3 \sinh(c + dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{a^3 - b^3 + 3b(a^2 - b^2)x^2 + 3(a-b)b^2x^4}{(1+x^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{b^3 \sinh(c + dx)}{d} + \frac{(a - b)^3 \operatorname{sech}^3(c + dx) \tanh(c + dx)}{4d} - \frac{\operatorname{Subst}\left(\int \frac{-3(a-b)}{\dots} dx, x, \sinh(c + dx)\right)}{4d} \\
&= \frac{b^3 \sinh(c + dx)}{d} + \frac{3(a - b)^2(a + 3b) \operatorname{sech}(c + dx) \tanh(c + dx)}{8d} + \frac{(a - b)^3}{4d} \\
&= \frac{3(a - b)(4b^2 + (a + b)^2) \tan^{-1}(\sinh(c + dx))}{8d} + \frac{b^3 \sinh(c + dx)}{d} + \frac{3(a - b)^3}{4d}
\end{aligned}$$

Mathematica [C] time = 9.93774, size = 472, normalized size = 4.58

$$\operatorname{csch}^5(c+dx) \left(256 \sinh^8(c+dx) (a+b \sinh^2(c+dx))^3 \operatorname{HypergeometricPFQ} \left(\left\{ \frac{3}{2}, 2, 2, 2, 2, 2 \right\}, \left\{ 1, 1, 1, 1, \frac{11}{2} \right\}, -\sinh^2(c+dx) \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] -(Csch[c + d*x]^5*(256*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3 + 384*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^2*(7*a + 5*b*Sinh[c + d*x]^2) - 21*(15*a*b^2*Sinh[c + d*x]^4*(36015 + 21529*Sinh[c + d*x]^2 + 1128*Sinh[c + d*x]^4) + 9*a^2*b*Sinh[c + d*x]^2*(72030 + 41615*Sinh[c + d*x]^2 + 2131*Sinh[c + d*x]^4) + b^3*Sinh[c + d*x]^6*(149460 + 90805*Sinh[c + d*x]^2 + 4887*Sinh[c + d*x]^4) + a^3*(252105 + 140965*Sinh[c + d*x]^2 + 8226*Sinh[c + d*x]^4)) + (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(a^3*(16807 + 15000*Sinh[c + d*x]^2 + 2187*Sinh[c + d*x]^4 - 62*Sinh[c + d*x]^6) + 9*a^2*b*Sinh[c + d*x]^2*(4802 + 4375*Sinh[c + d*x]^2 + 640*Sinh[c + d*x]^4 + 3*Sinh[c + d*x]^6) + b^3*Sinh[c + d*x]^6*(9964 + 9375*Sinh[c + d*x]^2 + 1458*Sinh[c + d*x]^4 + 7*Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]^4*(12005 + 11178*Sinh[c + d*x]^2 + 1701*Sinh[c + d*x]^4 + 8*Sinh[c + d*x]^6)))/Sqrt[-Sinh[c + d*x]^2]))/(60480*d)

Maple [B] time = 0.062, size = 376, normalized size = 3.7

$$\frac{a^3 \tanh(dx+c) (\operatorname{sech}(dx+c))^3}{4d} + \frac{3a^3 \operatorname{sech}(dx+c) \tanh(dx+c)}{8d} + \frac{3a^3 \arctan(e^{dx+c})}{4d} - \frac{a^2 b \sinh(dx+c)}{d (\cosh(dx+c))^4} + \frac{a^2 b \tanh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/4/d*a^3*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a^3*sech(d*x+c)*tanh(d*x+c)+3/4/d*a^3*arctan(exp(d*x+c))-1/d*a^2*b*sinh(d*x+c)/cosh(d*x+c)^4+1/4/d*a^2*b*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/4/d*a^2*b*arctan(exp(d*x+c))-3/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^4-3/d*a*b^2*sinh(d*x+c)/cosh(d*x+c)^4+3/4/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^3+9/8/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+9/4/d*a*b^2*arctan(exp(d*x+c))+1/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^4

$$c)^4 + 5/d*b^3*\sinh(d*x+c)^3/\cosh(d*x+c)^4 + 5/d*b^3*\sinh(d*x+c)/\cosh(d*x+c)^4 - 5/4/d*b^3*\tanh(d*x+c)*\operatorname{sech}(d*x+c)^3 - 15/8/d*b^3*\operatorname{sech}(d*x+c)*\tanh(d*x+c) - 15/4/d*b^3*\arctan(\exp(d*x+c))$$

Maxima [B] time = 1.71048, size = 660, normalized size = 6.41

$$\frac{1}{4} b^3 \left(\frac{15 \arctan(e^{(-dx-c)})}{d} - \frac{2e^{(-dx-c)}}{d} + \frac{17e^{(-2dx-2c)} + 13e^{(-4dx-4c)} + 7e^{(-6dx-6c)} - 7e^{(-8dx-8c)} + 2}{d(e^{(-dx-c)} + 4e^{(-3dx-3c)} + 6e^{(-5dx-5c)} + 4e^{(-7dx-7c)} + e^{(-9dx-9c)})} \right) - \frac{3}{4} ab^2 \left(\frac{3a}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{4} b^3 \left(\frac{15 \arctan(e^{(-dx-c)})}{d} - \frac{2e^{(-dx-c)}}{d} + \frac{17e^{(-2dx-2c)} + 13e^{(-4dx-4c)} + 7e^{(-6dx-6c)} - 7e^{(-8dx-8c)} + 2}{d(e^{(-dx-c)} + 4e^{(-3dx-3c)} + 6e^{(-5dx-5c)} + 4e^{(-7dx-7c)} + e^{(-9dx-9c)})} \right) - \frac{3}{4} ab^2 \left(\frac{3a}{d} \right)$

Fricas [B] time = 1.76507, size = 5520, normalized size = 53.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] $\frac{1}{4} (2b^3 \cosh(dx+c)^{10} + 20b^3 \cosh(dx+c) \sinh(dx+c)^9 + 2b^3 \sinh(dx+c)^{10} + 3(a^3 + a^2b - 5ab^2 + 5b^3) \cosh(dx+c)^8 + 3(30b^3 \cosh(dx+c)^2 + a^3 + a^2b - 5ab^2 + 5b^3) \sinh(dx+c)^8 + 24(10b^3 \cosh(dx+c)^3 + (a^3 + a^2b - 5ab^2 + 5b^3) \cosh(dx+c)) \sinh(dx+c)^7 + (11a^3 - 21a^2b + 9ab^2 + 5b^3) \cosh(dx+c)^6 + (4$

$$\begin{aligned}
& 20*b^3*\cosh(d*x + c)^4 + 11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3 + 84*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^6 + 6*(84*b^3*\cosh(d*x + c)^5 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + (11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 - (11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + (420*b^3*\cosh(d*x + c)^6 + 210*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 - 11*a^3 + 21*a^2*b - 9*a*b^2 - 5*b^3 + 15*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(60*b^3*\cosh(d*x + c)^7 + 42*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 + 5*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 - (11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 2*b^3 - 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*\cosh(d*x + c)^2 + 3*(30*b^3*\cosh(d*x + c)^8 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 5*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 - a^3 - a^2*b + 5*a*b^2 - 5*b^3 - 2*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 3*((a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^9 + 9*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^8 + (a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\sinh(d*x + c)^9 + 4*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^7 + 4*(a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 9*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^7 + 28*(3*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^3 + (a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^6 + 6*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^5 + 6*(21*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 14*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 2*(63*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^5 + 70*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^3 + 15*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^3 + 4*(21*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^6 + 35*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 15*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 12*(3*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^7 + 7*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^5 + 5*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^3 + (a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + (a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c) + (9*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^8 + 28*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^6 + 30*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 12*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) + 2*(10*b^3*\cosh(d*x + c)^9 + 12*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 3*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^5 - 2*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 - 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/(d*\cosh(d*x + c)^9 + 9*d*\cosh(d*x + c)*\sinh(d*x + c)^8 + d*\sinh(d*x + c)^9 + 4*d*\cosh(d*x + c)^7 + 4*(9*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^7 + 28*(3*d*\cosh(d*x + c)^3 + d*\cosh(d*x + c))*\sinh(d*x + c)^6 + 6*d*\cosh(d*x + c)^5 + 6*(21*d*\cosh(d*x + c)^4 + 14*d*\cosh(d*x + c)^2 + d)*\sinh(d*x + c)^5 + 2*(63*d*\cosh(d*x + c)^5 + 70*d*\cosh(d*x + c)^3 + 15*d*\cosh(d*x + c))*\sinh(d*x + c)^4 + 4*d*\cosh(d
\end{aligned}$$

$$\begin{aligned}
 & *x + c)^3 + 4*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x \\
 & + c)^2 + d)*sinh(d*x + c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5 \\
 & + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) \\
 & + (9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12* \\
 & d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
 \end{aligned}$$

Sympy [F-1] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.35793, size = 412, normalized size = 4.

$$\frac{b^3(e^{(dx+c)} - e^{(-dx-c)})}{2d} + \frac{3\left(\pi + 2 \arctan\left(\frac{1}{2}(e^{2dx+2c} - 1)e^{(-dx-c)}\right)\right)(a^3 + a^2b + 3ab^2 - 5b^3)}{16d} + \frac{3a^3(e^{(dx+c)} - e^{(-dx-c)})^3}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $1/2*b^3*(e^{(d*x + c)} - e^{(-d*x - c)})/d + 3/16*(\pi + 2*\arctan(1/2*(e^{(2*d*x + 2*c)} - 1)*e^{(-d*x - c)}))*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)/d + 1/4*(3*a^3*(e^{(d*x + c)} - e^{(-d*x - c)})^3 + 3*a^2*b*(e^{(d*x + c)} - e^{(-d*x - c)})^3 - 15*a*b^2*(e^{(d*x + c)} - e^{(-d*x - c)})^3 + 9*b^3*(e^{(d*x + c)} - e^{(-d*x - c)})^3 + 20*a^3*(e^{(d*x + c)} - e^{(-d*x - c)}) - 12*a^2*b*(e^{(d*x + c)} - e^{(-d*x - c)}) - 36*a*b^2*(e^{(d*x + c)} - e^{(-d*x - c)}) + 28*b^3*(e^{(d*x + c)} - e^{(-d*x - c)}))/(((e^{(d*x + c)} - e^{(-d*x - c)})^2 + 4)^2*d)$

3.313 $\int \operatorname{sech}^6(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=74

$$\frac{(a^3 - b^3) \tanh(c + dx)}{d} + \frac{(a - b)^3 \tanh^5(c + dx)}{5d} - \frac{(a - b)^2(2a + b) \tanh^3(c + dx)}{3d} + b^3 x$$

[Out] $b^3 x + ((a^3 - b^3) \operatorname{Tanh}[c + d x]) / d - ((a - b)^2 (2 a + b) \operatorname{Tanh}[c + d x]^3) / (3 d) + ((a - b)^3 \operatorname{Tanh}[c + d x]^5) / (5 d)$

Rubi [A] time = 0.0798852, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3191, 390, 206}

$$\frac{(a^3 - b^3) \tanh(c + dx)}{d} + \frac{(a - b)^3 \tanh^5(c + dx)}{5d} - \frac{(a - b)^2(2a + b) \tanh^3(c + dx)}{3d} + b^3 x$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[c + d x]^6 (a + b \operatorname{Sinh}[c + d x]^2)^3, x]$

[Out] $b^3 x + ((a^3 - b^3) \operatorname{Tanh}[c + d x]) / d - ((a - b)^2 (2 a + b) \operatorname{Tanh}[c + d x]^3) / (3 d) + ((a - b)^3 \operatorname{Tanh}[c + d x]^5) / (5 d)$

Rule 3191

$\operatorname{Int}[\cos[(e_.) + (f_.)(x_.)]^{(m_.)} ((a_.) + (b_.) \sin[(e_.) + (f_.)(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(a + (a + b) ff^2 x^2)^p / (1 + ff^2 x^2)^{(m/2 + p + 1)}], x], \operatorname{Tan}[e + f x] / ff], x] /;$ $\operatorname{FreeQ}\{a, b, e, f\}, x \ \&\& \operatorname{IntegerQ}[m/2] \ \&\& \operatorname{IntegerQ}[p]$

Rule 390

$\operatorname{Int}[(a_.) + (b_.)(x_.)^{(n_.)]^{(p_.)} ((c_.) + (d_.)(x_.)^{(n_.)})^{(q_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{PolynomialDivide}[(a + b x^n)^p, (c + d x^n)^{-q}], x] /;$ $\operatorname{FreeQ}\{a, b, c, d\}, x \ \&\& \operatorname{NeQ}[b c - a d, 0] \ \&\& \operatorname{IGtQ}[n, 0] \ \&\& \operatorname{IGtQ}[p, 0] \ \&\& \operatorname{ILtQ}[q, 0] \ \&\& \operatorname{GeQ}[p, -q]$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)(x_.)^2]^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1 * \operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2] * x) / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2] * \operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{Gt}$

Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^6(c+dx) (a+b \sinh^2(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a-(a-b)x^2)^3}{1-x^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(a^3 - b^3 - (a-b)^2(2a+b)x^2 + (a-b)^3x^4 + \frac{b^3}{1-x^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{(a^3 - b^3) \tanh(c+dx)}{d} - \frac{(a-b)^2(2a+b) \tanh^3(c+dx)}{3d} + \frac{(a-b)^3 \tanh^5(c+dx)}{5d} \\ &= b^3x + \frac{(a^3 - b^3) \tanh(c+dx)}{d} - \frac{(a-b)^2(2a+b) \tanh^3(c+dx)}{3d} + \frac{(a-b)^3 \tanh^5(c+dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.731567, size = 86, normalized size = 1.16

$$\frac{(a-b) \tanh(c+dx) \left((4a^2 + 7ab - 11b^2) \operatorname{sech}^2(c+dx) + 8a^2 + 3(a-b)^2 \operatorname{sech}^4(c+dx) + 14ab + 23b^2 \right) + 15b^3(c+dx)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (15*b^3*(c + d*x) + (a - b)*(8*a^2 + 14*a*b + 23*b^2 + (4*a^2 + 7*a*b - 11*b^2)*Sech[c + d*x]^2 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(15*d)

Maple [B] time = 0.058, size = 199, normalized size = 2.7

$$\frac{1}{d} \left(a^3 \left(\frac{8}{15} + \frac{(\operatorname{sech}(dx+c))^4}{5} + \frac{4(\operatorname{sech}(dx+c))^2}{15} \right) \tanh(dx+c) + 3a^2b \left(-\frac{1}{4} \frac{\sinh(dx+c)}{(\cosh(dx+c))^5} + \frac{1}{4} \left(\frac{8}{15} + \frac{1}{5} (\operatorname{sech}(dx+c))^4 \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2))

2)*tanh(d*x+c))+3*a*b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3-1/5*tanh(d*x+c)^5))

Maxima [B] time = 1.24156, size = 1112, normalized size = 15.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $\frac{1}{15}b^3(15x + 15c/d - 2(70e^{(-2dx - 2c)} + 140e^{(-4dx - 4c)} + 90e^{(-6dx - 6c)} + 45e^{(-8dx - 8c)} + 23)/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1))) + 16/15a^3(5e^{(-2dx - 2c)})/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1)) + 10e^{(-4dx - 4c)}/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1)) + 1/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1))) + 4/5a^2b(5e^{(-2dx - 2c)})/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1)) - 5e^{(-4dx - 4c)}/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1)) + 15e^{(-6dx - 6c)}/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1)) + 1/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1))) + 6/5ab^2(10e^{(-4dx - 4c)})/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1)) + 5e^{(-8dx - 8c)}/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1)) + 1/(d(5e^{(-2dx - 2c)} + 10e^{(-4dx - 4c)} + 10e^{(-6dx - 6c)} + 5e^{(-8dx - 8c)} + e^{(-10dx - 10c)} + 1)))$

Fricas [B] time = 1.56797, size = 1287, normalized size = 17.39

$(15b^3dx - 8a^3 - 6a^2b - 9ab^2 + 23b^3) \cosh(dx + c)^5 + 5(15b^3dx - 8a^3 - 6a^2b - 9ab^2 + 23b^3) \cosh(dx + c) \sinh(dx + c)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\frac{1}{15} \left((15b^3dx - 8a^3 - 6a^2b - 9ab^2 + 23b^3) \cosh(dx+c)^5 + 5(15b^3dx - 8a^3 - 6a^2b - 9ab^2 + 23b^3) \cosh(dx+c) \sinh(dx+c)^4 + (8a^3 + 6a^2b + 9ab^2 - 23b^3) \sinh(dx+c)^5 + 5(15b^3dx - 8a^3 - 6a^2b - 9ab^2 + 23b^3) \cosh(dx+c)^3 + 5(8a^3 + 6a^2b - 9ab^2 - 5b^3 + 2(8a^3 + 6a^2b + 9ab^2 - 23b^3) \cosh(dx+c)^2) \sinh(dx+c)^3 + 5(2(15b^3dx - 8a^3 - 6a^2b - 9ab^2 + 23b^3) \cosh(dx+c)^3 + 3(15b^3dx - 8a^3 - 6a^2b - 9ab^2 + 23b^3) \cosh(dx+c)) \sinh(dx+c)^2 + 10(15b^3dx - 8a^3 - 6a^2b - 9ab^2 + 23b^3) \cosh(dx+c) + 5((8a^3 + 6a^2b + 9ab^2 - 23b^3) \cosh(dx+c))^4 + 16a^3 - 24a^2b + 18ab^2 - 10b^3 + 3(8a^3 + 6a^2b - 9ab^2 - 5b^3) \cosh(dx+c)^2) \sinh(dx+c) \right) / (d \cosh(dx+c)^5 + 5d \cosh(dx+c) \sinh(dx+c)^4 + 5d \cosh(dx+c)^3 + 5(2d \cosh(dx+c)^3 + 3d \cosh(dx+c)) \sinh(dx+c)^2 + 10d \cosh(dx+c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.38027, size = 288, normalized size = 3.89

$$\frac{(dx+c)b^3}{d} - \frac{2(45ab^2e^{(8dx+8c)} - 45b^3e^{(8dx+8c)} + 90a^2be^{(6dx+6c)} - 90b^3e^{(6dx+6c)} + 80a^3e^{(4dx+4c)} - 30a^2be^{(4dx+4c)} + 90a^3e^{(4dx+4c)})}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$(dx+c)b^3/d - 2/15(45a^2b^2e^{(8dx+8c)} - 45b^3e^{(8dx+8c)} + 90a^2b^2e^{(6dx+6c)} - 90b^3e^{(6dx+6c)} + 80a^3e^{(4dx+4c)})$$

$$\begin{aligned} & - 30a^2b e^{(4dx + 4c)} + 90ab^2 e^{(4dx + 4c)} - 140b^3 e^{(4dx + 4c)} \\ & + 40a^3 e^{(2dx + 2c)} + 30a^2b e^{(2dx + 2c)} - 70b^3 e^{(2dx + 2c)} \\ & + 8a^3 + 6a^2b + 9ab^2 - 23b^3) / (d(e^{(2dx + 2c)} + 1)^5) \end{aligned}$$

3.314 $\int \operatorname{sech}^7(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=154

$$\frac{(a+b)(5a^2-2ab+5b^2)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{(a-b)(15a^2+14ab+15b^2)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d} + \frac{5(a^2-b^2)\tanh(c+dx)\operatorname{sech}(c+dx)^3}{24d} + \frac{5(a-b)\operatorname{sech}(c+dx)^5}{6d}$$

```
[Out] ((a + b)*(5*a^2 - 2*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((a - b)*(15*a^2 + 14*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*(a^2 - b^2)*Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + ((a - b)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rubi [A] time = 0.152918, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3190, 413, 526, 385, 203}

$$\frac{(a+b)(5a^2-2ab+5b^2)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{(a-b)(15a^2+14ab+15b^2)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d} + \frac{5(a^2-b^2)\tanh(c+dx)\operatorname{sech}(c+dx)^3}{24d} + \frac{5(a-b)\operatorname{sech}(c+dx)^5}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a + b)*(5*a^2 - 2*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((a - b)*(15*a^2 + 14*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*(a^2 - b^2)*Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + ((a - b)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + 1) + 1)), x], x]
```

+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 526

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && GtQ[q, 0]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^7(c+dx) (a+b \sinh^2(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^3}{(1+x^2)^4} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{(a-b)\operatorname{sech}^5(c+dx) (a+b \sinh^2(c+dx))^2 \tanh(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^3}{(1+x^2)^4} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{5(a^2-b^2)\operatorname{sech}^3(c+dx) (a+b \sinh^2(c+dx)) \tanh(c+dx)}{24d} + \frac{(a-b)\operatorname{sech}^5(c+dx) (a+b \sinh^2(c+dx))^2 \tanh(c+dx)}{6d} \\
&= \frac{(a-b)(15a^2+14ab+15b^2)\operatorname{sech}(c+dx) \tanh(c+dx)}{48d} + \frac{5(a^2-b^2)\operatorname{sech}^3(c+dx) (a+b \sinh^2(c+dx)) \tanh(c+dx)}{24d} \\
&= \frac{(a+b)(5a^2-2ab+5b^2)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{(a-b)(15a^2+14ab+15b^2)\operatorname{sech}(c+dx) \tanh(c+dx)}{48d}
\end{aligned}$$

Mathematica [C] time = 14.2621, size = 1192, normalized size = 7.74

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (Csch[c + d*x]^5*(-117228825*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] - 109265625*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 274542345*a^2*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 17069535*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 - 260465625*a^2*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 - 215549775*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 + 142065*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 41427855*a^2*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 207173295*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 58009455*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 210735*a^2*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8 - 33756345*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8 - 56109375*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^8 - 174825*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10 - 9261945*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10 - 48825*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^12 + 117228825*a^3*Sqrt[-Sinh[c + d*x]^2] + 4093425*a^3*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 168951510*a^2*b*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 215549775*a*b^2*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 9514449*a^2*b*Sinh[c + d*x]^6*Sqrt[-Sinh[c

```

+ d*x]^2] + 135323370*a*b^2*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 580094
55*b^3*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 7808535*a*b^2*Sinh[c + d*x]
^8*Sqrt[-Sinh[c + d*x]^2] + 36772890*b^3*Sinh[c + d*x]^8*Sqrt[-Sinh[c + d*x]
]^2] + 2160711*b^3*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*x]^2] - 70189350*a^3*(
-Sinh[c + d*x]^2)^(3/2) - 274542345*a^2*b*(-Sinh[c + d*x]^2)^(3/2) + 1024*a
^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[
c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2) + 3072*a^2*b*Hypergeom
etricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*
Sinh[c + d*x]^8*(-Sinh[c + d*x]^2)^(3/2) + 3072*a*b^2*HypergeometricPFQ[{3/
2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]
]^10*(-Sinh[c + d*x]^2)^(3/2) + 1024*b^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2
, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12*(-Sinh[
c + d*x]^2)^(3/2) + 1536*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1
, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2)*(a + b*
Sinh[c + d*x]^2)^2*(9*a + 7*b*Sinh[c + d*x]^2) + 256*HypergeometricPFQ[{3/2
, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[
c + d*x]^2)^(3/2)*(295*a^3 + 741*a^2*b*Sinh[c + d*x]^2 + 621*a*b^2*Sinh[c +
d*x]^4 + 175*b^3*Sinh[c + d*x]^6))/(725760*d*Sqrt[-Sinh[c + d*x]^2])

```

Maple [B] time = 0.062, size = 467, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x)
```

```

[Out] 1/6/d*a^3*tanh(d*x+c)*sech(d*x+c)^5+5/24/d*a^3*tanh(d*x+c)*sech(d*x+c)^3+5/
16/d*a^3*sech(d*x+c)*tanh(d*x+c)+5/8/d*a^3*arctan(exp(d*x+c))-3/5/d*a^2*b*s
inh(d*x+c)/cosh(d*x+c)^6+1/10/d*a^2*b*tanh(d*x+c)*sech(d*x+c)^5+1/8/d*a^2*b
*tanh(d*x+c)*sech(d*x+c)^3+3/16/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/8/d*a^2*b
*arctan(exp(d*x+c))-1/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^6-3/5/d*a*b^2*sinh(
d*x+c)/cosh(d*x+c)^6+1/10/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^5+1/8/d*a*b^2*tan
h(d*x+c)*sech(d*x+c)^3+3/16/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+3/8/d*a*b^2*arc
tan(exp(d*x+c))-1/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^6-5/3/d*b^3*sinh(d*x+c)^3
/cosh(d*x+c)^6-1/d*b^3*sinh(d*x+c)/cosh(d*x+c)^6+1/6/d*b^3*tanh(d*x+c)*sech
(d*x+c)^5+5/24/d*b^3*tanh(d*x+c)*sech(d*x+c)^3+5/16/d*b^3*sech(d*x+c)*tanh(
d*x+c)+5/8/d*b^3*arctan(exp(d*x+c))

```

Maxima [B] time = 1.83213, size = 872, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & -1/24*b^3*(15*\arctan(e^{-(d*x - c)})/d + (33*e^{-(d*x - c)} - 5*e^{-(3*d*x - 3*c)} \\ & + 90*e^{-(5*d*x - 5*c)} - 90*e^{-(7*d*x - 7*c)} + 5*e^{-(9*d*x - 9*c)} - 33*e^{-(11*d*x - 11*c)})/(d*(6*e^{-(2*d*x - 2*c)} + 15*e^{-(4*d*x - 4*c)} + 20*e^{-(6*d*x - 6*c)} + 15*e^{-(8*d*x - 8*c)} + 6*e^{-(10*d*x - 10*c)} + e^{-(12*d*x - 12*c)} + 1))) \\ & - 1/24*a^3*(15*\arctan(e^{-(d*x - c)})/d - (15*e^{-(d*x - c)} + 85*e^{-(3*d*x - 3*c)} + 198*e^{-(5*d*x - 5*c)} - 198*e^{-(7*d*x - 7*c)} - 85*e^{-(9*d*x - 9*c)} - 15*e^{-(11*d*x - 11*c)})/(d*(6*e^{-(2*d*x - 2*c)} + 15*e^{-(4*d*x - 4*c)} + 20*e^{-(6*d*x - 6*c)} + 15*e^{-(8*d*x - 8*c)} + 6*e^{-(10*d*x - 10*c)} + e^{-(12*d*x - 12*c)} + 1))) \\ & - 1/8*a^2*b*(3*\arctan(e^{-(d*x - c)})/d - (3*e^{-(d*x - c)} + 17*e^{-(3*d*x - 3*c)} - 114*e^{-(5*d*x - 5*c)} + 114*e^{-(7*d*x - 7*c)} - 17*e^{-(9*d*x - 9*c)} - 3*e^{-(11*d*x - 11*c)})/(d*(6*e^{-(2*d*x - 2*c)} + 15*e^{-(4*d*x - 4*c)} + 20*e^{-(6*d*x - 6*c)} + 15*e^{-(8*d*x - 8*c)} + 6*e^{-(10*d*x - 10*c)} + e^{-(12*d*x - 12*c)} + 1))) \\ & - 1/8*a*b^2*(3*\arctan(e^{-(d*x - c)})/d - (3*e^{-(d*x - c)} - 47*e^{-(3*d*x - 3*c)} + 78*e^{-(5*d*x - 5*c)} - 78*e^{-(7*d*x - 7*c)} + 47*e^{-(9*d*x - 9*c)} - 3*e^{-(11*d*x - 11*c)})/(d*(6*e^{-(2*d*x - 2*c)} + 15*e^{-(4*d*x - 4*c)} + 20*e^{-(6*d*x - 6*c)} + 15*e^{-(8*d*x - 8*c)} + 6*e^{-(10*d*x - 10*c)} + e^{-(12*d*x - 12*c)} + 1))) \end{aligned}$$

Fricas [B] time = 1.81393, size = 9044, normalized size = 58.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & 1/24*(3*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*\cosh(d*x + c)^{11} + 33*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^{10} + 3*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*\sinh(d*x + c)^{11} + (85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*\cosh(d*x + c)^9 + (85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3 + 165*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\cosh(d*x + c)^2*\sinh(d*x + c)^9 + 9*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\cosh(d*x + c)^3 + (85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^8 + 18*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + c)^7 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\cosh(d*x + c)^2*\sinh(d*x + c)^7 + 9*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\cosh(d*x + c)*\sinh(d*x + c)^6 + 18*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + c)^5 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\cosh(d*x + c)^2*\sinh(d*x + c)^5 + 9*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\sinh(d*x + c)^4 + 18*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + c)^3 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\cosh(d*x + c)*\sinh(d*x + c)^3 + 9*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\sinh(d*x + c)^2 + 18*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + c)^2 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\cosh(d*x + c)*\sinh(d*x + c)^2 + 9*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\sinh(d*x + c) + 18*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + c) + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*\sinh(d*x + c) + 9*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3))*1 \end{aligned}$$

$$\begin{aligned}
& 11*b^3)*\cosh(d*x + c)^4 + 11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3 + 2*(85*a^3 \\
& + 51*a^2*b - 141*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^7 + 42*(33*(\\
& 5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*\cosh(d*x + c)^5 + 2*(85*a^3 + 51*a^2*b \\
& - 141*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + 3*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5* \\
& b^3)*\cosh(d*x + c))*\sinh(d*x + c)^6 - 18*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5* \\
& b^3)*\cosh(d*x + c)^5 + 18*(77*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*\cosh(d*x \\
& + c)^6 + 7*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 - 11*a^ \\
& 3 + 19*a^2*b - 13*a*b^2 + 5*b^3 + 21*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3) \\
& *\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11* \\
& b^3)*\cosh(d*x + c)^7 + 7*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*\cosh(d*x + \\
& c)^5 + 35*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + c)^3 - 5*(11*a \\
& ^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 - (85*a^3 \\
& + 51*a^2*b - 141*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + (495*(5*a^3 + 3*a^2*b + 3 \\
& *a*b^2 - 11*b^3)*\cosh(d*x + c)^8 + 84*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^ \\
& 3)*\cosh(d*x + c)^6 + 630*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + \\
& c)^4 - 85*a^3 - 51*a^2*b + 141*a*b^2 - 5*b^3 - 180*(11*a^3 - 19*a^2*b + 13* \\
& a*b^2 - 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 3*(55*(5*a^3 + 3*a^2*b + \\
& 3*a*b^2 - 11*b^3)*\cosh(d*x + c)^9 + 12*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b \\
& ^3)*\cosh(d*x + c)^7 + 126*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + \\
& c)^5 - 60*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*\cosh(d*x + c)^3 - (85*a^3 \\
& + 51*a^2*b - 141*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 3*((5*a^3 \\
& + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c)^12 + 12*(5*a^3 + 3*a^2*b + 3*a* \\
& b^2 + 5*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^11 + (5*a^3 + 3*a^2*b + 3*a*b^2 + \\
& 5*b^3)*\sinh(d*x + c)^12 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + \\
& c)^10 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 11*(5*a^3 + 3*a^2*b + 3*a*b^ \\
& 2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^10 + 20*(11*(5*a^3 + 3*a^2*b + 3* \\
& a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + 3*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh \\
& (d*x + c))*\sinh(d*x + c)^9 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d* \\
& x + c)^8 + 15*(33*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 5*a \\
& ^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 18*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cos \\
& h(d*x + c)^2)*\sinh(d*x + c)^8 + 24*(33*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)* \\
& \cosh(d*x + c)^5 + 30*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c)^3 + \\
& 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 20*(\\
& 5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 4*(231*(5*a^3 + 3*a^2*b \\
& + 3*a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 315*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b \\
& ^3)*\cosh(d*x + c)^4 + 25*a^3 + 15*a^2*b + 15*a*b^2 + 25*b^3 + 105*(5*a^3 + \\
& 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 24*(33*(5*a^3 \\
& + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c)^7 + 63*(5*a^3 + 3*a^2*b + 3*a*b \\
& ^2 + 5*b^3)*\cosh(d*x + c)^5 + 35*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d \\
& *x + c)^3 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^5 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c)^4 + 15*(33*(5* \\
& a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cosh(d*x + c)^8 + 84*(5*a^3 + 3*a^2*b + 3* \\
& a*b^2 + 5*b^3)*\cosh(d*x + c)^6 + 70*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*\cos \\
& h(d*x + c)^4 + 5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 20*(5*a^3 + 3*a^2*b + 3* \\
& a*b^2 + 5*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 20*(11*(5*a^3 + 3*a^2*b +
\end{aligned}$$

$$\begin{aligned}
& 3ab^2 + 5b^3) \cosh(dx + c)^9 + 36(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^7 \\
& + 42(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^5 + 20(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^3 \\
& + 3(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c) \sinh(dx + c)^3 + 5a^3 + 3a^2b + 3ab^2 + 5b^3 \\
& + 6(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^2 + 6(11(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^{10} \\
& + 45(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^8 + 70(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^6 \\
& + 50(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^4 + 5a^3 + 3a^2b + 3ab^2 + 5b^3 + 15(5a^3 + 3a^2b + 3ab^2 + 5b^3) \\
& \cosh(dx + c)^2) \sinh(dx + c)^2 + 12((5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^{11} + 5(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^9 \\
& + 10(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^7 + 10(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^5 + 5(5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)^3 \\
& + (5a^3 + 3a^2b + 3ab^2 + 5b^3) \cosh(dx + c)) \operatorname{arctan}(\cosh(dx + c) + \sinh(dx + c)) - 3(5a^3 + 3a^2b + 3ab^2 - 11b^3) \cosh(dx + c) \\
& + 3(11(5a^3 + 3a^2b + 3ab^2 - 11b^3) \cosh(dx + c)^{10} + 3(85a^3 + 51a^2b - 141ab^2 + 5b^3) \cosh(dx + c)^8 + 42(11a^3 - 19a^2b + 13ab^2 - 5b^3) \cosh(dx + c)^6 \\
& - 30(11a^3 - 19a^2b + 13ab^2 - 5b^3) \cosh(dx + c)^4 - 5a^3 - 3a^2b - 3ab^2 + 11b^3 - (85a^3 + 51a^2b - 141ab^2 + 5b^3) \cosh(dx + c)^2) \sinh(dx + c) \\
& / (d \cosh(dx + c)^{12} + 12d \cosh(dx + c) \sinh(dx + c)^{11} + d \sinh(dx + c)^{12} + 6d \cosh(dx + c)^{10} + 6(11d \cosh(dx + c)^2 + d) \sinh(dx + c)^{10} \\
& + 20(11d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^9 + 15d \cosh(dx + c)^8 + 15(33d \cosh(dx + c)^4 + 18d \cosh(dx + c)^2 + d) \sinh(dx + c)^8 \\
& + 24(33d \cosh(dx + c)^5 + 30d \cosh(dx + c)^3 + 5d \cosh(dx + c)) \sinh(dx + c)^7 + 20d \cosh(dx + c)^6 + 4(231d \cosh(dx + c)^6 + 315d \cosh(dx + c)^4 \\
& + 105d \cosh(dx + c)^2 + 5d) \sinh(dx + c)^6 + 24(33d \cosh(dx + c)^7 + 63d \cosh(dx + c)^5 + 35d \cosh(dx + c)^3 + 5d \cosh(dx + c)) \sinh(dx + c)^5 \\
& + 15d \cosh(dx + c)^4 + 15(33d \cosh(dx + c)^8 + 84d \cosh(dx + c)^6 + 70d \cosh(dx + c)^4 + 20d \cosh(dx + c)^2 + d) \sinh(dx + c)^4 \\
& + 20(11d \cosh(dx + c)^9 + 36d \cosh(dx + c)^7 + 42d \cosh(dx + c)^5 + 20d \cosh(dx + c)^3 + 3d \cosh(dx + c)) \sinh(dx + c)^3 + 6d \cosh(dx + c)^2 \\
& + 6(11d \cosh(dx + c)^{10} + 45d \cosh(dx + c)^8 + 70d \cosh(dx + c)^6 + 50d \cosh(dx + c)^4 + 15d \cosh(dx + c)^2 + d) \sinh(dx + c)^2 + 12(d \cosh(dx + c)^{11} + 5d \cosh(dx + c)^9 + 10d \cosh(dx + c)^7 \\
& + 10d \cosh(dx + c)^5 + 5d \cosh(dx + c)^3 + d \cosh(dx + c)) \sinh(dx + c) + d)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**7*(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.29618, size = 518, normalized size = 3.36

$$\frac{\left(\pi + 2 \arctan\left(\frac{1}{2}\left(e^{2dx+2c} - 1\right)e^{-dx-c}\right)\right)\left(5a^3 + 3a^2b + 3ab^2 + 5b^3\right)}{32d} + \frac{15a^3\left(e^{dx+c} - e^{-dx-c}\right)^5 + 9a^2b\left(e^{dx+c} - e^{-dx-c}\right)}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $\frac{1}{32}(\pi + 2\arctan(1/2*(e^{2dx+2c} - 1)*e^{-dx-c}))*(5a^3 + 3a^2b + 3ab^2 + 5b^3)/d + 1/24*(15a^3*(e^{dx+c} - e^{-dx-c})^5 + 9a^2b*(e^{dx+c} - e^{-dx-c})^5 + 9a^2b^2*(e^{dx+c} - e^{-dx-c})^5 - 33b^3*(e^{dx+c} - e^{-dx-c})^5 + 160a^3*(e^{dx+c} - e^{-dx-c})^3 + 96a^2b*(e^{dx+c} - e^{-dx-c})^3 - 96ab^2*(e^{dx+c} - e^{-dx-c})^3 - 160b^3*(e^{dx+c} - e^{-dx-c})^3 + 528a^3*(e^{dx+c} - e^{-dx-c}) - 144a^2b*(e^{dx+c} - e^{-dx-c}) - 144ab^2*(e^{dx+c} - e^{-dx-c}) - 240b^3*(e^{dx+c} - e^{-dx-c}))/(((e^{dx+c} - e^{-dx-c})^2 + 4)^3*d)$

3.315 $\int \operatorname{sech}^8(c + dx) (a + b \sinh^2(c + dx))^3 dx$

Optimal. Leaf size=80

$$-\frac{a^2(a-b)\tanh^3(c+dx)}{d} + \frac{a^3\tanh(c+dx)}{d} - \frac{(a-b)^3\tanh^7(c+dx)}{7d} + \frac{3a(a-b)^2\tanh^5(c+dx)}{5d}$$

[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - b)*Tanh[c + d*x]^3)/d + (3*a*(a - b)^2*Tanh[c + d*x]^5)/(5*d) - ((a - b)^3*Tanh[c + d*x]^7)/(7*d)

Rubi [A] time = 0.0738987, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3191, 194}

$$-\frac{a^2(a-b)\tanh^3(c+dx)}{d} + \frac{a^3\tanh(c+dx)}{d} - \frac{(a-b)^3\tanh^7(c+dx)}{7d} + \frac{3a(a-b)^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - b)*Tanh[c + d*x]^3)/d + (3*a*(a - b)^2*Tanh[c + d*x]^5)/(5*d) - ((a - b)^3*Tanh[c + d*x]^7)/(7*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 194

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^8(c+dx) (a+b \sinh^2(c+dx))^3 dx &= \frac{\operatorname{Subst}\left(\int (a-(a-b)x^2)^3 dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int (a^3-3a^2(a-b)x^2+3a(a-b)^2x^4-(a-b)^3x^6) dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{a^3 \tanh(c+dx)}{d} - \frac{a^2(a-b) \tanh^3(c+dx)}{d} + \frac{3a(a-b)^2 \tanh^5(c+dx)}{5d} - \frac{(a-b)^3 \tanh^7(c+dx)}{7d} \end{aligned}$$

Mathematica [B] time = 0.569323, size = 163, normalized size = 2.04

$$\frac{\tanh(c+dx) \operatorname{sech}^6(c+dx) \left((232a^2b + 464a^3 - 246ab^2 + 75b^3) \cosh(2(c+dx)) + 2(32a^2b + 64a^3 + 24ab^2 - 15b^3) \cosh(c+dx) \right)}{1120d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((512*a^3 - 304*a^2*b + 192*a*b^2 - 50*b^3 + (464*a^3 + 232*a^2*b - 246*a*b^2 + 75*b^3)*Cosh[2*(c + d*x)] + 2*(64*a^3 + 32*a^2*b + 24*a*b^2 - 15*b^3)*Cosh[4*(c + d*x)] + 16*a^3*Cosh[6*(c + d*x)] + 8*a^2*b*Cosh[6*(c + d*x)] + 6*a*b^2*Cosh[6*(c + d*x)] + 5*b^3*Cosh[6*(c + d*x)])*Sech[c + d*x]^6*Tanh[c + d*x])/(1120*d)

Maple [B] time = 0.089, size = 289, normalized size = 3.6

$$\frac{1}{d} \left(a^3 \left(\frac{16}{35} + \frac{(\operatorname{sech}(dx+c))^6}{7} + \frac{6(\operatorname{sech}(dx+c))^4}{35} + \frac{8(\operatorname{sech}(dx+c))^2}{35} \right) \tanh(dx+c) + 3a^2b \left(-\frac{1}{6} \frac{\sinh(dx+c)}{(\cosh(dx+c))^7} + \frac{1}{6} \frac{\sinh(dx+c)}{(\cosh(dx+c))^5} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/d*(a^3*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/6*sinh(d*x+c)/cosh(d*x+c)^7+1/6*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^5/cosh(d*x+c)^7-5/8*sinh(d*x+c)^3/cosh(d*x+c)^7-5/16*sinh(d*x+c)/cosh(d*x+c)^5))

$$(d*x+c)^7+5/16*(16/35+1/7*\operatorname{sech}(d*x+c)^6+6/35*\operatorname{sech}(d*x+c)^4+8/35*\operatorname{sech}(d*x+c)^2)*\tanh(d*x+c))$$

Maxima [B] time = 1.28064, size = 2368, normalized size = 29.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$\begin{aligned} & 32/35*a^3*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} \\ & + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} \\ & + e^{(-14*d*x - 14*c)} + 1)) + 21*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} \\ & + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} \\ & + e^{(-14*d*x - 14*c)} + 1)) + 35*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35* \\ & e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} \\ & + e^{(-14*d*x - 14*c)} + 1)) + 1/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} \\ & + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))) \\ & + 16/35*a^2*b*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} \\ & + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) \\ & + 21*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} \\ & + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) \\ & - 35*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} \\ & + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) \\ & + 70*e^{(-8*d*x - 8*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} \\ & + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1)) \\ & + 1/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} \\ & + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} + e^{(-14*d*x - 14*c)} + 1))) \\ & + 12/35*a*b^2*(7*e^{(-2*d*x - 2*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} \\ & + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} \\ & + e^{(-14*d*x - 14*c)} + 1)) - 14*e^{(-4*d*x - 4*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} \\ & + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} \\ & + e^{(-14*d*x - 14*c)} + 1)) + 70*e^{(-6*d*x - 6*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} \\ & + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} \\ & + e^{(-14*d*x - 14*c)} + 1)) - 35*e^{(-8*d*x - 8*c)}/(d*(7*e^{(-2*d*x - 2*c)} + 21*e^{(-4*d*x - 4*c)} \\ & + 35*e^{(-6*d*x - 6*c)} + 35*e^{(-8*d*x - 8*c)} + 21*e^{(-10*d*x - 10*c)} + 7*e^{(-12*d*x - 12*c)} \\ & + e^{(-14*d*x - 14*c)} + 1)) \end{aligned}$$

$$\begin{aligned}
& *c) + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1 \\
&)) + 35e^{(-10dx - 10c)} / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 3 \\
& 5e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12 \\
& *dx - 12c)} + e^{(-14dx - 14c)} + 1)) + 1 / (d(7e^{(-2dx - 2c)} + 21e^{(- \\
& -4dx - 4c)} + 35e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - \\
& 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1))) + 2/7*b^3*(21e^{(- \\
& -4dx - 4c)} / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} \\
& + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} \\
& + e^{(-14dx - 14c)} + 1)) + 35e^{(-8dx - 8c)} / (d(7e^{(-2dx - 2c)} + 2 \\
& 1e^{(-4dx - 4c)} + 35e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10* \\
& dx - 10c)} + 7e^{(-12dx - 12c)} + e^{(-14dx - 14c)} + 1)) + 7e^{(-12dx \\
& x - 12c)} / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 35e^{(-6dx - 6c} \\
&) + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12dx - 12c)} + e^{ \\
& (-14dx - 14c)} + 1)) + 1 / (d(7e^{(-2dx - 2c)} + 21e^{(-4dx - 4c)} + 3 \\
& 5e^{(-6dx - 6c)} + 35e^{(-8dx - 8c)} + 21e^{(-10dx - 10c)} + 7e^{(-12 \\
& *dx - 12c)} + e^{(-14dx - 14c)} + 1)))
\end{aligned}$$

Fricas [B] time = 1.53974, size = 2063, normalized size = 25.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^8*(a+b*sinh(dx+c))^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -4/35*((8a^3 + 4a^2b + 3ab^2 + 20b^3)*\cosh(dx + c)^6 - 6*(8a^3 + 4a^2b \\
& + 3ab^2 + 20b^3)*\cosh(dx + c)*\sinh(dx + c)^5 + (8a^3 + 4a^2b \\
& + 3ab^2 + 20b^3)*\sinh(dx + c)^6 + 14*(4a^3 + 2a^2b + 9ab^2)*\cosh(dx \\
& *x + c)^4 + (56a^3 + 28a^2b + 126ab^2 + 15*(8a^3 + 4a^2b + 3ab^2 \\
& + 20b^3)*\cosh(dx + c)^2)*\sinh(dx + c)^4 - 4*(5*(8a^3 + 4a^2b + 3ab^2 \\
& - 15b^3)*\cosh(dx + c)^3 + 28*(2a^3 + a^2b - 3ab^2)*\cosh(dx + c))*s \\
& inh(dx + c)^3 + 280a^3 - 140a^2b + 210ab^2 + 7*(24a^3 + 52a^2b - 2 \\
& 1ab^2 + 20b^3)*\cosh(dx + c)^2 + (15*(8a^3 + 4a^2b + 3ab^2 + 20b^3 \\
&)*\cosh(dx + c)^4 + 168a^3 + 364a^2b - 147ab^2 + 140b^3 + 84*(4a^3 + \\
& 2a^2b + 9ab^2)*\cosh(dx + c)^2)*\sinh(dx + c)^2 - 2*(3*(8a^3 + 4a^2b \\
& b + 3ab^2 - 15b^3)*\cosh(dx + c)^5 + 56*(2a^3 + a^2b - 3ab^2)*\cosh(dx \\
& *x + c)^3 + 7*(24a^3 - 28a^2b + 9ab^2 - 5b^3)*\cosh(dx + c))*\sinh(dx \\
& + c)) / (d*\cosh(dx + c)^8 + 8*d*\cosh(dx + c)*\sinh(dx + c)^7 + d*\sinh(dx \\
& + c)^8 + 8*d*\cosh(dx + c)^6 + 4*(7*d*\cosh(dx + c)^2 + 2*d)*\sinh(dx + c)^ \\
& 6 + 4*(14*d*\cosh(dx + c)^3 + 9*d*\cosh(dx + c))*\sinh(dx + c)^5 + 28*d*\cos \\
& h(dx + c)^4 + 2*(35*d*\cosh(dx + c)^4 + 60*d*\cosh(dx + c)^2 + 14*d)*\sinh(\\
& dx + c)^4 + 8*(7*d*\cosh(dx + c)^5 + 15*d*\cosh(dx + c)^3 + 7*d*\cosh(dx +
\end{aligned}$$

$c))\sinh(dx + c)^3 + 56d\cosh(dx + c)^2 + 4(7d\cosh(dx + c)^6 + 30d\cosh(dx + c)^4 + 42d\cosh(dx + c)^2 + 14d)\sinh(dx + c)^2 + 4(2d\cosh(dx + c)^7 + 9d\cosh(dx + c)^5 + 14d\cosh(dx + c)^3 + 7d\cosh(dx + c))\sinh(dx + c) + 35d)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)**8*(a+b*sinh(dx+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.26696, size = 351, normalized size = 4.39

$2(35b^3e^{12dx+12c} + 210ab^2e^{10dx+10c} + 560a^2be^{8dx+8c} - 210ab^2e^{8dx+8c} + 175b^3e^{8dx+8c} + 560a^3e^{6dx+6c} - 280$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^8*(a+b*sinh(dx+c)^2)^3,x, algorithm="giac")

[Out]
$$\frac{-2/35(35b^3e^{12dx+12c} + 210ab^2e^{10dx+10c} + 560a^2be^{8dx+8c} - 210ab^2e^{8dx+8c} + 175b^3e^{8dx+8c} + 560a^3e^{6dx+6c} - 280a^2be^{6dx+6c} + 420ab^2e^{6dx+6c} + 336a^3e^{4dx+4c} + 168a^2be^{4dx+4c} - 84ab^2e^{4dx+4c} + 105b^3e^{4dx+4c} + 112a^3e^{2dx+2c} + 56a^2be^{2dx+2c} + 42ab^2e^{2dx+2c} + 16a^3 + 8a^2b + 6ab^2 + 5b^3)}{(d(e^{2dx+2c} + 1))^7}$$

$$3.316 \quad \int \frac{\cosh^7(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=108

$$\frac{(a^2 - 3ab + 3b^2) \sinh(c + dx)}{b^3 d} - \frac{(a - 3b) \sinh^3(c + dx)}{3b^2 d} - \frac{(a - b)^3 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab}^{7/2} d} + \frac{\sinh^5(c + dx)}{5bd}$$

[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d)) + ((a^2 - 3*a*b + 3*b^2)*Sinh[c + d*x])/(b^3*d) - ((a - 3*b)*Sinh[c + d*x]^3)/(3*b^2*d) + Sinh[c + d*x]^5/(5*b*d)

Rubi [A] time = 0.112928, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3190, 390, 205}

$$\frac{(a^2 - 3ab + 3b^2) \sinh(c + dx)}{b^3 d} - \frac{(a - 3b) \sinh^3(c + dx)}{3b^2 d} - \frac{(a - b)^3 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab}^{7/2} d} + \frac{\sinh^5(c + dx)}{5bd}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]

[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d)) + ((a^2 - 3*a*b + 3*b^2)*Sinh[c + d*x])/(b^3*d) - ((a - 3*b)*Sinh[c + d*x]^3)/(3*b^2*d) + Sinh[c + d*x]^5/(5*b*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 390

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 205

$\text{Int}[\frac{(a + b \cdot x^2)^{-1}}{a + b \sinh^2(c + dx)}, x_{\text{Symbol}}] := \text{Simp}[\frac{\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]]}{a}, x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\cosh^7(c + dx)}{a + b \sinh^2(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{a+bx^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{b^3} - \frac{(a-3b)x^2}{b^2} + \frac{x^4}{b} + \frac{-a^3+3a^2b-3ab^2+b^3}{b^3(a+bx^2)}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{(a^2 - 3ab + 3b^2) \sinh(c + dx)}{b^3 d} - \frac{(a - 3b) \sinh^3(c + dx)}{3b^2 d} + \frac{\sinh^5(c + dx)}{5bd} - \frac{(a - b)^3 \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= -\frac{(a - b)^3 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c + dx)}{\sqrt{a}}\right)}{\sqrt{ab}^{7/2} d} + \frac{(a^2 - 3ab + 3b^2) \sinh(c + dx)}{b^3 d} - \frac{(a - 3b) \sinh^3(c + dx)}{3b^2 d} + \end{aligned}$$

Mathematica [A] time = 0.476694, size = 117, normalized size = 1.08

$$\frac{30\sqrt{b}(8a^2 - 22ab + 19b^2) \sinh(c + dx) + 5b^{3/2}(9b - 4a) \sinh(3(c + dx)) + \frac{3\left(\sqrt{ab}^{5/2} \sinh(5(c+dx)) + 80(a-b)^3 \tan^{-1}\left(\frac{\sqrt{a} \text{csch}(c+dx)}{\sqrt{b}}\right)\right)}{\sqrt{a}}}{240b^{7/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2), x]

[Out] (30*Sqrt[b]*(8*a^2 - 22*a*b + 19*b^2)*Sinh[c + d*x] + 5*b^(3/2)*(-4*a + 9*b)*Sinh[3*(c + d*x)] + (3*(80*(a - b)^3*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + Sqrt[a]*b^(5/2)*Sinh[5*(c + d*x)]))/Sqrt[a]/(240*b^(7/2)*d)

Maple [B] time = 0.081, size = 1656, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cosh(dx+c)^7/(a+b*\sinh(dx+c)^2), x)$

[Out]
$$\begin{aligned} & 11/8/d/b/(\tanh(1/2*d*x+1/2*c)+1)^2-11/8/d/b/(\tanh(1/2*d*x+1/2*c)-1)^2-4/d*a \\ & /(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d \\ & *x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+3/d*a/b/((2*(-b*(a-b))^{(1/2)} \\ &)+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b) \\ & *a)^{(1/2)})-1/5/d/b/(\tanh(1/2*d*x+1/2*c)-1)^5-1/d/((2*(-b*(a-b))^{(1/2)}+a-2*b \\ &)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}) \\ &)+1/d/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/ \\ & ((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/d/b^3*a^4/(-b*(a-b))^{(1/2)}/((2*(-b*(\\ & a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)} \\ &)-a+2*b)*a)^{(1/2)})+1/d/b^3*a^4/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b \\ &)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}) \\ &)-1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^2*a+1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)-1) \\ & ^2*a+3/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)*a+3/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)*a-4/ \\ & d*a^3/b^2/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c) \\ & /((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-4/d*a^3/b^2/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b) \\ &)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}) \\ &)+6/d*a^2/b/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b) \\ &)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}) \\ &)+6/d*a^2/b/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b) \\ &)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}) \\ &)+1/d*b/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c) \\ & /((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+1/d*b/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b) \\ &)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}) \\ &)+1/3/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^3*a-1/d/b^3/(\tanh(1/2*d*x+1/2*c)+1)*a^2+1/3/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)^3*a-1/d/b^3/(\tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)^4+3/d*a^2/b^2/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/d*a^3/b^3/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-3/d*a^2/b^2/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-5/4/d/b/(\tanh(1/2*d*x+1/2*c)+1)^3-5/4/d/b/(\tanh(1/2*d*x+1/2*c)-1)^3-3/d/b/(\tanh(1/2*d*x+1/2*c)+1)-3/d/b/(\tanh(1/2*d*x+1/2*c)-1)-3/d*a/b/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-4/d*a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-1/5/d/b/(\tanh(1/2*d*x+1/2*c)+1)^5-1/d*a^3/b^3/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(3b^2e^{10dx+10c} - 3b^2 - 5(4abe^{8c} - 9b^2e^{8c}))e^{8dx} + 30(8a^2e^{6c} - 22abe^{6c} + 19b^2e^{6c})e^{6dx} - 30(8a^2e^{4c} - 22ab}{480b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/480*(3*b^2*e^(10*d*x + 10*c) - 3*b^2 - 5*(4*a*b*e^(8*c) - 9*b^2*e^(8*c))*e^(8*d*x) + 30*(8*a^2*e^(6*c) - 22*a*b*e^(6*c) + 19*b^2*e^(6*c))*e^(6*d*x) - 30*(8*a^2*e^(4*c) - 22*a*b*e^(4*c) + 19*b^2*e^(4*c))*e^(4*d*x) + 5*(4*a*b*e^(2*c) - 9*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*integrate(256*((a^3*e^(3*c) - 3*a^2*b*e^(3*c) + 3*a*b^2*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) + (a^3*e^c - 3*a^2*b*e^c + 3*a*b^2*e^c - b^3*e^c)*e^(d*x))/(b^4*e^(4*d*x + 4*c) + b^4 + 2*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)

Fricas [B] time = 1.85931, size = 7526, normalized size = 69.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] [1/480*(3*a*b^3*cosh(d*x + c)^10 + 30*a*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 3*a*b^3*sinh(d*x + c)^10 - 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^8 + 5*(27*a*b^3*cosh(d*x + c)^2 - 4*a^2*b^2 + 9*a*b^3)*sinh(d*x + c)^8 + 40*(9*a*b^3*cosh(d*x + c)^3 - (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^6 + 10*(63*a*b^3*cosh(d*x + c)^4 + 24*a^3*b - 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*a*b^3*cosh(d*x + c)^5 - 70*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 30*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^4 + 10*(63*a*b^3*cosh(d*x + c)^6 - 35*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4 - 24*a^3*b + 66*a^2*b^2 - 57*a*b^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 3*a*b^3 + 40*(9*a*b^3*cosh(d*x + c)^7 - 7*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 15*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^3 - 3*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2 + 5*(27*a*b^3*cosh(d*x + c)^8 - 28*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^6 + 90*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^4 - 30*(8*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2 + 19*b^2*e^(6*c))*e^(6*d*x) - 30*(8*a^2*e^(4*c) - 22*a*b*e^(4*c) + 19*b^2*e^(4*c))*e^(4*d*x) + 5*(4*a*b*e^(2*c) - 9*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*integrate(256*((a^3*e^(3*c) - 3*a^2*b*e^(3*c) + 3*a*b^2*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) + (a^3*e^c - 3*a^2*b*e^c + 3*a*b^2*e^c - b^3*e^c)*e^(d*x))/(b^4*e^(4*d*x + 4*c) + b^4 + 2*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)

$$\begin{aligned}
& 2*b^2 + 19*a*b^3)*\cosh(d*x + c)^4 + 4*a^2*b^2 - 9*a*b^3 - 36*(8*a^3*b - 22* \\
& a^2*b^2 + 19*a*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 240*((a^3 - 3*a^2*b \\
& + 3*a*b^2 - b^3)*\cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d \\
& *x + c)^4*\sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)^ \\
& 3*\sinh(d*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)^2*\sinh \\
& (d*x + c)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)*\sinh(d*x + c) \\
& ^4 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\sinh(d*x + c)^5)*\sqrt{-a*b}*\log((b*\cos \\
& h(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2 \\
& *a + b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 \\
& + 4*(b*\cosh(d*x + c)^3 - (2*a + b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(\cosh(\\
& d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d* \\
& x + c)^2 - 1)*\sinh(d*x + c) - \cosh(d*x + c))*\sqrt{-a*b} + b)/(b*\cosh(d*x + \\
& c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)* \\
& \cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b* \\
& \cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) + 10*(3*a*b^ \\
& 3*\cosh(d*x + c)^9 - 4*(4*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^7 + 18*(8*a^3*b - \\
& 22*a^2*b^2 + 19*a*b^3)*\cosh(d*x + c)^5 - 12*(8*a^3*b - 22*a^2*b^2 + 19*a*b \\
& ^3)*\cosh(d*x + c)^3 + (4*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/(\\
& a*b^4*d*\cosh(d*x + c)^5 + 5*a*b^4*d*\cosh(d*x + c)^4*\sinh(d*x + c) + 10*a*b^ \\
& 4*d*\cosh(d*x + c)^3*\sinh(d*x + c)^2 + 10*a*b^4*d*\cosh(d*x + c)^2*\sinh(d*x + \\
& c)^3 + 5*a*b^4*d*\cosh(d*x + c)*\sinh(d*x + c)^4 + a*b^4*d*\sinh(d*x + c)^5), \\
& 1/480*(3*a*b^3*\cosh(d*x + c)^10 + 30*a*b^3*\cosh(d*x + c)*\sinh(d*x + c)^9 + \\
& 3*a*b^3*\sinh(d*x + c)^10 - 5*(4*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^8 + 5*(27 \\
& *a*b^3*\cosh(d*x + c)^2 - 4*a^2*b^2 + 9*a*b^3)*\sinh(d*x + c)^8 + 40*(9*a*b^3 \\
& *\cosh(d*x + c)^3 - (4*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 3 \\
& 0*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*\cosh(d*x + c)^6 + 10*(63*a*b^3*\cosh(d*x \\
& + c)^4 + 24*a^3*b - 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 - 9*a*b^3)*\cosh(\\
& d*x + c)^2)*\sinh(d*x + c)^6 + 4*(189*a*b^3*\cosh(d*x + c)^5 - 70*(4*a^2*b^2 \\
& - 9*a*b^3)*\cosh(d*x + c)^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^5 - 30*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*\cosh(d*x + c)^ \\
& 4 + 10*(63*a*b^3*\cosh(d*x + c)^6 - 35*(4*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^4 \\
& - 24*a^3*b + 66*a^2*b^2 - 57*a*b^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)* \\
& \cosh(d*x + c)^2)*\sinh(d*x + c)^4 - 3*a*b^3 + 40*(9*a*b^3*\cosh(d*x + c)^7 - \\
& 7*(4*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^5 + 15*(8*a^3*b - 22*a^2*b^2 + 19*a*b \\
& ^3)*\cosh(d*x + c)^3 - 3*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*\cosh(d*x + c))*\si \\
& nh(d*x + c)^3 + 5*(4*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^2 + 5*(27*a*b^3*\cosh(\\
& d*x + c)^8 - 28*(4*a^2*b^2 - 9*a*b^3)*\cosh(d*x + c)^6 + 90*(8*a^3*b - 22*a^ \\
& 2*b^2 + 19*a*b^3)*\cosh(d*x + c)^4 + 4*a^2*b^2 - 9*a*b^3 - 36*(8*a^3*b - 22* \\
& a^2*b^2 + 19*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 480*((a^3 - 3*a^2*b \\
& + 3*a*b^2 - b^3)*\cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d \\
& *x + c)^4*\sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)^ \\
& 3*\sinh(d*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)^2*\sinh \\
& (d*x + c)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\cosh(d*x + c)*\sinh(d*x + c) \\
& ^4 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*\sinh(d*x + c)^5)*\sqrt{a*b}*\arctan(1/2* \\
& \sqrt{a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) - 480*((a^3 - 3*a^2*b + 3*a*b^
\end{aligned}$$

$$2 - b^3) \cosh(dx + c)^5 + 5(a^3 - 3a^2b + 3ab^2 - b^3) \cosh(dx + c)^4 \sinh(dx + c) + 10(a^3 - 3a^2b + 3ab^2 - b^3) \cosh(dx + c)^3 \sinh(dx + c)^2 + 10(a^3 - 3a^2b + 3ab^2 - b^3) \cosh(dx + c)^2 \sinh(dx + c)^3 + 5(a^3 - 3a^2b + 3ab^2 - b^3) \cosh(dx + c) \sinh(dx + c)^4 + (a^3 - 3a^2b + 3ab^2 - b^3) \sinh(dx + c)^5 \sqrt{ab} \arctan(1/2(b \cosh(dx + c)^3 + 3b \cosh(dx + c) \sinh(dx + c)^2 + b \sinh(dx + c)^3 + (4a - b) \cosh(dx + c) + (3b \cosh(dx + c)^2 + 4a - b) \sinh(dx + c)) \sqrt{ab} / (ab)) + 10(3ab^3 \cosh(dx + c)^9 - 4(4a^2b^2 - 9ab^3) \cosh(dx + c)^7 + 18(8a^3b - 22a^2b^2 + 19ab^3) \cosh(dx + c)^5 - 12(8a^3b - 22a^2b^2 + 19ab^3) \cosh(dx + c)^3 + (4a^2b^2 - 9ab^3) \cosh(dx + c)) \sinh(dx + c) / (ab^4 d \cosh(dx + c)^5 + 5ab^4 d \cosh(dx + c)^4 \sinh(dx + c) + 10ab^4 d \cosh(dx + c)^3 \sinh(dx + c)^2 + 10ab^4 d \cosh(dx + c)^2 \sinh(dx + c)^3 + 5ab^4 d \cosh(dx + c) \sinh(dx + c)^4 + ab^4 d \sinh(dx + c)^5)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)**7/(a+b*sinh(dx+c)**2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^7/(a+b*sinh(dx+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.317 \quad \int \frac{\cosh^6(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=121

$$\frac{x(8a^2 - 20ab + 15b^2)}{8b^3} - \frac{(a-b)^{5/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^3}d} - \frac{(4a-7b) \sinh(c+dx) \cosh(c+dx)}{8b^2d} + \frac{\sinh(c+dx) \cosh^3(c+dx)}{4bd}$$

[Out] ((8*a^2 - 20*a*b + 15*b^2)*x)/(8*b^3) - ((a - b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^3*d) - ((4*a - 7*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*b*d)

Rubi [A] time = 0.220512, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3191, 414, 527, 522, 206, 208}

$$\frac{x(8a^2 - 20ab + 15b^2)}{8b^3} - \frac{(a-b)^{5/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^3}d} - \frac{(4a-7b) \sinh(c+dx) \cosh(c+dx)}{8b^2d} + \frac{\sinh(c+dx) \cosh^3(c+dx)}{4bd}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]

[Out] ((8*a^2 - 20*a*b + 15*b^2)*x)/(8*b^3) - ((a - b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^3*d) - ((4*a - 7*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*b^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*b*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]]

, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(!IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a-(a-b)x^2)} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{-a+4b-3(a-b)x^2}{(1-x^2)^2(a+(-a+b)x^2)} dx, x, \tanh(c+dx)\right)}{4bd} \\
&= -\frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{4a^2-9ab+8b^2+}{(1-x^2)(a+)}\right)}{4bd} \\
&= -\frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} - \frac{(a-b)^3 \text{Subst}\left(\int \frac{1}{a+}\right)}{4bd} \\
&= \frac{(8a^2-20ab+15b^2)x}{8b^3} - \frac{(a-b)^{5/2} \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^3}d} - \frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d}
\end{aligned}$$

Mathematica [A] time = 0.316766, size = 106, normalized size = 0.88

$$\frac{\sqrt{a}\left(4(8a^2-20ab+15b^2)(c+dx) - 8b(a-2b)\sinh(2(c+dx)) + b^2\sinh(4(c+dx))\right) - 32(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{32\sqrt{ab^3}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2), x]

[Out] (-32*(a - b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*(4*(8*a^2 - 20*a*b + 15*b^2)*(c + d*x) - 8*(a - 2*b)*b*Sinh[2*(c + d*x)] + b^2*Sinh[4*(c + d*x)]))/(32*Sqrt[a]*b^3*d)

Maple [B] time = 0.063, size = 1497, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2), x)


```
[Out] 15/8/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-15/8/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-11/8/
d/b/(tanh(1/2*d*x+1/2*c)+1)^2+11/8/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+3/d*a/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*a/b/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2))+1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/
2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1
/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a+1/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2-1/
2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2-5/2
/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+5/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/
d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a^3/b^2/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2))-3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
))-1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/4/d/b/(tanh(1/2*d*x+1/2*c)-1)^4-1/4/d/b/
(tanh(1/2*d*x+1/2*c)+1)^4-3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*a^3/
b^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))
+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^3+9/8/d/
b/(tanh(1/2*d*x+1/2*c)+1)+9/8/d/b/(tanh(1/2*d*x+1/2*c)-1)+3/d*a/b/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2))+3/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d
*a^3/b^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.75907, size = 4566, normalized size = 37.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/64*(b^2*\cosh(d*x + c)^8 + 8*b^2*\cosh(d*x + c)*\sinh(d*x + c)^7 + b^2*\sinh \\ & (d*x + c)^8 + 8*(8*a^2 - 20*a*b + 15*b^2)*d*x*\cosh(d*x + c)^4 - 8*(a*b - 2* \\ & b^2)*\cosh(d*x + c)^6 + 4*(7*b^2*\cosh(d*x + c)^2 - 2*a*b + 4*b^2)*\sinh(d*x + \\ & c)^6 + 8*(7*b^2*\cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*\cosh(d*x + c))*\sinh(d*x \\ & + c)^5 + 2*(35*b^2*\cosh(d*x + c)^4 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x - 60*(\\ & a*b - 2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*b^2*\cosh(d*x + c)^5 + \\ & 4*(8*a^2 - 20*a*b + 15*b^2)*d*x*\cosh(d*x + c) - 20*(a*b - 2*b^2)*\cosh(d*x + \\ & c)^3)*\sinh(d*x + c)^3 + 8*(a*b - 2*b^2)*\cosh(d*x + c)^2 + 4*(7*b^2*\cosh(d* \\ & x + c)^6 + 12*(8*a^2 - 20*a*b + 15*b^2)*d*x*\cosh(d*x + c)^2 - 30*(a*b - 2*b \\ & ^2)*\cosh(d*x + c)^4 + 2*a*b - 4*b^2)*\sinh(d*x + c)^2 + 32*((a^2 - 2*a*b + b \\ & ^2)*\cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)^3*\sinh(d*x + c) + \\ & 6*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*(a^2 - 2*a*b + b \\ & ^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*\sinh(d*x + c)^4)*\text{sq} \\ & \text{rt}((a - b)/a)*\log((b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^ \\ & 3 + b^2*\sinh(d*x + c)^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*b^2*\cosh(d \\ & *x + c)^2 + 2*a*b - b^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*\cos \\ & h(d*x + c)^3 + (2*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(a*b*\cosh(d*x \\ & + c)^2 + 2*a*b*\cosh(d*x + c)*\sinh(d*x + c) + a*b*\sinh(d*x + c)^2 + 2*a^2 - \\ & a*b)*\text{sqrt}((a - b)/a))/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c) \\ & ^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c) \\ & ^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + \\ & c))*\sinh(d*x + c) + b)) - b^2 + 8*(b^2*\cosh(d*x + c)^7 + 4*(8*a^2 - 20*a*b \\ & + 15*b^2)*d*x*\cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*\cosh(d*x + c)^5 + 2*(a*b - \\ & 2*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/(b^3*d*\cosh(d*x + c)^4 + 4*b^3*d*\cosh \\ & (d*x + c)^3*\sinh(d*x + c) + 6*b^3*d*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*b^3 \\ & *d*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^3*d*\sinh(d*x + c)^4), 1/64*(b^2*\cosh(d \\ & *x + c)^8 + 8*b^2*\cosh(d*x + c)*\sinh(d*x + c)^7 + b^2*\sinh(d*x + c)^8 + 8*(\\ & 8*a^2 - 20*a*b + 15*b^2)*d*x*\cosh(d*x + c)^4 - 8*(a*b - 2*b^2)*\cosh(d*x + c) \\ &)^6 + 4*(7*b^2*\cosh(d*x + c)^2 - 2*a*b + 4*b^2)*\sinh(d*x + c)^6 + 8*(7*b^2* \\ & \cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*b^ \\ & 2*\cosh(d*x + c)^4 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x - 60*(a*b - 2*b^2)*\cosh \end{aligned}$$

```
(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 - 20*a*b
+ 15*b^2)*d*x*cosh(d*x + c) - 20*(a*b - 2*b^2)*cosh(d*x + c)^3)*sinh(d*x +
c)^3 + 8*(a*b - 2*b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8*
a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b - 2*b^2)*cosh(d*x + c)
^4 + 2*a*b - 4*b^2)*sinh(d*x + c)^2 + 64*((a^2 - 2*a*b + b^2)*cosh(d*x + c)
^4 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^2 - 2*a*b +
b^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^4)*sqrt(-(a - b)/a)*ar
ctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) - b^2 + 8*(b^2*cosh(d*x + c)^7
+ 4*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d
*x + c)^5 + 2*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x +
c)^4 + 4*b^3*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30117, size = 320, normalized size = 2.64

$$\frac{(8a^2 - 20ab + 15b^2)(dx + c)}{8b^3d} - \frac{(48a^2e^{(4dx+4c)} - 120abe^{(4dx+4c)} + 90b^2e^{(4dx+4c)} - 8abe^{(2dx+2c)} + 16b^2e^{(2dx+2c)} + b^2)e^{(4dx+4c)}}{64b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 - 20*a*b + 15*b^2)*(d*x + c)/(b^3*d) - 1/64*(48*a^2*e^(4*d*x + 4
*c) - 120*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 8*a*b*e^(2*d*x + 2
*c) + 16*b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/(b^3*d) - (a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a
```

$$b))/(\sqrt{-a^2 + a*b}*b^3*d) + 1/64*(b*d*e^{(4*d*x + 4*c)} - 8*a*d*e^{(2*d*x + 2*c)} + 16*b*d*e^{(2*d*x + 2*c)})/(b^2*d^2)$$

$$3.318 \quad \int \frac{\cosh^5(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=77

$$-\frac{(a-2b) \sinh(c+dx)}{b^2 d} + \frac{(a-b)^2 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab}^{5/2} d} + \frac{\sinh^3(c+dx)}{3bd}$$

[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - ((a - 2*b)*Sinh[c + d*x])/(b^2*d) + Sinh[c + d*x]^3/(3*b*d)

Rubi [A] time = 0.0933289, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3190, 390, 205}

$$-\frac{(a-2b) \sinh(c+dx)}{b^2 d} + \frac{(a-b)^2 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab}^{5/2} d} + \frac{\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2), x]

[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - ((a - 2*b)*Sinh[c + d*x])/(b^2*d) + Sinh[c + d*x]^3/(3*b*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 390

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-\frac{a-2b}{b^2} + \frac{x^2}{b} + \frac{a^2-2ab+b^2}{b^2(a+bx^2)}\right) dx, x, \sinh(c+dx)\right)}{d} \\ &= -\frac{(a-2b)\sinh(c+dx)}{b^2d} + \frac{\sinh^3(c+dx)}{3bd} + \frac{(a-b)^2 \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c+dx)\right)}{b^2d} \\ &= \frac{(a-b)^2 \tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d} - \frac{(a-2b)\sinh(c+dx)}{b^2d} + \frac{\sinh^3(c+dx)}{3bd} \end{aligned}$$

Mathematica [A] time = 0.244737, size = 79, normalized size = 1.03

$$\frac{3\sqrt{b}(7b-4a)\sinh(c+dx) - \frac{12(a-b)^2 \tan^{-1}\left(\frac{\sqrt{a}\text{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}} + b^{3/2}\sinh(3(c+dx))}{12b^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2), x]

[Out] ((-12*(a - b)^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 3*Sqrt[b]*(-4*a + 7*b)*Sinh[c + d*x] + b^(3/2)*Sinh[3*(c + d*x)]/(12*b^(5/2)*d)

Maple [B] time = 0.059, size = 1148, normalized size = 14.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2), x)

```
[Out] -1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a-2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d*a/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a^2/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-2/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/3/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-2/d/b/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(3(4ae^{4c} - 7be^{4c})e^{4dx} - 3(4ae^{2c} - 7be^{2c})e^{2dx} - be^{6dx+6c} + b)e^{-3dx-3c}}{24b^2d} + \frac{1}{32} \int \frac{64((a^2e^{3c} - 2abe^{3c} + b^2)e^{3dx} + (a^2e^c - 2ab^2e^c + b^3)e^{dx})}{b^3e^{4dx+4c} + b^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(4*a*e^(4*c) - 7*b*e^(4*c))*e^(4*d*x) - 3*(4*a*e^(2*c) - 7*b*e^(2*c))*e^(2*d*x) - b*e^(6*d*x + 6*c) + b)*e^(-3*d*x - 3*c)/(b^2*d) + 1/32*integrate(64*((a^2*e^(3*c) - 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^2*e^c - 2*a*b*e^c + b^2*e^c)*e^(d*x))/(b^3*e^(4*d*x + 4*c) + b^3 + 2*(2*a*b^2*e^c
```

$2*c) - b^3*e^{(2*c)}*e^{(2*d*x)}, x)$

Fricas [B] time = 1.71961, size = 3804, normalized size = 49.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $[1/24*(a*b^2*\cosh(d*x + c)^6 + 6*a*b^2*\cosh(d*x + c)*\sinh(d*x + c)^5 + a*b^2*\sinh(d*x + c)^6 - 3*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c)^4 + 3*(5*a*b^2*\cosh(d*x + c)^2 - 4*a^2*b + 7*a*b^2)*\sinh(d*x + c)^4 + 4*(5*a*b^2*\cosh(d*x + c)^3 - 3*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - a*b^2 + 3*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c)^2 + 3*(5*a*b^2*\cosh(d*x + c)^4 + 4*a^2*b - 7*a*b^2 - 6*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 12*((a^2 - 2*a*b + b^2)*\cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*\sinh(d*x + c)^3)*\sqrt{-a*b}*\log((b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a + b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a + b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c) - \cosh(d*x + c))*\sqrt{-a*b} + b)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b) + 6*(a*b^2*\cosh(d*x + c)^5 - 2*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c)^3 + (4*a^2*b - 7*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/(a*b^3*d*\cosh(d*x + c)^3 + 3*a*b^3*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a*b^3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*b^3*d*\sinh(d*x + c)^3), 1/24*(a*b^2*\cosh(d*x + c)^6 + 6*a*b^2*\cosh(d*x + c)*\sinh(d*x + c)^5 + a*b^2*\sinh(d*x + c)^6 - 3*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c)^4 + 3*(5*a*b^2*\cosh(d*x + c)^2 - 4*a^2*b + 7*a*b^2)*\sinh(d*x + c)^4 + 4*(5*a*b^2*\cosh(d*x + c)^3 - 3*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 - a*b^2 + 3*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c)^2 + 3*(5*a*b^2*\cosh(d*x + c)^4 + 4*a^2*b - 7*a*b^2 - 6*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 24*((a^2 - 2*a*b + b^2)*\cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*\sinh(d*x + c)^3)*\sqrt{a*b}*\arctan(1/2*\sqrt{a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) + 24*((a^2 - 2*a*b + b^2)*\cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*\sinh(d*x + c)^3)*\sqrt{a*b}*\arctan(1/2*(b*\cosh(d*x + c) + \sinh(d*x + c))/a) + 6*(a*b^2*\cosh(d*x + c)^5 - 2*(4*a^2*b - 7*a*b^2)*\cosh(d*x + c)^3 + (4*a^2*b - 7*a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)/(a*b^3*d*\cosh(d*x + c)^3 + 3*a*b^3*d*\cosh(d*x + c)^2*\sinh(d*x + c) + 3*a*b^3*d*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*b^3*d*\sinh(d*x + c)^3)$

$$d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) + 6*(a*b^2*cosh(d*x + c)^5 - 2*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^3 + (4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + a*b^3*d*sinh(d*x + c)^3)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.319 \quad \int \frac{\cosh^4(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=81

$$\frac{(a-b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^2d}} - \frac{x(2a-3b)}{2b^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2bd}$$

[Out] -((2*a - 3*b)*x)/(2*b^2) + ((a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)

Rubi [A] time = 0.132927, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3191, 414, 522, 206, 208}

$$\frac{(a-b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^2d}} - \frac{x(2a-3b)}{2b^2} + \frac{\sinh(c+dx) \cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2), x]

[Out] -((2*a - 3*b)*x)/(2*b^2) + ((a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,

d, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-(a-b)x^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{-a+2b+(-a+b)x^2}{(1-x^2)(a+(-a+b)x^2)} dx, x, \tanh(c+dx)\right)}{2bd} \\ &= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{(2a-3b)\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{2b^2d} + \frac{(a-b)^2\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{2b^2d} \\ &= -\frac{(2a-3b)x}{2b^2} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^2d}} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} \end{aligned}$$

Mathematica [A] time = 0.14953, size = 80, normalized size = 0.99

$$\frac{\sqrt{a}(b\sinh(2(c+dx)) - 2(2a-3b)(c+dx)) + 4(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{4\sqrt{ab^2d}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]

[Out] $(4*(a - b)^{3/2}*\text{ArcTanh}[(\text{Sqrt}[a - b]*\text{Tanh}[c + d*x])/\text{Sqrt}[a]] + \text{Sqrt}[a]*(-2*(2*a - 3*b)*(c + d*x) + b*\text{Sinh}[2*(c + d*x)]))/(4*\text{Sqrt}[a]*b^2*d)$

Maple [B] time = 0.054, size = 993, normalized size = 12.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)

[Out] $-1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(\tanh(1/2*d*x+1/2*c)+1)+3/2/d/b*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)+1/d*a^2/b^2/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-2/d*a/b/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+1/d/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/d*a^2/b/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+2/d*a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/d*b/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/d*a^2/b^2/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+2/d*a/b/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-1/d/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-1/d*a^2/b/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+2/d*a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-1/d*b/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(\tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)-1)-3/2/d/b*\ln(\tanh(1/2*d*x+1/2*c)-1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70902, size = 2222, normalized size = 27.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a - 3*b)*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x + c)^2 + 2*(a - b)*cosh(d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x + c)^2)*sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2), -1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a - 3*b)*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a - b)*cosh(d*x + c)^2 + 2*(a - b)*cosh(d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x + c)^2)*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) + 4*(2*(2*a - 3*b)*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)`

[Out] Timed out

Giac [B] time = 1.32998, size = 194, normalized size = 2.4

$$-\frac{(dx+c)(2a-3b)}{2b^2d} + \frac{e^{(2dx+2c)}}{8bd} + \frac{(4ae^{(2dx+2c)} - 6be^{(2dx+2c)} - b)e^{(-2dx-2c)}}{8b^2d} + \frac{(a^2 - 2ab + b^2) \arctan\left(\frac{be^{(2dx+2c)} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{\sqrt{-a^2 + ab}b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")`

[Out] $-1/2*(d*x + c)*(2*a - 3*b)/(b^2*d) + 1/8*e^{(2*d*x + 2*c)}/(b*d) + 1/8*(4*a*e^{(2*d*x + 2*c)} - 6*b*e^{(2*d*x + 2*c)} - b)*e^{(-2*d*x - 2*c)}/(b^2*d) + (a^2 - 2*a*b + b^2)*\arctan(1/2*(b*e^{(2*d*x + 2*c)} + 2*a - b)/\sqrt{-a^2 + a*b})/(\sqrt{-a^2 + a*b})*b^2*d)$

$$3.320 \quad \int \frac{\cosh^3(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=52

$$\frac{\sinh(c+dx)}{bd} - \frac{(a-b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^{3/2}d}}$$

[Out] -(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) + Sinh[c + d*x]/(b*d)

Rubi [A] time = 0.0704174, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3190, 388, 205}

$$\frac{\sinh(c+dx)}{bd} - \frac{(a-b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^{3/2}d}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2), x]

[Out] -(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) + Sinh[c + d*x]/(b*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 388

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\sinh(c+dx)}{bd} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c+dx)\right)}{bd} \\ &= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^3/2}d} + \frac{\sinh(c+dx)}{bd} \end{aligned}$$

Mathematica [A] time = 0.0434289, size = 50, normalized size = 0.96

$$\frac{\frac{\sinh(c+dx)}{b} - \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ab^3/2}}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2), x]

[Out] (-(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + Sinh[c + d*x]/b)/d

Maple [B] time = 0.049, size = 732, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2), x)

[Out] 1/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))

$$\begin{aligned}
& 2) + a - 2*b) * a)^{(1/2)} - 2/d * a / (-b * (a - b))^{(1/2)} / ((2 * (-b * (a - b))^{(1/2)} + a - 2*b) * a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (-b * (a - b))^{(1/2)} + a - 2*b) * a)^{(1/2)} + 1/d * a^2 / b / (-b * (a - b))^{(1/2)} / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} - 1/d * a / b / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} - 2/d * a / (-b * (a - b))^{(1/2)} / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} - 1/d / ((2 * (-b * (a - b))^{(1/2)} + a - 2*b) * a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (-b * (a - b))^{(1/2)} + a - 2*b) * a)^{(1/2)} + 1/d * b / (-b * (a - b))^{(1/2)} / ((2 * (-b * (a - b))^{(1/2)} + a - 2*b) * a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (-b * (a - b))^{(1/2)} + a - 2*b) * a)^{(1/2)} + 1/d / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} + 1/d * b / (-b * (a - b))^{(1/2)} / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2 * d * x + 1/2 * c)) / ((2 * (-b * (a - b))^{(1/2)} - a + 2*b) * a)^{(1/2)} - 1/d / b / (\tanh(1/2 * d * x + 1/2 * c) + 1) - 1/d / b / (\tanh(1/2 * d * x + 1/2 * c) - 1)
\end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(e^{(2dx+2c)} - 1)e^{(-dx-c)}}{2bd} - \frac{1}{8} \int \frac{16((ae^{(3c)} - be^{(3c)})e^{(3dx)} + (ae^c - be^c)e^{(dx)})}{b^2e^{(4dx+4c)} + b^2 + 2(2abe^{(2c)} - b^2e^{(2c)})e^{(2dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(b*d) - 1/8*integrate(16*((a*e^(3*c) - b*e^(3*c))*e^(3*d*x) + (a*e^c - b*e^c)*e^(d*x))/(b^2*e^(4*d*x + 4*c) + b^2 + 2*(2*a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)

Fricas [B] time = 1.6135, size = 1754, normalized size = 33.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] [1/2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + sqrt(-a*b)*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 -

$$2*(2*a + b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a + b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c) - \cosh(d*x + c))*\sqrt{-a*b} + b)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) - a*b)/(a*b^2*d*\cosh(d*x + c) + a*b^2*d*\sinh(d*x + c)), 1/2*(a*b*\cosh(d*x + c)^2 + 2*a*b*\cosh(d*x + c)*\sinh(d*x + c) + a*b*\sinh(d*x + c)^2 - 2*\sqrt{a*b}*((a - b)*\cosh(d*x + c) + (a - b)*\sinh(d*x + c))*\arctan(1/2*\sqrt{a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) - 2*\sqrt{a*b}*((a - b)*\cosh(d*x + c) + (a - b)*\sinh(d*x + c))*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 + (4*a - b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - b)*\sinh(d*x + c))*\sqrt{a*b}/(a*b)) - a*b)/(a*b^2*d*\cosh(d*x + c) + a*b^2*d*\sinh(d*x + c))]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.321 \quad \int \frac{\cosh^2(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=50

$$\frac{x}{b} - \frac{\sqrt{a-b} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{abd}}$$

[Out] x/b - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b*d)

Rubi [A] time = 0.0817953, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3191, 391, 206, 208}

$$\frac{x}{b} - \frac{\sqrt{a-b} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{abd}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2), x]

[Out] x/b - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 391

Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c+dx)\right)}{bd} - \frac{(a-b)\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2} dx, x, \tanh(c+dx)\right)}{bd} \\ &= \frac{x}{b} - \frac{\sqrt{a-b} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{abd}} \end{aligned}$$

Mathematica [A] time = 0.0816948, size = 50, normalized size = 1.

$$-\frac{\sqrt{a-b} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}} + c + dx$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2), x]
```

```
[Out] (c + d*x - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[
a])/(b*d)
```

Maple [B] time = 0.04, size = 577, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)`

[Out] $\frac{1}{d} \frac{1}{b} \ln(\tanh(\frac{1}{2}d*x + \frac{1}{2}c) + 1) - \frac{1}{d} \frac{a}{b} \frac{1}{((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2}} \operatorname{arctanh}\left(\frac{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)}{((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2}}\right) + \frac{1}{d} \frac{a}{(-b*(a-b))^{1/2}} \frac{1}{((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2}} \operatorname{arctanh}\left(\frac{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)}{((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2}}\right) + \frac{1}{d} \frac{a}{b} \frac{1}{((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2}} \operatorname{arctan}\left(\frac{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)}{((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2}}\right) + \frac{1}{d} \frac{a}{(-b*(a-b))^{1/2}} \frac{1}{((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2}} \operatorname{arctan}\left(\frac{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)}{((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2}}\right) + \frac{1}{d} \frac{1}{((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2}} \operatorname{arctanh}\left(\frac{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)}{((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2}}\right) - \frac{1}{d} \frac{b}{(-b*(a-b))^{1/2}} \frac{1}{((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2}} \operatorname{arctanh}\left(\frac{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)}{((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2}}\right) - \frac{1}{d} \frac{1}{((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2}} \operatorname{arctan}\left(\frac{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)}{((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2}}\right) + \frac{1}{d} \frac{b}{(-b*(a-b))^{1/2}} \frac{1}{((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2}} \operatorname{arctan}\left(\frac{a \tanh(\frac{1}{2}d*x + \frac{1}{2}c)}{((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2}}\right) - \frac{1}{d} \frac{1}{b} \ln(\tanh(\frac{1}{2}d*x + \frac{1}{2}c) - 1)$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

Fricas [B] time = 1.62478, size = 1088, normalized size = 21.76

$$\left[2 dx + \sqrt{\frac{a-b}{a}} \log \left(\frac{b^2 \cosh(dx+c)^4 + 4b^2 \cosh(dx+c) \sinh(dx+c)^3 + b^2 \sinh(dx+c)^4 + 2(2ab-b^2) \cosh(dx+c)^2 + 2(3b^2 \cosh(dx+c)^2 + 2ab-b^2) \sinh(dx+c)^2}{b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 + 2(2a-b) \cosh(dx+c)^2 + 2(3b^2 \cosh(dx+c)^2 + 2ab-b^2) \sinh(dx+c)^2} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")`

```
[Out] [1/2*(2*d*x + sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)
)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) +
4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x
+ c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a
- b)*cosh(d*x + c))*sinh(d*x + c) + b))] / (b*d), (d*x + sqrt(-(a - b)/a)*ar
ctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
+ c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)))/(b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28074, size = 93, normalized size = 1.86

$$-\frac{(a-b) \arctan\left(\frac{be^{2dx+2c}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}bd} + \frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-
a^2 + a*b)*b*d) + (d*x + c)/(b*d)
```

$$3.322 \quad \int \frac{\cosh(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{bd}}$$

[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

Rubi [A] time = 0.0365203, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3190, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{bd}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 205

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(c + dx)}{a + b \sinh^2(c + dx)} dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{bd}}$$

Mathematica [A] time = 0.0099342, size = 32, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{bd}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2), x]

[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)

Maple [A] time = 0.01, size = 24, normalized size = 0.8

$$\frac{1}{d} \arctan\left(b \sinh(dx + c) \frac{1}{\sqrt{ab}}\right) \frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2), x)

[Out] 1/d/(a*b)^(1/2)*arctan(sinh(d*x+c)*b/(a*b)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{b \sinh(dx + c)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)

Fricas [B] time = 1.57284, size = 1207, normalized size = 37.72

$$\left[\frac{\sqrt{-ab} \log \left(\frac{b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 - 2(2a+b) \cosh(dx+c)^2 + 2(3b \cosh(dx+c)^2 - 2a-b) \sinh(dx+c)^2 + 4(b \cosh(dx+c)^2 - 2a-b) \sinh(dx+c)}{b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 + 2(2a-b) \cosh(dx+c)^2 + 2(3b \cosh(dx+c)^2 - 2a-b) \sinh(dx+c)} \right)}{2ab} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*\sqrt{-a*b}*\log((b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 \\ & + b*\sinh(d*x + c)^4 - 2*(2*a + b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 \\ & - 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a + b)*\cosh(d*x + c \\ &))*\sinh(d*x + c) - 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 \\ & + (3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c) - \cosh(d*x + c))*\sqrt{-a*b} + b)/(b*\cosh(d*x + c)^4 \\ & + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 \\ & + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) \\ & + b))/(a*b*d), (\sqrt{a*b}*\arctan(1/2*\sqrt{a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) + \sqrt{a*b}*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 \\ & + b*\sinh(d*x + c)^3 + (4*a - b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - b)*\sinh(d*x + c))*\sqrt{a*b}/(a*b)))/(a*b*d)] \end{aligned}$$

Sympy [A] time = 5.01131, size = 128, normalized size = 4.

$$\left\{ \begin{array}{ll} \frac{\infty x \cosh(c)}{\sinh^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\sinh(c+dx)}{\sinh(c+dx)} & \text{for } b = 0 \\ \frac{ad}{1} & \text{for } a = 0 \\ \frac{bd \sinh(c+dx)}{x \cosh(c)} & \text{for } d = 0 \\ \frac{a+b \sinh^2(c)}{i \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + \sinh(c+dx)\right)} + \frac{i \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + \sinh(c+dx)\right)}{2\sqrt{abd}\sqrt{\frac{1}{b}}} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh
(c + d*x)/(a*d), Eq(b, 0)), (-1/(b*d*sinh(c + d*x)), Eq(a, 0)), (x*cosh(c)/
(a + b*sinh(c)**2), Eq(d, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x)
)/(2*sqrt(a)*b*d*sqrt(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))/(2
*sqrt(a)*b*d*sqrt(1/b)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.323 \quad \int \frac{\operatorname{sech}(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=59

$$\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a-b)}$$

[Out] ArcTan[Sinh[c + d*x]]/((a - b)*d) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)*d)

Rubi [A] time = 0.0667097, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$, Rules used = {3190, 391, 203, 205}

$$\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a-b)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2), x]

[Out] ArcTan[Sinh[c + d*x]]/((a - b)*d) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)*d)

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx)\right)}{(a-b)d} - \frac{b \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c+dx)\right)}{(a-b)d} \\ &= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)d} - \frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)d} \end{aligned}$$

Mathematica [A] time = 0.126033, size = 54, normalized size = 0.92

$$\frac{\frac{\sqrt{b} \tan^{-1}\left(\frac{\sqrt{a} \operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}} + 2 \tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad - bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2), x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 2*ArcTan[Tanh[(c + d*x)/2]])/(a*d - b*d)
```

Maple [B] time = 0.058, size = 481, normalized size = 8.2

$$\frac{ab}{d(a-b)} \operatorname{Arctanh}\left(a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \frac{1}{\sqrt{(2\sqrt{-b(a-b)} + a - 2b)a}}\right) \frac{1}{\sqrt{-b(a-b)}} \frac{1}{\sqrt{(2\sqrt{-b(a-b)} + a - 2b)a}} + \frac{b}{d(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x)`

[Out]
$$\frac{1}{d} \frac{b}{(a-b)} \frac{a}{(-b(a-b))^{1/2}} \frac{1}{((2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}} \operatorname{arctanh}\left(\frac{a \tanh(1/2 dx + 1/2 c)}{(2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}}\right) + \frac{1}{d} \frac{b}{(a-b)} \frac{1}{((2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}} \operatorname{arctanh}\left(\frac{a \tanh(1/2 dx + 1/2 c)}{(2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}}\right) - \frac{1}{d} \frac{b^2}{(a-b)} \frac{1}{(-b(a-b))^{1/2}} \frac{1}{((2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}} \operatorname{arctanh}\left(\frac{a \tanh(1/2 dx + 1/2 c)}{(2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}}\right) + \frac{1}{d} \frac{b}{(a-b)} \frac{a}{(-b(a-b))^{1/2}} \frac{1}{((2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}} \operatorname{arctan}\left(\frac{a \tanh(1/2 dx + 1/2 c)}{(2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}}\right) - \frac{1}{d} \frac{b}{(a-b)} \frac{1}{((2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}} \operatorname{arctan}\left(\frac{a \tanh(1/2 dx + 1/2 c)}{(2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}}\right) - \frac{1}{d} \frac{b^2}{(a-b)} \frac{1}{(-b(a-b))^{1/2}} \frac{1}{((2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}} \operatorname{arctan}\left(\frac{a \tanh(1/2 dx + 1/2 c)}{(2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}}\right) + \frac{2}{d} \frac{1}{(a-b)} \operatorname{arctan}(\tanh(1/2 dx + 1/2 c))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2 \arctan(e^{(dx+c)})}{ad - bd} - 2 \int \frac{be^{(3dx+3c)} + be^{(dx+c)}}{ab - b^2 + (abe^{(4c)} - b^2e^{(4c)})e^{(4dx)} + 2(2a^2e^{(2c)} - 3abe^{(2c)} + b^2e^{(2c)})e^{(2dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")`

[Out]
$$2 \operatorname{arctan}(e^{(dx+c)}) / (a*d - b*d) - 2 \operatorname{integrate}((b*e^{(3*d*x + 3*c)} + b*e^{(d*x + c)}) / (a*b - b^2 + (a*b*e^{(4*c)} - b^2*e^{(4*c)})*e^{(4*d*x)} + 2*(2*a^2*e^{(2*c)} - 3*a*b*e^{(2*c)} + b^2*e^{(2*c)})*e^{(2*d*x)}), x)$$

Fricas [B] time = 1.66076, size = 1341, normalized size = 22.73

$$\left[\sqrt{-\frac{b}{a}} \log \left(\frac{b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 - 2(2a+b) \cosh(dx+c)^2 + 2(3b \cosh(dx+c)^2 - 2a-b) \sinh(dx+c)^2 + 4(b \cosh(dx+c) - b \sinh(dx+c)) \cosh(dx+c) \sinh(dx+c)}{b \cosh(dx+c)^4 + 4b \cosh(dx+c) \sinh(dx+c)^3 + b \sinh(dx+c)^4 + 2(2a-b) \cosh(dx+c)^2} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/2*(\sqrt{-b/a})*\log((b*\cosh(d*x + c))^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a + b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c))^3 - (2*a + b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 - a*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 - a)*\sinh(d*x + c))*\sqrt{-b/a} + b)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c))^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b) - 4*\arctan(\cosh(d*x + c) + \sinh(d*x + c)))/((a - b)*d), \\ & -(\sqrt{b/a})*\arctan(1/2*\sqrt{b/a}*(\cosh(d*x + c) + \sinh(d*x + c))) + \sqrt{b/a}*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 + (4*a - b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - b)*\sinh(d*x + c))*\sqrt{b/a}/b) - 2*\arctan(\cosh(d*x + c) + \sinh(d*x + c)))/((a - b)*d)] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c + dx)}{a + b \sinh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2),x)

[Out] Integral(sech(c + d*x)/(a + b*sinh(c + d*x)**2), x)

Giac [B] time = 1.43074, size = 289, normalized size = 4.9

$$\frac{2\sqrt{abd}|b|\arctan\left(\frac{2\sqrt{\frac{1}{2}}(e^{(dx+c)}-e^{(-dx-c)})}{\sqrt{\frac{4ad+4bd-\sqrt{-64abd^2+16(ad+bd)^2}}{bd}}}\right)}{(ad-bd)^2b-(ab+b^2)d|-ad+bd|} - \frac{2bd\arctan\left(\frac{2\sqrt{\frac{1}{2}}(e^{(dx+c)}-e^{(-dx-c)})}{\sqrt{\frac{4ad+4bd+\sqrt{-64abd^2+16(ad+bd)^2}}{bd}}}\right)}{ad|-ad+bd|+bd|-ad+bd|+(ad-bd)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

```
[Out] -2*sqrt(a*b)*d*abs(b)*arctan(2*sqrt(1/2)*(e^(d*x + c) - e^(-d*x - c))/sqrt(
(4*a*d + 4*b*d - sqrt(-64*a*b*d^2 + 16*(a*d + b*d)^2))/(b*d)))/((a*d - b*d)
^2*b - (a*b + b^2)*d*abs(-a*d + b*d)) - 2*b*d*arctan(2*sqrt(1/2)*(e^(d*x +
c) - e^(-d*x - c))/sqrt((4*a*d + 4*b*d + sqrt(-64*a*b*d^2 + 16*(a*d + b*d)^
2))/(b*d)))/(a*d*abs(-a*d + b*d) + b*d*abs(-a*d + b*d) + (a*d - b*d)^2)
```

$$3.324 \quad \int \frac{\operatorname{sech}^2(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=60

$$\frac{\tanh(c+dx)}{d(a-b)} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a-b)^{3/2}}$$

[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d)) + Tanh[c + d*x]/((a - b)*d)

Rubi [A] time = 0.0788262, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3191, 388, 208}

$$\frac{\tanh(c+dx)}{d(a-b)} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2), x]

[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d)) + Tanh[c + d*x]/((a - b)*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 388

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1-x^2}{a-(a-b)x^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\tanh(c+dx)}{(a-b)d} - \frac{b \operatorname{Subst}\left(\int \frac{1}{a+(-a+b)x^2} dx, x, \tanh(c+dx)\right)}{(a-b)d} \\ &= -\frac{b \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}d} + \frac{\tanh(c+dx)}{(a-b)d} \end{aligned}$$

Mathematica [A] time = 0.154095, size = 60, normalized size = 1.

$$\frac{\tanh(c+dx)}{d(a-b)} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2), x]

[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d)) + Tanh[c + d*x]/((a - b)*d)

Maple [B] time = 0.06, size = 335, normalized size = 5.6

$$-\frac{b}{d(a-b)} \operatorname{Artanh}\left(a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \frac{1}{\sqrt{(2\sqrt{-b(a-b)} + a - 2b)a}}\right) \frac{1}{\sqrt{(2\sqrt{-b(a-b)} + a - 2b)a}} + \frac{b^2}{d(a-b)} \operatorname{Artanh}\left(a \tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \frac{1}{\sqrt{(2\sqrt{-b(a-b)} + a - 2b)a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2), x)

[Out] -1/d*b/(a-b)/(((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^2/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))

$$(2*(-b*(a-b))^{1/2}+a-2*b)*a^{1/2}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a^{1/2}))+1/d*b/(a-b)/((2*(-b*(a-b))^{1/2}-a+2*b)*a^{1/2})*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a^{1/2}))+1/d*b^2/(a-b)/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a^{1/2})*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a^{1/2}))+2/d/(a-b)*\operatorname{tanh}(1/2*d*x+1/2*c)/(\operatorname{tanh}(1/2*d*x+1/2*c)^2+1)$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.63066, size = 1770, normalized size = 29.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$[-1/2*((b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + b)*\sqrt{a^2 - a*b})*\log((b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*\sinh(d*x + c)^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*b^2*\cosh(d*x + c)^2 + 2*a*b - b^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*\cosh(d*x + c)^3 + (2*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{a^2 - a*b})/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) + 4*a^2 - 4*a*b)/((a^3 - 2*a^2*b + a*b^2)*d*\cosh(d*x + c)^2 + 2*(a^3 - 2*a^2*b + a*b^2)*d*\cosh(d*x + c)*\sinh(d*x + c) + (a^3 - 2*a^2*b + a*b^2)*d*\sinh(d*x + c)^2 + (a^3 - 2*a^2*b + a*b^2)*d), ((b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + b)*\sqrt{-a^2 + a*b})*\operatorname{arctan}(-1/2*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-a^2 + a*b})/(a^2 - a*b)) - 2*a^2 +$$

$$2ab/((a^3 - 2a^2b + ab^2)d \cosh(dx + c)^2 + 2(a^3 - 2a^2b + ab^2)d \cosh(dx + c) \sinh(dx + c) + (a^3 - 2a^2b + ab^2)d \sinh(dx + c)^2 + (a^3 - 2a^2b + ab^2)d)$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c + dx)}{a + b \sinh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)**2/(a+b*sinh(dx+c)**2),x)

[Out] Integral(sech(c + dx)**2/(a + b*sinh(c + dx)**2), x)

Giac [A] time = 1.22527, size = 111, normalized size = 1.85

$$-\frac{b \arctan\left(\frac{be^{2dx+2c}+2a-b}{2\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}(ad-bd)} - \frac{2}{(ad-bd)(e^{2dx+2c}+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^2/(a+b*sinh(dx+c)^2),x, algorithm="giac")

[Out] -b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*(a*d - b*d)) - 2/((a*d - b*d)*(e^(2*d*x + 2*c) + 1))

$$3.325 \quad \int \frac{\operatorname{sech}^3(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=92

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a-b)^2} + \frac{(a-3b) \tan^{-1}(\sinh(c+dx))}{2d(a-b)^2} + \frac{\tanh(c+dx) \operatorname{sech}(c+dx)}{2d(a-b)}$$

[Out] ((a - 3*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^2*d) + (b^(3/2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^2*d) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d)

Rubi [A] time = 0.10543, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3190, 414, 522, 203, 205}

$$\frac{b^{3/2} \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a-b)^2} + \frac{(a-3b) \tan^{-1}(\sinh(c+dx))}{2d(a-b)^2} + \frac{\tanh(c+dx) \operatorname{sech}(c+dx)}{2d(a-b)}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2), x]

[Out] ((a - 3*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^2*d) + (b^(3/2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^2*d) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]

, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(!IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-bx^2}{(1+x^2)(a+bx^2)} dx, x, \sinh(c+dx)\right)}{2(a-b)d} \\ &= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d} + \frac{(a-3b)\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx)\right)}{2(a-b)^2d} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c+dx)\right)}{2(a-b)^2d} \\ &= \frac{(a-3b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^2d} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^2d} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d} \end{aligned}$$

Mathematica [A] time = 0.222276, size = 91, normalized size = 0.99

$$\frac{-2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right) + 2\sqrt{a}(a-3b)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \sqrt{a}(a-b)\tanh(c+dx)\operatorname{sech}(c+dx)}{2\sqrt{ad}(a-b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2), x]

[Out] $(-2*b^{(3/2)}*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(a - 3*b)*ArcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/(2*Sqrt[a]*(a - b)^2*d)$

Maple [B] time = 0.068, size = 662, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2), x)

[Out] $-1/d*b^2/(a-b)^2*a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-1/d*b^2/(a-b)^2/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+1/d*b^3/(a-b)^2/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-1/d*b^2/(a-b)^2*a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/d*b^2/(a-b)^2/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/d*b^3/(a-b)^2/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*a+1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*b+1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*a-1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*b+1/d/(a-b)^2*arctan(tanh(1/2*d*x+1/2*c))*a-3/d/(a-b)^2*arctan(tanh(1/2*d*x+1/2*c))*b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(ae^c - 3be^c) \arctan(e^{(dx+c)})e^{(-c)}}{a^2d - 2abd + b^2d} + \frac{e^{(3dx+3c)} - e^{(dx+c)}}{ad - bd + (ade^{(4c)} - bde^{(4c)})e^{(4dx)} + 2(ade^{(2c)} - bde^{(2c)})e^{(2dx)}} + 8 \int \frac{1}{4(a^2b - 2ab^2 + b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] $(a e^{-c} - 3 b e^c) \arctan(e^{(d x + c)}) e^{-c} / (a^2 d - 2 a b d + b^2 d) + (e^{(3 d x + 3 c)} - e^{(d x + c)}) / (a d - b d + (a d e^{(4 c)} - b d e^{(4 c)}) e^{(4 d x)} + 2 (a d e^{(2 c)} - b d e^{(2 c)}) e^{(2 d x)}) + 8 \int (1/4 (b^2 e^{(3 d x + 3 c)} + b^2 e^{(d x + c)}) / (a^2 b - 2 a b^2 + b^3 + (a^2 b e^{(4 c)} - 2 a b^2 e^{(4 c)} + b^3 e^{(4 c)}) e^{(4 d x)} + 2 (2 a^3 e^{(2 c)} - 5 a^2 b e^{(2 c)} + 4 a b^2 e^{(2 c)} - b^3 e^{(2 c)}) e^{(2 d x)}), x$

Fricas [B] time = 1.92306, size = 4370, normalized size = 47.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] $[1/2(2(a - b)\cosh(dx + c)^3 + 6(a - b)\cosh(dx + c)\sinh(dx + c)^2 + 2(a - b)\sinh(dx + c)^3 + (b\cosh(dx + c)^4 + 4b\cosh(dx + c)\sinh(dx + c)^3 + b\sinh(dx + c)^4 + 2b\cosh(dx + c)^2 + 2(3b\cosh(dx + c)^2 + b)\sinh(dx + c)^2 + 4(b\cosh(dx + c)^3 + b\cosh(dx + c))\sinh(dx + c) + b)\sqrt{-b/a}\log((b\cosh(dx + c)^4 + 4b\cosh(dx + c)\sinh(dx + c)^3 + b\sinh(dx + c)^4 - 2(2a + b)\cosh(dx + c)^2 + 2(3b\cosh(dx + c)^2 - 2a - b)\sinh(dx + c)^2 + 4(b\cosh(dx + c)^3 - (2a + b)\cosh(dx + c))\sinh(dx + c) + 4(a\cosh(dx + c)^3 + 3a\cosh(dx + c)\sinh(dx + c)^2 + a\sinh(dx + c)^3 - a\cosh(dx + c) + (3a\cosh(dx + c)^2 - a)\sinh(dx + c))\sqrt{-b/a} + b) / (b\cosh(dx + c)^4 + 4b\cosh(dx + c)\sinh(dx + c)^3 + b\sinh(dx + c)^4 + 2(2a - b)\cosh(dx + c)^2 + 2(3b\cosh(dx + c)^2 + 2a - b)\sinh(dx + c)^2 + 4(b\cosh(dx + c)^3 + (2a - b)\cosh(dx + c))\sinh(dx + c) + b) + 2((a - 3b)\cosh(dx + c)^4 + 4(a - 3b)\cosh(dx + c)\sinh(dx + c)^3 + (a - 3b)\sinh(dx + c)^4 + 2(a - 3b)\cosh(dx + c)^2 + 2(3(a - 3b)\cosh(dx + c)^2 + a - 3b)\sinh(dx + c)^2 + 4((a - 3b)\cosh(dx + c)^3 + (a - 3b)\cosh(dx + c))\sinh(dx + c) + a - 3b)\arctan(\cosh(dx + c) + \sinh(dx + c)) - 2(a - b)\cosh(dx + c) + 2(3(a - b)\cosh(dx + c)^2 - a + b)\sinh(dx + c)) / ((a^2 - 2ab + b^2)d\cosh(dx + c)^4 + 4(a^2 - 2ab + b^2)d\cosh(dx + c)\sinh(dx + c)^3 + (a^2 - 2ab + b^2)d\sinh(dx + c)^4 + 2(a^2 - 2ab + b^2)d\cosh(dx + c)^2 + 2(3(a^2 - 2ab + b^2)d\cosh(dx + c)^2 + (a^2 - 2ab + b^2)d)\sinh(dx + c)^2 + (a^2 - 2ab + b^2)d + 4((a^2 - 2ab + b^2)d\cosh(dx + c)^3 + (a^2 - 2ab + b^2)d\cosh(dx + c))\sinh(dx + c)), ((a - b)\cosh(dx + c)^3 + 3(a - b)\cosh(dx + c)\sinh(dx + c)^2 + (a - b)\sinh(dx + c)^3 + (b\cosh(dx + c)^4 + 4b\cosh(dx + c)\sinh(dx + c)^3 + b\sinh(dx + c)^4$

```

4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(
b*cosh(d*x + c)^3 + b*cosh(d*x + c)*sinh(d*x + c) + b)*sqrt(b/a)*arctan(1/
2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + (b*cosh(d*x + c)^4 + 4*b*cos
h(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x
+ c))*sinh(d*x + c) + b)*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cos
h(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) +
(3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) + ((a - 3*b)*co
sh(d*x + c)^4 + 4*(a - 3*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - 3*b)*sinh(
d*x + c)^4 + 2*(a - 3*b)*cosh(d*x + c)^2 + 2*(3*(a - 3*b)*cosh(d*x + c)^2 +
a - 3*b)*sinh(d*x + c)^2 + 4*((a - 3*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d
*x + c))*sinh(d*x + c) + a - 3*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (
a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c))/
(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*d*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*d*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b
+ b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a
^2 - 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*d + 4*((a^2 - 2*
a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*
x + c))]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c + dx)}{a + b \sinh^2(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


$$3.326 \quad \int \frac{\operatorname{sech}^4(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=88

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a-b)^{5/2}} - \frac{\tanh^3(c+dx)}{3d(a-b)} + \frac{(a-2b) \tanh(c+dx)}{d(a-b)^2}$$

[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d) + ((a - 2*b)*Tanh[c + d*x])/((a - b)^2*d) - Tanh[c + d*x]^3/(3*(a - b)*d)

Rubi [A] time = 0.10538, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3191, 390, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a-b)^{5/2}} - \frac{\tanh^3(c+dx)}{3d(a-b)} + \frac{(a-2b) \tanh(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2), x]

[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d) + ((a - 2*b)*Tanh[c + d*x])/((a - b)^2*d) - Tanh[c + d*x]^3/(3*(a - b)*d)

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{a-(a-b)x^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a-2b}{(a-b)^2} - \frac{x^2}{a-b} + \frac{b^2}{(a-b)^2(a-(a-b)x^2)}\right) dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{(a-2b)\tanh(c+dx)}{(a-b)^2d} - \frac{\tanh^3(c+dx)}{3(a-b)d} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{a-(a-b)x^2} dx, x, \tanh(c+dx)\right)}{(a-b)^2d} \\ &= \frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}d} + \frac{(a-2b)\tanh(c+dx)}{(a-b)^2d} - \frac{\tanh^3(c+dx)}{3(a-b)d} \end{aligned}$$

Mathematica [A] time = 0.491389, size = 84, normalized size = 0.95

$$\frac{3b^2 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}} + \frac{\tanh(c+dx)\left((a-b)\operatorname{sech}^2(c+dx)+2a-5b\right)}{(a-b)^2}$$

3d

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2), x]

[Out] ((3*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2))) + ((2*a - 5*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^2/(3*d)

Maple [B] time = 0.07, size = 535, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2), x)

```
[Out] 1/d*b^2/(a-b)^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^3/(a-b)^2/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^2/(a-b)^2/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))-1/d*b^3/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d/(a-b)
^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a-4/d/(a-b)^2/(tanh(1/
2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*b+4/3/d/(a-b)^2/(tanh(1/2*d*x+1/2
*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*a-16/3/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)
^3*tanh(1/2*d*x+1/2*c)^3*b+2/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2
*d*x+1/2*c)*a-4/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.81998, size = 5762, normalized size = 65.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 - 8*a^3 + 28*a^2*b -
20*a*b^2 - 24*(a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - 3*a^2*b
+ 2*a*b^2 - 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^2*co
sh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 +
3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^4 +
3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sin
h(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 + 6*b^2*cosh(d*x + c)^2 + b^2)*sinh
(d*x + c)^2 + b^2 + 6*(b^2*cosh(d*x + c)^5 + 2*b^2*cosh(d*x + c)^3 + b^2*co
```


$$3*b + 3*a^2*b^2 - a*b^3)*d + 6*((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^5 + 2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [A] time = 1.41082, size = 192, normalized size = 2.18

$$\frac{b^2 \arctan\left(\frac{be^{(2dx+2c)+2a-b}}{2\sqrt{-a^2+ab}}\right)}{(a^2d - 2abd + b^2d)\sqrt{-a^2 + ab}} + \frac{2(3be^{(4dx+4c)} - 6ae^{(2dx+2c)} + 12be^{(2dx+2c)} - 2a + 5b)}{3(a^2d - 2abd + b^2d)(e^{(2dx+2c)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] $b^2*\arctan(1/2*(b*e^{(2*d*x + 2*c)} + 2*a - b)/\sqrt{-a^2 + a*b})/((a^2*d - 2*a*b*d + b^2*d)*\sqrt{-a^2 + a*b}) + 2/3*(3*b*e^{(4*d*x + 4*c)} - 6*a*e^{(2*d*x + 2*c)} + 12*b*e^{(2*d*x + 2*c)} - 2*a + 5*b)/((a^2*d - 2*a*b*d + b^2*d)*(e^{(2*d*x + 2*c)} + 1)^3)$

$$3.327 \quad \int \frac{\operatorname{sech}^5(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=138

$$\frac{(3a^2 - 10ab + 15b^2) \tan^{-1}(\sinh(c + dx))}{8d(a - b)^3} - \frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a - b)^3} + \frac{\tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d(a - b)} + \frac{(3a - 7b) \tanh(c + dx)}{8d(a - b)}$$

[Out] $((3*a^2 - 10*a*b + 15*b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*(a - b)^3*d) - (b^{5/2})*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[c + d*x])/\operatorname{Sqrt}[a]]/(\operatorname{Sqrt}[a]*(a - b)^3*d) + ((3*a - 7*b)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*(a - b)^2*d) + (\operatorname{Sech}[c + d*x]^3*\operatorname{Tanh}[c + d*x])/(4*(a - b)*d)$

Rubi [A] time = 0.161852, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{(3a^2 - 10ab + 15b^2) \tan^{-1}(\sinh(c + dx))}{8d(a - b)^3} - \frac{b^{5/2} \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a - b)^3} + \frac{\tanh(c + dx) \operatorname{sech}^3(c + dx)}{4d(a - b)} + \frac{(3a - 7b) \tanh(c + dx)}{8d(a - b)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[c + d*x]^5/(a + b*\operatorname{Sinh}[c + d*x]^2), x]$

[Out] $((3*a^2 - 10*a*b + 15*b^2)*\operatorname{ArcTan}[\operatorname{Sinh}[c + d*x]])/(8*(a - b)^3*d) - (b^{5/2})*\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[c + d*x])/\operatorname{Sqrt}[a]]/(\operatorname{Sqrt}[a]*(a - b)^3*d) + ((3*a - 7*b)*\operatorname{Sech}[c + d*x]*\operatorname{Tanh}[c + d*x])/(8*(a - b)^2*d) + (\operatorname{Sech}[c + d*x]^3*\operatorname{Tanh}[c + d*x])/(4*(a - b)*d)$

Rule 3190

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*ff^2*x^2)^p, x], x, \operatorname{Sin}[e + f*x]/ff], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 414

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^{(n_.)]^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)]^{(q_.)}, x_Symbol] \rightarrow -\operatorname{Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)})/(a*n*(p+1)*(b*c -$

$a*d)), x] + \text{Dist}[1/(a*n*(p + 1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p + 1)}*(c + d*x^n)^q * \text{Simp}[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, n, q\}, x \} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& \text{!(IntegerQ}[p] \&\& \text{IntegerQ}[q] \&\& \text{LtQ}[q, -1]) \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 527

$\text{Int}[(a_ + (b_)*(x_)^{(n_}))^{(p_)}*((c_ + (d_)*(x_)^{(n_}))^{(q_)}*((e_ + (f_)*(x_)^{(n_})), x_Symbol] :> -\text{Simp}[(b*e - a*f)*x*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q + 1)}/(a*n*(b*c - a*d)*(p + 1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p + 1)), \text{Int}[(a + b*x^n)^{(p + 1)}*(c + d*x^n)^q * \text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, f, n, q\}, x \} \&\& \text{LtQ}[p, -1]$

Rule 522

$\text{Int}[(e_ + (f_)*(x_)^{(n_}))/((a_ + (b_)*(x_)^{(n_}))*((c_ + (d_)*(x_)^{(n_}))), x_Symbol] :> \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^n), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[1/(c + d*x^n), x], x] /;$
 $\text{FreeQ}\{a, b, c, d, e, f, n\}, x]$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /;$
 $\text{FreeQ}\{a, b\}, x \} \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{GtQ}[b, 0])$

Rule 205

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /;$
 $\text{FreeQ}\{a, b\}, x \} \&\& \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^3(a+bx^2)} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} - \frac{\operatorname{Subst}\left(\int \frac{-3a+4b-3bx^2}{(1+x^2)^2(a+bx^2)} dx, x, \sinh(c+dx)\right)}{4(a-b)d} \\
&= \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^2d} + \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} + \frac{\operatorname{Subst}\left(\int \frac{3a^2-7ab+8b^2+(a+bx^2)}{(1+x^2)^2(a+bx^2)} dx, x, \sinh(c+dx)\right)}{8(a-b)d} \\
&= \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^2d} + \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} - \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c+dx)\right)}{(a-b)^3d} \\
&= \frac{(3a^2-10ab+15b^2)\tan^{-1}(\sinh(c+dx))}{8(a-b)^3d} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^3d} + \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)d}
\end{aligned}$$

Mathematica [A] time = 0.50705, size = 139, normalized size = 1.01

$$\frac{2\sqrt{a}(3a^2-10ab+15b^2)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \sqrt{a}(3a^2-10ab+7b^2)\tanh(c+dx)\operatorname{sech}(c+dx) + 8b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{a+b\sinh^2(c+dx)}}\right)}{8\sqrt{ad}(a-b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^5/(a + b*Sinh[c + d*x]^2), x]

[Out] (8*b^(5/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(3*a^2 - 10*a*b + 15*b^2)*ArcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(3*a^2 - 10*a*b + 7*b^2)*Sech[c + d*x]*Tanh[c + d*x] + 2*Sqrt[a]*(a - b)^2*Sech[c + d*x]^3*Tanh[c + d*x])/(8*Sqrt[a]*(a - b)^3*d)

Maple [B] time = 0.073, size = 1023, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2), x)


```
*x + c))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4 + (a^3*b*e^(4*c) - 3*a^2*b^2*e^(4*c) + 3*a*b^3*e^(4*c) - b^4*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 7*a^3*b*e^(2*c) + 9*a^2*b^2*e^(2*c) - 5*a*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)),
x)
```

Fricas [B] time = 2.41039, size = 13526, normalized size = 98.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*((3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 - 10*a*b + 7*b^2)
*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 - 10*a*b + 7*b^2)*sinh(d*x + c)^7 +
(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 - 10*a*b + 7*b^2)*
cosh(d*x + c)^2 + 11*a^2 - 26*a*b + 15*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 -
10*a*b + 7*b^2)*cosh(d*x + c)^3 + (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^4 - (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 + (35*(3*a^2
- 10*a*b + 7*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x
+ c)^2 - 11*a^2 + 26*a*b - 15*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 - 10*a*b +
7*b^2)*cosh(d*x + c)^5 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 - 3
*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^2*cosh(d*
x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*b^
2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 6*b^2
*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 30*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(
d*x + c)^4 + 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d*x + c)^5 + 10*b^2*cosh
(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^2*cosh(d*x + c)
^6 + 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2
+ b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*b^2*cosh(d*x + c)^5 + 3*b^2*cosh(d*x +
c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 +
4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d
*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d
*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 +
3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) +
(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4
+ 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + ((3*a^2 - 10*a
*b + 15*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)*si
nh(d*x + c)^7 + (3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 - 10*a
```

$$\begin{aligned}
& *b + 15*b^2)*\cosh(d*x + c)^6 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c) \\
& ^2 + 3*a^2 - 10*a*b + 15*b^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2 - 10*a*b + 15*b \\
& ^2)*\cosh(d*x + c)^3 + 3*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^5 + 6*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^4 + 2*(35*(3*a^2 - 10*a*b \\
& + 15*b^2)*\cosh(d*x + c)^4 + 30*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^2 + \\
& 9*a^2 - 30*a*b + 45*b^2)*\sinh(d*x + c)^4 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)* \\
& \cosh(d*x + c)^5 + 10*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^3 + 3*(3*a^2 - \\
& 10*a*b + 15*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(3*a^2 - 10*a*b + 15*b \\
& ^2)*\cosh(d*x + c)^2 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^6 + 15*(\\
& 3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^4 + 9*(3*a^2 - 10*a*b + 15*b^2)*\cosh \\
& (d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2)*\sinh(d*x + c)^2 + 3*a^2 - 10*a*b + 1 \\
& 5*b^2 + 8*((3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^7 + 3*(3*a^2 - 10*a*b + \\
& 15*b^2)*\cosh(d*x + c)^5 + 3*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^3 + (3* \\
& a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \\
& \sinh(d*x + c)) - (3*a^2 - 10*a*b + 7*b^2)*\cosh(d*x + c) + (7*(3*a^2 - 10*a \\
& *b + 7*b^2)*\cosh(d*x + c)^6 + 5*(11*a^2 - 26*a*b + 15*b^2)*\cosh(d*x + c)^4 \\
& - 3*(11*a^2 - 26*a*b + 15*b^2)*\cosh(d*x + c)^2 - 3*a^2 + 10*a*b - 7*b^2)*\si \\
& nh(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a^3 - \\
& 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^3 - 3*a^2*b + \\
& 3*a*b^2 - b^3)*d*\sinh(d*x + c)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos \\
& h(d*x + c)^6 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^2 + (a^ \\
& 3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*\sinh(d*x + c)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^ \\
& 2 - b^3)*d*\cosh(d*x + c)^4 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d* \\
& x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c) \\
& ^5 + 2*(35*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^4 + 30*(a^3 - 3* \\
& a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3 \\
&)*d)*\sinh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^2 \\
& + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^5 + 10*(a^3 - 3*a^2* \\
& b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d* \\
& \cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos \\
& h(d*x + c)^6 + 15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^4 + 9*(a^ \\
& 3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - \\
& b^3)*d)*\sinh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 8*((a^3 - 3* \\
& a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3 \\
&)*d*\cosh(d*x + c)^5 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^3 + \\
& (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/4*((3*a \\
& ^2 - 10*a*b + 7*b^2)*\cosh(d*x + c)^7 + 7*(3*a^2 - 10*a*b + 7*b^2)*\cosh(d*x \\
& + c)*\sinh(d*x + c)^6 + (3*a^2 - 10*a*b + 7*b^2)*\sinh(d*x + c)^7 + (11*a^2 - \\
& 26*a*b + 15*b^2)*\cosh(d*x + c)^5 + (21*(3*a^2 - 10*a*b + 7*b^2)*\cosh(d*x + \\
& c)^2 + 11*a^2 - 26*a*b + 15*b^2)*\sinh(d*x + c)^5 + 5*(7*(3*a^2 - 10*a*b + \\
& 7*b^2)*\cosh(d*x + c)^3 + (11*a^2 - 26*a*b + 15*b^2)*\cosh(d*x + c))*\sinh(d*x \\
& + c)^4 - (11*a^2 - 26*a*b + 15*b^2)*\cosh(d*x + c)^3 + (35*(3*a^2 - 10*a*b \\
& + 7*b^2)*\cosh(d*x + c)^4 + 10*(11*a^2 - 26*a*b + 15*b^2)*\cosh(d*x + c)^2 - \\
& 11*a^2 + 26*a*b - 15*b^2)*\sinh(d*x + c)^3 + (21*(3*a^2 - 10*a*b + 7*b^2)*co \\
& sh(d*x + c)^5 + 10*(11*a^2 - 26*a*b + 15*b^2)*\cosh(d*x + c)^3 - 3*(11*a^2 -
\end{aligned}$$

$$\begin{aligned}
& 26*a*b + 15*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^2 - 4*(b^2*\cosh(d*x + c))^8 + \\
& 8*b^2*\cosh(d*x + c)*\sinh(d*x + c)^7 + b^2*\sinh(d*x + c)^8 + 4*b^2*\cosh(d*x \\
& + c)^6 + 4*(7*b^2*\cosh(d*x + c)^2 + b^2)*\sinh(d*x + c)^6 + 6*b^2*\cosh(d*x \\
& + c)^4 + 8*(7*b^2*\cosh(d*x + c)^3 + 3*b^2*\cosh(d*x + c))*\sinh(d*x + c)^5 + \\
& 2*(35*b^2*\cosh(d*x + c)^4 + 30*b^2*\cosh(d*x + c)^2 + 3*b^2)*\sinh(d*x + c)^4 \\
& + 4*b^2*\cosh(d*x + c)^2 + 8*(7*b^2*\cosh(d*x + c)^5 + 10*b^2*\cosh(d*x + c)^ \\
& 3 + 3*b^2*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*b^2*\cosh(d*x + c)^6 + 15*b^ \\
& 2*\cosh(d*x + c)^4 + 9*b^2*\cosh(d*x + c)^2 + b^2)*\sinh(d*x + c)^2 + b^2 + 8* \\
& (b^2*\cosh(d*x + c)^7 + 3*b^2*\cosh(d*x + c)^5 + 3*b^2*\cosh(d*x + c)^3 + b^2* \\
& \cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/2*\sqrt{b/a}*(\cosh(d*x + c) \\
& + \sinh(d*x + c))) - 4*(b^2*\cosh(d*x + c))^8 + 8*b^2*\cosh(d*x + c)*\sinh(d*x \\
& + c)^7 + b^2*\sinh(d*x + c)^8 + 4*b^2*\cosh(d*x + c)^6 + 4*(7*b^2*\cosh(d*x + \\
& c)^2 + b^2)*\sinh(d*x + c)^6 + 6*b^2*\cosh(d*x + c)^4 + 8*(7*b^2*\cosh(d*x + c) \\
&)^3 + 3*b^2*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*b^2*\cosh(d*x + c)^4 + 30 \\
& *b^2*\cosh(d*x + c)^2 + 3*b^2)*\sinh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)^2 + 8*(\\
& 7*b^2*\cosh(d*x + c)^5 + 10*b^2*\cosh(d*x + c)^3 + 3*b^2*\cosh(d*x + c))*\sinh(\\
& d*x + c)^3 + 4*(7*b^2*\cosh(d*x + c)^6 + 15*b^2*\cosh(d*x + c)^4 + 9*b^2*\cosh \\
& (d*x + c)^2 + b^2)*\sinh(d*x + c)^2 + b^2 + 8*(b^2*\cosh(d*x + c)^7 + 3*b^2*c \\
& osh(d*x + c)^5 + 3*b^2*\cosh(d*x + c)^3 + b^2*\cosh(d*x + c))*\sinh(d*x + c))* \\
& \sqrt{b/a}*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 \\
& + b*\sinh(d*x + c)^3 + (4*a - b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a \\
& - b)*\sinh(d*x + c))*\sqrt{b/a}/b) + ((3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c) \\
&)^8 + 8*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2 - \\
& 10*a*b + 15*b^2)*\sinh(d*x + c)^8 + 4*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c) \\
&)^6 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^2 + 3*a^2 - 10*a*b + 15* \\
& b^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^3 + 3*(\\
& 3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 6*(3*a^2 - 10*a*b \\
& + 15*b^2)*\cosh(d*x + c)^4 + 2*(35*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^ \\
& 4 + 30*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^2 + 9*a^2 - 30*a*b + 45*b^2) \\
& *\sinh(d*x + c)^4 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^5 + 10*(3*a \\
& ^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^3 + 3*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d* \\
& x + c))*\sinh(d*x + c)^3 + 4*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^2 + 4*(\\
& 7*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^6 + 15*(3*a^2 - 10*a*b + 15*b^2)* \\
& \cosh(d*x + c)^4 + 9*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^2 + 3*a^2 - 10* \\
& a*b + 15*b^2)*\sinh(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2 + 8*((3*a^2 - 10*a* \\
& b + 15*b^2)*\cosh(d*x + c)^7 + 3*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^5 + \\
& 3*(3*a^2 - 10*a*b + 15*b^2)*\cosh(d*x + c)^3 + (3*a^2 - 10*a*b + 15*b^2)*\co \\
& sh(d*x + c))*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - (3*a^2 \\
& - 10*a*b + 7*b^2)*\cosh(d*x + c) + (7*(3*a^2 - 10*a*b + 7*b^2)*\cosh(d*x + c) \\
&)^6 + 5*(11*a^2 - 26*a*b + 15*b^2)*\cosh(d*x + c)^4 - 3*(11*a^2 - 26*a*b + 15 \\
& *b^2)*\cosh(d*x + c)^2 - 3*a^2 + 10*a*b - 7*b^2)*\sinh(d*x + c))/((a^3 - 3*a^ \\
& 2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)* \\
& d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\sinh(d* \\
& x + c)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a^3 \\
& - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b
\end{aligned}$$

$$\begin{aligned}
&^3)d*\sinh(dx + c)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(dx + c)^4 \\
&+ 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(dx + c)^3 + 3*(a^3 - 3*a^2 \\
&*b + 3*a*b^2 - b^3)*d*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(35*(a^3 - 3*a^2*b \\
&+ 3*a*b^2 - b^3)*d*\cosh(dx + c)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d* \\
&\cosh(dx + c)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*\sinh(dx + c)^4 + 4* \\
&(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(dx + c)^2 + 8*(7*(a^3 - 3*a^2*b + 3 \\
&*a*b^2 - b^3)*d*\cosh(dx + c)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh \\
&(dx + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(dx + c))*\sinh(dx + \\
&c)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(dx + c)^6 + 15*(a^3 - \\
&3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(dx + c)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b \\
&^3)*d*\cosh(dx + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*\sinh(dx + c)^2 \\
&+ (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d* \\
&\cosh(dx + c)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(dx + c)^5 + 3*(\\
&a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*\cosh(dx + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2 \\
&- b^3)*d*\cosh(dx + c))*\sinh(dx + c)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)**5/(a+b*sinh(dx+c)**2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^5/(a+b*sinh(dx+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.328 \quad \int \frac{\operatorname{sech}^6(c+dx)}{a+b \sinh^2(c+dx)} dx$$

Optimal. Leaf size=126

$$\frac{(a^2 - 3ab + 3b^2) \tanh(c + dx)}{d(a - b)^3} - \frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a - b)^{7/2}} + \frac{\tanh^5(c + dx)}{5d(a - b)} - \frac{(2a - 3b) \tanh^3(c + dx)}{3d(a - b)^2}$$

[Out] $-\left(\frac{b^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tanh}[c + d*x]}{\sqrt{a}}\right]}{\sqrt{a} (a - b)^{7/2}}\right) + \frac{(a^2 - 3ab + 3b^2) \operatorname{Tanh}[c + d*x]}{d(a - b)^3} - \frac{\operatorname{Tanh}[c + d*x]^5}{5d(a - b)} - \frac{(2a - 3b) \operatorname{Tanh}[c + d*x]^3}{3d(a - b)^2}$

Rubi [A] time = 0.147213, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3191, 390, 208}

$$\frac{(a^2 - 3ab + 3b^2) \tanh(c + dx)}{d(a - b)^3} - \frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{ad}(a - b)^{7/2}} + \frac{\tanh^5(c + dx)}{5d(a - b)} - \frac{(2a - 3b) \tanh^3(c + dx)}{3d(a - b)^2}$$

Antiderivative was successfully verified.

[In] `Int[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]`

[Out] $-\left(\frac{b^3 \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tanh}[c + d*x]}{\sqrt{a}}\right]}{\sqrt{a} (a - b)^{7/2}}\right) + \frac{(a^2 - 3ab + 3b^2) \operatorname{Tanh}[c + d*x]}{d(a - b)^3} - \frac{\operatorname{Tanh}[c + d*x]^5}{5d(a - b)} - \frac{(2a - 3b) \operatorname{Tanh}[c + d*x]^3}{3d(a - b)^2}$

Rule 3191

`Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]`

Rule 390

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]`

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{a-(a-b)x^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{(a-b)^3} - \frac{(2a-3b)x^2}{(a-b)^2} + \frac{x^4}{a-b} - \frac{b^3}{(a-b)^3(a-(a-b)x^2)}\right) dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3d} - \frac{(2a-3b)\tanh^3(c+dx)}{3(a-b)^2d} + \frac{\tanh^5(c+dx)}{5(a-b)d} - \frac{b^3 \operatorname{Subst}\left(\int \frac{1}{a-(a-b)x^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= -\frac{b^3 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}d} + \frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3d} - \frac{(2a-3b)\tanh^3(c+dx)}{3(a-b)^2d} + \end{aligned}$$

Mathematica [A] time = 0.893982, size = 119, normalized size = 0.94

$$\frac{\frac{\tanh(c+dx)\left((4a^2-13ab+9b^2)\operatorname{sech}^2(c+dx)+8a^2+3(a-b)^2\operatorname{sech}^4(c+dx)-26ab+33b^2\right)}{(a-b)^3} - \frac{15b^3 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}}}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2), x]

[Out] ((-15*b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2)) + ((8*a^2 - 26*a*b + 33*b^2 + (4*a^2 - 13*a*b + 9*b^2)*Sech[c + d*x]^2 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(a - b)^3)/(15*d)

Maple [B] time = 0.075, size = 907, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{sech}(d*x+c)^6/(a+b*\sinh(d*x+c)^2), x)$

[Out]
$$\begin{aligned} & -1/d*b^3/(a-b)^3/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+1/d*b^4/(a-b)^3/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\text{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+1/d*b^3/(a-b)^3/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+1/d*b^4/(a-b)^3/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\text{arctan}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+2/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^9*a^2-6/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^9*a*b+6/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^7*a^2-32/3/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^7*a*b+16/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^7*b^2+116/15/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^5*a^2-332/15/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^5*a*b+132/5/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^5*b^2+8/3/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^3*a^2-32/3/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^3*a*b+16/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)^3*b^2+2/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)*a^2-6/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)*a*b+6/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^{2+1})^5*\tanh(1/2*d*x+1/2*c)*b^2 \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\text{sech}(d*x+c)^6/(a+b*\sinh(d*x+c)^2), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [B] time = 2.13266, size = 14236, normalized size = 112.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/30*(60*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^8 + 480*(a^2*b^2 - a*b^3)*\cosh(d \\ & *x + c)*\sinh(d*x + c)^7 + 60*(a^2*b^2 - a*b^3)*\sinh(d*x + c)^8 - 120*(a^3*b \\ & - 4*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^6 - 120*(a^3*b - 4*a^2*b^2 + 3*a*b^3 \\ & - 14*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 240*(14*(a^2*b^2 \\ & - a*b^3)*\cosh(d*x + c)^3 - 3*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 \\ & + 40*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*\cosh(d*x + c)^4 + 40*(105*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^4 \\ & + 8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 \\ & + 32*a^4 - 136*a^3*b + 236*a^2*b^2 - 132*a*b^3 + 160*(21*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^5 - 15*(a^3*b - 4*a^2*b^2 \\ & + 3*a*b^3)*\cosh(d*x + c)^3 + (8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 \\ & + 40*(4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3)*\cosh(d*x + c)^2 + 40*(42*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^6 \\ & - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^4 + 4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3 + 6*(8*a^4 - 31*a^3*b \\ & + 47*a^2*b^2 - 24*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 15*(b^3*\cosh(d*x + c)^10 + 10*b^3*\cosh(d*x + c)*\sinh(d*x + c)^9 \\ & + b^3*\sinh(d*x + c)^10 + 5*b^3*\cosh(d*x + c)^8 + 10*b^3*\cosh(d*x + c)^6 + 5*(9*b^3*\cosh(d*x + c)^2 + b^3)*\sinh(d*x + c)^8 \\ & + 40*(3*b^3*\cosh(d*x + c)^3 + b^3*\cosh(d*x + c))*\sinh(d*x + c)^7 + 10*b^3*\cosh(d*x + c)^4 + 10*(21*b^3*\cosh(d*x + c)^4 \\ & + 14*b^3*\cosh(d*x + c)^2 + b^3)*\sinh(d*x + c)^6 + 4*(63*b^3*\cosh(d*x + c)^5 + 70*b^3*\cosh(d*x + c)^3 + 15*b^3*\cosh(d*x + c))*\sinh(d*x + c)^5 \\ & + 5*b^3*\cosh(d*x + c)^2 + 10*(21*b^3*\cosh(d*x + c)^6 + 35*b^3*\cosh(d*x + c)^4 + 15*b^3*\cosh(d*x + c)^2 + b^3)*\sinh(d*x + c)^4 \\ & + 40*(3*b^3*\cosh(d*x + c)^7 + 7*b^3*\cosh(d*x + c)^5 + 5*b^3*\cosh(d*x + c)^3 + b^3*\cosh(d*x + c))*\sinh(d*x + c)^3 + b^3 \\ & + 5*(9*b^3*\cosh(d*x + c)^8 + 28*b^3*\cosh(d*x + c)^6 + 30*b^3*\cosh(d*x + c)^4 + 12*b^3*\cosh(d*x + c)^2 + b^3)*\sinh(d*x + c)^2 \\ & + 10*(b^3*\cosh(d*x + c)^9 + 4*b^3*\cosh(d*x + c)^7 + 6*b^3*\cosh(d*x + c)^5 + 4*b^3*\cosh(d*x + c)^3 + b^3*\cosh(d*x + c))*\sinh(d*x + c) \\ & * \sqrt{a^2 - a*b} * \log((b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*\sinh(d*x + c)^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 \\ & + 2*(3*b^2*\cosh(d*x + c)^2 + 2*a*b - b^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*\cosh(d*x + c)^3 + (2*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c) \\ & - 4*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{a^2 - a*b}) / (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 \\ & + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) \\ & + b)) + 80*(6*(a^2*b^2 - a*b^3)*\cosh(d*x + c)^7 - 9*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^5 + 2*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*\cosh(d*x + c)^3 \\ & + (4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c) / ((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^10 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 \\ & + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\sinh(d*x + c)^10 + 5*(a^5 - 4*a^4*b + \end{aligned}$$

$$\begin{aligned}
& 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^8 + 5*(9*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^2 + (a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d) * \sinh(dx + c)^8 + 10*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^6 + 40*(3*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^3 + (a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)) * \sinh(dx + c)^7 + 10*(21*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^4 + 14*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^2 + (a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d) * \sinh(dx + c)^6 + 10*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^4 + 4*(63*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^5 + 70*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^3 + 15*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 10*(21*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^6 + 35*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^4 + 15*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^2 + (a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d) * \sinh(dx + c)^4 + 5*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^2 + 40*(3*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^7 + 7*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^5 + 5*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^3 + (a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 5*(9*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^8 + 28*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^6 + 30*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^4 + 12*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^2 + (a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d) * \sinh(dx + c)^2 + (a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d + 10*((a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^9 + 4*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^7 + 6*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^5 + 4*(a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)^3 + (a^5 - 4a^4b + 6a^3b^2 - 4a^2b^3 + ab^4) * d * \cosh(dx + c)) * \sinh(dx + c)), -1/15*(30*(a^2b^2 - ab^3) * \cosh(dx + c)^8 + 240*(a^2b^2 - ab^3) * \cosh(dx + c) * \sinh(dx + c)^7 + 30*(a^2b^2 - ab^3) * \sinh(dx + c)^8 - 60*(a^3b - 4a^2b^2 + 3ab^3) * \cosh(dx + c)^6 - 60*(a^3b - 4a^2b^2 + 3ab^3 - 14*(a^2b^2 - ab^3) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 120*(14*(a^2b^2 - ab^3) * \cosh(dx + c)^3 - 3*(a^3b - 4a^2b^2 + 3ab^3) * \cosh(dx + c)) * \sinh(dx + c)^5 + 20*(8a^4 - 31a^3b + 47a^2b^2 - 24ab^3) * \cosh(dx + c)^4 + 20*(105*(a^2b^2 - ab^3) * \cosh(dx + c)^4 + 8a^4 - 31a^3b + 47a^2b^2 - 24ab^3 - 45*(a^3b - 4a^2b^2 + 3ab^3) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 16a^4 - 68a^3b + 118a^2b^2 - 66ab^3 + 80*(21*(a^2b^2 - ab^3) * \cosh(dx + c)^5 - 15*(a^3b - 4a^2b^2 + 3ab^3) * \cosh(dx + c)^3 + (8a^4 - 31a^3b + 47a^2b^2 - 24ab^3) * \cosh(dx + c)) * \sinh(dx + c)^3 + 20*(4a^4 - 17a^3b + 28a^2b^2 - 15ab^3) * \cosh(dx + c)^2 + 20*(42*(a^2b^2 - ab^3) * \cosh(dx + c)^6 - 45*(a^3b - 4a^2b^2 + 3ab^3) * \cosh(dx + c)^4 + 4a^4 - 17a^3b + 28a^2b^2 - 1
\end{aligned}$$

$$\begin{aligned}
& 5*a*b^3 + 6*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*\cosh(d*x + c)^2*\sin \\
& h(d*x + c)^2 - 15*(b^3*\cosh(d*x + c)^{10} + 10*b^3*\cosh(d*x + c)*\sinh(d*x + c \\
&)^9 + b^3*\sinh(d*x + c)^{10} + 5*b^3*\cosh(d*x + c)^8 + 10*b^3*\cosh(d*x + c)^6 \\
& + 5*(9*b^3*\cosh(d*x + c)^2 + b^3)*\sinh(d*x + c)^8 + 40*(3*b^3*\cosh(d*x + c \\
&)^3 + b^3*\cosh(d*x + c))*\sinh(d*x + c)^7 + 10*b^3*\cosh(d*x + c)^4 + 10*(21* \\
& b^3*\cosh(d*x + c)^4 + 14*b^3*\cosh(d*x + c)^2 + b^3)*\sinh(d*x + c)^6 + 4*(63 \\
& *b^3*\cosh(d*x + c)^5 + 70*b^3*\cosh(d*x + c)^3 + 15*b^3*\cosh(d*x + c))*\sinh(\\
& d*x + c)^5 + 5*b^3*\cosh(d*x + c)^2 + 10*(21*b^3*\cosh(d*x + c)^6 + 35*b^3*co \\
& sh(d*x + c)^4 + 15*b^3*\cosh(d*x + c)^2 + b^3)*\sinh(d*x + c)^4 + 40*(3*b^3*c \\
& osh(d*x + c)^7 + 7*b^3*\cosh(d*x + c)^5 + 5*b^3*\cosh(d*x + c)^3 + b^3*\cosh(d \\
& *x + c))*\sinh(d*x + c)^3 + b^3 + 5*(9*b^3*\cosh(d*x + c)^8 + 28*b^3*\cosh(d*x \\
& + c)^6 + 30*b^3*\cosh(d*x + c)^4 + 12*b^3*\cosh(d*x + c)^2 + b^3)*\sinh(d*x + \\
& c)^2 + 10*(b^3*\cosh(d*x + c)^9 + 4*b^3*\cosh(d*x + c)^7 + 6*b^3*\cosh(d*x + \\
& c)^5 + 4*b^3*\cosh(d*x + c)^3 + b^3*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a^2 \\
& + a*b}*\arctan(-1/2*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b \\
& *\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-a^2 + a*b}/(a^2 - a*b)) + 40*(6*(a^2*b^2 \\
& - a*b^3)*\cosh(d*x + c)^7 - 9*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^5 \\
& + 2*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*\cosh(d*x + c)^3 + (4*a^4 - 1 \\
& 7*a^3*b + 28*a^2*b^2 - 15*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5 - 4*a^ \\
& 4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^{10} + 10*(a^5 - 4*a^4*b \\
& + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^5 - \\
& 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\sinh(d*x + c)^{10} + 5*(a^5 - 4*a^ \\
& 4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^8 + 5*(9*(a^5 - 4*a^4* \\
& b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a \\
& ^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*\sinh(d*x + c)^8 + 10*(a^5 - 4*a^4*b + 6*a^3* \\
& b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^6 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b \\
& ^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4* \\
& a^2*b^3 + a*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 10*(21*(a^5 - 4*a^4*b + \\
& 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^4 + 14*(a^5 - 4*a^4*b + 6*a \\
& ^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 \\
& - 4*a^2*b^3 + a*b^4)*d)*\sinh(d*x + c)^6 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4 \\
& *a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^4 + 4*(63*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4* \\
& a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^5 + 70*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2* \\
& b^3 + a*b^4)*d*\cosh(d*x + c)^3 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 \\
& + a*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 10*(21*(a^5 - 4*a^4*b + 6*a^3*b \\
& ^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^6 + 35*(a^5 - 4*a^4*b + 6*a^3*b^2 - \\
& 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^4 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a \\
& ^2*b^3 + a*b^4)*d*\cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 \\
& + a*b^4)*d)*\sinh(d*x + c)^4 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a* \\
& b^4)*d*\cosh(d*x + c)^2 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b \\
& ^4)*d*\cosh(d*x + c)^7 + 7*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d \\
& *\cosh(d*x + c)^5 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh \\
& (d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + \\
& c))*\sinh(d*x + c)^3 + 5*(9*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)* \\
& d*\cosh(d*x + c)^8 + 28*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*co
\end{aligned}$$

$$\begin{aligned} & \operatorname{sh}(d*x + c)^6 + 30*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d \\ & *x + c)^4 + 12*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + \\ & c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*\sinh(d*x + c)^2 \\ & + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d + 10*((a^5 - 4*a^4*b + \\ & 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^9 + 4*(a^5 - 4*a^4*b + 6*a^3 \\ & *b^2 - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^7 + 6*(a^5 - 4*a^4*b + 6*a^3*b^2 \\ & - 4*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^5 + 4*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a \\ & ^2*b^3 + a*b^4)*d*\cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 \\ & + a*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out

Giac [B] time = 1.42036, size = 348, normalized size = 2.76

$$\frac{b^3 \arctan\left(\frac{be^{(2dx+2c)+2a-b}}{2\sqrt{-a^2+ab}}\right)}{(a^3d - 3a^2bd + 3ab^2d - b^3d)\sqrt{-a^2 + ab}} - \frac{2(15b^2e^{(8dx+8c)} - 30abe^{(6dx+6c)} + 90b^2e^{(6dx+6c)} + 80a^2e^{(4dx+4c)} - 230abe^{(4dx+4c)} + 240b^2e^{(4dx+4c)} + 40a^2e^{(2dx+2c)} - 130ab^2e^{(2dx+2c)} + 150b^2e^{(2dx+2c)} + 8a^2 - 26ab + 33b^2)}{15(a^3d - 3a^2bd + 3ab^2d - b^3d)(e^{(2dx+2c)} + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] $-b^3*\arctan(1/2*(b*e^{(2*d*x + 2*c)} + 2*a - b)/\sqrt{-a^2 + a*b})/((a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d)*\sqrt{-a^2 + a*b}) - 2/15*(15*b^2*e^{(8*d*x + 8*c)} - 30*a*b*e^{(6*d*x + 6*c)} + 90*b^2*e^{(6*d*x + 6*c)} + 80*a^2*e^{(4*d*x + 4*c)} - 230*a*b*e^{(4*d*x + 4*c)} + 240*b^2*e^{(4*d*x + 4*c)} + 40*a^2*e^{(2*d*x + 2*c)} - 130*a*b*e^{(2*d*x + 2*c)} + 150*b^2*e^{(2*d*x + 2*c)} + 8*a^2 - 26*a*b + 33*b^2)/((a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d)*(e^{(2*d*x + 2*c)} + 1)^5)$

$$3.329 \quad \int \frac{\cosh^6(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$$

Optimal. Leaf size=158

$$\frac{(4a+b)(a-b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(2a-b)(a-b) \tanh(c+dx)}{2ab^2d(a-(a-b) \tanh^2(c+dx))} - \frac{x(4a-5b)}{2b^3} + \frac{\sinh(c+dx) \cosh(c+dx)}{2bd(a-(a-b) \tanh^2(c+dx))}$$

[Out] -((4*a - 5*b)*x)/(2*b^3) + ((a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((a - b)*(2*a - b)*Tanh[c + d*x])/(2*a*b^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.25785, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3191, 414, 527, 522, 206, 208}

$$\frac{(4a+b)(a-b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(2a-b)(a-b) \tanh(c+dx)}{2ab^2d(a-(a-b) \tanh^2(c+dx))} - \frac{x(4a-5b)}{2b^3} + \frac{\sinh(c+dx) \cosh(c+dx)}{2bd(a-(a-b) \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] -((4*a - 5*b)*x)/(2*b^3) + ((a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((a - b)*(2*a - b)*Tanh[c + d*x])/(2*a*b^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 414

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]

```

Rule 527

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

```

Rule 522

```

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^6(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-(a-b)x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd(a-(a-b)\tanh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{-a+2b-3(a-b)x^2}{(1-x^2)(a+(-a+b)x^2)^2} dx, x, \tanh(c+dx)\right)}{2bd} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd(a-(a-b)\tanh^2(c+dx))} + \frac{(a-b)(2a-b)\tanh(c+dx)}{2ab^2d(a-(a-b)\tanh^2(c+dx))} - \frac{\text{Subst}\left(\int \frac{2(2a^2-2)}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{2ab^2d} \\
&= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd(a-(a-b)\tanh^2(c+dx))} + \frac{(a-b)(2a-b)\tanh(c+dx)}{2ab^2d(a-(a-b)\tanh^2(c+dx))} - \frac{(4a-5b)\text{Subst}\left(\int \frac{2(2a^2-2)}{(1-x^2)^2} dx, x, \tanh(c+dx)\right)}{2ab^2d} \\
&= -\frac{(4a-5b)x}{2b^3} + \frac{(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd(a-(a-b)\tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.576941, size = 118, normalized size = 0.75

$$\frac{\frac{2(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + 2(5b-4a)(c+dx) + \frac{2b(a-b)^2\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)} + b\sinh(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (2*(-4*a + 5*b)*(c + d*x) + (2*(a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + b*Sinh[2*(c + d*x)] + (2*(a - b)^2*b*Sinh[2*(c + d*x)]/(a*(2*a - b + b*Cosh[2*(c + d*x)])))/(4*b^3*d)

Maple [B] time = 0.073, size = 1659, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cosh(dx+c)^6/(a+b*\sinh(dx+c)^2)^2,x)$

[Out]
$$\begin{aligned} & -2/d/b^3*a^2/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-7/2/d/b^2*a/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+7/2/d/b^2*a/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/d/b^2/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*a*\tanh(1/2*d*x+1/2*c)^3+1/d/b^2/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*a*\tanh(1/2*d*x+1/2*c)-1/2/d/a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})*b-2/d/b^2*a^2/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-2/d/b^2*a^2/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+7/2/d/b*a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+7/2/d/b*a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-1/2/d/a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})*b-2/d/b^3*\ln(\tanh(1/2*d*x+1/2*c)+1)*a+2/d/b^3*\ln(\tanh(1/2*d*x+1/2*c)-1)*a+1/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/a*\tanh(1/2*d*x+1/2*c)^3+1/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/a*\tanh(1/2*d*x+1/2*c)-1/d/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-1/d/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/2/d/a/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-1/2/d/a/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-2/d/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*\tanh(1/2*d*x+1/2*c)^3-2/d/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*\tanh(1/2*d*x+1/2*c)+1/d/b/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-1/d/b/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^2+5/2/d/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-5/2/d/b^2*\ln(\tanh(1/2*d*x+1/2*c)-1)+2/d/b^3*a^2/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)+1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)-1) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.90558, size = 8500, normalized size = 53.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/8*(a*b^2*\cosh(d*x + c)^8 + 8*a*b^2*\cosh(d*x + c)*\sinh(d*x + c)^7 + a*b^2 \\ & * \sinh(d*x + c)^8 + 2*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x \\ & + c)^6 + 2*(14*a*b^2*\cosh(d*x + c)^2 + 2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a* \\ & b^2)*d*x)*\sinh(d*x + c)^6 + 4*(14*a*b^2*\cosh(d*x + c)^3 + 3*(2*a^2*b - a*b^ \\ & 2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^5 - 8*(2*a^3 - \\ & 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*\cosh(d*x + c)^4 \\ & + 2*(35*a*b^2*\cosh(d*x + c)^4 - 8*a^3 + 20*a^2*b - 16*a*b^2 + 4*b^3 - 4*(8 \\ & *a^3 - 14*a^2*b + 5*a*b^2)*d*x + 15*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2 \\ &)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*a*b^2*\cosh(d*x + c)^5 + 5*(2 \\ & *a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c)^3 - 4*(2*a^3 - 5* \\ & a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*\cosh(d*x + c))*\si \\ & nh(d*x + c)^3 - a*b^2 - 2*(6*a^2*b - 9*a*b^2 + 4*b^3 + 2*(4*a^2*b - 5*a*b^2 \\ &)*d*x)*\cosh(d*x + c)^2 + 2*(14*a*b^2*\cosh(d*x + c)^6 + 15*(2*a^2*b - a*b^2 \\ & - 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c)^4 - 6*a^2*b + 9*a*b^2 - 4*b^3 - \\ & 2*(4*a^2*b - 5*a*b^2)*d*x - 24*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - \\ & 14*a^2*b + 5*a*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 2*((4*a^2*b - 3 \\ & *a*b^2 - b^3)*\cosh(d*x + c)^6 + 6*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)*s \\ & inh(d*x + c)^5 + (4*a^2*b - 3*a*b^2 - b^3)*\sinh(d*x + c)^6 + 2*(8*a^3 - 10* \\ & a^2*b + a*b^2 + b^3)*\cosh(d*x + c)^4 + (16*a^3 - 20*a^2*b + 2*a*b^2 + 2*b^3 \\ & + 15*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(5*(4* \\ & a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^3 + 2*(8*a^3 - 10*a^2*b + a*b^2 + b^3) \\ & *\cosh(d*x + c))*\sinh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^2 \\ & + (15*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^4 + 4*a^2*b - 3*a*b^2 - b^3 \\ & + 12*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2* \end{aligned}$$

$$\begin{aligned}
& (3*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^5 + 4*(8*a^3 - 10*a^2*b + a*b^2 \\
& + b^3)*\cosh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c))*\sinh(d*x \\
& + c))*\sqrt{(a - b)/a}*\log((b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d \\
& *x + c)^3 + b^2*\sinh(d*x + c)^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*b^ \\
& 2*\cosh(d*x + c)^2 + 2*a*b - b^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4* \\
& (b^2*\cosh(d*x + c)^3 + (2*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(a*b* \\
& \cosh(d*x + c)^2 + 2*a*b*\cosh(d*x + c)*\sinh(d*x + c) + a*b*\sinh(d*x + c)^2 + \\
& 2*a^2 - a*b)*\sqrt{(a - b)/a))/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(\\
& d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(\\
& d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\co \\
& sh(d*x + c))*\sinh(d*x + c) + b)) + 4*(2*a*b^2*\cosh(d*x + c)^7 + 3*(2*a^2*b \\
& - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c)^5 - 8*(2*a^3 - 5*a^2*b + \\
& 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*\cosh(d*x + c)^3 - (6*a^2 \\
& *b - 9*a*b^2 + 4*b^3 + 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + \\
& c))/(a*b^4*d*\cosh(d*x + c)^6 + 6*a*b^4*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + a \\
& *b^4*d*\sinh(d*x + c)^6 + a*b^4*d*\cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d* \\
& \cosh(d*x + c)^4 + (15*a*b^4*d*\cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d)*\si \\
& nh(d*x + c)^4 + 4*(5*a*b^4*d*\cosh(d*x + c)^3 + 2*(2*a^2*b^3 - a*b^4)*d*\cosh \\
& (d*x + c))*\sinh(d*x + c)^3 + (15*a*b^4*d*\cosh(d*x + c)^4 + a*b^4*d + 12*(2* \\
& a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(3*a*b^4*d*\cosh(d*x \\
& + c)^5 + a*b^4*d*\cosh(d*x + c) + 4*(2*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^3)* \\
& \sinh(d*x + c)), 1/8*(a*b^2*\cosh(d*x + c)^8 + 8*a*b^2*\cosh(d*x + c)*\sinh(d*x \\
& + c)^7 + a*b^2*\sinh(d*x + c)^8 + 2*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2 \\
&)*d*x)*\cosh(d*x + c)^6 + 2*(14*a*b^2*\cosh(d*x + c)^2 + 2*a^2*b - a*b^2 - 2* \\
& (4*a^2*b - 5*a*b^2)*d*x)*\sinh(d*x + c)^6 + 4*(14*a*b^2*\cosh(d*x + c)^3 + 3* \\
& (2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c)^ \\
& 5 - 8*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)* \\
& \cosh(d*x + c)^4 + 2*(35*a*b^2*\cosh(d*x + c)^4 - 8*a^3 + 20*a^2*b - 16*a*b^2 \\
& + 4*b^3 - 4*(8*a^3 - 14*a^2*b + 5*a*b^2)*d*x + 15*(2*a^2*b - a*b^2 - 2*(4* \\
& a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*a*b^2*\cosh(d \\
& x + c)^5 + 5*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c)^3 \\
& - 4*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*\co \\
& sh(d*x + c))*\sinh(d*x + c)^3 - a*b^2 - 2*(6*a^2*b - 9*a*b^2 + 4*b^3 + 2*(4* \\
& a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c)^2 + 2*(14*a*b^2*\cosh(d*x + c)^6 + 15*(2 \\
& *a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c)^4 - 6*a^2*b + 9*a \\
& *b^2 - 4*b^3 - 2*(4*a^2*b - 5*a*b^2)*d*x - 24*(2*a^3 - 5*a^2*b + 4*a*b^2 - \\
& b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - \\
& 4*((4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^6 + 6*(4*a^2*b - 3*a*b^2 - b^3)* \\
& \cosh(d*x + c)*\sinh(d*x + c)^5 + (4*a^2*b - 3*a*b^2 - b^3)*\sinh(d*x + c)^6 + \\
& 2*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*\cosh(d*x + c)^4 + (16*a^3 - 20*a^2*b + \\
& 2*a*b^2 + 2*b^3 + 15*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^4 + 4*(5*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^3 + 2*(8*a^3 - 10*a^2*b \\
& + a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)* \\
& \cosh(d*x + c)^2 + (15*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^4 + 4*a^2*b - \\
& 3*a*b^2 - b^3 + 12*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(
\end{aligned}$$

$$d*x + c)^2 + 2*(3*(4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c)^5 + 4*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*\cosh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-(a - b)/a}*\arctan(-1/2*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-(a - b)/a}/(a - b)) + 4*(2*a*b^2*\cosh(d*x + c)^7 + 3*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c)^5 - 8*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*\cosh(d*x + c)^3 - (6*a^2*b - 9*a*b^2 + 4*b^3 + 2*(4*a^2*b - 5*a*b^2)*d*x)*\cosh(d*x + c))*\sinh(d*x + c))/(a*b^4*d*\cosh(d*x + c)^6 + 6*a*b^4*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + a*b^4*d*\sinh(d*x + c)^6 + a*b^4*d*\cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^4 + (15*a*b^4*d*\cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d)*\sinh(d*x + c)^4 + 4*(5*a*b^4*d*\cosh(d*x + c)^3 + 2*(2*a^2*b^3 - a*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (15*a*b^4*d*\cosh(d*x + c)^4 + a*b^4*d + 12*(2*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(3*a*b^4*d*\cosh(d*x + c)^5 + a*b^4*d*\cosh(d*x + c) + 4*(2*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^3)*\sinh(d*x + c))]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.17041, size = 421, normalized size = 2.66

$$-\frac{(dx + c)(4a - 5b)}{2b^3d} + \frac{e^{(2dx+2c)}}{8b^2d} + \frac{(4a^3 - 7a^2b + 2ab^2 + b^3)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{2\sqrt{-a^2+ab}b^3d} + \frac{8a^2be^{(6dx+6c)} - 10ab^2e^{(6dx+6c)}}{2\sqrt{-a^2+ab}b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-\frac{1}{2}*(d*x + c)*(4*a - 5*b)/(b^3*d) + \frac{1}{8}*e^{(2*d*x + 2*c)}/(b^2*d) + \frac{1}{2}*(4*a^3 - 7*a^2*b + 2*a*b^2 + b^3)*\arctan(1/2*(b*e^{(2*d*x + 2*c)} + 2*a - b)/\sqrt{-(a^2 + a*b)})/(\sqrt{-(a^2 + a*b)}*a*b^3*d) + \frac{1}{24}*(8*a^2*b*e^{(6*d*x + 6*c)} - 10*a*b^2*e^{(6*d*x + 6*c)} - 16*a^3*e^{(4*d*x + 4*c)} + 64*a^2*b*e^{(4*d*x + 4*c)}$

$$\begin{aligned} &) - 79*a*b^2*e^{(4*d*x + 4*c)} + 24*b^3*e^{(4*d*x + 4*c)} - 28*a^2*b*e^{(2*d*x + 2*c)} \\ & + 44*a*b^2*e^{(2*d*x + 2*c)} - 24*b^3*e^{(2*d*x + 2*c)} - 3*a*b^2)/((b*e^{(6*d*x + 6*c)} \\ & + 4*a*e^{(4*d*x + 4*c)} - 2*b*e^{(4*d*x + 4*c)} + b*e^{(2*d*x + 2*c)})*a*b^3*d) \end{aligned}$$

$$3.330 \quad \int \frac{\cosh^5(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=104

$$-\frac{(3a^2 - 2ab - b^2) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{(a-b)^2 \sinh(c+dx)}{2ab^2d(a+b \sinh^2(c+dx))} + \frac{\sinh(c+dx)}{b^2d}$$

[Out] -((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(5/2)*d) + Sinh[c + d*x]/(b^2*d) + ((a - b)^2*Sinh[c + d*x])/(2*a*b^2*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.138491, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 390, 385, 205}

$$-\frac{(3a^2 - 2ab - b^2) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{(a-b)^2 \sinh(c+dx)}{2ab^2d(a+b \sinh^2(c+dx))} + \frac{\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] -((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(5/2)*d) + Sinh[c + d*x]/(b^2*d) + ((a - b)^2*Sinh[c + d*x])/(2*a*b^2*d*(a + b*Sinh[c + d*x]^2))

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,

0] && GeQ[p, -q]

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^5(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a^2-b^2+2(a-b)bx^2}{b^2(a+bx^2)^2}\right) dx, x, \sinh(c+dx)\right)}{d} \\
 &= \frac{\sinh(c+dx)}{b^2d} - \frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{(a+bx^2)^2} dx, x, \sinh(c+dx)\right)}{b^2d} \\
 &= \frac{\sinh(c+dx)}{b^2d} + \frac{(a-b)^2 \sinh(c+dx)}{2ab^2d(a+b\sinh^2(c+dx))} - \frac{((a-b)(3a+b)) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c+dx)\right)}{2ab^2d} \\
 &= -\frac{(a-b)(3a+b) \tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\sinh(c+dx)}{b^2d} + \frac{(a-b)^2 \sinh(c+dx)}{2ab^2d(a+b\sinh^2(c+dx))}
 \end{aligned}$$

Mathematica [A] time = 0.28311, size = 106, normalized size = 1.02

$$\frac{-\frac{(-3a^2+2ab+b^2) \tan^{-1}\left(\frac{\sqrt{a}\text{csch}(c+dx)}{\sqrt{b}}\right)}{a^{3/2}} + \frac{2\sqrt{b}(a-b)^2 \sinh(c+dx)}{a(2a+b\cosh(2(c+dx))-b)} + 2\sqrt{b} \sinh(c+dx)}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-((( -3*a^2 + 2*a*b + b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(3/2)
+ 2*Sqrt[b]*Sinh[c + d*x] + (2*(a - b)^2*Sqrt[b]*Sinh[c + d*x]))/(a*(2*a -
b + b*Cosh[2*(c + d*x)])))/(2*b^(5/2)*d)
```

Maple [B] time = 0.065, size = 1403, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/b^2/(tanh(1/2*d*x+1/2*c)+1)-1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)^3+2/d/b
/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2
*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/b^2/(tanh(
1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a
*tanh(1/2*d*x+1/2*c)-2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c
)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x
+1/2*c)+3/2/d/b^2*a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d/
b^2*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-5/2/d/b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))+3/2/d/b^2*a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
))-3/2/d/b^2*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/2/d/b*a/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))-1/d/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1
/2/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/a/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2))+1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
```

$$\frac{\operatorname{arctanh}(a \tanh(1/2 dx + 1/2 c)) / ((2(-b(a-b))^{1/2} + a - 2b) a)^{1/2} * b + 1/2 / d / a / ((2(-b(a-b))^{1/2} - a + 2b) a)^{1/2} * \operatorname{arctan}(a \tanh(1/2 dx + 1/2 c)) / ((2(-b(a-b))^{1/2} - a + 2b) a)^{1/2} + 1/2 / d / a / (-b(a-b))^{1/2} / ((2(-b(a-b))^{1/2} - a + 2b) a)^{1/2} * \operatorname{arctan}(a \tanh(1/2 dx + 1/2 c)) / ((2(-b(a-b))^{1/2} - a + 2b) a)^{1/2} * b - 1/d/b^2 / (\tanh(1/2 dx + 1/2 c) - 1)}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{abe^{(6dx+6c)} - ab + (6a^2e^{(4c)} - 7abe^{(4c)} + 2b^2e^{(4c)})e^{(4dx)} - (6a^2e^{(2c)} - 7abe^{(2c)} + 2b^2e^{(2c)})e^{(2dx)}}{2(ab^3de^{(5dx+5c)} + ab^3de^{(dx+c)} + 2(2a^2b^2de^{(3c)} - ab^3de^{(3c)})e^{(3dx)})} - \frac{1}{32} \int \frac{32((3a^2e^{(3c)} - ab^3e^{(3c)} - b^2e^{(3c)})e^{(3dx)} + (3a^2e^{(3c)} - 2ab^2e^{(3c)} - b^3e^{(3c)})e^{(3dx)} + (3a^2e^{(3c)} - 2ab^2e^{(3c)} - b^3e^{(3c)})e^{(3dx)})}{ab^3e^{(3c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^5/(a+b*sinh(dx+c)^2)^2,x, algorithm="maxima")

[Out] $\frac{1}{2} * (a * b * e^{(6 * dx + 6 * c)} - a * b + (6 * a^2 * e^{(4 * c)} - 7 * a * b * e^{(4 * c)} + 2 * b^2 * e^{(4 * c)}) * e^{(4 * dx)} - (6 * a^2 * e^{(2 * c)} - 7 * a * b * e^{(2 * c)} + 2 * b^2 * e^{(2 * c)}) * e^{(2 * dx)}) / (a * b^3 * d * e^{(5 * dx + 5 * c)} + a * b^3 * d * e^{(dx + c)} + 2 * (2 * a^2 * b^2 * d * e^{(3 * c)} - a * b^3 * d * e^{(3 * c)}) * e^{(3 * dx)}) - 1/32 * \operatorname{integrate}(32 * ((3 * a^2 * e^{(3 * c)} - 2 * a * b * e^{(3 * c)} - b^2 * e^{(3 * c)}) * e^{(3 * dx)} + (3 * a^2 * e^{(3 * c)} - 2 * a * b * e^{(3 * c)} - b^2 * e^{(3 * c)}) * e^{(3 * dx)} + (3 * a^2 * e^{(3 * c)} - 2 * a * b * e^{(3 * c)} - b^2 * e^{(3 * c)}) * e^{(3 * dx)}), x)$

Fricas [B] time = 1.82207, size = 6456, normalized size = 62.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^5/(a+b*sinh(dx+c)^2)^2,x, algorithm="fricas")

[Out] $[1/4 * (2 * a^2 * b^2 * \cosh(dx + c)^6 + 12 * a^2 * b^2 * \cosh(dx + c) * \sinh(dx + c)^5 + 2 * a^2 * b^2 * \sinh(dx + c)^6 + 2 * (6 * a^3 * b - 7 * a^2 * b^2 + 2 * a * b^3) * \cosh(dx + c)^4 + 2 * (15 * a^2 * b^2 * \cosh(dx + c)^2 + 6 * a^3 * b - 7 * a^2 * b^2 + 2 * a * b^3) * \sinh(dx + c)^4 - 2 * a^2 * b^2 + 8 * (5 * a^2 * b^2 * \cosh(dx + c)^3 + (6 * a^3 * b - 7 * a^2 * b^2 + 2 * a * b^3) * \cosh(dx + c)) * \sinh(dx + c)^3 - 2 * (6 * a^3 * b - 7 * a^2 * b^2 + 2 * a * b^3) * \cosh(dx + c)^2 + 2 * (15 * a^2 * b^2 * \cosh(dx + c)^4 - 6 * a^3 * b + 7 * a^2 * b^2 - 2 * a * b^3 + 6 * (6 * a^3 * b - 7 * a^2 * b^2 + 2 * a * b^3) * \cosh(dx + c)^2) * \sinh(dx + c)^2 + ((3 * a^2 * b - 2 * a * b^2 - b^3) * \cosh(dx + c)^5 + 5 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \sinh(dx + c)^5 + 5 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \cosh(dx + c)^4 + 10 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \sinh(dx + c)^4 + 5 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \cosh(dx + c)^3 + 10 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \sinh(dx + c)^3 + 5 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \cosh(dx + c)^2 + 10 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \sinh(dx + c)^2 + 5 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \cosh(dx + c) + 10 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \sinh(dx + c) + 5 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \cosh(dx + c) + 5 * (3 * a^2 * b - 2 * a * b^2 - b^3) * \sinh(dx + c)]$

$$\begin{aligned}
&^3) \cosh(dx + c) \sinh(dx + c)^4 + (3a^2b - 2ab^2 - b^3) \sinh(dx + c) \\
&^5 + 2(6a^3 - 7a^2b + b^3) \cosh(dx + c)^3 + 2(6a^3 - 7a^2b + b^3 + \\
&5(3a^2b - 2ab^2 - b^3) \cosh(dx + c)^2) \sinh(dx + c)^3 + 2(5(3a^2 \\
&*b - 2ab^2 - b^3) \cosh(dx + c)^3 + 3(6a^3 - 7a^2b + b^3) \cosh(dx + \\
&c)) \sinh(dx + c)^2 + (3a^2b - 2ab^2 - b^3) \cosh(dx + c) + (5(3a^2b \\
&- 2ab^2 - b^3) \cosh(dx + c)^4 + 3a^2b - 2ab^2 - b^3 + 6(6a^3 - 7 \\
&a^2b + b^3) \cosh(dx + c)^2) \sinh(dx + c) \sqrt{-ab} \log((b \cosh(dx + c) \\
&)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 - 2(2a + b) \c \\
&osh(dx + c)^2 + 2(3b \cosh(dx + c)^2 - 2a - b) \sinh(dx + c)^2 + 4(b \c \\
&osh(dx + c)^3 - (2a + b) \cosh(dx + c)) \sinh(dx + c) - 4(\cosh(dx + c)^ \\
&3 + 3 \cosh(dx + c) \sinh(dx + c)^2 + \sinh(dx + c)^3 + (3 \cosh(dx + c)^2 \\
&- 1) \sinh(dx + c) - \cosh(dx + c)) \sqrt{-ab} + b) / (b \cosh(dx + c)^4 + 4 \\
&b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 + 2(2a - b) \cosh(dx \\
&+ c)^2 + 2(3b \cosh(dx + c)^2 + 2a - b) \sinh(dx + c)^2 + 4(b \cosh(dx \\
&+ c)^3 + (2a - b) \cosh(dx + c)) \sinh(dx + c) + b) + 4(3a^2b^2 \cosh(d \\
&*x + c)^5 + 2(6a^3b - 7a^2b^2 + 2ab^3) \cosh(dx + c)^3 - (6a^3b - \\
&7a^2b^2 + 2ab^3) \cosh(dx + c)) \sinh(dx + c) / (a^2b^4 d \cosh(dx + c) \\
&^5 + 5a^2b^4 d \cosh(dx + c) \sinh(dx + c)^4 + a^2b^4 d \sinh(dx + c)^5 \\
&+ a^2b^4 d \cosh(dx + c) + 2(2a^3b^3 - a^2b^4) d \cosh(dx + c)^3 + 2(\\
&5a^2b^4 d \cosh(dx + c)^2 + (2a^3b^3 - a^2b^4) d) \sinh(dx + c)^3 + 2(\\
&5a^2b^4 d \cosh(dx + c)^3 + 3(2a^3b^3 - a^2b^4) d \cosh(dx + c)) \sin \\
&h(dx + c)^2 + (5a^2b^4 d \cosh(dx + c)^4 + a^2b^4 d + 6(2a^3b^3 - a^ \\
&2b^4) d \cosh(dx + c)^2) \sinh(dx + c)), 1/2(a^2b^2 \cosh(dx + c)^6 + 6 \\
&a^2b^2 \cosh(dx + c) \sinh(dx + c)^5 + a^2b^2 \sinh(dx + c)^6 + (6a^3b \\
&- 7a^2b^2 + 2ab^3) \cosh(dx + c)^4 + (15a^2b^2 \cosh(dx + c)^2 + 6a^ \\
&3b - 7a^2b^2 + 2ab^3) \sinh(dx + c)^4 - a^2b^2 + 4(5a^2b^2 \cosh(dx \\
&x + c)^3 + (6a^3b - 7a^2b^2 + 2ab^3) \cosh(dx + c)) \sinh(dx + c)^3 - \\
&(6a^3b - 7a^2b^2 + 2ab^3) \cosh(dx + c)^2 + (15a^2b^2 \cosh(dx + c) \\
&)^4 - 6a^3b + 7a^2b^2 - 2ab^3 + 6(6a^3b - 7a^2b^2 + 2ab^3) \cos \\
&h(dx + c)^2) \sinh(dx + c)^2 - ((3a^2b - 2ab^2 - b^3) \cosh(dx + c)^5 \\
&+ 5(3a^2b - 2ab^2 - b^3) \cosh(dx + c) \sinh(dx + c)^4 + (3a^2b - 2 \\
&a^2b^2 - b^3) \sinh(dx + c)^5 + 2(6a^3 - 7a^2b + b^3) \cosh(dx + c)^3 + \\
&2(6a^3 - 7a^2b + b^3 + 5(3a^2b - 2ab^2 - b^3) \cosh(dx + c)^2) \sin \\
&h(dx + c)^3 + 2(5(3a^2b - 2ab^2 - b^3) \cosh(dx + c)^3 + 3(6a^3 - \\
&7a^2b + b^3) \cosh(dx + c)) \sinh(dx + c)^2 + (3a^2b - 2ab^2 - b^3) \c \\
&osh(dx + c) + (5(3a^2b - 2ab^2 - b^3) \cosh(dx + c)^4 + 3a^2b - 2a \\
&>*b^2 - b^3 + 6(6a^3 - 7a^2b + b^3) \cosh(dx + c)^2) \sinh(dx + c) \sqrt{ \\
&(ab) \arctan(1/2 \sqrt{ab} (\cosh(dx + c) + \sinh(dx + c)) / a) - ((3a^2b - \\
&2ab^2 - b^3) \cosh(dx + c)^5 + 5(3a^2b - 2ab^2 - b^3) \cosh(dx + c) \\
&* \sinh(dx + c)^4 + (3a^2b - 2ab^2 - b^3) \sinh(dx + c)^5 + 2(6a^3 - 7 \\
&a^2b + b^3) \cosh(dx + c)^3 + 2(6a^3 - 7a^2b + b^3 + 5(3a^2b - 2a \\
&>*b^2 - b^3) \cosh(dx + c)^2) \sinh(dx + c)^3 + 2(5(3a^2b - 2ab^2 - b^ \\
&3) \cosh(dx + c)^3 + 3(6a^3 - 7a^2b + b^3) \cosh(dx + c)) \sinh(dx + c) \\
&^2 + (3a^2b - 2ab^2 - b^3) \cosh(dx + c) + (5(3a^2b - 2ab^2 - b^3) \\
&* \cosh(dx + c)^4 + 3a^2b - 2ab^2 - b^3 + 6(6a^3 - 7a^2b + b^3) \cosh
\end{aligned}$$

$$\begin{aligned} & (d*x + c)^2 * \sinh(d*x + c) * \sqrt{a*b} * \arctan\left(\frac{1}{2} * (b * \cosh(d*x + c)^3 + 3*b*c\right. \\ & \left. \cosh(d*x + c) * \sinh(d*x + c)^2 + b * \sinh(d*x + c)^3 + (4*a - b) * \cosh(d*x + c)\right. \\ & \left. + (3*b * \cosh(d*x + c)^2 + 4*a - b) * \sinh(d*x + c)) * \sqrt{a*b} / (a*b)\right) + 2 * (3*a^{\wedge} \\ & 2 * b^2 * \cosh(d*x + c)^5 + 2 * (6*a^3 * b - 7*a^2 * b^2 + 2*a * b^3) * \cosh(d*x + c)^3 - \\ & (6*a^3 * b - 7*a^2 * b^2 + 2*a * b^3) * \cosh(d*x + c)) * \sinh(d*x + c) / (a^2 * b^4 * d * c \\ & \cosh(d*x + c)^5 + 5*a^2 * b^4 * d * \cosh(d*x + c) * \sinh(d*x + c)^4 + a^2 * b^4 * d * \sinh \\ & (d*x + c)^5 + a^2 * b^4 * d * \cosh(d*x + c) + 2 * (2*a^3 * b^3 - a^2 * b^4) * d * \cosh(d*x \\ & + c)^3 + 2 * (5*a^2 * b^4 * d * \cosh(d*x + c)^2 + (2*a^3 * b^3 - a^2 * b^4) * d) * \sinh(d*x \\ & + c)^3 + 2 * (5*a^2 * b^4 * d * \cosh(d*x + c)^3 + 3 * (2*a^3 * b^3 - a^2 * b^4) * d * \cosh(d \\ & * x + c)) * \sinh(d*x + c)^2 + (5*a^2 * b^4 * d * \cosh(d*x + c)^4 + a^2 * b^4 * d + 6 * (2 * \\ & a^3 * b^3 - a^2 * b^4) * d * \cosh(d*x + c)^2) * \sinh(d*x + c)) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.331 \quad \int \frac{\cosh^4(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$$

Optimal. Leaf size=100

$$-\frac{\sqrt{a-b}(2a+b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b) \tanh(c+dx)}{2abd(a-(a-b) \tanh^2(c+dx))} + \frac{x}{b^2}$$

[Out] x/b^2 - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) - ((a - b)*Tanh[c + d*x])/(2*a*b*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.1384, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3191, 414, 522, 206, 208}

$$-\frac{\sqrt{a-b}(2a+b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b) \tanh(c+dx)}{2abd(a-(a-b) \tanh^2(c+dx))} + \frac{x}{b^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] x/b^2 - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^2*d) - ((a - b)*Tanh[c + d*x])/(2*a*b*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 414

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +

```
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^4(c + dx)}{(a + b \sinh^2(c + dx))^2} dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)^2} dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{(a-b) \tanh(c + dx)}{2abd(a - (a-b) \tanh^2(c + dx))} - \frac{\text{Subst}\left(\int \frac{-a-b+(-a+b)x^2}{(1-x^2)(a+(-a+b)x^2)} dx, x, \tanh(c + dx)\right)}{2abd}$$

$$= -\frac{(a-b) \tanh(c + dx)}{2abd(a - (a-b) \tanh^2(c + dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \tanh(c + dx)\right)}{b^2d} - \frac{((a-b)(2a+b))}{2abd(a - (a-b) \tanh^2(c + dx))}$$

$$= \frac{x}{b^2} - \frac{\sqrt{a-b}(2a+b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b) \tanh(c + dx)}{2abd(a - (a-b) \tanh^2(c + dx))}$$

Mathematica [A] time = 0.590935, size = 108, normalized size = 1.08

$$\frac{\frac{(2a^2-ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}} + \frac{b(b-a)\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)} + 2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] $(2*(c + d*x) - ((2*a^2 - a*b - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^{3/2}*Sqrt[a - b]) + (b*(-a + b)*Sinh[2*(c + d*x)]/(a*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*b^2*d)$

Maple [B] time = 0.062, size = 1116, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)

[Out] $1/d/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/d/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*\tanh(1/2*d*x+1/2*c)^3+1/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/a*\tanh(1/2*d*x+1/2*c)^3-1/d/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*\tanh(1/2*d*x+1/2*c)+1/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/a*\tanh(1/2*d*x+1/2*c)-1/d/b^2*a/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+1/d/b*a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+1/d/b^2*a/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+1/d/b*a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+1/2/d/b/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/2/d/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/2/d/b/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-1/2/d/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})$

$$\begin{aligned} & /2)) + 1/2/d/a/((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2*d*x + 1/2*c)) / ((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2} - 1/2/d/a/(-b*(a-b))^{1/2} / ((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2} * \operatorname{arctanh}(a * \tanh(1/2*d*x + 1/2*c)) / ((2*(-b*(a-b))^{1/2} + a - 2*b)*a)^{1/2} * b - 1/2/d/a/((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2*d*x + 1/2*c)) / ((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2} - 1/2/d/a/(-b*(a-b))^{1/2} / ((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2} * \operatorname{arctan}(a * \tanh(1/2*d*x + 1/2*c)) / ((2*(-b*(a-b))^{1/2} - a + 2*b)*a)^{1/2} * b - 1/d/b^2 * \ln(\tanh(1/2*d*x + 1/2*c) - 1) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.75171, size = 3749, normalized size = 37.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 \\ & + 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(2*a^2 - a*b)*d*x + 2*a^2 - \\ & 3*a*b + b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - a* \\ & b)*d*x + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x + c) \\ &)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d* \\ & x + c)^4 + 2*(4*a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + \\ & c)^2 + 4*a^2 - b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d \\ & *x + c)^3 + (4*a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a - b)/a)*log \\ & ((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x \\ & + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b \\ & - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2 \\ & *a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c \\ & osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a - b \end{aligned}$$

```

)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sin
h(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c
) + b)) + 4*a*b - 4*b^2 + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (2*(2*a^2 - a*b)*d
*x + 2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x +
c)^4 + 4*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^3*d*sinh(d*x + c)^4 +
a*b^3*d + 2*(2*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*a*b^3*d*cosh(d*x
+ c)^2 + (2*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + 4*(a*b^3*d*cosh(d*x + c)^
3 + (2*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*d*x*cos
h(d*x + c)^4 + 8*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*b*d*x*sinh(d*x
+ c)^4 + 2*a*b*d*x + 2*(2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a*b + b^2)*cosh(d*
x + c)^2 + 2*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a
*b + b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2
)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(4*a^2
- b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 4*a^2 - b^2)*
sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (4*a^2 -
b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*s
qrt(-(a - b)/a)/(a - b)) + 2*a*b - 2*b^2 + 4*(2*a*b*d*x*cosh(d*x + c)^3 + (
2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(a
*b^3*d*cosh(d*x + c)^4 + 4*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^3*d*
sinh(d*x + c)^4 + a*b^3*d + 2*(2*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*
a*b^3*d*cosh(d*x + c)^2 + (2*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + 4*(a*b^3
*d*cosh(d*x + c)^3 + (2*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.19301, size = 243, normalized size = 2.43

$$\frac{dx + c}{b^2 d} - \frac{(2a^2 - ab - b^2) \arctan\left(\frac{be^{2dx+2c} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{2\sqrt{-a^2 + ab} b^2 d} + \frac{2a^2 e^{(2dx+2c)} - 3abe^{(2dx+2c)} + b^2 e^{(2dx+2c)} + ab - b^2}{(be^{(4dx+4c)} + 4ae^{(2dx+2c)} - 2be^{(2dx+2c)} + b)ab^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")`

[Out] $(d*x + c)/(b^2*d) - 1/2*(2*a^2 - a*b - b^2)*\arctan(1/2*(b*e^{(2*d*x + 2*c)} + 2*a - b)/\sqrt{-a^2 + a*b})/(\sqrt{-a^2 + a*b})*a*b^2*d + (2*a^2*e^{(2*d*x + 2*c)} - 3*a*b*e^{(2*d*x + 2*c)} + b^2*e^{(2*d*x + 2*c)} + a*b - b^2)/((b*e^{(4*d*x + 4*c)} + 4*a*e^{(2*d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + b)*a*b^2*d)$

$$3.332 \quad \int \frac{\cosh^3(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$$

Optimal. Leaf size=77

$$\frac{(a+b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} - \frac{(a-b) \sinh(c+dx)}{2abd(a+b \sinh^2(c+dx))}$$

[Out] ((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - ((a - b)*Sinh[c + d*x])/(2*a*b*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.0781891, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3190, 385, 205}

$$\frac{(a+b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} - \frac{(a-b) \sinh(c+dx)}{2abd(a+b \sinh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - ((a - b)*Sinh[c + d*x])/(2*a*b*d*(a + b*Sinh[c + d*x]^2))

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

p, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= -\frac{(a-b)\sinh(c+dx)}{2abd(a+b\sinh^2(c+dx))} + \frac{(a+b)\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c+dx)\right)}{2abd} \\ &= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} - \frac{(a-b)\sinh(c+dx)}{2abd(a+b\sinh^2(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.327024, size = 75, normalized size = 0.97

$$\frac{\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} - \frac{(a-b)\sinh(c+dx)}{2ab(a+b\sinh^2(c+dx))}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) - ((a - b)*Sinh[c + d*x])/(2*a*b*(a + b*Sinh[c + d*x]^2)))/d

Maple [B] time = 0.057, size = 808, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cosh(dx+c)^3/(a+b*\sinh(dx+c)^2)^2,x)$

[Out] $1/d/b/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}*\tanh(1/2*d*x+1/2*c)^{3-1/d}/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}/a*\tanh(1/2*d*x+1/2*c)^{3-1/d}/b/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}})*\tanh(1/2*d*x+1/2*c)+1/d/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}/a*\tanh(1/2*d*x+1/2*c)-1/2/d/b*a/(-b*(a-b))^{(1/2)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-1/2/d/b/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-1/2/d/b*a/(-b*(a-b))^{(1/2)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/2/d/b/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-1/2/d/a/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+1/2/d/a/(-b*(a-b))^{(1/2)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})*b+1/2/d/a/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/2/d/a/(-b*(a-b))^{(1/2)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})*b$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{(ae^{3c} - be^{3c})e^{3dx} - (ae^c - be^c)e^{dx}}{ab^2de^{4dx+4c} + ab^2d + 2(2a^2bde^{2c} - ab^2de^{2c})e^{2dx}} + \frac{1}{8} \int \frac{8((ae^{3c} + be^{3c})e^{3dx} + (ae^c + be^c)e^{dx})}{ab^2e^{4dx+4c} + ab^2 + 2(2a^2be^{2c} - ab^2e^{2c})e^{2dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cosh(dx+c)^3/(a+b*\sinh(dx+c)^2)^2,x, \text{algorithm}="maxima")$

[Out] $-((a*e^{(3*c)} - b*e^{(3*c)})*e^{(3*d*x)} - (a*e^c - b*e^c)*e^{(d*x)})/(a*b^2*d*e^{(4*d*x + 4*c)} + a*b^2*d + 2*(2*a^2*b*d*e^{(2*c)} - a*b^2*d*e^{(2*c)})*e^{(2*d*x)}) + 1/8*\text{integrate}(8*((a*e^{(3*c)} + b*e^{(3*c)})*e^{(3*d*x)} + (a*e^c + b*e^c)*e^{(d*x)})/(a*b^2*e^{(4*d*x + 4*c)} + a*b^2 + 2*(2*a^2*b*e^{(2*c)} - a*b^2*e^{(2*c)})*e^{(2*d*x)}), x)$

Fricas [B] time = 1.65626, size = 3937, normalized size = 51.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*(4*(a^2*b - a*b^2)*\cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*\cosh(d*x + c) \\ & * \sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*\sinh(d*x + c)^3 + ((a*b + b^2)*\cosh(d*x + c)^4 \\ & + 4*(a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a*b + b^2)*\sinh(d*x + c)^4 \\ & + 2*(2*a^2 + a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*\cosh(d*x + c)^2 \\ & + 2*a^2 + a*b - b^2)*\sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)*\cosh(d*x + c)^3 \\ & + (2*a^2 + a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a*b}*\log((b*\cosh(d*x + c)^4 \\ & + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a + b)*\cosh(d*x + c)^2 \\ & + 2*(3*b*\cosh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a + b)*\cosh(d*x + c))*\sinh(d*x + c) \\ & - 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c) - \cosh(d*x + c))*\sqrt{-a*b} + b) \\ & / (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 \\ & + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b) \\ &) - 4*(a^2*b - a*b^2)*\cosh(d*x + c) - 4*(a^2*b - a*b^2 - 3*(a^2*b - a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c) \\ &) / (a^2*b^3*d*\cosh(d*x + c)^4 + 4*a^2*b^3*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*b^3*d*\sinh(d*x + c)^4 \\ & + a^2*b^3*d + 2*(2*a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)^2 + 2*(3*a^2*b^3*d*\cosh(d*x + c)^2 + (2*a^3*b^2 - a^2*b^3)*d)*\sinh(d*x + c)^2 \\ & + 4*(a^2*b^3*d*\cosh(d*x + c)^3 + (2*a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/2*(2*(a^2*b - a*b^2)*\cosh(d*x + c)^3 \\ & + 6*(a^2*b - a*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*(a^2*b - a*b^2)*\sinh(d*x + c)^3 - ((a*b + b^2)*\cosh(d*x + c)^4 \\ & + 4*(a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a*b + b^2)*\sinh(d*x + c)^4 + 2*(2*a^2 + a*b - b^2)*\cosh(d*x + c)^2 \\ & + 2*(3*(a*b + b^2)*\cosh(d*x + c)^2 + 2*a^2 + a*b - b^2)*\sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)*\cosh(d*x + c)^3 + (2*a^2 + a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a*b}*\arctan(1/2*\sqrt{a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) - ((a*b + b^2)*\cosh(d*x + c)^4 + 4*(a*b + b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (a*b + b^2)*\sinh(d*x + c)^4 + 2*(2*a^2 + a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*\cosh(d*x + c)^2 + 2*a^2 + a*b - b^2)*\sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)*\cosh(d*x + c)^3 + (2*a^2 + a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a*b}*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 + (4*a - b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - b)*\sinh(d*x + c))*\sqrt{a*b}/(a*b)) - 2*(a^2*b - a*b^2)*\cosh(d*x + c) - 2*(a^2*b - a*b^2 - 3*(a^2*b - a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c) \\ &) / (a^2*b^3*d*\cosh(d*x + c)^4 + 4*a^2*b^3*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*b^3*d*\sinh(d*x + c)^4 + a^2*b^3*d + 2*(2*a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)^2 + 2*(3*a^2*b^3*d*\cosh(d*x + c)^2 + (2*a^3*b^2 - a^2*b^3)*d)*\sinh(d*x + c)^2 + 4*(a^2*b^3*d*\cosh(d*x + c)^3 + (2*a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.333 \quad \int \frac{\cosh^2(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}} + \frac{\tanh(c+dx)}{2ad(a-(a-b)\tanh^2(c+dx))}$$

[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Tanh[c + d*x]/(2*a*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.0768361, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3191, 199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}} + \frac{\tanh(c+dx)}{2ad(a-(a-b)\tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Tanh[c + d*x]/(2*a*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

ator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^2(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a-(a-b)x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\ &= \frac{\tanh(c+dx)}{2ad(a-(a-b)\tanh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2} dx, x, \tanh(c+dx)\right)}{2ad} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}d} + \frac{\tanh(c+dx)}{2ad(a-(a-b)\tanh^2(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.206394, size = 78, normalized size = 0.99

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a}\sinh(2(c+dx))}{2a+b\cosh(2(c+dx))-b}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*Sinh[2*(c + d*x)]/(2*a - b + b*Cosh[2*(c + d*x)]))/(2*a^(3/2)*d)

Maple [B] time = 0.053, size = 404, normalized size = 5.1

$$\frac{1}{da} \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^3 \left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^4 a - 2 (\tanh(1/2 dx + c/2))^2 a + 4 (\tanh(1/2 dx + c/2))^2 b + a \right)^{-1} + \frac{1}{da} \tanh\left(\frac{dx}{2} + \frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)+1/2/d/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b-1/2/d/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.68031, size = 3321, normalized size = 42.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)))*sqrt(a^2 - a*b)*log((b^
```


$$\begin{aligned}
& 2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*\sinh(d*x + c) \\
& ^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*b^2*\cosh(d*x + c)^2 + 2*a*b - b \\
& ^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*\cosh(d*x + c)^3 + (2*a*b \\
& - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x \\
& + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{a^2 - a*b})/(b*\cosh(\\
& d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a \\
& - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + \\
& 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)))/((a^3 \\
& *b^2 - a^2*b^3)*d*\cosh(d*x + c)^4 + 4*(a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)*\s \\
& inh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*\sinh(d*x + c)^4 + 2*(2*a^4*b - 3*a^3 \\
& *b^2 + a^2*b^3)*d*\cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c \\
&)^2 + (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d)*\sinh(d*x + c)^2 + (a^3*b^2 - a^2*b \\
& ^3)*d + 4*((a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)^3 + (2*a^4*b - 3*a^3*b^2 + a \\
& ^2*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/2*(2*a^2*b - 2*a*b^2 + 2*(2*a^3 \\
& - 3*a^2*b + a*b^2)*\cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*\cosh(d*x \\
& + c)*\sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*\sinh(d*x + c)^2 + (b^2*\cos \\
& h(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*\sinh(d*x + c)^4 + \\
& 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*b^2*\cosh(d*x + c)^2 + 2*a*b - b^2)*\s \\
& inh(d*x + c)^2 + b^2 + 4*(b^2*\cosh(d*x + c)^3 + (2*a*b - b^2)*\cosh(d*x + c) \\
&)*\sinh(d*x + c))*\sqrt{-a^2 + a*b}*\arctan(-1/2*(b*\cosh(d*x + c)^2 + 2*b*\cosh \\
& (d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-a^2 + a*b})/(a^ \\
& 2 - a*b)))/((a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)^4 + 4*(a^3*b^2 - a^2*b^3)*d \\
& *\cosh(d*x + c)*\sinh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*\sinh(d*x + c)^4 + 2* \\
& (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d*\cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - a^2*b^3 \\
&)*d*\cosh(d*x + c)^2 + (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d)*\sinh(d*x + c)^2 + \\
& (a^3*b^2 - a^2*b^3)*d + 4*((a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)^3 + (2*a^4*b \\
& - 3*a^3*b^2 + a^2*b^3)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [A] time = 1.29874, size = 173, normalized size = 2.19

$$\frac{\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{2\sqrt{-a^2+ab}ad} - \frac{2ae^{(2dx+2c)} - be^{(2dx+2c)} + b}{(be^{(4dx+4c)} + 4ae^{(2dx+2c)} - 2be^{(2dx+2c)} + b)abd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*d) - (2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)*a*b*d)

$$3.334 \quad \int \frac{\cosh(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{bd}} + \frac{\sinh(c+dx)}{2ad(a+b\sinh^2(c+dx))}$$

[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Sinh[c + d*x]/(2*a*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.0466733, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3190, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{bd}} + \frac{\sinh(c+dx)}{2ad(a+b\sinh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]

[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Sinh[c + d*x]/(2*a*d*(a + b*Sinh[c + d*x]^2))

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 199

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin

ator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= \frac{\sinh(c+dx)}{2ad(a+b\sinh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c+dx)\right)}{2ad} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{bd}} + \frac{\sinh(c+dx)}{2ad(a+b\sinh^2(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.0456064, size = 64, normalized size = 0.97

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\sinh(c+dx)}{2a(a+b\sinh^2(c+dx))}}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2), x]

[Out] (ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sinh[c + d*x]/(2*a*(a + b*Sinh[c + d*x]^2)))/d

Maple [A] time = 0.013, size = 57, normalized size = 0.9

$$\frac{\sinh(dx+c)}{2da(a+b(\sinh(dx+c))^2)} + \frac{1}{2da} \arctan\left(b\sinh(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)`

[Out] $\frac{1}{2} \frac{\sinh(dx+c)}{a/d} \frac{1}{(a+b\sinh(dx+c)^2)} + \frac{1}{2} \frac{d/a}{(a*b)^{1/2}} \arctan(\sinh(dx+c)*b/(a*b)^{1/2})$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{(3dx+3c)} - e^{(dx+c)}}{abde^{(4dx+4c)} + abd + 2(2a^2de^{(2c)} - abde^{(2c)})e^{(2dx)}} + \frac{1}{2} \int \frac{2(e^{(3dx+3c)} + e^{(dx+c)})}{abe^{(4dx+4c)} + ab + 2(2a^2e^{(2c)} - abe^{(2c)})e^{(2dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")`

[Out] $(e^{(3d*x + 3*c)} - e^{(d*x + c)})/(a*b*d*e^{(4*d*x + 4*c)} + a*b*d + 2*(2*a^2*d*e^{(2*c)} - a*b*d*e^{(2*c)})e^{(2*d*x)}) + 1/2*integrate(2*(e^{(3*d*x + 3*c)} + e^{(d*x + c)})/(a*b*e^{(4*d*x + 4*c)} + a*b + 2*(2*a^2*e^{(2*c)} - a*b*e^{(2*c)})e^{(2*d*x)}), x)$

Fricas [B] time = 1.6321, size = 3372, normalized size = 51.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")`

[Out] $[1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b*sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)$

$$\begin{aligned} &^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) + 4*(3*a*b*\cosh(d*x + c)^2 - a*b)*\sinh(d*x + c))/(a^2*b^2*d*\cosh(d*x + c)^4 + 4*a^2*b^2*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*b^2*d*\sinh(d*x + c)^4 + a^2*b^2*d + 2*(2*a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 + 2*(3*a^2*b^2*d*\cosh(d*x + c)^2 + (2*a^3*b - a^2*b^2)*d)*\sinh(d*x + c)^2 + 4*(a^2*b^2*d*\cosh(d*x + c)^3 + (2*a^3*b - a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/2*(2*a*b*\cosh(d*x + c)^3 + 6*a*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + 2*a*b*\sinh(d*x + c)^3 - 2*a*b*\cosh(d*x + c) + (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)*\sqrt{a*b}*\arctan(1/2*\sqrt{a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) + (b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)*\sqrt{a*b}*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 + (4*a - b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - b)*\sinh(d*x + c))*\sqrt{a*b}/(a*b)) + 2*(3*a*b*\cosh(d*x + c)^2 - a*b)*\sinh(d*x + c))/(a^2*b^2*d*\cosh(d*x + c)^4 + 4*a^2*b^2*d*\cosh(d*x + c)*\sinh(d*x + c)^3 + a^2*b^2*d*\sinh(d*x + c)^4 + a^2*b^2*d + 2*(2*a^3*b - a^2*b^2)*d*\cosh(d*x + c)^2 + 2*(3*a^2*b^2*d*\cosh(d*x + c)^2 + (2*a^3*b - a^2*b^2)*d)*\sinh(d*x + c)^2 + 4*(a^2*b^2*d*\cosh(d*x + c)^3 + (2*a^3*b - a^2*b^2)*d*\cosh(d*x + c))*\sinh(d*x + c))]] \end{aligned}$$

Sympy [A] time = 31.6704, size = 428, normalized size = 6.48

$$\left\{ \begin{array}{l} \infty x \cosh(c) \\ \frac{\sinh^4(c)}{\sinh(c+dx)} \\ \frac{a^2 d}{1} \\ \frac{3b^2 d \sinh^3(c+dx)}{x \cosh(c)} \\ (a+b \sinh^2(c))^2 \end{array} \right. + \frac{2i\sqrt{ab}\sqrt{\frac{1}{b}} \sinh(c+dx)}{4ia^2 bd \sqrt{\frac{1}{b}} + 4ia^2 b^2 d \sqrt{\frac{1}{b}} \sinh^2(c+dx)} + \frac{a \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + \sinh(c+dx)\right)}{4ia^2 bd \sqrt{\frac{1}{b}} + 4ia^2 b^2 d \sqrt{\frac{1}{b}} \sinh^2(c+dx)} - \frac{a \log\left(i\sqrt{a}\sqrt{\frac{1}{b}} + \sinh(c+dx)\right)}{4ia^2 bd \sqrt{\frac{1}{b}} + 4ia^2 b^2 d \sqrt{\frac{1}{b}} \sinh^2(c+dx)} + \frac{b \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}} + \sinh(c+dx)\right)}{4ia^2 bd \sqrt{\frac{1}{b}} + 4ia^2 b^2 d \sqrt{\frac{1}{b}} \sinh^2(c+dx)} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a**2*d), Eq(b, 0)), (-1/(3*b**2*d*sinh(c + d*x)**3), Eq(a, 0)), (x*cosh(c)/(a + b*sinh(c)**2)**2, Eq(d, 0)), (2*I*sqrt(a)*b*sqrt(1/b)*sinh(

```

c + d*x)/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqrt(1/b)*sinh(c
+ d*x)**2) + a*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))/(4*I*a**(5/2)*b*d
*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2) - a*log(I*sqrt
(a)*sqrt(1/b) + sinh(c + d*x))/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b
**2*d*sqrt(1/b)*sinh(c + d*x)**2) + b*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d
*x))*sinh(c + d*x)**2/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqr
t(1/b)*sinh(c + d*x)**2) - b*log(I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))*sinh(
c + d*x)**2/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqrt(1/b)*sin
h(c + d*x)**2), True))

```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.335 \quad \int \frac{\operatorname{sech}(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=106

$$-\frac{\sqrt{b}(3a-b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2} - \frac{b \sinh(c+dx)}{2ad(a-b)(a+b \sinh^2(c+dx))} + \frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)^2}$$

[Out] ArcTan[Sinh[c + d*x]]/((a - b)^2*d) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^2*d) - (b*Sinh[c + d*x])/(2*a*(a - b)*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.107907, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3190, 414, 522, 203, 205}

$$-\frac{\sqrt{b}(3a-b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2} - \frac{b \sinh(c+dx)}{2ad(a-b)(a+b \sinh^2(c+dx))} + \frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2), x]

[Out] ArcTan[Sinh[c + d*x]]/((a - b)^2*d) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^2*d) - (b*Sinh[c + d*x])/(2*a*(a - b)*d*(a + b*Sinh[c + d*x]^2))

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +

$d*x^n)^q*$ Simp[$b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n$, x], x] /; FreeQ[{ a, b, c, d, n, q }, x] && NeQ[$b*c - a*d, 0$] && LtQ[$p, -1$] && !(IntegerQ[p] && IntegerQ[q] && LtQ[$q, -1$]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 522

Int[(($e_$) + ($f_$)*($x_$)^($n_$))/((($a_$) + ($b_$)*($x_$)^($n_$))*(($c_$) + ($d_$)*($x_$)^($n_$))), x _Symbol] := Dist[($b*e - a*f$)/($b*c - a*d$), Int[1/($a + b*x^n$), x], x] - Dist[($d*e - c*f$)/($b*c - a*d$), Int[1/($c + d*x^n$), x], x] /; FreeQ[{ a, b, c, d, e, f, n }, x]

Rule 203

Int[(($a_$) + ($b_$)*($x_$)^2)^(-1), x _Symbol] := Simp[(1*ArcTan[(Rt[$b, 2$]* x)/Rt[$a, 2$]])/(Rt[$a, 2$]*Rt[$b, 2$]), x] /; FreeQ[{ a, b }, x] && PosQ[a/b] && (GtQ[$a, 0$] || GtQ[$b, 0$])

Rule 205

Int[(($a_$) + ($b_$)*($x_$)^2)^(-1), x _Symbol] := Simp[(Rt[$a/b, 2$]*ArcTan[x /Rt[$a/b, 2$]])/a, x] /; FreeQ[{ a, b }, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\ &= -\frac{b\sinh(c+dx)}{2a(a-b)d(a+b\sinh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)} dx, x, \sinh(c+dx)\right)}{2a(a-b)d} \\ &= -\frac{b\sinh(c+dx)}{2a(a-b)d(a+b\sinh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx)\right)}{(a-b)^2d} - \frac{((3a-b)b)\operatorname{Subst}\left(\int \frac{x}{(1+x^2)(a+bx^2)} dx, x, \sinh(c+dx)\right)}{(a-b)^2d} \\ &= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)^2d} - \frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d} - \frac{b\sinh(c+dx)}{2a(a-b)d(a+b\sinh^2(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.377087, size = 174, normalized size = 1.64

$$\frac{\cosh(2(c + dx)) \left(4a^{3/2} b \tan^{-1} \left(\tanh \left(\frac{1}{2}(c + dx) \right) \right) - b^{3/2} (b - 3a) \tan^{-1} \left(\frac{\sqrt{a} \operatorname{csch}(c + dx)}{\sqrt{b}} \right) \right) + (2a - b) \left(4a^{3/2} \tan^{-1} \left(\tanh \left(\frac{1}{2}(c + dx) \right) \right) \right)}{2a^{3/2} d (a - b)^2 (2a + b \cosh(2(c + dx)) - b)}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((2*a - b)*(-(Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]]) + 4*a^(3/2)*ArcTan[Tanh[(c + d*x)/2]]) + (-b^(3/2)*(-3*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]]) + 4*a^(3/2)*b*ArcTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)] - 2*Sqrt[a]*(a - b)*b*Sinh[c + d*x])/(2*a^(3/2)*(a - b)^2*d*(2*a - b + b*Cosh[2*(c + d*x)]))

Maple [B] time = 0.082, size = 1080, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)

[Out] 1/d*b/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3-1/d*b^2/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3-1/d*b/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d*b^2/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)+3/2/d*b/(a-b)^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d*b/(a-b)^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-2/d*b^2/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d*b/(a-b)^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/2/d*b/(a-b)^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-2/d*b^2/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d*b^2/(a-b)^2/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/

$$\begin{aligned} & ((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)})+1/2/d*b^3/(a-b)^2/a/(-b*(a-b))^{(1/2)}/ \\ & ((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)}*\operatorname{arctanh}(a*\operatorname{tanh}(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{(1/2)+a-2*b}*a)^{(1/2)})+1/2/d*b^2/(a-b)^2/a/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}))+1/2/d*b^3/(a-b)^2/a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)})*\operatorname{arctan}(a*\operatorname{tanh}(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{(1/2)-a+2*b}*a)^{(1/2)}))+2/d/(a-b)^2*\operatorname{arctan}(\operatorname{tanh}(1/2*d*x+1/2*c)) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{be^{(3dx+3c)} - be^{(dx+c)}}{a^2bd - ab^2d + (a^2bde^{(4c)} - ab^2de^{(4c)})e^{(4dx)} + 2(2a^3de^{(2c)} - 3a^2bde^{(2c)} + ab^2de^{(2c)})e^{(2dx)}} + \frac{2 \operatorname{arctan}(e^{(dx+c)})}{a^2d - 2abd + b^2d} - 2 \int$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] $-(b*e^{(3*d*x + 3*c)} - b*e^{(d*x + c)})/(a^2*b*d - a*b^2*d + (a^2*b*d*e^{(4*c)} - a*b^2*d*e^{(4*c)})*e^{(4*d*x)} + 2*(2*a^3*d*e^{(2*c)} - 3*a^2*b*d*e^{(2*c)} + a*b^2*d*e^{(2*c)})*e^{(2*d*x)}) + 2*\operatorname{arctan}(e^{(d*x + c)})/(a^2*d - 2*a*b*d + b^2*d) - 2*\operatorname{integrate}(1/2*((3*a*b*e^{(3*c)} - b^2*e^{(3*c)})*e^{(3*d*x)} + (3*a*b*e^{(c)} - b^2*e^{(c)})*e^{(d*x)})/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^{(4*c)} - 2*a^2*b^2*e^{(4*c)} + a*b^3*e^{(4*c)})*e^{(4*d*x)} + 2*(2*a^4*e^{(2*c)} - 5*a^3*b*e^{(2*c)} + 4*a^2*b^2*e^{(2*c)} - a*b^3*e^{(2*c)})*e^{(2*d*x)}), x)$

Fricas [B] time = 2.00276, size = 5192, normalized size = 48.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] $[-1/4*(4*(a*b - b^2)*\cosh(d*x + c)^3 + 12*(a*b - b^2)*\cosh(d*x + c)*\sinh(d*x + c)^2 + 4*(a*b - b^2)*\sinh(d*x + c)^3 + ((3*a*b - b^2)*\cosh(d*x + c)^4 + 4*(3*a*b - b^2)*\cosh(d*x + c)*\sinh(d*x + c)^3 + (3*a*b - b^2)*\sinh(d*x + c)^4) + 2*(6*a^2 - 5*a*b + b^2)*\cosh(d*x + c)^2 + 2*(3*(3*a*b - b^2)*\cosh(d*x + c)^2 + 6*a^2 - 5*a*b + b^2)*\sinh(d*x + c)^2 + 3*a*b - b^2 + 4*((3*a*b - b^2)*\cosh(d*x + c)^3 + (6*a^2 - 5*a*b + b^2)*\cosh(d*x + c))*\sinh(d*x + c)*$

$$\begin{aligned}
& \sqrt{-b/a} \cdot \log((b \cosh(dx + c))^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 - 2(2a + b) \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 - 2a - b) \sinh(dx + c)^2 + 4(b \cosh(dx + c)^3 - (2a + b) \cosh(dx + c)) \sinh(dx + c) + 4(a \cosh(dx + c)^3 + 3a \cosh(dx + c) \sinh(dx + c)^2 + a \sinh(dx + c)^3 - a \cosh(dx + c) + (3a \cosh(dx + c)^2 - a) \sinh(dx + c)) \sqrt{-b/a} + b) / (b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 + 2(2a - b) \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 + 2a - b) \sinh(dx + c)^2 + 4(b \cosh(dx + c)^3 + (2a - b) \cosh(dx + c)) \sinh(dx + c) + b) - 8(a b \cosh(dx + c)^4 + 4a b \cosh(dx + c) \sinh(dx + c)^3 + a b \sinh(dx + c)^4 + 2(2a^2 - a b) \cosh(dx + c)^2 + 2(3a b \cosh(dx + c)^2 + 2a^2 - a b) \sinh(dx + c)^2 + a b + 4(a b \cosh(dx + c)^3 + (2a^2 - a b) \cosh(dx + c)) \sinh(dx + c)) \arctan(\cosh(dx + c) + \sinh(dx + c)) - 4(a b - b^2) \cosh(dx + c) + 4(3(a b - b^2) \cosh(dx + c)^2 - a b + b^2) \sinh(dx + c)) / ((a^3 b - 2a^2 b^2 + a b^3) d \cosh(dx + c)^4 + 4(a^3 b - 2a^2 b^2 + a b^3) d \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b - 2a^2 b^2 + a b^3) d \sinh(dx + c)^4 + 2(2a^4 - 5a^3 b + 4a^2 b^2 - a b^3) d \cosh(dx + c)^2 + 2(3(a^3 b - 2a^2 b^2 + a b^3) d \cosh(dx + c)^2 + (2a^4 - 5a^3 b + 4a^2 b^2 - a b^3) d) \sinh(dx + c)^2 + (a^3 b - 2a^2 b^2 + a b^3) d + 4((a^3 b - 2a^2 b^2 + a b^3) d \cosh(dx + c)^3 + (2a^4 - 5a^3 b + 4a^2 b^2 - a b^3) d \cosh(dx + c)) \sinh(dx + c)), -1/2(2(a b - b^2) \cosh(dx + c)^3 + 6(a b - b^2) \cosh(dx + c) \sinh(dx + c)^2 + 2(a b - b^2) \sinh(dx + c)^3 + ((3a b - b^2) \cosh(dx + c)^4 + 4(3a b - b^2) \cosh(dx + c) \sinh(dx + c)^3 + (3a b - b^2) \sinh(dx + c)^4 + 2(6a^2 - 5a b + b^2) \cosh(dx + c)^2 + 2(3(3a b - b^2) \cosh(dx + c)^2 + 6a^2 - 5a b + b^2) \sinh(dx + c)^2 + 3a b - b^2 + 4((3a b - b^2) \cosh(dx + c)^3 + (6a^2 - 5a b + b^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{b/a} \arctan(1/2 \sqrt{b/a} (\cosh(dx + c) + \sinh(dx + c))) + ((3a b - b^2) \cosh(dx + c)^4 + 4(3a b - b^2) \cosh(dx + c) \sinh(dx + c)^3 + (3a b - b^2) \sinh(dx + c)^4 + 2(6a^2 - 5a b + b^2) \cosh(dx + c)^2 + 2(3(3a b - b^2) \cosh(dx + c)^2 + 6a^2 - 5a b + b^2) \sinh(dx + c)^2 + 3a b - b^2 + 4((3a b - b^2) \cosh(dx + c)^3 + (6a^2 - 5a b + b^2) \cosh(dx + c)) \sinh(dx + c)) \sqrt{b/a} \arctan(1/2 (b \cosh(dx + c)^3 + 3b \cosh(dx + c) \sinh(dx + c)^2 + b \sinh(dx + c)^3 + (4a - b) \cosh(dx + c) + (3b \cosh(dx + c)^2 + 4a - b) \sinh(dx + c)) \sqrt{b/a} / b) - 4(a b \cosh(dx + c)^4 + 4a b \cosh(dx + c) \sinh(dx + c)^3 + a b \sinh(dx + c)^4 + 2(2a^2 - a b) \cosh(dx + c)^2 + 2(3a b \cosh(dx + c)^2 + 2a^2 - a b) \sinh(dx + c)^2 + a b + 4(a b \cosh(dx + c)^3 + (2a^2 - a b) \cosh(dx + c)) \sinh(dx + c)) \arctan(\cosh(dx + c) + \sinh(dx + c)) - 2(a b - b^2) \cosh(dx + c) + 2(3(a b - b^2) \cosh(dx + c)^2 - a b + b^2) \sinh(dx + c)) / ((a^3 b - 2a^2 b^2 + a b^3) d \cosh(dx + c)^4 + 4(a^3 b - 2a^2 b^2 + a b^3) d \cosh(dx + c) \sinh(dx + c)^3 + (a^3 b - 2a^2 b^2 + a b^3) d \sinh(dx + c)^4 + 2(2a^4 - 5a^3 b + 4a^2 b^2 - a b^3) d \cosh(dx + c)^2 + 2(3(a^3 b - 2a^2 b^2 + a b^3) d \cosh(dx + c)^2 + (2a^4 - 5a^3 b + 4a^2 b^2 - a b^3) d) \sinh(dx + c)^2 + (a^3 b - 2a^2 b^2 + a b^3) d + 4((a^3 b - 2a^2 b^2 + a b^3) d \cosh(dx + c)^3 + (2a^4 - 5a^3 b + 4a^2 b^2 - a b^3) d \cosh(dx + c)
\end{aligned}$$

```
) *sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.336 \quad \int \frac{\operatorname{sech}^2(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$$

Optimal. Leaf size=114

$$-\frac{b(4a-b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}} + \frac{b^2 \tanh(c+dx)}{2ad(a-b)^2(a-(a-b) \tanh^2(c+dx))} + \frac{\tanh(c+dx)}{d(a-b)^2}$$

[Out] -((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^(5/2)*d) + Tanh[c + d*x]/((a - b)^2*d) + (b^2*Tanh[c + d*x])/(2*a*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.179353, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3191, 390, 385, 208}

$$-\frac{b(4a-b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}} + \frac{b^2 \tanh(c+dx)}{2ad(a-b)^2(a-(a-b) \tanh^2(c+dx))} + \frac{\tanh(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] -((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^(5/2)*d) + Tanh[c + d*x]/((a - b)^2*d) + (b^2*Tanh[c + d*x])/(2*a*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 390

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] := Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,

0] && GeQ[p, -q]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}^2(c + dx)}{(a + b \sinh^2(c + dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{(a-(a-b)x^2)^2} dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a-b)^2} - \frac{(2a-b)b-2(a-b)bx^2}{(a-b)^2(a+(-a+b)x^2)^2}\right) dx, x, \tanh(c + dx)\right)}{d} \\
 &= \frac{\tanh(c + dx)}{(a-b)^2d} - \frac{\operatorname{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{(a+(-a+b)x^2)^2} dx, x, \tanh(c + dx)\right)}{(a-b)^2d} \\
 &= \frac{\tanh(c + dx)}{(a-b)^2d} + \frac{b^2 \tanh(c + dx)}{2a(a-b)^2d(a - (a-b)\tanh^2(c + dx))} - \frac{((4a-b)b) \operatorname{Subst}\left(\int \frac{1}{a+(-a+b)x^2} dx, x, \tanh(c + dx)\right)}{2a(a-b)^2d} \\
 &= -\frac{(4a-b)b \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d} + \frac{\tanh(c + dx)}{(a-b)^2d} + \frac{b^2 \tanh(c + dx)}{2a(a-b)^2d(a - (a-b)\tanh^2(c + dx))}
 \end{aligned}$$

Mathematica [A] time = 0.855237, size = 105, normalized size = 0.92

$$\frac{\frac{b^2 \sinh(2(c+dx))}{a(2a+b \cosh(2(c+dx))-b)} + 2 \tanh(c+dx)}{(a-b)^2} - \frac{b(4a-b) \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] $-\left(\frac{4a-b}{a}\right) \frac{b \operatorname{ArcTanh}\left[\frac{\sqrt{a-b} \operatorname{Tanh}[c+d*x]}{\sqrt{a}}\right]}{a^{3/2}(a-b)^{5/2}} + \frac{b^2 \operatorname{Sinh}[2(c+d*x)]}{a(2a-b+b \operatorname{Cosh}[2(c+d*x)])} + 2 \frac{\operatorname{Tanh}[c+d*x]}{(a-b)^2} \frac{1}{2d}$

Maple [B] time = 0.075, size = 798, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)

[Out] $\frac{1}{d} \frac{b^2}{(a-b)^2} \frac{1}{\operatorname{tanh}(1/2*d*x+1/2*c)^{4a-2} \operatorname{tanh}(1/2*d*x+1/2*c)^{2a+4} \operatorname{tanh}(1/2*d*x+1/2*c)^{2b+a}} \frac{1}{a \operatorname{tanh}(1/2*d*x+1/2*c)^3} + \frac{1}{d} \frac{b^2}{(a-b)^2} \frac{1}{\operatorname{tanh}(1/2*d*x+1/2*c)^{4a-2} \operatorname{tanh}(1/2*d*x+1/2*c)^{2a+4} \operatorname{tanh}(1/2*d*x+1/2*c)^{2b+a}} \frac{1}{a \operatorname{tanh}(1/2*d*x+1/2*c)^2} - \frac{2}{d} \frac{b}{(a-b)^2} \frac{1}{((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2} \operatorname{arctanh}(a \operatorname{tanh}(1/2*d*x+1/2*c))} \frac{1}{((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}} + \frac{2}{d} \frac{b^2}{(a-b)^2} \frac{1}{(-b*(a-b))^{1/2} \operatorname{arctanh}(a \operatorname{tanh}(1/2*d*x+1/2*c))} \frac{1}{((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}} + \frac{2}{d} \frac{b}{(a-b)^2} \frac{1}{((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2} \operatorname{arctan}(a \operatorname{tanh}(1/2*d*x+1/2*c))} \frac{1}{((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}} + \frac{2}{d} \frac{b^2}{(a-b)^2} \frac{1}{(-b*(a-b))^{1/2} \operatorname{arctan}(a \operatorname{tanh}(1/2*d*x+1/2*c))} \frac{1}{((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}} + \frac{1}{2} \frac{1}{d} \frac{b^2}{(a-b)^2} \frac{1}{a} \frac{1}{((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2} \operatorname{arctanh}(a \operatorname{tanh}(1/2*d*x+1/2*c))} \frac{1}{((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}} - \frac{1}{2} \frac{1}{d} \frac{b^3}{(a-b)^2} \frac{1}{a} \frac{1}{(-b*(a-b))^{1/2} \operatorname{arctanh}(a \operatorname{tanh}(1/2*d*x+1/2*c))} \frac{1}{((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}} - \frac{1}{2} \frac{1}{d} \frac{b^2}{(a-b)^2} \frac{1}{a} \frac{1}{((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2} \operatorname{arctan}(a \operatorname{tanh}(1/2*d*x+1/2*c))} \frac{1}{((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}} - \frac{1}{2} \frac{1}{d} \frac{b^3}{(a-b)^2} \frac{1}{a} \frac{1}{(-b*(a-b))^{1/2} \operatorname{arctan}(a \operatorname{tanh}(1/2*d*x+1/2*c))} \frac{1}{((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}} + \frac{2}{d} \frac{1}{(a-b)^2} \frac{1}{\operatorname{tanh}(1/2*d*x+1/2*c)^{2+1}}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08948, size = 7185, normalized size = 63.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^4 + 16*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(4*a^3*b - 5*a^2*b^2 + a*b^3)*sinh(d*x + c)^4 + 8*a^3*b - 4*a^2*b^2 - 4*a*b^3 + 8*(4*a^4 - 5*a^3*b + a^2*b^2)*cosh(d*x + c)^2 + 8*(4*a^4 - 5*a^3*b + a^2*b^2 + 3*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - b^3)*sinh(d*x + c)^6 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^2*b - 8*a*b^2 + b^3 + 15*(4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - b^3)*cosh(d*x + c)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*a*b^2 - b^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(4*a*b^2 - b^3)*cosh(d*x + c)^4 + 16*a^2*b - 8*a*b^2 + b^3 + 6*(16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*((4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 5*a^3*b + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*sinh(d*x + c)^6 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^4 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + 4
```

$$\begin{aligned}
&*(5*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*\cosh(d*x + c)^3 + (4*a^6 - \\
&13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c) \\
&^3 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*\cosh(d*x + c)^4 + 6*(4 \\
&*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^2 + (4* \\
&a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*\sinh(d*x + c)^2 + (a^ \\
&5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d + 2*(3*(a^5*b - 3*a^4*b^2 + 3*a^3* \\
&b^3 - a^2*b^4)*d*\cosh(d*x + c)^5 + 2*(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3* \\
&*b^3 + a^2*b^4)*d*\cosh(d*x + c)^3 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3* \\
&b^3 + a^2*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/2*(2*(4*a^3*b - 5*a^2*b^ \\
&2 + a*b^3)*\cosh(d*x + c)^4 + 8*(4*a^3*b - 5*a^2*b^2 + a*b^3)*\cosh(d*x + c)* \\
&\sinh(d*x + c)^3 + 2*(4*a^3*b - 5*a^2*b^2 + a*b^3)*\sinh(d*x + c)^4 + 4*a^3*b \\
&- 2*a^2*b^2 - 2*a*b^3 + 4*(4*a^4 - 5*a^3*b + a^2*b^2)*\cosh(d*x + c)^2 + 4* \\
&(4*a^4 - 5*a^3*b + a^2*b^2 + 3*(4*a^3*b - 5*a^2*b^2 + a*b^3)*\cosh(d*x + c)^ \\
&2)*\sinh(d*x + c)^2 - ((4*a*b^2 - b^3)*\cosh(d*x + c)^6 + 6*(4*a*b^2 - b^3)*c \\
&osh(d*x + c)*\sinh(d*x + c)^5 + (4*a*b^2 - b^3)*\sinh(d*x + c)^6 + (16*a^2*b \\
&- 8*a*b^2 + b^3)*\cosh(d*x + c)^4 + (16*a^2*b - 8*a*b^2 + b^3 + 15*(4*a*b^2 \\
&- b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - b^3)*\cosh(d*x + c \\
&)^3 + (16*a^2*b - 8*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*a*b^2 - \\
&b^3 + (16*a^2*b - 8*a*b^2 + b^3)*\cosh(d*x + c)^2 + (15*(4*a*b^2 - b^3)*cos \\
&h(d*x + c)^4 + 16*a^2*b - 8*a*b^2 + b^3 + 6*(16*a^2*b - 8*a*b^2 + b^3)*\cosh \\
&(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - b^3)*\cosh(d*x + c)^5 + 2*(16 \\
&*a^2*b - 8*a*b^2 + b^3)*\cosh(d*x + c)^3 + (16*a^2*b - 8*a*b^2 + b^3)*\cosh(d \\
&*x + c))*\sinh(d*x + c))*\sqrt{-a^2 + a*b}*\arctan(-1/2*(b*\cosh(d*x + c)^2 + 2 \\
&*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-a^2 + a \\
&*b)/(a^2 - a*b)) + 8*((4*a^3*b - 5*a^2*b^2 + a*b^3)*\cosh(d*x + c)^3 + (4*a^ \\
&4 - 5*a^3*b + a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5*b - 3*a^4*b^2 + \\
&3*a^3*b^3 - a^2*b^4)*d*\cosh(d*x + c)^6 + 6*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - \\
&a^2*b^4)*d*\cosh(d*x + c)*\sinh(d*x + c)^5 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 \\
&- a^2*b^4)*d*\sinh(d*x + c)^6 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + \\
&a^2*b^4)*d*\cosh(d*x + c)^4 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4) \\
&)*d*\cosh(d*x + c)^2 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)* \\
&d)*\sinh(d*x + c)^4 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)* \\
&d*\cosh(d*x + c)^2 + 4*(5*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*\cosh(d \\
&*x + c)^3 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*\cosh(d \\
&x + c))*\sinh(d*x + c)^3 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*c \\
&>osh(d*x + c)^4 + 6*(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d* \\
&cosh(d*x + c)^2 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)* \\
&\sinh(d*x + c)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d + 2*(3*(a^5*b \\
&- 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*\cosh(d*x + c)^5 + 2*(4*a^6 - 13*a^5*b \\
&+ 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c)^3 + (4*a^6 - 13*a^5*b \\
&+ 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [A] time = 2.44126, size = 200, normalized size = 1.75

$$\frac{4abe^{(4dx+4c)} - b^2e^{(4dx+4c)} + 8a^2e^{(2dx+2c)} - 2abe^{(2dx+2c)} + 2ab + b^2}{(a^3d - 2a^2bd + ab^2d)(be^{(6dx+6c)} + 4ae^{(4dx+4c)} - be^{(4dx+4c)} + 4ae^{(2dx+2c)} - be^{(2dx+2c)} + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] $-(4*a*b*e^{(4*d*x + 4*c)} - b^2*e^{(4*d*x + 4*c)} + 8*a^2*e^{(2*d*x + 2*c)} - 2*a*b*e^{(2*d*x + 2*c)} + 2*a*b + b^2)/((a^3*d - 2*a^2*b*d + a*b^2*d)*(b*e^{(6*d*x + 6*c)} + 4*a*e^{(4*d*x + 4*c)} - b*e^{(4*d*x + 4*c)} + 4*a*e^{(2*d*x + 2*c)} - b*e^{(2*d*x + 2*c)} + b))$

$$3.337 \quad \int \frac{\operatorname{sech}^3(c+dx)}{(a+b \sinh^2(c+dx))^2} dx$$

Optimal. Leaf size=157

$$\frac{b^{3/2}(5a-b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3} + \frac{b(a+b) \sinh(c+dx)}{2ad(a-b)^2(a+b \sinh^2(c+dx))} + \frac{(a-5b) \tan^{-1}(\sinh(c+dx))}{2d(a-b)^3} + \frac{\tanh(c+dx) \operatorname{sech}^2(c+dx)}{2d(a-b)(a+b \sinh^2(c+dx))}$$

[Out] ((a - 5*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^3*d) + ((5*a - b)*b^(3/2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (b*(a + b)*Sinh[c + d*x])/(2*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.19279, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}(5a-b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3} + \frac{b(a+b) \sinh(c+dx)}{2ad(a-b)^2(a+b \sinh^2(c+dx))} + \frac{(a-5b) \tan^{-1}(\sinh(c+dx))}{2d(a-b)^3} + \frac{\tanh(c+dx) \operatorname{sech}^2(c+dx)}{2d(a-b)(a+b \sinh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]

[Out] ((a - 5*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^3*d) + ((5*a - b)*b^(3/2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (b*(a + b)*Sinh[c + d*x])/(2*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]

```

Rule 527

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

```

Rule 522

```

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]

```

Rule 203

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

```

Rule 205

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^2} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d(a+b\sinh^2(c+dx))} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-3bx^2}{(1+x^2)(a+bx^2)^2} dx, x, \sinh(c+dx)\right)}{2(a-b)d} \\
&= \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d(a+b\sinh^2(c+dx))} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d(a+b\sinh^2(c+dx))} - \frac{\operatorname{Subst}\left(\int \frac{-2(a^2-4ab-3bx^2)}{(1+x^2)(a+bx^2)^2} dx, x, \sinh(c+dx)\right)}{2(a-b)d} \\
&= \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d(a+b\sinh^2(c+dx))} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d(a+b\sinh^2(c+dx))} + \frac{(a-5b)\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx)\right)}{2(a-b)d} \\
&= \frac{(a-5b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^3d} + \frac{(5a-b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d} + \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d(a+b\sinh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.874894, size = 230, normalized size = 1.46

$$\frac{(2a-b)\left(2a^{3/2}(a-5b)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + a^{3/2}(a-b)\tanh(c+dx)\operatorname{sech}(c+dx) + b^{3/2}(b-5a)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)\right)}{2a^{3/2}(a-b)^3d}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] (2*Sqrt[a]*(a - b)*b^2*Sinh[c + d*x] + (2*a - b)*(b^(3/2))*(-5*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2)*(a - 5*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh[c + d*x] + b*Cosh[2*(c + d*x)]*(b^(3/2))*(-5*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2)*(a - 5*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/((2*a^(3/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d*x)]))

Maple [B] time = 0.089, size = 1265, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\operatorname{sech}(d*x+c)^3/(a+b*\sinh(d*x+c)^2)^2, x)$

[Out]
$$\begin{aligned} & -1/d*b^2/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*\tanh(1/2*d*x+1/2*c)^3+1/d*b^3/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/a*\tanh(1/2*d*x+1/2*c)^3+1/d*b^2/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)*\tanh(1/2*d*x+1/2*c)-1/d*b^3/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/a*\tanh(1/2*d*x+1/2*c)-5/2/d*b^2/(a-b)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-5/2/d*b^2/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*b^3/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-5/2/d*b^2/(a-b)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+5/2/d*b^2/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/d*b^3/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d*b^4/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d*b^4/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*a+1/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)^3*b+1/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*a-1/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^2*\tanh(1/2*d*x+1/2*c)*b+1/d/(a-b)^3*\operatorname{arctan}(\tanh(1/2*d*x+1/2*c))*a-5/d/(a-b)^3*\operatorname{arctan}(\tanh(1/2*d*x+1/2*c))*b \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(\operatorname{sech}(d*x+c)^3/(a+b*\sinh(d*x+c)^2)^2, x, \operatorname{algorithm}="maxima")$

```
[Out] (a*e^c - 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^3*d - 3*a^2*b*d + 3*a*b^2*d
- b^3*d) + ((a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - 3*a*b
*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - 3*a*b*e^(3*c) + b^2*e^
(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b*d - 2*a^2*b^2*d + a*
b^3*d + (a^3*b*d*e^(8*c) - 2*a^2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x)
+ 4*(a^4*d*e^(6*c) - 2*a^3*b*d*e^(6*c) + a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2*
(4*a^4*d*e^(4*c) - 9*a^3*b*d*e^(4*c) + 6*a^2*b^2*d*e^(4*c) - a*b^3*d*e^(4*c
))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - 2*a^3*b*d*e^(2*c) + a^2*b^2*d*e^(2*c))*e^
(2*d*x)) + 8*integrate(1/8*((5*a*b^2*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) + (5*
a*b^2*e^c - b^3*e^c)*e^(d*x))/(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4 + (a^4
*b*e^(4*c) - 3*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c) - a*b^4*e^(4*c))*e^(4*d*
x) + 2*(2*a^5*e^(2*c) - 7*a^4*b*e^(2*c) + 9*a^3*b^2*e^(2*c) - 5*a^2*b^3*e^
(2*c) + a*b^4*e^(2*c))*e^(2*d*x)), x
```

Fricas [B] time = 3.00745, size = 15107, normalized size = 96.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b - b^3)*cosh(d*x + c)^7 + 28*(a^2*b - b^3)*cosh(d*x + c)*sinh
(d*x + c)^6 + 4*(a^2*b - b^3)*sinh(d*x + c)^7 + 4*(4*a^3 - 7*a^2*b + 4*a*b^
2 - b^3)*cosh(d*x + c)^5 + 4*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3 + 21*(a^2*b -
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b - b^3)*cosh(d*x + c)^
3 + (4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(4
*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 + 4*(35*(a^2*b - b^3)*cosh(
d*x + c)^4 - 4*a^3 + 7*a^2*b - 4*a*b^2 + b^3 + 10*(4*a^3 - 7*a^2*b + 4*a*b^
2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(a^2*b - b^3)*cosh(d*x +
c)^5 + 10*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^3 - 7*
a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((5*a*b^2 - b^3)*co
sh(d*x + c)^8 + 8*(5*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2
- b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(5*a^2*b -
a*b^2 + 7*(5*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2
- b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - b^3)*co
sh(d*x + c)^4 + 20*a^2*b - 9*a*b^2 + b^3 + 30*(5*a^2*b - a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b
- a*b^2)*cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c))*sinh(d
*x + c)^3 + 5*a*b^2 - b^3 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(5*a
*b^2 - b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - a*b^2)*cosh(d*x + c)^4 + 5*a^2*
```


$$\begin{aligned}
& b - a*b^2 + 3*(20*a^2*b - 9*a*b^2 + b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + \\
& 8*((5*a*b^2 - b^3)*\cosh(d*x + c)^7 + 3*(5*a^2*b - a*b^2)*\cosh(d*x + c)^5 + \\
& (20*a^2*b - 9*a*b^2 + b^3)*\cosh(d*x + c)^3 + (5*a^2*b - a*b^2)*\cosh(d*x + \\
& c))*\sinh(d*x + c))*\sqrt{-b/a}*\log((b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\si \\
& nh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a + b)*\cosh(d*x + c)^2 + 2*(3*b*\co \\
& sh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a + b) \\
& *\cosh(d*x + c))*\sinh(d*x + c) + 4*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\si \\
& nh(d*x + c)^2 + a*\sinh(d*x + c)^3 - a*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 \\
& - a)*\sinh(d*x + c))*\sqrt{-b/a} + b)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)* \\
& sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\c \\
& osh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - \\
& b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) + 4*((a^2*b - 5*a*b^2)*\cosh(d*x + c)^ \\
& 8 + 8*(a^2*b - 5*a*b^2)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^2*b - 5*a*b^2)*s \\
& inh(d*x + c)^8 + 4*(a^3 - 5*a^2*b)*\cosh(d*x + c)^6 + 4*(a^3 - 5*a^2*b + 7*(\\
& a^2*b - 5*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a^2*b - 5*a*b^2)* \\
& cosh(d*x + c)^3 + 3*(a^3 - 5*a^2*b)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(4*a \\
& ^3 - 21*a^2*b + 5*a*b^2)*\cosh(d*x + c)^4 + 2*(35*(a^2*b - 5*a*b^2)*\cosh(d*x \\
& + c)^4 + 4*a^3 - 21*a^2*b + 5*a*b^2 + 30*(a^3 - 5*a^2*b)*\cosh(d*x + c)^2)* \\
& sinh(d*x + c)^4 + 8*(7*(a^2*b - 5*a*b^2)*\cosh(d*x + c)^5 + 10*(a^3 - 5*a^2* \\
& b)*\cosh(d*x + c)^3 + (4*a^3 - 21*a^2*b + 5*a*b^2)*\cosh(d*x + c))*\sinh(d*x + \\
& c)^3 + a^2*b - 5*a*b^2 + 4*(a^3 - 5*a^2*b)*\cosh(d*x + c)^2 + 4*(7*(a^2*b - \\
& 5*a*b^2)*\cosh(d*x + c)^6 + 15*(a^3 - 5*a^2*b)*\cosh(d*x + c)^4 + a^3 - 5*a^ \\
& 2*b + 3*(4*a^3 - 21*a^2*b + 5*a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*(\\
& (a^2*b - 5*a*b^2)*\cosh(d*x + c)^7 + 3*(a^3 - 5*a^2*b)*\cosh(d*x + c)^5 + (4* \\
& a^3 - 21*a^2*b + 5*a*b^2)*\cosh(d*x + c)^3 + (a^3 - 5*a^2*b)*\cosh(d*x + c))* \\
& sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - 4*(a^2*b - b^3)*\cosh \\
& (d*x + c) + 4*(7*(a^2*b - b^3)*\cosh(d*x + c)^6 + 5*(4*a^3 - 7*a^2*b + 4*a*b \\
& ^2 - b^3)*\cosh(d*x + c)^4 - a^2*b + b^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^ \\
& 3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4) \\
& *d*\cosh(d*x + c)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*\cosh(d*x + \\
& c)*\sinh(d*x + c)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*\sinh(d*x + \\
& c)^8 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)^6 + 4*(7*(a^ \\
& 4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3 \\
& *a^3*b^2 - a^2*b^3)*d)*\sinh(d*x + c)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - \\
& 7*a^2*b^3 + a*b^4)*d*\cosh(d*x + c)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 \\
& - a*b^4)*d*\cosh(d*x + c)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cos \\
& h(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4) \\
& *d*\cosh(d*x + c)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cosh(d*x + \\
& c)^2 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d)*\sinh(d*x + c) \\
& ^4 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)^2 + 8*(7*(a^4* \\
& b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^5 + 10*(a^5 - 3*a^4*b + \\
& 3*a^3*b^2 - a^2*b^3)*d*\cosh(d*x + c)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7 \\
& *a^2*b^3 + a*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^4*b - 3*a^3*b^ \\
& 2 + 3*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - \\
& a^2*b^3)*d*\cosh(d*x + c)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 +
\end{aligned}$$

$$\begin{aligned}
& a*b^4)*d*\cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d)*\sinh(d \\
& *x + c)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d + 8*((a^4*b - 3*a^3*b \\
& ^2 + 3*a^2*b^3 - a*b^4)*d*\cosh(d*x + c)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - \\
& a^2*b^3)*d*\cosh(d*x + c)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a \\
& *b^4)*d*\cosh(d*x + c)^3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cosh(d*x \\
& + c))*\sinh(d*x + c)), 1/2*(2*(a^2*b - b^3)*\cosh(d*x + c)^7 + 14*(a^2*b - b^ \\
& 3)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 2*(a^2*b - b^3)*\sinh(d*x + c)^7 + 2*(4*a \\
& ^3 - 7*a^2*b + 4*a*b^2 - b^3)*\cosh(d*x + c)^5 + 2*(4*a^3 - 7*a^2*b + 4*a*b^ \\
& 2 - b^3 + 21*(a^2*b - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 10*(7*(a^2*b \\
& - b^3)*\cosh(d*x + c)^3 + (4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*\cosh(d*x + c))*s \\
& inh(d*x + c)^4 - 2*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*\cosh(d*x + c)^3 + 2*(3 \\
& 5*(a^2*b - b^3)*\cosh(d*x + c)^4 - 4*a^3 + 7*a^2*b - 4*a*b^2 + b^3 + 10*(4*a \\
& ^3 - 7*a^2*b + 4*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 2*(21*(a^2 \\
& *b - b^3)*\cosh(d*x + c)^5 + 10*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*\cosh(d*x + \\
& c)^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 \\
& + ((5*a*b^2 - b^3)*\cosh(d*x + c)^8 + 8*(5*a*b^2 - b^3)*\cosh(d*x + c)*\sinh(d \\
& *x + c)^7 + (5*a*b^2 - b^3)*\sinh(d*x + c)^8 + 4*(5*a^2*b - a*b^2)*\cosh(d*x \\
& + c)^6 + 4*(5*a^2*b - a*b^2 + 7*(5*a*b^2 - b^3)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c)^6 + 8*(7*(5*a*b^2 - b^3)*\cosh(d*x + c)^3 + 3*(5*a^2*b - a*b^2)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^5 + 2*(20*a^2*b - 9*a*b^2 + b^3)*\cosh(d*x + c)^4 + 2*(\\
& 35*(5*a*b^2 - b^3)*\cosh(d*x + c)^4 + 20*a^2*b - 9*a*b^2 + b^3 + 30*(5*a^2*b \\
& - a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(5*a*b^2 - b^3)*\cosh(d*x \\
& + c)^5 + 10*(5*a^2*b - a*b^2)*\cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2 + b^3)* \\
& \cosh(d*x + c))*\sinh(d*x + c)^3 + 5*a*b^2 - b^3 + 4*(5*a^2*b - a*b^2)*\cosh(d \\
& *x + c)^2 + 4*(7*(5*a*b^2 - b^3)*\cosh(d*x + c)^6 + 15*(5*a^2*b - a*b^2)*\cos \\
& h(d*x + c)^4 + 5*a^2*b - a*b^2 + 3*(20*a^2*b - 9*a*b^2 + b^3)*\cosh(d*x + c) \\
& ^2)*\sinh(d*x + c)^2 + 8*((5*a*b^2 - b^3)*\cosh(d*x + c)^7 + 3*(5*a^2*b - a*b \\
& ^2)*\cosh(d*x + c)^5 + (20*a^2*b - 9*a*b^2 + b^3)*\cosh(d*x + c)^3 + (5*a^2*b \\
& - a*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/2*\sqrt{b/a}*(\cos \\
& h(d*x + c) + \sinh(d*x + c))) + ((5*a*b^2 - b^3)*\cosh(d*x + c)^8 + 8*(5*a*b^ \\
& 2 - b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (5*a*b^2 - b^3)*\sinh(d*x + c)^8 + \\
& 4*(5*a^2*b - a*b^2)*\cosh(d*x + c)^6 + 4*(5*a^2*b - a*b^2 + 7*(5*a*b^2 - b^3 \\
&)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(5*a*b^2 - b^3)*\cosh(d*x + c)^3 + \\
& 3*(5*a^2*b - a*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(20*a^2*b - 9*a*b^2 \\
& + b^3)*\cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - b^3)*\cosh(d*x + c)^4 + 20*a^2*b \\
& - 9*a*b^2 + b^3 + 30*(5*a^2*b - a*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8 \\
& *(7*(5*a*b^2 - b^3)*\cosh(d*x + c)^5 + 10*(5*a^2*b - a*b^2)*\cosh(d*x + c)^3 \\
& + (20*a^2*b - 9*a*b^2 + b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 5*a*b^2 - b^3 \\
& + 4*(5*a^2*b - a*b^2)*\cosh(d*x + c)^2 + 4*(7*(5*a*b^2 - b^3)*\cosh(d*x + c) \\
& ^6 + 15*(5*a^2*b - a*b^2)*\cosh(d*x + c)^4 + 5*a^2*b - a*b^2 + 3*(20*a^2*b - \\
& 9*a*b^2 + b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((5*a*b^2 - b^3)*\cosh(\\
& d*x + c)^7 + 3*(5*a^2*b - a*b^2)*\cosh(d*x + c)^5 + (20*a^2*b - 9*a*b^2 + b^ \\
& 3)*\cosh(d*x + c)^3 + (5*a^2*b - a*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b \\
& /a}*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*s \\
& inh(d*x + c)^3 + (4*a - b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - b)*
\end{aligned}$$

$$\begin{aligned}
& \sinh(dx + c))\sqrt{b/a}/b) + 2*((a^2*b - 5*a*b^2)*\cosh(dx + c)^8 + 8*(a^2 \\
& *b - 5*a*b^2)*\cosh(dx + c)*\sinh(dx + c)^7 + (a^2*b - 5*a*b^2)*\sinh(dx + \\
& c)^8 + 4*(a^3 - 5*a^2*b)*\cosh(dx + c)^6 + 4*(a^3 - 5*a^2*b + 7*(a^2*b - 5* \\
& a*b^2)*\cosh(dx + c)^2)*\sinh(dx + c)^6 + 8*(7*(a^2*b - 5*a*b^2)*\cosh(dx + \\
& c)^3 + 3*(a^3 - 5*a^2*b)*\cosh(dx + c))*\sinh(dx + c)^5 + 2*(4*a^3 - 21*a^ \\
& 2*b + 5*a*b^2)*\cosh(dx + c)^4 + 2*(35*(a^2*b - 5*a*b^2)*\cosh(dx + c)^4 + \\
& 4*a^3 - 21*a^2*b + 5*a*b^2 + 30*(a^3 - 5*a^2*b)*\cosh(dx + c)^2)*\sinh(dx + \\
& c)^4 + 8*(7*(a^2*b - 5*a*b^2)*\cosh(dx + c)^5 + 10*(a^3 - 5*a^2*b)*\cosh(dx \\
& x + c)^3 + (4*a^3 - 21*a^2*b + 5*a*b^2)*\cosh(dx + c))*\sinh(dx + c)^3 + a^ \\
& 2*b - 5*a*b^2 + 4*(a^3 - 5*a^2*b)*\cosh(dx + c)^2 + 4*(7*(a^2*b - 5*a*b^2)* \\
& \cosh(dx + c)^6 + 15*(a^3 - 5*a^2*b)*\cosh(dx + c)^4 + a^3 - 5*a^2*b + 3*(4 \\
& *a^3 - 21*a^2*b + 5*a*b^2)*\cosh(dx + c)^2)*\sinh(dx + c)^2 + 8*((a^2*b - 5 \\
& *a*b^2)*\cosh(dx + c)^7 + 3*(a^3 - 5*a^2*b)*\cosh(dx + c)^5 + (4*a^3 - 21*a \\
& ^2*b + 5*a*b^2)*\cosh(dx + c)^3 + (a^3 - 5*a^2*b)*\cosh(dx + c))*\sinh(dx + \\
& c))*\arctan(\cosh(dx + c) + \sinh(dx + c)) - 2*(a^2*b - b^3)*\cosh(dx + c) \\
& + 2*(7*(a^2*b - b^3)*\cosh(dx + c)^6 + 5*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)* \\
& \cosh(dx + c)^4 - a^2*b + b^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*\cosh(dx \\
& x + c)^2)*\sinh(dx + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*\cosh(dx \\
& x + c)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*\cosh(dx + c)*\sinh(dx \\
& *x + c)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*\sinh(dx + c)^8 + 4*(\\
& a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cosh(dx + c)^6 + 4*(7*(a^4*b - 3*a^ \\
& 3*b^2 + 3*a^2*b^3 - a*b^4)*d*\cosh(dx + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - \\
& a^2*b^3)*d)*\sinh(dx + c)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 \\
& + a*b^4)*d*\cosh(dx + c)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)* \\
& d*\cosh(dx + c)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cosh(dx + c) \\
&)*\sinh(dx + c)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*\cosh(dx \\
& x + c)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cosh(dx + c)^2 + (4* \\
& a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d)*\sinh(dx + c)^4 + 4*(a^ \\
& 5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cosh(dx + c)^2 + 8*(7*(a^4*b - 3*a^3* \\
& b^2 + 3*a^2*b^3 - a*b^4)*d*\cosh(dx + c)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2 \\
& - a^2*b^3)*d*\cosh(dx + c)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + \\
& a*b^4)*d*\cosh(dx + c))*\sinh(dx + c)^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2* \\
& b^3 - a*b^4)*d*\cosh(dx + c)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d \\
& *\cosh(dx + c)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d* \\
& \cosh(dx + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d)*\sinh(dx + c)^2 \\
& + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d + 8*((a^4*b - 3*a^3*b^2 + 3*a^2 \\
& *b^3 - a*b^4)*d*\cosh(dx + c)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d \\
& *\cosh(dx + c)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d*co \\
& sh(dx + c)^3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*\cosh(dx + c))*\sinh \\
& (dx + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.61301, size = 1322, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out]
$$-1/2*(a^5*b*d - 12*a^4*b^2*d + 22*a^3*b^3*d - 12*a^2*b^4*d + a*b^5*d - a*b*\text{abs}(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d) - b^2*\text{abs}(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d))*\arctan(1/2*\sqrt{2}*(e^{(d*x + c)} - e^{(-d*x - c)})/\sqrt{((a^4*d - a^3*b*d - a^2*b^2*d + a*b^3*d + \sqrt{((a^4*d - a^3*b*d - a^2*b^2*d + a*b^3*d)^2 - 4*(a^4*d - 2*a^3*b*d + a^2*b^2*d)*(a^3*b*d - 2*a^2*b^2*d + a*b^3*d))})/(a^3*b*d - 2*a^2*b^2*d + a*b^3*d)))/(a^4*d*\text{abs}(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d) - a^3*b*d*\text{abs}(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d) + a*b^3*d*\text{abs}(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d) + (a^4*d - 3*a^3*b*d + 3*a^2*b^2*d - a*b^3*d)^2) - 1/2*((a^5 - 12*a^4*b + 22*a^3*b^2 - 12*a^2*b^3 + a*b^4)*\sqrt{a*b}*d*\text{abs}(b) + \sqrt{a*b}*(a + b)*\text{abs}(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d)*\text{abs}(b))*\arctan(1/2*\sqrt{2}*(e^{(d*x + c)} - e^{(-d*x - c)})/\sqrt{((a^4*d - a^3*b*d - a^2*b^2*d + a*b^3*d - \sqrt{((a^4*d - a^3*b*d - a^2*b^2*d + a*b^3*d)^2 - 4*(a^4*d - 2*a^3*b*d + a^2*b^2*d)*(a^3*b*d - 2*a^2*b^2*d + a*b^3*d))})/(a^3*b*d - 2*a^2*b^2*d + a*b^3*d)))/((a^4*d - 3*a^3*b*d + 3*a^2*b^2*d - a*b^3*d)^2*b - (a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*d*\text{abs}(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d)) + (a*b*(e^{(d*x + c)} - e^{(-d*x - c)})^3 + b^2*(e^{(d*x + c)} - e^{(-d*x - c)})^3 + 4*a^2*(e^{(d*x + c)} - e^{(-d*x - c)}) + 4*b^2*(e^{(d*x + c)} - e^{(-d*x - c)}))/((b*(e^{(d*x + c)} - e^{(-d*x - c)})^4 + 4*a*(e^{(d*x + c)} - e^{(-d*x - c)})^2 + 4*b*(e^{(d*x + c)} - e^{(-d*x - c)})^2 + 16*a)*(a^3*d - 2*a^2*b*d + a*b^2*d))$$

$$3.338 \quad \int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^2} dx$$

Optimal. Leaf size=143

$$\frac{b^2(6a-b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}} - \frac{b^3 \tanh(c+dx)}{2ad(a-b)^3(a-(a-b) \tanh^2(c+dx))} - \frac{\tanh^3(c+dx)}{3d(a-b)^2} + \frac{(a-3b) \tanh(c+dx)}{d(a-b)^3}$$

[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^(7/2)*d) + ((a - 3*b)*Tanh[c + d*x])/((a - b)^3*d) - Tanh[c + d*x]^3/(3*(a - b)^2*d) - (b^3*Tanh[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.207041, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3191, 390, 385, 208}

$$\frac{b^2(6a-b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}} - \frac{b^3 \tanh(c+dx)}{2ad(a-b)^3(a-(a-b) \tanh^2(c+dx))} - \frac{\tanh^3(c+dx)}{3d(a-b)^2} + \frac{(a-3b) \tanh(c+dx)}{d(a-b)^3}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^(7/2)*d) + ((a - 3*b)*Tanh[c + d*x])/((a - b)^3*d) - Tanh[c + d*x]^3/(3*(a - b)^2*d) - (b^3*Tanh[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\sinh^2(c+dx))^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{(a-(a-b)x^2)^2} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a-3b}{(a-b)^3} - \frac{x^2}{(a-b)^2} + \frac{(3a-b)b^2-3(a-b)b^2x^2}{(a-b)^3(a+(-a+b)x^2)^2}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{(a-3b)\tanh(c+dx)}{(a-b)^3d} - \frac{\tanh^3(c+dx)}{3(a-b)^2d} + \frac{\operatorname{Subst}\left(\int \frac{(3a-b)b^2-3(a-b)b^2x^2}{(a+(-a+b)x^2)^2} dx, x, \tanh(c+dx)\right)}{(a-b)^3d} \\
&= \frac{(a-3b)\tanh(c+dx)}{(a-b)^3d} - \frac{\tanh^3(c+dx)}{3(a-b)^2d} - \frac{b^3\tanh(c+dx)}{2a(a-b)^3d(a-(a-b)\tanh^2(c+dx))} + \frac{((6a-b)^2)}{2a(a-b)^3d} \\
&= \frac{(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d} + \frac{(a-3b)\tanh(c+dx)}{(a-b)^3d} - \frac{\tanh^3(c+dx)}{3(a-b)^2d} - \frac{b^3\tanh(c+dx)}{2a(a-b)^3d}
\end{aligned}$$

Mathematica [A] time = 1.89396, size = 130, normalized size = 0.91

$$\frac{3b^2(6a-b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{7/2}} + \frac{2 \tanh(c+dx) \left((a-b) \operatorname{sech}^2(c+dx) + 2(a-4b) \right) - \frac{3b^3 \sinh(2(c+dx))}{a(2a+b \cosh(2(c+dx))-b)}}{(a-b)^3}$$

$$6d$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]

[Out] ((3*(6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a - b)^(7/2)) + ((-3*b^3*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])) + 2*(2*(a - 4*b) + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x]/(a - b)^3)/(6*d)

Maple [B] time = 0.092, size = 998, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)

[Out]
$$\begin{aligned} & -1/d*b^3/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/a*\tanh(1/2*d*x+1/2*c)^3-1/d*b^3/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)/a*\tanh(1/2*d*x+1/2*c)+3/d*b^2/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*b^3/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*b^2/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*b^3/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d*b^4/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d*b^4/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^3*\tanh(1/2*d*x+1/2*c)^5*a \end{aligned}$$

$$-6/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^3*\tanh(1/2*d*x+1/2*c)^5*b+4/3/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^3*\tanh(1/2*d*x+1/2*c)^3*a-28/3/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^3*\tanh(1/2*d*x+1/2*c)^3*b+2/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^3*\tanh(1/2*d*x+1/2*c)*a-6/d/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^2+1)^3*\tanh(1/2*d*x+1/2*c)*b$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.66689, size = 17920, normalized size = 125.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/12*(12*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^8 + 96*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + 12*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*sinh(d*x + c)^8 + 24*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 + 24*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4 + 14*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 48*(14*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^3 + 3*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 16*a^4*b + 80*a^3*b^2 - 52*a^2*b^3 - 12*a*b^4 - 8*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^4 - 8*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3 - 105*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 - 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 + 15*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^3 - (24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(8*a^5 - 38*a^4*b + 25*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2 + 8*(42*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 - 8*a^5 + 38*a^4*b - 25*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 + 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3

$$\begin{aligned}
& + a*b^4)*\cosh(d*x + c)^4 - 6*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3) \\
& * \cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 3*((6*a*b^3 - b^4)*\cosh(d*x + c)^{10} + 1 \\
& 0*(6*a*b^3 - b^4)*\cosh(d*x + c)*\sinh(d*x + c)^9 + (6*a*b^3 - b^4)*\sinh(d*x \\
& + c)^{10} + (24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c)^8 + (24*a^2*b^2 + 2*a* \\
& b^3 - b^4 + 45*(6*a*b^3 - b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(6* \\
& a*b^3 - b^4)*\cosh(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c))* \\
& \sinh(d*x + c)^7 + 2*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c)^6 + 2*(105* \\
& (6*a*b^3 - b^4)*\cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 14*(24*a^2* \\
& b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - b^ \\
& 4)*\cosh(d*x + c)^5 + 14*(24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c)^3 + 3*(3 \\
& 6*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(36*a^2*b^2 \\
& - 12*a*b^3 + b^4)*\cosh(d*x + c)^4 + 2*(105*(6*a*b^3 - b^4)*\cosh(d*x + c)^6 \\
& + 35*(24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 + \\
& b^4 + 15*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + \\
& 6*a*b^3 - b^4 + 8*(15*(6*a*b^3 - b^4)*\cosh(d*x + c)^7 + 7*(24*a^2*b^2 + 2*a \\
& *b^3 - b^4)*\cosh(d*x + c)^5 + 5*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c) \\
& ^3 + (36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (24*a^2 \\
& *b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c)^2 + (45*(6*a*b^3 - b^4)*\cosh(d*x + c)^8 \\
& + 28*(24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c)^6 + 30*(36*a^2*b^2 - 12*a* \\
& b^3 + b^4)*\cosh(d*x + c)^4 + 24*a^2*b^2 + 2*a*b^3 - b^4 + 12*(36*a^2*b^2 - \\
& 12*a*b^3 + b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - b^4)*\cos \\
& h(d*x + c)^9 + 4*(24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c)^7 + 6*(36*a^2*b \\
& ^2 - 12*a*b^3 + b^4)*\cosh(d*x + c)^5 + 4*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh \\
& (d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\text{sq} \\
& \text{rt}(a^2 - a*b)*\log((b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^ \\
& 3 + b^2*\sinh(d*x + c)^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*b^2*\cosh(d \\
& *x + c)^2 + 2*a*b - b^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*\cos \\
& h(d*x + c)^3 + (2*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(b*\cosh(d*x + \\
& c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\text{sqrt} \\
& (a^2 - a*b))/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sin \\
& h(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - \\
& b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(\\
& d*x + c) + b)) + 16*(6*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*\cosh(d*x + c)^7 + 9* \\
& (6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*\cosh(d*x + c)^5 - 2*(24*a^5 - 106*a \\
& ^4*b + 95*a^3*b^2 - 13*a^2*b^3)*\cosh(d*x + c)^3 - (8*a^5 - 38*a^4*b + 25*a^ \\
& 3*b^2 + 2*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c))/((a^6*b - 4*a^5* \\
& b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^{10} + 10*(a^6*b - 4*a \\
& ^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + \\
& (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\sinh(d*x + c)^{10} + \\
& (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*\cosh(d*x + c)^8 + \\
& (45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^ \\
& 2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d)*\sinh(d*x + c) \\
& ^8 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)* \\
& d*\cosh(d*x + c)^6 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2* \\
& b^5)*d*\cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*
\end{aligned}$$

$$\begin{aligned}
& b^5) * d * \cosh(dx + c) * \sinh(dx + c)^7 + 2 * (105 * (a^6 * b - 4 * a^5 * b^2 + 6 * a^4 * b^3 - 4 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^4 + 14 * (4 * a^7 - 15 * a^6 * b + 20 * a^5 * b^2 - 10 * a^4 * b^3 + a^2 * b^5) * d * \cosh(dx + c)^2 + (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d) * \sinh(dx + c)^6 + 2 * (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^4 + 4 * (63 * (a^6 * b - 4 * a^5 * b^2 + 6 * a^4 * b^3 - 4 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^5 + 14 * (4 * a^7 - 15 * a^6 * b + 20 * a^5 * b^2 - 10 * a^4 * b^3 + a^2 * b^5) * d * \cosh(dx + c)^3 + 3 * (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2 * (105 * (a^6 * b - 4 * a^5 * b^2 + 6 * a^4 * b^3 - 4 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^6 + 35 * (4 * a^7 - 15 * a^6 * b + 20 * a^5 * b^2 - 10 * a^4 * b^3 + a^2 * b^5) * d * \cosh(dx + c)^4 + 15 * (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d) * \sinh(dx + c)^4 + (4 * a^7 - 15 * a^6 * b + 20 * a^5 * b^2 - 10 * a^4 * b^3 + a^2 * b^5) * d * \cosh(dx + c)^2 + 8 * (15 * (a^6 * b - 4 * a^5 * b^2 + 6 * a^4 * b^3 - 4 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^7 + 7 * (4 * a^7 - 15 * a^6 * b + 20 * a^5 * b^2 - 10 * a^4 * b^3 + a^2 * b^5) * d * \cosh(dx + c)^5 + 5 * (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^3 + (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + (45 * (a^6 * b - 4 * a^5 * b^2 + 6 * a^4 * b^3 - 4 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c)^8 + 28 * (4 * a^7 - 15 * a^6 * b + 20 * a^5 * b^2 - 10 * a^4 * b^3 + a^2 * b^5) * d * \cosh(dx + c)^6 + 30 * (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^4 + 12 * (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^2 + (4 * a^7 - 15 * a^6 * b + 20 * a^5 * b^2 - 10 * a^4 * b^3 + a^2 * b^5) * d) * \sinh(dx + c)^2 + (a^6 * b - 4 * a^5 * b^2 + 6 * a^4 * b^3 - 4 * a^3 * b^4 + a^2 * b^5) * d + 2 * (5 * (a^6 * b - 4 * a^5 * b^2 + 6 * a^4 * b^3 - 4 * a^3 * b^4 + a^2 * b^5) * d * \cosh(dx + c))^9 + 4 * (4 * a^7 - 15 * a^6 * b + 20 * a^5 * b^2 - 10 * a^4 * b^3 + a^2 * b^5) * d * \cosh(dx + c)^7 + 6 * (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^5 + 4 * (6 * a^7 - 25 * a^6 * b + 40 * a^5 * b^2 - 30 * a^4 * b^3 + 10 * a^3 * b^4 - a^2 * b^5) * d * \cosh(dx + c)^3 + (4 * a^7 - 15 * a^6 * b + 20 * a^5 * b^2 - 10 * a^4 * b^3 + a^2 * b^5) * d * \cosh(dx + c)) * \sinh(dx + c), 1/6 * (6 * (6 * a^3 * b^2 - 7 * a^2 * b^3 + a * b^4) * \cosh(dx + c)^8 + 48 * (6 * a^3 * b^2 - 7 * a^2 * b^3 + a * b^4) * \cosh(dx + c) * \sinh(dx + c)^7 + 6 * (6 * a^3 * b^2 - 7 * a^2 * b^3 + a * b^4) * \sinh(dx + c)^8 + 12 * (6 * a^4 * b - a^3 * b^2 - 6 * a^2 * b^3 + a * b^4) * \cosh(dx + c)^6 + 12 * (6 * a^4 * b - a^3 * b^2 - 6 * a^2 * b^3 + a * b^4 + 14 * (6 * a^3 * b^2 - 7 * a^2 * b^3 + a * b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^6 + 24 * (14 * (6 * a^3 * b^2 - 7 * a^2 * b^3 + a * b^4) * \cosh(dx + c)^3 + 3 * (6 * a^4 * b - a^3 * b^2 - 6 * a^2 * b^3 + a * b^4) * \cosh(dx + c)) * \sinh(dx + c)^5 - 8 * a^4 * b + 40 * a^3 * b^2 - 26 * a^2 * b^3 - 6 * a * b^4 - 4 * (24 * a^5 - 106 * a^4 * b + 95 * a^3 * b^2 - 13 * a^2 * b^3) * \cosh(dx + c)^4 - 4 * (24 * a^5 - 106 * a^4 * b + 95 * a^3 * b^2 - 13 * a^2 * b^3 - 105 * (6 * a^3 * b^2 - 7 * a^2 * b^3 + a * b^4) * \cosh(dx + c)^4 - 45 * (6 * a^4 * b - a^3 * b^2 - 6 * a^2 * b^3 + a * b^4) * \cosh(dx + c)^2) * \sinh(dx + c)^4 + 16 * (21 * (6 * a^3 * b^2 - 7 * a^2 * b^3 + a * b^4) * \cosh(dx + c)^5 + 15 * (6 * a^4 * b - a^3 * b^2 - 6 * a^2 * b^3 + a * b^4) * \cosh(dx + c)^3 - (24 * a^5 - 106 * a^4 * b + 95 * a^3 * b^2 - 13 * a^2 * b^3) * \cosh(dx + c)) * \sinh(dx + c)^3 - 4 * (8 * a^5 - 38 * a^4 * b + 25 * a^3 * b^2 + 2 * a^2 * b^3 + 3 * a * b^4) * \cosh(dx + c)^2 + 4 * (42 * (6 * a^3 * b^2 - 7 * a^2 *
\end{aligned}$$

$$\begin{aligned}
& *b^3 + a*b^4)*\cosh(d*x + c)^6 - 8*a^5 + 38*a^4*b - 25*a^3*b^2 - 2*a^2*b^3 - \\
& 3*a*b^4 + 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*\cosh(d*x + c)^4 - 6*(\\
& 24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c \\
&)^2 - 3*((6*a*b^3 - b^4)*\cosh(d*x + c)^{10} + 10*(6*a*b^3 - b^4)*\cosh(d*x + c \\
&)*\sinh(d*x + c)^9 + (6*a*b^3 - b^4)*\sinh(d*x + c)^{10} + (24*a^2*b^2 + 2*a*b^3 \\
& - b^4)*\cosh(d*x + c)^8 + (24*a^2*b^2 + 2*a*b^3 - b^4 + 45*(6*a*b^3 - b^4) \\
& *\cosh(d*x + c)^2)*\sinh(d*x + c)^8 + 8*(15*(6*a*b^3 - b^4)*\cosh(d*x + c)^3 + \\
& (24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(36*a^2*b^ \\
& 2 - 12*a*b^3 + b^4)*\cosh(d*x + c)^6 + 2*(105*(6*a*b^3 - b^4)*\cosh(d*x + c)^ \\
& 4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 14*(24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x \\
& + c)^2)*\sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - b^4)*\cosh(d*x + c)^5 + 14*(24*a^ \\
& 2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c)^3 + 3*(36*a^2*b^2 - 12*a*b^3 + b^4)*\co \\
& sh(d*x + c))*\sinh(d*x + c)^5 + 2*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c \\
&)^4 + 2*(105*(6*a*b^3 - b^4)*\cosh(d*x + c)^6 + 35*(24*a^2*b^2 + 2*a*b^3 - b \\
& ^4)*\cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 15*(36*a^2*b^2 - 12*a*b \\
& ^3 + b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 6*a*b^3 - b^4 + 8*(15*(6*a*b^3 \\
& - b^4)*\cosh(d*x + c)^7 + 7*(24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x + c)^5 + \\
& 5*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c)^3 + (36*a^2*b^2 - 12*a*b^3 + \\
& b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*\cosh(d*x \\
& + c)^2 + (45*(6*a*b^3 - b^4)*\cosh(d*x + c)^8 + 28*(24*a^2*b^2 + 2*a*b^3 - \\
& b^4)*\cosh(d*x + c)^6 + 30*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c)^4 + 2 \\
& 4*a^2*b^2 + 2*a*b^3 - b^4 + 12*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c)^ \\
& 2)*\sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - b^4)*\cosh(d*x + c)^9 + 4*(24*a^2*b^2 + \\
& 2*a*b^3 - b^4)*\cosh(d*x + c)^7 + 6*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x \\
& + c)^5 + 4*(36*a^2*b^2 - 12*a*b^3 + b^4)*\cosh(d*x + c)^3 + (24*a^2*b^2 + 2* \\
& a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a^2 + a*b}*\arctan(-1/2*(b* \\
& \cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a \\
& - b)*\sqrt{-a^2 + a*b}/(a^2 - a*b)) + 8*(6*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)* \\
& \cosh(d*x + c)^7 + 9*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*\cosh(d*x + c)^5 \\
& - 2*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*\cosh(d*x + c)^3 - (8*a^ \\
& 5 - 38*a^4*b + 25*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c))*\sinh(d*x + \\
& c))/((a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^ \\
& 10 + 10*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + \\
& c)*\sinh(d*x + c)^9 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)* \\
& d*\sinh(d*x + c)^{10} + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5) \\
& *d*\cosh(d*x + c)^8 + (45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b \\
& ^5)*d*\cosh(d*x + c)^2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b \\
& ^5)*d)*\sinh(d*x + c)^8 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10 \\
& *a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^6 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^ \\
& 3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 \\
& - 10*a^4*b^3 + a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^6*b - \\
& 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^4 + 14*(4*a^7 \\
& - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*\cosh(d*x + c)^2 + (6*a^7 \\
& - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d)*\sinh(d*x + \\
& c)^6 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^
\end{aligned}$$

$$\begin{aligned}
& 5)*d*\cosh(d*x + c)^4 + 4*(63*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^5 + 14*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 \\
& + a^2*b^5)*d*\cosh(d*x + c)^3 + 3*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^6*b \\
& - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^6 + 35*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*\cosh(d*x + c)^4 + 15*(6 \\
& *a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^2 + (6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b \\
& ^5)*d*\sinh(d*x + c)^4 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*\cosh(d*x + c)^2 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + \\
& a^2*b^5)*d*\cosh(d*x + c)^7 + 7*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*\cosh(d*x + c)^5 + 5*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 \\
& + 10*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^3 + (6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + \\
& (45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^8 + 28*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*\cosh(d*x + c) \\
&)^6 + 30*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^4 + 12*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a \\
& ^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*\sinh(d*x + c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a \\
& ^3*b^4 + a^2*b^5)*d + 2*(5*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^9 + 4*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a \\
& ^2*b^5)*d*\cosh(d*x + c)^7 + 6*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^5 + 4*(6*a^7 - 25*a^6*b + 40*a^5*b^2 \\
& - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out

Giac [B] time = 1.38008, size = 377, normalized size = 2.64

$$\frac{(6ab^2 - b^3) \arctan\left(\frac{be^{(2dx+2c)} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{2(a^4d - 3a^3bd + 3a^2b^2d - ab^3d)\sqrt{-a^2 + ab}} + \frac{2ab^2e^{(2dx+2c)} - b^3e^{(2dx+2c)} + b^3}{(a^4d - 3a^3bd + 3a^2b^2d - ab^3d)(be^{(4dx+4c)} + 4ae^{(2dx+2c)} - 2be^{(2dx+2c)} + 1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] 1/2*(6*a*b^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^4*d - 3*a^3*b*d + 3*a^2*b^2*d - a*b^3*d)*sqrt(-a^2 + a*b)) + (2*a*b^2*e^(2*d*x + 2*c) - b^3*e^(2*d*x + 2*c) + b^3)/((a^4*d - 3*a^3*b*d + 3*a^2*b^2*d - a*b^3*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)) + 4/3*(3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 9*b*e^(2*d*x + 2*c) - a + 4*b)/((a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d)*(e^(2*d*x + 2*c) + 1)^3)

$$3.339 \quad \int \frac{\cosh^6(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=160

$$\frac{\sqrt{a-b}(8a^2+4ab+3b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} - \frac{(a-b)(4a+3b) \tanh(c+dx)}{8a^2b^2d(a-(a-b) \tanh^2(c+dx))} - \frac{(a-b) \tanh(c+dx)}{4abd(a-(a-b) \tanh^2(c+dx))}$$

[Out] x/b^3 - (Sqrt[a - b]*(8*a^2 + 4*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) - ((a - b)*Tanh[c + d*x])/(4*a*b*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((a - b)*(4*a + 3*b)*Tanh[c + d*x])/(8*a^2*b^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.236404, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3191, 414, 527, 522, 206, 208}

$$\frac{\sqrt{a-b}(8a^2+4ab+3b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} - \frac{(a-b)(4a+3b) \tanh(c+dx)}{8a^2b^2d(a-(a-b) \tanh^2(c+dx))} - \frac{(a-b) \tanh(c+dx)}{4abd(a-(a-b) \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] x/b^3 - (Sqrt[a - b]*(8*a^2 + 4*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) - ((a - b)*Tanh[c + d*x])/(4*a*b*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((a - b)*(4*a + 3*b)*Tanh[c + d*x])/(8*a^2*b^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 414

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]

```

Rule 527

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

```

Rule 522

```

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]

```

Rule 206

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^6(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= -\frac{(a-b)\tanh(c+dx)}{4abd(a-(a-b)\tanh^2(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{-a-3b-3(a-b)x^2}{(1-x^2)(a+(-a+b)x^2)^2} dx, x, \tanh(c+dx)\right)}{4abd} \\
&= -\frac{(a-b)\tanh(c+dx)}{4abd(a-(a-b)\tanh^2(c+dx))^2} - \frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d(a-(a-b)\tanh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{4a^2-}{1-x^2}\right)}{8a^2b^2d} \\
&= -\frac{(a-b)\tanh(c+dx)}{4abd(a-(a-b)\tanh^2(c+dx))^2} - \frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d(a-(a-b)\tanh^2(c+dx))} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\right)}{8a^2b^2d} \\
&= \frac{x}{b^3} - \frac{\sqrt{a-b}(8a^2+4ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} - \frac{(a-b)\tanh(c+dx)}{4abd(a-(a-b)\tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 1.34049, size = 164, normalized size = 1.02

$$\frac{(-4a^2b+8a^3-ab^2-3b^3)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}} + \frac{3b(-2a^2+ab+b^2)\sinh(2(c+dx))}{a^2(2a+b\cosh(2(c+dx))-b)} + \frac{4b(a-b)^2\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)^2} + 8(c+dx)$$

$8b^3d$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (8*(c + d*x) - ((8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b])) + (4*(a - b)^2*b*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])^2) + (3*b*(-2*a^2 + a*b + b^2)*Sinh[2*(c + d*x)]/(a^2*(2*a - b + b*Cosh[2*(c + d*x)])))/(8*b^3*d)

Maple [B] time = 0.076, size = 2048, normalized size = 12.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cosh(d*x+c)^6/(a+b*\sinh(d*x+c)^2)^3,x)$

[Out]
$$\begin{aligned} & -1/d/b^3*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d/b^3*\ln(\tanh(1/2*d*x+1/2*c)+1)+1/d/b^2*a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-3/8/d*b/a^2/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+1/d/b^2*a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-3/8/d*b/a^2/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+3/d*b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*\tanh(1/2*d*x+1/2*c)^3+1/2/d/b^2/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/2/d/b^2/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-1/4/d/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^7-23/4/d/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^5-23/4/d/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^3-1/4/d/b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)+5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)-3/8/d/a^2/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+3/8/d/a^2/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+7/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^3+5/4/d/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^5-1/d/b^2/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*\tanh(1/2*d*x+1/2*c)+1/d/b^2/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*\tanh(1/2*d*x+1/2*c)^5+3/d*b/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*\tanh(1/2*d*x+1/2*c)^5+1/d/b^2/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*\tanh(1/2*d*x+1/2*c)^3-1/d/b^3*a/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})+1/d/b^3*a/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-1/d/b^2/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*\tanh(1/2*d*x+1/2*c)^7-1/8/d/a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/8/d/a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-1/8/d/b/a/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/8/d/b/a/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-1/8/d/b/a/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-1/8/d/b/a/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}) \end{aligned}$$

$$\begin{aligned} & -a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)* \\ & a)^{(1/2)}))+1/8/d/b/a/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2 \\ & *d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}))-1/2/d/b/(-b*(a-b))^{(1/2)}/ \\ & ((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b* \\ & (a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}))-1/2/d/b/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)} \\ & -a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a \\ &)^{(1/2)}) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.55732, size = 12741, normalized size = 79.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(16*a^2*b^2*d*x*cosh(d*x + c)^8 + 128*a^2*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + 16*a^2*b^2*d*x*sinh(d*x + c)^8 + 4*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^6 + 4*(112*a^2*b^2*d*x*cosh(d*x + c)^2 + 16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*sinh(d*x + c)^6 + 16*a^2*b^2*d*x + 8*(112*a^2*b^2*d*x*cosh(d*x + c)^3 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^4 + 4*(280*a^2*b^2*d*x*cosh(d*x + c)^4 + 48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x + 15*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 24*a^2*b^2 - 12*a*b^3 - 12*b^4 + 16*(56*a^2*b^2*d*x*cosh(d*x + c)^5 + 5*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^3 + (48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(

$$\begin{aligned}
& 2*b^5)*d*cosh(d*x + c)^2 + 8*(7*a^2*b^5*d*cosh(d*x + c)^5 + 10*(2*a^3*b^4 - \\
& a^2*b^5)*d*cosh(d*x + c)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d* \\
& x + c))*sinh(d*x + c)^3 + 4*(7*a^2*b^5*d*cosh(d*x + c)^6 + 15*(2*a^3*b^4 - \\
& a^2*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d \\
& *x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^2 + 8*(a^2*b^5*d*cosh(d* \\
& x + c)^7 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + (8*a^4*b^3 - 8*a^3*b \\
& ^4 + 3*a^2*b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))* \\
& sinh(d*x + c)), 1/8*(8*a^2*b^2*d*x*cosh(d*x + c)^8 + 64*a^2*b^2*d*x*cosh(d* \\
& x + c)*sinh(d*x + c)^7 + 8*a^2*b^2*d*x*sinh(d*x + c)^8 + 2*(16*a^3*b - 20*a \\
& ^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^6 + 2*(1 \\
& 12*a^2*b^2*d*x*cosh(d*x + c)^2 + 16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16 \\
& *(2*a^3*b - a^2*b^2)*d*x)*sinh(d*x + c)^6 + 8*a^2*b^2*d*x + 4*(112*a^2*b^2* \\
& d*x*cosh(d*x + c)^3 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3* \\
& b - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(48*a^4 - 72*a^3*b + 1 \\
& 8*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d* \\
& x + c)^4 + 2*(280*a^2*b^2*d*x*cosh(d*x + c)^4 + 48*a^4 - 72*a^3*b + 18*a^2* \\
& b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x + 15*(16*a^3*b \\
& - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^2 \\
&)*sinh(d*x + c)^4 + 12*a^2*b^2 - 6*a*b^3 - 6*b^4 + 8*(56*a^2*b^2*d*x*cosh(d \\
& *x + c)^5 + 5*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^ \\
& 2)*d*x)*cosh(d*x + c)^3 + (48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^ \\
& 4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2 \\
& *(32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*co \\
& sh(d*x + c)^2 + 2*(112*a^2*b^2*d*x*cosh(d*x + c)^6 + 15*(16*a^3*b - 20*a^2* \\
& b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^4 + 32*a^3*b \\
& - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x + 6*(48*a^4 \\
& - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2 \\
&)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b^2 + 4*a*b^3 + 3*b^4)*co \\
& sh(d*x + c)^8 + 8*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c) \\
& ^7 + (8*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b + 2*a*b^3 \\
& - 3*b^4)*cosh(d*x + c)^6 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 + 4 \\
& *a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 + 4*a*b^ \\
& 3 + 3*b^4)*cosh(d*x + c)^3 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))* \\
& sinh(d*x + c)^5 + 2*(64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cos \\
& h(d*x + c)^4 + 2*(35*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 64*a^4 \\
& - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30*(16*a^3*b + 2*a*b^3 - 3*b^ \\
& 4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 + 4*a*b^3 + 3*b^4 + 8*(7*(8 \\
& *a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(16*a^3*b + 2*a*b^3 - 3*b^ \\
& 4)*cosh(d*x + c)^3 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*co \\
& sh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c) \\
& ^2 + 4*(7*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b + 2* \\
& a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b + 2*a*b^3 - 3*b^4 + 3*(64*a^4 - 3 \\
& 2*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + \\
& 8*((8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(16*a^3*b + 2*a*b^3 - \\
& 3*b^4)*cosh(d*x + c)^5 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^
\end{aligned}$$

4)*cosh(d*x + c)^3 + (16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) + 4*(16*a^2*b^2*d*x*cosh(d*x + c)^7 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^5 + 2*(48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^3 + (32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^5*d*cosh(d*x + c)^8 + 8*a^2*b^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*b^5*d*sinh(d*x + c)^8 + a^2*b^5*d + 4*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*a^2*b^5*d*cosh(d*x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*a^2*b^5*d*cosh(d*x + c)^3 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*b^5*d*cosh(d*x + c)^4 + 30*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + 8*(7*a^2*b^5*d*cosh(d*x + c)^5 + 10*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^2*b^5*d*cosh(d*x + c)^6 + 15*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^2 + 8*(a^2*b^5*d*cosh(d*x + c)^7 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.19356, size = 481, normalized size = 3.01

$$\frac{dx+c}{b^3d} - \frac{(8a^3 - 4a^2b - ab^2 - 3b^3) \arctan\left(\frac{be^{2dx+2c} + 2a-b}{2\sqrt{-a^2+ab}}\right)}{8\sqrt{-a^2+ab}a^2b^3d} + \frac{16a^3be^{6dx+6c} - 20a^2b^2e^{6dx+6c} + ab^3e^{6dx+6c} + 3b^4e^{6dx+6c}}{8\sqrt{-a^2+ab}a^2b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] $(d*x + c)/(b^3*d) - 1/8*(8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*\arctan(1/2*(b*e^{2*d*x + 2*c} + 2*a - b)/\sqrt{-a^2 + a*b})/(\sqrt{-a^2 + a*b})*a^2*b^3*d + 1/4*(16*a^3*b*e^{6*d*x + 6*c} - 20*a^2*b^2*e^{6*d*x + 6*c} + a*b^3*e^{6*d*x + 6*c} + 3*b^4*e^{6*d*x + 6*c} + 48*a^4*e^{4*d*x + 4*c} - 72*a^3*b*e^{4*d*x + 4*c} + 18*a^2*b^2*e^{4*d*x + 4*c} + 15*a*b^3*e^{4*d*x + 4*c} - 9*b^4*e^{4*d*x + 4*c} + 32*a^3*b*e^{2*d*x + 2*c} - 28*a^2*b^2*e^{2*d*x + 2*c} - 13*a*b^3*e^{2*d*x + 2*c} + 9*b^4*e^{2*d*x + 2*c} + 6*a^2*b^2 - 3*a*b^3 - 3*b^4)/((b*e^{4*d*x + 4*c} + 4*a*e^{2*d*x + 2*c} - 2*b*e^{2*d*x + 2*c} + b)^2*a^2*b^3*d)$

$$3.340 \quad \int \frac{\cosh^5(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^3} dx$$

Optimal. Leaf size=133

$$\frac{3\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \sinh(c+dx)}{8d\left(a+b \sinh^2(c+dx)\right)} + \frac{(3a^2+2ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} - \frac{(a-b) \sinh(c+dx) \cosh^2(c+dx)}{4abd\left(a+b \sinh^2(c+dx)\right)^2}$$

[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(5/2)*d) - ((a - b)*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Sinh[c + d*x])/(8*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.124537, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 413, 385, 205}

$$\frac{3\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \sinh(c+dx)}{8d\left(a+b \sinh^2(c+dx)\right)} + \frac{(3a^2+2ab+3b^2) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} - \frac{(a-b) \sinh(c+dx) \cosh^2(c+dx)}{4abd\left(a+b \sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(5/2)*d) - ((a - b)*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Sinh[c + d*x])/(8*d*(a + b*Sinh[c + d*x]^2))

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^5(c + dx)}{(a + b \sinh^2(c + dx))^3} dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^2)^3} dx, x, \sinh(c + dx)\right)}{d}$$

$$= -\frac{(a-b) \cosh^2(c + dx) \sinh(c + dx)}{4abd (a + b \sinh^2(c + dx))^2} + \frac{\text{Subst}\left(\int \frac{a+3b+(3a+b)x^2}{(a+bx^2)^2} dx, x, \sinh(c + dx)\right)}{4abd}$$

$$= -\frac{(a-b) \cosh^2(c + dx) \sinh(c + dx)}{4abd (a + b \sinh^2(c + dx))^2} - \frac{3(a^2 - b^2) \sinh(c + dx)}{8a^2b^2d (a + b \sinh^2(c + dx))} + \frac{(3a^2 + 2ab + 3b^2)}{(a + b \sinh^2(c + dx))}$$

$$= \frac{(3a^2 + 2ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} - \frac{(a-b) \cosh^2(c + dx) \sinh(c + dx)}{4abd (a + b \sinh^2(c + dx))^2} - \frac{3(a^2 - b^2)}{8a^2b^2d (a + b \sinh^2(c + dx))}$$

Mathematica [A] time = 0.352346, size = 149, normalized size = 1.12

$$\frac{\frac{2\sqrt{a}\sqrt{b}(5a^2-2ab-3b^2) \sinh(c+dx)}{2a+b \cosh(2(c+dx))-b} - (3a^2 + 2ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{a}\text{csch}(c+dx)}{\sqrt{b}}\right) + \frac{8a^{3/2}\sqrt{b}(a-b)^2 \sinh(c+dx)}{(2a+b \cosh(2(c+dx))-b)^2}}{8a^{5/2}b^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]

[Out]
$$\begin{aligned} & -((3a^2 + 2ab + 3b^2) \operatorname{ArcTan}[\frac{\sqrt{a} \operatorname{Csch}[c + dx]}{\sqrt{b}}]) + (8a^{3/2} \\ & (a - b)^2 \sqrt{b} \operatorname{Sinh}[c + dx]) / (2a - b + b \operatorname{Cosh}[2(c + dx)])^2 - \\ & (2\sqrt{a} \sqrt{b} (5a^2 - 2ab - 3b^2) \operatorname{Sinh}[c + dx]) / (2a - b + b \operatorname{Cosh}[2(c + dx)]) \\ &) / (8a^{5/2} b^{5/2} d) \end{aligned}$$

Maple [B] time = 0.062, size = 1884, normalized size = 14.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x)

[Out]
$$\begin{aligned} & -3/8/d/b^2a/(-b(a-b))^{1/2}/((2(-b(a-b))^{1/2}-a+2b)a)^{1/2} \operatorname{arctan}(a \\ & * \operatorname{tanh}(1/2dx+1/2c)/((2(-b(a-b))^{1/2}-a+2b)a)^{1/2}) + 3/8/d/b/a^2/(-b \\ & (a-b))^{1/2}/((2(-b(a-b))^{1/2}-a+2b)a)^{1/2} \operatorname{arctan}(a \operatorname{tanh}(1/2dx+1/2 \\ & *c)/((2(-b(a-b))^{1/2}-a+2b)a)^{1/2}) - 3/8/d/b^2a/(-b(a-b))^{1/2}/((2 \\ & (-b(a-b))^{1/2}+a-2b)a)^{1/2} \operatorname{arctanh}(a \operatorname{tanh}(1/2dx+1/2c)/((2(-b(a-b) \\ &))^{1/2}+a-2b)a)^{1/2}) + 3/8/d/b/a^2/(-b(a-b))^{1/2}/((2(-b(a-b))^{1/2} \\ & +a-2b)a)^{1/2} \operatorname{arctanh}(a \operatorname{tanh}(1/2dx+1/2c)/((2(-b(a-b))^{1/2}+a-2b) \\ & a)^{1/2}) + 3/d/b/(\operatorname{tanh}(1/2dx+1/2c))^4 a - 2 \operatorname{tanh}(1/2dx+1/2c)^2 a + 4 \operatorname{tanh}(1 \\ & /2dx+1/2c)^2 b + a)^2/a^2 \operatorname{tanh}(1/2dx+1/2c)^3 - 3/8/d/b^2/((2(-b(a-b))^{1/2} \\ & +a-2b)a)^{1/2} \operatorname{arctanh}(a \operatorname{tanh}(1/2dx+1/2c)/((2(-b(a-b))^{1/2}+a-2 \\ & *b)a)^{1/2}) + 3/8/d/b^2/((2(-b(a-b))^{1/2}-a+2b)a)^{1/2} \operatorname{arctan}(a \operatorname{tanh}(\\ & 1/2dx+1/2c)/((2(-b(a-b))^{1/2}-a+2b)a)^{1/2}) + 1/2/d/b/(\operatorname{tanh}(1/2dx+ \\ & 1/2c))^4 a - 2 \operatorname{tanh}(1/2dx+1/2c)^2 a + 4 \operatorname{tanh}(1/2dx+1/2c)^2 b + a)^2 \operatorname{tanh}(1/ \\ & 2dx+1/2c)^7 + 7/2/d/b/(\operatorname{tanh}(1/2dx+1/2c))^4 a - 2 \operatorname{tanh}(1/2dx+1/2c)^2 a + 4 \\ & * \operatorname{tanh}(1/2dx+1/2c)^2 b + a)^2 \operatorname{tanh}(1/2dx+1/2c)^5 - 7/2/d/b/(\operatorname{tanh}(1/2dx+1 \\ & /2c))^4 a - 2 \operatorname{tanh}(1/2dx+1/2c)^2 a + 4 \operatorname{tanh}(1/2dx+1/2c)^2 b + a)^2 \operatorname{tanh}(1/2 \\ & *dx+1/2c)^3 - 1/2/d/b/(\operatorname{tanh}(1/2dx+1/2c))^4 a - 2 \operatorname{tanh}(1/2dx+1/2c)^2 a + 4 \\ & \operatorname{tanh}(1/2dx+1/2c)^2 b + a)^2 \operatorname{tanh}(1/2dx+1/2c) + 5/4/d/(\operatorname{tanh}(1/2dx+1/2c) \\ & ^4 a - 2 \operatorname{tanh}(1/2dx+1/2c)^2 a + 4 \operatorname{tanh}(1/2dx+1/2c)^2 b + a)^2/a \operatorname{tanh}(1/2d \\ & x+1/2c) + 3/8/d/a^2/((2(-b(a-b))^{1/2}-a+2b)a)^{1/2} \operatorname{arctan}(a \operatorname{tanh}(1/2d \\ & *x+1/2c)/((2(-b(a-b))^{1/2}-a+2b)a)^{1/2}) - 3/8/d/a^2/((2(-b(a-b))^{1/2} \\ & +a-2b)a)^{1/2} \operatorname{arctanh}(a \operatorname{tanh}(1/2dx+1/2c)/((2(-b(a-b))^{1/2}+a-2 \\ & b)a)^{1/2}) - 7/4/d/(\operatorname{tanh}(1/2dx+1/2c))^4 a - 2 \operatorname{tanh}(1/2dx+1/2c)^2 a + 4 \operatorname{tan} \\ & h(1/2dx+1/2c)^2 b + a)^2/a \operatorname{tanh}(1/2dx+1/2c)^3 - 5/4/d/(\operatorname{tanh}(1/2dx+1/2c) \\ &)^4 a - 2 \operatorname{tanh}(1/2dx+1/2c)^2 a + 4 \operatorname{tanh}(1/2dx+1/2c)^2 b + a)^2/a \operatorname{tanh}(1/2d \end{aligned}$$

$$\begin{aligned} & *x+1/2*c)^{7+7/4/d}/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}/a*\tanh(1/2*d*x+1/2*c)^{5-3/4/d/b^2}/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}/a^2*\tanh(1/2*d*x+1/2*c)^{5+9/4/d/b^2}/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}/a^2*\tanh(1/2*d*x+1/2*c)^{3+3/4/d/b^2}/(\tanh(1/2*d*x+1/2*c)^{4*a-2*\tanh(1/2*d*x+1/2*c)^{2*a+4*\tanh(1/2*d*x+1/2*c)^{2*b+a}}/a^2*\tanh(1/2*d*x+1/2*c)^{7-1/8/d/a}/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-1/8/d/a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+1/4/d/b/a/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-1/4/d/b/a/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+1/8/d/b/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+1/8/d/b/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}) \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(5a^2be^{7c} - 2ab^2e^{7c} - 3b^3e^{7c})e^{7dx} + (12a^3e^{5c} - 7a^2be^{5c} - 14ab^2e^{5c} + 9b^3e^{5c})e^{5dx} - (12a^3e^{3c} - 7a^2be^{3c})e^{3dx}}{4(a^2b^4de^{8dx+8c} + a^2b^4d + 4(2a^3b^3de^{6c} - a^2b^4de^{6c})e^{6dx} + 2(8a^4b^2de^{4c} - 8a^3b^3de^{4c} + 3a^2b^4de^{4c}))e^{4dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $-1/4*((5*a^2*b*e^{(7*c)} - 2*a*b^2*e^{(7*c)} - 3*b^3*e^{(7*c)})*e^{(7*d*x)} + (12*a^3*e^{(5*c)} - 7*a^2*b*e^{(5*c)} - 14*a*b^2*e^{(5*c)} + 9*b^3*e^{(5*c)})*e^{(5*d*x)} - (12*a^3*e^{(3*c)} - 7*a^2*b*e^{(3*c)} - 14*a*b^2*e^{(3*c)} + 9*b^3*e^{(3*c)})*e^{(3*d*x)} - (5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3*e^c)*e^{(d*x)})/(a^2*b^4*d*e^{(8*d*x + 8*c)} + a^2*b^4*d + 4*(2*a^3*b^3*d*e^{(6*c)} - a^2*b^4*d*e^{(6*c)})*e^{(6*d*x)} + 2*(8*a^4*b^2*d*e^{(4*c)} - 8*a^3*b^3*d*e^{(4*c)} + 3*a^2*b^4*d*e^{(4*c)})*e^{(4*d*x)} + 4*(2*a^3*b^3*d*e^{(2*c)} - a^2*b^4*d*e^{(2*c)})*e^{(2*d*x)}) + 1/32*\operatorname{integrate}(8*((3*a^2*e^{(3*c)} + 2*a*b*e^{(3*c)} + 3*b^2*e^{(3*c)})*e^{(3*d*x)} + (3*a^2*e^c + 2*a*b*e^c + 3*b^2*e^c)*e^{(d*x)})/(a^2*b^3*e^{(4*d*x + 4*c)} + a^2*b^3 + 2*(2*a^3*b^2*e^{(2*c)} - a^2*b^3*e^{(2*c)})*e^{(2*d*x)}), x)$

Fricas [B] time = 2.45113, size = 13445, normalized size = 101.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^7 + 28*(5*a^3*b^2 \\ & - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 4*(5*a^3*b^2 - 2*a^2 \\ & *b^3 - 3*a*b^4)*\sinh(d*x + c)^7 + 4*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9 \\ & *a*b^4)*\cosh(d*x + c)^5 + 4*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4 + \\ & 21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 20* \\ & (7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^3 + (12*a^4*b - 7*a^3*b^2 \\ & - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 4*(12*a^4*b - 7* \\ & a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^3 - 4*(12*a^4*b - 7*a^3*b^2 - \\ & 14*a^2*b^3 + 9*a*b^4 - 35*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^4 \\ & - 10*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^2)*\sinh(\\ & d*x + c)^3 + 4*(21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^5 + 10*(\\ & 12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^3 - 3*(12*a^4*b \\ & - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^2 + ((3*a^2 \\ & *b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)* \\ & \cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\sinh(d*x + c) \\ & ^8 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 4*(6*a^3*b + \\ & a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^2 \\ &)*\sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + 3 \\ & *(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(\\ & 24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^4 + 2*(35*(\\ & 3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^2 \\ & - 18*a*b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + \\ & c)^2)*\sinh(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a \\ & *b^3 + 3*b^4)*\cosh(d*x + c)^5 + 10*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\co \\ & sh(d*x + c)^3 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x \\ & + c))*\sinh(d*x + c)^3 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + \\ & c)^2 + 4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^6 + 15*(6*a^3*b + \\ & a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 6*a^3*b + a^2*b^2 + 4*a*b^3 - \\ & 3*b^4 + 3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^2 \\ &)*\sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^7 + 3*(\\ & 6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + (24*a^4 - 8*a^3*b + \\ & 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b \\ & ^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a*b}*\log((b*\cosh(d*x + c)^4 \\ & + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a + b)*\cosh \\ & (d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh \\ & (d*x + c)^3 - (2*a + b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(\cosh(d*x + c)^3 + \end{aligned}$$

$$\begin{aligned}
& 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 - 1) \\
&)*\sinh(d*x + c) - \cosh(d*x + c))*\sqrt{-a*b} + b)/(b*\cosh(d*x + c)^4 + 4*b*c \\
& \cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c \\
&)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c \\
&)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) - 4*(5*a^3*b^2 - 2*a^2*b \\
& ^3 - 3*a*b^4)*\cosh(d*x + c) + 4*(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d \\
& *x + c)^6 - 5*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4 + 5*(12*a^4*b - 7*a^3*b^2 - 14* \\
& a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^4 - 3*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + \\
& 9*a*b^4)*\cosh(d*x + c)^2*\sinh(d*x + c))/(a^3*b^5*d*\cosh(d*x + c)^8 + 8*a^ \\
& 3*b^5*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^3*b^5*d*\sinh(d*x + c)^8 + a^3*b^5 \\
& *d + 4*(2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^6 + 4*(7*a^3*b^5*d*\cosh(d*x + \\
& c)^2 + (2*a^4*b^4 - a^3*b^5)*d)*\sinh(d*x + c)^6 + 2*(8*a^5*b^3 - 8*a^4*b^4 \\
& + 3*a^3*b^5)*d*\cosh(d*x + c)^4 + 8*(7*a^3*b^5*d*\cosh(d*x + c)^3 + 3*(2*a^4* \\
& b^4 - a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*a^3*b^5*d*\cosh(d*x \\
& + c)^4 + 30*(2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^2 + (8*a^5*b^3 - 8*a^4*b^ \\
& 4 + 3*a^3*b^5)*d)*\sinh(d*x + c)^4 + 4*(2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c) \\
& ^2 + 8*(7*a^3*b^5*d*\cosh(d*x + c)^5 + 10*(2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + \\
& c)^3 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^ \\
& 3 + 4*(7*a^3*b^5*d*\cosh(d*x + c)^6 + 15*(2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + \\
& c)^4 + 3*(8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^2 + (2*a^4*b^4 \\
& - a^3*b^5)*d)*\sinh(d*x + c)^2 + 8*(a^3*b^5*d*\cosh(d*x + c)^7 + 3*(2*a^4*b^ \\
& 4 - a^3*b^5)*d*\cosh(d*x + c)^5 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*\cosh \\
& (d*x + c)^3 + (2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/8*(\\
& 2*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^7 + 14*(5*a^3*b^2 - 2*a^2 \\
& *b^3 - 3*a*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 2*(5*a^3*b^2 - 2*a^2*b^3 - \\
& 3*a*b^4)*\sinh(d*x + c)^7 + 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)* \\
& \cosh(d*x + c)^5 + 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4 + 21*(5*a^ \\
& 3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 10*(7*(5*a^ \\
& 3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^3 + (12*a^4*b - 7*a^3*b^2 - 14*a \\
& ^2*b^3 + 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 2*(12*a^4*b - 7*a^3*b^2 \\
& - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^3 - 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2* \\
& b^3 + 9*a*b^4 - 35*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^4 - 10*(\\
& 12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c) \\
& ^3 + 2*(21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^5 + 10*(12*a^4*b \\
& - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^3 - 3*(12*a^4*b - 7*a^3* \\
& b^2 - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^2 - ((3*a^2*b^2 + \\
& 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x \\
& + c)*\sinh(d*x + c)^7 + (3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\sinh(d*x + c)^8 + 4*(\\
& 6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 4*(6*a^3*b + a^2*b^2 \\
& + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(\\
& d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + 3*(6*a^3* \\
& b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(24*a^4 - \\
& 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^4 + 2*(35*(3*a^2*b^ \\
& 2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a \\
& *b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^2)*\si
\end{aligned}$$

$$\begin{aligned}
& \text{nh}(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^5 + 10*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^3 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^2 + 4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^6 + 15*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 + 3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^7 + 3*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a*b}*\arctan(1/2*\sqrt{a*b}*(\cosh(d*x + c) + \sinh(d*x + c)))/a - ((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\sinh(d*x + c)^8 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + 3*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^4 + 2*(35*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^5 + 10*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^3 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^2 + 4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^6 + 15*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 + 3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*\cosh(d*x + c)^7 + 3*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*\cosh(d*x + c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a*b}*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 + (4*a - b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - b)*\sinh(d*x + c)))*\sqrt{a*b}/(a*b)) - 2*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c) + 2*(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*\cosh(d*x + c)^6 - 5*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4 + 5*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^4 - 3*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(a^3*b^5*d*\cosh(d*x + c)^8 + 8*a^3*b^5*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^3*b^5*d*\sinh(d*x + c)^8 + a^3*b^5*d + 4*(2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^6 + 4*(7*a^3*b^5*d*\cosh(d*x + c)^2 + (2*a^4*b^4 - a^3*b^5)*d)*\sinh(d*x + c)^6 + 2*(8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^4 + 8*(7*a^3*b^5*d*\cosh(d*x + c)^3 + 3*(2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*a^3*b^5*d*\cosh(d*x + c)^4 + 30*(2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^2 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d)*\sinh(d*x + c)^4 + 4*(2*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^2 + 8*(7*a^3
\end{aligned}$$

$$*b^5*d*cosh(d*x + c)^5 + 10*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^5*d*cosh(d*x + c)^6 + 15*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + 8*(a^3*b^5*d*cosh(d*x + c)^7 + 3*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.341 \quad \int \frac{\cosh^4(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=114

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a-b}} + \frac{3 \tanh(c+dx)}{8a^2d(a-(a-b)\tanh^2(c+dx))} + \frac{\tanh(c+dx)}{4ad(a-(a-b)\tanh^2(c+dx))^2}$$

[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a - b]*d) + Tanh[c + d*x]/(4*a*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])/(8*a^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.0954025, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3191, 199, 208}

$$\frac{3 \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a-b}} + \frac{3 \tanh(c+dx)}{8a^2d(a-(a-b)\tanh^2(c+dx))} + \frac{\tanh(c+dx)}{4ad(a-(a-b)\tanh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a - b]*d) + Tanh[c + d*x]/(4*a*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])/(8*a^2*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3191

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 199

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x]

$(p + 1), x], x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ (\text{IntegerQ}[2*p] \ || \ (n == 2 \ \&\& \ \text{IntegerQ}[4*p]) \ || \ (n == 2 \ \&\& \ \text{IntegerQ}[3*p]) \ || \ \text{Denominator}[p + 1/n] < \text{Denominator}[p])$

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \ :> \ \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\cosh^4(c + dx)}{(a + b \sinh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a - (a-b)x^2)^3} dx, x, \tanh(c + dx)\right)}{d} \\ &= \frac{\tanh(c + dx)}{4ad(a - (a-b)\tanh^2(c + dx))^2} + \frac{3 \text{Subst}\left(\int \frac{1}{(a + (-a+b)x^2)^2} dx, x, \tanh(c + dx)\right)}{4ad} \\ &= \frac{\tanh(c + dx)}{4ad(a - (a-b)\tanh^2(c + dx))^2} + \frac{3 \tanh(c + dx)}{8a^2d(a - (a-b)\tanh^2(c + dx))} + \frac{3 \text{Subst}\left(\int \frac{1}{a + (-a+b)x^2} dx, x, \tanh(c + dx)\right)}{8a^2d} \\ &= \frac{3 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a-bd}} + \frac{\tanh(c + dx)}{4ad(a - (a-b)\tanh^2(c + dx))^2} + \frac{3 \tanh(c + dx)}{8a^2d(a - (a-b)\tanh^2(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.654372, size = 102, normalized size = 0.89

$$\frac{\frac{3 \tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a} \sinh(2(c+dx))(2a+3b) \cosh(2(c+dx))+8a-3b}{(2a+b \cosh(2(c+dx))-b)^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*(8*a - 3*b + (2*a + 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(2*a - b + b*Cosh[2*(c + d*x)]))^2)/(8*a^(5/2)*d)

Maple [B] time = 0.064, size = 664, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cosh(dx+c)^4 / (a+b\sinh(dx+c)^2)^3, x)$

[Out] $\frac{5}{4} \frac{d}{dx} \frac{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)^{4a-2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a})^2}{a \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{7+3/4} d} \frac{d}{dx} \frac{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)^{4a-2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a})^2}{a \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{5+3/4} d} \frac{d}{dx} \frac{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)^{4a-2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a})^2}{a^2 \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{5+3/4} d} \frac{d}{dx} \frac{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)^{4a-2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a})^2}{a \tanh(\frac{1}{2}dx+\frac{1}{2}c)^3} \frac{d}{dx} \frac{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)^{4a-2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a})^2}{a^2 \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{3+5/4} d} \frac{d}{dx} \frac{(\tanh(\frac{1}{2}dx+\frac{1}{2}c)^{4a-2} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2a+4} \tanh(\frac{1}{2}dx+\frac{1}{2}c)^{2b+a})^2}{a \tanh(\frac{1}{2}dx+\frac{1}{2}c)} + \frac{3}{8} \frac{d}{dx} \frac{1}{a^2} \frac{((2(-b(a-b))^{1/2} + a - 2b)a)^{1/2} \operatorname{arctanh}(a \tanh(\frac{1}{2}dx+\frac{1}{2}c))}{((2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}} - \frac{3}{8} \frac{d}{dx} \frac{1}{a^2} \frac{(-b(a-b))^{1/2}}{((2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}} \operatorname{arctanh}(a \tanh(\frac{1}{2}dx+\frac{1}{2}c))}{((2(-b(a-b))^{1/2} + a - 2b)a)^{1/2}} - \frac{3}{8} \frac{d}{dx} \frac{1}{a^2} \frac{((2(-b(a-b))^{1/2} - a + 2b)a)^{1/2} \operatorname{arctan}(a \tanh(\frac{1}{2}dx+\frac{1}{2}c))}{((2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}} - \frac{3}{8} \frac{d}{dx} \frac{1}{a^2} \frac{(-b(a-b))^{1/2}}{((2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}} \operatorname{arctan}(a \tanh(\frac{1}{2}dx+\frac{1}{2}c))}{((2(-b(a-b))^{1/2} - a + 2b)a)^{1/2}}$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\cosh(dx+c)^4 / (a+b\sinh(dx+c)^2)^3, x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

Fricas [B] time = 2.28789, size = 10175, normalized size = 89.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^6 + 24* \\ & (8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^5 + \\ & 4*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*\sinh(d*x + c)^6 + 8*a^3*b^2 \\ & + 4*a^2*b^3 - 12*a*b^4 + 4*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9* \\ & a*b^4)*\cosh(d*x + c)^4 + 4*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9* \\ & a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^2)*\sin \\ & h(d*x + c)^4 + 16*(5*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*\cosh(d*x + \\ & c)^3 + (16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*\cosh(d*x + c \\ &))*\sinh(d*x + c)^3 + 4*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*\cosh(d* \\ & x + c)^2 + 4*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4 + 15*(8*a^4*b - 8* \\ & a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^4 + 6*(16*a^5 - 8*a^4*b - 26*a \\ & ^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 - 3*(b^4*co \\ & sh(d*x + c)^8 + 8*b^4*\cosh(d*x + c)*\sinh(d*x + c)^7 + b^4*\sinh(d*x + c)^8 + \\ & 4*(2*a*b^3 - b^4)*\cosh(d*x + c)^6 + 4*(7*b^4*\cosh(d*x + c)^2 + 2*a*b^3 - b \\ & ^4)*\sinh(d*x + c)^6 + 8*(7*b^4*\cosh(d*x + c)^3 + 3*(2*a*b^3 - b^4)*\cosh(d*x \\ & + c))*\sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + \\ & 2*(35*b^4*\cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^3 + 3*b^4 + 30*(2*a*b^3 - b^4 \\ &)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + b^4 + 8*(7*b^4*\cosh(d*x + c)^5 + 10*(2 \\ & *a*b^3 - b^4)*\cosh(d*x + c)^3 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*\cosh(d*x + c \\ &)*\sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*\cosh(d*x + c)^2 + 4*(7*b^4*\cosh(d*x + \\ & c)^6 + 15*(2*a*b^3 - b^4)*\cosh(d*x + c)^4 + 2*a*b^3 - b^4 + 3*(8*a^2*b^2 - \\ & 8*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*(b^4*\cosh(d*x + c)^7 \\ & + 3*(2*a*b^3 - b^4)*\cosh(d*x + c)^5 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*\cosh(d \\ & *x + c)^3 + (2*a*b^3 - b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a^2 - a*b}*l \\ & og((b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*\sinh(d* \\ & x + c)^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*b^2*\cosh(d*x + c)^2 + 2*a \\ & *b - b^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*\cosh(d*x + c)^3 + \\ & (2*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(b*\cosh(d*x + c)^2 + 2*b*cos \\ & h(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{a^2 - a*b})/(b \\ & *\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + \\ & 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + \\ & c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) \\ & + 8*(3*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^5 + 2*(16* \\ & a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*\cosh(d*x + c)^3 + (8*a^4 \\ & *b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c))/((a^4* \\ & b^4 - a^3*b^5)*d*\cosh(d*x + c)^8 + 8*(a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)*\si \\ & nh(d*x + c)^7 + (a^4*b^4 - a^3*b^5)*d*\sinh(d*x + c)^8 + 4*(2*a^5*b^3 - 3*a^ \\ & 4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^6 + 4*(7*(a^4*b^4 - a^3*b^5)*d*\cosh(d*x + \\ & c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*\sinh(d*x + c)^6 + 2*(8*a^6*b^2 \\ & - 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*\cosh(d*x + c)^4 + 8*(7*(a^4*b^4 - \end{aligned}$$

$$\begin{aligned}
& a^3 b^5) d \cosh(dx + c)^3 + 3(2a^5 b^3 - 3a^4 b^4 + a^3 b^5) d \cosh(dx + c) \sinh(dx + c)^5 + 2(35(a^4 b^4 - a^3 b^5) d \cosh(dx + c)^4 + 30(2a^5 b^3 - 3a^4 b^4 + a^3 b^5) d \cosh(dx + c)^2 + (8a^6 b^2 - 16a^5 b^3 + 11a^4 b^4 - 3a^3 b^5) d) \sinh(dx + c)^4 + 4(2a^5 b^3 - 3a^4 b^4 + a^3 b^5) d \cosh(dx + c)^2 + 8(7(a^4 b^4 - a^3 b^5) d \cosh(dx + c)^5 + 10(2a^5 b^3 - 3a^4 b^4 + a^3 b^5) d \cosh(dx + c)^3 + (8a^6 b^2 - 16a^5 b^3 + 11a^4 b^4 - 3a^3 b^5) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^4 b^4 - a^3 b^5) d \cosh(dx + c)^6 + 15(2a^5 b^3 - 3a^4 b^4 + a^3 b^5) d \cosh(dx + c)^4 + 3(8a^6 b^2 - 16a^5 b^3 + 11a^4 b^4 - 3a^3 b^5) d \cosh(dx + c)^2 + (2a^5 b^3 - 3a^4 b^4 + a^3 b^5) d) \sinh(dx + c)^2 + (a^4 b^4 - a^3 b^5) d + 8((a^4 b^4 - a^3 b^5) d \cosh(dx + c)^7 + 3(2a^5 b^3 - 3a^4 b^4 + a^3 b^5) d \cosh(dx + c)^5 + (8a^6 b^2 - 16a^5 b^3 + 11a^4 b^4 - 3a^3 b^5) d \cosh(dx + c)^3 + (2a^5 b^3 - 3a^4 b^4 + a^3 b^5) d \cosh(dx + c)) \sinh(dx + c)), \\
& -1/8(2(8a^4 b - 8a^3 b^2 - 3a^2 b^3 + 3a b^4) \cosh(dx + c)^6 + 12(8a^4 b - 8a^3 b^2 - 3a^2 b^3 + 3a b^4) \cosh(dx + c) \sinh(dx + c)^5 + 2(8a^4 b - 8a^3 b^2 - 3a^2 b^3 + 3a b^4) \sinh(dx + c)^6 + 4a^3 b^2 + 2a^2 b^3 - 6a b^4 + 2(16a^5 - 8a^4 b - 26a^3 b^2 + 27a^2 b^3 - 9a b^4) \cosh(dx + c)^4 + 2(16a^5 - 8a^4 b - 26a^3 b^2 + 27a^2 b^3 - 9a b^4) \cosh(dx + c)^2 \sinh(dx + c)^4 + 8(5(8a^4 b - 8a^3 b^2 - 3a^2 b^3 + 3a b^4) \cosh(dx + c)^3 + (16a^5 - 8a^4 b - 26a^3 b^2 + 27a^2 b^3 - 9a b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 2(8a^4 b + 8a^3 b^2 - 25a^2 b^3 + 9a b^4) \cosh(dx + c)^2 + 2(8a^4 b + 8a^3 b^2 - 25a^2 b^3 + 9a b^4 + 15(8a^4 b - 8a^3 b^2 - 3a^2 b^3 + 3a b^4) \cosh(dx + c)^4 + 6(16a^5 - 8a^4 b - 26a^3 b^2 + 27a^2 b^3 - 9a b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 3(b^4 \cosh(dx + c)^8 + 8b^4 \cosh(dx + c) \sinh(dx + c))^7 + b^4 \sinh(dx + c)^8 + 4(2a b^3 - b^4) \cosh(dx + c)^6 + 4(7b^4 \cosh(dx + c)^2 + 2a b^3 - b^4) \sinh(dx + c)^6 + 8(7b^4 \cosh(dx + c)^3 + 3(2a b^3 - b^4) \cosh(dx + c)) \sinh(dx + c)^5 + 2(8a^2 b^2 - 8a b^3 + 3b^4) \cosh(dx + c)^4 + 2(35b^4 \cosh(dx + c)^4 + 8a^2 b^2 - 8a b^3 + 3b^4 + 30(2a b^3 - b^4) \cosh(dx + c)^2) \sinh(dx + c)^4 + b^4 + 8(7b^4 \cosh(dx + c)^5 + 10(2a b^3 - b^4) \cosh(dx + c)^3 + (8a^2 b^2 - 8a b^3 + 3b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(2a b^3 - b^4) \cosh(dx + c)^2 + 4(7b^4 \cosh(dx + c)^6 + 15(2a b^3 - b^4) \cosh(dx + c)^4 + 2a b^3 - b^4 + 3(8a^2 b^2 - 8a b^3 + 3b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 + 8(b^4 \cosh(dx + c)^7 + 3(2a b^3 - b^4) \cosh(dx + c)^5 + (8a^2 b^2 - 8a b^3 + 3b^4) \cosh(dx + c)^3 + (2a b^3 - b^4) \cosh(dx + c)) \sinh(dx + c)) \sqrt{-a^2 + a b} \arctan(-1/2(b \cosh(dx + c)^2 + 2b \cosh(dx + c) \sinh(dx + c) + b \sinh(dx + c)^2 + 2a - b) \sqrt{-a^2 + a b} / (a^2 - a b)) + 4(3(8a^4 b - 8a^3 b^2 - 3a^2 b^3 + 3a b^4) \cosh(dx + c)^5 + 2(16a^5 - 8a^4 b - 26a^3 b^2 + 27a^2 b^3 - 9a b^4) \cosh(dx + c)^3 + (8a^4 b + 8a^3 b^2 - 25a^2 b^3 + 9a b^4) \cosh(dx + c)) \sinh(dx + c)) / ((a^4 b^4 - a^3 b^5) d \cosh(dx + c)^8 + 8(a^4 b^4 - a^3 b^5) d \cosh(dx + c) \sinh(dx + c)^7 + (a^4 b^4 - a^3 b^5) d \sinh(dx + c)^8 + 4(2a^5 b^3 - 3a^4 b^4 + a^3 b^5) d \cosh(dx + c)^6 + 4(7(a^4 b^4 - a^3 b^5) d \cosh(dx + c)
\end{aligned}$$

$$\begin{aligned}
& + c)^2 + (2a^5b^3 - 3a^4b^4 + a^3b^5)d) \sinh(dx + c)^6 + 2(8a^6b^2 - 16a^5b^3 + 11a^4b^4 - 3a^3b^5)d \cosh(dx + c)^4 + 8(7(a^4b^4 - a^3b^5)d \cosh(dx + c)^3 + 3(2a^5b^3 - 3a^4b^4 + a^3b^5)d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35(a^4b^4 - a^3b^5)d \cosh(dx + c)^4 + 30(2a^5b^3 - 3a^4b^4 + a^3b^5)d \cosh(dx + c)^2 + (8a^6b^2 - 16a^5b^3 + 11a^4b^4 - 3a^3b^5)d) \sinh(dx + c)^4 + 4(2a^5b^3 - 3a^4b^4 + a^3b^5)d \cosh(dx + c)^2 + 8(7(a^4b^4 - a^3b^5)d \cosh(dx + c)^5 + 10(2a^5b^3 - 3a^4b^4 + a^3b^5)d \cosh(dx + c)^3 + (8a^6b^2 - 16a^5b^3 + 11a^4b^4 - 3a^3b^5)d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7(a^4b^4 - a^3b^5)d \cosh(dx + c)^6 + 15(2a^5b^3 - 3a^4b^4 + a^3b^5)d \cosh(dx + c)^4 + 3(8a^6b^2 - 16a^5b^3 + 11a^4b^4 - 3a^3b^5)d \cosh(dx + c)^2 + (2a^5b^3 - 3a^4b^4 + a^3b^5)d) \sinh(dx + c)^2 + (a^4b^4 - a^3b^5)d + 8((a^4b^4 - a^3b^5)d \cosh(dx + c)^7 + 3(2a^5b^3 - 3a^4b^4 + a^3b^5)d \cosh(dx + c)^5 + (8a^6b^2 - 16a^5b^3 + 11a^4b^4 - 3a^3b^5)d \cosh(dx + c)^3 + (2a^5b^3 - 3a^4b^4 + a^3b^5)d \cosh(dx + c)) \sinh(dx + c)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)**4/(a+b*sinh(dx+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.36082, size = 331, normalized size = 2.9

$$\frac{3 \arctan\left(\frac{be^{(2dx+2c)+2a-b}}{2\sqrt{-a^2+ab}}\right)}{8\sqrt{-a^2+ab}d} - \frac{8a^2be^{(6dx+6c)} - 3b^3e^{(6dx+6c)} + 16a^3e^{(4dx+4c)} + 8a^2be^{(4dx+4c)} - 18ab^2e^{(4dx+4c)} + 9b^3e^{(4dx+4c)}}{4\left(be^{(4dx+4c)} + 4ae^{(2dx+2c)} - 2be^{(2dx+2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^4/(a+b*sinh(dx+c)^2)^3,x, algorithm="giac")

[Out] 3/8*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a^2*d) - 1/4*(8*a^2*b*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 16*a^

$$\frac{3e^{(4dx + 4c)} + 8a^2be^{(4dx + 4c)} - 18ab^2e^{(4dx + 4c)} + 9b^3e^{(4dx + 4c)} + 8a^2be^{(2dx + 2c)} + 16ab^2e^{(2dx + 2c)} - 9b^3e^{(2dx + 2c)} + 2ab^2 + 3b^3}{(be^{(4dx + 4c)} + 4ae^{(2dx + 2c)} - 2be^{(2dx + 2c)} + b)^2 a^2 b^2 d}$$

$$3.342 \quad \int \frac{\cosh^3(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=117

$$\frac{(a+3b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} + \frac{(a+3b) \sinh(c+dx)}{8a^2bd(a+b \sinh^2(c+dx))} - \frac{(a-b) \sinh(c+dx)}{4abd(a+b \sinh^2(c+dx))^2}$$

[Out] ((a + 3*b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*d) - ((a - b)*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh[c + d*x]^2)^2) + ((a + 3*b)*Sinh[c + d*x])/(8*a^2*b*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.0925692, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 385, 199, 205}

$$\frac{(a+3b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} + \frac{(a+3b) \sinh(c+dx)}{8a^2bd(a+b \sinh^2(c+dx))} - \frac{(a-b) \sinh(c+dx)}{4abd(a+b \sinh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((a + 3*b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*d) - ((a - b)*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh[c + d*x]^2)^2) + ((a + 3*b)*Sinh[c + d*x])/(8*a^2*b*d*(a + b*Sinh[c + d*x]^2))

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 385

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b

c(n*(p + 1) + 1)/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(c + dx)}{(a + b \sinh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= -\frac{(a - b) \sinh(c + dx)}{4abd (a + b \sinh^2(c + dx))^2} + \frac{(a + 3b) \text{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, \sinh(c + dx)\right)}{4abd} \\ &= -\frac{(a - b) \sinh(c + dx)}{4abd (a + b \sinh^2(c + dx))^2} + \frac{(a + 3b) \sinh(c + dx)}{8a^2bd (a + b \sinh^2(c + dx))} + \frac{(a + 3b) \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c + dx)\right)}{8a^2bd} \\ &= \frac{(a + 3b) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} - \frac{(a - b) \sinh(c + dx)}{4abd (a + b \sinh^2(c + dx))^2} + \frac{(a + 3b) \sinh(c + dx)}{8a^2bd (a + b \sinh^2(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.665558, size = 114, normalized size = 0.97

$$\frac{(a + 3b) \left(\frac{\sinh(c+dx)(5a+3b \sinh^2(c+dx))}{8a^2(a+b \sinh^2(c+dx))^2} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}} \right) - \frac{\sinh(c+dx)}{(a+b \sinh^2(c+dx))^2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-(Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2) + (a + 3*b)*((3*ArcTan[(Sqrt[b]
*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b])) + (Sinh[c + d*x]*(5*a + 3*b*S
inh[c + d*x]^2))/(8*a^2*(a + b*Sinh[c + d*x]^2)^2))/(3*b*d)
```

Maple [B] time = 0.068, size = 1348, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7-3
/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+11/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-3
/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5+3/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-
11/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+3/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^
3-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-1/8
/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8/d/b/a/((2*(-b*(a-b)
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2))-1/4/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8
/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/8/d/b/a/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2))-1/4/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/
a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b)
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
```


$$+a-2*b)*a)^{(1/2)}+3/8/d/a^2/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}))+3/8/d*b/a^2/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(abe^{7c} + 3b^2e^{7c})e^{7dx} - (4a^2e^{5c} - 17abe^{5c} + 9b^2e^{5c})e^{5dx} + (4a^2e^{3c} - 17abe^{3c} + 9b^2e^{3c})e^{3dx} - 4(a^2b^3de^{8dx+8c} + a^2b^3d + 4(2a^3b^2de^{6c} - a^2b^3de^{6c})e^{6dx} + 2(8a^4bde^{4c} - 8a^3b^2de^{4c} + 3a^2b^3de^{4c})e^{4dx} + 4(2a^4b^2de^{2c} - 4a^3b^2de^{2c} + 2a^2b^3de^{2c})e^{2dx})}{4(a^2b^3de^{8dx+8c} + a^2b^3d + 4(2a^3b^2de^{6c} - a^2b^3de^{6c})e^{6dx} + 2(8a^4bde^{4c} - 8a^3b^2de^{4c} + 3a^2b^3de^{4c})e^{4dx} + 4(2a^4b^2de^{2c} - 4a^3b^2de^{2c} + 2a^2b^3de^{2c})e^{2dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/4*((a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) - (4*a^2*e^(5*c) - 17*a*b*e^(5*c) + 9*b^2*e^(5*c))*e^(5*d*x) + (4*a^2*e^(3*c) - 17*a*b*e^(3*c) + 9*b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^2*b^3*d*e^(8*d*x + 8*c) + a^2*b^3*d + 4*(2*a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*d*x) + 2*(8*a^4*b*d*e^(4*c) - 8*a^3*b^2*d*e^(4*c) + 3*a^2*b^3*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3*b^2*d*e^(2*c) - a^2*b^3*d*e^(2*c))*e^(2*d*x) + 1/8*integrate(2*((a*e^(3*c) + 3*b*e^(3*c))*e^(3*d*x) + (a*e^c + 3*b*e^c)*e^(d*x))/(a^2*b^2*e^(4*d*x + 4*c) + a^2*b^2 + 2*(2*a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)

Fricas [B] time = 2.33105, size = 11478, normalized size = 98.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 28*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^7 - 4*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^5 - 4*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3 - 21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 - (4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^3 + 4*(35*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 4*a^3*b - 17*a^2*b^2 + 9*a*b^3)

$$\begin{aligned}
& 3 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c)^3 + \\
& 4*(21*(a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^5 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a* \\
& b^3)*\cosh(d*x + c)^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 - \\
& ((a*b^2 + 3*b^3)*\cosh(d*x + c)^8 + 8*(a*b^2 + 3*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 + 3*b^3)*\sinh(d*x + c)^8 + 4*(2*a^2*b + 5* \\
& a*b^2 - 3*b^3)*\cosh(d*x + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3 + 7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b^3)*\cosh(d*x + c) \\
&)^3 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c)^4 + 2*(35*(a*b^2 + 3*b^3)*\cosh(d*x + c)^4 + 8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^5 + 10*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^3 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a*b^2 + 3*b^3 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^6 + 15*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^4 + 2*a^2*b + 5*a*b^2 - 3*b^3 + 3*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a*b^2 + 3*b^3)*\cosh(d*x + c)^7 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^5 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a*b}*\log((b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a + b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a + b)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(\cosh(d*x + c)^3 + 3*\cosh(d*x + c)*\sinh(d*x + c)^2 + \sinh(d*x + c)^3 + (3*\cosh(d*x + c)^2 - 1)*\sinh(d*x + c) - \cosh(d*x + c))*\sqrt{-a*b} + b)/(b*\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) - 4*(a^2*b^2 + 3*a*b^3)*\cosh(d*x + c) + 4*(7*(a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^6 - 5*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c)^4 - a^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c))/(a^3*b^4*d*\cosh(d*x + c)^8 + 8*a^3*b^4*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^3*b^4*d*\sinh(d*x + c)^8 + a^3*b^4*d + 4*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^6 + 4*(7*a^3*b^4*d*\cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x + c)^6 + 2*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^4 + 8*(7*a^3*b^4*d*\cosh(d*x + c)^3 + 3*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*a^3*b^4*d*\cosh(d*x + c)^4 + 30*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^2 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d)*\sinh(d*x + c)^4 + 4*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^2 + 8*(7*a^3*b^4*d*\cosh(d*x + c)^5 + 10*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^3 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*a^3*b^4*d*\cosh(d*x + c)^6 + 15*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^4 + 3*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x + c)^2 + 8*(a^3*b^4*d*\cosh(d*x + c)^7 + 3*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^5 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^3 + (2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)), 1/8*(2*(a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^7 + 14*(a^2*b^2
\end{aligned}$$

$$\begin{aligned}
& + 3*a*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 2*(a^2*b^2 + 3*a*b^3)*\sinh(d*x + c)^7 - 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c)^5 - 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3 - 21*(a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 10*(7*(a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^3 - (4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^4 + 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c)^3 + 2*(35*(a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^4 + 4*a^3*b - 17*a^2*b^2 + 9*a*b^3 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 2*(21*(a^2*b^2 + 3*a*b^3)*\cosh(d*x + c)^5 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c)^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^2 + ((a*b^2 + 3*b^3)*\cosh(d*x + c)^8 + 8*(a*b^2 + 3*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 + 3*b^3)*\sinh(d*x + c)^8 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^3 + 7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^3 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c)^4 + 2*(35*(a*b^2 + 3*b^3)*\cosh(d*x + c)^4 + 8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^5 + 10*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^3 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a*b^2 + 3*b^3 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^6 + 15*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^4 + 2*a^2*b + 5*a*b^2 - 3*b^3 + 3*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a*b^2 + 3*b^3)*\cosh(d*x + c)^7 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^5 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a*b}*\arctan(1/2*\sqrt{a*b}*(\cosh(d*x + c) + \sinh(d*x + c))/a) + ((a*b^2 + 3*b^3)*\cosh(d*x + c)^8 + 8*(a*b^2 + 3*b^3)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a*b^2 + 3*b^3)*\sinh(d*x + c)^8 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3 + 7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^3 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c)^4 + 2*(35*(a*b^2 + 3*b^3)*\cosh(d*x + c)^4 + 8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^5 + 10*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^3 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c))*\sinh(d*x + c)^3 + a*b^2 + 3*b^3 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3*b^3)*\cosh(d*x + c)^6 + 15*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^4 + 2*a^2*b + 5*a*b^2 - 3*b^3 + 3*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((a*b^2 + 3*b^3)*\cosh(d*x + c)^7 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c)^5 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*\cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 - 3*b^3)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a*b}*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3*b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 + (4*a - b)*\cosh(d*x + c) + (3*b*\cosh(d*x + c)^2 + 4*a - b)*\sinh(d*x + c))*\sqrt{a*b}/(a*b)) - 2*(a^2*b^2 + 3*a*b^3)*\cosh(d*x + c) + 2*(7*(a^2*b^2 + 3*a*b^3)*\cosh(d*x +
\end{aligned}$$

$$c)^6 - 5*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c)^4 - a^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*\cosh(d*x + c)^2*\sinh(d*x + c))/(a^3*b^4*d*\cosh(d*x + c)^8 + 8*a^3*b^4*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^3*b^4*d*\sinh(d*x + c)^8 + a^3*b^4*d + 4*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^6 + 4*(7*a^3*b^4*d*\cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x + c)^6 + 2*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^4 + 8*(7*a^3*b^4*d*\cosh(d*x + c)^3 + 3*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*a^3*b^4*d*\cosh(d*x + c)^4 + 30*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^2 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d)*\sinh(d*x + c)^4 + 4*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^2 + 8*(7*a^3*b^4*d*\cosh(d*x + c)^5 + 10*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^3 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*a^3*b^4*d*\cosh(d*x + c)^6 + 15*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^4 + 3*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*\sinh(d*x + c)^2 + 8*(a^3*b^4*d*\cosh(d*x + c)^7 + 3*(2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c)^5 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*\cosh(d*x + c)^3 + (2*a^4*b^3 - a^3*b^4)*d*\cosh(d*x + c))*\sinh(d*x + c))]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.343 \quad \int \frac{\cosh^2(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=143

$$\frac{(4a-3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{3/2}} + \frac{(4a-3b) \tanh(c+dx)}{8a^2d(a-b)(a-(a-b) \tanh^2(c+dx))} - \frac{b \tanh(c+dx)}{4ad(a-b)(a-(a-b) \tanh^2(c+dx))^2}$$

[Out] ((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^(3/2)*d) - (b*Tanh[c + d*x])/(4*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + ((4*a - 3*b)*Tanh[c + d*x])/(8*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.125523, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3191, 385, 199, 208}

$$\frac{(4a-3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{3/2}} + \frac{(4a-3b) \tanh(c+dx)}{8a^2d(a-b)(a-(a-b) \tanh^2(c+dx))} - \frac{b \tanh(c+dx)}{4ad(a-b)(a-(a-b) \tanh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^(3/2)*d) - (b*Tanh[c + d*x])/(4*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + ((4*a - 3*b)*Tanh[c + d*x])/(8*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[
((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[
{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)
)/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(
p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c + dx)}{(a + b \sinh^2(c + dx))^3} dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{(a-(a-b)x^2)^3} dx, x, \tanh(c + dx)\right)}{d}$$

$$= -\frac{b \tanh(c + dx)}{4a(a-b)d \left(a - (a-b) \tanh^2(c + dx)\right)^2} + \frac{(4a-3b) \text{Subst}\left(\int \frac{1}{(a+(-a+b)x^2)^2} dx, x, \tanh(c + dx)\right)}{4a(a-b)d}$$

$$= -\frac{b \tanh(c + dx)}{4a(a-b)d \left(a - (a-b) \tanh^2(c + dx)\right)^2} + \frac{(4a-3b) \tanh(c + dx)}{8a^2(a-b)d \left(a - (a-b) \tanh^2(c + dx)\right)} + \frac{(4a-3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{3/2}d} - \frac{b \tanh(c + dx)}{4a(a-b)d \left(a - (a-b) \tanh^2(c + dx)\right)^2} + \frac{(4a-3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^2(a-b)}$$

Mathematica [A] time = 0.888273, size = 124, normalized size = 0.87

$$\frac{\sqrt{a} \sinh(2(c+dx))(8a^2+b(2a-3b) \cosh(2(c+dx))-12ab+3b^2)}{(a-b)(2a+b \cosh(2(c+dx))-b)^2} + \frac{(4a-3b) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}}$$

$$8a^{5/2}d$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a - b)^(3/2) + (Sqrt[a]*(8*a^2 - 12*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)

Maple [B] time = 0.06, size = 1322, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a-b)*tanh(1/2*d*x+1/2*c)^7-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/2*c)^7*b-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a-b)*tanh(1/2*d*x+1/2*c)^5+13/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/2*c)^5*b-3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a-b)*tanh(1/2*d*x+1/2*c)^5*b^2-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a-b)*tanh(1/2*d*x+1/2*c)^3+13/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/2*c)^3*b-3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a-b)*tanh(1/2*d*x+1/2*c)^3*b^2+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a-b)*tanh(1/2*d*x+1/2*c)^5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/2*c)*b+1/2/d/a/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b-1/2/d/a/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/a/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b-3/8/d/a^2/(a-b)*b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d/a^2/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))

$$*b^2+3/8/d/a^2/(a-b)*b/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+3/8/d/a^2/(a-b)/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})*b^2$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.35308, size = 11760, normalized size = 82.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 - 8*a^3*b^2 + 20*a^2*b^3 - 12*a*b^4 - 4*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 - 4*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 - (16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(16*a^4*b - 44*a^3*b^2 + 37*a^2*b^3 - 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a*b^3 - 3*b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4 + 7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^4 + 2

$$\begin{aligned}
&*(35*(4*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - \\
&9*b^4 + 30*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 \\
&+ 4*a*b^3 - 3*b^4 + 8*(7*(4*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + 10*(8*a^2*b^2 \\
&- 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - \\
&9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*\co \\
&sh(d*x + c)^2 + 4*(7*(4*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 15*(8*a^2*b^2 - 10 \\
&)*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 8*a^2*b^2 - 10*a*b^3 + 3*b^4 + 3*(32*a^3* \\
&b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((4 \\
&)*a*b^3 - 3*b^4)*\cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x \\
&+ c)^5 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*\cosh(d*x + c)^3 + (8*a \\
&^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{a^2 - a*b}*lo \\
&g((b^2*\cosh(d*x + c)^4 + 4*b^2*\cosh(d*x + c)*\sinh(d*x + c)^3 + b^2*\sinh(d*x \\
&+ c)^4 + 2*(2*a*b - b^2)*\cosh(d*x + c)^2 + 2*(3*b^2*\cosh(d*x + c)^2 + 2*a* \\
&b - b^2)*\sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*\cosh(d*x + c)^3 + (\\
&2*a*b - b^2)*\cosh(d*x + c))*\sinh(d*x + c) - 4*(b*\cosh(d*x + c)^2 + 2*b*\cosh \\
&(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{a^2 - a*b}))/ (b* \\
&\cosh(d*x + c)^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2 \\
&)*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c \\
&)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) + \\
&8*(3*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^5 - 2*(16*a^5 - 56*a^ \\
&4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^3 - (16*a^4*b - 44*a \\
&^3*b^2 + 37*a^2*b^3 - 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5*b^3 - 2* \\
&a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^8 + 8*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d* \\
&\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*\sinh(d*x \\
&+ c)^8 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^6 \\
&+ 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^2 + (2*a^6*b^2 - 5*a \\
&^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*\sinh(d*x + c)^6 + 2*(8*a^7*b - 24*a^6*b^2 \\
&+ 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^4 + 8*(7*(a^5*b^3 - \\
&2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b \\
&^4 - a^3*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^5*b^3 - 2*a^4*b^4 \\
&+ a^3*b^5)*d*\cosh(d*x + c)^4 + 30*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3 \\
&)*b^5)*d*\cosh(d*x + c)^2 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + \\
&3*a^3*b^5)*d)*\sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3 \\
&)*b^5)*d*\cosh(d*x + c)^2 + 8*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + \\
&c)^5 + 10*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^3 \\
&+ (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + \\
&c))*\sinh(d*x + c)^3 + 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c) \\
&^6 + 15*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^4 + 3 \\
&)*(8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + \\
&c)^2 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*\sinh(d*x + c)^2 + (\\
&a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d + 8*((a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*\cos \\
&h(d*x + c)^7 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + \\
&c)^5 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*\cosh \\
&(d*x + c)^3 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c) \\
&)*\sinh(d*x + c)), 1/8*(2*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^6
\end{aligned}$$

$$\begin{aligned}
& + 12*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^5 + 2*(4 \\
& *a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*\sinh(d*x + c)^6 - 4*a^3*b^2 + 10*a^2*b^3 - \\
& 6*a*b^4 - 2*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*\cosh(d* \\
& x + c)^4 - 2*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4 - 15*(4 \\
& *a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 8*(5*(4* \\
& a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^3 - (16*a^5 - 56*a^4*b + 70*a^ \\
& 3*b^2 - 39*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 - 2*(16*a^4*b \\
& - 44*a^3*b^2 + 37*a^2*b^3 - 9*a*b^4)*\cosh(d*x + c)^2 - 2*(16*a^4*b - 44*a^3 \\
& *b^2 + 37*a^2*b^3 - 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*\cosh(d*x \\
& + c)^4 + 6*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*\cosh(d* \\
& x + c)^2)*\sinh(d*x + c)^2 - ((4*a*b^3 - 3*b^4)*\cosh(d*x + c)^8 + 8*(4*a*b^3 \\
& - 3*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (4*a*b^3 - 3*b^4)*\sinh(d*x + c)^8 \\
& + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^6 + 4*(8*a^2*b^2 - 10*a*b \\
& ^3 + 3*b^4 + 7*(4*a*b^3 - 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(4 \\
& *a*b^3 - 3*b^4)*\cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^5 + 2*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*\cosh(\\
& d*x + c)^4 + 2*(35*(4*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 32*a^3*b - 56*a^2*b^ \\
& 2 + 36*a*b^3 - 9*b^4 + 30*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*s \\
& inh(d*x + c)^4 + 4*a*b^3 - 3*b^4 + 8*(7*(4*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + \\
& 10*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + (32*a^3*b - 56*a^2*b^2 \\
& + 36*a*b^3 - 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 10*a*b \\
& ^3 + 3*b^4)*\cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 15*(\\
& 8*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 8*a^2*b^2 - 10*a*b^3 + 3*b^ \\
& 4 + 3*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x \\
& + c)^2 + 8*((4*a*b^3 - 3*b^4)*\cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 10*a*b^3 + 3 \\
& *b^4)*\cosh(d*x + c)^5 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*\cosh(d*x \\
& + c)^3 + (8*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{ \\
& (-a^2 + a*b)*\arctan(-1/2*(b*\cosh(d*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + \\
& c) + b*\sinh(d*x + c)^2 + 2*a - b)*\sqrt{-a^2 + a*b}/(a^2 - a*b)) + 4*(3*(4*a \\
& ^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*\cosh(d*x + c)^5 - 2*(16*a^5 - 56*a^4*b + 70*a \\
& ^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*\cosh(d*x + c)^3 - (16*a^4*b - 44*a^3*b^2 + 3 \\
& 7*a^2*b^3 - 9*a*b^4)*\cosh(d*x + c))*\sinh(d*x + c))/((a^5*b^3 - 2*a^4*b^4 + \\
& a^3*b^5)*d*\cosh(d*x + c)^8 + 8*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + \\
& c)*\sinh(d*x + c)^7 + (a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*\sinh(d*x + c)^8 + 4 \\
& *(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^6 + 4*(7*(a^ \\
& 5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^2 + (2*a^6*b^2 - 5*a^5*b^3 + 4 \\
& *a^4*b^4 - a^3*b^5)*d)*\sinh(d*x + c)^6 + 2*(8*a^7*b - 24*a^6*b^2 + 27*a^5*b \\
& ^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*\cosh(d*x + c)^4 + 8*(7*(a^5*b^3 - 2*a^4*b^4 \\
& + a^3*b^5)*d*\cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b \\
& ^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5 \\
&)*d*\cosh(d*x + c)^4 + 30*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*co \\
& sh(d*x + c)^2 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5 \\
&)*d)*\sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*co \\
& sh(d*x + c)^2 + 8*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*\cosh(d*x + c)^5 + 10 \\
& *(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*\cosh(d*x + c)^3 + (8*a^7*b
\end{aligned}$$

$$\begin{aligned}
& - 24a^6b^2 + 27a^5b^3 - 14a^4b^4 + 3a^3b^5) * d * \cosh(dx + c) * \sinh(dx + c)^3 + 4 * (7 * (a^5b^3 - 2a^4b^4 + a^3b^5) * d * \cosh(dx + c)^6 + 15 * (2 \\
& * a^6b^2 - 5a^5b^3 + 4a^4b^4 - a^3b^5) * d * \cosh(dx + c)^4 + 3 * (8a^7b \\
& - 24a^6b^2 + 27a^5b^3 - 14a^4b^4 + 3a^3b^5) * d * \cosh(dx + c)^2 + (2 * \\
& a^6b^2 - 5a^5b^3 + 4a^4b^4 - a^3b^5) * d) * \sinh(dx + c)^2 + (a^5b^3 - \\
& 2a^4b^4 + a^3b^5) * d + 8 * ((a^5b^3 - 2a^4b^4 + a^3b^5) * d * \cosh(dx + c) \\
& ^7 + 3 * (2a^6b^2 - 5a^5b^3 + 4a^4b^4 - a^3b^5) * d * \cosh(dx + c)^5 + (8 \\
& * a^7b - 24a^6b^2 + 27a^5b^3 - 14a^4b^4 + 3a^3b^5) * d * \cosh(dx + c)^3 \\
& + (2a^6b^2 - 5a^5b^3 + 4a^4b^4 - a^3b^5) * d * \cosh(dx + c) * \sinh(dx + c))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)**2/(a+b*sinh(dx+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.39818, size = 365, normalized size = 2.55

$$\frac{(4a - 3b) \arctan\left(\frac{be^{2dx+2c} + 2a - b}{2\sqrt{-a^2 + ab}}\right)}{8(a^3d - a^2bd)\sqrt{-a^2 + ab}} + \frac{4ab^2e^{6dx+6c} - 3b^3e^{6dx+6c} - 16a^3e^{4dx+4c} + 40a^2be^{4dx+4c} - 30ab^2e^{4dx+4c} + 4a^2b^2e^{4dx+4c}}{4(a^3bd - a^2b^2d)(be^{4dx+4c} + 4a^2b^2e^{4dx+4c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)^2/(a+b*sinh(dx+c)^2)^3,x, algorithm="giac")

[Out] 1/8*(4*a - 3*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^3*d - a^2*b*d)*sqrt(-a^2 + a*b)) + 1/4*(4*a*b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) - 16*a^3*e^(4*d*x + 4*c) + 40*a^2*b*e^(4*d*x + 4*c) - 30*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) - 16*a^2*b*e^(2*d*x + 2*c) + 2*8*a*b^2*e^(2*d*x + 2*c) - 9*b^3*e^(2*d*x + 2*c) - 2*a*b^2 + 3*b^3)/((a^3*b*d - a^2*b^2*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2)

$$3.344 \quad \int \frac{\cosh(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^3} dx$$

Optimal. Leaf size=96

$$\frac{3 \sinh(c+dx)}{8a^2d(a+b \sinh^2(c+dx))} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{bd}} + \frac{\sinh(c+dx)}{4ad(a+b \sinh^2(c+dx))^2}$$

[Out] (3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Sinh[c + d*x]/(4*a*d*(a + b*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/(8*a^2*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.0571966, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3190, 199, 205}

$$\frac{3 \sinh(c+dx)}{8a^2d(a+b \sinh^2(c+dx))} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{bd}} + \frac{\sinh(c+dx)}{4ad(a+b \sinh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Sinh[c + d*x]/(4*a*d*(a + b*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/(8*a^2*d*(a + b*Sinh[c + d*x]^2))

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 199

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer

$Q[2*p] \parallel (n == 2 \ \&\& \ \text{IntegerQ}[4*p]) \parallel (n == 2 \ \&\& \ \text{IntegerQ}[3*p]) \parallel \text{Denominator}[p + 1/n] < \text{Denominator}[p]$

Rule 205

$\text{Int}[(a_ + (b_ \cdot x)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2] \cdot \text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \ \&\& \ \text{PosQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{(a + b \sinh^2(c + dx))^3} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^3} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\sinh(c + dx)}{4ad (a + b \sinh^2(c + dx))^2} + \frac{3 \text{Subst}\left(\int \frac{1}{(a+bx^2)^2} dx, x, \sinh(c + dx)\right)}{4ad} \\ &= \frac{\sinh(c + dx)}{4ad (a + b \sinh^2(c + dx))^2} + \frac{3 \sinh(c + dx)}{8a^2d (a + b \sinh^2(c + dx))} + \frac{3 \text{Subst}\left(\int \frac{1}{a+bx^2} dx, x, \sinh(c + dx)\right)}{8a^2d} \\ &= \frac{3 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{bd}} + \frac{\sinh(c + dx)}{4ad (a + b \sinh^2(c + dx))^2} + \frac{3 \sinh(c + dx)}{8a^2d (a + b \sinh^2(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.170788, size = 79, normalized size = 0.82

$$\frac{\frac{\sqrt{a} \sinh(c+dx)(5a+3b \sinh^2(c+dx))}{(a+b \sinh^2(c+dx))^2} + \frac{3 \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/Sqrt[b] + (Sqrt[a]*Sinh[c + d*x]*(5*a + 3*b*Sinh[c + d*x]^2))/(a + b*Sinh[c + d*x]^2)^2)/(8*a^(5/2)*d)

Maple [A] time = 0.013, size = 85, normalized size = 0.9

$$\frac{\sinh(dx+c)}{4da(a+b(\sinh(dx+c))^2)^2} + \frac{3\sinh(dx+c)}{8da^2(a+b(\sinh(dx+c))^2)} + \frac{3}{8da^2} \arctan\left(b\sinh(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)

[Out] 1/4*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+b*sinh(d*x+c)^2)+3/8/d/a^2/(a*b)^(1/2)*arctan(sinh(d*x+c)*b/(a*b)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(20ae^{5c} - 9be^{5c})e^{5dx} - (20ae^{3c} - 9be^{3c})e^{3dx} + 3be^{7dx+7c} - 3be^{dx+c}}{4(a^2b^2de^{8dx+8c} + a^2b^2d + 4(2a^3bde^{6c} - a^2b^2de^{6c})e^{6dx} + 2(8a^4de^{4c} - 8a^3bde^{4c} + 3a^2b^2de^{4c})e^{4dx} + 4(2a^3bde^{2c} - a^2b^2de^{2c})e^{2dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] 1/4*((20*a*e^(5*c) - 9*b*e^(5*c))*e^(5*d*x) - (20*a*e^(3*c) - 9*b*e^(3*c))*e^(3*d*x) + 3*b*e^(7*d*x + 7*c) - 3*b*e^(d*x + c))/(a^2*b^2*d*e^(8*d*x + 8*c) + a^2*b^2*d + 4*(2*a^3*b*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2*(8*a^4*d*e^(4*c) - 8*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3*b*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(3/2*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)

Fricas [B] time = 2.18802, size = 9451, normalized size = 98.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(12*a*b^2*cosh(d*x + c)^7 + 84*a*b^2*cosh(d*x + c)*sinh(d*x + c)^6 + 12*a*b^2*sinh(d*x + c)^7 + 4*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^5 + 4*(63*a

$$\begin{aligned}
& *b^2 \cosh(dx + c)^2 + 20a^2b - 9a^2b^2) \sinh(dx + c)^5 + 20(21ab^2 \cosh(dx + c)^3 + (20a^2b - 9a^2b^2) \cosh(dx + c)) \sinh(dx + c)^4 - 12a^2b^2 \cosh(dx + c) - 4(20a^2b - 9a^2b^2) \cosh(dx + c)^3 + 4(105ab^2 \cosh(dx + c)^4 - 20a^2b + 9a^2b^2 + 10(20a^2b - 9a^2b^2) \cosh(dx + c)^2) \sinh(dx + c)^3 + 4(63ab^2 \cosh(dx + c)^5 + 10(20a^2b - 9a^2b^2) \cosh(dx + c)^3 - 3(20a^2b - 9a^2b^2) \cosh(dx + c)) \sinh(dx + c)^2 - 3(b^2 \cosh(dx + c)^8 + 8b^2 \cosh(dx + c) \sinh(dx + c)^7 + b^2 \sinh(dx + c)^8 + 4(2ab - b^2) \cosh(dx + c)^6 + 4(7b^2 \cosh(dx + c)^2 + 2ab - b^2) \sinh(dx + c)^6 + 8(7b^2 \cosh(dx + c)^3 + 3(2ab - b^2) \cosh(dx + c)) \sinh(dx + c)^5 + 2(8a^2 - 8ab + 3b^2) \cosh(dx + c)^4 + 2(35b^2 \cosh(dx + c)^4 + 30(2ab - b^2) \cosh(dx + c)^2 + 8a^2 - 8ab + 3b^2) \sinh(dx + c)^4 + 8(7b^2 \cosh(dx + c)^5 + 10(2ab - b^2) \cosh(dx + c)^3 + (8a^2 - 8ab + 3b^2) \cosh(dx + c)) \sinh(dx + c)^3 + 4(2ab - b^2) \cosh(dx + c)^2 + 4(7b^2 \cosh(dx + c)^6 + 15(2ab - b^2) \cosh(dx + c)^4 + 3(8a^2 - 8ab + 3b^2) \cosh(dx + c)^2 + 2ab - b^2) \sinh(dx + c)^2 + b^2 + 8(b^2 \cosh(dx + c)^7 + 3(2ab - b^2) \cosh(dx + c)^5 + (8a^2 - 8ab + 3b^2) \cosh(dx + c)^3 + (2ab - b^2) \cosh(dx + c)) \sinh(dx + c) \sqrt{-ab} \log((b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 - 2(2a + b) \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 - 2a - b) \sinh(dx + c)^2 + 4(b \cosh(dx + c)^3 - (2a + b) \cosh(dx + c)) \sinh(dx + c) - 4(\cosh(dx + c)^3 + 3 \cosh(dx + c) \sinh(dx + c)^2 + \sinh(dx + c)^3 + (3 \cosh(dx + c)^2 - 1) \sinh(dx + c) - \cosh(dx + c)) \sqrt{-ab} + b) / (b \cosh(dx + c)^4 + 4b \cosh(dx + c) \sinh(dx + c)^3 + b \sinh(dx + c)^4 + 2(2a - b) \cosh(dx + c)^2 + 2(3b \cosh(dx + c)^2 + 2a - b) \sinh(dx + c)^2 + 4(b \cosh(dx + c)^3 + (2a - b) \cosh(dx + c)) \sinh(dx + c) + b)) + 4(21ab^2 \cosh(dx + c)^6 + 5(20a^2b - 9a^2b^2) \cosh(dx + c)^4 - 3a^2b^2 - 3(20a^2b - 9a^2b^2) \cosh(dx + c)^2) \sinh(dx + c) / (a^3b^3d \cosh(dx + c)^8 + 8a^3b^3d \cosh(dx + c) \sinh(dx + c)^7 + a^3b^3d \sinh(dx + c)^8 + 4(2a^4b^2 - a^3b^3) d \cosh(dx + c)^6 + a^3b^3d + 4(7a^3b^3d \cosh(dx + c)^2 + (2a^4b^2 - a^3b^3) d) \sinh(dx + c)^6 + 2(8a^5b - 8a^4b^2 + 3a^3b^3) d \cosh(dx + c)^4 + 8(7a^3b^3d \cosh(dx + c)^3 + 3(2a^4b^2 - a^3b^3) d \cosh(dx + c)) \sinh(dx + c)^5 + 2(35a^3b^3d \cosh(dx + c)^4 + 30(2a^4b^2 - a^3b^3) d \cosh(dx + c)^2 + (8a^5b - 8a^4b^2 + 3a^3b^3) d) \sinh(dx + c)^4 + 4(2a^4b^2 - a^3b^3) d \cosh(dx + c)^2 + 8(7a^3b^3d \cosh(dx + c)^5 + 10(2a^4b^2 - a^3b^3) d \cosh(dx + c)^3 + (8a^5b - 8a^4b^2 + 3a^3b^3) d \cosh(dx + c)) \sinh(dx + c)^3 + 4(7a^3b^3d \cosh(dx + c)^6 + 15(2a^4b^2 - a^3b^3) d \cosh(dx + c)^4 + 3(8a^5b - 8a^4b^2 + 3a^3b^3) d \cosh(dx + c)^2 + (2a^4b^2 - a^3b^3) d) \sinh(dx + c)^2 + 8(a^3b^3d \cosh(dx + c)^7 + 3(2a^4b^2 - a^3b^3) d \cosh(dx + c)^5 + (8a^5b - 8a^4b^2 + 3a^3b^3) d \cosh(dx + c)^3 + (2a^4b^2 - a^3b^3) d \cosh(dx + c)) \sinh(dx + c)), 1/8(6a^2b^2 \cosh(dx + c)^7 + 42a^2b^2 \cosh(dx + c) \sinh(dx + c)^6 + 6a^2b^2 \sinh(dx + c)^7 + 2(20a^2b - 9a^2b^2) \cosh(dx + c)^5 + 2(63ab^2 \cosh(dx + c)^2 + 20a^2b - 9a^2b^2) \sinh(dx + c)^5 + 10(21ab^2 \cosh(dx + c)^3 + (20a^2b - 9a^2b^2) \cosh(dx + c)
\end{aligned}$$

$$\begin{aligned}
& + c)) * \sinh(dx + c)^4 - 6*a*b^2 * \cosh(dx + c) - 2*(20*a^2*b - 9*a*b^2) * \cosh(dx + c)^3 + 2*(105*a*b^2 * \cosh(dx + c)^4 - 20*a^2*b + 9*a*b^2 + 10*(20*a^2*b - 9*a*b^2) * \cosh(dx + c)^2) * \sinh(dx + c)^3 + 2*(63*a*b^2 * \cosh(dx + c)^5 + 10*(20*a^2*b - 9*a*b^2) * \cosh(dx + c)^3 - 3*(20*a^2*b - 9*a*b^2) * \cosh(dx + c)) * \sinh(dx + c)^2 + 3*(b^2 * \cosh(dx + c)^8 + 8*b^2 * \cosh(dx + c) * \sinh(dx + c)^7 + b^2 * \sinh(dx + c)^8 + 4*(2*a*b - b^2) * \cosh(dx + c)^6 + 4*(7*b^2 * \cosh(dx + c)^2 + 2*a*b - b^2) * \sinh(dx + c)^6 + 8*(7*b^2 * \cosh(dx + c)^3 + 3*(2*a*b - b^2) * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(8*a^2 - 8*a*b + 3*b^2) * \cosh(dx + c)^4 + 2*(35*b^2 * \cosh(dx + c)^4 + 30*(2*a*b - b^2) * \cosh(dx + c)^2 + 8*a^2 - 8*a*b + 3*b^2) * \sinh(dx + c)^4 + 8*(7*b^2 * \cosh(dx + c)^5 + 10*(2*a*b - b^2) * \cosh(dx + c)^3 + (8*a^2 - 8*a*b + 3*b^2) * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(2*a*b - b^2) * \cosh(dx + c)^2 + 4*(7*b^2 * \cosh(dx + c)^6 + 15*(2*a*b - b^2) * \cosh(dx + c)^4 + 3*(8*a^2 - 8*a*b + 3*b^2) * \cosh(dx + c)^2 + 2*a*b - b^2) * \sinh(dx + c)^2 + b^2 + 8*(b^2 * \cosh(dx + c)^7 + 3*(2*a*b - b^2) * \cosh(dx + c)^5 + (8*a^2 - 8*a*b + 3*b^2) * \cosh(dx + c)^3 + (2*a*b - b^2) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{a*b} * \arctan(1/2 * \sqrt{a*b} * (\cosh(dx + c) + \sinh(dx + c))/a) + 3*(b^2 * \cosh(dx + c)^8 + 8*b^2 * \cosh(dx + c) * \sinh(dx + c)^7 + b^2 * \sinh(dx + c)^8 + 4*(2*a*b - b^2) * \cosh(dx + c)^6 + 4*(7*b^2 * \cosh(dx + c)^2 + 2*a*b - b^2) * \sinh(dx + c)^6 + 8*(7*b^2 * \cosh(dx + c)^3 + 3*(2*a*b - b^2) * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(8*a^2 - 8*a*b + 3*b^2) * \cosh(dx + c)^4 + 2*(35*b^2 * \cosh(dx + c)^4 + 30*(2*a*b - b^2) * \cosh(dx + c)^2 + 8*a^2 - 8*a*b + 3*b^2) * \sinh(dx + c)^4 + 8*(7*b^2 * \cosh(dx + c)^5 + 10*(2*a*b - b^2) * \cosh(dx + c)^3 + (8*a^2 - 8*a*b + 3*b^2) * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(2*a*b - b^2) * \cosh(dx + c)^2 + 4*(7*b^2 * \cosh(dx + c)^6 + 15*(2*a*b - b^2) * \cosh(dx + c)^4 + 3*(8*a^2 - 8*a*b + 3*b^2) * \cosh(dx + c)^2 + 2*a*b - b^2) * \sinh(dx + c)^2 + b^2 + 8*(b^2 * \cosh(dx + c)^7 + 3*(2*a*b - b^2) * \cosh(dx + c)^5 + (8*a^2 - 8*a*b + 3*b^2) * \cosh(dx + c)^3 + (2*a*b - b^2) * \cosh(dx + c)) * \sinh(dx + c)) * \sqrt{a*b} * \arctan(1/2 * (b * \cosh(dx + c)^3 + 3*b * \cosh(dx + c) * \sinh(dx + c)^2 + b * \sinh(dx + c)^3 + (4*a - b) * \cosh(dx + c) + (3*b * \cosh(dx + c)^2 + 4*a - b) * \sinh(dx + c)) * \sqrt{a*b} / (a*b)) + 2*(21*a*b^2 * \cosh(dx + c)^6 + 5*(20*a^2*b - 9*a*b^2) * \cosh(dx + c)^4 - 3*a*b^2 - 3*(20*a^2*b - 9*a*b^2) * \cosh(dx + c)^2) * \sinh(dx + c)) / (a^3*b^3*d * \cosh(dx + c)^8 + 8*a^3*b^3*d * \cosh(dx + c) * \sinh(dx + c)^7 + a^3*b^3*d * \sinh(dx + c)^8 + 4*(2*a^4*b^2 - a^3*b^3) * d * \cosh(dx + c)^6 + a^3*b^3*d + 4*(7*a^3*b^3*d * \cosh(dx + c)^2 + (2*a^4*b^2 - a^3*b^3) * d) * \sinh(dx + c)^6 + 2*(8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3) * d * \cosh(dx + c)^4 + 8*(7*a^3*b^3*d * \cosh(dx + c)^3 + 3*(2*a^4*b^2 - a^3*b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^5 + 2*(35*a^3*b^3*d * \cosh(dx + c)^4 + 30*(2*a^4*b^2 - a^3*b^3) * d * \cosh(dx + c)^2 + (8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3) * d) * \sinh(dx + c)^4 + 4*(2*a^4*b^2 - a^3*b^3) * d * \cosh(dx + c)^2 + 8*(7*a^3*b^3*d * \cosh(dx + c)^5 + 10*(2*a^4*b^2 - a^3*b^3) * d * \cosh(dx + c)^3 + (8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3) * d * \cosh(dx + c)) * \sinh(dx + c)^3 + 4*(7*a^3*b^3*d * \cosh(dx + c)^6 + 15*(2*a^4*b^2 - a^3*b^3) * d * \cosh(dx + c)^4 + 3*(8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3) * d * \cosh(dx + c)^2 + (2*a^4*b^2 - a^3*b^3) * d) * \sinh(dx + c)^2 + 8*(a^3*b^3 * d * \cosh(dx + c)^7 + 3*(2*a^4*b^2 - a^3*b^3) * d * \cosh(dx + c)^5 + (8*a^5*b -
\end{aligned}$$

$$8a^4b^2 + 3a^3b^3)d \cosh(dx + c)^3 + (2a^4b^2 - a^3b^3)d \cosh(dx + c) \sinh(dx + c)]$$

Sympy [A] time = 175.048, size = 915, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(dx+c)/(a+b*sinh(dx+c)**2)**3,x)

[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh(c + dx)/(a**3*d), Eq(b, 0)), (-1/(5*b**3*d*sinh(c + dx)**5), Eq(a, 0)), (x*cosh(c)/(a + b*sinh(c)**2)**3, Eq(d, 0)), (10*I*a**(3/2)*b*sqrt(1/b)*sinh(c + dx)/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + dx)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + dx)**4) + 6*I*sqrt(a)*b**2*sqrt(1/b)*sinh(c + dx)**3/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + dx)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + dx)**4) + 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + dx))/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + dx)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + dx)**4) - 3*a**2*log(I*sqrt(a)*sqrt(1/b) + sinh(c + dx))/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + dx)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + dx)**4) + 6*a*b*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + dx))*sinh(c + dx)**2/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + dx)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + dx)**4) - 6*a*b*log(I*sqrt(a)*sqrt(1/b) + sinh(c + dx))*sinh(c + dx)**2/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + dx)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + dx)**4) + 3*b**2*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + dx))*sinh(c + dx)**4/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + dx)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + dx)**4) - 3*b**2*log(I*sqrt(a)*sqrt(1/b) + sinh(c + dx))*sinh(c + dx)**4/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + dx)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + dx)**4), True))

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.345 \quad \int \frac{\operatorname{sech}(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^3} dx$$

Optimal. Leaf size=159

$$\frac{\sqrt{b}(15a^2 - 10ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3} - \frac{b(7a-3b) \sinh(c+dx)}{8a^2d(a-b)^2(a+b \sinh^2(c+dx))} - \frac{b \sinh(c+dx)}{4ad(a-b)(a+b \sinh^2(c+dx))^2}$$

[Out] ArcTan[Sinh[c + d*x]]/((a - b)^3*d) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Sinh[c + d*x])/(4*a*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2) - ((7*a - 3*b)*b*Sinh[c + d*x])/(8*a^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))

Rubi [A] time = 0.180384, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{\sqrt{b}(15a^2 - 10ab + 3b^2) \tan^{-1}\left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3} - \frac{b(7a-3b) \sinh(c+dx)}{8a^2d(a-b)^2(a+b \sinh^2(c+dx))} - \frac{b \sinh(c+dx)}{4ad(a-b)(a+b \sinh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ArcTan[Sinh[c + d*x]]/((a - b)^3*d) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Sinh[c + d*x])/(4*a*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2) - ((7*a - 3*b)*b*Sinh[c + d*x])/(8*a^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]

```

Rule 527

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

```

Rule 522

```

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]

```

Rule 203

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

```

Rule 205

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= -\frac{b\sinh(c+dx)}{4a(a-b)d(a+b\sinh^2(c+dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2} dx, x, \sinh(c+dx)\right)}{4a(a-b)d} \\
&= -\frac{b\sinh(c+dx)}{4a(a-b)d(a+b\sinh^2(c+dx))^2} - \frac{(7a-3b)b\sinh(c+dx)}{8a^2(a-b)^2d(a+b\sinh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{8a^2}{1+x^2} dx, x, \sinh(c+dx)\right)}{8a^2(a-b)^2d(a+b\sinh^2(c+dx))} \\
&= -\frac{b\sinh(c+dx)}{4a(a-b)d(a+b\sinh^2(c+dx))^2} - \frac{(7a-3b)b\sinh(c+dx)}{8a^2(a-b)^2d(a+b\sinh^2(c+dx))} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c+dx)\right)}{8a^2(a-b)^2d(a+b\sinh^2(c+dx))} \\
&= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)^3d} - \frac{\sqrt{b}(15a^2-10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} - \frac{b\sinh(c+dx)}{4a(a-b)d(a+b\sinh^2(c+dx))}
\end{aligned}$$

Mathematica [B] time = 0.730706, size = 321, normalized size = 2.02

$$-2\sqrt{ab}(-35a^2b+18a^3+20ab^2-3b^3)\sinh(c+dx) + (b-2a)^2\left(\sqrt{b}(15a^2-10ab+3b^2)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)\right) + 16a^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((-2*a + b)^2*(Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]]) + (b^(5/2)*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 16*a^(5/2)*b^2*ArcTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)]^2 - 2*Sqrt[a]*b*(18*a^3 - 35*a^2*b + 20*a*b^2 - 3*b^3)*Sinh[c + d*x] - 2*b*Cosh[2*(c + d*x)]*(-((2*a - b)*(Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]])) + Sqrt[a]*b*(7*a^2 - 10*a*b + 3*b^2)*Sinh[c + d*x]))/(8*a^(5/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d*x)])^2)

Maple [B] time = 0.092, size = 2118, normalized size = 13.3

result too large to display

$$2*b+a)^2*\tanh(1/2*d*x+1/2*c)^7+35/2/d*b^2/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^5-35/2/d*b^2/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^3+7/2/d*b^2/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)+15/8/d*b/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-15/8/d*b/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out]
$$-1/4*((7*a*b^2*e^{(7*c)} - 3*b^3*e^{(7*c)})e^{(7*d*x)} + (36*a^2*b*e^{(5*c)} - 41*a*b^2*e^{(5*c)} + 9*b^3*e^{(5*c)})e^{(5*d*x)} - (36*a^2*b*e^{(3*c)} - 41*a*b^2*e^{(3*c)} + 9*b^3*e^{(3*c)})e^{(3*d*x)} - (7*a*b^2*e^c - 3*b^3*e^c)e^{(d*x)})/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^{(8*c)} - 2*a^3*b^3*d*e^{(8*c)} + a^2*b^4*d*e^{(8*c)})e^{(8*d*x)} + 4*(2*a^5*b*d*e^{(6*c)} - 5*a^4*b^2*d*e^{(6*c)} + 4*a^3*b^3*d*e^{(6*c)} - a^2*b^4*d*e^{(6*c)})e^{(6*d*x)} + 2*(8*a^6*d*e^{(4*c)} - 24*a^5*b*d*e^{(4*c)} + 27*a^4*b^2*d*e^{(4*c)} - 14*a^3*b^3*d*e^{(4*c)} + 3*a^2*b^4*d*e^{(4*c)})e^{(4*d*x)} + 4*(2*a^5*b*d*e^{(2*c)} - 5*a^4*b^2*d*e^{(2*c)} + 4*a^3*b^3*d*e^{(2*c)} - a^2*b^4*d*e^{(2*c)})e^{(2*d*x)}) + 2*\operatorname{arctan}(e^{(d*x + c)})/(a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d) - 2*\operatorname{integrate}(1/8*((15*a^2*b*e^{(3*c)} - 10*a*b^2*e^{(3*c)} + 3*b^3*e^{(3*c)})e^{(3*d*x)} + (15*a^2*b*e^c - 10*a*b^2*e^c + 3*b^3*e^c)e^{(d*x)})/(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4 + (a^5*b*e^{(4*c)} - 3*a^4*b^2*e^{(4*c)} + 3*a^3*b^3*e^{(4*c)} - a^2*b^4*e^{(4*c)})e^{(4*d*x)} + 2*(2*a^6*e^{(2*c)} - 7*a^5*b*e^{(2*c)} + 9*a^4*b^2*e^{(2*c)} - 5*a^3*b^3*e^{(2*c)} + a^2*b^4*e^{(2*c)})e^{(2*d*x)}), x)$$

Fricas [B] time = 3.15793, size = 18737, normalized size = 117.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^7 + 28*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^6 + 4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\sinh(d*x + c)^7 + 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*\cosh(d*x + c)^5 + 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4 + 21*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^5 + 20*(7*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + (36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^4 - 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*\cosh(d*x + c)^3 + 4*(35*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 - 36*a^3*b + 77*a^2*b^2 - 50*a*b^3 + 9*b^4 + 10*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^3 + 4*(21*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^5 + 10*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*\cosh(d*x + c)^3 - 3*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^2 + ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^8 + 8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)*\sinh(d*x + c)^7 + (15*a^2*b^2 - 10*a*b^3 + 3*b^4)*\sinh(d*x + c)^8 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*\cosh(d*x + c)^6 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^3 + 3*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*\cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^4 + 120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 15*a^2*b^2 - 10*a*b^3 + 3*b^4 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^5 + 10*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*\cosh(d*x + c)^3 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*\cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^6 + 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*\cosh(d*x + c)^4 + 30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3*(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^7 + 3*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*\cosh(d*x + c)^3 + (30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*\cosh(d*x + c))*\sinh(d*x + c)*\sqrt{-b/a}*\log((b*\cosh(d*x + c))^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 - 2*(2*a + b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 - 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 - (2*a + b)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(a*\cosh(d*x + c)^3 + 3*a*\cosh(d*x + c)*\sinh(d*x + c)^2 + a*\sinh(d*x + c)^3 - a*\cosh(d*x + c) + (3*a*\cosh(d*x + c)^2 - a)*\sinh(d*x + c))*\sqrt{-b/a} + b)/(b*\cosh(d*x + c))^4 + 4*b*\cosh(d*x + c)*\sinh(d*x + c)^3 + b*\sinh(d*x + c)^4 + 2*(2*a - b)*\cosh(d*x + c)^2 + 2*(3*b*\cosh(d*x + c)^2 + 2*a - b)*\sinh(d*x + c)^2 + 4*(b*\cosh(d*x + c)^3 + (2*a - b)*\cosh(d*x + c))*\sinh(d*x + c) + b)) - 32*(a^2*b^2*\cosh(d*x + c)^8 + 8*a^2*b^2*co$$

$$\begin{aligned}
& \text{sh}(d*x + c)*\sinh(d*x + c)^7 + a^2*b^2*\sinh(d*x + c)^8 + 4*(2*a^3*b - a^2*b^2) \\
& *2*\cosh(d*x + c)^6 + 4*(7*a^2*b^2*\cosh(d*x + c)^2 + 2*a^3*b - a^2*b^2)*\sinh \\
& (d*x + c)^6 + 8*(7*a^2*b^2*\cosh(d*x + c)^3 + 3*(2*a^3*b - a^2*b^2)*\cosh(d*x \\
& + c))*\sinh(d*x + c)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^4 + \\
& 2*(35*a^2*b^2*\cosh(d*x + c)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - \\
& a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + a^2*b^2 + 8*(7*a^2*b^2*\cosh(d* \\
& x + c)^5 + 10*(2*a^3*b - a^2*b^2)*\cosh(d*x + c)^3 + (8*a^4 - 8*a^3*b + 3*a^ \\
& 2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(2*a^3*b - a^2*b^2)*\cosh(d*x + c) \\
& ^2 + 4*(7*a^2*b^2*\cosh(d*x + c)^6 + 15*(2*a^3*b - a^2*b^2)*\cosh(d*x + c)^4 \\
& + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^2)*\sinh \\
& (d*x + c)^2 + 8*(a^2*b^2*\cosh(d*x + c)^7 + 3*(2*a^3*b - a^2*b^2)*\cosh(d*x + \\
& c)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*\cosh(d*x + c)^3 + (2*a^3*b - a^2*b^2) \\
& *\cosh(d*x + c))*\sinh(d*x + c))*\arctan(\cosh(d*x + c) + \sinh(d*x + c)) - 4*(7 \\
& *a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c) + 4*(7*(7*a^2*b^2 - 10*a*b^3 + 3 \\
& *b^4)*\cosh(d*x + c)^6 + 5*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*\cosh(d \\
& *x + c)^4 - 7*a^2*b^2 + 10*a*b^3 - 3*b^4 - 3*(36*a^3*b - 77*a^2*b^2 + 50*a* \\
& b^3 - 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3* \\
& b^4 - a^2*b^5)*d*\cosh(d*x + c)^8 + 8*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2 \\
& *b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - \\
& a^2*b^5)*d*\sinh(d*x + c)^8 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 \\
& + a^2*b^5)*d*\cosh(d*x + c)^6 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2 \\
& *b^5)*d*\cosh(d*x + c)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^ \\
& 2*b^5)*d)*\sinh(d*x + c)^6 + 2*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + \\
& 17*a^3*b^4 - 3*a^2*b^5)*d*\cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3* \\
& a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^3 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - \\
& 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(35*(a^5*b^2 - 3 \\
& *a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^4 + 30*(2*a^6*b - 7*a^5*b^2 \\
& + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^2 + (8*a^7 - 32*a^6*b + \\
& 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d)*\sinh(d*x + c)^4 + 4*(\\
& 2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^2 + \\
& 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^5 + 10*(2* \\
& a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^3 + (8 \\
& *a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*\cosh(\\
& d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5 \\
&)*d*\cosh(d*x + c)^6 + 15*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2 \\
& *b^5)*d*\cosh(d*x + c)^4 + 3*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 1 \\
& 7*a^3*b^4 - 3*a^2*b^5)*d*\cosh(d*x + c)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 \\
& - 5*a^3*b^4 + a^2*b^5)*d)*\sinh(d*x + c)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b \\
& ^4 - a^2*b^5)*d + 8*((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x \\
& + c)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cosh(\\
& d*x + c)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a \\
& ^2*b^5)*d*\cosh(d*x + c)^3 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + \\
& a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)), -1/8*(2*(7*a^2*b^2 - 10*a*b^3 + 3 \\
& *b^4)*\cosh(d*x + c)^7 + 14*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)*\sin \\
& h(d*x + c)^6 + 2*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\sinh(d*x + c)^7 + 2*(36*a^3
\end{aligned}$$

$$\begin{aligned}
& *b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4) * \cosh(d*x + c)^5 + 2*(36*a^3*b - 77*a^2* \\
& b^2 + 50*a*b^3 - 9*b^4 + 21*(7*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^2) \\
& * \sinh(d*x + c)^5 + 10*(7*(7*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^3 + (\\
& 36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4) * \cosh(d*x + c)) * \sinh(d*x + c)^4 - \\
& 2*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4) * \cosh(d*x + c)^3 + 2*(35*(7*a^2 \\
& *b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^4 - 36*a^3*b + 77*a^2*b^2 - 50*a*b^3 \\
& + 9*b^4 + 10*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4) * \cosh(d*x + c)^2) * s \\
& \sinh(d*x + c)^3 + 2*(21*(7*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^5 + 10* \\
& (36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4) * \cosh(d*x + c)^3 - 3*(36*a^3*b - \\
& 77*a^2*b^2 + 50*a*b^3 - 9*b^4) * \cosh(d*x + c)) * \sinh(d*x + c)^2 + ((15*a^2*b^ \\
& 2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^8 + 8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) * c \\
& \cosh(d*x + c) * \sinh(d*x + c)^7 + (15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \sinh(d*x + c \\
&)^8 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)^6 + 4*(30* \\
& a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) * c \\
& \cosh(d*x + c)^2) * \sinh(d*x + c)^6 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh \\
& (d*x + c)^3 + 3*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)) * s \\
& \sinh(d*x + c)^5 + 2*(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4) * c \\
& \cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^4 + 12 \\
& 0*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2* \\
& b^2 + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)^2) * \sinh(d*x + c)^4 + 15*a^2*b^2 - 10* \\
& a*b^3 + 3*b^4 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^5 + 10*(\\
& 30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)^3 + (120*a^4 - 200* \\
& a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4) * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 4* \\
& (30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)^2 + 4*(7*(15*a^2*b \\
& ^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^6 + 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b \\
& ^3 - 3*b^4) * \cosh(d*x + c)^4 + 30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3* \\
& (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4) * \cosh(d*x + c)^2) * \sin \\
& h(d*x + c)^2 + 8*((15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^7 + 3*(30*a \\
& ^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)^5 + (120*a^4 - 200*a^3* \\
& b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4) * \cosh(d*x + c)^3 + (30*a^3*b - 35*a^2*b^ \\
& 2 + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)) * \sinh(d*x + c)) * \sqrt{b/a} * \arctan(1/2*\sqrt{ \\
& b/a} * (\cosh(d*x + c) + \sinh(d*x + c))) + ((15*a^2*b^2 - 10*a*b^3 + 3*b^4) \\
& * \cosh(d*x + c)^8 + 8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c) * \sinh(d*x \\
& + c)^7 + (15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \sinh(d*x + c)^8 + 4*(30*a^3*b - 3 \\
& 5*a^2*b^2 + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)^6 + 4*(30*a^3*b - 35*a^2*b^2 + \\
& 16*a*b^3 - 3*b^4 + 7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^2) * \sinh(\\
& d*x + c)^6 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^3 + 3*(30*a \\
& ^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)) * \sinh(d*x + c)^5 + 2*(1 \\
& 20*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4) * \cosh(d*x + c)^4 + 2*(3 \\
& 5*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^4 + 120*a^4 - 200*a^3*b + 1 \\
& 49*a^2*b^2 - 54*a*b^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^ \\
& 4) * \cosh(d*x + c)^2) * \sinh(d*x + c)^4 + 15*a^2*b^2 - 10*a*b^3 + 3*b^4 + 8*(7* \\
& (15*a^2*b^2 - 10*a*b^3 + 3*b^4) * \cosh(d*x + c)^5 + 10*(30*a^3*b - 35*a^2*b^2 \\
& + 16*a*b^3 - 3*b^4) * \cosh(d*x + c)^3 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - \\
& 54*a*b^3 + 9*b^4) * \cosh(d*x + c)) * \sinh(d*x + c)^3 + 4*(30*a^3*b - 35*a^2*b^
\end{aligned}$$

$$\begin{aligned}
& 2 + 16*a*b^3 - 3*b^4)*\cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4) \\
&)*\cosh(d*x + c)^6 + 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*\cosh(d*x \\
& + c)^4 + 30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3*(120*a^4 - 200*a^3*b \\
& + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*((15 \\
& *a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^7 + 3*(30*a^3*b - 35*a^2*b^2 + 1 \\
& 6*a*b^3 - 3*b^4)*\cosh(d*x + c)^5 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54* \\
& a*b^3 + 9*b^4)*\cosh(d*x + c)^3 + (30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4) \\
& *\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{b/a}*\arctan(1/2*(b*\cosh(d*x + c)^3 + 3* \\
& b*\cosh(d*x + c)*\sinh(d*x + c)^2 + b*\sinh(d*x + c)^3 + (4*a - b)*\cosh(d*x + \\
& c) + (3*b*\cosh(d*x + c)^2 + 4*a - b)*\sinh(d*x + c))*\sqrt{b/a}/b) - 16*(a^2* \\
& b^2*\cosh(d*x + c)^8 + 8*a^2*b^2*\cosh(d*x + c)*\sinh(d*x + c)^7 + a^2*b^2*\sin \\
& h(d*x + c)^8 + 4*(2*a^3*b - a^2*b^2)*\cosh(d*x + c)^6 + 4*(7*a^2*b^2*\cosh(d* \\
& x + c)^2 + 2*a^3*b - a^2*b^2)*\sinh(d*x + c)^6 + 8*(7*a^2*b^2*\cosh(d*x + c)^ \\
& 3 + 3*(2*a^3*b - a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(8*a^4 - 8*a^3 \\
& *b + 3*a^2*b^2)*\cosh(d*x + c)^4 + 2*(35*a^2*b^2*\cosh(d*x + c)^4 + 8*a^4 - 8 \\
& *a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^ \\
& 4 + a^2*b^2 + 8*(7*a^2*b^2*\cosh(d*x + c)^5 + 10*(2*a^3*b - a^2*b^2)*\cosh(d* \\
& x + c)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4 \\
& *(2*a^3*b - a^2*b^2)*\cosh(d*x + c)^2 + 4*(7*a^2*b^2*\cosh(d*x + c)^6 + 15*(2 \\
& *a^3*b - a^2*b^2)*\cosh(d*x + c)^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b \\
& + 3*a^2*b^2)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 8*(a^2*b^2*\cosh(d*x + c)^7 \\
& + 3*(2*a^3*b - a^2*b^2)*\cosh(d*x + c)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*\cos \\
& h(d*x + c)^3 + (2*a^3*b - a^2*b^2)*\cosh(d*x + c))*\sinh(d*x + c))*\arctan(\cos \\
& h(d*x + c) + \sinh(d*x + c)) - 2*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c \\
&) + 2*(7*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*\cosh(d*x + c)^6 + 5*(36*a^3*b - 77* \\
& a^2*b^2 + 50*a*b^3 - 9*b^4)*\cosh(d*x + c)^4 - 7*a^2*b^2 + 10*a*b^3 - 3*b^4 \\
& - 3*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + \\
& c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^8 + 8*(a^5 \\
& *b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)*\sinh(d*x + c)^7 + (\\
& a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\sinh(d*x + c)^8 + 4*(2*a^6*b - \\
& 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^6 + 4*(7*(a^5 \\
& *b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^2 + (2*a^6*b - 7*a^ \\
& 5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d)*\sinh(d*x + c)^6 + 2*(8*a^7 - 32 \\
& *a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*\cosh(d*x + c)^ \\
& 4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^3 + 3* \\
& (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c))*\si \\
& nh(d*x + c)^5 + 2*(35*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d* \\
& x + c)^4 + 30*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cos \\
& h(d*x + c)^2 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3 \\
& *a^2*b^5)*d)*\sinh(d*x + c)^4 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b \\
& ^4 + a^2*b^5)*d*\cosh(d*x + c)^2 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a \\
& ^2*b^5)*d*\cosh(d*x + c)^5 + 10*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 \\
& + a^2*b^5)*d*\cosh(d*x + c)^3 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 \\
& + 17*a^3*b^4 - 3*a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + 4*(7*(a^5*b^2 \\
& - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^6 + 15*(2*a^6*b - 7*a^5
\end{aligned}$$

$$\begin{aligned}
& *b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^4 + 3*(8*a^7 - 32*a \\
& ^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*\cosh(d*x + c)^2 \\
& + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d)*\sinh(d*x + c)^ \\
& 2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d + 8*((a^5*b^2 - 3*a^4*b^3 \\
& + 3*a^3*b^4 - a^2*b^5)*d*\cosh(d*x + c)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4* \\
& b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b \\
& ^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*\cosh(d*x + c)^3 + (2*a^6*b - 7* \\
& a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*\cosh(d*x + c))*\sinh(d*x + c)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.346 \quad \int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b \sinh^2(c+dx)\right)^3} dx$$

Optimal. Leaf size=172

$$\frac{3b^2(4a-b) \tanh(c+dx)}{8a^2d(a-b)^3(a-(a-b) \tanh^2(c+dx))} - \frac{3b(8a^2-4ab+b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{7/2}} - \frac{b^3 \tanh(c+dx)}{4ad(a-b)^3(a-(a-b) \tanh^2(c+dx))}$$

[Out] $(-3*b*(8*a^2 - 4*a*b + b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tanh}[c + d*x])/\operatorname{Sqrt}[a]])/(8*a^{(5/2)}*(a - b)^{(7/2)}*d) + \operatorname{Tanh}[c + d*x]/((a - b)^3*d) - (b^3*\operatorname{Tanh}[c + d*x])/((4*a*(a - b)^3*d*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2)^2) + (3*(4*a - b)*b^2*\operatorname{Tanh}[c + d*x])/((8*a^2*(a - b)^3*d*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2))$

Rubi [A] time = 0.264594, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3191, 390, 1157, 385, 208}

$$\frac{3b^2(4a-b) \tanh(c+dx)}{8a^2d(a-b)^3(a-(a-b) \tanh^2(c+dx))} - \frac{3b(8a^2-4ab+b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{7/2}} - \frac{b^3 \tanh(c+dx)}{4ad(a-b)^3(a-(a-b) \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Sech}[c + d*x]^2/(a + b*\operatorname{Sinh}[c + d*x]^2)^3, x]$

[Out] $(-3*b*(8*a^2 - 4*a*b + b^2)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[a - b]*\operatorname{Tanh}[c + d*x])/\operatorname{Sqrt}[a]])/(8*a^{(5/2)}*(a - b)^{(7/2)}*d) + \operatorname{Tanh}[c + d*x]/((a - b)^3*d) - (b^3*\operatorname{Tanh}[c + d*x])/((4*a*(a - b)^3*d*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2)^2) + (3*(4*a - b)*b^2*\operatorname{Tanh}[c + d*x])/((8*a^2*(a - b)^3*d*(a - (a - b)*\operatorname{Tanh}[c + d*x]^2))$

Rule 3191

$\operatorname{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\operatorname{Tan}[e + f*x], x]\}, \operatorname{Dist}[ff/f, \operatorname{Subst}[\operatorname{Int}[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^{(m/2 + p + 1)}, x], x, \operatorname{Tan}[e + f*x]/ff], x]] /; \operatorname{FreeQ}\{a, b, e, f\}, x] \ \&\& \ \operatorname{IntegerQ}[m/2] \ \&\& \ \operatorname{IntegerQ}[p]$

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x],
0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{(a-(a-b)x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{(a-b)^3} - \frac{b(3a^2-3ab+b^2)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{(a-b)^3(a+(-a+b)x^2)^3}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{\operatorname{Subst}\left(\int \frac{b(3a^2-3ab+b^2)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{(a+(-a+b)x^2)^3} dx, x, \tanh(c+dx)\right)}{(a-b)^3d} \\
&= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3 \tanh(c+dx)}{4a(a-b)^3d(a-(a-b)\tanh^2(c+dx))^2} + \frac{\operatorname{Subst}\left(\int \frac{-3(2a-b)^2b+12a(a-b)b}{(a+(-a+b)x^2)^2} dx, x, \tanh(c+dx)\right)}{4a(a-b)^3d} \\
&= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3 \tanh(c+dx)}{4a(a-b)^3d(a-(a-b)\tanh^2(c+dx))^2} + \frac{3(4a-b)b^2 \tanh(c+dx)}{8a^2(a-b)^3d(a-(a-b)\tanh^2(c+dx))} \\
&= -\frac{3b(8a^2-4ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{7/2}d} + \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3 \tanh(c+dx)}{4a(a-b)^3d(a-(a-b)\tanh^2(c+dx))}
\end{aligned}$$

Mathematica [A] time = 1.07899, size = 165, normalized size = 0.96

$$\frac{3b(8a^2-4ab+b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{7/2}} + \frac{b^2(10a-3b)\sinh(2(c+dx))}{a^2(a-b)^3(2a+b\cosh(2(c+dx))-b)} + \frac{4b^2\sinh(2(c+dx))}{a(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} + \frac{8\tanh(c+dx)}{(a-b)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*(a - b)^(7/2)) + (4*b^2*Sinh[2*(c + d*x)]/(a*(a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)]^2)) + ((10*a - 3*b)*b^2*Sinh[2*(c + d*x)]/(a^2*(a - b)^3*(2*a - b + b*Cosh[2*(c + d*x)])) + (8*Tanh[c + d*x]/(a - b)^3)/(8*d)

Maple [B] time = 0.084, size = 1694, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{sech}(d*x+c)^2/(a+b*\sinh(d*x+c))^2)^3,x$

[Out]
$$\begin{aligned} & -5/4/d*b^3/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^7+3/d*b^2/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^7-3/d*b^2/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^5+45/4/d*b^3/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^5-3/d*b^2/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^3+45/4/d*b^3/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*\tanh(1/2*d*x+1/2*c)^3-5/4/d*b^3/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)+3/d*b^2/(a-b)^3/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)-3/d*b/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d*b^2/(a-b)^3/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*b^2/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/2/d*b^3/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b^4/(a-b)^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\arctanh(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*b/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/2/d*b^2/(a-b)^3/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/d*b^2/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/2/d*b^3/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b^4/(a-b)^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d/(a-b)^3*\tanh(1/2*d*x+1/2*c)/(\tanh(1/2*d*x+1/2*c)^2+1) \end{aligned}$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 2.84346, size = 21442, normalized size = 124.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(12*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*\cosh(d*x + c)^8 + 9 \\ & 6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*\cosh(d*x + c)*\sinh(d*x + c)^7 \\ & 7 + 12*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*\sinh(d*x + c)^8 + 24*(2 \\ & 4*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*\cosh(d*x + c)^6 + 24 \\ & *(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5 + 14*(8*a^4*b^2 - \\ & 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^6 + 32*a^4*b \\ & ^2 + 8*a^3*b^3 - 52*a^2*b^4 + 12*a*b^5 + 48*(14*(8*a^4*b^2 - 12*a^3*b^3 + 5 \\ & *a^2*b^4 - a*b^5)*\cosh(d*x + c)^3 + 3*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - \\ & 8*a^2*b^4 + a*b^5)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 8*(64*a^6 - 88*a^5*b + \\ & 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*\cosh(d*x + c)^4 + 8*(64*a^6 - 88*a^5*b + \\ & 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4 + 105*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 \\ & - a*b^5)*\cosh(d*x + c)^4 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2* \\ & b^4 + a*b^5)*\cosh(d*x + c)^2)*\sinh(d*x + c)^4 + 32*(21*(8*a^4*b^2 - 12*a^3* \\ & b^3 + 5*a^2*b^4 - a*b^5)*\cosh(d*x + c)^5 + 15*(24*a^5*b - 44*a^4*b^2 + 27*a \\ & ^3*b^3 - 8*a^2*b^4 + a*b^5)*\cosh(d*x + c)^3 + (64*a^6 - 88*a^5*b + 28*a^4*b \\ & ^2 - 3*a^3*b^3 - a^2*b^4)*\cosh(d*x + c))*\sinh(d*x + c)^3 + 8*(32*a^5*b - 16 \\ & *a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*\cosh(d*x + c)^2 + 8*(42*(8*a^ \\ & 4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*\cosh(d*x + c)^6 + 32*a^5*b - 16*a^4 \\ & *b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a \\ & ^3*b^3 - 8*a^2*b^4 + a*b^5)*\cosh(d*x + c)^4 + 6*(64*a^6 - 88*a^5*b + 28*a^4 \\ & *b^2 - 3*a^3*b^3 - a^2*b^4)*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 3*((8*a^2*b^ \\ & 3 - 4*a*b^4 + b^5)*\cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 4*a*b^4 + b^5)*\cosh(d \end{aligned}$$

$$\begin{aligned}
& *x + c) * \sinh(d*x + c)^9 + (8*a^2*b^3 - 4*a*b^4 + b^5) * \sinh(d*x + c)^{10} + (6 \\
& 4*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5) * \cosh(d*x + c)^8 + (64*a^3*b^2 - \\
& 56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 45*(8*a^2*b^3 - 4*a*b^4 + b^5) * \cosh(d*x + c) \\
&)^2) * \sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5) * \cosh(d*x + c)^3 + \\
& (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5) * \cosh(d*x + c)) * \sinh(d*x + c)^7 \\
& + 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5) * \cosh(d*x + c)^6 + \\
& 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5 + 105*(8*a^2*b^3 - 4 \\
& *a*b^4 + b^5) * \cosh(d*x + c)^4 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3* \\
& b^5) * \cosh(d*x + c)^2) * \sinh(d*x + c)^6 + 4*(63*(8*a^2*b^3 - 4*a*b^4 + b^5) * c \\
& osh(d*x + c)^5 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5) * \cosh(d*x + \\
& c)^3 + 3*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5) * \cosh(d*x + c \\
&)) * \sinh(d*x + c)^5 + 8*a^2*b^3 - 4*a*b^4 + b^5 + 2*(64*a^4*b - 64*a^3*b^2 + \\
& 32*a^2*b^3 - 8*a*b^4 + b^5) * \cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 4*a*b^4 \\
& + b^5) * \cosh(d*x + c)^6 + 64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5 \\
& + 35*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5) * \cosh(d*x + c)^4 + 15*(64 \\
& *a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5) * \cosh(d*x + c)^2) * \sinh(d*x \\
& + c)^4 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5) * \cosh(d*x + c)^7 + 7*(64*a^3*b^2 \\
& - 56*a^2*b^3 + 20*a*b^4 - 3*b^5) * \cosh(d*x + c)^5 + 5*(64*a^4*b - 64*a^3*b^ \\
& 2 + 32*a^2*b^3 - 8*a*b^4 + b^5) * \cosh(d*x + c)^3 + (64*a^4*b - 64*a^3*b^2 + \\
& 32*a^2*b^3 - 8*a*b^4 + b^5) * \cosh(d*x + c)) * \sinh(d*x + c)^3 + (64*a^3*b^2 - \\
& 56*a^2*b^3 + 20*a*b^4 - 3*b^5) * \cosh(d*x + c)^2 + (45*(8*a^2*b^3 - 4*a*b^4 + \\
& b^5) * \cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5) * cos \\
& h(d*x + c)^6 + 64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 30*(64*a^4*b - \\
& 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5) * \cosh(d*x + c)^4 + 12*(64*a^4*b - 6 \\
& 4*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5) * \cosh(d*x + c)^2) * \sinh(d*x + c)^2 + \\
& 2*(5*(8*a^2*b^3 - 4*a*b^4 + b^5) * \cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 56*a^2*b \\
& ^3 + 20*a*b^4 - 3*b^5) * \cosh(d*x + c)^7 + 6*(64*a^4*b - 64*a^3*b^2 + 32*a^2* \\
& b^3 - 8*a*b^4 + b^5) * \cosh(d*x + c)^5 + 4*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^ \\
& 3 - 8*a*b^4 + b^5) * \cosh(d*x + c)^3 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - \\
& 3*b^5) * \cosh(d*x + c)) * \sinh(d*x + c)) * \sqrt{a^2 - a*b} * \log((b^2 * \cosh(d*x + c) \\
& ^4 + 4*b^2 * \cosh(d*x + c) * \sinh(d*x + c)^3 + b^2 * \sinh(d*x + c)^4 + 2*(2*a*b - \\
& b^2) * \cosh(d*x + c)^2 + 2*(3*b^2 * \cosh(d*x + c)^2 + 2*a*b - b^2) * \sinh(d*x + \\
& c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2 * \cosh(d*x + c)^3 + (2*a*b - b^2) * \cosh(d* \\
& x + c)) * \sinh(d*x + c) - 4*(b * \cosh(d*x + c)^2 + 2*b * \cosh(d*x + c) * \sinh(d*x + \\
& c) + b * \sinh(d*x + c)^2 + 2*a - b) * \sqrt{a^2 - a*b}) / (b * \cosh(d*x + c)^4 + 4* \\
& b * \cosh(d*x + c) * \sinh(d*x + c)^3 + b * \sinh(d*x + c)^4 + 2*(2*a - b) * \cosh(d*x \\
& + c)^2 + 2*(3*b * \cosh(d*x + c)^2 + 2*a - b) * \sinh(d*x + c)^2 + 4*(b * \cosh(d*x \\
& + c)^3 + (2*a - b) * \cosh(d*x + c)) * \sinh(d*x + c) + b)) + 16*(6*(8*a^4*b^2 - \\
& 12*a^3*b^3 + 5*a^2*b^4 - a*b^5) * \cosh(d*x + c)^7 + 9*(24*a^5*b - 44*a^4*b^2 \\
& + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5) * \cosh(d*x + c)^5 + 2*(64*a^6 - 88*a^5*b + \\
& 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4) * \cosh(d*x + c)^3 + (32*a^5*b - 16*a^4*b^2 \\
& - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5) * \cosh(d*x + c)) * \sinh(d*x + c)) / ((a^7*b^ \\
& 2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6) * d * \cosh(d*x + c)^{10} + 10*(a \\
& ^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6) * d * \cosh(d*x + c) * \sinh(\\
& d*x + c)^9 + (a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6) * d * \sinh
\end{aligned}$$

$$\begin{aligned}
& (d*x + c)^{10} + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 \\
& - 3*a^3*b^6)*d*\cosh(d*x + c)^8 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4* \\
& a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - \\
& 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d)*\sinh(d*x + c)^8 + 2*(8*a^9 - 36*a^ \\
& 8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh(d* \\
& x + c)^6 + 8*(15*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d* \\
& \cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4* \\
& b^5 - 3*a^3*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^7*b^2 - 4*a^6 \\
& *b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^4 + 14*(8*a^8*b - 3 \\
& 5*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x + \\
& c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 \\
& + a^3*b^6)*d)*\sinh(d*x + c)^6 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6* \\
& b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^4 + 4*(63*(a^7*b^2 \\
& - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^5 + 14*(8*a^ \\
& 8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh \\
& (d*x + c)^3 + 3*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - \\
& 8*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^7*b^2 - 4 \\
& *a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^6 + 35*(8*a^8*b \\
& - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d* \\
& x + c)^4 + 15*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8* \\
& a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60* \\
& a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d)*\sinh(d*x + c)^4 + (8*a^8*b - \\
& 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x \\
& + c)^2 + 8*(15*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*co \\
& sh(d*x + c)^7 + 7*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4* \\
& b^5 - 3*a^3*b^6)*d*\cosh(d*x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60* \\
& a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^3 + (8*a^9 - 36 \\
& *a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh \\
& (d*x + c))*\sinh(d*x + c)^3 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b \\
& ^5 + a^3*b^6)*d*\cosh(d*x + c)^8 + 28*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 5 \\
& 0*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x + c)^6 + 30*(8*a^9 - 36*a^8* \\
& b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh(d*x \\
& + c)^4 + 12*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^ \\
& 4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 5 \\
& 0*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d)*\sinh(d*x + c)^2 + (a^7*b^2 - 4*a^6*b \\
& ^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d + 2*(5*(a^7*b^2 - 4*a^6*b^3 + 6*a^5 \\
& *b^4 - 4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^9 + 4*(8*a^8*b - 35*a^7*b^2 + 6 \\
& 0*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x + c)^7 + 6*(8*a \\
& ^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6) \\
& *d*\cosh(d*x + c)^5 + 4*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5 \\
& *b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60* \\
& a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x + c))*\sinh(d*x + \\
& c)), -1/8*(6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*\cosh(d*x + c)^8 + \\
& 48*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*\cosh(d*x + c)*\sinh(d*x + c \\
&)^7 + 6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*\sinh(d*x + c)^8 + 12*(
\end{aligned}$$

$$\begin{aligned}
& 24a^5b - 44a^4b^2 + 27a^3b^3 - 8a^2b^4 + ab^5) \cosh(dx + c)^6 + 1 \\
& 2(24a^5b - 44a^4b^2 + 27a^3b^3 - 8a^2b^4 + ab^5 + 14(8a^4b^2 - \\
& 12a^3b^3 + 5a^2b^4 - ab^5) \cosh(dx + c)^2) \sinh(dx + c)^6 + 16a^4b^2 \\
& + 4a^3b^3 - 26a^2b^4 + 6ab^5 + 24(14(8a^4b^2 - 12a^3b^3 + 5 \\
& a^2b^4 - ab^5) \cosh(dx + c)^3 + 3(24a^5b - 44a^4b^2 + 27a^3b^3 - \\
& 8a^2b^4 + ab^5) \cosh(dx + c)) \sinh(dx + c)^5 + 4(64a^6 - 88a^5b + \\
& 28a^4b^2 - 3a^3b^3 - a^2b^4) \cosh(dx + c)^4 + 4(64a^6 - 88a^5b + \\
& 28a^4b^2 - 3a^3b^3 - a^2b^4 + 105(8a^4b^2 - 12a^3b^3 + 5a^2b^4 \\
& - ab^5) \cosh(dx + c)^4 + 45(24a^5b - 44a^4b^2 + 27a^3b^3 - 8a^2b^4 \\
& + ab^5) \cosh(dx + c)^2) \sinh(dx + c)^4 + 16(21(8a^4b^2 - 12a^3b^3 \\
& + 5a^2b^4 - ab^5) \cosh(dx + c)^5 + 15(24a^5b - 44a^4b^2 + 27a^3b^3 - \\
& 8a^2b^4 + ab^5) \cosh(dx + c)^3 + (64a^6 - 88a^5b + 28a^4b^2 - \\
& 3a^3b^3 - a^2b^4) \cosh(dx + c)) \sinh(dx + c)^3 + 4(32a^5b - 16 \\
& a^4b^2 - 37a^3b^3 + 24a^2b^4 - 3ab^5) \cosh(dx + c)^2 + 4(42(8a^4b^2 - \\
& 12a^3b^3 + 5a^2b^4 - ab^5) \cosh(dx + c)^6 + 32a^5b - 16a^4b^2 - \\
& 37a^3b^3 + 24a^2b^4 - 3ab^5 + 45(24a^5b - 44a^4b^2 + 27a^3b^3 - \\
& 8a^2b^4 + ab^5) \cosh(dx + c)^4 + 6(64a^6 - 88a^5b + 28a^4b^2 - \\
& 3a^3b^3 - a^2b^4) \cosh(dx + c)^2) \sinh(dx + c)^2 - 3((8a^2b^3 - \\
& 4ab^4 + b^5) \cosh(dx + c)^{10} + 10(8a^2b^3 - 4ab^4 + b^5) \cosh(dx + c) \\
& \sinh(dx + c)^9 + (8a^2b^3 - 4ab^4 + b^5) \sinh(dx + c)^{10} + (6 \\
& 4a^3b^2 - 56a^2b^3 + 20ab^4 - 3b^5) \cosh(dx + c)^8 + (64a^3b^2 - \\
& 56a^2b^3 + 20ab^4 - 3b^5 + 45(8a^2b^3 - 4ab^4 + b^5) \cosh(dx + c) \\
&)^2) \sinh(dx + c)^8 + 8(15(8a^2b^3 - 4ab^4 + b^5) \cosh(dx + c)^3 + \\
& (64a^3b^2 - 56a^2b^3 + 20ab^4 - 3b^5) \cosh(dx + c)) \sinh(dx + c)^7 \\
& + 2(64a^4b - 64a^3b^2 + 32a^2b^3 - 8ab^4 + b^5) \cosh(dx + c)^6 + \\
& 2(64a^4b - 64a^3b^2 + 32a^2b^3 - 8ab^4 + b^5 + 105(8a^2b^3 - 4 \\
& ab^4 + b^5) \cosh(dx + c)^4 + 14(64a^3b^2 - 56a^2b^3 + 20ab^4 - 3 \\
& b^5) \cosh(dx + c)^2) \sinh(dx + c)^6 + 4(63(8a^2b^3 - 4ab^4 + b^5) \cosh(dx + c) \\
&)^5 + 14(64a^3b^2 - 56a^2b^3 + 20ab^4 - 3b^5) \cosh(dx + c) \\
&)^3 + 3(64a^4b - 64a^3b^2 + 32a^2b^3 - 8ab^4 + b^5) \cosh(dx + c) \\
&) \sinh(dx + c)^5 + 8a^2b^3 - 4ab^4 + b^5 + 2(64a^4b - 64a^3b^2 + \\
& 32a^2b^3 - 8ab^4 + b^5) \cosh(dx + c)^4 + 2(105(8a^2b^3 - 4ab^4 \\
& + b^5) \cosh(dx + c)^6 + 64a^4b - 64a^3b^2 + 32a^2b^3 - 8ab^4 + b^5 \\
& + 35(64a^3b^2 - 56a^2b^3 + 20ab^4 - 3b^5) \cosh(dx + c)^4 + 15(64 \\
& a^4b - 64a^3b^2 + 32a^2b^3 - 8ab^4 + b^5) \cosh(dx + c)^2) \sinh(dx \\
& + c)^4 + 8(15(8a^2b^3 - 4ab^4 + b^5) \cosh(dx + c)^7 + 7(64a^3b^2 \\
& - 56a^2b^3 + 20ab^4 - 3b^5) \cosh(dx + c)^5 + 5(64a^4b - 64a^3b^2 \\
& + 32a^2b^3 - 8ab^4 + b^5) \cosh(dx + c)^3 + (64a^4b - 64a^3b^2 + \\
& 32a^2b^3 - 8ab^4 + b^5) \cosh(dx + c)) \sinh(dx + c)^3 + (64a^3b^2 - \\
& 56a^2b^3 + 20ab^4 - 3b^5) \cosh(dx + c)^2 + (45(8a^2b^3 - 4ab^4 + \\
& b^5) \cosh(dx + c)^8 + 28(64a^3b^2 - 56a^2b^3 + 20ab^4 - 3b^5) \cos \\
& h(dx + c)^6 + 64a^3b^2 - 56a^2b^3 + 20ab^4 - 3b^5 + 30(64a^4b - \\
& 64a^3b^2 + 32a^2b^3 - 8ab^4 + b^5) \cosh(dx + c)^4 + 12(64a^4b - 6 \\
& 4a^3b^2 + 32a^2b^3 - 8ab^4 + b^5) \cosh(dx + c)^2) \sinh(dx + c)^2 + \\
& 2(5(8a^2b^3 - 4ab^4 + b^5) \cosh(dx + c)^9 + 4(64a^3b^2 - 56a^2b^3
\end{aligned}$$

$$\begin{aligned}
&^3 + 20*a*b^4 - 3*b^5)*\cosh(d*x + c)^7 + 6*(64*a^4*b - 64*a^3*b^2 + 32*a^2* \\
&b^3 - 8*a*b^4 + b^5)*\cosh(d*x + c)^5 + 4*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^ \\
&3 - 8*a*b^4 + b^5)*\cosh(d*x + c)^3 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - \\
&3*b^5)*\cosh(d*x + c))*\sinh(d*x + c))*\sqrt{-a^2 + a*b}*\arctan(-1/2*(b*\cosh(d \\
&*x + c)^2 + 2*b*\cosh(d*x + c)*\sinh(d*x + c) + b*\sinh(d*x + c)^2 + 2*a - b)* \\
&\sqrt{-a^2 + a*b}/(a^2 - a*b)) + 8*(6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - \\
&a*b^5)*\cosh(d*x + c)^7 + 9*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 \\
&+ a*b^5)*\cosh(d*x + c)^5 + 2*(64*a^6 - 88*a^5*b + 28*a^4*b^2 - 3*a^3*b^3 - \\
&a^2*b^4)*\cosh(d*x + c)^3 + (32*a^5*b - 16*a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4 \\
&- 3*a*b^5)*\cosh(d*x + c))*\sinh(d*x + c))/((a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 \\
&- 4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^10 + 10*(a^7*b^2 - 4*a^6*b^3 + 6*a^ \\
&5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)*\sinh(d*x + c)^9 + (a^7*b^2 - 4 \\
&*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*\sinh(d*x + c)^10 + (8*a^8*b - \\
&35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x \\
&+ c)^8 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*\cosh \\
&(d*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 \\
&- 3*a^3*b^6)*d)*\sinh(d*x + c)^8 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6 \\
&)*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^6 + 8*(15*(a^7*b^2 \\
&- 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^3 + (8*a^8* \\
&b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d \\
&*x + c))*\sinh(d*x + c)^7 + 2*(105*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4* \\
&b^5 + a^3*b^6)*d*\cosh(d*x + c)^4 + 14*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - \\
&50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x + c)^2 + (8*a^9 - 36*a^8*b \\
&+ 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d)*\sinh(d*x + \\
&c)^6 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4* \\
&b^5 + a^3*b^6)*d*\cosh(d*x + c)^4 + 4*(63*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - \\
&4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^5 + 14*(8*a^8*b - 35*a^7*b^2 + 60*a^6 \\
&)*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x + c)^3 + 3*(8*a^9 - \\
&36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*co \\
&sh(d*x + c))*\sinh(d*x + c)^5 + 2*(105*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4* \\
&a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^6 + 35*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^ \\
&3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x + c)^4 + 15*(8*a^9 - 36 \\
&)*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh \\
&(d*x + c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8* \\
&a^4*b^5 + a^3*b^6)*d)*\sinh(d*x + c)^4 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 \\
&- 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d*x + c)^2 + 8*(15*(a^7*b^2 - \\
&4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^7 + 7*(8*a^8* \\
&b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*\cosh(d \\
&*x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8* \\
&a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^3 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60* \\
&a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c))*\sinh(d*x + c)^ \\
&3 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*\cosh(d*x \\
&+ c)^8 + 28*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - \\
&3*a^3*b^6)*d*\cosh(d*x + c)^6 + 30*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b \\
&)^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*\cosh(d*x + c)^4 + 12*(8*a^9 - 36*a
\end{aligned}$$

$$\begin{aligned} & ^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d \\ & *x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - \\ & 3*a^3*b^6)*d)*sinh(d*x + c)^2 + (a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^ \\ & 5 + a^3*b^6)*d + 2*(5*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^ \\ & 6)*d*cosh(d*x + c)^9 + 4*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + \\ & 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^7 + 6*(8*a^9 - 36*a^8*b + 65*a^7*b^ \\ & 2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^5 + 4*(8 \\ & *a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^ \\ & 6)*d*cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20 \\ & *a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c))] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.398, size = 509, normalized size = 2.96

$$\frac{3(8a^2b - 4ab^2 + b^3) \arctan\left(\frac{be^{2dx+2c} + 2a - b}{2\sqrt{-a^2 + ab}}\right) - 16a^2b^2e^{(6dx+6c)} - 12ab^3e^{(6dx+6c)} + 3b^4e^{(6dx+6c)} + 80a^3be^{(4dx+4c)} - 10a^4b^2e^{(4dx+4c)}}{8(a^5d - 3a^4bd + 3a^3b^2d - a^2b^3d)\sqrt{-a^2 + ab}} - \frac{16a^2b^2e^{(6dx+6c)} - 12ab^3e^{(6dx+6c)} + 3b^4e^{(6dx+6c)} + 80a^3be^{(4dx+4c)} - 10a^4b^2e^{(4dx+4c)}}{4(a^5d - 3a^4bd + 3a^3b^2d - a^2b^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -3/8*(8*a^2*b - 4*a*b^2 + b^3)*arctan(1/2*(b*e^{(2*d*x + 2*c)} + 2*a - b)/sqrt \\ & t(-a^2 + a*b))/((a^5*d - 3*a^4*b*d + 3*a^3*b^2*d - a^2*b^3*d)*sqrt(-a^2 + a \\ & *b)) - 1/4*(16*a^2*b^2*e^{(6*d*x + 6*c)} - 12*a*b^3*e^{(6*d*x + 6*c)} + 3*b^4*e \\ & ^{(6*d*x + 6*c)} + 80*a^3*b*e^{(4*d*x + 4*c)} - 104*a^2*b^2*e^{(4*d*x + 4*c)} + 5 \\ & 4*a*b^3*e^{(4*d*x + 4*c)} - 9*b^4*e^{(4*d*x + 4*c)} + 64*a^2*b^2*e^{(2*d*x + 2*c)} \\ &) - 52*a*b^3*e^{(2*d*x + 2*c)} + 9*b^4*e^{(2*d*x + 2*c)} + 10*a*b^3 - 3*b^4)/((\\ & a^5*d - 3*a^4*b*d + 3*a^3*b^2*d - a^2*b^3*d)*(b*e^{(4*d*x + 4*c)} + 4*a*e^{(2* \\ & d*x + 2*c)} - 2*b*e^{(2*d*x + 2*c)} + b)^2) - 2/((a^3*d - 3*a^2*b*d + 3*a*b^2* \\ & d - b^3*d)*(e^{(2*d*x + 2*c)} + 1)) \end{aligned}$$

$$3.347 \quad \int \frac{\operatorname{sech}^3(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=217

$$\frac{b^{3/2} (35a^2 - 14ab + 3b^2) \tan^{-1} \left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}} \right)}{8a^{5/2} d(a-b)^4} + \frac{b(4a-b)(a+3b) \sinh(c+dx)}{8a^2 d(a-b)^3 (a+b \sinh^2(c+dx))} + \frac{b(2a+b) \sinh(c+dx)}{4ad(a-b)^2 (a+b \sinh^2(c+dx))^2}$$

[Out] ((a - 7*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^4*d) + (b^(3/2)*(35*a^2 - 14*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^4*d) + (b*(2*a + b)*Sinh[c + d*x])/(4*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)^2) + ((4*a - b)*b*(a + 3*b)*Sinh[c + d*x])/(8*a^2*(a - b)^3*d*(a + b*Sinh[c + d*x]^2)) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2)

Rubi [A] time = 0.303491, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{b^{3/2} (35a^2 - 14ab + 3b^2) \tan^{-1} \left(\frac{\sqrt{b} \sinh(c+dx)}{\sqrt{a}} \right)}{8a^{5/2} d(a-b)^4} + \frac{b(4a-b)(a+3b) \sinh(c+dx)}{8a^2 d(a-b)^3 (a+b \sinh^2(c+dx))} + \frac{b(2a+b) \sinh(c+dx)}{4ad(a-b)^2 (a+b \sinh^2(c+dx))^2}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((a - 7*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^4*d) + (b^(3/2)*(35*a^2 - 14*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^4*d) + (b*(2*a + b)*Sinh[c + d*x])/(4*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)^2) + ((4*a - b)*b*(a + 3*b)*Sinh[c + d*x])/(8*a^2*(a - b)^3*d*(a + b*Sinh[c + d*x]^2)) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/

ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^3} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d(a+b\sinh^2(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-5bx^2}{(1+x^2)(a+bx^2)^3} dx, x, \sinh(c+dx)\right)}{2(a-b)d} \\
&= \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d(a+b\sinh^2(c+dx))^2} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d(a+b\sinh^2(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{-2(2a^2}{\dots}\right)}{\dots} \\
&= \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d(a+b\sinh^2(c+dx))^2} + \frac{(4a-b)b(a+3b)\sinh(c+dx)}{8a^2(a-b)^3d(a+b\sinh^2(c+dx))} + \frac{\operatorname{sech}(c+dx)}{2(a-b)d} \\
&= \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d(a+b\sinh^2(c+dx))^2} + \frac{(4a-b)b(a+3b)\sinh(c+dx)}{8a^2(a-b)^3d(a+b\sinh^2(c+dx))} + \frac{\operatorname{sech}(c+dx)}{2(a-b)d} \\
&= \frac{(a-7b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^4d} + \frac{b^{3/2}(35a^2-14ab+3b^2)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^4d} + \frac{b}{4a(a-b)}
\end{aligned}$$

Mathematica [A] time = 1.79072, size = 222, normalized size = 1.02

$$\frac{2b^2(a-b)\sinh(c+dx)(26a^2+b(11a-3b)\cosh(2(c+dx))-21ab+3b^2)}{a^2(2a+b\cosh(2(c+dx))-b)^2} - \frac{3b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{5/2}} + \frac{14b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{3/2}} - \frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}} - \frac{b}{4a(a-b)}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] ((-35*b^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + (14*b^(5/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(3/2) - (3*b^(7/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(5/2) + 8*a*ArcTan[Tanh[(c + d*x)/2]] - 56*b*ArcTan[Tanh[(c + d*x)/2]] + (2*(a - b)*b^2*(26*a^2 - 21*a*b + 3*b^2 + (11*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(a^2*(2*a - b + b*Cosh[2*(c + d*x)])^2) + 4*(a - b)*Sech[c + d*x]*Tanh[c + d*x]/(8*(a - b)^4*d)

Maple [B] time = 0.104, size = 2307, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{sech}(d*x+c)^3/(a+b*\sinh(d*x+c)^2)^3, x)$

[Out]
$$\begin{aligned} & -13/4/d*b^2/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*\tanh(1/2*d*x+1/2*c)^7+39/4/d*b^2/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*\tanh(1/2*d*x+1/2*c)^5-39/4/d*b^2/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*\tanh(1/2*d*x+1/2*c)^3 \\ & +13/4/d*b^2/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*\tanh(1/2*d*x+1/2*c)+49/8/d*b^3/(a-b)^4/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}+49/8/d*b^3/(a-b)^4/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-5/4/d*b^4/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^7+71/4/d*b^4/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^5-3/d*b^5/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*\tanh(1/2*d*x+1/2*c)^5-71/4/d*b^4/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^3+3/d*b^5/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*\tanh(1/2*d*x+1/2*c)^3+5/4/d*b^4/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)+7/4/d*b^3/(a-b)^4/a/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-3/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-7/4/d*b^3/(a-b)^4/a/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+3/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-35/8/d*b^2/(a-b)^4*a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}+a-2*b)*a)^{1/2})-35/8/d*b^2/(a-b)^4*a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-17/8/d*b^4/(a-b)^4/a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})+3/8/d*b^5/(a-b)^4/a^2/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-17/8/d*b^4/(a-b)^4/a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2})-17/8/d*b^4/(a-b)^4/a/(-b*(a-b))^{1/2}/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}*\arctan(a*\tanh(1/2*d*x+1/2*c))/((2*(-b*(a-b))^{1/2}-a+2*b)*a)^{1/2}) \end{aligned}$$

$$\begin{aligned} &))^{(1/2)} / ((2*(-b*(a-b))^{(1/2)} + a - 2*b)*a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2*d*x + 1/2*c)) \\ &/ ((2*(-b*(a-b))^{(1/2)} + a - 2*b)*a)^{(1/2)} + 3/8/d*b^5/(a-b)^4/a^2/(-b*(a-b))^{(1/2)} \\ &/ ((2*(-b*(a-b))^{(1/2)} + a - 2*b)*a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2*d*x + 1/2*c)) / ((2*(-b*(a-b))^{(1/2)} + a - 2*b)*a)^{(1/2)} \\ &+ 1/d/(a-b)^4/(\tanh(1/2*d*x + 1/2*c)^2 + 1)^2 * a * \tanh(1/2*d*x + 1/2*c) - 9/2/d*b^3/(a-b)^4/(\tanh(1/2*d*x + 1/2*c)^4 * a - 2 * \tanh(1/2*d*x + 1/2*c)^2 * a + 4 * \tanh(1/2*d*x + 1/2*c)^2 * b + a)^2 * \tanh(1/2*d*x + 1/2*c) + 49/2/d*b^3/(a-b)^4/(\tanh(1/2*d*x + 1/2*c)^4 * a - 2 * \tanh(1/2*d*x + 1/2*c)^2 * a + 4 * \tanh(1/2*d*x + 1/2*c)^2 * b + a)^2 * \tanh(1/2*d*x + 1/2*c)^3 - 1/d/(a-b)^4/(\tanh(1/2*d*x + 1/2*c)^2 + 1)^2 * a * \tanh(1/2*d*x + 1/2*c)^3 - 1/d/(a-b)^4/(\tanh(1/2*d*x + 1/2*c)^2 + 1)^2 * \tanh(1/2*d*x + 1/2*c) * b + 1/d/(a-b)^4/(\tanh(1/2*d*x + 1/2*c)^2 + 1)^2 * \tanh(1/2*d*x + 1/2*c)^3 * b - 49/2/d*b^3/(a-b)^4/(\tanh(1/2*d*x + 1/2*c)^4 * a - 2 * \tanh(1/2*d*x + 1/2*c)^2 * a + 4 * \tanh(1/2*d*x + 1/2*c)^2 * b + a)^2 * \tanh(1/2*d*x + 1/2*c)^5 + 35/8/d*b^2/(a-b)^4/((2*(-b*(a-b))^{(1/2)} - a + 2*b)*a)^{(1/2)} * \operatorname{arctan}(a * \tanh(1/2*d*x + 1/2*c)) / ((2*(-b*(a-b))^{(1/2)} - a + 2*b)*a)^{(1/2)} + 9/2/d*b^3/(a-b)^4/(\tanh(1/2*d*x + 1/2*c)^4 * a - 2 * \tanh(1/2*d*x + 1/2*c)^2 * a + 4 * \tanh(1/2*d*x + 1/2*c)^2 * b + a)^2 * \tanh(1/2*d*x + 1/2*c)^7 - 35/8/d*b^2/(a-b)^4/((2*(-b*(a-b))^{(1/2)} + a - 2*b)*a)^{(1/2)} * \operatorname{arctanh}(a * \tanh(1/2*d*x + 1/2*c)) / ((2*(-b*(a-b))^{(1/2)} + a - 2*b)*a)^{(1/2)} - 7/d/(a-b)^4 * \operatorname{arctan}(\tanh(1/2*d*x + 1/2*c)) * b + 1/d/(a-b)^4 * \operatorname{arctan}(\tanh(1/2*d*x + 1/2*c)) * a \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] $(a * e^c - 7 * b * e^c) * \operatorname{arctan}(e^{(d * x + c)}) * e^{-c} / (a^4 * d - 4 * a^3 * b * d + 6 * a^2 * b^2 * d - 4 * a * b^3 * d + b^4 * d) + 1/4 * ((4 * a^2 * b^2 * e^{(11 * c)} + 11 * a * b^3 * e^{(11 * c)} - 3 * b^4 * e^{(11 * c)}) * e^{(11 * d * x)} + (32 * a^3 * b * e^{(9 * c)} + 32 * a^2 * b^2 * e^{(9 * c)} - 31 * a * b^3 * e^{(9 * c)} + 3 * b^4 * e^{(9 * c)}) * e^{(9 * d * x)} + 2 * (32 * a^4 * e^{(7 * c)} - 48 * a^3 * b * e^{(7 * c)} + 46 * a^2 * b^2 * e^{(7 * c)} - 21 * a * b^3 * e^{(7 * c)} + 3 * b^4 * e^{(7 * c)}) * e^{(7 * d * x)} - 2 * (32 * a^4 * e^{(5 * c)} - 48 * a^3 * b * e^{(5 * c)} + 46 * a^2 * b^2 * e^{(5 * c)} - 21 * a * b^3 * e^{(5 * c)} + 3 * b^4 * e^{(5 * c)}) * e^{(5 * d * x)} - (32 * a^3 * b * e^{(3 * c)} + 32 * a^2 * b^2 * e^{(3 * c)} - 31 * a * b^3 * e^{(3 * c)} + 3 * b^4 * e^{(3 * c)}) * e^{(3 * d * x)} - (4 * a^2 * b^2 * e^c + 11 * a * b^3 * e^c - 3 * b^4 * e^c) * e^{(d * x)}) / (a^5 * b^2 * d - 3 * a^4 * b^3 * d + 3 * a^3 * b^4 * d - a^2 * b^5 * d + (a^5 * b^2 * d * e^{(12 * c)} - 3 * a^4 * b^3 * d * e^{(12 * c)} + 3 * a^3 * b^4 * d * e^{(12 * c)} - a^2 * b^5 * d * e^{(12 * c)}) * e^{(12 * d * x)} + 2 * (4 * a^6 * b * d * e^{(10 * c)} - 13 * a^5 * b^2 * d * e^{(10 * c)} + 15 * a^4 * b^3 * d * e^{(10 * c)} - 7 * a^3 * b^4 * d * e^{(10 * c)} + a^2 * b^5 * d * e^{(10 * c)}) * e^{(10 * d * x)} + (16 * a^7 * d * e^{(8 * c)} - 48 * a^6 * b * d * e^{(8 * c)} + 47 * a^5 * b^2 * d * e^{(8 * c)} - 13 * a^4 * b^3 * d * e^{(8 * c)} - 3 * a^3 * b^4 * d * e^{(8 * c)} + a^2 * b^5 * d * e^{(8 * c)}) * e^{(8 * d * x)} + 4 * (8 * a^7 * d * e^{(6 * c)} - 28 * a^6 * b * d * e^{(6 * c)} + 37 * a^5 * b^2 * d * e^{(6 * c)} - 23 * a^4 * b^3 * d * e^{(6 * c)} +$

$$7a^3b^4de^{6c} - a^2b^5de^{6c})e^{6dx} + (16a^7de^{4c} - 48a^6bde^{4c} + 47a^5b^2de^{4c} - 13a^4b^3de^{4c} - 3a^3b^4de^{4c} + a^2b^5de^{4c})e^{4dx} + 2(4a^6bde^{2c} - 13a^5b^2de^{2c} + 15a^4b^3de^{2c} - 7a^3b^4de^{2c} + a^2b^5de^{2c})e^{2dx} + 8 \int \frac{1}{32} ((35a^2b^2e^{3c} - 14ab^3e^{3c} + 3b^4e^{3c})e^{3dx} + (35a^2b^2e^c - 14ab^3e^c + 3b^4e^c)e^{dx}) / (a^6b - 4a^5b^2 + 6a^4b^3 - 4a^3b^4 + a^2b^5 + (a^6be^{4c} - 4a^5b^2e^{4c} + 6a^4b^3e^{4c} - 4a^3b^4e^{4c} + a^2b^5e^{4c}))e^{4dx} + 2(2a^7e^{2c} - 9a^6be^{2c} + 16a^5b^2e^{2c} - 14a^4b^3e^{2c} + 6a^3b^4e^{2c} - a^2b^5e^{2c})e^{2dx}, x$$

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)^3/(a+b*sinh(dx+c)^2)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(dx+c)**3/(a+b*sinh(dx+c)**2)**3,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.348 \quad \int \frac{\operatorname{sech}^4(c+dx)}{(a+b \sinh^2(c+dx))^3} dx$$

Optimal. Leaf size=203

$$\frac{b^3(16a-3b) \tanh(c+dx)}{8a^2d(a-b)^4(a-(a-b) \tanh^2(c+dx))} + \frac{b^2(48a^2-16ab+3b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{9/2}} + \frac{b^4 \tanh(c+dx)}{4ad(a-b)^4(a-(a-b) \tanh^2(c+dx))}$$

[Out] (b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^(9/2)*d) + ((a - 4*b)*Tanh[c + d*x])/((a - b)^4*d) - Tanh[c + d*x]^3/(3*(a - b)^3*d) + (b^4*Tanh[c + d*x])/(4*a*(a - b)^4*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((16*a - 3*b)*b^3*Tanh[c + d*x])/(8*a^2*(a - b)^4*d*(a - (a - b)*Tanh[c + d*x]^2))

Rubi [A] time = 0.340881, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3191, 390, 1157, 385, 208}

$$\frac{b^3(16a-3b) \tanh(c+dx)}{8a^2d(a-b)^4(a-(a-b) \tanh^2(c+dx))} + \frac{b^2(48a^2-16ab+3b^2) \tanh^{-1}\left(\frac{\sqrt{a-b} \tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{9/2}} + \frac{b^4 \tanh(c+dx)}{4ad(a-b)^4(a-(a-b) \tanh^2(c+dx))}$$

Antiderivative was successfully verified.

[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]

[Out] (b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^(9/2)*d) + ((a - 4*b)*Tanh[c + d*x])/((a - b)^4*d) - Tanh[c + d*x]^3/(3*(a - b)^3*d) + (b^4*Tanh[c + d*x])/(4*a*(a - b)^4*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((16*a - 3*b)*b^3*Tanh[c + d*x])/(8*a^2*(a - b)^4*d*(a - (a - b)*Tanh[c + d*x]^2))

Rule 3191

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\sinh^2(c+dx))^3} dx &= \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^4}{(a-(a-b)x^2)^3} dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{\operatorname{Subst}\left(\int \left(\frac{a-4b}{(a-b)^4} - \frac{x^2}{(a-b)^3} + \frac{b^2(6a^2-4ab+b^2)-4(a-b)(3a-b)b^2x^2+6(a-b)^2b^2x^4}{(a-b)^4(a+(-a+b)x^2)^3}\right) dx, x, \tanh(c+dx)\right)}{d} \\
&= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d} - \frac{\tanh^3(c+dx)}{3(a-b)^3d} + \frac{\operatorname{Subst}\left(\int \frac{b^2(6a^2-4ab+b^2)-4(a-b)(3a-b)b^2x^2+6(a-b)^2b^2x^4}{(a+(-a+b)x^2)^3} dx, x, \tanh(c+dx)\right)}{(a-b)^4d} \\
&= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d} - \frac{\tanh^3(c+dx)}{3(a-b)^3d} + \frac{b^4\tanh(c+dx)}{4a(a-b)^4d(a-(a-b)\tanh^2(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{b^2(6a^2-4ab+b^2)-4(a-b)(3a-b)b^2x^2+6(a-b)^2b^2x^4}{(a+(-a+b)x^2)^3} dx, x, \tanh(c+dx)\right)}{8a^2d} \\
&= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d} - \frac{\tanh^3(c+dx)}{3(a-b)^3d} + \frac{b^4\tanh(c+dx)}{4a(a-b)^4d(a-(a-b)\tanh^2(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{b^2(6a^2-4ab+b^2)-4(a-b)(3a-b)b^2x^2+6(a-b)^2b^2x^4}{(a+(-a+b)x^2)^3} dx, x, \tanh(c+dx)\right)}{8a^2d} \\
&= \frac{b^2(48a^2-16ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{9/2}d} + \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d} - \frac{\tanh^3(c+dx)}{3(a-b)^3d}
\end{aligned}$$

Mathematica [A] time = 2.19955, size = 169, normalized size = 0.83

$$\frac{3b^2(48a^2-16ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{9/2}} + \frac{3b^3\sinh(2(c+dx))(-32a^2+b(3b-14a)\cosh(2(c+dx))+24ab-3b^2)}{a^2(2a+b\cosh(2(c+dx))-b)^2} + \frac{8\tanh(c+dx)((a-b)\operatorname{sech}^2(c+dx)+2a-11b)}{(a-b)^4}$$

$$24d$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3, x]

[Out] ((3*b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*(a - b)^(9/2)) + ((3*b^3*(-32*a^2 + 24*a*b - 3*b^2 + b*(-14*a + 3*b))*Cosh[2*(c + d*x)]*Sinh[2*(c + d*x)])/(a^2*(2*a - b + b*Cosh[2*(c + d*x)])^2) + 8*(2*a - 11*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^4)/(24*d)

Maple [B] time = 0.108, size = 1892, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (\operatorname{sech}(d*x+c)^4/(a+b*\sinh(d*x+c)^2)^3, x)$

[Out]
$$\begin{aligned} & -6/d*b^3/(a-b)^4/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-6/d*b^3/(a-b)^4/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+5/4/d*b^4/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^7-61/4/d*b^4/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^5+3/d*b^5/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*\tanh(1/2*d*x+1/2*c)^5-61/4/d*b^4/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)^3+3/d*b^5/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*\tanh(1/2*d*x+1/2*c)^3+5/4/d*b^4/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*\tanh(1/2*d*x+1/2*c)-2/d*b^3/(a-b)^4/a/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+3/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+2/d*b^3/(a-b)^4/a/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-3/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+2/d*b^4/(a-b)^4/a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})-3/8/d*b^5/(a-b)^4/a^2/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)}*\operatorname{arctan}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}-a+2*b)*a)^{(1/2)})+2/d*b^4/(a-b)^4/a/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})-3/8/d*b^5/(a-b)^4/a^2/(-b*(a-b))^{(1/2)}/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)}*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^{(1/2)}+a-2*b)*a)^{(1/2)})+2/d/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^2+1)^3*\tanh(1/2*d*x+1/2*c)^5*a-8/d/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^2+1)^3*b*\tanh(1/2*d*x+1/2*c)^5+4/3/d/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^2+1)^3*\tanh(1/2*d*x+1/2*c)^3*a-40/3/d/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^2+1)^3*b*\tanh(1/2*d*x+1/2*c)^3+2/d/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^2+1)^3*\tanh(1/2*d*x+1/2*c)*a-8/d/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^2+1)^3*b*\tanh(1/2*d*x+1/2*c)-4/d*b^3/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)+4/d*b^3/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c) \end{aligned}$$

$$2*c)^3+4/d*b^3/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^5-6/d*b^2/(a-b)^4/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*\arctan(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-4/d*b^3/(a-b)^4/(\tanh(1/2*d*x+1/2*c)^4*a-2*\tanh(1/2*d*x+1/2*c)^2*a+4*\tanh(1/2*d*x+1/2*c)^2*b+a)^2*\tanh(1/2*d*x+1/2*c)^7+6/d*b^2/(a-b)^4/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*\operatorname{arctanh}(a*\tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))$$

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out

Giac [B] time = 1.49803, size = 608, normalized size = 3.

$$\frac{(48 a^2 b^2 - 16 a b^3 + 3 b^4) \arctan\left(\frac{b e^{(2 d x + 2 c)} + 2 a - b}{2 \sqrt{-a^2 + a b}}\right)}{8 (a^6 d - 4 a^5 b d + 6 a^4 b^2 d - 4 a^3 b^3 d + a^2 b^4 d) \sqrt{-a^2 + a b}} + \frac{24 a^2 b^3 e^{(6 d x + 6 c)} - 16 a b^4 e^{(6 d x + 6 c)} + 3 b^5 e^{(6 d x + 6 c)} + 112 a^3 b^2 e^{(6 d x + 6 c)}}{4 (a^6 d - 4 a^5 b d + 6 a^4 b^2 d - 4 a^3 b^3 d + a^2 b^4 d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] 1/8*(48*a^2*b^2 - 16*a*b^3 + 3*b^4)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^6*d - 4*a^5*b*d + 6*a^4*b^2*d - 4*a^3*b^3*d + a^2*b^4*d)*sqrt(-a^2 + a*b)) + 1/4*(24*a^2*b^3*e^(6*d*x + 6*c) - 16*a*b^4*e^(6*d*x + 6*c) + 3*b^5*e^(6*d*x + 6*c) + 112*a^3*b^2*e^(4*d*x + 4*c) - 136*a^2*b^3*e^(4*d*x + 4*c) + 66*a*b^4*e^(4*d*x + 4*c) - 9*b^5*e^(4*d*x + 4*c) + 88*a^2*b^3*e^(2*d*x + 2*c) - 64*a*b^4*e^(2*d*x + 2*c) + 9*b^5*e^(2*d*x + 2*c) + 14*a*b^4 - 3*b^5)/((a^6*d - 4*a^5*b*d + 6*a^4*b^2*d - 4*a^3*b^3*d + a^2*b^4*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2) + 2/3*(9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) - 2*a + 11*b)/((a^4*d - 4*a^3*b*d + 6*a^2*b^2*d - 4*a*b^3*d + b^4*d)*(e^(2*d*x + 2*c) + 1)^3)

$$3.349 \quad \int \frac{\cosh^2(x)}{1-\sinh^2(x)} dx$$

Optimal. Leaf size=19

$$\sqrt{2} \tanh^{-1}\left(\sqrt{2} \tanh(x)\right) - x$$

[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]

Rubi [A] time = 0.0450643, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3191, 391, 206}

$$\sqrt{2} \tanh^{-1}\left(\sqrt{2} \tanh(x)\right) - x$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^2/(1 - Sinh[x]^2), x]

[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e + f*x]/ff], x] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{1 - \sinh^2(x)} dx &= \text{Subst} \left(\int \frac{1}{(1 - 2x^2)(1 - x^2)} dx, x, \tanh(x) \right) \\
&= 2 \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \tanh(x) \right) - \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \tanh(x) \right) \\
&= -x + \sqrt{2} \tanh^{-1} \left(\sqrt{2} \tanh(x) \right)
\end{aligned}$$

Mathematica [A] time = 0.0756311, size = 24, normalized size = 1.26

$$-2 \left(\frac{x}{2} - \frac{\tanh^{-1}(\sqrt{2} \tanh(x))}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^2/(1 - Sinh[x]^2), x]

[Out] -2*(x/2 - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2])

Maple [B] time = 0.023, size = 54, normalized size = 2.8

$$\sqrt{2} \text{Artanh} \left(\frac{\sqrt{2}}{4} (2 \tanh(x/2) + 2) \right) - \ln \left(\tanh \left(\frac{x}{2} \right) + 1 \right) + \sqrt{2} \text{Artanh} \left(\frac{\sqrt{2}}{4} (2 \tanh(x/2) - 2) \right) + \ln \left(\tanh \left(\frac{x}{2} \right) - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^2/(1-sinh(x)^2), x)

[Out] 2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))-ln(tanh(1/2*x)+1)+2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+ln(tanh(1/2*x)-1)

Maxima [B] time = 1.70328, size = 86, normalized size = 4.53

$$\frac{1}{2} \sqrt{2} \log \left(-\frac{\sqrt{2} - e^{(-x)} + 1}{\sqrt{2} + e^{(-x)} - 1} \right) - \frac{1}{2} \sqrt{2} \log \left(-\frac{\sqrt{2} - e^{(-x)} - 1}{\sqrt{2} + e^{(-x)} + 1} \right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="maxima")

[Out] $\frac{1}{2}\sqrt{2}\log\left(\frac{-(\sqrt{2}-e^{-x}+1)}{(\sqrt{2}+e^{-x}-1)}\right) - \frac{1}{2}\sqrt{2}\log\left(\frac{-(\sqrt{2}-e^{-x}-1)}{(\sqrt{2}+e^{-x}+1)}\right) - x$

Fricas [B] time = 1.56854, size = 220, normalized size = 11.58

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{3(2\sqrt{2}-3)\cosh(x)^2 - 4(3\sqrt{2}-4)\cosh(x)\sinh(x) + 3(2\sqrt{2}-3)\sinh(x)^2 - 2\sqrt{2} + 3}{\cosh(x)^2 + \sinh(x)^2 - 3}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="fricas")

[Out] $\frac{1}{2}\sqrt{2}\log\left(\frac{-(3(2\sqrt{2}-3)\cosh(x)^2 - 4(3\sqrt{2}-4)\cosh(x)\sinh(x) + 3(2\sqrt{2}-3)\sinh(x)^2 - 2\sqrt{2} + 3)}{(\cosh(x)^2 + \sinh(x)^2 - 3)}\right) - x$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**2/(1-sinh(x)**2),x)

[Out] Timed out

Giac [B] time = 1.17485, size = 55, normalized size = 2.89

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{|-4\sqrt{2} + 2e^{(2x)} - 6|}{|4\sqrt{2} + 2e^{(2x)} - 6|}\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x) - 6)) - x
```

$$3.350 \quad \int \frac{\cosh^3(x)}{1-\sinh^2(x)} dx$$

Optimal. Leaf size=10

$$2 \tanh^{-1}(\sinh(x)) - \sinh(x)$$

[Out] 2*ArcTanh[Sinh[x]] - Sinh[x]

Rubi [A] time = 0.0389517, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3190, 388, 206}

$$2 \tanh^{-1}(\sinh(x)) - \sinh(x)$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^3/(1 - Sinh[x]^2), x]

[Out] 2*ArcTanh[Sinh[x]] - Sinh[x]

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```


Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{1 - \sinh^2(x)} dx &= \text{Subst} \left(\int \frac{1 + x^2}{1 - x^2} dx, x, \sinh(x) \right) \\ &= -\sinh(x) + 2 \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sinh(x) \right) \\ &= 2 \tanh^{-1}(\sinh(x)) - \sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0095023, size = 14, normalized size = 1.4

$$-2 \left(\frac{\sinh(x)}{2} - \tanh^{-1}(\sinh(x)) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^3/(1 - Sinh[x]^2),x]

[Out] -2*(-ArcTanh[Sinh[x]] + Sinh[x]/2)

Maple [B] time = 0.028, size = 50, normalized size = 5.

$$-\ln \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 + 2 \tanh(x/2) - 1 \right) + \left(\tanh \left(\frac{x}{2} \right) + 1 \right)^{-1} + \ln \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 - 2 \tanh(x/2) - 1 \right) + \left(\tanh \left(\frac{x}{2} \right) - 1 \right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(x)^3/(1-sinh(x)^2),x)

[Out] -ln(tanh(1/2*x)^2+2*tanh(1/2*x)-1)+1/(tanh(1/2*x)+1)+ln(tanh(1/2*x)^2-2*tanh(1/2*x)-1)+1/(tanh(1/2*x)-1)

Maxima [B] time = 1.10616, size = 53, normalized size = 5.3

$$\frac{1}{2} e^{(-x)} - \frac{1}{2} e^x - \log(2 e^{(-x)} + e^{(-2x)} - 1) + \log(-2 e^{(-x)} + e^{(-2x)} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="maxima")

[Out] $\frac{1}{2}e^{-x} - \frac{1}{2}e^x - \log(2e^{-x} + e^{-2x} - 1) + \log(-2e^{-x} + e^{-2x} - 1)$

Fricas [B] time = 1.49507, size = 275, normalized size = 27.5

$$\frac{\cosh(x)^2 - 2(\cosh(x) + \sinh(x)) \log\left(\frac{2(\sinh(x)+1)}{\cosh(x)-\sinh(x)}\right) + 2(\cosh(x) + \sinh(x)) \log\left(\frac{2(\sinh(x)-1)}{\cosh(x)-\sinh(x)}\right) + 2 \cosh(x) \sinh(x)}{2(\cosh(x) + \sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="fricas")

[Out] $-\frac{1}{2}(\cosh(x)^2 - 2(\cosh(x) + \sinh(x)) \log(2(\sinh(x) + 1)/(\cosh(x) - \sinh(x))) + 2(\cosh(x) + \sinh(x)) \log(2(\sinh(x) - 1)/(\cosh(x) - \sinh(x))) + 2 \cosh(x) \sinh(x) + \sinh(x)^2 - 1)/(\cosh(x) + \sinh(x))$

Sympy [B] time = 2.30906, size = 129, normalized size = 12.9

$$\frac{\log\left(\tanh^2\left(\frac{x}{2}\right) - 2 \tanh\left(\frac{x}{2}\right) - 1\right) \tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right) - 1} - \frac{\log\left(\tanh^2\left(\frac{x}{2}\right) - 2 \tanh\left(\frac{x}{2}\right) - 1\right)}{\tanh^2\left(\frac{x}{2}\right) - 1} - \frac{\log\left(\tanh^2\left(\frac{x}{2}\right) + 2 \tanh\left(\frac{x}{2}\right) - 1\right) \tanh^2\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**3/(1-sinh(x)**2),x)

[Out] $\log(\tanh(x/2)**2 - 2*\tanh(x/2) - 1)*\tanh(x/2)**2/(\tanh(x/2)**2 - 1) - \log(\tanh(x/2)**2 - 2*\tanh(x/2) - 1)/(\tanh(x/2)**2 - 1) - \log(\tanh(x/2)**2 + 2*\tanh(x/2) - 1)*\tanh(x/2)**2/(\tanh(x/2)**2 - 1) + \log(\tanh(x/2)**2 + 2*\tanh(x/2) - 1)/(\tanh(x/2)**2 - 1) + 2*\tanh(x/2)/(\tanh(x/2)**2 - 1)$

Giac [B] time = 1.1708, size = 50, normalized size = 5.

$$\frac{1}{2}e^{(-x)} - \frac{1}{2}e^x + \log\left(\left|-e^{(-x)} + e^x + 2\right|\right) - \log\left(\left|-e^{(-x)} + e^x - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) - 1/2*e^x + log(abs(-e^(-x) + e^x + 2)) - log(abs(-e^(-x) + e^x - 2))
```

$$3.351 \quad \int \frac{\cosh^4(x)}{1-\sinh^2(x)} dx$$

Optimal. Leaf size=30

$$-\frac{5x}{2} + 2\sqrt{2} \tanh^{-1}\left(\sqrt{2} \tanh(x)\right) - \frac{1}{2} \sinh(x) \cosh(x)$$

[Out] $(-5*x)/2 + 2*\text{Sqrt}[2]*\text{ArcTanh}[\text{Sqrt}[2]*\text{Tanh}[x]] - (\text{Cosh}[x]*\text{Sinh}[x])/2$

Rubi [A] time = 0.0589306, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3191, 414, 522, 206}

$$-\frac{5x}{2} + 2\sqrt{2} \tanh^{-1}\left(\sqrt{2} \tanh(x)\right) - \frac{1}{2} \sinh(x) \cosh(x)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cosh}[x]^4/(1 - \text{Sinh}[x]^2), x]$

[Out] $(-5*x)/2 + 2*\text{Sqrt}[2]*\text{ArcTanh}[\text{Sqrt}[2]*\text{Tanh}[x]] - (\text{Cosh}[x]*\text{Sinh}[x])/2$

Rule 3191

$\text{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{With}\{\text{ff} = \text{FreeFactors}[\text{Tan}[e + f*x], x]\}, \text{Dist}[\text{ff}/f, \text{Subst}[\text{Int}[(a + (a + b)*\text{ff}^2*x^2)^p/(1 + \text{ff}^2*x^2)^{(m/2 + p + 1)}, x], x, \text{Tan}[e + f*x]/\text{ff}], x] \text{ /; FreeQ}\{a, b, e, f\}, x\} \&\& \text{IntegerQ}[m/2] \&\& \text{IntegerQ}[p]$

Rule 414

$\text{Int}[(a + (b_.)*(x_)^{(n_)})^{(p_)}*((c_) + (d_.)*(x_)^{(n_)})^{(q_)}, x_Symbol] \rightarrow -\text{Simp}[(b*x*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q + 1)})/(a*n*(p + 1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p + 1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p + 1)}*(c + d*x^n)^q * \text{Simp}[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x] \text{ /; FreeQ}\{a, b, c, d, n, q\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& !(!\text{IntegerQ}[p] \&\& \text{IntegerQ}[q] \&\& \text{LtQ}[q, -1]) \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 522

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^4(x)}{1 - \sinh^2(x)} dx &= \text{Subst} \left(\int \frac{1}{(1 - 2x^2)(1 - x^2)^2} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} \cosh(x) \sinh(x) - \frac{1}{2} \text{Subst} \left(\int \frac{-3 - 2x^2}{(1 - 2x^2)(1 - x^2)} dx, x, \tanh(x) \right) \\ &= -\frac{1}{2} \cosh(x) \sinh(x) - \frac{5}{2} \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \tanh(x) \right) + 4 \text{Subst} \left(\int \frac{1}{1 - 2x^2} dx, x, \tanh(x) \right) \\ &= -\frac{5x}{2} + 2\sqrt{2} \tanh^{-1}(\sqrt{2} \tanh(x)) - \frac{1}{2} \cosh(x) \sinh(x) \end{aligned}$$

Mathematica [A] time = 0.0686895, size = 32, normalized size = 1.07

$$-2 \left(\frac{5x}{4} + \frac{1}{8} \sinh(2x) - \sqrt{2} \tanh^{-1}(\sqrt{2} \tanh(x)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(1 - Sinh[x]^2), x]
```

```
[Out] -2*((5*x)/4 - Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + Sinh[2*x]/8)
```

Maple [B] time = 0.032, size = 98, normalized size = 3.3

$$2\sqrt{2} \text{Artanh} \left(\frac{1}{4} (2 \tanh(x/2) + 2) \sqrt{2} \right) + \frac{1}{2} \left(\tanh \left(\frac{x}{2} \right) + 1 \right)^{-2} - \frac{1}{2} \left(\tanh \left(\frac{x}{2} \right) + 1 \right)^{-1} - \frac{5}{2} \ln \left(\tanh \left(\frac{x}{2} \right) + 1 \right) + 2\sqrt{2} A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(x)^4/(1-sinh(x)^2),x)`

[Out] $2*2^{(1/2)}*\operatorname{arctanh}(1/4*(2*\tanh(1/2*x)+2)*2^{(1/2)})+1/2/(\tanh(1/2*x)+1)^2-1/2/(\tanh(1/2*x)+1)-5/2*\ln(\tanh(1/2*x)+1)+2*2^{(1/2)}*\operatorname{arctanh}(1/4*(2*\tanh(1/2*x)-2)*2^{(1/2)})-1/2/(\tanh(1/2*x)-1)^2-1/2/(\tanh(1/2*x)-1)+5/2*\ln(\tanh(1/2*x)-1)$

Maxima [B] time = 1.53231, size = 101, normalized size = 3.37

$$\sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \sqrt{2} \log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) - \frac{5}{2}x - \frac{1}{8}e^{(2x)} + \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="maxima")`

[Out] $\operatorname{sqrt}(2)*\log(-(\operatorname{sqrt}(2)-e^{(-x)}+1)/(\operatorname{sqrt}(2)+e^{(-x)}-1))-\operatorname{sqrt}(2)*\log(-(\operatorname{sqrt}(2)-e^{(-x)}-1)/(\operatorname{sqrt}(2)+e^{(-x)}+1))-5/2*x-1/8*e^{(2*x)}+1/8*e^{(-2*x)}$

Fricas [B] time = 1.51314, size = 552, normalized size = 18.4

$$\cosh(x)^4 + 4 \cosh(x) \sinh(x)^3 + \sinh(x)^4 + 20x \cosh(x)^2 + 2(3 \cosh(x)^2 + 10x) \sinh(x)^2 - 8(\sqrt{2} \cosh(x)^2 + 2\sqrt{2} \cosh(x) \sinh(x) + \sinh(x)^2)$$

$8(\cosh(x) \sinh(x) + \sinh(x)^2)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="fricas")`

[Out] $-1/8*(\cosh(x)^4 + 4*\cosh(x)*\sinh(x)^3 + \sinh(x)^4 + 20*x*\cosh(x)^2 + 2*(3*\cosh(x)^2 + 10*x)*\sinh(x)^2 - 8*(\operatorname{sqrt}(2)*\cosh(x)^2 + 2*\operatorname{sqrt}(2)*\cosh(x)*\sinh(x) + \operatorname{sqrt}(2)*\sinh(x)^2)*\log(-(3*(2*\operatorname{sqrt}(2)-3)*\cosh(x)^2 - 4*(3*\operatorname{sqrt}(2)-4)*\cosh(x)*\sinh(x) + 3*(2*\operatorname{sqrt}(2)-3)*\sinh(x)^2 - 2*\operatorname{sqrt}(2)+3)/(\cosh(x)^2 + \sinh(x)^2 - 3)) + 4*(\cosh(x)^3 + 10*x*\cosh(x))*\sinh(x) - 1)/(\cosh(x)^2 + 2*\cosh(x)*\sinh(x) + \sinh(x)^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)**4/(1-sinh(x)**2),x)

[Out] Timed out

Giac [B] time = 1.15401, size = 82, normalized size = 2.73

$$\frac{1}{8} (10 e^{2x} + 1) e^{-2x} - \sqrt{2} \log \left(\frac{|-4\sqrt{2} + 2e^{2x} - 6|}{|4\sqrt{2} + 2e^{2x} - 6|} \right) - \frac{5}{2}x - \frac{1}{8}e^{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="giac")

[Out] 1/8*(10*e^(2*x) + 1)*e^(-2*x) - sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6) / abs(4*sqrt(2) + 2*e^(2*x) - 6)) - 5/2*x - 1/8*e^(2*x)

$$3.352 \quad \int \cosh^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=117

$$-\frac{a(a-4b) \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{\sinh(e+fx)(a+b \sinh^2(e+fx))^{3/2}}{4bf} - \frac{(a-4b) \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8bf}$$

[Out] $-(a*(a - 4*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*b^(3/2)*f) - ((a - 4*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*b*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*b*f)$

Rubi [A] time = 0.113507, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3190, 388, 195, 217, 206}

$$-\frac{a(a-4b) \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{\sinh(e+fx)(a+b \sinh^2(e+fx))^{3/2}}{4bf} - \frac{(a-4b) \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8bf}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] $-(a*(a - 4*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*b^(3/2)*f) - ((a - 4*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*b*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*b*f)$

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 388

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,

$c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n*(p + 1) + 1, 0]$

Rule 195

$\text{Int}[\{(a_)+ (b_)*(x_)^{(n_)}\}^{(p_)}, x_Symbol] \rightarrow \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{(p - 1)}, x], x] /;$ FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_)+ (b_)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

$\text{Int}[\{(a_)+ (b_)*(x_)^2\}^{(-1)}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \cosh^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx &= \frac{\text{Subst}\left(\int (1 + x^2) \sqrt{a + bx^2} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{4bf} - \frac{(a - 4b) \text{Subst}\left(\int \sqrt{a + bx^2} dx, x, \sinh(e + fx)\right)}{4bf} \\ &= -\frac{(a - 4b) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{8bf} + \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{4bf} \\ &= -\frac{(a - 4b) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{8bf} + \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{4bf} \\ &= -\frac{a(a - 4b) \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}}\right)}{8b^{3/2}f} - \frac{(a - 4b) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{8bf} \end{aligned}$$

Mathematica [A] time = 0.663081, size = 124, normalized size = 1.06

$$\frac{\sqrt{a + b \sinh^2(e + fx)} \left(\sqrt{b} \sinh(e + fx) \sqrt{\frac{b \sinh^2(e + fx)}{a} + 1} (a + b \cosh(2(e + fx)) + 3b) - \sqrt{a} (a - 4b) \sinh^{-1} \left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a}} \right) \right)}{8b^{3/2} f \sqrt{\frac{b \sinh^2(e + fx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(-(Sqrt[a]*(a - 4*b)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]]) + Sqrt[b]*(a + 3*b + b*Cosh[2*(e + f*x)])*Sinh[e + f*x]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]))/(8*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Maple [C] time = 0.09, size = 52, normalized size = 0.4

$$\frac{1}{f} \int \frac{(b (\cosh(fx + e))^4 + (a - b) (\cosh(fx + e))^2) \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}} \sinh(fx + e)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] `int/indef0`(((b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \cosh^3(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^3, x)

Fricas [B] time = 2.45862, size = 8343, normalized size = 71.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/64*(2*((a^2 - 4*a*b)*\cosh(f*x + e)^4 + 4*(a^2 - 4*a*b)*\cosh(f*x + e)^3* \\ & \sinh(f*x + e) + 6*(a^2 - 4*a*b)*\cosh(f*x + e)^2*\sinh(f*x + e)^2 + 4*(a^2 - \\ & 4*a*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a^2 - 4*a*b)*\sinh(f*x + e)^4)*\sqrt{ \\ & b)*\log(-((a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3) \\ &)*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*\sinh(f*x + e)^8 + \\ & 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5 \\ & *a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^ \\ & 6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a* \\ & b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)* \\ & \cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^4 + 9*a^2*b - 1 \\ & 4*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^2)*\sin \\ & h(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^5 + 10*(a^3 - 4* \\ & a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cos \\ & h(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*\cosh(f*x + e)^2 + 2 \\ & *(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 \\ & - 2*b^3)*\cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3) \\ & *\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + \sqrt{2}*((a^2 - 2*a*b + b^2)*\cosh(f*x + \\ & e)^6 + 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (a^2 - 2*a*b \\ & + b^2)*\sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 + 3*(5*(a^2 \\ & - 2*a*b + b^2)*\cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*\sinh(f*x + e)^4 + 4*(5* \\ & (a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e))* \\ & \sinh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2) \\ & *\cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - 4*a*b + 3*b^2)* \\ & \sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^5 - 6*(a^2 - \\ & 2*a*b + b^2)*\cosh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x + e))*\sinh(f*x + e \\ &))*\sqrt{b)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x \\ & + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(2*(a^2*b - \\ & 2*a*b^2 + b^3)*\cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(\\ & f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - 2*b^ \\ & 3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(\\ & f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x \\ & + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e \end{aligned}$$

$$\begin{aligned}
&)^5 + \sinh(f*x + e)^6)) + 2*((a^2 - 4*a*b)*\cosh(f*x + e)^4 + 4*(a^2 - 4*a*b) \\
&)*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*(a^2 - 4*a*b)*\cosh(f*x + e)^2*\sinh(f*x \\
&+ e)^2 + 4*(a^2 - 4*a*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a^2 - 4*a*b)*\sinh \\
&(f*x + e)^4)*\sqrt{b}*\log((b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + \\
&e)^3 + b*\sinh(f*x + e)^4 + 2*a*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a \\
&)*\sinh(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e \\
&+ \sinh(f*x + e)^2 + 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^ \\
&2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + \\
&e)^2)) + 4*(b*\cosh(f*x + e)^3 + a*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f \\
&*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) - \sqrt{2}*(b^ \\
&2*\cosh(f*x + e)^6 + 6*b^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + b^2*\sinh(f*x + e) \\
&^6 + (2*a*b + 5*b^2)*\cosh(f*x + e)^4 + (15*b^2*\cosh(f*x + e)^2 + 2*a*b + 5* \\
&b^2)*\sinh(f*x + e)^4 + 4*(5*b^2*\cosh(f*x + e)^3 + (2*a*b + 5*b^2)*\cosh(f*x \\
&+ e))*\sinh(f*x + e)^3 - (2*a*b + 5*b^2)*\cosh(f*x + e)^2 + (15*b^2*\cosh(f*x \\
&+ e)^4 + 6*(2*a*b + 5*b^2)*\cosh(f*x + e)^2 - 2*a*b - 5*b^2)*\sinh(f*x + e)^2 \\
&- b^2 + 2*(3*b^2*\cosh(f*x + e)^5 + 2*(2*a*b + 5*b^2)*\cosh(f*x + e)^3 - (2* \\
&a*b + 5*b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh \\
&(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b^2*f*\cosh(f*x + e)^4 + 4*b^2*f*\cosh(f*x + e)^3*\sinh(f*x \\
&+ e) + 6*b^2*f*\cosh(f*x + e)^2*\sinh(f*x + e)^2 + 4*b^2*f*\cosh(f*x + e)*\sinh \\
&(f*x + e)^3 + b^2*f*\sinh(f*x + e)^4), 1/64*(4*((a^2 - 4*a*b)*\cosh(f*x + e) \\
&^4 + 4*(a^2 - 4*a*b)*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*(a^2 - 4*a*b)*\cosh(f \\
&*x + e)^2*\sinh(f*x + e)^2 + 4*(a^2 - 4*a*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + \\
&(a^2 - 4*a*b)*\sinh(f*x + e)^4)*\sqrt{-b}*\arctan(\sqrt{2}*((a - b)*\cosh(f*x + \\
&e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 + b \\
&)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x \\
&+ e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a*b - b^2)*\cosh \\
&(f*x + e)^4 + 4*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\sinh \\
&(f*x + e)^4 - (3*a*b - 2*b^2)*\cosh(f*x + e)^2 + (6*(a*b - b^2)*\cosh(f*x \\
&+ e)^2 - 3*a*b + 2*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*\cosh(f*x + \\
&e)^3 - (3*a*b - 2*b^2)*\cosh(f*x + e))*\sinh(f*x + e))) + 4*((a^2 - 4*a*b)*\cosh \\
&(f*x + e)^4 + 4*(a^2 - 4*a*b)*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*(a^2 - 4 \\
&a*b)*\cosh(f*x + e)^2*\sinh(f*x + e)^2 + 4*(a^2 - 4*a*b)*\cosh(f*x + e)*\sinh \\
&(f*x + e)^3 + (a^2 - 4*a*b)*\sinh(f*x + e)^4)*\sqrt{-b}*\arctan(\sqrt{2}*(\cosh(f \\
&*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1)*\sqrt{-b}*\sqrt{ \\
&((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2* \\
&\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\cosh \\
&(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e) \\
&^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e) \\
&^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) + \sqrt{2}*(b^2*\cosh(f*x + \\
&e)^6 + 6*b^2*\cosh(f*x + e)*\sinh(f*x + e)^5 + b^2*\sinh(f*x + e)^6 + (2*a*b \\
&+ 5*b^2)*\cosh(f*x + e)^4 + (15*b^2*\cosh(f*x + e)^2 + 2*a*b + 5*b^2)*\sinh(f*x \\
&+ e)^4 + 4*(5*b^2*\cosh(f*x + e)^3 + (2*a*b + 5*b^2)*\cosh(f*x + e))*\sinh(f \\
&*x + e)^3 - (2*a*b + 5*b^2)*\cosh(f*x + e)^2 + (15*b^2*\cosh(f*x + e)^4 + 6*(\\
&2*a*b + 5*b^2)*\cosh(f*x + e)^2 - 2*a*b - 5*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(
\end{aligned}$$

```

3*b^2*cosh(f*x + e)^5 + 2*(2*a*b + 5*b^2)*cosh(f*x + e)^3 - (2*a*b + 5*b^2)
*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(b^2*f*cosh(f*x + e)^4 + 4*b^2*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b^
2*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^3
+ b^2*f*sinh(f*x + e)^4)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \cosh^3(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^3, x)
```

$$3.353 \quad \int \cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=72

$$\frac{\sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2f} + \frac{a \tanh^{-1} \left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} \right)}{2\sqrt{bf}}$$

[Out] (a*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[b]*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)

Rubi [A] time = 0.0555239, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 195, 217, 206}

$$\frac{\sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2f} + \frac{a \tanh^{-1} \left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} \right)}{2\sqrt{bf}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] (a*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[b]*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 195

Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],

Denominator[p]])

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^2], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x \ \&\& \ !\text{GtQ}[a, 0]$

Rule 206

$\text{Int}[(a_) + (b_.)*(x_)^2]^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx &= \frac{\text{Subst}\left(\int \sqrt{a + bx^2} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2f} + \frac{a \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^2}} dx, x, \sinh(e + fx)\right)}{2f} \\ &= \frac{\sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2f} + \frac{a \text{Subst}\left(\int \frac{1}{1 - bx^2} dx, x, \frac{\sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}}\right)}{2f} \\ &= \frac{a \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}}\right)}{2\sqrt{b}f} + \frac{\sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2f} \end{aligned}$$

Mathematica [A] time = 0.245125, size = 96, normalized size = 1.33

$$\frac{a^{3/2} \sinh^{-1}\left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a}}\right) \sqrt{\frac{b \sinh^2(e + fx)}{a} + 1} + \sqrt{b} \sinh(e + fx) (a + b \sinh^2(e + fx))}{2\sqrt{b}f \sqrt{a + b \sinh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] (Sqrt[b]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2) + a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(2*Sqrt[b]*f*Sqrt[a

+ b*Sinh[e + f*x]^2])

Maple [A] time = 0.013, size = 62, normalized size = 0.9

$$\frac{\sinh(fx + e)}{2f} \sqrt{a + b(\sinh(fx + e))^2} + \frac{a}{2f} \ln \left(\sinh(fx + e) \sqrt{b} + \sqrt{a + b(\sinh(fx + e))^2} \right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] 1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/2/f*a/b^(1/2)*ln(sinh(f*x+e)*b^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \cosh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e), x)

Fricas [B] time = 2.09124, size = 6229, normalized size = 86.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [1/8*((a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4

$$\begin{aligned}
& a^2b + 5ab^2 - 2b^3 + 14(a^2b - 2ab^2 + b^3)\cosh(fx + e)^2\sinh \\
& (fx + e)^6 + 4(14(a^2b - 2ab^2 + b^3)\cosh(fx + e)^3 + 3(a^3 - 4a^2 \\
& 2b + 5ab^2 - 2b^3)\cosh(fx + e))\sinh(fx + e)^5 + (9a^2b - 14ab^2 \\
& + 6b^3)\cosh(fx + e)^4 + (70(a^2b - 2ab^2 + b^3)\cosh(fx + e)^4 + 9 \\
& a^2b - 14ab^2 + 6b^3 + 30(a^3 - 4a^2b + 5ab^2 - 2b^3)\cosh(fx + \\
& e)^2)\sinh(fx + e)^4 + 4(14(a^2b - 2ab^2 + b^3)\cosh(fx + e)^5 + 10 \\
& (a^3 - 4a^2b + 5ab^2 - 2b^3)\cosh(fx + e)^3 + (9a^2b - 14ab^2 + \\
& 6b^3)\cosh(fx + e))\sinh(fx + e)^3 + b^3 + 2(3ab^2 - 2b^3)\cosh(fx \\
& + e)^2 + 2(14(a^2b - 2ab^2 + b^3)\cosh(fx + e)^6 + 15(a^3 - 4a^2b \\
& + 5ab^2 - 2b^3)\cosh(fx + e)^4 + 3ab^2 - 2b^3 + 3(9a^2b - 14ab^2 \\
& + 6b^3)\cosh(fx + e)^2)\sinh(fx + e)^2 + \sqrt{2}((a^2 - 2ab + b^2)* \\
& \cosh(fx + e)^6 + 6(a^2 - 2ab + b^2)\cosh(fx + e)\sinh(fx + e)^5 + (a^2 \\
& - 2ab + b^2)\sinh(fx + e)^6 - 3(a^2 - 2ab + b^2)\cosh(fx + e)^4 + \\
& 3(5(a^2 - 2ab + b^2)\cosh(fx + e)^2 - a^2 + 2ab - b^2)\sinh(fx + e) \\
& ^4 + 4(5(a^2 - 2ab + b^2)\cosh(fx + e)^3 - 3(a^2 - 2ab + b^2)\cosh(\\
& fx + e))\sinh(fx + e)^3 - (4ab - 3b^2)\cosh(fx + e)^2 + (15(a^2 - 2 \\
& ab + b^2)\cosh(fx + e)^4 - 18(a^2 - 2ab + b^2)\cosh(fx + e)^2 - 4ab \\
& + 3b^2)\sinh(fx + e)^2 - b^2 + 2(3(a^2 - 2ab + b^2)\cosh(fx + e)^5 \\
& - 6(a^2 - 2ab + b^2)\cosh(fx + e)^3 - (4ab - 3b^2)\cosh(fx + e))\si \\
& nh(fx + e))\sqrt{b}\sqrt{(b\cosh(fx + e)^2 + b\sinh(fx + e)^2 + 2a - b) \\
& /(\cosh(fx + e)^2 - 2\cosh(fx + e)\sinh(fx + e) + \sinh(fx + e)^2)) + 4(\\
& 2(a^2b - 2ab^2 + b^3)\cosh(fx + e)^7 + 3(a^3 - 4a^2b + 5ab^2 - 2 \\
& b^3)\cosh(fx + e)^5 + (9a^2b - 14ab^2 + 6b^3)\cosh(fx + e)^3 + (3a \\
& b^2 - 2b^3)\cosh(fx + e))\sinh(fx + e))/(\cosh(fx + e)^6 + 6\cosh(fx + \\
& e)^5\sinh(fx + e) + 15\cosh(fx + e)^4\sinh(fx + e)^2 + 20\cosh(fx + e)^ \\
& 3\sinh(fx + e)^3 + 15\cosh(fx + e)^2\sinh(fx + e)^4 + 6\cosh(fx + e)*\si \\
& nh(fx + e)^5 + \sinh(fx + e)^6)) + (a\cosh(fx + e)^2 + 2a\cosh(fx + e)* \\
& \sinh(fx + e) + a\sinh(fx + e)^2)\sqrt{b}\log((b\cosh(fx + e)^4 + 4b\cos \\
& h(fx + e)\sinh(fx + e)^3 + b\sinh(fx + e)^4 + 2a\cosh(fx + e)^2 + 2(3 \\
& *b\cosh(fx + e)^2 + a)\sinh(fx + e)^2 + \sqrt{2}(\cosh(fx + e)^2 + 2\cosh \\
& (fx + e)\sinh(fx + e) + \sinh(fx + e)^2 + 1)\sqrt{b}\sqrt{(b\cosh(fx + e) \\
&)^2 + b\sinh(fx + e)^2 + 2a - b)/(\cosh(fx + e)^2 - 2\cosh(fx + e)\sinh(\\
& fx + e) + \sinh(fx + e)^2)) + 4(b\cosh(fx + e)^3 + a\cosh(fx + e))*\sinh \\
& (fx + e) + b)/(\cosh(fx + e)^2 + 2\cosh(fx + e)\sinh(fx + e) + \sinh(fx \\
& + e)^2)) + \sqrt{2}(b\cosh(fx + e)^2 + 2b\cosh(fx + e)\sinh(fx + e) + b \\
& *\sinh(fx + e)^2 - b)\sqrt{(b\cosh(fx + e)^2 + b\sinh(fx + e)^2 + 2a - b) \\
&)/(\cosh(fx + e)^2 - 2\cosh(fx + e)\sinh(fx + e) + \sinh(fx + e)^2))/((b \\
& f\cosh(fx + e)^2 + 2bf\cosh(fx + e)\sinh(fx + e) + bf*\sinh(fx + e)^2 \\
&), -1/8(2(a\cosh(fx + e)^2 + 2a\cosh(fx + e)\sinh(fx + e) + a\sinh(fx \\
& + e)^2)\sqrt{-b}\arctan(\sqrt{2}((a - b)\cosh(fx + e)^2 + 2(a - b)\cosh \\
& (fx + e)\sinh(fx + e) + (a - b)\sinh(fx + e)^2 + b)\sqrt{-b}\sqrt{(b\cos \\
& h(fx + e)^2 + b\sinh(fx + e)^2 + 2a - b)/(\cosh(fx + e)^2 - 2\cosh(fx + \\
& e)\sinh(fx + e) + \sinh(fx + e)^2)))/((ab - b^2)\cosh(fx + e)^4 + 4(ab \\
& - b^2)\cosh(fx + e)\sinh(fx + e)^3 + (ab - b^2)\sinh(fx + e)^4 - (3a \\
& b - 2b^2)\cosh(fx + e)^2 + (6(ab - b^2)\cosh(fx + e)^2 - 3ab + 2b^2
\end{aligned}$$

```
)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)
)*cosh(f*x + e))*sinh(f*x + e))) + 2*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)
*sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(
3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*
a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*
b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e)
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^2 + 2*b*f*cosh(f*x + e)*si
nh(f*x + e) + b*f*sinh(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \cosh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e), x)
```

3.354 $\int \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b} \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f}$$

[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2])/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2])/f

Rubi [A] time = 0.0920647, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3190, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a-b} \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2])/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2])/f

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 402

Int[((a_.) + (b_.)*(x_)^2)^(p_.)/((c_.) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&

GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2} dx, x, \sinh(e+fx)\right)}{f} \\ &= \frac{(a-b) \operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} + \frac{b \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\ &= \frac{(a-b) \operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2} dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} \\ &= \frac{\sqrt{a-b} \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} \end{aligned}$$

Mathematica [A] time = 0.710105, size = 130, normalized size = 1.53

$$\frac{\sqrt{a}\sqrt{b} \sinh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a}}\right) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} + \sqrt{a-b} \tan^{-1}\left(\frac{\sqrt{2a-2b} \sinh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right)}{\sqrt{2a+b \cosh(2(e+fx))-b}} \cdot f$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Sqrt[a - b]*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + (Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]]*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/f

Maple [C] time = 0.091, size = 51, normalized size = 0.6

$$\frac{1}{f} \int \frac{-b(\sinh(fx+e))^2 - a}{(\cosh(fx+e))^2} \frac{1}{\sqrt{a+b(\sinh(fx+e))^2}} \sinh(fx+e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2), x)

[Out] `int/indef0`((-b*sinh(f*x+e)^2-a)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2), sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx+e) + a} \operatorname{sech}(fx+e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e), x)

Fricas [B] time = 2.644, size = 13256, normalized size = 155.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(\sqrt{b})\log(-((a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*\sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*\cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + \sqrt{2}*((a^2 - 2*a*b + b^2)*\cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*\sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*\sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - 4*a*b + 3*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b})\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(2*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + 2*\sqrt{-a + b})\log(((a - 2*b)*\cosh(f*x + e)^4 + 4*(a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a - 2*b)*\sinh(f*x + e)^4 - 2*(3*a - 2*b)*\cosh(f*x + e)^2 + 2*(3*(a - 2*b)*\cosh(f*x + e)^2 - 3*a + 2*b)*\sinh(f*x + e)^2 + 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1))*\sqrt{-a + b})\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) \end{aligned}$$

$$\begin{aligned}
& + \sinh(f*x + e)^2)) + 4*((a - 2*b)*\cosh(f*x + e)^3 - (3*a - 2*b)*\cosh(f*x + e))*\sinh(f*x + e) + a - 2*b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) + \sqrt{b}*\log((b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*a*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a)*\sinh(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1))*\sqrt{b}*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(b*\cosh(f*x + e)^3 + a*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/f, 1/4*(4*\sqrt{a - b}*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1))*\sqrt{a - b}*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))}/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) + \sqrt{b}*\log(-((a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*\sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*\cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + \sqrt{2}*((a^2 - 2*a*b + b^2)*\cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*\sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*\sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - 4*a*b + 3*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(2*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f
\end{aligned}$$

$$\begin{aligned}
& *x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + \\
& e)^5 + \sinh(f*x + e)^6)) + \sqrt{b}*\log((b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + \\
& e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*a*\cosh(f*x + e)^2 + 2*(3*b*\cosh \\
& (f*x + e)^2 + a)*\sinh(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + \\
& e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b \\
& *\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e \\
&) + \sinh(f*x + e)^2)) + 4*(b*\cosh(f*x + e)^3 + a*\cosh(f*x + e))*\sinh(f*x + \\
& e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2) \\
&))/f, -1/2*(\sqrt{-b}*\arctan(\sqrt{2}*((a - b)*\cosh(f*x + e)^2 + 2*(a - b)*\co \\
& sh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 + b)*\sqrt{-b}*\sqrt{(b*\c \\
& osh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x \\
& + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a*b - b^2)*\cosh(f*x + e)^4 + 4*(a \\
& *b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\sinh(f*x + e)^4 - (3* \\
& a*b - 2*b^2)*\cosh(f*x + e)^2 + (6*(a*b - b^2)*\cosh(f*x + e)^2 - 3*a*b + 2*b \\
& ^2)*\sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*\cosh(f*x + e)^3 - (3*a*b - 2*b \\
& ^2)*\cosh(f*x + e))*\sinh(f*x + e))) + \sqrt{-b}*\arctan(\sqrt{2}*(\cosh(f*x + e) \\
& ^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1)*\sqrt{-b}*\sqrt{(b* \\
& cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f* \\
& x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x \\
& + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2* \\
& (3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2 \\
& *a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) - \sqrt{-a + b}*\log(((a - 2*b)*\co \\
& sh(f*x + e)^4 + 4*(a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a - 2*b)*\sinh(\\
& f*x + e)^4 - 2*(3*a - 2*b)*\cosh(f*x + e)^2 + 2*(3*(a - 2*b)*\cosh(f*x + e)^2 \\
& - 3*a + 2*b)*\sinh(f*x + e)^2 + 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e \\
&)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{-a + b}*\sqrt{(b*\cosh(f*x + e)^2 + \\
& b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x \\
& + e) + \sinh(f*x + e)^2)) + 4*((a - 2*b)*\cosh(f*x + e)^3 - (3*a - 2*b)*\cosh \\
& (f*x + e))*\sinh(f*x + e) + a - 2*b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh \\
& (f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + \\
& 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1) \\
&))/f, 1/2*(2*\sqrt{a - b}*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)* \\
& \sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{a - b}*\sqrt{(b*\cosh(f*x + e)^2 + \\
& b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + \\
& e) + \sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e) \\
& ^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e) \\
& ^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + \\
& e))*\sinh(f*x + e) + b)) - \sqrt{-b}*\arctan(\sqrt{2}*((a - b)*\cosh(f*x + e)^2 \\
& + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 + b)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a*b - b^2)*\cosh(f*x + e)^4 + 4*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\sinh(f*x + e)^4 - (3*a*b - 2*b^2)*\cosh(f*x + e)^2 + (6*(a*b - b^2)*\cosh(f*x + e)^2 - 3*a*b + 2*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*\cosh(f*x + e)^3 - (3*a*b - 2*b^2)*\cosh(f*x + e))*\sinh(f*x + e))) - \sqrt{-b}*\arctan(\sqrt{2}
\end{aligned}$$


```
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \operatorname{sech}(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \operatorname{sech}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e), x)
```

$$3.355 \quad \int \operatorname{sech}^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=86

$$\frac{a \tan^{-1} \left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} \right)}{2f\sqrt{a-b}} + \frac{\tanh(e+fx) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{2f}$$

[Out] (a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[a - b]*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*f)

Rubi [A] time = 0.0964859, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3190, 378, 377, 203}

$$\frac{a \tan^{-1} \left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} \right)}{2f\sqrt{a-b}} + \frac{\tanh(e+fx) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] (a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[a - b]*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 378

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre

$eQ[\{a, b, c, d, n, p\}, x] \ \&\& \ NeQ[b*c - a*d, 0] \ \&\& \ EqQ[n*(p + q + 1) + 1, 0]$
 $\ \&\& \ GtQ[q, 0] \ \&\& \ NeQ[p, -1]$

Rule 377

$Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] \ :> \ Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] \ ; \ FreeQ[\{a, b, c, d\}, x] \ \&\& \ NeQ[b*c - a*d, 0] \ \&\& \ EqQ[n*p + 1, 0] \ \&\& \ IntegerQ[n]$

Rule 203

$Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] \ :> \ Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] \ ; \ FreeQ[\{a, b\}, x] \ \&\& \ PosQ[a/b] \ \&\& \ (GtQ[a, 0] \ || \ GtQ[b, 0])$

Rubi steps

$$\int \operatorname{sech}^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^2} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{2f} + \frac{a \operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}} dx, x, \sinh(e + fx)\right)}{2f}$$

$$= \frac{\operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{2f} + \frac{a \operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2} dx, x, \sinh(e + fx)\right)}{2f}$$

$$= \frac{a \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2\sqrt{a-b}f} + \frac{\operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{2f}$$

Mathematica [B] time = 1.51378, size = 175, normalized size = 2.03

$$\frac{\sinh(e + fx) \left(\sqrt{2a} \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \tanh^{-1} \left(\frac{\sqrt{\frac{(a-b) \sinh^2(e+fx)}{a}}}{\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1}} \right) + \operatorname{sech}^2(e + fx) \sqrt{-\frac{(a-b) \sinh^2(e+fx)}{a}} (2a + b \cosh(2(e + fx))) \right)}{4f \sqrt{-\frac{(a-b) \sinh^2(e+fx)}{a}} \sqrt{a + b \sinh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] (Sinh[e + f*x]*(Sqrt[2]*a*ArcTanh[Sqrt[-((a - b)*Sinh[e + f*x]^2)/a]]/Sqrt[1 + (b*Sinh[e + f*x]^2)/a])*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a] + (2*a - b + b*Cosh[2*(e + f*x)]*Sech[e + f*x]^2*Sqrt[-((a - b)*Sinh[e + f*x]^2)/a]))/(4*f*Sqrt[-((a - b)*Sinh[e + f*x]^2)/a])*Sqrt[a + b*Sinh[e + f*x]^2])

Maple [C] time = 0.085, size = 35, normalized size = 0.4

$$\frac{1}{f} \int \frac{1}{(\cosh(fx + e))^4} \sqrt{a + b(\sinh(fx + e))^2} \, \sinh(fx + e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] `int/indef0` (1/cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \operatorname{sech}(fx + e)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^3, x)

Fricas [B] time = 1.97148, size = 3534, normalized size = 41.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4*((a*\cosh(f*x + e)^4 + 4*a*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*\sinh(f*x + e)^4 + 2*a*\cosh(f*x + e)^2 + 2*(3*a*\cosh(f*x + e)^2 + a)*\sinh(f*x + e)^2 + 4*(a*\cosh(f*x + e)^3 + a*\cosh(f*x + e))*\sinh(f*x + e) + a)*\sqrt{-a + b}*1 \\ & \log(((a - 2*b)*\cosh(f*x + e)^4 + 4*(a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a - 2*b)*\sinh(f*x + e)^4 - 2*(3*a - 2*b)*\cosh(f*x + e)^2 + 2*(3*(a - 2*b) \\ & *\cosh(f*x + e)^2 - 3*a + 2*b)*\sinh(f*x + e)^2 - 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{-a + b}*\sqrt{(b \\ & *\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*((a - 2*b)*\cosh(f*x + e)^3 - \\ & (3*a - 2*b)*\cosh(f*x + e))*\sinh(f*x + e) + a - 2*b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) - 2*\sqrt{2}*((a - b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 - a + b)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a - b)*f*\cosh(f*x + e)^4 + 4*(a - b)*f*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a - b)*f*\sinh(f*x + e)^4 + 2*(a - b)*f*\cosh(f*x + e)^2 + 2*(3*(a - b)*f*\cosh(f*x + e)^2 + (a - b)*f)*\sinh(f*x + e)^2 + (a - b)*f + 4*((a - b)*f*\cosh(f*x + e)^3 + (a - b)*f*\cosh(f*x + e))*\sinh(f*x + e)), 1/2*((a*\cosh(f*x + e)^4 + 4*a*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*\sinh(f*x + e)^4 + 2*a*\cosh(f*x + e)^2 + 2*(3*a*\cosh(f*x + e)^2 + a)*\sinh(f*x + e)^2 + 4*(a*\cosh(f*x + e)^3 + a*\cosh(f*x + e))*\sinh(f*x + e) + a)*\sqrt{a - b}*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{a - b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) + \sqrt{2}*((a - b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 - a + b)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a - b)*f*\cosh(f*x + e)^4 + 4*(a - b)*f*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a - b)*f*\sinh(f*x + e)^4 + 2*(a - b)*f*\cosh(f*x + e)^2 + 2*(3*(a - b)*f*\cosh(f*x + e)^2 + (a - b)*f)*\sinh(f*x + e)^2 + (a - b)*f + 4*((a - b)*f*\cosh(f*x + e)^3 + (a - b)*f*\cosh(f*x + e))*\sinh(f*x + e)]] \end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \operatorname{sech}^3(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)`

[Out] `Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**3, x)`

Giac [B] time = 1.29754, size = 938, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")`

[Out]
$$\begin{aligned} & a \arctan\left(\frac{-1/2(\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b}) + \sqrt{b}}{\sqrt{a-b}}\right) / (\sqrt{a-b}) * \\ & f - 2 * ((\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b})^3 a - 2 * (\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b})^3 b - 5 * \\ & (\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b})^2 a \sqrt{b} + 2 * (\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b})^2 b^{3/2} \\ & - 4 * (\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b}) * a^2 - (\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b}) * a * b + 2 * (\sqrt{b} \\ &) * e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b}) * b^2 - 4 * a^2 \sqrt{b} + 5 * a * b^{3/2} - 2 * b^{5/2}) / (((\sqrt{b} \\ &) * e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b})^2 + 2 * (\sqrt{b}e^{2fx+2e} - \sqrt{be^{4fx+4e} + 4ae^{2fx+2e} - 2be^{2fx+2e} + b}) * \sqrt{b} + 4 * a - 3 * b)^{2 * f} \end{aligned}$$

$$3.356 \quad \int \operatorname{sech}^5(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=151

$$\frac{a(3a - 4b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{8f(a-b)^{3/2}} + \frac{\tanh(e+fx) \operatorname{sech}^3(e+fx) (a + b \sinh^2(e+fx))^{3/2}}{4f(a-b)} + \frac{(3a - 4b) \tanh(e+fx) \operatorname{sech}^2(e+fx)}{8f}$$

[Out] (a*(3*a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*(a - b)^(3/2)*f) + ((3*a - 4*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(8*(a - b)*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(4*(a - b)*f)

Rubi [A] time = 0.143462, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3190, 382, 378, 377, 203}

$$\frac{a(3a - 4b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{8f(a-b)^{3/2}} + \frac{\tanh(e+fx) \operatorname{sech}^3(e+fx) (a + b \sinh^2(e+fx))^{3/2}}{4f(a-b)} + \frac{(3a - 4b) \tanh(e+fx) \operatorname{sech}^2(e+fx)}{8f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (a*(3*a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*(a - b)^(3/2)*f) + ((3*a - 4*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(8*(a - b)*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(4*(a - b)*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^(m/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 382

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c -

```
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
  := -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^5(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^3} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\operatorname{sech}^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} \tanh(e+fx)}{4(a-b)f} + \frac{(3a-4b) \operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^3} dx, x, \sinh(e+fx)\right)}{4f} \\
&= \frac{(3a-4b) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{8(a-b)f} + \frac{\operatorname{sech}^3(e+fx)}{8(a-b)f} \\
&= \frac{(3a-4b) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{8(a-b)f} + \frac{\operatorname{sech}^3(e+fx)}{8(a-b)f} \\
&= \frac{a(3a-4b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{8(a-b)^{3/2}f} + \frac{(3a-4b) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8(a-b)f}
\end{aligned}$$

Mathematica [C] time = 12.4229, size = 684, normalized size = 4.53

$$\operatorname{tanh}(e+fx) \operatorname{sech}^3(e+fx) \left(\frac{b \sinh^2(e+fx)}{a} + 1\right) \left(10b \sinh^2(e+fx) \sqrt{\frac{(a-b) \operatorname{tanh}^2(e+fx) \operatorname{sech}^2(e+fx) (a+b \sinh^2(e+fx))}{a^2}} + 15a \sqrt{\frac{(a-b) \operatorname{tanh}^2(e+fx) \operatorname{sech}^2(e+fx) (a+b \sinh^2(e+fx))}{a^2}}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] $-(\operatorname{Sech}[e + f*x]^3(1 + (b \operatorname{Sinh}[e + f*x]^2)/a) \operatorname{Tanh}[e + f*x] (-15a \operatorname{ArcSin}[\operatorname{Sqrt}[(a-b) \operatorname{Tanh}[e + f*x]^2/a]] - 10b \operatorname{ArcSin}[\operatorname{Sqrt}[(a-b) \operatorname{Tanh}[e + f*x]^2/a]] \operatorname{Sinh}[e + f*x]^2 - 30a \operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2(a + b \operatorname{Sinh}[e + f*x]^2))/a] * ((a-b) \operatorname{Tanh}[e + f*x]^2/a)^{3/2} - 20b \operatorname{Sinh}[e + f*x]^2 \operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2(a + b \operatorname{Sinh}[e + f*x]^2))/a] * ((a-b) \operatorname{Tanh}[e + f*x]^2/a)^{3/2} - 32a \operatorname{Hypergeometric2F1}[2, 4, 7/2, ((a-b) \operatorname{Tanh}[e + f*x]^2/a) \operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2(a + b \operatorname{Sinh}[e + f*x]^2))/a] * ((a-b) \operatorname{Tanh}[e + f*x]^2/a)^{5/2}] - 32b \operatorname{Hypergeometric2F1}[2, 4, 7/2, ((a-b) \operatorname{Tanh}[e + f*x]^2/a) \operatorname{Sinh}[e + f*x]^2 \operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2(a + b \operatorname{Sinh}[e + f*x]^2))/a] * ((a-b) \operatorname{Tanh}[e + f*x]^2/a)^{5/2}] + 32a \operatorname{Hypergeometric2F1}[2, 4, 7/2, ((a-b) \operatorname{Tanh}[e + f*x]^2/a) \operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2(a + b \operatorname{Sinh}[e + f*x]^2))/a] * ((a-b) \operatorname{Tanh}[e + f*x]^2/a)^{5/2}])$

$$+ f*x]^2)/a)^{7/2} + 32*b*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^{7/2} + 15*a*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2] + 10*b*Sinh[e + f*x]^2*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2]))/(40*f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^{3/2})$$

Maple [C] time = 0.104, size = 35, normalized size = 0.2

$$\frac{1}{f} \int \frac{1}{(\cosh(fx + e))^6} \sqrt{a + b(\sinh(fx + e))^2} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] `int/indef0`(1/cosh(f*x+e)^6*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \operatorname{sech}^5(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^5, x)

Fricas [B] time = 3.42903, size = 9495, normalized size = 62.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

```
[Out] [-1/16*(((3*a^2 - 4*a*b)*cosh(f*x + e)^8 + 8*(3*a^2 - 4*a*b)*cosh(f*x + e)*
sinh(f*x + e)^7 + (3*a^2 - 4*a*b)*sinh(f*x + e)^8 + 4*(3*a^2 - 4*a*b)*cosh(
f*x + e)^6 + 4*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x
+ e)^6 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x
+ e))*sinh(f*x + e)^5 + 6*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 2*(35*(3*a^2 -
4*a*b)*cosh(f*x + e)^4 + 30*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 9*a^2 - 12*a
*b)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^5 + 10*(3*a^2 - 4*
a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 4
*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 4*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 1
5*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 9*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a
^2 - 4*a*b)*sinh(f*x + e)^2 + 3*a^2 - 4*a*b + 8*((3*a^2 - 4*a*b)*cosh(f*x +
e)^7 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)^5 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)
^3 + (3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2
*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)
*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x
+ e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2))) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)
*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)
*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x +
e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e)
+ 1)) - 2*sqrt(2)*((3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6 + 6*(3*a^2 - 5
*a*b + 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b + 2*b^2)*sinh(
f*x + e)^6 + (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^4 + (15*(3*a^2 - 5*a*
b + 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 21*a*b + 10*b^2)*sinh(f*x + e)^4 + 4*
(5*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 21*a*b + 10*b^2)*cos
h(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^2 +
(15*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 21*a*b + 10*b^2)*
cosh(f*x + e)^2 - 11*a^2 + 21*a*b - 10*b^2)*sinh(f*x + e)^2 - 3*a^2 + 5*a*b
- 2*b^2 + 2*(3*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 21*a*
b + 10*b^2)*cosh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x
+ e)^7 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^8 + 4*(a^2 - 2*a*b + b^2)*f*co
sh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b +
b^2)*f)*sinh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 8*(7*(
a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)
))*sinh(f*x + e)^5 + 2*(35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 30*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^2 + 3*(a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^4
+ 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 8*(7*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^5 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^6 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 9*(a^2 - 2*a*b + b^2)*
```

$$\begin{aligned}
& f \cosh(fx + e)^2 + (a^2 - 2ab + b^2)f \sinh(fx + e)^2 + (a^2 - 2ab + b^2)f \\
& + 8((a^2 - 2ab + b^2)f \cosh(fx + e)^7 + 3(a^2 - 2ab + b^2)f \cosh(fx + e)^5 + 3(a^2 - 2ab + b^2)f \cosh(fx + e)^3 + (a^2 - 2ab + b^2)f \cosh(fx + e)) \sinh(fx + e), \\
& 1/8(((3a^2 - 4ab) \cosh(fx + e)^8 + 8(3a^2 - 4ab) \cosh(fx + e) \sinh(fx + e)^7 + (3a^2 - 4ab) \sinh(fx + e)^8 + 4(3a^2 - 4ab) \cosh(fx + e)^6 + 4(7(3a^2 - 4ab) \cosh(fx + e)^2 + 3a^2 - 4ab) \sinh(fx + e)^6 + 8(7(3a^2 - 4ab) \cosh(fx + e)^3 + 3(3a^2 - 4ab) \cosh(fx + e)) \sinh(fx + e)^5 + 6(3a^2 - 4ab) \cosh(fx + e)^4 + 2(35(3a^2 - 4ab) \cosh(fx + e)^4 + 30(3a^2 - 4ab) \cosh(fx + e)^2 + 9a^2 - 12ab) \sinh(fx + e)^4 + 8(7(3a^2 - 4ab) \cosh(fx + e)^5 + 10(3a^2 - 4ab) \cosh(fx + e)^3 + 3(3a^2 - 4ab) \cosh(fx + e)) \sinh(fx + e)^3 + 4(3a^2 - 4ab) \cosh(fx + e)^2 + 4(7(3a^2 - 4ab) \cosh(fx + e)^6 + 15(3a^2 - 4ab) \cosh(fx + e)^4 + 9(3a^2 - 4ab) \cosh(fx + e)^2 + 3a^2 - 4ab) \sinh(fx + e)^2 + 3a^2 - 4ab + 8((3a^2 - 4ab) \cosh(fx + e)^7 + 3(3a^2 - 4ab) \cosh(fx + e)^5 + 3(3a^2 - 4ab) \cosh(fx + e)^3 + (3a^2 - 4ab) \cosh(fx + e)) \sinh(fx + e)) \sqrt{a - b} \arctan(\sqrt{2}(\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 - 1) \sqrt{a - b} \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)}) / (b \cosh(fx + e)^4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 + 2(2a - b) \cosh(fx + e)^2 + 2(3b \cosh(fx + e)^2 + 2a - b) \sinh(fx + e)^2 + 4(b \cosh(fx + e)^3 + (2a - b) \cosh(fx + e)) \sinh(fx + e) + b)) + \sqrt{2}((3a^2 - 5ab + 2b^2) \cosh(fx + e)^6 + 6(3a^2 - 5ab + 2b^2) \cosh(fx + e) \sinh(fx + e)^5 + (3a^2 - 5ab + 2b^2) \sinh(fx + e)^6 + (11a^2 - 21ab + 10b^2) \cosh(fx + e)^4 + (15(3a^2 - 5ab + 2b^2) \cosh(fx + e)^2 + 11a^2 - 21ab + 10b^2) \sinh(fx + e)^4 + 4(5(3a^2 - 5ab + 2b^2) \cosh(fx + e)^3 + (11a^2 - 21ab + 10b^2) \cosh(fx + e)) \sinh(fx + e)^3 - (11a^2 - 21ab + 10b^2) \cosh(fx + e)^2 + (15(3a^2 - 5ab + 2b^2) \cosh(fx + e)^4 + 6(11a^2 - 21ab + 10b^2) \cosh(fx + e)^2 - 11a^2 + 21ab - 10b^2) \sinh(fx + e)^2 - 3a^2 + 5ab - 2b^2 + 2(3(3a^2 - 5ab + 2b^2) \cosh(fx + e)^5 + 2(11a^2 - 21ab + 10b^2) \cosh(fx + e)^3 - (11a^2 - 21ab + 10b^2) \cosh(fx + e)) \sinh(fx + e)) \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)}) / ((a^2 - 2ab + b^2)f \cosh(fx + e)^8 + 8(a^2 - 2ab + b^2)f \cosh(fx + e) \sinh(fx + e)^7 + (a^2 - 2ab + b^2)f \sinh(fx + e)^8 + 4(a^2 - 2ab + b^2)f \cosh(fx + e)^6 + 4(7(a^2 - 2ab + b^2)f \cosh(fx + e)^2 + (a^2 - 2ab + b^2)f) \sinh(fx + e)^6 + 6(a^2 - 2ab + b^2)f \cosh(fx + e)^4 + 8(7(a^2 - 2ab + b^2)f \cosh(fx + e)^3 + 3(a^2 - 2ab + b^2)f \cosh(fx + e)) \sinh(fx + e)^5 + 2(35(a^2 - 2ab + b^2)f \cosh(fx + e)^4 + 30(a^2 - 2ab + b^2)f \cosh(fx + e)^2 + 3(a^2 - 2ab + b^2)f) \sinh(fx + e)^4 + 4(a^2 - 2ab + b^2)f \cosh(fx + e)^2 + 8(7(a^2 - 2ab + b^2)f \cosh(fx + e)^5 + 10(a^2 - 2ab + b^2)f \cosh(fx + e)^3 + 3(a^2 - 2ab + b^2)f \cosh(fx + e)) \sinh(fx + e)^3 + 4(7(a^2 - 2ab + b^2)f \cosh(fx + e)^6 + 15(a^2 - 2ab + b^2)f \cosh(fx + e)^4 + 9(a^2
\end{aligned}$$

$$- 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f + 8*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [B] time = 1.41716, size = 3009, normalized size = 19.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{4}*(3*a^2 - 4*a*b)*\arctan\left(\frac{-1/2*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b) + \sqrt{b}}{\sqrt{a - b}}\right) - \frac{1}{2}*(3*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b))^7*a^2 - 4*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b))^7*a*b + 21*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b))^6*a^2*\sqrt{b} - 60*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b))^6*a*b^{3/2} + 32*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b))^6*b^{5/2} + 44*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b))^5*a^3 - 253*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b))^5*a^2*b + 252*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b))^5*a*b^2 - 64*(\sqrt{b}*e^{2*f*x + 2*e} - \sqrt{b*e^{4*f*x + 4*e} + 4*a*e^{2*f*x + 2*e} - 2*b*e^{2*f*x + 2*e} + b))^5$

$$\begin{aligned}
& ^5b^3 - 292*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^4*a^3*\sqrt{b} + 317*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^4*a^2*b^{(3/2)} - 28*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^4*a*b^{(5/2)} - 32*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^4*b^{(7/2)} - 176*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^3*a^4 - 168*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^3*a^3*b + 737*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^3*a^2*b^2 - 556*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^3*a*b^3 + 128*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^3*b^4 - 528*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^2*a^4*\sqrt{b} + 1176*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^2*a^3*b^{(3/2)} - 937*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^2*a^2*b^{(5/2)} + 300*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^2*a*b^{(7/2)} - 32*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^2*b^{(9/2)} - 192*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))*a^5 + 304*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))*a^4*b + 124*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))*a^3*b^2 - 487*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))*a^2*b^3 + 308*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))*a*b^4 - 64*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))*b^5 - 192*a^5*\sqrt{b} + 656*a^4*b^{(3/2)} - 884*a^3*b^{(5/2)} + 599*a^2*b^{(7/2)} - 212*a*b^{(9/2)} + 32*b^{(11/2)})/(((\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))^2 + 2*(\sqrt{b}*e^{(2*f*x + 2*e)} - \sqrt{b*e^{(4*f*x + 4*e)} + 4*a*e^{(2*f*x + 2*e)} - 2*b*e^{(2*f*x + 2*e)} + b))*\sqrt{b} + 4*a - 3*b)^4*(a*f - b*f))
\end{aligned}$$

$$3.357 \quad \int \cosh^4(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=301

$$\frac{(a - 9b) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) + (2a^2 - 7ab - 3b^2) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15b^2 f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

[Out] $(-2*(a - 3*b)*\operatorname{Cosh}[e + f*x]*\operatorname{Sinh}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(15*b*f) + (\operatorname{Cosh}[e + f*x]*\operatorname{Sinh}[e + f*x]*(a + b*\operatorname{Sinh}[e + f*x]^2)^{(3/2)})/(5*b*f) + ((2*a^2 - 7*a*b - 3*b^2)*\operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(15*b^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) - ((a - 9*b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(15*b*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) - ((2*a^2 - 7*a*b - 3*b^2)*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2]*\operatorname{Tanh}[e + f*x])/(15*b^2*f)$

Rubi [A] time = 0.297575, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3192, 416, 528, 531, 418, 492, 411}

$$\frac{(2a^2 - 7ab - 3b^2) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15b^2 f} + \frac{(2a^2 - 7ab - 3b^2) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} E\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{15b^2 f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Cosh}[e + f*x]^4*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2], x]$

[Out] $(-2*(a - 3*b)*\operatorname{Cosh}[e + f*x]*\operatorname{Sinh}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(15*b*f) + (\operatorname{Cosh}[e + f*x]*\operatorname{Sinh}[e + f*x]*(a + b*\operatorname{Sinh}[e + f*x]^2)^{(3/2)})/(5*b*f) + ((2*a^2 - 7*a*b - 3*b^2)*\operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(15*b^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) - ((a - 9*b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(15*b*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) - ((2*a^2 - 7*a*b - 3*b^2)*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2]*\operatorname{Tanh}[e + f*x])/(15*b^2*f)$

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```


$a*d, 0] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c] \&\& !\text{SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^{(3/2)}, x_Symbol] \text{ :> Sim}$
 $\text{p}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[$
 $d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /;$ FreeQ
 $\{a, b, c, d\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\int \cosh^4(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx = \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int (1 + x^2)^{3/2} \sqrt{a + bx^2} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{\cosh(e + fx) \sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{5bf} + \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int (1 + x^2)^{3/2} \sqrt{a + bx^2} dx, x, \sinh(e + fx)\right)}{f}$$

$$= -\frac{2(a - 3b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} + \frac{\cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf}$$

$$= -\frac{2(a - 3b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} + \frac{\cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf}$$

$$= -\frac{2(a - 3b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} + \frac{\cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf}$$

$$= -\frac{2(a - 3b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf} + \frac{\cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15bf}$$

Mathematica [C] time = 1.37619, size = 211, normalized size = 0.7

$$\frac{-32ia(a^2 - 4ab + 3b^2) \sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} \operatorname{EllipticF}\left(i(e + fx), \frac{b}{a}\right) + \sqrt{2}b \sinh(2(e + fx)) (8a^2 + 4b(4a + 3b) \cosh(2(e + fx)))}{240b^2 f \sqrt{2a + b \cosh(2(e + fx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((16*I)*a*(2*a^2 - 7*a*b - 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*E
llipticE[I*(e + f*x), b/a] - (32*I)*a*(a^2 - 4*a*b + 3*b^2)*Sqrt[(2*a - b +
b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 + 3
2*a*b - 15*b^2 + 4*b*(4*a + 3*b)*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x)
])*Sinh[2*(e + f*x)]/(240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.106, size = 521, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+4*(-1/a*b)^(1/2)*a*b*c
osh(f*x+e)^4*sinh(f*x+e)+((-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-3*(-1/a*b
)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))+2
*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2-2*
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+
e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^2+7*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b+3*(
b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a \cosh^2(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh (fx + e)^2 + a} \cosh (fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh (fx + e)^2 + a} \cosh (fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)

$$3.358 \quad \int \cosh^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=223

$$\frac{2 \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{3f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{(a + b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3bf}$$

[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b*f)

Rubi [A] time = 0.203341, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3192, 417, 531, 418, 492, 411}

$$\frac{(a + b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3bf} + \frac{\sinh(e + fx) \cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{2 \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b*f)

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a

+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 417

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(x*(a + b*x^n)^p*(c + d*x^n)^q)/(n*(p + q) + 1), x] + Dist[n/(n*(p + q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 531

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \cosh^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \sqrt{1 + x^2} \sqrt{a + bx^2} dx, x, \sinh(e + fx)\right)}{f} \\
&= \frac{\cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{\left(2\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right)}{3f} \\
&= \frac{\cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{\left(2a\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right)}{3f} \\
&= \frac{\cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{2F\left(\tan^{-1}(\sinh(e + fx))\right)}{3f\sqrt{\operatorname{sech}(e + fx)}} \\
&= \frac{\cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{(a + b)E\left(\tan^{-1}(\sinh(e + fx))\right)}{3bf\sqrt{\operatorname{sech}(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 0.697644, size = 168, normalized size = 0.75

$$\frac{2i\sqrt{2a(a-b)}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b) - 2i\sqrt{2a(a+b)}}{6bf\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((-2*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.085, size = 339, normalized size = 1.5

$$\frac{1}{3f\cosh(fx + e)} \left(\sqrt{\frac{-b}{a}} b \sinh(fx + e) (\cosh(fx + e))^4 + \left(\sqrt{\frac{-b}{a}} a - \sqrt{\frac{-b}{a}} b \right) (\cosh(fx + e))^2 \sinh(fx + e) + a \sqrt{\frac{b}{a}} (\cosh(fx + e))^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] $\frac{1}{3} \left((-1/a*b)^{(1/2)} * b * \sinh(f*x+e) * \cosh(f*x+e)^4 + (-1/a*b)^{(1/2)} * a - (-1/a*b)^{(1/2)} * b * \cosh(f*x+e)^2 * \sinh(f*x+e) + a * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) - (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b + (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a + (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b \right) / (-1/a*b)^{(1/2)} / \cosh(f*x+e) / (a+b*\sinh(f*x+e)^2)^{(1/2)} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a \cosh^2(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\sqrt{b \sinh^2(fx + e) + a \cosh^2(fx + e)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \cosh^2(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)`

$$3.359 \quad \int \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=60

$$-\frac{i\sqrt{a + b \sinh^2(e + fx)}E\left(ie + ifx \left| \frac{b}{a} \right.\right)}{f\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1}}$$

[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rubi [A] time = 0.036322, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a + b \sinh^2(e + fx)}E\left(ie + ifx \left| \frac{b}{a} \right.\right)}{f\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rule 3178

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[1 + (b*Sinh[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sinh[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3177

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\int \sqrt{a + b \sinh^2(e + fx)} dx = \frac{\sqrt{a + b \sinh^2(e + fx)} \int \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}} dx}{\sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}}$$

$$= -\frac{iE\left(i e + i f x \left| \frac{b}{a} \right. \right) \sqrt{a + b \sinh^2(e + fx)}}{f \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}}$$

Mathematica [A] time = 0.0788632, size = 69, normalized size = 1.15

$$-\frac{ia \sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} E\left(i(e + fx) \left| \frac{b}{a} \right. \right)}{f \sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a]/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0., size = 140, normalized size = 2.3

$$\frac{1}{\cosh(fx + e) f} \sqrt{\frac{a + b (\sinh(fx + e))^2}{a}} \sqrt{(\cosh(fx + e))^2} \left(a \text{EllipticF}\left(\sinh(fx + e) \sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) - b \text{EllipticF}\left(\sinh(fx + e) \sqrt{\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^(1/2), x)

[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2)))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh^2(fx + e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.360 $\int \operatorname{sech}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=70

$$\frac{\operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} E\left(\tan^{-1}(\sinh(e + fx)) \middle| 1 - \frac{b}{a}\right)}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])

Rubi [A] time = 0.0844919, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {3192, 411}

$$\frac{\operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} E\left(\tan^{-1}(\sinh(e + fx)) \middle| 1 - \frac{b}{a}\right)}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 411

Int[Sqrt[(a_.) + (b_.)*(x_.)^2]/((c_.) + (d_.)*(x_.)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ

[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\int \operatorname{sech}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx = \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{E\left(\tan^{-1}(\sinh(e+fx)) \mid 1 - \frac{b}{a}\right) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{f \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Mathematica [C] time = 0.484483, size = 148, normalized size = 2.11

$$\frac{-2ia \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2} \tanh(e+fx)(2a+b \cosh(2(e+fx))-b) + 2ia \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}}}{2f \sqrt{2a+b \cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] - (2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)]*Tanh[e + f*x])/(2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.101, size = 177, normalized size = 2.5

$$\frac{1}{f \cosh(fx+e)} \left(\sqrt{-\frac{b}{a}} b (\sinh(fx+e))^3 + b \sqrt{\frac{a+b(\sinh(fx+e))^2}{a}} \sqrt{(\cosh(fx+e))^2} \operatorname{EllipticF}\left(\sinh(fx+e) \sqrt{-\frac{b}{a}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2), x)

[Out] $((-1/a*b)^{(1/2)}*b*\sinh(f*x+e)^3+b*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-b*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})+(-1/a*b)^{(1/2)}*a*\sinh(f*x+e))/(-1/a*b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \operatorname{sech}^2(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh^2(fx + e) + a} \operatorname{sech}^2(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \operatorname{sech}^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)`

[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \operatorname{sech}(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)

3.361 $\int \operatorname{sech}^4(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$

Optimal. Leaf size=206

$$\frac{b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{3f(a - b) \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{\tanh(e + fx) \operatorname{sech}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f}$$

```
[Out] ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]
^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(3*f)
```

Rubi [A] time = 0.166938, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3192, 412, 525, 418, 411}

$$\frac{\tanh(e + fx) \operatorname{sech}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} F\left(\tan^{-1}(\sinh(e + fx)) \mid 1 - \frac{b}{a}\right)}{3f(a - b) \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2], x]
```

```
[Out] ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]
^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(3*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a
```

+ b*ff²*x²)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]

Rule 412

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*xⁿ)^(p + 1)*(c + d*xⁿ)^q/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*xⁿ)^(p + 1)*(c + d*xⁿ)^(q - 1)*Simp[c*(n*(p + 1)
+ 1) + d*(n*(p + q + 1) + 1)*xⁿ, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
d, n, p, q, x]

Rule 525

Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^(
3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x²]*S
qrt[c + d*x²]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x²]
/(c + d*x²)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]

Rule 418

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x²]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*R
t[d/c, 2]*Sqrt[c + d*x²]*Sqrt[(c*(a + b*x²))/(a*(c + d*x²))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x²]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(c*Rt[
d/c, 2]*Sqrt[c + d*x²]*Sqrt[(c*(a + b*x²))/(a*(c + d*x²))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^4(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right) \operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\operatorname{sech}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{3f} - \frac{\left(\sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right)}{3f} \\
&= \frac{\operatorname{sech}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{3f} + \frac{\left((2a-b) \sqrt{\cosh^2(e+fx)}\right)}{3f} \\
&= \frac{(2a-b) E\left(\tan^{-1}(\sinh(e+fx)) \mid 1 - \frac{b}{a}\right) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3(a-b) f \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 2.83669, size = 204, normalized size = 0.99

$$\frac{-16ia(a-b) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2} \tanh(e+fx) \operatorname{sech}^2(e+fx) \left((8a^2-4b^2) \cosh(2(e+fx))\right)}{24f(a-b) \sqrt{2a+b \cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((8*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] - (16*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*((8*a^2 - 4*b^2)*Cosh[2*(e + f*x)] + (2*a - b)*(8*a - 5*b + b*Cosh[4*(e + f*x)]))*Sech[e + f*x]^2*Tanh[e + f*x]/(24*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.132, size = 318, normalized size = 1.5

$$\frac{1}{3 (\cosh(fx+e))^3 (a-b) f} \left(\left(2 \sqrt{-\frac{b}{a} ab} - \sqrt{-\frac{b}{a} b^2} \right) \sinh(fx+e) (\cosh(fx+e))^4 + \left(2 \sqrt{-\frac{b}{a} a^2} - 2 \sqrt{-\frac{b}{a} ab} \right) (\cosh(fx+e))^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+(2
*(-1/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b)*cosh(f*x+e)^2*sinh(f*x+e)+((-1/a*
b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)+(cosh(f*x
+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2)))-2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+b*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2)/cosh(f*x+e)^3/(a-b)/(-1/
a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \operatorname{sech}^4(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{b \sinh^2(fx + e) + a} \operatorname{sech}^4(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \operatorname{sech}^4(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)

$$3.362 \quad \int \cosh^3(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=157

$$-\frac{a^2(a-6b) \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{\sinh(e+fx) (a+b \sinh^2(e+fx))^{5/2}}{6bf} - \frac{(a-6b) \sinh(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{24bf}$$

```
[Out] -(a^2*(a - 6*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(16*b^(3/2)*f) - (a*(a - 6*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(16*b*f) - ((a - 6*b)*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*b*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(5/2))/(6*b*f)
```

Rubi [A] time = 0.128896, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3190, 388, 195, 217, 206}

$$-\frac{a^2(a-6b) \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{\sinh(e+fx) (a+b \sinh^2(e+fx))^{5/2}}{6bf} - \frac{(a-6b) \sinh(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(a^2*(a - 6*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(16*b^(3/2)*f) - (a*(a - 6*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(16*b*f) - ((a - 6*b)*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*b*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(5/2))/(6*b*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

$(p + 1) + 1) / (b * (n * (p + 1) + 1))$, Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \cosh^3(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{\text{Subst}\left(\int (1 + x^2) (a + bx^2)^{3/2} dx, x, \sinh(e + fx)\right)}{f} \\
&= \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{5/2}}{6bf} - \frac{(a - 6b) \text{Subst}\left(\int (a + bx^2)^{3/2} dx\right)}{6bf} \\
&= -\frac{(a - 6b) \sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{24bf} + \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{6bf} \\
&= -\frac{a(a - 6b) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{16bf} - \frac{(a - 6b) \sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{24bf} \\
&= -\frac{a(a - 6b) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{16bf} - \frac{(a - 6b) \sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{24bf} \\
&= -\frac{a^2(a - 6b) \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}}\right)}{16b^{3/2}f} - \frac{a(a - 6b) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{16bf}
\end{aligned}$$

Mathematica [A] time = 1.19984, size = 149, normalized size = 0.95

$$\frac{\sqrt{a + b \sinh^2(e + fx)} \left(\sqrt{b} \sinh(e + fx) \sqrt{\frac{b \sinh^2(e + fx)}{a} + 1} (2b(7a + 6b) \sinh^2(e + fx) + 3a(a + 10b) + 8b^2 \sinh^4(e + fx)) \right)}{48b^{3/2}f \sqrt{\frac{b \sinh^2(e + fx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(-3*a^(3/2)*(a - 6*b)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]] + Sqrt[b]*Sinh[e + f*x]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]*(3*a*(a + 10*b) + 2*b*(7*a + 6*b)*Sinh[e + f*x]^2 + 8*b^2*Sinh[e + f*x]^4)))/(48*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Maple [C] time = 0.085, size = 77, normalized size = 0.5

$$\frac{1}{f} \int \frac{\int \int \int \int \int \int \left((b^2 (\sinh(fx + e))^6 + (2ab + b^2) (\sinh(fx + e))^4 + (a^2 + 2ab) (\sinh(fx + e))^2 + a^2) \sqrt{a + b (\sinh(fx + e))^2} \right)}{\sqrt{a + b (\sinh(fx + e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out] `\`int/indef0`((b^2*sinh(f*x+e)^6+(2*a*b+b^2)*sinh(f*x+e)^4+(a^2+2*a*b)*sinh(f*x+e)^2+a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \cosh(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^3, x)`

Fricas [B] time = 3.63228, size = 11570, normalized size = 73.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `[-1/384*(6*((a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 - 6*a^2*b)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 - 6*a^2*b)*sinh(f*x + e)^6)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*`

$$\begin{aligned}
& b^3 \cosh(fx + e)^6 + 2(a^3 - 4a^2b + 5ab^2 - 2b^3 + 14(a^2b - 2ab^2 + b^3) \cosh(fx + e)^2) \sinh(fx + e)^6 + 4(14(a^2b - 2ab^2 + b^3) \cosh(fx + e)^3 + 3(a^3 - 4a^2b + 5ab^2 - 2b^3) \cosh(fx + e)) \sinh(fx + e)^5 + (9a^2b - 14ab^2 + 6b^3) \cosh(fx + e)^4 + (70(a^2b - 2ab^2 + b^3) \cosh(fx + e)^4 + 9a^2b - 14ab^2 + 6b^3 + 30(a^3 - 4a^2b + 5ab^2 - 2b^3) \cosh(fx + e)^2) \sinh(fx + e)^4 + 4(14(a^2b - 2ab^2 + b^3) \cosh(fx + e)^5 + 10(a^3 - 4a^2b + 5ab^2 - 2b^3) \cosh(fx + e)^3 + (9a^2b - 14ab^2 + 6b^3) \cosh(fx + e)) \sinh(fx + e)^3 + b^3 + 2(3ab^2 - 2b^3) \cosh(fx + e)^2 + 2(14(a^2b - 2ab^2 + b^3) \cosh(fx + e)^6 + 15(a^3 - 4a^2b + 5ab^2 - 2b^3) \cosh(fx + e)^4 + 3ab^2 - 2b^3 + 3(9a^2b - 14ab^2 + 6b^3) \cosh(fx + e)^2) \sinh(fx + e)^2 + \sqrt{2}((a^2 - 2ab + b^2) \cosh(fx + e)^6 + 6(a^2 - 2ab + b^2) \cosh(fx + e) \sinh(fx + e)^5 + (a^2 - 2ab + b^2) \sinh(fx + e)^6 - 3(a^2 - 2ab + b^2) \cosh(fx + e)^4 + 3(5(a^2 - 2ab + b^2) \cosh(fx + e)^2 - a^2 + 2ab - b^2) \sinh(fx + e)^4 + 4(5(a^2 - 2ab + b^2) \cosh(fx + e)^3 - 3(a^2 - 2ab + b^2) \cosh(fx + e)) \sinh(fx + e)^3 - (4ab - 3b^2) \cosh(fx + e)^2 + (15(a^2 - 2ab + b^2) \cosh(fx + e)^4 - 18(a^2 - 2ab + b^2) \cosh(fx + e)^2 - 4ab + 3b^2) \sinh(fx + e)^2 - b^2 + 2(3(a^2 - 2ab + b^2) \cosh(fx + e)^5 - 6(a^2 - 2ab + b^2) \cosh(fx + e)^3 - (4ab - 3b^2) \cosh(fx + e)) \sinh(fx + e)) \sqrt{b} \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)) + 4(2(a^2b - 2ab^2 + b^3) \cosh(fx + e)^7 + 3(a^3 - 4a^2b + 5ab^2 - 2b^3) \cosh(fx + e)^5 + (9a^2b - 14ab^2 + 6b^3) \cosh(fx + e)^3 + (3ab^2 - 2b^3) \cosh(fx + e)) \sinh(fx + e) / (\cosh(fx + e)^6 + 6 \cosh(fx + e)^5 \sinh(fx + e) + 15 \cosh(fx + e)^4 \sinh(fx + e)^2 + 20 \cosh(fx + e)^3 \sinh(fx + e)^3 + 15 \cosh(fx + e)^2 \sinh(fx + e)^4 + 6 \cosh(fx + e) \sinh(fx + e)^5 + \sinh(fx + e)^6)) + 6((a^3 - 6a^2b) \cosh(fx + e)^6 + 6(a^3 - 6a^2b) \cosh(fx + e)^5 \sinh(fx + e) + 15(a^3 - 6a^2b) \cosh(fx + e)^4 \sinh(fx + e)^2 + 20(a^3 - 6a^2b) \cosh(fx + e)^3 \sinh(fx + e)^3 + 15(a^3 - 6a^2b) \cosh(fx + e)^2 \sinh(fx + e)^4 + 6(a^3 - 6a^2b) \cosh(fx + e) \sinh(fx + e)^5 + (a^3 - 6a^2b) \sinh(fx + e)^6) \sqrt{b} \log((b \cosh(fx + e)^4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 + 2a \cosh(fx + e)^2 + 2(3b \cosh(fx + e)^2 + a) \sinh(fx + e)^2 + \sqrt{2}(\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 + 1) \sqrt{b} \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)) + 4(b \cosh(fx + e)^3 + a \cosh(fx + e)) \sinh(fx + e) + b) / (\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)) - \sqrt{2}(b^3 \cosh(fx + e)^{10} + 10b^3 \cosh(fx + e) \sinh(fx + e)^9 + b^3 \sinh(fx + e)^{10} + (7ab^2 + b^3) \cosh(fx + e)^8 + (45b^3 \cosh(fx + e)^2 + 7ab^2 + b^3) \sinh(fx + e)^8 + 8(15b^3 \cosh(fx + e)^3 + (7ab^2 + b^3) \cosh(fx + e)) \sinh(fx + e)^7 + (6a^2b + 39ab^2 - 8b^3) \cosh(fx + e)^6 + (210b^3 \cosh(fx + e)^4 + 6a^2b + 39ab^2 - 8b^3 + 28(7ab^2 + b^3) \cosh(fx + e)^2) \sinh(fx + e)^6 + 2(126b^3 \cosh(fx + e)^5 + 28(7ab^2 + b^3) \cosh(fx + e)^3 + 3(6a^2b + 39ab^2 - 8b^3) \cosh(fx
\end{aligned}$$

$$\begin{aligned}
& + e))\sinh(f*x + e)^5 - (6*a^2*b + 39*a*b^2 - 8*b^3)\cosh(f*x + e)^4 + (21 \\
& 0*b^3\cosh(f*x + e)^6 + 70*(7*a*b^2 + b^3)\cosh(f*x + e)^4 - 6*a^2*b - 39*a \\
& *b^2 + 8*b^3 + 15*(6*a^2*b + 39*a*b^2 - 8*b^3)\cosh(f*x + e)^2)\sinh(f*x + \\
& e)^4 + 4*(30*b^3\cosh(f*x + e)^7 + 14*(7*a*b^2 + b^3)\cosh(f*x + e)^5 + 5*(\\
& 6*a^2*b + 39*a*b^2 - 8*b^3)\cosh(f*x + e)^3 - (6*a^2*b + 39*a*b^2 - 8*b^3)* \\
& \cosh(f*x + e))\sinh(f*x + e)^3 - b^3 - (7*a*b^2 + b^3)\cosh(f*x + e)^2 + (4 \\
& 5*b^3\cosh(f*x + e)^8 + 28*(7*a*b^2 + b^3)\cosh(f*x + e)^6 + 15*(6*a^2*b + \\
& 39*a*b^2 - 8*b^3)\cosh(f*x + e)^4 - 7*a*b^2 - b^3 - 6*(6*a^2*b + 39*a*b^2 - \\
& 8*b^3)\cosh(f*x + e)^2)\sinh(f*x + e)^2 + 2*(5*b^3\cosh(f*x + e)^9 + 4*(7* \\
& a*b^2 + b^3)\cosh(f*x + e)^7 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)\cosh(f*x + e) \\
& ^5 - 2*(6*a^2*b + 39*a*b^2 - 8*b^3)\cosh(f*x + e)^3 - (7*a*b^2 + b^3)\cosh(\\
& f*x + e))\sinh(f*x + e))\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a \\
& - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))/ \\
& (b^2*f*\cosh(f*x + e)^6 + 6*b^2*f*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*b^2*f*c \\
& osh(f*x + e)^4*\sinh(f*x + e)^2 + 20*b^2*f*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + \\
& 15*b^2*f*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*b^2*f*\cosh(f*x + e)*\sinh(f*x \\
& + e)^5 + b^2*f*\sinh(f*x + e)^6), 1/384*(12*((a^3 - 6*a^2*b)*\cosh(f*x + e)^6 \\
& + 6*(a^3 - 6*a^2*b)*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cos \\
& h(f*x + e)^4*\sinh(f*x + e)^2 + 20*(a^3 - 6*a^2*b)*\cosh(f*x + e)^3*\sinh(f*x \\
& + e)^3 + 15*(a^3 - 6*a^2*b)*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*(a^3 - 6*a^ \\
& 2*b)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (a^3 - 6*a^2*b)*\sinh(f*x + e)^6)\sqrt{ \\
& -b}\arctan(\sqrt{2}*((a - b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(\\
& f*x + e) + (a - b)*\sinh(f*x + e)^2 + b)\sqrt{-b})\sqrt{(b*\cosh(f*x + e)^2 + \\
& b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + \\
& e) + \sinh(f*x + e)^2))/((a*b - b^2)*\cosh(f*x + e)^4 + 4*(a*b - b^2)*\cosh(f* \\
& x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\sinh(f*x + e)^4 - (3*a*b - 2*b^2)*\cosh \\
& (f*x + e)^2 + (6*(a*b - b^2)*\cosh(f*x + e)^2 - 3*a*b + 2*b^2)*\sinh(f*x + e) \\
& ^2 - b^2 + 2*(2*(a*b - b^2)*\cosh(f*x + e)^3 - (3*a*b - 2*b^2)*\cosh(f*x + e) \\
&)*\sinh(f*x + e))) + 12*((a^3 - 6*a^2*b)*\cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b) \\
& *\cosh(f*x + e)^5*\sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*\cosh(f*x + e)^4*\sinh(f* \\
& x + e)^2 + 20*(a^3 - 6*a^2*b)*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*(a^3 - 6 \\
& *a^2*b)*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*\cosh(f*x + e)* \\
& \sinh(f*x + e)^5 + (a^3 - 6*a^2*b)*\sinh(f*x + e)^6)\sqrt{-b}\arctan(\sqrt{2}*(\\
& \cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1)\sqrt{ \\
& -b})\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^ \\
& 2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))/(b*\cosh(f*x + e)^4 + \\
& 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f* \\
& x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f* \\
& x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) + \sqrt{2}*(b^3*\cosh \\
& (f*x + e)^10 + 10*b^3*\cosh(f*x + e)*\sinh(f*x + e)^9 + b^3*\sinh(f*x + e)^10 \\
& + (7*a*b^2 + b^3)*\cosh(f*x + e)^8 + (45*b^3*\cosh(f*x + e)^2 + 7*a*b^2 + b^3) \\
&)*\sinh(f*x + e)^8 + 8*(15*b^3*\cosh(f*x + e)^3 + (7*a*b^2 + b^3)*\cosh(f*x + \\
& e))\sinh(f*x + e)^7 + (6*a^2*b + 39*a*b^2 - 8*b^3)\cosh(f*x + e)^6 + (210*b \\
& ^3*\cosh(f*x + e)^4 + 6*a^2*b + 39*a*b^2 - 8*b^3 + 28*(7*a*b^2 + b^3)*\cosh(f \\
& *x + e)^2)*\sinh(f*x + e)^6 + 2*(126*b^3*\cosh(f*x + e)^5 + 28*(7*a*b^2 + b^3
\end{aligned}$$

```

)*cosh(f*x + e)^3 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e))*sinh(f*x
+ e)^5 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*x +
e)^6 + 70*(7*a*b^2 + b^3)*cosh(f*x + e)^4 - 6*a^2*b - 39*a*b^2 + 8*b^3 + 1
5*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(30*b^3
*cosh(f*x + e)^7 + 14*(7*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b + 39*a*b
^2 - 8*b^3)*cosh(f*x + e)^3 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e))*s
inh(f*x + e)^3 - b^3 - (7*a*b^2 + b^3)*cosh(f*x + e)^2 + (45*b^3*cosh(f*x +
e)^8 + 28*(7*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b + 39*a*b^2 - 8*b^3
))*cosh(f*x + e)^4 - 7*a*b^2 - b^3 - 6*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*a*b^2 + b^3)*cos
h(f*x + e)^7 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^5 - 2*(6*a^2*b
+ 39*a*b^2 - 8*b^3)*cosh(f*x + e)^3 - (7*a*b^2 + b^3)*cosh(f*x + e))*sinh(f
*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x
+ e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*cosh(f*x + e)^4*s
inh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*b^2*f*cosh(f
*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*f*s
inh(f*x + e)^6)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \cosh(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^3, x)

$$3.363 \quad \int \cosh(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=104

$$\frac{3a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} \right)}{8\sqrt{bf}} + \frac{3a \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8f} + \frac{\sinh(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{4f}$$

[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*Sqrt[b]*f) + (3*a*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*f)

Rubi [A] time = 0.0687584, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 195, 217, 206}

$$\frac{3a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} \right)}{8\sqrt{bf}} + \frac{3a \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8f} + \frac{\sinh(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{4f}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*Sqrt[b]*f) + (3*a*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 195

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&

IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
Denominator[p]])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \cosh(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{\text{Subst} \left(\int (a + bx^2)^{3/2} dx, x, \sinh(e + fx) \right)}{f} \\ &= \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{4f} + \frac{(3a) \text{Subst} \left(\int \sqrt{a + bx^2} dx, x, \sinh(e + fx) \right)}{4f} \\ &= \frac{3a \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{8f} + \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{4f} \\ &= \frac{3a \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{8f} + \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{4f} \\ &= \frac{3a^2 \tanh^{-1} \left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} \right)}{8\sqrt{b}f} + \frac{3a \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{8f} + \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{4f} \end{aligned}$$

Mathematica [A] time = 0.485459, size = 93, normalized size = 0.89

$$\frac{\sqrt{a + b \sinh^2(e + fx)} \left(\frac{3a^{3/2} \sinh^{-1} \left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a}} \right)}{\sqrt{b} \sqrt{\frac{b \sinh^2(e + fx)}{a} + 1}} + 5a \sinh(e + fx) + 2b \sinh^3(e + fx) \right)}{8f}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(5*a*Sinh[e + f*x] + 2*b*Sinh[e + f*x]^3 + (3*a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])))/(8*f)

Maple [A] time = 0.011, size = 90, normalized size = 0.9

$$\frac{\sinh(fx + e)}{4f} \left(a + b(\sinh(fx + e))^2 \right)^{\frac{3}{2}} + \frac{3a \sinh(fx + e)}{8f} \sqrt{a + b(\sinh(fx + e))^2} + \frac{3a^2}{8f} \ln \left(\sinh(fx + e) \sqrt{b} + \sqrt{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] 1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+3/8*a*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+3/8/f*a^2/b^(1/2)*ln(sinh(f*x+e)*b^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \cosh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e), x)

Fricas [B] time = 2.75815, size = 8070, normalized size = 77.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/64*(6*(a^2*\cosh(f*x + e)^4 + 4*a^2*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*a^2 \\ & *\cosh(f*x + e)^2*\sinh(f*x + e)^2 + 4*a^2*\cosh(f*x + e)*\sinh(f*x + e)^3 + a^2 \\ & *2*\sinh(f*x + e)^4)*\sqrt{b}*\log(-((a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^8 + \\ & 8*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 \\ & + b^3)*\sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^6 \\ & + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x \\ & + e)^2)*\sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^3 + \\ & 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b \\ & - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x \\ & + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b \\ & ^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x \\ & + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + (9*a^2*b \\ & - 14*a*b^2 + 6*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2* \\ & b^3)*\cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^6 + 15*(\\ & a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a \\ & ^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + \sqrt{2}*((a^2 - \\ & 2*a*b + b^2)*\cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)*\sinh(f*x \\ & + e)^5 + (a^2 - 2*a*b + b^2)*\sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*\cosh \\ & (f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - a^2 + 2*a*b - b^2) \\ & *\sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - 3*(a^2 - 2*a* \\ & b + b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x + e)^2 + \\ & (15*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*\cosh(f*x \\ & + e)^2 - 4*a*b + 3*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*\co \\ & sh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - (4*a*b - 3*b^2)*\cos \\ & h(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e \\ &)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x \\ & + e)^2))} + 4*(2*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b \\ & + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x \\ & + e)^3 + (3*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + \\ & 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20* \\ & \cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\co \\ & sh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + 6*(a^2*\cosh(f*x + e)^4 + \\ & 4*a^2*\cosh(f*x + e)^3*\sinh(f*x + e) + 6*a^2*\cosh(f*x + e)^2*\sinh(f*x + e)^2 \\ & + 4*a^2*\cosh(f*x + e)*\sinh(f*x + e)^3 + a^2*\sinh(f*x + e)^4)*\sqrt{b}*\log((\\ & b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + \\ & 2*a*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a)*\sinh(f*x + e)^2 + \sqrt{2} \\ &)*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1)*\s \\ & \sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e \\ &)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))} + 4*(b*\cosh(f*x + e \\ &)^3 + a*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e \\ &)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + \sqrt{2}*(b^2*\cosh(f*x + e)^6 + 6*b^2* \\ & \cosh(f*x + e)*\sinh(f*x + e)^5 + b^2*\sinh(f*x + e)^6 + (10*a*b - 3*b^2)*\cosh \end{aligned}$$

$$\begin{aligned}
& (f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*a*b - 3*b^2)*sinh(f*x + e)^4 + 4 \\
& *(5*b^2*cosh(f*x + e)^3 + (10*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - \\
& (10*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(10*a*b - 3 \\
& *b^2)*cosh(f*x + e)^2 - 10*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*b^2*co \\
& sh(f*x + e)^5 + 2*(10*a*b - 3*b^2)*cosh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(\\
& f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a \\
& - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/ \\
& (b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b*f*cosh(f*x \\
& + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + b*f*sinh(f*x \\
& + e)^4), -1/64*(12*(a^2*cosh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x \\
& + e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x \\
& + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + \\
& e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b) \\
& *sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x \\
& + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a*b - b^2)*cos \\
& h(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*si \\
& nh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + \\
& e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + \\
& e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + 12*(a^2*cosh(f*x + \\
& e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x \\
& + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(- \\
& b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f \\
& *x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - \\
& b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b \\
& *cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + \\
& 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + \\
& e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) \\
& - sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2* \\
& sinh(f*x + e)^6 + (10*a*b - 3*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^ \\
& 2 + 10*a*b - 3*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b - \\
& 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(f*x + e)^2 + \\
& (15*b^2*cosh(f*x + e)^4 + 6*(10*a*b - 3*b^2)*cosh(f*x + e)^2 - 10*a*b + 3*b \\
& ^2)*sinh(f*x + e)^2 - b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 3*b^2)*c \\
& osh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cos \\
& h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + \\
& e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f* \\
& x + e)^3*sinh(f*x + e) + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh \\
& (f*x + e)*sinh(f*x + e)^3 + b*f*sinh(f*x + e)^4)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \cosh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e), x)`

3.364 $\int \operatorname{sech}(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$

Optimal. Leaf size=125

$$\frac{b \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2f} + \frac{(a - b)^{3/2} \tan^{-1} \left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} \right)}{f} + \frac{\sqrt{b}(3a - 2b) \tanh^{-1} \left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} \right)}{2f}$$

[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*f) + (b*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)

Rubi [A] time = 0.138583, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3190, 416, 523, 217, 206, 377, 203}

$$\frac{b \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2f} + \frac{(a - b)^{3/2} \tan^{-1} \left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} \right)}{f} + \frac{\sqrt{b}(3a - 2b) \tanh^{-1} \left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} \right)}{2f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*f) + (b*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 416

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),

```
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x
_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{1+x^2} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{b \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{2f} + \frac{\operatorname{Subst}\left(\int \frac{a(2a-b)+(3a-2b)bx^2}{(1+x^2)\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{2f} \\
&= \frac{b \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{2f} + \frac{(a-b)^2 \operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{b \sinh(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{2f} + \frac{(a-b)^2 \operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{(a-b)^{3/2} \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} + \frac{(3a-2b)\sqrt{b} \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2f}
\end{aligned}$$

Mathematica [A] time = 0.45291, size = 142, normalized size = 1.14

$$\frac{b \sinh(e+fx) \sqrt{4a+2b \cosh(2(e+fx))-2b} + 4(a-b)^{3/2} \tan^{-1}\left(\frac{\sqrt{2a-2b} \sinh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right) + 2\sqrt{b}(3a-2b) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b} \sinh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right)}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (4*(a - b)^(3/2)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + 2*(3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + b*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]*Sinh[e + f*x]/(4*f)

Maple [C] time = 0.085, size = 63, normalized size = 0.5

$$\frac{1}{f} \operatorname{int}/\operatorname{indef}0 \left(\frac{b^2 (\sinh(fx+e))^4 + 2ab (\sinh(fx+e))^2 + a^2}{(\cosh(fx+e))^2} \frac{1}{\sqrt{a+b (\sinh(fx+e))^2}}, \sinh(fx+e) \right)$$

$$\begin{aligned}
& f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^4 + 3*a*b^2 \\
& - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^2*\sinh(f*x + e)^2 \\
& - \sqrt{2}*((a^2 - 2*a*b + b^2)*\cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*\cosh \\
& (f*x + e)*\sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*\sinh(f*x + e)^6 - 3*(a^2 - \\
& 2*a*b + b^2)*\cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - a \\
& ^2 + 2*a*b - b^2)*\sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^ \\
& 3 - 3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 - (4*a*b - 3*b^2)* \\
& \cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 - 18*(a^2 - 2*a*b \\
& + b^2)*\cosh(f*x + e)^2 - 4*a*b + 3*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 \\
& - 2*a*b + b^2)*\cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - (4 \\
& *a*b - 3*b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 \\
& + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x \\
& + e) + \sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^7 + \\
& 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + \\
& 6*b^3)*\cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e))/ \\
& (\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh \\
& (f*x + e)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh \\
& (f*x + e)^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + 4*((a - \\
& b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f \\
& *x + e)^2)*\sqrt{-a + b}*\log(((a - 2*b)*\cosh(f*x + e)^4 + 4*(a - 2*b)*\cosh(f \\
& *x + e)*\sinh(f*x + e)^3 + (a - 2*b)*\sinh(f*x + e)^4 - 2*(3*a - 2*b)*\cosh(f* \\
& x + e)^2 + 2*(3*(a - 2*b)*\cosh(f*x + e)^2 - 3*a + 2*b)*\sinh(f*x + e)^2 - 2* \\
& \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 \\
& - 1)*\sqrt{-a + b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\c \\
& osh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*((a \\
& - 2*b)*\cosh(f*x + e)^3 - (3*a - 2*b)*\cosh(f*x + e))*\sinh(f*x + e) + a - 2*b \\
&)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2* \\
& (3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + \\
& e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) + ((3*a - 2*b)*\cosh(f*x + e)^2 + \\
& 2*(3*a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e) + (3*a - 2*b)*\sinh(f*x + e)^2)*\s \\
& \sqrt{b}*\log((b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh \\
& (f*x + e)^4 + 2*a*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a)*\sinh(f*x + e \\
&)^2 - \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + \\
& e)^2 + 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\c \\
& osh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(b* \\
& \cosh(f*x + e)^3 + a*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2* \\
& \cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) - \sqrt{2}*(b*\cosh(f*x + e)^ \\
& 2 + 2*b*\cosh(f*x + e)*\sinh(f*x + e) + b*\sinh(f*x + e)^2 - b)*\sqrt{(b*\cosh(f \\
& *x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e) \\
& *\sinh(f*x + e) + \sinh(f*x + e)^2)))/(f*\cosh(f*x + e)^2 + 2*f*\cosh(f*x + e)* \\
& \sinh(f*x + e) + f*\sinh(f*x + e)^2), 1/8*(8*((a - b)*\cosh(f*x + e)^2 + 2*(a \\
& - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2)*\sqrt{a - b}*\arc \\
& \tan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e \\
&)^2 - 1)*\sqrt{a - b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b) \\
& /(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*co
\end{aligned}$$

$$\begin{aligned}
& \text{sh}(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(\\
& 2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^ \\
& 2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) - (\\
& (3*a - 2*b)*\cosh(f*x + e)^2 + 2*(3*a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e) + (\\
& 3*a - 2*b)*\sinh(f*x + e)^2)*\sqrt{b}*\log(-((a^2*b - 2*a*b^2 + b^3)*\cosh(f*x \\
& + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^2*b - \\
& 2*a*b^2 + b^3)*\sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(\\
& f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3 \\
&)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x \\
& + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^ \\
& 5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b \\
& ^3)*\cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a* \\
& b^2 - 2*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^ \\
& 3)*\cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + \\
& (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a \\
& *b^2 - 2*b^3)*\cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e) \\
& ^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 \\
& + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 - \sqrt{2} \\
&)*((a^2 - 2*a*b + b^2)*\cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e) \\
&)*\sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*\sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + \\
& b^2)*\cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - a^2 + 2*a \\
& *b - b^2)*\sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - 3*(a \\
& ^2 - 2*a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x \\
& + e)^2 + (15*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)* \\
& \cosh(f*x + e)^2 - 4*a*b + 3*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b \\
& + b^2)*\cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - (4*a*b - 3 \\
& *b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sin \\
& h(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \\
& \sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^7 + 3*(a^3 - \\
& 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)* \\
& \cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x \\
& + e)^6 + 6*\cosh(f*x + e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e) \\
&)^2 + 20*\cosh(f*x + e)^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e) \\
& ^4 + 6*\cosh(f*x + e)*\sinh(f*x + e)^5 + \sinh(f*x + e)^6)) - ((3*a - 2*b)*\cos \\
& h(f*x + e)^2 + 2*(3*a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e) + (3*a - 2*b)*\sinh \\
& (f*x + e)^2)*\sqrt{b}*\log((b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + \\
& e)^3 + b*\sinh(f*x + e)^4 + 2*a*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + a \\
&)*\sinh(f*x + e)^2 - \sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) \\
&) + \sinh(f*x + e)^2 + 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^ \\
& 2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + \\
& e)^2)) + 4*(b*\cosh(f*x + e)^3 + a*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f \\
& *x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + \sqrt{2}*(b* \\
& \cosh(f*x + e)^2 + 2*b*\cosh(f*x + e)*\sinh(f*x + e) + b*\sinh(f*x + e)^2 - b)* \\
& \sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2 \\
& *\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(f*\cosh(f*x + e)^2 + 2*f*
\end{aligned}$$

$$\begin{aligned} & \cosh(f*x + e)*\sinh(f*x + e) + f*\sinh(f*x + e)^2), -1/8*(2*((3*a - 2*b)*\cosh \\ & (f*x + e)^2 + 2*(3*a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e) + (3*a - 2*b)*\sinh \\ & (f*x + e)^2)*\sqrt{-b}*\arctan(\sqrt{2}*((a - b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh \\ & (f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 + b)*\sqrt{-b}*\sqrt{(b*\cosh \\ & (f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x \\ & + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a*b - b^2)*\cosh(f*x + e)^4 + 4*(a \\ & *b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\sinh(f*x + e)^4 - (3* \\ & a*b - 2*b^2)*\cosh(f*x + e)^2 + (6*(a*b - b^2)*\cosh(f*x + e)^2 - 3*a*b + 2*b \\ & ^2)*\sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*\cosh(f*x + e)^3 - (3*a*b - 2*b \\ & ^2)*\cosh(f*x + e))*\sinh(f*x + e))) + 2*((3*a - 2*b)*\cosh(f*x + e)^2 + 2*(3* \\ & a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e) + (3*a - 2*b)*\sinh(f*x + e)^2)*\sqrt{-b} \\ &)*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f* \\ & x + e)^2 + 1)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - \\ & b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b* \\ & \cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2 \\ & *(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e \\ &)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) + \\ & 4*((a - b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - \\ & b)*\sinh(f*x + e)^2)*\sqrt{-a + b}*\log(((a - 2*b)*\cosh(f*x + e)^4 + 4*(a - 2* \\ & b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a - 2*b)*\sinh(f*x + e)^4 - 2*(3*a - 2*b) \\ &)*\cosh(f*x + e)^2 + 2*(3*(a - 2*b)*\cosh(f*x + e)^2 - 3*a + 2*b)*\sinh(f*x + \\ & e)^2 - 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f* \\ & x + e)^2 - 1)*\sqrt{-a + b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2* \\ & a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) \\ & + 4*((a - 2*b)*\cosh(f*x + e)^3 - (3*a - 2*b)*\cosh(f*x + e))*\sinh(f*x + e) \\ & + a - 2*b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + \\ & e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\cosh \\ & (f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) - \sqrt{2}*(b*\cosh(f*x + \\ & e)^2 + 2*b*\cosh(f*x + e)*\sinh(f*x + e) + b*\sinh(f*x + e)^2 - b)*\sqrt{(b*\cosh \\ & (f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x \\ & + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(f*\cosh(f*x + e)^2 + 2*f*\cosh(f*x + \\ & e)*\sinh(f*x + e) + f*\sinh(f*x + e)^2), 1/8*(8*((a - b)*\cosh(f*x + e)^2 + 2 \\ & *(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2)*\sqrt{a - b} \\ &)*\arctan(\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x \\ & + e)^2 - 1)*\sqrt{a - b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a \\ & - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(\\ & b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + \\ & 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + \\ & e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) \\ & - 2*((3*a - 2*b)*\cosh(f*x + e)^2 + 2*(3*a - 2*b)*\cosh(f*x + e)*\sinh(f*x + \\ & e) + (3*a - 2*b)*\sinh(f*x + e)^2)*\sqrt{-b}*\arctan(\sqrt{2}*((a - b)*\cosh(f*x \\ & + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 + \\ & b)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f \\ & *x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a*b - b^2)* \\ & \cosh(f*x + e)^4 + 4*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2) \end{aligned}$$

```

*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*
x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x
 + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) - 2*((3*a - 2*b)*c
osh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*si
nh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))
*sinh(f*x + e) + b)) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(
f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x
 + e)^2)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{sech}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e), x)

3.365 $\int \operatorname{sech}^3(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=133

$$\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} + \frac{\sqrt{a-b}(a+2b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2f} + \frac{(a-b) \tanh(e+fx) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{2f}$$

[Out] (Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/f + ((a - b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*f)

Rubi [A] time = 0.146103, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3190, 413, 523, 217, 206, 377, 203}

$$\frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} + \frac{\sqrt{a-b}(a+2b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2f} + \frac{(a-b) \tanh(e+fx) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/f + ((a - b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 413

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1))

```
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1+x^2)^2} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{2f} + \frac{\operatorname{Subst}\left(\int \frac{a(a+bx^2)}{(1+x^2)^2} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{2f} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{2f} + \frac{b^2 \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\sqrt{a-b}(a+2b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2f} + \frac{b^{3/2} \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f}
\end{aligned}$$

Mathematica [A] time = 0.727985, size = 150, normalized size = 1.13

$$\frac{4b^{3/2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b} \sinh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right) + 2\sqrt{a-b}(a+2b) \tan^{-1}\left(\frac{\sqrt{2a-2b} \sinh(e+fx)}{\sqrt{2a+b \cosh(2(e+fx))-b}}\right) + (a-b) \tanh(e+fx) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (2*Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + 4*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + (a - b)*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]*Sech[e + f*x]*Tanh[e + f*x])/(4*f)

Maple [C] time = 0.091, size = 63, normalized size = 0.5

$$\frac{1}{f} \operatorname{int}/\operatorname{indef}0 \left(\frac{b^2 (\sinh(fx+e))^4 + 2ab (\sinh(fx+e))^2 + a^2}{(\cosh(fx+e))^4} \frac{1}{\sqrt{a+b (\sinh(fx+e))^2}}, \sinh(fx+e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{sech}(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^3, x)
```

Fricas [B] time = 4.64181, size = 19290, normalized size = 145.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)
```

$$\begin{aligned}
&) * \sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3) * \cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3) * \cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3) * \cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3) * \cosh(f*x + e)^2) * \sinh(f*x + e)^2 + \sqrt{2} * ((a^2 - 2*a*b + b^2) * \cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2) * \cosh(f*x + e) * \sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2) * \sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2) * \cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2) * \cosh(f*x + e)^2 - a^2 + 2*a*b - b^2) * \sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2) * \cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^3 - (4*a*b - 3*b^2) * \cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2) * \cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2) * \cosh(f*x + e)^2 - 4*a*b + 3*b^2) * \sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2) * \cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2) * \cosh(f*x + e)^3 - (4*a*b - 3*b^2) * \cosh(f*x + e)) * \sinh(f*x + e)) * \sqrt{b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3) * \cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3) * \cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3) * \cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3) * \cosh(f*x + e)) * \sinh(f*x + e) / (\cosh(f*x + e)^6 + 6 * \cosh(f*x + e)^5 * \sinh(f*x + e) + 15 * \cosh(f*x + e)^4 * \sinh(f*x + e)^2 + 20 * \cosh(f*x + e)^3 * \sinh(f*x + e)^3 + 15 * \cosh(f*x + e)^2 * \sinh(f*x + e)^4 + 6 * \cosh(f*x + e) * \sinh(f*x + e)^5 + \sinh(f*x + e)^6) + ((a + 2*b) * \cosh(f*x + e)^4 + 4*(a + 2*b) * \cosh(f*x + e) * \sinh(f*x + e)^3 + (a + 2*b) * \sinh(f*x + e)^4 + 2*(a + 2*b) * \cosh(f*x + e)^2 + 2*(3*(a + 2*b) * \cosh(f*x + e)^2 + a + 2*b) * \sinh(f*x + e)^2 + 4*((a + 2*b) * \cosh(f*x + e)^3 + (a + 2*b) * \cosh(f*x + e)) * \sinh(f*x + e) + a + 2*b) * \sqrt{-a + b} * \log(((a - 2*b) * \cosh(f*x + e)^4 + 4*(a - 2*b) * \cosh(f*x + e) * \sinh(f*x + e)^3 + (a - 2*b) * \sinh(f*x + e)^4 - 2*(3*a - 2*b) * \cosh(f*x + e)^2 + 2*(3*(a - 2*b) * \cosh(f*x + e)^2 - 3*a + 2*b) * \sinh(f*x + e)^2 + 2 * \sqrt{2} * (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2 - 1) * \sqrt{-a + b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*((a - 2*b) * \cosh(f*x + e)^3 - (3*a - 2*b) * \cosh(f*x + e)) * \sinh(f*x + e) + a - 2*b) / (\cosh(f*x + e)^4 + 4 * \cosh(f*x + e) * \sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3 * \cosh(f*x + e)^2 + 1) * \sinh(f*x + e)^2 + 2 * \cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e)) * \sinh(f*x + e) + 1)) + (b * \cosh(f*x + e)^4 + 4*b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2*b * \cosh(f*x + e)^2 + 2*(3*b * \cosh(f*x + e)^2 + b) * \sinh(f*x + e)^2 + 4*(b * \cosh(f*x + e)^3 + b * \cosh(f*x + e)) * \sinh(f*x + e) + b) * \sqrt{b} * \log((b * \cosh(f*x + e)^4 + 4*b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2*a * \cosh(f*x + e)^2 + 2*(3*b * \cosh(f*x + e)^2 + a) * \sinh(f*x + e)^2 + \sqrt{2} * (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2 + 1) * \sqrt{b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(b * \cosh(f*x + e)^3 + a * \cosh(f*x + e)) * \sinh(f*x + e) + b) / (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 2 * \sqrt{2} * ((a - b) * \cosh(f*x + e)^2 + 2*(a - b) * \cosh(f*x + e) * \sinh(f*x + e) + (a - b) * \sinh(f*x + e)^2 - a + b) * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) / (f * \cosh(f*x +
\end{aligned}$$

$$\begin{aligned}
& e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 + 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 + f*cosh(f*x + e))*sinh(f*x + e) + f), \\
& 1/4*(2*((a + 2*b)*cosh(f*x + e)^4 + 4*(a + 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + 2*b)*sinh(f*x + e)^4 + 2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e)^2 + a + 2*b)*sinh(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*cosh(f*x + e))*sinh(f*x + e) + a + 2*b)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + (b*cosh(f*x + e)^4 + 4*b*cosh(
\end{aligned}$$

$$\begin{aligned}
& f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*b*\cosh(f*x + e)^2 + 2*(3*b \\
& *cosh(f*x + e)^2 + b)*\sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + \\
& e))*\sinh(f*x + e) + b)*\sqrt{b}*\log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)* \\
& \sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x \\
& + e)^2 + a)*\sinh(f*x + e)^2 + \sqrt{2}*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s \\
& inh(f*x + e) + \sinh(f*x + e)^2 + 1)*\sqrt{b}*\sqrt{(b*cosh(f*x + e)^2 + b*\sin \\
& h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*\sinh(f*x + e) + \\
& \sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*\sinh(f*x + e) + \\
& b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + \\
& 2*\sqrt{2}*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*\sinh(f*x + e) \\
& + (a - b)*\sinh(f*x + e)^2 - a + b)*\sqrt{(b*cosh(f*x + e)^2 + b*\sinh(f*x + e) \\
&)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x \\
& + e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*\sinh(f*x + e)^3 + f*\sinh(f \\
& *x + e)^4 + 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x + e)^2 + f)*\sinh(f*x + e) \\
& ^2 + 4*(f*cosh(f*x + e)^3 + f*cosh(f*x + e))*\sinh(f*x + e) + f), -1/4*(2*(b \\
& *cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + \\
& 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*\sinh(f*x + e)^2 + 4*(b*co \\
& sh(f*x + e)^3 + b*cosh(f*x + e))*\sinh(f*x + e) + b)*\sqrt{-b}*\arctan(\sqrt{2} \\
& *((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*\sinh(f*x + e) + (a - b) \\
& *\sinh(f*x + e)^2 + b)*\sqrt{-b}*\sqrt{(b*cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 \\
& + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e) \\
& ^2)))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*\sinh(f*x + \\
& e)^3 + (a*b - b^2)*\sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(\\
& a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(2*(a \\
& *b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*\sinh(f*x + e)) \\
& + 2*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e) \\
&)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*\sinh(f*x + e)^2 + 4 \\
& *(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*\sinh(f*x + e) + b)*\sqrt{-b}*\arctan(s \\
& \sqrt{2}*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + \\
& 1)*\sqrt{-b}*\sqrt{(b*cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f \\
& *x + e)^2 - 2*cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*cosh(f*x + \\
& e)^4 + 4*b*cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b) \\
& *cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b \\
& *cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*\sinh(f*x + e) + b)) - ((a + 2*b) \\
& *cosh(f*x + e)^4 + 4*(a + 2*b)*cosh(f*x + e)*\sinh(f*x + e)^3 + (a + 2*b)*s \\
& inh(f*x + e)^4 + 2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e) \\
& ^2 + a + 2*b)*\sinh(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*co \\
& sh(f*x + e))*\sinh(f*x + e) + a + 2*b)*\sqrt{-a + b}*\log(((a - 2*b)*cosh(f*x \\
& + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*\sinh(f*x + e)^3 + (a - 2*b)*\sinh(f*x + e) \\
&)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a \\
& + 2*b)*\sinh(f*x + e)^2 + 2*\sqrt{2}*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*\sinh(\\
& f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{-a + b}*\sqrt{(b*cosh(f*x + e)^2 + b*si \\
& nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*\sinh(f*x + e) + \\
& \sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + \\
& e))*\sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*\sinh(f*x +
\end{aligned}$$

$$\begin{aligned}
& e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh \\
& (f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) - 2*s \\
& \text{qrt}(2)*((a - b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (\\
& a - b)*\sinh(f*x + e)^2 - a + b)*\text{sqrt}((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 \\
& + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e \\
&)^2)))/(f*\cosh(f*x + e)^4 + 4*f*\cosh(f*x + e)*\sinh(f*x + e)^3 + f*\sinh(f*x \\
& + e)^4 + 2*f*\cosh(f*x + e)^2 + 2*(3*f*\cosh(f*x + e)^2 + f)*\sinh(f*x + e)^2 \\
& + 4*(f*\cosh(f*x + e)^3 + f*\cosh(f*x + e))*\sinh(f*x + e) + f), 1/2*((a + 2* \\
& b)*\cosh(f*x + e)^4 + 4*(a + 2*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a + 2*b)* \\
& \sinh(f*x + e)^4 + 2*(a + 2*b)*\cosh(f*x + e)^2 + 2*(3*(a + 2*b)*\cosh(f*x + e \\
&)^2 + a + 2*b)*\sinh(f*x + e)^2 + 4*((a + 2*b)*\cosh(f*x + e)^3 + (a + 2*b)*\c \\
& \text{osh}(f*x + e))*\sinh(f*x + e) + a + 2*b)*\text{sqrt}(a - b)*\arctan(\text{sqrt}(2)*(\cosh(f*x \\
& + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\text{sqrt}(a - b)* \\
& \text{sqrt}((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2 \\
& *\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\c \\
& \text{osh}(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e \\
&)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e \\
&)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) - (b*\cosh(f*x + e)^4 + 4 \\
& *b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*b*\cosh(f*x + e)^2 \\
& + 2*(3*b*\cosh(f*x + e)^2 + b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + b*\c \\
& \text{osh}(f*x + e))*\sinh(f*x + e) + b)*\text{sqrt}(-b)*\arctan(\text{sqrt}(2)*((a - b)*\cosh(f*x + \\
& e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 + b \\
&)*\text{sqrt}(-b)*\text{sqrt}((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x \\
& + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a*b - b^2)*\c \\
& \text{osh}(f*x + e)^4 + 4*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\s \\
& \text{inh}(f*x + e)^4 - (3*a*b - 2*b^2)*\cosh(f*x + e)^2 + (6*(a*b - b^2)*\cosh(f*x \\
& + e)^2 - 3*a*b + 2*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*\cosh(f*x + \\
& e)^3 - (3*a*b - 2*b^2)*\cosh(f*x + e))*\sinh(f*x + e))) - (b*\cosh(f*x + e)^4 \\
& + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*b*\cosh(f*x + e \\
&)^2 + 2*(3*b*\cosh(f*x + e)^2 + b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + \\
& b*\cosh(f*x + e))*\sinh(f*x + e) + b)*\text{sqrt}(-b)*\arctan(\text{sqrt}(2)*(\cosh(f*x + e)^ \\
& 2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1)*\text{sqrt}(-b)*\text{sqrt}((b*\c \\
& \text{osh}(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x \\
& + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + \\
& e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(\\
& 3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2* \\
& a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) + \text{sqrt}(2)*((a - b)*\cosh(f*x + e)^ \\
& 2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 - a + b \\
&)*\text{sqrt}((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - \\
& 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(f*\cosh(f*x + e)^4 + 4* \\
& f*\cosh(f*x + e)*\sinh(f*x + e)^3 + f*\sinh(f*x + e)^4 + 2*f*\cosh(f*x + e)^2 + \\
& 2*(3*f*\cosh(f*x + e)^2 + f)*\sinh(f*x + e)^2 + 4*(f*\cosh(f*x + e)^3 + f*\c \\
& \text{osh}(f*x + e))*\sinh(f*x + e) + f)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{sech}(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^3, x)

3.366 $\int \operatorname{sech}^5(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=126

$$\frac{3a^2 \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{8f\sqrt{a-b}} + \frac{\tanh(e+fx) \operatorname{sech}^3(e+fx) (a + b \sinh^2(e+fx))^{3/2}}{4f} + \frac{3a \tanh(e+fx) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8f}$$

[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*Sqrt[a - b]*f) + (3*a*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(8*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(4*f)

Rubi [A] time = 0.120855, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3190, 378, 377, 203}

$$\frac{3a^2 \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{8f\sqrt{a-b}} + \frac{\tanh(e+fx) \operatorname{sech}^3(e+fx) (a + b \sinh^2(e+fx))^{3/2}}{4f} + \frac{3a \tanh(e+fx) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*Sqrt[a - b]*f) + (3*a*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(8*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(4*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 378

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c

q)/(a(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \operatorname{sech}^5(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1+x^2)^3} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\operatorname{sech}^3(e + fx) (a + b \sinh^2(e + fx))^{3/2} \tanh(e + fx)}{4f} + \frac{(3a) \operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^3} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{3a \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{8f} + \frac{\operatorname{sech}^3(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{8f} \\ &= \frac{3a \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{8f} + \frac{\operatorname{sech}^3(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{8f} \\ &= \frac{3a^2 \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{8\sqrt{a-b}f} + \frac{3a \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{8f} \end{aligned}$$

Mathematica [C] time = 0.108615, size = 66, normalized size = 0.52

$$\frac{a^2 \sinh(e + fx) {}_2F_1\left(\frac{1}{2}, 3; \frac{3}{2}; -\frac{(a-b) \sinh^2(e+fx)}{b \sinh^2(e+fx)+a}\right)}{f \sqrt{a + b \sinh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (a^2*Hypergeometric2F1[1/2, 3, 3/2, -(((a - b)*Sinh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2))]*Sinh[e + f*x])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Maple [C] time = 0.107, size = 63, normalized size = 0.5

$$\frac{1}{f} \int \frac{b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^2 + a^2}{(\cosh(fx + e))^6 \sqrt{a + b (\sinh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e)^2 + a)^{\frac{3}{2}} \operatorname{sech}(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^5, x)

Fricas [B] time = 3.80103, size = 7839, normalized size = 62.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/16*(3*(a^2*\cosh(f*x + e)^8 + 8*a^2*\cosh(f*x + e)*\sinh(f*x + e)^7 + a^2* \\ & \sinh(f*x + e)^8 + 4*a^2*\cosh(f*x + e)^6 + 4*(7*a^2*\cosh(f*x + e)^2 + a^2)*\sinh(f*x + e)^6 + 6*a^2*\cosh(f*x + e)^4 + 8*(7*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(35*a^2*\cosh(f*x + e)^4 + 30*a^2*\cosh(f*x + e)^2 + 3*a^2)*\sinh(f*x + e)^4 + 4*a^2*\cosh(f*x + e)^2 + 8*(7*a^2*\cosh(f*x + e)^5 + 10*a^2*\cosh(f*x + e)^3 + 3*a^2*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(7*a^2*\cosh(f*x + e)^6 + 15*a^2*\cosh(f*x + e)^4 + 9*a^2*\cosh(f*x + e)^2 + a^2)*\sinh(f*x + e)^2 + a^2 + 8*(a^2*\cosh(f*x + e)^7 + 3*a^2*\cosh(f*x + e)^5 + 3*a^2*\cosh(f*x + e)^3 + a^2*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-a + b} \\ & \log(((a - 2*b)*\cosh(f*x + e)^4 + 4*(a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a - 2*b)*\sinh(f*x + e)^4 - 2*(3*a - 2*b)*\cosh(f*x + e)^2 + 2*(3*(a - 2*b))*\cosh(f*x + e)^2 - 3*a + 2*b)*\sinh(f*x + e)^2 - 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{-a + b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*((a - 2*b)*\cosh(f*x + e)^3 - (3*a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e) + a - 2*b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) - 2*\sqrt{2}*((3*a^2 - a*b - 2*b^2)*\cosh(f*x + e)^6 + 6*(3*a^2 - a*b - 2*b^2)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (3*a^2 - a*b - 2*b^2)*\sinh(f*x + e)^6 + (11*a^2 - 17*a*b + 6*b^2)*\cosh(f*x + e)^4 + (15*(3*a^2 - a*b - 2*b^2)*\cosh(f*x + e)^2 + 11*a^2 - 17*a*b + 6*b^2)*\sinh(f*x + e)^4 + 4*(5*(3*a^2 - a*b - 2*b^2)*\cosh(f*x + e)^3 + (11*a^2 - 17*a*b + 6*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*\cosh(f*x + e)^2 + (15*(3*a^2 - a*b - 2*b^2)*\cosh(f*x + e)^4 + 6*(11*a^2 - 17*a*b + 6*b^2)*\cosh(f*x + e)^2 - 11*a^2 + 17*a*b - 6*b^2)*\sinh(f*x + e)^2 - 3*a^2 + a*b + 2*b^2 + 2*(3*(3*a^2 - a*b - 2*b^2)*\cosh(f*x + e)^5 + 2*(11*a^2 - 17*a*b + 6*b^2)*\cosh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a - b)*f*\cosh(f*x + e)^8 + 8*(a - b)*f*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a - b)*f*\sinh(f*x + e)^8 + 4*(a - b)*f*\cosh(f*x + e)^6 + 4*(7*(a - b)*f*\cosh(f*x + e)^2 + (a - b)*f)*\sinh(f*x + e)^6 + 6*(a - b)*f*\cosh(f*x + e)^4 + 8*(7*(a - b)*f*\cosh(f*x + e)^3 + 3*(a - b)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(35*(a - b)*f*\cosh(f*x + e)^4 + 30*(a - b)*f*\cosh(f*x + e)^2 + 3*(a - b)*f)*\sinh(f*x + e)^4 + 4*(a - b)*f*\cosh(f*x + e)^2 + 8*(7*(a - b)*f*\cosh(f*x + e)^5 + 10*(a - b)*f*\cosh(f*x + e)^3 + 3*(a - b)*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(7*(a - b)*f*\cosh(f*x + e)^6 + 15*(a - b)*f*\cosh(f*x + e)^4 + 9*(a - b)*f*\cosh(f*x + e)^2 + (a - b)*f)*\sinh(f*x + e)^2 + (a - b)*f + 8*((a - b)*f*\cosh(f*x + e)^7 + 3*(a - b)*f*\cosh(f*x + e)^5 + 3*(a - b)*f*\cosh(f*x + e)^3 + (a - b)*f*\cosh(f*x + e))*\sinh(f*x + e)), 1/8*(3*(a^2*\cosh(f*x + e)^8 + 8*a^2*\cosh(f*x + e)*\sinh(f*x + e)^7 + a^2*\sinh(f*x + e)^8 + 4*a^2*\cosh(f*x + e)^6 + 4*(7*a^2* \end{aligned}$$

$$\begin{aligned} & \cosh(f*x + e)^2 + a^2) * \sinh(f*x + e)^6 + 6*a^2 * \cosh(f*x + e)^4 + 8*(7*a^2 * \cosh(f*x + e)^3 + 3*a^2 * \cosh(f*x + e)) * \sinh(f*x + e)^5 + 2*(35*a^2 * \cosh(f*x + e)^4 + 30*a^2 * \cosh(f*x + e)^2 + 3*a^2) * \sinh(f*x + e)^4 + 4*a^2 * \cosh(f*x + e)^2 + 8*(7*a^2 * \cosh(f*x + e)^5 + 10*a^2 * \cosh(f*x + e)^3 + 3*a^2 * \cosh(f*x + e)) * \sinh(f*x + e)^3 + 4*(7*a^2 * \cosh(f*x + e)^6 + 15*a^2 * \cosh(f*x + e)^4 + 9*a^2 * \cosh(f*x + e)^2 + a^2) * \sinh(f*x + e)^2 + a^2 + 8*(a^2 * \cosh(f*x + e)^7 + 3*a^2 * \cosh(f*x + e)^5 + 3*a^2 * \cosh(f*x + e)^3 + a^2 * \cosh(f*x + e)) * \sinh(f*x + e) * \sqrt{a - b} * \arctan(\sqrt{2} * (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2 - 1) * \sqrt{a - b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)}) / (b * \cosh(f*x + e)^4 + 4*b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2*(2*a - b) * \cosh(f*x + e)^2 + 2*(3*b * \cosh(f*x + e)^2 + 2*a - b) * \sinh(f*x + e)^2 + 4*(b * \cosh(f*x + e)^3 + (2*a - b) * \cosh(f*x + e)) * \sinh(f*x + e) + b)) + \sqrt{2} * ((3*a^2 - a*b - 2*b^2) * \cosh(f*x + e)^6 + 6 * (3*a^2 - a*b - 2*b^2) * \cosh(f*x + e) * \sinh(f*x + e)^5 + (3*a^2 - a*b - 2*b^2) * \sinh(f*x + e)^6 + (11*a^2 - 17*a*b + 6*b^2) * \cosh(f*x + e)^4 + (15*(3*a^2 - a*b - 2*b^2) * \cosh(f*x + e)^2 + 11*a^2 - 17*a*b + 6*b^2) * \sinh(f*x + e)^4 + 4*(5*(3*a^2 - a*b - 2*b^2) * \cosh(f*x + e)^3 + (11*a^2 - 17*a*b + 6*b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2) * \cosh(f*x + e)^2 + (15*(3*a^2 - a*b - 2*b^2) * \cosh(f*x + e)^4 + 6*(11*a^2 - 17*a*b + 6*b^2) * \cosh(f*x + e)^2 - 11*a^2 + 17*a*b - 6*b^2) * \sinh(f*x + e)^2 - 3*a^2 + a*b + 2*b^2 + 2*(3*(3*a^2 - a*b - 2*b^2) * \cosh(f*x + e)^5 + 2*(11*a^2 - 17*a*b + 6*b^2) * \cosh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2) * \cosh(f*x + e)) * \sinh(f*x + e) * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)}) / ((a - b) * f * \cosh(f*x + e)^8 + 8*(a - b) * f * \cosh(f*x + e) * \sinh(f*x + e)^7 + (a - b) * f * \sinh(f*x + e)^8 + 4*(a - b) * f * \cosh(f*x + e)^6 + 4*(7*(a - b) * f * \cosh(f*x + e)^2 + (a - b) * f) * \sinh(f*x + e)^6 + 6*(a - b) * f * \cosh(f*x + e)^4 + 8*(7*(a - b) * f * \cosh(f*x + e)^3 + 3*(a - b) * f * \cosh(f*x + e)) * \sinh(f*x + e)^5 + 2*(35*(a - b) * f * \cosh(f*x + e)^4 + 30*(a - b) * f * \cosh(f*x + e)^2 + 3*(a - b) * f) * \sinh(f*x + e)^4 + 4*(a - b) * f * \cosh(f*x + e)^2 + 8*(7*(a - b) * f * \cosh(f*x + e)^5 + 10*(a - b) * f * \cosh(f*x + e)^3 + 3*(a - b) * f * \cosh(f*x + e)) * \sinh(f*x + e)^3 + 4*(7*(a - b) * f * \cosh(f*x + e)^6 + 15*(a - b) * f * \cosh(f*x + e)^4 + 9*(a - b) * f * \cosh(f*x + e)^2 + (a - b) * f) * \sinh(f*x + e)^2 + (a - b) * f + 8*((a - b) * f * \cosh(f*x + e)^7 + 3*(a - b) * f * \cosh(f*x + e)^5 + 3*(a - b) * f * \cosh(f*x + e)^3 + (a - b) * f * \cosh(f*x + e)) * \sinh(f*x + e))] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{sech}(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^5, x)
```

3.367 $\int \operatorname{sech}^7(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx$

Optimal. Leaf size=205

$$\frac{a^2(5a - 6b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b} \sinh^2(e+fx)}\right)}{16f(a-b)^{3/2}} + \frac{\tanh(e+fx) \operatorname{sech}^5(e+fx) (a + b \sinh^2(e+fx))^{5/2}}{6f(a-b)} + \frac{(5a - 6b) \tanh(e+fx) \operatorname{sech}^7(e+fx)}{24f(a-b)^{3/2}}$$

[Out] (a^2*(5*a - 6*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(16*(a - b)^(3/2)*f) + (a*(5*a - 6*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(16*(a - b)*f) + ((5*a - 6*b)*Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(24*(a - b)*f) + (Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(5/2)*Tanh[e + f*x])/(6*(a - b)*f)

Rubi [A] time = 0.172294, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3190, 382, 378, 377, 203}

$$\frac{a^2(5a - 6b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b} \sinh^2(e+fx)}\right)}{16f(a-b)^{3/2}} + \frac{\tanh(e+fx) \operatorname{sech}^5(e+fx) (a + b \sinh^2(e+fx))^{5/2}}{6f(a-b)} + \frac{(5a - 6b) \tanh(e+fx) \operatorname{sech}^7(e+fx)}{24f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (a^2*(5*a - 6*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(16*(a - b)^(3/2)*f) + (a*(5*a - 6*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(16*(a - b)*f) + ((5*a - 6*b)*Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(24*(a - b)*f) + (Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(5/2)*Tanh[e + f*x])/(6*(a - b)*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 382

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)], I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]

```

Rule 378

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
&& GtQ[q, 0] && NeQ[p, -1]

```

Rule 377

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

```

Rule 203

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])

```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^7(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1+x^2)^4} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\operatorname{sech}^5(e+fx) (a+b \sinh^2(e+fx))^{5/2} \tanh(e+fx)}{6(a-b)f} + \frac{(5a-6b) \operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1+x^2)^4} dx, x, \sinh(e+fx)\right)}{6(a-b)f} \\
&= \frac{(5a-6b) \operatorname{sech}^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} \tanh(e+fx)}{24(a-b)f} + \frac{\operatorname{sech}^5(e+fx) (a+b \sinh^2(e+fx))^{5/2} \tanh(e+fx)}{24(a-b)f} \\
&= \frac{a(5a-6b) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{16(a-b)f} + \frac{(5a-6b) \operatorname{sech}^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} \tanh(e+fx)}{16(a-b)f} \\
&= \frac{a(5a-6b) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{16(a-b)f} + \frac{(5a-6b) \operatorname{sech}^3(e+fx) (a+b \sinh^2(e+fx))^{3/2} \tanh(e+fx)}{16(a-b)f} \\
&= \frac{a^2(5a-6b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{16(a-b)^{3/2}f} + \frac{a(5a-6b) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{16(a-b)f}
\end{aligned}$$

Mathematica [C] time = 15.325, size = 959, normalized size = 4.68

$$a^2 \operatorname{sech}^3(e+fx) \left(\frac{b \sinh^2(e+fx)}{a} + 1\right)^2 \tanh(e+fx) \left(256b {}_2F_1\left(2, 5; \frac{7}{2}; \frac{(a-b) \tanh^2(e+fx)}{a}\right) \sinh^2(e+fx) \sqrt{\frac{\operatorname{sech}^2(e+fx) (b \sinh^2(e+fx))^{3/2}}{a}}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (a^2*Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)^2*Tanh[e + f*x]*(45*a*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2 + 210*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + 140*b*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) - 120*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tan

$$\frac{h[e + f*x]^2}{a} * \text{Sqrt}[(\text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2)) / a] * (((a - b) * \text{Tanh}[e + f*x]^2) / a)^{5/2} - 80 * b * \text{Sinh}[e + f*x]^2 * \text{Sqrt}[(\text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2)) / a] * (((a - b) * \text{Tanh}[e + f*x]^2) / a)^{5/2} + 256 * b * \text{Hypergeometric2F1}[2, 5, 7/2, ((a - b) * \text{Tanh}[e + f*x]^2) / a] * \text{Sinh}[e + f*x]^2 * \text{Sqrt}[(\text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2)) / a] * (((a - b) * \text{Tanh}[e + f*x]^2) / a)^{5/2} - 512 * a * \text{Hypergeometric2F1}[2, 5, 7/2, ((a - b) * \text{Tanh}[e + f*x]^2) / a] * \text{Sqrt}[(\text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2)) / a] * (((a - b) * \text{Tanh}[e + f*x]^2) / a)^{7/2} - 512 * b * \text{Hypergeometric2F1}[2, 5, 7/2, ((a - b) * \text{Tanh}[e + f*x]^2) / a] * \text{Sinh}[e + f*x]^2 * \text{Sqrt}[(\text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2)) / a] * (((a - b) * \text{Tanh}[e + f*x]^2) / a)^{7/2} + 256 * a * \text{Hypergeometric2F1}[2, 5, 7/2, ((a - b) * \text{Tanh}[e + f*x]^2) / a] * \text{Sqrt}[(\text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2)) / a] * (((a - b) * \text{Tanh}[e + f*x]^2) / a)^{9/2} + 256 * b * \text{Hypergeometric2F1}[2, 5, 7/2, ((a - b) * \text{Tanh}[e + f*x]^2) / a] * \text{Sinh}[e + f*x]^2 * \text{Sqrt}[(\text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2)) / a] * (((a - b) * \text{Tanh}[e + f*x]^2) / a)^{9/2} - 45 * a * \text{Sqrt}[(a - b) * \text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2) * \text{Tanh}[e + f*x]^2] / a^2 - 30 * b * \text{Sinh}[e + f*x]^2 * \text{Sqrt}[(a - b) * \text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2) * \text{Tanh}[e + f*x]^2] / a^2) / (240 * f * (a + b * \text{Sinh}[e + f*x]^2)^{3/2} * \text{Sqrt}[(\text{Sech}[e + f*x]^2 * (a + b * \text{Sinh}[e + f*x]^2)) / a] * (((a - b) * \text{Tanh}[e + f*x]^2) / a)^{3/2})$$

Maple [C] time = 0.154, size = 63, normalized size = 0.3

$$\frac{1}{f} \int \frac{b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^2 + a^2}{(\cosh(fx + e))^8} \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] `int/indef0` ((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^8/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e)^2 + a)^{\frac{3}{2}} \text{sech}(fx + e)^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^7, x)

Fricas [B] time = 10.1145, size = 19302, normalized size = 94.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/96*(3*((5*a^3 - 6*a^2*b)*\cosh(f*x + e)^{12} + 12*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)*\sinh(f*x + e)^{11} + (5*a^3 - 6*a^2*b)*\sinh(f*x + e)^{12} + 6*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^{10} + 6*(5*a^3 - 6*a^2*b + 11*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^{10} + 20*(11*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^3 + 3*(5*a^3 - 6*a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^9 + 15*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^8 + 15*(33*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 18*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^8 + 24*(33*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^5 + 30*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^3 + 5*(5*a^3 - 6*a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^7 + 20*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^6 + 4*(231*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^6 + 315*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^4 + 25*a^3 - 30*a^2*b + 105*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 24*(33*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^7 + 63*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^5 + 35*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^3 + 5*(5*a^3 - 6*a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^5 + 15*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^4 + 15*(33*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^8 + 84*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^6 + 70*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 20*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 20*(11*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^9 + 36*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^7 + 42*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^5 + 20*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^3 + 3*(5*a^3 - 6*a^2*b)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 5*a^3 - 6*a^2*b + 6*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^2 + 6*(11*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^{10} + 45*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^8 + 70*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^6 + 50*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 15*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 12*((5*a^3 - 6*a^2*b)*\cosh(f*x + e)^{11} + 5*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^9 + 10*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^7 + 10*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^5 + 5*(5*a^3 - 6*a^2*b)*\cosh(f*x + e)^3 + (5*a^3 - 6*a^2*b)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-a + b}*\log(((a - 2*b)*\cosh(f*x + e)^4 + 4*(a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a - 2*b)*\sinh(f*x + e)^4 - 2*(3*a - 2*b)*\cosh(f*x + e)^2 + 2*(3*(a - 2*b)*\cosh(f*x + e)^2 - 3*a + 2*b)*\sinh(f*x + e)^2 - 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1))*\sqrt{-a + b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e))} \end{aligned}$$

$$\begin{aligned}
& x + e) + \sinh(f*x + e)^2)) + 4*((a - 2*b)*\cosh(f*x + e)^3 - (3*a - 2*b)*\cos \\
& h(f*x + e))*\sinh(f*x + e) + a - 2*b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sin \\
& h(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 \\
& + 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1 \\
&)) - 2*\sqrt{2}*((15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)^10 + 10 \\
& *(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)*\sinh(f*x + e)^9 + (15* \\
& a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*\sinh(f*x + e)^10 + (85*a^3 - 133*a^2*b + \\
& 20*a*b^2 + 28*b^3)*\cosh(f*x + e)^8 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^ \\
& 3 + 45*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e) \\
& ^8 + 8*(15*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)^3 + (85*a^3 \\
& - 133*a^2*b + 20*a*b^2 + 28*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^7 + 2*(99*a^3 \\
& - 247*a^2*b + 200*a*b^2 - 52*b^3)*\cosh(f*x + e)^6 + 2*(105*(15*a^3 - 23*a^ \\
& 2*b + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)^4 + 99*a^3 - 247*a^2*b + 200*a*b^2 - 5 \\
& 2*b^3 + 14*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*\cosh(f*x + e)^2)*\sinh(f \\
& *x + e)^6 + 4*(63*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)^5 + 1 \\
& 4*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*\cosh(f*x + e)^3 + 3*(99*a^3 - 24 \\
& 7*a^2*b + 200*a*b^2 - 52*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 - 2*(99*a^3 - \\
& 247*a^2*b + 200*a*b^2 - 52*b^3)*\cosh(f*x + e)^4 + 2*(105*(15*a^3 - 23*a^2*b \\
& + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)^6 + 35*(85*a^3 - 133*a^2*b + 20*a*b^2 + 2 \\
& 8*b^3)*\cosh(f*x + e)^4 - 99*a^3 + 247*a^2*b - 200*a*b^2 + 52*b^3 + 15*(99*a \\
& ^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 8*(\\
& 15*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)^7 + 7*(85*a^3 - 133* \\
& a^2*b + 20*a*b^2 + 28*b^3)*\cosh(f*x + e)^5 + 5*(99*a^3 - 247*a^2*b + 200*a* \\
& b^2 - 52*b^3)*\cosh(f*x + e)^3 - (99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*\c \\
& osh(f*x + e))*\sinh(f*x + e)^3 - 15*a^3 + 23*a^2*b - 4*a*b^2 - 4*b^3 - (85*a \\
& ^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*\cosh(f*x + e)^2 + (45*(15*a^3 - 23*a^2* \\
& b + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)^8 + 28*(85*a^3 - 133*a^2*b + 20*a*b^2 + \\
& 28*b^3)*\cosh(f*x + e)^6 + 30*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*\cosh \\
& (f*x + e)^4 - 85*a^3 + 133*a^2*b - 20*a*b^2 - 28*b^3 - 12*(99*a^3 - 247*a^2 \\
& *b + 200*a*b^2 - 52*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 2*(5*(15*a^3 - \\
& 23*a^2*b + 4*a*b^2 + 4*b^3)*\cosh(f*x + e)^9 + 4*(85*a^3 - 133*a^2*b + 20*a* \\
& b^2 + 28*b^3)*\cosh(f*x + e)^7 + 6*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3) \\
& *\cosh(f*x + e)^5 - 4*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*\cosh(f*x + e \\
&)^3 - (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*\cosh(f*x + e))*\sinh(f*x + e) \\
&)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - \\
& 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f* \\
& \cosh(f*x + e)^12 + 12*(a^2 - 2*a*b + b^2)*f*\cosh(f*x + e)*\sinh(f*x + e)^11 \\
& + (a^2 - 2*a*b + b^2)*f*\sinh(f*x + e)^12 + 6*(a^2 - 2*a*b + b^2)*f*\cosh(f*x \\
& + e)^10 + 6*(11*(a^2 - 2*a*b + b^2)*f*\cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2 \\
&)*f)*\sinh(f*x + e)^10 + 15*(a^2 - 2*a*b + b^2)*f*\cosh(f*x + e)^8 + 20*(11*(\\
& a^2 - 2*a*b + b^2)*f*\cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*\cosh(f*x + e \\
&))*\sinh(f*x + e)^9 + 15*(33*(a^2 - 2*a*b + b^2)*f*\cosh(f*x + e)^4 + 18*(a^2 \\
& - 2*a*b + b^2)*f*\cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*\sinh(f*x + e)^8 \\
& + 20*(a^2 - 2*a*b + b^2)*f*\cosh(f*x + e)^6 + 24*(33*(a^2 - 2*a*b + b^2)*f*\c \\
& osh(f*x + e)^5 + 30*(a^2 - 2*a*b + b^2)*f*\cosh(f*x + e)^3 + 5*(a^2 - 2*a*b
\end{aligned}$$

$$\begin{aligned}
& + b^2) * f * \cosh(f * x + e)) * \sinh(f * x + e)^7 + 4 * (231 * (a^2 - 2 * a * b + b^2) * f * \cosh \\
& (f * x + e)^6 + 315 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^4 + 105 * (a^2 - 2 * a * b \\
& + b^2) * f * \cosh(f * x + e)^2 + 5 * (a^2 - 2 * a * b + b^2) * f) * \sinh(f * x + e)^6 + 15 * (a \\
& ^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^4 + 24 * (33 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x \\
& + e)^7 + 63 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^5 + 35 * (a^2 - 2 * a * b + b^2) \\
& * f * \cosh(f * x + e)^3 + 5 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)) * \sinh(f * x + e)^5 \\
& + 15 * (33 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^8 + 84 * (a^2 - 2 * a * b + b^2) * f * \\
& \cosh(f * x + e)^6 + 70 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^4 + 20 * (a^2 - 2 * a * \\
& b + b^2) * f * \cosh(f * x + e)^2 + (a^2 - 2 * a * b + b^2) * f) * \sinh(f * x + e)^4 + 6 * (a^2 \\
& - 2 * a * b + b^2) * f * \cosh(f * x + e)^2 + 20 * (11 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x \\
& + e)^9 + 36 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^7 + 42 * (a^2 - 2 * a * b + b^2) * \\
& f * \cosh(f * x + e)^5 + 20 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^3 + 3 * (a^2 - 2 * a \\
& * b + b^2) * f * \cosh(f * x + e)) * \sinh(f * x + e)^3 + 6 * (11 * (a^2 - 2 * a * b + b^2) * f * \cosh \\
& (f * x + e)^10 + 45 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^8 + 70 * (a^2 - 2 * a * b \\
& + b^2) * f * \cosh(f * x + e)^6 + 50 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^4 + 15 * (\\
& a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^2 + (a^2 - 2 * a * b + b^2) * f) * \sinh(f * x + e) \\
& ^2 + (a^2 - 2 * a * b + b^2) * f + 12 * ((a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^11 + 5 \\
& * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^9 + 10 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x \\
& + e)^7 + 10 * (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)^5 + 5 * (a^2 - 2 * a * b + b^2) * f \\
& * \cosh(f * x + e)^3 + (a^2 - 2 * a * b + b^2) * f * \cosh(f * x + e)) * \sinh(f * x + e)), 1/4 \\
& 8 * (3 * ((5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^12 + 12 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + \\
& e) * \sinh(f * x + e)^11 + (5 * a^3 - 6 * a^2 * b) * \sinh(f * x + e)^12 + 6 * (5 * a^3 - 6 * a^2 \\
& * b) * \cosh(f * x + e)^10 + 6 * (5 * a^3 - 6 * a^2 * b + 11 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + \\
& e)^2) * \sinh(f * x + e)^10 + 20 * (11 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^3 + 3 * (5 * a \\
& ^3 - 6 * a^2 * b) * \cosh(f * x + e)) * \sinh(f * x + e)^9 + 15 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * \\
& x + e)^8 + 15 * (33 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^4 + 5 * a^3 - 6 * a^2 * b + 18 * \\
& (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^2) * \sinh(f * x + e)^8 + 24 * (33 * (5 * a^3 - 6 * a^2 * \\
& b) * \cosh(f * x + e)^5 + 30 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^3 + 5 * (5 * a^3 - 6 * a^2 \\
& * b) * \cosh(f * x + e)) * \sinh(f * x + e)^7 + 20 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^6 \\
& + 4 * (231 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^6 + 315 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x \\
& + e)^4 + 25 * a^3 - 30 * a^2 * b + 105 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^2) * \sinh(f * \\
& x + e)^6 + 24 * (33 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^7 + 63 * (5 * a^3 - 6 * a^2 * b) \\
& * \cosh(f * x + e)^5 + 35 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^3 + 5 * (5 * a^3 - 6 * a^2 * \\
& b) * \cosh(f * x + e)) * \sinh(f * x + e)^5 + 15 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^4 + \\
& 15 * (33 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^8 + 84 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + \\
& e)^6 + 70 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^4 + 5 * a^3 - 6 * a^2 * b + 20 * (5 * a^3 - \\
& 6 * a^2 * b) * \cosh(f * x + e)^2) * \sinh(f * x + e)^4 + 20 * (11 * (5 * a^3 - 6 * a^2 * b) * \cosh(\\
& f * x + e)^9 + 36 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^7 + 42 * (5 * a^3 - 6 * a^2 * b) * \cosh \\
& (f * x + e)^5 + 20 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^3 + 3 * (5 * a^3 - 6 * a^2 * b) * \\
& \cosh(f * x + e)) * \sinh(f * x + e)^3 + 5 * a^3 - 6 * a^2 * b + 6 * (5 * a^3 - 6 * a^2 * b) * \cosh \\
& (f * x + e)^2 + 6 * (11 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^10 + 45 * (5 * a^3 - 6 * a^2 * \\
& b) * \cosh(f * x + e)^8 + 70 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^6 + 50 * (5 * a^3 - 6 * a \\
& ^2 * b) * \cosh(f * x + e)^4 + 5 * a^3 - 6 * a^2 * b + 15 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + \\
& e)^2) * \sinh(f * x + e)^2 + 12 * ((5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^11 + 5 * (5 * a^3 - \\
& 6 * a^2 * b) * \cosh(f * x + e)^9 + 10 * (5 * a^3 - 6 * a^2 * b) * \cosh(f * x + e)^7 + 10 * (5 * a^3
\end{aligned}$$

$$\begin{aligned}
& - 6a^2b \cosh(fx + e)^5 + 5(5a^3 - 6a^2b) \cosh(fx + e)^3 + (5a^3 - 6a^2b) \cosh(fx + e) \sinh(fx + e) \sqrt{a - b} \arctan(\sqrt{2} (\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 - 1) \sqrt{a - b}) \\
& \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)} / (b \cosh(fx + e)^4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 + 2(2a - b) \cosh(fx + e)^2 + 2(3b \cosh(fx + e)^2 + 2a - b) \sinh(fx + e)^2 + 4(b \cosh(fx + e)^3 + (2a - b) \cosh(fx + e)) \sinh(fx + e) + b) + \sqrt{2} ((15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e)^{10} + 10(15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e) \sinh(fx + e)^9 + (15a^3 - 23a^2b + 4ab^2 + 4b^3) \sinh(fx + e)^{10} + (85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)^8 + (85a^3 - 133a^2b + 20ab^2 + 28b^3 + 45(15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e)^2) \sinh(fx + e)^8 + 8(15(15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e)^3 + (85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)) \sinh(fx + e)^7 + 2(99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)^6 + 2(105(15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e)^4 + 99a^3 - 247a^2b + 200ab^2 - 52b^3 + 14(85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)^2) \sinh(fx + e)^6 + 4(63(15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e)^5 + 14(85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)^3 + 3(99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)) \sinh(fx + e)^5 - 2(99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)^4 + 2(105(15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e)^6 + 35(85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)^4 - 99a^3 + 247a^2b - 200ab^2 + 52b^3 + 15(99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)^2) \sinh(fx + e)^4 + 8(15(15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e)^7 + 7(85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)^5 + 5(99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)^3 - (99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)) \sinh(fx + e)^3 - 15a^3 + 23a^2b - 4ab^2 - 4b^3 - (85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)^2 + (45(15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e)^8 + 28(85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)^6 + 30(99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)^4 - 85a^3 + 133a^2b - 20ab^2 - 28b^3 - 12(99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)^2) \sinh(fx + e)^2 + 2(5(15a^3 - 23a^2b + 4ab^2 + 4b^3) \cosh(fx + e)^9 + 4(85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)^7 + 6(99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)^5 - 4(99a^3 - 247a^2b + 200ab^2 - 52b^3) \cosh(fx + e)^3 - (85a^3 - 133a^2b + 20ab^2 + 28b^3) \cosh(fx + e)) \sinh(fx + e) \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)} / ((a^2 - 2ab + b^2) f \cosh(fx + e)^{12} + 12(a^2 - 2ab + b^2) f \cosh(fx + e) \sinh(fx + e)^{11} + (a^2 - 2ab + b^2) f \sinh(fx + e)^{12} + 6(a^2 - 2ab + b^2) f \cosh(fx + e)^{10} + 6(11(a^2 - 2ab + b^2) f \cosh(fx + e)^2 + (a^2 - 2ab + b^2) f) \sinh(fx + e)^{10} + 15(a^2 - 2ab + b^2) f \cosh(fx + e)^8 + 20(11(a^2 - 2ab + b^2) f \cosh(fx + e)^3 + 3(a^2 - 2ab + b^2) f \cosh(fx + e)) \sinh(fx + e)^9 + 15(33
\end{aligned}$$

```

*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 18*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^8 + 20*(a^2 - 2*a*b + b^2)*f*
cosh(f*x + e)^6 + 24*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 30*(a^2 -
2*a*b + b^2)*f*cosh(f*x + e)^3 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sin
h(f*x + e)^7 + 4*(231*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 315*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^4 + 105*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 +
5*(a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^6 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*
x + e)^4 + 24*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 63*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^5 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 5*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(33*(a^2 - 2*a*b + b^
2)*f*cosh(f*x + e)^8 + 84*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 70*(a^2 -
2*a*b + b^2)*f*cosh(f*x + e)^4 + 20*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2
+ (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^4 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^2 + 20*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^9 + 36*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^7 + 42*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 20*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*
sinh(f*x + e)^3 + 6*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^10 + 45*(a^2 -
2*a*b + b^2)*f*cosh(f*x + e)^8 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 +
50*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f
*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f
+ 12*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^11 + 5*(a^2 - 2*a*b + b^2)*f*cosh
(f*x + e)^9 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 10*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2
*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{sech}(fx + e)^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^7, x)
```

$$3.368 \quad \int \cosh^4(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=357

$$\frac{(a^2 - 18ab + b^2) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) - 2(a + b)(a^2 - 6ab + b^2) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35bf \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

```
[Out] ((a^2 + 9*a*b - 2*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b*f) + (2*(4*a - b)*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]^5*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(7*f) + (2*(a + b)*(a^2 - 6*a*b + b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 18*a*b + b^2)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*(a^2 - 6*a*b + b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(35*b^2*f)
```

Rubi [A] time = 0.392957, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3192, 416, 528, 531, 418, 492, 411}

$$\frac{2(a + b)(a^2 - 6ab + b^2) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35b^2f} + \frac{(a^2 + 9ab - 2b^2) \sinh(e + fx) \cosh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] ((a^2 + 9*a*b - 2*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b*f) + (2*(4*a - b)*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]^5*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(7*f) + (2*(a + b)*(a^2 - 6*a*b + b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 18*a*b + b^2)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*(a^2 - 6*a*b + b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(35*b^2*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 416

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol]
:= Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((e_.) + (
f_.)*(x_.)^(n_.)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((e_.) + (
f_.)*(x_.)^(n_.)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_.)^2]*Sqrt[(c_.) + (d_.)*(x_.)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_.) + (b_.)*(x_.)^2]*Sqrt[(c_.) + (d_.)*(x_.)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
```

+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
 \int \cosh^4(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int (1 + x^2)^{3/2} (a + bx^2)^{3/2} dx, x, \sinh(e + fx)\right)}{f} \\
 &= \frac{b \cosh^5(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{7f} + \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int (1 + x^2)^{3/2} (a + bx^2)^{3/2} dx, x, \sinh(e + fx)\right)}{f} \\
 &= \frac{2(4a - b) \cosh^3(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35f} + \frac{b \cosh^5(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{7f} \\
 &= \frac{(a^2 + 9ab - 2b^2) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35bf} + \frac{2(4a - b) \cosh^3(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35f} \\
 &= \frac{(a^2 + 9ab - 2b^2) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35bf} + \frac{2(4a - b) \cosh^3(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35f} \\
 &= \frac{(a^2 + 9ab - 2b^2) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35bf} + \frac{2(4a - b) \cosh^3(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35f} \\
 &= \frac{(a^2 + 9ab - 2b^2) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35bf} + \frac{2(4a - b) \cosh^3(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{35f}
 \end{aligned}$$

, (a/b)^(1/2))*b^3)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \cosh(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \cosh(fx + e)^4 \sinh(fx + e)^2 + a \cosh(fx + e)^4\right) \sqrt{b \sinh(fx + e)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral((b*cosh(f*x + e)^4*sinh(f*x + e)^2 + a*cosh(f*x + e)^4)*sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \cosh(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^4, x)
```

$$3.369 \quad \int \cosh^2(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=299

$$\frac{(9a - b)\operatorname{sech}(e + fx)\sqrt{a + b \sinh^2(e + fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) + (3a^2 + 7ab - 2b^2)\tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{15f\sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{(3a^2 + 7ab - 2b^2)\tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{15bf}$$

```
[Out] (2*(3*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f) + (b*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) - ((3*a^2 + 7*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((9*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a^2 + 7*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(15*b*f)
```

Rubi [A] time = 0.290854, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3192, 416, 528, 531, 418, 492, 411}

$$\frac{(3a^2 + 7ab - 2b^2)\tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{15bf} - \frac{(3a^2 + 7ab - 2b^2)\operatorname{sech}(e + fx)\sqrt{a + b \sinh^2(e + fx)}E\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{15bf\sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] (2*(3*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f) + (b*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) - ((3*a^2 + 7*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((9*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a^2 + 7*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(15*b*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

$a*d, 0] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c] \ \&\& \ !\text{SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^{(3/2)}, x_Symbol] \ :> \ \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned} \int \cosh^2(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \sqrt{1 + x^2} (a + bx^2)^{3/2} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{b \cosh^3(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{5f} + \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \sqrt{1 + x^2} (a + bx^2)^{3/2} dx, x, \sinh(e + fx)\right)}{5f} \\ &= \frac{2(3a - b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15f} + \frac{b \cosh^3(e + fx)}{5f} \\ &= \frac{2(3a - b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15f} + \frac{b \cosh^3(e + fx)}{5f} \\ &= \frac{2(3a - b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15f} + \frac{b \cosh^3(e + fx)}{5f} \\ &= \frac{2(3a - b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{15f} + \frac{b \cosh^3(e + fx)}{5f} \end{aligned}$$

Mathematica [C] time = 1.32657, size = 213, normalized size = 0.71

$$\frac{16ia(3a^2 - 2ab - b^2) \sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} \operatorname{EllipticF}\left(i(e + fx), \frac{b}{a}\right) + \sqrt{2}b \sinh(2(e + fx)) (48a^2 + 4b(9a - 2b) \cosh(2(e + fx)))}{240bf \sqrt{2a + b \cosh(2(e + fx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] ((-16*I)*a*(3*a^2 + 7*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*
EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 2*a*b - b^2)*Sqrt[(2*a - b
+ b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48*a^2 -
28*a*b + 5*b^2 + 4*(9*a - 2*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x
)])*Sinh[2*(e + f*x)]/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.12, size = 535, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+(9*(-1/a*b)^(1/2)*a*b-
5*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(6*(-1/a*b)^(1/2)*a^2-8*(-1
/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+6*a^2*(b/a*
cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2), (a/b)^(1/2))-8*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b+2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b
)^(1/2), (a/b)^(1/2))*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^2+7*(b/a*cosh(f
x+e)^2+(a-b)/a)^(1/2)(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2), (a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2)/(-1/a*b)^(1/2
) / cosh(f*x+e) / (a+b*sinh(f*x+e)^2)^(1/2) / f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \cosh(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \cosh(fx + e)^2 \sinh(fx + e)^2 + a \cosh(fx + e)^2\right) \sqrt{b \sinh(fx + e)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral((b*cosh(f*x + e)^2*sinh(f*x + e)^2 + a*cosh(f*x + e)^2)*sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}} \cosh(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^2, x)

$$3.370 \quad \int (a + b \sinh^2(e + fx))^{3/2} dx$$

Optimal. Leaf size=174

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}} + 1\text{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{a+b\sinh^2(e+fx)}}$$

[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.189529, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}} + 1F\left(ie + ifx \left| \frac{b}{a} \right. \right)}{3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}}{3f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3180

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[1/(2*p), Int[(a + b*Sinh[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && GtQ[p, 1]

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]^2], x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Simp[(1*El
lipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{1}{3} \int \frac{a(3a - b) + 2(2a - b)b \sinh^2}{\sqrt{a + b \sinh^2(e + fx)}} \\
&= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{1}{3}(a(a - b)) \int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} \\
&= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{\left(2(2a - b) \sqrt{a + b \sinh^2(e + fx)}\right)}{3 \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}} \\
&= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{2i(2a - b)E\left(ie + ifx \left| \frac{b}{a} \right.\right) \sqrt{a + b \sinh^2(e + fx)}}{3f \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.633349, size = 169, normalized size = 0.97

$$\frac{2i\sqrt{2a(a-b)}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)-4i\sqrt{2a(2a-b)}}{6f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0., size = 416, normalized size = 2.4

$$\frac{1}{3 \cosh(fx + e) f} \left(\sqrt{\frac{b}{a}} b^2 \sinh(fx + e) (\cosh(fx + e))^4 + \left(\sqrt{\frac{b}{a}} ab - \sqrt{\frac{b}{a}} b^2 \right) (\cosh(fx + e))^2 \sinh(fx + e) + 3 a^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out] $\frac{1}{3} \left((-1/ab)^{1/2} b^2 \sinh(fx+e) \cosh(fx+e)^4 + (-1/ab)^{1/2} ab - (-1/ab)^{1/2} b^2 \cosh(fx+e)^2 \sinh(fx+e) + 3a^2 (b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) \right) - 5a (b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + 2(b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + b^2 + 4(b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticE}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + ab - 2(b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticE}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + b^2 \right) / (-1/ab)^{1/2} / \cosh(fx+e) / (a+b \sinh(fx+e)^2)^{1/2} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^(3/2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.371 $\int \operatorname{sech}^2(e + fx) \left(a + b \sinh^2(e + fx)\right)^{3/2} dx$

Optimal. Leaf size=210

$$\frac{b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) - (a - 2b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

```
[Out] ((a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
+ (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sin
h[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a
- 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f + ((a - b)*Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rubi [A] time = 0.201249, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3192, 413, 531, 418, 492, 411}

$$-\frac{(a - 2b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a - b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] ((a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
+ (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sin
h[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a
- 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f + ((a - b)*Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
```

+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 531

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right) \operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1+x^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{(a-b)\sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right)}{f} \\
&= \frac{(a-b)\sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{f} + \frac{\left(ab\sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right)}{f} \\
&= \frac{bF\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right) \operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} - \frac{(a-b)}{f} \\
&= \frac{(a-2b)E\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right) \operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 0.939001, size = 160, normalized size = 0.76

$$\frac{(a-b)\left(\sqrt{2} \tanh(e+fx)(2a+b \cosh(2(e+fx)))-b\right)-2ia\sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right)+2ia(a-2b)\sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}}}{2f\sqrt{2a+b \cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((2*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)]]/a)*EllipticE[I*(e + f*x), b/a] + (a - b)*((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)]]/a)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)]*Tanh[e + f*x]))/(2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.115, size = 334, normalized size = 1.6

$$\frac{1}{f \cosh(fx+e)} \left(\sqrt{-\frac{b}{a}} ab (\sinh(fx+e))^3 - \sqrt{-\frac{b}{a}} b^2 (\sinh(fx+e))^3 + 2a \sqrt{\frac{a+b (\sinh(fx+e))^2}{a}} \sqrt{(\cosh(fx+e))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out] $((-1/a*b)^{(1/2)}*a*b*sinh(f*x+e)^3-(-1/a*b)^{(1/2)}*b^2*sinh(f*x+e)^3+2*a*((a+b*sinh(f*x+e)^2)/a)^{(1/2)}*(cosh(f*x+e)^2)^{(1/2)}*EllipticF(sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b-2*((a+b*sinh(f*x+e)^2)/a)^{(1/2)}*(cosh(f*x+e)^2)^{(1/2)}*EllipticF(sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^2-((a+b*sinh(f*x+e)^2)/a)^{(1/2)}*(cosh(f*x+e)^2)^{(1/2)}*EllipticE(sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a*b+2*((a+b*sinh(f*x+e)^2)/a)^{(1/2)}*(cosh(f*x+e)^2)^{(1/2)}*EllipticE(sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^2+(-1/a*b)^{(1/2)}*a^2*sinh(f*x+e)-(-1/a*b)^{(1/2)}*a*b*sinh(f*x+e))/(-1/a*b)^{(1/2)}/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^{\frac{3}{2}} \operatorname{sech}^2(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \operatorname{sech}^2(fx + e) \sinh^2(fx + e) + a \operatorname{sech}^2(fx + e)\right) \sqrt{b \sinh^2(fx + e) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*sech(f*x + e)^2*sinh(f*x + e)^2 + a*sech(f*x + e)^2)*sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{sech}(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^2, x)

$$3.372 \quad \int \operatorname{sech}^4(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=193

$$\frac{b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{3f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{(a - b) \tanh(e + fx) \operatorname{sech}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f}$$

[Out] (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a - b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)

Rubi [A] time = 0.187938, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3192, 413, 525, 418, 411}

$$\frac{(a - b) \tanh(e + fx) \operatorname{sech}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{b \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} F\left(\tan^{-1}(\sinh(e + fx))\right)}{3f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a - b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^(
3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt
[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^4(e+fx) (a+b \sinh^2(e+fx))^{3/2} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right) \operatorname{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1+x^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{(a-b) \operatorname{sech}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right)}{3f} \\
&= \frac{(a-b) \operatorname{sech}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} \tanh(e+fx)}{3f} - \frac{\left(ab \sqrt{\cosh^2(e+fx)} \operatorname{sech}(e+fx)\right)}{3f} \\
&= \frac{2(a+b) E\left(\tan^{-1}(\sinh(e+fx)) \mid 1 - \frac{b}{a}\right) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3f \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 1.96538, size = 197, normalized size = 1.02

$$\frac{-2ia(2a+b) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \frac{\tanh(e+fx) \operatorname{sech}^2(e+fx) \left((4a^2+6ab-2b^2) \cosh(2(e+fx))+8a^2+b(a+b) \cosh(4(e+fx))\right)}{\sqrt{2}}}{6f \sqrt{2a+b \cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] ((4*I)*a*(a+b)*Sqrt[(2*a-b+b*Cosh[2*(e+f*x)])/a]*EllipticE[I*(e+f*x), b/a] - (2*I)*a*(2*a+b)*Sqrt[(2*a-b+b*Cosh[2*(e+f*x)])/a]*EllipticF[I*(e+f*x), b/a] + ((8*a^2-3*a*b+b^2+(4*a^2+6*a*b-2*b^2)*Cosh[2*(e+f*x)]+b*(a+b)*Cosh[4*(e+f*x)])*Sech[e+f*x]^2*Tanh[e+f*x])/Sqrt[2])/(6*f*Sqrt[2*a-b+b*Cosh[2*(e+f*x)]])

Maple [A] time = 0.116, size = 324, normalized size = 1.7

$$\frac{1}{3 (\cosh(fx+e))^3 f} \left(\left(2 \sqrt{-\frac{b}{a}} ab + 2 \sqrt{-\frac{b}{a}} b^2 \right) \sinh(fx+e) (\cosh(fx+e))^4 + \left(2 \sqrt{-\frac{b}{a}} a^2 + \sqrt{-\frac{b}{a}} ab - 3 \sqrt{-\frac{b}{a}} b^2 \right) (\cosh(fx+e))^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out] $\frac{1}{3} \left((2(-1/ab)^{1/2}ab + 2(-1/ab)^{1/2}b^2) \sinh(fx+e) \cosh(fx+e)^4 + (2(-1/ab)^{1/2}a^2 + (-1/ab)^{1/2}ab - 3(-1/ab)^{1/2}b^2) \cosh(fx+e)^2 \sinh(fx+e) + ((-1/ab)^{1/2}a^2 - 2(-1/ab)^{1/2}ab + (-1/ab)^{1/2}b^2) \sinh(fx+e) + (\cosh(fx+e)^2)^{1/2} (b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} b (a \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + 2b \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2})) - 2 \operatorname{EllipticE}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) \right) a - 2b \operatorname{EllipticE}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) \cosh(fx+e)^2 / \cosh(fx+e)^3 / (-1/ab)^{1/2} / (a+b \sinh(fx+e)^2)^{1/2} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^{\frac{3}{2}} \operatorname{sech}^4(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^4, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\left(b \operatorname{sech}^4(fx + e) \sinh^2(fx + e) + a \operatorname{sech}^4(fx + e) \right) \sqrt{b \sinh^2(fx + e) + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*sech(f*x + e)^4*sinh(f*x + e)^2 + a*sech(f*x + e)^4)*sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \operatorname{sech}(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^4, x)`

$$3.373 \quad \int \frac{\cosh^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=79

$$\frac{\sinh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{2bf} - \frac{(a-2b) \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2b^{3/2}f}$$

[Out] -((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*b^(3/2)*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b*f)

Rubi [A] time = 0.0897847, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3190, 388, 217, 206}

$$\frac{\sinh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{2bf} - \frac{(a-2b) \tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*b^(3/2)*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 388

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
 &= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf} - \frac{(a-2b)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{2bf} \\
 &= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf} - \frac{(a-2b)\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2bf} \\
 &= -\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}
 \end{aligned}$$

Mathematica [A] time = 0.102935, size = 77, normalized size = 0.97

$$\frac{\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2b} - \frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}}}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (-((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*b^(3/2)) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b))/f

Maple [C] time = 0.073, size = 35, normalized size = 0.4

$$\frac{1}{f} \int \frac{\cosh^2(fx + e)}{\sqrt{a + b(\sinh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] `int/indef0` (cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(fx + e)}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(cosh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [B] time = 2.48203, size = 6375, normalized size = 80.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [-1/8*(((a - 2*b)*cosh(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b

$$\begin{aligned}
&^3) \cosh(f*x + e)^2 * \sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3) \cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3) \cosh(f*x + e)) * \sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3) \cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3) \cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3) \cosh(f*x + e)^2) * \sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3) \cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3) \cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3) \cosh(f*x + e)) * \sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3) \cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3) \cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3) \cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3) \cosh(f*x + e)^2) * \sinh(f*x + e)^2 + \sqrt{2}*((a^2 - 2*a*b + b^2) \cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2) \cosh(f*x + e) * \sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2) * \sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^2 - a^2 + 2*a*b - b^2) * \sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2) \cosh(f*x + e)) * \sinh(f*x + e)^3 - (4*a*b - 3*b^2) \cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2) * \cosh(f*x + e)^2 - 4*a*b + 3*b^2) * \sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^3 - (4*a*b - 3*b^2) \cosh(f*x + e)) * \sinh(f*x + e)) * \sqrt{b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3) \cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3) \cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3) * \cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3) \cosh(f*x + e)) * \sinh(f*x + e)) / (\cosh(f*x + e)^6 + 6 * \cosh(f*x + e)^5 * \sinh(f*x + e) + 15 * \cosh(f*x + e)^4 * \sinh(f*x + e)^2 + 20 * \cosh(f*x + e)^3 * \sinh(f*x + e)^3 + 15 * \cosh(f*x + e)^2 * \sinh(f*x + e)^4 + 6 * \cosh(f*x + e) * \sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + ((a - 2*b) * \cosh(f*x + e)^2 + 2*(a - 2*b) * \cosh(f*x + e) * \sinh(f*x + e) + (a - 2*b) * \sinh(f*x + e)^2) * \sqrt{b} * \log((b * \cosh(f*x + e)^4 + 4*b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2*a * \cosh(f*x + e)^2 + 2*(3*b * \cosh(f*x + e)^2 + a) * \sinh(f*x + e)^2 + \sqrt{2} * (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2 + 1) * \sqrt{b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*(b * \cosh(f*x + e)^3 + a * \cosh(f*x + e)) * \sinh(f*x + e) + b) / (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) - \sqrt{2} * (b * \cosh(f*x + e)^2 + 2*b * \cosh(f*x + e) * \sinh(f*x + e) + b * \sinh(f*x + e)^2 - b) * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) / (b^2 * f * \cosh(f*x + e)^2 + 2*b^2 * f * \cosh(f*x + e) * \sinh(f*x + e) + b^2 * f * \sinh(f*x + e)^2), 1/8 * (2 * ((a - 2*b) * \cosh(f*x + e)^2 + 2*(a - 2*b) * \cosh(f*x + e) * \sinh(f*x + e) + (a - 2*b) * \sinh(f*x + e)^2) * \sqrt{-b} * \arctan(\sqrt{2} * ((a - b) * \cosh(f*x + e)^2 + 2*(a - b) * \cosh(f*x + e) * \sinh(f*x + e) + (a - b) * \sinh(f*x + e)^2 + b) * \sqrt{-b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) / ((a*b - b^2) * \cosh(f*x + e)^4 + 4*(a*b - b^2) * \cosh(f*x + e) * \sinh(f*x + e)^3 + (a*b - b^2) * \sinh(f*x + e)^4 - (3*a*b - 2*b^2) * \cosh(f*x + e)^2 + (6*(a*b - b^2) * \cosh(f*x + e)^2 - 3*a*b + 2*
\end{aligned}$$

```

b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*
b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a - 2*b)*cosh(f*x + e)^2 + 2*(a -
2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh(f*x + e)^2)*sqrt(-b)*arc
tan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a
- b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 +
4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt
(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^
2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x +
e)^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(fx + e)}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(cosh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.374 \quad \int \frac{\cosh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)

Rubi [A] time = 0.0465059, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3190, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 217

Int[1/Sqrt[(a_.) + (b_.)*(x_.)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_.) + (b_.)*(x_.)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cosh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f} \\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{\sqrt{b}f} \end{aligned}$$

Mathematica [A] time = 0.0158504, size = 38, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)

Maple [A] time = 0.013, size = 34, normalized size = 0.9

$$\frac{1}{f} \ln\left(\sinh(fx + e) \sqrt{b} + \sqrt{a + b(\sinh(fx + e))^2}\right) \frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2), x)

[Out] $1/f*\ln(\sinh(f*x+e)*b^{(1/2)}+(a+b*\sinh(f*x+e)^2)^{(1/2)})/b^{(1/2)}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(cosh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [B] time = 2.10497, size = 5052, normalized size = 132.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] $[1/4*(\sqrt{b})*\log(-((a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*\sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*\cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + \sqrt{2}*((a^2 - 2*a*b + b^2)*\cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*\sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*\sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*\cosh$

$$\begin{aligned}
& (f*x + e))*\sinh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x + e)^2 + (15*(a^2 - 2 \\
& *a*b + b^2)*\cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^2 - 4*a* \\
& b + 3*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^5 \\
& - 6*(a^2 - 2*a*b + b^2)*\cosh(f*x + e)^3 - (4*a*b - 3*b^2)*\cosh(f*x + e))*s \\
& \sinh(f*x + e))*\sqrt{b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b \\
&)}/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)) + 4* \\
& (2*(a^2*b - 2*a*b^2 + b^3)*\cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2 \\
& *b^3)*\cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*\cosh(f*x + e)^3 + (3*a \\
& *b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e))/(\cosh(f*x + e)^6 + 6*\cosh(f*x + \\
& e)^5*\sinh(f*x + e) + 15*\cosh(f*x + e)^4*\sinh(f*x + e)^2 + 20*\cosh(f*x + e) \\
& ^3*\sinh(f*x + e)^3 + 15*\cosh(f*x + e)^2*\sinh(f*x + e)^4 + 6*\cosh(f*x + e)*s \\
& \sinh(f*x + e)^5 + \sinh(f*x + e)^6)) + \sqrt{b}*\log((b*\cosh(f*x + e)^4 + 4*b*c \\
& osh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*a*\cosh(f*x + e)^2 + 2* \\
& (3*b*\cosh(f*x + e)^2 + a)*\sinh(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2*c \\
& osh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1)*\sqrt{b}*\sqrt{(b*\cosh(f*x + \\
& e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)}/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sin \\
& h(f*x + e) + \sinh(f*x + e)^2)) + 4*(b*\cosh(f*x + e)^3 + a*\cosh(f*x + e))*\si \\
& nh(f*x + e) + b)/(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f* \\
& x + e)^2)))/(b*f), -1/2*(\sqrt{-b}*\arctan(\sqrt{2}*((a - b)*\cosh(f*x + e)^2 + \\
& 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 + b)*\sqrt{ \\
& -b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)}/(\cosh(f*x + e)^2 \\
& - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a*b - b^2)*\cosh(f*x \\
& + e)^4 + 4*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\sinh(f*x \\
& + e)^4 - (3*a*b - 2*b^2)*\cosh(f*x + e)^2 + (6*(a*b - b^2)*\cosh(f*x + e)^2 \\
& - 3*a*b + 2*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*\cosh(f*x + e)^3 - \\
& (3*a*b - 2*b^2)*\cosh(f*x + e))*\sinh(f*x + e))) + \sqrt{-b}*\arctan(\sqrt{2}*(\\
& \cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 + 1)*\sqrt{ \\
& -b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)}/(\cosh(f*x + e)^ \\
& 2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + \\
& 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f* \\
& x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f* \\
& x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)))/(b*f)]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(cosh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.375 \quad \int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b]*f)

Rubi [A] time = 0.0688678, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3190, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b]*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 377

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)/((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\ &= \frac{\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2} dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{a-b}f} \end{aligned}$$

Mathematica [A] time = 0.0307613, size = 46, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b]*f)

Maple [C] time = 0.079, size = 35, normalized size = 0.8

$$\frac{1}{f} \operatorname{int}/\operatorname{indef}0 \left(\frac{1}{(\cosh(fx+e))^2} \frac{1}{\sqrt{a+b(\sinh(fx+e))^2}}, \sinh(fx+e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx + e)}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 1.87393, size = 1615, normalized size = 35.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1))/((a - b)*f), arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*cosh(f*x + e)^4 + 4*b*c
```

```
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))/(sqrt(a - b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [B] time = 1.43875, size = 113, normalized size = 2.46

$$\frac{2 \arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)} - \sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b + \sqrt{b}}}{2\sqrt{a-b}}\right)}{\sqrt{a-bf}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/sqrt(a - b)*f)
```

$$3.376 \quad \int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=97

$$\frac{(a-2b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2f(a-b)^{3/2}} + \frac{\tanh(e+fx) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{2f(a-b)}$$

[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*(a - b)^(3/2)*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*(a - b)*f)

Rubi [A] time = 0.107228, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3190, 382, 377, 203}

$$\frac{(a-2b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2f(a-b)^{3/2}} + \frac{\tanh(e+fx) \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{2f(a-b)}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*(a - b)^(3/2)*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*(a - b)*f)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 382

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -

$a*d)), x] + \text{Dist}[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p + 1)*(c + d*x^n)^q}, x], x] /; \text{FreeQ}\{a, b, c, d, n, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*(p + q + 2) + 1, 0] \&\& (\text{LtQ}[p, -1] \|\| !\text{LtQ}[q, -1]) \&\& \text{NeQ}[p, -1]$

Rule 377

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}/((c_ + (d_)*(x_)^{(n_)}), x_Symbol] :> \text{Subst}[\text{Int}[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^{(1/n)}] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*p + 1, 0] \&\& \text{IntegerQ}[n]$

Rule 203

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] :> \text{Simp}[(1*\text{ArcTan}[\text{Rt}[b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a/b] \&\& (\text{GtQ}[a, 0] \|\| \text{GtQ}[b, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\text{sech}^3(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2 \sqrt{a+bx^2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\text{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{2(a - b)f} + \frac{(a - 2b) \text{Subst}\left(\int \frac{1}{(1+x^2) \sqrt{a+bx^2}} dx, x, \frac{\tanh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}}\right)}{2(a - b)f} \\ &= \frac{\text{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{2(a - b)f} + \frac{(a - 2b) \text{Subst}\left(\int \frac{1}{1 - (-a+b)x^2} dx, x, \frac{\tanh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}}\right)}{2(a - b)f} \\ &= \frac{(a - 2b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2(a - b)^{3/2}f} + \frac{\text{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{2(a - b)f} \end{aligned}$$

Mathematica [C] time = 9.47891, size = 443, normalized size = 4.57

$$\tanh(e + fx) \text{sech}^3(e + fx) \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right) \left(-30b \sinh^2(e + fx) \sqrt{\frac{\tanh^2(e + fx) \text{sech}^2(e + fx) (a^2 + ab(\sinh^2(e + fx) - 1) - b^2 \sinh^2(e + fx))}{a^2}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] (Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)*Tanh[e + f*x]*(45*a*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2 + 16*a*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) + 16*b*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) - 45*a*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2] - 30*b*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2])/(30*a*f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))

Maple [C] time = 0.096, size = 35, normalized size = 0.4

$$\frac{1}{f} \int \frac{1}{(\cosh(fx + e))^4 \sqrt{a + b(\sinh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] `int/indef0` (1/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx + e)^3}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sech(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [B] time = 2.36256, size = 4010, normalized size = 41.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/4 * (((a - 2*b) * \cosh(f*x + e)^4 + 4*(a - 2*b) * \cosh(f*x + e) * \sinh(f*x + e) \\ & ^3 + (a - 2*b) * \sinh(f*x + e)^4 + 2*(a - 2*b) * \cosh(f*x + e)^2 + 2*(3*(a - 2*b) \\ & * \cosh(f*x + e)^2 + a - 2*b) * \sinh(f*x + e)^2 + 4*((a - 2*b) * \cosh(f*x + e)^3 \\ & + (a - 2*b) * \cosh(f*x + e)) * \sinh(f*x + e) + a - 2*b) * \sqrt{-a + b} * \log(((a - 2*b) \\ & * \cosh(f*x + e)^4 + 4*(a - 2*b) * \cosh(f*x + e) * \sinh(f*x + e)^3 + (a - 2*b) \\ & * \sinh(f*x + e)^4 - 2*(3*a - 2*b) * \cosh(f*x + e)^2 + 2*(3*(a - 2*b) * \cosh(f \\ & * x + e)^2 - 3*a + 2*b) * \sinh(f*x + e)^2 - 2*\sqrt{2} * (\cosh(f*x + e)^2 + 2*\cos \\ & h(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2 - 1) * \sqrt{-a + b} * \sqrt{(b * \cosh(f \\ & * x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) \\ & * \sinh(f*x + e) + \sinh(f*x + e)^2)) + 4*((a - 2*b) * \cosh(f*x + e)^3 - (3*a - \\ & 2*b) * \cosh(f*x + e)) * \sinh(f*x + e) + a - 2*b) / (\cosh(f*x + e)^4 + 4 * \cosh(f*x \\ & + e) * \sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3 * \cosh(f*x + e)^2 + 1) * \sinh(f*x \\ & + e)^2 + 2 * \cosh(f*x + e)^2 + 4 * (\cosh(f*x + e)^3 + \cosh(f*x + e)) * \sinh(f*x \\ & + e) + 1)) - 2 * \sqrt{2} * ((a - b) * \cosh(f*x + e)^2 + 2 * (a - b) * \cosh(f*x + e) * \s \\ & inh(f*x + e) + (a - b) * \sinh(f*x + e)^2 - a + b) * \sqrt{(b * \cosh(f*x + e)^2 + b \\ & * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e \\ &) + \sinh(f*x + e)^2)) / ((a^2 - 2*a*b + b^2) * f * \cosh(f*x + e)^4 + 4 * (a^2 - 2* \\ & a*b + b^2) * f * \cosh(f*x + e) * \sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2) * f * \sinh(f*x \\ & + e)^4 + 2 * (a^2 - 2*a*b + b^2) * f * \cosh(f*x + e)^2 + 2 * (3 * (a^2 - 2*a*b + b^2) \\ &) * f * \cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2) * f) * \sinh(f*x + e)^2 + (a^2 - 2*a*b \\ & + b^2) * f + 4 * ((a^2 - 2*a*b + b^2) * f * \cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2) * \\ & f * \cosh(f*x + e)) * \sinh(f*x + e)), 1/2 * (((a - 2*b) * \cosh(f*x + e)^4 + 4*(a - 2 \\ & * b) * \cosh(f*x + e) * \sinh(f*x + e)^3 + (a - 2*b) * \sinh(f*x + e)^4 + 2*(a - 2*b) \\ & * \cosh(f*x + e)^2 + 2*(3*(a - 2*b) * \cosh(f*x + e)^2 + a - 2*b) * \sinh(f*x + e)^ \\ & 2 + 4*((a - 2*b) * \cosh(f*x + e)^3 + (a - 2*b) * \cosh(f*x + e)) * \sinh(f*x + e) + \\ & a - 2*b) * \sqrt{a - b} * \arctan(\sqrt{2} * (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sin \\ & h(f*x + e) + \sinh(f*x + e)^2 - 1) * \sqrt{a - b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \s \\ & inh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) \\ & + \sinh(f*x + e)^2)) / (b * \cosh(f*x + e)^4 + 4 * b * \cosh(f*x + e) * \sinh(f*x + e)^3 \\ & + b * \sinh(f*x + e)^4 + 2 * (2*a - b) * \cosh(f*x + e)^2 + 2 * (3 * b * \cosh(f*x + e)^2 \\ & + 2*a - b) * \sinh(f*x + e)^2 + 4 * (b * \cosh(f*x + e)^3 + (2*a - b) * \cosh(f*x + e) \\ &) * \sinh(f*x + e) + b)) + \sqrt{2} * ((a - b) * \cosh(f*x + e)^2 + 2 * (a - b) * \cosh(f \end{aligned}$$

```
*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f
*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^
2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*
b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [B] time = 2.01015, size = 976, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] f*((a - 2*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/((
a*f^2 - b*f^2)*sqrt(a - b)) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a - 2*(sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))^3*b - 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) + 2*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^2*b^(3/2) - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2 - (sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
```


$$\begin{aligned}
& *e) + b)) * a * b + 2 * (\text{sqrt}(b) * e^{(2*f*x + 2*e)} - \text{sqrt}(b * e^{(4*f*x + 4*e)} + 4*a * e \\
& ^{(2*f*x + 2*e)} - 2*b * e^{(2*f*x + 2*e)} + b)) * b^2 - 4*a^2 * \text{sqrt}(b) + 5*a * b^{(3/2)} \\
&) - 2*b^{(5/2)}) / ((a*f^2 - b*f^2) * ((\text{sqrt}(b) * e^{(2*f*x + 2*e)} - \text{sqrt}(b * e^{(4*f*x \\
& + 4*e)} + 4*a * e^{(2*f*x + 2*e)} - 2*b * e^{(2*f*x + 2*e)} + b))^2 + 2 * (\text{sqrt}(b) * e^{ \\
& (2*f*x + 2*e)} - \text{sqrt}(b * e^{(4*f*x + 4*e)} + 4*a * e^{(2*f*x + 2*e)} - 2*b * e^{(2*f*x \\
& + 2*e)} + b)) * \text{sqrt}(b) + 4*a - 3*b)^2))
\end{aligned}$$

$$3.377 \quad \int \frac{\cosh^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=241

$$\frac{(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3abf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} - \frac{2(a-2b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3b^2f}$$

[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f)

Rubi [A] time = 0.218549, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3192, 416, 531, 418, 492, 411}

$$\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3b^2f} + \frac{2(a-2b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))\right)}{3b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f)

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt
[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{3bf} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(2(a-2b)\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{3bf} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{(a-3b)F\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3abf\sqrt{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{2(a-2b)E\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3b^2f\sqrt{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}}
\end{aligned}$$

Mathematica [C] time = 0.821807, size = 179, normalized size = 0.74

$$\frac{-2i\sqrt{2}(2a^2 - 5ab + 3b^2)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b) + 4b^2\sqrt{4a+2b\cosh(2(e+fx))-2b}}{6b^2f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((4*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)]*Sinh[2*(e + f*x)])/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.109, size = 344, normalized size = 1.4

$$\frac{1}{3b \cosh(fx + e) f} \left(\sqrt{\frac{b}{a}} \sinh(fx + e) (\cosh(fx + e))^4 + \left(\sqrt{\frac{b}{a}} a - \sqrt{\frac{b}{a}} b \right) (\cosh(fx + e))^2 \sinh(fx + e) - 2 \sqrt{\frac{b}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] $\frac{1}{3} \left((-1/a*b)^{(1/2)} * b * \sinh(f*x+e) * \cosh(f*x+e)^4 + ((-1/a*b)^{(1/2)} * a - (-1/a*b)^{(1/2)} * b) * \cosh(f*x+e)^2 * \sinh(f*x+e) - 2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a + 4 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b + a * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) - (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b \right) / b / (-1/a*b)^{(1/2)} / \cosh(f*x+e) / (a+b*sinh(f*x+e)^2)^{(1/2)} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\cosh(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

$$3.378 \quad \int \frac{\cosh^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=177

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{bf} - \frac{\operatorname{sech}(e+fx)}{bf}$$

[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(b*f)

Rubi [A] time = 0.14809, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3192, 422, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{bf} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} - \frac{\operatorname{sech}(e+fx)}{bf}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(b*f)

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x]

&& IntegerQ[m/2] && !IntegerQ[p]

Rule 422

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a]

Rule 418

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right)}{f} \\
&= \frac{F\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right) \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{\sqrt{a+b\sinh^2(e+fx)}}{bf} \\
&= -\frac{E\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right) \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{F\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{bf}
\end{aligned}$$

Mathematica [C] time = 0.24074, size = 95, normalized size = 0.54

$$-\frac{i\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\left((b-a)\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + aE\left(i(e+fx)\left|\frac{b}{a}\right.\right)\right)}{bf\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*(a*EllipticE[I*(e + f*x), b/a] + (-a + b)*EllipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.082, size = 86, normalized size = 0.5

$$\frac{1}{f\cosh(fx+e)}\sqrt{\frac{a+b(\sinh(fx+e))^2}{a}}\sqrt{(\cosh(fx+e))^2}\operatorname{EllipticE}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}\sqrt{a+b(\sinh(fx+e))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] $((a+b\sinh(fx+e)^2)/a)^{1/2}*(\cosh(fx+e)^2)^{1/2}*\text{EllipticE}(\sinh(fx+e))*(-1/a*b)^{1/2},(a/b)^{1/2})/(-1/a*b)^{1/2}/\cosh(fx+e)/(a+b\sinh(fx+e)^2)^{1/2}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^2}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.379 \quad \int \frac{1}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=60

$$\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1 \operatorname{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{f\sqrt{a+b \sinh^2(e+fx)}}$$

[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.0383432, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3183, 3182}

$$\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1F\left(ie + ifx \left| \frac{b}{a} \right. \right)}{f\sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3183

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(1*EllipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx = \frac{\sqrt{1 + \frac{b \sinh^2(e+fx)}{a}} \int \frac{1}{\sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}} dx}{\sqrt{a + b \sinh^2(e + fx)}}$$

$$= -\frac{iF\left(ie + ifx \left| \frac{b}{a} \right. \right) \sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}}{f \sqrt{a + b \sinh^2(e + fx)}}$$

Mathematica [A] time = 0.0690047, size = 68, normalized size = 1.13

$$\frac{i \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \text{EllipticF}\left(i(e+fx), \frac{b}{a}\right)}{f \sqrt{2a+b \cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/ (f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0., size = 86, normalized size = 1.4

$$\frac{1}{\cosh(fx+e) f} \sqrt{\frac{a+b(\sinh(fx+e))^2}{a}} \sqrt{(\cosh(fx+e))^2} \text{EllipticF}\left(\sinh(fx+e) \sqrt{\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) \frac{1}{\sqrt{\frac{b}{a}}} \frac{1}{\sqrt{a+b(\sinh(fx+e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{1}{\sqrt{b \sinh^2(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

$$3.380 \quad \int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=160

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} - \frac{b\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/((a*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])

Rubi [A] time = 0.180525, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3192, 414, 21, 422, 418, 492, 411}

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} - \frac{b\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/((a*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x]

&& IntegerQ[m/2] && !IntegerQ[p]

Rule 414

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(!IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 21

Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplrQ[c + d*x, a + b*x])

Rule 422

Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a]

Rule 418

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ

[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
 \int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
 &= \frac{\sqrt{a+b\sinh^2(e+fx)} \tanh(e+fx)}{(a-b)f} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{b+bx^2}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
 &= \frac{\sqrt{a+b\sinh^2(e+fx)} \tanh(e+fx)}{(a-b)f} + \frac{\left(b\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
 &= \frac{\sqrt{a+b\sinh^2(e+fx)} \tanh(e+fx)}{(a-b)f} + \frac{\left(b\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
 &= -\frac{bF\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right) \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{a(a-b)f} \\
 &= \frac{E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right) \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{bF\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right) \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f}
 \end{aligned}$$

Mathematica [C] time = 0.622135, size = 159, normalized size = 0.99

$$\frac{-2i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2} \tanh(e+fx)(2a+b\cosh(2(e+fx))-b) + 2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}{2f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)]*Tanh[e + f*x])/(2*(a -

b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])

Maple [A] time = 0.138, size = 133, normalized size = 0.8

$$-\frac{1}{(a-b)\cosh(fx+e)f}\left(-\sqrt{\frac{b}{a}}b(\sinh(fx+e))^3+b\sqrt{\frac{a+b(\sinh(fx+e))^2}{a}}\sqrt{(\cosh(fx+e))^2}\operatorname{EllipticE}\left(\sinh(fx+e)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] $-\left(-\frac{1}{a*b}\right)^{1/2}*b*\sinh(f*x+e)^3+b*\left(\frac{a+b*\sinh(f*x+e)^2}{a}\right)^{1/2}*(\cosh(f*x+e)^2)^{1/2}*EllipticE(\sinh(f*x+e)*\left(-\frac{1}{a*b}\right)^{1/2},\left(\frac{a}{b}\right)^{1/2})-\left(-\frac{1}{a*b}\right)^{1/2}*a*\sinh(f*x+e)/(a-b)/\left(-\frac{1}{a*b}\right)^{1/2}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{1/2}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx + e)^2}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

$$3.381 \quad \int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=219

$$\frac{b(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right) + \frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3f(a-b)}}{3af(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

[Out] (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*f)

Rubi [A] time = 0.194929, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3192, 414, 525, 418, 411}

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3f(a-b)} - \frac{b(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))\right)}{3af(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*f)

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[

$\text{Cos}[e + f*x]^2]/(f*\text{Cos}[e + f*x]), \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*ff^2*x^2)^p, x], x, \text{Sin}[e + f*x]/ff], x] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[m/2] \&\& \text{!IntegerQ}[p]$

Rule 414

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \rightarrow -\text{Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)}]/(a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& \text{!(IntegerQ}[p] \&\& \text{IntegerQ}[q] \&\& \text{LtQ}[q, -1]) \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 525

$\text{Int}[(e_ + (f_)*(x_)^2)/(\text{Sqrt}[(a_ + (b_)*(x_)^2)*((c_ + (d_)*(x_)^2)^{(3/2)}), x_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(\text{Sqrt}[a + b*x^2]*\text{Sqrt}[c + d*x^2]), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[\text{Sqrt}[a + b*x^2]/(c + d*x^2)^{(3/2)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rule 418

$\text{Int}[1/(\text{Sqrt}[(a_ + (b_)*(x_)^2)*\text{Sqrt}[(c_ + (d_)*(x_)^2]), x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[d/c] \&\& \text{PosQ}[b/a] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_ + (b_)*(x_)^2)/((c_ + (d_)*(x_)^2)^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{(1+x^2)^{5/2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{(1+x^2)^{5/2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{3(a-b)f} \\
&= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} + \frac{\left(2(a-2b)\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{(1+x^2)^{5/2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{3(a-b)f} \\
&= \frac{2(a-2b)E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{(a-3b)bF\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 2.1519, size = 219, normalized size = 1.

$$\frac{-2i(2a^2 - 5ab + 3b^2)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)((4a^2-6ab-2b^2)\cosh(2(e+fx))+8a^2+b(a-b))}{\sqrt{2}}}{6f(a-b)^2\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((4*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + ((8*a^2 - 15*a*b + 4*b^2 + (4*a^2 - 6*a*b - 2*b^2)*Cosh[2*(e + f*x)] + (a - 2*b)*b*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e + f*x]/Sqrt[2])/(6*(a - b)^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.22, size = 343, normalized size = 1.6

$$\frac{1}{3(\cosh(fx+e))^3(a^2-2ab+b^2)f}\sqrt{(a+b(\sinh(fx+e))^2)(\cosh(fx+e))^2}\left(2(\cosh(fx+e))^4\sqrt{-\frac{b}{a}}b(a-2b)\operatorname{sech}(e+fx)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] $\frac{1}{3} \left(\frac{(a+b \sinh(fx+e))^2 \cosh(fx+e)^2}{\cosh(fx+e)^3} \left(-\frac{1}{a} \right)^{1/2} / \left(b \cosh(fx+e)^4 + (a-b) \cosh(fx+e)^2 \right)^{1/2} / (a^2 - 2ab + b^2) \right. \\ \left. + 2 \cosh(fx+e)^4 \left(-\frac{1}{a} \right)^{1/2} + b(a-2b) \sinh(fx+e) + \cosh(fx+e)^2 \left(-\frac{1}{a} \right)^{1/2} + (2a^2 - 5ab + 3b^2) \sinh(fx+e) + \left(-\frac{1}{a} \right)^{1/2} (a^2 - 2ab + b^2) \sinh(fx+e) + \frac{b}{a} \cosh(fx+e)^2 + \frac{(a-b)}{a} \right)^{1/2} \\ \left(\cosh(fx+e)^2 \right)^{1/2} + b \operatorname{EllipticF}(\sinh(fx+e), \left(-\frac{1}{a} \right)^{1/2}, (a/b)^{1/2}) - b \operatorname{EllipticF}(\sinh(fx+e), \left(-\frac{1}{a} \right)^{1/2}, (a/b)^{1/2}) - 2 \operatorname{EllipticE}(\sinh(fx+e), \left(-\frac{1}{a} \right)^{1/2}, (a/b)^{1/2}) + a + 4b \operatorname{EllipticE}(\sinh(fx+e), \left(-\frac{1}{a} \right)^{1/2}, (a/b)^{1/2}) \right) \cosh(fx+e)^2 / (a+b \sinh(fx+e)^2)^{1/2} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx+e)^4}{\sqrt{b \sinh(fx+e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{\operatorname{sech}(fx+e)^4}{\sqrt{b \sinh(fx+e)^2 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^4(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(sech(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.382 \quad \int \frac{\cosh^3(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{b^{3/2} f} - \frac{(a-b) \sinh(e+fx)}{abf \sqrt{a+b \sinh^2(e+fx)}}$$

[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(3/2)*f) - ((a - b)*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.0985304, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3190, 385, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{b^{3/2} f} - \frac{(a-b) \sinh(e+fx)}{abf \sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(3/2)*f) - ((a - b)*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b

c(n*(p + 1) + 1)/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^3(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^{3/2}} dx, x, \sinh(e + fx)\right)}{f} \\
 &= -\frac{(a-b) \sinh(e + fx)}{abf \sqrt{a + b \sinh^2(e + fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \sinh(e + fx)\right)}{bf} \\
 &= -\frac{(a-b) \sinh(e + fx)}{abf \sqrt{a + b \sinh^2(e + fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{bf} \\
 &= \frac{\tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{b^{3/2} f} - \frac{(a-b) \sinh(e + fx)}{abf \sqrt{a + b \sinh^2(e + fx)}}
 \end{aligned}$$

Mathematica [A] time = 0.169992, size = 89, normalized size = 1.16

$$\frac{a^{3/2} \sinh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a}}\right) \sqrt{\frac{b \sinh^2(e+fx)}{a} + 1} + \sqrt{b}(b-a) \sinh(e + fx)}{ab^{3/2} f \sqrt{a + b \sinh^2(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(-a + b)*Sinh[e + f*x] + a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a*b^(3/2)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] time = 0.065, size = 35, normalized size = 0.5

$$\frac{1}{f} \int \frac{\cosh^2(fx + e) \left(a + b \sinh^2(fx + e) \right)^{-\frac{3}{2}}}{\sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0` (cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^3(fx + e)}{\left(b \sinh^2(fx + e) + a \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.72556, size = 7876, normalized size = 102.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh
(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 +
2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)
*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*
b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*co
sh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 +
b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x +
e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2
+ b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5
*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 +
b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^
3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(
3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*
b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqr
t(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 +
2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3
*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(
f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^
2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b
- 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^
3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(
f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x
+ e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + (a*b*cosh(f*x
+ e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*
a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f
*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sin
h(f*x + e))*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)
*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*
x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 4*sqrt(2)*((
a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a
*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
```

$$\begin{aligned}
& + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a*b^3*f*\cosh(f*x + e)^4 + 4*a*b^3*f*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*b^3*f*\sinh(f*x + e)^4 + a*b^3*f + 2*(2*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^2 + 2*(3*a*b^3*f*\cosh(f*x + e)^2 + (2*a^2*b^2 - a*b^3)*f)*\sinh(f*x + e)^2 + 4*(a*b^3*f*\cosh(f*x + e)^3 + (2*a^2*b^2 - a*b^3)*f*\cosh(f*x + e))*\sinh(f*x + e)), -1/2*((a*b*\cosh(f*x + e)^4 + 4*a*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*b*\sinh(f*x + e)^4 + 2*(2*a^2 - a*b)*\cosh(f*x + e)^2 + 2*(3*a*b*\cosh(f*x + e)^2 + 2*a^2 - a*b)*\sinh(f*x + e)^2 + a*b + 4*(a*b*\cosh(f*x + e)^3 + (2*a^2 - a*b)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-b}*\arctan(\sqrt{2}*((a - b)*\cosh(f*x + e)^2 + 2*(a - b)*\cosh(f*x + e)*\sinh(f*x + e) + (a - b)*\sinh(f*x + e)^2 + b)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*b - b^2)*\cosh(f*x + e)^4 + 4*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a*b - b^2)*\sinh(f*x + e)^4 - (3*a*b - 2*b^2)*\cosh(f*x + e)^2 + (6*(a*b - b^2)*\cosh(f*x + e)^2 - 3*a*b + 2*b^2)*\sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*\cosh(f*x + e)^3 - (3*a*b - 2*b^2)*\cosh(f*x + e))*\sinh(f*x + e))) + (a*b*\cosh(f*x + e)^4 + 4*a*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*b*\sinh(f*x + e)^4 + 2*(2*a^2 - a*b)*\cosh(f*x + e)^2 + 2*(3*a*b*\cosh(f*x + e)^2 + 2*a^2 - a*b)*\sinh(f*x + e)^2 + a*b + 4*(a*b*\cosh(f*x + e)^3 + (2*a^2 - a*b)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-b}*\arctan(\sqrt{2}*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*\sqrt{-b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) + b)) + 2*\sqrt{2}*((a*b - b^2)*\cosh(f*x + e)^2 + 2*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e) + (a*b - b^2)*\sinh(f*x + e)^2 - a*b + b^2)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*b^3*f*\cosh(f*x + e)^4 + 4*a*b^3*f*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*b^3*f*\sinh(f*x + e)^4 + a*b^3*f + 2*(2*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^2 + 2*(3*a*b^3*f*\cosh(f*x + e)^2 + (2*a^2*b^2 - a*b^3)*f)*\sinh(f*x + e)^2 + 4*(a*b^3*f*\cosh(f*x + e)^3 + (2*a^2*b^2 - a*b^3)*f*\cosh(f*x + e))*\sinh(f*x + e))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^3}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(cosh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.383 \quad \int \frac{\cosh(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=29

$$\frac{\sinh(e+fx)}{af\sqrt{a+b \sinh^2(e+fx)}}$$

[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.0442715, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3190, 191}

$$\frac{\sinh(e+fx)}{af\sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 191

```
Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\cosh(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{\sinh(e + fx)}{af\sqrt{a + b \sinh^2(e + fx)}}$$

Mathematica [A] time = 0.0279143, size = 29, normalized size = 1.

$$\frac{\sinh(e + fx)}{af\sqrt{a + b \sinh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])

Maple [A] time = 0.012, size = 28, normalized size = 1.

$$\frac{\sinh(fx + e)}{af} \frac{1}{\sqrt{a + b(\sinh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2), x)

[Out] sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)

Maxima [B] time = 1.65636, size = 319, normalized size = 11.

$$\frac{b^2 e^{(-6fx-6e)} + 2ab - b^2 + (8a^2 - 8ab + 3b^2)e^{(-2fx-2e)} + 3(2ab - b^2)e^{(-4fx-4e)}}{2(a^2 - ab)\left(2(2a - b)e^{(-2fx-2e)} + be^{(-4fx-4e)} + b\right)^{\frac{3}{2}} f} - \frac{b^2 + 3(2ab - b^2)e^{(-2fx-2e)} + (8a^2 - 8ab + 3b^2)e^{(-4fx-4e)}}{2(a^2 - ab)\left(2(2a - b)e^{(-2fx-2e)} + be^{(-4fx-4e)} + b\right)^{\frac{3}{2}} f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{2}(b^2 e^{-6fx-6e} + 2ab - b^2 + (8a^2 - 8ab + 3b^2)e^{-2fx-2e} + 3(2ab - b^2)e^{-4fx-4e}) / ((a^2 - ab)(2(2a - b)e^{-2fx-2e} + b e^{-4fx-4e} + b)^{3/2} f) - \frac{1}{2}(b^2 + 3(2ab - b^2)e^{-2fx-2e} + (8a^2 - 8ab + 3b^2)e^{-4fx-4e} + (2ab - b^2)e^{-6fx-6e}) / ((a^2 - ab)(2(2a - b)e^{-2fx-2e} + b e^{-4fx-4e} + b)^{3/2} f)$

Fricas [B] time = 1.82896, size = 641, normalized size = 22.1

$$\frac{\sqrt{2}(\cosh(fx+e)^2 + 2 \cosh(fx+e) \sinh(fx+e) + \sinh(fx+e)^2)}{abf \cosh(fx+e)^4 + 4abf \cosh(fx+e) \sinh(fx+e)^3 + abf \sinh(fx+e)^4 + 2(2a^2 - ab)f \cosh(fx+e)^2 + abf + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] $\frac{\sqrt{2}(\cosh(fx+e)^2 + 2 \cosh(fx+e) \sinh(fx+e) + \sinh(fx+e)^2 - 1) \sqrt{(b \cosh(fx+e)^2 + b \sinh(fx+e)^2 + 2a - b) / (\cosh(fx+e)^2 - 2 \cosh(fx+e) \sinh(fx+e) + \sinh(fx+e)^2)}}{(abf \cosh(fx+e)^4 + 4abf \cosh(fx+e) \sinh(fx+e)^3 + abf \sinh(fx+e)^4 + 2(2a^2 - ab)f \cosh(fx+e)^2 + abf + 2) \sinh(fx+e)}$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [B] time = 1.2445, size = 211, normalized size = 7.28

$$\frac{\frac{(a^3f-2a^2bf+ab^2f)e^{(2fx+2e)}}{a^4b^3-2a^3b^4+a^2b^5} - \frac{a^3f-2a^2bf+ab^2f}{a^4b^3-2a^3b^4+a^2b^5}}{256\sqrt{be^{(4fx+4e)} + 4ae^{(2fx+2e)} - 2be^{(2fx+2e)} + b}} + \frac{f}{256ab^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] 1/256*((a^3*f - 2*a^2*b*f + a*b^2*f)*e^(2*f*x + 2*e)/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5) - (a^3*f - 2*a^2*b*f + a*b^2*f)/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + 1/256*f/(a*b^(7/2))

$$3.384 \quad \int \frac{\operatorname{sech}(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f(a-b)^{3/2}} - \frac{b \sinh(e+fx)}{af(a-b)\sqrt{a+b \sinh^2(e+fx)}}$$

[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(3/2)*f) - (b*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.0965584, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f(a-b)^{3/2}} - \frac{b \sinh(e+fx)}{af(a-b)\sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(3/2)*f) - (b*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 382

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I

```
Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x] /; FreeQ[{a, b, c, d, n, q}, x]
&& NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\operatorname{sech}(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\ &= -\frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{(a-b)f} \\ &= -\frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2} dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)f} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} \end{aligned}$$

Mathematica [C] time = 7.55193, size = 315, normalized size = 3.71

$$\tanh(e+fx)\operatorname{sech}^7(e+fx)\sqrt{a+b\sinh^2(e+fx)}\left(4(a-b)^2\sinh^4(e+fx)(a+b\sinh^2(e+fx)){}_2F_1\left(2, 2; \frac{7}{2}; \frac{(a-b)\tanh^2(e+fx)}{a}\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sech[e + f*x]^7*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]*(4*(a - b)^2*Hypergeometric2F1[2, 2, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2/a^2] + 15*a*Cosh[e + f*x]^2*(3*a + 2*b*Sinh[e + f*x]^2)*(-ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*(a + b*Sinh[e + f*x]^2)) + a*Cosh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2/a^2])))/(15*a^5*f*((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2/a^2)^(3/2))
```

Maple [C] time = 0.115, size = 101, normalized size = 1.2

$$\frac{1}{f} \int \frac{-b(\sinh(fx+e))^2 - a}{-b^2(\sinh(fx+e))^6 + (-2ab - b^2)(\sinh(fx+e))^4 + (-a^2 - 2ab)(\sinh(fx+e))^2 - a^2} \sqrt{a + b(\sinh(fx+e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((-b*sinh(f*x+e)^2-a)/(-b^2*sinh(f*x+e)^6+(-2*a*b-b^2)*sinh(f*x+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx+e)}{(b \sinh(fx+e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

[Out] integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [B] time = 2.48192, size = 4262, normalized size = 50.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\frac{1}{2} \left((a*b*\cosh(f*x + e)^4 + 4*a*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*b*\sinh(f*x + e)^4 + 2*(2*a^2 - a*b)*\cosh(f*x + e)^2 + 2*(3*a*b*\cosh(f*x + e)^2 + 2*a^2 - a*b)*\sinh(f*x + e)^2 + a*b + 4*(a*b*\cosh(f*x + e)^3 + (2*a^2 - a*b)*\cosh(f*x + e))*\sinh(f*x + e) \right) * \sqrt{-a + b} * \log\left(\frac{(a - 2*b)*\cosh(f*x + e)^4 + 4*(a - 2*b)*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a - 2*b)*\sinh(f*x + e)^4 - 2*(3*a - 2*b)*\cosh(f*x + e)^2 + 2*(3*(a - 2*b)*\cosh(f*x + e)^2 - 3*a + 2*b)*\sinh(f*x + e)^2 + 2*\sqrt{2}*(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1)*\sqrt{-a + b}}{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)} \right) + 4*((a - 2*b)*\cosh(f*x + e)^3 - (3*a - 2*b)*\cosh(f*x + e))*\sinh(f*x + e) + a - 2*b \Big/ \left(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1 \right) - 2*\sqrt{2} * \left((a*b - b^2)*\cosh(f*x + e)^2 + 2*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e) + (a*b - b^2)*\sinh(f*x + e)^2 - a*b + b^2 \right) * \sqrt{\frac{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)}{(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)}} \Big/ \left((a^3*b - 2*a^2*b^2 + a*b^3)*f*\cosh(f*x + e)^4 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*f*\cosh(f*x + e)*\sinh(f*x + e)^3 + (a^3*b - 2*a^2*b^2 + a*b^3)*f*\sinh(f*x + e)^4 + 2*(2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*f*\cosh(f*x + e)^2 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f)*\sinh(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*f*\cosh(f*x + e)^3 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*\cosh(f*x + e))*\sinh(f*x + e) \right), \left((a*b*\cosh(f*x + e)^4 + 4*a*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*b*\sinh(f*x + e)^4 + 2*(2*a^2 - a*b)*\cosh(f*x + e)^2 + 2*(3*a*b*\cosh(f*x + e)^2 + 2*a^2 - a*b)*\sinh(f*x + e)^2 + a*b + 4*(a*b*\cosh(f*x + e)^3 + (2*a^2 - a*b)*\cosh(f*x + e))*\sinh(f*x + e) \right) * \sqrt{a - b} * \arctan\left(\sqrt{2} * \left(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1 \right) * \sqrt{a - b} * \sqrt{\frac{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)}{(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)}} \right) \Big/ \left((b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(2*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^2 + 4*(b*\cosh(f*x + e)^3 + (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) \right)$$

(f*x + e) + b)) - sqrt(2)*((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^4 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^3*b - 2*a^2*b^2 + a*b^3)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^2 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*f + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^3 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e)))]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2), x)

[Out] Integral(sech(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)

Giac [B] time = 1.47401, size = 385, normalized size = 4.53

$$-\frac{\frac{(a^4bf-3a^3b^2f+3a^2b^3f-ab^4f)e^{(2fx+2e)}}{a^4b^3-2a^3b^4+a^2b^5} - \frac{a^4bf-3a^3b^2f+3a^2b^3f-ab^4f}{a^4b^3-2a^3b^4+a^2b^5}}{8\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}} + \frac{2 \arctan\left(-\frac{\sqrt{be^{(2fx+2e)}}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b+\sqrt{b}}}{2\sqrt{a-b}}\right)}{(af-bf)\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="giac")

[Out] -1/8*((a^4*b*f - 3*a^3*b^2*f + 3*a^2*b^3*f - a*b^4*f)*e^(2*f*x + 2*e)/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5) - (a^4*b*f - 3*a^3*b^2*f + 3*a^2*b^3*f - a*b^4*f)/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) -

$$\frac{\sqrt{b e^{4 f x + 4 e} + 4 a e^{2 f x + 2 e} - 2 b e^{2 f x + 2 e} + b} + \sqrt[3]{b}}{\sqrt{a - b}} \left((a f - b f) \sqrt{a - b} - \frac{1}{8} (a \sqrt{b} f - b^{3/2}) \right) / (a b^3)$$

$$3.385 \quad \int \frac{\operatorname{sech}^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=142

$$\frac{b(a+2b) \sinh(e+fx)}{2af(a-b)^2 \sqrt{a+b \sinh^2(e+fx)}} + \frac{(a-4b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2f(a-b)^{5/2}} + \frac{\tanh(e+fx) \operatorname{sech}(e+fx)}{2f(a-b) \sqrt{a+b \sinh^2(e+fx)}}$$

[Out] ((a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*(a - b)^(5/2)*f) + (b*(a + 2*b)*Sinh[e + f*x])/(2*a*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (Sech[e + f*x]*Tanh[e + f*x])/(2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.164915, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3190, 414, 527, 12, 377, 203}

$$\frac{b(a+2b) \sinh(e+fx)}{2af(a-b)^2 \sqrt{a+b \sinh^2(e+fx)}} + \frac{(a-4b) \tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{2f(a-b)^{5/2}} + \frac{\tanh(e+fx) \operatorname{sech}(e+fx)}{2f(a-b) \sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*(a - b)^(5/2)*f) + (b*(a + 2*b)*Sinh[e + f*x])/(2*a*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (Sech[e + f*x]*Tanh[e + f*x])/(2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{2(a-b)f} \\
&= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{a(a-4b)}{(1+x^2)\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{2(a-b)f} \\
&= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a-4b)\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(e+fx)\right)}{2(a-b)f} \\
&= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a-4b)\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(e+fx)\right)}{2(a-b)f} \\
&= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a-4b)\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(e+fx)\right)}{2(a-b)f} \\
&= \frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{5/2}f} + \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 5.36355, size = 231, normalized size = 1.63

$$\tanh(e+fx)\operatorname{sech}^5(e+fx)\left(16(a-b)\sinh^2(e+fx)(a+b\sinh^2(e+fx))^2\operatorname{HypergeometricPFQ}\left(\{2,2,3\},\left\{1,\frac{9}{2}\right\},\frac{(a-b)}{2}\right)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (Sech[e + f*x]^5*(16*(a - b)*HypergeometricPFQ[{2, 2, 3}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^2 + 16*(a - b)*Hypergeometric2F1[2, 3, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*(4*a^2 + 7*a*b*Sinh[e + f*x]^2 + 3*b^2*Sinh[e + f*x]^4) + 7*a*Cosh[e + f*x]^2*Hypergeometric2F1[1, 2, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*(15*a^2 + 20*

$a*b*\text{Sinh}[e + f*x]^2 + 8*b^2*\text{Sinh}[e + f*x]^4)*\text{Tanh}[e + f*x]/(105*a^4*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Maple [C] time = 0.149, size = 95, normalized size = 0.7

$$\frac{1}{f} \int \frac{(\cosh(fx + e))^2}{-b^2 (\cosh(fx + e))^{10} + (-2ab + 2b^2) (\cosh(fx + e))^8 + (-a^2 + 2ab - b^2) (\cosh(fx + e))^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2), x)`

[Out] `\int/undef0^(-(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*cosh(f*x+e)^10+(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{sech}(fx + e)^3}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="maxima")`

[Out] `integrate(sech(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)`

Fricas [B] time = 4.65664, size = 11344, normalized size = 79.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="fricas")`

```
[Out] [1/4*((a^2*b - 4*a*b^2)*cosh(f*x + e)^8 + 8*(a^2*b - 4*a*b^2)*cosh(f*x + e)
)*sinh(f*x + e)^7 + (a^2*b - 4*a*b^2)*sinh(f*x + e)^8 + 4*(a^3 - 4*a^2*b)*c
osh(f*x + e)^6 + 4*(a^3 - 4*a^2*b + 7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^2)*si
nh(f*x + e)^6 + 8*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b)*
cosh(f*x + e))*sinh(f*x + e)^5 + 2*(4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x +
e)^4 + 2*(35*(a^2*b - 4*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 17*a^2*b + 4*a*b^2
+ 30*(a^3 - 4*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b - 4*a*
b^2)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b)*cosh(f*x + e)^3 + (4*a^3 - 17*a^2
*b + 4*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b - 4*a*b^2 + 4*(a^3 - 4
*a^2*b)*cosh(f*x + e)^2 + 4*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^6 + 15*(a^3
- 4*a^2*b)*cosh(f*x + e)^4 + a^3 - 4*a^2*b + 3*(4*a^3 - 17*a^2*b + 4*a*b^2)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b - 4*a*b^2)*cosh(f*x + e)^7 +
3*(a^3 - 4*a^2*b)*cosh(f*x + e)^5 + (4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x +
e)^3 + (a^3 - 4*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a
- 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2
*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f
*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a -
2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x
+ e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x
+ e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
+ e) + 1)) + 2*sqrt(2)*((a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 6*(a^2*b
+ a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b + a*b^2 - 2*b^3)*si
nh(f*x + e)^6 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + (4*a^
3 - 7*a^2*b + 5*a*b^2 - 2*b^3 + 15*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^2)
*sinh(f*x + e)^4 + 4*(5*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (4*a^3 -
7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - a^2*b - a*b^2 +
2*b^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + (15*(a^2*b +
a*b^2 - 2*b^3)*cosh(f*x + e)^4 - 4*a^3 + 7*a^2*b - 5*a*b^2 + 2*b^3 + 6*(4*
a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a
^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^5 + 2*(4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e)^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*f*sinh(f*x + e)^8 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f
*cosh(f*x + e)^6 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x
+ e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^6 + 2*(4*a^
5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^
4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^3 + 3*(a^5 - 3*a^4*b +
3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b
```

$$\begin{aligned}
&^2 - a^2b^3) * f * \cosh(f*x + e)^2 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4) * f * \sinh(f*x + e)^4 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3) * f * \cosh(f*x + e)^2 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4) * f * \cosh(f*x + e)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3) * f * \cosh(f*x + e)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4) * f * \cosh(f*x + e)) * \sinh(f*x + e)^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4) * f * \cosh(f*x + e)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3) * f * \cosh(f*x + e)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4) * f * \cosh(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3) * f) * \sinh(f*x + e)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4) * f + 8*((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4) * f * \cosh(f*x + e)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3) * f * \cosh(f*x + e)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4) * f * \cosh(f*x + e)^3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3) * f * \cosh(f*x + e)) * \sinh(f*x + e)), 1/2*(((a^2*b - 4*a*b^2) * \cosh(f*x + e)^8 + 8*(a^2*b - 4*a*b^2) * \cosh(f*x + e) * \sinh(f*x + e)^7 + (a^2*b - 4*a*b^2) * \sinh(f*x + e)^8 + 4*(a^3 - 4*a^2*b) * \cosh(f*x + e)^6 + 4*(a^3 - 4*a^2*b + 7*(a^2*b - 4*a*b^2) * \cosh(f*x + e)^2) * \sinh(f*x + e)^6 + 8*(7*(a^2*b - 4*a*b^2) * \cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b) * \cosh(f*x + e)) * \sinh(f*x + e)^5 + 2*(4*a^3 - 17*a^2*b + 4*a*b^2) * \cosh(f*x + e)^4 + 2*(35*(a^2*b - 4*a*b^2) * \cosh(f*x + e)^4 + 4*a^3 - 17*a^2*b + 4*a*b^2 + 30*(a^3 - 4*a^2*b) * \cosh(f*x + e)^2) * \sinh(f*x + e)^4 + 8*(7*(a^2*b - 4*a*b^2) * \cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b) * \cosh(f*x + e)^3 + (4*a^3 - 17*a^2*b + 4*a*b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^3 + a^2*b - 4*a*b^2 + 4*(a^3 - 4*a^2*b) * \cosh(f*x + e)^2 + 4*(7*(a^2*b - 4*a*b^2) * \cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b) * \cosh(f*x + e)^4 + a^3 - 4*a^2*b + 3*(4*a^3 - 17*a^2*b + 4*a*b^2) * \cosh(f*x + e)^2) * \sinh(f*x + e)^2 + 8*((a^2*b - 4*a*b^2) * \cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b) * \cosh(f*x + e)^5 + (4*a^3 - 17*a^2*b + 4*a*b^2) * \cosh(f*x + e)^3 + (a^3 - 4*a^2*b) * \cosh(f*x + e)) * \sinh(f*x + e)) * \sqrt{a - b} * \arctan(\sqrt{2} * (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2 - 1) * \sqrt{a - b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)}) / (b * \cosh(f*x + e)^4 + 4*b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2*(2*a - b) * \cosh(f*x + e)^2 + 2*(3*b * \cosh(f*x + e)^2 + 2*a - b) * \sinh(f*x + e)^2 + 4*(b * \cosh(f*x + e)^3 + (2*a - b) * \cosh(f*x + e)) * \sinh(f*x + e) + b)) + \sqrt{2} * ((a^2*b + a*b^2 - 2*b^3) * \cosh(f*x + e)^6 + 6*(a^2*b + a*b^2 - 2*b^3) * \cosh(f*x + e) * \sinh(f*x + e)^5 + (a^2*b + a*b^2 - 2*b^3) * \sinh(f*x + e)^6 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3) * \cosh(f*x + e)^4 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3 + 15*(a^2*b + a*b^2 - 2*b^3) * \cosh(f*x + e)^2) * \sinh(f*x + e)^4 + 4*(5*(a^2*b + a*b^2 - 2*b^3) * \cosh(f*x + e)^3 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3) * \cosh(f*x + e)) * \sinh(f*x + e)^3 - a^2*b - a*b^2 + 2*b^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3) * \cosh(f*x + e)^2 + (15*(a^2*b + a*b^2 - 2*b^3) * \cosh(f*x + e)^4 - 4*a^3 + 7*a^2*b - 5*a*b^2 + 2*b^3 + 6*(4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3) * \cosh(f*x + e)^2) * \sinh(f*x + e)^2 + 2*(3*(a^2*b + a*b^2 - 2*b^3) * \cosh(f*x + e)^5 + 2*(4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3) * \cosh(f*x + e)^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3) * \cosh(f*x + e)) * \sinh(f*x + e)) * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f
\end{aligned}$$

```

*x + e)^2)))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^8 + 8
*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 +
(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*sinh(f*x + e)^8 + 4*(a^5 - 3*a^4*
b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*f*cosh(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
*sinh(f*x + e)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*
cosh(f*x + e)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x +
e)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x +
e)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 3
0*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^2 + (4*a^5 - 13*a^4
*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f)*sinh(f*x + e)^4 + 4*(a^5 - 3*a^4*b
+ 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*
b^3 - a*b^4)*f*cosh(f*x + e)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f
*cosh(f*x + e)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*co
sh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*f*cosh(f*x + e)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x +
e)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e
)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^2 + (a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f + 8*((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4
)*f*cosh(f*x + e)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x +
e)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^
3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e))]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2), x)

[Out] Integral(sech(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx + e)^3}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

$$3.386 \quad \int \frac{\cosh^6(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=325

$$\frac{2(2a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3ab^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{(8a^2-13ab+3b^2)\tanh(e+fx)}{3ab^3f}$$

```
[Out] -(((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*b^2*f) + ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1
- b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^3*f*Sqrt[(Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(2*a - 3*b)*EllipticF[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^2*f
*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 - 13*a*b + 3*
b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*b^3*f)
```

Rubi [A] time = 0.300737, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3192, 413, 528, 531, 418, 492, 411}

$$-\frac{(8a^2-13ab+3b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^3f} + \frac{(8a^2-13ab+3b^2)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3ab^3f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*b^2*f) + ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1
- b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^3*f*Sqrt[(Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(2*a - 3*b)*EllipticF[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^2*f
*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 - 13*a*b + 3*
b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*b^3*f)
```

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 413

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 528

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((e_.) + (f_.)*(x_.)^(n_.)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]

Rule 531

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((e_.) + (f_.)*(x_.)^(n_.)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_.) + (b_.)*(x_.)^2]*Sqrt[(c_.) + (d_.)*(x_.)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_.) + (b_.)*(x_)^2]*Sqrt[(c_.) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -

$a*d, 0]$ && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^6(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{5/2}}{(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\ &= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{abf} \\ &= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^2f} \\ &= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^2f} \\ &= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^2f} \\ &= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^2f} \\ &= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^2f} \end{aligned}$$

Mathematica [C] time = 1.06951, size = 196, normalized size = 0.6

$$\frac{-4ia(8a^2 - 17ab + 9b^2)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2}b\sinh(2(e+fx))(8a^2 + ab\cosh(2(e+fx)) - 1)}{12ab^3f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] ((4*I)*a*(8*a^2 - 13*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] - (4*I)*a*(8*a^2 - 17*a*b + 9*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 - 13*a*b + 6*b^2 + a*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]/(12*a*b^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.118, size = 498, normalized size = 1.5

$$\frac{1}{3ab^2 \cosh(fx + e) f} \left(\sqrt{\frac{b}{a}} ab (\cosh(fx + e))^4 \sinh(fx + e) + \left(4 \sqrt{\frac{b}{a}} a^2 - 7 \sqrt{\frac{b}{a}} ab + 3 \sqrt{\frac{b}{a}} b^2 \right) (\cosh(fx + e))^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] 1/3*((-1/a*b)^(1/2)*a*b*cosh(f*x+e)^4*sinh(f*x+e)+(4*(-1/a*b)^(1/2)*a^2-7*(-1/a*b)^(1/2)*a*b+3*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+4*a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-7*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2/b^2/(-1/a*b)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^6}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \cosh^6(fx + e)}{b^2 \sinh^4(fx + e) + 2ab \sinh^2(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^6(fx + e)}{\left(b \sinh^2(fx + e) + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

$$3.387 \quad \int \frac{\cosh^4(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=244

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{abf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{(2a-b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{ab^2f}$$

```
[Out] -(((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(a*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
)/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*b^2*f)
```

Rubi [A] time = 0.218062, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3192, 413, 531, 418, 492, 411}

$$\frac{(2a-b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{ab^2f} - \frac{(2a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)}{ab^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] -(((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(a*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
)/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*b^2*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
```


$\text{Cos}[e + f*x]^2)/(f*\text{Cos}[e + f*x]), \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*ff^2*x^2)^p, x], x, \text{Sin}[e + f*x]/ff], x] /;$ FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 413

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)*((c_ + (d_)*(x_)^{(n_)})^{(q_)}, x_Symbol] :> \text{Simp}[(a*d - c*b)*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)}]/(a*b*n*(p+1)), x] - \text{Dist}[1/(a*b*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-2)}*\text{Simp}[c*(a*d - c*b*(n*(p+1) + 1)) + d*(a*d*(n*(q-1) + 1) - b*c*(n*(p+q) + 1))*x^n, x], x], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 531

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)*((c_ + (d_)*(x_)^{(n_)})^{(q_)*((e_ + (f_)*(x_)^{(n_)}, x_Symbol] :> \text{Dist}[e, \text{Int}[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + \text{Dist}[f, \text{Int}[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;$ FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

$\text{Int}[1/(\text{Sqrt}[a_ + (b_)*(x_)^2]*\text{Sqrt}[(c_ + (d_)*(x_)^2]), x_Symbol] :> \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /;$ FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

$\text{Int}[(x_)^2/(\text{Sqrt}[a_ + (b_)*(x_)^2]*\text{Sqrt}[(c_ + (d_)*(x_)^2]), x_Symbol] :> \text{Simp}[(x*\text{Sqrt}[a + b*x^2])/(b*\text{Sqrt}[c + d*x^2]), x] - \text{Dist}[c/b, \text{Int}[\text{Sqrt}[a + b*x^2]/(c + d*x^2)^{(3/2)}, x], x] /;$ FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

$\text{Int}[\text{Sqrt}[(a_ + (b_)*(x_)^2)/((c_ + (d_)*(x_)^2)^{(3/2)}, x_Symbol] :> \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /;$ FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a+(2a-b)x^2}{\sqrt{1+x^2}\sqrt{a}} dx, x, \sinh(e+fx)\right)}{abf} \\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a}} dx, x, \sinh(e+fx)\right)}{bf} \\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{F\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{abf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{ab^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 0.602297, size = 155, normalized size = 0.64

$$\frac{(a-b)\left(-\sqrt{2}b\sinh(2(e+fx)) + 4ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right)\right) - 2ia(2a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx), \frac{b}{a}\right)}{2ab^2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((-2*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (a - b)*((4*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.116, size = 322, normalized size = 1.3

$$\frac{1}{ab \cosh(fx + e) f} \left(\left(\sqrt{\frac{b}{a}} - \sqrt{\frac{b}{a}} \right) (\cosh(fx + e))^2 \sinh(fx + e) + a \sqrt{\frac{b (\cosh(fx + e))^2}{a} + \frac{a - b}{a}} \sqrt{(\cosh(fx + e))^2 + \frac{a - b}{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2), x)

[Out] -(((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b/b/(-1/a*b)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^4}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="maxima")

[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh(fx + e)^2 + a} \cosh(fx + e)^4}{b^2 \sinh(fx + e)^4 + 2ab \sinh(fx + e)^2 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^4}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

$$3.388 \quad \int \frac{\cosh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=91

$$\frac{\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\left|1-\frac{a}{b}\right.\right)}{\sqrt{a}\sqrt{b}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

[Out] (Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]) / (Sqrt[a]*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.1012, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {3192, 411}

$$\frac{\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\left|1-\frac{a}{b}\right.\right)}{\sqrt{a}\sqrt{b}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b]) / (Sqrt[a]*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3192

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\cosh^2(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx = \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{(a+bx^2)^{3/2}} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{\cosh(e + fx) E\left(\tan^{-1}\left(\frac{\sqrt{b} \sinh(e + fx)}{\sqrt{a}}\right) \middle| 1 - \frac{a}{b}\right)}{\sqrt{a} \sqrt{b} f \sqrt{\frac{a \cosh^2(e + fx)}{a + b \sinh^2(e + fx)}} \sqrt{a + b \sinh^2(e + fx)}}$$

Mathematica [C] time = 0.306912, size = 143, normalized size = 1.57

$$\frac{-i\sqrt{2a}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + i\sqrt{2a}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx) \middle| \frac{b}{a}\right) + b\sinh(2(e+fx))}{abf\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] (I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x),
b/a] - I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e
+ f*x), b/a] + b*Sinh[2*(e + f*x)]/(a*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e +
f*x)]])
```

Maple [A] time = 0.097, size = 181, normalized size = 2.

$$\frac{1}{a \cosh(fx + e) f} \left(\sqrt{-\frac{b}{a}} \sinh(fx + e) (\cosh(fx + e))^2 + \sqrt{\frac{b (\cosh(fx + e))^2}{a} + \frac{a - b}{a}} \sqrt{(\cosh(fx + e))^2} \operatorname{EllipticF}\left(\operatorname{si}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out] $((-1/a*b)^{(1/2)}*\sinh(f*x+e)*\cosh(f*x+e)^2+(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)}))/(-1/a*b)^{(1/2)}/a/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(fx + e)}{(b \sinh^2(fx + e) + a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sinh^2(fx + e) + a} \cosh^2(fx + e)}{b^2 \sinh^4(fx + e) + 2ab \sinh^2(fx + e) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^2}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.389 \quad \int \frac{1}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=115

$$\frac{b \sinh(e+fx) \cosh(e+fx)}{af(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{i\sqrt{a+b \sinh^2(e+fx)}E\left(ie+ifx \left| \frac{b}{a} \right. \right)}{af(a-b)\sqrt{\frac{b \sinh^2(e+fx)}{a}+1}}$$

[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rubi [A] time = 0.0611369, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3184, 21, 3178, 3177}

$$\frac{b \sinh(e+fx) \cosh(e+fx)}{af(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{i\sqrt{a+b \sinh^2(e+fx)}E\left(ie+ifx \left| \frac{b}{a} \right. \right)}{af(a-b)\sqrt{\frac{b \sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]

[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rule 3184

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a + b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e
+ f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \sinh^2(e + fx))^{3/2}} dx &= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} - \frac{\int \frac{-a - b \sinh^2(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx}{a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} + \frac{\int \sqrt{a + b \sinh^2(e + fx)} dx}{a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} + \frac{\sqrt{a + b \sinh^2(e + fx)} \int \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}} dx}{a(a - b)\sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} - \frac{iE\left(ie + ifx \left| \frac{b}{a} \right. \right) \sqrt{a + b \sinh^2(e + fx)}}{a(a - b)f\sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.143378, size = 100, normalized size = 0.87

$$\frac{-\sqrt{2}b \sinh(2(e + fx)) - 2ia\sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} E\left(ie + ifx \left| \frac{b}{a} \right. \right)}{2af(a - b)\sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]

[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.001, size = 252, normalized size = 2.2

$$\frac{1}{a(a-b)\cosh(fx+e)f} \left(-\sqrt{-\frac{b}{a}}b\sinh(fx+e)(\cosh(fx+e))^2 + a\sqrt{\frac{b(\cosh(fx+e))^2}{a} + \frac{a-b}{a}}\sqrt{(\cosh(fx+e))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^2+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b/a/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sinh(fx+e)^2+a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a}}{b^2 \sinh^4(fx + e) + 2ab \sinh^2(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh^2(fx + e) + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)

$$3.390 \quad \int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=217

$$\frac{2b \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{af(a-b)^2 \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{\tanh(e+fx)}{f(a-b) \sqrt{a+b \sinh^2(e+fx)}} + \frac{\sqrt{b(a+b)}}{\sqrt{a}}$$

[Out] (Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(Sqrt[a]*(a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.202535, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3192, 414, 525, 418, 411}

$$\frac{\tanh(e+fx)}{f(a-b) \sqrt{a+b \sinh^2(e+fx)}} + \frac{\sqrt{b(a+b)} \cosh(e+fx) E\left(\tan^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a}}\right) \middle| 1-\frac{a}{b}\right)}{\sqrt{a} f(a-b)^2 \sqrt{a+b \sinh^2(e+fx)} \sqrt{\frac{a \cosh^2(e+fx)}{a+b \sinh^2(e+fx)}}} - \frac{2b \operatorname{sech}(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{af(a-b)^2 \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(Sqrt[a]*(a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3192

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[

$\text{Cos}[e + f*x]^2]/(f*\text{Cos}[e + f*x]), \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*ff^2*x^2)^p, x], x, \text{Sin}[e + f*x]/ff], x] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[m/2] \&\& \text{!IntegerQ}[p]$

Rule 414

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \rightarrow -\text{Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)}]/(a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& \text{!(IntegerQ}[p] \&\& \text{IntegerQ}[q] \&\& \text{LtQ}[q, -1]) \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 525

$\text{Int}[(e_ + (f_)*(x_)^2)/(\text{Sqrt}[(a_ + (b_)*(x_)^2)*((c_ + (d_)*(x_)^2)^{(3/2)}), x_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(\text{Sqrt}[a + b*x^2]*\text{Sqrt}[c + d*x^2]), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[\text{Sqrt}[a + b*x^2]/(c + d*x^2)^{(3/2)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rule 418

$\text{Int}[1/(\text{Sqrt}[(a_ + (b_)*(x_)^2)*\text{Sqrt}[(c_ + (d_)*(x_)^2]), x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[d/c] \&\& \text{PosQ}[b/a] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_ + (b_)*(x_)^2)/((c_ + (d_)*(x_)^2)^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{b-bx^2}{\sqrt{1+x^2}(a+bx^2)} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
&= \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2b\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{(a-b)(-a+b)f} \\
&= \frac{\sqrt{b}(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\left|1-\frac{a}{b}\right.\right)}{\sqrt{a}(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{2bF\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{a}{b}\right.\right)}{a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 1.17733, size = 178, normalized size = 0.82

$$\frac{-i\sqrt{2a(a-b)}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \tanh(e+fx)(2a^2 + b(a+b)\cosh(2(e+fx)) - ab + b^2) + i}{af(a-b)^2\sqrt{4a+2b\cosh(2(e+fx))}-2b}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (I*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] - I*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a *EllipticF[I*(e + f*x), b/a] + (2*a^2 - a*b + b^2 + b*(a + b)*Cosh[2*(e + f*x)])*Tanh[e + f*x]/(a*(a - b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)])]

Maple [A] time = 0.148, size = 345, normalized size = 1.6

$$-\frac{1}{a(a-b)^2\cosh(fx+e)f}\left[-\sqrt{\frac{b}{a}}ab(\sinh(fx+e))^3 - \sqrt{-\frac{b}{a}}b^2(\sinh(fx+e))^3 + a\sqrt{\frac{a+b(\sinh(fx+e))^2}{a}}\sqrt{(\cosh(fx+e))^2 - a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out]
$$-(-(-1/a*b)^{(1/2)}*a*b*\sinh(f*x+e)^3-(-1/a*b)^{(1/2)}*b^2*\sinh(f*x+e)^3+a*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b-((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^2+((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a*b+((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^2-(-1/a*b)^{(1/2)}*a^2*\sinh(f*x+e)-(-1/a*b)^{(1/2)}*b^2*\sinh(f*x+e))/(a-b)^2/a/(-1/a*b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx+e)^2}{(b \sinh(fx+e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b \sinh(fx+e)^2 + a} \operatorname{sech}(fx+e)^2}{b^2 \sinh(fx+e)^4 + 2ab \sinh(fx+e)^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2), x)

[Out] Integral(sech(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx + e)^2}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="giac")

[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.391 \quad \int \frac{\cosh^5(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=134

$$-\frac{(a-b)(3a+2b) \sinh(e+fx)}{3a^2b^2f\sqrt{a+b \sinh^2(e+fx)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a-b) \sinh(e+fx) \cosh^2(e+fx)}{3abf(a+b \sinh^2(e+fx))^{3/2}}$$

[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(5/2)*f) - ((a - b)*Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - ((a - b)*(3*a + 2*b)*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.141492, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3190, 413, 385, 217, 206}

$$-\frac{(a-b)(3a+2b) \sinh(e+fx)}{3a^2b^2f\sqrt{a+b \sinh^2(e+fx)}} + \frac{\tanh^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a-b) \sinh(e+fx) \cosh^2(e+fx)}{3abf(a+b \sinh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(5/2)*f) - ((a - b)*Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - ((a - b)*(3*a + 2*b)*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+2b+3ax^2}{(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{3abf} \\
&= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{3abf} \\
&= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \sinh(e+fx)\right)}{3abf} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.863662, size = 126, normalized size = 0.94

$$\frac{2\sqrt{2}(b-a)\sinh(e+fx)(3a^2+b(2a+b)\cosh(2(e+fx))+ab-b^2)}{3a^2b^2(2a+b\cosh(2(e+fx))-b)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{b^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] (ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/b^(5/2) + (2*Sqrt[2]*(-a + b)*(3*a^2 + a*b - b^2 + b*(2*a + b)*Cosh[2*(e + f*x)])*Sinh[e + f*x]/(3*a^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)))/f

Maple [C] time = 0.082, size = 65, normalized size = 0.5

$$\frac{1}{f} \int \frac{(\cosh(fx + e))^4}{b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^2 + a^2} \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] `int/indef0` (cosh(f*x+e)^4/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^5}{(b \sinh(fx + e)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(cosh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [B] time = 5.85418, size = 15815, normalized size = 118.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [1/12*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)

$$\begin{aligned}
&^4 + 8a^4 - 8a^3b + 3a^2b^2 + 30(2a^3b - a^2b^2)\cosh(fx + e)^2) * \\
&\sinh(fx + e)^4 + a^2b^2 + 8(7a^2b^2\cosh(fx + e)^5 + 10(2a^3b - a^2b^2) * \\
&\cosh(fx + e)^3 + (8a^4 - 8a^3b + 3a^2b^2)\cosh(fx + e))\sinh(fx + e)^3 + 4(2a^3b - a^2b^2) * \\
&\cosh(fx + e)^2 + 4(7a^2b^2\cosh(fx + e)^6 + 15(2a^3b - a^2b^2)\cosh(fx + e)^4 + 2a^3b - a^2b^2 + 3(8a^4 - 8a^3b + 3a^2b^2) * \\
&\cosh(fx + e)^2)\sinh(fx + e)^2 + 8(a^2b^2\cosh(fx + e)^7 + 3(2a^3b - a^2b^2)\cosh(fx + e)^5 + (8a^4 - 8a^3b + 3a^2b^2) * \\
&\cosh(fx + e)^3 + (2a^3b - a^2b^2)\cosh(fx + e))\sinh(fx + e))\sqrt{b}\log(-((a^2b - 2ab^2 + b^3)\cosh(fx + e)^8 + 8(a^2b - 2ab^2 + b^3) * \\
&\cosh(fx + e)\sinh(fx + e)^7 + (a^2b - 2ab^2 + b^3)\sinh(fx + e)^8 + 2(a^3 - 4a^2b + 5ab^2 - 2b^3)\cosh(fx + e)^6 + 2(a^3 - 4a^2b + 5ab^2 - 2b^3 + 14(a^2b - 2ab^2 + b^3)\cosh(fx + e)^2)\sinh(fx + e)^6 + 4(14(a^2b - 2ab^2 + b^3)\cosh(fx + e)^3 + 3(a^3 - 4a^2b + 5ab^2 - 2b^3)\cosh(fx + e))\sinh(fx + e)^5 + (9a^2b - 14ab^2 + 6b^3)\cosh(fx + e)^4 + (70(a^2b - 2ab^2 + b^3)\cosh(fx + e)^4 + 9a^2b - 14ab^2 + 6b^3 + 30(a^3 - 4a^2b + 5ab^2 - 2b^3)\cosh(fx + e)^2)\sinh(fx + e)^4 + 4(14(a^2b - 2ab^2 + b^3)\cosh(fx + e)^5 + 10(a^3 - 4a^2b + 5ab^2 - 2b^3)\cosh(fx + e)^3 + (9a^2b - 14ab^2 + 6b^3)\cosh(fx + e))\sinh(fx + e)^3 + b^3 + 2(3ab^2 - 2b^3)\cosh(fx + e)^2 + 2(14(a^2b - 2ab^2 + b^3)\cosh(fx + e)^6 + 15(a^3 - 4a^2b + 5ab^2 - 2b^3)\cosh(fx + e)^4 + 3ab^2 - 2b^3 + 3(9a^2b - 14ab^2 + 6b^3)\cosh(fx + e)^2)\sinh(fx + e)^2 + \sqrt{2}((a^2 - 2ab + b^2)\cosh(fx + e)^6 + 6(a^2 - 2ab + b^2)\cosh(fx + e)\sinh(fx + e)^5 + (a^2 - 2ab + b^2)\sinh(fx + e)^6 - 3(a^2 - 2ab + b^2)\cosh(fx + e)^4 + 3(5(a^2 - 2ab + b^2)\cosh(fx + e)^2 - a^2 + 2ab - b^2)\sinh(fx + e)^4 + 4(5(a^2 - 2ab + b^2)\cosh(fx + e)^3 - 3(a^2 - 2ab + b^2)\cosh(fx + e))\sinh(fx + e)^3 - (4ab - 3b^2)\cosh(fx + e)^2 + (15(a^2 - 2ab + b^2)\cosh(fx + e)^4 - 18(a^2 - 2ab + b^2)\cosh(fx + e)^2 - 4ab + 3b^2)\sinh(fx + e)^2 - b^2 + 2(3(a^2 - 2ab + b^2)\cosh(fx + e)^5 - 6(a^2 - 2ab + b^2)\cosh(fx + e)^3 - (4ab - 3b^2)\cosh(fx + e))\sinh(fx + e))\sqrt{b}\sqrt{(b\cosh(fx + e)^2 + b\sinh(fx + e)^2 + 2a - b)/(cosh(fx + e)^2 - 2\cosh(fx + e)\sinh(fx + e) + \sinh(fx + e)^2)) + 4(2(a^2b - 2ab^2 + b^3)\cosh(fx + e)^7 + 3(a^3 - 4a^2b + 5ab^2 - 2b^3)\cosh(fx + e)^5 + (9a^2b - 14ab^2 + 6b^3)\cosh(fx + e)^3 + (3ab^2 - 2b^3)\cosh(fx + e))\sinh(fx + e))/(cosh(fx + e)^6 + 6\cosh(fx + e)^5\sinh(fx + e) + 15\cosh(fx + e)^4\sinh(fx + e)^2 + 20\cosh(fx + e)^3\sinh(fx + e)^3 + 15\cosh(fx + e)^2\sinh(fx + e)^4 + 6\cosh(fx + e)\sinh(fx + e)^5 + \sinh(fx + e)^6)) + 3(a^2b^2\cosh(fx + e)^8 + 8a^2b^2\cosh(fx + e)\sinh(fx + e)^7 + a^2b^2\sinh(fx + e)^8 + 4(2a^3b - a^2b^2)\cosh(fx + e)^6 + 4(7a^2b^2\cosh(fx + e)^2 + 2a^3b - a^2b^2)\sinh(fx + e)^6 + 8(7a^2b^2\cosh(fx + e)^3 + 3(2a^3b - a^2b^2)\cosh(fx + e))\sinh(fx + e)^5 + 2(8a^4 - 8a^3b + 3a^2b^2)\cosh(fx + e)^4 + 2(35a^2b^2\cosh(fx + e)^4 + 8a^4 - 8a^3b + 3a^2b^2 + 30(2a^3b - a^2b^2)\cosh(fx + e)^2)\sinh(fx + e)^4 + a^2b^2 + 8(7a^2b^2\cosh(fx + e)^5 + 10(2a^3b - a^2b^2)\cosh(fx + e)^3 + (8a^4 - 8a^3b + 3a
\end{aligned}$$

$$\begin{aligned}
& ^2b^2) \cosh(f*x + e)) \sinh(f*x + e)^3 + 4*(2a^3b - a^2b^2) \cosh(f*x + e) \\
&)^2 + 4*(7a^2b^2 \cosh(f*x + e)^6 + 15*(2a^3b - a^2b^2) \cosh(f*x + e)^4 \\
& + 2a^3b - a^2b^2 + 3*(8a^4 - 8a^3b + 3a^2b^2) \cosh(f*x + e)^2) \sin \\
& h(f*x + e)^2 + 8*(a^2b^2 \cosh(f*x + e)^7 + 3*(2a^3b - a^2b^2) \cosh(f*x \\
& + e)^5 + (8a^4 - 8a^3b + 3a^2b^2) \cosh(f*x + e)^3 + (2a^3b - a^2b^2 \\
&) \cosh(f*x + e)) \sinh(f*x + e) \sqrt{b} \log((b \cosh(f*x + e)^4 + 4b \cosh(f \\
& *x + e) \sinh(f*x + e)^3 + b \sinh(f*x + e)^4 + 2a \cosh(f*x + e)^2 + 2*(3b \\
& \cosh(f*x + e)^2 + a) \sinh(f*x + e)^2 + \sqrt{2}*(\cosh(f*x + e)^2 + 2 \cosh(f* \\
& x + e) \sinh(f*x + e) + \sinh(f*x + e)^2 + 1) \sqrt{b} \sqrt{(b \cosh(f*x + e)^2 \\
& + b \sinh(f*x + e)^2 + 2a - b) / (\cosh(f*x + e)^2 - 2 \cosh(f*x + e) \sinh(f*x \\
& + e) + \sinh(f*x + e)^2)) + 4*(b \cosh(f*x + e)^3 + a \cosh(f*x + e) \sinh(f* \\
& x + e) + b) / (\cosh(f*x + e)^2 + 2 \cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e \\
&)^2)) - 8 \sqrt{2} * ((2a^2b^2 - ab^3 - b^4) \cosh(f*x + e)^6 + 6*(2a^2b^2 \\
& - ab^3 - b^4) \cosh(f*x + e) \sinh(f*x + e)^5 + (2a^2b^2 - ab^3 - b^4) s \\
& inh(f*x + e)^6 + 3*(2a^3b - 2a^2b^2 - ab^3 + b^4) \cosh(f*x + e)^4 + 3* \\
& (2a^3b - 2a^2b^2 - ab^3 + b^4 + 5*(2a^2b^2 - ab^3 - b^4) \cosh(f*x + \\
& e)^2) \sinh(f*x + e)^4 - 2a^2b^2 + ab^3 + b^4 + 4*(5*(2a^2b^2 - ab^3 \\
& - b^4) \cosh(f*x + e)^3 + 3*(2a^3b - 2a^2b^2 - ab^3 + b^4) \cosh(f*x + e \\
&)) \sinh(f*x + e)^3 - 3*(2a^3b - 2a^2b^2 - ab^3 + b^4) \cosh(f*x + e)^2 \\
& + 3*(5*(2a^2b^2 - ab^3 - b^4) \cosh(f*x + e)^4 - 2a^3b + 2a^2b^2 + a \\
& b^3 - b^4 + 6*(2a^3b - 2a^2b^2 - ab^3 + b^4) \cosh(f*x + e)^2) \sinh(f*x \\
& + e)^2 + 6*((2a^2b^2 - ab^3 - b^4) \cosh(f*x + e)^5 + 2*(2a^3b - 2a^2 \\
& *b^2 - ab^3 + b^4) \cosh(f*x + e)^3 - (2a^3b - 2a^2b^2 - ab^3 + b^4) \c \\
& osh(f*x + e)) \sinh(f*x + e) \sqrt{(b \cosh(f*x + e)^2 + b \sinh(f*x + e)^2 + \\
& 2a - b) / (\cosh(f*x + e)^2 - 2 \cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e)^2 \\
&))} / (a^2b^5 f \cosh(f*x + e)^8 + 8a^2b^5 f \cosh(f*x + e) \sinh(f*x + e)^7 \\
& + a^2b^5 f \sinh(f*x + e)^8 + a^2b^5 f + 4*(2a^3b^4 - a^2b^5) f \cosh(f* \\
& x + e)^6 + 4*(7a^2b^5 f \cosh(f*x + e)^2 + (2a^3b^4 - a^2b^5) f) \sinh(f \\
& *x + e)^6 + 2*(8a^4b^3 - 8a^3b^4 + 3a^2b^5) f \cosh(f*x + e)^4 + 8*(7a \\
& ^2b^5 f \cosh(f*x + e)^3 + 3*(2a^3b^4 - a^2b^5) f \cosh(f*x + e)) \sinh(f \\
& *x + e)^5 + 2*(35a^2b^5 f \cosh(f*x + e)^4 + 30*(2a^3b^4 - a^2b^5) f \co \\
& sh(f*x + e)^2 + (8a^4b^3 - 8a^3b^4 + 3a^2b^5) f) \sinh(f*x + e)^4 + 4* \\
& (2a^3b^4 - a^2b^5) f \cosh(f*x + e)^2 + 8*(7a^2b^5 f \cosh(f*x + e)^5 + \\
& 10*(2a^3b^4 - a^2b^5) f \cosh(f*x + e)^3 + (8a^4b^3 - 8a^3b^4 + 3a^2 \\
& *b^5) f \cosh(f*x + e)) \sinh(f*x + e)^3 + 4*(7a^2b^5 f \cosh(f*x + e)^6 + 1 \\
& 5*(2a^3b^4 - a^2b^5) f \cosh(f*x + e)^4 + 3*(8a^4b^3 - 8a^3b^4 + 3a^ \\
& 2b^5) f \cosh(f*x + e)^2 + (2a^3b^4 - a^2b^5) f) \sinh(f*x + e)^2 + 8*(a^ \\
& 2b^5 f \cosh(f*x + e)^7 + 3*(2a^3b^4 - a^2b^5) f \cosh(f*x + e)^5 + (8a^ \\
& 4b^3 - 8a^3b^4 + 3a^2b^5) f \cosh(f*x + e)^3 + (2a^3b^4 - a^2b^5) f \\
& \cosh(f*x + e) \sinh(f*x + e), -1/6*(3*(a^2b^2 \cosh(f*x + e)^8 + 8a^2b^2 \\
& * \cosh(f*x + e) \sinh(f*x + e)^7 + a^2b^2 \sinh(f*x + e)^8 + 4*(2a^3b - a^2 \\
& *b^2) \cosh(f*x + e)^6 + 4*(7a^2b^2 \cosh(f*x + e)^2 + 2a^3b - a^2b^2) s \\
& inh(f*x + e)^6 + 8*(7a^2b^2 \cosh(f*x + e)^3 + 3*(2a^3b - a^2b^2) \cosh(\\
& f*x + e)) \sinh(f*x + e)^5 + 2*(8a^4 - 8a^3b + 3a^2b^2) \cosh(f*x + e)^4 \\
& + 2*(35a^2b^2 \cosh(f*x + e)^4 + 8a^4 - 8a^3b + 3a^2b^2 + 30*(2a^3*
\end{aligned}$$

$$\begin{aligned}
& b - a^2 b^2) \cosh(fx + e)^2) \sinh(fx + e)^4 + a^2 b^2 + 8(7a^2 b^2 \cosh \\
& (fx + e)^5 + 10(2a^3 b - a^2 b^2) \cosh(fx + e)^3 + (8a^4 - 8a^3 b + 3 \\
& a^2 b^2) \cosh(fx + e)) \sinh(fx + e)^3 + 4(2a^3 b - a^2 b^2) \cosh(fx + \\
& e)^2 + 4(7a^2 b^2 \cosh(fx + e)^6 + 15(2a^3 b - a^2 b^2) \cosh(fx + e) \\
& ^4 + 2a^3 b - a^2 b^2 + 3(8a^4 - 8a^3 b + 3a^2 b^2) \cosh(fx + e)^2) \sinh \\
& (fx + e)^2 + 8(a^2 b^2 \cosh(fx + e)^7 + 3(2a^3 b - a^2 b^2) \cosh(fx \\
& + e)^5 + (8a^4 - 8a^3 b + 3a^2 b^2) \cosh(fx + e)^3 + (2a^3 b - a^2 b \\
& ^2) \cosh(fx + e)) \sinh(fx + e)) \sqrt{-b} \arctan(\sqrt{2}((a - b) \cosh(fx \\
& + e)^2 + 2(a - b) \cosh(fx + e) \sinh(fx + e) + (a - b) \sinh(fx + e)^2 + \\
& b) \sqrt{-b} \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(f \\
& x + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)) / ((a b - b^2) \\
& \cosh(fx + e)^4 + 4(a b - b^2) \cosh(fx + e) \sinh(fx + e)^3 + (a b - b^2) \\
& * \sinh(fx + e)^4 - (3a b - 2b^2) \cosh(fx + e)^2 + (6(a b - b^2) \cosh(fx \\
& + e)^2 - 3a b + 2b^2) \sinh(fx + e)^2 - b^2 + 2(2(a b - b^2) \cosh(fx \\
& + e)^3 - (3a b - 2b^2) \cosh(fx + e) \sinh(fx + e))) + 3(a^2 b^2 \cosh(\\
& fx + e)^8 + 8a^2 b^2 \cosh(fx + e) \sinh(fx + e)^7 + a^2 b^2 \sinh(fx + e) \\
&)^8 + 4(2a^3 b - a^2 b^2) \cosh(fx + e)^6 + 4(7a^2 b^2 \cosh(fx + e)^2 \\
& + 2a^3 b - a^2 b^2) \sinh(fx + e)^6 + 8(7a^2 b^2 \cosh(fx + e)^3 + 3(2a \\
& ^3 b - a^2 b^2) \cosh(fx + e)) \sinh(fx + e)^5 + 2(8a^4 - 8a^3 b + 3a^2 \\
& b^2) \cosh(fx + e)^4 + 2(35a^2 b^2 \cosh(fx + e)^4 + 8a^4 - 8a^3 b + \\
& 3a^2 b^2 + 30(2a^3 b - a^2 b^2) \cosh(fx + e)^2) \sinh(fx + e)^4 + a^2 b \\
& ^2 + 8(7a^2 b^2 \cosh(fx + e)^5 + 10(2a^3 b - a^2 b^2) \cosh(fx + e)^3 \\
& + (8a^4 - 8a^3 b + 3a^2 b^2) \cosh(fx + e)) \sinh(fx + e)^3 + 4(2a^3 b \\
& - a^2 b^2) \cosh(fx + e)^2 + 4(7a^2 b^2 \cosh(fx + e)^6 + 15(2a^3 b - \\
& a^2 b^2) \cosh(fx + e)^4 + 2a^3 b - a^2 b^2 + 3(8a^4 - 8a^3 b + 3a^2 b \\
& ^2) \cosh(fx + e)^2) \sinh(fx + e)^2 + 8(a^2 b^2 \cosh(fx + e)^7 + 3(2a^3 \\
& b - a^2 b^2) \cosh(fx + e)^5 + (8a^4 - 8a^3 b + 3a^2 b^2) \cosh(fx + e) \\
&)^3 + (2a^3 b - a^2 b^2) \cosh(fx + e)) \sinh(fx + e)) \sqrt{-b} \arctan(\sqrt{2} \\
& ((\cosh(fx + e)^2 + 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2 + 1 \\
&) \sqrt{-b} \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx \\
& + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)) / (b \cosh(fx + e) \\
&)^4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 + 2(2a - b) \c \\
& osh(fx + e)^2 + 2(3b \cosh(fx + e)^2 + 2a - b) \sinh(fx + e)^2 + 4(b \c \\
& osh(fx + e)^3 + (2a - b) \cosh(fx + e)) \sinh(fx + e) + b)) + 4 \sqrt{2} (\\
& (2a^2 b^2 - a b^3 - b^4) \cosh(fx + e)^6 + 6(2a^2 b^2 - a b^3 - b^4) \cos \\
& h(fx + e) \sinh(fx + e)^5 + (2a^2 b^2 - a b^3 - b^4) \sinh(fx + e)^6 + 3 \\
& (2a^3 b - 2a^2 b^2 - a b^3 + b^4) \cosh(fx + e)^4 + 3(2a^3 b - 2a^2 b^2 \\
& - a b^3 + b^4 + 5(2a^2 b^2 - a b^3 - b^4) \cosh(fx + e)^2) \sinh(fx + e) \\
&)^4 - 2a^2 b^2 + a b^3 + b^4 + 4(5(2a^2 b^2 - a b^3 - b^4) \cosh(fx + e) \\
&)^3 + 3(2a^3 b - 2a^2 b^2 - a b^3 + b^4) \cosh(fx + e)) \sinh(fx + e)^3 \\
& - 3(2a^3 b - 2a^2 b^2 - a b^3 + b^4) \cosh(fx + e)^2 + 3(5(2a^2 b^2 - \\
& a b^3 - b^4) \cosh(fx + e)^4 - 2a^3 b + 2a^2 b^2 + a b^3 - b^4 + 6(2a^2 \\
& b^2 - 2a^2 b^2 - a b^3 + b^4) \cosh(fx + e)^2) \sinh(fx + e)^2 + 6((2a^2 \\
& b^2 - a b^3 - b^4) \cosh(fx + e)^5 + 2(2a^3 b - 2a^2 b^2 - a b^3 + b^4) \\
& * \cosh(fx + e)^3 - (2a^3 b - 2a^2 b^2 - a b^3 + b^4) \cosh(fx + e)) \sinh(
\end{aligned}$$


```
f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b^5*f*cosh
(f*x + e)^8 + 8*a^2*b^5*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^5*f*sinh(f*
x + e)^8 + a^2*b^5*f + 4*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^6 + 4*(7*a^2
*b^5*f*cosh(f*x + e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^
4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*a^2*b^5*f*cosh(f*x
+ e)^3 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a
^2*b^5*f*cosh(f*x + e)^4 + 30*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^2 + (8*
a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b^4 - a^2*b^
5)*f*cosh(f*x + e)^2 + 8*(7*a^2*b^5*f*cosh(f*x + e)^5 + 10*(2*a^3*b^4 - a^2
*b^5)*f*cosh(f*x + e)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x +
e))*sinh(f*x + e)^3 + 4*(7*a^2*b^5*f*cosh(f*x + e)^6 + 15*(2*a^3*b^4 - a^2*
b^5)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x +
e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f*x + e)^2 + 8*(a^2*b^5*f*cosh(f*x +
e)^7 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 + (8*a^4*b^3 - 8*a^3*b^4 +
3*a^2*b^5)*f*cosh(f*x + e)^3 + (2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e))*sinh
(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^5(fx + e)}{\left(b \sinh^2(fx + e) + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

$$3.392 \quad \int \frac{\cosh^3(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=73

$$\frac{2 \sinh(e+fx)}{3a^2 f \sqrt{a+b \sinh^2(e+fx)}} + \frac{\sinh(e+fx) \cosh^2(e+fx)}{3af (a+b \sinh^2(e+fx))^{3/2}}$$

[Out] (Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*Sinh[e + f*x])/(3*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.0929841, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3190, 378, 191}

$$\frac{2 \sinh(e+fx)}{3a^2 f \sqrt{a+b \sinh^2(e+fx)}} + \frac{\sinh(e+fx) \cosh^2(e+fx)}{3af (a+b \sinh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] (Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*Sinh[e + f*x])/(3*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol]
:> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^(m - 1)/2*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

$\text{Int}[(a + (b \cdot x)^n)^{p+1}, x_Symbol] \rightarrow \text{Simp}[(x \cdot (a + b \cdot x^n)^{p+1})/a, x] /; \text{FreeQ}\{a, b, n, p\}, x] \ \&\& \ \text{EqQ}[1/n + p + 1, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^{5/2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\cosh^2(e + fx) \sinh(e + fx)}{3af (a + b \sinh^2(e + fx))^{3/2}} + \frac{2 \text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}} dx, x, \sinh(e + fx)\right)}{3af} \\ &= \frac{\cosh^2(e + fx) \sinh(e + fx)}{3af (a + b \sinh^2(e + fx))^{3/2}} + \frac{2 \sinh(e + fx)}{3a^2 f \sqrt{a + b \sinh^2(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.105302, size = 50, normalized size = 0.68

$$\frac{(a + 2b) \sinh^3(e + fx) + 3a \sinh(e + fx)}{3a^2 f (a + b \sinh^2(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]

[Out] (3*a*Sinh[e + f*x] + (a + 2*b)*Sinh[e + f*x]^3)/(3*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2))

Maple [C] time = 0.082, size = 65, normalized size = 0.9

$$\frac{1}{f} \int \frac{\cosh^2(fx + e)}{b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^2 + a^2} \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\cosh(f*x+e)^3/(a+b*\sinh(f*x+e)^2)^{(5/2)},x)$

[Out] $\text{\`int/indef0}(\cosh(f*x+e)^2/(b^2*\sinh(f*x+e)^4+2*a*b*\sinh(f*x+e)^2+a^2)/(a+b*\sinh(f*x+e)^2)^{(1/2),\sinh(f*x+e))/f$

Maxima [B] time = 1.75748, size = 1251, normalized size = 17.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\cosh(f*x+e)^3/(a+b*\sinh(f*x+e)^2)^{(5/2)},x, \text{algorithm}=\text{"maxima"})$

[Out]
$$\begin{aligned} & -1/12*(b^4*e^{(-10*f*x - 10*e)} - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^{(-2*f*x - 2*e)} + 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^{(-4*f*x - 4*e)} + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^{(-6*f*x - 6*e)} + 5*(2*a*b^3 - b^4)*e^{(-8*f*x - 8*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f}) + 1/4*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^{(-2*f*x - 2*e)} + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^{(-4*f*x - 4*e)} + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^{(-6*f*x - 6*e)} + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^{(-8*f*x - 8*e)} + (2*a*b^3 - b^4)*e^{(-10*f*x - 10*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f}) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^{(-2*f*x - 2*e)} + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^{(-4*f*x - 4*e)} + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^{(-6*f*x - 6*e)} + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^{(-8*f*x - 8*e)} + (2*a^2*b^2 - 2*a*b^3 + b^4)*e^{(-10*f*x - 10*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f}) + 1/12*(b^4 + 5*(2*a*b^3 - b^4)*e^{(-2*f*x - 2*e)} + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^{(-4*f*x - 4*e)} + 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^{(-6*f*x - 6*e)} - (16*a^4 - 32*a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^{(-8*f*x - 8*e)} - (4*a^3*b - 6*a^2*b^2 + b^4)*e^{(-10*f*x - 10*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f}) \end{aligned}$$

Fricas [B] time = 3.10231, size = 2267, normalized size = 31.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*((a + 2*b)*cosh(f*x + e)^6 + 6*(a + 2*b)*cosh(f*x + e)*sinh(f*x
+ e)^5 + (a + 2*b)*sinh(f*x + e)^6 + 3*(3*a - 2*b)*cosh(f*x + e)^4 + 3*(5*
(a + 2*b)*cosh(f*x + e)^2 + 3*a - 2*b)*sinh(f*x + e)^4 + 4*(5*(a + 2*b)*cos
h(f*x + e)^3 + 3*(3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a - 2*b)
*cosh(f*x + e)^2 + 3*(5*(a + 2*b)*cosh(f*x + e)^4 + 6*(3*a - 2*b)*cosh(f*x
+ e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 6*((a + 2*b)*cosh(f*x + e)^5 + 2*(3*a
- 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) - a - 2*
b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a^2*b^2*f*cosh(f*x + e
)^8 + 8*a^2*b^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*f*sinh(f*x + e)^8
+ 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^2*f*cosh(f*x + e)^2
+ (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)
*f*cosh(f*x + e)^4 + 8*(7*a^2*b^2*f*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)
*f*cosh(f*x + e))*sinh(f*x + e)^5 + a^2*b^2*f + 2*(35*a^2*b^2*f*cosh(f*x +
e)^4 + 30*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (8*a^4 - 8*a^3*b + 3*a^2*
b^2)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^
2*b^2*f*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (8*a^4
- 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^2*b^2*f*c
osh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^4 - 8*a^
3*b + 3*a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^2
+ 8*(a^2*b^2*f*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^5 +
(8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*f*co
sh(f*x + e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.63833, size = 801, normalized size = 10.97

$$\left(\left(\frac{(a^7 b^2 f^3 - 2 a^6 b^3 f^3 - 2 a^5 b^4 f^3 + 8 a^4 b^5 f^3 - 7 a^3 b^6 f^3 + 2 a^2 b^7 f^3) e^{(2fx+2e)}}{a^8 b^2 f^4 - 4 a^7 b^3 f^4 + 6 a^6 b^4 f^4 - 4 a^5 b^5 f^4 + a^4 b^6 f^4} + \frac{3(3 a^7 b^2 f^3 - 14 a^6 b^3 f^3 + 26 a^5 b^4 f^3 - 24 a^4 b^5 f^3 + 11 a^3 b^6 f^3 - 2 a^2 b^7 f^3)}{a^8 b^2 f^4 - 4 a^7 b^3 f^4 + 6 a^6 b^4 f^4 - 4 a^5 b^5 f^4 + a^4 b^6 f^4} \right) e^{(2fx+2e)} - 2 \right) \frac{3 \left(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 \right)}{3 \left(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] $\frac{1}{3} \left(\left(\frac{(a^7 b^2 f^3 - 2 a^6 b^3 f^3 - 2 a^5 b^4 f^3 + 8 a^4 b^5 f^3 - 7 a^3 b^6 f^3 + 2 a^2 b^7 f^3) e^{(2fx+2e)}}{a^8 b^2 f^4 - 4 a^7 b^3 f^4 + 6 a^6 b^4 f^4 - 4 a^5 b^5 f^4 + a^4 b^6 f^4} + \frac{3(3 a^7 b^2 f^3 - 14 a^6 b^3 f^3 + 26 a^5 b^4 f^3 - 24 a^4 b^5 f^3 + 11 a^3 b^6 f^3 - 2 a^2 b^7 f^3)}{a^8 b^2 f^4 - 4 a^7 b^3 f^4 + 6 a^6 b^4 f^4 - 4 a^5 b^5 f^4 + a^4 b^6 f^4} \right) e^{(2fx+2e)} - 2 \right) \frac{3 \left(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 \right)}{3 \left(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 \right)}$

$$3.393 \quad \int \frac{\cosh(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{5/2}} dx$$

Optimal. Leaf size=65

$$\frac{2 \sinh(e+fx)}{3a^2 f \sqrt{a+b \sinh^2(e+fx)}} + \frac{\sinh(e+fx)}{3af (a+b \sinh^2(e+fx))^{3/2}}$$

[Out] Sinh[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*Sinh[e + f*x])/(3*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.0579766, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3190, 192, 191}

$$\frac{2 \sinh(e+fx)}{3a^2 f \sqrt{a+b \sinh^2(e+fx)}} + \frac{\sinh(e+fx)}{3af (a+b \sinh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] Sinh[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*Sinh[e + f*x])/(3*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3190

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 192

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cosh(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^{5/2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\sinh(e + fx)}{3af (a + b \sinh^2(e + fx))^{3/2}} + \frac{2 \text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}} dx, x, \sinh(e + fx)\right)}{3af} \\ &= \frac{\sinh(e + fx)}{3af (a + b \sinh^2(e + fx))^{3/2}} + \frac{2 \sinh(e + fx)}{3a^2 f \sqrt{a + b \sinh^2(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.0449284, size = 47, normalized size = 0.72

$$\frac{\sinh(e + fx) (3a + 2b \sinh^2(e + fx))}{3a^2 f (a + b \sinh^2(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] (Sinh[e + f*x]*(3*a + 2*b*Sinh[e + f*x]^2))/(3*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2))

Maple [A] time = 0.011, size = 56, normalized size = 0.9

$$\frac{1}{f} \left(\frac{\sinh(fx + e)}{3a} (a + b (\sinh(fx + e))^2)^{-\frac{3}{2}} + \frac{2 \sinh(fx + e)}{3a^2} \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2), x)

[Out] $1/f*(1/3*\sinh(f*x+e)/a/(a+b*\sinh(f*x+e)^2)^{(3/2)}+2/3/a^2*\sinh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)})$

Maxima [B] time = 1.6551, size = 655, normalized size = 10.08

$$\frac{2a^2b^2 - 2ab^3 + b^4 + 5(4a^3b - 6a^2b^2 + 4ab^3 - b^4)e^{(-2fx-2e)} + 2(24a^4 - 48a^3b + 49a^2b^2 - 25ab^3 + 5b^4)e^{(-4fx-4e)} + 3(a^4 - 2a^3b + a^2b^2)(2(2a - b)e^{(-2fx-2e)} + \dots)}{3(a^4 - 2a^3b + a^2b^2)(2(2a - b)e^{(-2fx-2e)} + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] $1/3*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^{(-2*f*x - 2*e)} + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^{(-4*f*x - 4*e)} + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^{(-6*f*x - 6*e)} + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^{(-8*f*x - 8*e)} + (2*a*b^3 - b^4)*e^{(-10*f*x - 10*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f} - 1/3*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^{(-2*f*x - 2*e)} + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^{(-4*f*x - 4*e)} + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^{(-6*f*x - 6*e)} + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^{(-8*f*x - 8*e)} + (2*a^2*b^2 - 2*a*b^3 + b^4)*e^{(-10*f*x - 10*e)})/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^{(-2*f*x - 2*e)} + b*e^{(-4*f*x - 4*e)} + b)^{(5/2)*f})$

Fricas [B] time = 3.09934, size = 2161, normalized size = 33.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")`

[Out] $2/3*\sqrt{2}*(b*\cosh(f*x + e)^6 + 6*b*\cosh(f*x + e)*\sinh(f*x + e)^5 + b*\sinh(f*x + e)^6 + 3*(2*a - b)*\cosh(f*x + e)^4 + 3*(5*b*\cosh(f*x + e)^2 + 2*a - b)*\sinh(f*x + e)^4 + 4*(5*b*\cosh(f*x + e)^3 + 3*(2*a - b)*\cosh(f*x + e))*\sinh(f*x + e)^3 - 3*(2*a - b)*\cosh(f*x + e)^2 + 3*(5*b*\cosh(f*x + e)^4 + 6*(2*a - b)*\cosh(f*x + e)^2 - 2*a + b)*\sinh(f*x + e)^2 + 6*(b*\cosh(f*x + e)^5 + 2*(2*a - b)*\cosh(f*x + e)^3 - (2*a - b)*\cosh(f*x + e))*\sinh(f*x + e) - b)*$

$$\sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)} / (a^2 b^2 f \cosh(fx + e)^8 + 8 a^2 b^2 f \cosh(fx + e) \sinh(fx + e)^7 + a^2 b^2 f \sinh(fx + e)^8 + 4(2 a^3 b - a^2 b^2) f \cosh(fx + e)^6 + 4(7 a^2 b^2 f \cosh(fx + e)^2 + (2 a^3 b - a^2 b^2) f) \sinh(fx + e)^6 + 2(8 a^4 - 8 a^3 b + 3 a^2 b^2) f \cosh(fx + e)^4 + 8(7 a^2 b^2 f \cosh(fx + e)^3 + 3(2 a^3 b - a^2 b^2) f \cosh(fx + e)) \sinh(fx + e)^5 + a^2 b^2 f + 2(35 a^2 b^2 f \cosh(fx + e)^4 + 30(2 a^3 b - a^2 b^2) f \cosh(fx + e)^2 + (8 a^4 - 8 a^3 b + 3 a^2 b^2) f) \sinh(fx + e)^4 + 4(2 a^3 b - a^2 b^2) f \cosh(fx + e)^2 + 8(7 a^2 b^2 f \cosh(fx + e)^5 + 10(2 a^3 b - a^2 b^2) f \cosh(fx + e)^3 + (8 a^4 - 8 a^3 b + 3 a^2 b^2) f \cosh(fx + e)) \sinh(fx + e)^3 + 4(7 a^2 b^2 f \cosh(fx + e)^6 + 15(2 a^3 b - a^2 b^2) f \cosh(fx + e)^4 + 3(8 a^4 - 8 a^3 b + 3 a^2 b^2) f \cosh(fx + e)^2 + (2 a^3 b - a^2 b^2) f) \sinh(fx + e)^2 + 8(a^2 b^2 f \cosh(fx + e)^7 + 3(2 a^3 b - a^2 b^2) f \cosh(fx + e)^5 + (8 a^4 - 8 a^3 b + 3 a^2 b^2) f \cosh(fx + e)^3 + (2 a^3 b - a^2 b^2) f \cosh(fx + e)) \sinh(fx + e))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [B] time = 1.69479, size = 702, normalized size = 10.8

$$2 \left(\left(\frac{(a^6 b^3 f - 4 a^5 b^4 f + 6 a^4 b^5 f - 4 a^3 b^6 f + a^2 b^7 f) e^{(2fx+2e)}}{a^8 b^2 f^2 - 4 a^7 b^3 f^2 + 6 a^6 b^4 f^2 - 4 a^5 b^5 f^2 + a^4 b^6 f^2} + \frac{3(2 a^7 b^2 f - 9 a^6 b^3 f + 16 a^5 b^4 f - 14 a^4 b^5 f + 6 a^3 b^6 f - a^2 b^7 f)}{a^8 b^2 f^2 - 4 a^7 b^3 f^2 + 6 a^6 b^4 f^2 - 4 a^5 b^5 f^2 + a^4 b^6 f^2} \right) e^{(2fx+2e)} - \frac{3(2 a^7 b^2 f - 9 a^6 b^3 f + 16 a^5 b^4 f - 14 a^4 b^5 f + 6 a^3 b^6 f - a^2 b^7 f)}{a^8 b^2 f^2 - 4 a^7 b^3 f^2 + 6 a^6 b^4 f^2 - 4 a^5 b^5 f^2 + a^4 b^6 f^2} \right) \frac{3 \left(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 b e^{(2fx+2e)} + b \right)^{\frac{3}{2}}}{3 \left(b e^{(4fx+4e)} + 4 a e^{(2fx+2e)} - 2 b e^{(2fx+2e)} + b \right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] 2/3*(((a^6*b^3*f - 4*a^5*b^4*f + 6*a^4*b^5*f - 4*a^3*b^6*f + a^2*b^7*f)*e^(2*f*x + 2*e))/(a^8*b^2*f^2 - 4*a^7*b^3*f^2 + 6*a^6*b^4*f^2 - 4*a^5*b^5*f^2

$$\begin{aligned}
& + a^4 b^6 f^2) + 3(2a^7 b^2 f - 9a^6 b^3 f + 16a^5 b^4 f - 14a^4 b^5 f \\
& + 6a^3 b^6 f - a^2 b^7 f)/(a^8 b^2 f^2 - 4a^7 b^3 f^2 + 6a^6 b^4 f^2 - \\
& 4a^5 b^5 f^2 + a^4 b^6 f^2))e^{(2fx + 2e)} - 3(2a^7 b^2 f - 9a^6 b^3 f \\
& + 16a^5 b^4 f - 14a^4 b^5 f + 6a^3 b^6 f - a^2 b^7 f)/(a^8 b^2 f^2 - 4 \\
& a^7 b^3 f^2 + 6a^6 b^4 f^2 - 4a^5 b^5 f^2 + a^4 b^6 f^2))e^{(2fx + 2e)} \\
& - (a^6 b^3 f - 4a^5 b^4 f + 6a^4 b^5 f - 4a^3 b^6 f + a^2 b^7 f)/(a^8 b^2 f^2 - 4a^7 b^3 f^2 + 6a^6 b^4 f^2 - 4a^5 b^5 f^2 + a^4 b^6 f^2))/(b e^{(4fx + 4e)} + 4a e^{(2fx + 2e)} - 2b e^{(2fx + 2e)} + b)^{(3/2)} + 2/3/(a^2 \sqrt{b} f)
\end{aligned}$$

$$3.394 \quad \int \frac{\operatorname{sech}(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=134

$$-\frac{b(5a-2b) \sinh(e+fx)}{3a^2 f(a-b)^2 \sqrt{a+b \sinh^2(e+fx)}} - \frac{b \sinh(e+fx)}{3af(a-b)(a+b \sinh^2(e+fx))^{3/2}} + \frac{\tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(5/2)*f) - (b*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - ((5*a - 2*b)*b*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.148909, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3190, 414, 527, 12, 377, 203}

$$-\frac{b(5a-2b) \sinh(e+fx)}{3a^2 f(a-b)^2 \sqrt{a+b \sinh^2(e+fx)}} - \frac{b \sinh(e+fx)}{3af(a-b)(a+b \sinh^2(e+fx))^{3/2}} + \frac{\tan^{-1}\left(\frac{\sqrt{a-b} \sinh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(5/2)*f) - (b*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - ((5*a - 2*b)*b*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{b\sinh(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\operatorname{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{3a(a-b)f} \\
&= -\frac{b\sinh(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(e+fx)\right)}{3a(a-b)f} \\
&= -\frac{b\sinh(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(e+fx)\right)}{3a(a-b)f} \\
&= -\frac{b\sinh(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(e+fx)\right)}{3a(a-b)f} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\sinh(e+fx)}{3a(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 9.30254, size = 1331, normalized size = 9.93

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] (Sech[e + f*x]*Tanh[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] + (2100*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2)/a + (840*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^4)/a^2 - (3150*(a - b)*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^2)/a - (4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2*Tanh[e + f*x]^2)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^4*Tanh[e + f*x]^2)/a^3 + (1575*(a - b)^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^4)/a^2 + (2100*(a - b)^2*b*ArcS

```

in[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2*Tanh[e + f*x]^4/a^3
+ (840*(a - b)^2*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]
]^4*Tanh[e + f*x]^4/a^4 + 2100*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + (2800*b*Sinh[e + f*x]^2*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(
3/2))/a + (1120*b^2*Sinh[e + f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f
*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))/a^2 + 96*Hypergeometric2F1[
2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 24*HypergeometricPFQ[{
2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a +
b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + (168*b*Hyperg
eometric2F1[2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7
/2))/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x
]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*((
(a - b)*Tanh[e + f*x]^2)/a)^(7/2))/a + (72*b^2*Hypergeometric2F1[2, 2, 9/2
, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2))/a^2 + (24*b^2*Hy
pergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e +
f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e +
f*x]^2)/a)^(7/2))/a^2 - 1575*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e +
f*x]^2)*Tanh[e + f*x]^2)/a^2] - (2100*b*Sinh[e + f*x]^2*Sqrt[((a - b)*Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2])/a - (840*b^2*Sinh
[e + f*x]^4*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e +
f*x]^2)/a^2])/a^2)/(315*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e +
f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(1 + (b*Sinh[e + f*x]^2)/a)*(((a - b)*Ta
nh[e + f*x]^2)/a)^(5/2))

```

Maple [C] time = 0.133, size = 169, normalized size = 1.3

$$\frac{1}{f} \int \frac{-b^2 (\sinh(fx + e))^4 - 2ab (\sinh(fx + e))^3 - 2a^2b^2 (\sinh(fx + e))^2 - 2a^3b^3 (\sinh(fx + e)) - 2a^4b^4}{-b^4 (\sinh(fx + e))^{10} + (-4ab^3 - b^4) (\sinh(fx + e))^8 + (-6a^2b^2 - 4ab^3) (\sinh(fx + e))^6 + (-4a^3b - 6a^2b^2) (\sinh(fx + e))^4 + (-a^4 - 4a^3b) (\sinh(fx + e))^2 - a^4}{(a + b \sinh(fx + e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] `int/indef0`((-b^2*sinh(f*x+e)^4-2*a*b*sinh(f*x+e)^2-a^2)/(-b^4*sinh(f*x+e)^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*x+e)^6+(-4*a^3*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4)/(a+b*sinh(f*x+e))^2)

$+e)^2)^{(1/2)}, \sinh(f*x+e))/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx + e)}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [B] time = 5.13512, size = 12320, normalized size = 91.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/6*(3*(a^2*b^2*\cosh(f*x + e)^8 + 8*a^2*b^2*\cosh(f*x + e)*\sinh(f*x + e)^7 \\ & + a^2*b^2*\sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*\cosh(f*x + e)^6 + 4*(7*a^2*b^2* \\ & \cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*\sinh(f*x + e)^6 + 8*(7*a^2*b^2* \\ & \cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2* \\ & (8*a^4 - 8*a^3*b + 3*a^2*b^2)*\cosh(f*x + e)^4 + 2*(35*a^2*b^2*\cosh(f*x + e) \\ & ^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*\cosh(f*x + e)^2)* \\ & \sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*\cosh(f*x + e)^5 + 10*(2*a^3*b - a^2* \\ & b^2)*\cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*\cosh(f*x + e))*\sinh(\\ & f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*\cosh(f*x + e)^2 + 4*(7*a^2*b^2*\cosh(f*x \\ & + e)^6 + 15*(2*a^3*b - a^2*b^2)*\cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8* \\ & a^4 - 8*a^3*b + 3*a^2*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 8*(a^2*b^2*\cosh \\ & (f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*\cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b + \\ & 3*a^2*b^2)*\cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*\cosh(f*x + e))*\sinh(f*x + \\ & e))*\sqrt{-a + b}*\log(((a - 2*b)*\cosh(f*x + e)^4 + 4*(a - 2*b)*\cosh(f*x + e) \\ & *\sinh(f*x + e)^3 + (a - 2*b)*\sinh(f*x + e)^4 - 2*(3*a - 2*b)*\cosh(f*x + e)^2 \\ & + 2*(3*(a - 2*b)*\cosh(f*x + e)^2 - 3*a + 2*b)*\sinh(f*x + e)^2 - 2*\sqrt{2} \\ & *(\cosh(f*x + e)^2 + 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2 - 1))*\sqrt{2} \end{aligned}$$

$$\begin{aligned}
& 5) * f * \cosh(f*x + e)) * \sinh(f*x + e)), \frac{1}{3} * (3 * (a^2 * b^2 * \cosh(f*x + e))^8 + 8 * a^2 * b^2 * \cosh(f*x + e) * \sinh(f*x + e)^7 + a^2 * b^2 * \sinh(f*x + e)^8 + 4 * (2 * a^3 * b - a^2 * b^2) * \cosh(f*x + e)^6 + 4 * (7 * a^2 * b^2 * \cosh(f*x + e)^2 + 2 * a^3 * b - a^2 * b^2) * \sinh(f*x + e)^6 + 8 * (7 * a^2 * b^2 * \cosh(f*x + e)^3 + 3 * (2 * a^3 * b - a^2 * b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^5 + 2 * (8 * a^4 - 8 * a^3 * b + 3 * a^2 * b^2) * \cosh(f*x + e)^4 + 2 * (35 * a^2 * b^2 * \cosh(f*x + e)^4 + 8 * a^4 - 8 * a^3 * b + 3 * a^2 * b^2 + 30 * (2 * a^3 * b - a^2 * b^2) * \cosh(f*x + e)^2) * \sinh(f*x + e)^4 + a^2 * b^2 + 8 * (7 * a^2 * b^2 * \cosh(f*x + e)^5 + 10 * (2 * a^3 * b - a^2 * b^2) * \cosh(f*x + e)^3 + (8 * a^4 - 8 * a^3 * b + 3 * a^2 * b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^3 + 4 * (2 * a^3 * b - a^2 * b^2) * \cosh(f*x + e)^2 + 4 * (7 * a^2 * b^2 * \cosh(f*x + e)^6 + 15 * (2 * a^3 * b - a^2 * b^2) * \cosh(f*x + e)^4 + 2 * a^3 * b - a^2 * b^2 + 3 * (8 * a^4 - 8 * a^3 * b + 3 * a^2 * b^2) * \cosh(f*x + e)^2) * \sinh(f*x + e)^2 + 8 * (a^2 * b^2 * \cosh(f*x + e)^7 + 3 * (2 * a^3 * b - a^2 * b^2) * \cosh(f*x + e)^5 + (8 * a^4 - 8 * a^3 * b + 3 * a^2 * b^2) * \cosh(f*x + e)^3 + (2 * a^3 * b - a^2 * b^2) * \cosh(f*x + e)) * \sinh(f*x + e)) * \sqrt{a - b} * \arctan(\sqrt{2} * (\cosh(f*x + e)^2 + 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2 - 1)) * \sqrt{a - b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2 * a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)}) / (b * \cosh(f*x + e)^4 + 4 * b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2 * (2 * a - b) * \cosh(f*x + e)^2 + 2 * (3 * b * \cosh(f*x + e)^2 + 2 * a - b) * \sinh(f*x + e)^2 + 4 * (b * \cosh(f*x + e)^3 + (2 * a - b) * \cosh(f*x + e)) * \sinh(f*x + e) + b)) - \sqrt{2} * ((5 * a^2 * b^2 - 7 * a * b^3 + 2 * b^4) * \cosh(f*x + e)^6 + 6 * (5 * a^2 * b^2 - 7 * a * b^3 + 2 * b^4) * \cosh(f*x + e) * \sinh(f*x + e)^5 + (5 * a^2 * b^2 - 7 * a * b^3 + 2 * b^4) * \sinh(f*x + e)^6 + 3 * (8 * a^3 * b - 17 * a^2 * b^2 + 11 * a * b^3 - 2 * b^4) * \cosh(f*x + e)^4 + 3 * (8 * a^3 * b - 17 * a^2 * b^2 + 11 * a * b^3 - 2 * b^4 + 5 * (5 * a^2 * b^2 - 7 * a * b^3 + 2 * b^4) * \cosh(f*x + e)^2) * \sinh(f*x + e)^4 - 5 * a^2 * b^2 + 7 * a * b^3 - 2 * b^4 + 4 * (5 * (5 * a^2 * b^2 - 7 * a * b^3 + 2 * b^4) * \cosh(f*x + e)^3 + 3 * (8 * a^3 * b - 17 * a^2 * b^2 + 11 * a * b^3 - 2 * b^4) * \cosh(f*x + e)) * \sinh(f*x + e)^3 - 3 * (8 * a^3 * b - 17 * a^2 * b^2 + 11 * a * b^3 - 2 * b^4) * \cosh(f*x + e)^2 + 3 * (5 * (5 * a^2 * b^2 - 7 * a * b^3 + 2 * b^4) * \cosh(f*x + e)^4 - 8 * a^3 * b + 17 * a^2 * b^2 - 11 * a * b^3 + 2 * b^4 + 6 * (8 * a^3 * b - 17 * a^2 * b^2 + 11 * a * b^3 - 2 * b^4) * \cosh(f*x + e)^2) * \sinh(f*x + e)^2 + 6 * ((5 * a^2 * b^2 - 7 * a * b^3 + 2 * b^4) * \cosh(f*x + e)^5 + 2 * (8 * a^3 * b - 17 * a^2 * b^2 + 11 * a * b^3 - 2 * b^4) * \cosh(f*x + e)^3 - (8 * a^3 * b - 17 * a^2 * b^2 + 11 * a * b^3 - 2 * b^4) * \cosh(f*x + e)) * \sinh(f*x + e)) * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2 * a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2))} / ((a^5 * b^2 - 3 * a^4 * b^3 + 3 * a^3 * b^4 - a^2 * b^5) * f * \cosh(f*x + e)^8 + 8 * (a^5 * b^2 - 3 * a^4 * b^3 + 3 * a^3 * b^4 - a^2 * b^5) * f * \cosh(f*x + e) * \sinh(f*x + e)^7 + (a^5 * b^2 - 3 * a^4 * b^3 + 3 * a^3 * b^4 - a^2 * b^5) * f * \sinh(f*x + e)^8 + 4 * (2 * a^6 * b - 7 * a^5 * b^2 + 9 * a^4 * b^3 - 5 * a^3 * b^4 + a^2 * b^5) * f * \cosh(f*x + e)^6 + 4 * (7 * (a^5 * b^2 - 3 * a^4 * b^3 + 3 * a^3 * b^4 - a^2 * b^5) * f * \cosh(f*x + e)^2 + (2 * a^6 * b - 7 * a^5 * b^2 + 9 * a^4 * b^3 - 5 * a^3 * b^4 + a^2 * b^5) * f) * \sinh(f*x + e)^6 + 2 * (8 * a^7 - 32 * a^6 * b + 51 * a^5 * b^2 - 41 * a^4 * b^3 + 17 * a^3 * b^4 - 3 * a^2 * b^5) * f * \cosh(f*x + e)^4 + 8 * (7 * (a^5 * b^2 - 3 * a^4 * b^3 + 3 * a^3 * b^4 - a^2 * b^5) * f * \cosh(f*x + e)^3 + 3 * (2 * a^6 * b - 7 * a^5 * b^2 + 9 * a^4 * b^3 - 5 * a^3 * b^4 + a^2 * b^5) * f * \cosh(f*x + e)) * \sinh(f*x + e)^5 + 2 * (35 * (a^5 * b^2 - 3 * a^4 * b^3 + 3 * a^3 * b^4 - a^2 * b^5) * f * \cosh(f*x + e)^4 + 30 * (2 * a^6 * b - 7 * a^5 * b^2 + 9 * a^4 * b^3 - 5 * a^3 * b^4 + a^2 * b^5) * f * \cosh(f*x + e)^2 + (8 * a^7 - 32 * a
\end{aligned}$$

$$\begin{aligned} &^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f)*\sinh(f*x + e)^4 \\ &+ 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*\cosh(f*x + e) \\ &)^2 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*\cosh(f*x + e)^5 + \\ &10*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*\cosh(f*x + e)^ \\ &3 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f \\ &*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a \\ &^2*b^5)*f*\cosh(f*x + e)^6 + 15*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 \\ &+ a^2*b^5)*f*\cosh(f*x + e)^4 + 3*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b \\ &^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*\cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a \\ &^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f)*\sinh(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3 \\ &*a^3*b^4 - a^2*b^5)*f + 8*((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*\co \\ &sh(f*x + e)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f \\ &*\cosh(f*x + e)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 \\ &- 3*a^2*b^5)*f*\cosh(f*x + e)^3 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3* \\ &b^4 + a^2*b^5)*f*\cosh(f*x + e))*\sinh(f*x + e))] \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [B] time = 3.25782, size = 1831, normalized size = 13.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out]
$$-1/3*(5*a*b - 2*b^2)/(a^4*\sqrt{b}*f - 2*a^3*b^{(3/2)}*f + a^2*b^{(5/2)}*f) - 1/3*(((5*a^{13}*b^4*f^3 - 52*a^{12}*b^5*f^3 + 245*a^{11}*b^6*f^3 - 690*a^{10}*b^7*f^3 + 1290*a^9*b^8*f^3 - 1680*a^8*b^9*f^3 + 1554*a^7*b^{10}*f^3 - 1020*a^6*b^{11}*f^3 + 465*a^5*b^{12}*f^3 - 140*a^4*b^{13}*f^3 + 25*a^3*b^{14}*f^3 - 2*a^2*b^{15}*f^3)*e^{(2*f*x + 2*e)})/(a^{16}*b^2*f^4 - 12*a^{15}*b^3*f^4 + 66*a^{14}*b^4*f^4 - 220$$

$$\begin{aligned}
& a^{13}b^5f^4 + 495a^{12}b^6f^4 - 792a^{11}b^7f^4 + 924a^{10}b^8f^4 - 792a^9b^9f^4 + 495a^8b^{10}f^4 - 220a^7b^{11}f^4 + 66a^6b^{12}f^4 - 12a^5b^{13}f^4 + a^4b^{14}f^4) + 3(8a^{14}b^3f^3 - 89a^{13}b^4f^3 + 452a^{12}b^5f^3 - 1385a^{11}b^6f^3 + 2850a^{10}b^7f^3 - 4146a^9b^8f^3 + 4368a^8b^9f^3 - 3354a^7b^{10}f^3 + 1860a^6b^{11}f^3 - 725a^5b^{12}f^3 + 188a^4b^{13}f^3 - 29a^3b^{14}f^3 + 2a^2b^{15}f^3)/(a^{16}b^2f^4 - 12a^{15}b^3f^4 + 66a^{14}b^4f^4 - 220a^{13}b^5f^4 + 495a^{12}b^6f^4 - 792a^{11}b^7f^4 + 924a^{10}b^8f^4 - 792a^9b^9f^4 + 495a^8b^{10}f^4 - 220a^7b^{11}f^4 + 66a^6b^{12}f^4 - 12a^5b^{13}f^4 + a^4b^{14}f^4))e^{(2fx + 2e)} - 3(8a^{14}b^3f^3 - 89a^{13}b^4f^3 + 452a^{12}b^5f^3 - 1385a^{11}b^6f^3 + 2850a^{10}b^7f^3 - 4146a^9b^8f^3 + 4368a^8b^9f^3 - 3354a^7b^{10}f^3 + 1860a^6b^{11}f^3 - 725a^5b^{12}f^3 + 188a^4b^{13}f^3 - 29a^3b^{14}f^3 + 2a^2b^{15}f^3)/(a^{16}b^2f^4 - 12a^{15}b^3f^4 + 66a^{14}b^4f^4 - 220a^{13}b^5f^4 + 495a^{12}b^6f^4 - 792a^{11}b^7f^4 + 924a^{10}b^8f^4 - 792a^9b^9f^4 + 495a^8b^{10}f^4 - 220a^7b^{11}f^4 + 66a^6b^{12}f^4 - 12a^5b^{13}f^4 + a^4b^{14}f^4))e^{(2fx + 2e)} - (5a^{13}b^4f^3 - 52a^{12}b^5f^3 + 245a^{11}b^6f^3 - 690a^{10}b^7f^3 + 1290a^9b^8f^3 - 1680a^8b^9f^3 + 1554a^7b^{10}f^3 - 1020a^6b^{11}f^3 + 465a^5b^{12}f^3 - 140a^4b^{13}f^3 + 25a^3b^{14}f^3 - 2a^2b^{15}f^3)/(a^{16}b^2f^4 - 12a^{15}b^3f^4 + 66a^{14}b^4f^4 - 220a^{13}b^5f^4 + 495a^{12}b^6f^4 - 792a^{11}b^7f^4 + 924a^{10}b^8f^4 - 792a^9b^9f^4 + 495a^8b^{10}f^4 - 220a^7b^{11}f^4 + 66a^6b^{12}f^4 - 12a^5b^{13}f^4 + a^4b^{14}f^4))/(b^{4fx + 4e} + 4ae^{(2fx + 2e)} - 2be^{(2fx + 2e)} + b)^{(3/2)} + 2\arctan(-1/2*(\sqrt{b})e^{(2fx + 2e)} - \sqrt{be^{(4fx + 4e)} + 4ae^{(2fx + 2e)}} - 2be^{(2fx + 2e)} + b) + \sqrt{b})/\sqrt{a - b})/((a^2f - 2abf + b^2f)*\sqrt{a - b})
\end{aligned}$$

$$3.395 \quad \int \frac{\cosh^6(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=330

$$\frac{(4a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^2b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{(8a^2-3ab-2b^2)\tanh(e+fx)\sqrt{a}}{3a^2b^3f}$$

```
[Out] -((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(a - b)*(2*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((4*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((8*a^2 - 3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*b^3*f)
```

Rubi [A] time = 0.331467, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3192, 413, 526, 531, 418, 492, 411}

$$\frac{(8a^2-3ab-2b^2)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2b^3f} - \frac{2(a-b)(2a+b)\sinh(e+fx)\cosh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^2b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(a - b)*(2*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((4*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((8*a^2 - 3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*b^3*f)
```

Rule 3192

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 526

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p + 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && GtQ[q, 0]

Rule 531

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -

$a*d, 0] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c] \&\& !\text{SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^{(3/2)}, x_Symbol] \text{ :> Sim}$
 $p[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[$
 $d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /;$ FreeQ
 $[a, b, c, d], x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\int \frac{\cosh^6(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx = \frac{\left(\sqrt{\cosh^2(e+fx)\text{sech}(e+fx)}\right) \text{Subst}\left(\int \frac{(1+x^2)^{5/2}}{(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)\text{sech}(e+fx)}\right) \text{Subst}\left(\int \frac{\sqrt{1+x^2}}{\dots} dx, x, \sinh(e+fx)\right)}{3abf}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} - \dots$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \dots$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \dots$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \dots$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} - \dots$$

Mathematica [C] time = 2.13341, size = 206, normalized size = 0.62

$$\frac{\frac{1}{2}(a-b)\left(-2\sqrt{2}b\sinh(2(e+fx))(8a^2+b(5a+2b)\cosh(2(e+fx))+ab-2b^2)+4ia^2(8a+b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\right)}{6a^2b^3f(2a+b\cosh(2(e+fx))-b)}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]

[Out]
$$\frac{((-2*I)*a^2*(8*a^2 - 3*a*b - 2*b^2)*((2*a - b + b*\text{Cosh}[2*(e + f*x)]))/a)^{(3/2)}* \text{EllipticE}[I*(e + f*x), b/a] + ((a - b)*((4*I)*a^2*(8*a + b)*((2*a - b + b*\text{Cosh}[2*(e + f*x)]))/a)^{(3/2)}* \text{EllipticF}[I*(e + f*x), b/a] - 2*\text{Sqrt}[2]*b*(8*a^2 + a*b - 2*b^2 + b*(5*a + 2*b)*\text{Cosh}[2*(e + f*x)])*\text{Sinh}[2*(e + f*x)]/2}{(6*a^2*b^3*f*(2*a - b + b*\text{Cosh}[2*(e + f*x)])^{(3/2)}}$$

Maple [B] time = 0.133, size = 812, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out]
$$\begin{aligned} & -1/3*((5*(-1/a*b)^{(1/2)}*a^2*b-3*(-1/a*b)^{(1/2)}*a*b^2-2*(-1/a*b)^{(1/2)}*b^3)* \\ & \sinh(f*x+e)*\cosh(f*x+e)^4+(4*(-1/a*b)^{(1/2)}*a^3-6*(-1/a*b)^{(1/2)}*a^2*b+2*(-1/a*b)^{(1/2)}*b^3)* \\ & \cosh(f*x+e)^2*\sinh(f*x+e)+(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}* \\ & b*(4*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)}))*a^2-2*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & a*b-2*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*b^2-8*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & a^2+3*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})*a*b+2*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & b^2*\cosh(f*x+e)^2+4*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & a^3-6*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & a^2*b+2*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & b^3-8*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & a^3+11*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & a^2*b-(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & a*b^2-2*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)}, (a/b)^{(1/2)})* \\ & b^3/a^2/(a+b*\sinh(f*x+e)^2)^{(3/2)}/(-1/a*b)^{(1/2)}/b^2/\cosh(f*x+e)/f \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^6}{(b \sinh(fx + e)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh(fx + e)^2 + a} \cosh(fx + e)^6}{b^3 \sinh(fx + e)^6 + 3ab^2 \sinh(fx + e)^4 + 3a^2b \sinh(fx + e)^2 + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^6}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

$$3.396 \quad \int \frac{\cosh^4(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=223

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3a^2bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{2(a+b) \cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right)}{3a^{3/2}b^{3/2}f\sqrt{a+b \sinh^2(e+fx)}\sqrt{\frac{a \cosh^2(e+fx)}{a+b \sinh^2(e+fx)}}$$

```
[Out] -((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b*f*Sqrt[(Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rubi [A] time = 0.210649, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3192, 413, 525, 418, 411}

$$\frac{2(a+b) \cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right)\left|1-\frac{a}{b}\right|}{3a^{3/2}b^{3/2}f\sqrt{a+b \sinh^2(e+fx)}\sqrt{\frac{a \cosh^2(e+fx)}{a+b \sinh^2(e+fx)}}} - \frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))\right)\left|1-\frac{b}{a}\right|}{3a^2bf\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2), x]
```

```
[Out] -((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b*f*Sqrt[(Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
```

$\text{Cos}[e + f*x]^2]/(f*\text{Cos}[e + f*x]), \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*ff^2*x^2)^p, x], x, \text{Sin}[e + f*x]/ff], x] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[m/2] \&\& \text{!IntegerQ}[p]$

Rule 413

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \rightarrow \text{Simp}[(a*d - c*b)*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)}/(a*b*n*(p+1)), x] - \text{Dist}[1/(a*b*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-2)}*\text{Simp}[c*(a*d - c*b*(n*(p+1) + 1)) + d*(a*d*(n*(q-1) + 1) - b*c*(n*(p+q) + 1))*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& \text{GtQ}[q, 1] \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 525

$\text{Int}[(e_ + (f_)*(x_)^2)/(\text{Sqrt}[a_ + (b_)*(x_)^2]*((c_ + (d_)*(x_)^2)^{(3/2)})), x_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(\text{Sqrt}[a + b*x^2]*\text{Sqrt}[c + d*x^2]), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[\text{Sqrt}[a + b*x^2]/(c + d*x^2)^{(3/2)}, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rule 418

$\text{Int}[1/(\text{Sqrt}[a_ + (b_)*(x_)^2]*\text{Sqrt}[c_ + (d_)*(x_)^2]), x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[d/c] \&\& \text{PosQ}[b/a] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[a_ + (b_)*(x_)^2]/((c_ + (d_)*(x_)^2)^{(3/2)}), x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a+2b+bx^2}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{3abf} \\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{3abf} \\
&= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} + \frac{2(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right)}{3a^{3/2}b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 1.3781, size = 178, normalized size = 0.8

$$\frac{-ia^2(2a+b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2}b\sinh(2(e+fx))(a^2+b(a+b)\cosh(2(e+fx))+2ab)}{3a^2b^2f(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ((2*I)*a^2*(a+b)*((2*a-b+b*Cosh[2*(e+f*x)])/a)^(3/2)*EllipticE[I*(e+f*x), b/a] - I*a^2*(2*a+b)*((2*a-b+b*Cosh[2*(e+f*x)])/a)^(3/2)*EllipticF[I*(e+f*x), b/a] + Sqrt[2]*b*(a^2+2*a*b-b^2+b*(a+b)*Cosh[2*(e+f*x)]*Sinh[2*(e+f*x)])/(3*a^2*b^2*f*(2*a-b+b*Cosh[2*(e+f*x)]))^(3/2))

Maple [B] time = 0.253, size = 597, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] $\frac{1}{3} \left(2(-1/ab)^{1/2}ab + 2(-1/ab)^{1/2}b^2 \right) \sinh(fx+e) \cosh(fx+e)^4 + (-1/ab)^{1/2}a^2 + (-1/ab)^{1/2}ab - 2(-1/ab)^{1/2}b^2 \cosh(fx+e)^2 \sinh(fx+e) + (\cosh(fx+e)^2)^{1/2} (b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} b (a \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + 2b \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) - 2 \operatorname{EllipticE}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2})) a - 2b \operatorname{EllipticE}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2})) \cosh(fx+e)^2 + a^2 (b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + a (b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + b - 2(b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticF}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + b^2 - 2(b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticE}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + a^2 + 2(b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} \operatorname{EllipticE}(\sinh(fx+e) (-1/ab)^{1/2}, (a/b)^{1/2}) + b^2 / a^2 / (a+b \sinh(fx+e)^2)^{3/2} / (-1/ab)^{1/2} / b / \cosh(fx+e) / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx+e)^4}{(b \sinh(fx+e)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{\sqrt{b \sinh(fx+e)^2 + a} \cosh(fx+e)^4}{b^3 \sinh(fx+e)^6 + 3ab^2 \sinh(fx+e)^4 + 3a^2b \sinh(fx+e)^2 + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2), x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^4(fx + e)}{\left(b \sinh^2(fx + e) + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2), x, algorithm="giac")`

[Out] `integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)`

$$3.397 \quad \int \frac{\cosh^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=228

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3a^2 f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{(a-2b) \cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a}}\right)\right)}{3a^{3/2}\sqrt{b}f(a-b)\sqrt{a+b \sinh^2(e+fx)}\sqrt{\frac{a \cosh^2(e+fx)}{a+b \sinh^2(e+fx)}}$$

[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((a - 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*(a - b)*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])

Rubi [A] time = 0.20429, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3192, 412, 525, 418, 411}

$$\frac{(a-2b) \cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b} \sinh(e+fx)}{\sqrt{a}}\right)\right)\left[1-\frac{a}{b}\right]}{3a^{3/2}\sqrt{b}f(a-b)\sqrt{a+b \sinh^2(e+fx)}\sqrt{\frac{a \cosh^2(e+fx)}{a+b \sinh^2(e+fx)}}} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))\right)\left[1-\frac{b}{a}\right]}{3a^2 f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((a - 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*(a - b)*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])

Rule 3192

Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[

$\text{Cos}[e + f*x]^2)/(f*\text{Cos}[e + f*x]), \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*ff^2*x^2)^p, x], x, \text{Sin}[e + f*x]/ff], x] /; \text{FreeQ}\{a, b, e, f, p\}, x$
 $\&\& \text{IntegerQ}[m/2] \&\& \text{!IntegerQ}[p]$

Rule 412

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol]$
 $:= -\text{Simp}[(x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^q)/(a*n*(p+1)), x] + \text{Dist}[1/(a*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)}*\text{Simp}[c*(n*(p+1) + 1) + d*(n*(p+q+1) + 1)*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x$
 $\&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& \text{LtQ}[0, q, 1] \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 525

$\text{Int}[(e_ + (f_)*(x_)^2)/(\text{Sqrt}[a_ + (b_)*(x_)^2]*((c_ + (d_)*(x_)^2)^{(3/2)}), x_Symbol]$
 $:= \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(\text{Sqrt}[a + b*x^2]*\text{Sqrt}[c + d*x^2]), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[\text{Sqrt}[a + b*x^2]/(c + d*x^2)^{(3/2)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rule 418

$\text{Int}[1/(\text{Sqrt}[a_ + (b_)*(x_)^2]*\text{Sqrt}[(c_ + (d_)*(x_)^2]), x_Symbol] := \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{PosQ}[d/c] \&\& \text{PosQ}[b/a] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[a_ + (b_)*(x_)^2]/((c_ + (d_)*(x_)^2)^{(3/2)}, x_Symbol] := \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] /; \text{FreeQ}\{a, b, c, d\}, x \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{-2-x^2}{\sqrt{1+x^2}(a+bx^2)^{3/2}} dx\right)}{3af} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a+bx^2}} dx\right)}{3a(a-b)f} \\
&= \frac{\cosh(e+fx)\sinh(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right)\left[1-\frac{a}{b}\right]}{3a^{3/2}(a-b)\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} +
\end{aligned}$$

Mathematica [C] time = 1.40191, size = 193, normalized size = 0.85

$$\frac{-2ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \sqrt{2}b\sinh(2(e+fx))(-4a^2 - b(a-2b)\cosh(2(e+fx)) + 7a^2)}{6a^2bf(a-b)(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-4*a^2 + 7*a*b - 2*b^2 - (a - 2*b)*b*Cosh[2*(e + f*x)]*Sinh[2*(e + f*x)])/(6*a^2*(a - b)*b*f*(2*a - b + b*Cosh[2*(e + f*x)]^(3/2))

Maple [B] time = 0.151, size = 662, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)`

[Out] $\frac{1}{3} \left((-1/ab)^{1/2} ab \sinh(fx+e)^5 - 2(-1/ab)^{1/2} b^2 \sinh(fx+e)^5 + 2 \left(\frac{a+b \sinh(fx+e)^2}{a} \right)^{1/2} (\cosh(fx+e)^2)^{1/2} \text{EllipticF}(\sinh(fx+e) * (-1/ab)^{1/2}, (a/b)^{1/2}) ab \sinh(fx+e)^2 - 2 \left(\frac{a+b \sinh(fx+e)^2}{a} \right)^{1/2} (\cosh(fx+e)^2)^{1/2} \text{EllipticF}(\sinh(fx+e) * (-1/ab)^{1/2}, (a/b)^{1/2}) b^2 \sinh(fx+e)^2 - \left(\frac{a+b \sinh(fx+e)^2}{a} \right)^{1/2} (\cosh(fx+e)^2)^{1/2} \text{EllipticE}(\sinh(fx+e) * (-1/ab)^{1/2}, (a/b)^{1/2}) ab \sinh(fx+e)^2 + 2 \left(\frac{a+b \sinh(fx+e)^2}{a} \right)^{1/2} (\cosh(fx+e)^2)^{1/2} \text{EllipticE}(\sinh(fx+e) * (-1/ab)^{1/2}, (a/b)^{1/2}) b^2 \sinh(fx+e)^2 + 2(-1/ab)^{1/2} a^2 \sinh(fx+e)^3 - 2(-1/ab)^{1/2} ab \sinh(fx+e)^3 - 2(-1/ab)^{1/2} b^2 \sinh(fx+e)^3 + 2a^2 \left(\frac{a+b \sinh(fx+e)^2}{a} \right)^{1/2} (\cosh(fx+e)^2)^{1/2} \text{EllipticF}(\sinh(fx+e) * (-1/ab)^{1/2}, (a/b)^{1/2}) - 2a \left(\frac{a+b \sinh(fx+e)^2}{a} \right)^{1/2} (\cosh(fx+e)^2)^{1/2} \text{EllipticF}(\sinh(fx+e) * (-1/ab)^{1/2}, (a/b)^{1/2}) b - \left(\frac{a+b \sinh(fx+e)^2}{a} \right)^{1/2} (\cosh(fx+e)^2)^{1/2} \text{EllipticE}(\sinh(fx+e) * (-1/ab)^{1/2}, (a/b)^{1/2}) a^2 + 2 \left(\frac{a+b \sinh(fx+e)^2}{a} \right)^{1/2} (\cosh(fx+e)^2)^{1/2} \text{EllipticE}(\sinh(fx+e) * (-1/ab)^{1/2}, (a/b)^{1/2}) ab + 2(-1/ab)^{1/2} a^2 \sinh(fx+e) - 3(-1/ab)^{1/2} ab \sinh(fx+e) \right) / a^2 / (a-b) / (a+b \sinh(fx+e)^2)^{3/2} / (-1/ab)^{1/2} / \cosh(fx+e) / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh^2(fx+e)}{\left(b \sinh^2(fx+e) + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx+e) + a} \cosh^2(fx+e)}{b^3 \sinh^6(fx+e) + 3ab^2 \sinh^4(fx+e) + 3a^2b \sinh^2(fx+e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx + e)^2}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

$$3.398 \quad \int \frac{1}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=251

$$\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1 \operatorname{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{3af(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{2b(2a-b) \sinh(e+fx) \cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b \sinh^2(e+fx)}} - \frac{2i(2a-b)\sqrt{a+b \sinh^2(e+fx)}E\left(i\sqrt{\frac{b \sinh^2(e+fx)}{a}}\right)}{3a^2f(a-b)^2\sqrt{\frac{b \sinh^2(e+fx)}{a}}}$$

[Out] $-(b*\operatorname{Cosh}[e+fx]*\operatorname{Sinh}[e+fx])/(3*a*(a-b)*f*(a+b*\operatorname{Sinh}[e+fx]^2)^{(3/2)}) - (2*(2*a-b)*b*\operatorname{Cosh}[e+fx]*\operatorname{Sinh}[e+fx])/(3*a^2*(a-b)^2*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2]) - (((2*I)/3)*(2*a-b)*\operatorname{EllipticE}[I*e+I*fx, b/a]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2])/(a^2*(a-b)^2*f*\operatorname{Sqrt}[1+(b*\operatorname{Sinh}[e+fx]^2)/a]) + ((I/3)*\operatorname{EllipticF}[I*e+I*fx, b/a]*\operatorname{Sqrt}[1+(b*\operatorname{Sinh}[e+fx]^2)/a])/(a*(a-b)*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2])$

Rubi [A] time = 0.280601, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{2b(2a-b) \sinh(e+fx) \cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b \sinh^2(e+fx)}} - \frac{2i(2a-b)\sqrt{a+b \sinh^2(e+fx)}E\left(ie + ifx \left| \frac{b}{a} \right.\right)}{3a^2f(a-b)^2\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1} - \frac{b \sinh(e+fx) \cosh(e+fx)}{3af(a-b)(a+b \sinh^2(e+fx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a+b*\operatorname{Sinh}[e+fx]^2)^{-5/2}, x]$

[Out] $-(b*\operatorname{Cosh}[e+fx]*\operatorname{Sinh}[e+fx])/(3*a*(a-b)*f*(a+b*\operatorname{Sinh}[e+fx]^2)^{(3/2)}) - (2*(2*a-b)*b*\operatorname{Cosh}[e+fx]*\operatorname{Sinh}[e+fx])/(3*a^2*(a-b)^2*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2]) - (((2*I)/3)*(2*a-b)*\operatorname{EllipticE}[I*e+I*fx, b/a]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2])/(a^2*(a-b)^2*f*\operatorname{Sqrt}[1+(b*\operatorname{Sinh}[e+fx]^2)/a]) + ((I/3)*\operatorname{EllipticF}[I*e+I*fx, b/a]*\operatorname{Sqrt}[1+(b*\operatorname{Sinh}[e+fx]^2)/a])/(a*(a-b)*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2])$

Rule 3184

$\operatorname{Int}[(a_+ + (b_+)*\sin[(e_+) + (f_+)*(x_+)]^2)^{(p_+), x_Symbol] := -\operatorname{Simp}[(b_+*\operatorname{Cos}[e_+ + f_+*x_+]*\operatorname{Sin}[e_+ + f_+*x_+]*(a_+ + b_+*\operatorname{Sin}[e_+ + f_+*x_+]^2)^{(p_+ + 1)})/(2*a_+*f_+(p_+ + 1)*(a_+ +$

b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]

Rule 3173

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]

Rule 3172

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])^2]/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3178

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2], x_Symbol] := Dist[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3177

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rule 3183

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2], x_Symbol] := Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2], x_Symbol] := Simp[(1*EllipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \sinh^2(e + fx))^{5/2}} dx &= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{\int \frac{-3a+2b+b \sinh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx}{3a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{\int \frac{-a(3a+b \sinh^2(e+fx))}{(a+b \sinh^2(e+fx))^{3/2}} dx}{3a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{\int \frac{2a+b \sinh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx}{3a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} + \frac{\int \frac{2(2a-b) \sinh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx}{3a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{2i(2a - b)}{3a(a - b)}
\end{aligned}$$

Mathematica [A] time = 1.26375, size = 190, normalized size = 0.76

$$\frac{ia^2(a - b) \left(\frac{2a + b \cosh(2(e + fx)) - b}{a} \right)^{3/2} \text{EllipticF} \left(i(e + fx), \frac{b}{a} \right) + \sqrt{2}b \sinh(2(e + fx)) (-5a^2 + b(b - 2a) \cosh(2(e + fx)) + 5a^2)}{3a^2 f (a - b)^2 (2a + b \cosh(2(e + fx)) - b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2), x]

[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b)*Cosh[2*(e + f*x)]*Sinh[2*(e + f*x)]/(3*a^2*(a - b)^2*f*(2*a - b + b*Cosh[2*(e + f*x)]))^(3/2))

Maple [A] time = 0., size = 406, normalized size = 1.6

$$\frac{1}{\cosh(fx+e)f} \sqrt{\left(a+b(\sinh(fx+e))^2\right) (\cosh(fx+e))^2} \left(-\frac{\sinh(fx+e)}{3ab(a-b)} \sqrt{\left(a+b(\sinh(fx+e))^2\right) (\cosh(fx+e))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh(fx+e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh(fx+e)^2 + a}}{b^3 \sinh(fx+e)^6 + 3ab^2 \sinh(fx+e)^4 + 3a^2b \sinh(fx+e)^2 + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*
x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

$$3.399 \quad \int \frac{\operatorname{sech}^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=292

$$\frac{b(9a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right) + \sqrt{b}(3a^2+7ab-2b^2)\cosh(e+fx)}{3a^2f(a-b)^3\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{\sqrt{b}(3a^2+7ab-2b^2)\cosh(e+fx)}{3a^{3/2}f(a-b)^3\sqrt{a+b \sinh^2(e+fx)}}$$

[Out] (b*(3*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (Sqrt[b]*(3*a^2 + 7*a*b - 2*b^2)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*(a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - ((9*a - b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2))

Rubi [A] time = 0.313725, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3192, 414, 527, 525, 418, 411}

$$\frac{\sqrt{b}(3a^2+7ab-2b^2)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\left|1-\frac{a}{b}\right.\right)}{3a^{3/2}f(a-b)^3\sqrt{a+b \sinh^2(e+fx)}\sqrt{\frac{a \cosh^2(e+fx)}{a+b \sinh^2(e+fx)}}} - \frac{b(9a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3a^2f(a-b)^3\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] (b*(3*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (Sqrt[b]*(3*a^2 + 7*a*b - 2*b^2)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*(a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - ((9*a - b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2))

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 414

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 527

Int[((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.))^(q_.)*((e_.) + (f_.)*(x_.)^(n_.)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 525

Int[((e_.) + (f_.)*(x_.)^2)/(Sqrt[(a_.) + (b_.)*(x_.)^2]*((c_.) + (d_.)*(x_.)^2)^(3/2)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] && PosQ[d/c]

Rule 418

Int[1/(Sqrt[(a_.) + (b_.)*(x_.)^2]*Sqrt[(c_.) + (d_.)*(x_.)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_.) + (b_.)*(x_.)^2]/((c_.) + (d_.)*(x_.)^2)^(3/2), x_Symbol] := Sim

$p[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned} \int \frac{\text{sech}^2(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e + fx)\text{sech}(e + fx)}\right) \text{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}(a+bx^2)^{5/2}} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\tanh(e + fx)}{(a - b)f(a + b \sinh^2(e + fx))^{3/2}} + \frac{\left(\sqrt{\cosh^2(e + fx)\text{sech}(e + fx)}\right) \text{Subst}\left(\int \frac{b-3bx^2}{\sqrt{1+x^2}(a+bx)} dx, x, \sinh(e + fx)\right)}{(-a + b)f} \\ &= \frac{b(3a + b) \cosh(e + fx) \sinh(e + fx)}{3a(a - b)^2 f (a + b \sinh^2(e + fx))^{3/2}} + \frac{\tanh(e + fx)}{(a - b)f(a + b \sinh^2(e + fx))^{3/2}} + \frac{\left(\sqrt{\cosh^2(e + fx)\text{sech}(e + fx)}\right) \text{Subst}\left(\int \frac{b-3bx^2}{\sqrt{1+x^2}(a+bx)} dx, x, \sinh(e + fx)\right)}{(-a + b)f} \\ &= \frac{b(3a + b) \cosh(e + fx) \sinh(e + fx)}{3a(a - b)^2 f (a + b \sinh^2(e + fx))^{3/2}} + \frac{\tanh(e + fx)}{(a - b)f(a + b \sinh^2(e + fx))^{3/2}} + \frac{\left((9a - b)b\sqrt{c}\right) \text{Subst}\left(\int \frac{b-3bx^2}{\sqrt{1+x^2}(a+bx)} dx, x, \sinh(e + fx)\right)}{(-a + b)f} \\ &= \frac{b(3a + b) \cosh(e + fx) \sinh(e + fx)}{3a(a - b)^2 f (a + b \sinh^2(e + fx))^{3/2}} + \frac{\sqrt{b}(3a^2 + 7ab - 2b^2) \cosh(e + fx) E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}}\right)\right)}{3a^{3/2}(a - b)^3 f \sqrt{\frac{a \cosh^2(e + fx)}{a + b \sinh^2(e + fx)}} \sqrt{a + b \sinh^2(e + fx)}} \end{aligned}$$

Mathematica [C] time = 3.45983, size = 260, normalized size = 0.89

$$\frac{-2ia^2(3a^2 - 2ab - b^2) \left(\frac{2a+b \cosh(2(e+fx))-b}{a}\right)^{3/2} \text{EllipticF}\left(i(e + fx), \frac{b}{a}\right) + \frac{\tanh(e+fx)(4ab(6a^2+5ab-3b^2) \cosh(2(e+fx))+b^2(3a^2+7ab-2b^2))}{\sqrt{2}}}{6a^2 f(a - b)^3 (2a + b \cosh(2(e + fx)))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ((2*I)*a^2*(3*a^2 + 7*a*b - 2*b^2)*((2*a - b + b*Cosh[2*(e + f*x)]))/a)^(3/2)*EllipticE[I*(e + f*x), b/a] - (2*I)*a^2*(3*a^2 - 2*a*b - b^2)*((2*a - b + b*Cosh[2*(e + f*x)]))/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + ((24*a^4 - 24*a^3*b + 41*a^2*b^2 - 19*a*b^3 + 2*b^4 + 4*a*b*(6*a^2 + 5*a*b - 3*b^2)*Cosh[

$$2*(e + f*x)] + b^2*(3*a^2 + 7*a*b - 2*b^2)*\text{Cosh}[4*(e + f*x)]*\text{Tanh}[e + f*x] \\)/\text{Sqrt}[2]]/(6*a^2*(a - b)^3*f*(2*a - b + b*\text{Cosh}[2*(e + f*x)])^(3/2))$$

Maple [B] time = 0.166, size = 1002, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{sech}(f*x+e)^2/(a+b*\sinh(f*x+e)^2)^(5/2), x)$

[Out] $\frac{1}{3}*(3*(-1/a*b)^(1/2)*a^2*b^2*\sinh(f*x+e)^5+7*(-1/a*b)^(1/2)*a*b^3*\sinh(f*x+e)^5-2*(-1/a*b)^(1/2)*b^4*\sinh(f*x+e)^5-6*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^2*b^2*\sinh(f*x+e)^2+8*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b^3*\sinh(f*x+e)^2-2*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^4*\sinh(f*x+e)^2-3*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^2*b^2*\sinh(f*x+e)^2-7*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b^3*\sinh(f*x+e)^2+2*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^4*\sinh(f*x+e)^2+6*(-1/a*b)^(1/2)*a^3*b*\sinh(f*x+e)^3+8*(-1/a*b)^(1/2)*a^2*b^2*\sinh(f*x+e)^3+4*(-1/a*b)^(1/2)*a*b^3*\sinh(f*x+e)^3-2*(-1/a*b)^(1/2)*b^4*\sinh(f*x+e)^3-6*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^3*b+8*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^2*b^2-2*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b^3-3*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^3*b-7*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a^2*b^2+2*((a+b*\sinh(f*x+e)^2)/a)^(1/2)*(\cosh(f*x+e)^2)^(1/2)*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b^3+3*(-1/a*b)^(1/2)*a^4*\sinh(f*x+e)+8*(-1/a*b)^(1/2)*a^2*b^2*\sinh(f*x+e)-3*(-1/a*b)^(1/2)*a*b^3*\sinh(f*x+e))/(-1/a*b)^(1/2)/(a+b*\sinh(f*x+e)^2)^(3/2)/a^2/(a-b)^3/\cosh(f*x+e)/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx + e)^2}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b \sinh(fx + e)^2 + a} \operatorname{sech}(fx + e)^2}{b^3 \sinh(fx + e)^6 + 3ab^2 \sinh(fx + e)^4 + 3a^2b \sinh(fx + e)^2 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx + e)^2}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.400 $\int (d \cosh(e + fx))^m (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=117

$$\frac{d \sinh(e + fx) \cosh^2(e + fx)^{\frac{1-m}{2}} (d \cosh(e + fx))^{m-1} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{1-m}{2}, -p; \frac{3}{2}; -\sinh^2 \right)}{f}$$

[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*(d*Cosh[e + f*x])^(-1 + m)*(Cosh[e + f*x]^2)^((1 - m)/2)*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.110778, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3193, 430, 429}

$$\frac{d \sinh(e + fx) \cosh^2(e + fx)^{\frac{1-m}{2}} (d \cosh(e + fx))^{m-1} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{1-m}{2}, -p; \frac{3}{2}; -\sinh^2 \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*(d*Cosh[e + f*x])^(-1 + m)*(Cosh[e + f*x]^2)^((1 - m)/2)*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3193

Int[(cos[(e_.) + (f_.)*(x_.)]*(d_.))^m]*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^p, x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Cos[e + f*x])^(2*FracPart[(m - 1)/2]))/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && !IntegerQ[m]

Rule 430

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],

Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
 :-> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\int (d \cosh(e + fx))^m (a + b \sinh^2(e + fx))^p dx = \frac{\left(d(d \cosh(e + fx))^{2\left(-\frac{1}{2} + \frac{m}{2}\right)} \cosh^2(e + fx)^{\frac{1}{2} - \frac{m}{2}} \right) \text{Subst} \left(\int (1 + x^2)^{\frac{1}{2}(-1 + p)} dx \right)}{f}$$

$$= \frac{\left(d(d \cosh(e + fx))^{2\left(-\frac{1}{2} + \frac{m}{2}\right)} \cosh^2(e + fx)^{\frac{1}{2} - \frac{m}{2}} (a + b \sinh^2(e + fx))^p \right)}{d \cosh(e + fx)}$$

$$= \frac{dF_1 \left(\frac{1}{2}; \frac{1-m}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right) (d \cosh(e + fx))^{-1 + p}}{d \cosh(e + fx)}$$

Mathematica [F] time = 8.18999, size = 0, normalized size = 0.

$$\int (d \cosh(e + fx))^m (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.51, size = 0, normalized size = 0.

$$\int (d \cosh(fx + e))^m (a + b (\sinh(fx + e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \left(d \cosh(fx + e) \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \left(d \cosh(fx + e)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*cosh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p (d \cosh(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)

3.401 $\int \cosh^5(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=214

$$\frac{(3a^2 - 2ab(2p + 5) + b^2(4p^2 + 16p + 15)) \sinh(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \sinh^2(e + fx)}{a}\right)}{b^2 f (2p + 3)(2p + 5)}$$

[Out] -(((3*a - b*(7 + 2*p))*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b^2*f*(3 + 2*p)*(5 + 2*p))) + (Cosh[e + f*x]^2*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 2*a*b*(5 + 2*p) + b^2*(15 + 16*p + 4*p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.20449, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3190, 416, 388, 246, 245}

$$\frac{(3a^2 - 2ab(2p + 5) + b^2(4p^2 + 16p + 15)) \sinh(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \sinh^2(e + fx)}{a}\right)}{b^2 f (2p + 3)(2p + 5)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] -(((3*a - b*(7 + 2*p))*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b^2*f*(3 + 2*p)*(5 + 2*p))) + (Cosh[e + f*x]^2*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 2*a*b*(5 + 2*p) + b^2*(15 + 16*p + 4*p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simp
lify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}
\int \cosh^5(e+fx) (a+b\sinh^2(e+fx))^p dx &= \frac{\text{Subst}\left(\int (1+x^2)^2 (a+bx^2)^p dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\cosh^2(e+fx) \sinh(e+fx) (a+b\sinh^2(e+fx))^{1+p}}{bf(5+2p)} + \frac{\text{Subst}\left(\int (a+bx^2)^p dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{(3a-b(7+2p)) \sinh(e+fx) (a+b\sinh^2(e+fx))^{1+p}}{b^2 f(3+2p)(5+2p)} + \frac{\cosh^2(e+fx) \sinh(e+fx) (a+b\sinh^2(e+fx))^{1+p}}{bf(5+2p)} \\
&= -\frac{(3a-b(7+2p)) \sinh(e+fx) (a+b\sinh^2(e+fx))^{1+p}}{b^2 f(3+2p)(5+2p)} + \frac{\cosh^2(e+fx) \sinh(e+fx) (a+b\sinh^2(e+fx))^{1+p}}{bf(5+2p)} \\
&= -\frac{(3a-b(7+2p)) \sinh(e+fx) (a+b\sinh^2(e+fx))^{1+p}}{b^2 f(3+2p)(5+2p)} + \frac{\cosh^2(e+fx) \sinh(e+fx) (a+b\sinh^2(e+fx))^{1+p}}{bf(5+2p)}
\end{aligned}$$

Mathematica [F] time = 10.606, size = 0, normalized size = 0.

$$\int \cosh^5(e+fx) (a+b\sinh^2(e+fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.645, size = 0, normalized size = 0.

$$\int (\cosh(fx+e))^5 (a+b(\sinh(fx+e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)

[Out] int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^p \cosh (fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh (fx + e)^2 + a\right)^p \cosh (fx + e)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")

[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^p \cosh (fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```


$$3.402 \quad \int \cosh^3(e + fx) \left(a + b \sinh^2(e + fx)\right)^p dx$$

Optimal. Leaf size=125

$$\frac{\sinh(e + fx) \left(a + b \sinh^2(e + fx)\right)^{p+1}}{bf(2p + 3)} - \frac{(a - b(2p + 3)) \sinh(e + fx) \left(a + b \sinh^2(e + fx)\right)^p \left(\frac{b \sinh^2(e + fx)}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -\right)}{bf(2p + 3)}$$

[Out] (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a - b*(3 + 2*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(b*f*(3 + 2*p)*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.102418, antiderivative size = 119, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 388, 246, 245}

$$\frac{\left(1 - \frac{a}{2bp+3b}\right) \sinh(e + fx) \left(a + b \sinh^2(e + fx)\right)^p \left(\frac{b \sinh^2(e + fx)}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \sinh^2(e + fx)}{a}\right)}{f} + \frac{\sinh(e + fx) \left(a + b \sinh^2(e + fx)\right)^p}{bf(2p + 3)}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) + ((1 - a/(3*b + 2*b*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 388

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,

c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
/; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simpli
fy[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \cosh^3(e + fx) (a + b \sinh^2(e + fx))^p dx &= \frac{\text{Subst}\left(\int (1 + x^2) (a + bx^2)^p dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{1+p}}{bf(3 + 2p)} + \frac{\left(1 - \frac{a}{3b+2bp}\right) \text{Subst}\left(\int (a + bx^2)^p dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{1+p}}{bf(3 + 2p)} + \frac{\left(1 - \frac{a}{3b+2bp}\right) (a + b \sinh^2(e + fx))^p}{bf(3 + 2p)} \\ &= \frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^{1+p}}{bf(3 + 2p)} + \frac{\left(1 - \frac{a}{3b+2bp}\right) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \sinh^2(e + fx)}{a}\right)}{bf(3 + 2p)} \end{aligned}$$

Mathematica [A] time = 0.20196, size = 120, normalized size = 0.96

$$\frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1\right)^{-p} \left((b(2p + 3) - a) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \sinh^2(e + fx)}{a}\right) + (a + b \sinh^2(e + fx))^p\right)}{bf(2p + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

[Out] $(\text{Sinh}[e + f*x]*(a + b*\text{Sinh}[e + f*x]^2)^p*((-a + b*(3 + 2*p))*\text{Hypergeometric} 2F1[1/2, -p, 3/2, -((b*\text{Sinh}[e + f*x]^2)/a)] + (a + b*\text{Sinh}[e + f*x]^2)*(1 + (b*\text{Sinh}[e + f*x]^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*\text{Sinh}[e + f*x]^2)/a)^p)$

Maple [F] time = 0.453, size = 0, normalized size = 0.

$$\int (\cosh(fx + e))^3 \left(a + b(\sinh(fx + e))^2 \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \cosh(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \cosh(fx + e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \cosh(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)

3.403 $\int \cosh(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=67

$$\frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} {}_2F_1 \left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.0462517, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3190, 246, 245}

$$\frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} {}_2F_1 \left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 246

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \cosh(e + fx) (a + b \sinh^2(e + fx))^p dx &= \frac{\text{Subst}\left(\int (a + bx^2)^p dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\left((a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e + fx)}{a}\right)^{-p}\right) \text{Subst}\left(\int \left(1 + \frac{bx^2}{a}\right)^p dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{{}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \sinh^2(e + fx)}{a}\right) \sinh(e + fx) (a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e + fx)}{a}\right)^{-p}}{f} \end{aligned}$$

Mathematica [A] time = 0.0255693, size = 67, normalized size = 1.

$$\frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{b \sinh^2(e + fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a
+ b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int \cosh(fx + e) \left(a + b (\sinh(fx + e))^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

[Out] `int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \cosh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \cosh(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \cosh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```


3.404 $\int \operatorname{sech}(e + fx) \left(a + b \sinh^2(e + fx)\right)^p dx$

Optimal. Leaf size=78

$$\frac{\sinh(e + fx) \left(a + b \sinh^2(e + fx)\right)^p \left(\frac{b \sinh^2(e + fx)}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; 1, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a}\right)}{f}$$

[Out] (AppellF1[1/2, 1, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.0757751, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3190, 430, 429}

$$\frac{\sinh(e + fx) \left(a + b \sinh^2(e + fx)\right)^p \left(\frac{b \sinh^2(e + fx)}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; 1, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]

[Out] (AppellF1[1/2, 1, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3190

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 430

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \operatorname{sech}(e + fx) (a + b \sinh^2(e + fx))^p dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{1+x^2} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{\left((a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e+fx)}{a}\right)^{-p}\right) \operatorname{Subst}\left(\int \frac{\left(1 + \frac{bx^2}{a}\right)^p}{1+x^2} dx, x, \sinh(e + fx)\right)}{f} \\ &= \frac{F_1\left(\frac{1}{2}; 1, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e+fx)}{a}\right) \sinh(e + fx) (a + b \sinh^2(e + fx))^p}{f} \end{aligned}$$

Mathematica [F] time = 3.3539, size = 0, normalized size = 0.

$$\int \operatorname{sech}(e + fx) (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]

[Out] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int \operatorname{sech}(fx + e) \left(a + b (\sinh(fx + e))^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p, x)

[Out] `int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{sech}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \operatorname{sech}(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{sech}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

3.405 $\int \operatorname{sech}^3(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=78

$$\frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; 2, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

[Out] (AppellF1[1/2, 2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.0819552, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3190, 430, 429}

$$\frac{\sinh(e + fx) (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; 2, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[1/2, 2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol]
:> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]
/; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*((c_.) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x]
/; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(e + fx) (a + b \sinh^2(e + fx))^p dx = \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{(1+x^2)^2} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{\left((a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e+fx)}{a}\right)^{-p}\right) \operatorname{Subst}\left(\int \frac{\left(1 + \frac{bx^2}{a}\right)^p}{(1+x^2)^2} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2}; 2, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e+fx)}{a}\right) \sinh(e + fx) (a + b \sinh^2(e + fx))^p}{f}$$

Mathematica [F] time = 5.45435, size = 0, normalized size = 0.

$$\int \operatorname{sech}^3(e + fx) (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int (\operatorname{sech}(fx + e))^3 (a + b (\sinh(fx + e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

[Out] `int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{sech}(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \operatorname{sech}(fx + e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^p \operatorname{sech} (fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```


$$3.406 \quad \int \cosh^4(e + fx) \left(a + b \sinh^2(e + fx)\right)^p dx$$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -\frac{3}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a}\right)}{f}$$

[Out] (AppellF1[1/2, -3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.0856548, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -\frac{3}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[1/2, -3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 430

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}

, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \cosh^4(e+fx) (a+b \sinh^2(e+fx))^p dx &= \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int (1+x^2)^{3/2} (a+bx^2)^p dx, x, \sinh(e+fx)\right)}{f} \\ &= \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)} (a+b \sinh^2(e+fx))^p \left(1 + \frac{b \sinh^2(e+fx)}{a}\right)^{-p}\right)}{f} \\ &= \frac{F_1\left(\frac{1}{2}; -\frac{3}{2}, -p; \frac{3}{2}; -\sinh^2(e+fx), -\frac{b \sinh^2(e+fx)}{a}\right) \sqrt{\cosh^2(e+fx)} (a+b \sinh^2(e+fx))^p}{f} \end{aligned}$$

Mathematica [F] time = 9.8837, size = 0, normalized size = 0.

$$\int \cosh^4(e+fx) (a+b \sinh^2(e+fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]

[Out] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.482, size = 0, normalized size = 0.

$$\int (\cosh(fx+e))^4 \left(a+b(\sinh(fx+e))^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \cosh(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \cosh(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \cosh(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)

$$3.407 \quad \int \cosh^2(e + fx) \left(a + b \sinh^2(e + fx) \right)^p dx$$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} \left(a + b \sinh^2(e + fx) \right)^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; -\frac{1}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

[Out] (AppellF1[1/2, -1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.0822861, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} \left(a + b \sinh^2(e + fx) \right)^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; -\frac{1}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[1/2, -1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^(m - 1)/2]*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 430

Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}

, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\begin{aligned} \int \cosh^2(e+fx) (a+b \sinh^2(e+fx))^p dx &= \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \sqrt{1+x^2} (a+bx^2)^p dx, x, \sinh(e+fx)\right)}{f} \\ &= \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)} (a+b \sinh^2(e+fx))^p \left(1 + \frac{b \sinh^2(e+fx)}{a}\right)^{-p}\right)}{f} \\ &= \frac{F_1\left(\frac{1}{2}; -\frac{1}{2}, -p; \frac{3}{2}; -\sinh^2(e+fx), -\frac{b \sinh^2(e+fx)}{a}\right) \sqrt{\cosh^2(e+fx) (a+b \sinh^2(e+fx))^p}}{f} \end{aligned}$$

Mathematica [F] time = 10.0742, size = 0, normalized size = 0.

$$\int \cosh^2(e+fx) (a+b \sinh^2(e+fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]

[Out] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int (\cosh(fx+e))^2 (a+b(\sinh(fx+e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \cosh(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \cosh(fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^p \cosh (fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)

3.408 $\int (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{1}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

[Out] (AppellF1[1/2, 1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.0515143, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3185, 430, 429}

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{1}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[1/2, 1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3185

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(a + b*ff^2*x^2)^p/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && !IntegerQ[p]

Rule 430

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (a + b \sinh^2(e + fx))^p dx = \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{(a + bx^2)^p}{\sqrt{1+x^2}} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} (a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e + fx)}{a}\right)^{-p}\right) \operatorname{Subst}\left(\int \frac{\left(1 + \frac{bx}{a}\right)^p}{\sqrt{1+x^2}} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2}; \frac{1}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a}\right) \sqrt{\cosh^2(e + fx)} (a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e + fx)}{a}\right)^{-p}}{f}$$

Mathematica [F] time = 2.13693, size = 0, normalized size = 0.

$$\int (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^p, x]

[Out] Integrate[(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int (a + b (\sinh(fx + e))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^p, x)

[Out] `int((a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh^2(fx + e) + a\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p, x)
```

3.409 $\int \operatorname{sech}^2(e + fx) (a + b \sinh^2(e + fx))^p dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{3}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

[Out] (AppellF1[1/2, 3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.0828345, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{3}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[1/2, 3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 430

Int[((a_.) + (b_.)*(x_)^(n_.))^p]*((c_.) + (d_.)*(x_)^(n_.))^q, x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}

, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{sech}^2(e + fx) (a + b \sinh^2(e + fx))^p dx = \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{(a + bx^2)^p}{(1+x^2)^{3/2}} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} (a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e + fx)}{a}\right)^{-p}\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2}; \frac{3}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a}\right) \sqrt{\cosh^2(e + fx)} (a + b \sinh^2(e + fx))^p}{f}$$

Mathematica [F] time = 4.18865, size = 0, normalized size = 0.

$$\int \operatorname{sech}^2(e + fx) (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]

[Out] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.257, size = 0, normalized size = 0.

$$\int \left(\operatorname{sech}(fx + e)\right)^2 \left(a + b \left(\sinh(fx + e)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{sech}(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \operatorname{sech}(fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{sech}(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)

$$3.410 \quad \int \operatorname{sech}^4(e + fx) (a + b \sinh^2(e + fx))^p dx$$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{5}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

[Out] (AppellF1[1/2, 5/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.0818326, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e + fx) \tanh(e + fx)} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{1}{2}; \frac{5}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] (AppellF1[1/2, 5/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sqrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3192

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 430

Int[((a_.) + (b_.)*(x_)^(n_.))^p]*((c_.) + (d_.)*(x_)^(n_.))^q, x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}

, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{sech}^4(e + fx) (a + b \sinh^2(e + fx))^p dx = \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{(a + bx^2)^p}{(1+x^2)^{5/2}} dx, x, \sinh(e + fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} (a + b \sinh^2(e + fx))^p \left(1 + \frac{b \sinh^2(e + fx)}{a}\right)^{-p}\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2}; \frac{5}{2}, -p; \frac{3}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a}\right) \sqrt{\cosh^2(e + fx)} (a + b \sinh^2(e + fx))^p}{f}$$

Mathematica [F] time = 7.95743, size = 0, normalized size = 0.

$$\int \operatorname{sech}^4(e + fx) (a + b \sinh^2(e + fx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]

[Out] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]

Maple [F] time = 0.256, size = 0, normalized size = 0.

$$\int \left(\operatorname{sech}(fx + e)\right)^4 \left(a + b \left(\sinh(fx + e)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

[Out] `int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{sech}(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \operatorname{sech}(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \operatorname{sech}(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)

$$3.411 \quad \int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}} dx$$

Optimal. Leaf size=259

$$\frac{2a^2 \sinh^{\frac{7}{2}}(c+dx)}{7b^3d} - \frac{a^3 \sinh^3(c+dx)}{3b^4d} + \frac{2(a^4+2b^4) \sinh^{\frac{5}{2}}(c+dx)}{5b^5d} - \frac{a(a^4+2b^4) \sinh^2(c+dx)}{2b^6d} + \frac{2a^2(a^4+2b^4) \sinh(c+dx)}{3b^7d}$$

```
[Out] (-2*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*(a^4 + b^4)^2*Sqrt[Sinh[c + d*x]])/(b^9*d) - (a^3*(a^4 + 2*b^4)*Sinh[c + d*x])/(b^8*d) + (2*a^2*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) - (a*(a^4 + 2*b^4)*Sinh[c + d*x]^2)/(2*b^6*d) + (2*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2))/(5*b^5*d) - (a^3*Sinh[c + d*x]^3)/(3*b^4*d) + (2*a^2*Sinh[c + d*x]^(7/2))/(7*b^3*d) - (a*Sinh[c + d*x]^4)/(4*b^2*d) + (2*Sinh[c + d*x]^(9/2))/(9*b*d)
```

Rubi [A] time = 0.296316, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3223, 1890, 1620}

$$\frac{2a^2 \sinh^{\frac{7}{2}}(c+dx)}{7b^3d} - \frac{a^3 \sinh^3(c+dx)}{3b^4d} + \frac{2(a^4+2b^4) \sinh^{\frac{5}{2}}(c+dx)}{5b^5d} - \frac{a(a^4+2b^4) \sinh^2(c+dx)}{2b^6d} + \frac{2a^2(a^4+2b^4) \sinh(c+dx)}{3b^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*(a^4 + b^4)^2*Sqrt[Sinh[c + d*x]])/(b^9*d) - (a^3*(a^4 + 2*b^4)*Sinh[c + d*x])/(b^8*d) + (2*a^2*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) - (a*(a^4 + 2*b^4)*Sinh[c + d*x]^2)/(2*b^6*d) + (2*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2))/(5*b^5*d) - (a^3*Sinh[c + d*x]^3)/(3*b^4*d) + (2*a^2*Sinh[c + d*x]^(7/2))/(7*b^3*d) - (a*Sinh[c + d*x]^4)/(4*b^2*d) + (2*Sinh[c + d*x]^(9/2))/(9*b*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```


$$\int^3 + 360*a^2*b^7*\text{Sinh}[c + d*x]^{(7/2)} - 315*a*b^8*\text{Sinh}[c + d*x]^4 + 280*b^9*\text{Sinh}[c + d*x]^{(9/2)} / (1260*b^{10}*d)$$

Maple [C] time = 0.105, size = 780, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x)`

[Out]
$$-a/d/b^2*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)-1/4*a/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^4+a^7/d/b^8/(\tanh(1/2*d*x+1/2*c)-1)+2*a^3/d/b^4/(\tanh(1/2*d*x+1/2*c)-1)+a^7/d/b^8/(\tanh(1/2*d*x+1/2*c)+1)+1/2*a^5/d/b^6/(\tanh(1/2*d*x+1/2*c)+1)+2*a^3/d/b^4/(\tanh(1/2*d*x+1/2*c)+1)+1/3*a^3/d/b^4/(\tanh(1/2*d*x+1/2*c)-1)^3-1/2*a^5/d/b^6/(\tanh(1/2*d*x+1/2*c)-1)^2+1/2*a^3/d/b^4/(\tanh(1/2*d*x+1/2*c)-1)^2+a^9/d/b^{10}*\ln(\tanh(1/2*d*x+1/2*c)-1)+\int/\int \text{ind} \text{ef}0^(-\cosh(d*x+c)^4*b*\sinh(d*x+c)^{(1/2)} / (-b^2*\sinh(d*x+c)+a^2), \sinh(d*x+c)) / d-9/8/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^2*a-9/8/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)^2*a+7/8/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)*a-7/8/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)*a+1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^3*a-1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)^3*a-1/2*a^5/d/b^6/(\tanh(1/2*d*x+1/2*c)-1)+2*a^5/d/b^6*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/3*a^3/d/b^4/(\tanh(1/2*d*x+1/2*c)+1)^3-1/2*a^5/d/b^6/(\tanh(1/2*d*x+1/2*c)+1)^2-1/2*a^3/d/b^4/(\tanh(1/2*d*x+1/2*c)+1)^2+a^9/d/b^{10}*\ln(\tanh(1/2*d*x+1/2*c)+1)+2*a^5/d/b^6*\ln(\tanh(1/2*d*x+1/2*c)+1)-a^9/d/b^{10}*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)-2*a^5/d/b^6*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)-1/4*a/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)^4$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^5}{b\sqrt{\sinh(dx+c)+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)`

$$\begin{aligned}
& ^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8)*c)*\cosh(d*x + c)^3 - 9*(4*a^7*b^2 + 7*a \\
& ^3*b^6)*\cosh(d*x + c)^2 + 3*(2*a^5*b^4 + 3*a*b^8)*\cosh(d*x + c))*\sinh(d*x + \\
& c) - 8*(35*b^9*\cosh(d*x + c)^8 + 35*b^9*\sinh(d*x + c)^8 + 90*a^2*b^7*\cosh(\\
& d*x + c)^7 - 90*a^2*b^7*\cosh(d*x + c) + 35*b^9 + 10*(28*b^9*\cosh(d*x + c) + \\
& 9*a^2*b^7)*\sinh(d*x + c)^7 + 28*(9*a^4*b^5 + 13*b^9)*\cosh(d*x + c)^6 + 14* \\
& (70*b^9*\cosh(d*x + c)^2 + 45*a^2*b^7*\cosh(d*x + c) + 18*a^4*b^5 + 26*b^9)*\s \\
& inh(d*x + c)^6 + 30*(28*a^6*b^3 + 47*a^2*b^7)*\cosh(d*x + c)^5 + 2*(980*b^9* \\
& \cosh(d*x + c)^3 + 945*a^2*b^7*\cosh(d*x + c)^2 + 420*a^6*b^3 + 705*a^2*b^7 + \\
& 84*(9*a^4*b^5 + 13*b^9)*\cosh(d*x + c))*\sinh(d*x + c)^5 + 42*(120*a^8*b + 2 \\
& 28*a^4*b^5 + 101*b^9)*\cosh(d*x + c)^4 + 2*(1225*b^9*\cosh(d*x + c)^4 + 1575* \\
& a^2*b^7*\cosh(d*x + c)^3 + 2520*a^8*b + 4788*a^4*b^5 + 2121*b^9 + 210*(9*a^4 \\
& *b^5 + 13*b^9)*\cosh(d*x + c)^2 + 75*(28*a^6*b^3 + 47*a^2*b^7)*\cosh(d*x + c) \\
&)*\sinh(d*x + c)^4 - 30*(28*a^6*b^3 + 47*a^2*b^7)*\cosh(d*x + c)^3 + 2*(980*b \\
& ^9*\cosh(d*x + c)^5 + 1575*a^2*b^7*\cosh(d*x + c)^4 - 420*a^6*b^3 - 705*a^2*b \\
& ^7 + 280*(9*a^4*b^5 + 13*b^9)*\cosh(d*x + c)^3 + 150*(28*a^6*b^3 + 47*a^2*b^ \\
& 7)*\cosh(d*x + c)^2 + 84*(120*a^8*b + 228*a^4*b^5 + 101*b^9)*\cosh(d*x + c))* \\
& \sinh(d*x + c)^3 + 28*(9*a^4*b^5 + 13*b^9)*\cosh(d*x + c)^2 + 2*(490*b^9*\cosh \\
& (d*x + c)^6 + 945*a^2*b^7*\cosh(d*x + c)^5 + 126*a^4*b^5 + 182*b^9 + 210*(9* \\
& a^4*b^5 + 13*b^9)*\cosh(d*x + c)^4 + 150*(28*a^6*b^3 + 47*a^2*b^7)*\cosh(d*x \\
& + c)^3 + 126*(120*a^8*b + 228*a^4*b^5 + 101*b^9)*\cosh(d*x + c)^2 - 45*(28*a \\
& ^6*b^3 + 47*a^2*b^7)*\cosh(d*x + c))*\sinh(d*x + c)^2 + 2*(140*b^9*\cosh(d*x + \\
& c)^7 + 315*a^2*b^7*\cosh(d*x + c)^6 - 45*a^2*b^7 + 84*(9*a^4*b^5 + 13*b^9)* \\
& \cosh(d*x + c)^5 + 75*(28*a^6*b^3 + 47*a^2*b^7)*\cosh(d*x + c)^4 + 84*(120*a^ \\
& 8*b + 228*a^4*b^5 + 101*b^9)*\cosh(d*x + c)^3 - 45*(28*a^6*b^3 + 47*a^2*b^7) \\
& *\cosh(d*x + c)^2 + 28*(9*a^4*b^5 + 13*b^9)*\cosh(d*x + c))*\sinh(d*x + c))*\sq \\
& rt(\sinh(d*x + c)))/(b^10*d*\cosh(d*x + c)^4 + 4*b^10*d*\cosh(d*x + c)^3*\sinh(\\
& d*x + c) + 6*b^10*d*\cosh(d*x + c)^2*\sinh(d*x + c)^2 + 4*b^10*d*\cosh(d*x + c \\
&)*\sinh(d*x + c)^3 + b^10*d*\sinh(d*x + c)^4)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2)),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)^5}{b\sqrt{\sinh(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)
```

$$3.412 \quad \int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}} dx$$

Optimal. Leaf size=136

$$\frac{2a^2 \sinh^{\frac{3}{2}}(c+dx)}{3b^3d} - \frac{a^3 \sinh(c+dx)}{b^4d} + \frac{2(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^5d} - \frac{2a(a^4+b^4)\log(a+b\sqrt{\sinh(c+dx)})}{b^6d} - \frac{a \sinh^2(c+dx)}{2b^2d}$$

[Out] $(-2*a*(a^4 + b^4)*\text{Log}[a + b*\text{Sqrt}[\text{Sinh}[c + d*x]])/(b^6*d) + (2*(a^4 + b^4)*\text{Sqrt}[\text{Sinh}[c + d*x]])/(b^5*d) - (a^3*\text{Sinh}[c + d*x])/(b^4*d) + (2*a^2*\text{Sinh}[c + d*x]^{(3/2)})/(3*b^3*d) - (a*\text{Sinh}[c + d*x]^2)/(2*b^2*d) + (2*\text{Sinh}[c + d*x]^{(5/2)})/(5*b*d)$

Rubi [A] time = 0.156701, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3223, 1890, 1620}

$$\frac{2a^2 \sinh^{\frac{3}{2}}(c+dx)}{3b^3d} - \frac{a^3 \sinh(c+dx)}{b^4d} + \frac{2(a^4+b^4)\sqrt{\sinh(c+dx)}}{b^5d} - \frac{2a(a^4+b^4)\log(a+b\sqrt{\sinh(c+dx)})}{b^6d} - \frac{a \sinh^2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cosh}[c + d*x]^3/(a + b*\text{Sqrt}[\text{Sinh}[c + d*x]]), x]$

[Out] $(-2*a*(a^4 + b^4)*\text{Log}[a + b*\text{Sqrt}[\text{Sinh}[c + d*x]])/(b^6*d) + (2*(a^4 + b^4)*\text{Sqrt}[\text{Sinh}[c + d*x]])/(b^5*d) - (a^3*\text{Sinh}[c + d*x])/(b^4*d) + (2*a^2*\text{Sinh}[c + d*x]^{(3/2)})/(3*b^3*d) - (a*\text{Sinh}[c + d*x]^2)/(2*b^2*d) + (2*\text{Sinh}[c + d*x]^{(5/2)})/(5*b*d)$

Rule 3223

$\text{Int}[\cos[(e_.) + (f_.)*(x_)]^{(m_.)}*((a_.) + (b_.)*((c_.)*\sin[(e_.) + (f_.)*(x_)]))^{(n_.)}]^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f*x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*(c*ff*x)^n)^p, x], x, \text{Sin}[e + f*x]/ff], x]] /;$ $\text{FreeQ}[\{a, b, c, e, f, n, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2] \ \&\& \ (\text{EqQ}[n, 4] \ || \ \text{GtQ}[m, 0] \ || \ \text{IGtQ}[p, 0] \ || \ \text{IntegersQ}[m, p])$

Rule 1890

$\text{Int}[(Pq_)*((a_.) + (b_.)*(x_))^{(n_.)}]^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{g = \text{Denominator}[n]\}, \text{Dist}[g, \text{Subst}[\text{Int}[x^{(g-1)}*(Pq /. x \rightarrow x^g)*(a + b*x^{(g*n)})^p, x], x$

, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
 :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
 , d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
 xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(c + dx)}{a + b\sqrt{\sinh(c + dx)}} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b\sqrt{x}} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \frac{x(1+x^4)}{a+bx} dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \left(\frac{a^4+b^4}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a(a^4+b^4)}{b^5(a+bx)}\right) dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= -\frac{2a(a^4 + b^4) \log(a + b\sqrt{\sinh(c + dx)})}{b^6d} + \frac{2(a^4 + b^4) \sqrt{\sinh(c + dx)}}{b^5d} - \frac{a^3 \sinh(c + dx)}{b^4d} + \frac{2a^2b^3 \sinh^3(c + dx) - 30a^3b^2 \sinh(c + dx) + 60b(a^4 + b^4) \sqrt{\sinh(c + dx)} - 60a(a^4 + b^4) \log(a + b\sqrt{\sinh(c + dx)}) - 15a^2b^3}{30b^6d} \end{aligned}$$

Mathematica [A] time = 0.150835, size = 117, normalized size = 0.86

$$\frac{20a^2b^3 \sinh^3(c + dx) - 30a^3b^2 \sinh(c + dx) + 60b(a^4 + b^4) \sqrt{\sinh(c + dx)} - 60a(a^4 + b^4) \log(a + b\sqrt{\sinh(c + dx)}) - 15a^2b^3}{30b^6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]]), x]

[Out] (-60*a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]) + 60*b*(a^4 + b^4)*Sqrt[Sinh[c + d*x]] - 30*a^3*b^2*Sinh[c + d*x] + 20*a^2*b^3*Sinh[c + d*x]^(3/2) - 15*a*b^4*Sinh[c + d*x]^2 + 12*b^5*Sinh[c + d*x]^(5/2))/(30*b^6*d)

Maple [C] time = 0.1, size = 359, normalized size = 2.6

$$-\frac{a^5}{db^6} \ln\left(a^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 + 2b^2 \tanh\left(\frac{1}{2} dx + \frac{c}{2}\right) - a^2\right) - \frac{a}{db^2} \ln\left(a^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 + 2b^2 \tanh\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x)`

[Out]
$$-a^5/d/b^6*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)-a/d/b^2*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)-1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^2*a+a^3/d/b^4/(\tanh(1/2*d*x+1/2*c)+1)+1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)*a+a^5/d/b^6*\ln(\tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)-1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)^2*a+a^3/d/b^4/(\tanh(1/2*d*x+1/2*c)-1)-1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)*a+a^5/d/b^6*\ln(\tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*\ln(\tanh(1/2*d*x+1/2*c)-1)+\int/undef0^(-\cosh(d*x+c)^2*b*\sinh(d*x+c)^(1/2)/(-b^2*\sinh(d*x+c)+a^2),\sinh(d*x+c))/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3}{b\sqrt{\sinh(dx+c)+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)`

Fricas [B] time = 7.32631, size = 2182, normalized size = 16.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")`

[Out]
$$-1/120*(15*a*b^4*\cosh(d*x + c)^4 + 15*a*b^4*\sinh(d*x + c)^4 + 60*a^3*b^2*\cosh(d*x + c)^3 - 60*a^3*b^2*\cosh(d*x + c) + 15*a*b^4 + 60*(a*b^4*\cosh(d*x + c) + a^3*b^2)*\sinh(d*x + c)^3 - 120*((a^5 + a*b^4)*d*x + (a^5 + a*b^4)*c)*\cosh(d*x + c)^2 + 30*(3*a*b^4*\cosh(d*x + c)^2 + 6*a^3*b^2*\cosh(d*x + c) - 4*(a^5 + a*b^4)*d*x - 4*(a^5 + a*b^4)*c)*\sinh(d*x + c)^2 - 120*((a^5 + a*b^4)*\cosh(d*x + c)^2 + 2*(a^5 + a*b^4)*\cosh(d*x + c)*\sinh(d*x + c) + (a^5 + a*b^4)*\sinh(d*x + c)^2)*\log((b^2*\cosh(d*x + c)^2 + b^2*\sinh(d*x + c)^2 + 2*a^2$$

```
*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*c
osh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2
+ b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) -
a^2)*sinh(d*x + c)) + 120*((a^5 + a*b^4)*cosh(d*x + c)^2 + 2*(a^5 + a*b^4
)*cosh(d*x + c)*sinh(d*x + c) + (a^5 + a*b^4)*sinh(d*x + c)^2)*log(2*(b^2*s
inh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 60*(a*b^4*cosh(d*x +
c)^3 + 3*a^3*b^2*cosh(d*x + c)^2 - a^3*b^2 - 4*((a^5 + a*b^4)*d*x + (a^5 +
a*b^4)*c)*cosh(d*x + c))*sinh(d*x + c) - 4*(3*b^5*cosh(d*x + c)^4 + 3*b^5*
sinh(d*x + c)^4 + 10*a^2*b^3*cosh(d*x + c)^3 - 10*a^2*b^3*cosh(d*x + c) + 3
*b^5 + 2*(6*b^5*cosh(d*x + c) + 5*a^2*b^3)*sinh(d*x + c)^3 + 6*(10*a^4*b +
9*b^5)*cosh(d*x + c)^2 + 6*(3*b^5*cosh(d*x + c)^2 + 5*a^2*b^3*cosh(d*x + c)
+ 10*a^4*b + 9*b^5)*sinh(d*x + c)^2 + 2*(6*b^5*cosh(d*x + c)^3 + 15*a^2*b^
3*cosh(d*x + c)^2 - 5*a^2*b^3 + 6*(10*a^4*b + 9*b^5)*cosh(d*x + c))*sinh(d*
x + c))*sqrt(sinh(d*x + c)))/(b^6*d*cosh(d*x + c)^2 + 2*b^6*d*cosh(d*x + c)
*sinh(d*x + c) + b^6*d*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)^3}{b\sqrt{\sinh(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)
```

$$3.413 \quad \int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}} dx$$

Optimal. Leaf size=43

$$\frac{2\sqrt{\sinh(c+dx)}}{bd} - \frac{2a \log(a + b\sqrt{\sinh(c+dx)})}{b^2d}$$

[Out] $(-2*a*\text{Log}[a + b*\text{Sqrt}[\text{Sinh}[c + d*x]])/(b^2*d) + (2*\text{Sqrt}[\text{Sinh}[c + d*x]])/(b*d)$

Rubi [A] time = 0.050232, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3223, 190, 43}

$$\frac{2\sqrt{\sinh(c+dx)}}{bd} - \frac{2a \log(a + b\sqrt{\sinh(c+dx)})}{b^2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cosh}[c + d*x]/(a + b*\text{Sqrt}[\text{Sinh}[c + d*x]]), x]$

[Out] $(-2*a*\text{Log}[a + b*\text{Sqrt}[\text{Sinh}[c + d*x]])/(b^2*d) + (2*\text{Sqrt}[\text{Sinh}[c + d*x]])/(b*d)$

Rule 3223

$\text{Int}[\cos[(e_.) + (f_.)*(x_.)]^{(m_.)*((a_.) + (b_.)*((c_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] := \text{With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f*x], x]\}, \text{Dist}[ff/f, \text{Subst}[\text{Int}[(1 - ff^2*x^2)^{(m-1)/2}*(a + b*(c*ff*x)^n)^p, x], x, \text{Sin}[e + f*x]/ff], x]] /; \text{FreeQ}\{a, b, c, e, f, n, p\}, x] \&\& \text{IntegerQ}[(m-1)/2] \&\& (\text{EqQ}[n, 4] \parallel \text{GtQ}[m, 0] \parallel \text{IGtQ}[p, 0] \parallel \text{IntegersQ}[m, p])$

Rule 190

$\text{Int}[(a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(1/n-1)*(a + b*x)^p}, x], x, x^n], x] /; \text{FreeQ}\{a, b, p\}, x] \&\& \text{FractionQ}[n] \&\& \text{IntegerQ}[1/n]$

Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)]^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\},$

$x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\ !\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{a + b\sqrt{\sinh(c + dx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b\sqrt{x}} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \frac{x}{a+bx} dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= -\frac{2a \log(a + b\sqrt{\sinh(c + dx)})}{b^2 d} + \frac{2\sqrt{\sinh(c + dx)}}{bd} \end{aligned}$$

Mathematica [A] time = 0.0274741, size = 41, normalized size = 0.95

$$\frac{2 \left(\frac{\sqrt{\sinh(c+dx)}}{b} - \frac{a \log(a+b\sqrt{\sinh(c+dx)})}{b^2} \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]), x]

[Out] (2*(-((a*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^2) + Sqrt[Sinh[c + d*x]]/b))/d

Maple [B] time = 0.013, size = 89, normalized size = 2.1

$$2 \frac{\sqrt{\sinh(dx + c)}}{bd} + \frac{a}{db^2} \ln\left(b\sqrt{\sinh(dx + c)} - a\right) - \frac{a}{db^2} \ln\left(a + b\sqrt{\sinh(dx + c)}\right) - \frac{a \ln\left(b^2 \sinh(dx + c) - a^2\right)}{db^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)), x)

[Out] 2*sinh(d*x+c)^(1/2)/b/d+1/d/b^2*a*ln(b*sinh(d*x+c)^(1/2)-a)-a*ln(a+b*sinh(d*x+c)^(1/2))/b^2/d-1/d*a*ln(b^2*sinh(d*x+c)-a^2)/b^2

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{b\sqrt{\sinh(dx+c)+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)

Fricas [B] time = 6.3547, size = 563, normalized size = 13.09

$$adx + a \log \left(\frac{b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 + 2a^2 \cosh(dx+c) - b^2 + 2(b^2 \cosh(dx+c) + a^2) \sinh(dx+c) - 4(ab \cosh(dx+c) + ab \sinh(dx+c)) \sqrt{\sinh(dx+c)}}{b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 - 2a^2 \cosh(dx+c) - b^2 + 2(b^2 \cosh(dx+c) - a^2) \sinh(dx+c)} \right) \frac{1}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] (a*d*x + a*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh(d*x + c))) - a*log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 2*b*sqrt(sinh(d*x + c)))/(b^2*d)

Sympy [A] time = 1.90048, size = 68, normalized size = 1.58

$$\begin{cases} \frac{x \cosh(c)}{\sinh(c+dx)} & \text{for } b = 0 \wedge d = 0 \\ \frac{a}{\sinh(c+dx)} & \text{for } b = 0 \\ \frac{ad}{x \cosh(c)} & \text{for } d = 0 \\ -\frac{a+b\sqrt{\sinh(c)}}{2a \log\left(\frac{a}{b} + \sqrt{\sinh(c+dx)}\right)} + \frac{2\sqrt{\sinh(c+dx)}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)

[Out] Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a*d), Eq(b, 0)), (x*cosh(c)/(a + b*sqrt(sinh(c))), Eq(d, 0)), (-2*a*log(a/b + sqrt(sinh(c + d*x)))/(b**2*d) + 2*sqrt(sinh(c + d*x))/(b*d), True))

Giac [A] time = 1.23697, size = 92, normalized size = 2.14

$$-\frac{2a \log\left(\left|b\sqrt{\frac{1}{2}e^{dx+c}} - \frac{1}{2}e^{-dx-c}\right| + a\right)}{b^2d} + \frac{2\sqrt{\frac{1}{2}e^{dx+c}} - \frac{1}{2}e^{-dx-c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")

[Out] -2*a*log(abs(b*sqrt(1/2*e^(d*x + c) - 1/2*e^(-d*x - c)) + a))/(b^2*d) + 2*sqrt(1/2*e^(d*x + c) - 1/2*e^(-d*x - c))/(b*d)

$$3.414 \quad \int \frac{\operatorname{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}} dx$$

Optimal. Leaf size=286

$$\frac{2ab^2 \log(a + b\sqrt{\sinh(c+dx)})}{d(a^4 + b^4)} - \frac{b(a^2 + b^2) \log(\sinh(c+dx) - \sqrt{2}\sqrt{\sinh(c+dx)} + 1)}{2\sqrt{2}d(a^4 + b^4)} + \frac{b(a^2 + b^2) \log(\sinh(c+dx) + \sqrt{2}\sqrt{\sinh(c+dx)})}{2\sqrt{2}d(a^4 + b^4)}$$

```
[Out] (b*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^4 + b^4)*d) - (b*(a^2 - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^4 + b^4)*d) + (a^3*ArcTan[Sinh[c + d*x]])/((a^4 + b^4)*d) + (a*b^2*Log[Cosh[c + d*x]])/((a^4 + b^4)*d) - (2*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(a^4 + b^4)*d - (b*(a^2 + b^2)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d) + (b*(a^2 + b^2)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d)
```

Rubi [A] time = 0.482466, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3223, 6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$\frac{2ab^2 \log(a + b\sqrt{\sinh(c+dx)})}{d(a^4 + b^4)} - \frac{b(a^2 + b^2) \log(\sinh(c+dx) - \sqrt{2}\sqrt{\sinh(c+dx)} + 1)}{2\sqrt{2}d(a^4 + b^4)} + \frac{b(a^2 + b^2) \log(\sinh(c+dx) + \sqrt{2}\sqrt{\sinh(c+dx)})}{2\sqrt{2}d(a^4 + b^4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]), x]
```

```
[Out] (b*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^4 + b^4)*d) - (b*(a^2 - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^4 + b^4)*d) + (a^3*ArcTan[Sinh[c + d*x]])/((a^4 + b^4)*d) + (a*b^2*Log[Cosh[c + d*x]])/((a^4 + b^4)*d) - (2*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(a^4 + b^4)*d - (b*(a^2 + b^2)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d) + (b*(a^2 + b^2)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)]))^(n_.)]^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[m - 1]
```

)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rule 1876

Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rule 1168

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]

Rule 1162

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 617

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1165

```
Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Free
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol]
:= Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(c + dx)}{a + b\sqrt{\sinh(c + dx)}} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+b\sqrt{x})(1+x^2)} dx, x, \sinh(c + dx)\right)}{d} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{x}{(a+bx)(1+x^4)} dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\
&= \frac{2 \operatorname{Subst}\left(\int \left(-\frac{ab^3}{(a^4+b^4)(a+bx)} + \frac{b^3+a^3x-a^2bx^2+ab^2x^3}{(a^4+b^4)(1+x^4)}\right) dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\
&= -\frac{2ab^2 \log(a + b\sqrt{\sinh(c + dx)})}{(a^4 + b^4)d} + \frac{2 \operatorname{Subst}\left(\int \frac{b^3+a^3x-a^2bx^2+ab^2x^3}{1+x^4} dx, x, \sqrt{\sinh(c + dx)}\right)}{(a^4 + b^4)d} \\
&= -\frac{2ab^2 \log(a + b\sqrt{\sinh(c + dx)})}{(a^4 + b^4)d} + \frac{2 \operatorname{Subst}\left(\int \left(\frac{b^3-a^2bx^2}{1+x^4} + \frac{x(a^3+ab^2x^2)}{1+x^4}\right) dx, x, \sqrt{\sinh(c + dx)}\right)}{(a^4 + b^4)d} \\
&= -\frac{2ab^2 \log(a + b\sqrt{\sinh(c + dx)})}{(a^4 + b^4)d} + \frac{2 \operatorname{Subst}\left(\int \frac{b^3-a^2bx^2}{1+x^4} dx, x, \sqrt{\sinh(c + dx)}\right)}{(a^4 + b^4)d} + \frac{2 \operatorname{Subst}\left(\int \frac{x(a^3+ab^2x^2)}{1+x^4} dx, x, \sqrt{\sinh(c + dx)}\right)}{(a^4 + b^4)d} \\
&= -\frac{2ab^2 \log(a + b\sqrt{\sinh(c + dx)})}{(a^4 + b^4)d} + \frac{\operatorname{Subst}\left(\int \frac{a^3+ab^2x}{1+x^2} dx, x, \sinh(c + dx)\right)}{(a^4 + b^4)d} - \frac{(b(a^2 - b^2)) \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{(a^4 + b^4)d} \\
&= -\frac{2ab^2 \log(a + b\sqrt{\sinh(c + dx)})}{(a^4 + b^4)d} + \frac{a^3 \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{(a^4 + b^4)d} + \frac{(ab^2) \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{(a^4 + b^4)d} \\
&= \frac{a^3 \tan^{-1}(\sinh(c + dx))}{(a^4 + b^4)d} + \frac{ab^2 \log(\cosh(c + dx))}{(a^4 + b^4)d} - \frac{2ab^2 \log(a + b\sqrt{\sinh(c + dx)})}{(a^4 + b^4)d} - \frac{b(a^2 + b^2) \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sinh(c + dx)\right)}{(a^4 + b^4)d} \\
&= \frac{b(a^2 - b^2) \tan^{-1}(1 - \sqrt{2}\sqrt{\sinh(c + dx)})}{\sqrt{2}(a^4 + b^4)d} - \frac{b(a^2 - b^2) \tan^{-1}(1 + \sqrt{2}\sqrt{\sinh(c + dx)})}{\sqrt{2}(a^4 + b^4)d} + \frac{a^3 \tan^{-1}(\sinh(c + dx))}{(a^4 + b^4)d}
\end{aligned}$$

Mathematica [C] time = 0.248221, size = 229, normalized size = 0.8

$$3(4a^3 \tan^{-1}(\sinh(c + dx)) - 8ab^2 \log(a + b\sqrt{\sinh(c + dx)}) + 4ab^2 \log(\cosh(c + dx)) - \sqrt{2}b^3 \log(\sinh(c + dx) - \sqrt{2}\sqrt{\sinh(c + dx)}))$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]), x]

[Out] $(3*(-2*\sqrt{2}*b^3*\text{ArcTan}[1 - \sqrt{2}*\sqrt{\text{Sinh}[c + d*x]})] + 2*\sqrt{2}*b^3*\text{ArcTan}[1 + \sqrt{2}*\sqrt{\text{Sinh}[c + d*x]})] + 4*a^3*\text{ArcTan}[\text{Sinh}[c + d*x]] + 4*a*b^2*\text{Log}[\text{Cosh}[c + d*x]] - 8*a*b^2*\text{Log}[a + b*\sqrt{\text{Sinh}[c + d*x]})] - \sqrt{2}*b^3*\text{Log}[1 - \sqrt{2}*\sqrt{\text{Sinh}[c + d*x]} + \text{Sinh}[c + d*x]] + \sqrt{2}*b^3*\text{Log}[1 + \sqrt{2}*\sqrt{\text{Sinh}[c + d*x]} + \text{Sinh}[c + d*x]]) - 8*a^2*b*\text{Hypergeometric2F1}[3/4, 1, 7/4, -\text{Sinh}[c + d*x]^2]*\text{Sinh}[c + d*x]^{(3/2)}/(12*(a^4 + b^4)*d)$

Maple [C] time = 0.108, size = 206, normalized size = 0.7

$$-\frac{ab^2}{d(a^4 + b^4)} \ln\left(a^2 \left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 + 2b^2 \tanh\left(\frac{1}{2}dx + \frac{c}{2}\right) - a^2\right) + \frac{ab^2}{d(a^4 + b^4)} \ln\left(\left(\tanh\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2 + 1\right) + 2\frac{a^3}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x)`

[Out] $-a/d*b^2/(a^4+b^4)*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)+a/d/(a^4+b^4)*b^2*\ln(\tanh(1/2*d*x+1/2*c)^2+1)+2*a^3/d/(a^4+b^4)*\arctan(\tanh(1/2*d*x+1/2*c))+\int/\text{indef0}^{\wedge}(b*\sinh(d*x+c)^{(1/2)}*(-b^2*\sinh(d*x+c)+a^2)/(2*a^2*b^2*\sinh(d*x+c)*\cosh(d*x+c)^2-b^4*\cosh(d*x+c)^4+(-a^4+b^4)*\cosh(d*x+c)^2),\sinh(d*x+c))/d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{sech}(dx + c)}{b\sqrt{\sinh(dx + c) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c + dx)}{a + b\sqrt{\sinh(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx + c)}{b\sqrt{\sinh(dx + c)} + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```


$$3.415 \quad \int \frac{\cosh^5(c+dx)}{(a+b\sqrt{\sinh(c+dx)})^2} dx$$

Optimal. Leaf size=270

$$\frac{a^2 \sinh^3(c+dx)}{b^4 d} - \frac{8a^3 \sinh^{\frac{5}{2}}(c+dx)}{5b^5 d} + \frac{(5a^4 + 2b^4) \sinh^2(c+dx)}{2b^6 d} - \frac{4a(3a^4 + 2b^4) \sinh^{\frac{3}{2}}(c+dx)}{3b^7 d} + \frac{a^2(7a^4 + 6b^4) \sinh(c+dx)}{b^8 d}$$

[Out] (2*(a^4 + b^4)*(9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*a*(a^4 + b^4)^2)/(b^10*d*(a + b*Sqrt[Sinh[c + d*x]])) - (16*a^3*(a^4 + b^4)*Sqrt[Sinh[c + d*x]])/(b^9*d) + (a^2*(7*a^4 + 6*b^4)*Sinh[c + d*x])/(b^8*d) - (4*a*(3*a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) + ((5*a^4 + 2*b^4)*Sinh[c + d*x]^2)/(2*b^6*d) - (8*a^3*Sinh[c + d*x]^(5/2))/(5*b^5*d) + (a^2*Sinh[c + d*x]^3)/(b^4*d) - (4*a*Sinh[c + d*x]^(7/2))/(7*b^3*d) + Sinh[c + d*x]^4/(4*b^2*d)

Rubi [A] time = 0.323083, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3223, 1890, 1620}

$$\frac{a^2 \sinh^3(c+dx)}{b^4 d} - \frac{8a^3 \sinh^{\frac{5}{2}}(c+dx)}{5b^5 d} + \frac{(5a^4 + 2b^4) \sinh^2(c+dx)}{2b^6 d} - \frac{4a(3a^4 + 2b^4) \sinh^{\frac{3}{2}}(c+dx)}{3b^7 d} + \frac{a^2(7a^4 + 6b^4) \sinh(c+dx)}{b^8 d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]

[Out] (2*(a^4 + b^4)*(9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*a*(a^4 + b^4)^2)/(b^10*d*(a + b*Sqrt[Sinh[c + d*x]])) - (16*a^3*(a^4 + b^4)*Sqrt[Sinh[c + d*x]])/(b^9*d) + (a^2*(7*a^4 + 6*b^4)*Sinh[c + d*x])/(b^8*d) - (4*a*(3*a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) + ((5*a^4 + 2*b^4)*Sinh[c + d*x]^2)/(2*b^6*d) - (8*a^3*Sinh[c + d*x]^(5/2))/(5*b^5*d) + (a^2*Sinh[c + d*x]^3)/(b^4*d) - (4*a*Sinh[c + d*x]^(7/2))/(7*b^3*d) + Sinh[c + d*x]^4/(4*b^2*d)

Rule 3223

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_.)])^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,

$\text{Sin}[e + f*x]/ff], x]] /; \text{FreeQ}\{a, b, c, e, f, n, p\}, x\} \&\& \text{IntegerQ}\{(m - 1)/2\} \&\& (\text{EqQ}[n, 4] \parallel \text{GtQ}[m, 0] \parallel \text{IGtQ}[p, 0] \parallel \text{IntegersQ}[m, p])$

Rule 1890

$\text{Int}[(\text{Pq}_*)*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] := \text{With}\{g = \text{Denominator}[n]\}, \text{Dist}[g, \text{Subst}[\text{Int}[x^{(g-1)}*(\text{Pq} /. x \rightarrow x^g)*(a + b*x^{(g*n)})^p, x], x, x^{(1/g)}], x]] /; \text{FreeQ}\{a, b, p\}, x\} \&\& \text{PolyQ}[\text{Pq}, x] \&\& \text{FractionQ}[n]$

Rule 1620

$\text{Int}[(\text{Px}_*)*((a_.) + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \text{Int}[\text{ExpandIntegrand}[\text{Px}*(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x\} \&\& \text{PolyQ}[\text{Px}, x] \&\& (\text{IntegersQ}[m, n] \parallel \text{IGtQ}[m, -2]) \&\& \text{GtQ}[\text{Expon}[\text{Px}, x], 2]$

Rubi steps

$$\begin{aligned} \int \frac{\cosh^5(c + dx)}{(a + b\sqrt{\sinh(c + dx)})^2} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+b\sqrt{x})^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \frac{x(1+x^4)^2}{(a+bx)^2} dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \left(-\frac{8a^3(a^4+b^4)}{b^9} + \frac{a^2(7a^4+6b^4)x}{b^8} - \frac{2a(3a^4+2b^4)x^2}{b^7} + \frac{(5a^4+2b^4)x^3}{b^6} - \frac{4a^3x^4}{b^5} + \frac{3a^2x^5}{b^4} - \frac{2ax^6}{b^3} + \frac{2x^7}{b^2}\right) dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= \frac{2(a^4 + b^4)(9a^4 + b^4) \log(a + b\sqrt{\sinh(c + dx)})}{b^{10}d} + \frac{2a(a^4 + b^4)^2}{b^{10}d(a + b\sqrt{\sinh(c + dx)})} - \frac{16a^3(a^4 + b^4)}{b^{10}d} \end{aligned}$$

Mathematica [A] time = 0.618676, size = 288, normalized size = 1.07

$$\frac{180a^2b^7 \sinh^{\frac{7}{2}}(c + dx) - 252a^3b^6 \sinh^3(c + dx) + 42b^5(9a^4 + 10b^4) \sinh^{\frac{5}{2}}(c + dx) - 70ab^4(9a^4 + 10b^4) \sinh^2(c + dx) + 16a^3(a^4 + b^4)}{b^{10}d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]

```
[Out] (840*a*(a^4 + b^4)*(a^4 + b^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]]
+ 840*b*(a^4 + b^4)*(-8*a^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])
)*Sqrt[Sinh[c + d*x]] - 420*a^3*b^2*(9*a^4 + 10*b^4)*Sinh[c + d*x] + 140*
a^2*b^3*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(3/2) - 70*a*b^4*(9*a^4 + 10*b^4)*Si
nh[c + d*x]^2 + 42*b^5*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(5/2) - 252*a^3*b^6*S
inh[c + d*x]^3 + 180*a^2*b^7*Sinh[c + d*x]^(7/2) - 135*a*b^8*Sinh[c + d*x]^
4 + 105*b^9*Sinh[c + d*x]^(9/2))/(420*b^10*d*(a + b*Sqrt[Sinh[c + d*x]]))
```

Maple [C] time = 0.218, size = 955, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

```
[Out] 9/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2+9/8/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2-7/d
/b^8/(tanh(1/2*d*x+1/2*c)-1)*a^6+5/2/d/b^6/(tanh(1/2*d*x+1/2*c)-1)*a^4-6/d/
b^4/(tanh(1/2*d*x+1/2*c)-1)*a^2-1/d/b^4/(tanh(1/2*d*x+1/2*c)-1)^3*a^2+5/2/
/b^6/(tanh(1/2*d*x+1/2*c)-1)^2*a^4-3/2/d/b^4/(tanh(1/2*d*x+1/2*c)-1)^2*a^2-
9/d/b^10*ln(tanh(1/2*d*x+1/2*c)-1)*a^8-10/d/b^6*ln(tanh(1/2*d*x+1/2*c)-1)*a
^4+9/d/b^10*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^8
+10/d/b^6*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^4-7
/d/b^8/(tanh(1/2*d*x+1/2*c)+1)*a^6-5/2/d/b^6/(tanh(1/2*d*x+1/2*c)+1)*a^4-6/
d/b^4/(tanh(1/2*d*x+1/2*c)+1)*a^2-1/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^3*a^2+5/2
/d/b^6/(tanh(1/2*d*x+1/2*c)+1)^2*a^4+3/2/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^2*a^
2-9/d/b^10*ln(tanh(1/2*d*x+1/2*c)+1)*a^8-10/d/b^6*ln(tanh(1/2*d*x+1/2*c)+1)
*a^4-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-4/
d*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-
a^2)-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^3+1/4/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^
4+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3+1/d/b^2*ln(a^2*tanh(1/2*d*x+1/2*c)^2+
2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+1/4/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^4-7/8/d/b^
2/(tanh(1/2*d*x+1/2*c)+1)+7/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)+`int/undef0`(-2
*cosh(d*x+c)^4*a*b*sinh(d*x+c)^(1/2)/(b^4*sinh(d*x+c)^2-2*a^2*b^2*sinh(d*x+
c)+a^4),sinh(d*x+c))/d-8/d/b^4*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)
^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^4-4/d/b^8*tanh(1/2*d*x+1/2*c)/(a^2*tanh
(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^8
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 8.8936, size = 12573, normalized size = 46.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(105*b^10*cosh(d*x + c)^10 + 105*b^10*sinh(d*x + c)^10 + 630*a^2*b^8
*cosh(d*x + c)^9 + 630*a^2*b^8*cosh(d*x + c) - 105*b^10 + 210*(5*b^10*cosh(
d*x + c) + 3*a^2*b^8)*sinh(d*x + c)^9 + 105*(24*a^4*b^6 + 11*b^10)*cosh(d*x
+ c)^8 + 105*(45*b^10*cosh(d*x + c)^2 + 54*a^2*b^8*cosh(d*x + c) + 24*a^4*
b^6 + 11*b^10)*sinh(d*x + c)^8 + 840*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c
)^7 + 840*(15*b^10*cosh(d*x + c)^3 + 27*a^2*b^8*cosh(d*x + c)^2 + 18*a^6*b^
4 + 17*a^2*b^8 + (24*a^4*b^6 + 11*b^10)*cosh(d*x + c))*sinh(d*x + c)^7 - 42
0*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d
*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^6 + 210*(105*b^10*
cosh(d*x + c)^4 + 252*a^2*b^8*cosh(d*x + c)^3 - 224*a^8*b^2 - 188*a^4*b^6 -
6*b^10 - 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 14*(24*a^4*b^6 + 11*b^10
)*cosh(d*x + c)^2 - 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c + 28*(18*a^6*b^4 +
17*a^2*b^8)*cosh(d*x + c))*sinh(d*x + c)^6 - 1680*(16*a^10 + 60*a^6*b^4 +
37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4
+ a^2*b^8)*c)*cosh(d*x + c)^5 + 420*(63*b^10*cosh(d*x + c)^5 + 189*a^2*b^8
*cosh(d*x + c)^4 - 64*a^10 - 240*a^6*b^4 - 148*a^2*b^8 + 14*(24*a^4*b^6 + 1
1*b^10)*cosh(d*x + c)^3 + 32*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x + 42*(18*a
^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^2 + 32*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c
- 6*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10
)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c))*sinh(d*x + c)^
5 + 420*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b
^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^4 + 210*(105
*b^10*cosh(d*x + c)^6 + 378*a^2*b^8*cosh(d*x + c)^5 + 224*a^8*b^2 + 188*a^4
*b^6 + 6*b^10 + 35*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^4 + 140*(18*a^6*b^4
+ 17*a^2*b^8)*cosh(d*x + c)^3 + 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x - 3
0*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d
*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^2 + 32*(9*a^8*b^2
+ 10*a^4*b^6 + b^10)*c - 40*(16*a^10 + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a^10
```

$$\begin{aligned}
& + 10a^6b^4 + a^2b^8)dx - 8(9a^{10} + 10a^6b^4 + a^2b^8)c \cosh(dx + c) \sinh(dx + c)^4 + 840(18a^6b^4 + 17a^2b^8) \cosh(dx + c)^3 + 840(15b^{10} \cosh(dx + c)^7 + 63a^2b^8 \cosh(dx + c)^6 + 18a^6b^4 + 17a^2b^8 + 7(24a^4b^6 + 11b^{10}) \cosh(dx + c)^5 + 35(18a^6b^4 + 17a^2b^8) \cosh(dx + c)^4 - 10(112a^8b^2 + 94a^4b^6 + 3b^{10} + 16(9a^8b^2 + 10a^4b^6 + b^{10}))dx + 16(9a^8b^2 + 10a^4b^6 + b^{10})c \cosh(dx + c)^3 - 20(16a^{10} + 60a^6b^4 + 37a^2b^8 - 8(9a^{10} + 10a^6b^4 + a^2b^8))dx - 8(9a^{10} + 10a^6b^4 + a^2b^8)c \cosh(dx + c)^2 + 2(112a^8b^2 + 94a^4b^6 + 3b^{10} + 16(9a^8b^2 + 10a^4b^6 + b^{10}))dx + 16(9a^8b^2 + 10a^4b^6 + b^{10})c \cosh(dx + c) \sinh(dx + c)^3 - 105(24a^4b^6 + 11b^{10}) \cosh(dx + c)^2 + 105(45b^{10} \cosh(dx + c)^8 + 216a^2b^8 \cosh(dx + c)^7 - 24a^4b^6 - 11b^{10} + 28(24a^4b^6 + 11b^{10}) \cosh(dx + c)^6 + 168(18a^6b^4 + 17a^2b^8) \cosh(dx + c)^5 - 60(112a^8b^2 + 94a^4b^6 + 3b^{10} + 16(9a^8b^2 + 10a^4b^6 + b^{10}))dx + 16(9a^8b^2 + 10a^4b^6 + b^{10})c \cosh(dx + c)^4 - 160(16a^{10} + 60a^6b^4 + 37a^2b^8 - 8(9a^{10} + 10a^6b^4 + a^2b^8))dx - 8(9a^{10} + 10a^6b^4 + a^2b^8)c \cosh(dx + c)^3 + 24(112a^8b^2 + 94a^4b^6 + 3b^{10} + 16(9a^8b^2 + 10a^4b^6 + b^{10}))dx + 16(9a^8b^2 + 10a^4b^6 + b^{10})c \cosh(dx + c)^2 + 24(18a^6b^4 + 17a^2b^8) \cosh(dx + c) \sinh(dx + c)^2 + 6720((9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^6 + (9a^8b^2 + 10a^4b^6 + b^{10}) \sinh(dx + c)^6 - 2(9a^{10} + 10a^6b^4 + a^2b^8) \cosh(dx + c)^5 - 2(9a^{10} + 10a^6b^4 + a^2b^8 - 3(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)) \sinh(dx + c)^5 - (9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^4 - (9a^8b^2 + 10a^4b^6 + b^{10} - 15(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^2 + 10(9a^{10} + 10a^6b^4 + a^2b^8) \cosh(dx + c)) \sinh(dx + c)^4 + 4(5(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^3 - 5(9a^{10} + 10a^6b^4 + a^2b^8) \cosh(dx + c)^2 - (9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)) \sinh(dx + c)^3 + (15(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^4 - 20(9a^{10} + 10a^6b^4 + a^2b^8) \cosh(dx + c)^3 - 6(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^2) \sinh(dx + c)^2 + 2(3(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^5 - 5(9a^{10} + 10a^6b^4 + a^2b^8) \cosh(dx + c)^4 - 2(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^3) \sinh(dx + c) \log(-(b^2 \cosh(dx + c)^2 + b^2 \sinh(dx + c)^2 + 2a^2 \cosh(dx + c) - b^2 + 2(b^2 \cosh(dx + c) + a^2) \sinh(dx + c) + 4(a b \cosh(dx + c) + a b \sinh(dx + c)) \sqrt{\sinh(dx + c)})) / (b^2 \cosh(dx + c)^2 + b^2 \sinh(dx + c)^2 - 2a^2 \cosh(dx + c) - b^2 + 2(b^2 \cosh(dx + c) - a^2) \sinh(dx + c))) + 6720((9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^6 + (9a^8b^2 + 10a^4b^6 + b^{10}) \sinh(dx + c)^6 - 2(9a^{10} + 10a^6b^4 + a^2b^8) \cosh(dx + c)^5 - 2(9a^{10} + 10a^6b^4 + a^2b^8 - 3(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)) \sinh(dx + c)^5 - (9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^4 - (9a^8b^2 + 10a^4b^6 + b^{10} - 15(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^2 + 10(9a^{10} + 10a^6b^4 + a^2b^8) \cosh(dx + c)) \sinh(dx + c)^4 + 4(5(9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)^3 - 5(9a^{10} + 10a^6b^4 + a^2b^8) \cosh(dx + c)^2 - (9a^8b^2 + 10a^4b^6 + b^{10}) \cosh(dx + c)) \sinh(dx + c)^3 + (15(9a^8
\end{aligned}$$

$$\begin{aligned}
& *b^2 + 10*a^4*b^6 + b^{10}) * \cosh(dx + c)^4 - 20*(9*a^{10} + 10*a^6*b^4 + a^2*b^8) * \cosh(dx + c)^3 - 6*(9*a^8*b^2 + 10*a^4*b^6 + b^{10}) * \cosh(dx + c)^2 * \sinh(dx + c)^2 + 2*(3*(9*a^8*b^2 + 10*a^4*b^6 + b^{10}) * \cosh(dx + c)^5 - 5*(9*a^{10} + 10*a^6*b^4 + a^2*b^8) * \cosh(dx + c)^4 - 2*(9*a^8*b^2 + 10*a^4*b^6 + b^{10}) * \cosh(dx + c)^3) * \sinh(dx + c) * \log(2*(b^2 * \sinh(dx + c) - a^2) / (\cosh(dx + c) - \sinh(dx + c))) + 210*(5*b^{10} * \cosh(dx + c)^9 + 27*a^2*b^8 * \cosh(dx + c)^8 + 3*a^2*b^8 + 4*(24*a^4*b^6 + 11*b^{10}) * \cosh(dx + c)^7 + 28*(18*a^6*b^4 + 17*a^2*b^8) * \cosh(dx + c)^6 - 12*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^{10} + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^{10}) * dx + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^{10}) * c) * \cosh(dx + c)^5 - 40*(16*a^{10} + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a^{10} + 10*a^6*b^4 + a^2*b^8) * dx - 8*(9*a^{10} + 10*a^6*b^4 + a^2*b^8) * c) * \cosh(dx + c)^4 + 8*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^{10} + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^{10}) * dx + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^{10}) * c) * \cosh(dx + c)^3 + 12*(18*a^6*b^4 + 17*a^2*b^8) * \cosh(dx + c)^2 - (24*a^4*b^6 + 11*b^{10}) * \cosh(dx + c) * \sinh(dx + c) - 32*(15*a*b^9 * \cosh(dx + c)^9 + 15*a*b^9 * \sinh(dx + c)^9 + 54*a^3*b^7 * \cosh(dx + c)^8 - 54*a^3*b^7 * \cosh(dx + c)^2 + 15*a*b^9 * \cosh(dx + c) + 27*(5*a*b^9 * \cosh(dx + c) + 2*a^3*b^7) * \sinh(dx + c)^8 + 4*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)^7 + 4*(135*a*b^9 * \cosh(dx + c)^2 + 108*a^3*b^7 * \cosh(dx + c) + 63*a^5*b^5 + 55*a*b^9) * \sinh(dx + c)^7 + 2*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)^6 + 2*(630*a*b^9 * \cosh(dx + c)^3 + 756*a^3*b^7 * \cosh(dx + c)^2 + 1260*a^7*b^3 + 1319*a^3*b^7 + 14*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)) * \sinh(dx + c)^6 - 2*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9) * \cosh(dx + c)^5 + 2*(945*a*b^9 * \cosh(dx + c)^4 + 1512*a^3*b^7 * \cosh(dx + c)^3 - 3780*a^9*b - 4452*a^5*b^5 - 655*a*b^9 + 42*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)^2 + 6*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)) * \sinh(dx + c)^5 - 2*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)^4 + 2*(945*a*b^9 * \cosh(dx + c)^5 + 1890*a^3*b^7 * \cosh(dx + c)^4 - 1260*a^7*b^3 - 1319*a^3*b^7 + 70*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)^3 + 15*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)^2 - 5*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9) * \cosh(dx + c)) * \sinh(dx + c)^4 + 4*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)^3 + 4*(315*a*b^9 * \cosh(dx + c)^6 + 756*a^3*b^7 * \cosh(dx + c)^5 + 63*a^5*b^5 + 55*a*b^9 + 35*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)^4 + 10*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)^3 - 5*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9) * \cosh(dx + c)^2 - 2*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)) * \sinh(dx + c)^3 + 2*(270*a*b^9 * \cosh(dx + c)^7 + 756*a^3*b^7 * \cosh(dx + c)^6 - 27*a^3*b^7 + 42*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)^5 + 15*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)^4 - 10*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9) * \cosh(dx + c)^3 - 6*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)^2 + 6*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)) * \sinh(dx + c)^2 + (135*a*b^9 * \cosh(dx + c)^8 + 432*a^3*b^7 * \cosh(dx + c)^7 - 108*a^3*b^7 * \cosh(dx + c) + 15*a*b^9 + 28*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)^6 + 12*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)^5 - 10*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9) * \cosh(dx + c)^4 - 8*(1260*a^7*b^3 + 1319*a^3*b^7) * \cosh(dx + c)^3 + 12*(63*a^5*b^5 + 55*a*b^9) * \cosh(dx + c)^2) * \sinh(dx + c)) * \sqrt{\sinh(dx + c)}) / (b^{12} * d * \cosh(dx + c)^6 + b^{12} * d * \sinh(dx + c)^6 - 2*a^2*b^{10} * d * \cosh(dx + c)^5 - b^{12} * d * \cosh(dx
\end{aligned}$$

$$x + c)^4 + 2*(3*b^{12}*d*\cosh(d*x + c) - a^2*b^{10}*d)*\sinh(d*x + c)^5 + (15*b^{12}*d*\cosh(d*x + c)^2 - 10*a^2*b^{10}*d*\cosh(d*x + c) - b^{12}*d)*\sinh(d*x + c)^4 + 4*(5*b^{12}*d*\cosh(d*x + c)^3 - 5*a^2*b^{10}*d*\cosh(d*x + c)^2 - b^{12}*d*\cosh(d*x + c))*\sinh(d*x + c)^3 + (15*b^{12}*d*\cosh(d*x + c)^4 - 20*a^2*b^{10}*d*\cosh(d*x + c)^3 - 6*b^{12}*d*\cosh(d*x + c)^2)*\sinh(d*x + c)^2 + 2*(3*b^{12}*d*\cosh(d*x + c)^5 - 5*a^2*b^{10}*d*\cosh(d*x + c)^4 - 2*b^{12}*d*\cosh(d*x + c)^3)*\sinh(d*x + c)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2))**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.416 \quad \int \frac{\cosh^3(c+dx)}{(a+b\sqrt{\sinh(c+dx)})^2} dx$$

Optimal. Leaf size=142

$$-\frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d} + \frac{3a^2\sinh(c+dx)}{b^4d} + \frac{2a(a^4+b^4)}{b^6d(a+b\sqrt{\sinh(c+dx)})} + \frac{2(5a^4+b^4)\log(a+b\sqrt{\sinh(c+dx)})}{b^6d} - \frac{4a\sinh^{\frac{3}{2}}(c+dx)}{3b^3d}$$

[Out] (2*(5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*a*(a^4 + b^4))/(b^6*d*(a + b*Sqrt[Sinh[c + d*x]])) - (8*a^3*Sqrt[Sinh[c + d*x]])/(b^5*d) + (3*a^2*Sinh[c + d*x])/(b^4*d) - (4*a*Sinh[c + d*x]^(3/2))/(3*b^3*d) + Sinh[c + d*x]^2/(2*b^2*d)

Rubi [A] time = 0.163606, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3223, 1890, 1620}

$$-\frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d} + \frac{3a^2\sinh(c+dx)}{b^4d} + \frac{2a(a^4+b^4)}{b^6d(a+b\sqrt{\sinh(c+dx)})} + \frac{2(5a^4+b^4)\log(a+b\sqrt{\sinh(c+dx)})}{b^6d} - \frac{4a\sinh^{\frac{3}{2}}(c+dx)}{3b^3d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]

[Out] (2*(5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*a*(a^4 + b^4))/(b^6*d*(a + b*Sqrt[Sinh[c + d*x]])) - (8*a^3*Sqrt[Sinh[c + d*x]])/(b^5*d) + (3*a^2*Sinh[c + d*x])/(b^4*d) - (4*a*Sinh[c + d*x]^(3/2))/(3*b^3*d) + Sinh[c + d*x]^2/(2*b^2*d)

Rule 3223

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])

Rule 1890

Int[(Pq)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x

, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
 :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
 , d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
 xpon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(c + dx)}{(a + b\sqrt{\sinh(c + dx)})^2} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+b\sqrt{x})^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \frac{x(1+x^4)}{(a+bx)^2} dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \left(-\frac{4a^3}{b^5} + \frac{3a^2x}{b^4} - \frac{2ax^2}{b^3} + \frac{x^3}{b^2} - \frac{a(a^4+b^4)}{b^5(a+bx)^2} + \frac{5a^4+b^4}{b^5(a+bx)}\right) dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= \frac{2(5a^4 + b^4) \log(a + b\sqrt{\sinh(c + dx)})}{b^6d} + \frac{2a(a^4 + b^4)}{b^6d(a + b\sqrt{\sinh(c + dx)})} - \frac{8a^3\sqrt{\sinh(c + dx)}}{b^5d} \end{aligned}$$

Mathematica [A] time = 0.464117, size = 123, normalized size = 0.87

$$\frac{18a^2b^2 \sinh(c + dx) + 12\left(\frac{a(a^4+b^4)}{a+b\sqrt{\sinh(c+dx)}} + (5a^4 + b^4) \log(a + b\sqrt{\sinh(c + dx)})\right) - 48a^3b\sqrt{\sinh(c + dx)} - 8ab^3 \sinh^{\frac{3}{2}}(c + dx)}{6b^6d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]

[Out] (12*((5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] + (a*(a^4 + b^4))/(a + b*Sqrt[Sinh[c + d*x]])) - 48*a^3*b*Sqrt[Sinh[c + d*x]] + 18*a^2*b^2*Sinh[c + d*x] - 8*a*b^3*Sinh[c + d*x]^(3/2) + 3*b^4*Sinh[c + d*x]^2)/(6*b^6*d)

Maple [C] time = 0.207, size = 481, normalized size = 3.4

$$-4 \frac{\tanh(1/2 dx + c/2) a^4}{db^4 (a^2 (\tanh(1/2 dx + c/2))^2 + 2 b^2 \tanh(1/2 dx + c/2) - a^2)} - 4 \frac{\tanh(1/2 dx + c/2)}{d (a^2 (\tanh(1/2 dx + c/2))^2 + 2 b^2 \tanh(1/2 dx + c/2) - a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x)

[Out]
$$\begin{aligned} & -4/d/b^4*\tanh(1/2*d*x+1/2*c)/(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+ \\ & 1/2*c)-a^2)*a^4-4/d*\tanh(1/2*d*x+1/2*c)/(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+ \\ & 1/2*c)-a^2)+5/d/b^6*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+ \\ & 1/2*c)-a^2)*a^4+1/d/b^2*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+ \\ & 1/2*c)-a^2)+1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)^2-3/d/b^4/(\tanh(1/2*d*x+1/2*c) \\ & +1)*a^2-1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)+1)-5/d/b^6*\ln(\tanh(1/2*d*x+1/2*c)+ \\ & 1)*a^4-1/d/b^2*\ln(\tanh(1/2*d*x+1/2*c)+1)+1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)^2-3/d/b^4/(\tanh(1/2*d*x+1/2*c)-1)*a^2+1/2/d/b^2/(\tanh(1/2*d*x+1/2*c)-1)-5/d \\ & /b^6*\ln(\tanh(1/2*d*x+1/2*c)-1)*a^4-1/d/b^2*\ln(\tanh(1/2*d*x+1/2*c)-1)+\int/ \\ & \text{int/} \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3}{(b\sqrt{\sinh(dx+c)}+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a)^2, x)

Fricas [B] time = 7.24589, size = 5027, normalized size = 35.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] $\frac{1}{24} \cdot (3b^6 \cosh(dx+c)^6 + 3b^6 \sinh(dx+c)^6 + 30a^2b^4 \cosh(dx+c)^5 + 30a^2b^4 \cosh(dx+c) - 3b^6 + 6(3b^6 \cosh(dx+c) + 5a^2b^4) \sinh(dx+c)^5 - 3(24a^4b^2 + b^6 + 8(5a^4b^2 + b^6)dx + 8(5a^4b^2 + b^6)c) \cosh(dx+c)^4 + 3(15b^6 \cosh(dx+c)^2 + 50a^2b^4 \cosh(dx+c) - 24a^4b^2 - b^6 - 8(5a^4b^2 + b^6)dx - 8(5a^4b^2 + b^6)c) \sinh(dx+c)^4 - 24(4a^6 + 7a^2b^4 - 2(5a^6 + a^2b^4)dx - 2(5a^6 + a^2b^4)c) \cosh(dx+c)^3 + 12(5b^6 \cosh(dx+c)^3 + 25a^2b^4 \cosh(dx+c)^2 - 8a^6 - 14a^2b^4 + 4(5a^6 + a^2b^4)dx + 4(5a^6 + a^2b^4)c - (24a^4b^2 + b^6 + 8(5a^4b^2 + b^6)dx + 8(5a^4b^2 + b^6)c) \cosh(dx+c)) \sinh(dx+c)^3 + 3(24a^4b^2 + b^6 + 8(5a^4b^2 + b^6)dx + 8(5a^4b^2 + b^6)c) \cosh(dx+c)^2 + 3(15b^6 \cosh(dx+c)^4 + 100a^2b^4 \cosh(dx+c)^3 + 24a^4b^2 + b^6 + 8(5a^4b^2 + b^6)dx - 6(24a^4b^2 + b^6 + 8(5a^4b^2 + b^6)dx + 8(5a^4b^2 + b^6)c) \cosh(dx+c)^2 + 8(5a^4b^2 + b^6)c - 24(4a^6 + 7a^2b^4 - 2(5a^6 + a^2b^4)dx - 2(5a^6 + a^2b^4)c) \cosh(dx+c)) \sinh(dx+c)^2 + 24((5a^4b^2 + b^6) \cosh(dx+c)^4 + (5a^4b^2 + b^6) \sinh(dx+c)^4 - 2(5a^6 + a^2b^4) \cosh(dx+c)^3 - 2(5a^6 + a^2b^4 - 2(5a^4b^2 + b^6) \cosh(dx+c)) \sinh(dx+c)^3 - (5a^4b^2 + b^6) \cosh(dx+c)^2 - (5a^4b^2 + b^6 - 6(5a^4b^2 + b^6) \cosh(dx+c)^2 + 6(5a^6 + a^2b^4) \cosh(dx+c)) \sinh(dx+c)^2 + 2(2(5a^4b^2 + b^6) \cosh(dx+c)^3 - 3(5a^6 + a^2b^4) \cosh(dx+c)^2 - (5a^4b^2 + b^6) \cosh(dx+c)) \sinh(dx+c)) \log(-b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 + 2a^2 \cosh(dx+c) - b^2 + 2(b^2 \cosh(dx+c) + a^2) \sinh(dx+c) + 4(ab \cosh(dx+c) + a b \sinh(dx+c)) \sqrt{\sinh(dx+c)}) / (b^2 \cosh(dx+c)^2 + b^2 \sinh(dx+c)^2 - 2a^2 \cosh(dx+c) - b^2 + 2(b^2 \cosh(dx+c) - a^2) \sinh(dx+c))) + 24((5a^4b^2 + b^6) \cosh(dx+c)^4 + (5a^4b^2 + b^6) \sinh(dx+c)^4 - 2(5a^6 + a^2b^4) \cosh(dx+c)^3 - 2(5a^6 + a^2b^4 - 2(5a^4b^2 + b^6) \cosh(dx+c)) \sinh(dx+c)^3 - (5a^4b^2 + b^6) \cosh(dx+c)^2 - (5a^4b^2 + b^6 - 6(5a^4b^2 + b^6) \cosh(dx+c)^2 + 6(5a^6 + a^2b^4) \cosh(dx+c)) \sinh(dx+c)^2 + 2(2(5a^4b^2 + b^6) \cosh(dx+c)^3 - 3(5a^6 + a^2b^4) \cosh(dx+c)^2 - (5a^4b^2 + b^6) \cosh(dx+c)) \sinh(dx+c)) \log(2(b^2 \sinh(dx+c) - a^2) / (\cosh(dx+c) - \sinh(dx+c))) + 6(3b^6 \cosh(dx+c)^5 + 25a^2b^4 \cosh(dx+c)^4 + 5a^2b^4 - 2(24a^4b^2 + b^6 + 8(5a^4b^2 + b^6)dx + 8(5a^4b^2 + b^6)c) \cosh(dx+c)^3 - 12(4a^6 + 7a^2b^4 - 2(5a^6 + a^2b^4)dx - 2(5a^6 + a^2b^4)c) \cosh(dx+c)^2 + (24a^4b^2 + b^6 + 8(5a^4b^2 + b^6)dx + 8(5a^4b^2 + b^6)c) \sinh(dx+c) - 16(ab^5 \cosh(dx+c)^5 + ab^5 \sinh(dx+c)^5 + 10a^3b^3 \cosh(dx+c)^4 - 10a^3b^3 \cosh(dx+c)^2 + ab^5 \cosh(dx+c) + 5(ab^5 \cosh(dx+c) + 2a^3b^3) \sinh(dx+c)^4 - 2(15a^5b + 4ab^5) \cosh(dx+c)^3 + 2(5ab^5 \cosh(dx+c)^2 + 20a^3b^3 \cosh(dx+c) - 15a^5b - 4ab^5) \sinh(dx+c)^3 + 2(5ab^5 \cosh(dx+c)^3 + 30a^3b^3 \cosh(dx+c)^3$

$$2 - 5a^3b^3 - 3(15a^5b + 4ab^5)\cosh(dx + c)\sinh(dx + c)^2 + (5ab^5\cosh(dx + c)^4 + 40a^3b^3\cosh(dx + c)^3 - 20a^3b^3\cosh(dx + c) + ab^5 - 6(15a^5b + 4ab^5)\cosh(dx + c)^2)\sinh(dx + c)\sqrt{\sinh(dx + c)} / (b^8d\cosh(dx + c)^4 + b^8d\sinh(dx + c)^4 - 2a^2b^6d\cosh(dx + c)^3 - b^8d\cosh(dx + c)^2 + 2(2b^8d\cosh(dx + c) - a^2b^6d)\sinh(dx + c)^3 + (6b^8d\cosh(dx + c)^2 - 6a^2b^6d\cosh(dx + c) - b^8d)\sinh(dx + c)^2 + 2(2b^8d\cosh(dx + c)^3 - 3a^2b^6d\cosh(dx + c)^2 - b^8d\cosh(dx + c))\sinh(dx + c))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2))**2,x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.417 \quad \int \frac{\cosh(c+dx)}{(a+b\sqrt{\sinh(c+dx)})^2} dx$$

Optimal. Leaf size=49

$$\frac{2a}{b^2d(a+b\sqrt{\sinh(c+dx)})} + \frac{2\log(a+b\sqrt{\sinh(c+dx)})}{b^2d}$$

[Out] (2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*a)/(b^2*d*(a + b*Sqrt[Sinh[c + d*x]]))

Rubi [A] time = 0.0545739, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3223, 190, 43}

$$\frac{2a}{b^2d(a+b\sqrt{\sinh(c+dx)})} + \frac{2\log(a+b\sqrt{\sinh(c+dx)})}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]

[Out] (2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*a)/(b^2*d*(a + b*Sqrt[Sinh[c + d*x]]))

Rule 3223

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh(c + dx)}{(a + b\sqrt{\sinh(c + dx)})^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\sqrt{x})^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \frac{x}{(a+bx)^2} dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) dx, x, \sqrt{\sinh(c + dx)}\right)}{d} \\ &= \frac{2 \log(a + b\sqrt{\sinh(c + dx)})}{b^2 d} + \frac{2a}{b^2 d (a + b\sqrt{\sinh(c + dx)})} \end{aligned}$$

Mathematica [A] time = 0.0609125, size = 42, normalized size = 0.86

$$\frac{2 \left(\frac{a}{a+b\sqrt{\sinh(c+dx)}} + \log(a + b\sqrt{\sinh(c + dx)}) \right)}{b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2, x]
```

```
[Out] (2*(Log[a + b*Sqrt[Sinh[c + d*x]]] + a/(a + b*Sqrt[Sinh[c + d*x]])))/(b^2*d)
```

Maple [B] time = 0.021, size = 144, normalized size = 2.9

$$-2 \frac{a^2}{d (b^2 \sinh(dx + c) - a^2) b^2} + \frac{\ln(b^2 \sinh(dx + c) - a^2)}{db^2} + \frac{a}{db^2} (b\sqrt{\sinh(dx + c)} - a)^{-1} - \frac{1}{db^2} \ln(b\sqrt{\sinh(dx + c)} - a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x)`

[Out]
$$-2/d*a^2/(b^2*\sinh(d*x+c)-a^2)/b^2+1/d*\ln(b^2*\sinh(d*x+c)-a^2)/b^2+1/d*a/b^2/(b*\sinh(d*x+c)^(1/2)-a)-1/d/b^2*\ln(b*\sinh(d*x+c)^(1/2)-a)+a/b^2/d/(a+b*\sinh(d*x+c)^(1/2))+\ln(a+b*\sinh(d*x+c)^(1/2))/b^2/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{(b\sqrt{\sinh(dx+c)}+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")`

[Out] `integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)`

Fricas [B] time = 1.77643, size = 1370, normalized size = 27.96

$$b^2 dx + b^2 c - (b^2 dx + b^2 c) \cosh(dx+c)^2 - (b^2 dx + b^2 c) \sinh(dx+c)^2 + 2(a^2 dx + a^2 c - 2a^2) \cosh(dx+c) + (b^2 \cosh$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")`

[Out]
$$(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*\cosh(d*x + c)^2 - (b^2*d*x + b^2*c)*\sinh(d*x + c)^2 + 2*(a^2*d*x + a^2*c - 2*a^2)*\cosh(d*x + c) + (b^2*\cosh(d*x + c)^2 + b^2*\sinh(d*x + c)^2 - 2*a^2*\cosh(d*x + c) - b^2 + 2*(b^2*\cosh(d*x + c) - a^2)*\sinh(d*x + c))*\log((b^2*\cosh(d*x + c)^2 + b^2*\sinh(d*x + c)^2 + 2*a^2*\cosh(d*x + c) - b^2 + 2*(b^2*\cosh(d*x + c) + a^2)*\sinh(d*x + c) + 4*(a*b*\cosh(d*x + c) + a*b*\sinh(d*x + c))*\sqrt{\sinh(d*x + c)}))/(b^2*\cosh(d*x + c)^2 + b^2*\sinh(d*x + c)^2 - 2*a^2*\cosh(d*x + c) - b^2 + 2*(b^2*\cosh(d*x + c) - a^2)*\sinh(d*x + c)) + (b^2*\cosh(d*x + c)^2 + b^2*\sinh(d*x + c)^2 - 2*a^2*\cosh(d*x + c) - b^2 + 2*(b^2*\cosh(d*x + c) - a^2)*\sinh(d*x + c))*\log(2*(b^2*\sinh(d*x + c) - a^2)/(\cosh(d*x + c) - \sinh(d*x + c))) + 2*(a^2*d*x + a^2*c - 2*a^2 - (b^2*d*x + b^2*c)*\cosh(d*x + c))*\sinh(d*x + c) + 4*(a*b*\cosh(d*x + c) + a*b*\sinh(d*x + c))*\sqrt{\sinh(d*x + c)}))/(b^4*d*\cosh(d*x + c)^2$$

+ b⁴*d*sinh(d*x + c)² - 2*a²*b²*d*cosh(d*x + c) - b⁴*d + 2*(b⁴*d*cos
h(d*x + c) - a²*b²*d)*sinh(d*x + c))

Sympy [A] time = 4.92605, size = 151, normalized size = 3.08

$$\begin{cases} \frac{x \cosh(c)}{a^2 \sinh(c+dx)} & \text{for } b = 0 \wedge d = 0 \\ \frac{a^2 d}{x \cosh(c)} & \text{for } b = 0 \\ \frac{(a+b\sqrt{\sinh(c)})^2}{2a \log\left(\frac{a}{b} + \sqrt{\sinh(c+dx)}\right)} + \frac{2a}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} + \frac{2b \log\left(\frac{a}{b} + \sqrt{\sinh(c+dx)}\right)\sqrt{\sinh(c+dx)}}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)

[Out] Piecewise((x*cosh(c)/a**2, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a**2*d), Eq(b, 0)), (x*cosh(c)/(a + b*sqrt(sinh(c)))**2, Eq(d, 0)), (2*a*log(a/b + sqrt(sinh(c + d*x)))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))) + 2*a/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))) + 2*b*log(a/b + sqrt(sinh(c + d*x)))*sqrt(sinh(c + d*x))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))), True))

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.418 \quad \int \frac{\operatorname{sech}(c+dx)}{(a+b\sqrt{\sinh(c+dx)})^2} dx$$

Optimal. Leaf size=384

$$\frac{2ab^2}{d(a^4 + b^4)(a + b\sqrt{\sinh(c + dx)})} - \frac{ab(2a^2b^2 + a^4 - b^4) \log(\sinh(c + dx) - \sqrt{2}\sqrt{\sinh(c + dx)} + 1)}{\sqrt{2}d(a^4 + b^4)^2} + \frac{ab(2a^2b^2 + a^4 - b^4)}{\sqrt{2}d(a^4 + b^4)^2}$$

```
[Out] (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]]
)/((a^4 + b^4)^2*d) - (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 + Sqrt
[2]*Sqrt[Sinh[c + d*x]]])/((a^4 + b^4)^2*d) + (a^2*(a^4 - 3*b^4)*ArcTan[Sin
h[c + d*x]])/((a^4 + b^4)^2*d) + (b^2*(3*a^4 - b^4)*Log[Cosh[c + d*x]])/((a
^4 + b^4)^2*d) - (2*b^2*(3*a^4 - b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^4
+ b^4)^2*d) - (a*b*(a^4 + 2*a^2*b^2 - b^4)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d
*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 + b^4)^2*d) + (a*b*(a^4 + 2*a^2*b^2 -
b^4)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 +
b^4)^2*d) + (2*a*b^2)/((a^4 + b^4)*d*(a + b*Sqrt[Sinh[c + d*x]]))
```

Rubi [A] time = 0.630777, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {3223, 6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$\frac{2ab^2}{d(a^4 + b^4)(a + b\sqrt{\sinh(c + dx)})} - \frac{ab(2a^2b^2 + a^4 - b^4) \log(\sinh(c + dx) - \sqrt{2}\sqrt{\sinh(c + dx)} + 1)}{\sqrt{2}d(a^4 + b^4)^2} + \frac{ab(2a^2b^2 + a^4 - b^4)}{\sqrt{2}d(a^4 + b^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]]
)/((a^4 + b^4)^2*d) - (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 + Sqrt
[2]*Sqrt[Sinh[c + d*x]]])/((a^4 + b^4)^2*d) + (a^2*(a^4 - 3*b^4)*ArcTan[Sin
h[c + d*x]])/((a^4 + b^4)^2*d) + (b^2*(3*a^4 - b^4)*Log[Cosh[c + d*x]])/((a
^4 + b^4)^2*d) - (2*b^2*(3*a^4 - b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^4
+ b^4)^2*d) - (a*b*(a^4 + 2*a^2*b^2 - b^4)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d
*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 + b^4)^2*d) + (a*b*(a^4 + 2*a^2*b^2 -
b^4)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 +
b^4)^2*d) + (2*a*b^2)/((a^4 + b^4)*d*(a + b*Sqrt[Sinh[c + d*x]]))
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = Sum[(x^ii*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1}]}], Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + Dist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

a, 0] || LtQ[b, 0])

Rule 1165

Int[((d_) + (e_)*(x_)^2)/((a_) + (c_)*(x_)^4), x_Symbol] := With[{q = Rt[(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1248

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x]

Rule 635

Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{\operatorname{sech}(c+dx)}{(a+b\sqrt{\sinh(c+dx)})^2} dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{(a+b\sqrt{x})^2(1+x^2)} dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{2 \operatorname{Subst}\left(\int \frac{x}{(a+bx)^2(1+x^4)} dx, x, \sqrt{\sinh(c+dx)}\right)}{d} \\
&= \frac{2 \operatorname{Subst}\left(\int \left(-\frac{ab^3}{(a^4+b^4)(a+bx)^2} + \frac{-3a^4b^3+b^7}{(a^4+b^4)^2(a+bx)} + \frac{4a^3b^3+a^2(a^4-3b^4)x-2ab(a^4-b^4)x^2+b^2(3a^4-b^4)x^3}{(a^4+b^4)^2(1+x^4)}\right) dx, x, \sqrt{\sinh(c+dx)}\right)}{d} \\
&= -\frac{2b^2(3a^4-b^4)\log(a+b\sqrt{\sinh(c+dx)})}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)d(a+b\sqrt{\sinh(c+dx)})} + \frac{2 \operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\sinh(c+dx)}\right)}{d} \\
&= -\frac{2b^2(3a^4-b^4)\log(a+b\sqrt{\sinh(c+dx)})}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)d(a+b\sqrt{\sinh(c+dx)})} + \frac{2 \operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\sinh(c+dx)}\right)}{d} \\
&= -\frac{2b^2(3a^4-b^4)\log(a+b\sqrt{\sinh(c+dx)})}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)d(a+b\sqrt{\sinh(c+dx)})} + \frac{2 \operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\sinh(c+dx)}\right)}{d} \\
&= -\frac{2b^2(3a^4-b^4)\log(a+b\sqrt{\sinh(c+dx)})}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)d(a+b\sqrt{\sinh(c+dx)})} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^4} dx, x, \sqrt{\sinh(c+dx)}\right)}{d} \\
&= -\frac{2b^2(3a^4-b^4)\log(a+b\sqrt{\sinh(c+dx)})}{(a^4+b^4)^2 d} + \frac{2ab^2}{(a^4+b^4)d(a+b\sqrt{\sinh(c+dx)})} + \frac{(a^2(a^4-b^4)\tan^{-1}(\sqrt{2}\sqrt{\sinh(c+dx)}) - b^2(3a^4-b^4)\log(\cosh(c+dx)))}{(a^4+b^4)^2 d} \\
&= \frac{a^2(a^4-3b^4)\tan^{-1}(\sinh(c+dx))}{(a^4+b^4)^2 d} + \frac{b^2(3a^4-b^4)\log(\cosh(c+dx))}{(a^4+b^4)^2 d} - \frac{2b^2(3a^4-b^4)\log(a+b\sqrt{\sinh(c+dx)})}{(a^4+b^4)^2 d} \\
&= \frac{\sqrt{2}ab(a^4-2a^2b^2-b^4)\tan^{-1}(1-\sqrt{2}\sqrt{\sinh(c+dx)})}{(a^4+b^4)^2 d} - \frac{\sqrt{2}ab(a^4-2a^2b^2-b^4)\tan^{-1}(1+\sqrt{2}\sqrt{\sinh(c+dx)})}{(a^4+b^4)^2 d}
\end{aligned}$$

Mathematica [C] time = 0.732946, size = 280, normalized size = 0.73

$$-4ab(a^4-b^4)\sinh^{\frac{3}{2}}(c+dx) {}_2F_1\left(\frac{3}{4}, 1; \frac{7}{4}; -\sinh^2(c+dx)\right) + \frac{6ab^2(a^4+b^4)}{a+b\sqrt{\sinh(c+dx)}} - 3\sqrt{2}a^3b^3\left(\log(\sinh(c+dx)) - \sqrt{2}\sqrt{\sinh(c+dx)}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]

[Out] $(-6\sqrt{2}a^3b^3(\text{ArcTan}[1 - \sqrt{2}\sqrt{\text{Sinh}[c + dx]})] - \text{ArcTan}[1 + \sqrt{2}\sqrt{\text{Sinh}[c + dx]}]) + 3a^2(a^4 - 3b^4)\text{ArcTan}[\text{Sinh}[c + dx]] - 3b^2(-3a^4 + b^4)\text{Log}[\text{Cosh}[c + dx]] + 6b^2(-3a^4 + b^4)\text{Log}[a + b\sqrt{\text{Sinh}[c + dx]}] - 3\sqrt{2}a^3b^3(\text{Log}[1 - \sqrt{2}\sqrt{\text{Sinh}[c + dx]} + \text{Sinh}[c + dx]] - \text{Log}[1 + \sqrt{2}\sqrt{\text{Sinh}[c + dx]} + \text{Sinh}[c + dx]]) + (6ab^2(a^4 + b^4))/(a + b\sqrt{\text{Sinh}[c + dx]}) - 4ab(a^4 - b^4)\text{Hypergeometric2F1}[3/4, 1, 7/4, -\text{Sinh}[c + dx]^2*\text{Sinh}[c + dx]^{3/2}]/(3(a^4 + b^4)^2d)$

Maple [C] time = 0.234, size = 567, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x)

[Out] $-4/d*b^4/(a^4+b^4)^2*\tanh(1/2*d*x+1/2*c)/(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)*a^4-4/d*b^8/(a^4+b^4)^2*\tanh(1/2*d*x+1/2*c)/(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)-3/d*b^2/(a^4+b^4)^2*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)*a^4+1/d*b^6/(a^4+b^4)^2*\ln(a^2*\tanh(1/2*d*x+1/2*c)^2+2*b^2*\tanh(1/2*d*x+1/2*c)-a^2)+3/d/(a^8+2*a^4*b^4+b^8)*\ln(\tanh(1/2*d*x+1/2*c)^2+1)*a^4*b^2-1/d/(a^8+2*a^4*b^4+b^8)*\ln(\tanh(1/2*d*x+1/2*c)^2+1)*b^6+2/d/(a^8+2*a^4*b^4+b^8)*\arctan(\tanh(1/2*d*x+1/2*c))*a^6-6/d/(a^8+2*a^4*b^4+b^8)*\arctan(\tanh(1/2*d*x+1/2*c))*a^2*b^4+\int/undef0(2*a*b*sinh(d*x+c)^(1/2)*(b^4*sinh(d*x+c)^2-2*a^2*b^2*sinh(d*x+c)+a^4)/(4*a^2*b^6*sinh(d*x+c)*cosh(d*x+c)^4+(4*a^6*b^2-4*a^2*b^6)*cosh(d*x+c)^2*sinh(d*x+c)-b^8*cosh(d*x+c)^6+(-6*a^4*b^4+2*b^8)*cosh(d*x+c)^4+(-a^8+6*a^4*b^4-b^8)*cosh(d*x+c)^2),sinh(d*x+c))/d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\text{sech}(dx + c)}{(b\sqrt{\sinh(dx + c)} + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] Timed out

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c + dx)}{(a + b\sqrt{\sinh(c + dx)})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)

[Out] Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x)))**2, x)

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.419 \quad \int \frac{\cosh^5(c+dx)}{a+b \sinh^n(c+dx)} dx$$

Optimal. Leaf size=130

$$\frac{\sinh^5(c+dx) {}_2F_1\left(1, \frac{5}{n}; \frac{n+5}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{5ad} + \frac{2 \sinh^3(c+dx) {}_2F_1\left(1, \frac{3}{n}; \frac{n+3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3ad} + \frac{\sinh(c+dx) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{ad}$$

[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a*d) + (2*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a*d) + (Hypergeometric2F1[1, 5/n, (5 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a*d)

Rubi [A] time = 0.140401, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^5(c+dx) {}_2F_1\left(1, \frac{5}{n}; \frac{n+5}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{5ad} + \frac{2 \sinh^3(c+dx) {}_2F_1\left(1, \frac{3}{n}; \frac{n+3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3ad} + \frac{\sinh(c+dx) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n), x]

[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a*d) + (2*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a*d) + (Hypergeometric2F1[1, 5/n, (5 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a*d)

Rule 3223

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])

Rule 1893

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly

Q[Pq, x^n])

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^5(c + dx)}{a + b \sinh^n(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^n} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a+bx^n} + \frac{2x^2}{a+bx^n} + \frac{x^4}{a+bx^n}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^n} dx, x, \sinh(c + dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^n} dx, x, \sinh(c + dx)\right)}{d} + \frac{2 \text{Subst}\left(\int \frac{x^2}{a+bx^n} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh(c + dx)}{ad} + \frac{{}_2F_1\left(1, \frac{3}{n}; \frac{3+n}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh^3(c + dx)}{3ad} + \frac{{}_2F_1\left(1, \frac{5}{n}; \frac{5+n}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh^5(c + dx)}{5ad} \end{aligned}$$

Mathematica [A] time = 0.141965, size = 119, normalized size = 0.92

$$\frac{3 \sinh^5(c + dx) {}_2F_1\left(1, \frac{5}{n}; \frac{5+n}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) + 10 \sinh^3(c + dx) {}_2F_1\left(1, \frac{3}{n}; \frac{3+n}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) + 15 \sinh(c + dx) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{15ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n), x]
```

```
[Out] (15*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x] + 10*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[1, 5/n, (5 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/15
```


)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[1, 5/n, (5 + n)/n, -(b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a*d)

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx + c))^5}{a + b(\sinh(dx + c))^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)

[Out] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)^5}{b \sinh(dx + c)^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c)^5}{b \sinh(dx + c)^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")

[Out] integral(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^5}{b \sinh(dx+c)^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="giac")

[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)

$$3.420 \quad \int \frac{\cosh^3(c+dx)}{a+b \sinh^n(c+dx)} dx$$

Optimal. Leaf size=84

$$\frac{\sinh^3(c+dx) {}_2F_1\left(1, \frac{3}{n}; \frac{n+3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3ad} + \frac{\sinh(c+dx) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{ad}$$

[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a*d) + (Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a*d)

Rubi [A] time = 0.103662, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^3(c+dx) {}_2F_1\left(1, \frac{3}{n}; \frac{n+3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3ad} + \frac{\sinh(c+dx) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n), x]

[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a*d) + (Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a*d)

Rule 3223

Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)]^(n_.))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])

Rule 1893

Int[(Pq_)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n])

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\cosh^3(c + dx)}{a + b \sinh^n(c + dx)} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^n} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{a+bx^n} + \frac{x^2}{a+bx^n}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{a+bx^n} dx, x, \sinh(c + dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^n} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh(c + dx)}{ad} + \frac{{}_2F_1\left(1, \frac{3}{n}; \frac{3+n}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh^3(c + dx)}{3ad} \end{aligned}$$

Mathematica [A] time = 0.0547093, size = 82, normalized size = 0.98

$$\frac{\sinh^3(c+dx) {}_2F_1\left(1, \frac{3}{n}; 1 + \frac{3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3a} + \frac{\sinh(c+dx) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{a}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n), x]
```

```
[Out] ((Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/a + (Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a))/d
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx + c))^3}{a + b(\sinh(dx + c))^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)

[Out] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)^3}{b \sinh(dx + c)^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")

[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c)^3}{b \sinh(dx + c)^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")

[Out] integral(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)^3}{b \sinh(dx + c)^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="giac")`

[Out] `integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)`

$$3.421 \quad \int \frac{\cosh(c+dx)}{a+b \sinh^n(c+dx)} dx$$

Optimal. Leaf size=37

$$\frac{\sinh(c+dx) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{ad}$$

[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a*d)

Rubi [A] time = 0.0434645, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3223, 245}

$$\frac{\sinh(c+dx) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n), x]

[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a*d)

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\cosh(c + dx)}{a + b \sinh^n(c + dx)} dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^n} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{{}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh(c + dx)}{ad}$$

Mathematica [A] time = 0.0088907, size = 37, normalized size = 1.

$$\frac{\sinh(c + dx) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n), x]

[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a*d)

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{a + b (\sinh(dx + c))^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n), x)

[Out] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{b \sinh(dx + c)^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")`

[Out] `integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)}{b \sinh(dx+c)^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")`

[Out] `integral(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{b \sinh(dx+c)^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="giac")`

[Out] `integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)`

$$3.422 \quad \int \frac{\cosh^5(c+dx)}{(a+b \sinh^n(c+dx))^2} dx$$

Optimal. Leaf size=130

$$\frac{\sinh^5(c+dx) {}_2F_1\left(2, \frac{5}{n}; \frac{n+5}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{5a^2d} + \frac{2 \sinh^3(c+dx) {}_2F_1\left(2, \frac{3}{n}; \frac{n+3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3a^2d} + \frac{\sinh(c+dx) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}\right)}{a^2d}$$

[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a^2*d) + (2*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d) + (Hypergeometric2F1[2, 5/n, (5 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a^2*d)

Rubi [A] time = 0.13324, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^5(c+dx) {}_2F_1\left(2, \frac{5}{n}; \frac{n+5}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{5a^2d} + \frac{2 \sinh^3(c+dx) {}_2F_1\left(2, \frac{3}{n}; \frac{n+3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3a^2d} + \frac{\sinh(c+dx) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}\right)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]

[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a^2*d) + (2*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d) + (Hypergeometric2F1[2, 5/n, (5 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a^2*d)

Rule 3223

Int[cos[(e_.) + (f_.)*(x_.)]^(m_.)*((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])

Rule 1893

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly

Q[Pq, x^n])

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \frac{\cosh^5(c + dx)}{(a + b \sinh^n(c + dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^n)^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+bx^n)^2} + \frac{2x^2}{(a+bx^n)^2} + \frac{x^4}{(a+bx^n)^2}\right) dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2} dx, x, \sinh(c + dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^4}{(a+bx^n)^2} dx, x, \sinh(c + dx)\right)}{d} + \frac{2 \text{Subst}\left(\int \frac{x^2}{(a+bx^n)^2} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{{}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh(c + dx)}{a^2 d} + \frac{{}_2F_1\left(2, \frac{3}{n}; \frac{3+n}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh^3(c + dx)}{3a^2 d} \end{aligned}$$

Mathematica [A] time = 0.110634, size = 119, normalized size = 0.92

$$\frac{3 \sinh^5(c + dx) {}_2F_1\left(2, \frac{5}{n}; \frac{n+5}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) + 10 \sinh^3(c + dx) {}_2F_1\left(2, \frac{3}{n}; \frac{n+3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) + 15 \sinh(c + dx) {}_2F_1\left(2, \frac{1}{n}; \frac{n+1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{15a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]

```
[Out] (15*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh
[c + d*x] + 10*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a
)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[2, 5/n, (5 + n)/n, -((b*Sinh[c + d
*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a^2*d)
```

Maple [F] time = 5.743, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx + c))^5}{(a + b(\sinh(dx + c))^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(2^n e^{(cn+10dx+10c)} + 3 \cdot 2^n e^{(cn+8dx+8c)} + 2^{n+1} e^{(cn+6dx+6c)} - 2^{n+1} e^{(cn+4dx+4c)} - 3 \cdot 2^n e^{(cn+2dx+2c)} - 2^n e^{(cn)}) e^{(dnx)}}{32 \left(2^n a^2 d n e^{(dnx+cn+5dx+5c)} + a b d n e^{(5dx+n \log(e^{(dx+c)+1})+n \log(e^{(dx+c)-1})+5c)} \right)} + \frac{1}{32} \int \frac{(2^n e^{(cn+10dx+10c)} + 3 \cdot 2^n e^{(cn+8dx+8c)} + 2^{n+1} e^{(cn+6dx+6c)} - 2^{n+1} e^{(cn+4dx+4c)} - 3 \cdot 2^n e^{(cn+2dx+2c)} - 2^n e^{(cn)}) e^{(dnx)}}{32 \left(2^n a^2 d n e^{(dnx+cn+5dx+5c)} + a b d n e^{(5dx+n \log(e^{(dx+c)+1})+n \log(e^{(dx+c)-1})+5c)} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(2^n*e^(c*n + 10*d*x + 10*c) + 3*2^n*e^(c*n + 8*d*x + 8*c) + 2^(n + 1)
*e^(c*n + 6*d*x + 6*c) - 2^(n + 1)*e^(c*n + 4*d*x + 4*c) - 3*2^n*e^(c*n + 2
*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 5*d*x +
5*c) + a*b*d*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
5*c)) + 1/32*integrate((2^n*n*e^(c*n) - 5*2^n*e^(c*n) + (2^n*n*e^(c*n) - 5
*2^n*e^(c*n))*e^(10*d*x + 10*c) + (5*2^n*n*e^(c*n) - 9*2^n*e^(c*n))*e^(8*d*
x + 8*c) + (5*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(6*d*x + 6*c) + (5
*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(4*d*x + 4*c) + (5*2^n*n*e^(c*n)
) - 9*2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 5
*d*x + 5*c) + a*b*d*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) -
1) + 5*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^5}{b^2 \sinh(dx+c)^{2n} + 2ab \sinh(dx+c)^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")

[Out] integral(cosh(d*x + c)^5/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n + a^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^5}{(b \sinh(dx+c)^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")

[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a)^2, x)

$$3.423 \quad \int \frac{\cosh^3(c+dx)}{(a+b \sinh^n(c+dx))^2} dx$$

Optimal. Leaf size=84

$$\frac{\sinh^3(c+dx) {}_2F_1\left(2, \frac{3}{n}; \frac{n+3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3a^2d} + \frac{\sinh(c+dx) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{a^2d}$$

[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a^2*d) + (Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d)

Rubi [A] time = 0.102242, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^3(c+dx) {}_2F_1\left(2, \frac{3}{n}; \frac{n+3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3a^2d} + \frac{\sinh(c+dx) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{a^2d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]

[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a^2*d) + (Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d)

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n])
```

Rule 245

Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 364

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/((c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\cosh^3(c + dx)}{(a + b \sinh^n(c + dx))^2} dx &= \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^n)^2} dx, x, \sinh(c + dx)\right)}{d} \\
 &= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+bx^n)^2} + \frac{x^2}{(a+bx^n)^2}\right) dx, x, \sinh(c + dx)\right)}{d} \\
 &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2} dx, x, \sinh(c + dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx^n)^2} dx, x, \sinh(c + dx)\right)}{d} \\
 &= \frac{{}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh(c + dx)}{a^2 d} + \frac{{}_2F_1\left(2, \frac{3}{n}; \frac{3+n}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh^3(c + dx)}{3a^2 d}
 \end{aligned}$$

Mathematica [A] time = 0.0516983, size = 82, normalized size = 0.98

$$\frac{\sinh^3(c+dx) {}_2F_1\left(2, \frac{3}{n}; 1 + \frac{3}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{3a^2} + \frac{\sinh(c+dx) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]

[Out] ((Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/a^2 + (Hypergeometric2F1[2, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^3)/(3*a^2))/d

Maple [F] time = 5.335, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx + c))^3}{(a + b(\sinh(dx + c))^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)

[Out] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(2^n e^{(cn+6dx+6c)} + 2^n e^{(cn+4dx+4c)} - 2^n e^{(cn+2dx+2c)} - 2^n e^{(cn)})e^{(dnx)}}{8(2^n a^2 d n e^{(dnx+cn+3dx+3c)} + a b d n e^{(3dx+n \log(e^{(dx+c)}+1)+n \log(e^{(dx+c)}-1)+3c))} + \frac{1}{8} \int \frac{(2^n n e^{(cn)} - 3 \cdot 2^n e^{(cn)} + (2^n n e^{(cn)} - 3 \cdot 2^n e^{(cn)} - 3 \cdot 2^n e^{(cn)}))e^{(dnx)}}{2^n a^2 n e^{(dnx+cn+3dx+3c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")

[Out] 1/8*(2^n*e^(c*n + 6*d*x + 6*c) + 2^n*e^(c*n + 4*d*x + 4*c) - 2^n*e^(c*n + 2*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 3*d*x + 3*c) + a*b*d*n*e^(3*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) + 3*c)) + 1/8*integrate((2^n*n*e^(c*n) - 3*2^n*e^(c*n) + (2^n*n*e^(c*n) - 3*2^n*e^(c*n))*e^(6*d*x + 6*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(4*d*x + 4*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n*e^(d*n*x + c*n + 3*d*x + 3*c) + a*b*n*e^(3*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) + 3*c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c)^3}{b^2 \sinh(dx + c)^{2n} + 2ab \sinh(dx + c)^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n +
a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3}{(b \sinh(dx+c)^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a)^2, x)
```

$$3.424 \quad \int \frac{\cosh(c+dx)}{(a+b \sinh^n(c+dx))^2} dx$$

Optimal. Leaf size=37

$$\frac{\sinh(c+dx) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{a^2 d}$$

[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a^2*d)

Rubi [A] time = 0.0439955, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3223, 245}

$$\frac{\sinh(c+dx) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{a^2 d}$$

Antiderivative was successfully verified.

[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]

[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a^2*d)

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_.) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 245

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\cosh(c + dx)}{(a + b \sinh^n(c + dx))^2} dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2} dx, x, \sinh(c + dx)\right)}{d}$$

$$= \frac{{}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right) \sinh(c + dx)}{a^2 d}$$

Mathematica [A] time = 0.0084442, size = 37, normalized size = 1.

$$\frac{\sinh(c + dx) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{b \sinh^n(c+dx)}{a}\right)}{a^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]

[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x])/(a^2*d)

Maple [F] time = 5.571, size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(a + b(\sinh(dx + c))^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)

[Out] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(2^n e^{(cn+2dx+2c)} - 2^n e^{(cn)})e^{(dnx)}}{2\left(2^n a^2 d n e^{(dnx+cn+dx+c)} + ab d n e^{(dx+n \log(e^{(dx+c)+1})+n \log(e^{(dx+c)-1})+c)}\right)} + \frac{1}{2} \int \frac{(2^n n e^{(cn)} - 2^n e^{(cn)} + (2^n n e^{(cn)} - 2^n e^{(cn)})e^{(2c)}}{2^n a^2 n e^{(dnx+cn+dx+c)} + ab n e^{(dx+n \log(e^{(dx+c)+1})+n \log(e^{(dx+c)-1})+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")

[Out] $\frac{1}{2} * (2^n * e^{(c*n + 2*d*x + 2*c)} - 2^n * e^{(c*n)}) * e^{(d*n*x)} / (2^n * a^{2*d*n} * e^{(d*n*x + c*n + d*x + c)} + a*b*d*n * e^{(d*x + n*\log(e^{(d*x + c)} + 1) + n*\log(e^{(d*x + c)} - 1) + c))} + \frac{1}{2} * \text{integrate}((2^n * n * e^{(c*n)} - 2^n * e^{(c*n)} + (2^n * n * e^{(c*n)} - 2^n * e^{(c*n)}) * e^{(2*d*x + 2*c)}) * e^{(d*n*x)} / (2^n * a^{2*n} * e^{(d*n*x + c*n + d*x + c)} + a*b*n * e^{(d*x + n*\log(e^{(d*x + c)} + 1) + n*\log(e^{(d*x + c)} - 1) + c))), x)$

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx + c)}{b^2 \sinh(dx + c)^{2n} + 2ab \sinh(dx + c)^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")

[Out] integral(cosh(d*x + c)/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n + a^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx + c)}{(b \sinh(dx + c)^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a)^2, x)
```

$$3.425 \quad \int \frac{\coth(x)}{1-\sinh^2(x)} dx$$

Optimal. Leaf size=17

$$\log(\sinh(x)) - \frac{1}{2} \log(1 - \sinh^2(x))$$

[Out] Log[Sinh[x]] - Log[1 - Sinh[x]^2]/2

Rubi [A] time = 0.0324463, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3194, 36, 31, 29}

$$\log(\sinh(x)) - \frac{1}{2} \log(1 - \sinh^2(x))$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/(1 - Sinh[x]^2),x]

[Out] Log[Sinh[x]] - Log[1 - Sinh[x]^2]/2

Rule 3194

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 36

```
Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth(x)}{1 - \sinh^2(x)} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1-x)x} dx, x, \sinh^2(x) \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x} dx, x, \sinh^2(x) \right) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x} dx, x, \sinh^2(x) \right) \\ &= \log(\sinh(x)) - \frac{1}{2} \log(1 - \sinh^2(x)) \end{aligned}$$

Mathematica [A] time = 0.013551, size = 23, normalized size = 1.35

$$-2 \left(\frac{1}{4} \log(1 - \sinh^2(x)) - \frac{1}{2} \log(\sinh(x)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 - Sinh[x]^2), x]
```

```
[Out] -2*(-Log[Sinh[x]]/2 + Log[1 - Sinh[x]^2]/4)
```

Maple [B] time = 0.029, size = 41, normalized size = 2.4

$$-\frac{1}{2} \ln \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 - 2 \tanh(x/2) - 1 \right) - \frac{1}{2} \ln \left(\left(\tanh \left(\frac{x}{2} \right) \right)^2 + 2 \tanh(x/2) - 1 \right) + \ln \left(\tanh \left(\frac{x}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1-sinh(x)^2), x)
```

```
[Out] -1/2*ln(tanh(1/2*x)^2-2*tanh(1/2*x)-1)-1/2*ln(tanh(1/2*x)^2+2*tanh(1/2*x)-1)+ln(tanh(1/2*x))
```

Maxima [B] time = 1.109, size = 61, normalized size = 3.59

$$-\frac{1}{2} \log(2e^{-x} + e^{-2x} - 1) + \log(e^{-x} + 1) + \log(e^{-x} - 1) - \frac{1}{2} \log(-2e^{-x} + e^{-2x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="maxima")

[Out] -1/2*log(2*e^(-x) + e^(-2*x) - 1) + log(e^(-x) + 1) + log(e^(-x) - 1) - 1/2*log(-2*e^(-x) + e^(-2*x) - 1)

Fricas [B] time = 1.78169, size = 165, normalized size = 9.71

$$-\frac{1}{2} \log\left(\frac{2(\cosh(x)^2 + \sinh(x)^2 - 3)}{\cosh(x)^2 - 2\cosh(x)\sinh(x) + \sinh(x)^2}\right) + \log\left(\frac{2\sinh(x)}{\cosh(x) - \sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="fricas")

[Out] -1/2*log(2*(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + log(2*sinh(x)/(cosh(x) - sinh(x)))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\coth(x)}{\sinh^2(x) - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(1-sinh(x)**2),x)

[Out] -Integral(coth(x)/(sinh(x)**2 - 1), x)

Giac [A] time = 1.1704, size = 34, normalized size = 2.

$$-\frac{1}{2} \log(|e^{4x} - 6e^{2x} + 1|) + \log(|e^{2x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(e^(4*x) - 6*e^(2*x) + 1)) + log(abs(e^(2*x) - 1))
```

$$3.426 \quad \int \sqrt{a + a \sinh^2(e + fx)} \tanh^5(e + fx) dx$$

Optimal. Leaf size=63

$$-\frac{a^2}{3f(a \cosh^2(e + fx))^{3/2}} + \frac{2a}{f\sqrt{a \cosh^2(e + fx)}} + \frac{\sqrt{a \cosh^2(e + fx)}}{f}$$

[Out] $-a^2/(3*f*(a*\text{Cosh}[e + f*x]^2)^{(3/2)}) + (2*a)/(f*\text{Sqrt}[a*\text{Cosh}[e + f*x]^2]) + \text{Sqrt}[a*\text{Cosh}[e + f*x]^2]/f$

Rubi [A] time = 0.128025, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3205, 16, 43}

$$-\frac{a^2}{3f(a \cosh^2(e + fx))^{3/2}} + \frac{2a}{f\sqrt{a \cosh^2(e + fx)}} + \frac{\sqrt{a \cosh^2(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[a + a*\text{Sinh}[e + f*x]^2]*\text{Tanh}[e + f*x]^5, x]$

[Out] $-a^2/(3*f*(a*\text{Cosh}[e + f*x]^2)^{(3/2)}) + (2*a)/(f*\text{Sqrt}[a*\text{Cosh}[e + f*x]^2]) + \text{Sqrt}[a*\text{Cosh}[e + f*x]^2]/f$

Rule 3176

$\text{Int}[(u_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u*(a*\cos[e + f*x]^2)^p], x] /; \text{FreeQ}\{a, b, e, f, p\}, x] \ \&\& \ \text{EqQ}[a + b, 0]$

Rule 3205

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]^{(n_.)} \tan[(e_.) + (f_.)*(x_)]^{(m_.)}, x_Symbol] \rightarrow \text{With}\{ff = \text{FreeFactors}[\text{Sin}[e + f*x]^2, x]\}, \text{Dist}[ff^{(m+1)/2}/(2*f), \text{Subst}[\text{Int}[(x^{(m-1)/2}*(b*ff^{(n/2)}*x^{(n/2)})^p]/(1 - ff*x)^{(m+1)/2}, x], x, \text{Sin}[e + f*x]^2/ff], x] /; \text{FreeQ}\{b, e, f, p\}, x] \ \&\& \ \text{IntegerQ}[(m-1)/2] \ \&\& \ \text{IntegerQ}[n/2]$

Rule 16

`Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]`

Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

Rubi steps

$$\begin{aligned}
 \int \sqrt{a + a \sinh^2(e + fx)} \tanh^5(e + fx) dx &= \int \sqrt{a \cosh^2(e + fx)} \tanh^5(e + fx) dx \\
 &= \frac{\text{Subst}\left(\int \frac{(1-x)^2 \sqrt{ax}}{x^3} dx, x, \cosh^2(e + fx)\right)}{2f} \\
 &= \frac{a^3 \text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{5/2}} dx, x, \cosh^2(e + fx)\right)}{2f} \\
 &= \frac{a^3 \text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}} - \frac{2}{a(ax)^{3/2}} + \frac{1}{a^2 \sqrt{ax}}\right) dx, x, \cosh^2(e + fx)\right)}{2f} \\
 &= -\frac{a^2}{3f (a \cosh^2(e + fx))^{3/2}} + \frac{2a}{f \sqrt{a \cosh^2(e + fx)}} + \frac{\sqrt{a \cosh^2(e + fx)}}{f}
 \end{aligned}$$

Mathematica [A] time = 0.100224, size = 51, normalized size = 0.81

$$\frac{(3 \cosh^4(e + fx) + 6 \cosh^2(e + fx) - 1) \operatorname{sech}^4(e + fx) \sqrt{a \cosh^2(e + fx)}}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]

[Out] (Sqrt[a*Cosh[e + f*x]^2]*(-1 + 6*Cosh[e + f*x]^2 + 3*Cosh[e + f*x]^4)*Sech[e + f*x]^4)/(3*f)

Maple [C] time = 0.145, size = 42, normalized size = 0.7

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^5 a}{(\cosh(fx + e))^4 \sqrt{a (\cosh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x)

[Out] `int/indef0` (sinh(f*x+e)^5*a/cosh(f*x+e)^4/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [B] time = 1.79859, size = 394, normalized size = 6.25

$$\frac{6\sqrt{a}e^{(-2fx-2e)}}{f(e^{(-fx-e)} + 3e^{(-3fx-3e)} + 3e^{(-5fx-5e)} + e^{(-7fx-7e)})} + \frac{25\sqrt{a}e^{(-4fx-4e)}}{3f(e^{(-fx-e)} + 3e^{(-3fx-3e)} + 3e^{(-5fx-5e)} + e^{(-7fx-7e)})} + \frac{6\sqrt{a}e^{(-6fx-6e)}}{f(e^{(-fx-e)} + 3e^{(-3fx-3e)} + 3e^{(-5fx-5e)} + e^{(-7fx-7e)})} + \frac{1}{2}\sqrt{a}e^{(-8fx-8e)}/(f(e^{(-fx-e)} + 3e^{(-3fx-3e)} + 3e^{(-5fx-5e)} + e^{(-7fx-7e)})) + \frac{1}{2}\sqrt{a}/(f(e^{(-fx-e)} + 3e^{(-3fx-3e)} + 3e^{(-5fx-5e)} + e^{(-7fx-7e)}))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")

[Out] 6*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 25/3*sqrt(a)*e^(-4*f*x - 4*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 6*sqrt(a)*e^(-6*f*x - 6*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 1/2*sqrt(a)*e^(-8*f*x - 8*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 1/2*sqrt(a)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e)))

Fricas [B] time = 1.8592, size = 2361, normalized size = 37.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(24*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + 3*e^(f*x + e)*sinh(f*x + e)^8 + 12*(7*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^6 + 24*(7*cosh(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 10*(21*cosh(f*x + e)^4 + 54*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(21*cosh(f*x + e)^5 + 90*cosh(f*x + e)^3 + 25*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 12*(7*cosh(f*x + e)^6 + 45*cosh(f*x + e)^4 + 25*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(3*cosh(f*x + e)^7 + 27*cosh(f*x + e)^5 + 25*cosh(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x + e)^8 + 36*cosh(f*x + e)^6 + 50*cosh(f*x + e)^4 + 36*cosh(f*x + e)^2 + 3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^7 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 7*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^6 + 3*f*cosh(f*x + e)^5 + 3*(7*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (7*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 3*f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 + 30*f*cosh(f*x + e)^2 + (35*f*cosh(f*x + e)^4 + 30*f*cosh(f*x + e)^2 + 3*f)*e^(2*f*x + 2*e) + 3*f)*sinh(f*x + e)^3 + 3*(7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^7 + 3*f*cosh(f*x + e)^5 + 3*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (7*f*cosh(f*x + e)^6 + 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 + 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30062, size = 108, normalized size = 1.71

$$\frac{\sqrt{a} \left(\frac{8 \left(3e^{(5fx+5e)} + 4e^{(3fx+3e)} + 3e^{(fx+e)} \right)}{\left(e^{(2fx+2e)} + 1 \right)^3} + 3e^{(fx+e)} + 3e^{(-fx-e)} \right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")

[Out] 1/6*sqrt(a)*(8*(3*e^(5*f*x + 5*e) + 4*e^(3*f*x + 3*e) + 3*e^(f*x + e))/(e^(2*f*x + 2*e) + 1)^3 + 3*e^(f*x + e) + 3*e^(-f*x - e))/f

$$3.427 \quad \int \sqrt{a + a \sinh^2(e + fx)} \tanh^3(e + fx) dx$$

Optimal. Leaf size=38

$$\frac{a}{f\sqrt{a \cosh^2(e + fx)}} + \frac{\sqrt{a \cosh^2(e + fx)}}{f}$$

[Out] a/(f*Sqrt[a*Cosh[e + f*x]^2]) + Sqrt[a*Cosh[e + f*x]^2]/f

Rubi [A] time = 0.113288, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{f\sqrt{a \cosh^2(e + fx)}} + \frac{\sqrt{a \cosh^2(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]

[Out] a/(f*Sqrt[a*Cosh[e + f*x]^2]) + Sqrt[a*Cosh[e + f*x]^2]/f

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_)*sin[(e_) + (f_)*(x_)]^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^(m + 1)/2), x], x, Sin[e + f*x]^2/ff, x] /; FreeQ[{b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_)^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
 \int \sqrt{a + a \sinh^2(e + fx)} \tanh^3(e + fx) dx &= \int \sqrt{a \cosh^2(e + fx)} \tanh^3(e + fx) dx \\
 &= -\frac{\text{Subst}\left(\int \frac{(1-x)\sqrt{ax}}{x^2} dx, x, \cosh^2(e + fx)\right)}{2f} \\
 &= -\frac{a^2 \text{Subst}\left(\int \frac{1-x}{(ax)^{3/2}} dx, x, \cosh^2(e + fx)\right)}{2f} \\
 &= -\frac{a^2 \text{Subst}\left(\int \left(\frac{1}{(ax)^{3/2}} - \frac{1}{a\sqrt{ax}}\right) dx, x, \cosh^2(e + fx)\right)}{2f} \\
 &= \frac{a}{f\sqrt{a \cosh^2(e + fx)}} + \frac{\sqrt{a \cosh^2(e + fx)}}{f}
 \end{aligned}$$

Mathematica [A] time = 0.0874479, size = 29, normalized size = 0.76

$$\frac{a(\cosh^2(e + fx) + 1)}{f\sqrt{a \cosh^2(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] (a*(1 + Cosh[e + f*x]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```


Maple [C] time = 0.105, size = 42, normalized size = 1.1

$$\frac{1}{f} \int \frac{(\sinh(fx+e))^3 a}{(\cosh(fx+e))^2 \sqrt{a(\cosh(fx+e))^2}} \operatorname{tanh}(fx+e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x)`

[Out] `\`int/indef0\` (sinh(f*x+e)^3*a/cosh(f*x+e)^2/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [B] time = 1.79138, size = 143, normalized size = 3.76

$$\frac{3\sqrt{a}e^{(-2fx-2e)}}{f(e^{(-fx-e)} + e^{(-3fx-3e)})} + \frac{\sqrt{a}e^{(-4fx-4e)}}{2f(e^{(-fx-e)} + e^{(-3fx-3e)})} + \frac{\sqrt{a}}{2f(e^{(-fx-e)} + e^{(-3fx-3e)})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")`

[Out] `3*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqrt(a)*e^(-4*f*x - 4*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqrt(a)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e)))`

Fricas [B] time = 1.89306, size = 829, normalized size = 21.82

$$\frac{(4 \cosh(fx+e)e^{(fx+e)} \sinh(fx+e)^3 + e^{(fx+e)} \sinh(fx+e)^4 + 6(\cosh(fx+e)^2 + 1)e^{(fx+e)} \sinh(fx+e)^2 + 4(\cosh(fx+e)^3 + (fe^{(2fx+2e)} + f) \sinh(fx+e)^3 + 3(fe^{(2fx+2e)} + f \cosh(fx+e)) \sinh(fx+e)^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")`

[Out] `1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 6*(cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)`

)³ + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)⁴ + 6*cosh(f*x + e)² + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)³ + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)³ + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)² + f*cosh(f*x + e) + (f*cosh(f*x + e)³ + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (3*f*cosh(f*x + e)² + (3*f*cosh(f*x + e)² + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sinh^2(e + fx) + 1)} \tanh^3(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)

[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**3, x)

Giac [A] time = 1.25468, size = 72, normalized size = 1.89

$$\frac{\sqrt{a} \left(\frac{(5e^{2fx+2e} + 1)e^{-e}}{e^{(3fx+2e)+e(fx)}} + e^{(fx+e)} \right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")

[Out] 1/2*sqrt(a)*((5*e^(2*f*x + 2*e) + 1)*e^(-e)/(e^(3*f*x + 2*e) + e^(f*x)) + e^(f*x + e))/f

$$3.428 \quad \int \sqrt{a + a \sinh^2(e + fx)} \tanh(e + fx) dx$$

Optimal. Leaf size=18

$$\frac{\sqrt{a \cosh^2(e + fx)}}{f}$$

[Out] Sqrt[a*Cosh[e + f*x]^2]/f

Rubi [A] time = 0.0693726, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3176, 3205, 16, 32}

$$\frac{\sqrt{a \cosh^2(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x], x]

[Out] Sqrt[a*Cosh[e + f*x]^2]/f

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^p], x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_)*sin[(e_) + (f_)*(x_)]^n)^p]*tan[(e_) + (f_)*(x_)]^m, x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]

Rule 16

Int[(u_)*(v_)^m*((b_)*(v_))^n], x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 32

$\text{Int}[(a + b \cdot x)^m, x_Symbol] \rightarrow \text{Simp}[(a + b \cdot x)^{m+1}/(b \cdot (m+1)), x] /; \text{FreeQ}\{a, b, m, x\} \ \&\& \ \text{NeQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \sinh^2(e + fx)} \tanh(e + fx) dx &= \int \sqrt{a \cosh^2(e + fx)} \tanh(e + fx) dx \\ &= \frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{x} dx, x, \cosh^2(e + fx)\right)}{2f} \\ &= \frac{a \text{Subst}\left(\int \frac{1}{\sqrt{ax}} dx, x, \cosh^2(e + fx)\right)}{2f} \\ &= \frac{\sqrt{a \cosh^2(e + fx)}}{f} \end{aligned}$$

Mathematica [A] time = 0.0402495, size = 18, normalized size = 1.

$$\frac{\sqrt{a \cosh^2(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x],x]

[Out] Sqrt[a*Cosh[e + f*x]^2]/f

Maple [A] time = 0.026, size = 19, normalized size = 1.1

$$\frac{1}{f} \sqrt{a + a (\sinh(fx + e))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x)

[Out] $1/f*(a+a*\sinh(f*x+e)^2)^{(1/2)}$

Maxima [A] time = 1.75767, size = 43, normalized size = 2.39

$$\frac{\sqrt{ae^{(fx+e)}}}{2f} + \frac{\sqrt{ae^{(-fx-e)}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="maxima")`

[Out] $1/2*\sqrt{a}*e^{(f*x + e)}/f + 1/2*\sqrt{a}*e^{(-f*x - e)}/f$

Fricas [B] time = 1.80631, size = 365, normalized size = 20.28

$$\frac{\left(2 \cosh(fx + e)e^{(fx+e)} \sinh(fx + e) + e^{(fx+e)} \sinh(fx + e)^2 + (\cosh(fx + e)^2 + 1)e^{(fx+e)}\right) \sqrt{ae^{(4fx+4e)} + 2ae^{(2fx+2e)}}}{2\left(f \cosh(fx + e)e^{(2fx+2e)} + f \cosh(fx + e) + (fe^{(2fx+2e)} + f) \sinh(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="fricas")`

[Out] $1/2*(2*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e) + e^{(f*x + e)}*\sinh(f*x + e)^2 + (\cosh(f*x + e)^2 + 1)*e^{(f*x + e)})*\sqrt{a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a}*e^{(-f*x - e)}/(f*\cosh(f*x + e)*e^{(2*f*x + 2*e)} + f*\cosh(f*x + e) + (f*e^{(2*f*x + 2*e)} + f)*\sinh(f*x + e))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sinh^2(e + fx) + 1)} \tanh(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)`

[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x), x)

Giac [A] time = 1.15893, size = 35, normalized size = 1.94

$$\frac{\sqrt{a}\left(e^{(fx+e)} + e^{(-fx-e)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")

[Out] 1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e))/f

$$3.429 \quad \int \coth(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx$$

Optimal. Leaf size=50

$$\frac{\sqrt{a \cosh^2(e + fx)}}{f} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e + fx)}}{\sqrt{a}}\right)}{f}$$

[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a*Cosh[e + f*x]^2]/f

Rubi [A] time = 0.0977005, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3176, 3205, 50, 63, 206}

$$\frac{\sqrt{a \cosh^2(e + fx)}}{f} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e + fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a*Cosh[e + f*x]^2]/f

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^p], x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_)*sin[(e_) + (f_)*(x_)]^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \coth(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx &= \int \sqrt{a \cosh^2(e + fx)} \coth(e + fx) dx \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x} dx, x, \cosh^2(e + fx)\right)}{2f} \\
&= \frac{\sqrt{a \cosh^2(e + fx)}}{f} - \frac{a \text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}} dx, x, \cosh^2(e + fx)\right)}{2f} \\
&= \frac{\sqrt{a \cosh^2(e + fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a \cosh^2(e + fx)}\right)}{f} \\
&= -\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e + fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a \cosh^2(e + fx)}}{f}
\end{aligned}$$

Mathematica [A] time = 0.0588644, size = 42, normalized size = 0.84

$$\frac{\operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)} \left(\cosh(e + fx) + \log \left(\tanh \left(\frac{1}{2}(e + fx) \right) \right) \right)}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] (Sqrt[a*Cosh[e + f*x]^2]*(Cosh[e + f*x] + Log[Tanh[(e + f*x)/2]])*Sech[e + f*x])/f

Maple [C] time = 0.089, size = 42, normalized size = 0.8

$$\frac{1}{f}, \int/\operatorname{indef}0 \left(\frac{a (\cosh(fx + e))^2}{\sinh(fx + e)} \frac{1}{\sqrt{a (\cosh(fx + e))^2}}, \sinh(fx + e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2), x)

[Out] `int/indef0` (a*cosh(f*x+e)^2/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2), sinh(f*x+e))/f

Maxima [A] time = 1.781, size = 92, normalized size = 1.84

$$\frac{\left(\sqrt{a} e^{(-2fx-2e)} + \sqrt{a} \right) e^{(fx+e)}}{2f} - \frac{\sqrt{a} \log \left(e^{(-fx-e)} + 1 \right)}{f} + \frac{\sqrt{a} \log \left(e^{(-fx-e)} - 1 \right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2), x, algorithm="maxima")

[Out] 1/2*(sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a))*e^(f*x + e)/f - sqrt(a)*log(e^(-f*x - e) + 1)/f + sqrt(a)*log(e^(-f*x - e) - 1)/f

Fricas [B] time = 1.8532, size = 549, normalized size = 10.98

$$\frac{\left(2 \cosh (f x+e) e^{(f x+e)} \sinh (f x+e)+e^{(f x+e)} \sinh (f x+e)^2+\left(\cosh (f x+e)^2+1\right) e^{(f x+e)}+2\left(\cosh (f x+e) e^{(f x+e)}+e^{(f x+e)}\right)\right)}{2\left(f \cosh (f x+e) e^{(2 f x+2 e)}+f \cosh (f x+e)\right)+\left(f e^{(2 f x+2 e)}+f\right) \sinh (f x+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e) + 2*(cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e) + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \left(\sinh^2(e + f x) + 1\right)} \coth(e + f x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x), x)

Giac [A] time = 1.20647, size = 69, normalized size = 1.38

$$\frac{\sqrt{a}\left(e^{(f x+e)}+e^{(-f x-e)}-2 \log \left(e^{(f x+e)}+1\right)+2 \log \left(\left|e^{(f x+e)}-1\right|\right)\right)}{2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

```
[Out] 1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e) - 2*log(e^(f*x + e) + 1) + 2*log(abs(e^(f*x + e) - 1)))/f
```

$$3.430 \quad \int \coth^3(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx$$

Optimal. Leaf size=87

$$\frac{3\sqrt{a \cosh^2(e + fx)}}{2f} - \frac{3\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e + fx)}}{\sqrt{a}}\right)}{2f} - \frac{\operatorname{csch}^2(e + fx) (a \cosh^2(e + fx))^{3/2}}{2af}$$

[Out] $(-3\sqrt{a} \operatorname{ArcTanh}[\sqrt{a \cosh^2(e + fx)}/\sqrt{a}])/(2f) + (3\sqrt{a \cosh^2(e + fx)})/(2f) - ((a \cosh^2(e + fx))^{3/2} \operatorname{Csch}[e + fx]^2)/(2af)$

Rubi [A] time = 0.142148, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3176, 3205, 16, 47, 50, 63, 206}

$$\frac{3\sqrt{a \cosh^2(e + fx)}}{2f} - \frac{3\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e + fx)}}{\sqrt{a}}\right)}{2f} - \frac{\operatorname{csch}^2(e + fx) (a \cosh^2(e + fx))^{3/2}}{2af}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[e + fx]^3 \sqrt{a + a \operatorname{Sinh}[e + fx]^2}, x]$

[Out] $(-3\sqrt{a} \operatorname{ArcTanh}[\sqrt{a \cosh^2(e + fx)}/\sqrt{a}])/(2f) + (3\sqrt{a \cosh^2(e + fx)})/(2f) - ((a \cosh^2(e + fx))^{3/2} \operatorname{Csch}[e + fx]^2)/(2af)$

Rule 3176

$\operatorname{Int}[(u_.) * ((a_.) + (b_.) * \sin[(e_.) + (f_.) * (x_)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ActivateTrig}[u * (a \cos[e + fx]^2)^p], x] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \ \&\& \ \operatorname{EqQ}[a + b, 0]$

Rule 3205

$\operatorname{Int}[(b_.) * \sin[(e_.) + (f_.) * (x_)]^{(n_.)} \tan[(e_.) + (f_.) * (x_)]^{(m_.)}, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\sin[e + fx]^2, x]\}, \operatorname{Dist}[ff^{(m + 1)/2} / (2f), \operatorname{Subst}[\operatorname{Int}[(x^{(m - 1)/2} * (b * ff^{(n/2)} * x^{(n/2)})^p] / (1 - ff * x)^{(m + 1)/2}, x], x, \sin[e + fx]^2 / ff], x] /; \operatorname{FreeQ}\{b, e, f, p\}, x] \ \&\& \ \operatorname{IntegerQ}[(m - 1)/2] \ \&\& \ \operatorname{IntegerQ}[n/2]$

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) & IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \coth^3(e+fx) \sqrt{a+a \sinh^2(e+fx)} dx &= \int \sqrt{a \cosh^2(e+fx)} \coth^3(e+fx) dx \\
&= \frac{\text{Subst}\left(\int \frac{x\sqrt{ax}}{(1-x)^2} dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \frac{(ax)^{3/2}}{(1-x)^2} dx, x, \cosh^2(e+fx)\right)}{2af} \\
&= -\frac{(a \cosh^2(e+fx))^{3/2} \text{csch}^2(e+fx)}{2af} - \frac{3 \text{Subst}\left(\int \frac{\sqrt{ax}}{1-x} dx, x, \cosh^2(e+fx)\right)}{4f} \\
&= \frac{3\sqrt{a \cosh^2(e+fx)}}{2f} - \frac{(a \cosh^2(e+fx))^{3/2} \text{csch}^2(e+fx)}{2af} - \frac{(3a) \text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \cosh^2(e+fx)\right)}{4f} \\
&= \frac{3\sqrt{a \cosh^2(e+fx)}}{2f} - \frac{(a \cosh^2(e+fx))^{3/2} \text{csch}^2(e+fx)}{2af} - \frac{3 \text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \cosh^2(e+fx)\right)}{4f} \\
&= -\frac{3\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{3\sqrt{a \cosh^2(e+fx)}}{2f} - \frac{(a \cosh^2(e+fx))^{3/2} \text{csch}^2(e+fx)}{2af}
\end{aligned}$$

Mathematica [A] time = 0.260867, size = 77, normalized size = 0.89

$$\frac{\text{sech}(e+fx) \sqrt{a \cosh^2(e+fx)} \left(8 \cosh(e+fx) - \text{csch}^2\left(\frac{1}{2}(e+fx)\right) - \text{sech}^2\left(\frac{1}{2}(e+fx)\right) + 12 \log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right) \right)}{8f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^3*Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] (Sqrt[a*Cosh[e + f*x]^2]*(8*Cosh[e + f*x] - Csch[(e + f*x)/2]^2 + 12*Log[Tanh[(e + f*x)/2]] - Sech[(e + f*x)/2]^2)*Sech[e + f*x])/(8*f)

Maple [C] time = 0.092, size = 54, normalized size = 0.6

$$\frac{1}{f} \int \frac{a (\cosh(fx+e))^4}{\sinh(fx+e) \left((\cosh(fx+e))^2 - 1 \right) \sqrt{a (\cosh(fx+e))^2}} dx, \sinh(fx+e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x)`

[Out] `\`int/indef0` (a*cosh(f*x+e)^4/sinh(f*x+e)/(cosh(f*x+e)^2-1)/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [A] time = 1.94735, size = 170, normalized size = 1.95

$$\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)}{2f} + \frac{3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)}{2f} - \frac{3\sqrt{ae}^{(-2fx-2e)} + 3\sqrt{ae}^{(-4fx-4e)} - \sqrt{ae}^{(-6fx-6e)} - \sqrt{a}}{2f\left(e^{(-fx-e)} - 2e^{(-3fx-3e)} + e^{(-5fx-5e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `-3/2*sqrt(a)*log(e^(-f*x - e) + 1)/f + 3/2*sqrt(a)*log(e^(-f*x - e) - 1)/f - 1/2*(3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) - sqrt(a)*e^(-6*f*x - 6*e) - sqrt(a))/(f*(e^(-f*x - e) - 2*e^(-3*f*x - 3*e) + e^(-5*f*x - 5*e)))`

Fricas [B] time = 1.97631, size = 2056, normalized size = 23.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^5 + e^(f*x + e)*sinh(f*x + e)^6 + 3*(5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^4 + 4*(5*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 3*(5*cosh(f*x + e)^4 - 6*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 6*(cosh(f*x + e)^5 - 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^6 - 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 + 1)*e^(f*x + e) + 3*(5*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^5 + 2*(5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4 - 6*cosh`

```
(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 - 2*cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1
)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f
*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^5 + (f*e^(2*f*x + 2*e) + f)*si
nh(f*x + e)^5 + 5*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(
f*x + e)^4 - 2*f*cosh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^2 + (5*f*cosh(f*x +
e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^3 -
3*f*cosh(f*x + e) + (5*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^5 - 2*f*cosh(f*x +
e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (5*f*cosh(f*x + e)^4 - 6*f*cosh(
f*x + e)^2 + (5*f*cosh(f*x + e)^4 - 6*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e
) + f)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25896, size = 120, normalized size = 1.38

$$\frac{\sqrt{a} \left(\frac{2 \left(e^{(3fx+3e)+e^{(fx+e)}} \right)}{\left(e^{(2fx+2e)} - 1 \right)^2} - e^{(fx+e)} - e^{(-fx-e)} + 3 \log \left(e^{(fx+e)} + 1 \right) - 3 \log \left(\left| e^{(fx+e)} - 1 \right| \right) \right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(2*(e^(3*f*x + 3*e) + e^(f*x + e))/(e^(2*f*x + 2*e) - 1)^2 - e
^(f*x + e) - e^(-f*x - e) + 3*log(e^(f*x + e) + 1) - 3*log(abs(e^(f*x + e)
- 1)))/f
```


$$3.431 \quad \int \sqrt{a + a \sinh^2(e + fx)} \tanh^6(e + fx) dx$$

Optimal. Leaf size=120

$$\frac{\tanh^5(e + fx)\sqrt{a \cosh^2(e + fx)}}{4f} - \frac{5 \tanh^3(e + fx)\sqrt{a \cosh^2(e + fx)}}{8f} + \frac{15 \tanh(e + fx)\sqrt{a \cosh^2(e + fx)}}{8f} - \frac{15 \operatorname{sech}(e + fx)}{8f}$$

[Out] (-15*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(8*f) + (15*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(8*f) - (5*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^3)/(8*f) - (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^5)/(4*f)

Rubi [A] time = 0.129995, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3176, 3207, 2592, 288, 321, 203}

$$\frac{\tanh^5(e + fx)\sqrt{a \cosh^2(e + fx)}}{4f} - \frac{5 \tanh^3(e + fx)\sqrt{a \cosh^2(e + fx)}}{8f} + \frac{15 \tanh(e + fx)\sqrt{a \cosh^2(e + fx)}}{8f} - \frac{15 \operatorname{sech}(e + fx)}{8f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]

[Out] (-15*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(8*f) + (15*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(8*f) - (5*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^3)/(8*f) - (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^5)/(4*f)

Rule 3176

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^p], x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_.)]^n)^p], x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;]

FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]]

Rule 2592

Int[((a_)*sin[(e_) + (f_)*(x_)])^(m_)*tan[(e_) + (f_)*(x_)]^(n_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sinh^2(e + fx)} \tanh^6(e + fx) dx &= \int \sqrt{a \cosh^2(e + fx)} \tanh^6(e + fx) dx \\
&= \left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \int \sinh(e + fx) \tanh^5(e + fx) dx \\
&= \frac{\left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{x^6}{(1+x^2)^3} dx, x, \sinh(e + fx) \right)}{f} \\
&= -\frac{\sqrt{a \cosh^2(e + fx)} \tanh^5(e + fx)}{4f} + \frac{\left(5\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{x^4}{(1+x^2)^3} dx, x, \sinh(e + fx) \right)}{4f} \\
&= -\frac{5\sqrt{a \cosh^2(e + fx)} \tanh^3(e + fx)}{8f} - \frac{\sqrt{a \cosh^2(e + fx)} \tanh^5(e + fx)}{4f} + \frac{\left(15\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{x^2}{(1+x^2)^3} dx, x, \sinh(e + fx) \right)}{8f} \\
&= \frac{15\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{8f} - \frac{5\sqrt{a \cosh^2(e + fx)} \tanh^3(e + fx)}{8f} - \frac{\sqrt{a \cosh^2(e + fx)} \tanh^5(e + fx)}{4f} \\
&= -\frac{15 \tan^{-1}(\sinh(e + fx)) \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx)}{8f} + \frac{15\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{8f} - \frac{5\sqrt{a \cosh^2(e + fx)} \tanh^3(e + fx)}{8f} - \frac{\sqrt{a \cosh^2(e + fx)} \tanh^5(e + fx)}{4f}
\end{aligned}$$

Mathematica [A] time = 0.34292, size = 75, normalized size = 0.62

$$\frac{\operatorname{sech}^5(e + fx) \sqrt{a \cosh^2(e + fx)} (-5 \sinh(e + fx) - 15 \sinh(3(e + fx)) - 2 \sinh(5(e + fx)) + 60 \cosh^4(e + fx) \tan^{-1}(\sinh(e + fx)))}{32f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]

[Out] -(Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]^5*(60*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x]^4 - 5*Sinh[e + f*x] - 15*Sinh[3*(e + f*x)] - 2*Sinh[5*(e + f*x)]))/(32*f)

Maple [A] time = 0.109, size = 85, normalized size = 0.7

$$\frac{a \left(15 \arctan(\sinh(fx + e)) (\cosh(fx + e))^4 - 8 (\cosh(fx + e))^4 \sinh(fx + e) - 9 (\cosh(fx + e))^2 \sinh(fx + e) \right)}{8 (\cosh(fx + e))^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+a*\sinh(f*x+e)^2)^{(1/2)}*\tanh(f*x+e)^6,x)$

[Out] $-1/8*a*(15*\arctan(\sinh(f*x+e))*\cosh(f*x+e)^4-8*\cosh(f*x+e)^4*\sinh(f*x+e)-9*\cosh(f*x+e)^2*\sinh(f*x+e)+2*\sinh(f*x+e))/\cosh(f*x+e)^3/(a*\cosh(f*x+e)^2)^{(1/2)}/f$

Maxima [B] time = 1.90429, size = 1203, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((a+a*\sinh(f*x+e)^2)^{(1/2)}*\tanh(f*x+e)^6,x, \text{algorithm}=\text{"maxima"})$

[Out] $315/128*\sqrt{a}*\arctan(e^{-f*x - e})/f + 1/128*(105*\sqrt{a}*\arctan(e^{-f*x - e}) + (279*\sqrt{a}*e^{-f*x - e} + 511*\sqrt{a}*e^{-3*f*x - 3*e} + 385*\sqrt{a}*e^{-5*f*x - 5*e} + 105*\sqrt{a}*e^{-7*f*x - 7*e}))/ (4*e^{-2*f*x - 2*e} + 6*e^{-4*f*x - 4*e} + 4*e^{-6*f*x - 6*e} + e^{-8*f*x - 8*e} + 1))/f + 1/128*(105*\sqrt{a}*\arctan(e^{-f*x - e}) - (105*\sqrt{a}*e^{-f*x - e} + 385*\sqrt{a}*e^{-3*f*x - 3*e} + 511*\sqrt{a}*e^{-5*f*x - 5*e} + 279*\sqrt{a}*e^{-7*f*x - 7*e}))/ (4*e^{-2*f*x - 2*e} + 6*e^{-4*f*x - 4*e} + 4*e^{-6*f*x - 6*e} + e^{-8*f*x - 8*e} + 1))/f - 5/256*(15*\sqrt{a}*\arctan(e^{-f*x - e}) - (15*\sqrt{a}*e^{-f*x - e} + 55*\sqrt{a}*e^{-3*f*x - 3*e} + 73*\sqrt{a}*e^{-5*f*x - 5*e} - 15*\sqrt{a}*e^{-7*f*x - 7*e}))/ (4*e^{-2*f*x - 2*e} + 6*e^{-4*f*x - 4*e} + 4*e^{-6*f*x - 6*e} + e^{-8*f*x - 8*e} + 1))/f - 5/256*(15*\sqrt{a}*\arctan(e^{-f*x - e}) - (15*\sqrt{a}*e^{-f*x - e} - 73*\sqrt{a}*e^{-3*f*x - 3*e} - 55*\sqrt{a}*e^{-5*f*x - 5*e} - 15*\sqrt{a}*e^{-7*f*x - 7*e}))/ (4*e^{-2*f*x - 2*e} + 6*e^{-4*f*x - 4*e} + 4*e^{-6*f*x - 6*e} + e^{-8*f*x - 8*e} + 1))/f + 5/64*(3*\sqrt{a}*\arctan(e^{-f*x - e}) - (3*\sqrt{a}*e^{-f*x - e} + 11*\sqrt{a}*e^{-3*f*x - 3*e} - 11*\sqrt{a}*e^{-5*f*x - 5*e} - 3*\sqrt{a}*e^{-7*f*x - 7*e}))/ (4*e^{-2*f*x - 2*e} + 6*e^{-4*f*x - 4*e} + 4*e^{-6*f*x - 6*e} + e^{-8*f*x - 8*e} + 1))/f + 1/256*(837*\sqrt{a}*e^{-2*f*x - 2*e} + 1533*\sqrt{a}*e^{-4*f*x - 4*e} + 1155*\sqrt{a}*e^{-6*f*x - 6*e} + 315*\sqrt{a}*e^{-8*f*x - 8*e} + 128*\sqrt{a}))/ (f*(e^{-f*x - e} + 4*e^{-3*f*x - 3*e} + 6*e^{-5*f*x - 5*e} + 4*e^{-7*f*x - 7*e} + e^{-9*f*x - 9*e})) - 1/256*(315*\sqrt{a}*e^{-f*x - e} + 1155*\sqrt{a}*e^{-3*f*x - 3*e} + 1533*\sqrt{a}*e^{-5*f*x - 5*e} + 837*\sqrt{a}*e^{-7*f*x - 7*e} + 128*\sqrt{a}*e^{-9*f*x - 9*e}))/ (f*(4*e^{-2*f*x - 2*e} + 6*e^{-4*f*x - 4*e} + 4*e^{-6*f*x - 6*e} + e^{-8*f*x - 8*e} + 1))$

+ e)^4 + 15*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^3 + 12*(3*f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 + f*cosh(f*x + e) + (3*f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^9 + 4*f*cosh(f*x + e)^7 + 6*f*cosh(f*x + e)^5 + 4*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (9*f*cosh(f*x + e)^8 + 28*f*cosh(f*x + e)^6 + 30*f*cosh(f*x + e)^4 + 12*f*cosh(f*x + e)^2 + (9*f*cosh(f*x + e)^8 + 28*f*cosh(f*x + e)^6 + 30*f*cosh(f*x + e)^4 + 12*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**6,x)

[Out] Timed out

Giac [A] time = 1.40308, size = 134, normalized size = 1.12

$$\frac{\sqrt{a} \left(\frac{9e^{(7fx+7e)} + e^{(5fx+5e)} - e^{(3fx+3e)} - 9e^{(fx+e)}}{(e^{(2fx+2e)} + 1)^4} - 15 \arctan(e^{(fx+e)}) + 2e^{(fx+e)} - 2e^{(-fx-e)} \right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="giac")

[Out] 1/4*sqrt(a)*((9*e^(7*f*x + 7*e) + e^(5*f*x + 5*e) - e^(3*f*x + 3*e) - 9*e^(f*x + e))/(e^(2*f*x + 2*e) + 1)^4 - 15*arctan(e^(f*x + e)) + 2*e^(f*x + e) - 2*e^(-f*x - e))/f

$$3.432 \quad \int \sqrt{a + a \sinh^2(e + fx)} \tanh^4(e + fx) dx$$

Optimal. Leaf size=91

$$-\frac{\tanh^3(e + fx)\sqrt{a \cosh^2(e + fx)}}{2f} + \frac{3 \tanh(e + fx)\sqrt{a \cosh^2(e + fx)}}{2f} - \frac{3 \operatorname{sech}(e + fx)\sqrt{a \cosh^2(e + fx)} \tan^{-1}(\sinh(e + fx))}{2f}$$

[Out] (-3*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(2*f) + (3*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(2*f) - (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^3)/(2*f)

Rubi [A] time = 0.124577, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3176, 3207, 2592, 288, 321, 203}

$$-\frac{\tanh^3(e + fx)\sqrt{a \cosh^2(e + fx)}}{2f} + \frac{3 \tanh(e + fx)\sqrt{a \cosh^2(e + fx)}}{2f} - \frac{3 \operatorname{sech}(e + fx)\sqrt{a \cosh^2(e + fx)} \tan^{-1}(\sinh(e + fx))}{2f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]

[Out] (-3*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(2*f) + (3*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(2*f) - (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^3)/(2*f)

Rule 3176

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^p], x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_.)]^n)^p], x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;]

FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]]

Rule 2592

Int[((a_)*sin[(e_.) + (f_)*(x_)])^(m_)*tan[(e_.) + (f_)*(x_)]^(n_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]

Rule 288

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[m + n*(p + 1) + 1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sinh^2(e + fx)} \tanh^4(e + fx) dx &= \int \sqrt{a \cosh^2(e + fx)} \tanh^4(e + fx) dx \\
&= \left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \int \sinh(e + fx) \tanh^3(e + fx) dx \\
&= \frac{\left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{x^4}{(1+x^2)^2} dx, x, \sinh(e + fx) \right)}{f} \\
&= -\frac{\sqrt{a \cosh^2(e + fx)} \tanh^3(e + fx)}{2f} + \frac{\left(3\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, \sinh(e + fx) \right)}{2f} \\
&= \frac{3\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{2f} - \frac{\sqrt{a \cosh^2(e + fx)} \tanh^3(e + fx)}{2f} - \frac{\left(3\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, \sinh(e + fx) \right)}{2f} \\
&= -\frac{3 \tan^{-1}(\sinh(e + fx)) \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx)}{2f} + \frac{3\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{2f}
\end{aligned}$$

Mathematica [A] time = 0.194464, size = 55, normalized size = 0.6

$$\frac{a \left((\cosh(2(e + fx)) + 2) \tanh(e + fx) - 3 \cosh(e + fx) \tan^{-1}(\sinh(e + fx)) \right)}{2f \sqrt{a \cosh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]

[Out] (a*(-3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (2 + Cosh[2*(e + f*x)]))*Tanh[e + f*x])/(2*f*Sqrt[a*Cosh[e + f*x]^2])

Maple [A] time = 0.098, size = 69, normalized size = 0.8

$$\frac{a \left(3 \arctan(\sinh(fx + e)) (\cosh(fx + e))^2 - 2 (\cosh(fx + e))^2 \sinh(fx + e) - \sinh(fx + e) \right)}{2f \cosh(fx + e)} \frac{1}{\sqrt{a (\cosh(fx + e))}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x)

[Out] $-1/2*a*(3*\arctan(\sinh(f*x+e))*\cosh(f*x+e)^2-2*\cosh(f*x+e)^2*\sinh(f*x+e)-\sinh(f*x+e))/\cosh(f*x+e)/(a*\cosh(f*x+e)^2)^(1/2)/f$

Maxima [B] time = 1.67472, size = 522, normalized size = 5.74

$$\frac{15\sqrt{a}\arctan\left(e^{(-fx-e)}\right)}{8f} + \frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right) + \frac{5\sqrt{a}e^{(-fx-e)} + 3\sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)} + e^{(-4fx-4e)} + 1}}{4f} + \frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right) - \frac{3\sqrt{a}e^{(-fx-e)} + 5\sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)} + e^{(-4fx-4e)} + 1}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")

[Out] $15/8*\sqrt{a}*\arctan(e^{-fx - e})/f + 1/4*(3*\sqrt{a}*\arctan(e^{-fx - e}) + (5*\sqrt{a}*e^{-fx - e} + 3*\sqrt{a}*e^{-3fx - 3e}))/ (2*e^{-2fx - 2e} + e^{-4fx - 4e} + 1))/f + 1/4*(3*\sqrt{a}*\arctan(e^{-fx - e}) - (3*\sqrt{a}*e^{-fx - e} + 5*\sqrt{a}*e^{-3fx - 3e}))/ (2*e^{-2fx - 2e} + e^{-4fx - 4e} + 1))/f - 3/8*(\sqrt{a}*\arctan(e^{-fx - e}) - (\sqrt{a}*e^{-fx - e} - \sqrt{a}*e^{-3fx - 3e}))/ (2*e^{-2fx - 2e} + e^{-4fx - 4e} + 1))/f + 1/16*(25*\sqrt{a}*e^{-2fx - 2e} + 15*\sqrt{a}*e^{-4fx - 4e} + 8*\sqrt{a}))/ (f*(e^{-fx - e} + 2*e^{-3fx - 3e} + e^{-5fx - 5e})) - 1/16*(15*\sqrt{a}*e^{-fx - e} + 25*\sqrt{a}*e^{-3fx - 3e} + 8*\sqrt{a}*e^{-5fx - 5e}))/ (f*(2*e^{-2fx - 2e} + e^{-4fx - 4e} + 1))$

Fricas [B] time = 1.96191, size = 2003, normalized size = 22.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")

[Out] $1/2*(6*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e)^5 + e^{(f*x + e)}*\sinh(f*x + e)^6 + 3*(5*\cosh(f*x + e)^2 + 1)*e^{(f*x + e)}*\sinh(f*x + e)^4 + 4*(5*\cosh(f*x + e)^3 + 3*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^3 + 3*(5*\cosh(f*x + e)^4 + 6*\cosh(f*x + e)^2 - 1)*e^{(f*x + e)}*\sinh(f*x + e)^2 + 6*(\cosh(f*x + e)^$

```

5 + 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) - 6*(5*cos
h(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^5 + 2*(5
*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x + e)^3 +
3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4 + 6*cosh(
f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 + 2*cosh(f*x +
e)^3 + cosh(f*x + e))*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) +
(cosh(f*x + e)^6 + 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 - 1)*e^(f*x + e))
*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x
+ e)^5 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 5*(f*cosh(f*x + e)*e^(2
*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^4 + 2*f*cosh(f*x + e)^3 + 2*(5
*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*
x + e)^3 + 2*(5*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (5*f*cosh(f*x + e)^
3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) +
(f*cosh(f*x + e)^5 + 2*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e
) + (5*f*cosh(f*x + e)^4 + 6*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^4 + 6*f
*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sinh^2(e + fx) + 1)} \tanh^4(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)

[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**4, x)

Giac [A] time = 1.28528, size = 100, normalized size = 1.1

$$\frac{\sqrt{a} \left(\frac{2 \left(e^{3fx+3e} - e^{fx+e} \right)}{\left(e^{2fx+2e} + 1 \right)^2} - 6 \arctan \left(e^{fx+e} \right) + e^{fx+e} - e^{-fx-e} \right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")

```
[Out] 1/2*sqrt(a)*(2*(e^(3*f*x + 3*e) - e^(f*x + e))/(e^(2*f*x + 2*e) + 1)^2 - 6*  
arctan(e^(f*x + e)) + e^(f*x + e) - e^(-f*x - e))/f
```

$$3.433 \quad \int \sqrt{a + a \sinh^2(e + fx)} \tanh^2(e + fx) dx$$

Optimal. Leaf size=57

$$\frac{\tanh(e + fx)\sqrt{a \cosh^2(e + fx)}}{f} - \frac{\operatorname{sech}(e + fx)\sqrt{a \cosh^2(e + fx)} \tan^{-1}(\sinh(e + fx))}{f}$$

[Out] $-\left(\frac{\operatorname{ArcTan}[\sinh(e + fx)] \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx)}{f}\right) + \left(\frac{\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{f}\right)$

Rubi [A] time = 0.108551, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3176, 3207, 2592, 321, 203}

$$\frac{\tanh(e + fx)\sqrt{a \cosh^2(e + fx)}}{f} - \frac{\operatorname{sech}(e + fx)\sqrt{a \cosh^2(e + fx)} \tan^{-1}(\sinh(e + fx))}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]

[Out] $-\left(\frac{\operatorname{ArcTan}[\sinh(e + fx)] \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx)}{f}\right) + \left(\frac{\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{f}\right)$

Rule 3176

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p], x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^n)^p], x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.)] /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])

Rule 2592

```
Int[((a_)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(n_), x_
Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + a \sinh^2(e + fx)} \tanh^2(e + fx) dx &= \int \sqrt{a \cosh^2(e + fx)} \tanh^2(e + fx) dx \\
&= \left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \int \sinh(e + fx) \tanh(e + fx) dx \\
&= \frac{\left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, \sinh(e + fx) \right)}{f} \\
&= \frac{\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{f} - \frac{\left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{x^2}{1+x^2} dx, x, \sinh(e + fx) \right)}{f} \\
&= -\frac{\tan^{-1}(\sinh(e + fx)) \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx)}{f} + \frac{\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.0526466, size = 40, normalized size = 0.7

$$\frac{\operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)} (\sinh(e + fx) - \tan^{-1}(\sinh(e + fx)))}{f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]

[Out] (Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*(-ArcTan[Sinh[e + f*x]] + Sinh[e + f*x]))/f

Maple [A] time = 0.088, size = 41, normalized size = 0.7

$$-\frac{a \cosh(fx + e) (-\sinh(fx + e) + \arctan(\sinh(fx + e)))}{f} \frac{1}{\sqrt{a} (\cosh(fx + e))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x)

[Out] -a*cosh(f*x+e)*(-sinh(f*x+e)+arctan(sinh(f*x+e)))/(a*cosh(f*x+e)^2)^(1/2)/f

Maxima [A] time = 1.76125, size = 68, normalized size = 1.19

$$\frac{2\sqrt{a} \arctan(e^{(-fx-e)})}{f} + \frac{\sqrt{ae}^{(fx+e)}}{2f} - \frac{\sqrt{ae}^{(-fx-e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")

[Out] 2*sqrt(a)*arctan(e^(-f*x - e))/f + 1/2*sqrt(a)*e^(f*x + e)/f - 1/2*sqrt(a)*e^(-f*x - e)/f

Fricas [B] time = 1.97095, size = 497, normalized size = 8.72

$$\frac{\left(2 \cosh(fx + e) e^{(fx+e)} \sinh(fx + e) + e^{(fx+e)} \sinh(fx + e)\right)^2 - 4 \left(\cosh(fx + e) e^{(fx+e)} + e^{(fx+e)} \sinh(fx + e)\right) \arctan\left(\frac{e^{(fx+e)} \sinh(fx + e)}{\cosh(fx + e) e^{(fx+e)} + e^{(fx+e)} \sinh(fx + e)}\right)}{2 \left(f \cosh(fx + e) e^{(2fx+2e)} + f \cosh(fx + e) + \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")

[Out] $\frac{1}{2}*(2*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e) + e^{(f*x + e)}*\sinh(f*x + e))^2 - 4*(\cosh(f*x + e)*e^{(f*x + e)} + e^{(f*x + e)}*\sinh(f*x + e))*\arctan(\cosh(f*x + e) + \sinh(f*x + e)) + (\cosh(f*x + e)^2 - 1)*e^{(f*x + e)}*\sqrt{a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a}*e^{(-f*x - e)}/(f*\cosh(f*x + e)*e^{(2*f*x + 2*e)} + f*\cosh(f*x + e) + (f*e^{(2*f*x + 2*e)} + f)*\sinh(f*x + e))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sinh^2(e + fx) + 1)} \tanh^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)

[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**2, x)

Giac [A] time = 1.26655, size = 51, normalized size = 0.89

$$-\frac{\sqrt{a}\left(4 \arctan\left(e^{(fx+e)}\right) - e^{(fx+e)} + e^{(-fx-e)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")

[Out] $-1/2*\sqrt{a}*(4*\arctan(e^{(f*x + e)}) - e^{(f*x + e)} + e^{(-f*x - e)})/f$

$$3.434 \quad \int \coth^2(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx$$

Optimal. Leaf size=56

$$\frac{\tanh(e + fx) \sqrt{a \cosh^2(e + fx)}}{f} - \frac{\operatorname{csch}(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{f}$$

[Out] $-\left(\frac{\sqrt{a \cosh^2(e + fx)} \operatorname{csch}(e + fx) \operatorname{sech}(e + fx)}{f}\right) + \left(\frac{\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{f}\right)$

Rubi [A] time = 0.113688, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2590, 14}

$$\frac{\tanh(e + fx) \sqrt{a \cosh^2(e + fx)}}{f} - \frac{\operatorname{csch}(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In] `Int[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2],x]`

[Out] $-\left(\frac{\sqrt{a \cosh^2(e + fx)} \operatorname{csch}(e + fx) \operatorname{sech}(e + fx)}{f}\right) + \left(\frac{\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{f}\right)$

Rule 3176

`Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]`

Rule 3207

`Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sinh[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.)] /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])`

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]
```

Rubi steps

$$\begin{aligned}
\int \coth^2(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx &= \int \sqrt{a \cosh^2(e + fx)} \coth^2(e + fx) dx \\
&= \left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \int \cosh(e + fx) \coth^2(e + fx) dx \\
&= -\frac{\left(i \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{1-x^2}{x^2} dx, x, -i \sinh(e + fx) \right)}{f} \\
&= -\frac{\left(i \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \left(-1 + \frac{1}{x^2} \right) dx, x, -i \sinh(e + fx) \right)}{f} \\
&= -\frac{\sqrt{a \cosh^2(e + fx)} \operatorname{csch}(e + fx) \operatorname{sech}(e + fx)}{f} + \frac{\sqrt{a \cosh^2(e + fx)} \tanh(e + fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.0754151, size = 35, normalized size = 0.62

$$-\frac{\tanh(e + fx) (\operatorname{csch}^2(e + fx) - 1) \sqrt{a \cosh^2(e + fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2], x]
```

```
[Out] -((Sqrt[a*Cosh[e + f*x]^2]*(-1 + Csch[e + f*x]^2)*Tanh[e + f*x])/f)
```

Maple [A] time = 0.085, size = 42, normalized size = 0.8

$$\frac{\cosh(fx + e) a \left(-1 + (\sinh(fx + e))^2\right)}{\sinh(fx + e) f} \frac{1}{\sqrt{a (\cosh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x)`

[Out] `cosh(f*x+e)*a*(-1+sinh(f*x+e)^2)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f`

Maxima [B] time = 1.73373, size = 169, normalized size = 3.02

$$\frac{\sqrt{ae}^{-fx-e}}{f(e^{(-2fx-2e)} - 1)} - \frac{2\sqrt{ae}^{(-2fx-2e)} - \sqrt{a}}{2f(e^{(-fx-e)} - e^{(-3fx-3e)})} + \frac{2\sqrt{ae}^{(-fx-e)} - \sqrt{ae}^{(-3fx-3e)}}{2f(e^{(-2fx-2e)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `sqrt(a)*e^(-f*x - e)/(f*(e^(-2*f*x - 2*e) - 1)) - 1/2*(2*sqrt(a)*e^(-2*f*x - 2*e) - sqrt(a))/(f*(e^(-f*x - e) - e^(-3*f*x - 3*e))) + 1/2*(2*sqrt(a)*e^(-f*x - e) - sqrt(a)*e^(-3*f*x - 3*e))/(f*(e^(-2*f*x - 2*e) - 1))`

Fricas [B] time = 1.78849, size = 829, normalized size = 14.8

$$\frac{\left(4 \cosh(fx + e) e^{(fx+e)} \sinh(fx + e)^3 + e^{(fx+e)} \sinh(fx + e)^4 + 6 \left(\cosh(fx + e)^2 - 1\right) e^{(fx+e)} \sinh(fx + e)^2 + 4 \left(\cosh(fx + e) e^{(fx+e)} \sinh(fx + e)^3 + e^{(fx+e)} \sinh(fx + e)^4\right)\right)}{2 \left(f \cosh(fx + e)^3 + \left(f e^{(2fx+2e)} + f\right) \sinh(fx + e)^3 + 3 \left(f \cosh(fx + e) e^{(2fx+2e)} + f \cosh(fx + e)\right) \sinh(fx + e)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 6*(cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)`

)³ - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)⁴ - 6*cosh(f*x + e)² + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)³ + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)³ + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)² - f*cosh(f*x + e) + (f*cosh(f*x + e)³ - f*cosh(f*x + e))*e^(2*f*x + 2*e) + (3*f*cosh(f*x + e)² + (3*f*cosh(f*x + e)² - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a(\sinh^2(e + fx) + 1)} \coth^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**2*(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**2, x)

Giac [A] time = 1.29183, size = 77, normalized size = 1.38

$$\frac{\sqrt{a} \left(\frac{(5e^{(2fx+2e)} - 1)e^{(-e)}}{e^{(3fx+2e)} - e^{(fx)}} - e^{(fx+e)} \right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -1/2*sqrt(a)*((5*e^(2*f*x + 2*e) - 1)*e^(-e)/(e^(3*f*x + 2*e) - e^(f*x)) - e^(f*x + e))/f

$$3.435 \quad \int \coth^4(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx$$

Optimal. Leaf size=91

$$\frac{\tanh(e + fx) \sqrt{a \cosh^2(e + fx)}}{f} - \frac{\operatorname{csch}^3(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{3f} - \frac{2 \operatorname{csch}(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{f}$$

[Out] $(-2 \sqrt{a \cosh^2(e + fx)} \operatorname{csch}(e + fx) \operatorname{sech}(e + fx))/f - (\sqrt{a \cosh^2(e + fx)} \operatorname{csch}^3(e + fx) \operatorname{sech}(e + fx))/(3f) + (\sqrt{a \cosh^2(e + fx)} \operatorname{csch}(e + fx) \operatorname{sech}(e + fx))/f$

Rubi [A] time = 0.121694, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2590, 270}

$$\frac{\tanh(e + fx) \sqrt{a \cosh^2(e + fx)}}{f} - \frac{\operatorname{csch}^3(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{3f} - \frac{2 \operatorname{csch}(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[e + fx]^4 \sqrt{a + a \sinh^2[e + fx]^2}, x]$

[Out] $(-2 \sqrt{a \cosh^2(e + fx)} \operatorname{csch}(e + fx) \operatorname{sech}(e + fx))/f - (\sqrt{a \cosh^2(e + fx)} \operatorname{csch}^3(e + fx) \operatorname{sech}(e + fx))/(3f) + (\sqrt{a \cosh^2(e + fx)} \operatorname{csch}(e + fx) \operatorname{sech}(e + fx))/f$

Rule 3176

$\text{Int}[(u_.) * ((a_.) + (b_.) * \sin[(e_.) + (f_.) * (x_)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u * (a \cos[e + fx]^2)^p], x] /;$ FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

$\text{Int}[(u_.) * ((b_.) * \sin[(e_.) + (f_.) * (x_)]^n)^{(p_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\sin[e + fx], x]\}, \text{Dist}[(b * ff^n)^{\text{IntPart}[p]} * (b * \sin[e + fx]^n)^{\text{FracPart}[p]}] / (\sin[e + fx] / ff)^{(n * \text{FracPart}[p])}, \text{Int}[\text{ActivateTrig}[u * (\sin[e + fx] / ff)^{n * p}], x], x] /;$ FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.) * (trig_)[e + fx]^m_.) /;

FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]]

Rule 2590

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
 := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[Exp
 andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
 IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \coth^4(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx &= \int \sqrt{a \cosh^2(e + fx)} \coth^4(e + fx) dx \\
 &= \left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \int \cosh(e + fx) \coth^4(e + fx) dx \\
 &= \frac{\left(i \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{(1-x^2)^2}{x^4} dx, x, -i \sinh(e + fx) \right)}{f} \\
 &= \frac{\left(i \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \left(1 + \frac{1}{x^4} - \frac{2}{x^2} \right) dx, x, -i \sinh(e + fx) \right)}{f} \\
 &= -\frac{2 \sqrt{a \cosh^2(e + fx)} \operatorname{csch}(e + fx) \operatorname{sech}(e + fx)}{f} - \frac{\sqrt{a \cosh^2(e + fx)} \operatorname{csch}^3(e + fx)}{3f}
 \end{aligned}$$

Mathematica [A] time = 0.0729251, size = 47, normalized size = 0.52

$$-\frac{\tanh(e + fx) \left(\operatorname{csch}^4(e + fx) + 6 \operatorname{csch}^2(e + fx) - 3 \right) \sqrt{a \cosh^2(e + fx)}}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^4*Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] $-(\text{Sqrt}[a*\text{Cosh}[e + f*x]^2]*(-3 + 6*\text{Csch}[e + f*x]^2 + \text{Csch}[e + f*x]^4)*\text{Tanh}[e + f*x])/(3*f)$

Maple [A] time = 0.092, size = 55, normalized size = 0.6

$$\frac{\cosh(fx + e) a \left(3 \left(\sinh(fx + e) \right)^4 - 6 \left(\sinh(fx + e) \right)^2 - 1 \right)}{3 \left(\sinh(fx + e) \right)^3 f} \frac{1}{\sqrt{a \left(\cosh(fx + e) \right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{coth}(f*x+e)^4*(a+a*\sinh(f*x+e)^2)^{(1/2)}, x)$

[Out] $1/3*\cosh(f*x+e)*a*(3*\sinh(f*x+e)^4-6*\sinh(f*x+e)^2-1)/\sinh(f*x+e)^3/(a*\cosh(f*x+e)^2)^{(1/2)}/f$

Maxima [B] time = 1.78277, size = 657, normalized size = 7.22

$$\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)-3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)-\frac{2\left(9\sqrt{ae}^{(-fx-e)}-8\sqrt{ae}^{(-3fx-3e)}+3\sqrt{ae}^{(-5fx-5e)}\right)}{3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1}}{12f} + \frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)-3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)}{12f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\text{coth}(f*x+e)^4*(a+a*\sinh(f*x+e)^2)^{(1/2)}, x, \text{algorithm}=\text{"maxima"})$

[Out] $-1/12*(3*\text{sqrt}(a)*\log(e^{(-f*x - e)} + 1) - 3*\text{sqrt}(a)*\log(e^{(-f*x - e)} - 1) - 2*(9*\text{sqrt}(a)*e^{(-f*x - e)} - 8*\text{sqrt}(a)*e^{(-3*f*x - 3*e)} + 3*\text{sqrt}(a)*e^{(-5*f*x - 5*e)})/(3*e^{(-2*f*x - 2*e)} - 3*e^{(-4*f*x - 4*e)} + e^{(-6*f*x - 6*e)} - 1)) / f + 1/12*(3*\text{sqrt}(a)*\log(e^{(-f*x - e)} + 1) - 3*\text{sqrt}(a)*\log(e^{(-f*x - e)} - 1) + 2*(3*\text{sqrt}(a)*e^{(-f*x - e)} - 8*\text{sqrt}(a)*e^{(-3*f*x - 3*e)} + 9*\text{sqrt}(a)*e^{(-5*f*x - 5*e)})/(3*e^{(-2*f*x - 2*e)} - 3*e^{(-4*f*x - 4*e)} + e^{(-6*f*x - 6*e)} - 1))/f + \text{sqrt}(a)*e^{(-3*f*x - 3*e)}/(f*(3*e^{(-2*f*x - 2*e)} - 3*e^{(-4*f*x - 4*e)} + e^{(-6*f*x - 6*e)} - 1)) - 1/12*(33*\text{sqrt}(a)*e^{(-2*f*x - 2*e)} - 40*\text{sqrt}(a)*e^{(-4*f*x - 4*e)} + 15*\text{sqrt}(a)*e^{(-6*f*x - 6*e)} - 6*\text{sqrt}(a))/(f*(e^{(-f*x - e)} - 3*e^{(-3*f*x - 3*e)} + 3*e^{(-5*f*x - 5*e)} - e^{(-7*f*x - 7*e)})) + 1/12*(15*\text{sqrt}(a)*e^{(-f*x - e)} - 40*\text{sqrt}(a)*e^{(-3*f*x - 3*e)} + 33*\text{sqrt}(a)*e^{(-5*f*x - 5*e)} - 6*\text{sqrt}(a)*e^{(-7*f*x - 7*e)})/(f*(3*e^{(-2*f*x - 2*e)} - 3*e^{(-4*f*x - 4*e)} + e^{(-6*f*x - 6*e)} - 1))$

- 4*e) + e^(-6*f*x - 6*e) - 1))

Fricas [B] time = 1.97043, size = 2361, normalized size = 25.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\frac{1}{6} \cdot (24 \cdot \cosh(fx + e) \cdot e^{fx + e} \cdot \sinh(fx + e)^7 + 3 \cdot e^{fx + e} \cdot \sinh(fx + e)^8 + 12 \cdot (7 \cdot \cosh(fx + e)^2 - 3) \cdot e^{fx + e} \cdot \sinh(fx + e)^6 + 24 \cdot (7 \cdot \cosh(fx + e)^3 - 9 \cdot \cosh(fx + e)) \cdot e^{fx + e} \cdot \sinh(fx + e)^5 + 10 \cdot (21 \cdot \cosh(fx + e)^4 - 54 \cdot \cosh(fx + e)^2 + 5) \cdot e^{fx + e} \cdot \sinh(fx + e)^4 + 8 \cdot (21 \cdot \cosh(fx + e)^5 - 90 \cdot \cosh(fx + e)^3 + 25 \cdot \cosh(fx + e)) \cdot e^{fx + e} \cdot \sinh(fx + e)^3 + 12 \cdot (7 \cdot \cosh(fx + e)^6 - 45 \cdot \cosh(fx + e)^4 + 25 \cdot \cosh(fx + e)^2 - 3) \cdot e^{fx + e} \cdot \sinh(fx + e)^2 + 8 \cdot (3 \cdot \cosh(fx + e)^7 - 27 \cdot \cosh(fx + e)^5 + 25 \cdot \cosh(fx + e)^3 - 9 \cdot \cosh(fx + e)) \cdot e^{fx + e} \cdot \sinh(fx + e) + (3 \cdot \cosh(fx + e)^8 - 36 \cdot \cosh(fx + e)^6 + 50 \cdot \cosh(fx + e)^4 - 36 \cdot \cosh(fx + e)^2 + 3) \cdot e^{fx + e}) \cdot \sqrt{a \cdot e^{4fx + 4e} + 2 \cdot a \cdot e^{2fx + 2e} + a} \cdot e^{-fx - e} / (f \cdot \cosh(fx + e)^7 + (f \cdot e^{2fx + 2e} + f) \cdot \sinh(fx + e)^7 + 7 \cdot (f \cdot \cosh(fx + e) \cdot e^{2fx + 2e} + f \cdot \cosh(fx + e)) \cdot \sinh(fx + e)^6 - 3 \cdot f \cdot \cosh(fx + e)^5 + 3 \cdot (7 \cdot f \cdot \cosh(fx + e)^2 + (7 \cdot f \cdot \cosh(fx + e)^2 - f) \cdot e^{2fx + 2e} - f) \cdot \sinh(fx + e)^5 + 5 \cdot (7 \cdot f \cdot \cosh(fx + e)^3 - 3 \cdot f \cdot \cosh(fx + e) + (7 \cdot f \cdot \cosh(fx + e)^3 - 3 \cdot f \cdot \cosh(fx + e)) \cdot e^{2fx + 2e}) \cdot \sinh(fx + e)^4 + 3 \cdot f \cdot \cosh(fx + e)^3 + (35 \cdot f \cdot \cosh(fx + e)^4 - 30 \cdot f \cdot \cosh(fx + e)^2 + (35 \cdot f \cdot \cosh(fx + e)^4 - 30 \cdot f \cdot \cosh(fx + e)^2 + 3 \cdot f) \cdot e^{2fx + 2e} + 3 \cdot f) \cdot \sinh(fx + e)^3 + 3 \cdot (7 \cdot f \cdot \cosh(fx + e)^5 - 10 \cdot f \cdot \cosh(fx + e)^3 + 3 \cdot f \cdot \cosh(fx + e) + (7 \cdot f \cdot \cosh(fx + e)^5 - 10 \cdot f \cdot \cosh(fx + e)^3 + 3 \cdot f \cdot \cosh(fx + e)) \cdot e^{2fx + 2e}) \cdot \sinh(fx + e)^2 - f \cdot \cosh(fx + e) + (f \cdot \cosh(fx + e)^7 - 3 \cdot f \cdot \cosh(fx + e)^5 + 3 \cdot f \cdot \cosh(fx + e)^3 - f \cdot \cosh(fx + e)) \cdot e^{2fx + 2e} + (7 \cdot f \cdot \cosh(fx + e)^6 - 15 \cdot f \cdot \cosh(fx + e)^4 + 9 \cdot f \cdot \cosh(fx + e)^2 + (7 \cdot f \cdot \cosh(fx + e)^6 - 15 \cdot f \cdot \cosh(fx + e)^4 + 9 \cdot f \cdot \cosh(fx + e)^2 - f) \cdot e^{2fx + 2e} - f) \cdot \sinh(fx + e))$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**4*(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.27173, size = 108, normalized size = 1.19

$$\frac{\sqrt{a} \left(\frac{8 \left(3e^{(5fx+5e)} - 4e^{(3fx+3e)} + 3e^{(fx+e)} \right)}{\left(e^{(2fx+2e)} - 1 \right)^3} - 3e^{(fx+e)} + 3e^{(-fx-e)} \right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -1/6*sqrt(a)*(8*(3*e^(5*f*x + 5*e) - 4*e^(3*f*x + 3*e) + 3*e^(f*x + e))/(e^(2*f*x + 2*e) - 1)^3 - 3*e^(f*x + e) + 3*e^(-f*x - e))/f

$$3.436 \quad \int \coth^6(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx$$

Optimal. Leaf size=124

$$\frac{\tanh(e + fx) \sqrt{a \cosh^2(e + fx)}}{f} - \frac{\operatorname{csch}^5(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{5f} - \frac{\operatorname{csch}^3(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{f}$$

[Out] (-3*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/f - (Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]^5*Sech[e + f*x])/(5*f) + (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/f

Rubi [A] time = 0.124526, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2590, 270}

$$\frac{\tanh(e + fx) \sqrt{a \cosh^2(e + fx)}}{f} - \frac{\operatorname{csch}^5(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{5f} - \frac{\operatorname{csch}^3(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{f}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] (-3*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/f - (Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]^5*Sech[e + f*x])/(5*f) + (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/f

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^p], x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_)*((b_)*sin[(e_) + (f_)*(x_)]^n)^p], x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \coth^6(e + fx) \sqrt{a + a \sinh^2(e + fx)} dx &= \int \sqrt{a \cosh^2(e + fx)} \coth^6(e + fx) dx \\
&= \left(\sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \int \cosh(e + fx) \coth^6(e + fx) dx \\
&= -\frac{\left(i \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \frac{(1-x^2)^3}{x^6} dx, x, -i \sinh(e + fx) \right)}{f} \\
&= -\frac{\left(i \sqrt{a \cosh^2(e + fx)} \operatorname{sech}(e + fx) \right) \operatorname{Subst} \left(\int \left(-1 + \frac{1}{x^6} - \frac{3}{x^4} + \frac{3}{x^2} \right) dx, x, -i \sinh(e + fx) \right)}{f} \\
&= -\frac{3 \sqrt{a \cosh^2(e + fx)} \operatorname{csch}(e + fx) \operatorname{sech}(e + fx)}{f} - \frac{\sqrt{a \cosh^2(e + fx)} \operatorname{csch}^3(e + fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.193013, size = 67, normalized size = 0.54

$$\frac{(235 \cosh(2(e + fx)) - 90 \cosh(4(e + fx)) + 5 \cosh(6(e + fx)) - 182) \operatorname{csch}^5(e + fx) \operatorname{sech}(e + fx) \sqrt{a \cosh^2(e + fx)}}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

[Out] $(\text{Sqrt}[a*\text{Cosh}[e + f*x]^2]*(-182 + 235*\text{Cosh}[2*(e + f*x)] - 90*\text{Cosh}[4*(e + f*x)]) + 5*\text{Cosh}[6*(e + f*x)])*\text{Csch}[e + f*x]^5*\text{Sech}[e + f*x]/(160*f)$

Maple [A] time = 0.092, size = 65, normalized size = 0.5

$$\frac{\cosh(fx + e) a \left(5 \left(\sinh(fx + e) \right)^6 - 15 \left(\sinh(fx + e) \right)^4 - 5 \left(\sinh(fx + e) \right)^2 - 1 \right)}{5 \left(\sinh(fx + e) \right)^5 f} \frac{1}{\sqrt{a \left(\cosh(fx + e) \right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(\text{coth}(f*x+e)^6*(a+a*\sinh(f*x+e)^2)^{(1/2)}, x)$

[Out] $1/5*\cosh(f*x+e)*a*(5*\sinh(f*x+e)^6-15*\sinh(f*x+e)^4-5*\sinh(f*x+e)^2-1)/\sinh(f*x+e)^5/(a*\cosh(f*x+e)^2)^{(1/2)}/f$

Maxima [B] time = 1.78392, size = 1418, normalized size = 11.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(\text{coth}(f*x+e)^6*(a+a*\sinh(f*x+e)^2)^{(1/2)}, x, \text{algorithm}="maxima")$

[Out] $-1/320*(105*\text{sqrt}(a)*\log(e^{-f*x - e} + 1) - 105*\text{sqrt}(a)*\log(e^{-f*x - e} - 1) - 2*(375*\text{sqrt}(a)*e^{-f*x - e} - 790*\text{sqrt}(a)*e^{-3*f*x - 3*e} + 896*\text{sqrt}(a)*e^{-5*f*x - 5*e} - 490*\text{sqrt}(a)*e^{-7*f*x - 7*e} + 105*\text{sqrt}(a)*e^{-9*f*x - 9*e}))/ (5*e^{-2*f*x - 2*e} - 10*e^{-4*f*x - 4*e} + 10*e^{-6*f*x - 6*e} - 5*e^{-8*f*x - 8*e} + e^{-10*f*x - 10*e} - 1))/f + 1/320*(105*\text{sqrt}(a)*\log(e^{-f*x - e} + 1) - 105*\text{sqrt}(a)*\log(e^{-f*x - e} - 1) + 2*(105*\text{sqrt}(a)*e^{-f*x - e} - 490*\text{sqrt}(a)*e^{-3*f*x - 3*e} + 896*\text{sqrt}(a)*e^{-5*f*x - 5*e} - 790*\text{sqrt}(a)*e^{-7*f*x - 7*e} + 375*\text{sqrt}(a)*e^{-9*f*x - 9*e}))/ (5*e^{-2*f*x - 2*e} - 10*e^{-4*f*x - 4*e} + 10*e^{-6*f*x - 6*e} - 5*e^{-8*f*x - 8*e} + e^{-10*f*x - 10*e} - 1))/f + 1/256*(15*\text{sqrt}(a)*\log(e^{-f*x - e} + 1) - 15*\text{sqrt}(a)*\log(e^{-f*x - e} - 1) + 2*(15*\text{sqrt}(a)*e^{-f*x - e} + 250*\text{sqrt}(a)*e^{-3*f*x - 3*e} - 128*\text{sqrt}(a)*e^{-5*f*x - 5*e} + 70*\text{sqrt}(a)*e^{-7*f*x - 7*e} - 15*\text{sqrt}(a)*e^{-9*f*x - 9*e}))/ (5*e^{-2*f*x - 2*e} - 10*e^{-4*f*x - 4*e} + 10*e^{-6*f*x - 6*e} - 5*e^{-8*f*x - 8*e} + e^{-10*f*x - 10*e} - 1))/f - 1/256*(15*\text{sqrt}(a)*\log(e^{-f*x - e} + 1) - 15*\text{sqrt}(a)*\log(e^{-f*x - e} - 1) + 2*(15*\text{sqrt}(a)*e^{-f*x - e} + 250*\text{sqrt}(a)*e^{-3*f*x - 3*e} - 128*\text{sqrt}(a)*e^{-5*f*x - 5*e} + 70*\text{sqrt}(a)*e^{-7*f*x - 7*e} - 15*\text{sqrt}(a)*e^{-9*f*x - 9*e}))/ (5*e^{-2*f*x - 2*e} - 10*e^{-4*f*x - 4*e} + 10*e^{-6*f*x - 6*e} - 5*e^{-8*f*x - 8*e} + e^{-10*f*x - 10*e} - 1))/f$

$$\begin{aligned} & \text{rt}(a)*e^{(-f*x - e)} - 70*\text{sqrt}(a)*e^{(-3*f*x - 3*e)} + 128*\text{sqrt}(a)*e^{(-5*f*x - 5*e)} - 250*\text{sqrt}(a)*e^{(-7*f*x - 7*e)} - 15*\text{sqrt}(a)*e^{(-9*f*x - 9*e)} / (5*e^{(-2*f*x - 2*e)} - 10*e^{(-4*f*x - 4*e)} + 10*e^{(-6*f*x - 6*e)} - 5*e^{(-8*f*x - 8*e)} + e^{(-10*f*x - 10*e)} - 1) / f + 2*\text{sqrt}(a)*e^{(-5*f*x - 5*e)} / (f*(5*e^{(-2*f*x - 2*e)} - 10*e^{(-4*f*x - 4*e)} + 10*e^{(-6*f*x - 6*e)} - 5*e^{(-8*f*x - 8*e)} + e^{(-10*f*x - 10*e)} - 1)) - 1/640*(2895*\text{sqrt}(a)*e^{(-2*f*x - 2*e)} - 7110*\text{sqrt}(a)*e^{(-4*f*x - 4*e)} + 8064*\text{sqrt}(a)*e^{(-6*f*x - 6*e)} - 4410*\text{sqrt}(a)*e^{(-8*f*x - 8*e)} + 945*\text{sqrt}(a)*e^{(-10*f*x - 10*e)} - 320*\text{sqrt}(a)) / (f*(e^{(-f*x - e)} - 5*e^{(-3*f*x - 3*e)} + 10*e^{(-5*f*x - 5*e)} - 10*e^{(-7*f*x - 7*e)} + 5*e^{(-9*f*x - 9*e)} - e^{(-11*f*x - 11*e)})) + 1/640*(945*\text{sqrt}(a)*e^{(-f*x - e)} - 4410*\text{sqrt}(a)*e^{(-3*f*x - 3*e)} + 8064*\text{sqrt}(a)*e^{(-5*f*x - 5*e)} - 7110*\text{sqrt}(a)*e^{(-7*f*x - 7*e)} + 2895*\text{sqrt}(a)*e^{(-9*f*x - 9*e)} - 320*\text{sqrt}(a)*e^{(-11*f*x - 11*e)}) / (f*(5*e^{(-2*f*x - 2*e)} - 10*e^{(-4*f*x - 4*e)} + 10*e^{(-6*f*x - 6*e)} - 5*e^{(-8*f*x - 8*e)} + e^{(-10*f*x - 10*e)} - 1)) \end{aligned}$$

Fricas [B] time = 2.11362, size = 4725, normalized size = 38.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] $1/10*(60*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e)^{11} + 5*e^{(f*x + e)}*\sinh(f*x + e)^{12} + 30*(11*\cosh(f*x + e)^2 - 3)*e^{(f*x + e)}*\sinh(f*x + e)^{10} + 100*(11*\cosh(f*x + e)^3 - 9*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^9 + 5*(495*\cosh(f*x + e)^4 - 810*\cosh(f*x + e)^2 + 47)*e^{(f*x + e)}*\sinh(f*x + e)^8 + 40*(99*\cosh(f*x + e)^5 - 270*\cosh(f*x + e)^3 + 47*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^7 + 28*(165*\cosh(f*x + e)^6 - 675*\cosh(f*x + e)^4 + 235*\cosh(f*x + e)^2 - 13)*e^{(f*x + e)}*\sinh(f*x + e)^6 + 8*(495*\cosh(f*x + e)^7 - 2835*\cosh(f*x + e)^5 + 1645*\cosh(f*x + e)^3 - 273*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^5 + 5*(495*\cosh(f*x + e)^8 - 3780*\cosh(f*x + e)^6 + 3290*\cosh(f*x + e)^4 - 1092*\cosh(f*x + e)^2 + 47)*e^{(f*x + e)}*\sinh(f*x + e)^4 + 20*(55*\cosh(f*x + e)^9 - 540*\cosh(f*x + e)^7 + 658*\cosh(f*x + e)^5 - 364*\cosh(f*x + e)^3 + 47*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^3 + 10*(33*\cosh(f*x + e)^10 - 405*\cosh(f*x + e)^8 + 658*\cosh(f*x + e)^6 - 546*\cosh(f*x + e)^4 + 141*\cosh(f*x + e)^2 - 9)*e^{(f*x + e)}*\sinh(f*x + e)^2 + 4*(15*\cosh(f*x + e)^11 - 225*\cosh(f*x + e)^9 + 470*\cosh(f*x + e)^7 - 546*\cosh(f*x + e)^5 + 235*\cosh(f*x + e)^3 - 45*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e) + (5*\cosh(f*x + e)^12 - 90*\cosh(f*x + e)^10 + 235*\cosh(f*x + e)^8 - 364*\cosh(f*x + e)^6 + 235*\cosh(f*x + e)^4 - 90*\cosh(f*x + e)^2 + 5)*e^{(f*x + e)}*\text{sqrt}(a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a)*e^{(-f*x - e)} / (f*\cosh(f*x + e)^{11} + ($

```

f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^11 + 11*(f*cosh(f*x + e)*e^(2*f*x + 2*
e) + f*cosh(f*x + e))*sinh(f*x + e)^10 - 5*f*cosh(f*x + e)^9 + 5*(11*f*cosh
(f*x + e)^2 + (11*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)
^9 + 15*(11*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e) + (11*f*cosh(f*x + e)^3 -
3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*f*cosh(f*x + e)^7
+ 10*(33*f*cosh(f*x + e)^4 - 18*f*cosh(f*x + e)^2 + (33*f*cosh(f*x + e)^4
- 18*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 14*(33*f
*cosh(f*x + e)^5 - 30*f*cosh(f*x + e)^3 + 5*f*cosh(f*x + e) + (33*f*cosh(f*
x + e)^5 - 30*f*cosh(f*x + e)^3 + 5*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(
f*x + e)^6 - 10*f*cosh(f*x + e)^5 + 2*(231*f*cosh(f*x + e)^6 - 315*f*cosh(f
*x + e)^4 + 105*f*cosh(f*x + e)^2 + (231*f*cosh(f*x + e)^6 - 315*f*cosh(f*x
+ e)^4 + 105*f*cosh(f*x + e)^2 - 5*f)*e^(2*f*x + 2*e) - 5*f)*sinh(f*x + e)
^5 + 10*(33*f*cosh(f*x + e)^7 - 63*f*cosh(f*x + e)^5 + 35*f*cosh(f*x + e)^3
- 5*f*cosh(f*x + e) + (33*f*cosh(f*x + e)^7 - 63*f*cosh(f*x + e)^5 + 35*f*
cosh(f*x + e)^3 - 5*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 5*f
*cosh(f*x + e)^3 + 5*(33*f*cosh(f*x + e)^8 - 84*f*cosh(f*x + e)^6 + 70*f*co
sh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + (33*f*cosh(f*x + e)^8 - 84*f*cosh(f*
x + e)^6 + 70*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e)
+ f)*sinh(f*x + e)^3 + 5*(11*f*cosh(f*x + e)^9 - 36*f*cosh(f*x + e)^7 + 42
*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (11*f*cosh(
f*x + e)^9 - 36*f*cosh(f*x + e)^7 + 42*f*cosh(f*x + e)^5 - 20*f*cosh(f*x +
e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 - f*cosh(f*x + e
) + (f*cosh(f*x + e)^11 - 5*f*cosh(f*x + e)^9 + 10*f*cosh(f*x + e)^7 - 10*f
*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e) +
(11*f*cosh(f*x + e)^10 - 45*f*cosh(f*x + e)^8 + 70*f*cosh(f*x + e)^6 - 50*
f*cosh(f*x + e)^4 + 15*f*cosh(f*x + e)^2 + (11*f*cosh(f*x + e)^10 - 45*f*co
sh(f*x + e)^8 + 70*f*cosh(f*x + e)^6 - 50*f*cosh(f*x + e)^4 + 15*f*cosh(f*x
+ e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**6*(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.29864, size = 140, normalized size = 1.13

$$\frac{\sqrt{a} \left(\frac{4 \left(15 e^{(9fx+9e)} - 40 e^{(7fx+7e)} + 66 e^{(5fx+5e)} - 40 e^{(3fx+3e)} + 15 e^{(fx+e)} \right)}{\left(e^{(2fx+2e)} - 1 \right)^5} - 5 e^{(fx+e)} + 5 e^{(-fx-e)} \right)}{10f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")`

[Out] `-1/10*sqrt(a)*(4*(15*e^(9*f*x + 9*e) - 40*e^(7*f*x + 7*e) + 66*e^(5*f*x + 5*e) - 40*e^(3*f*x + 3*e) + 15*e^(f*x + e))/(e^(2*f*x + 2*e) - 1)^5 - 5*e^(f*x + e) + 5*e^(-f*x - e))/f`

$$3.437 \quad \int \frac{\tanh^5(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=66

$$-\frac{a^2}{5f(a \cosh^2(e+fx))^{5/2}} + \frac{2a}{3f(a \cosh^2(e+fx))^{3/2}} - \frac{1}{f\sqrt{a \cosh^2(e+fx)}}$$

[Out] $-a^2/(5*f*(a*Cosh[e + f*x]^2)^{(5/2)}) + (2*a)/(3*f*(a*Cosh[e + f*x]^2)^{(3/2)}) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])$

Rubi [A] time = 0.129055, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3205, 16, 43}

$$-\frac{a^2}{5f(a \cosh^2(e+fx))^{5/2}} + \frac{2a}{3f(a \cosh^2(e+fx))^{3/2}} - \frac{1}{f\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] `Int[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2], x]`

[Out] $-a^2/(5*f*(a*Cosh[e + f*x]^2)^{(5/2)}) + (2*a)/(3*f*(a*Cosh[e + f*x]^2)^{(3/2)}) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])$

Rule 3176

`Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^{(p_)}, x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]`

Rule 3205

`Int[((b_)*sin[(e_) + (f_)*(x_)]^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]`

Rule 16

$\text{Int}[(u_.)*(v_.)^{(m_.)}*((b_.)*(v_.)^{(n_.)}), x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\& \ \text{IntegerQ}[m]$

Rule 43

$\text{Int}[(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx &= \int \frac{\tanh^5(e+fx)}{\sqrt{a\cosh^2(e+fx)}} dx \\ &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{ax}} dx, x, \cosh^2(e+fx)\right)}{2f} \\ &= \frac{a^3 \text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{7/2}} dx, x, \cosh^2(e+fx)\right)}{2f} \\ &= \frac{a^3 \text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}} - \frac{2}{a(ax)^{5/2}} + \frac{1}{a^2(ax)^{3/2}}\right) dx, x, \cosh^2(e+fx)\right)}{2f} \\ &= -\frac{a^2}{5f(a\cosh^2(e+fx))^{5/2}} + \frac{2a}{3f(a\cosh^2(e+fx))^{3/2}} - \frac{1}{f\sqrt{a\cosh^2(e+fx)}} \end{aligned}$$

Mathematica [A] time = 0.0896333, size = 43, normalized size = 0.65

$$\frac{-3\text{sech}^4(e+fx) + 10\text{sech}^2(e+fx) - 15}{15f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] (-15 + 10*Sech[e + f*x]^2 - 3*Sech[e + f*x]^4)/(15*f*Sqrt[a*Cosh[e + f*x]^2])

Maple [C] time = 0.102, size = 41, normalized size = 0.6

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^5}{(\cosh(fx + e))^6} \frac{1}{\sqrt{a(\cosh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x)

[Out] `int/indef0` (sinh(f*x+e)^5/cosh(f*x+e)^6/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [B] time = 2.08391, size = 602, normalized size = 9.12

$$\frac{2e^{(-fx-e)}}{\left(5\sqrt{ae}^{(-2fx-2e)} + 10\sqrt{ae}^{(-4fx-4e)} + 10\sqrt{ae}^{(-6fx-6e)} + 5\sqrt{ae}^{(-8fx-8e)} + \sqrt{ae}^{(-10fx-10e)} + \sqrt{a}\right)f} - \frac{3\left(5\sqrt{ae}^{(-2fx-2e)} + \dots\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out]
$$\begin{aligned} & -2e^{(-fx-e)} / \left((5\sqrt{a}e^{(-2fx-2e)} + 10\sqrt{a}e^{(-4fx-4e)} + 10\sqrt{a}e^{(-6fx-6e)} + 5\sqrt{a}e^{(-8fx-8e)} + \sqrt{a}e^{(-10fx-10e)} + \sqrt{a})f \right) \\ & - \frac{8/3e^{(-3fx-3e)} / \left((5\sqrt{a}e^{(-2fx-2e)} + 10\sqrt{a}e^{(-4fx-4e)} + 10\sqrt{a}e^{(-6fx-6e)} + 5\sqrt{a}e^{(-8fx-8e)} + \sqrt{a}e^{(-10fx-10e)} + \sqrt{a})f \right) - 116/15e^{(-5fx-5e)} / \left((5\sqrt{a}e^{(-2fx-2e)} + 10\sqrt{a}e^{(-4fx-4e)} + 10\sqrt{a}e^{(-6fx-6e)} + 5\sqrt{a}e^{(-8fx-8e)} + \sqrt{a}e^{(-10fx-10e)} + \sqrt{a})f \right) - 8/3e^{(-7fx-7e)} / \left((5\sqrt{a}e^{(-2fx-2e)} + 10\sqrt{a}e^{(-4fx-4e)} + 10\sqrt{a}e^{(-6fx-6e)} + 5\sqrt{a}e^{(-8fx-8e)} + \sqrt{a}e^{(-10fx-10e)} + \sqrt{a})f \right) - 2e^{(-9fx-9e)} / \left((5\sqrt{a}e^{(-2fx-2e)} + 10\sqrt{a}e^{(-4fx-4e)} + 10\sqrt{a}e^{(-6fx-6e)} + 5\sqrt{a}e^{(-8fx-8e)} + \sqrt{a}e^{(-10fx-10e)} + \sqrt{a})f \right) \end{aligned}$$

Fricas [B] time = 1.89517, size = 3776, normalized size = 57.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out]
$$-2/15*(135*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e)^8 + 15*e^{(f*x + e)}*\sinh(f*x + e)^9 + 20*(27*\cosh(f*x + e)^2 + 1)*e^{(f*x + e)}*\sinh(f*x + e)^7 + 140*(9*\cosh(f*x + e)^3 + \cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^6 + 2*(945*\cosh(f*x + e)^4 + 210*\cosh(f*x + e)^2 + 29)*e^{(f*x + e)}*\sinh(f*x + e)^5 + 10*(189*\cosh(f*x + e)^5 + 70*\cosh(f*x + e)^3 + 29*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^4 + 20*(63*\cosh(f*x + e)^6 + 35*\cosh(f*x + e)^4 + 29*\cosh(f*x + e)^2 + 1)*e^{(f*x + e)}*\sinh(f*x + e)^3 + 20*(27*\cosh(f*x + e)^7 + 21*\cosh(f*x + e)^5 + 29*\cosh(f*x + e)^3 + 3*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^2 + 5*(27*\cosh(f*x + e)^8 + 28*\cosh(f*x + e)^6 + 58*\cosh(f*x + e)^4 + 12*\cosh(f*x + e)^2 + 3)*e^{(f*x + e)}*\sinh(f*x + e) + (15*\cosh(f*x + e)^9 + 20*\cosh(f*x + e)^7 + 58*\cosh(f*x + e)^5 + 20*\cosh(f*x + e)^3 + 15*\cosh(f*x + e))*e^{(f*x + e)}*\sqrt{a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a}*e^{(-f*x - e)}/(a*f*\cosh(f*x + e)^{10} + (a*f*e^{(2*f*x + 2*e)} + a*f)*\sinh(f*x + e)^{10} + 5*a*f*\cosh(f*x + e)^8 + 10*(a*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)} + a*f*\cosh(f*x + e)*\sinh(f*x + e)^9 + 5*(9*a*f*\cosh(f*x + e)^2 + a*f + (9*a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^8 + 10*a*f*\cosh(f*x + e)^6 + 40*(3*a*f*\cosh(f*x + e)^3 + a*f*\cosh(f*x + e) + (3*a*f*\cosh(f*x + e)^3 + a*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^7 + 10*(21*a*f*\cosh(f*x + e)^4 + 14*a*f*\cosh(f*x + e)^2 + a*f + (21*a*f*\cosh(f*x + e)^4 + 14*a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^6 + 10*a*f*\cosh(f*x + e)^4 + 4*(63*a*f*\cosh(f*x + e)^5 + 70*a*f*\cosh(f*x + e)^3 + 15*a*f*\cosh(f*x + e) + (63*a*f*\cosh(f*x + e)^5 + 70*a*f*\cosh(f*x + e)^3 + 15*a*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^5 + 10*(21*a*f*\cosh(f*x + e)^6 + 35*a*f*\cosh(f*x + e)^4 + 15*a*f*\cosh(f*x + e)^2 + a*f + (21*a*f*\cosh(f*x + e)^6 + 35*a*f*\cosh(f*x + e)^4 + 15*a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^4 + 5*a*f*\cosh(f*x + e)^2 + 40*(3*a*f*\cosh(f*x + e)^7 + 7*a*f*\cosh(f*x + e)^5 + 5*a*f*\cosh(f*x + e)^3 + a*f*\cosh(f*x + e) + (3*a*f*\cosh(f*x + e)^7 + 7*a*f*\cosh(f*x + e)^5 + 5*a*f*\cosh(f*x + e)^3 + a*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^3 + 5*(9*a*f*\cosh(f*x + e)^8 + 28*a*f*\cosh(f*x + e)^6 + 30*a*f*\cosh(f*x + e)^4 + 12*a*f*\cosh(f*x + e)^2 + a*f + (9*a*f*\cosh(f*x + e)^8 + 28*a*f*\cosh(f*x + e)^6 + 30*a*f*\cosh(f*x + e)^4 + 12*a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^2 + a*f + (a*f*\cosh(f*x + e)^{10} + 5*a*f*\cosh(f*x + e)^8 + 10*a*f*\cosh(f*x + e)^6 + 10*a*f*\cosh(f*x + e)^4 + 5*a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)} + 10*(a*f*\cosh(f*x + e)^9 + 4*a*f*\cosh(f*x + e)^7 + 6*a*f*\cosh(f*x + e)^5 + 4*a*f*\cosh(f*x + e)^3 + a*f*\cosh(f*x + e) + (a*f*\cosh(f*x + e)^9 + 4*a*f*\cosh(f*x + e)^7 + 6*a$$

```
*f*cosh(f*x + e)^5 + 4*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(e + fx)}{\sqrt{a(\sinh^2(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.39911, size = 128, normalized size = 1.94

$$\frac{2 \left(15 \sqrt{ae}^{(9fx+9e)} + 20 \sqrt{ae}^{(7fx+7e)} + 58 \sqrt{ae}^{(5fx+5e)} + 20 \sqrt{ae}^{(3fx+3e)} + 15 \sqrt{ae}^{(fx+e)} \right)}{15af \left(e^{(2fx+2e)} + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(a)*e^(9*f*x + 9*e) + 20*sqrt(a)*e^(7*f*x + 7*e) + 58*sqrt(a)*e^(5*f*x + 5*e) + 20*sqrt(a)*e^(3*f*x + 3*e) + 15*sqrt(a)*e^(f*x + e))/(a*f*(e^(2*f*x + 2*e) + 1)^5)
```

$$3.438 \quad \int \frac{\tanh^3(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=42

$$\frac{a}{3f(a \cosh^2(e+fx))^{3/2}} - \frac{1}{f\sqrt{a \cosh^2(e+fx)}}$$

[Out] a/(3*f*(a*Cosh[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.117641, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{3f(a \cosh^2(e+fx))^{3/2}} - \frac{1}{f\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]

[Out] a/(3*f*(a*Cosh[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_)^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx &= \int \frac{\tanh^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}} dx \\
&= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{ax}} dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= -\frac{a^2 \text{Subst}\left(\int \frac{1-x}{(ax)^{5/2}} dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= -\frac{a^2 \text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}} - \frac{1}{a(ax)^{3/2}}\right) dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= \frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}} - \frac{1}{f\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0701354, size = 31, normalized size = 0.74

$$\frac{\text{sech}^2(e+fx) - 3}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2], x]
```

```
[Out] (-3 + Sech[e + f*x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [C] time = 0.098, size = 41, normalized size = 1.

$$\frac{1}{f} \int \frac{\left(\frac{(\sinh(fx+e))^3}{(\cosh(fx+e))^4} \frac{1}{\sqrt{a}(\cosh(fx+e))^2}, \sinh(fx+e) \right)}{\sinh(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x)

[Out] `int/indef0` (sinh(f*x+e)^3/cosh(f*x+e)^4/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [B] time = 1.89918, size = 248, normalized size = 5.9

$$\frac{2e^{(-fx-e)}}{\left(3\sqrt{ae^{(-2fx-2e)}} + 3\sqrt{ae^{(-4fx-4e)}} + \sqrt{ae^{(-6fx-6e)}} + \sqrt{a}\right)f} - \frac{4e^{(-3fx-3e)}}{3\left(3\sqrt{ae^{(-2fx-2e)}} + 3\sqrt{ae^{(-4fx-4e)}} + \sqrt{ae^{(-6fx-6e)}} + \sqrt{a}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] $-2e^{(-fx-e)}/((3\sqrt{a})e^{(-2fx-2e)} + 3\sqrt{a})e^{(-4fx-4e)} + \sqrt{a})e^{(-6fx-6e)} + \sqrt{a})f - 4/3e^{(-3fx-3e)}/((3\sqrt{a})e^{(-2fx-2e)} + 3\sqrt{a})e^{(-4fx-4e)} + \sqrt{a})e^{(-6fx-6e)} + \sqrt{a})f - 2e^{(-5fx-5e)}/((3\sqrt{a})e^{(-2fx-2e)} + 3\sqrt{a})e^{(-4fx-4e)} + \sqrt{a})e^{(-6fx-6e)} + \sqrt{a})f$

Fricas [B] time = 1.76068, size = 1692, normalized size = 40.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] $-2/3*(15*\cosh(f*x+e)*e^{(f*x+e)}*\sinh(f*x+e)^4 + 3*e^{(f*x+e)}*\sinh(f*x+e)^5 + 2*(15*\cosh(f*x+e)^2 + 1)*e^{(f*x+e)}*\sinh(f*x+e)^3 + 6*(5*\cos$

$$\begin{aligned}
& h(f*x + e)^3 + \cosh(f*x + e)) * e^{(f*x + e)} * \sinh(f*x + e)^2 + 3 * (5 * \cosh(f*x + e) \\
& e)^4 + 2 * \cosh(f*x + e)^2 + 1) * e^{(f*x + e)} * \sinh(f*x + e) + (3 * \cosh(f*x + e) \\
& ^5 + 2 * \cosh(f*x + e)^3 + 3 * \cosh(f*x + e)) * e^{(f*x + e)} * \sqrt{a * e^{(4 * f*x + 4 * e)} + 2 * a * e^{(2 * f*x + 2 * e)} + a} * e^{(-f*x - e)} / (a * f * \cosh(f*x + e)^6 + (a * f * e^{(2 * f*x + 2 * e)} + a * f) * \sinh(f*x + e)^6 + 3 * a * f * \cosh(f*x + e)^4 + 6 * (a * f * \cosh(f*x + e) * e^{(2 * f*x + 2 * e)} + a * f * \cosh(f*x + e)) * \sinh(f*x + e)^5 + 3 * (5 * a * f * \cosh(f*x + e)^2 + a * f + (5 * a * f * \cosh(f*x + e)^2 + a * f) * e^{(2 * f*x + 2 * e)}) * \sinh(f*x + e)^4 + 3 * a * f * \cosh(f*x + e)^2 + 4 * (5 * a * f * \cosh(f*x + e)^3 + 3 * a * f * \cosh(f*x + e) + (5 * a * f * \cosh(f*x + e)^3 + 3 * a * f * \cosh(f*x + e)) * e^{(2 * f*x + 2 * e)}) * \sinh(f*x + e)^3 + 3 * (5 * a * f * \cosh(f*x + e)^4 + 6 * a * f * \cosh(f*x + e)^2 + a * f + (5 * a * f * \cosh(f*x + e)^4 + 6 * a * f * \cosh(f*x + e)^2 + a * f) * e^{(2 * f*x + 2 * e)}) * \sinh(f*x + e)^2 + a * f + (a * f * \cosh(f*x + e)^6 + 3 * a * f * \cosh(f*x + e)^4 + 3 * a * f * \cosh(f*x + e)^2 + a * f) * e^{(2 * f*x + 2 * e)} + 6 * (a * f * \cosh(f*x + e)^5 + 2 * a * f * \cosh(f*x + e)^3 + a * f * \cosh(f*x + e) + (a * f * \cosh(f*x + e)^5 + 2 * a * f * \cosh(f*x + e)^3 + a * f * \cosh(f*x + e)) * e^{(2 * f*x + 2 * e)}) * \sinh(f*x + e))
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(e + fx)}{\sqrt{a(\sinh^2(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(tanh(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)

Giac [A] time = 1.34448, size = 88, normalized size = 2.1

$$\frac{2 \left(3 \sqrt{a} e^{(5fx+5e)} + 2 \sqrt{a} e^{(3fx+3e)} + 3 \sqrt{a} e^{(fx+e)} \right)}{3 a f \left(e^{(2fx+2e)} + 1 \right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -2/3*(3*sqrt(a)*e^(5*f*x + 5*e) + 2*sqrt(a)*e^(3*f*x + 3*e) + 3*sqrt(a)*e^(f*x + e))/(a*f*(e^(2*f*x + 2*e) + 1)^3)

$$3.439 \quad \int \frac{\tanh(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=19

$$-\frac{1}{f\sqrt{a \cosh^2(e+fx)}}$$

[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))

Rubi [A] time = 0.0696189, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3176, 3205, 16, 32}

$$-\frac{1}{f\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_)*sin[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\tanh(e + fx)}{\sqrt{a + a \sinh^2(e + fx)}} dx &= \int \frac{\tanh(e + fx)}{\sqrt{a \cosh^2(e + fx)}} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{ax}} dx, x, \cosh^2(e + fx)\right)}{2f} \\ &= \frac{a \text{Subst}\left(\int \frac{1}{(ax)^{3/2}} dx, x, \cosh^2(e + fx)\right)}{2f} \\ &= -\frac{1}{f\sqrt{a \cosh^2(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.0317421, size = 19, normalized size = 1.

$$-\frac{1}{f\sqrt{a \cosh^2(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2], x]
```

```
[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Maple [A] time = 0.024, size = 20, normalized size = 1.1

$$-\frac{1}{f\sqrt{a + a(\sinh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x)`

[Out] $-1/f/(a+a*\sinh(f*x+e)^2)^{(1/2)}$

Maxima [A] time = 1.77428, size = 45, normalized size = 2.37

$$\frac{2e^{(-fx-e)}}{\left(\sqrt{ae^{(-2fx-2e)}} + \sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] $-2*e^{(-f*x - e)/((\text{sqrt}(a)*e^{(-2*f*x - 2*e)} + \text{sqrt}(a))*f)}$

Fricas [B] time = 1.75363, size = 428, normalized size = 22.53

$$\frac{2\sqrt{ae^{(4fx+4e)} + 2ae^{(2fx+2e)} + a}\left(\cosh(fx+e)e^{(fx+e)} + e^{(fx+e)}\sinh(fx+e)\right)e^{(-fx-e)}}{af\cosh(fx+e)^2 + \left(afe^{(2fx+2e)} + af\right)\sinh(fx+e)^2 + af + \left(af\cosh(fx+e)^2 + af\right)e^{(2fx+2e)} + 2\left(af\cosh(fx+e)e^{(2fx+2e)} + a*fe^{(2fx+2e)} + a*f\cosh(fx+e)\right)*\sinh(fx+e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] $-2*\text{sqrt}(a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a)*(\cosh(f*x + e)*e^{(f*x + e)} + e^{(f*x + e)}*\sinh(f*x + e))*e^{(-f*x - e)}/(a*f*\cosh(f*x + e)^2 + (a*f*e^{(2*f*x + 2*e)} + a*f)*\sinh(f*x + e)^2 + a*f + (a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)} + 2*(a*f*\cosh(f*x + e)*e^{(2*f*x + 2*e)} + a*f*\cosh(f*x + e))*\sinh(f*x + e)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(e + fx)}{\sqrt{a(\sinh^2(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(tanh(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)

Giac [A] time = 1.25999, size = 39, normalized size = 2.05

$$-\frac{2e^{(fx+e)}}{\sqrt{a}f\left(e^{(2fx+2e)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -2*e^(f*x + e)/(sqrt(a)*f*(e^(2*f*x + 2*e) + 1))

$$3.440 \quad \int \frac{\coth(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a*\text{Cosh}[e + f*x]^2]/\text{Sqrt}[a]]/(\text{Sqrt}[a]*f))$

Rubi [A] time = 0.0856261, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3176, 3205, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[e + f*x]/\text{Sqrt}[a + a*\text{Sinh}[e + f*x]^2], x]$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a*\text{Cosh}[e + f*x]^2]/\text{Sqrt}[a]]/(\text{Sqrt}[a]*f))$

Rule 3176

$\text{Int}[(u_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ActivateTrig}[u*(a*\cos[e + f*x]^2)^p], x] /;$ FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)}]^{(p_.)}*\tan[(e_.) + (f_.)*(x_.)]^{(m_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f*x]^2, x]\}, \text{Dist}[ff^{(m+1)/2}/(2*f), \text{Subst}[\text{Int}[(x^{(m-1)/2}*(b*ff^{(n/2)}*x^{(n/2)})^p]/(1 - ff*x)^{(m+1)/2}, x], x, \text{Sin}[e + f*x]^2/ff], x] /;$ FreeQ[{b, e, f, p}, x] && IntegerQ[(m-1)/2] && IntegerQ[n/2]

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/
Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx &= \int \frac{\coth(e+fx)}{\sqrt{a\cosh^2(e+fx)}} dx \\ &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}} dx, x, \cosh^2(e+fx)\right)}{2f} \\ &= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{af} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} \end{aligned}$$

Mathematica [A] time = 0.053196, size = 49, normalized size = 1.58

$$\frac{\cosh(e+fx) \left(\log\left(\sinh\left(\frac{1}{2}(e+fx)\right)\right) - \log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right) \right)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2], x]
```

```
[Out] (Cosh[e + f*x]*(-Log[Cosh[(e + f*x)/2]] + Log[Sinh[(e + f*x)/2]]))/(f*Sqrt[
a*Cosh[e + f*x]^2])
```

Maple [C] time = 0.07, size = 33, normalized size = 1.1

$$\frac{1}{f} \int \frac{1}{\sinh(fx+e) \sqrt{a(\cosh(fx+e))^2}} \sinh(fx+e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x)`

[Out] `\int/indf0\ (1/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [A] time = 1.86608, size = 54, normalized size = 1.74

$$-\frac{\log(e^{-fx-e} + 1)}{\sqrt{af}} + \frac{\log(e^{-fx-e} - 1)}{\sqrt{af}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `-log(e^(-f*x - e) + 1)/(sqrt(a)*f) + log(e^(-f*x - e) - 1)/(sqrt(a)*f)`

Fricas [B] time = 1.84154, size = 455, normalized size = 14.68

$$\left[\frac{\sqrt{ae^{4fx+4e} + 2ae^{2fx+2e} + a} \log\left(\frac{\cosh(fx+e)+\sinh(fx+e)-1}{\cosh(fx+e)+\sinh(fx+e)+1}\right)}{afe^{2fx+2e} + af}, 2\sqrt{-a} \arctan\left(\frac{\sqrt{ae^{4fx+4e} + 2ae^{2fx+2e} + a}\sqrt{-a}}{a \cosh(fx+e)e^{2fx+2e} + a \cosh(fx+e) + (ae^{2fx+2e} + a)}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `[sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*log((cosh(f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1))/(a*f*e^(2*f*x + 2*e) +`

```
a*f), 2*sqrt(-a)*arctan(sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*
sqrt(-a)/(a*cosh(f*x + e)*e^(2*f*x + 2*e) + a*cosh(f*x + e) + (a*e^(2*f*x +
2*e) + a)*sinh(f*x + e)))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(e + fx)}{\sqrt{a(\sinh^2(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.32173, size = 49, normalized size = 1.58

$$-\frac{\frac{\log(e^{(fx+e)}+1)}{\sqrt{a}} - \frac{\log\left(\left|e^{(fx+e)}-1\right|\right)}{\sqrt{a}}}{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(e^(f*x + e) + 1)/sqrt(a) - log(abs(e^(f*x + e) - 1))/sqrt(a))/f
```


$$3.441 \quad \int \frac{\coth^3(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=66

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{af}} - \frac{\operatorname{csch}^2(e+fx)\sqrt{a \cosh^2(e+fx)}}{2af}$$

[Out] -ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*Sqrt[a]*f) - (Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]^2)/(2*a*f)

Rubi [A] time = 0.129395, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3176, 3205, 16, 47, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{af}} - \frac{\operatorname{csch}^2(e+fx)\sqrt{a \cosh^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] -ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*Sqrt[a]*f) - (Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]^2)/(2*a*f)

Rule 3176

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{b, e, f, p}, x] && Integ

erQ[(m - 1)/2] && IntegerQ[n/2]

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx &= \int \frac{\coth^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}} dx \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{ax}} dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{(1-x)^2} dx, x, \cosh^2(e+fx)\right)}{2af} \\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\text{csch}^2(e+fx)}{2af} - \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}} dx, x, \cosh^2(e+fx)\right)}{4f} \\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\text{csch}^2(e+fx)}{2af} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{2af} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a\cosh^2(e+fx)}\text{csch}^2(e+fx)}{2af}
\end{aligned}$$

Mathematica [A] time = 0.122417, size = 65, normalized size = 0.98

$$\frac{\cosh(e+fx)\left(\text{csch}^2\left(\frac{1}{2}(e+fx)\right) + \text{sech}^2\left(\frac{1}{2}(e+fx)\right) - 4\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] -(Cosh[e + f*x]*(Csch[(e + f*x)/2]^2 - 4*Log[Tanh[(e + f*x)/2]] + Sech[(e + f*x)/2]^2))/(8*f*Sqrt[a*Cosh[e + f*x]^2])

Maple [C] time = 0.086, size = 42, normalized size = 0.6

$$\frac{1}{f} \text{ int/indef0 } \left(\left((\sinh(fx+e))^{-1} + (\sinh(fx+e))^{-3} \right) \frac{1}{\sqrt{a(\cosh(fx+e))^2}}, \sinh(fx+e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x)`

[Out] ``int/indef0`(((1/sinh(f*x+e)+1/sinh(f*x+e)^3)/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [A] time = 1.90444, size = 135, normalized size = 2.05

$$-\frac{\log\left(e^{(-fx-e)} + 1\right)}{2\sqrt{a}f} + \frac{\log\left(e^{(-fx-e)} - 1\right)}{2\sqrt{a}f} + \frac{e^{(-fx-e)} + e^{(-3fx-3e)}}{\left(2\sqrt{a}e^{(-2fx-2e)} - \sqrt{a}e^{(-4fx-4e)} - \sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `-1/2*log(e^(-f*x - e) + 1)/(sqrt(a)*f) + 1/2*log(e^(-f*x - e) - 1)/(sqrt(a)*f) + (e^(-f*x - e) + e^(-3*f*x - 3*e))/(2*sqrt(a)*e^(-2*f*x - 2*e) - sqrt(a)*e^(-4*f*x - 4*e) - sqrt(a))*f`

Fricas [B] time = 1.94943, size = 1409, normalized size = 21.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `-1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x + e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^4 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 + 4*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^3 +`

$2*(3*a*f*\cosh(f*x + e)^2 - a*f + (3*a*f*\cosh(f*x + e)^2 - a*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^2 + a*f + (a*f*\cosh(f*x + e)^4 - 2*a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)} + 4*(a*f*\cosh(f*x + e)^3 - a*f*\cosh(f*x + e) + (a*f*\cosh(f*x + e)^3 - a*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)})*\sinh(f*x + e)$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(e + fx)}{\sqrt{a(\sinh^2(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2), x)

[Out] Integral(coth(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)

Giac [A] time = 1.36714, size = 109, normalized size = 1.65

$$\frac{\frac{\log(e^{(fx+e)}+1)}{\sqrt{a}} - \frac{\log\left(\left|e^{(fx+e)}-1\right|\right)}{\sqrt{a}} + \frac{2\left(\sqrt{ae}^{(3fx+3e)}+\sqrt{ae}^{(fx+e)}\right)}{a\left(e^{(2fx+2e)}-1\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2), x, algorithm="giac")

[Out] $-1/2*(\log(e^{(f*x + e)} + 1)/\sqrt{a} - \log(\text{abs}(e^{(f*x + e)} - 1))/\sqrt{a}) + 2*(\sqrt{a}*e^{(3*f*x + 3*e)} + \sqrt{a}*e^{(f*x + e)})/(a*(e^{(2*f*x + 2*e)} - 1)^2)/f$

$$3.442 \quad \int \frac{\tanh^4(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=91

$$-\frac{\tanh^3(e+fx)}{4f\sqrt{a \cosh^2(e+fx)}} - \frac{3 \tanh(e+fx)}{8f\sqrt{a \cosh^2(e+fx)}} + \frac{3 \cosh(e+fx) \tan^{-1}(\sinh(e+fx))}{8f\sqrt{a \cosh^2(e+fx)}}$$

[Out] (3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - (3*Tanh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[e + f*x]^3/(4*f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.144237, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2611, 3770}

$$-\frac{\tanh^3(e+fx)}{4f\sqrt{a \cosh^2(e+fx)}} - \frac{3 \tanh(e+fx)}{8f\sqrt{a \cosh^2(e+fx)}} + \frac{3 \cosh(e+fx) \tan^{-1}(\sinh(e+fx))}{8f\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] (3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - (3*Tanh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[e + f*x]^3/(4*f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^p], x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_)*((b_)*sin[(e_) + (f_)*(x_)]^n)^p], x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_)*(trig_)[e + f*x])^(m_) /;]

FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]]

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^4(e + fx)}{\sqrt{a + a \sinh^2(e + fx)}} dx &= \int \frac{\tanh^4(e + fx)}{\sqrt{a \cosh^2(e + fx)}} dx \\
 &= \frac{\cosh(e + fx) \int \operatorname{sech}(e + fx) \tanh^4(e + fx) dx}{\sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{\tanh^3(e + fx)}{4f\sqrt{a \cosh^2(e + fx)}} + \frac{(3 \cosh(e + fx)) \int \operatorname{sech}(e + fx) \tanh^2(e + fx) dx}{4\sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{3 \tanh(e + fx)}{8f\sqrt{a \cosh^2(e + fx)}} - \frac{\tanh^3(e + fx)}{4f\sqrt{a \cosh^2(e + fx)}} + \frac{(3 \cosh(e + fx)) \int \operatorname{sech}(e + fx) dx}{8\sqrt{a \cosh^2(e + fx)}} \\
 &= \frac{3 \tan^{-1}(\sinh(e + fx)) \cosh(e + fx)}{8f\sqrt{a \cosh^2(e + fx)}} - \frac{3 \tanh(e + fx)}{8f\sqrt{a \cosh^2(e + fx)}} - \frac{\tanh^3(e + fx)}{4f\sqrt{a \cosh^2(e + fx)}}
 \end{aligned}$$

Mathematica [A] time = 0.132891, size = 66, normalized size = 0.73

$$\frac{\tanh(e + fx) (-8 \tanh^2(e + fx) - 6 \operatorname{sech}^2(e + fx) + 3) + 3 \cosh(e + fx) \tan^{-1}(\sinh(e + fx))}{8f\sqrt{a \cosh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] $(3 \operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]] * \operatorname{Cosh}[e + f*x] + \operatorname{Tanh}[e + f*x] * (3 - 6 * \operatorname{Sech}[e + f*x]^2 - 8 * \operatorname{Tanh}[e + f*x]^2)) / (8 * f * \operatorname{Sqrt}[a * \operatorname{Cosh}[e + f*x]^2])$

Maple [A] time = 0.108, size = 68, normalized size = 0.8

$$\frac{3 \arctan(\sinh(fx + e)) (\cosh(fx + e))^4 - 5 (\cosh(fx + e))^2 \sinh(fx + e) + 2 \sinh(fx + e)}{8 (\cosh(fx + e))^3 f} \frac{1}{\sqrt{a (\cosh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x)`

[Out] $1/8 * (3 * \arctan(\sinh(f*x+e)) * \cosh(f*x+e)^4 - 5 * \cosh(f*x+e)^2 * \sinh(f*x+e) + 2 * \sinh(f*x+e)) / \cosh(f*x+e)^3 / (a * \cosh(f*x+e)^2)^{(1/2)} / f$

Maxima [B] time = 1.95509, size = 848, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] $1/48 * (15 * \arctan(e^{-f*x - e}) / \operatorname{sqrt}(a) - (15 * e^{-f*x - e} + 55 * e^{-3*f*x - 3*e} + 73 * e^{-5*f*x - 5*e} - 15 * e^{-7*f*x - 7*e})) / (4 * \operatorname{sqrt}(a) * e^{-2*f*x - 2*e} + 6 * \operatorname{sqrt}(a) * e^{-4*f*x - 4*e} + 4 * \operatorname{sqrt}(a) * e^{-6*f*x - 6*e} + \operatorname{sqrt}(a) * e^{-8*f*x - 8*e} + \operatorname{sqrt}(a))) / f + 1/48 * (15 * \arctan(e^{-f*x - e}) / \operatorname{sqrt}(a) - (15 * e^{-f*x - e} - 73 * e^{-3*f*x - 3*e} - 55 * e^{-5*f*x - 5*e} - 15 * e^{-7*f*x - 7*e})) / (4 * \operatorname{sqrt}(a) * e^{-2*f*x - 2*e} + 6 * \operatorname{sqrt}(a) * e^{-4*f*x - 4*e} + 4 * \operatorname{sqrt}(a) * e^{-6*f*x - 6*e} + \operatorname{sqrt}(a) * e^{-8*f*x - 8*e} + \operatorname{sqrt}(a))) / f - 3/32 * (3 * \arctan(e^{-f*x - e}) / \operatorname{sqrt}(a) - (3 * e^{-f*x - e} + 11 * e^{-3*f*x - 3*e} - 11 * e^{-5*f*x - 5*e} - 3 * e^{-7*f*x - 7*e})) / (4 * \operatorname{sqrt}(a) * e^{-2*f*x - 2*e} + 6 * \operatorname{sqrt}(a) * e^{-4*f*x - 4*e} + 4 * \operatorname{sqrt}(a) * e^{-6*f*x - 6*e} + \operatorname{sqrt}(a) * e^{-8*f*x - 8*e} + \operatorname{sqrt}(a))) / f - 35/32 * \arctan(e^{-f*x - e}) / (\operatorname{sqrt}(a) * f) - 1/192 * (279 * e^{-f*x - e} + 511 * e^{-3*f*x - 3*e} + 385 * e^{-5*f*x - 5*e} + 105 * e^{-7*f*x - 7*e}) / ((4 * \operatorname{sqrt}(a) * e^{-2*f*x - 2*e} + 6 * \operatorname{sqrt}(a) * e^{-4*f*x - 4*e} + 4 * \operatorname{sqrt}(a) * e^{-6*f*x - 6*e} + \operatorname{sqrt}(a) * e^{-8*f*x - 8*e} + \operatorname{sqrt}(a)) * f) + 1/192 * (105 * e^{-f*x - e} + 385 * e^{-3*f*x - 3*e} + 511 * e^{-5*f*x - 5*e} + 279 * e^{-7*f*x - 7*e}) / ((4 * \operatorname{sqrt}(a) * e^{-2*f*x - 2*e} + 6 * \operatorname{sqrt}(a) * e^{-4*f*x - 4*e} + 4 * \operatorname{sqrt}(a) * e^{-6*f*x - 6*e} + \operatorname{sqrt}(a) * e^{-8*f*x - 8*e} + \operatorname{sqrt}(a)) * f)$

) $e^{(-2*f*x - 2*e)}$ + 6*sqrt(a) $e^{(-4*f*x - 4*e)}$ + 4*sqrt(a) $e^{(-6*f*x - 6*e)}$
) + sqrt(a) $e^{(-8*f*x - 8*e)}$ + sqrt(a))*f)

Fricas [B] time = 2.04753, size = 3584, normalized size = 39.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/4*(35*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e)^6 + 5*e^{(f*x + e)}*\sinh(f*x \\ & + e)^7 + 3*(35*\cosh(f*x + e)^2 - 1)*e^{(f*x + e)}*\sinh(f*x + e)^5 + 5*(35*\cosh \\ & \cosh(f*x + e)^3 - 3*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^4 + (175*\cosh(f* \\ & x + e)^4 - 30*\cosh(f*x + e)^2 + 3)*e^{(f*x + e)}*\sinh(f*x + e)^3 + 3*(35*\cosh \\ & (f*x + e)^5 - 10*\cosh(f*x + e)^3 + 3*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + \\ & e)^2 + (35*\cosh(f*x + e)^6 - 15*\cosh(f*x + e)^4 + 9*\cosh(f*x + e)^2 - 5)*e^{ \\ & (f*x + e)}*\sinh(f*x + e) - 3*(8*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e)^7 + \\ & e^{(f*x + e)}*\sinh(f*x + e)^8 + 4*(7*\cosh(f*x + e)^2 + 1)*e^{(f*x + e)}*\sinh(f* \\ & x + e)^6 + 8*(7*\cosh(f*x + e)^3 + 3*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e \\ &)^5 + 2*(35*\cosh(f*x + e)^4 + 30*\cosh(f*x + e)^2 + 3)*e^{(f*x + e)}*\sinh(f*x \\ & + e)^4 + 8*(7*\cosh(f*x + e)^5 + 10*\cosh(f*x + e)^3 + 3*\cosh(f*x + e))*e^{(f* \\ & x + e)}*\sinh(f*x + e)^3 + 4*(7*\cosh(f*x + e)^6 + 15*\cosh(f*x + e)^4 + 9*\cosh \\ & (f*x + e)^2 + 1)*e^{(f*x + e)}*\sinh(f*x + e)^2 + 8*(\cosh(f*x + e)^7 + 3*\cosh(\\ & f*x + e)^5 + 3*\cosh(f*x + e)^3 + \cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e) + \\ & (\cosh(f*x + e)^8 + 4*\cosh(f*x + e)^6 + 6*\cosh(f*x + e)^4 + 4*\cosh(f*x + e) \\ & ^2 + 1)*e^{(f*x + e)}*\arctan(\cosh(f*x + e) + \sinh(f*x + e)) + (5*\cosh(f*x + \\ & e)^7 - 3*\cosh(f*x + e)^5 + 3*\cosh(f*x + e)^3 - 5*\cosh(f*x + e))*e^{(f*x + e)} \\ &)*\sqrt{a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a}*e^{(-f*x - e)}/(a*f*\cosh(\\ & f*x + e)^8 + (a*f*e^{(2*f*x + 2*e)} + a*f)*\sinh(f*x + e)^8 + 4*a*f*\cosh(f*x + \\ & e)^6 + 8*(a*f*\cosh(f*x + e)*e^{(2*f*x + 2*e)} + a*f*\cosh(f*x + e))*\sinh(f*x \\ & + e)^7 + 4*(7*a*f*\cosh(f*x + e)^2 + a*f + (7*a*f*\cosh(f*x + e)^2 + a*f)*e^{(\\ & 2*f*x + 2*e)})*\sinh(f*x + e)^6 + 6*a*f*\cosh(f*x + e)^4 + 8*(7*a*f*\cosh(f*x + \\ & e)^3 + 3*a*f*\cosh(f*x + e) + (7*a*f*\cosh(f*x + e)^3 + 3*a*f*\cosh(f*x + e)) \\ & *e^{(2*f*x + 2*e)})*\sinh(f*x + e)^5 + 2*(35*a*f*\cosh(f*x + e)^4 + 30*a*f*\cosh \\ & (f*x + e)^2 + 3*a*f + (35*a*f*\cosh(f*x + e)^4 + 30*a*f*\cosh(f*x + e)^2 + 3* \\ & a*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^4 + 4*a*f*\cosh(f*x + e)^2 + 8*(7*a*f*\cosh \\ & \cosh(f*x + e)^5 + 10*a*f*\cosh(f*x + e)^3 + 3*a*f*\cosh(f*x + e) + (7*a*f*\cosh(\\ & f*x + e)^5 + 10*a*f*\cosh(f*x + e)^3 + 3*a*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)} \\ &)*\sinh(f*x + e)^3 + 4*(7*a*f*\cosh(f*x + e)^6 + 15*a*f*\cosh(f*x + e)^4 + 9*a* \\ & f*\cosh(f*x + e)^2 + a*f + (7*a*f*\cosh(f*x + e)^6 + 15*a*f*\cosh(f*x + e)^4 + \\ & 9*a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^2 + a*f + (a*f \end{aligned}$$

```
*cosh(f*x + e)^8 + 4*a*f*cosh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 4*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 8*(a*f*cosh(f*x + e)^7 + 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^7 + 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(e + fx)}{\sqrt{a(\sinh^2(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**4/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.37038, size = 130, normalized size = 1.43

$$\frac{\frac{3 \arctan\left(e^{(fx+e)}\right)}{\sqrt{a}} - \frac{5\sqrt{ae}^{(7fx+7e)} - 3\sqrt{ae}^{(5fx+5e)} + 3\sqrt{ae}^{(3fx+3e)} - 5\sqrt{ae}^{(fx+e)}}{a\left(e^{(2fx+2e)} + 1\right)^4}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*arctan(e^(f*x + e))/sqrt(a) - (5*sqrt(a)*e^(7*f*x + 7*e) - 3*sqrt(a)*e^(5*f*x + 5*e) + 3*sqrt(a)*e^(3*f*x + 3*e) - 5*sqrt(a)*e^(f*x + e))/(a*(e^(2*f*x + 2*e) + 1)^4))/f
```

$$3.443 \quad \int \frac{\tanh^2(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=62

$$\frac{\cosh(e+fx) \tan^{-1}(\sinh(e+fx))}{2f\sqrt{a \cosh^2(e+fx)}} - \frac{\tanh(e+fx)}{2f\sqrt{a \cosh^2(e+fx)}}$$

[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(2*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[e + f*x]/(2*f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.120652, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2611, 3770}

$$\frac{\cosh(e+fx) \tan^{-1}(\sinh(e+fx))}{2f\sqrt{a \cosh^2(e+fx)}} - \frac{\tanh(e+fx)}{2f\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]

[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(2*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[e + f*x]/(2*f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p], x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^n)^p], x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x]^m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx &= \int \frac{\tanh^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}} dx \\
&= \frac{\cosh(e+fx) \int \operatorname{sech}(e+fx) \tanh^2(e+fx) dx}{\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}} + \frac{\cosh(e+fx) \int \operatorname{sech}(e+fx) dx}{2\sqrt{a\cosh^2(e+fx)}} \\
&= \frac{\tan^{-1}(\sinh(e+fx)) \cosh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}} - \frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0574041, size = 44, normalized size = 0.71

$$\frac{\cosh(e+fx) \tan^{-1}(\sinh(e+fx)) - \tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2], x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] - Tanh[e + f*x])/((2*f*Sqrt[a*Cosh[e +
f*x]^2])
```

Maple [A] time = 0.224, size = 51, normalized size = 0.8

$$\frac{1}{f \cosh(fx + e)} \left(\frac{\arctan(\sinh(fx + e)) (\cosh(fx + e))^2}{2} - \frac{\sinh(fx + e)}{2} \right) \frac{1}{\sqrt{a} (\cosh(fx + e))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2), x)

[Out] (1/2*arctan(sinh(f*x+e))*cosh(f*x+e)^2-1/2*sinh(f*x+e))/cosh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f

Maxima [B] time = 1.80806, size = 293, normalized size = 4.73

$$\frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}} - \frac{e^{(-fx-e)} - e^{(-3fx-3e)}}{2\sqrt{ae^{(-2fx-2e)} + \sqrt{ae^{(-4fx-4e)} + \sqrt{a}}}} - \frac{3 \arctan\left(e^{(-fx-e)}\right)}{2\sqrt{a}f} - \frac{5e^{(-fx-e)} + 3e^{(-3fx-3e)}}{4\left(2\sqrt{ae^{(-2fx-2e)} + \sqrt{ae^{(-4fx-4e)} + \sqrt{a}}}\right)f} + \frac{1}{4\left(2\sqrt{ae^{(-2fx-2e)} + \sqrt{ae^{(-4fx-4e)} + \sqrt{a}}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2), x, algorithm="maxima")

[Out] 1/2*(arctan(e^(-f*x - e))/sqrt(a) - (e^(-f*x - e) - e^(-3*f*x - 3*e))/(2*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a)))/f - 3/2*arctan(e^(-f*x - e))/(sqrt(a)*f) - 1/4*(5*e^(-f*x - e) + 3*e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a))*f) + 1/4*(3*e^(-f*x - e) + 5*e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a))*f)

Fricas [B] time = 1.84082, size = 1343, normalized size = 21.66

$$\frac{\left(3 \cosh(fx + e) e^{(fx+e)} \sinh(fx + e)^2 + e^{(fx+e)} \sinh(fx + e)^3 + \left(3 \cosh(fx + e)^2 - 1\right) e^{(fx+e)} \sinh(fx + e) - \left(4 \cosh(fx + e) e^{(fx+e)} \sinh(fx + e) + 4 \cosh(fx + e) e^{(fx+e)} \sinh(fx + e)\right)\right)}{af \cosh(fx + e)^4 + \left(af e^{(2fx+2e)} + af\right) \sinh(fx + e)^4 + 2af \cosh(fx + e)^2 + 4\left(af \cosh(fx + e) e^{(fx+e)} \sinh(fx + e) + af \cosh(fx + e) e^{(fx+e)} \sinh(fx + e)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out]
$$-(3*\cosh(f*x + e)*e^{(f*x + e)*\sinh(f*x + e)^2} + e^{(f*x + e)*\sinh(f*x + e)^3} + (3*\cosh(f*x + e)^2 - 1)*e^{(f*x + e)*\sinh(f*x + e)} - (4*\cosh(f*x + e)*e^{(f*x + e)*\sinh(f*x + e)^3} + e^{(f*x + e)*\sinh(f*x + e)^4} + 2*(3*\cosh(f*x + e)^2 + 1)*e^{(f*x + e)*\sinh(f*x + e)^2} + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*e^{(f*x + e)*\sinh(f*x + e)} + (\cosh(f*x + e)^4 + 2*\cosh(f*x + e)^2 + 1)*e^{(f*x + e)})*\arctan(\cosh(f*x + e) + \sinh(f*x + e)) + (\cosh(f*x + e)^3 - \cosh(f*x + e))*e^{(f*x + e)}*\sqrt{a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a}*e^{(-f*x - e)/(a*f*\cosh(f*x + e)^4 + (a*f*e^{(2*f*x + 2*e)} + a*f)*\sinh(f*x + e)^4 + 2*a*f*\cosh(f*x + e)^2 + 4*(a*f*\cosh(f*x + e)*e^{(2*f*x + 2*e)} + a*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + 2*(3*a*f*\cosh(f*x + e)^2 + a*f + (3*a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^2 + a*f + (a*f*\cosh(f*x + e)^4 + 2*a*f*\cosh(f*x + e)^2 + a*f)*e^{(2*f*x + 2*e)} + 4*(a*f*\cosh(f*x + e)^3 + a*f*\cosh(f*x + e) + (a*f*\cosh(f*x + e)^3 + a*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)})*\sinh(f*x + e))$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(e + fx)}{\sqrt{a(\sinh^2(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(tanh(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)

Giac [A] time = 1.31437, size = 85, normalized size = 1.37

$$\frac{\frac{\arctan\left(e^{(fx+e)}\right)}{\sqrt{a}} - \frac{\sqrt{a}e^{(3fx+3e)} - \sqrt{a}e^{(fx+e)}}{a\left(e^{(2fx+2e)} + 1\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

```
[Out] (arctan(e^(f*x + e))/sqrt(a) - (sqrt(a)*e^(3*f*x + 3*e) - sqrt(a)*e^(f*x + e))/(a*(e^(2*f*x + 2*e) + 1)^2))/f
```

$$3.444 \quad \int \frac{\coth^2(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=25

$$-\frac{\coth(e+fx)}{f\sqrt{a \cosh^2(e+fx)}}$$

[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))

Rubi [A] time = 0.104443, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2606, 8}

$$-\frac{\coth(e+fx)}{f\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]

[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))

Rule 3176

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^p], x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^n)^p], x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sinh[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 2606


```
Int[((a_.)*sec[(e_.) + (f_.)*(x_.)]^(m_.))*((b_.)*tan[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^2(e + fx)}{\sqrt{a + a \sinh^2(e + fx)}} dx &= \int \frac{\coth^2(e + fx)}{\sqrt{a \cosh^2(e + fx)}} dx \\
 &= \frac{\cosh(e + fx) \int \coth(e + fx) \operatorname{csch}(e + fx) dx}{\sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{(i \cosh(e + fx)) \operatorname{Subst}(\int 1 dx, x, -i \operatorname{csch}(e + fx))}{f \sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{\coth(e + fx)}{f \sqrt{a \cosh^2(e + fx)}}
 \end{aligned}$$

Mathematica [A] time = 0.0373885, size = 25, normalized size = 1.

$$-\frac{\coth(e + fx)}{f \sqrt{a \cosh^2(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2], x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Maple [A] time = 0.07, size = 32, normalized size = 1.3

$$-\frac{\cosh(fx + e)}{\sinh(fx + e) f \sqrt{a (\cosh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x)`

[Out] `-cosh(f*x+e)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f`

Maxima [B] time = 1.84918, size = 136, normalized size = 5.44

$$\frac{\frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}} + \frac{\sqrt{a}e^{(-fx-e)}}{ae^{(-2fx-2e)}-a}}{f} - \frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}f} + \frac{\sqrt{a}e^{(-fx-e)}}{\left(ae^{(-2fx-2e)}-a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `(arctan(e^(-f*x - e))/sqrt(a) + sqrt(a)*e^(-f*x - e)/(a*e^(-2*f*x - 2*e) - a))/f - arctan(e^(-f*x - e))/(sqrt(a)*f) + sqrt(a)*e^(-f*x - e)/((a*e^(-2*f*x - 2*e) - a)*f)`

Fricas [B] time = 1.87446, size = 428, normalized size = 17.12

$$\frac{2\sqrt{ae^{(4fx+4e)} + 2ae^{(2fx+2e)} + a}\left(\cosh(fx+e)e^{(fx+e)} + e^{(fx+e)}\sinh(fx+e)\right)e^{(-fx-e)}}{af\cosh(fx+e)^2 + \left(af e^{(2fx+2e)} + af\right)\sinh(fx+e)^2 - af + \left(af\cosh(fx+e)^2 - af\right)e^{(2fx+2e)} + 2\left(af\cosh(fx+e) + af\sinh(fx+e)\right)e^{(fx+e)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `-2*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*e^(-f*x - e)/(a*f*cosh(f*x + e)^2 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 2*(a*f*cosh(f*x + e) + a*f*sinh(f*x + e))*e^(f*x + e)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(e + fx)}{\sqrt{a(\sinh^2(e + fx) + 1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(coth(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)

Giac [A] time = 1.37947, size = 39, normalized size = 1.56

$$-\frac{2e^{(fx+e)}}{\sqrt{a}f(e^{(2fx+2e)} - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -2*e^(f*x + e)/(sqrt(a)*f*(e^(2*f*x + 2*e) - 1))

$$3.445 \quad \int \frac{\coth^4(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=61

$$-\frac{\coth(e+fx)}{f\sqrt{a \cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a \cosh^2(e+fx)}}$$

[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (Coth[e + f*x]*Csch[e + f*x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.114711, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3176, 3207, 2606}

$$-\frac{\coth(e+fx)}{f\sqrt{a \cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (Coth[e + f*x]*Csch[e + f*x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_)*((b_)*sin[(e_) + (f_)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p]]/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_)*(trig_)[e + f*x])^(m_) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^4(e + fx)}{\sqrt{a + a \sinh^2(e + fx)}} dx &= \int \frac{\coth^4(e + fx)}{\sqrt{a \cosh^2(e + fx)}} dx \\
 &= \frac{\cosh(e + fx) \int \coth^3(e + fx) \operatorname{csch}(e + fx) dx}{\sqrt{a \cosh^2(e + fx)}} \\
 &= \frac{(i \cosh(e + fx)) \operatorname{Subst}\left(\int (-1 + x^2) dx, x, -i \operatorname{csch}(e + fx)\right)}{f \sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{\coth(e + fx)}{f \sqrt{a \cosh^2(e + fx)}} - \frac{\coth(e + fx) \operatorname{csch}^2(e + fx)}{3f \sqrt{a \cosh^2(e + fx)}}
 \end{aligned}$$

Mathematica [A] time = 0.0614957, size = 37, normalized size = 0.61

$$-\frac{\coth(e + fx) (\operatorname{csch}^2(e + fx) + 3)}{3f \sqrt{a \cosh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] -(Coth[e + f*x]*(3 + Csch[e + f*x]^2))/(3*f*Sqrt[a*Cosh[e + f*x]^2])

Maple [A] time = 0.098, size = 44, normalized size = 0.7

$$-\frac{\cosh(fx + e) \left(3 (\sinh(fx + e))^2 + 1\right)}{3 (\sinh(fx + e))^3 f} \frac{1}{\sqrt{a (\cosh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*cosh(f*x+e)*(3*sinh(f*x+e)^2+1)/sinh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 2.00025, size = 751, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*arctan(e^(-f*x - e))/sqrt(a) + 3*log(e^(-f*x - e) + 1)/sqrt(a) - 3*log(e^(-f*x - e) - 1)/sqrt(a) + 4*(3*sqrt(a)*e^(-f*x - e) - sqrt(a)*e^(-3*f*x - 3*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a))/f + 1/12*(6*arctan(e^(-f*x - e))/sqrt(a) - 3*log(e^(-f*x - e) + 1)/sqrt(a) + 3*log(e^(-f*x - e) - 1)/sqrt(a) - 4*(sqrt(a)*e^(-3*f*x - 3*e) - 3*sqrt(a)*e^(-5*f*x - 5*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a))/f - 1/4*(3*arctan(e^(-f*x - e))/sqrt(a) + (3*sqrt(a)*e^(-f*x - e) - 10*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*f*x - 5*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a))/f - 1/4*arctan(e^(-f*x - e))/(sqrt(a)*f) + 1/24*(27*sqrt(a)*e^(-f*x - e) - 38*sqrt(a)*e^(-3*f*x - 3*e) + 15*sqrt(a)*e^(-5*f*x - 5*e))/((3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a)*f) + 1/24*(15*sqrt(a)*e^(-f*x - e) - 38*sqrt(a)*e^(-3*f*x - 3*e) + 27*sqrt(a)*e^(-5*f*x - 5*e))/((3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a)*f)
```

Fricas [B] time = 1.75004, size = 1692, normalized size = 27.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(15*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + 3*e^(f*x + e)*sinh(f*x + e)^5 + 2*(15*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 6*(5*cos
```

```

h(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 3*(5*cosh(f*x +
e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x + e)
^5 - 2*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*
e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^6 + (a*f*e^(2
*f*x + 2*e) + a*f)*sinh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4 + 6*(a*f*cosh(f*
x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*(5*a*f*cosh
(f*x + e)^2 - a*f + (5*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*x
+ e)^4 + 3*a*f*cosh(f*x + e)^2 + 4*(5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x
+ e) + (5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh
(f*x + e)^3 + 3*(5*a*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f + (5*a
*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
+ e)^2 - a*f + (a*f*cosh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4 + 3*a*f*cosh(f
*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 6*(a*f*cosh(f*x + e)^5 - 2*a*f*cosh(f*x
+ e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^5 - 2*a*f*cosh(f*x + e)^3 +
a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.49729, size = 88, normalized size = 1.44

$$-\frac{2\left(3\sqrt{ae}^{(5fx+5e)} - 2\sqrt{ae}^{(3fx+3e)} + 3\sqrt{ae}^{(fx+e)}\right)}{3af\left(e^{(2fx+2e)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -2/3*(3*sqrt(a)*e^(5*f*x + 5*e) - 2*sqrt(a)*e^(3*f*x + 3*e) + 3*sqrt(a)*e^(f*x + e))/(a*f*(e^(2*f*x + 2*e) - 1)^3)

$$3.446 \quad \int \frac{\coth^6(e+fx)}{\sqrt{a+a \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=96

$$-\frac{\coth(e+fx)}{f\sqrt{a \cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5f\sqrt{a \cosh^2(e+fx)}} - \frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a \cosh^2(e+fx)}}$$

[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (2*Coth[e + f*x]*Csch[e + f*x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x]^4)/(5*f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.122724, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2606, 194}

$$-\frac{\coth(e+fx)}{f\sqrt{a \cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5f\sqrt{a \cosh^2(e+fx)}} - \frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (2*Coth[e + f*x]*Csch[e + f*x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x]^4)/(5*f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;

FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])

Rule 2606

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}} dx &= \int \frac{\coth^6(e+fx)}{\sqrt{a\cosh^2(e+fx)}} dx \\
 &= \frac{\cosh(e+fx) \int \coth^5(e+fx) \operatorname{csch}(e+fx) dx}{\sqrt{a\cosh^2(e+fx)}} \\
 &= -\frac{(i\cosh(e+fx)) \operatorname{Subst}\left(\int (-1+x^2)^2 dx, x, -i\operatorname{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}} \\
 &= -\frac{(i\cosh(e+fx)) \operatorname{Subst}\left(\int (1-2x^2+x^4) dx, x, -i\operatorname{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}} \\
 &= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}} - \frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5f\sqrt{a\cosh^2(e+fx)}}
 \end{aligned}$$

Mathematica [A] time = 0.0819224, size = 49, normalized size = 0.51

$$\frac{\coth(e+fx) (3\operatorname{csch}^4(e+fx) + 10\operatorname{csch}^2(e+fx) + 15)}{15f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2], x]

[Out] $-(\text{Coth}[e + f*x]*(15 + 10*\text{Csch}[e + f*x]^2 + 3*\text{Csch}[e + f*x]^4))/(15*f*\text{Sqrt}[a*\text{Cosh}[e + f*x]^2])$

Maple [A] time = 0.113, size = 54, normalized size = 0.6

$$\frac{\cosh(fx + e) \left(15 (\sinh(fx + e))^4 + 10 (\sinh(fx + e))^2 + 3 \right)}{15 (\sinh(fx + e))^5 f} \frac{1}{\sqrt{a (\cosh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x)`

[Out] $-1/15*\cosh(f*x+e)*(15*\sinh(f*x+e)^4+10*\sinh(f*x+e)^2+3)/\sinh(f*x+e)^5/(a*\cosh(f*x+e)^2)^(1/2)/f$

Maxima [B] time = 2.24241, size = 1662, normalized size = 17.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] $-1/256*(120*\arctan(e^{-f*x - e})/\text{sqrt}(a) + 45*\log(e^{-f*x - e} + 1)/\text{sqrt}(a) - 45*\log(e^{-f*x - e} - 1)/\text{sqrt}(a) + 2*(105*\text{sqrt}(a)*e^{-f*x - e} - 530*\text{sqrt}(a)*e^{-3*f*x - 3*e} + 328*\text{sqrt}(a)*e^{-5*f*x - 5*e} - 110*\text{sqrt}(a)*e^{-7*f*x - 7*e} + 15*\text{sqrt}(a)*e^{-9*f*x - 9*e}))/ (5*a*e^{-2*f*x - 2*e} - 10*a*e^{-4*f*x - 4*e} + 10*a*e^{-6*f*x - 6*e} - 5*a*e^{-8*f*x - 8*e} + a*e^{-10*f*x - 10*e} - a))/f - 1/256*(120*\arctan(e^{-f*x - e})/\text{sqrt}(a) - 45*\log(e^{-f*x - e} + 1)/\text{sqrt}(a) + 45*\log(e^{-f*x - e} - 1)/\text{sqrt}(a) + 2*(15*\text{sqrt}(a)*e^{-f*x - e} - 110*\text{sqrt}(a)*e^{-3*f*x - 3*e} + 328*\text{sqrt}(a)*e^{-5*f*x - 5*e} - 530*\text{sqrt}(a)*e^{-7*f*x - 7*e} + 105*\text{sqrt}(a)*e^{-9*f*x - 9*e}))/ (5*a*e^{-2*f*x - 2*e} - 10*a*e^{-4*f*x - 4*e} + 10*a*e^{-6*f*x - 6*e} - 5*a*e^{-8*f*x - 8*e} + a*e^{-10*f*x - 10*e} - a))/f + 1/320*(60*\arctan(e^{-f*x - e})/\text{sqrt}(a) + 75*\log(e^{-f*x - e} + 1)/\text{sqrt}(a) - 75*\log(e^{-f*x - e} - 1)/\text{sqrt}(a) + 2*(105*\text{sqrt}(a)*e^{-f*x - e} + 130*\text{sqrt}(a)*e^{-3*f*x - 3*e} - 284*\text{sqrt}(a)*e^{-5*f*x - 5*e} + 190*\text{sqrt}(a)*e^{-7*f*x - 7*e} - 45*\text{sqrt}(a)*e^{-9*f*x - 9*e}))/ (5*a*e^{-2*f*x - 2*e} - 10*a*e^{-4*f*x - 4*e} + 10*a*e^{-6*f*x - 6*e} - 5*a*e^{-8*f*x - 8*e} + a*e^{-10*f*x - 10*e} - a))/f$

```

*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/320*(60*arctan(e^(-f*x - e))
/sqrt(a) - 75*log(e^(-f*x - e) + 1)/sqrt(a) + 75*log(e^(-f*x - e) - 1)/sqrt
(a) - 2*(45*sqrt(a)*e^(-f*x - e) - 190*sqrt(a)*e^(-3*f*x - 3*e) + 284*sqrt(
a)*e^(-5*f*x - 5*e) - 130*sqrt(a)*e^(-7*f*x - 7*e) - 105*sqrt(a)*e^(-9*f*x
- 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*
e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/24*(15*arctan(
e^(-f*x - e))/sqrt(a) + (15*sqrt(a)*e^(-f*x - e) - 80*sqrt(a)*e^(-3*f*x - 3
*e) + 178*sqrt(a)*e^(-5*f*x - 5*e) - 80*sqrt(a)*e^(-7*f*x - 7*e) + 15*sqrt(
a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e
^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f - 1/1
6*arctan(e^(-f*x - e))/(sqrt(a)*f) + 1/1920*(2685*sqrt(a)*e^(-f*x - e) - 73
70*sqrt(a)*e^(-3*f*x - 3*e) + 8632*sqrt(a)*e^(-5*f*x - 5*e) - 4790*sqrt(a)*
e^(-7*f*x - 7*e) + 1035*sqrt(a)*e^(-9*f*x - 9*e))/((5*a*e^(-2*f*x - 2*e) -
10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^
(-10*f*x - 10*e) - a)*f) + 1/1920*(1035*sqrt(a)*e^(-f*x - e) - 4790*sqrt(a)
*e^(-3*f*x - 3*e) + 8632*sqrt(a)*e^(-5*f*x - 5*e) - 7370*sqrt(a)*e^(-7*f*x
- 7*e) + 2685*sqrt(a)*e^(-9*f*x - 9*e))/((5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4
*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x -
10*e) - a)*f)

```

Fricas [B] time = 1.89456, size = 3776, normalized size = 39.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

```

[Out] -2/15*(135*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + 15*e^(f*x + e)*sinh(
f*x + e)^9 + 20*(27*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^7 + 140*
(9*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 2*(945*co
sh(f*x + e)^4 - 210*cosh(f*x + e)^2 + 29)*e^(f*x + e)*sinh(f*x + e)^5 + 10*
(189*cosh(f*x + e)^5 - 70*cosh(f*x + e)^3 + 29*cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e)^4 + 20*(63*cosh(f*x + e)^6 - 35*cosh(f*x + e)^4 + 29*cosh(f*x
+ e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 20*(27*cosh(f*x + e)^7 - 21*cosh(
f*x + e)^5 + 29*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(27*cosh(f*x + e)^8 - 28*cosh(f*x + e)^6 + 58*cosh(f*x + e)^4 - 12*
cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e) + (15*cosh(f*x + e)^9 - 20*c
osh(f*x + e)^7 + 58*cosh(f*x + e)^5 - 20*cosh(f*x + e)^3 + 15*cosh(f*x + e)
)*e^(f*x + e)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a*f*cosh(f*x + e)^10 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^10 - 5
*a*f*cosh(f*x + e)^8 + 10*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x

```

```

+ e))*sinh(f*x + e)^9 + 5*(9*a*f*cosh(f*x + e)^2 - a*f + (9*a*f*cosh(f*x +
e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 40
*(3*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e)^3 - a*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*(21*a*f*cosh(f*x + e)^
4 - 14*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^4 - 14*a*f*cosh(f*
x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 - 10*a*f*cosh(f*x + e)^4 +
4*(63*a*f*cosh(f*x + e)^5 - 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e)
+ (63*a*f*cosh(f*x + e)^5 - 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e))*
e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 10*(21*a*f*cosh(f*x + e)^6 - 35*a*f*cosh
(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 - a*f + (21*a*f*cosh(f*x + e)^6 - 35*a
*f*cosh(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*
x + e)^4 + 5*a*f*cosh(f*x + e)^2 + 40*(3*a*f*cosh(f*x + e)^7 - 7*a*f*cosh(f
*x + e)^5 + 5*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e
)^7 - 7*a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*e^
(2*f*x + 2*e))*sinh(f*x + e)^3 + 5*(9*a*f*cosh(f*x + e)^8 - 28*a*f*cosh(f*x
+ e)^6 + 30*a*f*cosh(f*x + e)^4 - 12*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*co
sh(f*x + e)^8 - 28*a*f*cosh(f*x + e)^6 + 30*a*f*cosh(f*x + e)^4 - 12*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x
+ e)^10 - 5*a*f*cosh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 - 10*a*f*cosh(f*x
+ e)^4 + 5*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 10*(a*f*cosh(f*x +
e)^9 - 4*a*f*cosh(f*x + e)^7 + 6*a*f*cosh(f*x + e)^5 - 4*a*f*cosh(f*x + e)
^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^9 - 4*a*f*cosh(f*x + e)^7 + 6*a
*f*cosh(f*x + e)^5 - 4*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e))

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [A] time = 1.56128, size = 128, normalized size = 1.33

$$\frac{2 \left(15 \sqrt{ae}^{(9fx+9e)} - 20 \sqrt{ae}^{(7fx+7e)} + 58 \sqrt{ae}^{(5fx+5e)} - 20 \sqrt{ae}^{(3fx+3e)} + 15 \sqrt{ae}^{(fx+e)} \right)}{15af \left(e^{(2fx+2e)} - 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(a)*e^(9*f*x + 9*e) - 20*sqrt(a)*e^(7*f*x + 7*e) + 58*sqrt(a)
*e^(5*f*x + 5*e) - 20*sqrt(a)*e^(3*f*x + 3*e) + 15*sqrt(a)*e^(f*x + e))/(a*
f*(e^(2*f*x + 2*e) - 1)^5)
```

$$3.447 \quad \int \frac{\tanh^5(e+fx)}{\left(a+a \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=68

$$-\frac{a^2}{7f(a \cosh^2(e+fx))^{7/2}} + \frac{2a}{5f(a \cosh^2(e+fx))^{5/2}} - \frac{1}{3f(a \cosh^2(e+fx))^{3/2}}$$

[Out] $-a^2/(7*f*(a*Cosh[e + f*x]^2)^{(7/2)}) + (2*a)/(5*f*(a*Cosh[e + f*x]^2)^{(5/2)}) - 1/(3*f*(a*Cosh[e + f*x]^2)^{(3/2)})$

Rubi [A] time = 0.141147, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3205, 16, 43}

$$-\frac{a^2}{7f(a \cosh^2(e+fx))^{7/2}} + \frac{2a}{5f(a \cosh^2(e+fx))^{5/2}} - \frac{1}{3f(a \cosh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] $-a^2/(7*f*(a*Cosh[e + f*x]^2)^{(7/2)}) + (2*a)/(5*f*(a*Cosh[e + f*x]^2)^{(5/2)}) - 1/(3*f*(a*Cosh[e + f*x]^2)^{(3/2)})$

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_)*sin[(e_) + (f_)*(x_)]^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]

Rule 16

$\text{Int}[(u_.)*(v_)^{(m_.)}*((b_)*(v_))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\& \ \text{IntegerQ}[m]$

Rule 43

$\text{Int}[(a_. + (b_.)*(x_))^{(m_.)}*((c_.) + (d_.)*(x_))^{(n_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[(a + b*x)^m*(c + d*x)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[m, 0] \ \&\& \ (\! \text{IntegerQ}[n] \ || \ (\text{EqQ}[c, 0] \ \&\& \ \text{LeQ}[7*m + 4*n + 4, 0]) \ || \ \text{LtQ}[9*m + 5*(n + 1), 0] \ || \ \text{GtQ}[m + n + 2, 0])$

Rubi steps

$$\begin{aligned} \int \frac{\tanh^5(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}} dx &= \int \frac{\tanh^5(e+fx)}{(a\cosh^2(e+fx))^{3/2}} dx \\ &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(ax)^{3/2}} dx, x, \cosh^2(e+fx)\right)}{2f} \\ &= \frac{a^3 \text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{9/2}} dx, x, \cosh^2(e+fx)\right)}{2f} \\ &= \frac{a^3 \text{Subst}\left(\int \left(\frac{1}{(ax)^{9/2}} - \frac{2}{a(ax)^{7/2}} + \frac{1}{a^2(ax)^{5/2}}\right) dx, x, \cosh^2(e+fx)\right)}{2f} \\ &= -\frac{a^2}{7f(a\cosh^2(e+fx))^{7/2}} + \frac{2a}{5f(a\cosh^2(e+fx))^{5/2}} - \frac{1}{3f(a\cosh^2(e+fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.112383, size = 51, normalized size = 0.75

$$\frac{(-35 \cosh^4(e+fx) + 42 \cosh^2(e+fx) - 15) \text{sech}^4(e+fx)}{105f(a\cosh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((-15 + 42*Cosh[e + f*x]^2 - 35*Cosh[e + f*x]^4)*Sech[e + f*x]^4)/(105*f*(a*Cosh[e + f*x]^2)^(3/2))

Maple [C] time = 0.107, size = 44, normalized size = 0.7

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^5}{(\cosh(fx + e))^8} \frac{1}{a \sqrt{a (\cosh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x)`

[Out] ``int/indef0` (sinh(f*x+e)^5/cosh(f*x+e)^8/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [B] time = 2.21789, size = 791, normalized size = 11.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `-8/3*e^(-3*f*x - 3*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) + 32/15*e^(-5*f*x - 5*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) - 304/35*e^(-7*f*x - 7*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) + 32/15*e^(-9*f*x - 9*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) - 8/3*e^(-11*f*x - 11*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f)`

Fricas [B] time = 2.14094, size = 6649, normalized size = 97.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & -8/105*(385*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e)^{10} + 35*e^{(f*x + e)}*\sin \\ & h(f*x + e)^{11} + 7*(275*\cosh(f*x + e)^2 - 4)*e^{(f*x + e)}*\sinh(f*x + e)^9 + 2 \\ & 1*(275*\cosh(f*x + e)^3 - 12*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^8 + 6* \\ & (1925*\cosh(f*x + e)^4 - 168*\cosh(f*x + e)^2 + 19)*e^{(f*x + e)}*\sinh(f*x + e) \\ & ^7 + 42*(385*\cosh(f*x + e)^5 - 56*\cosh(f*x + e)^3 + 19*\cosh(f*x + e))*e^{(f* \\ & x + e)}*\sinh(f*x + e)^6 + 14*(1155*\cosh(f*x + e)^6 - 252*\cosh(f*x + e)^4 + 1 \\ & 71*\cosh(f*x + e)^2 - 2)*e^{(f*x + e)}*\sinh(f*x + e)^5 + 14*(825*\cosh(f*x + e) \\ & ^7 - 252*\cosh(f*x + e)^5 + 285*\cosh(f*x + e)^3 - 10*\cosh(f*x + e))*e^{(f*x + \\ & e)}*\sinh(f*x + e)^4 + 7*(825*\cosh(f*x + e)^8 - 336*\cosh(f*x + e)^6 + 570*\co \\ & sh(f*x + e)^4 - 40*\cosh(f*x + e)^2 + 5)*e^{(f*x + e)}*\sinh(f*x + e)^3 + 7*(27 \\ & 5*\cosh(f*x + e)^9 - 144*\cosh(f*x + e)^7 + 342*\cosh(f*x + e)^5 - 40*\cosh(f*x \\ & + e)^3 + 15*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^2 + 7*(55*\cosh(f*x + \\ & e)^{10} - 36*\cosh(f*x + e)^8 + 114*\cosh(f*x + e)^6 - 20*\cosh(f*x + e)^4 + 15* \\ & \cosh(f*x + e)^2)*e^{(f*x + e)}*\sinh(f*x + e) + (35*\cosh(f*x + e)^{11} - 28*\cosh \\ & (f*x + e)^9 + 114*\cosh(f*x + e)^7 - 28*\cosh(f*x + e)^5 + 35*\cosh(f*x + e)^3 \\ &)*e^{(f*x + e)}*\sqrt{a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a}*e^{(-f*x - \\ & e)/(a^2*f*\cosh(f*x + e)^{14} + 7*a^2*f*\cosh(f*x + e)^{12} + (a^2*f*e^{(2*f*x + 2 \\ & *e)} + a^2*f)*\sinh(f*x + e)^{14} + 14*(a^2*f*\cosh(f*x + e)*e^{(2*f*x + 2*e)} + a \\ & ^2*f*\cosh(f*x + e))*\sinh(f*x + e)^{13} + 21*a^2*f*\cosh(f*x + e)^{10} + 7*(13*a^ \\ & 2*f*\cosh(f*x + e)^2 + a^2*f + (13*a^2*f*\cosh(f*x + e)^2 + a^2*f)*e^{(2*f*x + \\ & 2*e)})*\sinh(f*x + e)^{12} + 28*(13*a^2*f*\cosh(f*x + e)^3 + 3*a^2*f*\cosh(f*x + \\ & e) + (13*a^2*f*\cosh(f*x + e)^3 + 3*a^2*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)})*\s \\ & inh(f*x + e)^{11} + 35*a^2*f*\cosh(f*x + e)^8 + 7*(143*a^2*f*\cosh(f*x + e)^4 + \\ & 66*a^2*f*\cosh(f*x + e)^2 + 3*a^2*f + (143*a^2*f*\cosh(f*x + e)^4 + 66*a^2*f \\ & *\cosh(f*x + e)^2 + 3*a^2*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^{10} + 14*(143*a^2 \\ & *f*\cosh(f*x + e)^5 + 110*a^2*f*\cosh(f*x + e)^3 + 15*a^2*f*\cosh(f*x + e) + (\\ & 143*a^2*f*\cosh(f*x + e)^5 + 110*a^2*f*\cosh(f*x + e)^3 + 15*a^2*f*\cosh(f*x + \\ & e))*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^9 + 35*a^2*f*\cosh(f*x + e)^6 + 7*(429*a \\ & ^2*f*\cosh(f*x + e)^6 + 495*a^2*f*\cosh(f*x + e)^4 + 135*a^2*f*\cosh(f*x + e)^ \\ & 2 + 5*a^2*f + (429*a^2*f*\cosh(f*x + e)^6 + 495*a^2*f*\cosh(f*x + e)^4 + 135* \\ & a^2*f*\cosh(f*x + e)^2 + 5*a^2*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^8 + 8*(429* \\ & a^2*f*\cosh(f*x + e)^7 + 693*a^2*f*\cosh(f*x + e)^5 + 315*a^2*f*\cosh(f*x + e) \\ & ^3 + 35*a^2*f*\cosh(f*x + e) + (429*a^2*f*\cosh(f*x + e)^7 + 693*a^2*f*\cosh(f \\ & *x + e)^5 + 315*a^2*f*\cosh(f*x + e)^3 + 35*a^2*f*\cosh(f*x + e))*e^{(2*f*x + \\ & 2*e)})*\sinh(f*x + e)^7 + 21*a^2*f*\cosh(f*x + e)^4 + 7*(429*a^2*f*\cosh(f*x + \\ & e)^8 + 924*a^2*f*\cosh(f*x + e)^6 + 630*a^2*f*\cosh(f*x + e)^4 + 140*a^2*f*\co \end{aligned}$$

$$\begin{aligned} & \operatorname{sh}(f*x + e)^2 + 5*a^2*f + (429*a^2*f*\cosh(f*x + e)^8 + 924*a^2*f*\cosh(f*x + \\ & e)^6 + 630*a^2*f*\cosh(f*x + e)^4 + 140*a^2*f*\cosh(f*x + e)^2 + 5*a^2*f)*e^ \\ & (2*f*x + 2*e))*\sinh(f*x + e)^6 + 14*(143*a^2*f*\cosh(f*x + e)^9 + 396*a^2*f* \\ & \cosh(f*x + e)^7 + 378*a^2*f*\cosh(f*x + e)^5 + 140*a^2*f*\cosh(f*x + e)^3 + 1 \\ & 5*a^2*f*\cosh(f*x + e) + (143*a^2*f*\cosh(f*x + e)^9 + 396*a^2*f*\cosh(f*x + e \\ &)^7 + 378*a^2*f*\cosh(f*x + e)^5 + 140*a^2*f*\cosh(f*x + e)^3 + 15*a^2*f*\cosh \\ & (f*x + e))*e^(2*f*x + 2*e))*\sinh(f*x + e)^5 + 7*a^2*f*\cosh(f*x + e)^2 + 7*(\\ & 143*a^2*f*\cosh(f*x + e)^10 + 495*a^2*f*\cosh(f*x + e)^8 + 630*a^2*f*\cosh(f*x \\ & + e)^6 + 350*a^2*f*\cosh(f*x + e)^4 + 75*a^2*f*\cosh(f*x + e)^2 + 3*a^2*f + \\ & (143*a^2*f*\cosh(f*x + e)^10 + 495*a^2*f*\cosh(f*x + e)^8 + 630*a^2*f*\cosh(f* \\ & x + e)^6 + 350*a^2*f*\cosh(f*x + e)^4 + 75*a^2*f*\cosh(f*x + e)^2 + 3*a^2*f)* \\ & e^(2*f*x + 2*e))*\sinh(f*x + e)^4 + 28*(13*a^2*f*\cosh(f*x + e)^11 + 55*a^2*f \\ & *\cosh(f*x + e)^9 + 90*a^2*f*\cosh(f*x + e)^7 + 70*a^2*f*\cosh(f*x + e)^5 + 25 \\ & *a^2*f*\cosh(f*x + e)^3 + 3*a^2*f*\cosh(f*x + e) + (13*a^2*f*\cosh(f*x + e)^11 \\ & + 55*a^2*f*\cosh(f*x + e)^9 + 90*a^2*f*\cosh(f*x + e)^7 + 70*a^2*f*\cosh(f*x \\ & + e)^5 + 25*a^2*f*\cosh(f*x + e)^3 + 3*a^2*f*\cosh(f*x + e))*e^(2*f*x + 2*e)) \\ & *\sinh(f*x + e)^3 + a^2*f + 7*(13*a^2*f*\cosh(f*x + e)^12 + 66*a^2*f*\cosh(f*x \\ & + e)^10 + 135*a^2*f*\cosh(f*x + e)^8 + 140*a^2*f*\cosh(f*x + e)^6 + 75*a^2*f \\ & *\cosh(f*x + e)^4 + 18*a^2*f*\cosh(f*x + e)^2 + a^2*f + (13*a^2*f*\cosh(f*x + \\ & e)^12 + 66*a^2*f*\cosh(f*x + e)^10 + 135*a^2*f*\cosh(f*x + e)^8 + 140*a^2*f*c \\ & osh(f*x + e)^6 + 75*a^2*f*\cosh(f*x + e)^4 + 18*a^2*f*\cosh(f*x + e)^2 + a^2*f \\ &)*e^(2*f*x + 2*e))*\sinh(f*x + e)^2 + (a^2*f*\cosh(f*x + e)^14 + 7*a^2*f*cos \\ & h(f*x + e)^12 + 21*a^2*f*\cosh(f*x + e)^10 + 35*a^2*f*\cosh(f*x + e)^8 + 35*a \\ & ^2*f*\cosh(f*x + e)^6 + 21*a^2*f*\cosh(f*x + e)^4 + 7*a^2*f*\cosh(f*x + e)^2 + \\ & a^2*f)*e^(2*f*x + 2*e) + 14*(a^2*f*\cosh(f*x + e)^13 + 6*a^2*f*\cosh(f*x + e \\ &)^11 + 15*a^2*f*\cosh(f*x + e)^9 + 20*a^2*f*\cosh(f*x + e)^7 + 15*a^2*f*\cosh \\ & (f*x + e)^5 + 6*a^2*f*\cosh(f*x + e)^3 + a^2*f*\cosh(f*x + e) + (a^2*f*\cosh(f* \\ & x + e)^13 + 6*a^2*f*\cosh(f*x + e)^11 + 15*a^2*f*\cosh(f*x + e)^9 + 20*a^2*f* \\ & \cosh(f*x + e)^7 + 15*a^2*f*\cosh(f*x + e)^5 + 6*a^2*f*\cosh(f*x + e)^3 + a^2*f \\ & *\cosh(f*x + e))*e^(2*f*x + 2*e))*\sinh(f*x + e) \end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [A] time = 1.73257, size = 132, normalized size = 1.94

$$\frac{8 \left(35 \sqrt{ae}^{(11fx+11e)} - 28 \sqrt{ae}^{(9fx+9e)} + 114 \sqrt{ae}^{(7fx+7e)} - 28 \sqrt{ae}^{(5fx+5e)} + 35 \sqrt{ae}^{(3fx+3e)} \right)}{105 a^2 f \left(e^{(2fx+2e)} + 1 \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] -8/105*(35*sqrt(a)*e^(11*f*x + 11*e) - 28*sqrt(a)*e^(9*f*x + 9*e) + 114*sqrt(a)*e^(7*f*x + 7*e) - 28*sqrt(a)*e^(5*f*x + 5*e) + 35*sqrt(a)*e^(3*f*x + 3*e))/(a^2*f*(e^(2*f*x + 2*e) + 1)^7)

$$3.448 \quad \int \frac{\tanh^3(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=44

$$\frac{a}{5f(a \cosh^2(e+fx))^{5/2}} - \frac{1}{3f(a \cosh^2(e+fx))^{3/2}}$$

[Out] a/(5*f*(a*Cosh[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))

Rubi [A] time = 0.126134, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{5f(a \cosh^2(e+fx))^{5/2}} - \frac{1}{3f(a \cosh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] a/(5*f*(a*Cosh[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_)*sin[(e_) + (f_)*(x_)]^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]

Rule 16

`Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]`

Rule 43

`Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])`

Rubi steps

$$\begin{aligned} \int \frac{\tanh^3(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx &= \int \frac{\tanh^3(e + fx)}{(a \cosh^2(e + fx))^{3/2}} dx \\ &= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2(ax)^{3/2}} dx, x, \cosh^2(e + fx)\right)}{2f} \\ &= -\frac{a^2 \text{Subst}\left(\int \frac{1-x}{(ax)^{7/2}} dx, x, \cosh^2(e + fx)\right)}{2f} \\ &= -\frac{a^2 \text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}} - \frac{1}{a(ax)^{5/2}}\right) dx, x, \cosh^2(e + fx)\right)}{2f} \\ &= \frac{a}{5f(a \cosh^2(e + fx))^{5/2}} - \frac{1}{3f(a \cosh^2(e + fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.122513, size = 34, normalized size = 0.77

$$\frac{a(3 - 5 \cosh^2(e + fx))}{15f(a \cosh^2(e + fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]

[Out] (a*(3 - 5*Cosh[e + f*x]^2))/(15*f*(a*Cosh[e + f*x]^2)^(5/2))

Maple [C] time = 0.098, size = 44, normalized size = 1.

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^3}{(\cosh(fx + e))^6} \frac{1}{a \sqrt{a (\cosh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x)

[Out] `int/indef0` (sinh(f*x+e)^3/cosh(f*x+e)^6/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [B] time = 1.87989, size = 362, normalized size = 8.23

$$\frac{8e^{(-3fx-3e)}}{3\left(5a^{\frac{3}{2}}e^{(-2fx-2e)} + 10a^{\frac{3}{2}}e^{(-4fx-4e)} + 10a^{\frac{3}{2}}e^{(-6fx-6e)} + 5a^{\frac{3}{2}}e^{(-8fx-8e)} + a^{\frac{3}{2}}e^{(-10fx-10e)} + a^{\frac{3}{2}}\right)f} + \frac{1}{15\left(5a^{\frac{3}{2}}e^{(-2fx-2e)} + 10a^{\frac{3}{2}}e^{(-4fx-4e)} + 10a^{\frac{3}{2}}e^{(-6fx-6e)} + 5a^{\frac{3}{2}}e^{(-8fx-8e)} + a^{\frac{3}{2}}e^{(-10fx-10e)} + a^{\frac{3}{2}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out]
$$\frac{-8/3e^{(-3fx-3e)}}{\left(\left(5a^{(3/2)}e^{(-2fx-2e)} + 10a^{(3/2)}e^{(-4fx-4e)} + 10a^{(3/2)}e^{(-6fx-6e)} + 5a^{(3/2)}e^{(-8fx-8e)} + a^{(3/2)}e^{(-10fx-10e)} + a^{(3/2)}\right)f\right)} + \frac{16/15e^{(-5fx-5e)}}{\left(\left(5a^{(3/2)}e^{(-2fx-2e)} + 10a^{(3/2)}e^{(-4fx-4e)} + 10a^{(3/2)}e^{(-6fx-6e)} + 5a^{(3/2)}e^{(-8fx-8e)} + a^{(3/2)}e^{(-10fx-10e)} + a^{(3/2)}\right)f\right)} - \frac{8/3e^{(-7fx-7e)}}{\left(\left(5a^{(3/2)}e^{(-2fx-2e)} + 10a^{(3/2)}e^{(-4fx-4e)} + 10a^{(3/2)}e^{(-6fx-6e)} + 5a^{(3/2)}e^{(-8fx-8e)} + a^{(3/2)}e^{(-10fx-10e)} + a^{(3/2)}\right)f\right)}$$

Fricas [B] time = 1.89151, size = 3571, normalized size = 81.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

```
[Out] -8/15*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*x + e)^7 + (105*cosh(f*x + e)^2 - 2)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*cosh(f*x + e)^3 - 2*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 5*(35*cosh(f*x + e)^4 - 4*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 5*(21*cosh(f*x + e)^5 - 4*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 5*(7*cosh(f*x + e)^6 - 2*cosh(f*x + e)^4 + 3*cosh(f*x + e)^2)*e^(f*x + e)*sinh(f*x + e) + (5*cosh(f*x + e)^7 - 2*cosh(f*x + e)^5 + 5*cosh(f*x + e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^10 + 5*a^2*f*cosh(f*x + e)^8 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^10 + 10*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^6 + 5*(9*a^2*f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 40*(3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*a^2*f*cosh(f*x + e)^4 + 10*(21*a^2*f*cosh(f*x + e)^4 + 14*a^2*f*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^4 + 14*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 4*(63*a^2*f*cosh(f*x + e)^5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (63*a^2*f*cosh(f*x + e)^5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^2 + 10*(21*a^2*f*cosh(f*x + e)^6 + 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^6 + 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 40*(3*a^2*f*cosh(f*x + e)^7 + 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (3*a^2*f*cosh(f*x + e)^7 + 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*f + 5*(9*a^2*f*cosh(f*x + e)^8 + 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 + 12*a^2*f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^8 + 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 + 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^10 + 5*a^2*f*cosh(f*x + e)^8 + 10*a^2*f*cosh(f*x + e)^6 + 10*a^2*f*cosh(f*x + e)^4 + 5*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 10*(a^2*f*cosh(f*x + e)^9 + 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^9 + 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(e + fx)}{(a(\sinh^2(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)`

[Out] `Integral(tanh(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)`

Giac [A] time = 1.53619, size = 92, normalized size = 2.09

$$-\frac{8\left(5\sqrt{ae}^{(7fx+7e)} - 2\sqrt{ae}^{(5fx+5e)} + 5\sqrt{ae}^{(3fx+3e)}\right)}{15a^2f\left(e^{(2fx+2e)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

[Out] `-8/15*(5*sqrt(a)*e^(7*f*x + 7*e) - 2*sqrt(a)*e^(5*f*x + 5*e) + 5*sqrt(a)*e^(3*f*x + 3*e))/(a^2*f*(e^(2*f*x + 2*e) + 1)^5)`

$$3.449 \quad \int \frac{\tanh(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=21

$$-\frac{1}{3f(a \cosh^2(e+fx))^{3/2}}$$

[Out] -1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))

Rubi [A] time = 0.0774936, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3176, 3205, 16, 32}

$$-\frac{1}{3f(a \cosh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]

[Out] -1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_)*sin[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]

Rule 16

`Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]`

Rule 32

`Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m+1)/(b*(m+1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]`

Rubi steps

$$\begin{aligned} \int \frac{\tanh(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}} dx &= \int \frac{\tanh(e+fx)}{(a\cosh^2(e+fx))^{3/2}} dx \\ &= \frac{\text{Subst}\left(\int \frac{1}{x(ax)^{3/2}} dx, x, \cosh^2(e+fx)\right)}{2f} \\ &= \frac{a \text{Subst}\left(\int \frac{1}{(ax)^{5/2}} dx, x, \cosh^2(e+fx)\right)}{2f} \\ &= -\frac{1}{3f(a\cosh^2(e+fx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.0388074, size = 21, normalized size = 1.

$$-\frac{1}{3f(a\cosh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] -1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))

Maple [A] time = 0.018, size = 20, normalized size = 1.

$$-\frac{1}{3f} \left(a + a (\sinh(fx + e))^2 \right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x)`

[Out] $-1/3/f/(a+a*\sinh(f*x+e)^2)^{(3/2)}$

Maxima [B] time = 1.8976, size = 82, normalized size = 3.9

$$-\frac{8e^{(-3fx-3e)}}{3\left(3a^{\frac{3}{2}}e^{(-2fx-2e)} + 3a^{\frac{3}{2}}e^{(-4fx-4e)} + a^{\frac{3}{2}}e^{(-6fx-6e)} + a^{\frac{3}{2}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] $-8/3*e^{(-3*f*x - 3*e)}/((3*a^{(3/2)}*e^{(-2*f*x - 2*e)} + 3*a^{(3/2)}*e^{(-4*f*x - 4*e)} + a^{(3/2)}*e^{(-6*f*x - 6*e)} + a^{(3/2)})*f)$

Fricas [B] time = 1.84687, size = 1511, normalized size = 71.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] $-8/3*(\cosh(f*x + e)^3*e^{(f*x + e)} + 3*\cosh(f*x + e)^2*e^{(f*x + e)}*\sinh(f*x + e) + 3*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e)^2 + e^{(f*x + e)}*\sinh(f*x + e)^3)*\sqrt{a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a}*e^{(-f*x - e)}/(a^2*f*\cosh(f*x + e)^6 + 3*a^2*f*\cosh(f*x + e)^4 + (a^2*f*e^{(2*f*x + 2*e)} + a^2*f)*\sinh(f*x + e)^6 + 6*(a^2*f*\cosh(f*x + e)*e^{(2*f*x + 2*e)} + a^2*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + 3*a^2*f*\cosh(f*x + e)^2 + 3*(5*a^2*f*\cosh(f*x + e))^2 + a^2*f + (5*a^2*f*\cosh(f*x + e)^2 + a^2*f)*e^{(2*f*x + 2*e)}*\sinh(f*x + e)^4 + 4*(5*a^2*f*\cosh(f*x + e)^3 + 3*a^2*f*\cosh(f*x + e) + (5*a^2*f*\cosh(f*x + e)^3 + 3*a^2*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^3 + a^2*f + 3*(5*a^2*f*\cosh(f*x + e)^4 + 6*a^2*f*\cosh(f*x + e)^2 + a^2*f + (5*a^2*f*\cosh(f*x + e)^4 + 6*a^2*f*\cosh(f*x + e)^2 + a^2*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^2 + (a^2*f*\cosh(f*x + e)^6 + 3*a^2*f*\cosh(f*x + e)^4 + 3*a^2*f*\cosh(f*x + e)^2 + a^2*f)*e^{(2*f*x + 2*e)} + 6*(a^2*f*\cosh(f*x + e)^5 + 2*a^2*f*\cosh(f*x + e)^3 + a^2*f*\cosh(f*x + e) + (a^2*f*\cosh(f*x + e)^5 + 2*a^2*f*\cosh$

$h(f*x + e)^3 + a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e))*sinh(f*x + e))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(e + fx)}{(a(\sinh^2(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)

[Out] Integral(tanh(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)

Giac [A] time = 1.31568, size = 43, normalized size = 2.05

$$-\frac{8e^{(3fx+3e)}}{3a^{\frac{3}{2}}f(e^{(2fx+2e)}+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] -8/3*e^(3*f*x + 3*e)/(a^(3/2)*f*(e^(2*f*x + 2*e) + 1)^3)

$$3.450 \quad \int \frac{\coth(e+fx)}{\left(a+a \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=53

$$\frac{1}{af\sqrt{a \cosh^2(e+fx)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.107651, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3176, 3205, 51, 63, 206}

$$\frac{1}{af\sqrt{a \cosh^2(e+fx)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_)*sin[(e_) + (f_)*(x_)]^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && Integ

erQ[(m - 1)/2] && IntegerQ[n/2]

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}} dx &= \int \frac{\coth(e+fx)}{(a\cosh^2(e+fx))^{3/2}} dx \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(ax)^{3/2}} dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= \frac{1}{af\sqrt{a\cosh^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}} dx, x, \cosh^2(e+fx)\right)}{2af} \\
&= \frac{1}{af\sqrt{a\cosh^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{a^2f} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0621527, size = 41, normalized size = 0.77

$$\frac{\cosh(e+fx) \log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right) + 1}{af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] (1 + Cosh[e + f*x]*Log[Tanh[(e + f*x)/2]])/(a*f*Sqrt[a*Cosh[e + f*x]^2])

Maple [C] time = 0.066, size = 44, normalized size = 0.8

$$\frac{1}{f} \int \frac{1}{(\cosh(fx+e))^2 \sinh(fx+e) a \sqrt{a(\cosh(fx+e))^2}} dx, \sinh(fx+e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x)`

[Out] ``int/indef0`(1/cosh(f*x+e)^2/sinh(f*x+e)/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [A] time = 1.79287, size = 103, normalized size = 1.94

$$\frac{2\sqrt{ae}^{-fx-e}}{(a^2e^{-2fx-2e} + a^2)f} - \frac{\log(e^{-fx-e} + 1)}{a^{\frac{3}{2}}f} + \frac{\log(e^{-fx-e} - 1)}{a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `2*sqrt(a)*e^(-f*x - e)/((a^2*e^(-2*f*x - 2*e) + a^2)*f) - log(e^(-f*x - e) + 1)/(a^(3/2)*f) + log(e^(-f*x - e) - 1)/(a^(3/2)*f)`

Fricas [B] time = 1.85301, size = 706, normalized size = 13.32

$$\frac{\sqrt{ae^{4fx+4e} + 2ae^{2fx+2e} + a} \left(2 \cosh(fx + e) e^{fx+e} + \left(2 \cosh(fx + e) e^{fx+e} \sinh(fx + e) + e^{fx+e} \sinh(fx + e) \right)^2 \right)}{a^2 f \cosh(fx + e)^2 + a^2 f + \left(a^2 f e^{2fx+2e} + a^2 f \right) \sinh(fx + e)^2 + \left(a^2 f \cosh(fx + e)^2 + a^2 f \right) e^{2fx+2e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(2*cosh(f*x + e)*e^(f*x + e) + (2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e))^2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)) + 2*e^(f*x + e)*sinh(f*x + e)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^2 + a^2*f + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 2*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e))`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(e + fx)}{(a(\sinh^2(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)

[Out] Integral(coth(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)

Giac [A] time = 1.35105, size = 84, normalized size = 1.58

$$\frac{\frac{\log(e^{(fx+e)}+1)}{a^{\frac{3}{2}}} - \frac{\log\left(\left|e^{(fx+e)}-1\right|\right)}{a^{\frac{3}{2}}} - \frac{2e^{(fx+e)}}{a^{\frac{3}{2}}(e^{(2fx+2e)}+1)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] $-(\log(e^{(f*x + e)} + 1)/a^{(3/2)} - \log(\text{abs}(e^{(f*x + e)} - 1))/a^{(3/2)} - 2*e^{(f*x + e)}/(a^{(3/2)}*(e^{(2*f*x + 2*e)} + 1)))/f$

$$3.451 \quad \int \frac{\coth^3(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\operatorname{csch}^2(e+fx)\sqrt{a \cosh^2(e+fx)}}{2a^2f}$$

[Out] ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*a^(3/2)*f) - (Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]^2)/(2*a^2*f)

Rubi [A] time = 0.139467, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3176, 3205, 16, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a \cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\operatorname{csch}^2(e+fx)\sqrt{a \cosh^2(e+fx)}}{2a^2f}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*a^(3/2)*f) - (Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]^2)/(2*a^2*f)

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3205

Int[((b_)*sin[(e_) + (f_)*(x_)])^(n_)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{b, e, f, p}, x] && Integ

erQ[(m - 1)/2] && IntegerQ[n/2]

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_)^(n_)), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}} dx &= \int \frac{\coth^3(e+fx)}{(a\cosh^2(e+fx))^{3/2}} dx \\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(ax)^{3/2}} dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)^2\sqrt{ax}} dx, x, \cosh^2(e+fx)\right)}{2af} \\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\text{csch}^2(e+fx)}{2a^2f} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}} dx, x, \cosh^2(e+fx)\right)}{4af} \\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\text{csch}^2(e+fx)}{2a^2f} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}} dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{2a^2f} \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\sqrt{a\cosh^2(e+fx)}\text{csch}^2(e+fx)}{2a^2f}
\end{aligned}$$

Mathematica [A] time = 0.131715, size = 67, normalized size = 1.02

$$-\frac{\cosh^3(e+fx)\left(\text{csch}^2\left(\frac{1}{2}(e+fx)\right) + \text{sech}^2\left(\frac{1}{2}(e+fx)\right) + 4\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] -(Cosh[e + f*x]^3*(Csch[(e + f*x)/2]^2 + 4*Log[Tanh[(e + f*x)/2]] + Sech[(e + f*x)/2]^2))/(8*f*(a*Cosh[e + f*x]^2)^(3/2))

Maple [C] time = 0.069, size = 36, normalized size = 0.6

$$\frac{1}{f}, \text{int/indef0} \left(\frac{1}{(\sinh(fx+e))^3} \frac{1}{a\sqrt{a(\cosh(fx+e))^2}}, \sinh(fx+e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x)`

[Out] ``int/indef0`(1/sinh(f*x+e)^3/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [A] time = 2.11193, size = 135, normalized size = 2.05

$$\frac{e^{(-fx-e)} + e^{(-3fx-3e)}}{\left(2a^{\frac{3}{2}}e^{(-2fx-2e)} - a^{\frac{3}{2}}e^{(-4fx-4e)} - a^{\frac{3}{2}}\right)f} + \frac{\log\left(e^{(-fx-e)} + 1\right)}{2a^{\frac{3}{2}}f} - \frac{\log\left(e^{(-fx-e)} - 1\right)}{2a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `(e^(-f*x - e) + e^(-3*f*x - 3*e))/((2*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - a^(3/2))*f) + 1/2*log(e^(-f*x - e) + 1)/(a^(3/2)*f) - 1/2*log(e^(-f*x - e) - 1)/(a^(3/2)*f)`

Fricas [B] time = 1.87461, size = 1458, normalized size = 22.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `-1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x + e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) + 1)/(cosh(f*x + e) + sinh(f*x + e) - 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^4 + 4*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f + (3*a^2*f*cosh(f*x`

```
+ e)^2 - a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^4 -
  2*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 4*(a^2*f*cosh(f*x + e)^
3 - a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*e^(
2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(e + fx)}{(a(\sinh^2(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.452 \quad \int \frac{\tanh^2(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=106

$$\frac{\tanh(e+fx)}{8af\sqrt{a \cosh^2(e+fx)}} + \frac{\cosh(e+fx) \tan^{-1}(\sinh(e+fx))}{8af\sqrt{a \cosh^2(e+fx)}} - \frac{\tanh(e+fx) \operatorname{sech}^2(e+fx)}{4af\sqrt{a \cosh^2(e+fx)}}$$

[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) + Tanh[e + f*x]/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Sech[e + f*x]^2*Tanh[e + f*x])/(4*a*f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.159819, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3176, 3207, 2611, 3768, 3770}

$$\frac{\tanh(e+fx)}{8af\sqrt{a \cosh^2(e+fx)}} + \frac{\cosh(e+fx) \tan^{-1}(\sinh(e+fx))}{8af\sqrt{a \cosh^2(e+fx)}} - \frac{\tanh(e+fx) \operatorname{sech}^2(e+fx)}{4af\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) + Tanh[e + f*x]/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Sech[e + f*x]^2*Tanh[e + f*x])/(4*a*f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^p, x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]

&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])

Rule 2611

Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] && NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^2(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx &= \int \frac{\tanh^2(e + fx)}{(a \cosh^2(e + fx))^{3/2}} dx \\
 &= \frac{\cosh(e + fx) \int \operatorname{sech}^3(e + fx) \tanh^2(e + fx) dx}{a \sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{\operatorname{sech}^2(e + fx) \tanh(e + fx)}{4af \sqrt{a \cosh^2(e + fx)}} + \frac{\cosh(e + fx) \int \operatorname{sech}^3(e + fx) dx}{4a \sqrt{a \cosh^2(e + fx)}} \\
 &= \frac{\tanh(e + fx)}{8af \sqrt{a \cosh^2(e + fx)}} - \frac{\operatorname{sech}^2(e + fx) \tanh(e + fx)}{4af \sqrt{a \cosh^2(e + fx)}} + \frac{\cosh(e + fx) \int \operatorname{sech}(e + fx) dx}{8a \sqrt{a \cosh^2(e + fx)}} \\
 &= \frac{\tan^{-1}(\sinh(e + fx)) \cosh(e + fx)}{8af \sqrt{a \cosh^2(e + fx)}} + \frac{\tanh(e + fx)}{8af \sqrt{a \cosh^2(e + fx)}} - \frac{\operatorname{sech}^2(e + fx) \tanh(e + fx)}{4af \sqrt{a \cosh^2(e + fx)}}
 \end{aligned}$$

Mathematica [A] time = 0.093846, size = 58, normalized size = 0.55

$$\frac{\tanh(e + fx) (1 - 2\operatorname{sech}^2(e + fx)) + \cosh(e + fx) \tan^{-1}(\sinh(e + fx))}{8af\sqrt{a \cosh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (1 - 2*Sech[e + f*x]^2)*Tanh[e + f*x])/((8*a*f*Sqrt[a*Cosh[e + f*x]^2]))

Maple [A] time = 0.142, size = 69, normalized size = 0.7

$$\frac{\arctan(\sinh(fx + e)) (\cosh(fx + e))^4 + (\cosh(fx + e))^2 \sinh(fx + e) - 2 \sinh(fx + e)}{8a (\cosh(fx + e))^3 f} \frac{1}{\sqrt{a (\cosh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2), x)

[Out] 1/8/a*(arctan(sinh(f*x+e))*cosh(f*x+e)^4+cosh(f*x+e)^2*sinh(f*x+e)-2*sinh(f*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f

Maxima [B] time = 1.89891, size = 498, normalized size = 4.7

$$\frac{3e^{(-fx-e)} + 11e^{(-3fx-3e)} - 11e^{(-5fx-5e)} - 3e^{(-7fx-7e)}}{4a^{\frac{3}{2}}e^{(-2fx-2e)} + 6a^{\frac{3}{2}}e^{(-4fx-4e)} + 4a^{\frac{3}{2}}e^{(-6fx-6e)} + a^{\frac{3}{2}}e^{(-8fx-8e)} + a^{\frac{3}{2}}} - \frac{3 \arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}} + \frac{15e^{(-fx-e)} + 55e^{(-3fx-3e)} + 73e^{(-5fx-5e)}}{48\left(4a^{\frac{3}{2}}e^{(-2fx-2e)} + 6a^{\frac{3}{2}}e^{(-4fx-4e)} + 4a^{\frac{3}{2}}e^{(-6fx-6e)} + a^{\frac{3}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2), x, algorithm="maxima")

[Out] -1/8*((3*e^(-f*x - e) + 11*e^(-3*f*x - 3*e) - 11*e^(-5*f*x - 5*e) - 3*e^(-7*f*x - 7*e))/(4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(3/2)*e^(-4*f*x - 4*e) + 4*a

$$\begin{aligned} & \left(a^{3/2} e^{-6fx - 6e} + a^{3/2} e^{-8fx - 8e} + a^{3/2} \right) - 3 \arctan\left(\frac{e^{-fx - e}}{a^{3/2}} \right) / f + 1/48 \left(15 e^{-fx - e} + 55 e^{-3fx - 3e} + 73 e^{-5fx - 5e} - 15 e^{-7fx - 7e} \right) / \left(4 a^{3/2} e^{-2fx - 2e} + 6 a^{3/2} e^{-4fx - 4e} + 4 a^{3/2} e^{-6fx - 6e} + a^{3/2} e^{-8fx - 8e} + a^{3/2} \right) * f \\ & + 1/48 \left(15 e^{-fx - e} - 73 e^{-3fx - 3e} - 55 e^{-5fx - 5e} - 15 e^{-7fx - 7e} \right) / \left(4 a^{3/2} e^{-2fx - 2e} + 6 a^{3/2} e^{-4fx - 4e} + 4 a^{3/2} e^{-6fx - 6e} + a^{3/2} e^{-8fx - 8e} + a^{3/2} \right) * f - 5/8 \arctan\left(\frac{e^{-fx - e}}{a^{3/2}} \right) / \left(a^{3/2} * f \right) \end{aligned}$$

Fricas [B] time = 2.00091, size = 3698, normalized size = 34.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] $1/4 * (7 * \cosh(f*x + e) * e^{f*x + e} * \sinh(f*x + e)^6 + e^{f*x + e} * \sinh(f*x + e)^7 + 7 * (3 * \cosh(f*x + e)^2 - 1) * e^{f*x + e} * \sinh(f*x + e)^5 + 35 * (\cosh(f*x + e)^3 - \cosh(f*x + e)) * e^{f*x + e} * \sinh(f*x + e)^4 + 7 * (5 * \cosh(f*x + e)^4 - 10 * \cosh(f*x + e)^2 + 1) * e^{f*x + e} * \sinh(f*x + e)^3 + 7 * (3 * \cosh(f*x + e)^5 - 10 * \cosh(f*x + e)^3 + 3 * \cosh(f*x + e)) * e^{f*x + e} * \sinh(f*x + e)^2 + (7 * \cosh(f*x + e)^6 - 35 * \cosh(f*x + e)^4 + 21 * \cosh(f*x + e)^2 - 1) * e^{f*x + e} * \sinh(f*x + e) + (8 * \cosh(f*x + e) * e^{f*x + e} * \sinh(f*x + e)^7 + e^{f*x + e} * \sinh(f*x + e)^8 + 4 * (7 * \cosh(f*x + e)^2 + 1) * e^{f*x + e} * \sinh(f*x + e)^6 + 8 * (7 * \cosh(f*x + e)^3 + 3 * \cosh(f*x + e)) * e^{f*x + e} * \sinh(f*x + e)^5 + 2 * (35 * \cosh(f*x + e)^4 + 30 * \cosh(f*x + e)^2 + 3) * e^{f*x + e} * \sinh(f*x + e)^4 + 8 * (7 * \cosh(f*x + e)^5 + 10 * \cosh(f*x + e)^3 + 3 * \cosh(f*x + e)) * e^{f*x + e} * \sinh(f*x + e)^3 + 4 * (7 * \cosh(f*x + e)^6 + 15 * \cosh(f*x + e)^4 + 9 * \cosh(f*x + e)^2 + 1) * e^{f*x + e} * \sinh(f*x + e)^2 + 8 * (\cosh(f*x + e)^7 + 3 * \cosh(f*x + e)^5 + 3 * \cosh(f*x + e)^3 + \cosh(f*x + e)) * e^{f*x + e} * \sinh(f*x + e) + (\cosh(f*x + e)^8 + 4 * \cosh(f*x + e)^6 + 6 * \cosh(f*x + e)^4 + 4 * \cosh(f*x + e)^2 + 1) * e^{f*x + e} * \arctan(\cosh(f*x + e) + \sinh(f*x + e)) + (\cosh(f*x + e)^7 - 7 * \cosh(f*x + e)^5 + 7 * \cosh(f*x + e)^3 - \cosh(f*x + e)) * e^{f*x + e} * \sqrt{a * e^{4 * f * x + 4 * e} + 2 * a * e^{2 * f * x + 2 * e} + a} * e^{-f * x - e} / (a^2 * f * \cosh(f*x + e)^8 + 4 * a^2 * f * \cosh(f*x + e)^6 + (a^2 * f * e^{2 * f * x + 2 * e} + a^2 * f) * \sinh(f*x + e)^8 + 8 * (a^2 * f * \cosh(f*x + e) * e^{2 * f * x + 2 * e} + a^2 * f * \cosh(f*x + e)) * \sinh(f*x + e)^7 + 6 * a^2 * f * \cosh(f*x + e)^4 + 4 * (7 * a^2 * f * \cosh(f*x + e)^2 + a^2 * f + (7 * a^2 * f * \cosh(f*x + e)^2 + a^2 * f) * e^{2 * f * x + 2 * e}) * \sinh(f*x + e)^6 + 8 * (7 * a^2 * f * \cosh(f*x + e)^3 + 3 * a^2 * f * \cosh(f*x + e) + (7 * a^2 * f * \cosh(f*x + e)^3 + 3 * a^2 * f * \cosh(f*x + e)) * e^{2 * f * x + 2 * e}) * \sinh(f*x + e)^5 + 4 * a^2 * f * \cosh(f*x + e)^2 + 2 * (35 * a^2 * f * \cosh(f*x + e)^4 + 30 * a^2 * f * \cosh(f*x + e)^2 + 3 * a^2 * f + (35 * a^2$

```
*f*cosh(f*x + e)^4 + 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*s
inh(f*x + e)^4 + 8*(7*a^2*f*cosh(f*x + e)^5 + 10*a^2*f*cosh(f*x + e)^3 + 3*
a^2*f*cosh(f*x + e) + (7*a^2*f*cosh(f*x + e)^5 + 10*a^2*f*cosh(f*x + e)^3 +
3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*f + 4*(7*a^2
*f*cosh(f*x + e)^6 + 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f*x + e)^2 + a
^2*f + (7*a^2*f*cosh(f*x + e)^6 + 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f
*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^
8 + 4*a^2*f*cosh(f*x + e)^6 + 6*a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e) + 8*(a^2*f*cosh(f*x + e)^7 + 3*a^2*f*cosh(f*x
+ e)^5 + 3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x +
e)^7 + 3*a^2*f*cosh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x
+ e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(e + fx)}{(a(\sinh^2(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)

[Out] Integral(tanh(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)

Giac [A] time = 1.42199, size = 126, normalized size = 1.19

$$\frac{\frac{\arctan\left(e^{(fx+e)}\right)}{a^{\frac{3}{2}}} + \frac{\sqrt{ae^{(7fx+7e)}} - 7\sqrt{ae^{(5fx+5e)}} + 7\sqrt{ae^{(3fx+3e)}} - \sqrt{ae^{(fx+e)}}}{a^2\left(e^{(2fx+2e)} + 1\right)^4}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] 1/4*(arctan(e^(f*x + e))/a^(3/2) + (sqrt(a)*e^(7*f*x + 7*e) - 7*sqrt(a)*e^(5*f*x + 5*e) + 7*sqrt(a)*e^(3*f*x + 3*e) - sqrt(a)*e^(f*x + e))/(a^2*(e^(2*f*x + 2*e) + 1)^4))/f

$$3.453 \quad \int \frac{\coth^2(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=64

$$\frac{\coth(e+fx)}{af\sqrt{a \cosh^2(e+fx)}} - \frac{\cosh(e+fx) \tan^{-1}(\sinh(e+fx))}{af\sqrt{a \cosh^2(e+fx)}}$$

[Out] -((ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(a*f*Sqrt[a*Cosh[e + f*x]^2])) - Coth[e + f*x]/(a*f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.131732, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3176, 3207, 2621, 321, 207}

$$\frac{\coth(e+fx)}{af\sqrt{a \cosh^2(e+fx)}} - \frac{\cosh(e+fx) \tan^{-1}(\sinh(e+fx))}{af\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] -((ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(a*f*Sqrt[a*Cosh[e + f*x]^2])) - Coth[e + f*x]/(a*f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

```
Int[(u_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(p_), x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_)^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]
```

Rule 2621

Int[(csc[(e_.) + (f_.)*(x_.)]*(a_.))^(m_.)*sec[(e_.) + (f_.)*(x_.)]^(n_.), x_Symbol] := -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^(n + 1)/2], x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])

Rule 321

Int[((c_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 207

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^2(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx &= \int \frac{\coth^2(e + fx)}{(a \cosh^2(e + fx))^{3/2}} dx \\
 &= \frac{\cosh(e + fx) \int \operatorname{csch}^2(e + fx) \operatorname{sech}(e + fx) dx}{a \sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{(i \cosh(e + fx)) \operatorname{Subst}\left(\int \frac{x^2}{-1+x^2} dx, x, -i \operatorname{csch}(e + fx)\right)}{af \sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{\coth(e + fx)}{af \sqrt{a \cosh^2(e + fx)}} - \frac{(i \cosh(e + fx)) \operatorname{Subst}\left(\int \frac{1}{-1+x^2} dx, x, -i \operatorname{csch}(e + fx)\right)}{af \sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{\tan^{-1}(\sinh(e + fx)) \cosh(e + fx)}{af \sqrt{a \cosh^2(e + fx)}} - \frac{\coth(e + fx)}{af \sqrt{a \cosh^2(e + fx)}}
 \end{aligned}$$

Mathematica [C] time = 0.0787, size = 46, normalized size = 0.72

$$\frac{\coth(e + fx) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; -\sinh^2(e + fx)\right)}{af\sqrt{a\cosh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] -((Coth[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[e + f*x]^2])/(a*f*Sqrt[a*Cosh[e + f*x]^2]))

Maple [A] time = 0.093, size = 51, normalized size = 0.8

$$\frac{\cosh(fx + e) \left(\arctan(\sinh(fx + e)) \sinh(fx + e) + 1 \right)}{a \sinh(fx + e) f} \frac{1}{\sqrt{a (\cosh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2), x)

[Out] -1/a*cosh(f*x+e)*(arctan(sinh(f*x+e))*sinh(f*x+e)+1)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f

Maxima [B] time = 1.71757, size = 433, normalized size = 6.77

$$\frac{\frac{3\sqrt{ae}^{-fx-e} + 2\sqrt{ae}^{-3fx-3e} + 3\sqrt{ae}^{-5fx-5e}}{a^2e^{-2fx-2e} - a^2e^{-4fx-4e} - a^2e^{-6fx-6e} + a^2} - \frac{3\arctan(e^{-fx-e})}{a^{\frac{3}{2}}}}{2f} - \frac{5\sqrt{ae}^{-fx-e} + 6\sqrt{ae}^{-3fx-3e} - 3\sqrt{ae}^{-5fx-5e}}{4(a^2e^{-2fx-2e} - a^2e^{-4fx-4e} - a^2e^{-6fx-6e} + a^2)f} + \frac{3\sqrt{ae}^{-fx-e}}{4(a^2e^{-2fx-2e} - a^2e^{-4fx-4e} - a^2e^{-6fx-6e} + a^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2), x, algorithm="maxima")

[Out] -1/2*((3*sqrt(a)*e^(-f*x - e) + 2*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*f*x - 5*e))/(a^2*e^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x

$- 6*e) + a^2) - 3*\arctan(e^{(-f*x - e)}/a^{(3/2)})/f - 1/4*(5*\sqrt{a}*e^{(-f*x - e)} + 6*\sqrt{a}*e^{(-3*f*x - 3*e)} - 3*\sqrt{a}*e^{(-5*f*x - 5*e)})/((a^2*e^{(-2*f*x - 2*e)} - a^2*e^{(-4*f*x - 4*e)} - a^2*e^{(-6*f*x - 6*e)} + a^2)*f) + 1/4*(3*\sqrt{a}*e^{(-f*x - e)} - 6*\sqrt{a}*e^{(-3*f*x - 3*e)} - 5*\sqrt{a}*e^{(-5*f*x - 5*e)})/((a^2*e^{(-2*f*x - 2*e)} - a^2*e^{(-4*f*x - 4*e)} - a^2*e^{(-6*f*x - 6*e)} + a^2)*f) + 1/2*\arctan(e^{(-f*x - e)}/(a^{(3/2)}*f)$

Fricas [B] time = 1.80605, size = 652, normalized size = 10.19

$$\frac{2 \left(\left(2 \cosh (fx + e) e^{(fx+e)} \sinh (fx + e) + e^{(fx+e)} \sinh (fx + e) \right)^2 + \left(\cosh (fx + e) \right)^2 - 1 \right) e^{(fx+e)} \arctan (\cosh (fx + e))}{a^2 f \cosh (fx + e)^2 - a^2 f + \left(a^2 f e^{(2fx+2e)} + a^2 f \right) \sinh (fx + e)^2 + \left(a^2 f \cosh (fx + e) \right)^2 - a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] $-2*((2*\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e) + e^{(f*x + e)}*\sinh(f*x + e))^2 + (\cosh(f*x + e)^2 - 1)*e^{(f*x + e)})*\arctan(\cosh(f*x + e) + \sinh(f*x + e)) + \cosh(f*x + e)*e^{(f*x + e)} + e^{(f*x + e)}*\sinh(f*x + e))*\sqrt{a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a}*e^{(-f*x - e)}/(a^2*f*\cosh(f*x + e)^2 - a^2*f + (a^2*f*e^{(2*f*x + 2*e)} + a^2*f)*\sinh(f*x + e)^2 + (a^2*f*\cosh(f*x + e))^2 - a^2*f)*e^{(2*f*x + 2*e)} + 2*(a^2*f*\cosh(f*x + e)*e^{(2*f*x + 2*e)} + a^2*f*\cosh(f*x + e))*\sinh(f*x + e))$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(e + fx)}{(a(\sinh^2(e + fx) + 1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)

[Out] Integral(coth(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)

Giac [A] time = 1.42971, size = 58, normalized size = 0.91

$$-\frac{2\left(\frac{\arctan\left(e^{(fx+e)}\right)}{a^{\frac{3}{2}}} + \frac{e^{(fx+e)}}{a^{\frac{3}{2}}\left(e^{(2fx+2e)}-1\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] -2*(arctan(e^(f*x + e))/a^(3/2) + e^(f*x + e)/(a^(3/2)*(e^(2*f*x + 2*e) - 1)))/f

$$3.454 \quad \int \frac{\coth^4(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=38

$$-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.126204, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2606, 30}

$$-\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

Int[(u_)*((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_), x_Symbol] := Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_)*((b_)*sin[(e_) + (f_)*(x_)]^(n_))^(p_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sinh[e + f*x]^n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin[e + f*x]/ff)^(n*p), x], x] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p] && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_)*(trig_)[e + f*x])^(m_) /; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig])]

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\coth^4(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx &= \int \frac{\coth^4(e + fx)}{(a \cosh^2(e + fx))^{3/2}} dx \\ &= \frac{\cosh(e + fx) \int \coth(e + fx) \operatorname{csch}^3(e + fx) dx}{a \sqrt{a \cosh^2(e + fx)}} \\ &= \frac{(i \cosh(e + fx)) \operatorname{Subst}\left(\int x^2 dx, x, -i \operatorname{csch}(e + fx)\right)}{af \sqrt{a \cosh^2(e + fx)}} \\ &= -\frac{\coth(e + fx) \operatorname{csch}^2(e + fx)}{3af \sqrt{a \cosh^2(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.0546857, size = 29, normalized size = 0.76

$$\frac{\coth^3(e + fx)}{3f (a \cosh^2(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] -Coth[e + f*x]^3/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.078, size = 35, normalized size = 0.9

$$\frac{\cosh(fx + e)}{3a(\sinh(fx + e))^3} \frac{1}{f\sqrt{a(\cosh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x)`

[Out] `-1/3*cosh(f*x+e)/a/sinh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f`

Maxima [B] time = 1.98643, size = 1111, normalized size = 29.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `1/12*((21*e^(-f*x - e) - 16*e^(-3*f*x - 3*e) + 34*e^(-5*f*x - 5*e) + 8*e^(-7*f*x - 7*e) - 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2)) + 3*arctan(e^(-f*x - e))/a^(3/2) + 9*log(e^(-f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/12*((15*e^(-f*x - e) - 8*e^(-3*f*x - 3*e) - 34*e^(-5*f*x - 5*e) + 16*e^(-7*f*x - 7*e) - 21*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2)) - 3*arctan(e^(-f*x - e))/a^(3/2) + 9*log(e^(-f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/8*((15*e^(-f*x - e) - 20*e^(-3*f*x - 3*e) - 22*e^(-5*f*x - 5*e) - 20*e^(-7*f*x - 7*e) + 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2)) + 15*arctan(e^(-f*x - e))/a^(3/2))/f + 1/48*(45*e^(-f*x - e) - 52*e^(-3*f*x - 3*e) - 74*e^(-5*f*x - 5*e) + 92*e^(-7*f*x - 7*e) + 21*e^(-9*f*x - 9*e))/((a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2))*f) + 1/48*(21*e^(-f*x - e) + 92*e^(-3*f*x - 3*e) - 74*e^(-5*f*x - 5*e) - 52*e^(-7*f*x - 7*e) + 45*e^(-9*f*x - 9*e))/((a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2))*f) + 11`

$$/8*\arctan(e^{(-f*x - e)})/(a^{(3/2)*f})$$

Fricas [B] time = 1.78055, size = 1511, normalized size = 39.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(f*x + e)^3*e^(f*x + e) + 3*cosh(f*x + e)^2*e^(f*x + e)*sinh(f*x + e) + 3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x + e)^3)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^6 + 6*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^2 + 3*(5*a^2*f*cosh(f*x + e)^2 - a^2*f + (5*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 4*(5*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e) + (5*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 - a^2*f + 3*(5*a^2*f*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f + (5*a^2*f*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 + 3*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e) + 6*(a^2*f*cosh(f*x + e)^5 - 2*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^5 - 2*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.65161, size = 43, normalized size = 1.13

$$-\frac{8e^{(3fx+3e)}}{3a^{\frac{3}{2}}f\left(e^{(2fx+2e)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] -8/3*e^(3*f*x + 3*e)/(a^(3/2)*f*(e^(2*f*x + 2*e) - 1)^3)

$$3.455 \quad \int \frac{\coth^6(e+fx)}{\left(a+a \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=77

$$\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a \cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a \cosh^2(e+fx)}}$$

[Out] $-(\operatorname{Coth}[e + f*x]*\operatorname{Csch}[e + f*x]^2)/(3*a*f*\operatorname{Sqrt}[a*\operatorname{Cosh}[e + f*x]^2]) - (\operatorname{Coth}[e + f*x]*\operatorname{Csch}[e + f*x]^4)/(5*a*f*\operatorname{Sqrt}[a*\operatorname{Cosh}[e + f*x]^2])$

Rubi [A] time = 0.143745, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2606, 14}

$$\frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a \cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[e + f*x]^6/(a + a*\operatorname{Sinh}[e + f*x]^2)^{(3/2)}, x]$

[Out] $-(\operatorname{Coth}[e + f*x]*\operatorname{Csch}[e + f*x]^2)/(3*a*f*\operatorname{Sqrt}[a*\operatorname{Cosh}[e + f*x]^2]) - (\operatorname{Coth}[e + f*x]*\operatorname{Csch}[e + f*x]^4)/(5*a*f*\operatorname{Sqrt}[a*\operatorname{Cosh}[e + f*x]^2])$

Rule 3176

$\operatorname{Int}[(u_.)*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^2)^{(p_.)}, x_Symbol] \rightarrow \operatorname{Int}[\operatorname{ActivateTrig}[u*(a*\cos[e + f*x]^2)^p], x] /;$ $\operatorname{FreeQ}\{a, b, e, f, p\}, x \ \&\& \ \operatorname{EqQ}[a + b, 0]$

Rule 3207

$\operatorname{Int}[(u_.)*((b_.)*\sin[(e_.) + (f_.)*(x_)]^n)^{(p_.)}, x_Symbol] \rightarrow \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\sin[e + f*x], x]\}, \operatorname{Dist}[(b*ff^n)^{\operatorname{IntPart}[p]}*(b*\sin[e + f*x]^n)^{\operatorname{FracPart}[p]}]/(\sin[e + f*x]/ff)^{(n*\operatorname{FracPart}[p])}, \operatorname{Int}[\operatorname{ActivateTrig}[u]*(\sin[e + f*x]/ff)^{(n*p)}, x], x]\} /;$ $\operatorname{FreeQ}\{b, e, f, n, p\}, x \ \&\& \ !\operatorname{IntegerQ}[p] \ \&\& \ \operatorname{IntegerQ}[n] \ \&\& \ (\operatorname{EqQ}[u, 1] \ || \ \operatorname{MatchQ}[u, ((d_.)*(\operatorname{trig}_)[e + f*x])^{(m_.)}] /;$ $\operatorname{FreeQ}\{d, m\}, x \ \&\& \ \operatorname{MemberQ}\{\sin, \cos, \tan, \cot, \sec, \csc\}, \operatorname{trig}]]$

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\coth^6(e + fx)}{(a + a \sinh^2(e + fx))^{3/2}} dx &= \int \frac{\coth^6(e + fx)}{(a \cosh^2(e + fx))^{3/2}} dx \\
 &= \frac{\cosh(e + fx) \int \coth^3(e + fx) \operatorname{csch}^3(e + fx) dx}{a \sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{(i \cosh(e + fx)) \operatorname{Subst}\left(\int x^2 (-1 + x^2) dx, x, -i \operatorname{csch}(e + fx)\right)}{af \sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{(i \cosh(e + fx)) \operatorname{Subst}\left(\int (-x^2 + x^4) dx, x, -i \operatorname{csch}(e + fx)\right)}{af \sqrt{a \cosh^2(e + fx)}} \\
 &= -\frac{\coth(e + fx) \operatorname{csch}^2(e + fx)}{3af \sqrt{a \cosh^2(e + fx)}} - \frac{\coth(e + fx) \operatorname{csch}^4(e + fx)}{5af \sqrt{a \cosh^2(e + fx)}}
 \end{aligned}$$

Mathematica [A] time = 0.108842, size = 41, normalized size = 0.53

$$-\frac{\coth^3(e + fx) (3 \operatorname{csch}^2(e + fx) + 5)}{15f (a \cosh^2(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^6/(a + a*Sinh[e + f*x]^2)^(3/2),x]

[Out] -(Coth[e + f*x]^3*(5 + 3*CsCh[e + f*x]^2))/(15*f*(a*Cosh[e + f*x]^2)^(3/2))

Maple [A] time = 0.132, size = 67, normalized size = 0.9

$$\frac{\cosh(fx + e) \left(5 \left(\cosh(fx + e) \right)^2 - 2 \right)}{15 \left(\cosh(fx + e) - 1 \right)^2 \left(\cosh(fx + e) + 1 \right)^2 a \sinh(fx + e) f \sqrt{a \left(\cosh(fx + e) \right)^2}} 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x)`

[Out] `-1/15*cosh(f*x+e)*(5*cosh(f*x+e)^2-2)/(cosh(f*x+e)-1)^2/(cosh(f*x+e)+1)^2/a/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f`

Maxima [B] time = 2.32633, size = 2067, normalized size = 26.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `-3/256*(2*(105*e^(-f*x - e) - 300*e^(-3*f*x - 3*e) + 81*e^(-5*f*x - 5*e) - 248*e^(-7*f*x - 7*e) + 51*e^(-9*f*x - 9*e) + 100*e^(-11*f*x - 11*e) - 45*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) + 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log(e^(-f*x - e) + 1)/a^(3/2) - 75*log(e^(-f*x - e) - 1)/a^(3/2))/f + 1/48*((105*e^(-f*x - e) - 350*e^(-3*f*x - 3*e) + 231*e^(-5*f*x - 5*e) + 412*e^(-7*f*x - 7*e) + 231*e^(-9*f*x - 9*e) - 350*e^(-11*f*x - 11*e) + 105*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) + 105*arctan(e^(-f*x - e))/a^(3/2))/f + 3/256*(2*(45*e^(-f*x - e) - 100*e^(-3*f*x - 3*e) - 51*e^(-5*f*x - 5*e) + 248*e^(-7*f*x - 7*e) - 81*e^(-9*f*x - 9*e) + 300*e^(-11*f*x - 11*e) - 105*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) - 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log(e^(-f*x - e) + 1)/a^(3/2) - 75*log(e^(-f*x - e) - 1)/a^(3/2))/f - 3/320*(4*(45*e^(-f*x - e)`

$$\begin{aligned}
& - 135e^{(-3fx - 3e)} + 54e^{(-5fx - 5e)} + 198e^{(-7fx - 7e)} - 211e^{(-9fx - 9e)} - 15e^{(-11fx - 11e)}) / (3a^{(3/2)}e^{(-2fx - 2e)} - a^{(3/2)}e^{(-4fx - 4e)} - 5a^{(3/2)}e^{(-6fx - 6e)} + 5a^{(3/2)}e^{(-8fx - 8e)} + a^{(3/2)}e^{(-10fx - 10e)} - 3a^{(3/2)}e^{(-12fx - 12e)} + a^{(3/2)}e^{(-14fx - 14e)} - a^{(3/2)}) + 90\arctan(e^{(-fx - e)})/a^{(3/2)} + 45\log(e^{(-fx - e)} + 1)/a^{(3/2)} - 45\log(e^{(-fx - e)} - 1)/a^{(3/2)})/f + 3/320*(4*(15e^{(-3fx - 3e)} + 211e^{(-5fx - 5e)} - 198e^{(-7fx - 7e)} - 54e^{(-9fx - 9e)} + 135e^{(-11fx - 11e)} - 45e^{(-13fx - 13e)}) / (3a^{(3/2)}e^{(-2fx - 2e)} - a^{(3/2)}e^{(-4fx - 4e)} - 5a^{(3/2)}e^{(-6fx - 6e)} + 5a^{(3/2)}e^{(-8fx - 8e)} + a^{(3/2)}e^{(-10fx - 10e)} - 3a^{(3/2)}e^{(-12fx - 12e)} - a^{(3/2)}e^{(-14fx - 14e)} - a^{(3/2)}) - 90\arctan(e^{(-fx - e)})/a^{(3/2)} + 45\log(e^{(-fx - e)} + 1)/a^{(3/2)} - 45\log(e^{(-fx - e)} - 1)/a^{(3/2)})/f + 1/1920*(1155e^{(-fx - e)} + 1460e^{(-3fx - 3e)} - 4173e^{(-5fx - 5e)} + 2024e^{(-7fx - 7e)} + 1857e^{(-9fx - 9e)} - 2140e^{(-11fx - 11e)} + 585e^{(-13fx - 13e)}) / ((3a^{(3/2)}e^{(-2fx - 2e)} - a^{(3/2)}e^{(-4fx - 4e)} - 5a^{(3/2)}e^{(-6fx - 6e)} + 5a^{(3/2)}e^{(-8fx - 8e)} + a^{(3/2)}e^{(-10fx - 10e)} - 3a^{(3/2)}e^{(-12fx - 12e)} + a^{(3/2)}e^{(-14fx - 14e)} - a^{(3/2)}) * f) + 1/1920*(585e^{(-fx - e)} - 2140e^{(-3fx - 3e)} + 1857e^{(-5fx - 5e)} + 2024e^{(-7fx - 7e)} - 4173e^{(-9fx - 9e)} + 1460e^{(-11fx - 11e)} + 1155e^{(-13fx - 13e)}) / ((3a^{(3/2)}e^{(-2fx - 2e)} - a^{(3/2)}e^{(-4fx - 4e)} - 5a^{(3/2)}e^{(-6fx - 6e)} + 5a^{(3/2)}e^{(-8fx - 8e)} + a^{(3/2)}e^{(-10fx - 10e)} - 3a^{(3/2)}e^{(-12fx - 12e)} + a^{(3/2)}e^{(-14fx - 14e)} - a^{(3/2)}) * f) + 29/32\arctan(e^{(-fx - e)}) / (a^{(3/2)} * f)
\end{aligned}$$

Fricas [B] time = 1.94361, size = 3571, normalized size = 46.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned}
& -8/15*(35\cosh(f*x + e)*e^{(f*x + e)}*\sinh(f*x + e)^6 + 5e^{(f*x + e)}*\sinh(f*x + e)^7 + (105\cosh(f*x + e)^2 + 2)*e^{(f*x + e)}*\sinh(f*x + e)^5 + 5*(35\cosh(f*x + e)^3 + 2*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^4 + 5*(35\cosh(f*x + e)^4 + 4*\cosh(f*x + e)^2 + 1)*e^{(f*x + e)}*\sinh(f*x + e)^3 + 5*(21\cosh(f*x + e)^5 + 4*\cosh(f*x + e)^3 + 3*\cosh(f*x + e))*e^{(f*x + e)}*\sinh(f*x + e)^2 + 5*(7*\cosh(f*x + e)^6 + 2*\cosh(f*x + e)^4 + 3*\cosh(f*x + e)^2)*e^{(f*x + e)}*\sinh(f*x + e) + (5*\cosh(f*x + e)^7 + 2*\cosh(f*x + e)^5 + 5*\cosh(f*x + e)^3)*e^{(f*x + e)}*\sqrt{a*e^{(4*f*x + 4*e)} + 2*a*e^{(2*f*x + 2*e)} + a}*e^{(-f*x - e)} / (a^2*f*\cosh(f*x + e)^{10} - 5*a^2*f*\cosh(f*x + e)^8 + (a^2*f*e^{(2*f*x
\end{aligned}$$

$$\begin{aligned}
& + 2*e) + a^2*f)*\sinh(f*x + e)^{10} + 10*(a^2*f*\cosh(f*x + e)*e^{(2*f*x + 2*e)} \\
& + a^2*f*\cosh(f*x + e))*\sinh(f*x + e)^9 + 10*a^2*f*\cosh(f*x + e)^6 + 5*(9*a^2*f*\cosh(f*x + e)^2 - a^2*f + (9*a^2*f*\cosh(f*x + e)^2 - a^2*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^8 + 40*(3*a^2*f*\cosh(f*x + e)^3 - a^2*f*\cosh(f*x + e) + (3*a^2*f*\cosh(f*x + e)^3 - a^2*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^7 - 10*a^2*f*\cosh(f*x + e)^4 + 10*(21*a^2*f*\cosh(f*x + e)^4 - 14*a^2*f*\cosh(f*x + e)^2 + a^2*f + (21*a^2*f*\cosh(f*x + e)^4 - 14*a^2*f*\cosh(f*x + e)^2 + a^2*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^6 + 4*(63*a^2*f*\cosh(f*x + e)^5 - 70*a^2*f*\cosh(f*x + e)^3 + 15*a^2*f*\cosh(f*x + e) + (63*a^2*f*\cosh(f*x + e)^5 - 70*a^2*f*\cosh(f*x + e)^3 + 15*a^2*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^5 + 5*a^2*f*\cosh(f*x + e)^2 + 10*(21*a^2*f*\cosh(f*x + e)^6 - 35*a^2*f*\cosh(f*x + e)^4 + 15*a^2*f*\cosh(f*x + e)^2 - a^2*f + (21*a^2*f*\cosh(f*x + e)^6 - 35*a^2*f*\cosh(f*x + e)^4 + 15*a^2*f*\cosh(f*x + e)^2 - a^2*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^4 + 40*(3*a^2*f*\cosh(f*x + e)^7 - 7*a^2*f*\cosh(f*x + e)^5 + 5*a^2*f*\cosh(f*x + e)^3 - a^2*f*\cosh(f*x + e))^e^{(2*f*x + 2*e)}*\sinh(f*x + e)^3 - a^2*f + 5*(9*a^2*f*\cosh(f*x + e)^8 - 28*a^2*f*\cosh(f*x + e)^6 + 30*a^2*f*\cosh(f*x + e)^4 - 12*a^2*f*\cosh(f*x + e)^2 + a^2*f + (9*a^2*f*\cosh(f*x + e)^8 - 28*a^2*f*\cosh(f*x + e)^6 + 30*a^2*f*\cosh(f*x + e)^4 - 12*a^2*f*\cosh(f*x + e)^2 + a^2*f)*e^{(2*f*x + 2*e)})*\sinh(f*x + e)^2 + (a^2*f*\cosh(f*x + e)^{10} - 5*a^2*f*\cosh(f*x + e)^8 + 10*a^2*f*\cosh(f*x + e)^6 - 10*a^2*f*\cosh(f*x + e)^4 + 5*a^2*f*\cosh(f*x + e)^2 - a^2*f)*e^{(2*f*x + 2*e)} + 10*(a^2*f*\cosh(f*x + e)^9 - 4*a^2*f*\cosh(f*x + e)^7 + 6*a^2*f*\cosh(f*x + e)^5 - 4*a^2*f*\cosh(f*x + e)^3 + a^2*f*\cosh(f*x + e) + (a^2*f*\cosh(f*x + e)^9 - 4*a^2*f*\cosh(f*x + e)^7 + 6*a^2*f*\cosh(f*x + e)^5 - 4*a^2*f*\cosh(f*x + e)^3 + a^2*f*\cosh(f*x + e))*e^{(2*f*x + 2*e)}))*\sinh(f*x + e))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [A] time = 1.80338, size = 92, normalized size = 1.19

$$\frac{8 \left(5 \sqrt{ae}^{(7fx+7e)} + 2 \sqrt{ae}^{(5fx+5e)} + 5 \sqrt{ae}^{(3fx+3e)} \right)}{15 a^2 f \left(e^{(2fx+2e)} - 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] -8/15*(5*sqrt(a)*e^(7*f*x + 7*e) + 2*sqrt(a)*e^(5*f*x + 5*e) + 5*sqrt(a)*e^(3*f*x + 3*e))/(a^2*f*(e^(2*f*x + 2*e) - 1)^5)

$$3.456 \quad \int \frac{\coth^8(e+fx)}{(a+a \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=115

$$\frac{\coth(e+fx)\operatorname{csch}^6(e+fx)}{7af\sqrt{a \cosh^2(e+fx)}} - \frac{2 \coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a \cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a \cosh^2(e+fx)}}$$

[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (2*Coth[e + f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x]^6)/(7*a*f*Sqrt[a*Cosh[e + f*x]^2])

Rubi [A] time = 0.149823, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3176, 3207, 2606, 270}

$$\frac{\coth(e+fx)\operatorname{csch}^6(e+fx)}{7af\sqrt{a \cosh^2(e+fx)}} - \frac{2 \coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a \cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a \cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2), x]

[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (2*Coth[e + f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x]^6)/(7*a*f*Sqrt[a*Cosh[e + f*x]^2])

Rule 3176

Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[ActivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ[a + b, 0]

Rule 3207

Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Ssin[e + f*x]^n)^FracPart[p]]/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u*(Sin[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]

```
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] := Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
&& !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^8(e+fx)}{(a+a\sinh^2(e+fx))^{3/2}} dx &= \int \frac{\coth^8(e+fx)}{(a\cosh^2(e+fx))^{3/2}} dx \\
&= \frac{\cosh(e+fx) \int \coth^5(e+fx) \operatorname{csch}^3(e+fx) dx}{a\sqrt{a\cosh^2(e+fx)}} \\
&= \frac{(i\cosh(e+fx)) \operatorname{Subst}\left(\int x^2(-1+x^2)^2 dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}} \\
&= \frac{(i\cosh(e+fx)) \operatorname{Subst}\left(\int (x^2-2x^4+x^6) dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}} - \frac{2\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^6(e+fx)}{7af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.136423, size = 51, normalized size = 0.44

$$\frac{\coth^3(e+fx)(15\operatorname{csch}^4(e+fx) + 42\operatorname{csch}^2(e+fx) + 35)}{105f(a\cosh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2),x]

[Out] $-(\text{Coth}[e + f*x]^3(35 + 42*\text{Csch}[e + f*x]^2 + 15*\text{Csch}[e + f*x]^4))/(105*f*(a*\text{Cosh}[e + f*x]^2)^(3/2))$

Maple [A] time = 0.105, size = 57, normalized size = 0.5

$$\frac{\cosh(fx + e) \left(35 (\cosh(fx + e))^4 - 28 (\cosh(fx + e))^2 + 8 \right)}{105 a (\sinh(fx + e))^7 f} \frac{1}{\sqrt{a (\cosh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x)

[Out] $-1/105*\cosh(f*x+e)*(35*\cosh(f*x+e)^4-28*\cosh(f*x+e)^2+8)/a/\sinh(f*x+e)^7/(a*\cosh(f*x+e)^2)^(1/2)/f$

Maxima [B] time = 2.60066, size = 2992, normalized size = 26.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{3840} * (2 * (4095 * e^{(-f*x - e)} - 20090 * e^{(-3*f*x - 3*e)} + 31654 * e^{(-5*f*x - 5*e)} - 850 * e^{(-7*f*x - 7*e)} - 51148 * e^{(-9*f*x - 9*e)} + 51090 * e^{(-11*f*x - 11*e)} - 2646 * e^{(-13*f*x - 13*e)} + 4410 * e^{(-15*f*x - 15*e)} - 1155 * e^{(-17*f*x - 17*e)}) / (5 * a^{(3/2)} * e^{(-2*f*x - 2*e)} - 8 * a^{(3/2)} * e^{(-4*f*x - 4*e)} + 14 * a^{(3/2)} * e^{(-8*f*x - 8*e)} - 14 * a^{(3/2)} * e^{(-10*f*x - 10*e)} + 8 * a^{(3/2)} * e^{(-14*f*x - 14*e)} - 5 * a^{(3/2)} * e^{(-16*f*x - 16*e)} + a^{(3/2)} * e^{(-18*f*x - 18*e)} - a^{(3/2)}) + 2940 * \arctan(e^{(-f*x - e)}) / a^{(3/2)} + 2625 * \log(e^{(-f*x - e)} + 1) / a^{(3/2)} - 2625 * \log(e^{(-f*x - e)} - 1) / a^{(3/2)}) / f - 1/8960 * (2 * (4095 * e^{(-f*x - e)} - 21630 * e^{(-3*f*x - 3*e)} + 39354 * e^{(-5*f*x - 5*e)} - 13830 * e^{(-7*f*x - 7*e)} - 47848 * e^{(-9*f*x - 9*e)} + 66950 * e^{(-11*f*x - 11*e)} - 22106 * e^{(-13*f*x - 13*e)} - 18690 * e^{(-15*f*x - 15*e)} + 3465 * e^{(-17*f*x - 17*e)}) / (5 * a^{(3/2)} * e^{(-2*f*x - 2*e)} - 8 * a^{(3/2)} * e^{(-4*f*x - 4*e)} + 14 * a^{(3/2)} * e^{(-8*f*x - 8*e)} - 14 * a^{(3/2)} * e^{(-10*f*x - 10*e)} + 8 * a^{(3/2)} * e^{(-14*f*x - 14*e)} - 5 * a^{(3/2)} * e^{(-16*f*x - 16*e)} + a^{(3/2)} * e^{(-18*f*x - 18*e)} - a^{(3/2)})$

$$\begin{aligned}
& f*x - 16*e) + a^{(3/2)}*e^{(-18*f*x - 18*e) - a^{(3/2)}} + 7560*\arctan(e^{(-f*x - e)})/a^{(3/2)} + 315*\log(e^{(-f*x - e) + 1})/a^{(3/2)} - 315*\log(e^{(-f*x - e) - 1})/a^{(3/2)}/f - 1/8960*(2*(3465*e^{(-f*x - e)} - 18690*e^{(-3*f*x - 3*e)} - 22106*e^{(-5*f*x - 5*e)} + 66950*e^{(-7*f*x - 7*e)} - 47848*e^{(-9*f*x - 9*e)} - 13830*e^{(-11*f*x - 11*e)} + 39354*e^{(-13*f*x - 13*e)} - 21630*e^{(-15*f*x - 15*e)} + 4095*e^{(-17*f*x - 17*e)})/(5*a^{(3/2)}*e^{(-2*f*x - 2*e)} - 8*a^{(3/2)}*e^{(-4*f*x - 4*e)} + 14*a^{(3/2)}*e^{(-8*f*x - 8*e)} - 14*a^{(3/2)}*e^{(-10*f*x - 10*e)} + 8*a^{(3/2)}*e^{(-14*f*x - 14*e)} - 5*a^{(3/2)}*e^{(-16*f*x - 16*e)} + a^{(3/2)}*e^{(-18*f*x - 18*e) - a^{(3/2)}} + 7560*\arctan(e^{(-f*x - e)})/a^{(3/2)} - 315*\log(e^{(-f*x - e) + 1})/a^{(3/2)} + 315*\log(e^{(-f*x - e) - 1})/a^{(3/2)}/f - 1/3840*(2*(1155*e^{(-f*x - e)} - 4410*e^{(-3*f*x - 3*e)} + 2646*e^{(-5*f*x - 5*e)} - 51090*e^{(-7*f*x - 7*e)} + 51148*e^{(-9*f*x - 9*e)} + 850*e^{(-11*f*x - 11*e)} - 31654*e^{(-13*f*x - 13*e)} + 20090*e^{(-15*f*x - 15*e)} - 4095*e^{(-17*f*x - 17*e)})/(5*a^{(3/2)}*e^{(-2*f*x - 2*e)} - 8*a^{(3/2)}*e^{(-4*f*x - 4*e)} + 14*a^{(3/2)}*e^{(-8*f*x - 8*e)} - 14*a^{(3/2)}*e^{(-10*f*x - 10*e)} + 8*a^{(3/2)}*e^{(-14*f*x - 14*e)} - 5*a^{(3/2)}*e^{(-16*f*x - 16*e)} + a^{(3/2)}*e^{(-18*f*x - 18*e) - a^{(3/2)}} - 2940*\arctan(e^{(-f*x - e)})/a^{(3/2)} + 2625*\log(e^{(-f*x - e) + 1})/a^{(3/2)} - 2625*\log(e^{(-f*x - e) - 1})/a^{(3/2)}/f + 1/768*(2*(1155*e^{(-f*x - e)} - 5670*e^{(-3*f*x - 3*e)} + 8946*e^{(-5*f*x - 5*e)} - 270*e^{(-7*f*x - 7*e)} + 4696*e^{(-9*f*x - 9*e)} - 2930*e^{(-11*f*x - 11*e)} - 658*e^{(-13*f*x - 13*e)} + 1190*e^{(-15*f*x - 15*e)} - 315*e^{(-17*f*x - 17*e)})/(5*a^{(3/2)}*e^{(-2*f*x - 2*e)} - 8*a^{(3/2)}*e^{(-4*f*x - 4*e)} + 14*a^{(3/2)}*e^{(-8*f*x - 8*e)} - 14*a^{(3/2)}*e^{(-10*f*x - 10*e)} + 8*a^{(3/2)}*e^{(-14*f*x - 14*e)} - 5*a^{(3/2)}*e^{(-16*f*x - 16*e)} + a^{(3/2)}*e^{(-18*f*x - 18*e) - a^{(3/2)}} + 840*\arctan(e^{(-f*x - e)})/a^{(3/2)} + 735*\log(e^{(-f*x - e) + 1})/a^{(3/2)} - 735*\log(e^{(-f*x - e) - 1})/a^{(3/2)}/f - 1/768*(2*(315*e^{(-f*x - e)} - 1190*e^{(-3*f*x - 3*e)} + 658*e^{(-5*f*x - 5*e)} + 2930*e^{(-7*f*x - 7*e)} - 4696*e^{(-9*f*x - 9*e)} + 270*e^{(-11*f*x - 11*e)} - 8946*e^{(-13*f*x - 13*e)} + 5670*e^{(-15*f*x - 15*e)} - 1155*e^{(-17*f*x - 17*e)})/(5*a^{(3/2)}*e^{(-2*f*x - 2*e)} - 8*a^{(3/2)}*e^{(-4*f*x - 4*e)} + 14*a^{(3/2)}*e^{(-8*f*x - 8*e)} - 14*a^{(3/2)}*e^{(-10*f*x - 10*e)} + 8*a^{(3/2)}*e^{(-14*f*x - 14*e)} - 5*a^{(3/2)}*e^{(-16*f*x - 16*e)} + a^{(3/2)}*e^{(-18*f*x - 18*e) - a^{(3/2)}} - 840*\arctan(e^{(-f*x - e)})/a^{(3/2)} + 735*\log(e^{(-f*x - e) + 1})/a^{(3/2)} - 735*\log(e^{(-f*x - e) - 1})/a^{(3/2)}/f - 1/128*((315*e^{(-f*x - e)} - 1680*e^{(-3*f*x - 3*e)} + 3108*e^{(-5*f*x - 5*e)} - 1200*e^{(-7*f*x - 7*e)} - 3646*e^{(-9*f*x - 9*e)} - 1200*e^{(-11*f*x - 11*e)} + 3108*e^{(-13*f*x - 13*e)} - 1680*e^{(-15*f*x - 15*e)} + 315*e^{(-17*f*x - 17*e)})/(5*a^{(3/2)}*e^{(-2*f*x - 2*e)} - 8*a^{(3/2)}*e^{(-4*f*x - 4*e)} + 14*a^{(3/2)}*e^{(-8*f*x - 8*e)} - 14*a^{(3/2)}*e^{(-10*f*x - 10*e)} + 8*a^{(3/2)}*e^{(-14*f*x - 14*e)} - 5*a^{(3/2)}*e^{(-16*f*x - 16*e)} + a^{(3/2)}*e^{(-18*f*x - 18*e) - a^{(3/2)}} + 315*\arctan(e^{(-f*x - e)})/a^{(3/2)}/f + 1/2688*(1155*e^{(-f*x - e)} + 1393*e^{(-3*f*x - 3*e)} - 4865*e^{(-5*f*x - 5*e)} + 3965*e^{(-7*f*x - 7*e)} + 825*e^{(-9*f*x - 9*e)} - 3245*e^{(-11*f*x - 11*e)} + 1925*e^{(-13*f*x - 13*e)} - 385*e^{(-15*f*x - 15*e)})/((5*a^{(3/2)}*e^{(-2*f*x - 2*e)} - 8*a^{(3/2)}*e^{(-4*f*x - 4*e)} + 14*a^{(3/2)}*e^{(-8*f*x - 8*e)} - 14*a^{(3/2)}*e^{(-10*f*x - 10*e)} + 8*a^{(3/2)}*e^{(-14*f*x - 14*e)} - 5*a^{(3/2)}*e^{(-16*f*x - 16*e)} + a^{(3/2)}*e^{(-18*f*x - 18*e) - a^{(3/2)}})*f) - 1/2688*(385*e^{(-3*f*x - 3*e)} - 1925*e^{(-
\end{aligned}$$

$$5f*x - 5e) + 3245e^{(-7f*x - 7e)} - 825e^{(-9f*x - 9e)} - 3965e^{(-11f*x - 11e)} + 4865e^{(-13f*x - 13e)} - 1393e^{(-15f*x - 15e)} - 1155e^{(-17f*x - 17e)} / ((5a^{(3/2)}e^{(-2f*x - 2e)} - 8a^{(3/2)}e^{(-4f*x - 4e)} + 14a^{(3/2)}e^{(-8f*x - 8e)} - 14a^{(3/2)}e^{(-10f*x - 10e)} + 8a^{(3/2)}e^{(-14f*x - 14e)} - 5a^{(3/2)}e^{(-16f*x - 16e)} + a^{(3/2)}e^{(-18f*x - 18e)} - a^{(3/2)}) * f) + 55/128 * \arctan(e^{(-f*x - e)}) / (a^{(3/2)} * f)$$

Fricas [B] time = 2.14295, size = 6649, normalized size = 57.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out]
$$-8/105 * (385 * \cosh(f*x + e) * e^{(f*x + e)} * \sinh(f*x + e)^{10} + 35 * e^{(f*x + e)} * \sinh(f*x + e)^{11} + 7 * (275 * \cosh(f*x + e)^2 + 4) * e^{(f*x + e)} * \sinh(f*x + e)^9 + 21 * (275 * \cosh(f*x + e)^3 + 12 * \cosh(f*x + e)) * e^{(f*x + e)} * \sinh(f*x + e)^8 + 6 * (1925 * \cosh(f*x + e)^4 + 168 * \cosh(f*x + e)^2 + 19) * e^{(f*x + e)} * \sinh(f*x + e)^7 + 42 * (385 * \cosh(f*x + e)^5 + 56 * \cosh(f*x + e)^3 + 19 * \cosh(f*x + e)) * e^{(f*x + e)} * \sinh(f*x + e)^6 + 14 * (1155 * \cosh(f*x + e)^6 + 252 * \cosh(f*x + e)^4 + 171 * \cosh(f*x + e)^2 + 2) * e^{(f*x + e)} * \sinh(f*x + e)^5 + 14 * (825 * \cosh(f*x + e)^7 + 252 * \cosh(f*x + e)^5 + 285 * \cosh(f*x + e)^3 + 10 * \cosh(f*x + e)) * e^{(f*x + e)} * \sinh(f*x + e)^4 + 7 * (825 * \cosh(f*x + e)^8 + 336 * \cosh(f*x + e)^6 + 570 * \cosh(f*x + e)^4 + 40 * \cosh(f*x + e)^2 + 5) * e^{(f*x + e)} * \sinh(f*x + e)^3 + 7 * (275 * \cosh(f*x + e)^9 + 144 * \cosh(f*x + e)^7 + 342 * \cosh(f*x + e)^5 + 40 * \cosh(f*x + e)^3 + 15 * \cosh(f*x + e)) * e^{(f*x + e)} * \sinh(f*x + e)^2 + 7 * (55 * \cosh(f*x + e)^{10} + 36 * \cosh(f*x + e)^8 + 114 * \cosh(f*x + e)^6 + 20 * \cosh(f*x + e)^4 + 15 * \cosh(f*x + e)^2) * e^{(f*x + e)} * \sinh(f*x + e) + (35 * \cosh(f*x + e)^{11} + 28 * \cosh(f*x + e)^9 + 114 * \cosh(f*x + e)^7 + 28 * \cosh(f*x + e)^5 + 35 * \cosh(f*x + e)^3) * e^{(f*x + e)} * \sqrt{a * e^{(4 * f * x + 4 * e)} + 2 * a * e^{(2 * f * x + 2 * e)} + a} * e^{(-f * x - e)} / (a^2 * f * \cosh(f*x + e)^{14} - 7 * a^2 * f * \cosh(f*x + e)^{12} + (a^2 * f * e^{(2 * f * x + 2 * e)} + a^2 * f) * \sinh(f*x + e)^{14} + 14 * (a^2 * f * \cosh(f*x + e) * e^{(2 * f * x + 2 * e)} + a^2 * f * \cosh(f*x + e)) * \sinh(f*x + e)^{13} + 21 * a^2 * f * \cosh(f*x + e)^{10} + 7 * (13 * a^2 * f * \cosh(f*x + e)^2 - a^2 * f + (13 * a^2 * f * \cosh(f*x + e)^2 - a^2 * f) * e^{(2 * f * x + 2 * e)}) * \sinh(f*x + e)^{12} + 28 * (13 * a^2 * f * \cosh(f*x + e)^3 - 3 * a^2 * f * \cosh(f*x + e) + (13 * a^2 * f * \cosh(f*x + e)^3 - 3 * a^2 * f * \cosh(f*x + e)) * e^{(2 * f * x + 2 * e)}) * \sinh(f*x + e)^{11} - 35 * a^2 * f * \cosh(f*x + e)^8 + 7 * (143 * a^2 * f * \cosh(f*x + e)^4 - 66 * a^2 * f * \cosh(f*x + e)^2 + 3 * a^2 * f + (143 * a^2 * f * \cosh(f*x + e)^4 - 66 * a^2 * f * \cosh(f*x + e)^2 + 3 * a^2 * f) * e^{(2 * f * x + 2 * e)}) * \sinh(f*x + e)^{10} + 14 * (143 * a^2 * f * \cosh(f*x + e)^5 - 110 * a^2 * f * \cosh(f*x + e)^3 + 15 * a^2 * f * \cosh(f*x + e) + (143 * a^2 * f * \cosh(f*x + e)^5 - 110 * a^2 * f * \cosh(f*x + e)^3 + 15 * a^2 * f * \cosh(f*x + e)$$

$$\begin{aligned}
& e))e^{(2fx + 2e)}\sinh(fx + e)^9 + 35a^2f\cosh(fx + e)^6 + 7(429a^2f\cosh(fx + e)^6 - 495a^2f\cosh(fx + e)^4 + 135a^2f\cosh(fx + e)^2 - 5a^2f + (429a^2f\cosh(fx + e)^6 - 495a^2f\cosh(fx + e)^4 + 135a^2f\cosh(fx + e)^2 - 5a^2f)e^{(2fx + 2e)})\sinh(fx + e)^8 + 8(429a^2f\cosh(fx + e)^7 - 693a^2f\cosh(fx + e)^5 + 315a^2f\cosh(fx + e)^3 - 35a^2f\cosh(fx + e) + (429a^2f\cosh(fx + e)^7 - 693a^2f\cosh(fx + e)^5 + 315a^2f\cosh(fx + e)^3 - 35a^2f\cosh(fx + e))e^{(2fx + 2e)})\sinh(fx + e)^7 - 21a^2f\cosh(fx + e)^4 + 7(429a^2f\cosh(fx + e)^8 - 924a^2f\cosh(fx + e)^6 + 630a^2f\cosh(fx + e)^4 - 140a^2f\cosh(fx + e)^2 + 5a^2f + (429a^2f\cosh(fx + e)^8 - 924a^2f\cosh(fx + e)^6 + 630a^2f\cosh(fx + e)^4 - 140a^2f\cosh(fx + e)^2 + 5a^2f)e^{(2fx + 2e)})\sinh(fx + e)^6 + 14(143a^2f\cosh(fx + e)^9 - 396a^2f\cosh(fx + e)^7 + 378a^2f\cosh(fx + e)^5 - 140a^2f\cosh(fx + e)^3 + 15a^2f\cosh(fx + e) + (143a^2f\cosh(fx + e)^9 - 396a^2f\cosh(fx + e)^7 + 378a^2f\cosh(fx + e)^5 - 140a^2f\cosh(fx + e)^3 + 15a^2f\cosh(fx + e))e^{(2fx + 2e)})\sinh(fx + e)^5 + 7a^2f\cosh(fx + e)^2 + 7(143a^2f\cosh(fx + e)^10 - 495a^2f\cosh(fx + e)^8 + 630a^2f\cosh(fx + e)^6 - 350a^2f\cosh(fx + e)^4 + 75a^2f\cosh(fx + e)^2 - 3a^2f + (143a^2f\cosh(fx + e)^10 - 495a^2f\cosh(fx + e)^8 + 630a^2f\cosh(fx + e)^6 - 350a^2f\cosh(fx + e)^4 + 75a^2f\cosh(fx + e)^2 - 3a^2f)e^{(2fx + 2e)})\sinh(fx + e)^4 + 28(13a^2f\cosh(fx + e)^11 - 55a^2f\cosh(fx + e)^9 + 90a^2f\cosh(fx + e)^7 - 70a^2f\cosh(fx + e)^5 + 25a^2f\cosh(fx + e)^3 - 3a^2f\cosh(fx + e) + (13a^2f\cosh(fx + e)^11 - 55a^2f\cosh(fx + e)^9 + 90a^2f\cosh(fx + e)^7 - 70a^2f\cosh(fx + e)^5 + 25a^2f\cosh(fx + e)^3 - 3a^2f\cosh(fx + e))e^{(2fx + 2e)})\sinh(fx + e)^3 - a^2f + 7(13a^2f\cosh(fx + e)^12 - 66a^2f\cosh(fx + e)^10 + 135a^2f\cosh(fx + e)^8 - 140a^2f\cosh(fx + e)^6 + 75a^2f\cosh(fx + e)^4 - 18a^2f\cosh(fx + e)^2 + a^2f + (13a^2f\cosh(fx + e)^12 - 66a^2f\cosh(fx + e)^10 + 135a^2f\cosh(fx + e)^8 - 140a^2f\cosh(fx + e)^6 + 75a^2f\cosh(fx + e)^4 - 18a^2f\cosh(fx + e)^2 + a^2f)e^{(2fx + 2e)})\sinh(fx + e)^2 + (a^2f\cosh(fx + e)^14 - 7a^2f\cosh(fx + e)^12 + 21a^2f\cosh(fx + e)^10 - 35a^2f\cosh(fx + e)^8 + 35a^2f\cosh(fx + e)^6 - 21a^2f\cosh(fx + e)^4 + 7a^2f\cosh(fx + e)^2 - a^2f)e^{(2fx + 2e)} + 14(a^2f\cosh(fx + e)^13 - 6a^2f\cosh(fx + e)^11 + 15a^2f\cosh(fx + e)^9 - 20a^2f\cosh(fx + e)^7 + 15a^2f\cosh(fx + e)^5 - 6a^2f\cosh(fx + e)^3 + a^2f\cosh(fx + e) + (a^2f\cosh(fx + e)^13 - 6a^2f\cosh(fx + e)^11 + 15a^2f\cosh(fx + e)^9 - 20a^2f\cosh(fx + e)^7 + 15a^2f\cosh(fx + e)^5 - 6a^2f\cosh(fx + e)^3 + a^2f\cosh(fx + e))e^{(2fx + 2e)})\sinh(fx + e))
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)**8/(a+a*sinh(f*x+e)**2)**(3/2),x)`

[Out] Timed out

Giac [A] time = 1.94312, size = 132, normalized size = 1.15

$$\frac{8 \left(35 \sqrt{ae}^{(11fx+11e)} + 28 \sqrt{ae}^{(9fx+9e)} + 114 \sqrt{ae}^{(7fx+7e)} + 28 \sqrt{ae}^{(5fx+5e)} + 35 \sqrt{ae}^{(3fx+3e)} \right)}{105 a^2 f \left(e^{(2fx+2e)} - 1 \right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

[Out] `-8/105*(35*sqrt(a)*e^(11*f*x + 11*e) + 28*sqrt(a)*e^(9*f*x + 9*e) + 114*sqrt(a)*e^(7*f*x + 7*e) + 28*sqrt(a)*e^(5*f*x + 5*e) + 35*sqrt(a)*e^(3*f*x + 3*e))/(a^2*f*(e^(2*f*x + 2*e) - 1)^7)`

$$3.457 \quad \int \sqrt{a + b \sinh^2(e + fx)} \tanh^5(e + fx) dx$$

Optimal. Leaf size=187

$$\frac{(8a^2 - 24ab + 15b^2) \sqrt{a + b \sinh^2(e + fx)}}{8f(a - b)^2} - \frac{(8a^2 - 24ab + 15b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{8f(a - b)^{3/2}} - \frac{\operatorname{sech}^4(e + fx)(a + b \sinh^2(e + fx))}{4f(a - b)}$$

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(8*(a - b)^(3/2)*f) + ((8*a^2 - 24*a*b + 15*b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*(a - b)^2*f) + ((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*(a - b)*f)
```

Rubi [A] time = 0.22461, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{(8a^2 - 24ab + 15b^2) \sqrt{a + b \sinh^2(e + fx)}}{8f(a - b)^2} - \frac{(8a^2 - 24ab + 15b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{8f(a - b)^{3/2}} - \frac{\operatorname{sech}^4(e + fx)(a + b \sinh^2(e + fx))}{4f(a - b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(8*(a - b)^(3/2)*f) + ((8*a^2 - 24*a*b + 15*b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*(a - b)^2*f) + ((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*(a - b)*f)
```

Rule 3194

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^2]^(p_))*tan[(e_) + (f_)*(x_)^2]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))2*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := Simp[((b*c - a*d)2*(c + d*x)(n + 1)*(e + f*x)(p + 1))/(d2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d2*(d*e - c*f)*(n + 1)), Int[(c + d*x)(n + 1)*(e + f*x)p*Simp[a2*d2*f*(n + p + 2) + b2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)(n + 1)*(e + f*x)(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)n*(e + f*x)(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))(m_.)*((c_.) + (d_.)*(x_))(n_.), x_Symbol] := Simp[((a + b*x)(m + 1)*(c + d*x)n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)m*(c + d*x)(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))(m_.)*((c_.) + (d_.)*(x_))(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x(p*(m + 1) - 1)*(c - (a*d)/b + (d*xp)/b)n, x], x, (a + b*x)(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)2)(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + b \sinh^2(e + fx)} \tanh^5(e + fx) dx &= \frac{\text{Subst}\left(\int \frac{x^2 \sqrt{a+bx}}{(1+x)^3} dx, x, \sinh^2(e + fx)\right)}{2f} \\
&= -\frac{\text{sech}^4(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{4(a - b)f} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-4a+3b)+2(a-b)x\right)\sqrt{a+bx}}{(1+x)^2} dx, x, \sinh^2(e + fx)\right)}{4(a - b)f} \\
&= \frac{(8a - 7b)\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{8(a - b)^2 f} - \frac{\text{sech}^4(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{4(a - b)f} \\
&= \frac{(8a^2 - 24ab + 15b^2) \sqrt{a + b \sinh^2(e + fx)}}{8(a - b)^2 f} + \frac{(8a - 7b)\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{8(a - b)^2 f} \\
&= \frac{(8a^2 - 24ab + 15b^2) \sqrt{a + b \sinh^2(e + fx)}}{8(a - b)^2 f} + \frac{(8a - 7b)\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{8(a - b)^2 f} \\
&= -\frac{(8a^2 - 24ab + 15b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a - b)^{3/2} f} + \frac{(8a^2 - 24ab + 15b^2) \sqrt{a + b \sinh^2(e + fx)}}{8(a - b)^2 f}
\end{aligned}$$

Mathematica [A] time = 0.571949, size = 151, normalized size = 0.81

$$\frac{(8a^2 - 24ab + 15b^2) \left(\sqrt{a - b} \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}} \right) - \sqrt{a + b \sinh^2(e + fx)} \right) + 2(a - b) \text{sech}^4(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{8f(a - b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]

[Out] -((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2)) + 2*(a - b)*Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2) + (8*a^2 - 24*a*b + 15*b^2)*(Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] - Sqrt[a + b*Sinh[e + f*x]^2])/((8*(a - b)^2*f)

Maple [C] time = 0.341, size = 43, normalized size = 0.2

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^5}{(\cosh(fx + e))^6} \sqrt{a + b(\sinh(fx + e))^2} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x)

[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^5/cosh(f*x+e)^6,sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \tanh^5(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^5, x)

Fricas [B] time = 9.01519, size = 12141, normalized size = 64.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")

[Out] [-1/16*(((8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^9 + 9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^9 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 14*(8*a^2 - 24*a*b + 15*b

$$\begin{aligned}
& ^2) \cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2) \sinh(f*x + e)^5 + 2*(63*(8*a \\
& ^2 - 24*a*b + 15*b^2) \cosh(f*x + e)^5 + 70*(8*a^2 - 24*a*b + 15*b^2) \cosh(f \\
& *x + e)^3 + 15*(8*a^2 - 24*a*b + 15*b^2) \cosh(f*x + e) \sinh(f*x + e)^4 + 4 \\
& *(8*a^2 - 24*a*b + 15*b^2) \cosh(f*x + e)^3 + 4*(21*(8*a^2 - 24*a*b + 15*b^2 \\
&) \cosh(f*x + e)^6 + 35*(8*a^2 - 24*a*b + 15*b^2) \cosh(f*x + e)^4 + 15*(8*a^ \\
& 2 - 24*a*b + 15*b^2) \cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2) \sinh(f*x + \\
& e)^3 + 12*(3*(8*a^2 - 24*a*b + 15*b^2) \cosh(f*x + e)^7 + 7*(8*a^2 - 24*a*b \\
& + 15*b^2) \cosh(f*x + e)^5 + 5*(8*a^2 - 24*a*b + 15*b^2) \cosh(f*x + e)^3 + (\\
& 8*a^2 - 24*a*b + 15*b^2) \cosh(f*x + e) \sinh(f*x + e)^2 + (8*a^2 - 24*a*b + \\
& 15*b^2) \cosh(f*x + e) + (9*(8*a^2 - 24*a*b + 15*b^2) \cosh(f*x + e)^8 + 28* \\
& (8*a^2 - 24*a*b + 15*b^2) \cosh(f*x + e)^6 + 30*(8*a^2 - 24*a*b + 15*b^2) \co \\
& sh(f*x + e)^4 + 12*(8*a^2 - 24*a*b + 15*b^2) \cosh(f*x + e)^2 + 8*a^2 - 24*a \\
& *b + 15*b^2) \sinh(f*x + e) \sqrt{a - b} \log((b \cosh(f*x + e)^4 + 4*b \cosh(f \\
& *x + e) \sinh(f*x + e)^3 + b \sinh(f*x + e)^4 + 2*(4*a - 3*b) \cosh(f*x + e)^2 \\
& + 2*(3*b \cosh(f*x + e)^2 + 4*a - 3*b) \sinh(f*x + e)^2 + 4*\sqrt{2}*\sqrt{a - \\
& b}*\sqrt{(b \cosh(f*x + e)^2 + b \sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 \\
& - 2*\cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e)^2)) * (\cosh(f*x + e) + \sinh(\\
& f*x + e)) + 4*(b \cosh(f*x + e)^3 + (4*a - 3*b) \cosh(f*x + e) \sinh(f*x + e) \\
& + b)/(cosh(f*x + e)^4 + 4*\cosh(f*x + e) \sinh(f*x + e)^3 + \sinh(f*x + e)^4 \\
& + 2*(3*\cosh(f*x + e)^2 + 1) \sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(cosh(f \\
& *x + e)^3 + \cosh(f*x + e) \sinh(f*x + e) + 1)) - 4*\sqrt{2}*(2*(a^2 - 2*a*b \\
& + b^2) \cosh(f*x + e)^8 + 16*(a^2 - 2*a*b + b^2) \cosh(f*x + e) \sinh(f*x + e) \\
& ^7 + 2*(a^2 - 2*a*b + b^2) \sinh(f*x + e)^8 + (16*a^2 - 33*a*b + 17*b^2) \cos \\
& h(f*x + e)^6 + (56*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^2 + 16*a^2 - 33*a*b + \\
& 17*b^2) \sinh(f*x + e)^6 + 2*(56*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^3 + 3*(16 \\
& *a^2 - 33*a*b + 17*b^2) \cosh(f*x + e) \sinh(f*x + e)^5 + 2*(10*a^2 - 21*a*b \\
& + 11*b^2) \cosh(f*x + e)^4 + (140*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^4 + 15* \\
& (16*a^2 - 33*a*b + 17*b^2) \cosh(f*x + e)^2 + 20*a^2 - 42*a*b + 22*b^2) \sinh \\
& (f*x + e)^4 + 4*(28*(a^2 - 2*a*b + b^2) \cosh(f*x + e)^5 + 5*(16*a^2 - 33*a* \\
& b + 17*b^2) \cosh(f*x + e)^3 + 2*(10*a^2 - 21*a*b + 11*b^2) \cosh(f*x + e) \sinh \\
& (f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2) \cosh(f*x + e)^2 + (56*(a^2 - 2* \\
& a*b + b^2) \cosh(f*x + e)^6 + 15*(16*a^2 - 33*a*b + 17*b^2) \cosh(f*x + e)^4 \\
& + 12*(10*a^2 - 21*a*b + 11*b^2) \cosh(f*x + e)^2 + 16*a^2 - 33*a*b + 17*b^2) \\
& * \sinh(f*x + e)^2 + 2*a^2 - 4*a*b + 2*b^2 + 2*(8*(a^2 - 2*a*b + b^2) \cosh(f* \\
& x + e)^7 + 3*(16*a^2 - 33*a*b + 17*b^2) \cosh(f*x + e)^5 + 4*(10*a^2 - 21*a* \\
& b + 11*b^2) \cosh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2) \cosh(f*x + e) \sinh \\
& (f*x + e) \sqrt{(b \cosh(f*x + e)^2 + b \sinh(f*x + e)^2 + 2*a - b)/(cosh(f* \\
& x + e)^2 - 2*\cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + e)^2)))/((a^2 - 2*a*b \\
& + b^2) * f * \cosh(f*x + e)^9 + 9*(a^2 - 2*a*b + b^2) * f * \cosh(f*x + e) \sinh(f*x \\
& + e)^8 + (a^2 - 2*a*b + b^2) * f * \sinh(f*x + e)^9 + 4*(a^2 - 2*a*b + b^2) * f * \co \\
& sh(f*x + e)^7 + 4*(9*(a^2 - 2*a*b + b^2) * f * \cosh(f*x + e)^2 + (a^2 - 2*a*b + \\
& b^2) * f) * \sinh(f*x + e)^7 + 6*(a^2 - 2*a*b + b^2) * f * \cosh(f*x + e)^5 + 28*(3* \\
& (a^2 - 2*a*b + b^2) * f * \cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2) * f * \cosh(f*x + e) \\
&) * \sinh(f*x + e)^6 + 6*(21*(a^2 - 2*a*b + b^2) * f * \cosh(f*x + e)^4 + 14*(a^2 - \\
& 2*a*b + b^2) * f * \cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2) * f) * \sinh(f*x + e)^5 +
\end{aligned}$$

$$\begin{aligned}
& 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 2*(63*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e) + 12*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 28*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 12*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)), -1/8*(((8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^9 + 9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^9 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 14*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^5 + 2*(63*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 70*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + 15*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^6 + 35*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 15*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^3 + 12*(3*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 7*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e) + (9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^8 + 28*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^6 + 30*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 12*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*(2*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^8 + 16*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + 2*(a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^6 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 16*a^2 - 33*a*b + 17*b^2)*sinh(f*x + e)^6 + 2*(56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^4 + (140*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 15*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + 20*a^2 - 42*a*b + 22*b^2)*sinh(f*x + e)^4 + 4*(28*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 5*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^3 + 2*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 15*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^4 + 12*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^2 + 16*a^2
\end{aligned}$$


```

- 33*a*b + 17*b^2)*sinh(f*x + e)^2 + 2*a^2 - 4*a*b + 2*b^2 + 2*(8*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^7 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^5
+ 4*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)
*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^9 + 9*(a^2 - 2*a*b + b^2)*f*cosh
(f*x + e)*sinh(f*x + e)^8 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^9 + 4*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^7 + 4*(9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)
^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^7 + 6*(a^2 - 2*a*b + b^2)*f*cosh(
f*x + e)^5 + 28*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b
^2)*f*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^4 + 14*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)
*sinh(f*x + e)^5 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 2*(63*(a^2 - 2
*a*b + b^2)*f*cosh(f*x + e)^5 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 +
15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(21*(a^2 - 2*a*
b + b^2)*f*cosh(f*x + e)^6 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e
)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e) + 12*(3*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^7 + 7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b
^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^
2 + (9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 28*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^6 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 12*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \tanh^5(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^5, x)
```

$$3.458 \quad \int \sqrt{a + b \sinh^2(e + fx)} \tanh^3(e + fx) dx$$

Optimal. Leaf size=126

$$\frac{(2a - 3b)\sqrt{a + b \sinh^2(e + fx)}}{2f(a - b)} - \frac{(2a - 3b) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{2f\sqrt{a - b}} + \frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))^{3/2}}{2f(a - b)}$$

[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*Sqrt[a - b]*f) + ((2*a - 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f) + (Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(2*(a - b)*f)

Rubi [A] time = 0.125283, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a - 3b)\sqrt{a + b \sinh^2(e + fx)}}{2f(a - b)} - \frac{(2a - 3b) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{2f\sqrt{a - b}} + \frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))^{3/2}}{2f(a - b)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]

[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*Sqrt[a - b]*f) + ((2*a - 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f) + (Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(2*(a - b)*f)

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^2]^(p_))*tan[(e_) + (f_)*(x_)^(m_)], x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 78

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f_)*(x_)^(p_)), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + b \sinh^2(e + fx)} \tanh^3(e + fx) dx &= \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{(1+x)^2} dx, x, \sinh^2(e + fx)\right)}{2f} \\
&= \frac{\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{2(a - b)f} + \frac{(2a - 3b) \text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x} dx, x, \sinh^2(e + fx)\right)}{4(a - b)f} \\
&= \frac{(2a - 3b)\sqrt{a + b \sinh^2(e + fx)}}{2(a - b)f} + \frac{\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{2(a - b)f} \\
&= \frac{(2a - 3b)\sqrt{a + b \sinh^2(e + fx)}}{2(a - b)f} + \frac{\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{2(a - b)f} \\
&= -\frac{(2a - 3b) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{2\sqrt{a - b}f} + \frac{(2a - 3b)\sqrt{a + b \sinh^2(e + fx)}}{2(a - b)f}
\end{aligned}$$

Mathematica [A] time = 0.405799, size = 88, normalized size = 0.7

$$\frac{(\cosh(2(e + fx)) + 2)\text{sech}^2(e + fx)\sqrt{a + b \sinh^2(e + fx)} - \frac{(2a - 3b) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{\sqrt{a - b}}}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]

[Out] (-(((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]))/Sqrt[a - b]) + (2 + Cosh[2*(e + f*x)])*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)

Maple [C] time = 0.102, size = 43, normalized size = 0.3

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^3}{(\cosh(fx + e))^4} \sqrt{a + b(\sinh(fx + e))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^3/cosh(f*x+e)^4,sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \tanh^3(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^3, x)
```

Fricas [B] time = 7.67511, size = 4352, normalized size = 34.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - 3*b)*cosh(f*x + e)^5 + 5*(2*a - 3*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a - 3*b)*sinh(f*x + e)^5 + 2*(2*a - 3*b)*cosh(f*x + e)^3 + 2*(5*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e)^3 + 2*(5*(2*a - 3*b)*cosh(f*x + e)^3 + 3*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a - 3*b)*cosh(f*x + e) + (5*(2*a - 3*b)*cosh(f*x + e)^4 + 6*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f
```

```

*x + e)^4 + 4*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 + 2*a
- 2*b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 + 2*(a - b)*cosh(f*x +
e))*sinh(f*x + e) + a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
)/((a - b)*f*cosh(f*x + e)^5 + 5*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^4 +
(a - b)*f*sinh(f*x + e)^5 + 2*(a - b)*f*cosh(f*x + e)^3 + 2*(5*(a - b)*f*co
sh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^3 + (a - b)*f*cosh(f*x + e) + 2*(5
*(a - b)*f*cosh(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (
5*(a - b)*f*cosh(f*x + e)^4 + 6*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh
(f*x + e)), -1/2*(((2*a - 3*b)*cosh(f*x + e)^5 + 5*(2*a - 3*b)*cosh(f*x + e
)*sinh(f*x + e)^4 + (2*a - 3*b)*sinh(f*x + e)^5 + 2*(2*a - 3*b)*cosh(f*x +
e)^3 + 2*(5*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e)^3 + 2*(5
*(2*a - 3*b)*cosh(f*x + e)^3 + 3*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^2
+ (2*a - 3*b)*cosh(f*x + e) + (5*(2*a - 3*b)*cosh(f*x + e)^4 + 6*(2*a - 3*
b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt
(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)
*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*((a - b)*cosh(f*x + e)^4
+ 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f*x + e)^4 + 4*(a
- b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 + 2*a - 2*b)*sinh(f*x
+ e)^2 + 4*((a - b)*cosh(f*x + e)^3 + 2*(a - b)*cosh(f*x + e))*sinh(f*x + e
) + a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cos
h(f*x + e)^5 + 5*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^4 + (a - b)*f*sinh(f
*x + e)^5 + 2*(a - b)*f*cosh(f*x + e)^3 + 2*(5*(a - b)*f*cosh(f*x + e)^2 +
(a - b)*f)*sinh(f*x + e)^3 + (a - b)*f*cosh(f*x + e) + 2*(5*(a - b)*f*cosh(
f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (5*(a - b)*f*cosh
(f*x + e)^4 + 6*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e))]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \tanh^3(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)

[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**3, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \tanh^3(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^3, x)
```


$$3.459 \quad \int \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx) dx$$

Optimal. Leaf size=62

$$\frac{\sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{\sqrt{a - b} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{f}$$

[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a + b*Sinh[e + f*x]^2]/f

Rubi [A] time = 0.0635905, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{\sqrt{a - b} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x], x]

[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[a + b*Sinh[e + f*x]^2]/f

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !IGtQ

```
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0])) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx) dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x} dx, x, \sinh^2(e + fx)\right)}{2f} \\
&= \frac{\sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a - b) \text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}} dx, x, \sinh^2(e + fx)\right)}{2f} \\
&= \frac{\sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a - b) \text{Subst}\left(\int \frac{1}{1 - \frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^2(e + fx)}\right)}{bf} \\
&= -\frac{\sqrt{a - b} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{f} + \frac{\sqrt{a + b \sinh^2(e + fx)}}{f}
\end{aligned}$$

Mathematica [A] time = 0.0523067, size = 65, normalized size = 1.05

$$\frac{\sqrt{a + b \cosh^2(e + fx) - b} - \sqrt{a - b} \tanh^{-1}\left(\frac{\sqrt{a + b \cosh^2(e + fx) - b}}{\sqrt{a - b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x], x]
```

[Out] $(-\text{Sqrt}[a - b] \cdot \text{ArcTanh}[\text{Sqrt}[a - b + b \cdot \text{Cosh}[e + f \cdot x]^2] / \text{Sqrt}[a - b]]) + \text{Sqrt}[a - b + b \cdot \text{Cosh}[e + f \cdot x]^2] / f$

Maple [C] time = 0.079, size = 41, normalized size = 0.7

$$\frac{1}{f} \int \frac{\sinh(fx + e)}{(\cosh(fx + e))^2} \sqrt{a + b(\sinh(fx + e))^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e), x)`

[Out] `\`int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)/cosh(f*x+e)^2, sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \tanh(fx + e) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e), x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e), x)`

Fricas [B] time = 7.16423, size = 1705, normalized size = 27.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e), x, algorithm="fricas")`

[Out] `[1/2*(sqrt(a - b)*(cosh(f*x + e) + sinh(f*x + e))*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(`

```
f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)
)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(
f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x +
e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sin
h(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f
*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 +
4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e
)), -1/2*(2*sqrt(-a + b)*(cosh(f*x + e) + sinh(f*x + e))*arctan(-1/2*sqrt(2)
)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*co
sh(f*x + e) + (a - b)*sinh(f*x + e)) - sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.460 \quad \int \coth(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=54

$$\frac{\sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{f}$$

[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Si
nh[e + f*x]^2]/f

Rubi [A] time = 0.0742229, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Si
nh[e + f*x]^2]/f

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ

```
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0])) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \coth(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, \sinh^2(e + fx)\right)}{2f} \\
&= \frac{\sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \sinh^2(e + fx)\right)}{2f} \\
&= \frac{\sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{a \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^2(e + fx)}\right)}{bf} \\
&= -\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a + b \sinh^2(e + fx)}}{f}
\end{aligned}$$

Mathematica [A] time = 0.0485787, size = 53, normalized size = 0.98

$$-\frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right) - \sqrt{a + b \sinh^2(e + fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2], x]
```

[Out] $-\left(\frac{\sqrt{a} \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \sinh(e + f x)^2}}{\sqrt{a}}\right] - \sqrt{a + b \sinh(e + f x)^2}}{f}\right)$

Maple [C] time = 0.085, size = 46, normalized size = 0.9

$$\frac{1}{f} \int \left(b \sinh(fx + e) + \frac{a}{\sinh(fx + e)} \right) \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] ``int/indef0`((b*sinh(f*x+e)+a/sinh(f*x+e))/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \operatorname{coth}(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e), x)`

Fricas [B] time = 3.9074, size = 1656, normalized size = 30.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] $\frac{1}{2} \left(\sqrt{a} (\cosh(fx + e) + \sinh(fx + e)) \log\left((b \cosh(fx + e)^4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sinh(fx + e)^4 + 2(4a - b) \cosh(fx + e) \right) \right)$

```
e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*
x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*
(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x +
e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e)), 1/2*(2*sqrt
(-a)*(cosh(f*x + e) + sinh(f*x + e))*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*cosh(f*x + e) + a*sinh(f*x + e))
) + sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x +
e) + f*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \coth(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


$$3.461 \quad \int \coth^3(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=106

$$\frac{(2a + b)\sqrt{a + b \sinh^2(e + fx)}}{2af} - \frac{(2a + b) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{\operatorname{csch}^2(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{2af}$$

[Out] $-\left(\frac{(2a + b) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right]}{2\sqrt{a}f}\right) + \left(\frac{(2a + b) \sqrt{a + b \sinh^2(e + fx)}}{2af}\right) - \left(\frac{\operatorname{Csch}[e + fx]^2 (a + b \sinh^2(e + fx))^{3/2}}{2af}\right)$

Rubi [A] time = 0.121803, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a + b)\sqrt{a + b \sinh^2(e + fx)}}{2af} - \frac{(2a + b) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{\operatorname{csch}^2(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{2af}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[e + fx]^3 \sqrt{a + b \sinh^2[e + fx]^2}, x]$

[Out] $-\left(\frac{(2a + b) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right]}{2\sqrt{a}f}\right) + \left(\frac{(2a + b) \sqrt{a + b \sinh^2(e + fx)}}{2af}\right) - \left(\frac{\operatorname{Csch}[e + fx]^2 (a + b \sinh^2(e + fx))^{3/2}}{2af}\right)$

Rule 3194

$\operatorname{Int}[(a + b \sin[e + fx])^2 \tan[e + fx]^p, x] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\sin[e + fx]^2, x]\}, \operatorname{Dist}[ff^{(m+1)/2} / (2f), \operatorname{Subst}[\operatorname{Int}[(x^{(m-1)/2} (a + bffx)^p] / (1 - ffx)^{(m+1)/2}, x], x, \sin[e + fx]^2 / ff, x]] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \ \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 78

$\operatorname{Int}[(a + b \sin[e + fx])^2 \tan[e + fx]^p, x] \rightarrow -\operatorname{Simp}[(b \sin[e + fx] - a) (c + d \sin[e + fx])^{n+1} (e + f \sin[e + fx])^{p+1} / ($

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \coth^3(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= \frac{\text{Subst}\left(\int \frac{(1+x)\sqrt{a+bx}}{x^2} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{csch}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{2af} + \frac{(2a+b) \text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, \sinh^2(e+fx)\right)}{4af} \\
&= \frac{(2a+b) \sqrt{a+b \sinh^2(e+fx)}}{2af} - \frac{\text{csch}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{2af} + \dots \\
&= \frac{(2a+b) \sqrt{a+b \sinh^2(e+fx)}}{2af} - \frac{\text{csch}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{2af} + \dots \\
&= -\frac{(2a+b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} + \frac{(2a+b) \sqrt{a+b \sinh^2(e+fx)}}{2af} - \dots
\end{aligned}$$

Mathematica [A] time = 0.36274, size = 69, normalized size = 0.65

$$-\frac{(2a+b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right) + (\text{csch}^2(e+fx) - 2) \sqrt{a+b \sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -(((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (-2 + Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)

Maple [C] time = 0.214, size = 58, normalized size = 0.6

$$\frac{1}{f} \int \frac{b \sinh(fx+e) + \frac{a+b}{\sinh(fx+e)} + \frac{a}{(\sinh(fx+e))^3}}{\sqrt{a+b(\sinh(fx+e))^2}} \sinh(fx+e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] ``int/indef0`((b*sinh(f*x+e)+(a+b)/sinh(f*x+e)+a/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \coth^3(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^3, x)`

Fricas [B] time = 4.97737, size = 3903, normalized size = 36.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `[1/4*(((2*a + b)*cosh(f*x + e)^5 + 5*(2*a + b)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a + b)*sinh(f*x + e)^5 - 2*(2*a + b)*cosh(f*x + e)^3 + 2*(5*(2*a + b)*cosh(f*x + e)^2 - 2*a - b)*sinh(f*x + e)^3 + 2*(5*(2*a + b)*cosh(f*x + e)^3 - 3*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a + b)*cosh(f*x + e) + (5*(2*a + b)*cosh(f*x + e)^4 - 6*(2*a + b)*cosh(f*x + e)^2 + 2*a + b)*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e)*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh(f*x + e)^4 - 4*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2`

$$\begin{aligned}
& - 2*a)*\sinh(f*x + e)^2 + 4*(a*\cosh(f*x + e)^3 - 2*a*\cosh(f*x + e))*\sinh(f*x \\
& + e) + a)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x \\
& + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a*f*\cosh(f*x \\
& + e)^5 + 5*a*f*\cosh(f*x + e)*\sinh(f*x + e)^4 + a*f*\sinh(f*x + e)^5 - 2*a*f* \\
& \cosh(f*x + e)^3 + 2*(5*a*f*\cosh(f*x + e)^2 - a*f)*\sinh(f*x + e)^3 + a*f*\cosh \\
& h(f*x + e) + 2*(5*a*f*\cosh(f*x + e)^3 - 3*a*f*\cosh(f*x + e))*\sinh(f*x + e)^2 \\
& + (5*a*f*\cosh(f*x + e)^4 - 6*a*f*\cosh(f*x + e)^2 + a*f)*\sinh(f*x + e)), 1 \\
& /2*((2*a + b)*\cosh(f*x + e)^5 + 5*(2*a + b)*\cosh(f*x + e)*\sinh(f*x + e)^4 \\
& + (2*a + b)*\sinh(f*x + e)^5 - 2*(2*a + b)*\cosh(f*x + e)^3 + 2*(5*(2*a + b)* \\
& \cosh(f*x + e)^2 - 2*a - b)*\sinh(f*x + e)^3 + 2*(5*(2*a + b)*\cosh(f*x + e)^3 \\
& - 3*(2*a + b)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (2*a + b)*\cosh(f*x + e) + (\\
& 5*(2*a + b)*\cosh(f*x + e)^4 - 6*(2*a + b)*\cosh(f*x + e)^2 + 2*a + b)*\sinh(f \\
& *x + e))*\sqrt{-a}*\arctan(1/2*\sqrt{2})*\sqrt{-a}*\sqrt{(b*\cosh(f*x + e)^2 + b*s \\
& inh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) \\
& + \sinh(f*x + e)^2)))/(a*\cosh(f*x + e) + a*\sinh(f*x + e))) + \sqrt{2}*(a*\cosh(\\
& f*x + e)^4 + 4*a*\cosh(f*x + e)*\sinh(f*x + e)^3 + a*\sinh(f*x + e)^4 - 4*a*\cosh \\
& sh(f*x + e)^2 + 2*(3*a*\cosh(f*x + e)^2 - 2*a)*\sinh(f*x + e)^2 + 4*(a*\cosh(f \\
& *x + e)^3 - 2*a*\cosh(f*x + e))*\sinh(f*x + e) + a)*\sqrt{(b*\cosh(f*x + e)^2 + \\
& b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + \\
& e) + \sinh(f*x + e)^2)))/(a*f*\cosh(f*x + e)^5 + 5*a*f*\cosh(f*x + e)*\sinh(f* \\
& x + e)^4 + a*f*\sinh(f*x + e)^5 - 2*a*f*\cosh(f*x + e)^3 + 2*(5*a*f*\cosh(f*x \\
& + e)^2 - a*f)*\sinh(f*x + e)^3 + a*f*\cosh(f*x + e) + 2*(5*a*f*\cosh(f*x + e)^ \\
& 3 - 3*a*f*\cosh(f*x + e))*\sinh(f*x + e)^2 + (5*a*f*\cosh(f*x + e)^4 - 6*a*f*c \\
& osh(f*x + e)^2 + a*f)*\sinh(f*x + e))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.462 \quad \int \coth^5(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=167

$$\frac{(8a^2 + 8ab - b^2) \sqrt{a + b \sinh^2(e + fx)}}{8a^2 f} - \frac{(8a^2 + 8ab - b^2) \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}} \right)}{8a^{3/2} f} - \frac{(8a - b) \operatorname{csch}^2(e + fx) (a + b \sinh^2(e + fx))}{8a^2 f}$$

[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*a^(3/2)*f) + ((8*a^2 + 8*a*b - b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a^2*f) - ((8*a - b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(8*a^2*f) - (Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*a*f)

Rubi [A] time = 0.179832, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{(8a^2 + 8ab - b^2) \sqrt{a + b \sinh^2(e + fx)}}{8a^2 f} - \frac{(8a^2 + 8ab - b^2) \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}} \right)}{8a^{3/2} f} - \frac{(8a - b) \operatorname{csch}^2(e + fx) (a + b \sinh^2(e + fx))}{8a^2 f}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*a^(3/2)*f) + ((8*a^2 + 8*a*b - b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a^2*f) - ((8*a - b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(8*a^2*f) - (Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*a*f)

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 89

```

Int[((a_.) + (b_.)*(x_))2*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := Simp[((b*c - a*d)2*(c + d*x)(n + 1)*(e + f*x)(p + 1))/(d2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d2*(d*e - c*f)*(n + 1)), Int[(c + d*x)(n + 1)*(e + f*x)p*Simp[a2*d2*f*(n + p + 2) + b2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

```

Rule 78

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)(n + 1)*(e + f*x)(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)n*(e + f*x)(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

```

Rule 50

```

Int[((a_.) + (b_.)*(x_))(m_)*((c_.) + (d_.)*(x_))(n_), x_Symbol] := Simp[((a + b*x)(m + 1)*(c + d*x)n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)m*(c + d*x)(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))(m_)*((c_.) + (d_.)*(x_))(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x(p*(m + 1) - 1)*(c - (a*d)/b + (d*xp)/b)n, x], x, (a + b*x)(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)2)(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \coth^5(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^2 \sqrt{a+bx}}{x^3} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{csch}^4(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{4af} + \frac{\text{Subst}\left(\int \frac{(\frac{1}{2}(8a-b)+2ax) \sqrt{a+bx}}{x^2} dx, \right)}{4af} \\
&= -\frac{(8a-b) \text{csch}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{8a^2 f} - \frac{\text{csch}^4(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{4af} \\
&= \frac{(8a^2+8ab-b^2) \sqrt{a+b \sinh^2(e+fx)}}{8a^2 f} - \frac{(8a-b) \text{csch}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{8a^2 f} \\
&= \frac{(8a^2+8ab-b^2) \sqrt{a+b \sinh^2(e+fx)}}{8a^2 f} - \frac{(8a-b) \text{csch}^2(e+fx) (a+b \sinh^2(e+fx))^{3/2}}{8a^2 f} \\
&= -\frac{(8a^2+8ab-b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2} f} + \frac{(8a^2+8ab-b^2) \sqrt{a+b \sinh^2(e+fx)}}{8a^2 f}
\end{aligned}$$

Mathematica [A] time = 0.564118, size = 102, normalized size = 0.61

$$\frac{(-8a^2 - 8ab + b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right) - \sqrt{a} \sqrt{a+b \sinh^2(e+fx)} ((8a+b) \text{csch}^2(e+fx) + 2a \text{csch}^4(e+fx) - 8a)}{8a^{3/2} f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-8*a^2 - 8*a*b + b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] - Sqrt[a]*(-8*a + (8*a + b)*Csch[e + f*x]^2 + 2*a*Csch[e + f*x]^4)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a^(3/2)*f)

Maple [C] time = 0.131, size = 80, normalized size = 0.5

$$\frac{1}{f} \int \frac{(\cosh(fx+e))^4 (a-b+b(\cosh(fx+e))^2)}{\sinh(fx+e) \left((\cosh(fx+e))^4 - 2(\cosh(fx+e))^2 + 1 \right) \sqrt{a+b(\sinh(fx+e))^2}} dx, \sinh(fx+e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4*(a-b+b*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \coth^5(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^5, x)
```

Fricas [B] time = 5.96195, size = 9559, normalized size = 57.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^9 + 9*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 + 8*a*b - b^2)*sinh(f*x + e)^9 - 4*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 - 14*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*sinh(f*x + e)^5 + 2*(63*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 - 70*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 + 15*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^4 - 4*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^6 - 35*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 + 15*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^3 + 12*(3*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 - 7*(8*a^2 + 8*a*b - b^2)*cos
```

$$\begin{aligned} & h(f*x + e)^5 + 5*(8*a^2 + 8*a*b - b^2)*\cosh(f*x + e)^3 - (8*a^2 + 8*a*b - b \\ & ^2)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (8*a^2 + 8*a*b - b^2)*\cosh(f*x + e) + \\ & (9*(8*a^2 + 8*a*b - b^2)*\cosh(f*x + e)^8 - 28*(8*a^2 + 8*a*b - b^2)*\cosh(f* \\ & x + e)^6 + 30*(8*a^2 + 8*a*b - b^2)*\cosh(f*x + e)^4 - 12*(8*a^2 + 8*a*b - b \\ & ^2)*\cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*\sinh(f*x + e))*\sqrt{a}*\log((b*\co \\ & sh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(\\ & 4*a - b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 4*a - b)*\sinh(f*x + e)^ \\ & 2 + 4*\sqrt{2}*\sqrt{a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b \\ &)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))*(\cos \\ & h(f*x + e) + \sinh(f*x + e)) + 4*(b*\cosh(f*x + e)^3 + (4*a - b)*\cosh(f*x + e \\ &))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \\ & \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 - 1)*\sinh(f*x + e)^2 - 2*\cosh(f*x + \\ & e)^2 + 4*(\cosh(f*x + e)^3 - \cosh(f*x + e))*\sinh(f*x + e) + 1)) - 4*\sqrt{2}* \\ & (2*a^2*\cosh(f*x + e)^8 + 16*a^2*\cosh(f*x + e)*\sinh(f*x + e)^7 + 2*a^2*\sinh(\\ & f*x + e)^8 - (16*a^2 + a*b)*\cosh(f*x + e)^6 + (56*a^2*\cosh(f*x + e)^2 - 16* \\ & a^2 - a*b)*\sinh(f*x + e)^6 + 2*(56*a^2*\cosh(f*x + e)^3 - 3*(16*a^2 + a*b)*\c \\ & osh(f*x + e))*\sinh(f*x + e)^5 + 2*(10*a^2 + a*b)*\cosh(f*x + e)^4 + (140*a^2 \\ & *\cosh(f*x + e)^4 - 15*(16*a^2 + a*b)*\cosh(f*x + e)^2 + 20*a^2 + 2*a*b)*\sinh \\ & (f*x + e)^4 + 4*(28*a^2*\cosh(f*x + e)^5 - 5*(16*a^2 + a*b)*\cosh(f*x + e)^3 \\ & + 2*(10*a^2 + a*b)*\cosh(f*x + e))*\sinh(f*x + e)^3 - (16*a^2 + a*b)*\cosh(f*x \\ & + e)^2 + (56*a^2*\cosh(f*x + e)^6 - 15*(16*a^2 + a*b)*\cosh(f*x + e)^4 + 12* \\ & (10*a^2 + a*b)*\cosh(f*x + e)^2 - 16*a^2 - a*b)*\sinh(f*x + e)^2 + 2*a^2 + 2* \\ & (8*a^2*\cosh(f*x + e)^7 - 3*(16*a^2 + a*b)*\cosh(f*x + e)^5 + 4*(10*a^2 + a*b \\ &)*\cosh(f*x + e)^3 - (16*a^2 + a*b)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{(b*\co \\ & sh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x \\ & + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a^2*f*\cosh(f*x + e)^9 + 9*a^2*f*\co \\ & sh(f*x + e)*\sinh(f*x + e)^8 + a^2*f*\sinh(f*x + e)^9 - 4*a^2*f*\cosh(f*x + e) \\ & ^7 + 6*a^2*f*\cosh(f*x + e)^5 + 4*(9*a^2*f*\cosh(f*x + e)^2 - a^2*f)*\sinh(f*x \\ & + e)^7 + 28*(3*a^2*f*\cosh(f*x + e)^3 - a^2*f*\cosh(f*x + e))*\sinh(f*x + e)^ \\ & 6 - 4*a^2*f*\cosh(f*x + e)^3 + 6*(21*a^2*f*\cosh(f*x + e)^4 - 14*a^2*f*\cosh(f \\ & *x + e)^2 + a^2*f)*\sinh(f*x + e)^5 + 2*(63*a^2*f*\cosh(f*x + e)^5 - 70*a^2*f \\ & *\cosh(f*x + e)^3 + 15*a^2*f*\cosh(f*x + e))*\sinh(f*x + e)^4 + a^2*f*\cosh(f*x \\ & + e) + 4*(21*a^2*f*\cosh(f*x + e)^6 - 35*a^2*f*\cosh(f*x + e)^4 + 15*a^2*f*\c \\ & osh(f*x + e)^2 - a^2*f)*\sinh(f*x + e)^3 + 12*(3*a^2*f*\cosh(f*x + e)^7 - 7*a \\ & ^2*f*\cosh(f*x + e)^5 + 5*a^2*f*\cosh(f*x + e)^3 - a^2*f*\cosh(f*x + e))*\sinh \\ & (f*x + e)^2 + (9*a^2*f*\cosh(f*x + e)^8 - 28*a^2*f*\cosh(f*x + e)^6 + 30*a^2*f \\ & *\cosh(f*x + e)^4 - 12*a^2*f*\cosh(f*x + e)^2 + a^2*f)*\sinh(f*x + e)), 1/8*((\\ & (8*a^2 + 8*a*b - b^2)*\cosh(f*x + e)^9 + 9*(8*a^2 + 8*a*b - b^2)*\cosh(f*x + \\ & e)*\sinh(f*x + e)^8 + (8*a^2 + 8*a*b - b^2)*\sinh(f*x + e)^9 - 4*(8*a^2 + 8*a \\ & *b - b^2)*\cosh(f*x + e)^7 + 4*(9*(8*a^2 + 8*a*b - b^2)*\cosh(f*x + e)^2 - 8* \\ & a^2 - 8*a*b + b^2)*\sinh(f*x + e)^7 + 28*(3*(8*a^2 + 8*a*b - b^2)*\cosh(f*x + \\ & e)^3 - (8*a^2 + 8*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e)^6 + 6*(8*a^2 + 8 \\ & *a*b - b^2)*\cosh(f*x + e)^5 + 6*(21*(8*a^2 + 8*a*b - b^2)*\cosh(f*x + e)^4 - \\ & 14*(8*a^2 + 8*a*b - b^2)*\cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*\sinh(f*x + \\ & e)^5 + 2*(63*(8*a^2 + 8*a*b - b^2)*\cosh(f*x + e)^5 - 70*(8*a^2 + 8*a*b - b \end{aligned}$$

$$\begin{aligned}
&^2) * \cosh(f*x + e)^3 + 15*(8*a^2 + 8*a*b - b^2) * \cosh(f*x + e) * \sinh(f*x + e) \\
&^4 - 4*(8*a^2 + 8*a*b - b^2) * \cosh(f*x + e)^3 + 4*(21*(8*a^2 + 8*a*b - b^2) * \\
&\cosh(f*x + e)^6 - 35*(8*a^2 + 8*a*b - b^2) * \cosh(f*x + e)^4 + 15*(8*a^2 + 8* \\
&a*b - b^2) * \cosh(f*x + e)^2 - 8*a^2 - 8*a*b + b^2) * \sinh(f*x + e)^3 + 12*(3*(\\
&8*a^2 + 8*a*b - b^2) * \cosh(f*x + e)^7 - 7*(8*a^2 + 8*a*b - b^2) * \cosh(f*x + e) \\
&)^5 + 5*(8*a^2 + 8*a*b - b^2) * \cosh(f*x + e)^3 - (8*a^2 + 8*a*b - b^2) * \cosh(\\
&f*x + e) * \sinh(f*x + e)^2 + (8*a^2 + 8*a*b - b^2) * \cosh(f*x + e) + (9*(8*a^2 \\
&+ 8*a*b - b^2) * \cosh(f*x + e)^8 - 28*(8*a^2 + 8*a*b - b^2) * \cosh(f*x + e)^6 \\
&+ 30*(8*a^2 + 8*a*b - b^2) * \cosh(f*x + e)^4 - 12*(8*a^2 + 8*a*b - b^2) * \cosh(\\
&f*x + e)^2 + 8*a^2 + 8*a*b - b^2) * \sinh(f*x + e) * \sqrt{-a} * \arctan(1/2 * \sqrt{2} \\
&) * \sqrt{-a} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x \\
&+ e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) / (a * \cosh(f*x + e \\
&) + a * \sinh(f*x + e))) + 2 * \sqrt{2} * (2*a^2 * \cosh(f*x + e)^8 + 16*a^2 * \cosh(f*x \\
&+ e) * \sinh(f*x + e)^7 + 2*a^2 * \sinh(f*x + e)^8 - (16*a^2 + a*b) * \cosh(f*x + e) \\
&^6 + (56*a^2 * \cosh(f*x + e)^2 - 16*a^2 - a*b) * \sinh(f*x + e)^6 + 2 * (56*a^2 * \cosh \\
&sh(f*x + e)^3 - 3 * (16*a^2 + a*b) * \cosh(f*x + e)) * \sinh(f*x + e)^5 + 2 * (10*a^2 \\
&+ a*b) * \cosh(f*x + e)^4 + (140*a^2 * \cosh(f*x + e)^4 - 15 * (16*a^2 + a*b) * \cosh \\
&(f*x + e)^2 + 20*a^2 + 2*a*b) * \sinh(f*x + e)^4 + 4 * (28*a^2 * \cosh(f*x + e)^5 - \\
&5 * (16*a^2 + a*b) * \cosh(f*x + e)^3 + 2 * (10*a^2 + a*b) * \cosh(f*x + e)) * \sinh(f* \\
&x + e)^3 - (16*a^2 + a*b) * \cosh(f*x + e)^2 + (56*a^2 * \cosh(f*x + e)^6 - 15 * (1 \\
&6*a^2 + a*b) * \cosh(f*x + e)^4 + 12 * (10*a^2 + a*b) * \cosh(f*x + e)^2 - 16*a^2 - \\
&a*b) * \sinh(f*x + e)^2 + 2*a^2 + 2 * (8*a^2 * \cosh(f*x + e)^7 - 3 * (16*a^2 + a*b) \\
&* \cosh(f*x + e)^5 + 4 * (10*a^2 + a*b) * \cosh(f*x + e)^3 - (16*a^2 + a*b) * \cosh(f \\
&*x + e)) * \sinh(f*x + e) * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - \\
&b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)) / (\\
&a^2 * f * \cosh(f*x + e)^9 + 9*a^2 * f * \cosh(f*x + e) * \sinh(f*x + e)^8 + a^2 * f * \sinh(\\
&f*x + e)^9 - 4*a^2 * f * \cosh(f*x + e)^7 + 6*a^2 * f * \cosh(f*x + e)^5 + 4 * (9*a^2 * f \\
&* \cosh(f*x + e)^2 - a^2 * f) * \sinh(f*x + e)^7 + 28 * (3*a^2 * f * \cosh(f*x + e)^3 - a \\
&^2 * f * \cosh(f*x + e)) * \sinh(f*x + e)^6 - 4*a^2 * f * \cosh(f*x + e)^3 + 6 * (21*a^2 * f \\
&* \cosh(f*x + e)^4 - 14*a^2 * f * \cosh(f*x + e)^2 + a^2 * f) * \sinh(f*x + e)^5 + 2 * (6 \\
&3*a^2 * f * \cosh(f*x + e)^5 - 70*a^2 * f * \cosh(f*x + e)^3 + 15*a^2 * f * \cosh(f*x + e) \\
&) * \sinh(f*x + e)^4 + a^2 * f * \cosh(f*x + e) + 4 * (21*a^2 * f * \cosh(f*x + e)^6 - 35* \\
&a^2 * f * \cosh(f*x + e)^4 + 15*a^2 * f * \cosh(f*x + e)^2 - a^2 * f) * \sinh(f*x + e)^3 + \\
&12 * (3*a^2 * f * \cosh(f*x + e)^7 - 7*a^2 * f * \cosh(f*x + e)^5 + 5*a^2 * f * \cosh(f*x + \\
&e)^3 - a^2 * f * \cosh(f*x + e)) * \sinh(f*x + e)^2 + (9*a^2 * f * \cosh(f*x + e)^8 - 2 \\
&8*a^2 * f * \cosh(f*x + e)^6 + 30*a^2 * f * \cosh(f*x + e)^4 - 12*a^2 * f * \cosh(f*x + e) \\
&^2 + a^2 * f) * \sinh(f*x + e))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \coth^5(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^5, x)
```

$$3.463 \quad \int \sqrt{a + b \sinh^2(e + fx)} \tanh^4(e + fx) dx$$

Optimal. Leaf size=292

$$\frac{(3a - 4b)\operatorname{sech}(e + fx)\sqrt{a + b \sinh^2(e + fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) - \tanh^3(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{3f(a - b)\sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

```
[Out] -((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*f) - ((3*a - 4*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*f) - (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.306428, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3196, 467, 578, 531, 418, 492, 411}

$$\frac{\tanh^3(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{(7a - 8b) \tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{3f(a - b)} - \frac{(3a - 4b) \tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{3f(a - b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] -((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*f) - ((3*a - 4*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*f) - (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3)/(3*f)
```

Rule 3196

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
```

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 467

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 578

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(g^(n - 1)*(b*e - a*f)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + b \sinh^2(e + fx)} \tanh^4(e + fx) dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{x^4 \sqrt{a + bx^2}}{(1+x^2)^{5/2}} dx, x, \sinh(e + fx)\right)}{f} \\
&= -\frac{\sqrt{a + b \sinh^2(e + fx)} \tanh^3(e + fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{x^2 \sqrt{a + bx^2}}{(1+x^2)^{5/2}} dx, x, \sinh(e + fx)\right)}{f} \\
&= -\frac{(3a - 4b)\sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{3(a - b)f} - \frac{\sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{3f} \\
&= -\frac{(3a - 4b)\sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{3(a - b)f} - \frac{\sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{3f} \\
&= \frac{(3a - 4b)F\left(\tan^{-1}(\sinh(e + fx))\left|1 - \frac{b}{a}\right.\right) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3(a - b)f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} \\
&= -\frac{(7a - 8b)E\left(\tan^{-1}(\sinh(e + fx))\left|1 - \frac{b}{a}\right.\right) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3(a - b)f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 1.94724, size = 214, normalized size = 0.73

$$\frac{8ia(a - b)\sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} \operatorname{EllipticF}\left(i(e + fx), \frac{b}{a}\right) - \frac{\tanh(e + fx) \operatorname{sech}^2(e + fx) (4(4a^2 - 6ab + b^2) \cosh(2(e + fx)) + 8a^2 + b(4a - 5b) \cosh(4(e + fx)))}{2\sqrt{2}}}{6f(a - b)\sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4, x]
```



```
[Out] ((-2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] - ((8*a^2 - 12*a*b + b^2 + 4*(4*a^2 - 6*a*b + b^2)*Cosh[2*(e + f*x)] + (4*a - 5*b)*b*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e + f*x])/(2*Sqrt[2]))/(6*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.177, size = 369, normalized size = 1.3

$$-\frac{1}{3 (\cosh(fx + e))^3 (a - b) f} \left(\left(4 \sqrt{\frac{b}{a}} ab - 5 \sqrt{\frac{b}{a}} b^2 \right) \sinh(fx + e) (\cosh(fx + e))^4 + \left(4 \sqrt{\frac{b}{a}} a^2 - 10 \sqrt{\frac{b}{a}} ab + 6 \sqrt{\frac{b}{a}} b^2 \right) \cosh(fx + e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x)
```

```
[Out] -1/3*((4*(-1/a*b)^(1/2)*a*b-5*(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+(4*(-1/a*b)^(1/2)*a^2-10*(-1/a*b)^(1/2)*a*b+6*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+(-(-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)-(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-11*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2)/cosh(f*x+e)^3/(a-b)/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \tanh^4(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh^2(fx + e) + a} \tanh^4(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \tanh^4(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)

[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**4, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \tanh^4(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)

$$3.464 \quad \int \sqrt{a + b \sinh^2(e + fx)} \tanh^2(e + fx) dx$$

Optimal. Leaf size=168

$$\frac{\operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{\tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{2 \operatorname{sech}(e + fx)}{f}$$

[Out] (-2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f

Rubi [A] time = 0.177173, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3196, 467, 531, 418, 492, 411}

$$\frac{\tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{\operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} F\left(\tan^{-1}(\sinh(e + fx)) \middle| 1 - \frac{b}{a}\right)}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} - \frac{2 \operatorname{sech}(e + fx)}{f}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]

[Out] (-2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^(p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + b \sinh^2(e + fx)} \tanh^2(e + fx) dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} \right) \operatorname{Subst} \left(\int \frac{x^2 \sqrt{a + bx^2}}{(1+x^2)^{3/2}} dx, x, \sinh(e + fx) \right)}{f} \\
&= -\frac{\sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{f} + \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} \right) \operatorname{Subst} \left(\int \frac{x^2 \sqrt{a + bx^2}}{(1+x^2)^{3/2}} dx, x, \sinh(e + fx) \right)}{f} \\
&= -\frac{\sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{f} + \frac{\left(a \sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)} \right) \operatorname{Subst} \left(\int \frac{x^2 \sqrt{a + bx^2}}{(1+x^2)^{3/2}} dx, x, \sinh(e + fx) \right)}{f} \\
&= \frac{F \left(\tan^{-1}(\sinh(e + fx)) \middle| 1 - \frac{b}{a} \right) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{\sqrt{a + b \sinh^2(e + fx)}}{f} \\
&= -\frac{2E \left(\tan^{-1}(\sinh(e + fx)) \middle| 1 - \frac{b}{a} \right) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{F \left(\tan^{-1}(\sinh(e + fx)) \middle| 1 - \frac{b}{a} \right) \sqrt{a + b \sinh^2(e + fx)}}{f}
\end{aligned}$$

Mathematica [C] time = 0.493876, size = 150, normalized size = 0.89

$$\frac{i\sqrt{2a} \sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} \operatorname{EllipticF} \left(i(e + fx), \frac{b}{a} \right) + \tanh(e + fx)(-2a - b \cosh(2(e + fx)) + b) - 2i\sqrt{2a} \sqrt{\frac{2a + b \cosh(2(e + fx))}{a}}}{f \sqrt{4a + 2b \cosh(2(e + fx)) - 2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]

[Out] ((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + (-2*a + b - b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.135, size = 233, normalized size = 1.4

$$-\frac{1}{f \cosh(fx + e)} \left(\sqrt{-\frac{b}{a}} b (\sinh(fx + e))^3 - a \sqrt{\frac{a + b (\sinh(fx + e))^2}{a}} \sqrt{(\cosh(fx + e))^2} \operatorname{EllipticF} \left(\sinh(fx + e) \sqrt{\frac{a + b (\sinh(fx + e))^2}{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x)`

[Out]
$$-\left(-\frac{1}{a*b}\right)^{\frac{1}{2}}*b*\sinh(f*x+e)^3-a*\left(\frac{a+b*\sinh(f*x+e)^2}{a}\right)^{\frac{1}{2}}*\cosh(f*x+e)^2)^{\frac{1}{2}}*\text{EllipticF}\left(\sinh(f*x+e)*\left(-\frac{1}{a*b}\right)^{\frac{1}{2}},\left(\frac{a}{b}\right)^{\frac{1}{2}}\right)+2*b*\left(\frac{a+b*\sinh(f*x+e)^2}{a}\right)^{\frac{1}{2}}*\cosh(f*x+e)^2)^{\frac{1}{2}}*\text{EllipticF}\left(\sinh(f*x+e)*\left(-\frac{1}{a*b}\right)^{\frac{1}{2}},\left(\frac{a}{b}\right)^{\frac{1}{2}}\right)-2*b*\left(\frac{a+b*\sinh(f*x+e)^2}{a}\right)^{\frac{1}{2}}*\cosh(f*x+e)^2)^{\frac{1}{2}}*\text{EllipticE}\left(\sinh(f*x+e)*\left(-\frac{1}{a*b}\right)^{\frac{1}{2}},\left(\frac{a}{b}\right)^{\frac{1}{2}}\right)+\left(-\frac{1}{a*b}\right)^{\frac{1}{2}}*a*\sinh(f*x+e)\right)/\left(-\frac{1}{a*b}\right)^{\frac{1}{2}}/\cosh(f*x+e)/\left(a+b*\sinh(f*x+e)^2\right)^{\frac{1}{2}}/f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \tanh^2(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh^2(fx + e) + a} \tanh^2(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \tanh^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)

[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**2, x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \tanh(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)

$$3.465 \quad \int \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=60

$$\frac{i\sqrt{a + b \sinh^2(e + fx)} E\left(ie + ifx \middle| \frac{b}{a}\right)}{f\sqrt{\frac{b \sinh^2(e + fx)}{a} + 1}}$$

[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rubi [A] time = 0.0380248, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3178, 3177}

$$\frac{i\sqrt{a + b \sinh^2(e + fx)} E\left(ie + ifx \middle| \frac{b}{a}\right)}{f\sqrt{\frac{b \sinh^2(e + fx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Dist[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{a + b \sinh^2(e + fx)} dx = \frac{\sqrt{a + b \sinh^2(e + fx)} \int \sqrt{1 + \frac{b \sinh^2(e+fx)}{a}} dx}{\sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}}$$

$$= -\frac{iE\left(i e + i f x \left| \frac{b}{a} \right. \right) \sqrt{a + b \sinh^2(e + fx)}}{f \sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.0841224, size = 69, normalized size = 1.15

$$\frac{ia \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} E\left(i(e+fx) \left| \frac{b}{a} \right. \right)}{f \sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a])/ (f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0., size = 140, normalized size = 2.3

$$\frac{1}{\cosh(fx + e) f} \sqrt{\frac{a + b (\sinh(fx + e))^2}{a}} \sqrt{(\cosh(fx + e))^2} \left(a \text{EllipticF}\left(\sinh(fx + e) \sqrt{-\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) - b \text{EllipticF}\left(\sinh(fx + e) \sqrt{\frac{b}{a}}, \sqrt{-\frac{a}{b}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2)))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh (fx + e)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh (fx + e)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2 (e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh (fx + e)^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

$$3.466 \quad \int \coth^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=202

$$\frac{(a + b)\operatorname{sech}(e + fx)\sqrt{a + b \sinh^2(e + fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{2 \tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{f}$$

[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f

Rubi [A] time = 0.198898, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3196, 473, 531, 418, 492, 411}

$$\frac{2 \tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{f} - \frac{\coth(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a + b)\operatorname{sech}(e + fx)\sqrt{a + b \sinh^2(e + fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2]^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^

$p)/(1 - ff^2*x^2)^{(m+1)/2}, x], x, \text{Sin}[e + f*x]/ff], x]] /; \text{FreeQ}\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[m/2] \&\& !\text{IntegerQ}[p]$

Rule 473

$\text{Int}[(e_*)*(x_)^{(m_*)}*((a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_)^{(n_*)})^{(q_*)}, x_Symbol] \rightarrow \text{Simp}[(e*x)^{(m+1)}*(a + b*x^n)^p*(c + d*x^n)^q/(e*(m+1)), x] - \text{Dist}[n/(e^n*(m+1)), \text{Int}[(e*x)^{(m+n)}*(a + b*x^n)^{(p-1)}*(c + d*x^n)^{(q-1)}*\text{Simp}[b*c*p + a*d*q + b*d*(p+q)*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[q, 0] \&\& \text{LtQ}[m, -1] \&\& \text{GtQ}[p, 0] \&\& \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 531

$\text{Int}[(a_*) + (b_*)*(x_)^{(n_*)})^{(p_*)}*((c_*) + (d_*)*(x_)^{(n_*)})^{(q_*)}*((e_*) + (f_*)*(x_)^{(n_*)}), x_Symbol] \rightarrow \text{Dist}[e, \text{Int}[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + \text{Dist}[f, \text{Int}[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p, q\}, x]$

Rule 418

$\text{Int}[1/(\text{Sqrt}[a_*) + (b_*)*(x_)^2]*\text{Sqrt}[(c_*) + (d_*)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticF}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{PosQ}[d/c] \&\& \text{PosQ}[b/a] \&\& !\text{SimplerSqrtQ}[b/a, d/c]$

Rule 492

$\text{Int}[(x_)^2/(\text{Sqrt}[a_*) + (b_*)*(x_)^2]*\text{Sqrt}[(c_*) + (d_*)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[(x*\text{Sqrt}[a + b*x^2])/(b*\text{Sqrt}[c + d*x^2]), x] - \text{Dist}[c/b, \text{Int}[\text{Sqrt}[a + b*x^2]/(c + d*x^2)^{(3/2)}, x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c] \&\& !\text{SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[a_*) + (b_*)*(x_)^2]/((c_*) + (d_*)*(x_)^2)^{(3/2)}, x_Symbol] \rightarrow \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned}
\int \coth^2(e+fx) \sqrt{a+b \sinh^2(e+fx)} dx &= \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2} \sqrt{a+bx^2}}{x^2} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\coth(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{\left(2\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2} \sqrt{a+bx^2}}{x^2} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\coth(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{\left(2b\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2} \sqrt{a+bx^2}}{x^2} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\coth(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{(a+b)F\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b)}{a}}} \\
&= -\frac{\coth(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{f} - \frac{2E\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right) \operatorname{sech}(e+fx)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b)}{a}}}
\end{aligned}$$

Mathematica [C] time = 0.570496, size = 154, normalized size = 0.76

$$\frac{i\sqrt{2}(a-b)\sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \coth(e+fx)(-2a-b \cosh(2(e+fx))+b) - 2i\sqrt{2}a\sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}}}{f\sqrt{4a+2b \cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.142, size = 215, normalized size = 1.1

$$-\frac{1}{\cosh(fx+e) \sinh(fx+e) f} \left(-\sinh(fx+e) \sqrt{\frac{b(\cosh(fx+e))^2}{a} + \frac{a-b}{a} \sqrt{(\cosh(fx+e))^2}} \left(a \operatorname{EllipticF}\left(\sinh(fx+e), \frac{b}{a}\right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out]
$$-\left(-\sinh(fx+e)\left(\frac{b}{a}\cosh(fx+e)^2+\frac{a-b}{a}\right)^{1/2}\cosh(fx+e)^2\right)^{1/2}\left(a\operatorname{EllipticF}\left(\sinh(fx+e)\left(-\frac{1}{a}b\right)^{1/2},\left(\frac{a}{b}\right)^{1/2}\right)-b\operatorname{EllipticF}\left(\sinh(fx+e)\left(-\frac{1}{a}b\right)^{1/2},\left(\frac{a}{b}\right)^{1/2}\right)+2b\operatorname{EllipticE}\left(\sinh(fx+e)\left(-\frac{1}{a}b\right)^{1/2},\left(\frac{a}{b}\right)^{1/2}\right)\right)+\left(-\frac{1}{a}b\right)^{1/2}b\cosh(fx+e)^4+\left(-\frac{1}{a}b\right)^{1/2}a-\left(-\frac{1}{a}b\right)^{1/2}b\cosh(fx+e)^2\right)/\sinh(fx+e)/\left(-\frac{1}{a}b\right)^{1/2}/\cosh(fx+e)/(a+b\sinh(fx+e)^2)^{1/2}/f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \coth^2(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\sqrt{b \sinh^2(fx + e) + a} \coth^2(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2(e + fx)} \coth^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```


$$3.467 \quad \int \coth^4(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx$$

Optimal. Leaf size=270

$$\frac{(3a + 5b) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right)}{3af \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{(7a + b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af}$$

```
[Out] -((3*a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rubi [A] time = 0.295758, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3196, 473, 580, 531, 418, 492, 411}

$$\frac{(7a + b) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af} - \frac{\coth^3(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{(3a + b) \coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2], x]
```

```
[Out] -((3*a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rule 3196

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
```

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 473

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 580

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned} \int \coth^4(e + fx) \sqrt{a + b \sinh^2(e + fx)} dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2} \sqrt{a+bx^2}}{x^4} dx, x, \sinh(e + fx)\right)}{f} \\ &= -\frac{\coth^3(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{\left(2\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) S}{3f} \\ &= -\frac{(3a + b) \coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af} - \frac{\coth^3(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} \\ &= -\frac{(3a + b) \coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af} - \frac{\coth^3(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} \\ &= -\frac{(3a + b) \coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af} - \frac{\coth^3(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} \\ &= -\frac{(3a + b) \coth(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3af} - \frac{\coth^3(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} \end{aligned}$$

Mathematica [C] time = 3.06738, size = 210, normalized size = 0.78

$$\frac{8ia(a-b)\sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\coth(e+fx) \operatorname{csch}^2(e+fx) (4(4a^2-2ab-b^2) \cosh(2(e+fx))-8a^2+b(4a+b) \cosh(4(e+fx)))}{2\sqrt{2}}}{6af\sqrt{2a+b \cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2], x]

```
[Out] (-((-8*a^2 + 4*a*b + 3*b^2 + 4*(4*a^2 - 2*a*b - b^2)*Cosh[2*(e + f*x)] + b*(4*a + b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/(2*Sqrt[2]) - (2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a*EllipticE[I*(e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a*EllipticF[I*(e + f*x), b/a]/(6*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)])]
```

Maple [A] time = 0.17, size = 519, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*sinh(f*x+e)^3+2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*sinh(f*x+e)^3+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2*sinh(f*x+e)^3-7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2*sinh(f*x+e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+6*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2+2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2/a/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh^2(fx + e) + a} \coth^4(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh(fx + e)^2 + a} \coth(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(fx + e)^2 + a} \coth(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)

3.468 $\int (a + b \sinh^2(e + fx))^{3/2} \tanh^5(e + fx) dx$

Optimal. Leaf size=232

$$\frac{(8a^2 - 40ab + 35b^2)(a + b \sinh^2(e + fx))^{3/2}}{24f(a - b)^2} + \frac{(8a^2 - 40ab + 35b^2)\sqrt{a + b \sinh^2(e + fx)}}{8f(a - b)} - \frac{(8a^2 - 40ab + 35b^2) \tanh^5(e + fx)}{8f\sqrt{a - b}}$$

```
[Out] -((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(8*Sqrt[a - b]*f) + ((8*a^2 - 40*a*b + 35*b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*(a - b)*f) + ((8*a^2 - 40*a*b + 35*b^2)*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*(a - b)^2*f) + ((8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4*(a - b)*f)
```

Rubi [A] time = 0.290255, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{(8a^2 - 40ab + 35b^2)(a + b \sinh^2(e + fx))^{3/2}}{24f(a - b)^2} + \frac{(8a^2 - 40ab + 35b^2)\sqrt{a + b \sinh^2(e + fx)}}{8f(a - b)} - \frac{(8a^2 - 40ab + 35b^2) \tanh^5(e + fx)}{8f\sqrt{a - b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5, x]
```

```
[Out] -((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(8*Sqrt[a - b]*f) + ((8*a^2 - 40*a*b + 35*b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*(a - b)*f) + ((8*a^2 - 40*a*b + 35*b^2)*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*(a - b)^2*f) + ((8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4*(a - b)*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(p_.)*tan[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
```

rQ[(m - 1)/2]

Rule 89

```
Int[((a_.) + (b_.)*(x_))2*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := Simp[((b*c - a*d)2*(c + d*x)(n + 1)*(e + f*x)(p + 1))/(d2*(d*e - c*f)(n + 1)), x] - Dist[1/(d2*(d*e - c*f)(n + 1)), Int[(c + d*x)(n + 1)*(e + f*x)p*Simp[a2*d2*f*(n + p + 2) + b2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)(n + 1)*(e + f*x)(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)n*(e + f*x)(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))(m_.)*((c_.) + (d_.)*(x_))(n_.), x_Symbol] := Simp[((a + b*x)(m + 1)*(c + d*x)n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)m*(c + d*x)(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))(m_.)*((c_.) + (d_.)*(x_))(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x(p*(m + 1) - 1)*(c - (a*d)/b + (d*xp)/b)n, x], x, (a + b*x)(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)2)(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^2(e + fx))^{3/2} \tanh^5(e + fx) dx &= \frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{(1+x)^3} dx, x, \sinh^2(e + fx)\right)}{2f} \\
&= -\frac{\text{sech}^4(e + fx) (a + b \sinh^2(e + fx))^{5/2}}{4(a - b)f} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-4a+5b)+2(a-b)x\right)(a+b)}{(1+x)^2} dx, x, \sinh^2(e + fx)\right)}{4(a - b)f} \\
&= \frac{(8a - 9b)\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{5/2}}{8(a - b)^2 f} - \frac{\text{sech}^4(e + fx) (a + b \sinh^2(e + fx))^{5/2}}{4(a - b)f} \\
&= \frac{(8a^2 - 40ab + 35b^2) (a + b \sinh^2(e + fx))^{3/2}}{24(a - b)^2 f} + \frac{(8a - 9b)\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{5/2}}{8(a - b)f} \\
&= \frac{(8a^2 - 40ab + 35b^2) \sqrt{a + b \sinh^2(e + fx)}}{8(a - b)f} + \frac{(8a^2 - 40ab + 35b^2) (a + b \sinh^2(e + fx))^{3/2}}{24(a - b)^2 f} \\
&= \frac{(8a^2 - 40ab + 35b^2) \sqrt{a + b \sinh^2(e + fx)}}{8(a - b)f} + \frac{(8a^2 - 40ab + 35b^2) (a + b \sinh^2(e + fx))^{3/2}}{24(a - b)^2 f} \\
&= -\frac{(8a^2 - 40ab + 35b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8\sqrt{a-b}f} + \frac{(8a^2 - 40ab + 35b^2) \sqrt{a + b \sinh^2(e + fx)}}{24(a - b)^2 f}
\end{aligned}$$

Mathematica [A] time = 1.63614, size = 169, normalized size = 0.73

$$\frac{-\left(8a^2 - 40ab + 35b^2\right) \left(\sqrt{a + b \sinh^2(e + fx)} (4a + b \sinh^2(e + fx) - 3b) - 3(a - b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)\right) + 6(a - b)^{3/2}}{24f(a - b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5,x]

[Out] -(-3*(8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + 6*(a - b)*Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2) - (8*a^2 - 40*a*b + 35*b^2)*(-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a + b*Sinh[e + f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2)))/(24*(a - b)^2*f)

Maple [C] time = 0.122, size = 71, normalized size = 0.3

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^5 (b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^2 + a^2)}{(\cosh(fx + e))^6} \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x)

[Out] `int/indef0` (sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e)^2 + a)^{\frac{3}{2}} \tanh(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^5, x)

Fricas [B] time = 9.4306, size = 16471, normalized size = 71.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="fricas")

[Out] [1/48*(3*((8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^11 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2 + 32*a^2 - 160*a*b + 140*b^2)*sinh(f*x + e)^9 + 3*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + 12*(8*a^2 - 40*a*b + 35

$$\begin{aligned}
& *b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^8 + 6*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x \\
& + e)^7 + 6*(55*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^4 + 24*(8*a^2 - 40* \\
& a*b + 35*b^2) * \cosh(f*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2) * \sinh(f*x + e)^7 + \\
& 42*(11*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^5 + 8*(8*a^2 - 40*a*b + 35*b \\
& ^2) * \cosh(f*x + e)^3 + (8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)) * \sinh(f*x + e \\
&)^6 + 4*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^5 + 2*(231*(8*a^2 - 40*a*b \\
& + 35*b^2) * \cosh(f*x + e)^6 + 252*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^4 + \\
& 63*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^2 + 16*a^2 - 80*a*b + 70*b^2) * s \\
& \sinh(f*x + e)^5 + 2*(165*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^7 + 252*(8* \\
& a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^5 + 105*(8*a^2 - 40*a*b + 35*b^2) * \cosh \\
& (f*x + e)^3 + 10*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^4 + \\
& (8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^3 + (165*(8*a^2 - 40*a*b + 35*b^2) \\
& * \cosh(f*x + e)^8 + 336*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^6 + 210*(8*a \\
& ^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^4 + 40*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f \\
& *x + e)^2 + 8*a^2 - 40*a*b + 35*b^2) * \sinh(f*x + e)^3 + (55*(8*a^2 - 40*a*b \\
& + 35*b^2) * \cosh(f*x + e)^9 + 144*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^7 + \\
& 126*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^5 + 40*(8*a^2 - 40*a*b + 35*b^ \\
& 2) * \cosh(f*x + e)^3 + 3*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)) * \sinh(f*x + \\
& e)^2 + (11*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^10 + 36*(8*a^2 - 40*a*b \\
& + 35*b^2) * \cosh(f*x + e)^8 + 42*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^6 + \\
& 20*(8*a^2 - 40*a*b + 35*b^2) * \cosh(f*x + e)^4 + 3*(8*a^2 - 40*a*b + 35*b^2) * \\
& \cosh(f*x + e)^2) * \sinh(f*x + e)) * \sqrt{a - b} * \log((b * \cosh(f*x + e)^4 + 4*b * co \\
& sh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2*(4*a - 3*b) * \cosh(f*x + \\
& e)^2 + 2*(3*b * \cosh(f*x + e)^2 + 4*a - 3*b) * \sinh(f*x + e)^2 - 4 * \sqrt{2} * \sqrt{ \\
& a - b} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + \\
& e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)}) * (\cosh(f*x + e) + s \\
& \sinh(f*x + e)) + 4*(b * \cosh(f*x + e)^3 + (4*a - 3*b) * \cosh(f*x + e)) * \sinh(f*x \\
& + e) + b) / (\cosh(f*x + e)^4 + 4 * \cosh(f*x + e) * \sinh(f*x + e)^3 + \sinh(f*x + e \\
&)^4 + 2*(3 * \cosh(f*x + e)^2 + 1) * \sinh(f*x + e)^2 + 2 * \cosh(f*x + e)^2 + 4*(co \\
& sh(f*x + e)^3 + \cosh(f*x + e)) * \sinh(f*x + e) + 1)) + 2 * \sqrt{2} * ((a*b - b^2) \\
& * \cosh(f*x + e)^12 + 12*(a*b - b^2) * \cosh(f*x + e) * \sinh(f*x + e)^11 + (a*b - \\
& b^2) * \sinh(f*x + e)^12 + 2*(8*a^2 - 25*a*b + 17*b^2) * \cosh(f*x + e)^10 + 2*(3 \\
& 3*(a*b - b^2) * \cosh(f*x + e)^2 + 8*a^2 - 25*a*b + 17*b^2) * \sinh(f*x + e)^10 + \\
& 20*(11*(a*b - b^2) * \cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2) * \cosh(f*x + \\
& e)) * \sinh(f*x + e)^9 + (112*a^2 - 335*a*b + 223*b^2) * \cosh(f*x + e)^8 + (495* \\
& (a*b - b^2) * \cosh(f*x + e)^4 + 90*(8*a^2 - 25*a*b + 17*b^2) * \cosh(f*x + e)^2 \\
& + 112*a^2 - 335*a*b + 223*b^2) * \sinh(f*x + e)^8 + 8*(99*(a*b - b^2) * \cosh(f*x \\
& + e)^5 + 30*(8*a^2 - 25*a*b + 17*b^2) * \cosh(f*x + e)^3 + (112*a^2 - 335*a*b \\
& + 223*b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^7 + 8*(18*a^2 - 59*a*b + 41*b^2) * c \\
& osh(f*x + e)^6 + 4*(231*(a*b - b^2) * \cosh(f*x + e)^6 + 105*(8*a^2 - 25*a*b + \\
& 17*b^2) * \cosh(f*x + e)^4 + 7*(112*a^2 - 335*a*b + 223*b^2) * \cosh(f*x + e)^2 \\
& + 36*a^2 - 118*a*b + 82*b^2) * \sinh(f*x + e)^6 + 8*(99*(a*b - b^2) * \cosh(f*x + \\
& e)^7 + 63*(8*a^2 - 25*a*b + 17*b^2) * \cosh(f*x + e)^5 + 7*(112*a^2 - 335*a*b \\
& + 223*b^2) * \cosh(f*x + e)^3 + 6*(18*a^2 - 59*a*b + 41*b^2) * \cosh(f*x + e)) * s \\
& \sinh(f*x + e)^5 + (112*a^2 - 335*a*b + 223*b^2) * \cosh(f*x + e)^4 + (495*(a*b
\end{aligned}$$

$$\begin{aligned}
& - b^2) \cosh(f*x + e)^8 + 420*(8*a^2 - 25*a*b + 17*b^2) \cosh(f*x + e)^6 + 70 \\
& *(112*a^2 - 335*a*b + 223*b^2) \cosh(f*x + e)^4 + 120*(18*a^2 - 59*a*b + 41* \\
& b^2) \cosh(f*x + e)^2 + 112*a^2 - 335*a*b + 223*b^2) \sinh(f*x + e)^4 + 4*(55 \\
& *(a*b - b^2) \cosh(f*x + e)^9 + 60*(8*a^2 - 25*a*b + 17*b^2) \cosh(f*x + e)^7 \\
& + 14*(112*a^2 - 335*a*b + 223*b^2) \cosh(f*x + e)^5 + 40*(18*a^2 - 59*a*b + \\
& 41*b^2) \cosh(f*x + e)^3 + (112*a^2 - 335*a*b + 223*b^2) \cosh(f*x + e) \sin \\
& h(f*x + e)^3 + 2*(8*a^2 - 25*a*b + 17*b^2) \cosh(f*x + e)^2 + 2*(33*(a*b - b \\
& ^2) \cosh(f*x + e)^10 + 45*(8*a^2 - 25*a*b + 17*b^2) \cosh(f*x + e)^8 + 14*(1 \\
& 12*a^2 - 335*a*b + 223*b^2) \cosh(f*x + e)^6 + 60*(18*a^2 - 59*a*b + 41*b^2) \\
& * \cosh(f*x + e)^4 + 3*(112*a^2 - 335*a*b + 223*b^2) \cosh(f*x + e)^2 + 8*a^2 \\
& - 25*a*b + 17*b^2) \sinh(f*x + e)^2 + a*b - b^2 + 4*(3*(a*b - b^2) \cosh(f*x \\
& + e)^11 + 5*(8*a^2 - 25*a*b + 17*b^2) \cosh(f*x + e)^9 + 2*(112*a^2 - 335*a* \\
& b + 223*b^2) \cosh(f*x + e)^7 + 12*(18*a^2 - 59*a*b + 41*b^2) \cosh(f*x + e)^ \\
& 5 + (112*a^2 - 335*a*b + 223*b^2) \cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^ \\
& 2) \cosh(f*x + e) \sinh(f*x + e) \sqrt{(b \cosh(f*x + e)^2 + b \sinh(f*x + e)^ \\
& 2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 \cosh(f*x + e) \sinh(f*x + e) + \sinh(f*x + \\
& e)^2)) / ((a - b) * f * \cosh(f*x + e)^11 + 11*(a - b) * f * \cosh(f*x + e) \sinh(f*x + \\
& e)^10 + (a - b) * f * \sinh(f*x + e)^11 + 4*(a - b) * f * \cosh(f*x + e)^9 + (55*(a \\
& - b) * f * \cosh(f*x + e)^2 + 4*(a - b) * f) \sinh(f*x + e)^9 + 6*(a - b) * f * \cosh(f* \\
& x + e)^7 + 3*(55*(a - b) * f * \cosh(f*x + e)^3 + 12*(a - b) * f * \cosh(f*x + e) \sin \\
& h(f*x + e)^8 + 6*(55*(a - b) * f * \cosh(f*x + e)^4 + 24*(a - b) * f * \cosh(f*x + e \\
&)^2 + (a - b) * f) \sinh(f*x + e)^7 + 4*(a - b) * f * \cosh(f*x + e)^5 + 42*(11*(a \\
& - b) * f * \cosh(f*x + e)^5 + 8*(a - b) * f * \cosh(f*x + e)^3 + (a - b) * f * \cosh(f*x + \\
& e) \sinh(f*x + e)^6 + 2*(231*(a - b) * f * \cosh(f*x + e)^6 + 252*(a - b) * f * \cos \\
& h(f*x + e)^4 + 63*(a - b) * f * \cosh(f*x + e)^2 + 2*(a - b) * f) \sinh(f*x + e)^5 \\
& + (a - b) * f * \cosh(f*x + e)^3 + 2*(165*(a - b) * f * \cosh(f*x + e)^7 + 252*(a - b \\
&) * f * \cosh(f*x + e)^5 + 105*(a - b) * f * \cosh(f*x + e)^3 + 10*(a - b) * f * \cosh(f*x \\
& + e) \sinh(f*x + e)^4 + (165*(a - b) * f * \cosh(f*x + e)^8 + 336*(a - b) * f * \cos \\
& h(f*x + e)^6 + 210*(a - b) * f * \cosh(f*x + e)^4 + 40*(a - b) * f * \cosh(f*x + e)^2 \\
& + (a - b) * f) \sinh(f*x + e)^3 + (55*(a - b) * f * \cosh(f*x + e)^9 + 144*(a - b) \\
& * f * \cosh(f*x + e)^7 + 126*(a - b) * f * \cosh(f*x + e)^5 + 40*(a - b) * f * \cosh(f*x \\
& + e)^3 + 3*(a - b) * f * \cosh(f*x + e) \sinh(f*x + e)^2 + (11*(a - b) * f * \cosh(f* \\
& x + e)^10 + 36*(a - b) * f * \cosh(f*x + e)^8 + 42*(a - b) * f * \cosh(f*x + e)^6 + 2 \\
& 0*(a - b) * f * \cosh(f*x + e)^4 + 3*(a - b) * f * \cosh(f*x + e)^2) \sinh(f*x + e)), \\
& -1/24*(3*((8*a^2 - 40*a*b + 35*b^2) \cosh(f*x + e)^11 + 11*(8*a^2 - 40*a*b + \\
& 35*b^2) \cosh(f*x + e) \sinh(f*x + e)^10 + (8*a^2 - 40*a*b + 35*b^2) \sinh(f* \\
& x + e)^11 + 4*(8*a^2 - 40*a*b + 35*b^2) \cosh(f*x + e)^9 + (55*(8*a^2 - 40*a* \\
& *b + 35*b^2) \cosh(f*x + e)^2 + 32*a^2 - 160*a*b + 140*b^2) \sinh(f*x + e)^9 \\
& + 3*(55*(8*a^2 - 40*a*b + 35*b^2) \cosh(f*x + e)^3 + 12*(8*a^2 - 40*a*b + 35 \\
& *b^2) \cosh(f*x + e) \sinh(f*x + e)^8 + 6*(8*a^2 - 40*a*b + 35*b^2) \cosh(f*x \\
& + e)^7 + 6*(55*(8*a^2 - 40*a*b + 35*b^2) \cosh(f*x + e)^4 + 24*(8*a^2 - 40* \\
& a*b + 35*b^2) \cosh(f*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2) \sinh(f*x + e)^7 + \\
& 42*(11*(8*a^2 - 40*a*b + 35*b^2) \cosh(f*x + e)^5 + 8*(8*a^2 - 40*a*b + 35*b \\
& ^2) \cosh(f*x + e)^3 + (8*a^2 - 40*a*b + 35*b^2) \cosh(f*x + e) \sinh(f*x + e \\
&)^6 + 4*(8*a^2 - 40*a*b + 35*b^2) \cosh(f*x + e)^5 + 2*(231*(8*a^2 - 40*a*b
\end{aligned}$$

$$\begin{aligned}
& + 35*b^2)*\cosh(f*x + e)^6 + 252*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^4 + \\
& 63*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^2 + 16*a^2 - 80*a*b + 70*b^2)*s \\
& \sinh(f*x + e)^5 + 2*(165*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^7 + 252*(8* \\
& a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^5 + 105*(8*a^2 - 40*a*b + 35*b^2)*\cosh \\
& (f*x + e)^3 + 10*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^4 + \\
& (8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^3 + (165*(8*a^2 - 40*a*b + 35*b^2) \\
& *\cosh(f*x + e)^8 + 336*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^6 + 210*(8*a \\
& ^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^4 + 40*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f \\
& *x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*\sinh(f*x + e)^3 + (55*(8*a^2 - 40*a*b \\
& + 35*b^2)*\cosh(f*x + e)^9 + 144*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^7 + \\
& 126*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^5 + 40*(8*a^2 - 40*a*b + 35*b^ \\
& 2)*\cosh(f*x + e)^3 + 3*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e))*\sinh(f*x + \\
& e)^2 + (11*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^10 + 36*(8*a^2 - 40*a*b \\
& + 35*b^2)*\cosh(f*x + e)^8 + 42*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^6 + \\
& 20*(8*a^2 - 40*a*b + 35*b^2)*\cosh(f*x + e)^4 + 3*(8*a^2 - 40*a*b + 35*b^2)* \\
& \cosh(f*x + e)^2)*\sinh(f*x + e))*\sqrt{-a + b}*\arctan(-1/2*\sqrt{2}*\sqrt{-a + \\
& b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 \\
& - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) \\
& + (a - b)*sinh(f*x + e))) - \sqrt{2}*((a*b - b^2)*\cosh(f*x + e)^12 + 12*(a*b \\
& - b^2)*\cosh(f*x + e)*sinh(f*x + e)^11 + (a*b - b^2)*sinh(f*x + e)^12 + 2*(\\
& 8*a^2 - 25*a*b + 17*b^2)*\cosh(f*x + e)^10 + 2*(33*(a*b - b^2)*\cosh(f*x + e) \\
& ^2 + 8*a^2 - 25*a*b + 17*b^2)*sinh(f*x + e)^10 + 20*(11*(a*b - b^2)*\cosh(f* \\
& x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*\cosh(f*x + e))*sinh(f*x + e)^9 + (112* \\
& a^2 - 335*a*b + 223*b^2)*\cosh(f*x + e)^8 + (495*(a*b - b^2)*\cosh(f*x + e)^4 \\
& + 90*(8*a^2 - 25*a*b + 17*b^2)*\cosh(f*x + e)^2 + 112*a^2 - 335*a*b + 223*b \\
& ^2)*sinh(f*x + e)^8 + 8*(99*(a*b - b^2)*\cosh(f*x + e)^5 + 30*(8*a^2 - 25*a* \\
& b + 17*b^2)*\cosh(f*x + e)^3 + (112*a^2 - 335*a*b + 223*b^2)*\cosh(f*x + e))* \\
& \sinh(f*x + e)^7 + 8*(18*a^2 - 59*a*b + 41*b^2)*\cosh(f*x + e)^6 + 4*(231*(a* \\
& b - b^2)*\cosh(f*x + e)^6 + 105*(8*a^2 - 25*a*b + 17*b^2)*\cosh(f*x + e)^4 + \\
& 7*(112*a^2 - 335*a*b + 223*b^2)*\cosh(f*x + e)^2 + 36*a^2 - 118*a*b + 82*b^2 \\
&)*\sinh(f*x + e)^6 + 8*(99*(a*b - b^2)*\cosh(f*x + e)^7 + 63*(8*a^2 - 25*a*b \\
& + 17*b^2)*\cosh(f*x + e)^5 + 7*(112*a^2 - 335*a*b + 223*b^2)*\cosh(f*x + e)^3 \\
& + 6*(18*a^2 - 59*a*b + 41*b^2)*\cosh(f*x + e))*sinh(f*x + e)^5 + (112*a^2 - \\
& 335*a*b + 223*b^2)*\cosh(f*x + e)^4 + (495*(a*b - b^2)*\cosh(f*x + e)^8 + 42 \\
& 0*(8*a^2 - 25*a*b + 17*b^2)*\cosh(f*x + e)^6 + 70*(112*a^2 - 335*a*b + 223*b \\
& ^2)*\cosh(f*x + e)^4 + 120*(18*a^2 - 59*a*b + 41*b^2)*\cosh(f*x + e)^2 + 112* \\
& a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^4 + 4*(55*(a*b - b^2)*\cosh(f*x + e)^ \\
& 9 + 60*(8*a^2 - 25*a*b + 17*b^2)*\cosh(f*x + e)^7 + 14*(112*a^2 - 335*a*b + \\
& 223*b^2)*\cosh(f*x + e)^5 + 40*(18*a^2 - 59*a*b + 41*b^2)*\cosh(f*x + e)^3 + \\
& (112*a^2 - 335*a*b + 223*b^2)*\cosh(f*x + e))*sinh(f*x + e)^3 + 2*(8*a^2 - 2 \\
& 5*a*b + 17*b^2)*\cosh(f*x + e)^2 + 2*(33*(a*b - b^2)*\cosh(f*x + e)^10 + 45*(\\
& 8*a^2 - 25*a*b + 17*b^2)*\cosh(f*x + e)^8 + 14*(112*a^2 - 335*a*b + 223*b^2) \\
& *\cosh(f*x + e)^6 + 60*(18*a^2 - 59*a*b + 41*b^2)*\cosh(f*x + e)^4 + 3*(112*a \\
& ^2 - 335*a*b + 223*b^2)*\cosh(f*x + e)^2 + 8*a^2 - 25*a*b + 17*b^2)*sinh(f*x \\
& + e)^2 + a*b - b^2 + 4*(3*(a*b - b^2)*\cosh(f*x + e)^11 + 5*(8*a^2 - 25*a*b
\end{aligned}$$

```

+ 17*b^2)*cosh(f*x + e)^9 + 2*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^
7 + 12*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^5 + (112*a^2 - 335*a*b + 22
3*b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x
+ e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x
+ e)^11 + 11*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^10 + (a - b)*f*sinh(f*x
+ e)^11 + 4*(a - b)*f*cosh(f*x + e)^9 + (55*(a - b)*f*cosh(f*x + e)^2 + 4*
(a - b)*f)*sinh(f*x + e)^9 + 6*(a - b)*f*cosh(f*x + e)^7 + 3*(55*(a - b)*f*
cosh(f*x + e)^3 + 12*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(55*(a -
b)*f*cosh(f*x + e)^4 + 24*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x +
e)^7 + 4*(a - b)*f*cosh(f*x + e)^5 + 42*(11*(a - b)*f*cosh(f*x + e)^5 + 8*
(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(2
31*(a - b)*f*cosh(f*x + e)^6 + 252*(a - b)*f*cosh(f*x + e)^4 + 63*(a - b)*f
*cosh(f*x + e)^2 + 2*(a - b)*f)*sinh(f*x + e)^5 + (a - b)*f*cosh(f*x + e)^3
+ 2*(165*(a - b)*f*cosh(f*x + e)^7 + 252*(a - b)*f*cosh(f*x + e)^5 + 105*(
a - b)*f*cosh(f*x + e)^3 + 10*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^4 + (1
65*(a - b)*f*cosh(f*x + e)^8 + 336*(a - b)*f*cosh(f*x + e)^6 + 210*(a - b)*
f*cosh(f*x + e)^4 + 40*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)
^3 + (55*(a - b)*f*cosh(f*x + e)^9 + 144*(a - b)*f*cosh(f*x + e)^7 + 126*(a
- b)*f*cosh(f*x + e)^5 + 40*(a - b)*f*cosh(f*x + e)^3 + 3*(a - b)*f*cosh(f
*x + e))*sinh(f*x + e)^2 + (11*(a - b)*f*cosh(f*x + e)^10 + 36*(a - b)*f*co
sh(f*x + e)^8 + 42*(a - b)*f*cosh(f*x + e)^6 + 20*(a - b)*f*cosh(f*x + e)^4
+ 3*(a - b)*f*cosh(f*x + e)^2)*sinh(f*x + e))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**5,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \tanh(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^5, x)
```

$$3.469 \quad \int (a + b \sinh^2(e + fx))^{3/2} \tanh^3(e + fx) dx$$

Optimal. Leaf size=156

$$\frac{(2a - 5b)(a + b \sinh^2(e + fx))^{3/2}}{6f(a - b)} + \frac{(2a - 5b)\sqrt{a + b \sinh^2(e + fx)}}{2f} - \frac{(2a - 5b)\sqrt{a - b} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{2f} + \frac{\operatorname{sech}^2(e + fx)}{2f}$$

```
[Out] -((2*a - 5*b)*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])
/(2*f) + ((2*a - 5*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a - 5*b)*(a
+ b*Sinh[e + f*x]^2)^(3/2))/(6*(a - b)*f) + (Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2)^(5/2))/(2*(a - b)*f)
```

Rubi [A] time = 0.158917, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a - 5b)(a + b \sinh^2(e + fx))^{3/2}}{6f(a - b)} + \frac{(2a - 5b)\sqrt{a + b \sinh^2(e + fx)}}{2f} - \frac{(2a - 5b)\sqrt{a - b} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{2f} + \frac{\operatorname{sech}^2(e + fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]
```

```
[Out] -((2*a - 5*b)*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])
/(2*f) + ((2*a - 5*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a - 5*b)*(a
+ b*Sinh[e + f*x]^2)^(3/2))/(6*(a - b)*f) + (Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2)^(5/2))/(2*(a - b)*f)
```

Rule 3194

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
+ 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

```

Rule 50

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^2(e + fx))^{3/2} \tanh^3(e + fx) dx &= \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{(1+x)^2} dx, x, \sinh^2(e + fx)\right)}{2f} \\
&= \frac{\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{5/2}}{2(a - b)f} + \frac{(2a - 5b) \text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x} dx\right)}{4(a - b)f} \\
&= \frac{(2a - 5b) (a + b \sinh^2(e + fx))^{3/2}}{6(a - b)f} + \frac{\text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{5/2}}{2(a - b)f} \\
&= \frac{(2a - 5b) \sqrt{a + b \sinh^2(e + fx)}}{2f} + \frac{(2a - 5b) (a + b \sinh^2(e + fx))^{3/2}}{6(a - b)f} + \dots \\
&= \frac{(2a - 5b) \sqrt{a + b \sinh^2(e + fx)}}{2f} + \frac{(2a - 5b) (a + b \sinh^2(e + fx))^{3/2}}{6(a - b)f} + \dots \\
&= -\frac{(2a - 5b) \sqrt{a - b} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{2f} + \frac{(2a - 5b) \sqrt{a + b \sinh^2(e + fx)}}{2f}
\end{aligned}$$

Mathematica [A] time = 0.54235, size = 122, normalized size = 0.78

$$\frac{(2a - 5b) \left(\sqrt{a + b \sinh^2(e + fx)} (4a + b \sinh^2(e + fx) - 3b) - 3(a - b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right) \right) + 3 \text{sech}^2(e + fx) (a + b \sinh^2(e + fx))^{5/2}}{6f(a - b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]

[Out] (3*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + (2*a - 5*b)*(-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a + b*Sinh[e + f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2))/(6*(a - b)*f)

Maple [C] time = 0.125, size = 71, normalized size = 0.5

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^3 (b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^2 + a^2)}{(\cosh(fx + e))^4} \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x)`

[Out] ``int/indef0`(sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \tanh(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^3, x)`

Fricas [B] time = 7.38749, size = 6527, normalized size = 41.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="fricas")`

[Out] `[-1/24*(6*((2*a - 5*b)*cosh(f*x + e)^7 + 7*(2*a - 5*b)*cosh(f*x + e)*sinh(f*x + e)^6 + (2*a - 5*b)*sinh(f*x + e)^7 + 2*(2*a - 5*b)*cosh(f*x + e)^5 + (21*(2*a - 5*b)*cosh(f*x + e)^2 + 4*a - 10*b)*sinh(f*x + e)^5 + 5*(7*(2*a - 5*b)*cosh(f*x + e)^3 + 2*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a - 5*b)*cosh(f*x + e)^3 + (35*(2*a - 5*b)*cosh(f*x + e)^4 + 20*(2*a - 5*b)*cosh(f*x + e)^2 + 2*a - 5*b)*sinh(f*x + e)^3 + (21*(2*a - 5*b)*cosh(f*x + e)^5 + 20*(2*a - 5*b)*cosh(f*x + e)^3 + 3*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (7*(2*a - 5*b)*cosh(f*x + e)^6 + 10*(2*a - 5*b)*cosh(f*x + e)^4 + 3*(2*a - 5*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))*(co`

$$\begin{aligned}
& \text{sh}(f*x + e) + \sinh(f*x + e)) + 4*(b*\cosh(f*x + e)^3 + (4*a - 3*b)*\cosh(f*x \\
& + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 \\
& + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x \\
& + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) - \sqrt{2} \\
& *(b*\cosh(f*x + e)^8 + 8*b*\cosh(f*x + e)*\sinh(f*x + e)^7 + b*\sinh(f*x + e)^8 \\
& + 8*(2*a - 3*b)*\cosh(f*x + e)^6 + 4*(7*b*\cosh(f*x + e)^2 + 4*a - 6*b)*\sinh \\
& (f*x + e)^6 + 8*(7*b*\cosh(f*x + e)^3 + 6*(2*a - 3*b)*\cosh(f*x + e))*\sinh(f* \\
& x + e)^5 + 2*(28*a - 37*b)*\cosh(f*x + e)^4 + 2*(35*b*\cosh(f*x + e)^4 + 60*(\\
& 2*a - 3*b)*\cosh(f*x + e)^2 + 28*a - 37*b)*\sinh(f*x + e)^4 + 8*(7*b*\cosh(f*x \\
& + e)^5 + 20*(2*a - 3*b)*\cosh(f*x + e)^3 + (28*a - 37*b)*\cosh(f*x + e))*\sin \\
& h(f*x + e)^3 + 8*(2*a - 3*b)*\cosh(f*x + e)^2 + 4*(7*b*\cosh(f*x + e)^6 + 30* \\
& (2*a - 3*b)*\cosh(f*x + e)^4 + 3*(28*a - 37*b)*\cosh(f*x + e)^2 + 4*a - 6*b)* \\
& \sinh(f*x + e)^2 + 8*(b*\cosh(f*x + e)^7 + 6*(2*a - 3*b)*\cosh(f*x + e)^5 + (2 \\
& 8*a - 37*b)*\cosh(f*x + e)^3 + 2*(2*a - 3*b)*\cosh(f*x + e))*\sinh(f*x + e) + \\
& b)*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 \\
& - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(f*\cosh(f*x + e)^7 + 7 \\
& *f*\cosh(f*x + e)*\sinh(f*x + e)^6 + f*\sinh(f*x + e)^7 + 2*f*\cosh(f*x + e)^5 \\
& + (21*f*\cosh(f*x + e)^2 + 2*f)*\sinh(f*x + e)^5 + 5*(7*f*\cosh(f*x + e)^3 + 2 \\
& *f*\cosh(f*x + e))*\sinh(f*x + e)^4 + f*\cosh(f*x + e)^3 + (35*f*\cosh(f*x + e) \\
& ^4 + 20*f*\cosh(f*x + e)^2 + f)*\sinh(f*x + e)^3 + (21*f*\cosh(f*x + e)^5 + 20 \\
& *f*\cosh(f*x + e)^3 + 3*f*\cosh(f*x + e))*\sinh(f*x + e)^2 + (7*f*\cosh(f*x + e) \\
&)^6 + 10*f*\cosh(f*x + e)^4 + 3*f*\cosh(f*x + e)^2)*\sinh(f*x + e)), -1/24*(12 \\
& *((2*a - 5*b)*\cosh(f*x + e)^7 + 7*(2*a - 5*b)*\cosh(f*x + e)*\sinh(f*x + e)^6 \\
& + (2*a - 5*b)*\sinh(f*x + e)^7 + 2*(2*a - 5*b)*\cosh(f*x + e)^5 + (21*(2*a - \\
& 5*b)*\cosh(f*x + e)^2 + 4*a - 10*b)*\sinh(f*x + e)^5 + 5*(7*(2*a - 5*b)*\cosh \\
& (f*x + e)^3 + 2*(2*a - 5*b)*\cosh(f*x + e))*\sinh(f*x + e)^4 + (2*a - 5*b)*\co \\
& sh(f*x + e)^3 + (35*(2*a - 5*b)*\cosh(f*x + e)^4 + 20*(2*a - 5*b)*\cosh(f*x + \\
& e)^2 + 2*a - 5*b)*\sinh(f*x + e)^3 + (21*(2*a - 5*b)*\cosh(f*x + e)^5 + 20*(\\
& 2*a - 5*b)*\cosh(f*x + e)^3 + 3*(2*a - 5*b)*\cosh(f*x + e))*\sinh(f*x + e)^2 + \\
& (7*(2*a - 5*b)*\cosh(f*x + e)^6 + 10*(2*a - 5*b)*\cosh(f*x + e)^4 + 3*(2*a - \\
& 5*b)*\cosh(f*x + e)^2)*\sinh(f*x + e))*\sqrt{-a + b}*\arctan(-1/2*\sqrt{2}*\sqrt{ \\
& (-a + b)*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + \\
& e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))}/((a - b)*\cosh(f*x \\
& + e) + (a - b)*\sinh(f*x + e))) - \sqrt{2}*(b*\cosh(f*x + e)^8 + 8*b*\cosh(f*x \\
& + e)*\sinh(f*x + e)^7 + b*\sinh(f*x + e)^8 + 8*(2*a - 3*b)*\cosh(f*x + e)^6 + \\
& 4*(7*b*\cosh(f*x + e)^2 + 4*a - 6*b)*\sinh(f*x + e)^6 + 8*(7*b*\cosh(f*x + e) \\
& ^3 + 6*(2*a - 3*b)*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(28*a - 37*b)*\cosh(f* \\
& x + e)^4 + 2*(35*b*\cosh(f*x + e)^4 + 60*(2*a - 3*b)*\cosh(f*x + e)^2 + 28*a \\
& - 37*b)*\sinh(f*x + e)^4 + 8*(7*b*\cosh(f*x + e)^5 + 20*(2*a - 3*b)*\cosh(f*x \\
& + e)^3 + (28*a - 37*b)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 8*(2*a - 3*b)*\cosh(\\
& f*x + e)^2 + 4*(7*b*\cosh(f*x + e)^6 + 30*(2*a - 3*b)*\cosh(f*x + e)^4 + 3*(2 \\
& 8*a - 37*b)*\cosh(f*x + e)^2 + 4*a - 6*b)*\sinh(f*x + e)^2 + 8*(b*\cosh(f*x + \\
& e)^7 + 6*(2*a - 3*b)*\cosh(f*x + e)^5 + (28*a - 37*b)*\cosh(f*x + e)^3 + 2*(2 \\
& *a - 3*b)*\cosh(f*x + e))*\sinh(f*x + e) + b)*\sqrt{((b*\cosh(f*x + e)^2 + b*\sin \\
& h(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) +
\end{aligned}$$

```
sinh(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 +
  f*sinh(f*x + e)^7 + 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 + 2*f)*sin
h(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 2*f*cosh(f*x + e))*sinh(f*x + e)^4
+ f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 + 20*f*cosh(f*x + e)^2 + f)*sin
h(f*x + e)^3 + (21*f*cosh(f*x + e)^5 + 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x
+ e))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 + 10*f*cosh(f*x + e)^4 + 3*f*c
osh(f*x + e)^2)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^{\frac{3}{2}} \tanh^3(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^3, x)
```

$$3.470 \quad \int (a + b \sinh^2(e + fx))^{3/2} \tanh(e + fx) dx$$

Optimal. Leaf size=90

$$\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a+b\sinh^2(e+fx))^{3/2}}{3f} - \frac{(a-b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + ((a - b)*Sqrt[a + b*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)

Rubi [A] time = 0.090232, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 50, 63, 208}

$$\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a+b\sinh^2(e+fx))^{3/2}}{3f} - \frac{(a-b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x],x]

[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + ((a - b)*Sqrt[a + b*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ

`[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0])) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 63

`Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rubi steps

$$\begin{aligned}
 \int (a + b \sinh^2(e + fx))^{3/2} \tanh(e + fx) dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x} dx, x, \sinh^2(e + fx)\right)}{2f} \\
 &= \frac{(a + b \sinh^2(e + fx))^{3/2}}{3f} + \frac{(a - b) \text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x} dx, x, \sinh^2(e + fx)\right)}{2f} \\
 &= \frac{(a - b) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a + b \sinh^2(e + fx))^{3/2}}{3f} + \frac{(a - b)^2 \text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x} dx, x, \sinh^2(e + fx)\right)}{2f} \\
 &= \frac{(a - b) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a + b \sinh^2(e + fx))^{3/2}}{3f} + \frac{(a - b)^2 \text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x} dx, x, \sinh^2(e + fx)\right)}{2f} \\
 &= -\frac{(a - b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(a - b) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a + b \sinh^2(e + fx))^{3/2}}{3f}
 \end{aligned}$$

Mathematica [A] time = 0.146198, size = 86, normalized size = 0.96

$$\frac{(4a + b \cosh^2(e + fx) - 4b) \sqrt{a + b \cosh^2(e + fx) - b} - 3(a - b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b \cosh^2(e+fx)-b}}{\sqrt{a-b}}\right)}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x], x]

[Out] $(-3*(a - b)^{(3/2)}*ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]] + (4*a - 4*b + b*Cosh[e + f*x]^2)*Sqrt[a - b + b*Cosh[e + f*x]^2])/(3*f)$

Maple [C] time = 0.296, size = 69, normalized size = 0.8

$$\frac{1}{f} \int \frac{\sinh(fx + e) \left(b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^2 + a^2 \right)}{(\cosh(fx + e))^2} \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e), x)

[Out] $\int (a + b \sinh(fx + e)^2)^{3/2} \tanh(fx + e) dx$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{3/2} \tanh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e), x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e), x)

Fricas [B] time = 6.38133, size = 2827, normalized size = 31.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e), x, algorithm="fricas")

```
[Out] [-1/24*(12*((a - b)*cosh(f*x + e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x + e) + 3*(a - b)*cosh(f*x + e)*sinh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - 7*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - 7*b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - 7*b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3), -1/24*(24*((a - b)*cosh(f*x + e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x + e) + 3*(a - b)*cosh(f*x + e)*sinh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - 7*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - 7*b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - 7*b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e), x)
```

```
[Out] Timed out
```


Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \tanh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e), x)
```

$$3.471 \quad \int \coth(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=78

$$-\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{(a+b \sinh^2(e+fx))^{3/2}}{3f}$$

[Out] $-\left(\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right]}{f}\right) + \frac{a \sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{(a+b \sinh^2(e+fx))^{3/2}}{3f}$

Rubi [A] time = 0.0924509, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 50, 63, 208}

$$-\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{(a+b \sinh^2(e+fx))^{3/2}}{3f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[e + fx] * (a + b \sinh^2[e + fx])^{3/2}, x]$

[Out] $-\left(\frac{a^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right]}{f}\right) + \frac{a \sqrt{a+b \sinh^2(e+fx)}}{f} + \frac{(a+b \sinh^2(e+fx))^{3/2}}{3f}$

Rule 3194

$\operatorname{Int}[(a + b \sin^2(e + f x))^{p/2} \tan(e + f x), x] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\sin^2[e + f x], x]\}, \operatorname{Dist}[ff^{(m+1)/2} / (2f), \operatorname{Subst}[\operatorname{Int}[(x^{(m-1)/2} (a + b ff x)^p) / (1 - ff x)^{(m+1)/2}, x], x, \sin[e + f x]^2 / ff, x]] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 50

$\operatorname{Int}[(a + b x)^m (c + d x)^n, x] \rightarrow \operatorname{Simp}[(a + b x)^{m+1} (c + d x)^n / (b(m+n+1)), x] + \operatorname{Dist}[(n(b c - a d)) / (b(m+n+1)), \operatorname{Int}[(a + b x)^m (c + d x)^{n-1}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b c - a d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[m + n + 1, 0] \&\& \operatorname{!}(\operatorname{IGtQ}$

`[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0])) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 208

`Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rubi steps

$$\begin{aligned}
 \int \coth(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, \sinh^2(e + fx)\right)}{2f} \\
 &= \frac{(a + b \sinh^2(e + fx))^{3/2}}{3f} + \frac{a \text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, \sinh^2(e + fx)\right)}{2f} \\
 &= \frac{a\sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a + b \sinh^2(e + fx))^{3/2}}{3f} + \frac{a^2 \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \sinh^2(e + fx)\right)}{2f} \\
 &= \frac{a\sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a + b \sinh^2(e + fx))^{3/2}}{3f} + \frac{a^2 \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sinh^2(e + fx)\right)}{2f} \\
 &= -\frac{a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{(a + b \sinh^2(e + fx))^{3/2}}{3f}
 \end{aligned}$$

Mathematica [A] time = 0.122469, size = 69, normalized size = 0.88

$$\frac{\sqrt{a + b \sinh^2(e + fx)} (4a + b \sinh^2(e + fx)) - 3a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{3f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (-3*a^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sinh[e + f*x]^2]*(4*a + b*Sinh[e + f*x]^2))/(3*f)

Maple [C] time = 0.078, size = 62, normalized size = 0.8

$$\frac{1}{f} \int \frac{b^2 (\sinh(fx + e))^3 + 2ab \sinh(fx + e) + \frac{a^2}{\sinh(fx + e)}}{\sqrt{a + b (\sinh(fx + e))^2}} \sinh(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] `int/indef0`((b^2*sinh(f*x+e)^3+2*a*b*sinh(f*x+e)+a^2/sinh(f*x+e))/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(fx + e)^2 + a)^{\frac{3}{2}} \coth(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e), x)

Fricas [B] time = 3.79683, size = 2696, normalized size = 34.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

```
[Out] [1/24*(12*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*x + e) + 3*a*cosh
(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(a)*log((b*cosh(f*x + e)
^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*co
sh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(
2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e)
+ sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e)*sinh(f*x
+ e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(c
osh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*(b*cosh(f*x +
e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - b)*sinh(f*x + e)^2 + 4*(b
*cosh(f*x + e)^3 + (8*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)
^2*sinh(f*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3),
1/24*(24*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*x + e) + 3*a*cosh
(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(-a)*arctan(1/2*sqrt(2)*
sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*cosh(f*x + e)
+ a*sinh(f*x + e))) + sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f
*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f
*x + e)^2 + 8*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - b)*cos
h(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f*x
+ e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.472 \quad \int \coth^3(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=140

$$\frac{(2a + 3b) \left(a + b \sinh^2(e + fx) \right)^{3/2}}{6af} + \frac{(2a + 3b) \sqrt{a + b \sinh^2(e + fx)}}{2f} - \frac{\sqrt{a}(2a + 3b) \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}} \right)}{2f} - \frac{\operatorname{csch}^2(e + fx)}{2f}$$

```
[Out] -(Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*f) +
((2*a + 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a + 3*b)*(a + b*Sinh
[e + f*x]^2)^(3/2))/(6*a*f) - (Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2
))/(2*a*f)
```

Rubi [A] time = 0.147753, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a + 3b) \left(a + b \sinh^2(e + fx) \right)^{3/2}}{6af} + \frac{(2a + 3b) \sqrt{a + b \sinh^2(e + fx)}}{2f} - \frac{\sqrt{a}(2a + 3b) \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}} \right)}{2f} - \frac{\operatorname{csch}^2(e + fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] -(Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*f) +
((2*a + 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a + 3*b)*(a + b*Sinh
[e + f*x]^2)^(3/2))/(6*a*f) - (Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2
))/(2*a*f)
```

Rule 3194

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
+ 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

```

Rule 50

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \coth^3(e+fx) (a+b\sinh^2(e+fx))^{3/2} dx &= \frac{\text{Subst}\left(\int \frac{(1+x)(a+bx)^{3/2}}{x^2} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{csch}^2(e+fx) (a+b\sinh^2(e+fx))^{5/2}}{2af} + \frac{(2a+3b) \text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, \sinh^2(e+fx)\right)}{4af} \\
&= \frac{(2a+3b) (a+b\sinh^2(e+fx))^{3/2}}{6af} - \frac{\text{csch}^2(e+fx) (a+b\sinh^2(e+fx))^{5/2}}{2af} \\
&= \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a+3b) (a+b\sinh^2(e+fx))^{3/2}}{6af} - \frac{\text{csch}^2(e+fx) (a+b\sinh^2(e+fx))^{5/2}}{2af} \\
&= \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a+3b) (a+b\sinh^2(e+fx))^{3/2}}{6af} - \frac{\text{csch}^2(e+fx) (a+b\sinh^2(e+fx))^{5/2}}{2af} \\
&= -\frac{\sqrt{a}(2a+3b) \tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f}
\end{aligned}$$

Mathematica [A] time = 0.44647, size = 90, normalized size = 0.64

$$\frac{\sqrt{a+b\sinh^2(e+fx)} (-3a\text{csch}^2(e+fx) + 8a + b\cosh(2(e+fx)) + 5b) - 3\sqrt{a}(2a+3b) \tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{6f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] (-3*Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + (8*a + 5*b + b*Cosh[2*(e + f*x)] - 3*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(6*f)

Maple [C] time = 0.124, size = 84, normalized size = 0.6

$$\frac{1}{f} \int \frac{b^2 (\sinh(fx+e))^3 + (2ab + b^2) \sinh(fx+e) + \frac{a^2 + 2ab}{\sinh(fx+e)} + \frac{a^2}{(\sinh(fx+e))^3}}{\sqrt{a+b\sinh^2(fx+e)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(((b^2*sinh(f*x+e)^3+(2*a*b+b^2)*sinh(f*x+e)+(a^2+2*a*b)/sinh(f*x+e)+a^2/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \coth(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^3, x)
```

Fricas [B] time = 5.47209, size = 6418, normalized size = 45.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(6*((2*a + 3*b)*cosh(f*x + e)^7 + 7*(2*a + 3*b)*cosh(f*x + e)*sinh(f*x + e)^6 + (2*a + 3*b)*sinh(f*x + e)^7 - 2*(2*a + 3*b)*cosh(f*x + e)^5 + (2*1*(2*a + 3*b)*cosh(f*x + e)^2 - 4*a - 6*b)*sinh(f*x + e)^5 + 5*(7*(2*a + 3*b)*cosh(f*x + e)^3 - 2*(2*a + 3*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a + 3*b)*cosh(f*x + e)^3 + (35*(2*a + 3*b)*cosh(f*x + e)^4 - 20*(2*a + 3*b)*cosh(f*x + e)^2 + 2*a + 3*b)*sinh(f*x + e)^3 + (21*(2*a + 3*b)*cosh(f*x + e)^5 - 20*(2*a + 3*b)*cosh(f*x + e)^3 + 3*(2*a + 3*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (7*(2*a + 3*b)*cosh(f*x + e)^6 - 10*(2*a + 3*b)*cosh(f*x + e)^4 + 3*(2*a + 3*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) +
```

$$\begin{aligned}
& \sinh(f*x + e)) + 4*(b*\cosh(f*x + e)^3 + (4*a - b)*\cosh(f*x + e))*\sinh(f*x + \\
& e) + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e) \\
& ^4 + 2*(3*\cosh(f*x + e)^2 - 1)*\sinh(f*x + e)^2 - 2*\cosh(f*x + e)^2 + 4*(\cos \\
& h(f*x + e)^3 - \cosh(f*x + e))*\sinh(f*x + e) + 1)) + \sqrt{2}*(b*\cosh(f*x + e) \\
&)^8 + 8*b*\cosh(f*x + e)*\sinh(f*x + e)^7 + b*\sinh(f*x + e)^8 + 8*(2*a + b)*\c \\
& osh(f*x + e)^6 + 4*(7*b*\cosh(f*x + e)^2 + 4*a + 2*b)*\sinh(f*x + e)^6 + 8*(7 \\
& *b*\cosh(f*x + e)^3 + 6*(2*a + b)*\cosh(f*x + e))*\sinh(f*x + e)^5 - 2*(28*a + \\
& 9*b)*\cosh(f*x + e)^4 + 2*(35*b*\cosh(f*x + e)^4 + 60*(2*a + b)*\cosh(f*x + e) \\
&)^2 - 28*a - 9*b)*\sinh(f*x + e)^4 + 8*(7*b*\cosh(f*x + e)^5 + 20*(2*a + b)*\c \\
& osh(f*x + e)^3 - (28*a + 9*b)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 8*(2*a + b)* \\
& cosh(f*x + e)^2 + 4*(7*b*\cosh(f*x + e)^6 + 30*(2*a + b)*\cosh(f*x + e)^4 - 3 \\
& *(28*a + 9*b)*\cosh(f*x + e)^2 + 4*a + 2*b)*\sinh(f*x + e)^2 + 8*(b*\cosh(f*x \\
& + e)^7 + 6*(2*a + b)*\cosh(f*x + e)^5 - (28*a + 9*b)*\cosh(f*x + e)^3 + 2*(2* \\
& a + b)*\cosh(f*x + e))*\sinh(f*x + e) + b)*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f \\
& *x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sin \\
& h(f*x + e)^2)))/(f*\cosh(f*x + e)^7 + 7*f*\cosh(f*x + e)*\sinh(f*x + e)^6 + f* \\
& sinh(f*x + e)^7 - 2*f*\cosh(f*x + e)^5 + (21*f*\cosh(f*x + e)^2 - 2*f)*\sinh(f \\
& *x + e)^5 + 5*(7*f*\cosh(f*x + e)^3 - 2*f*\cosh(f*x + e))*\sinh(f*x + e)^4 + f \\
& *cosh(f*x + e)^3 + (35*f*\cosh(f*x + e)^4 - 20*f*\cosh(f*x + e)^2 + f)*\sinh(f \\
& *x + e)^3 + (21*f*\cosh(f*x + e)^5 - 20*f*\cosh(f*x + e)^3 + 3*f*\cosh(f*x + e) \\
&))*\sinh(f*x + e)^2 + (7*f*\cosh(f*x + e)^6 - 10*f*\cosh(f*x + e)^4 + 3*f*\cosh \\
& (f*x + e)^2)*\sinh(f*x + e)), 1/24*(12*((2*a + 3*b)*\cosh(f*x + e)^7 + 7*(2*a \\
& + 3*b)*\cosh(f*x + e)*\sinh(f*x + e)^6 + (2*a + 3*b)*\sinh(f*x + e)^7 - 2*(2* \\
& a + 3*b)*\cosh(f*x + e)^5 + (21*(2*a + 3*b)*\cosh(f*x + e)^2 - 4*a - 6*b)*\sin \\
& h(f*x + e)^5 + 5*(7*(2*a + 3*b)*\cosh(f*x + e)^3 - 2*(2*a + 3*b)*\cosh(f*x + \\
& e))*\sinh(f*x + e)^4 + (2*a + 3*b)*\cosh(f*x + e)^3 + (35*(2*a + 3*b)*\cosh(f* \\
& x + e)^4 - 20*(2*a + 3*b)*\cosh(f*x + e)^2 + 2*a + 3*b)*\sinh(f*x + e)^3 + (2 \\
& 1*(2*a + 3*b)*\cosh(f*x + e)^5 - 20*(2*a + 3*b)*\cosh(f*x + e)^3 + 3*(2*a + 3 \\
& *b)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (7*(2*a + 3*b)*\cosh(f*x + e)^6 - 10*(2 \\
& *a + 3*b)*\cosh(f*x + e)^4 + 3*(2*a + 3*b)*\cosh(f*x + e)^2)*\sinh(f*x + e))*\s \\
& qrt(-a)*\arctan(1/2*\sqrt{2}*\sqrt{-a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + \\
& e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x \\
& + e)^2)))/(a*\cosh(f*x + e) + a*\sinh(f*x + e))) + \sqrt{2}*(b*\cosh(f*x + e)^8 \\
& + 8*b*\cosh(f*x + e)*\sinh(f*x + e)^7 + b*\sinh(f*x + e)^8 + 8*(2*a + b)*\cosh \\
& (f*x + e)^6 + 4*(7*b*\cosh(f*x + e)^2 + 4*a + 2*b)*\sinh(f*x + e)^6 + 8*(7*b* \\
& cosh(f*x + e)^3 + 6*(2*a + b)*\cosh(f*x + e))*\sinh(f*x + e)^5 - 2*(28*a + 9* \\
& b)*\cosh(f*x + e)^4 + 2*(35*b*\cosh(f*x + e)^4 + 60*(2*a + b)*\cosh(f*x + e)^2 \\
& - 28*a - 9*b)*\sinh(f*x + e)^4 + 8*(7*b*\cosh(f*x + e)^5 + 20*(2*a + b)*\cosh \\
& (f*x + e)^3 - (28*a + 9*b)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 8*(2*a + b)*\cos \\
& h(f*x + e)^2 + 4*(7*b*\cosh(f*x + e)^6 + 30*(2*a + b)*\cosh(f*x + e)^4 - 3*(2 \\
& 8*a + 9*b)*\cosh(f*x + e)^2 + 4*a + 2*b)*\sinh(f*x + e)^2 + 8*(b*\cosh(f*x + e) \\
&)^7 + 6*(2*a + b)*\cosh(f*x + e)^5 - (28*a + 9*b)*\cosh(f*x + e)^3 + 2*(2*a + \\
& b)*\cosh(f*x + e))*\sinh(f*x + e) + b)*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f*x \\
& + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f \\
& *x + e)^2)))/(f*\cosh(f*x + e)^7 + 7*f*\cosh(f*x + e)*\sinh(f*x + e)^6 + f*\sin
\end{aligned}$$

$$h(f*x + e)^7 - 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 - 2*f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 - 2*f*cosh(f*x + e))*sinh(f*x + e)^4 + f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^3 + (21*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 - 10*f*cosh(f*x + e)^4 + 3*f*cosh(f*x + e)^2)*sinh(f*x + e)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.473 \quad \int \coth^5(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=203

$$\frac{(8a^2 + 3b(8a + b)) (a + b \sinh^2(e + fx))^{3/2}}{24a^2 f} + \frac{(8a^2 + 3b(8a + b)) \sqrt{a + b \sinh^2(e + fx)}}{8af} - \frac{(8a^2 + 3b(8a + b)) \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}} \right)}{8\sqrt{af}}$$

[Out] -((8*a^2 + 3*b*(8*a + b))*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*Sqrt[a]*f) + ((8*a^2 + 3*b*(8*a + b))*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a*f) + ((8*a^2 + 3*b*(8*a + b))*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*a^2*f) - ((8*a + b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4*a*f)

Rubi [A] time = 0.23248, antiderivative size = 199, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{\left(\frac{3b(8a+b)}{a^2} + 8 \right) (a + b \sinh^2(e + fx))^{3/2}}{24f} + \frac{(8a^2 + 3b(8a + b)) \sqrt{a + b \sinh^2(e + fx)}}{8af} - \frac{(8a^2 + 3b(8a + b)) \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}} \right)}{8\sqrt{af}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] -((8*a^2 + 3*b*(8*a + b))*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*Sqrt[a]*f) + ((8*a^2 + 3*b*(8*a + b))*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a*f) + ((8 + (3*b*(8*a + b))/a^2)*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*f) - ((8*a + b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4*a*f)

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)^2]^(p_))*tan[(e_) + (f_)*(x_)^2]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 89

```
Int[((a_.) + (b_.)*(x_))2*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := Simp[((b*c - a*d)2*(c + d*x)(n + 1)*(e + f*x)(p + 1))/(d2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d2*(d*e - c*f)*(n + 1)), Int[(c + d*x)(n + 1)*(e + f*x)p*Simp[a2*d2*f*(n + p + 2) + b2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))(n_.)*((e_.) + (f_.)*(x_))(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)(n + 1)*(e + f*x)(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)n*(e + f*x)(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))(m_.)*((c_.) + (d_.)*(x_))(n_.), x_Symbol] := Simp[((a + b*x)(m + 1)*(c + d*x)n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)m*(c + d*x)(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))(m_.)*((c_.) + (d_.)*(x_))(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x(p*(m + 1) - 1)*(c - (a*d)/b + (d*xp)/b)n, x], x, (a + b*x)(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)2)(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \coth^5(e+fx)(a+b\sinh^2(e+fx))^{3/2} dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^2(a+bx)^{3/2}}{x^3} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{csch}^4(e+fx)(a+b\sinh^2(e+fx))^{5/2}}{4af} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(8a+b)+2ax\right)(a+bx)^{3/2}}{x^2} dx, x, \sinh^2(e+fx)\right)}{4af} \\
&= -\frac{(8a+b)\text{csch}^2(e+fx)(a+b\sinh^2(e+fx))^{5/2}}{8a^2f} - \frac{\text{csch}^4(e+fx)(a+b\sinh^2(e+fx))^{5/2}}{4af} \\
&= \frac{(8a^2+3b(8a+b))(a+b\sinh^2(e+fx))^{3/2}}{24a^2f} - \frac{(8a+b)\text{csch}^2(e+fx)(a+b\sinh^2(e+fx))^{5/2}}{8a^2f} \\
&= \frac{(8a^2+3b(8a+b))\sqrt{a+b\sinh^2(e+fx)}}{8af} + \frac{(8a^2+3b(8a+b))(a+b\sinh^2(e+fx))^{3/2}}{24a^2f} \\
&= \frac{(8a^2+3b(8a+b))\sqrt{a+b\sinh^2(e+fx)}}{8af} + \frac{(8a^2+3b(8a+b))(a+b\sinh^2(e+fx))^{3/2}}{24a^2f} \\
&= -\frac{(8a^2+3b(8a+b))\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f} + \frac{(8a^2+3b(8a+b))\sqrt{a+b\sinh^2(e+fx)}}{8af}
\end{aligned}$$

Mathematica [A] time = 0.789817, size = 123, normalized size = 0.61

$$\frac{\sqrt{a}\sqrt{a+b\sinh^2(e+fx)}(8(4a+b\sinh^2(e+fx)+6b)-3(8a+5b)\text{csch}^2(e+fx)-6a\text{csch}^4(e+fx))-3(8a^2+24ab)}{24\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (-3*(8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sqrt[a]*Sqrt[a + b*Sinh[e + f*x]^2]*(-3*(8*a + 5*b)*Csch[e + f*x]^2 - 6*a*Csch[e + f*x]^4 + 8*(4*a + 6*b + b*Sinh[e + f*x]^2)))/(24*Sqrt[a]*f)

Maple [C] time = 0.254, size = 113, normalized size = 0.6

$$\frac{1}{f} \int \frac{\left(\cosh(fx + e) \right)^4 \left(b^2 \left(\cosh(fx + e) \right)^4 + 2ab \left(\cosh(fx + e) \right)^2 - 2b^2 \left(\cosh(fx + e) \right)^2 + a^2 - 2ab + \dots \right)}{\sinh(fx + e) \left(\left(\cosh(fx + e) \right)^4 - 2 \left(\cosh(fx + e) \right)^2 + 1 \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)`

[Out] ``int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4*(b^2*cosh(f*x+e)^4+2*a*b*cosh(f*x+e)^2-2*b^2*cosh(f*x+e)^2+a^2-2*a*b+b^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \coth(fx + e)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^5, x)`

Fricas [B] time = 6.13721, size = 14438, normalized size = 71.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `[1/48*(3*((8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 + 24*a*b + 3*b^2)*sinh(f*x + e)^11 - 4*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2 - 32*a^2 - 96*a*b - 12*b^2)*sinh(f*x + e)^9 + 3*(55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 12*(8*a^2 + 24*a*b + 3*b^2)*cos`

$$\begin{aligned}
& h(f*x + e)) * \sinh(f*x + e)^8 + 6*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^7 + \\
& 6*(55*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^4 - 24*(8*a^2 + 24*a*b + 3*b^2) \\
&) * \cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3*b^2) * \sinh(f*x + e)^7 + 42*(11*(8*a^2 \\
& + 24*a*b + 3*b^2) * \cosh(f*x + e)^5 - 8*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + \\
& e)^3 + (8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^6 - 4*(8*a^2 + \\
& 24*a*b + 3*b^2) * \cosh(f*x + e)^5 + 2*(231*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x \\
& + e)^6 - 252*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^4 + 63*(8*a^2 + 24*a*b \\
& + 3*b^2) * \cosh(f*x + e)^2 - 16*a^2 - 48*a*b - 6*b^2) * \sinh(f*x + e)^5 + 2*(1 \\
& 65*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^7 - 252*(8*a^2 + 24*a*b + 3*b^2) * \\
& \cosh(f*x + e)^5 + 105*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^3 - 10*(8*a^2 \\
& + 24*a*b + 3*b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^4 + (8*a^2 + 24*a*b + 3*b^2) \\
& * \cosh(f*x + e)^3 + (165*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^8 - 336*(8*a \\
& ^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^6 + 210*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f* \\
& x + e)^4 - 40*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3 \\
& *b^2) * \sinh(f*x + e)^3 + (55*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^9 - 144* \\
& (8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^7 + 126*(8*a^2 + 24*a*b + 3*b^2) * \cos \\
& h(f*x + e)^5 - 40*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^3 + 3*(8*a^2 + 24* \\
& a*b + 3*b^2) * \cosh(f*x + e)) * \sinh(f*x + e)^2 + (11*(8*a^2 + 24*a*b + 3*b^2) * \\
& \cosh(f*x + e)^10 - 36*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^8 + 42*(8*a^2 \\
& + 24*a*b + 3*b^2) * \cosh(f*x + e)^6 - 20*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + \\
& e)^4 + 3*(8*a^2 + 24*a*b + 3*b^2) * \cosh(f*x + e)^2) * \sinh(f*x + e)) * \sqrt{a} * \log((b * \cosh(f*x + e)^4 + 4*b * \cosh(f*x + e) * \sinh(f*x + e)^3 + b * \sinh(f*x + e)^4 + 2*(4*a - b) * \cosh(f*x + e)^2 + 2*(3*b * \cosh(f*x + e)^2 + 4*a - b) * \sinh(f*x + e)^2 - 4*\sqrt{2} * \sqrt{a} * \sqrt{(b * \cosh(f*x + e)^2 + b * \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2 * \cosh(f*x + e) * \sinh(f*x + e) + \sinh(f*x + e)^2)}) * (\cosh(f*x + e) + \sinh(f*x + e)) + 4*(b * \cosh(f*x + e)^3 + (4*a - b) * \cosh(f*x + e)) * \sinh(f*x + e) + b) / (\cosh(f*x + e)^4 + 4 * \cosh(f*x + e) * \sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3 * \cosh(f*x + e)^2 - 1) * \sinh(f*x + e)^2 - 2 * \cosh(f*x + e)^2 + 4 * (\cosh(f*x + e)^3 - \cosh(f*x + e)) * \sinh(f*x + e) + 1)) + 2 * \sqrt{2} * (a * b * \cosh(f*x + e)^12 + 12 * a * b * \cosh(f*x + e) * \sinh(f*x + e)^11 + a * b * \sinh(f*x + e)^12 + 2 * (8 * a^2 + 9 * a * b) * \cosh(f*x + e)^10 + 2 * (33 * a * b * \cosh(f*x + e)^2 + 8 * a^2 + 9 * a * b) * \sinh(f*x + e)^10 + 20 * (11 * a * b * \cosh(f*x + e)^3 + (8 * a^2 + 9 * a * b) * \cosh(f*x + e)) * \sinh(f*x + e)^9 - (112 * a^2 + 111 * a * b) * \cosh(f*x + e)^8 + (495 * a * b * \cosh(f*x + e)^4 + 90 * (8 * a^2 + 9 * a * b) * \cosh(f*x + e)^2 - 112 * a^2 - 111 * a * b) * \sinh(f*x + e)^8 + 8 * (99 * a * b * \cosh(f*x + e)^5 + 30 * (8 * a^2 + 9 * a * b) * \cosh(f*x + e)^3 - (112 * a^2 + 111 * a * b) * \cosh(f*x + e)) * \sinh(f*x + e)^7 + 8 * (18 * a^2 + 23 * a * b) * \cosh(f*x + e)^6 + 4 * (231 * a * b * \cosh(f*x + e)^6 + 105 * (8 * a^2 + 9 * a * b) * \cosh(f*x + e)^4 - 7 * (112 * a^2 + 111 * a * b) * \cosh(f*x + e)^2 + 36 * a^2 + 46 * a * b) * \sinh(f*x + e)^6 + 8 * (99 * a * b * \cosh(f*x + e)^7 + 63 * (8 * a^2 + 9 * a * b) * \cosh(f*x + e)^5 - 7 * (112 * a^2 + 111 * a * b) * \cosh(f*x + e)^3 + 6 * (18 * a^2 + 23 * a * b) * \cosh(f*x + e)) * \sinh(f*x + e)^5 - (112 * a^2 + 111 * a * b) * \cosh(f*x + e)^4 + (495 * a * b * \cosh(f*x + e)^8 + 420 * (8 * a^2 + 9 * a * b) * \cosh(f*x + e)^6 - 70 * (112 * a^2 + 111 * a * b) * \cosh(f*x + e)^4 + 120 * (18 * a^2 + 23 * a * b) * \cosh(f*x + e)^2 - 112 * a^2 - 111 * a * b) * \sinh(f*x + e)^4 + 4 * (55 * a * b * \cosh(f*x + e)^9 + 60 * (8 * a^2 + 9 * a * b) * \cosh(f*x + e)^7 - 14 * (112 * a^2 + 111 * a * b) * \cosh(f*x + e)^5 + 40 * (18
\end{aligned}$$

$$\begin{aligned}
& a^2 + 23ab) \cosh(fx + e)^3 - (112a^2 + 111ab) \cosh(fx + e) \sinh(fx + e)^3 + 2(8a^2 + 9ab) \cosh(fx + e)^2 + 2(33ab \cosh(fx + e)^{10} + \\
& 45(8a^2 + 9ab) \cosh(fx + e)^8 - 14(112a^2 + 111ab) \cosh(fx + e)^6 + 60(18a^2 + 23ab) \cosh(fx + e)^4 - 3(112a^2 + 111ab) \cosh(fx + e)^2 + \\
& 8a^2 + 9ab) \sinh(fx + e)^2 + ab + 4(3ab \cosh(fx + e)^{11} + 5(8a^2 + 9ab) \cosh(fx + e)^9 - 2(112a^2 + 111ab) \cosh(fx + e)^7 + \\
& 12(18a^2 + 23ab) \cosh(fx + e)^5 - (112a^2 + 111ab) \cosh(fx + e)^3 + (8a^2 + 9ab) \cosh(fx + e) \sinh(fx + e)) \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(fx + e)^2)) / (af \cosh(fx + e)^{11} + 11af \cosh(fx + e) \sinh(fx + e)^{10} + af \sinh(fx + e)^{11} - 4af \cosh(fx + e)^9 + (55af \cosh(fx + e)^2 - 4af) \sinh(fx + e)^9 + 6af \cosh(fx + e)^7 + 3(55af \cosh(fx + e)^3 - 12af \cosh(fx + e) \sinh(fx + e)^8 + 6(55af \cosh(fx + e)^4 - 24af \cosh(fx + e)^2 + af) \sinh(fx + e)^7 - 4af \cosh(fx + e)^5 + 42(11af \cosh(fx + e)^5 - 8af \cosh(fx + e)^3 + af \cosh(fx + e)) \sinh(fx + e)^6 + 2(231af \cosh(fx + e)^6 - 252af \cosh(fx + e)^4 + 63af \cosh(fx + e)^2 - 2af) \sinh(fx + e)^5 + af \cosh(fx + e)^3 + 2(165af \cosh(fx + e)^7 - 252af \cosh(fx + e)^5 + 105af \cosh(fx + e)^3 - 10af \cosh(fx + e)) \sinh(fx + e)^4 + (165af \cosh(fx + e)^8 - 336af \cosh(fx + e)^6 + 210af \cosh(fx + e)^4 - 40af \cosh(fx + e)^2 + af) \sinh(fx + e)^3 + (55af \cosh(fx + e)^9 - 144af \cosh(fx + e)^7 + 126af \cosh(fx + e)^5 - 40af \cosh(fx + e)^3 + 3af \cosh(fx + e)) \sinh(fx + e)^2 + (11af \cosh(fx + e)^{10} - 36af \cosh(fx + e)^8 + 42af \cosh(fx + e)^6 - 20af \cosh(fx + e)^4 + 3af \cosh(fx + e)^2) \sinh(fx + e)), \\
& 1/24(3((8a^2 + 24ab + 3b^2) \cosh(fx + e)^{11} + 11(8a^2 + 24ab + 3b^2) \cosh(fx + e) \sinh(fx + e)^{10} + (8a^2 + 24ab + 3b^2) \sinh(fx + e)^{11} - 4(8a^2 + 24ab + 3b^2) \cosh(fx + e)^9 + (55(8a^2 + 24ab + 3b^2) \cosh(fx + e)^2 - 32a^2 - 96ab - 12b^2) \sinh(fx + e)^9 + 3(55(8a^2 + 24ab + 3b^2) \cosh(fx + e)^3 - 12(8a^2 + 24ab + 3b^2) \cosh(fx + e) \sinh(fx + e)^8 + 6(8a^2 + 24ab + 3b^2) \cosh(fx + e)^7 + 6(55(8a^2 + 24ab + 3b^2) \cosh(fx + e)^4 - 24(8a^2 + 24ab + 3b^2) \cosh(fx + e)^2 + 8a^2 + 24ab + 3b^2) \sinh(fx + e)^7 + 42(11(8a^2 + 24ab + 3b^2) \cosh(fx + e)^5 - 8(8a^2 + 24ab + 3b^2) \cosh(fx + e)^3 + (8a^2 + 24ab + 3b^2) \cosh(fx + e)) \sinh(fx + e)^6 - 4(8a^2 + 24ab + 3b^2) \cosh(fx + e)^5 + 2(231(8a^2 + 24ab + 3b^2) \cosh(fx + e)^6 - 252(8a^2 + 24ab + 3b^2) \cosh(fx + e)^4 + 63(8a^2 + 24ab + 3b^2) \cosh(fx + e)^2 - 16a^2 - 48ab - 6b^2) \sinh(fx + e)^5 + 2(165(8a^2 + 24ab + 3b^2) \cosh(fx + e)^7 - 252(8a^2 + 24ab + 3b^2) \cosh(fx + e)^5 + 105(8a^2 + 24ab + 3b^2) \cosh(fx + e)^3 - 10(8a^2 + 24ab + 3b^2) \cosh(fx + e) \sinh(fx + e)^4 + (8a^2 + 24ab + 3b^2) \cosh(fx + e)^3 + (165(8a^2 + 24ab + 3b^2) \cosh(fx + e)^8 - 336(8a^2 + 24ab + 3b^2) \cosh(fx + e)^6 + 210(8a^2 + 24ab + 3b^2) \cosh(fx + e)^4 - 40(8a^2 + 24ab + 3b^2) \cosh(fx + e)^2 + 8a^2 + 24ab + 3b^2) \sinh(fx + e)^3 + (55(8a^2 + 24ab + 3b^2) \cosh(fx + e)^9 - 144(8a^2 + 24ab + 3b^2) \cosh(fx + e)^7 + 126(8a^2 + 24ab + 3b^2) \cos
\end{aligned}$$

$$\begin{aligned}
& h(f*x + e)^5 - 40*(8*a^2 + 24*a*b + 3*b^2)*\cosh(f*x + e)^3 + 3*(8*a^2 + 24* \\
& a*b + 3*b^2)*\cosh(f*x + e)*\sinh(f*x + e)^2 + (11*(8*a^2 + 24*a*b + 3*b^2)* \\
& \cosh(f*x + e)^{10} - 36*(8*a^2 + 24*a*b + 3*b^2)*\cosh(f*x + e)^8 + 42*(8*a^2 \\
& + 24*a*b + 3*b^2)*\cosh(f*x + e)^6 - 20*(8*a^2 + 24*a*b + 3*b^2)*\cosh(f*x + \\
& e)^4 + 3*(8*a^2 + 24*a*b + 3*b^2)*\cosh(f*x + e)^2*\sinh(f*x + e))*\sqrt{-a}* \\
& \arctan(1/2*\sqrt{2}*\sqrt{-a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2 \\
& *a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)} \\
&)/(a*\cosh(f*x + e) + a*\sinh(f*x + e))) + \sqrt{2}*(a*b*\cosh(f*x + e)^{12} + 12 \\
& *a*b*\cosh(f*x + e)*\sinh(f*x + e)^{11} + a*b*\sinh(f*x + e)^{12} + 2*(8*a^2 + 9*a \\
& *b)*\cosh(f*x + e)^{10} + 2*(33*a*b*\cosh(f*x + e)^2 + 8*a^2 + 9*a*b)*\sinh(f*x \\
& + e)^{10} + 20*(11*a*b*\cosh(f*x + e)^3 + (8*a^2 + 9*a*b)*\cosh(f*x + e))*\sinh(\\
& f*x + e)^9 - (112*a^2 + 111*a*b)*\cosh(f*x + e)^8 + (495*a*b*\cosh(f*x + e)^4 \\
& + 90*(8*a^2 + 9*a*b)*\cosh(f*x + e)^2 - 112*a^2 - 111*a*b)*\sinh(f*x + e)^8 \\
& + 8*(99*a*b*\cosh(f*x + e)^5 + 30*(8*a^2 + 9*a*b)*\cosh(f*x + e)^3 - (112*a^2 \\
& + 111*a*b)*\cosh(f*x + e))*\sinh(f*x + e)^7 + 8*(18*a^2 + 23*a*b)*\cosh(f*x + \\
& e)^6 + 4*(231*a*b*\cosh(f*x + e)^6 + 105*(8*a^2 + 9*a*b)*\cosh(f*x + e)^4 - \\
& 7*(112*a^2 + 111*a*b)*\cosh(f*x + e)^2 + 36*a^2 + 46*a*b)*\sinh(f*x + e)^6 + \\
& 8*(99*a*b*\cosh(f*x + e)^7 + 63*(8*a^2 + 9*a*b)*\cosh(f*x + e)^5 - 7*(112*a^2 \\
& + 111*a*b)*\cosh(f*x + e)^3 + 6*(18*a^2 + 23*a*b)*\cosh(f*x + e))*\sinh(f*x + \\
& e)^5 - (112*a^2 + 111*a*b)*\cosh(f*x + e)^4 + (495*a*b*\cosh(f*x + e)^8 + 42 \\
& 0*(8*a^2 + 9*a*b)*\cosh(f*x + e)^6 - 70*(112*a^2 + 111*a*b)*\cosh(f*x + e)^4 \\
& + 120*(18*a^2 + 23*a*b)*\cosh(f*x + e)^2 - 112*a^2 - 111*a*b)*\sinh(f*x + e)^ \\
& 4 + 4*(55*a*b*\cosh(f*x + e)^9 + 60*(8*a^2 + 9*a*b)*\cosh(f*x + e)^7 - 14*(11 \\
& 2*a^2 + 111*a*b)*\cosh(f*x + e)^5 + 40*(18*a^2 + 23*a*b)*\cosh(f*x + e)^3 - (\\
& 112*a^2 + 111*a*b)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 2*(8*a^2 + 9*a*b)*\cosh(\\
& f*x + e)^2 + 2*(33*a*b*\cosh(f*x + e)^{10} + 45*(8*a^2 + 9*a*b)*\cosh(f*x + e)^ \\
& 8 - 14*(112*a^2 + 111*a*b)*\cosh(f*x + e)^6 + 60*(18*a^2 + 23*a*b)*\cosh(f*x \\
& + e)^4 - 3*(112*a^2 + 111*a*b)*\cosh(f*x + e)^2 + 8*a^2 + 9*a*b)*\sinh(f*x + \\
& e)^2 + a*b + 4*(3*a*b*\cosh(f*x + e)^{11} + 5*(8*a^2 + 9*a*b)*\cosh(f*x + e)^9 \\
& - 2*(112*a^2 + 111*a*b)*\cosh(f*x + e)^7 + 12*(18*a^2 + 23*a*b)*\cosh(f*x + e \\
&)^5 - (112*a^2 + 111*a*b)*\cosh(f*x + e)^3 + (8*a^2 + 9*a*b)*\cosh(f*x + e))* \\
& \sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh \\
& (f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a*f*\cosh(\\
& f*x + e)^{11} + 11*a*f*\cosh(f*x + e)*\sinh(f*x + e)^{10} + a*f*\sinh(f*x + e)^{11} \\
& - 4*a*f*\cosh(f*x + e)^9 + (55*a*f*\cosh(f*x + e)^2 - 4*a*f)*\sinh(f*x + e)^9 \\
& + 6*a*f*\cosh(f*x + e)^7 + 3*(55*a*f*\cosh(f*x + e)^3 - 12*a*f*\cosh(f*x + e)) \\
& *\sinh(f*x + e)^8 + 6*(55*a*f*\cosh(f*x + e)^4 - 24*a*f*\cosh(f*x + e)^2 + a*f \\
&)*\sinh(f*x + e)^7 - 4*a*f*\cosh(f*x + e)^5 + 42*(11*a*f*\cosh(f*x + e)^5 - 8* \\
& a*f*\cosh(f*x + e)^3 + a*f*\cosh(f*x + e))*\sinh(f*x + e)^6 + 2*(231*a*f*\cosh(\\
& f*x + e)^6 - 252*a*f*\cosh(f*x + e)^4 + 63*a*f*\cosh(f*x + e)^2 - 2*a*f)*\sinh \\
& (f*x + e)^5 + a*f*\cosh(f*x + e)^3 + 2*(165*a*f*\cosh(f*x + e)^7 - 252*a*f*\co \\
& sh(f*x + e)^5 + 105*a*f*\cosh(f*x + e)^3 - 10*a*f*\cosh(f*x + e))*\sinh(f*x + \\
& e)^4 + (165*a*f*\cosh(f*x + e)^8 - 336*a*f*\cosh(f*x + e)^6 + 210*a*f*\cosh(f* \\
& x + e)^4 - 40*a*f*\cosh(f*x + e)^2 + a*f)*\sinh(f*x + e)^3 + (55*a*f*\cosh(f*x \\
& + e)^9 - 144*a*f*\cosh(f*x + e)^7 + 126*a*f*\cosh(f*x + e)^5 - 40*a*f*\cosh(f
\end{aligned}$$

```
*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x + e)^2 + (11*a*f*cosh(f*x + e)^10
- 36*a*f*cosh(f*x + e)^8 + 42*a*f*cosh(f*x + e)^6 - 20*a*f*cosh(f*x + e)^4
+ 3*a*f*cosh(f*x + e)^2)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.474 \quad \int (a + b \sinh^2(e + fx))^{3/2} \tanh^4(e + fx) dx$$

Optimal. Leaf size=305

$$\frac{(3a - 8b) \operatorname{sech}(e + fx) \sqrt{a + b \sinh^2(e + fx)} \operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) \tanh^3(e + fx) (a + b \sinh^2(e + fx))}{3f \sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} - \frac{\tanh^3(e + fx) (a + b \sinh^2(e + fx))}{3f}$$

```
[Out] -((3*a - 8*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f)
) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f) + ((a - 2*b)*Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f - ((a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.379704, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {3196, 467, 577, 582, 531, 418, 492, 411}

$$\frac{\tanh^3(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{3f} + \frac{(a - 2b) \sinh^2(e + fx) \tanh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{f} + \frac{8(a - 2b) \tanh^3(e + fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4,x]
```

```
[Out] -((3*a - 8*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f)
) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f) + ((a - 2*b)*Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f - ((a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3)/(3*f)
```

Rule 3196

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 467

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n*q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

Rule 582

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(f*g^(n - 1)*(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1) + 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^2(e + fx))^{3/2} \tanh^4(e + fx) dx &= \frac{\left(\sqrt{\cosh^2(e + fx)\operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{x^4(a+bx^2)^{3/2}}{(1+x^2)^{5/2}} dx, x, \sinh(e + fx)\right)}{f} \\
&= -\frac{(a + b \sinh^2(e + fx))^{3/2} \tanh^3(e + fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e + fx)\operatorname{sech}(e + fx)}\right)}{f} \\
&= \frac{(a - 2b) \sinh^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{f} - \frac{(a + b \sinh^2(e + fx))^{3/2} \tanh^3(e + fx)}{3f} \\
&= -\frac{(3a - 8b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{(a - 2b) \sinh^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{f} \\
&= -\frac{(3a - 8b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{(a - 2b) \sinh^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{f} \\
&= -\frac{(3a - 8b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{(3a - 8b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} \\
&= -\frac{(3a - 8b) \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{8(a - 2b) \sinh^2(e + fx) \sqrt{a + b \sinh^2(e + fx)} \tanh(e + fx)}{3f}
\end{aligned}$$

Mathematica [C] time = 2.79562, size = 224, normalized size = 0.73

$$\frac{4ia(5a - 8b) \sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} \operatorname{EllipticF}\left(i(e + fx), \frac{b}{a}\right) - \frac{\tanh(e + fx) \operatorname{sech}^2(e + fx) ((64a^2 - 160ab + 17b^2) \cosh(2(e + fx)) + 32a^2 + 2b(6a - 17b) \cosh(2(e + fx)))}{4\sqrt{2}}}{12f \sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4, x]

[Out] ((-32*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (4*I)*a*(5*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] - ((32*a^2 - 108*a*b + 18*b^2 + (64*a^2 - 160*a*b + 17*b^2)*Cosh[2*(e + f*x)] + 2*(6*a - 17*b)*b*Cosh[4*(e + f*x)] - b^2*

$\text{Cosh}[6*(e + f*x)]*\text{Sech}[e + f*x]^2*\text{Tanh}[e + f*x]/(4*\text{Sqrt}[2])/(12*f*\text{Sqrt}[2]*a - b + b*\text{Cosh}[2*(e + f*x)])$

Maple [A] time = 0.167, size = 389, normalized size = 1.3

$$-\frac{1}{3 (\cosh (fx + e))^3 f} \left(-\sqrt{\frac{b}{a}} b^2 \sinh (fx + e) (\cosh (fx + e))^6 + \left(3 \sqrt{\frac{b}{a}} ab - 7 \sqrt{\frac{b}{a}} b^2 \right) (\cosh (fx + e))^4 \sinh (fx + e) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x)

[Out] $-1/3*(-(-1/a*b)^{(1/2)}*b^2*\sinh(f*x+e)*\cosh(f*x+e)^6+(3*(-1/a*b)^{(1/2)}*a*b-7*(-1/a*b)^{(1/2)}*b^2)*\cosh(f*x+e)^4*\sinh(f*x+e)+(4*(-1/a*b)^{(1/2)}*a^2-13*(-1/a*b)^{(1/2)}*a*b+9*(-1/a*b)^{(1/2)}*b^2)*\cosh(f*x+e)^2*\sinh(f*x+e)+(-(-1/a*b)^{(1/2)}*a^2+2*(-1/a*b)^{(1/2)}*a*b-(-1/a*b)^{(1/2)}*b^2)*\sinh(f*x+e)-(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*(3*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a^2-16*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a*b+16*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^2+8*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a*b-16*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^2)*\cosh(f*x+e)^2)/(-1/a*b)^{(1/2)}/\cosh(f*x+e)^3/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (fx + e)^2 + a \right)^{\frac{3}{2}} \tanh (fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}} \tanh(fx + e)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="fricas")

[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}} \tanh(fx + e)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)

$$3.475 \quad \int \left(a + b \sinh^2(e + fx) \right)^{3/2} \tanh^2(e + fx) dx$$

Optimal. Leaf size=260

$$\frac{(3a - 4b)\operatorname{sech}(e + fx)\sqrt{a + b\sinh^2(e + fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) - \frac{\tanh(e + fx)(a + b\sinh^2(e + fx))}{f}}{3f\sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b\sinh^2(e + fx))}{a}}}$$

```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a
- 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*
Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f) - ((a
+ b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/f
```

Rubi [A] time = 0.245338, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3196, 467, 528, 531, 418, 492, 411}

$$\frac{\tanh(e + fx)(a + b\sinh^2(e + fx))^{3/2}}{f} + \frac{(7a - 8b)\tanh(e + fx)\sqrt{a + b\sinh^2(e + fx)}}{3f} + \frac{4b\sinh(e + fx)\cosh(e + fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2, x]
```

```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a
- 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*
Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f) - ((a
+ b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/f
```

Rule 3196

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_)*tan[(e_) + (f_)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
```

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 467

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q - 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 528

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^2(e + fx))^{3/2} \tanh^2(e + fx) dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{x^2(a + bx^2)^{3/2}}{(1+x^2)^{3/2}} dx, x, \sinh(e + fx)\right)}{f} \\
&= -\frac{(a + b \sinh^2(e + fx))^{3/2} \tanh(e + fx)}{f} + \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right)}{f} \\
&= \frac{4b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{(a + b \sinh^2(e + fx))^{3/2}}{f} \\
&= \frac{4b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{(a + b \sinh^2(e + fx))^{3/2}}{f} \\
&= \frac{4b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{(3a - 4b)F\left(\tan^{-1}\left(\frac{\sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}}\right)\right)}{f} \\
&= \frac{4b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{(7a - 8b)E\left(\tan^{-1}\left(\frac{\sinh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}}\right)\right)}{f}
\end{aligned}$$

Mathematica [C] time = 2.88685, size = 188, normalized size = 0.72

$$\frac{32ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2}\tanh(e+fx)\left(-24a^2 - 4b(2a-3b)\cosh(2(e+fx)) + 40ab\right)}{24f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2, x]
```

```
[Out] ((-8*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*(-24*a^2 + 40*a*b - 13*b^2 - 4*(2*a - 3*b)*b*Cosh[2*(e + f*x)] + b^2*Cosh[4*(e + f*x)])*Tanh[e + f*x]/(24*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.148, size = 413, normalized size = 1.6

$$\frac{1}{3f \cosh(fx + e)} \left(\sqrt{\frac{b}{a}} b^2 (\sinh(fx + e))^5 - 2 \sqrt{\frac{b}{a}} ab (\sinh(fx + e))^3 + 4 \sqrt{\frac{b}{a}} b^2 (\sinh(fx + e))^3 + 3a^2 \sqrt{a + b} (\sinh(fx + e))^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5-2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3+4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-11*a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-3*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)+4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \tanh(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}} \tanh(fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="fricas")

[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}} \tanh(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)

$$3.476 \quad \int \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=174

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a} + 1}\text{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{3f\sqrt{a + b\sinh^2(e + fx)}} + \frac{b\sinh(e + fx)\cosh(e + fx)\sqrt{a + b\sinh^2(e + fx)}}{3f} - \frac{2i(2a - b)\sqrt{a + b\sinh^2(e + fx)}}{3f}$$

[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.182364, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{b\sinh(e + fx)\cosh(e + fx)\sqrt{a + b\sinh^2(e + fx)}}{3f} + \frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a} + 1}F\left(ie + ifx \middle| \frac{b}{a}\right)}{3f\sqrt{a + b\sinh^2(e + fx)}} - \frac{2i(2a - b)\sqrt{a + b\sinh^2(e + fx)}}{3f\sqrt{\frac{b\sinh^2(e + fx)}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3180

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :- Simp[(b*Cosh[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[1/(2*p), Int[(a + b*Sinh[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && GtQ[p, 1]

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}
\int (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{1}{3} \int \frac{a(3a - b) + 2(2a - b)b \sinh^2(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx \\
&= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{1}{3}(a(a - b)) \int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx \\
&= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{\left(2(2a - b) \sqrt{a + b \sinh^2(e + fx)}\right) \int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx}{3 \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}} \\
&= \frac{b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{2i(2a - b)E\left(ie + ifx \left| \frac{b}{a} \right.\right) \sqrt{a + b \sinh^2(e + fx)}}{3f \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.641269, size = 169, normalized size = 0.97

$$\frac{2i\sqrt{2a(a-b)}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)-4i\sqrt{2a(2a-b)}}{6f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0., size = 416, normalized size = 2.4

$$\frac{1}{3 \cosh(fx + e) f} \left[\sqrt{-\frac{b}{a}} b^2 \sinh(fx + e) (\cosh(fx + e))^4 + \left(\sqrt{-\frac{b}{a}} ab - \sqrt{-\frac{b}{a}} b^2 \right) (\cosh(fx + e))^2 \sinh(fx + e) + 3a^2 \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] $\frac{1}{3} * ((-1/a*b)^{(1/2)} * b^2 * \sinh(f*x+e) * \cosh(f*x+e)^4 + ((-1/a*b)^{(1/2)} * a*b - (-1/a*b)^{(1/2)} * b^2) * \cosh(f*x+e)^2 * \sinh(f*x+e) + 3*a^2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)})) - 5*a * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^2 + 2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a*b - 2 * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^2) / (-1/a*b)^{(1/2)} / \cosh(f*x+e) / (a+b*\sinh(f*x+e)^2)^{(1/2)} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\left(b \sinh^2(fx + e) + a \right)^{\frac{3}{2}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

$$3.477 \quad \int \coth^2(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=256

$$\frac{(3a + 5b)\operatorname{sech}(e + fx)\sqrt{a + b\sinh^2(e + fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) + (7a + b)\tanh(e + fx)\sqrt{a + b\sinh^2(e + fx)}}{3f\sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b\sinh^2(e + fx))}{a}}} + \frac{(7a + b)\tanh(e + fx)\sqrt{a + b\sinh^2(e + fx)}}{3f}$$

[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/f - ((7*a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)

Rubi [A] time = 0.271529, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3196, 473, 528, 531, 418, 492, 411}

$$\frac{(7a + b)\tanh(e + fx)\sqrt{a + b\sinh^2(e + fx)}}{3f} + \frac{4b\sinh(e + fx)\cosh(e + fx)\sqrt{a + b\sinh^2(e + fx)}}{3f} - \frac{\coth(e + fx)(a + b\sinh^2(e + fx))^{3/2}}{f}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/f - ((7*a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 473

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 528

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/((c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2)])), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned} \int \coth^2(e + fx) (a + b \sinh^2(e + fx))^{3/2} dx &= \frac{\left(\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2(a+bx^2)}^{3/2}}{x^2} dx, x, \sinh(e + fx)\right)}{f} \\ &= -\frac{\coth(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{f} + \frac{\left(2\sqrt{\cosh^2(e + fx) \operatorname{sech}(e + fx)}\right)}{f} \\ &= \frac{4b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{\coth(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{f} \\ &= \frac{4b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{\coth(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{f} \\ &= \frac{4b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{\coth(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{f} \\ &= \frac{4b \cosh(e + fx) \sinh(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{\coth(e + fx) (a + b \sinh^2(e + fx))^{3/2}}{f} \end{aligned}$$

Mathematica [C] time = 2.12637, size = 184, normalized size = 0.72

$$\frac{32ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2}\coth(e+fx)\left(-24a^2 - 4b(2a+b)\cosh(2(e+fx)) + 8ab\right)}{24f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] (Sqrt[2]*(-24*a^2 + 8*a*b + 3*b^2 - 4*b*(2*a + b)*Cosh[2*(e + f*x)] + b^2*Cosh[4*(e + f*x)])*Coth[e + f*x] - (8*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a]*EllipticE[I*(e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a]*EllipticF[I*(e + f*x), b/a])/(24*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.156, size = 327, normalized size = 1.3

$$\frac{1}{3 \cosh(fx + e) \sinh(fx + e) f} \left(\sinh(fx + e) \sqrt{(\cosh(fx + e))^2} \sqrt{\frac{b(\cosh(fx + e))^2}{a} + \frac{a-b}{a}} \left(3 \operatorname{EllipticF} \left(\sinh(fx + e) \right) \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)+(-1/a*b)^(1/2)*b^2*cosh(f*x+e)^6+(-2*(-1/a*b)^(1/2)*a*b-2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4+(-3*(-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2)/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}} \coth(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^2, x)
```


Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \coth (fx + e)^2 \sinh (fx + e)^2 + a \coth (fx + e)^2\right) \sqrt{b \sinh (fx + e)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral((b*coth(f*x + e)^2*sinh(f*x + e)^2 + a*coth(f*x + e)^2)*sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)`

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.478 \quad \int \coth^4(e + fx) \left(a + b \sinh^2(e + fx) \right)^{3/2} dx$$

Optimal. Leaf size=306

$$\frac{(3a + b)(a + 3b)\operatorname{sech}(e + fx)\sqrt{a + b \sinh^2(e + fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e + fx)), 1 - \frac{b}{a}\right) + 8(a + b)\tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{3af\sqrt{\frac{\operatorname{sech}^2(e + fx)(a + b \sinh^2(e + fx))}{a}}} + \frac{8(a + b)\tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{3f}$$

[Out] -(((a + b)*Cosh[e + f*x]^2*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) + ((3*a + 5*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2))/(3*f) - (8*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + b)*(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)

Rubi [A] time = 0.358856, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {3196, 473, 580, 528, 531, 418, 492, 411}

$$\frac{8(a + b)\tanh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{3f} + \frac{(3a + 5b)\sinh(e + fx)\cosh(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{3f} - \frac{\coth^3(e + fx)}{3f}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] -(((a + b)*Cosh[e + f*x]^2*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) + ((3*a + 5*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2))/(3*f) - (8*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + b)*(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)

Rule 3196

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 473

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 528

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
```

```
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
  :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \coth^4(e+fx)(a+b\sinh^2(e+fx))^{3/2} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}(a+bx^2)^{3/2}}{x^4} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\coth^3(e+fx)(a+b\sinh^2(e+fx))^{3/2}}{3f} + \frac{\left(2\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right)}{f} \\
&= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{\coth^3(e+fx)}{3f} \\
&= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a+5b)\coth(e+fx)}{3f} \\
&= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a+5b)\coth(e+fx)}{3f} \\
&= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a+5b)\coth(e+fx)}{3f} \\
&= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a+5b)\coth(e+fx)}{3f}
\end{aligned}$$

Mathematica [C] time = 4.70133, size = 229, normalized size = 0.75

$$\frac{4i(5a^2 - 2ab - 3b^2) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\coth(e+fx) \operatorname{csch}^2(e+fx) ((64a^2+32ab-79b^2) \cosh(2(e+fx))-32a^2+2b^2)}{4\sqrt{2}}}{12f\sqrt{2a+b \cosh(2(e+fx))}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (-((-32*a^2 - 44*a*b + 58*b^2 + (64*a^2 + 32*a*b - 79*b^2)*Cosh[2*(e + f*x)] + 2*b*(6*a + 11*b)*Cosh[4*(e + f*x)] - b^2*Cosh[6*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/(4*sqrt[2]) - (32*I)*a*(a + b)*sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a*EllipticE[I*(e + f*x), b/a] + (4*I)*(5*a^2 - 2*a*b - 3*b^2)

```
*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a]/(12*f
*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.288, size = 540, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^8+3*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+
3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*sinh(f*
x+e)^3-2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*sinh(f*x+e)^3+5*((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)
^(1/2))*b^2*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(
1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*a*b*sinh(f*x+e)^3-8*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-
1/a*b)^(1/2), (a/b)^(1/2))*b^2*sinh(f*x+e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+
e)^4+8*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+
5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2+5*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a
*b)^(1/2)*a^2)/(-1/a*b)^(1/2)/sinh(f*x+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh^2(fx + e) + a \right)^{\frac{3}{2}} \coth^4(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \coth(fx + e)^4 \sinh(fx + e)^2 + a \coth(fx + e)^4\right) \sqrt{b \sinh(fx + e)^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral((b*coth(f*x + e)^4*sinh(f*x + e)^2 + a*coth(f*x + e)^4)*sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.479 \quad \int \frac{\tanh^5(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=142

$$\frac{(8a^2 - 8ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{5/2}} - \frac{\operatorname{sech}^4(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{4f(a-b)} + \frac{(8a-5b)\operatorname{sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{8f(a-b)^2}$$

[Out] -((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(8*(a - b)^(5/2)*f) + ((8*a - 5*b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(8*(a - b)^2*f) - (Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*(a - b)*f)

Rubi [A] time = 0.193336, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 89, 78, 63, 208}

$$\frac{(8a^2 - 8ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{5/2}} - \frac{\operatorname{sech}^4(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{4f(a-b)} + \frac{(8a-5b)\operatorname{sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{8f(a-b)^2}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] -((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(8*(a - b)^(5/2)*f) + ((8*a - 5*b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(8*(a - b)^2*f) - (Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*(a - b)*f)

Rule 3194

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 89


```

Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^(2*(c + d*x)^(n + 1))*(e + f*x)^(p + 1))/(d^(2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^(2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

```

Rule 78

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^(1/p), x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 208

```

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3\sqrt{a+bx}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a+b)+2(a-b)x}{(1+x)^2\sqrt{a+bx}} dx, x, \sinh^2(e+fx)\right)}{4(a-b)f} \\
&= \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} - \frac{\text{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4(a-b)f} + \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} \\
&= \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} - \frac{\text{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4(a-b)f} + \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} \\
&= -\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}f} + \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f}
\end{aligned}$$

Mathematica [A] time = 0.462506, size = 116, normalized size = 0.82

$$\frac{(-8a^2 + 8ab - 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right) + \sqrt{a-b} \text{sech}^2(e+fx) \sqrt{a+b\sinh^2(e+fx)} (-2(a-b)\text{sech}^2(e+fx) + 8a)}{8f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a - b]*Sech[e + f*x]^2*(8*a - 5*b - 2*(a - b)*Sech[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*(a - b)^(5/2)*f)

Maple [C] time = 0.125, size = 43, normalized size = 0.3

$$\frac{1}{f} \int \frac{(\sinh(fx+e))^5}{(\cosh(fx+e))^6 \sqrt{a+b(\sinh(fx+e))^2}} dx, \sinh(fx+e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] ``int/undef0`(sinh(f*x+e)^5/cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^5}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(tanh(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [B] time = 3.74004, size = 9879, normalized size = 69.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `[1/16*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 + 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 10*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 15*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^7 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5`

$$\begin{aligned}
& + 3*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*\cosh \\
& (f*x + e))*\sinh(f*x + e))*\sqrt{a - b}*\log((b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x \\
& + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(4*a - 3*b)*\cosh(f*x + e)^2 + \\
& 2*(3*b*\cosh(f*x + e)^2 + 4*a - 3*b)*\sinh(f*x + e)^2 - 4*\sqrt{2}*\sqrt{a - b} \\
&)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - \\
& 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))*(\cosh(f*x + e) + \sinh(f* \\
& x + e)) + 4*(b*\cosh(f*x + e)^3 + (4*a - 3*b)*\cosh(f*x + e))*\sinh(f*x + e) + \\
& b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e)^4 + \\
& 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x \\
& + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) + 4*\sqrt{2}*((8*a^2 - 13*a*b + \\
& 5*b^2)*\cosh(f*x + e)^5 + 5*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e)*\sinh(f*x \\
& + e)^4 + (8*a^2 - 13*a*b + 5*b^2)*\sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2 \\
&)*\cosh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e)^2 + 4*a^2 - \\
& 5*a*b + b^2)*\sinh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e) \\
& ^3 + 3*(4*a^2 - 5*a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (8*a^2 - 13*a \\
& *b + 5*b^2)*\cosh(f*x + e) + (5*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e)^4 + 6 \\
& *(4*a^2 - 5*a*b + b^2)*\cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*\sinh(f*x + \\
& e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^ \\
& 2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a^3 - 3*a^2*b + 3* \\
& a*b^2 - b^3)*f*\cosh(f*x + e)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f \\
& *x + e)*\sinh(f*x + e)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\sinh(f*x + e)^8 \\
& + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^6 + 4*(7*(a^3 - 3*a^2*b \\
& + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*\s \\
& \sinh(f*x + e)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^4 + 8*(7 \\
& *(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^3 + 3*(a^3 - 3*a^2*b + 3*a \\
& *b^2 - b^3)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(35*(a^3 - 3*a^2*b + 3*a*b \\
& ^2 - b^3)*f*\cosh(f*x + e)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x \\
& + e)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*\sinh(f*x + e)^4 + 4*(a^3 - 3 \\
& *a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^2 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - \\
& b^3)*f*\cosh(f*x + e)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e \\
&)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + \\
& 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^6 + 15*(a^3 - 3*a^2*b \\
& + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\co \\
& sh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*\sinh(f*x + e)^2 + (a^3 - \\
& 3*a^2*b + 3*a*b^2 - b^3)*f + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x \\
& + e)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^5 + 3*(a^3 - 3* \\
& a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)* \\
& f*\cosh(f*x + e))*\sinh(f*x + e)), -1/8*((8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + \\
& e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)*\sinh(f*x + e)^7 + (8*a^2 - 8 \\
& *a*b + 3*b^2)*\sinh(f*x + e)^8 + 4*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^6 + \\
& 4*(7*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*\sinh \\
& (f*x + e)^6 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^3 + 3*(8*a^2 - 8*a \\
& *b + 3*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*\cosh \\
& (f*x + e)^4 + 2*(35*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^4 + 30*(8*a^2 - 8 \\
& *a*b + 3*b^2)*\cosh(f*x + e)^2 + 24*a^2 - 24*a*b + 9*b^2)*\sinh(f*x + e)^4 +
\end{aligned}$$

$$\begin{aligned}
& 8*(7*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^5 + 10*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 + \\
& 4*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^6 + 15*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^4 + 9*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*\sinh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^7 + 3*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^5 + 3*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-a + b}*\arctan(-1/2*\sqrt{2}*\sqrt{-a + b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)})))/((a - b)*\cosh(f*x + e) + (a - b)*\sinh(f*x + e)) - 2*\sqrt{2}*((8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e)^5 + 5*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e)*\sinh(f*x + e)^4 + (8*a^2 - 13*a*b + 5*b^2)*\sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2)*\cosh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e)^2 + 4*a^2 - 5*a*b + b^2)*\sinh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e)^3 + 3*(4*a^2 - 5*a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e) + (5*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e))^4 + 6*(4*a^2 - 5*a*b + b^2)*\cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)})))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\sinh(f*x + e)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^6 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*\sinh(f*x + e)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^4 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(35*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*\sinh(f*x + e)^4 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^2 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^6 + 15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*\sinh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^5 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e)^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*\cosh(f*x + e))*\sinh(f*x + e))]
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(tanh(e + f*x)**5/sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(fx + e)}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.480 \quad \int \frac{\tanh^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=89

$$\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{2f(a-b)} - \frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{3/2}}$$

[Out] $-\left(\frac{(2a-b)\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right]}{2f(a-b)^{3/2}}\right) + \frac{\operatorname{Sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{2f(a-b)}$

Rubi [A] time = 0.112257, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3194, 78, 63, 208}

$$\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{2f(a-b)} - \frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[e+fx]^3/\operatorname{Sqrt}[a+b \operatorname{Sinh}[e+fx]^2], x]$

[Out] $-\left(\frac{(2a-b)\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right]}{2f(a-b)^{3/2}}\right) + \frac{\operatorname{Sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{2f(a-b)}$

Rule 3194

$\operatorname{Int}[\left((a_{\cdot}) + (b_{\cdot})\sin[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]^2\right)^{(p_{\cdot})}\tan[(e_{\cdot}) + (f_{\cdot})(x_{\cdot})]^{(m_{\cdot})}, x_{\text{Symbol}}] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\sin[e+fx]^2, x]\}, \operatorname{Dist}[ff^{(m+1)/2}/(2f), \operatorname{Subst}[\operatorname{Int}[(x^{(m-1)/2}(a+bffx)^p)/(1-ffx)^{(m+1)/2}], x], x, \sin[e+fx]^2/ff], x]] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x] \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 78

$\operatorname{Int}[\left((a_{\cdot}) + (b_{\cdot})(x_{\cdot})\right)\left((c_{\cdot}) + (d_{\cdot})(x_{\cdot})\right)^{(n_{\cdot})}\left((e_{\cdot}) + (f_{\cdot})(x_{\cdot})\right)^{(p_{\cdot})}, x_{\text{Symbol}}] \rightarrow -\operatorname{Simp}[\left((b_{\cdot}e - a_{\cdot}f)(c + dx)^{(n+1)}(e+fx)^{(p+1)}\right)/(f(p+1)(c^2 - d^2e)), x] - \operatorname{Dist}[(a_{\cdot}d_{\cdot}f_{\cdot}(n+p+2) - b_{\cdot}(d_{\cdot}e_{\cdot}(n+1) + c_{\cdot}f_{\cdot}))], x]$

$(p + 1)) / (f * (p + 1) * (c * f - d * e))$, Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || ! (EqQ[e, 0] || ! (EqQ[c, 0] || LtQ[p, n]))))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\tanh^3(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx &= \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2 \sqrt{a+bx}} dx, x, \sinh^2(e + fx)\right)}{2f} \\ &= \frac{\text{sech}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2(a - b)f} + \frac{(2a - b) \text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}} dx, x, \sinh^2(e + fx)\right)}{4(a - b)f} \\ &= \frac{\text{sech}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2(a - b)f} + \frac{(2a - b) \text{Subst}\left(\int \frac{1}{1 - \frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^2(e + fx)}\right)}{2(a - b)bf} \\ &= -\frac{(2a - b) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a - b}}\right)}{2(a - b)^{3/2}f} + \frac{\text{sech}^2(e + fx) \sqrt{a + b \sinh^2(e + fx)}}{2(a - b)f} \end{aligned}$$

Mathematica [A] time = 0.116503, size = 85, normalized size = 0.96

$$-\frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} - \frac{\text{sech}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{a-b}}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] -(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2) - (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(a - b))/(2*f)

Maple [C] time = 0.129, size = 43, normalized size = 0.5

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^3}{(\cosh(fx + e))^4 \sqrt{a + b(\sinh(fx + e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] `int/indef0` (sinh(f*x+e)^3/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^3}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(tanh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [B] time = 2.74653, size = 3425, normalized size = 38.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

```
[Out] [1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)]]]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^3}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

$$3.481 \quad \int \frac{\tanh(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))

Rubi [A] time = 0.0583869, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3194, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))

Rule 3194

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 63

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}} dx, x, \sinh^2(e+fx)\right)}{2f} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{bf} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f} \end{aligned}$$

Mathematica [A] time = 0.0423987, size = 44, normalized size = 1.07

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(e+fx)-b}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -(ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))

Maple [C] time = 0.093, size = 41, normalized size = 1.

$$\frac{1}{f} \int \frac{\sinh(fx+e)}{(\cosh(fx+e))^2} \frac{1}{\sqrt{a+b(\sinh(fx+e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] ``int/indef0`(sinh(f*x+e)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(tanh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [B] time = 2.35449, size = 1168, normalized size = 28.49

$$\left[\log \left(\frac{b \cosh(fx+e)^4 + 4b \cosh(fx+e) \sinh(fx+e)^3 + b \sinh(fx+e)^4 + 2(4a-3b) \cosh(fx+e)^2 + 2(3b \cosh(fx+e)^2 + 4a-3b) \sinh(fx+e)^2 - 4\sqrt{2}\sqrt{a-b} \sqrt{\frac{1}{\cosh(fx+e)}}}{\cosh(fx+e)^4 + 4 \cosh(fx+e) \sinh(fx+e)^3 + \sinh(fx+e)^4 + 2(3 \cosh(fx+e)^2 + 1) \sinh(fx+e)} \right) \right] \frac{1}{2\sqrt{a-b}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `[1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*`

```
x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
+ e) + 1))/(sqrt(a - b)*f), -sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*cosh(f*x + e) +
(a - b)*sinh(f*x + e)))/((a - b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

$$3.482 \quad \int \frac{\coth(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=33

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))

Rubi [A] time = 0.070531, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3194, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))

Rule 3194

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 63

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```


Rule 208

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]`

Rubi steps

$$\begin{aligned} \int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \sinh^2(e+fx)\right)}{2f} \\ &= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{bf} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f} \end{aligned}$$

Mathematica [A] time = 0.0353481, size = 33, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))

Maple [C] time = 0.073, size = 35, normalized size = 1.1

$$\frac{1}{f} \int \frac{1}{\sinh(fx+e) \sqrt{a+b(\sinh(fx+e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] ``int/indef0`(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(coth(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [B] time = 2.22778, size = 1111, normalized size = 33.67

$$\log \left(\frac{b \cosh(fx+e)^4 + 4b \cosh(fx+e) \sinh(fx+e)^3 + b \sinh(fx+e)^4 + 2(4a-b) \cosh(fx+e)^2 + 2(3b \cosh(fx+e)^2 + 4a-b) \sinh(fx+e)^2 - 4\sqrt{2}\sqrt{a} \sqrt{\frac{b \cosh(fx+e)^2}{\cosh(fx+e)^2 - 2}}}{\cosh(fx+e)^4 + 4 \cosh(fx+e) \sinh(fx+e)^3 + \sinh(fx+e)^4 + 2(3 \cosh(fx+e)^2 - 1) \sinh(fx+e)^2 - 2\sqrt{a}f} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `[1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)`

```
)/(sqrt(a)*f), sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2
+ b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e)))/(a*f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.483 \quad \int \frac{\coth^3(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=77

$$-\frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\operatorname{csch}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{2af}$$

[Out] $-\left(\frac{(2a-b)\operatorname{ArcTanh}\left[\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right]}{2a^{3/2}f}\right) - \left(\frac{\operatorname{Csch}[e+fx]^2\sqrt{a+b\sinh^2(e+fx)}}{2af}\right)$

Rubi [A] time = 0.102198, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3194, 78, 63, 208}

$$-\frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\operatorname{csch}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

[In] `Int[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2], x]`

[Out] $-\left(\frac{(2a-b)\operatorname{ArcTanh}\left[\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right]}{2a^{3/2}f}\right) - \left(\frac{\operatorname{Csch}[e+fx]^2\sqrt{a+b\sinh^2(e+fx)}}{2af}\right)$

Rule 3194

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 78

```
Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

$(p + 1)))/(f*(p + 1)*(c*f - d*e)), \text{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n\}, x\} \&\& \text{LtQ}[p, -1] \&\& (!\text{LtQ}[n, -1] \|\| \text{IntegerQ}[p] \|\| !(\text{IntegerQ}[n] \|\| !(\text{EqQ}[e, 0] \|\| !(\text{EqQ}[c, 0] \|\| \text{LtQ}[p, n])))$

Rule 63

$\text{Int}[(a + b*x)^m*(c + d*x)^n, x_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m + 1) - 1}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a + b*x)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\coth^3(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx &= \frac{\text{Subst}\left(\int \frac{1+x}{x^2\sqrt{a+bx}} dx, x, \sinh^2(e + fx)\right)}{2f} \\ &= -\frac{\text{csch}^2(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{2af} + \frac{(2a - b) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \sinh^2(e + fx)\right)}{4af} \\ &= -\frac{\text{csch}^2(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{2af} + \frac{(2a - b) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^2(e + fx)}\right)}{2abf} \\ &= -\frac{(2a - b) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\text{csch}^2(e + fx)\sqrt{a + b \sinh^2(e + fx)}}{2af} \end{aligned}$$

Mathematica [A] time = 0.201515, size = 72, normalized size = 0.94

$$-\frac{(2a-b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\text{csch}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{a}}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] -(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2) + (Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/a)/(2*f)

Maple [C] time = 0.107, size = 44, normalized size = 0.6

$$\frac{1}{f} \int \frac{((\sinh(fx + e))^{-1} + (\sinh(fx + e))^{-3}) \frac{1}{\sqrt{a + b(\sinh(fx + e))^2}} \sinh(fx + e)}{\sinh(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] `int/indef0`(((1/sinh(f*x+e)+1/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^3}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [B] time = 2.49188, size = 2957, normalized size = 38.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

```
[Out] [-1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (2*a - b)*sinh(f*x + e)^4 - 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a -
b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^
3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a)*log((b*cosh(f
*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a
- b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 +
4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*
x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*s
inh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh
(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2
+ 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(a*c
osh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2
*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x
+ e)^2 - a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x
+ e))*sinh(f*x + e)), 1/2*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f
*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 - 2*(2*a - b)*cosh(f*x
+ e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 4*((2*
a - b)*cosh(f*x + e)^3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*
sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) - 2*sqrt(2)*(a*cosh(f*x + e
) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2
*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*f*sinh(f*x
+ e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^
2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh
(f*x + e)]]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2), x)
```

```
[Out] Integral(coth(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^3}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```


$$3.484 \quad \int \frac{\coth^5(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=126

$$\frac{(8a^2 - 8ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} - \frac{(8a - 3b) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8a^2f} - \frac{\operatorname{csch}^4(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{4af}$$

[Out] $-\left(\frac{(8a^2 - 8ab + 3b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right]}{8a^{5/2}f} - \frac{(8a - 3b) \operatorname{Csch}[e+fx]^2 \sqrt{a+b \sinh^2(e+fx)}}{8a^2f} - \frac{\operatorname{Csch}[e+fx]^4 \sqrt{a+b \sinh^2(e+fx)}}{4af}\right)$

Rubi [A] time = 0.147265, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 89, 78, 63, 208}

$$\frac{(8a^2 - 8ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} - \frac{(8a - 3b) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{8a^2f} - \frac{\operatorname{csch}^4(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{4af}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[e+fx]^5 / \sqrt{a+b \sinh^2(e+fx)}, x]$

[Out] $-\left(\frac{(8a^2 - 8ab + 3b^2) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right]}{8a^{5/2}f} - \frac{(8a - 3b) \operatorname{Csch}[e+fx]^2 \sqrt{a+b \sinh^2(e+fx)}}{8a^2f} - \frac{\operatorname{Csch}[e+fx]^4 \sqrt{a+b \sinh^2(e+fx)}}{4af}\right)$

Rule 3194

$\operatorname{Int}[(a_+ + (b_+ \sin[e_+ + (f_+)(x_+)]^2)^{(p_+)} \tan[e_+ + (f_+)(x_+)]^{(m_+)}, x_Symbol] :> \operatorname{With}[\{ff = \operatorname{FreeFactors}[\sin[e+fx]^2, x]\}, \operatorname{Dist}[ff^{(m+1)/2} / (2f), \operatorname{Subst}[\operatorname{Int}[(x^{(m-1)/2} (a + bffx)^p] / (1 - ffx)^{(m+1)/2}, x], x, \sin[e+fx]^2 / ff, x]] /; \operatorname{FreeQ}\{a, b, e, f, p\}, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 89

$\operatorname{Int}[(a_+ + (b_+)(x_+))^{2*} ((c_+ + (d_+)(x_+))^{(n_+)}) ((e_+ + (f_+)(x_+))^{(p_+)}, x_Symbol] :> \operatorname{Simp}[(b*c - a*d)^2 (c + d*x)^{(n+1)} (e + f*x)^{(p+1)}$

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
+ 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3\sqrt{a+bx}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-3b)+2ax}{x^2\sqrt{a+bx}} dx, x, \sinh^2(e+fx)\right)}{4af} \\
&= -\frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af} + \\
&= -\frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af} + \\
&= -\frac{(8a^2-8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} - \frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}
\end{aligned}$$

Mathematica [A] time = 0.362039, size = 100, normalized size = 0.79

$$\frac{(-8a^2 + 8ab - 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right) + \sqrt{a}\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}(-2a\text{csch}^2(e+fx) - 8a + 3b)}{8a^{5/2}f}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] ((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sqrt[a]*Csch[e + f*x]^2*(-8*a + 3*b - 2*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a^(5/2)*f)

Maple [C] time = 0.102, size = 54, normalized size = 0.4

$$\frac{1}{f} \int \frac{\left(\left(\sinh(fx+e)\right)^{-1} + 2\left(\sinh(fx+e)\right)^{-3} + \left(\sinh(fx+e)\right)^{-5}\right) \frac{1}{\sqrt{a+b\left(\sinh(fx+e)\right)^2}}}{\sqrt{a+b\left(\sinh(fx+e)\right)^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(((1/sinh(f*x+e)+2/sinh(f*x+e)^3+1/sinh(f*x+e)^5)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^5}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.84606, size = 7682, normalized size = 60.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)
*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 -
4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*co
sh(f*x + e)^2 - 8*a^2 + 8*a*b - 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e)^3 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(
f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 - 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 +
24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^5 - 10*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(
f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 - 15*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8
*a^2 + 8*a*b - 3*b^2)*sinh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 -
8*a*b + 3*b^2)*cosh(f*x + e)^7 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5
```

$$\begin{aligned}
&+ 3(8a^2 - 8ab + 3b^2)\cosh(fx + e)^3 - (8a^2 - 8ab + 3b^2)\cosh \\
&(fx + e)*\sinh(fx + e)*\sqrt{a}*\log((b*\cosh(fx + e)^4 + 4b*\cosh(fx + e) \\
&)*\sinh(fx + e)^3 + b*\sinh(fx + e)^4 + 2*(4a - b)*\cosh(fx + e)^2 + 2*(3* \\
&b*\cosh(fx + e)^2 + 4a - b)*\sinh(fx + e)^2 - 4*\sqrt{2}*\sqrt{a}*\sqrt{(b*\cosh \\
&\cosh(fx + e)^2 + b*\sinh(fx + e)^2 + 2a - b)/(\cosh(fx + e)^2 - 2*\cosh(fx \\
&+ e)*\sinh(fx + e) + \sinh(fx + e)^2))*(\cosh(fx + e) + \sinh(fx + e)) + 4* \\
&(b*\cosh(fx + e)^3 + (4a - b)*\cosh(fx + e))*\sinh(fx + e) + b)/(\cosh(fx \\
&+ e)^4 + 4*\cosh(fx + e)*\sinh(fx + e)^3 + \sinh(fx + e)^4 + 2*(3*\cosh(fx \\
&+ e)^2 - 1)*\sinh(fx + e)^2 - 2*\cosh(fx + e)^2 + 4*(\cosh(fx + e)^3 - \cosh \\
&(fx + e))*\sinh(fx + e) + 1)) - 4*\sqrt{2}*((8a^2 - 3ab)*\cosh(fx + e)^5 \\
&+ 5*(8a^2 - 3ab)*\cosh(fx + e)*\sinh(fx + e)^4 + (8a^2 - 3ab)*\sinh(f \\
&*x + e)^5 - 2*(4a^2 - 3ab)*\cosh(fx + e)^3 + 2*(5*(8a^2 - 3ab)*\cosh(f \\
&*x + e)^2 - 4a^2 + 3ab)*\sinh(fx + e)^3 + 2*(5*(8a^2 - 3ab)*\cosh(fx \\
&+ e)^3 - 3*(4a^2 - 3ab)*\cosh(fx + e))*\sinh(fx + e)^2 + (8a^2 - 3ab) \\
&*\cosh(fx + e) + (5*(8a^2 - 3ab)*\cosh(fx + e)^4 - 6*(4a^2 - 3ab)*\cos \\
&h(fx + e)^2 + 8a^2 - 3ab)*\sinh(fx + e))*\sqrt{(b*\cosh(fx + e)^2 + b*\si \\
&\sinh(fx + e)^2 + 2a - b)/(\cosh(fx + e)^2 - 2*\cosh(fx + e)*\sinh(fx + e) + \\
&\sinh(fx + e)^2)))/(a^3*f*\cosh(fx + e)^8 + 8a^3*f*\cosh(fx + e)*\sinh(fx \\
&+ e)^7 + a^3*f*\sinh(fx + e)^8 - 4a^3*f*\cosh(fx + e)^6 + 6a^3*f*\cosh(f \\
&x + e)^4 + 4*(7a^3*f*\cosh(fx + e)^2 - a^3*f)*\sinh(fx + e)^6 - 4a^3*f*\co \\
&sh(fx + e)^2 + 8*(7a^3*f*\cosh(fx + e)^3 - 3a^3*f*\cosh(fx + e))*\sinh(f \\
&x + e)^5 + 2*(35a^3*f*\cosh(fx + e)^4 - 30a^3*f*\cosh(fx + e)^2 + 3a^3*f \\
&)*\sinh(fx + e)^4 + a^3*f + 8*(7a^3*f*\cosh(fx + e)^5 - 10a^3*f*\cosh(fx \\
&+ e)^3 + 3a^3*f*\cosh(fx + e))*\sinh(fx + e)^3 + 4*(7a^3*f*\cosh(fx + e)^ \\
&6 - 15a^3*f*\cosh(fx + e)^4 + 9a^3*f*\cosh(fx + e)^2 - a^3*f)*\sinh(fx + \\
&e)^2 + 8*(a^3*f*\cosh(fx + e)^7 - 3a^3*f*\cosh(fx + e)^5 + 3a^3*f*\cosh(f \\
&x + e)^3 - a^3*f*\cosh(fx + e))*\sinh(fx + e)), 1/8*(((8a^2 - 8ab + 3b^ \\
&2)*\cosh(fx + e)^8 + 8*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)*\sinh(fx + e)^ \\
&7 + (8a^2 - 8ab + 3b^2)*\sinh(fx + e)^8 - 4*(8a^2 - 8ab + 3b^2)*\cos \\
&h(fx + e)^6 + 4*(7*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)^2 - 8a^2 + 8ab \\
&- 3b^2)*\sinh(fx + e)^6 + 8*(7*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)^3 - \\
&3*(8a^2 - 8ab + 3b^2)*\cosh(fx + e))*\sinh(fx + e)^5 + 6*(8a^2 - 8ab \\
&+ 3b^2)*\cosh(fx + e)^4 + 2*(35*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)^4 - \\
&30*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)^2 + 24a^2 - 24ab + 9b^2)*\sinh \\
&(fx + e)^4 + 8*(7*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)^5 - 10*(8a^2 - 8 \\
&a*b + 3b^2)*\cosh(fx + e)^3 + 3*(8a^2 - 8ab + 3b^2)*\cosh(fx + e))*\sin \\
&h(fx + e)^3 - 4*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)^2 + 4*(7*(8a^2 - 8 \\
&a*b + 3b^2)*\cosh(fx + e)^6 - 15*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)^4 + \\
&9*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)^2 - 8a^2 + 8ab - 3b^2)*\sinh(f \\
&x + e)^2 + 8a^2 - 8ab + 3b^2 + 8*((8a^2 - 8ab + 3b^2)*\cosh(fx + e) \\
&^7 - 3*(8a^2 - 8ab + 3b^2)*\cosh(fx + e)^5 + 3*(8a^2 - 8ab + 3b^2)* \\
&\cosh(fx + e)^3 - (8a^2 - 8ab + 3b^2)*\cosh(fx + e))*\sinh(fx + e))*\sqrt{ \\
&\sqrt{-a}*\arctan(1/2*\sqrt{2}*\sqrt{-a}*\sqrt{(b*\cosh(fx + e)^2 + b*\sinh(fx + e) \\
&^2 + 2a - b)/(\cosh(fx + e)^2 - 2*\cosh(fx + e)*\sinh(fx + e) + \sinh(fx + \\
&e)^2)))/(a*\cosh(fx + e) + a*\sinh(fx + e))} - 2*\sqrt{2}*((8a^2 - 3ab)*c
\end{aligned}$$

```

osh(f*x + e)^5 + 5*(8*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (8*a^2 -
3*a*b)*sinh(f*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(8*a^2 -
3*a*b)*cosh(f*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 3*
a*b)*cosh(f*x + e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (
8*a^2 - 3*a*b)*cosh(f*x + e) + (5*(8*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^
2 - 3*a*b)*cosh(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2)))/(a^3*f*cosh(f*x + e)^8 + 8*a^3*f*cosh(f*x
+ e)*sinh(f*x + e)^7 + a^3*f*sinh(f*x + e)^8 - 4*a^3*f*cosh(f*x + e)^6 + 6
*a^3*f*cosh(f*x + e)^4 + 4*(7*a^3*f*cosh(f*x + e)^2 - a^3*f)*sinh(f*x + e)^
6 - 4*a^3*f*cosh(f*x + e)^2 + 8*(7*a^3*f*cosh(f*x + e)^3 - 3*a^3*f*cosh(f*x
+ e))*sinh(f*x + e)^5 + 2*(35*a^3*f*cosh(f*x + e)^4 - 30*a^3*f*cosh(f*x +
e)^2 + 3*a^3*f)*sinh(f*x + e)^4 + a^3*f + 8*(7*a^3*f*cosh(f*x + e)^5 - 10*a
^3*f*cosh(f*x + e)^3 + 3*a^3*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*f*
cosh(f*x + e)^6 - 15*a^3*f*cosh(f*x + e)^4 + 9*a^3*f*cosh(f*x + e)^2 - a^3*f
)*sinh(f*x + e)^2 + 8*(a^3*f*cosh(f*x + e)^7 - 3*a^3*f*cosh(f*x + e)^5 + 3
*a^3*f*cosh(f*x + e)^3 - a^3*f*cosh(f*x + e))*sinh(f*x + e))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^5}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

$$3.485 \quad \int \frac{\tanh^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=219

$$\frac{(3a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3f(a-b)}$$

[Out] $(-2*(2*a - b)*\operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*(a - b)^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) + ((3*a - b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*(a - b)^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) + (\operatorname{Sech}[e + f*x]^2*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2]*\operatorname{Tanh}[e + f*x])/(3*(a - b)*f)$

Rubi [A] time = 0.196234, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3196, 470, 525, 418, 411}

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3f(a-b)} + \frac{(3a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))\right)}{3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[e + f*x]^4/\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2], x]$

[Out] $(-2*(2*a - b)*\operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*(a - b)^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) + ((3*a - b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*(a - b)^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) + (\operatorname{Sech}[e + f*x]^2*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2]*\operatorname{Tanh}[e + f*x])/(3*(a - b)*f)$

Rule 3196

$\operatorname{Int}[(a + (b + \sin[e + f*x])^2)^{p + \tan[e + f*x]}(m), x_Symbol] := \operatorname{With}\{\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[(ff^m +$

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 470

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 525

Int[((e_) + (f_)*(x_)^2)/(Sqrt[(a_) + (b_)*(x_)^2]*((c_) + (d_)*(x_)^2)^(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] && PosQ[d/c]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] :> Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)^{5/2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)^{5/2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{3(a-b)f} \\
&= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} - \frac{\left(2(2a-b)\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)^{5/2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{3(a-b)f} \\
&= -\frac{2(2a-b)E\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{(3a-b)F\left(\tan^{-1}(\sinh(e+fx))\left|1-\frac{b}{a}\right.\right)}{3(a-b)^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 2.10477, size = 206, normalized size = 0.94

$$\frac{2ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)(2(4a^2-3ab+b^2)\cosh(2(e+fx))+(2a-b)(2a+b\cosh(4(e+fx))))}{\sqrt{2}}}{6f(a-b)^2\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] $((-4*I)*a*(2*a - b)*\operatorname{Sqrt}[(2*a - b + b*\operatorname{Cosh}[2*(e + f*x)])]/a)*\operatorname{EllipticE}[I*(e + f*x), b/a] + (2*I)*a*(a - b)*\operatorname{Sqrt}[(2*a - b + b*\operatorname{Cosh}[2*(e + f*x)])]/a*\operatorname{EllipticF}[I*(e + f*x), b/a] - ((2*(4*a^2 - 3*a*b + b^2)*\operatorname{Cosh}[2*(e + f*x)] + (2*a - b)*(2*a + b + b*\operatorname{Cosh}[4*(e + f*x)]))*\operatorname{Sech}[e + f*x]^2*\operatorname{Tanh}[e + f*x])/(\operatorname{Sqrt}[2])/((6*(a - b)^2*f*\operatorname{Sqrt}[2*a - b + b*\operatorname{Cosh}[2*(e + f*x)]])$

Maple [A] time = 0.289, size = 366, normalized size = 1.7

$$\frac{1}{3(\cosh(fx+e))^3(a-b)^2f}\left(\left(-4\sqrt{\frac{b}{a}}ab+2\sqrt{\frac{b}{a}}b^2\right)\sinh(fx+e)(\cosh(fx+e))^4+\left(-4\sqrt{\frac{b}{a}}a^2+7\sqrt{\frac{b}{a}}ab-3\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] $\frac{1}{3} * ((-4 * (-1/a*b)^{(1/2)} * a*b + 2 * (-1/a*b)^{(1/2)} * b^2) * \sinh(f*x+e) * \cosh(f*x+e)^4 + (-4 * (-1/a*b)^{(1/2)} * a^2 + 7 * (-1/a*b)^{(1/2)} * a*b - 3 * (-1/a*b)^{(1/2)} * b^2) * \cosh(f*x+e)^2 * \sinh(f*x+e) + ((-1/a*b)^{(1/2)} * a^2 - 2 * (-1/a*b)^{(1/2)} * a*b + (-1/a*b)^{(1/2)} * b^2) * \sinh(f*x+e) + (\cosh(f*x+e)^2)^{(1/2)} * (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (3 * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a^2 - 5 * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a*b + 2 * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^2 + 4 * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a*b - 2 * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^2) * \cosh(f*x+e)^2) / \cosh(f*x+e)^3 / (a-b)^2 / (-1/a*b)^{(1/2)} / (a+b * \sinh(f*x+e)^2)^{(1/2)} / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\tanh(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(tanh(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(fx + e)}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.486 \quad \int \frac{\tanh^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=156

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rubi [A] time = 0.181255, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3196, 471, 422, 418, 492, 411}

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2], x]
```

```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 3196

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^(p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)^{3/2}\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
&= -\frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} - \frac{\left(a\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
&= \frac{F\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&= -\frac{E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} + \frac{F\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 0.382078, size = 109, normalized size = 0.7

$$\frac{\sqrt{2}\tanh(e+fx)(-2a-b\cosh(2(e+fx))+b)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)}{2f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] $((-2*I)*a*\sqrt{(2*a - b + b*\cosh[2*(e + f*x)])}/a)*\operatorname{EllipticE}[I*(e + f*x), b/a] + \sqrt{2}*(-2*a + b - b*\cosh[2*(e + f*x)])*\operatorname{Tanh}[e + f*x]/(2*(a - b)*f*\sqrt{2*a - b + b*\cosh[2*(e + f*x)])}$

Maple [A] time = 0.161, size = 239, normalized size = 1.5

$$\frac{1}{(a-b)\cosh(fx+e)f}\left(-\sqrt{-\frac{b}{a}}b(\sinh(fx+e))^3 + a\sqrt{\frac{a+b(\sinh(fx+e))^2}{a}}\sqrt{(\cosh(fx+e))^2}\operatorname{EllipticF}\left(\sinh(fx+e)\middle|\frac{b}{a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out]
$$\begin{aligned} &(-(-1/a*b)^{(1/2)}*b*\sinh(f*x+e)^3+a*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-b*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})+b*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-(-1/a*b)^{(1/2)}*a*\sinh(f*x+e))/(a-b)/(-1/a*b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(fx + e)}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\tanh^2(fx + e)}{\sqrt{b \sinh^2(fx + e) + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(tanh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(fx + e)}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.487 \quad \int \frac{1}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=60

$$-\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1 \operatorname{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{f\sqrt{a+b \sinh^2(e+fx)}}$$

[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.0365019, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1F\left(ie + ifx \left| \frac{b}{a} \right. \right)}{f\sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3183

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] :> Simp[(1*EllipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx = \frac{\sqrt{1 + \frac{b \sinh^2(e+fx)}{a}} \int \frac{1}{\sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}} dx}{\sqrt{a + b \sinh^2(e + fx)}}$$

$$= -\frac{iF\left(ie + ifx \left| \frac{b}{a} \right. \right) \sqrt{1 + \frac{b \sinh^2(e+fx)}{a}}}{f \sqrt{a + b \sinh^2(e + fx)}}$$

Mathematica [A] time = 0.0707229, size = 68, normalized size = 1.13

$$\frac{i \sqrt{\frac{2a + b \cosh(2(e+fx)) - b}{a}} \text{EllipticF}\left(i(e + fx), \frac{b}{a}\right)}{f \sqrt{2a + b \cosh(2(e + fx)) - b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out] ((-1)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a])/ (f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)])]

Maple [A] time = 0.001, size = 86, normalized size = 1.4

$$\frac{1}{\cosh(fx + e) f} \sqrt{\frac{a + b (\sinh(fx + e))^2}{a}} \sqrt{(\cosh(fx + e))^2} \text{EllipticF}\left(\sinh(fx + e) \sqrt{\frac{b}{a}}, \sqrt{\frac{a}{b}}\right) \frac{1}{\sqrt{\frac{b}{a}}} \frac{1}{\sqrt{a + b (\sinh(fx + e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b \sinh^2(fx + e) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(1/sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.488 \quad \int \frac{\coth^2(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=207

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{af} - \frac{\coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{af}$$

[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*f)

Rubi [A] time = 0.197208, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3196, 475, 422, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{af} - \frac{\coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{af} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*f)

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 475

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e^(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1) + b*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 422

Int[Sqrt[(a_) + (b_)*(x_)^2]/Sqrt[(c_) + (d_)*(x_)^2], x_Symbol] := Dist[a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2\sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{af} \\
&= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{F\left(\tan^{-1}(\sinh(e+fx))\right)\left[1-\frac{b}{a}\right]\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} \\
&= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} - \frac{E\left(\tan^{-1}(\sinh(e+fx))\right)\left[1-\frac{b}{a}\right]\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}
\end{aligned}$$

Mathematica [C] time = 0.388523, size = 105, normalized size = 0.51

$$\frac{\sqrt{2}\coth(e+fx)(-2a-b\cosh(2(e+fx))+b)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)}{2af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2], x]

[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a]/(2*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.179, size = 217, normalized size = 1.1

$$-\frac{1}{a\sinh(fx+e)\cosh(fx+e)f}\left(-\sinh(fx+e)\sqrt{(\cosh(fx+e))^2}\sqrt{\frac{b(\cosh(fx+e))^2}{a}}+\frac{a-b}{a}\left(a\operatorname{EllipticF}\left(\sinh\left(\frac{e+fx}{\sqrt{a+b\sinh^2(e+fx)}}\right)\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)`

[Out] $-\left(-\sinh(fx+e)\left(\cosh(fx+e)^2\right)^{1/2}\left(\frac{b}{a}\cosh(fx+e)^2+\frac{a-b}{a}\right)^{1/2}\left(a\operatorname{EllipticF}\left(\sinh(fx+e)\left(-\frac{1}{a}b\right)^{1/2},\left(\frac{a}{b}\right)^{1/2}\right)-b\operatorname{EllipticF}\left(\sinh(fx+e)\left(-\frac{1}{a}b\right)^{1/2},\left(\frac{a}{b}\right)^{1/2}\right)+b\operatorname{EllipticE}\left(\sinh(fx+e)\left(-\frac{1}{a}b\right)^{1/2},\left(\frac{a}{b}\right)^{1/2}\right)\right)+\left(-\frac{1}{a}b\right)^{1/2}b\cosh(fx+e)^4+\left(-\frac{1}{a}b\right)^{1/2}a-\left(-\frac{1}{a}b\right)^{1/2}b\right)\cosh(fx+e)^2/\left(-\frac{1}{a}b\right)^{1/2}/a/\sinh(fx+e)/\cosh(fx+e)/(a+b\sinh(fx+e)^2)^{1/2}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\coth(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")`

[Out] `integral(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Integral(coth(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(fx + e)}{\sqrt{b \sinh^2(fx + e) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.489 \quad \int \frac{\coth^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx$$

Optimal. Leaf size=285

$$\frac{(3a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{2(2a-b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^2f}$$

[Out] $(-2*(2*a - b)*\operatorname{Coth}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*a^2*f) - (\operatorname{Coth}[e + f*x]*\operatorname{Csch}[e + f*x]^2*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*a*f) - (2*(2*a - b)*\operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*a^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) + ((3*a - b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*a^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) + (2*(2*a - b)*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2]*\operatorname{Tanh}[e + f*x])/(3*a^2*f)$

Rubi [A] time = 0.304744, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3196, 474, 583, 531, 418, 492, 411}

$$\frac{2(2a-b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^2f} - \frac{2(2a-b)\coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^2f} + \frac{(3a-b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^2f}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[e + f*x]^4/\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2], x]$

[Out] $(-2*(2*a - b)*\operatorname{Coth}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*a^2*f) - (\operatorname{Coth}[e + f*x]*\operatorname{Csch}[e + f*x]^2*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*a*f) - (2*(2*a - b)*\operatorname{EllipticE}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*a^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) + ((3*a - b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e + f*x]], 1 - b/a]*\operatorname{Sech}[e + f*x]*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2])/(3*a^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e + f*x]^2*(a + b*\operatorname{Sinh}[e + f*x]^2))/a]) + (2*(2*a - b)*\operatorname{Sqrt}[a + b*\operatorname{Sinh}[e + f*x]^2]*\operatorname{Tanh}[e + f*x])/(3*a^2*f)$

Rule 3196

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 474

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1) + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

$a*d, 0] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c] \ \&\& \ \text{!SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_) + (b_) * (x_)^2] / ((c_) + (d_) * (x_)^2)^{(3/2)}, x_Symbol] \ :> \ \text{Simp}[(\text{Sqrt}[a + b*x^2] * \text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)]) / (c * \text{Rt}[d/c, 2] * \text{Sqrt}[c + d*x^2] * \text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2)]), x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned} \int \frac{\coth^4(e+fx)}{\sqrt{a+b \sinh^2(e+fx)}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4 \sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\ &= -\frac{\coth(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cosh^2(e+fx) \operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4 \sqrt{a+bx^2}} dx, x, \sinh(e+fx)\right)}{f} \\ &= -\frac{2(2a-b) \coth(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3a^2 f} - \frac{\coth(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3af} \\ &= -\frac{2(2a-b) \coth(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3a^2 f} - \frac{\coth(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3af} \\ &= -\frac{2(2a-b) \coth(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3a^2 f} - \frac{\coth(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3af} \\ &= -\frac{2(2a-b) \coth(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3a^2 f} - \frac{\coth(e+fx) \operatorname{csch}^2(e+fx) \sqrt{a+b \sinh^2(e+fx)}}{3af} \end{aligned}$$

Mathematica [C] time = 3.54866, size = 208, normalized size = 0.73

$$\frac{2ia(a-b) \sqrt{\frac{2a+b \cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\coth(e+fx) \operatorname{csch}^2(e+fx) (2(4a^2-5ab+2b^2) \cosh(2(e+fx)) - (2a-b)(2a-b \cosh(4(e+fx))))}{\sqrt{2}}}{6a^2 f \sqrt{2a+b \cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]

[Out]
$$\begin{aligned} & -\left(\left(2(4a^2 - 5ab + 2b^2)\cosh[2(e + fx)] - (2a - b)(2a - 3b - b\right. \right. \\ & \left. \left. \cosh[4(e + fx)])\right)\coth[e + fx]\operatorname{Csch}[e + fx]^2/\sqrt{2}\right) - (4I)a(2a \\ & - b)\sqrt{(2a - b + b\cosh[2(e + fx)])}/a\operatorname{EllipticE}[I(e + fx), b/a] + \\ & (2I)a(a - b)\sqrt{(2a - b + b\cosh[2(e + fx)])}/a\operatorname{EllipticF}[I(e + f \\ & *x), b/a]/(6a^2f\sqrt{2a - b + b\cosh[2(e + fx)])} \end{aligned}$$

Maple [A] time = 0.187, size = 522, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)

[Out]
$$\begin{aligned} & -1/3(4(-1/ab)^{1/2}ab\sinh(fx+e)^6-2(-1/ab)^{1/2}b^2\sinh(fx+e)^6 \\ & -3a^2((a+b\sinh(fx+e)^2)/a)^{1/2}(\cosh(fx+e)^2)^{1/2}\operatorname{EllipticF}(\sinh(f \\ & *x+e)(-1/ab)^{1/2}, (a/b)^{1/2})\sinh(fx+e)^3+5b((a+b\sinh(fx+e)^2)/a \\ & ^{1/2}(\cosh(fx+e)^2)^{1/2}\operatorname{EllipticF}(\sinh(fx+e)(-1/ab)^{1/2}, (a/b)^{1/2}) \\ &)a\sinh(fx+e)^3-2((a+b\sinh(fx+e)^2)/a)^{1/2}(\cosh(fx+e)^2)^{1/2}E \\ & \operatorname{llipticF}(\sinh(fx+e)(-1/ab)^{1/2}, (a/b)^{1/2})b^2\sinh(fx+e)^3-4((a+b \\ & \sinh(fx+e)^2)/a)^{1/2}(\cosh(fx+e)^2)^{1/2}\operatorname{EllipticE}(\sinh(fx+e)(-1/ab \\ &)^{1/2}, (a/b)^{1/2})ab\sinh(fx+e)^3+2((a+b\sinh(fx+e)^2)/a)^{1/2}(\cos \\ & h(fx+e)^2)^{1/2}\operatorname{EllipticE}(\sinh(fx+e)(-1/ab)^{1/2}, (a/b)^{1/2})b^2\sin \\ & h(fx+e)^3+4(-1/ab)^{1/2}a^2\sinh(fx+e)^4+3(-1/ab)^{1/2}ab\sinh(fx \\ & +e)^4-2(-1/ab)^{1/2}b^2\sinh(fx+e)^4+5(-1/ab)^{1/2}a^2\sinh(fx+e)^2 \\ & -(-1/ab)^{1/2}ab\sinh(fx+e)^2+(-1/ab)^{1/2}a^2/(-1/ab)^{1/2}/a^2/\sinh(fx+e)^3/\cosh(fx+e)/(a+b\sinh(fx+e)^2)^{1/2}/f \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\coth(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^4}{\sqrt{b \sinh(fx + e)^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)

$$3.490 \quad \int \frac{\tanh^5(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=187

$$\frac{8a^2 + 8ab - b^2}{8f(a-b)^3 \sqrt{a+b \sinh^2(e+fx)}} - \frac{(8a^2 + 8ab - b^2) \tanh^{-1} \left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}} \right)}{8f(a-b)^{7/2}} - \frac{\operatorname{sech}^4(e+fx)}{4f(a-b) \sqrt{a+b \sinh^2(e+fx)}} + \frac{1}{8f(a-b)^3 \sqrt{a+b \sinh^2(e+fx)}}$$

[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(8*(a - b)^(7/2)*f) + (8*a^2 + 8*a*b - b^2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2]) + ((8*a - 3*b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Sech[e + f*x]^4/(4*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.251366, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2 + 8ab - b^2}{8f(a-b)^3 \sqrt{a+b \sinh^2(e+fx)}} - \frac{(8a^2 + 8ab - b^2) \tanh^{-1} \left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}} \right)}{8f(a-b)^{7/2}} - \frac{\operatorname{sech}^4(e+fx)}{4f(a-b) \sqrt{a+b \sinh^2(e+fx)}} + \frac{1}{8f(a-b)^3 \sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(8*(a - b)^(7/2)*f) + (8*a^2 + 8*a*b - b^2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2]) + ((8*a - 3*b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Sech[e + f*x]^4/(4*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^(m + 1))/2, x], x, Sin[e + f*x]^2/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 89

```
Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a-b)+2(a-b)x}{(1+x)^2(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{4(a-b)f} \\
&= \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a^2+8ab-b^2)}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} \\
&= \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} \\
&= \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} \\
&= -\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f} + \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{sech}^4(e+fx)}{8(a-b)f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.467676, size = 113, normalized size = 0.6

$$\frac{(8a^2+8ab-b^2) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b\sinh^2(e+fx)+a}{a-b}\right) + \frac{1}{2}(a-b)\text{sech}^4(e+fx)((8a-3b)\cosh(2(e+fx)) + 4a+b)}{8f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] ((8*a^2 + 8*a*b - b^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x]^2)/(a - b)] + ((a - b)*(4*a + b + (8*a - 3*b)*Cosh[2*(e + f*x)])*Sech[e + f*x]^4)/2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2])

Maple [C] time = 0.225, size = 103, normalized size = 0.6

$$\frac{1}{f} \int \frac{(\sinh(fx+e))^5 (\cosh(fx+e))^4}{-b^2 (\cosh(fx+e))^{14} + (-2ab+2b^2) (\cosh(fx+e))^{12} + (-a^2+2ab-b^2) (\cosh(fx+e))^{10}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] `int/indef0` (-sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^4/(-b^2*cosh(f*x+e)^14+(-2*a*b+2*b^2)*cosh(f*x+e)^12+(-a^2+2*a*b-b^2)*cosh(f*x+e)^10),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx+e)^5}{(b \sinh(fx+e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [B] time = 6.76644, size = 24184, normalized size = 129.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/16*(((8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b + 8*a*b^2 - b^3)*sinh(f*x + e)^12 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^10 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3 + 33*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^2

$$\begin{aligned}
&)*\sinh(f*x + e)^{10} + 20*(11*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^3 + (16 \\
& *a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e)^9 + (128*a^3 \\
& + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^8 + (495*(8*a^2*b + 8*a*b^2 - b \\
& ^3)*\cosh(f*x + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 90*(16*a^3 + 2 \\
& 4*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^8 + 8*(99*(8*a^2*b \\
& + 8*a*b^2 - b^3)*\cosh(f*x + e)^5 + 30*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\c \\
& osh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e))*\sinh \\
& (f*x + e)^7 + 4*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e)^6 + 4*(2 \\
& 31*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^6 + 105*(16*a^3 + 24*a^2*b + 6*a \\
& *b^2 - b^3)*\cosh(f*x + e)^4 + 48*a^3 + 40*a^2*b - 14*a*b^2 + b^3 + 7*(128*a \\
& ^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 8*(99*(\\
& 8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^7 + 63*(16*a^3 + 24*a^2*b + 6*a*b^2 \\
& - b^3)*\cosh(f*x + e)^5 + 7*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x \\
& + e)^3 + 3*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e \\
&)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^4 + (495*(8*a^2* \\
& b + 8*a*b^2 - b^3)*\cosh(f*x + e)^8 + 420*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3 \\
&)*\cosh(f*x + e)^6 + 70*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e \\
& ^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 60*(48*a^3 + 40*a^2*b - 14*a*b^ \\
& 2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(55*(8*a^2*b + 8*a*b^2 - b^3) \\
& *\cosh(f*x + e)^9 + 60*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^7 + \\
& 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^5 + 20*(48*a^3 + 4 \\
& 0*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 \\
& + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 8*a^2*b + 8*a*b^2 - b^3 + 2*(16*a^ \\
& 3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^2 + 2*(33*(8*a^2*b + 8*a*b^2 - \\
& b^3)*\cosh(f*x + e)^10 + 45*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e \\
&)^8 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^6 + 30*(48*a^ \\
& 3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e)^4 + 16*a^3 + 24*a^2*b + 6*a*b^ \\
& 2 - b^3 + 3*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f* \\
& x + e)^2 + 4*(3*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^11 + 5*(16*a^3 + 24 \\
& *a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^9 + 2*(128*a^3 + 120*a^2*b - 24*a*b^2 \\
& + b^3)*\cosh(f*x + e)^7 + 6*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + \\
& e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^3 + (16*a^3 + \\
& 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{a - b}*\log((b* \\
& \cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2 \\
& *(4*a - 3*b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 4*a - 3*b)*\sinh(f*x \\
& + e)^2 - 4*\sqrt{2}*\sqrt{a - b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 \\
& + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e \\
&)^2)}*(\cosh(f*x + e) + \sinh(f*x + e)) + 4*(b*\cosh(f*x + e)^3 + (4*a - 3*b)* \\
& \cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f \\
& *x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2 \\
& *\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) \\
& + 4*\sqrt{2}*((8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^9 + 9*(8*a^3 - 9*a*b^2 + \\
& b^3)*\cosh(f*x + e)*\sinh(f*x + e)^8 + (8*a^3 - 9*a*b^2 + b^3)*\sinh(f*x + e) \\
& ^9 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^7 + 4*(16*a^3 - \\
& 19*a^2*b + 5*a*b^2 - 2*b^3 + 9*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sin
\end{aligned}$$

$$\begin{aligned}
& h(f*x + e)^7 + 28*(3*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^3 + (16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^6 + 2*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^4 + 40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3 + 42*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^5 + 70*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^4 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + 4*(21*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^6 + 35*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^4 + 16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^3 + 4*(9*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^7 + 21*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^5 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e)^3 + 3*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e) + (9*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^8 + 28*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^6 + 10*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e)^4 + 8*a^3 - 9*a*b^2 + b^3 + 12*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e))*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^12 + 12*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)*\sinh(f*x + e)^11 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\sinh(f*x + e)^12 + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^10 + 2*(33*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f)*\sinh(f*x + e)^10 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^8 + 20*(11*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^3 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e))*\sinh(f*x + e)^9 + (495*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^4 + 90*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^2 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f)*\sinh(f*x + e)^8 + 4*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*\cosh(f*x + e)^6 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^5 + 30*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e))*\sinh(f*x + e)^7 + 4*(231*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^6 + 105*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^4 + 7*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^2 + (6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f)*\sinh(f*x + e)^6 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^4 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^7 + 63*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^5 + 7*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a
\end{aligned}$$

$$\begin{aligned}
&^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^3 + 3*(6*a^5 - 25*a^4*b + 40*a^3*b \\
&^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + (495*(\\
&a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^8 + 420*(2*a \\
&^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^6 + 7 \\
&0*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f* \\
&x + e)^4 + 60*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5) \\
&*f*\cosh(f*x + e)^2 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b \\
&^4 - b^5)*f)*\sinh(f*x + e)^4 + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - \\
&2*a*b^4 + b^5)*f*\cosh(f*x + e)^2 + 4*(55*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - \\
&4*a*b^4 + b^5)*f*\cosh(f*x + e)^9 + 60*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2* \\
&b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^7 + 14*(16*a^5 - 65*a^4*b + 100*a^3*b^ \\
&2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^5 + 20*(6*a^5 - 25*a^4*b + \\
&40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*\cosh(f*x + e)^3 + (16*a^5 - 65 \\
&a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e))*\sinh(f \\
&*x + e)^3 + 2*(33*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f* \\
&x + e)^10 + 45*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f* \\
&\cosh(f*x + e)^8 + 14*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b \\
&^4 - b^5)*f*\cosh(f*x + e)^6 + 30*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^ \\
&3 + 10*a*b^4 - b^5)*f*\cosh(f*x + e)^4 + 3*(16*a^5 - 65*a^4*b + 100*a^3*b^2 \\
&- 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3 \\
&*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f)*\sinh(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + \\
&6*a^2*b^3 - 4*a*b^4 + b^5)*f + 4*(3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b \\
&^4 + b^5)*f*\cosh(f*x + e)^11 + 5*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - \\
&2*a*b^4 + b^5)*f*\cosh(f*x + e)^9 + 2*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70 \\
&a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^7 + 6*(6*a^5 - 25*a^4*b + 40*a^3 \\
&*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*\cosh(f*x + e)^5 + (16*a^5 - 65*a^4*b \\
&+ 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^3 + (2*a^5 - 7 \\
&a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e))*\sinh(f*x + \\
&e)), -1/8*(((8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^12 + 12*(8*a^2*b + 8*a \\
&*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e)^11 + (8*a^2*b + 8*a*b^2 - b^3)*\sinh \\
&(f*x + e)^12 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^10 + 2*(\\
&16*a^3 + 24*a^2*b + 6*a*b^2 - b^3 + 33*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + \\
&e)^2)*\sinh(f*x + e)^10 + 20*(11*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^3 \\
&+ (16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e)^9 + (128 \\
&a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^8 + (495*(8*a^2*b + 8*a*b^ \\
&2 - b^3)*\cosh(f*x + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 90*(16*a^ \\
&3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^8 + 8*(99*(8*a \\
&^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^5 + 30*(16*a^3 + 24*a^2*b + 6*a*b^2 - b \\
&^3)*\cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)) \\
&*\sinh(f*x + e)^7 + 4*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e)^6 + \\
&4*(231*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^6 + 105*(16*a^3 + 24*a^2*b \\
&+ 6*a*b^2 - b^3)*\cosh(f*x + e)^4 + 48*a^3 + 40*a^2*b - 14*a*b^2 + b^3 + 7*(\\
&128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 8* \\
&(99*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^7 + 63*(16*a^3 + 24*a^2*b + 6*a \\
&*b^2 - b^3)*\cosh(f*x + e)^5 + 7*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh
\end{aligned}$$

$$\begin{aligned}
& (f*x + e)^3 + 3*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e)*\sinh(f*x + e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^4 + (495*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^8 + 420*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^6 + 70*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 60*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(55*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^9 + 60*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^7 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^5 + 20*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 8*a^2*b + 8*a*b^2 - b^3 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^2 + 2*(33*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^10 + 45*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^8 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^6 + 30*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e)^4 + 16*a^3 + 24*a^2*b + 6*a*b^2 - b^3 + 3*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 4*(3*(8*a^2*b + 8*a*b^2 - b^3)*\cosh(f*x + e)^11 + 5*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e)^9 + 2*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^7 + 6*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*\cosh(f*x + e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*\cosh(f*x + e)^3 + (16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-a + b}*\arctan(-1/2*\sqrt{2}*\sqrt{-a + b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)})/((a - b)*\cosh(f*x + e) + (a - b)*\sinh(f*x + e))) - 2*\sqrt{2}*((8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^9 + 9*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)*\sinh(f*x + e)^8 + (8*a^3 - 9*a*b^2 + b^3)*\sinh(f*x + e)^9 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^7 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3 + 9*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^7 + 28*(3*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^3 + (16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^6 + 2*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^4 + 40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3 + 42*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^5 + 70*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^4 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^3 + 4*(21*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^6 + 35*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^4 + 16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^3 + 4*(9*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^7 + 21*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^5 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e)^3 + 3*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e) + (9*(8*a^3 - 9*a*b^2 + b^3)*\cosh(f*x + e)^8 + 28*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^6 + 10*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*\cosh(f*x + e)^4 + 8*a^3 - 9*a*b^2 + b^3 + 12*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)}/((a - b)*\cosh(f*x + e) + (a - b)*\sinh(f*x + e)))
\end{aligned}$$

$$\begin{aligned}
& x + e)^2 + b \sinh(f*x + e)^2 + 2*a - b) / (\cosh(f*x + e)^2 - 2*\cosh(f*x + e)* \\
& \sinh(f*x + e) + \sinh(f*x + e)^2)) / ((a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + \\
& 4 + b^5)*f*\cosh(f*x + e)^{12} + 12*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + \\
& b^5)*f*\cosh(f*x + e)*\sinh(f*x + e)^{11} + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4 \\
& *a*b^4 + b^5)*f*\sinh(f*x + e)^{12} + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b \\
& ^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^{10} + 2*(33*(a^4*b - 4*a^3*b^2 + 6*a^2*b \\
& ^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^ \\
& 2*b^3 - 2*a*b^4 + b^5)*f)*\sinh(f*x + e)^{10} + (16*a^5 - 65*a^4*b + 100*a^3*b \\
& ^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^8 + 20*(11*(a^4*b - 4*a^3 \\
& *b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^3 + (2*a^5 - 7*a^4*b + 8* \\
& a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e))*\sinh(f*x + e)^9 + (49 \\
& 5*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^4 + 90*(2 \\
& *a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^2 + \\
& (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f)*\sinh(f* \\
& x + e)^8 + 4*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)* \\
& f*\cosh(f*x + e)^6 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f \\
& *\cosh(f*x + e)^5 + 30*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + \\
& b^5)*f*\cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20 \\
& *a*b^4 - b^5)*f*\cosh(f*x + e))*\sinh(f*x + e)^7 + 4*(231*(a^4*b - 4*a^3*b^2 \\
& + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^6 + 105*(2*a^5 - 7*a^4*b + 8*a \\
& ^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^4 + 7*(16*a^5 - 65*a^4* \\
& b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^2 + (6*a^5 - \\
& 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f)*\sinh(f*x + e)^6 + \\
& (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x \\
& + e)^4 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + \\
& e)^7 + 63*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh \\
& (f*x + e)^5 + 7*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - \\
& b^5)*f*\cosh(f*x + e)^3 + 3*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10 \\
& *a*b^4 - b^5)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + (495*(a^4*b - 4*a^3*b^2 + \\
& 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^8 + 420*(2*a^5 - 7*a^4*b + 8*a^3 \\
& *b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^6 + 70*(16*a^5 - 65*a^4*b \\
& + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e)^4 + 60*(6*a^5 \\
& - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*\cosh(f*x + e)^2 + \\
& (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f)*\sinh(f* \\
& x + e)^4 + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*co \\
& sh(f*x + e)^2 + 4*(55*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos \\
& h(f*x + e)^9 + 60*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5) \\
& *f*\cosh(f*x + e)^7 + 14*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20* \\
& a*b^4 - b^5)*f*\cosh(f*x + e)^5 + 20*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2 \\
& *b^3 + 10*a*b^4 - b^5)*f*\cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 \\
& - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + 2*(33*(a \\
& ^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*\cosh(f*x + e)^{10} + 45*(2*a^ \\
& 5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*\cosh(f*x + e)^8 + 14 \\
& *(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*\cosh(f*x \\
& + e)^6 + 30*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*
\end{aligned}$$

```
f*cosh(f*x + e)^4 + 3*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*
b^4 - b^5)*f*cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2
*a*b^4 + b^5)*f)*sinh(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4
+ b^5)*f + 4*(3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x
+ e)^11 + 5*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*co
sh(f*x + e)^9 + 2*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4
- b^5)*f*cosh(f*x + e)^7 + 6*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 +
10*a*b^4 - b^5)*f*cosh(f*x + e)^5 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a
^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^3 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 -
2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^5}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.491 \quad \int \frac{\tanh^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=122

$$\frac{2a+b}{2f(a-b)^2 \sqrt{a+b \sinh^2(e+fx)}} - \frac{(2a+b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}} + \frac{\operatorname{sech}^2(e+fx)}{2f(a-b) \sqrt{a+b \sinh^2(e+fx)}}$$

[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^(5/2)*f) + (2*a + b)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + Sech[e + f*x]^2/(2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.139628, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a+b}{2f(a-b)^2 \sqrt{a+b \sinh^2(e+fx)}} - \frac{(2a+b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}} + \frac{\operatorname{sech}^2(e+fx)}{2f(a-b) \sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^(5/2)*f) + (2*a + b)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + Sech[e + f*x]^2/(2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 78

```

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

```

Rule 51

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= \frac{\text{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{4(a-b)f} \\
&= \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{4(a-b)f} \\
&= \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{4(a-b)f} \\
&= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f} + \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.117849, size = 79, normalized size = 0.65

$$\frac{(2a+b) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b\sinh^2(e+fx)+a}{a-b}\right) + (a-b)\text{sech}^2(e+fx)}{2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] ((2*a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x]^2)/(a - b)] + (a - b)*Sech[e + f*x]^2)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2])

Maple [C] time = 0.195, size = 103, normalized size = 0.8

$$\frac{1}{f} \int \frac{(\sinh(fx+e))^3 (\cosh(fx+e))^2}{-b^2 (\cosh(fx+e))^{10} + (-2ab+2b^2) (\cosh(fx+e))^8 + (-a^2+2ab-b^2) (\cosh(fx+e))^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*cosh(f*x+e)^10+(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^3}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 3.54302, size = 9603, normalized size = 78.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a*b + b^2)*cosh(f*x + e)^8 + 8*(2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a*b + b^2)*sinh(f*x + e)^8 + 4*(2*a^2 + a*b)*cosh(f*x + e)^6 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*sinh(f*x + e)^6 + 8*(7*(2*a*b + b^2)*cosh(f*x + e)^3 + 3*(2*a^2 + a*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b + b^2)*cosh(f*x + e)^4 + 30*(2*a^2 + a*b)*cosh(f*x + e)^2 + 8*a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 8*(7*(2*a*b + b^2)*cosh(f*x + e)^5 + 10*(2*a^2 + a*b)*cosh(f*x + e)^3 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 + a*b)*cosh(f*x + e)^2 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^6 + 15*(2*a^2 + a*b)*cosh(f*x + e)^4 + 3*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*sinh(f*x + e)^2 + 2*a*b + b^2 + 8*((2*a*b + b^2)*cosh(f*x + e)^7 + 3*(2*a^2
```

$$\begin{aligned}
& + a*b)*\cosh(f*x + e)^5 + (8*a^2 + 2*a*b - b^2)*\cosh(f*x + e)^3 + (2*a^2 + a \\
& *b)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{a - b}*\log((b*\cosh(f*x + e)^4 + 4*b* \\
& \cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(4*a - 3*b)*\cosh(f*x \\
& + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 4*a - 3*b)*\sinh(f*x + e)^2 - 4*\sqrt{2}*\sqrt{a - b})*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x \\
& + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2))*(\cosh(f*x + e) + \\
& \sinh(f*x + e)) + 4*(b*\cosh(f*x + e)^3 + (4*a - 3*b)*\cosh(f*x + e))*\sinh(f*x \\
& + e) + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + \\
& e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\\
& \cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) + 4*\sqrt{2}*((2*a^2 - \\
& a*b - b^2)*\cosh(f*x + e)^5 + 5*(2*a^2 - a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + \\
& e)^4 + (2*a^2 - a*b - b^2)*\sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2)*\cosh(\\
& f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*\cosh(f*x + e)^2 + 4*a^2 - 5*a*b + b^2) \\
&)*\sinh(f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*\cosh(f*x + e)^3 + 3*(4*a^2 - 5 \\
& *a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (2*a^2 - a*b - b^2)*\cosh(f*x + \\
& e) + (5*(2*a^2 - a*b - b^2)*\cosh(f*x + e)^4 + 6*(4*a^2 - 5*a*b + b^2)*\cosh \\
& (f*x + e)^2 + 2*a^2 - a*b - b^2)*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b \\
& *\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e \\
&) + \sinh(f*x + e)^2)))/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e) \\
& ^8 + 8*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)*\sinh(f*x + e)^7 \\
& + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\sinh(f*x + e)^8 + 4*(a^4 - 3*a^3*b \\
& + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^6 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 \\
& - b^4)*f*\cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*\sinh(f*x \\
& + e)^6 + 2*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*\cosh(f*x + e)^4 \\
& + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^3 + 3*(a^4 - 3 \\
& *a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(35*(a^3*b \\
& - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^4 + 30*(a^4 - 3*a^3*b + 3*a^2 \\
& *b^2 - a*b^3)*f*\cosh(f*x + e)^2 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 \\
& + b^4)*f)*\sinh(f*x + e)^4 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f* \\
& x + e)^2 + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^5 + 10* \\
& (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^3 + (4*a^4 - 13*a^3*b + \\
& 15*a^2*b^2 - 7*a*b^3 + b^4)*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(7*(a^3*b \\
& - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^6 + 15*(a^4 - 3*a^3*b + 3*a^2 \\
& *b^2 - a*b^3)*f*\cosh(f*x + e)^4 + 3*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^ \\
& 3 + b^4)*f*\cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*\sinh(f* \\
& x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f + 8*((a^3*b - 3*a^2*b^2 + \\
& 3*a*b^3 - b^4)*f*\cosh(f*x + e)^7 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f* \\
& \cosh(f*x + e)^5 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*\cosh(f* \\
& x + e)^3 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e))*\sinh(f*x + \\
& e)), -1/2*(((2*a*b + b^2)*\cosh(f*x + e)^8 + 8*(2*a*b + b^2)*\cosh(f*x + e)*\s \\
& inh(f*x + e)^7 + (2*a*b + b^2)*\sinh(f*x + e)^8 + 4*(2*a^2 + a*b)*\cosh(f*x + \\
& e)^6 + 4*(7*(2*a*b + b^2)*\cosh(f*x + e)^2 + 2*a^2 + a*b)*\sinh(f*x + e)^6 + \\
& 8*(7*(2*a*b + b^2)*\cosh(f*x + e)^3 + 3*(2*a^2 + a*b)*\cosh(f*x + e))*\sinh(f \\
& *x + e)^5 + 2*(8*a^2 + 2*a*b - b^2)*\cosh(f*x + e)^4 + 2*(35*(2*a*b + b^2)*\c \\
& osh(f*x + e)^4 + 30*(2*a^2 + a*b)*\cosh(f*x + e)^2 + 8*a^2 + 2*a*b - b^2)*\si
\end{aligned}$$

$$\begin{aligned}
& \text{nh}(f*x + e)^4 + 8*(7*(2*a*b + b^2)*\cosh(f*x + e)^5 + 10*(2*a^2 + a*b)*\cosh(f*x + e)^3 + (8*a^2 + 2*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(2*a^2 + a*b)*\cosh(f*x + e)^2 + 4*(7*(2*a*b + b^2)*\cosh(f*x + e)^6 + 15*(2*a^2 + a*b)*\cosh(f*x + e)^4 + 3*(8*a^2 + 2*a*b - b^2)*\cosh(f*x + e)^2 + 2*a^2 + a*b)*\sinh(f*x + e)^2 + 2*a*b + b^2 + 8*((2*a*b + b^2)*\cosh(f*x + e)^7 + 3*(2*a^2 + a*b)*\cosh(f*x + e)^5 + (8*a^2 + 2*a*b - b^2)*\cosh(f*x + e)^3 + (2*a^2 + a*b)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-a + b}*\arctan(-1/2*\sqrt{2}*\sqrt{(-a + b)*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)}}/(a - b)*\cosh(f*x + e) + (a - b)*\sinh(f*x + e))) - 2*\sqrt{2}*((2*a^2 - a*b - b^2)*\cosh(f*x + e)^5 + 5*(2*a^2 - a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^4 + (2*a^2 - a*b - b^2)*\sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2)*\cosh(f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*\cosh(f*x + e)^2 + 4*a^2 - 5*a*b + b^2)*\sinh(f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*\cosh(f*x + e)^3 + 3*(4*a^2 - 5*a*b + b^2)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (2*a^2 - a*b - b^2)*\cosh(f*x + e) + (5*(2*a^2 - a*b - b^2)*\cosh(f*x + e)^4 + 6*(4*a^2 - 5*a*b + b^2)*\cosh(f*x + e)^2 + 2*a^2 - a*b - b^2)*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)}}/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^8 + 8*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\sinh(f*x + e)^8 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^6 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*\sinh(f*x + e)^6 + 2*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*\cosh(f*x + e)^4 + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^3 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(35*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^4 + 30*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^2 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f)*\sinh(f*x + e)^4 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^2 + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^5 + 10*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^3 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^6 + 15*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^4 + 3*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*\cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*\sinh(f*x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f + 8*((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*\cosh(f*x + e)^7 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e)^5 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*\cosh(f*x + e)^3 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*\cosh(f*x + e))*\sinh(f*x + e)]
\end{aligned}$$

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Integral(tanh(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(fx + e)}{(b \sinh^2(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.492 \quad \int \frac{\tanh(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=69

$$\frac{1}{f(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2]/\text{Sqrt}[a - b]]/((a - b)^{(3/2)*f})) + 1/((a - b)*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Rubi [A] time = 0.0738817, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 51, 63, 208}

$$\frac{1}{f(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tanh}[e + f*x]/(a + b*\text{Sinh}[e + f*x]^2)^{(3/2)}, x]$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2]/\text{Sqrt}[a - b]]/((a - b)^{(3/2)*f})) + 1/((a - b)*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Rule 3194

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^2]^{(p_.)}*\tan[(e_.) + (f_.)*(x_)]^{(m_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f*x]^2, x]\}, \text{Dist}[ff^{((m + 1)/2)/(2*f)}, \text{Subst}[\text{Int}[(x^{((m - 1)/2)*(a + b*ff*x)^p})/(1 - ff*x)^{(m + 1)/2}), x], x, \text{Sin}[e + f*x]^2/ff, x]] /; \text{FreeQ}\{a, b, e, f, p\}, x \&\& \text{IntegerQ}[(m - 1)/2]$

Rule 51

$\text{Int}[(a_.) + (b_.)*(x_)]^{(m_.)}*((c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n + 1)}]/((b*c - a*d)*(m + 1)), x] - \text{Dist}[(d*($


```

m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}} dx, x, \sinh^2(e+fx)\right)}{2(a-b)f} \\
&= \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{(a-b)bf} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.0781408, size = 58, normalized size = 0.84

$$-\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b\cosh^2(e+fx)}{a-b} + 1\right)}{f(b-a)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/((-a + b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2]))

Maple [C] time = 0.123, size = 93, normalized size = 1.4

$$\frac{1}{f} \int \frac{\sinh(fx + e)}{-b^2 (\sinh(fx + e))^6 + (-2ab - b^2) (\sinh(fx + e))^4 + (-a^2 - 2ab) (\sinh(fx + e))^2 - a^2} \sqrt{a + b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] `int/indef0` (-sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(-b^2*sinh(f*x+e)^6+(-2*a*b-b^2)*sinh(f*x+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [B] time = 2.67186, size = 3475, normalized size = 50.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^4 + 4*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f*sinh(f*x + e)^4 + 2*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^2 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f)*sinh(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f + 4*((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^3 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)), -((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^4 + 4*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f*sinh(f*x + e)^4 + 2*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^2 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f)*sinh(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f + 4*((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^3 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)]]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Integral(tanh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.493 \quad \int \frac{\coth(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{3/2}} dx$$

Optimal. Leaf size=57

$$\frac{1}{af\sqrt{a+b \sinh^2(e+fx)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2]/\text{Sqrt}[a]]/(a^{(3/2)*f})) + 1/(a*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Rubi [A] time = 0.0840825, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 51, 63, 208}

$$\frac{1}{af\sqrt{a+b \sinh^2(e+fx)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[e + f*x]/(a + b*\text{Sinh}[e + f*x]^2)^{(3/2)}, x]$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2]/\text{Sqrt}[a]]/(a^{(3/2)*f})) + 1/(a*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Rule 3194

$\text{Int}[\left((a_{.}) + (b_{.})*\sin[(e_{.}) + (f_{.})*(x_{.})]^2\right)^{(p_{.})}*\tan[(e_{.}) + (f_{.})*(x_{.})]^{(m_{.})}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f*x]^2, x]\}, \text{Dist}[ff^{((m + 1)/2)/(2*f)}, \text{Subst}[\text{Int}[(x^{(m - 1)/2}*(a + b*ff*x)^p]/(1 - ff*x)^{(m + 1)/2}), x], x, \text{Sin}[e + f*x]^2/ff, x]] \text{ /; } \text{FreeQ}[\{a, b, e, f, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

Rule 51

$\text{Int}[\left((a_{.}) + (b_{.})*(x_{.})\right)^{(m_{.})}*\left((c_{.}) + (d_{.})*(x_{.})\right)^{(n_{.})}, x_Symbol] \rightarrow \text{Simp}[\left((a + b*x)^{(m + 1)}*(c + d*x)^{(n + 1)}\right)/\left((b*c - a*d)*(m + 1)\right), x] - \text{Dist}[(d*($

$m + n + 2)) / ((b*c - a*d)*(m + 1)), \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x]$, x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
 [n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
 ntLinearQ[a, b, c, d, m, n, x]

Rule 63

$\text{Int}[(a_.) + (b_.)*(x_)^{(m_)}*((c_.) + (d_.)*(x_)^{(n_)}, x_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 208

$\text{Int}[(a_.) + (b_.)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b]$

Rubi steps

$$\begin{aligned} \int \frac{\coth(e + fx)}{(a + b \sinh^2(e + fx))^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, \sinh^2(e + fx)\right)}{2f} \\ &= \frac{1}{af\sqrt{a + b \sinh^2(e + fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \sinh^2(e + fx)\right)}{2af} \\ &= \frac{1}{af\sqrt{a + b \sinh^2(e + fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^2(e + fx)}\right)}{abf} \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a + b \sinh^2(e + fx)}} \end{aligned}$$

Mathematica [C] time = 0.0595502, size = 46, normalized size = 0.81

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b \sinh^2(e+fx)}{a} + 1\right)}{af\sqrt{a + b \sinh^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])

Maple [C] time = 0.063, size = 35, normalized size = 0.6

$$\frac{1}{f} \int \frac{1}{\sinh(fx + e) \left(a + b (\sinh(fx + e))^2 \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] `int/indef0`(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)}{\left(b \sinh(fx + e)^2 + a \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [B] time = 2.3862, size = 2955, normalized size = 51.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x + e)^4 + 4*a^2*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*b*f*sinh(f*x + e)^4 + a^2*b*f + 2*(2*a^3 - a^2*b)*f*cosh(f*x + e)^2 + 2*(3*a^2*b*f*cosh(f*x + e)^2 + (2*a^3 - a^2*b)*f)*sinh(f*x + e)^2 + 4*(a^2*b*f*cosh(f*x + e)^3 + (2*a^3 - a^2*b)*f*cosh(f*x + e))*sinh(f*x + e)), ((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x + e)^4 + 4*a^2*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*b*f*sinh(f*x + e)^4 + a^2*b*f + 2*(2*a^3 - a^2*b)*f*cosh(f*x + e)^2 + 2*(3*a^2*b*f*cosh(f*x + e)^2 + (2*a^3 - a^2*b)*f)*sinh(f*x + e)^2 + 4*(a^2*b*f*cosh(f*x + e)^3 + (2*a^3 - a^2*b)*f*cosh(f*x + e))*sinh(f*x + e)]]

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)


```
[Out] Integral(coth(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.494 \quad \int \frac{\coth^3(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=110

$$\frac{2a-3b}{2a^2 f \sqrt{a+b \sinh^2(e+fx)}} - \frac{(2a-3b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2} f} - \frac{\operatorname{csch}^2(e+fx)}{2af \sqrt{a+b \sinh^2(e+fx)}}$$

[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) + (2*a - 3*b)/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Csch[e + f*x]^2/(2*a*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.135962, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-3b}{2a^2 f \sqrt{a+b \sinh^2(e+fx)}} - \frac{(2a-3b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2} f} - \frac{\operatorname{csch}^2(e+fx)}{2af \sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) + (2*a - 3*b)/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Csch[e + f*x]^2/(2*a*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{1+x}{x^2(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a-3b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{4af} \\
&= \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a-3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \sinh^2(e+fx)\right)}{4a^2f} \\
&= \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a-3b)\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sinh^2(e+fx)\right)}{2a^2b} \\
&= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} + \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.102635, size = 69, normalized size = 0.63

$$\frac{(2a-3b) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b\sinh^2(e+fx)}{a} + 1\right) - \text{acsch}^2(e+fx)}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] $(-(a*\text{Csch}[e + f*x]^2) + (2*a - 3*b)*\text{Hypergeometric2F1}[-1/2, 1, 1/2, 1 + (b*\text{Sinh}[e + f*x]^2)/a])/(2*a^2*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Maple [C] time = 0.104, size = 43, normalized size = 0.4

$$\frac{1}{f} \int \frac{\cosh^2(fx+e)}{\sinh^3(fx+e)} \left(a + b(\sinh(fx+e))^2\right)^{-\frac{3}{2}} dx, \sinh(fx+e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0` (cosh(f*x+e)^2/sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^3}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 3.05572, size = 7960, normalized size = 72.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a*b - 3*b^2)*cosh(f*x + e)^8 + 8*(2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (2*a*b - 3*b^2)*sinh(f*x + e)^8 + 4*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b - 3*b^2)*cosh(f*x + e)^4 + 30*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 18*a*b - 9*b^2)*sinh(f*x + e)^4 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^5 + 10*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^3 - (8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^6 + 15*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^4 - 3*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^2 + 2*a*b - 3*b^2 + 8*((2*a*
```

$$\begin{aligned}
& b - 3b^2) \cosh(fx + e)^7 + 3(2a^2 - 5ab + 3b^2) \cosh(fx + e)^5 - (8 \\
& a^2 - 18ab + 9b^2) \cosh(fx + e)^3 + (2a^2 - 5ab + 3b^2) \cosh(fx + \\
& e) \sinh(fx + e) \sqrt{a} \log((b \cosh(fx + e)^4 + 4b \cosh(fx + e) \sinh \\
& (fx + e)^3 + b \sinh(fx + e)^4 + 2(4a - b) \cosh(fx + e)^2 + 2(3b \cosh \\
& (fx + e)^2 + 4a - b) \sinh(fx + e)^2 + 4\sqrt{2} \sqrt{a} \sqrt{(b \cosh(fx \\
& + e)^2 + b \sinh(fx + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh \\
& (fx + e) + \sinh(fx + e)^2)) * (\cosh(fx + e) + \sinh(fx + e)) + 4(b \cosh \\
& (fx + e)^3 + (4a - b) \cosh(fx + e) \sinh(fx + e) + b) / (\cosh(fx + e)^4 \\
& + 4 \cosh(fx + e) \sinh(fx + e)^3 + \sinh(fx + e)^4 + 2(3 \cosh(fx + e)^2 \\
& - 1) \sinh(fx + e)^2 - 2 \cosh(fx + e)^2 + 4(\cosh(fx + e)^3 - \cosh(fx + \\
& e)) \sinh(fx + e) + 1) - 4\sqrt{2} * ((2a^2 - 3ab) \cosh(fx + e)^5 + 5(\\
& 2a^2 - 3ab) \cosh(fx + e) \sinh(fx + e)^4 + (2a^2 - 3ab) \sinh(fx + e \\
&)^5 - 2(4a^2 - 3ab) \cosh(fx + e)^3 + 2(5(2a^2 - 3ab) \cosh(fx + e \\
&)^2 - 4a^2 + 3ab) \sinh(fx + e)^3 + 2(5(2a^2 - 3ab) \cosh(fx + e)^3 \\
& - 3(4a^2 - 3ab) \cosh(fx + e)) \sinh(fx + e)^2 + (2a^2 - 3ab) \cosh(\\
& fx + e) + (5(2a^2 - 3ab) \cosh(fx + e)^4 - 6(4a^2 - 3ab) \cosh(fx \\
& + e)^2 + 2a^2 - 3ab) \sinh(fx + e) \sqrt{(b \cosh(fx + e)^2 + b \sinh(fx \\
& + e)^2 + 2a - b) / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh(\\
& fx + e)^2)) / (a^3 b f \cosh(fx + e)^8 + 8a^3 b f \cosh(fx + e) \sinh(fx + \\
& e)^7 + a^3 b f \sinh(fx + e)^8 + 4(a^4 - a^3 b) f \cosh(fx + e)^6 + 4(7a \\
& ^3 b f \cosh(fx + e)^2 + (a^4 - a^3 b) f) \sinh(fx + e)^6 - 2(4a^4 - 3a \\
& ^3 b) f \cosh(fx + e)^4 + 8(7a^3 b f \cosh(fx + e)^3 + 3(a^4 - a^3 b) f f \\
& \cosh(fx + e)) \sinh(fx + e)^5 + a^3 b f + 2(35a^3 b f \cosh(fx + e)^4 + \\
& 30(a^4 - a^3 b) f \cosh(fx + e)^2 - (4a^4 - 3a^3 b) f) \sinh(fx + e)^4 + \\
& 4(a^4 - a^3 b) f \cosh(fx + e)^2 + 8(7a^3 b f \cosh(fx + e)^5 + 10(a^4 \\
& - a^3 b) f \cosh(fx + e)^3 - (4a^4 - 3a^3 b) f \cosh(fx + e)) \sinh(fx + \\
& e)^3 + 4(7a^3 b f \cosh(fx + e)^6 + 15(a^4 - a^3 b) f \cosh(fx + e)^4 - \\
& 3(4a^4 - 3a^3 b) f \cosh(fx + e)^2 + (a^4 - a^3 b) f) \sinh(fx + e)^2 + \\
& 8(a^3 b f \cosh(fx + e)^7 + 3(a^4 - a^3 b) f \cosh(fx + e)^5 - (4a^4 - \\
& 3a^3 b) f \cosh(fx + e)^3 + (a^4 - a^3 b) f \cosh(fx + e)) \sinh(fx + e), \\
& 1/2 * (((2ab - 3b^2) \cosh(fx + e)^8 + 8(2ab - 3b^2) \cosh(fx + e) \sinh \\
& (fx + e)^7 + (2ab - 3b^2) \sinh(fx + e)^8 + 4(2a^2 - 5ab + 3b^2) \\
& \cosh(fx + e)^6 + 4(7(2ab - 3b^2) \cosh(fx + e)^2 + 2a^2 - 5ab + 3 \\
& b^2) \sinh(fx + e)^6 + 8(7(2ab - 3b^2) \cosh(fx + e)^3 + 3(2a^2 - 5 \\
& ab + 3b^2) \cosh(fx + e)) \sinh(fx + e)^5 - 2(8a^2 - 18ab + 9b^2) \c \\
& osh(fx + e)^4 + 2(35(2ab - 3b^2) \cosh(fx + e)^4 + 30(2a^2 - 5ab \\
& + 3b^2) \cosh(fx + e)^2 - 8a^2 + 18ab - 9b^2) \sinh(fx + e)^4 + 8(7(\\
& 2ab - 3b^2) \cosh(fx + e)^5 + 10(2a^2 - 5ab + 3b^2) \cosh(fx + e)^3 \\
& - (8a^2 - 18ab + 9b^2) \cosh(fx + e)) \sinh(fx + e)^3 + 4(2a^2 - 5a \\
& b + 3b^2) \cosh(fx + e)^2 + 4(7(2ab - 3b^2) \cosh(fx + e)^6 + 15(2a \\
& ^2 - 5ab + 3b^2) \cosh(fx + e)^4 - 3(8a^2 - 18ab + 9b^2) \cosh(fx \\
& + e)^2 + 2a^2 - 5ab + 3b^2) \sinh(fx + e)^2 + 2ab - 3b^2 + 8((2ab \\
& - 3b^2) \cosh(fx + e)^7 + 3(2a^2 - 5ab + 3b^2) \cosh(fx + e)^5 - (8a \\
& ^2 - 18ab + 9b^2) \cosh(fx + e)^3 + (2a^2 - 5ab + 3b^2) \cosh(fx + \\
& e)) \sinh(fx + e) \sqrt{-a} \arctan(1/2 \sqrt{2}) \sqrt{-a} \sqrt{(b \cosh(fx +
\end{aligned}$$

```

e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt
(2)*((2*a^2 - 3*a*b)*cosh(f*x + e)^5 + 5*(2*a^2 - 3*a*b)*cosh(f*x + e)*sinh
(f*x + e)^4 + (2*a^2 - 3*a*b)*sinh(f*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x
+ e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e
)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e
))*sinh(f*x + e)^2 + (2*a^2 - 3*a*b)*cosh(f*x + e) + (5*(2*a^2 - 3*a*b)*cos
h(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b)*sinh(f*x
+ e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^3*b*f*cosh(f*x +
e)^8 + 8*a^3*b*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^3*b*f*sinh(f*x + e)^8 +
4*(a^4 - a^3*b)*f*cosh(f*x + e)^6 + 4*(7*a^3*b*f*cosh(f*x + e)^2 + (a^4 -
a^3*b)*f)*sinh(f*x + e)^6 - 2*(4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^4 + 8*(7*a^
3*b*f*cosh(f*x + e)^3 + 3*(a^4 - a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)^5 +
a^3*b*f + 2*(35*a^3*b*f*cosh(f*x + e)^4 + 30*(a^4 - a^3*b)*f*cosh(f*x + e)^
2 - (4*a^4 - 3*a^3*b)*f)*sinh(f*x + e)^4 + 4*(a^4 - a^3*b)*f*cosh(f*x + e)^
2 + 8*(7*a^3*b*f*cosh(f*x + e)^5 + 10*(a^4 - a^3*b)*f*cosh(f*x + e)^3 - (4*
a^4 - 3*a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*b*f*cosh(f*x + e
)^6 + 15*(a^4 - a^3*b)*f*cosh(f*x + e)^4 - 3*(4*a^4 - 3*a^3*b)*f*cosh(f*x +
e)^2 + (a^4 - a^3*b)*f)*sinh(f*x + e)^2 + 8*(a^3*b*f*cosh(f*x + e)^7 + 3*(
a^4 - a^3*b)*f*cosh(f*x + e)^5 - (4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^3 + (a^4
- a^3*b)*f*cosh(f*x + e))*sinh(f*x + e))]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2), x)

[Out] Integral(coth(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(fx + e)}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```


$$3.495 \quad \int \frac{\coth^5(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=167

$$\frac{8a^2 - 24ab + 15b^2}{8a^3 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{(8a^2 - 24ab + 15b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{8a^{7/2} f} - \frac{(8a - 5b) \operatorname{csch}^2(e + fx)}{8a^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{\operatorname{csch}^4(e + fx)}{4af \sqrt{a + b \sinh^2(e + fx)}}$$

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*a^(7/2)*f) + (8*a^2 - 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((8*a - 5*b)*Csch[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Csch[e + f*x]^4/(4*a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.201072, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2 - 24ab + 15b^2}{8a^3 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{(8a^2 - 24ab + 15b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{8a^{7/2} f} - \frac{(8a - 5b) \operatorname{csch}^2(e + fx)}{8a^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{\operatorname{csch}^4(e + fx)}{4af \sqrt{a + b \sinh^2(e + fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*a^(7/2)*f) + (8*a^2 - 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((8*a - 5*b)*Csch[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Csch[e + f*x]^4/(4*a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^5(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-5b)+2ax}{x^2(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{4af} \\
&= -\frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a^2-24ab+15b^2)\text{Subst}}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} \\
&= \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \\
&= \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \\
&= -\frac{(8a^2-24ab+15b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.327152, size = 94, normalized size = 0.56

$$\frac{(8a^2 - 24ab + 15b^2) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b\sinh^2(e+fx)}{a} + 1\right) + \text{acsch}^2(e+fx)(-2\text{acsch}^2(e+fx) - 8a + 5b)}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (a*Csch[e + f*x]^2*(-8*a + 5*b - 2*a*Csch[e + f*x]^2) + (8*a^2 - 24*a*b + 15*b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a])/(8*a^3*f*Sqrt[a + b*Sinh[e + f*x]^2])

Maple [C] time = 0.09, size = 43, normalized size = 0.3

$$\frac{1}{f} \int \frac{\cosh^4(fx+e)}{\sinh^5(fx+e)} \left(a + b \left(\sinh(fx+e) \right)^2 \right)^{-\frac{3}{2}}, \sinh(fx+e)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] `int/indef0` (cosh(f*x+e)^4/sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^5(fx+e)}{\left(b \sinh^2(fx+e) + a \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [B] time = 4.58169, size = 19003, normalized size = 113.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [1/16*(((8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b - 24*a*b^2 + 15*b^3)*sinh(f*x + e)^12 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^10 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3 + 33*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(8*a^2*b - 24*a*b^2 +

$$\begin{aligned}
& 15*b^3)*\cosh(f*x + e)^3 + (16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*\cosh(f* \\
& x + e))*\sinh(f*x + e)^9 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*\cosh(\\
& f*x + e)^8 + (495*(8*a^2*b - 24*a*b^2 + 15*b^3))*\cosh(f*x + e)^4 - 128*a^3 + \\
& 504*a^2*b - 600*a*b^2 + 225*b^3 + 90*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b \\
& ^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^8 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3))* \\
& \cosh(f*x + e)^5 + 30*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*\cosh(f*x + e) \\
& ^3 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*\cosh(f*x + e))*\sinh(f*x + \\
& e)^7 + 4*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*\cosh(f*x + e)^6 + 4*(231 \\
& *(8*a^2*b - 24*a*b^2 + 15*b^3))*\cosh(f*x + e)^6 + 105*(16*a^3 - 72*a^2*b + 1 \\
& 02*a*b^2 - 45*b^3))*\cosh(f*x + e)^4 + 48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^ \\
& 3 - 7*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3))*\cosh(f*x + e)^2)*\sinh(f*x \\
& + e)^6 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3))*\cosh(f*x + e)^7 + 63*(16*a^3 \\
& - 72*a^2*b + 102*a*b^2 - 45*b^3))*\cosh(f*x + e)^5 - 7*(128*a^3 - 504*a^2*b + \\
& 600*a*b^2 - 225*b^3))*\cosh(f*x + e)^3 + 3*(48*a^3 - 184*a^2*b + 210*a*b^2 - \\
& 75*b^3))*\cosh(f*x + e))*\sinh(f*x + e)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2 \\
& - 225*b^3))*\cosh(f*x + e)^4 + (495*(8*a^2*b - 24*a*b^2 + 15*b^3))*\cosh(f*x + \\
& e)^8 + 420*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3))*\cosh(f*x + e)^6 - 70*(1 \\
& 28*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3))*\cosh(f*x + e)^4 - 128*a^3 + 504*a \\
& ^2*b - 600*a*b^2 + 225*b^3 + 60*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3))*c \\
& osh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(55*(8*a^2*b - 24*a*b^2 + 15*b^3))*\cosh(\\
& f*x + e)^9 + 60*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3))*\cosh(f*x + e)^7 - \\
& 14*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3))*\cosh(f*x + e)^5 + 20*(48*a^3 \\
& - 184*a^2*b + 210*a*b^2 - 75*b^3))*\cosh(f*x + e)^3 - (128*a^3 - 504*a^2*b + \\
& 600*a*b^2 - 225*b^3))*\cosh(f*x + e))*\sinh(f*x + e)^3 + 8*a^2*b - 24*a*b^2 + \\
& 15*b^3 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3))*\cosh(f*x + e)^2 + 2*(3 \\
& 3*(8*a^2*b - 24*a*b^2 + 15*b^3))*\cosh(f*x + e)^10 + 45*(16*a^3 - 72*a^2*b + \\
& 102*a*b^2 - 45*b^3))*\cosh(f*x + e)^8 - 14*(128*a^3 - 504*a^2*b + 600*a*b^2 - \\
& 225*b^3))*\cosh(f*x + e)^6 + 30*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3))*c \\
& osh(f*x + e)^4 + 16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3 - 3*(128*a^3 - 504*a \\
& ^2*b + 600*a*b^2 - 225*b^3))*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 4*(3*(8*a^2* \\
& b - 24*a*b^2 + 15*b^3))*\cosh(f*x + e)^11 + 5*(16*a^3 - 72*a^2*b + 102*a*b^2 \\
& - 45*b^3))*\cosh(f*x + e)^9 - 2*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3))*c \\
& osh(f*x + e)^7 + 6*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3))*\cosh(f*x + e)^ \\
& 5 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3))*\cosh(f*x + e)^3 + (16*a^3 - \\
& 72*a^2*b + 102*a*b^2 - 45*b^3))*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{a}*\log((\\
& b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e))*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + \\
& 2*(4*a - b))*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 4*a - b))*\sinh(f*x + \\
& e)^2 - 4*\sqrt{2}*\sqrt{a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a \\
& - b))/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e))*\sinh(f*x + e) + \sinh(f*x + e)^2))* \\
& (\cosh(f*x + e) + \sinh(f*x + e)) + 4*(b*\cosh(f*x + e)^3 + (4*a - b))*\cosh(f*x \\
& + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e))*\sinh(f*x + e)^ \\
& 3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 - 1))*\sinh(f*x + e)^2 - 2*\cosh(f* \\
& x + e)^2 + 4*(\cosh(f*x + e)^3 - \cosh(f*x + e))*\sinh(f*x + e) + 1)) + 4*\sqrt{ \\
& 2}*((8*a^3 - 24*a^2*b + 15*a*b^2))*\cosh(f*x + e)^9 + 9*(8*a^3 - 24*a^2*b + \\
& 15*a*b^2))*\cosh(f*x + e))*\sinh(f*x + e)^8 + (8*a^3 - 24*a^2*b + 15*a*b^2))*\sin
\end{aligned}$$

$$\begin{aligned}
& h(f*x + e)^9 - 4*(16*a^3 - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e)^7 - 4*(16*a^3 \\
& - 29*a^2*b + 15*a*b^2 - 9*(8*a^3 - 24*a^2*b + 15*a*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e)^7 + 28*(3*(8*a^3 - 24*a^2*b + 15*a*b^2)*\cosh(f*x + e)^3 - (16*a^3 \\
& - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^6 + 2*(40*a^3 - 92*a^2*b + 45*a*b^2)*\cosh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)*\cosh(f*x + e)^4 + 40*a^3 - 92*a^2*b + 45*a*b^2 - 42*(16*a^3 - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)*\cosh(f*x + e)^5 - 70*(16*a^3 - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e)^3 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^4 - 4*(16*a^3 - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e)^3 + 4*(21*(8*a^3 - 24*a^2*b + 15*a*b^2)*\cosh(f*x + e)^6 - 35*(16*a^3 - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e)^4 - 16*a^3 + 29*a^2*b - 15*a*b^2 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e)^3 + 4*(9*(8*a^3 - 24*a^2*b + 15*a*b^2)*\cosh(f*x + e)^7 - 21*(16*a^3 - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e)^5 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*\cosh(f*x + e)^3 - 3*(16*a^3 - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^2 + (8*a^3 - 24*a^2*b + 15*a*b^2)*\cosh(f*x + e) + (9*(8*a^3 - 24*a^2*b + 15*a*b^2)*\cosh(f*x + e)^8 - 28*(16*a^3 - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e)^6 + 10*(40*a^3 - 92*a^2*b + 45*a*b^2)*\cosh(f*x + e)^4 + 8*a^3 - 24*a^2*b + 15*a*b^2 - 12*(16*a^3 - 29*a^2*b + 15*a*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a^4*b*f*\cosh(f*x + e)^12 + 12*a^4*b*f*\cosh(f*x + e)*\sinh(f*x + e)^11 + a^4*b*f*\sinh(f*x + e)^12 + 2*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^10 + 2*(33*a^4*b*f*\cosh(f*x + e)^2 + (2*a^5 - 3*a^4*b)*f)*\sinh(f*x + e)^10 - (16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^8 + 20*(11*a^4*b*f*\cosh(f*x + e)^3 + (2*a^5 - 3*a^4*b)*f*\cosh(f*x + e))*\sinh(f*x + e)^9 + (495*a^4*b*f*\cosh(f*x + e)^4 + 90*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*\sinh(f*x + e)^8 + 4*(6*a^5 - 5*a^4*b)*f*\cosh(f*x + e)^6 + 8*(99*a^4*b*f*\cosh(f*x + e)^5 + 30*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*\cosh(f*x + e))*\sinh(f*x + e)^7 + 4*(231*a^4*b*f*\cosh(f*x + e)^6 + 105*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^4 - 7*(16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^2 + (6*a^5 - 5*a^4*b)*f)*\sinh(f*x + e)^6 + a^4*b*f - (16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^4 + 8*(99*a^4*b*f*\cosh(f*x + e)^7 + 63*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^5 - 7*(16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^3 + 3*(6*a^5 - 5*a^4*b)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + (495*a^4*b*f*\cosh(f*x + e)^8 + 420*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^6 - 70*(16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^4 + 60*(6*a^5 - 5*a^4*b)*f*\cosh(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*\sinh(f*x + e)^4 + 2*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^2 + 4*(55*a^4*b*f*\cosh(f*x + e)^9 + 60*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^7 - 14*(16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^5 + 20*(6*a^5 - 5*a^4*b)*f*\cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + 2*(33*a^4*b*f*\cosh(f*x + e)^10 + 45*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^8 - 14*(16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^6 + 30*(6*a^5 - 5*a^4*b)*f*\cosh(f*x + e)^4 - 3*(16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^2 + (2*a^5 - 3*a^4*b)*f)*\sinh(f*x + e)^2 + 4*(3*a^4*b*f*\cosh(f*x + e)^11 + 5*(2*a^5 - 3*a^4*b)*f*\cosh(f*x + e)^9 - 2*(16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^7 + 6*(6*a^5 - 5*a^4*b)*f*\cosh(f*x + e)^5 - 2*(16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^3 + 2*(6*a^5 - 5*a^4*b)*f*\cosh(f*x + e) - (16*a^5 - 15*a^4*b)*f)*\sinh(f*x + e)
\end{aligned}$$

$$\begin{aligned}
& 4*b)*f*\cosh(f*x + e)^5 - (16*a^5 - 15*a^4*b)*f*\cosh(f*x + e)^3 + (2*a^5 - 3 \\
& *a^4*b)*f*\cosh(f*x + e))*\sinh(f*x + e)), 1/8*(((8*a^2*b - 24*a*b^2 + 15*b^3 \\
&)*\cosh(f*x + e)^{12} + 12*(8*a^2*b - 24*a*b^2 + 15*b^3)*\cosh(f*x + e)*\sinh(f* \\
& x + e)^{11} + (8*a^2*b - 24*a*b^2 + 15*b^3)*\sinh(f*x + e)^{12} + 2*(16*a^3 - 72 \\
& *a^2*b + 102*a*b^2 - 45*b^3)*\cosh(f*x + e)^{10} + 2*(16*a^3 - 72*a^2*b + 102* \\
& a*b^2 - 45*b^3 + 33*(8*a^2*b - 24*a*b^2 + 15*b^3)*\cosh(f*x + e)^2)*\sinh(f*x \\
& + e)^{10} + 20*(11*(8*a^2*b - 24*a*b^2 + 15*b^3)*\cosh(f*x + e)^3 + (16*a^3 - \\
& 72*a^2*b + 102*a*b^2 - 45*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^9 - (128*a^3 - \\
& 504*a^2*b + 600*a*b^2 - 225*b^3)*\cosh(f*x + e)^8 + (495*(8*a^2*b - 24*a*b^ \\
& 2 + 15*b^3)*\cosh(f*x + e)^4 - 128*a^3 + 504*a^2*b - 600*a*b^2 + 225*b^3 + 9 \\
& 0*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^8 \\
& + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3)*\cosh(f*x + e)^5 + 30*(16*a^3 - 72*a^ \\
& 2*b + 102*a*b^2 - 45*b^3)*\cosh(f*x + e)^3 - (128*a^3 - 504*a^2*b + 600*a*b^ \\
& 2 - 225*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^7 + 4*(48*a^3 - 184*a^2*b + 210*a \\
& *b^2 - 75*b^3)*\cosh(f*x + e)^6 + 4*(231*(8*a^2*b - 24*a*b^2 + 15*b^3)*\cosh(\\
& f*x + e)^6 + 105*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*\cosh(f*x + e)^4 + \\
& 48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3 - 7*(128*a^3 - 504*a^2*b + 600*a*b \\
& ^2 - 225*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 8*(99*(8*a^2*b - 24*a*b^2 \\
& + 15*b^3)*\cosh(f*x + e)^7 + 63*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*\cos \\
& h(f*x + e)^5 - 7*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*\cosh(f*x + e)^ \\
& 3 + 3*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*\cosh(f*x + e))*\sinh(f*x + e \\
&)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*\cosh(f*x + e)^4 + (495*(8 \\
& *a^2*b - 24*a*b^2 + 15*b^3)*\cosh(f*x + e)^8 + 420*(16*a^3 - 72*a^2*b + 102* \\
& a*b^2 - 45*b^3)*\cosh(f*x + e)^6 - 70*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225 \\
& *b^3)*\cosh(f*x + e)^4 - 128*a^3 + 504*a^2*b - 600*a*b^2 + 225*b^3 + 60*(48* \\
& a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4* \\
& (55*(8*a^2*b - 24*a*b^2 + 15*b^3)*\cosh(f*x + e)^9 + 60*(16*a^3 - 72*a^2*b + \\
& 102*a*b^2 - 45*b^3)*\cosh(f*x + e)^7 - 14*(128*a^3 - 504*a^2*b + 600*a*b^2 \\
& - 225*b^3)*\cosh(f*x + e)^5 + 20*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*\c \\
& osh(f*x + e)^3 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*\cosh(f*x + e)) \\
& *\sinh(f*x + e)^3 + 8*a^2*b - 24*a*b^2 + 15*b^3 + 2*(16*a^3 - 72*a^2*b + 102 \\
& *a*b^2 - 45*b^3)*\cosh(f*x + e)^2 + 2*(33*(8*a^2*b - 24*a*b^2 + 15*b^3)*\cosh \\
& (f*x + e)^{10} + 45*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*\cosh(f*x + e)^8 \\
& - 14*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*\cosh(f*x + e)^6 + 30*(48*a \\
& ^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*\cosh(f*x + e)^4 + 16*a^3 - 72*a^2*b + \\
& 102*a*b^2 - 45*b^3 - 3*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*\cosh(f*x \\
& + e)^2)*\sinh(f*x + e)^2 + 4*(3*(8*a^2*b - 24*a*b^2 + 15*b^3)*\cosh(f*x + e) \\
& ^{11} + 5*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*\cosh(f*x + e)^9 - 2*(128*a \\
& ^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*\cosh(f*x + e)^7 + 6*(48*a^3 - 184*a^2 \\
& *b + 210*a*b^2 - 75*b^3)*\cosh(f*x + e)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2 \\
& - 225*b^3)*\cosh(f*x + e)^3 + (16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*\cosh \\
& (f*x + e))*\sinh(f*x + e))*\sqrt{-a}*\arctan(1/2*\sqrt{2}*\sqrt{-a}*\sqrt{(b*\cosh \\
& (f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + \\
& e)*\sinh(f*x + e) + \sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*\sinh(f*x + e))) + \\
& 2*\sqrt{2}*((8*a^3 - 24*a^2*b + 15*a*b^2)*\cosh(f*x + e)^9 + 9*(8*a^3 - 24*a
\end{aligned}$$


```
4*b)*f*cosh(f*x + e)^9 - 2*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^7 + 6*(6*a^5
- 5*a^4*b)*f*cosh(f*x + e)^5 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^3 + (2*
a^5 - 3*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^5}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

$$3.496 \quad \int \frac{\tanh^4(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=275

$$\frac{(3a+5b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f(a-b)^3\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}} - \frac{4a\tanh(e+fx)}{3f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}} +$$

[Out] $-(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*(7*a+b)*\operatorname{Cosh}[e+f*x]*\operatorname{EllipticE}[\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[e+f*x])/\operatorname{Sqrt}[a]],1-a/b])/((3*(a-b)^3*f*\operatorname{Sqrt}[(a*\operatorname{Cosh}[e+f*x]^2)/(a+b*\operatorname{Sinh}[e+f*x]^2)]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])) + ((3*a+5*b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+f*x]],1-b/a]*\operatorname{Sech}[e+f*x]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])/((3*(a-b)^3*f*\operatorname{Sqrt}[(\operatorname{Sech}[e+f*x]^2*(a+b*\operatorname{Sinh}[e+f*x]^2))/a])) - (4*a*\operatorname{Tanh}[e+f*x])/((3*(a-b)^2*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])) + (\operatorname{Sech}[e+f*x]^2*\operatorname{Tanh}[e+f*x])/((3*(a-b)*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2]))$

Rubi [A] time = 0.275938, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3196, 470, 527, 525, 418, 411}

$$-\frac{4a\tanh(e+fx)}{3f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}} + \frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)}{3f(a-b)\sqrt{a+b\sinh^2(e+fx)}} - \frac{\sqrt{a}\sqrt{b}(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right)}{3f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[e+f*x]^4/(a+b*\operatorname{Sinh}[e+f*x]^2)^{(3/2)},x]$

[Out] $-(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*(7*a+b)*\operatorname{Cosh}[e+f*x]*\operatorname{EllipticE}[\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[e+f*x])/\operatorname{Sqrt}[a]],1-a/b])/((3*(a-b)^3*f*\operatorname{Sqrt}[(a*\operatorname{Cosh}[e+f*x]^2)/(a+b*\operatorname{Sinh}[e+f*x]^2)]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])) + ((3*a+5*b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+f*x]],1-b/a]*\operatorname{Sech}[e+f*x]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])/((3*(a-b)^3*f*\operatorname{Sqrt}[(\operatorname{Sech}[e+f*x]^2*(a+b*\operatorname{Sinh}[e+f*x]^2))/a])) - (4*a*\operatorname{Tanh}[e+f*x])/((3*(a-b)^2*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])) + (\operatorname{Sech}[e+f*x]^2*\operatorname{Tanh}[e+f*x])/((3*(a-b)*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2]))$

Rule 3196

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 470

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_)*(x_)^2)/(Sqrt[(a_) + (b_)*(x_)^2]*((c_) + (d_)*(x_)^2)^(3/2)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
```

d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))], x] /; FreeQ
 [{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx = \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)^{5/2}(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a-3ax^2}{(1+x^2)^{3/2}(a+bx^2)} dx, x, \sinh(e+fx)\right)}{3(a-b)f}$$

$$= -\frac{4a\tanh(e+fx)}{3(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{ab(7a+b)\sqrt{\cos}}{(1+x^2)^{3/2}(a+bx^2)} dx, x, \sinh(e+fx)\right)}{3(a-b)f}$$

$$= -\frac{4a\tanh(e+fx)}{3(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(ab(7a+b)\sqrt{\cos}\right) \operatorname{Subst}\left(\int \frac{1}{(1+x^2)^{3/2}(a+bx^2)} dx, x, \sinh(e+fx)\right)}{3(a-b)f}$$

$$= -\frac{\sqrt{a}\sqrt{b}(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\left|1-\frac{a}{b}\right.\right)}{3(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a+5b)F\left(\tan^{-1}(\sinh(e+fx))\right)}{3(a-b)}$$

Mathematica [C] time = 2.1843, size = 212, normalized size = 0.77

$$\frac{8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)(4(4a^2+3ab+b^2)\cosh(2(e+fx))+8a^2+b(7a+b)\cosh(4(e+fx)))}{2\sqrt{2}}}{6f(a-b)^3\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((-2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] - ((8*a^2 + 21*a*b - 5*b^2 + 4*(4*a^2 + 3*a*b + b^2)*Cosh[2*(e + f*x)] + b*(7*a + b)*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e

+ f*x])/(2*sqrt[2]))/(6*(a - b)^3*f*sqrt[2*a - b + b*Cosh[2*(e + f*x)])]

Maple [A] time = 0.204, size = 354, normalized size = 1.3

$$-\frac{1}{3 (\cosh(fx + e))^3 (a - b)^3 f} \left(\left(7 \sqrt{\frac{-b}{a}} ab + \sqrt{\frac{-b}{a}} b^2 \right) \sinh(fx + e) (\cosh(fx + e))^4 + \left(4 \sqrt{\frac{-b}{a}} a^2 - 4 \sqrt{\frac{-b}{a}} ab \right) (\cosh(fx + e))^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2), x)

[Out]
$$-1/3 * ((7 * (-1/a*b)^{(1/2)} * a*b + (-1/a*b)^{(1/2)} * b^2) * \sinh(f*x+e) * \cosh(f*x+e)^4 + (4 * (-1/a*b)^{(1/2)} * a^2 - 4 * (-1/a*b)^{(1/2)} * a*b) * \cosh(f*x+e)^2 * \sinh(f*x+e) + (-(-1/a*b)^{(1/2)} * a^2 + 2 * (-1/a*b)^{(1/2)} * a*b - (-1/a*b)^{(1/2)} * b^2) * \sinh(f*x+e) - (b/a * \cosh(f*x+e)^2 + (a-b)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * (3 * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a^2 - 2 * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a*b - \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^2 + 7 * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a*b + \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^2) * \cosh(f*x+e)^2) / \cosh(f*x+e)^3 / (-1/a*b)^{(1/2)} / (a-b)^3 / (a+b * \sinh(f*x+e)^2)^{(1/2)} / f$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^4}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2), x, algorithm="maxima")

[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \tanh^4(fx + e)}{b^2 \sinh^4(fx + e) + 2ab \sinh^2(fx + e) + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Integral(tanh(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(fx + e)}{(b \sinh^2(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.497 \quad \int \frac{\tanh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=217

$$\frac{(a+b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{af(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}-\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\sqrt{a+b\sinh^2(e+fx)}}{af(a-b)^2}$$

[Out] $(-2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*\operatorname{Cosh}[e+f*x]*\operatorname{EllipticE}[\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[e+f*x])/ \operatorname{Sqrt}[a]],1-a/b])/((a-b)^2*f*\operatorname{Sqrt}[(a*\operatorname{Cosh}[e+f*x]^2)/(a+b*\operatorname{Sinh}[e+f*x]^2)]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])+(a+b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+f*x]],1-b/a]*\operatorname{Sech}[e+f*x]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])/(a*(a-b)^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e+f*x]^2*(a+b*\operatorname{Sinh}[e+f*x]^2))/a])-\operatorname{Tanh}[e+f*x]/((a-b)*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])$

Rubi [A] time = 0.212858, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3196, 471, 525, 418, 411}

$$-\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\left|1-\frac{a}{b}\right.\right)}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{(a+b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[e+f*x]^2/(a+b*\operatorname{Sinh}[e+f*x]^2)^{(3/2)},x]$

[Out] $(-2*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[b]*\operatorname{Cosh}[e+f*x]*\operatorname{EllipticE}[\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[e+f*x])/ \operatorname{Sqrt}[a]],1-a/b])/((a-b)^2*f*\operatorname{Sqrt}[(a*\operatorname{Cosh}[e+f*x]^2)/(a+b*\operatorname{Sinh}[e+f*x]^2)]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])+(a+b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+f*x]],1-b/a]*\operatorname{Sech}[e+f*x]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])/(a*(a-b)^2*f*\operatorname{Sqrt}[(\operatorname{Sech}[e+f*x]^2*(a+b*\operatorname{Sinh}[e+f*x]^2))/a])-\operatorname{Tanh}[e+f*x]/((a-b)*f*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+f*x]^2])$

Rule 3196

$\operatorname{Int}[(a_+ + (b_+)*\sin[(e_+) + (f_+)*(x_+)]^2)^{(p_+)}*\tan[(e_+) + (f_+)*(x_+)]^m, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\operatorname{Sin}[e + f*x], x]\}, \operatorname{Dist}[(ff^m +$

1)*Sqrt[Cos[e + f*x]^2]/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 471

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 525

Int[((e_) + (f_)*(x_)^2)/(Sqrt[(a_) + (b_)*(x_)^2]*((c_) + (d_)*(x_)^2)^(3/2)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] && PosQ[d/c]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 411

Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)^{3/2}(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a-bx^2}{\sqrt{1+x^2}(a+bx^2)} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
&= -\frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2ab\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{(a+bx^2)} dx, x, \sinh(e+fx)\right)}{(a-b)(-a+b)f} \\
&= -\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\left|1-\frac{a}{b}\right.\right)}{(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a+b)F\left(\tan^{-1}(\sinh(e+fx))\right)}{a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 1.28214, size = 158, normalized size = 0.73

$$\frac{i\sqrt{2}(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - 2\tanh(e+fx)(a+b\cosh(2(e+fx))) - 2i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))}{a}}}{f(a-b)^2\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] ((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticF[I*(e + f*x), b/a] - 2*(a + b*Cosh[2*(e + f*x)])*Tanh[e + f*x]/((a - b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])

Maple [A] time = 0.185, size = 256, normalized size = 1.2

$$-\frac{1}{(a-b)^2\cosh(fx+e)f}\left(2\sqrt{-\frac{b}{a}}(\sinh(fx+e))^3 - a\sqrt{\frac{a+b(\sinh(fx+e))^2}{a}}\sqrt{(\cosh(fx+e))^2}\operatorname{EllipticF}\left(\sinh(fx+e), \frac{b}{a}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] $-(2*(-1/a*b)^{(1/2)}*b*\sinh(f*x+e)^3-a*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})+b*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-2*b*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})+(-1/a*b)^{(1/2)}*a*\sinh(f*x+e)+b*\sinh(f*x+e)*(-1/a*b)^{(1/2)})/(a-b)^2/(-1/a*b)^{(1/2)}/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^2}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sinh(fx + e)^2 + a} \tanh(fx + e)^2}{b^2 \sinh(fx + e)^4 + 2ab \sinh(fx + e)^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Integral(tanh(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(fx + e)}{(b \sinh^2(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.498 \quad \int \frac{1}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=115

$$\frac{b \sinh(e+fx) \cosh(e+fx)}{af(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{i\sqrt{a+b \sinh^2(e+fx)}E\left(ie+ifx \left| \frac{b}{a} \right. \right)}{af(a-b)\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1}}$$

[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rubi [A] time = 0.067648, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$, Rules used = {3184, 21, 3178, 3177}

$$\frac{b \sinh(e+fx) \cosh(e+fx)}{af(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{i\sqrt{a+b \sinh^2(e+fx)}E\left(ie+ifx \left| \frac{b}{a} \right. \right)}{af(a-b)\sqrt{\frac{b \sinh^2(e+fx)}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2), x]

[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sinh[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a + b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sinh[e + f*x]^2)^(p + 1)*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 21

```
Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :=
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)]^2], x_Symbol] := Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \sinh^2(e + fx))^{3/2}} dx &= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} - \frac{\int \frac{-a - b \sinh^2(e + fx)}{\sqrt{a + b \sinh^2(e + fx)}} dx}{a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} + \frac{\int \sqrt{a + b \sinh^2(e + fx)} dx}{a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} + \frac{\sqrt{a + b \sinh^2(e + fx)} \int \sqrt{1 + \frac{b \sinh^2(e + fx)}{a}} dx}{a(a - b)\sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{a(a - b)f\sqrt{a + b \sinh^2(e + fx)}} - \frac{iE\left(ie + ifx \left| \frac{b}{a} \right. \right) \sqrt{a + b \sinh^2(e + fx)}}{a(a - b)f\sqrt{1 + \frac{b \sinh^2(e + fx)}{a}}}
\end{aligned}$$

Mathematica [A] time = 0.149781, size = 100, normalized size = 0.87

$$\frac{-\sqrt{2}b \sinh(2(e + fx)) - 2ia\sqrt{\frac{2a + b \cosh(2(e + fx)) - b}{a}} E\left(ie + ifx \left| \frac{b}{a} \right. \right)}{2af(a - b)\sqrt{2a + b \cosh(2(e + fx))} - b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]

[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])]/a)*EllipticE[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])

Maple [A] time = 0., size = 252, normalized size = 2.2

$$\frac{1}{a(a-b)\cosh(fx+e)f} \left(-\sqrt{\frac{b}{a}} b \sinh(fx+e) (\cosh(fx+e))^2 + a \sqrt{\frac{b(\cosh(fx+e))^2}{a} + \frac{a-b}{a}} \sqrt{(\cosh(fx+e))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^2+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b/a/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b \sinh(fx+e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh (fx + e)^2 + a}}{b^2 \sinh (fx + e)^4 + 2 ab \sinh (fx + e)^2 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh (fx + e)^2 + a\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)

$$3.499 \quad \int \frac{\coth^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=237

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{a^2 f \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{2 \tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{a^2 f} - \frac{2 \coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{a^2 f}$$

```
[Out] Coth[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*Coth[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(a^2*f) - (2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]
*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a
+ b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sec
h[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2
*f)
```

Rubi [A] time = 0.270323, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3196, 469, 583, 531, 418, 492, 411}

$$\frac{2 \tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{a^2 f} - \frac{2 \coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{a^2 f} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{a^2 f \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2), x]
```

```
[Out] Coth[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*Coth[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(a^2*f) - (2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]
*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a
+ b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sec
h[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2
*f)
```

Rule 3196


```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 469

```
Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
```

$a*d, 0] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c] \ \&\& \ \text{!SimplerSqrtQ}[b/a, d/c]$

Rule 411

$\text{Int}[\text{Sqrt}[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^{(3/2)}, x_Symbol] \ :> \ \text{Simp}[(\text{Sqrt}[a + b*x^2]*\text{EllipticE}[\text{ArcTan}[\text{Rt}[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*\text{Rt}[d/c, 2]*\text{Sqrt}[c + d*x^2]*\text{Sqrt}[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] \ /; \ \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{PosQ}[b/a] \ \&\& \ \text{PosQ}[d/c]$

Rubi steps

$$\begin{aligned} \int \frac{\coth^2(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\text{sech}(e+fx)}\right) \text{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\ &= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)\text{sech}(e+fx)}\right) \text{Subst}\left(\int \frac{-2-x^2}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x\right)}{af} \\ &= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \frac{\left(\sqrt{\cosh^2(e+fx)\text{sech}(e+fx)}\right) \text{Subst}\left(\int \frac{-2-x^2}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x\right)}{af} \\ &= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \frac{\left(\sqrt{\cosh^2(e+fx)\text{sech}(e+fx)}\right) \text{Subst}\left(\int \frac{-2-x^2}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}} dx, x\right)}{af} \\ &= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \frac{F\left(\tan^{-1}(\sinh(e+fx)), \frac{b}{a}\right)}{a^2f} \\ &= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} - \frac{2E\left(\tan^{-1}(\sinh(e+fx)), \frac{b}{a}\right)}{a^2f} \end{aligned}$$

Mathematica [C] time = 0.835849, size = 153, normalized size = 0.65

$$\frac{i\sqrt{2a}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - 2\coth(e+fx)(a+b\cosh(2(e+fx))-b) - 2i\sqrt{2a}\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}{a^2f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]

[Out] $(-2*(a - b + b*\text{Cosh}[2*(e + f*x)])*\text{Coth}[e + f*x] - (2*I)*\text{Sqrt}[2]*a*\text{Sqrt}[(2*a - b + b*\text{Cosh}[2*(e + f*x)])]/a)*\text{EllipticE}[I*(e + f*x), b/a] + I*\text{Sqrt}[2]*a*\text{Sqrt}[(2*a - b + b*\text{Cosh}[2*(e + f*x)])]/a*\text{EllipticF}[I*(e + f*x), b/a]/(a^2*f*\text{Sqrt}[4*a - 2*b + 2*b*\text{Cosh}[2*(e + f*x)]])$

Maple [A] time = 0.282, size = 219, normalized size = 0.9

$$-\frac{1}{\sinh(fx + e) a^2 \cosh(fx + e) f} \left(-\sinh(fx + e) \sqrt{(\cosh(fx + e))^2} \sqrt{\frac{b(\cosh(fx + e))^2}{a} + \frac{a-b}{a}} \left(a \text{EllipticF} \left(\sinh \right. \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] $(-\sinh(f*x+e)*(\cosh(f*x+e)^2)^{(1/2)}*(b/a*\cosh(f*x+e)^2+(a-b)/a)^{(1/2)}*(a*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-2*b*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})+2*b*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)}))+2*(-1/a*b)^{(1/2)}*b*\cosh(f*x+e)^4+((-1/a*b)^{(1/2)}*a-2*(-1/a*b)^{(1/2)}*b)*\cosh(f*x+e)^2)/(-1/a*b)^{(1/2)}/\sinh(f*x+e)/a^2/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^2}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sinh^2(fx + e) + a} \coth^2(fx + e)}{b^2 \sinh^4(fx + e) + 2ab \sinh^2(fx + e) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(e + fx)}{(a + b \sinh^2(e + fx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)`

[Out] `Integral(coth(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)`

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(fx + e)}{(b \sinh^2(fx + e) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")`

[Out] `integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)`

$$3.500 \quad \int \frac{\coth^4(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx$$

Optimal. Leaf size=341

$$\frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3a^3f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{(7a-8b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^3f}$$

```
[Out] -(((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) - (((7*a - 8*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f) + (
(3*a - 4*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a
^2*b*f) - (((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*
Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(
a + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh
[e + f*x])/(3*a^3*f)
```

Rubi [A] time = 0.40835, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$, Rules used = {3196, 468, 583, 531, 418, 492, 411}

$$\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^3f} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^3f} + \frac{(3a-4b)\coth(e+fx)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^3f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) - (((7*a - 8*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f) + (
(3*a - 4*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a
^2*b*f) - (((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*
Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(
a + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh
[e + f*x])/(3*a^3*f)
```

$(e + f*x)/(3*a^3*f)$

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 468

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e*x)^(m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 583

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^(n*(m + 1))), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
  :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
  + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
  a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
  p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
  d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
  [{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{3/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{3a-4b+(1+x^2)}{x^4\sqrt{1+x^2}} dx, x, \sinh(e+fx)\right)}{abf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a-4b)\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2bf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f} + \frac{(3a-4b)\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2bf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f} + \frac{(3a-4b)\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2bf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f} + \frac{(3a-4b)\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2bf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f} + \frac{(3a-4b)\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2bf}
\end{aligned}$$

Mathematica [C] time = 3.21993, size = 214, normalized size = 0.63

$$\frac{8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)(4(4a^2-11ab+8b^2)\cosh(2(e+fx))-8a^2+b(7a-8b)\cosh(4(e+fx)))}{2\sqrt{2}}}{6a^3f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2), x]

[Out] (-((-8*a^2 + 37*a*b - 24*b^2 + 4*(4*a^2 - 11*a*b + 8*b^2)*Cosh[2*(e + f*x)] + (7*a - 8*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/(2*sqrt[


```
2]) - (2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE
[I*(e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a
]*EllipticF[I*(e + f*x), b/a])/(6*a^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
)
```

Maple [A] time = 0.195, size = 522, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(7*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-8*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6
-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*sinh(f*x+e)^3+11*b*((a+b*sinh(f*x+e)^2)/a
)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1
/2))*a*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2*sinh(f*x+e)^3-7*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*
b)^(1/2), (a/b)^(1/2))*a*b*sinh(f*x+e)^3+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2), (a/b)^(1/2))*b^2*si
nh(f*x+e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+3*(-1/a*b)^(1/2)*a*b*sinh(f*
x+e)^4-8*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^
2-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2)/(-1/a*b)^(1/2)/sin
h(f*x+e)^3/a^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^4}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh(fx + e)^2 + a} \coth(fx + e)^4}{b^2 \sinh(fx + e)^4 + 2ab \sinh(fx + e)^2 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x + e)^2 + a^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^4}{(b \sinh(fx + e)^2 + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)

$$3.501 \quad \int \frac{\tanh^5(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{5/2}} dx$$

Optimal. Leaf size=232

$$\frac{8a^2 + 24ab + 3b^2}{8f(a-b)^4 \sqrt{a+b \sinh^2(e+fx)}} + \frac{8a^2 + 24ab + 3b^2}{24f(a-b)^3 (a+b \sinh^2(e+fx))^{3/2}} - \frac{(8a^2 + 24ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{9/2}}$$

[Out] -((8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(8*(a - b)^(9/2)*f) + (8*a^2 + 24*a*b + 3*b^2)/(24*(a - b)^3*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((8*a - b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - Sech[e + f*x]^4/(4*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (8*a^2 + 24*a*b + 3*b^2)/(8*(a - b)^4*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.315846, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2 + 24ab + 3b^2}{8f(a-b)^4 \sqrt{a+b \sinh^2(e+fx)}} + \frac{8a^2 + 24ab + 3b^2}{24f(a-b)^3 (a+b \sinh^2(e+fx))^{3/2}} - \frac{(8a^2 + 24ab + 3b^2) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] -((8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(8*(a - b)^(9/2)*f) + (8*a^2 + 24*a*b + 3*b^2)/(24*(a - b)^3*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((8*a - b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - Sech[e + f*x]^4/(4*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (8*a^2 + 24*a*b + 3*b^2)/(8*(a - b)^4*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^(m

+ 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1)/2)], x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 89

Int[((a_.) + (b_.)*(x_))^(2*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.))*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_.))*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.))*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{sech}^4(e+fx)}{4(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a-3b)+2(a-b)x}{(1+x)^2(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{4(a-b)f} \\
&= \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{sech}^4(e+fx)}{4(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(8a^2+24ab)}{24(a-b)^3f(a+b\sinh^2(e+fx))^{3/2}} \\
&= \frac{8a^2+24ab+3b^2}{24(a-b)^3f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{1}{4(a-b)f} \\
&= \frac{8a^2+24ab+3b^2}{24(a-b)^3f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{1}{4(a-b)f} \\
&= \frac{8a^2+24ab+3b^2}{24(a-b)^3f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{1}{4(a-b)f} \\
&= \frac{8a^2+24ab+3b^2}{24(a-b)^3f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{1}{4(a-b)f} \\
&= -\frac{(8a^2+24ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{9/2}f} + \frac{8a^2+24ab+3b^2}{24(a-b)^3f(a+b\sinh^2(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.58949, size = 114, normalized size = 0.49

$$\frac{2(8a^2+24ab+3b^2) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b\sinh^2(e+fx)+a}{a-b}\right) + 3(a-b)\text{sech}^4(e+fx)((8a-b)\cosh(2(e+fx)) + 4a + 3b)}{48f(a-b)^3(a+b\sinh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] $(2*(8*a^2 + 24*a*b + 3*b^2)*\text{Hypergeometric2F1}[-3/2, 1, -1/2, (a + b*\text{Sinh}[e + f*x]^2)/(a - b)] + 3*(a - b)*(4*a + 3*b + (8*a - b)*\text{Cosh}[2*(e + f*x)])*\text{Sech}[e + f*x]^4)/(48*(a - b)^3*f*(a + b*\text{Sinh}[e + f*x]^2)^{(3/2)})$

Maple [C] time = 0.298, size = 213, normalized size = 0.9

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^5 (b^2 (\sinh(fx + e))^2 + a^2)}{-b^4 (\cosh(fx + e))^{18} + (-4ab^3 + 4b^4) (\cosh(fx + e))^{16} + (-6a^2b^2 + 12ab^3 - 6b^4) (\cosh(fx + e))^{14} + (-4a^3b + 12a^2b^2 - 12ab^3 + 4b^4) (\cosh(fx + e))^{12} + (-a^4 + 4a^3b - 6a^2b^2 + 4ab^3 - b^4) (\cosh(fx + e))^{10}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)`

[Out] `\int/indef0`(-sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)*cosh(f*x+e)^4/(-b^4*cosh(f*x+e)^18+(-4*a*b^3+4*b^4)*cosh(f*x+e)^16+(-6*a^2*b^2+12*a*b^3-6*b^4)*cosh(f*x+e)^14+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cosh(f*x+e)^12+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^10)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^5}{(b \sinh(fx + e)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)`

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^5}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

$$3.502 \quad \int \frac{\tanh^3(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=163

$$\frac{2a+3b}{2f(a-b)^3 \sqrt{a+b \sinh^2(e+fx)}} + \frac{2a+3b}{6f(a-b)^2 (a+b \sinh^2(e+fx))^{3/2}} - \frac{(2a+3b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}} + \frac{s}{2f(a-b)}$$

[Out] $-\left(\frac{(2a+3b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right]}{2(a-b)^{7/2} f} + \frac{(2a+3b)}{6(a-b)^2 f (a+b \sinh^2(e+fx))^{3/2}} + \operatorname{Sech}\left[\frac{e+fx}{2}\right] \frac{1}{2(a-b) f (a+b \sinh^2(e+fx))^{3/2}} + \frac{(2a+3b)}{2(a-b)^3 f \sqrt{a+b \sinh^2(e+fx)}}\right)$

Rubi [A] time = 0.165729, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a+3b}{2f(a-b)^3 \sqrt{a+b \sinh^2(e+fx)}} + \frac{2a+3b}{6f(a-b)^2 (a+b \sinh^2(e+fx))^{3/2}} - \frac{(2a+3b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}} + \frac{s}{2f(a-b)}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[e+fx]^3 / (a+b \sinh^2(e+fx))^{5/2}, x]$

[Out] $-\left(\frac{(2a+3b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right]}{2(a-b)^{7/2} f} + \frac{(2a+3b)}{6(a-b)^2 f (a+b \sinh^2(e+fx))^{3/2}} + \operatorname{Sech}\left[\frac{e+fx}{2}\right] \frac{1}{2(a-b) f (a+b \sinh^2(e+fx))^{3/2}} + \frac{(2a+3b)}{2(a-b)^3 f \sqrt{a+b \sinh^2(e+fx)}}\right)$

Rule 3194

$\operatorname{Int}[(a_+ + (b_+ \sin[e_+ + (f_+)(x_+)]^2)^{p_+}) \tan[e_+ + (f_+)(x_+)]^{m_+}, x_Symbol] \rightarrow \operatorname{With}\{ff = \operatorname{FreeFactors}[\sin[e+fx]^2, x]\}, \operatorname{Dist}[ff^{((m+1)/2)/(2f)}, \operatorname{Subst}[\operatorname{Int}[(x^{((m-1)/2})(a+bffx)^p] / (1-ffx)^{(m+1)/2}), x], x, \sin[e+fx]^2/ff], x] \;/; \operatorname{FreeQ}\{a, b, e, f, p\}, x\} \&\& \operatorname{IntegerQ}[(m-1)/2]$

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^3(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= \frac{\text{sech}^2(e+fx)}{2(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{4(a-b)f} \\
&= \frac{2a+3b}{6(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{(2a+3b)\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{4(a-b)f} \\
&= \frac{2a+3b}{6(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{2}{2(a-b)^3f\sqrt{a-b}} \\
&= \frac{2a+3b}{6(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{2}{2(a-b)^3f\sqrt{a-b}} \\
&= -\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f} + \frac{2a+3b}{6(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} + \frac{2}{2(a-b)f\sqrt{a-b}}
\end{aligned}$$

Mathematica [C] time = 0.122481, size = 82, normalized size = 0.5

$$\frac{(2a+3b) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b\sinh^2(e+fx)+a}{a-b}\right) + 3(a-b)\text{sech}^2(e+fx)}{6f(a-b)^2(a+b\sinh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ((2*a + 3*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sinh[e + f*x]^2)/(a - b)] + 3*(a - b)*Sech[e + f*x]^2)/(6*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2))

Maple [C] time = 0.266, size = 213, normalized size = 1.3

$$\frac{1}{f} \int \frac{(\sinh(fx + e))^3 (b^2 (\sinh(fx + e))^3 + \dots)}{-b^4 (\cosh(fx + e))^{14} + (-4ab^3 + 4b^4) (\cosh(fx + e))^{12} + (-6a^2b^2 + 12ab^3 - 6b^4) (\cosh(fx + e))^{10} + \dots} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] `int/indef0` (-sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)*cosh(f*x+e)^2/(-b^4*cosh(f*x+e)^14+(-4*a*b^3+4*b^4)*cosh(f*x+e)^12+(-6*a^2*b^2+12*a*b^3-6*b^4)*cosh(f*x+e)^10+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cosh(f*x+e)^8+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^6)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^3}{(b \sinh(fx + e)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [B] time = 6.75029, size = 24677, normalized size = 151.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [-1/12*(3*((2*a*b^2 + 3*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 + 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (2*a*b^2 + 3*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b

$$\begin{aligned}
& + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^{10} + 2*(8*a^2*b + 10*a*b^2 - 3*b^3 + 33*(\\
& 2*a*b^2 + 3*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^{10} + 20*(11*(2*a*b^2 + 3*b^ \\
& 3)*\cosh(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e))*\sinh(f*x + \\
& e)^9 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^8 + (495*(2*a*b \\
& ^2 + 3*b^3)*\cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3 + 90*(8*a \\
& ^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^8 + 8*(99*(2*a*b^2 \\
& + 3*b^3)*\cosh(f*x + e)^5 + 30*(8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^3 \\
& + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^7 + 4* \\
& (16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*\cosh(f*x + e)^6 + 4*(231*(2*a*b^2 + \\
& 3*b^3)*\cosh(f*x + e)^6 + 105*(8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^4 + \\
& 16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3 + 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3* \\
& b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 8*(99*(2*a*b^2 + 3*b^3)*\cosh(f*x + \\
& e)^7 + 63*(8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^5 + 7*(32*a^3 + 48*a^2 \\
& *b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^3 + 3*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3 \\
& *b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3 \\
&)*\cosh(f*x + e)^4 + (495*(2*a*b^2 + 3*b^3)*\cosh(f*x + e)^8 + 420*(8*a^2*b + \\
& 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^6 + 70*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^ \\
& 3)*\cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3 + 60*(16*a^3 + 16* \\
& a^2*b - 10*a*b^2 + 3*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(55*(2*a*b^2 \\
& + 3*b^3)*\cosh(f*x + e)^9 + 60*(8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^7 \\
& + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^5 + 20*(16*a^3 + \\
& 16*a^2*b - 10*a*b^2 + 3*b^3)*\cosh(f*x + e)^3 + (32*a^3 + 48*a^2*b - 2*a*b^2 \\
& - 3*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 2*a*b^2 + 3*b^3 + 2*(8*a^2*b + 1 \\
& 0*a*b^2 - 3*b^3)*\cosh(f*x + e)^2 + 2*(33*(2*a*b^2 + 3*b^3)*\cosh(f*x + e)^10 \\
& + 45*(8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^8 + 14*(32*a^3 + 48*a^2*b \\
& - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^6 + 30*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b \\
& ^3)*\cosh(f*x + e)^4 + 8*a^2*b + 10*a*b^2 - 3*b^3 + 3*(32*a^3 + 48*a^2*b - 2 \\
& *a*b^2 - 3*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 4*(3*(2*a*b^2 + 3*b^3)*c \\
& osh(f*x + e)^11 + 5*(8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^9 + 2*(32*a^ \\
& 3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^7 + 6*(16*a^3 + 16*a^2*b - 10 \\
& *a*b^2 + 3*b^3)*\cosh(f*x + e)^5 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cos \\
& h(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e))*\sinh(f*x + e))*s \\
& \text{qrt}(a - b)*\log((b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*s \\
& \text{inh}(f*x + e)^4 + 2*(4*a - 3*b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 4 \\
& *a - 3*b)*\sinh(f*x + e)^2 + 4*\text{sqrt}(2)*\text{sqrt}(a - b)*\text{sqrt}((b*\cosh(f*x + e)^2 + \\
& b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + \\
& e) + \sinh(f*x + e)^2))*(\cosh(f*x + e) + \sinh(f*x + e)) + 4*(b*\cosh(f*x + e \\
&)^3 + (4*a - 3*b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^4 + 4*co \\
& sh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*s \\
& \text{inh}(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\text{si} \\
& \text{nh}(f*x + e) + 1)) - 4*\text{sqrt}(2)*(3*(2*a^2*b + a*b^2 - 3*b^3)*\cosh(f*x + e)^9 \\
& + 27*(2*a^2*b + a*b^2 - 3*b^3)*\cosh(f*x + e)*\sinh(f*x + e)^8 + 3*(2*a^2*b + \\
& a*b^2 - 3*b^3)*\sinh(f*x + e)^9 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*co \\
& sh(f*x + e)^7 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 27*(2*a^2*b + a*b^2 \\
& - 3*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^7 + 28*(9*(2*a^2*b + a*b^2 - 3*b^3
\end{aligned}$$

$$\begin{aligned}
&)\cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)\cosh(f*x + e)*\sinh(f*x + e)^6 + 2*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)\cosh(f*x + e)^5 + 2 \\
& *(189*(2*a^2*b + a*b^2 - 3*b^3)\cosh(f*x + e)^4 + 56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3 + 42*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)\cosh(f*x + e)^2)*\sinh \\
& (f*x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)\cosh(f*x + e)^5 + 70*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)\cosh(f*x + e)^3 + 5*(56*a^3 - 70*a^2*b + 17*a \\
& *b^2 - 3*b^3)\cosh(f*x + e))*\sinh(f*x + e)^4 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)\cosh(f*x + e)^3 + 4*(63*(2*a^2*b + a*b^2 - 3*b^3)\cosh(f*x + e)^ \\
& 6 + 35*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)\cosh(f*x + e)^4 + 8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 5*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)\cosh(f*x + \\
& e)^2)*\sinh(f*x + e)^3 + 4*(27*(2*a^2*b + a*b^2 - 3*b^3)\cosh(f*x + e)^7 + 21*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)\cosh(f*x + e)^5 + 5*(56*a^3 - 70*a^ \\
& 2*b + 17*a*b^2 - 3*b^3)\cosh(f*x + e)^3 + 3*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)\cosh(f*x + e))*\sinh(f*x + e)^2 + 3*(2*a^2*b + a*b^2 - 3*b^3)\cosh(f*x \\
& + e) + (27*(2*a^2*b + a*b^2 - 3*b^3)\cosh(f*x + e)^8 + 28*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)\cosh(f*x + e)^6 + 10*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3 \\
& *b^3)\cosh(f*x + e)^4 + 6*a^2*b + 3*a*b^2 - 9*b^3 + 12*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)\cosh(f*x + e)^2)*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + \\
& b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)* \\
& f*\cosh(f*x + e)^12 + 12*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f \\
& *\cosh(f*x + e)*\sinh(f*x + e)^11 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\sinh(f*x + e)^12 + 2*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2 \\
& *b^4 + 8*a*b^5 - b^6)*f*\cosh(f*x + e)^10 + 2*(33*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b^2 + 28*a^3* \\
& b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f)*\sinh(f*x + e)^10 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*\cosh(f*x + e)^8 + \\
& 20*(11*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^3 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*\cosh(\\
& f*x + e))*\sinh(f*x + e)^9 + (495*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^4 + 90*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2* \\
& b^4 + 8*a*b^5 - b^6)*f*\cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*\sinh(f*x + e)^8 + 4*(8*a^6 - 36 \\
& *a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*\cosh(f*x + e)^6 + 8*(99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x \\
& + e)^5 + 30*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*\cosh(f*x + e)^3 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2 \\
& *b^4 + 4*a*b^5 - b^6)*f*\cosh(f*x + e))*\sinh(f*x + e)^7 + 4*(231*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^6 + 105*(4*a^5*b - 1 \\
& 7*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*\cosh(f*x + e)^4 + 7* \\
& (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)* \\
& f*\cosh(f*x + e)^2 + (8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f)*\sinh(f*x + e)^6 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 6 \\
& 0*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*\cosh(f*x + e)^4 + 8*(99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^7 + 63*(4*a^5*b -
\end{aligned}$$

$$\begin{aligned}
& 17a^4b^2 + 28a^3b^3 - 22a^2b^4 + 8ab^5 - b^6) * f * \cosh(fx + e)^5 + 7 \\
& * (16a^6 - 64a^5b + 95a^4b^2 - 60a^3b^3 + 10a^2b^4 + 4ab^5 - b^6) \\
& * f * \cosh(fx + e)^3 + 3 * (8a^6 - 36a^5b + 65a^4b^2 - 60a^3b^3 + 30a^2 \\
& * b^4 - 8ab^5 + b^6) * f * \cosh(fx + e) * \sinh(fx + e)^5 + (495 * (a^4b^2 - 4 \\
& a^3b^3 + 6a^2b^4 - 4ab^5 + b^6) * f * \cosh(fx + e)^8 + 420 * (4a^5b - 17 \\
& a^4b^2 + 28a^3b^3 - 22a^2b^4 + 8ab^5 - b^6) * f * \cosh(fx + e)^6 + 70 * (\\
& 16a^6 - 64a^5b + 95a^4b^2 - 60a^3b^3 + 10a^2b^4 + 4ab^5 - b^6) * f \\
& * \cosh(fx + e)^4 + 60 * (8a^6 - 36a^5b + 65a^4b^2 - 60a^3b^3 + 30a^2 * \\
& b^4 - 8ab^5 + b^6) * f * \cosh(fx + e)^2 + (16a^6 - 64a^5b + 95a^4b^2 - \\
& 60a^3b^3 + 10a^2b^4 + 4ab^5 - b^6) * f) * \sinh(fx + e)^4 + 2 * (4a^5b - \\
& 17a^4b^2 + 28a^3b^3 - 22a^2b^4 + 8ab^5 - b^6) * f * \cosh(fx + e)^2 + 4 \\
& * (55 * (a^4b^2 - 4a^3b^3 + 6a^2b^4 - 4ab^5 + b^6) * f * \cosh(fx + e)^9 + \\
& 60 * (4a^5b - 17a^4b^2 + 28a^3b^3 - 22a^2b^4 + 8ab^5 - b^6) * f * \cosh(\\
& fx + e)^7 + 14 * (16a^6 - 64a^5b + 95a^4b^2 - 60a^3b^3 + 10a^2b^4 + \\
& 4ab^5 - b^6) * f * \cosh(fx + e)^5 + 20 * (8a^6 - 36a^5b + 65a^4b^2 - 60 \\
& a^3b^3 + 30a^2b^4 - 8ab^5 + b^6) * f * \cosh(fx + e)^3 + (16a^6 - 64a^5 * \\
& b + 95a^4b^2 - 60a^3b^3 + 10a^2b^4 + 4ab^5 - b^6) * f * \cosh(fx + e) * \\
& \sinh(fx + e)^3 + 2 * (33 * (a^4b^2 - 4a^3b^3 + 6a^2b^4 - 4ab^5 + b^6) * f \\
& * \cosh(fx + e)^10 + 45 * (4a^5b - 17a^4b^2 + 28a^3b^3 - 22a^2b^4 + 8 \\
& ab^5 - b^6) * f * \cosh(fx + e)^8 + 14 * (16a^6 - 64a^5b + 95a^4b^2 - 60a^ \\
& 3b^3 + 10a^2b^4 + 4ab^5 - b^6) * f * \cosh(fx + e)^6 + 30 * (8a^6 - 36a^5 * \\
& b + 65a^4b^2 - 60a^3b^3 + 30a^2b^4 - 8ab^5 + b^6) * f * \cosh(fx + e)^4 \\
& + 3 * (16a^6 - 64a^5b + 95a^4b^2 - 60a^3b^3 + 10a^2b^4 + 4ab^5 - \\
& b^6) * f * \cosh(fx + e)^2 + (4a^5b - 17a^4b^2 + 28a^3b^3 - 22a^2b^4 + \\
& 8ab^5 - b^6) * f) * \sinh(fx + e)^2 + (a^4b^2 - 4a^3b^3 + 6a^2b^4 - 4a * \\
& b^5 + b^6) * f + 4 * (3 * (a^4b^2 - 4a^3b^3 + 6a^2b^4 - 4ab^5 + b^6) * f * \cos \\
& h(fx + e)^11 + 5 * (4a^5b - 17a^4b^2 + 28a^3b^3 - 22a^2b^4 + 8ab^5 \\
& - b^6) * f * \cosh(fx + e)^9 + 2 * (16a^6 - 64a^5b + 95a^4b^2 - 60a^3b^3 \\
& + 10a^2b^4 + 4ab^5 - b^6) * f * \cosh(fx + e)^7 + 6 * (8a^6 - 36a^5b + 65 \\
& a^4b^2 - 60a^3b^3 + 30a^2b^4 - 8ab^5 + b^6) * f * \cosh(fx + e)^5 + (16 \\
& a^6 - 64a^5b + 95a^4b^2 - 60a^3b^3 + 10a^2b^4 + 4ab^5 - b^6) * f * \cos \\
& h(fx + e)^3 + (4a^5b - 17a^4b^2 + 28a^3b^3 - 22a^2b^4 + 8ab^5 - \\
& b^6) * f * \cosh(fx + e) * \sinh(fx + e)), -1/6 * (3 * ((2ab^2 + 3b^3) * \cosh(fx \\
& + e)^12 + 12 * (2ab^2 + 3b^3) * \cosh(fx + e) * \sinh(fx + e)^11 + (2ab^2 + \\
& 3b^3) * \sinh(fx + e)^12 + 2 * (8a^2b + 10ab^2 - 3b^3) * \cosh(fx + e)^10 + \\
& 2 * (8a^2b + 10ab^2 - 3b^3 + 33 * (2ab^2 + 3b^3) * \cosh(fx + e)^2) * \sinh \\
& (fx + e)^10 + 20 * (11 * (2ab^2 + 3b^3) * \cosh(fx + e)^3 + (8a^2b + 10ab \\
& ^2 - 3b^3) * \cosh(fx + e)) * \sinh(fx + e)^9 + (32a^3 + 48a^2b - 2ab^2 - \\
& 3b^3) * \cosh(fx + e)^8 + (495 * (2ab^2 + 3b^3) * \cosh(fx + e)^4 + 32a^3 + \\
& 48a^2b - 2ab^2 - 3b^3 + 90 * (8a^2b + 10ab^2 - 3b^3) * \cosh(fx + e) \\
& ^2) * \sinh(fx + e)^8 + 8 * (99 * (2ab^2 + 3b^3) * \cosh(fx + e)^5 + 30 * (8a^2b \\
& + 10ab^2 - 3b^3) * \cosh(fx + e)^3 + (32a^3 + 48a^2b - 2ab^2 - 3b^3) \\
&) * \cosh(fx + e)) * \sinh(fx + e)^7 + 4 * (16a^3 + 16a^2b - 10ab^2 + 3b^3) \\
& * \cosh(fx + e)^6 + 4 * (231 * (2ab^2 + 3b^3) * \cosh(fx + e)^6 + 105 * (8a^2b \\
& + 10ab^2 - 3b^3) * \cosh(fx + e)^4 + 16a^3 + 16a^2b - 10ab^2 + 3b^3
\end{aligned}$$

$$\begin{aligned}
& + 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^2*\sinh(f*x + e)^6 \\
& + 8*(99*(2*a*b^2 + 3*b^3)*\cosh(f*x + e)^7 + 63*(8*a^2*b + 10*a*b^2 - 3*b^3) \\
& *\cosh(f*x + e)^5 + 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^3 \\
& + 3*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + \\
& (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^4 + (495*(2*a*b^2 + 3* \\
& b^3)*\cosh(f*x + e)^8 + 420*(8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^6 + 7 \\
& 0*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b \\
& - 2*a*b^2 - 3*b^3 + 60*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*\cosh(f*x + e \\
&)^2)*\sinh(f*x + e)^4 + 4*(55*(2*a*b^2 + 3*b^3)*\cosh(f*x + e)^9 + 60*(8*a^2*b \\
& b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^7 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3 \\
& *b^3)*\cosh(f*x + e)^5 + 20*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*\cosh(f*x \\
& + e)^3 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e))*\sinh(f*x + e) \\
& ^3 + 2*a*b^2 + 3*b^3 + 2*(8*a^2*b + 10*a*b^2 - 3*b^3)*\cosh(f*x + e)^2 + 2*(\\
& 33*(2*a*b^2 + 3*b^3)*\cosh(f*x + e)^10 + 45*(8*a^2*b + 10*a*b^2 - 3*b^3)*\cos \\
& h(f*x + e)^8 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^6 + 3 \\
& 0*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*\cosh(f*x + e)^4 + 8*a^2*b + 10*a*b \\
& ^2 - 3*b^3 + 3*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^2)*\sinh(\\
& f*x + e)^2 + 4*(3*(2*a*b^2 + 3*b^3)*\cosh(f*x + e)^11 + 5*(8*a^2*b + 10*a*b^ \\
& 2 - 3*b^3)*\cosh(f*x + e)^9 + 2*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f \\
& *x + e)^7 + 6*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*\cosh(f*x + e)^5 + (32* \\
& a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*\cosh(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3 \\
& *b^3)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-a + b}*\arctan(-1/2*\sqrt{2}*\sqrt{- \\
& a + b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e \\
&)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)})/((a - b)*\cosh(f*x + \\
& e) + (a - b)*\sinh(f*x + e))) - 2*\sqrt{2}*(3*(2*a^2*b + a*b^2 - 3*b^3)*\cosh \\
& (f*x + e)^9 + 27*(2*a^2*b + a*b^2 - 3*b^3)*\cosh(f*x + e)*\sinh(f*x + e)^8 + \\
& 3*(2*a^2*b + a*b^2 - 3*b^3)*\sinh(f*x + e)^9 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 \\
& + 3*b^3)*\cosh(f*x + e)^7 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 27*(2*a \\
& ^2*b + a*b^2 - 3*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^7 + 28*(9*(2*a^2*b + a \\
& *b^2 - 3*b^3)*\cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*\cosh(f \\
& *x + e))*\sinh(f*x + e)^6 + 2*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*\cosh(f* \\
& x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*\cosh(f*x + e)^4 + 56*a^3 - 70*a \\
& ^2*b + 17*a*b^2 - 3*b^3 + 42*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*\cosh(f*x \\
& + e)^2)*\sinh(f*x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*\cosh(f*x + e)^5 \\
& + 70*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*\cosh(f*x + e)^3 + 5*(56*a^3 - 70* \\
& a^2*b + 17*a*b^2 - 3*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^4 + 4*(8*a^3 + 2*a^2 \\
& *b - 13*a*b^2 + 3*b^3)*\cosh(f*x + e)^3 + 4*(63*(2*a^2*b + a*b^2 - 3*b^3)*\co \\
& sh(f*x + e)^6 + 35*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*\cosh(f*x + e)^4 + 8 \\
& *a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 5*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3 \\
&)*\cosh(f*x + e)^2)*\sinh(f*x + e)^3 + 4*(27*(2*a^2*b + a*b^2 - 3*b^3)*\cosh(f \\
& *x + e)^7 + 21*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*\cosh(f*x + e)^5 + 5*(56 \\
& *a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*\cosh(f*x + e)^3 + 3*(8*a^3 + 2*a^2*b - \\
& 13*a*b^2 + 3*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^2 + 3*(2*a^2*b + a*b^2 - 3*b \\
& ^3)*\cosh(f*x + e) + (27*(2*a^2*b + a*b^2 - 3*b^3)*\cosh(f*x + e)^8 + 28*(8*a \\
& ^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*\cosh(f*x + e)^6 + 10*(56*a^3 - 70*a^2*b +
\end{aligned}$$

$$\begin{aligned}
& 17*a*b^2 - 3*b^3)*\cosh(f*x + e)^4 + 6*a^2*b + 3*a*b^2 - 9*b^3 + 12*(8*a^3 + \\
& 2*a^2*b - 13*a*b^2 + 3*b^3)*\cosh(f*x + e)^2*\sinh(f*x + e)*\sqrt{(b*\cosh(f* \\
& *x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e) \\
& *\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a \\
& *b^5 + b^6)*f*\cosh(f*x + e)^{12} + 12*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a* \\
& b^5 + b^6)*f*\cosh(f*x + e)*\sinh(f*x + e)^{11} + (a^4*b^2 - 4*a^3*b^3 + 6*a^2* \\
& b^4 - 4*a*b^5 + b^6)*f*\sinh(f*x + e)^{12} + 2*(4*a^5*b - 17*a^4*b^2 + 28*a^3* \\
& b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*\cosh(f*x + e)^{10} + 2*(33*(a^4*b^2 - 4*a \\
& ^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b \\
& ^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f)*\sinh(f*x + e)^{10} + (16*a^6 \\
& - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*\cosh(\\
& f*x + e)^8 + 20*(11*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cos \\
& h(f*x + e)^3 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - \\
& b^6)*f*\cosh(f*x + e))*\sinh(f*x + e)^9 + (495*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b \\
& ^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^4 + 90*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b \\
& ^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*\cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 9 \\
& 5*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*\sinh(f*x + e)^8 + 4 \\
& *(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)* \\
& f*\cosh(f*x + e)^6 + 8*(99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6) \\
& *f*\cosh(f*x + e)^5 + 30*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8 \\
& *a*b^5 - b^6)*f*\cosh(f*x + e)^3 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3* \\
& b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*\cosh(f*x + e))*\sinh(f*x + e)^7 + 4*(231 \\
& *(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^6 + 105* \\
& (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*\cosh(f*x \\
& + e)^4 + 7*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a \\
& *b^5 - b^6)*f*\cosh(f*x + e)^2 + (8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 \\
& + 30*a^2*b^4 - 8*a*b^5 + b^6)*f)*\sinh(f*x + e)^6 + (16*a^6 - 64*a^5*b + 95 \\
& *a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*\cosh(f*x + e)^4 + 8*(\\
& 99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^7 + 63 \\
& *(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*\cosh(f* \\
& x + e)^5 + 7*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4* \\
& a*b^5 - b^6)*f*\cosh(f*x + e)^3 + 3*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3* \\
& b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + (495*(\\
& a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f*x + e)^8 + 420*(4 \\
& *a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*\cosh(f*x + \\
& e)^6 + 70*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a* \\
& b^5 - b^6)*f*\cosh(f*x + e)^4 + 60*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b \\
& ^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*\cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 9 \\
& 5*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*\sinh(f*x + e)^4 + 2 \\
& *(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*\cosh(f* \\
& x + e)^2 + 4*(55*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*\cosh(f \\
& *x + e)^9 + 60*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - \\
& b^6)*f*\cosh(f*x + e)^7 + 14*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + \\
& 10*a^2*b^4 + 4*a*b^5 - b^6)*f*\cosh(f*x + e)^5 + 20*(8*a^6 - 36*a^5*b + 65*a \\
& ^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*\cosh(f*x + e)^3 + (16*a
\end{aligned}$$


```

^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cos
h(f*x + e))*sinh(f*x + e)^3 + 2*(33*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*
b^5 + b^6)*f*cosh(f*x + e)^10 + 45*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*
a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^8 + 14*(16*a^6 - 64*a^5*b + 95*a^4
*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^6 + 30*(8*a
^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cos
h(f*x + e)^4 + 3*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4
+ 4*a*b^5 - b^6)*f*cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 2
2*a^2*b^4 + 8*a*b^5 - b^6)*f)*sinh(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^
2*b^4 - 4*a*b^5 + b^6)*f + 4*(3*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
+ b^6)*f*cosh(f*x + e)^11 + 5*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b
^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^9 + 2*(16*a^6 - 64*a^5*b + 95*a^4*b^2 -
60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^7 + 6*(8*a^6 - 36
*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x +
e)^5 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5
- b^6)*f*cosh(f*x + e)^3 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4
+ 8*a*b^5 - b^6)*f*cosh(f*x + e))*sinh(f*x + e))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)^3}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)

$$3.503 \quad \int \frac{\tanh(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{5/2}} dx$$

Optimal. Leaf size=99

$$\frac{1}{f(a-b)^2 \sqrt{a+b \sinh^2(e+fx)}} + \frac{1}{3f(a-b) \left(a+b \sinh^2(e+fx)\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2]/\text{Sqrt}[a - b]]/((a - b)^{(5/2)*f})) + 1/(3*(a - b)*f*(a + b*\text{Sinh}[e + f*x]^2)^{(3/2)}) + 1/((a - b)^2*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Rubi [A] time = 0.0929154, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 51, 63, 208}

$$\frac{1}{f(a-b)^2 \sqrt{a+b \sinh^2(e+fx)}} + \frac{1}{3f(a-b) \left(a+b \sinh^2(e+fx)\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Tanh}[e + f*x]/(a + b*\text{Sinh}[e + f*x]^2)^{(5/2)}, x]$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2]/\text{Sqrt}[a - b]]/((a - b)^{(5/2)*f})) + 1/(3*(a - b)*f*(a + b*\text{Sinh}[e + f*x]^2)^{(3/2)}) + 1/((a - b)^2*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Rule 3194

$\text{Int}[(a + (b \cdot \sin[(e + f \cdot x)]^2)^{p}) \cdot \tan[(e + f \cdot x)]^{m}, x_{\text{Symbol}}] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\sin[e + f \cdot x]^2, x]\}, \text{Dist}[ff^{((m + 1)/2)/(2 \cdot f)}, \text{Subst}[\text{Int}[(x^{((m - 1)/2}) \cdot (a + b \cdot ff \cdot x)^p]/(1 - ff \cdot x)^{(m + 1)/2}), x], x, \sin[e + f \cdot x]^2/ff], x]] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \ \&\& \ \text{IntegerQ}[(m - 1)/2]$

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= \frac{1}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}} dx, x, \sinh^2(e+fx)\right)}{2(a-b)f} \\
&= \frac{1}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}} dx, x, \sinh^2(e+fx)\right)}{2(a-b)f} \\
&= \frac{1}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{1}{2}bx} dx, x, \sinh^2(e+fx)\right)}{2(a-b)f} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{1}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{1}{(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.0865591, size = 60, normalized size = 0.61

$$\frac{{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b \cosh^2(e+fx)}{a-b} + 1\right)}{3f(a-b)\left(a + b \cosh^2(e+fx) - b\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))

Maple [C] time = 0.218, size = 173, normalized size = 1.8

$$\frac{1}{f} \int \frac{\sinh(fx + e) \left(b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^3 + (-4ab^3 - b^4) (\sinh(fx + e))^2 + (-6a^2b^2 - 4ab^3) (\sinh(fx + e)) + (-4a^3b - 6a^2b^2) \right)}{-b^4 (\sinh(fx + e))^{10} + (-4ab^3 - b^4) (\sinh(fx + e))^8 + (-6a^2b^2 - 4ab^3) (\sinh(fx + e))^6 + (-4a^3b - 6a^2b^2) (\sinh(fx + e))^4 + (-a^4 - 4a^3b) (\sinh(fx + e))^2 - a^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2), x)

[Out] `int/indef0` (-sinh(f*x+e)*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(-b^4*sinh(f*x+e)^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*x+e)^6+(-4*a^3*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4)/(a+b*sinh(f*x+e)^2)^(1/2), sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2), x, algorithm="maxima")

[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [B] time = 3.47963, size = 9900, normalized size = 100.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/6*(3*(b^2*\cosh(f*x + e)^8 + 8*b^2*\cosh(f*x + e)*\sinh(f*x + e)^7 + b^2*\sinh(f*x + e)^8 + 4*(2*a*b - b^2)*\cosh(f*x + e)^6 + 4*(7*b^2*\cosh(f*x + e)^2 + 2*a*b - b^2)*\sinh(f*x + e)^6 + 8*(7*b^2*\cosh(f*x + e)^3 + 3*(2*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^4 + 2*(35*b^2*\cosh(f*x + e)^4 + 30*(2*a*b - b^2)*\cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*\sinh(f*x + e)^4 + 8*(7*b^2*\cosh(f*x + e)^5 + 10*(2*a*b - b^2)*\cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 4*(2*a*b - b^2)*\cosh(f*x + e)^2 + 4*(7*b^2*\cosh(f*x + e)^6 + 15*(2*a*b - b^2)*\cosh(f*x + e)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^2 + 2*a*b - b^2)*\sinh(f*x + e)^2 + b^2 + 8*(b^2*\cosh(f*x + e)^7 + 3*(2*a*b - b^2)*\cosh(f*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)*\cosh(f*x + e)^3 + (2*a*b - b^2)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{a - b}*\log((b*\cosh(f*x + e)^4 + 4*b*\cosh(f*x + e)*\sinh(f*x + e)^3 + b*\sinh(f*x + e)^4 + 2*(4*a - 3*b)*\cosh(f*x + e)^2 + 2*(3*b*\cosh(f*x + e)^2 + 4*a - 3*b)*\sinh(f*x + e)^2 - 4*\sqrt{2}*\sqrt{a - b}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)}*(\cosh(f*x + e) + \sinh(f*x + e)) + 4*(b*\cosh(f*x + e)^3 + (4*a - 3*b)*\cosh(f*x + e))*\sinh(f*x + e) + b)/(\cosh(f*x + e)^4 + 4*\cosh(f*x + e)*\sinh(f*x + e)^3 + \sinh(f*x + e)^4 + 2*(3*\cosh(f*x + e)^2 + 1)*\sinh(f*x + e)^2 + 2*\cosh(f*x + e)^2 + 4*(\cosh(f*x + e)^3 + \cosh(f*x + e))*\sinh(f*x + e) + 1)) + 4*\sqrt{2}*(3*(a*b - b^2)*\cosh(f*x + e)^5 + 15*(a*b - b^2)*\cosh(f*x + e)*\sinh(f*x + e)^4 + 3*(a*b - b^2)*\sinh(f*x + e)^5 + 2*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e)^3 + 2*(15*(a*b - b^2)*\cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*\sinh(f*x + e)^3 + 6*(5*(a*b - b^2)*\cosh(f*x + e)^3 + (8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^2 + 3*(a*b - b^2)*\cosh(f*x + e) + 3*(5*(a*b - b^2)*\cosh(f*x + e)^4 + 2*(8*a^2 - 13*a*b + 5*b^2)*\cosh(f*x + e)^2 + a*b - b^2)*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*\cosh(f*x + e)^8 + 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*\cosh(f*x + e)*\sinh(f*x + e)^7 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*\sinh(f*x + e)^8 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*\cosh(f*x + e)^6 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*\cosh(f*x + e)^2 + (2*a^4$$

$$\begin{aligned}
& *b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*\sinh(f*x + e)^6 + 2*(8*a^5 - \\
& 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*\cosh(f*x + e)^4 + \\
& 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*\cosh(f*x + e)^3 + 3*(2*a^4*b \\
& - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + \\
& 2*(35*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*\cosh(f*x + e)^4 + 30*(2*a^4*b \\
& b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*\cosh(f*x + e)^2 + (8*a^5 - 32* \\
& a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f)*\sinh(f*x + e)^4 + 4* \\
& (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*\cosh(f*x + e)^2 + 8*(7* \\
& (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*\cosh(f*x + e)^5 + 10*(2*a^4*b - 7*a \\
& ^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*\cosh(f*x + e)^3 + (8*a^5 - 32*a^4*b + \\
& 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*\cosh(f*x + e))*\sinh(f*x + e) \\
& ^3 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*\cosh(f*x + e)^6 + 15*(2*a \\
& ^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*\cosh(f*x + e)^4 + 3*(8*a^5 \\
& - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*\cosh(f*x + e)^2 \\
& + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*\sinh(f*x + e)^2 + (a \\
& ^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f + 8*((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - \\
& b^5)*f*\cosh(f*x + e)^7 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^ \\
& 5)*f*\cosh(f*x + e)^5 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b \\
& ^4 - 3*b^5)*f*\cosh(f*x + e)^3 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 \\
& + b^5)*f*\cosh(f*x + e))*\sinh(f*x + e)), -1/3*(3*(b^2*cosh(f*x + e)^8 + 8*b^ \\
& 2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*sinh(f*x + e)^8 + 4*(2*a*b - b^2)*cos \\
& h(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^6 + 8* \\
& (7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2 \\
& *(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(\\
& 2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(\\
& 7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + \\
& 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a*b - b^2)*cosh(f*x + e)^2 + \\
& 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8* \\
& a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2* \\
& cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2) \\
& *cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b) \\
& *arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e) \\
& ^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + \\
& e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*(3*(a* \\
& b - b^2)*cosh(f*x + e)^5 + 15*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + 3 \\
& *(a*b - b^2)*sinh(f*x + e)^5 + 2*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^3 + \\
& 2*(15*(a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^ \\
& 3 + 6*(5*(a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + \\
& e))*sinh(f*x + e)^2 + 3*(a*b - b^2)*cosh(f*x + e) + 3*(5*(a*b - b^2)*cosh(f \\
& *x + e)^4 + 2*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 + a*b - b^2)*sinh(f* \\
& x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + \\
& e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b^2 - 3*a^2 \\
& *b^3 + 3*a*b^4 - b^5)*f*\cosh(f*x + e)^8 + 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 \\
& - b^5)*f*\sinh(f*x + e)^8 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)
\end{aligned}$$

```

*f*cosh(f*x + e)^6 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x
+ e)^2 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(f*x + e)
^6 + 2*(8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*co
sh(f*x + e)^4 + 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^
3 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*si
nh(f*x + e)^5 + 2*(35*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)
^4 + 30*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2
+ (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f)*sinh(
f*x + e)^4 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x
+ e)^2 + 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^5 + 10
*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (8*a
^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)
)*sinh(f*x + e)^3 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x +
e)^6 + 15*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)
)^4 + 3*(8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*c
osh(f*x + e)^2 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(
f*x + e)^2 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f + 8*((a^3*b^2 - 3*a^2*
b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^7 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3
- 5*a*b^4 + b^5)*f*cosh(f*x + e)^5 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a
^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^3 + (2*a^4*b - 7*a^3*b^2 + 9*a^2
*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e))]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx + e)}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```


$$3.504 \quad \int \frac{\coth(e+fx)}{\left(a+b \sinh^2(e+fx)\right)^{5/2}} dx$$

Optimal. Leaf size=83

$$\frac{1}{a^2 f \sqrt{a+b \sinh^2(e+fx)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2} f} + \frac{1}{3af (a+b \sinh^2(e+fx))^{3/2}}$$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2]/\text{Sqrt}[a]]/(a^{(5/2)*f})) + 1/(3*a*f*(a + b*\text{Sinh}[e + f*x]^2)^{(3/2)}) + 1/(a^2*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Rubi [A] time = 0.0995702, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 51, 63, 208}

$$\frac{1}{a^2 f \sqrt{a+b \sinh^2(e+fx)}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2} f} + \frac{1}{3af (a+b \sinh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[e + f*x]/(a + b*\text{Sinh}[e + f*x]^2)^{(5/2)}, x]$

[Out] $-(\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2]/\text{Sqrt}[a]]/(a^{(5/2)*f})) + 1/(3*a*f*(a + b*\text{Sinh}[e + f*x]^2)^{(3/2)}) + 1/(a^2*f*\text{Sqrt}[a + b*\text{Sinh}[e + f*x]^2])$

Rule 3194

$\text{Int}[(a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_)]^2]^{(p_.)}*\tan[(e_.) + (f_.)*(x_)]^{(m_.)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f*x]^2, x]\}, \text{Dist}[ff^{(m+1)/2}/(2*f), \text{Subst}[\text{Int}[(x^{(m-1)/2}*(a + b*ff*x)^p]/(1 - ff*x)^{(m+1)/2}, x], x, \text{Sin}[e + f*x]^2/ff], x]] /; \text{FreeQ}[\{a, b, e, f, p\}, x] \&\& \text{IntegerQ}[(m-1)/2]$

Rule 51

$\text{Int}[(a_.) + (b_.)*(x_)]^{(m_.)}*((c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(a + b*x)^{(m+1)}*(c + d*x)^{(n+1)}/((b*c - a*d)*(m+1)), x] - \text{Dist}[(d*($

```
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(e + fx)}{(a + b \sinh^2(e + fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}} dx, x, \sinh^2(e + fx)\right)}{2f} \\
&= \frac{1}{3af(a + b \sinh^2(e + fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, \sinh^2(e + fx)\right)}{2af} \\
&= \frac{1}{3af(a + b \sinh^2(e + fx))^{3/2}} + \frac{1}{a^2f\sqrt{a + b \sinh^2(e + fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \sinh^2(e + fx)\right)}{2a^2f} \\
&= \frac{1}{3af(a + b \sinh^2(e + fx))^{3/2}} + \frac{1}{a^2f\sqrt{a + b \sinh^2(e + fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^2(e + fx)}\right)}{a^2bf} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af(a + b \sinh^2(e + fx))^{3/2}} + \frac{1}{a^2f\sqrt{a + b \sinh^2(e + fx)}}
\end{aligned}$$

Mathematica [C] time = 0.0653222, size = 49, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b \sinh^2(e+fx)}{a} + 1\right)}{3af(a + b \sinh^2(e + fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2))

Maple [C] time = 0.086, size = 65, normalized size = 0.8

$$\frac{1}{f} \int \frac{1}{\left(b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^2 + a^2\right) \sinh(fx + e) \sqrt{a + b (\sinh(fx + e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2), x)

[Out] `int/indef0` (1/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2), sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)}{\left(b \sinh(fx + e)^2 + a\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2), x, algorithm="maxima")

[Out] integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)

$$\begin{aligned}
& f*x + e)^3 + 4*(7*a^3*b^2*f*cosh(f*x + e)^6 + 15*(2*a^4*b - a^3*b^2)*f*cosh \\
& (f*x + e)^4 + 3*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (2*a^4*b \\
& - a^3*b^2)*f)*sinh(f*x + e)^2 + 8*(a^3*b^2*f*cosh(f*x + e)^7 + 3*(2*a^4*b - \\
& a^3*b^2)*f*cosh(f*x + e)^5 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e) \\
& ^3 + (2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/3*(3*(b^2*cosh(\\
& f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*sinh(f*x + e)^8 + 4* \\
& (2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sin \\
& h(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e))*si \\
& nh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*b^2*cosh(\\
& f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh \\
& (f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)*cosh(f*x + e)^3 + \\
& (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a*b - b^2)*c \\
& osh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b^2)*cosh(f*x + e)^ \\
& 4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^ \\
& 2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f*x + e)^5 + (8*a^2 \\
& - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + \\
& e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(\\
& f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si \\
& nh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt(2)*(3*a*b*cos \\
& h(f*x + e)^5 + 15*a*b*cosh(f*x + e)*sinh(f*x + e)^4 + 3*a*b*sinh(f*x + e)^5 \\
& + 2*(8*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(15*a*b*cosh(f*x + e)^2 + 8*a^2 - \\
& 3*a*b)*sinh(f*x + e)^3 + 3*a*b*cosh(f*x + e) + 6*(5*a*b*cosh(f*x + e)^3 + (\\
& 8*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(5*a*b*cosh(f*x + e)^4 + \\
& 2*(8*a^2 - 3*a*b)*cosh(f*x + e)^2 + a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + \\
& e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh \\
& (f*x + e) + sinh(f*x + e)^2)))/(a^3*b^2*f*cosh(f*x + e)^8 + 8*a^3*b^2*f*cos \\
& h(f*x + e)*sinh(f*x + e)^7 + a^3*b^2*f*sinh(f*x + e)^8 + 4*(2*a^4*b - a^3*b \\
& ^2)*f*cosh(f*x + e)^6 + 4*(7*a^3*b^2*f*cosh(f*x + e)^2 + (2*a^4*b - a^3*b^2 \\
&)*f)*sinh(f*x + e)^6 + a^3*b^2*f + 2*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f \\
& *x + e)^4 + 8*(7*a^3*b^2*f*cosh(f*x + e)^3 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f \\
& *x + e))*sinh(f*x + e)^5 + 2*(35*a^3*b^2*f*cosh(f*x + e)^4 + 30*(2*a^4*b - \\
& a^3*b^2)*f*cosh(f*x + e)^2 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f)*sinh(f*x + e) \\
& ^4 + 4*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^3*b^2*f*cosh(f*x + e) \\
& ^5 + 10*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^3 + (8*a^5 - 8*a^4*b + 3*a^3*b^ \\
& 2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*b^2*f*cosh(f*x + e)^6 + 15*(\\
& 2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*co \\
& sh(f*x + e)^2 + (2*a^4*b - a^3*b^2)*f)*sinh(f*x + e)^2 + 8*(a^3*b^2*f*cosh(\\
& f*x + e)^7 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^5 + (8*a^5 - 8*a^4*b + 3 \\
& *a^3*b^2)*f*cosh(f*x + e)^3 + (2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x \\
& + e))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.505 \quad \int \frac{\coth^3(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=143

$$\frac{2a-5b}{2a^3 f \sqrt{a+b \sinh^2(e+fx)}} + \frac{2a-5b}{6a^2 f (a+b \sinh^2(e+fx))^{3/2}} - \frac{(2a-5b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2} f} - \frac{\operatorname{csch}^2(e+fx)}{2af (a+b \sinh^2(e+fx))}$$

[Out] -((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) + (2*a - 5*b)/(6*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - Csch[e + f*x]^2/(2*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a - 5*b)/(2*a^3*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rubi [A] time = 0.152358, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-5b}{2a^3 f \sqrt{a+b \sinh^2(e+fx)}} + \frac{2a-5b}{6a^2 f (a+b \sinh^2(e+fx))^{3/2}} - \frac{(2a-5b) \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2} f} - \frac{\operatorname{csch}^2(e+fx)}{2af (a+b \sinh^2(e+fx))}$$

Antiderivative was successfully verified.

[In] Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]

[Out] -((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) + (2*a - 5*b)/(6*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - Csch[e + f*x]^2/(2*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a - 5*b)/(2*a^3*f*Sqrt[a + b*Sinh[e + f*x]^2])

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^(m + 1)]/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{1+x}{x^2(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{csch}^2(e+fx)}{2af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(2a-5b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{4af} \\
&= \frac{2a-5b}{6a^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{csch}^2(e+fx)}{2af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(2a-5b)\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{4af} \\
&= \frac{2a-5b}{6a^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{csch}^2(e+fx)}{2af(a+b\sinh^2(e+fx))^{3/2}} + \frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+fx)}} \\
&= \frac{2a-5b}{6a^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{csch}^2(e+fx)}{2af(a+b\sinh^2(e+fx))^{3/2}} + \frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+fx)}} \\
&= -\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} + \frac{2a-5b}{6a^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{csch}^2(e+fx)}{2af(a+b\sinh^2(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.30026, size = 69, normalized size = 0.48

$$\frac{(5b-2a) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b\sinh^2(e+fx)}{a} + 1\right) + 3a\text{csch}^2(e+fx)}{6a^2f(a+b\sinh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] -(3*a*Csch[e + f*x]^2 + (-2*a + 5*b)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sinh[e + f*x]^2)/a])/(6*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2))

Maple [C] time = 0.118, size = 73, normalized size = 0.5

$$\frac{1}{f} \int \frac{(\cosh(fx+e))^2}{(b^2(\sinh(fx+e))^4 + 2ab(\sinh(fx+e))^2 + a^2)(\sinh(fx+e))^3 \sqrt{a+b(\sinh(fx+e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] `int/indef0` (cosh(f*x+e)^2/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx+e)^3}{(b \sinh(fx+e)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [B] time = 4.73291, size = 18669, normalized size = 130.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] [-1/12*(3*((2*a*b^2 - 5*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 - 5*b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (2*a*b^2 - 5*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^10 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3 + 33*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f

$$\begin{aligned}
& x + e)^9 + (32a^3 - 144a^2b + 190ab^2 - 75b^3) \cosh(fx + e)^8 + (495 \\
& * (2ab^2 - 5b^3) \cosh(fx + e)^4 + 32a^3 - 144a^2b + 190ab^2 - 75b^3 \\
& + 90(8a^2b - 26ab^2 + 15b^3) \cosh(fx + e)^2) \sinh(fx + e)^8 + 8(\\
& 99(2ab^2 - 5b^3) \cosh(fx + e)^5 + 30(8a^2b - 26ab^2 + 15b^3) \cos \\
& h(fx + e)^3 + (32a^3 - 144a^2b + 190ab^2 - 75b^3) \cosh(fx + e)) \sin \\
& h(fx + e)^7 - 4(16a^3 - 64a^2b + 70ab^2 - 25b^3) \cosh(fx + e)^6 + \\
& 4(231(2ab^2 - 5b^3) \cosh(fx + e)^6 + 105(8a^2b - 26ab^2 + 15b^3) \\
&) \cosh(fx + e)^4 - 16a^3 + 64a^2b - 70ab^2 + 25b^3 + 7(32a^3 - 144 \\
& a^2b + 190ab^2 - 75b^3) \cosh(fx + e)^2) \sinh(fx + e)^6 + 8(99(2a \\
& b^2 - 5b^3) \cosh(fx + e)^7 + 63(8a^2b - 26ab^2 + 15b^3) \cosh(fx + \\
& e)^5 + 7(32a^3 - 144a^2b + 190ab^2 - 75b^3) \cosh(fx + e)^3 - 3(16 \\
& a^3 - 64a^2b + 70ab^2 - 25b^3) \cosh(fx + e)) \sinh(fx + e)^5 + (32a^ \\
& 3 - 144a^2b + 190ab^2 - 75b^3) \cosh(fx + e)^4 + (495(2ab^2 - 5b^3) \\
&) \cosh(fx + e)^8 + 420(8a^2b - 26ab^2 + 15b^3) \cosh(fx + e)^6 + 70 \\
& (32a^3 - 144a^2b + 190ab^2 - 75b^3) \cosh(fx + e)^4 + 32a^3 - 144a^ \\
& 2b + 190ab^2 - 75b^3 - 60(16a^3 - 64a^2b + 70ab^2 - 25b^3) \cosh(\\
& fx + e)^2) \sinh(fx + e)^4 + 4(55(2ab^2 - 5b^3) \cosh(fx + e)^9 + 60 \\
& (8a^2b - 26ab^2 + 15b^3) \cosh(fx + e)^7 + 14(32a^3 - 144a^2b + 19 \\
& 0ab^2 - 75b^3) \cosh(fx + e)^5 - 20(16a^3 - 64a^2b + 70ab^2 - 25b \\
& ^3) \cosh(fx + e)^3 + (32a^3 - 144a^2b + 190ab^2 - 75b^3) \cosh(fx + \\
& e)) \sinh(fx + e)^3 + 2ab^2 - 5b^3 + 2(8a^2b - 26ab^2 + 15b^3) \cos \\
& h(fx + e)^2 + 2(33(2ab^2 - 5b^3) \cosh(fx + e)^10 + 45(8a^2b - 26 \\
& ab^2 + 15b^3) \cosh(fx + e)^8 + 14(32a^3 - 144a^2b + 190ab^2 - 75b \\
& ^3) \cosh(fx + e)^6 - 30(16a^3 - 64a^2b + 70ab^2 - 25b^3) \cosh(fx + \\
& e)^4 + 8a^2b - 26ab^2 + 15b^3 + 3(32a^3 - 144a^2b + 190ab^2 - 7 \\
& 5b^3) \cosh(fx + e)^2) \sinh(fx + e)^2 + 4(3(2ab^2 - 5b^3) \cosh(fx + \\
& e)^11 + 5(8a^2b - 26ab^2 + 15b^3) \cosh(fx + e)^9 + 2(32a^3 - 144 \\
& a^2b + 190ab^2 - 75b^3) \cosh(fx + e)^7 - 6(16a^3 - 64a^2b + 70ab \\
& ^2 - 25b^3) \cosh(fx + e)^5 + (32a^3 - 144a^2b + 190ab^2 - 75b^3) \co \\
& sh(fx + e)^3 + (8a^2b - 26ab^2 + 15b^3) \cosh(fx + e)) \sinh(fx + e)) \\
& * \sqrt{a} * \log((b \cosh(fx + e))^4 + 4b \cosh(fx + e) \sinh(fx + e)^3 + b \sin \\
& h(fx + e)^4 + 2(4a - b) \cosh(fx + e)^2 + 2(3b \cosh(fx + e)^2 + 4a - \\
& b) \sinh(fx + e)^2 + 4\sqrt{2} \sqrt{a} \sqrt{(b \cosh(fx + e))^2 + b \sinh(fx \\
& + e)^2 + 2a - b} / (\cosh(fx + e)^2 - 2 \cosh(fx + e) \sinh(fx + e) + \sinh \\
& (fx + e)^2)) * (\cosh(fx + e) + \sinh(fx + e)) + 4(b \cosh(fx + e))^3 + (4a \\
& - b) \cosh(fx + e) \sinh(fx + e) + b) / (\cosh(fx + e)^4 + 4 \cosh(fx + e) \\
& \sinh(fx + e)^3 + \sinh(fx + e)^4 + 2(3 \cosh(fx + e)^2 - 1) \sinh(fx + e) \\
& ^2 - 2 \cosh(fx + e)^2 + 4(\cosh(fx + e)^3 - \cosh(fx + e)) \sinh(fx + e) \\
& + 1) - 4\sqrt{2} * (3(2a^2b - 5ab^2) \cosh(fx + e)^9 + 27(2a^2b - 5 \\
& ab^2) \cosh(fx + e) \sinh(fx + e)^8 + 3(2a^2b - 5ab^2) \sinh(fx + e)^ \\
& 9 + 4(8a^3 - 26a^2b + 15ab^2) \cosh(fx + e)^7 + 4(8a^3 - 26a^2b + \\
& 15ab^2 + 27(2a^2b - 5ab^2) \cosh(fx + e)^2) \sinh(fx + e)^7 + 28(9 \\
& * (2a^2b - 5ab^2) \cosh(fx + e)^3 + (8a^3 - 26a^2b + 15ab^2) \cosh(f \\
& x + e)) \sinh(fx + e)^6 - 2(56a^3 - 98a^2b + 45ab^2) \cosh(fx + e)^5 \\
& + 2(189(2a^2b - 5ab^2) \cosh(fx + e)^4 - 56a^3 + 98a^2b - 45ab^2
\end{aligned}$$

$$\begin{aligned}
& 2 + 42*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e)^2*\sinh(f*x + e)^5 + 2*(\\
& 189*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)^5 + 70*(8*a^3 - 26*a^2*b + 15*a*b^2)* \\
& \cosh(f*x + e)^3 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*\cosh(f*x + e))*\sinh(f*x \\
& + e)^4 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e)^3 + 4*(63*(2*a^2*b - \\
& 5*a*b^2)*\cosh(f*x + e)^6 + 35*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e)^ \\
& 4 + 8*a^3 - 26*a^2*b + 15*a*b^2 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*\cosh(f*x \\
& + e)^2)*\sinh(f*x + e)^3 + 4*(27*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)^7 + 21*(\\
& 8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e)^5 - 5*(56*a^3 - 98*a^2*b + 45*a* \\
& b^2)*\cosh(f*x + e)^3 + 3*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e))*\sinh(\\
& f*x + e)^2 + 3*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e) + (27*(2*a^2*b - 5*a*b^2)* \\
& \cosh(f*x + e)^8 + 28*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e)^6 - 10*(56 \\
& *a^3 - 98*a^2*b + 45*a*b^2)*\cosh(f*x + e)^4 + 6*a^2*b - 15*a*b^2 + 12*(8*a^ \\
& 3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e))*\sqrt{(b*\cosh(f*x + \\
& e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh(f*x + e)^2 - 2*\cosh(f*x + e)*\sin \\
& h(f*x + e) + \sinh(f*x + e)^2)))/(a^4*b^2*f*\cosh(f*x + e)^12 + 12*a^4*b^2*f* \\
& \cosh(f*x + e)*\sinh(f*x + e)^11 + a^4*b^2*f*\sinh(f*x + e)^12 + 2*(4*a^5*b - \\
& 3*a^4*b^2)*f*\cosh(f*x + e)^10 + 2*(33*a^4*b^2*f*\cosh(f*x + e)^2 + (4*a^5*b \\
& - 3*a^4*b^2)*f)*\sinh(f*x + e)^10 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(\\
& f*x + e)^8 + 20*(11*a^4*b^2*f*\cosh(f*x + e)^3 + (4*a^5*b - 3*a^4*b^2)*f*\cos \\
& h(f*x + e))*\sinh(f*x + e)^9 + (495*a^4*b^2*f*\cosh(f*x + e)^4 + 90*(4*a^5*b \\
& - 3*a^4*b^2)*f*\cosh(f*x + e)^2 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f)*\sinh(f \\
& *x + e)^8 - 4*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*\cosh(f*x + e)^6 + 8*(99*a^4* \\
& b^2*f*\cosh(f*x + e)^5 + 30*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^3 + (16*a^ \\
& 6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e))*\sinh(f*x + e)^7 + a^4*b^2*f + 4 \\
& *(231*a^4*b^2*f*\cosh(f*x + e)^6 + 105*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e) \\
& ^4 + 7*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^2 - (8*a^6 - 12*a^5 \\
& *b + 5*a^4*b^2)*f)*\sinh(f*x + e)^6 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cos \\
& h(f*x + e)^4 + 8*(99*a^4*b^2*f*\cosh(f*x + e)^7 + 63*(4*a^5*b - 3*a^4*b^2)*f \\
& *\cosh(f*x + e)^5 + 7*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^3 - 3 \\
& *(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*\cosh(f*x + e))*\sinh(f*x + e)^5 + (495*a^4 \\
& *b^2*f*\cosh(f*x + e)^8 + 420*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^6 + 70*(\\
& 16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^4 - 60*(8*a^6 - 12*a^5*b + \\
& 5*a^4*b^2)*f*\cosh(f*x + e)^2 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f)*\sinh(f*x \\
& + e)^4 + 2*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^2 + 4*(55*a^4*b^2*f*\cosh(\\
& f*x + e)^9 + 60*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^7 + 14*(16*a^6 - 32*a \\
& ^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^5 - 20*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f* \\
& \cosh(f*x + e)^3 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e))*\sinh(f* \\
& x + e)^3 + 2*(33*a^4*b^2*f*\cosh(f*x + e)^10 + 45*(4*a^5*b - 3*a^4*b^2)*f*\cos \\
& h(f*x + e)^8 + 14*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^6 - 30* \\
& (8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*\cosh(f*x + e)^4 + 3*(16*a^6 - 32*a^5*b + 1 \\
& 5*a^4*b^2)*f*\cosh(f*x + e)^2 + (4*a^5*b - 3*a^4*b^2)*f)*\sinh(f*x + e)^2 + 4 \\
& *(3*a^4*b^2*f*\cosh(f*x + e)^11 + 5*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^9 \\
& + 2*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^7 - 6*(8*a^6 - 12*a^5* \\
& b + 5*a^4*b^2)*f*\cosh(f*x + e)^5 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(\\
& f*x + e)^3 + (4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e))*\sinh(f*x + e)), 1/6*(3*
\end{aligned}$$

$$\begin{aligned}
& ((2*a*b^2 - 5*b^3)*\cosh(f*x + e)^{12} + 12*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)*\sinh(f*x + e)^{11} + (2*a*b^2 - 5*b^3)*\sinh(f*x + e)^{12} + 2*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^{10} + 2*(8*a^2*b - 26*a*b^2 + 15*b^3 + 33*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^{10} + 20*(11*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^9 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^8 + (495*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^4 + 32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3 + 90*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^8 + 8*(99*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^5 + 30*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^7 - 4*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*\cosh(f*x + e)^6 + 4*(231*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^6 + 105*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^4 - 16*a^3 + 64*a^2*b - 70*a*b^2 + 25*b^3 + 7*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^6 + 8*(99*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^7 + 63*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^5 + 7*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^3 - 3*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^5 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^4 + (495*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^8 + 420*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^6 + 70*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^4 + 32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3 - 60*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^4 + 4*(55*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^9 + 60*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^7 + 14*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^5 - 20*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*\cosh(f*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e))*\sinh(f*x + e)^3 + 2*a*b^2 - 5*b^3 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^2 + 2*(33*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^10 + 45*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^8 + 14*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^6 - 30*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*\cosh(f*x + e)^4 + 8*a^2*b - 26*a*b^2 + 15*b^3 + 3*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^2)*\sinh(f*x + e)^2 + 4*(3*(2*a*b^2 - 5*b^3)*\cosh(f*x + e)^11 + 5*(8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e)^9 + 2*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^7 - 6*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*\cosh(f*x + e)^5 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*\cosh(f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*\cosh(f*x + e))*\sinh(f*x + e))*\sqrt{-a}*\arctan(1/2*\sqrt{2}*\sqrt{-a}*\sqrt{(b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)})/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*\sqrt{2}*(3*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)^9 + 27*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)*\sinh(f*x + e)^8 + 3*(2*a^2*b - 5*a*b^2)*\sinh(f*x + e)^9 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e)^7 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2 + 27*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e)^7 + 28*(9*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)^3 + (8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^6 - 2*(56*a^3 - 98*a^2*b + 45*a*b^2)*\cosh(f*x + e)^5 + 2*(189*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)^4 - 56*a^3 + 98*a^2*b - 45*a*b^2 + 42*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(
\end{aligned}$$

$$\begin{aligned}
& f*x + e)^2)*\sinh(f*x + e)^5 + 2*(189*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)^5 + \\
& 70*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e)^3 - 5*(56*a^3 - 98*a^2*b + 4 \\
& 5*a*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^4 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*c \\
& \cosh(f*x + e)^3 + 4*(63*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)^6 + 35*(8*a^3 - 26 \\
& *a^2*b + 15*a*b^2)*\cosh(f*x + e)^4 + 8*a^3 - 26*a^2*b + 15*a*b^2 - 5*(56*a^ \\
& 3 - 98*a^2*b + 45*a*b^2)*\cosh(f*x + e)^2)*\sinh(f*x + e)^3 + 4*(27*(2*a^2*b \\
& - 5*a*b^2)*\cosh(f*x + e)^7 + 21*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e) \\
& ^5 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*\cosh(f*x + e)^3 + 3*(8*a^3 - 26*a^2*b \\
& + 15*a*b^2)*\cosh(f*x + e))*\sinh(f*x + e)^2 + 3*(2*a^2*b - 5*a*b^2)*\cosh(f* \\
& x + e) + (27*(2*a^2*b - 5*a*b^2)*\cosh(f*x + e)^8 + 28*(8*a^3 - 26*a^2*b + 1 \\
& 5*a*b^2)*\cosh(f*x + e)^6 - 10*(56*a^3 - 98*a^2*b + 45*a*b^2)*\cosh(f*x + e)^ \\
& 4 + 6*a^2*b - 15*a*b^2 + 12*(8*a^3 - 26*a^2*b + 15*a*b^2)*\cosh(f*x + e)^2)* \\
& \sinh(f*x + e))*\sqrt{((b*\cosh(f*x + e)^2 + b*\sinh(f*x + e)^2 + 2*a - b)/(\cosh \\
& (f*x + e)^2 - 2*\cosh(f*x + e)*\sinh(f*x + e) + \sinh(f*x + e)^2)))/(a^4*b^2*f \\
& *\cosh(f*x + e)^12 + 12*a^4*b^2*f*\cosh(f*x + e)*\sinh(f*x + e)^11 + a^4*b^2*f \\
& *\sinh(f*x + e)^12 + 2*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^10 + 2*(33*a^4* \\
& b^2*f*\cosh(f*x + e)^2 + (4*a^5*b - 3*a^4*b^2)*f)*\sinh(f*x + e)^10 + (16*a^6 \\
& - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^8 + 20*(11*a^4*b^2*f*\cosh(f*x + e) \\
&)^3 + (4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e))*\sinh(f*x + e)^9 + (495*a^4*b^2 \\
& *f*\cosh(f*x + e)^4 + 90*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^2 + (16*a^6 - \\
& 32*a^5*b + 15*a^4*b^2)*f)*\sinh(f*x + e)^8 - 4*(8*a^6 - 12*a^5*b + 5*a^4*b^ \\
& 2)*f*\cosh(f*x + e)^6 + 8*(99*a^4*b^2*f*\cosh(f*x + e)^5 + 30*(4*a^5*b - 3*a^ \\
& 4*b^2)*f*\cosh(f*x + e)^3 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e) \\
&)*\sinh(f*x + e)^7 + a^4*b^2*f + 4*(231*a^4*b^2*f*\cosh(f*x + e)^6 + 105*(4*a \\
& ^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^4 + 7*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f* \\
& \cosh(f*x + e)^2 - (8*a^6 - 12*a^5*b + 5*a^4*b^2)*f)*\sinh(f*x + e)^6 + (16*a \\
& ^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^4 + 8*(99*a^4*b^2*f*\cosh(f*x + \\
& e)^7 + 63*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^5 + 7*(16*a^6 - 32*a^5*b + \\
& 15*a^4*b^2)*f*\cosh(f*x + e)^3 - 3*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*\cosh(f*x \\
& + e))*\sinh(f*x + e)^5 + (495*a^4*b^2*f*\cosh(f*x + e)^8 + 420*(4*a^5*b - 3* \\
& a^4*b^2)*f*\cosh(f*x + e)^6 + 70*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x \\
& + e)^4 - 60*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*\cosh(f*x + e)^2 + (16*a^6 - 3 \\
& 2*a^5*b + 15*a^4*b^2)*f)*\sinh(f*x + e)^4 + 2*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f \\
& *x + e)^2 + 4*(55*a^4*b^2*f*\cosh(f*x + e)^9 + 60*(4*a^5*b - 3*a^4*b^2)*f*co \\
& sh(f*x + e)^7 + 14*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^5 - 20* \\
& (8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*\cosh(f*x + e)^3 + (16*a^6 - 32*a^5*b + 15* \\
& a^4*b^2)*f*\cosh(f*x + e))*\sinh(f*x + e)^3 + 2*(33*a^4*b^2*f*\cosh(f*x + e)^1 \\
& 0 + 45*(4*a^5*b - 3*a^4*b^2)*f*\cosh(f*x + e)^8 + 14*(16*a^6 - 32*a^5*b + 15 \\
& *a^4*b^2)*f*\cosh(f*x + e)^6 - 30*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*\cosh(f*x \\
& + e)^4 + 3*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^2 + (4*a^5*b - \\
& 3*a^4*b^2)*f)*\sinh(f*x + e)^2 + 4*(3*a^4*b^2*f*\cosh(f*x + e)^11 + 5*(4*a^5* \\
& b - 3*a^4*b^2)*f*\cosh(f*x + e)^9 + 2*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cos \\
& h(f*x + e)^7 - 6*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*\cosh(f*x + e)^5 + (16*a^6 \\
& - 32*a^5*b + 15*a^4*b^2)*f*\cosh(f*x + e)^3 + (4*a^5*b - 3*a^4*b^2)*f*\cosh(\\
& f*x + e))*\sinh(f*x + e))]
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3(fx + e)}{\left(b \sinh^2(fx + e) + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)

$$3.506 \quad \int \frac{\coth^5(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=208

$$\frac{8a^2 - 40ab + 35b^2}{8a^4 f \sqrt{a + b \sinh^2(e + fx)}} + \frac{8a^2 - 40ab + 35b^2}{24a^3 f (a + b \sinh^2(e + fx))^{3/2}} - \frac{(8a^2 - 40ab + 35b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{8a^{9/2} f} - \frac{(8a - 7b)}{8a^2 f (a + b \sinh^2(e + fx))^{5/2}}$$

[Out] $-\left(\left(8a^2 - 40ab + 35b^2\right) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right]\right) / \left(8a^{9/2} f\right) + \left(8a^2 - 40ab + 35b^2\right) / \left(24a^3 f (a + b \sinh^2(e + fx))^{3/2}\right) - \left(\left(8a - 7b\right) \operatorname{Csch}[e + fx]^2\right) / \left(8a^2 f (a + b \sinh^2(e + fx))^{3/2}\right) - \operatorname{Csch}[e + fx]^4 / \left(4a f (a + b \sinh^2(e + fx))^{3/2}\right) + \left(8a^2 - 40ab + 35b^2\right) / \left(8a^4 f \sqrt{a + b \sinh^2(e + fx)}\right)$

Rubi [A] time = 0.224905, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2 - 40ab + 35b^2}{8a^4 f \sqrt{a + b \sinh^2(e + fx)}} + \frac{8a^2 - 40ab + 35b^2}{24a^3 f (a + b \sinh^2(e + fx))^{3/2}} - \frac{(8a^2 - 40ab + 35b^2) \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right)}{8a^{9/2} f} - \frac{(8a - 7b)}{8a^2 f (a + b \sinh^2(e + fx))^{5/2}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Coth}[e + fx]^5 / (a + b \sinh^2(e + fx))^{5/2}, x]$

[Out] $-\left(\left(8a^2 - 40ab + 35b^2\right) \operatorname{ArcTanh}\left[\frac{\sqrt{a + b \sinh^2(e + fx)}}{\sqrt{a}}\right]\right) / \left(8a^{9/2} f\right) + \left(8a^2 - 40ab + 35b^2\right) / \left(24a^3 f (a + b \sinh^2(e + fx))^{3/2}\right) - \left(\left(8a - 7b\right) \operatorname{Csch}[e + fx]^2\right) / \left(8a^2 f (a + b \sinh^2(e + fx))^{3/2}\right) - \operatorname{Csch}[e + fx]^4 / \left(4a f (a + b \sinh^2(e + fx))^{3/2}\right) + \left(8a^2 - 40ab + 35b^2\right) / \left(8a^4 f \sqrt{a + b \sinh^2(e + fx)}\right)$

Rule 3194

$\operatorname{Int}[\left((a_{-}) + (b_{-}) \sin[(e_{-}) + (f_{-})(x_{-})]^2\right)^{(p_{-})} \tan[(e_{-}) + (f_{-})(x_{-})]^{(m_{-})}, x_{\text{Symbol}}] \rightarrow \operatorname{With}[\{ff = \operatorname{FreeFactors}[\sin[e + fx]^2, x]\}, \operatorname{Dist}[ff^{((m + 1)/2)} / (2f), \operatorname{Subst}[\operatorname{Int}[(x^{((m - 1)/2)} (a + bffx)^p] / (1 - ffx)^{(m + 1)}$

)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 89

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^(2*(n + 1))*(e + f*x)^(p + 1))/(d^(2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^(2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n + 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned}
\int \frac{\coth^5(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{2f} \\
&= -\frac{\text{csch}^4(e+fx)}{4af(a+b\sinh^2(e+fx))^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-7b)+2ax}{x^2(a+bx)^{5/2}} dx, x, \sinh^2(e+fx)\right)}{4af} \\
&= -\frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{csch}^4(e+fx)}{4af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(8a^2-40ab+35b^2)\text{csch}^4(e+fx)}{8a^2f(a+b\sinh^2(e+fx))^{3/2}} \\
&= \frac{8a^2-40ab+35b^2}{24a^3f(a+b\sinh^2(e+fx))^{3/2}} - \frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{csch}^4(e+fx)}{4af(a+b\sinh^2(e+fx))^{3/2}} \\
&= \frac{8a^2-40ab+35b^2}{24a^3f(a+b\sinh^2(e+fx))^{3/2}} - \frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{csch}^4(e+fx)}{4af(a+b\sinh^2(e+fx))^{3/2}} \\
&= \frac{8a^2-40ab+35b^2}{24a^3f(a+b\sinh^2(e+fx))^{3/2}} - \frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{csch}^4(e+fx)}{4af(a+b\sinh^2(e+fx))^{3/2}} \\
&= \frac{(8a^2-40ab+35b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{8a^2-40ab+35b^2}{24a^3f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\text{csch}^4(e+fx)}{8a^2f(a+b\sinh^2(e+fx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.423643, size = 117, normalized size = 0.56

$$\frac{\text{csch}^2(e+fx)\left((-8a^2+40ab-35b^2) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b\sinh^2(e+fx)}{a} + 1\right) + 3a\text{csch}^2(e+fx)(2a\text{csch}^2(e+fx) + 8a - 7b)\right)}{24a^3f\sqrt{a+b\sinh^2(e+fx)}(a\text{csch}^2(e+fx) + b)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] -(Csch[e + f*x]^2*(3*a*Csch[e + f*x]^2*(8*a - 7*b + 2*a*Csch[e + f*x]^2) + (-8*a^2 + 40*a*b - 35*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sinh[e + f*x]^2/a)])/(24*a^3*f*sqrt(a + b*Sinh[e + f*x]^2)*(a*Csch[e + f*x]^2 + b))

$f*x]^2/a)))/(24*a^3*f*(b + a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2]$
 $)$

Maple [C] time = 0.122, size = 73, normalized size = 0.4

$$\frac{1}{f} \int \frac{(\cosh(fx + e))^4}{\left(b^2 (\sinh(fx + e))^4 + 2ab (\sinh(fx + e))^2 + a^2\right) (\sinh(fx + e))^5 \sqrt{a + b (\sinh(fx + e))^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] `int/indef0` (cosh(f*x+e)^4/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^5}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^5}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")`

[Out] `integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)`

$$3.507 \quad \int \frac{\tanh^4(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=333

$$\frac{(3a+b)(a+3b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3af(a-b)^4\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}}-\frac{2(2a+b)\tanh(e+fx)}{3f(a-b)^2(a+b \sinh^2(e+fx))}$$

```
[Out] -(b*(5*a + 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^3*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (8*Sqrt[a]*Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^4*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + b)*(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*(a - b)^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(2*a + b)*Tanh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (Sech[e + f*x]^2*Tanh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.399476, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3196, 470, 527, 525, 418, 411}

$$-\frac{2(2a+b)\tanh(e+fx)}{3f(a-b)^2(a+b \sinh^2(e+fx))^{3/2}}-\frac{b(5a+3b)\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^3(a+b \sinh^2(e+fx))^{3/2}}+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)}{3f(a-b)(a+b \sinh^2(e+fx))^{3/2}}-\frac{8\sqrt{a}\sqrt{b}(a+b)\cosh(e+fx)\operatorname{EllipticE}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}, 1-\frac{a}{b}\right)}{3(a-b)^4f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}+\frac{(3a+b)(a+3b)\operatorname{EllipticF}\left(\operatorname{ArcTan}[\sinh(e+fx)], 1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a(a-b)^4f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}-\frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}}+\frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2), x]
```

```
[Out] -(b*(5*a + 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^3*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (8*Sqrt[a]*Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^4*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + b)*(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*(a - b)^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(2*a + b)*Tanh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (Sech[e + f*x]^2*Tanh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

$e + f*x]^2)^{(3/2)}$

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 470

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 527

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

Rule 525

Int[((e_) + (f_)*(x_)^2)/(Sqrt[(a_) + (b_)*(x_)^2]*((c_) + (d_)*(x_)^2)^(3/2)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] && PosQ[d/c]

Rule 418

Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^4}{(1+x^2)^{5/2}(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a+(-3)}{(1+x^2)^{3/2}}\right)}{3(a-b)f} \\
&= -\frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{a+(-3)}{(1+x^2)^{3/2}}\right)}{3(a-b)f} \\
&= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} \\
&= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f(a+b\sinh^2(e+fx))^{3/2}} \\
&= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f(a+b\sinh^2(e+fx))^{3/2}} - \frac{8\sqrt{a}\sqrt{b}(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)\right)}{3(a-b)^4f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 3.36955, size = 252, normalized size = 0.76

$$i\left(2ab\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\left((-5a^2+2ab+3b^2)\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right)+8a(a+b)E\left(i(e+fx)\left|\frac{b}{a}\right.\right)\right)-i\sqrt{2b}(2ab(a+b)\sqrt{a+b\sinh^2(e+fx)})\right)$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]

[Out] $\left(\frac{-1}{6}\right) \cdot \left(\frac{2ab((2a-b+b\cosh[2(e+fx)])/a)^{3/2}(8a(a+b)\text{EllipticE}[I(e+fx), b/a] + (-5a^2+2ab+3b^2)\text{EllipticF}[I(e+fx), b/a]) - I\sqrt{2}b(2a(a-b)b\sinh[2(e+fx)] + 4b(a+b)(2a-b+b\cosh[2(e+fx)])\sinh[2(e+fx)] + 4(a+b)(2a-b+b\cosh[2(e+fx)])^2\tanh[e+fx] - (a-b)(2a-b+b\cosh[2(e+fx)])^2\text{sech}[e+fx]^2\tanh[e+fx])}{(a-b)^4bf(2a-b+b\cosh[2(e+fx)])^{3/2}}\right)$

Maple [A] time = 0.217, size = 661, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] $\frac{1}{3} \left((-8(-1/ab)^{1/2}ab^2 - 8(-1/ab)^{1/2}b^3) \sinh(fx+e) \cosh(fx+e)^6 + (-13(-1/ab)^{1/2}a^2b + 2(-1/ab)^{1/2}ab^2 + 11(-1/ab)^{1/2}b^3) \cosh(fx+e)^4 \sinh(fx+e) + (-4(-1/ab)^{1/2}a^3 + 6(-1/ab)^{1/2}a^2b - 2(-1/ab)^{1/2}b^3) \cosh(fx+e)^2 \sinh(fx+e) + ((-1/ab)^{1/2}a^3 - 3(-1/ab)^{1/2}a^2b + 3(-1/ab)^{1/2}ab^2 - (-1/ab)^{1/2}b^3) \sinh(fx+e) + (b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} b (3\text{EllipticF}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) a^2 + 2\text{EllipticF}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) ab - 5\text{EllipticF}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) b^2 + 8\text{EllipticE}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) ab + 8\text{EllipticE}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) b^2) \cosh(fx+e)^4 + (b/a \cosh(fx+e)^2 + (a-b)/a)^{1/2} (\cosh(fx+e)^2)^{1/2} (3\text{EllipticF}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) a^3 - \text{EllipticF}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) a^2b - 7\text{EllipticF}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) ab^2 + 5\text{EllipticF}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) b^3 + 8\text{EllipticE}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) a^2b - 8\text{EllipticE}(\sinh(fx+e), (-1/ab)^{1/2}), (a/b)^{1/2}) b^3) \cosh(fx+e)^2) / \cosh(fx+e)^3 / (-1/ab)^{1/2} / (a+b\sinh(fx+e)^2)^{3/2} / (a-b)^{4/2}$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx+e)^4}{(b \sinh(fx+e)^2 + a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] `integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \tanh^4(fx + e)}{b^3 \sinh^6(fx + e) + 3ab^2 \sinh^4(fx + e) + 3a^2b \sinh^2(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")`

[Out] `integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4(fx + e)}{\left(b \sinh^2(fx + e) + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

$$3.508 \quad \int \frac{\tanh^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=274

$$\frac{(3a+5b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3af(a-b)^3\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}-\frac{\tanh(e+fx)}{f(a-b)(a+b\sinh^2(e+fx))^{3/2}}$$

[Out] $(-4*b*\operatorname{Cosh}[e+fx]*\operatorname{Sinh}[e+fx])/(3*(a-b)^2*f*(a+b*\operatorname{Sinh}[e+fx]^2)^{(3/2)}) - (\operatorname{Sqrt}[b]*(7*a+b)*\operatorname{Cosh}[e+fx]*\operatorname{EllipticE}[\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[e+fx])/ \operatorname{Sqrt}[a]],1-a/b])/(3*\operatorname{Sqrt}[a]*(a-b)^3*f*\operatorname{Sqrt}[(a*\operatorname{Cosh}[e+fx]^2)/(a+b*\operatorname{Sinh}[e+fx]^2)]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2]) + ((3*a+5*b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]],1-b/a]*\operatorname{Sech}[e+fx]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2])/(3*a*(a-b)^3*f*\operatorname{Sqrt}[(\operatorname{Sech}[e+fx]^2*(a+b*\operatorname{Sinh}[e+fx]^2))/a]) - \operatorname{Tanh}[e+fx]/((a-b)*f*(a+b*\operatorname{Sinh}[e+fx]^2)^{(3/2)})$

Rubi [A] time = 0.28018, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$, Rules used = {3196, 471, 527, 525, 418, 411}

$$\frac{\tanh(e+fx)}{f(a-b)(a+b\sinh^2(e+fx))^{3/2}} - \frac{4b\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^2(a+b\sinh^2(e+fx))^{3/2}} - \frac{\sqrt{b}(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right)}{3\sqrt{a}f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[\operatorname{Tanh}[e+fx]^2/(a+b*\operatorname{Sinh}[e+fx]^2)^{(5/2)},x]$

[Out] $(-4*b*\operatorname{Cosh}[e+fx]*\operatorname{Sinh}[e+fx])/(3*(a-b)^2*f*(a+b*\operatorname{Sinh}[e+fx]^2)^{(3/2)}) - (\operatorname{Sqrt}[b]*(7*a+b)*\operatorname{Cosh}[e+fx]*\operatorname{EllipticE}[\operatorname{ArcTan}[(\operatorname{Sqrt}[b]*\operatorname{Sinh}[e+fx])/ \operatorname{Sqrt}[a]],1-a/b])/(3*\operatorname{Sqrt}[a]*(a-b)^3*f*\operatorname{Sqrt}[(a*\operatorname{Cosh}[e+fx]^2)/(a+b*\operatorname{Sinh}[e+fx]^2)]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2]) + ((3*a+5*b)*\operatorname{EllipticF}[\operatorname{ArcTan}[\operatorname{Sinh}[e+fx]],1-b/a]*\operatorname{Sech}[e+fx]*\operatorname{Sqrt}[a+b*\operatorname{Sinh}[e+fx]^2])/(3*a*(a-b)^3*f*\operatorname{Sqrt}[(\operatorname{Sech}[e+fx]^2*(a+b*\operatorname{Sinh}[e+fx]^2))/a]) - \operatorname{Tanh}[e+fx]/((a-b)*f*(a+b*\operatorname{Sinh}[e+fx]^2)^{(3/2)})$

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_) + (f_.)*(x_)]^2)^(p_)*tan[(e_) + (f_.)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 471

```
Int[((e_)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m - n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n + 1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^(3/2)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] && PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Simp[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
```

[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rubi steps

$$\begin{aligned}
 \int \frac{\tanh^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^2}{(1+x^2)^{3/2}(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
 &= -\frac{\tanh(e+fx)}{(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^{a-3}}{\sqrt{1+x^2}(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
 &= -\frac{4b \cosh(e+fx) \sinh(e+fx)}{3(a-b)^2 f (a+b\sinh^2(e+fx))^{3/2}} - \frac{\tanh(e+fx)}{(a-b)f(a+b\sinh^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^{a-3}}{\sqrt{1+x^2}(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
 &= -\frac{4b \cosh(e+fx) \sinh(e+fx)}{3(a-b)^2 f (a+b\sinh^2(e+fx))^{3/2}} - \frac{\tanh(e+fx)}{(a-b)f(a+b\sinh^2(e+fx))^{3/2}} + \frac{\left(b(7a+b)\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{x^{a-3}}{\sqrt{1+x^2}(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{(-a+b)f} \\
 &= -\frac{4b \cosh(e+fx) \sinh(e+fx)}{3(a-b)^2 f (a+b\sinh^2(e+fx))^{3/2}} - \frac{\sqrt{b}(7a+b) \cosh(e+fx) E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right)}{3\sqrt{a}(a-b)^3 f \sqrt{\frac{a \cosh^2(e+fx)}{a+b\sinh^2(e+fx)}} \sqrt{a+b\sinh^2(e+fx)}}
 \end{aligned}$$

Mathematica [C] time = 2.6007, size = 215, normalized size = 0.78

$$\frac{8ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\tanh(e+fx)(-4a^2b+24a^3+b^2(7a+b)\cosh(4(e+fx))+5ab^2+4ab(11a-3b)\cosh(2(e+fx)))}{\sqrt{2}}}{6af(a-b)^3(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] ((-2*I)*a^2*(7*a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - ((24*a^3 - 4*a^2*b + 5*a*b^2 - b^3 + 4*a*(11*a - 3*b)*b*Cosh[2*(e + f*x)] + b^2*(7*a + b)*Cosh[4*(e + f*x)])*Tanh[e + f*x])/Sqrt[2])/(6*a*(a - b)^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))

Maple [B] time = 0.192, size = 799, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)`

[Out]
$$\begin{aligned} & \frac{1}{3} * (-7 * (-1/a*b)^{(1/2)} * a*b^2 * \sinh(f*x+e)^5 - (-1/a*b)^{(1/2)} * b^3 * \sinh(f*x+e)^5 \\ & + 3 * ((a+b*\sinh(f*x+e)^2)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) \\ &) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a^2 * b * \sinh(f*x+e)^2 - 2 * ((a+b*\sinh(f*x+e)^2)/a) \\ & ^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) \\ &) * a*b^2 * \sinh(f*x+e)^2 - ((a+b*\sinh(f*x+e)^2)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} \\ & * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^3 * \sinh(f*x+e)^2 + 7 * ((a+ \\ & b*\sinh(f*x+e)^2)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a \\ & *b)^{(1/2)}, (a/b)^{(1/2)}) * a*b^2 * \sinh(f*x+e)^2 + ((a+b*\sinh(f*x+e)^2)/a)^{(1/2)} * (\c \\ & \cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * b^3 * s \\ & \sinh(f*x+e)^2 - 11 * (-1/a*b)^{(1/2)} * a^2 * b * \sinh(f*x+e)^3 - 4 * (-1/a*b)^{(1/2)} * a*b^2 * s \\ & \sinh(f*x+e)^3 - (-1/a*b)^{(1/2)} * b^3 * \sinh(f*x+e)^3 + 3 * ((a+b*\sinh(f*x+e)^2)/a)^{(1/2)} * \\ & (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * \\ & a^3 - 2 * ((a+b*\sinh(f*x+e)^2)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f* \\ & x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a^2 * b - ((a+b*\sinh(f*x+e)^2)/a)^{(1/2)} * (\cosh \\ & (f*x+e)^2)^{(1/2)} * \text{EllipticF}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a*b^2 + 7 * (\\ & (a+b*\sinh(f*x+e)^2)/a)^{(1/2)} * (\cosh(f*x+e)^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (- \\ & 1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a^2 * b + ((a+b*\sinh(f*x+e)^2)/a)^{(1/2)} * (\cosh(f*x+e) \\ & ^2)^{(1/2)} * \text{EllipticE}(\sinh(f*x+e) * (-1/a*b)^{(1/2)}, (a/b)^{(1/2)}) * a*b^2 - 3 * (-1/a*b) \\ & ^{(1/2)} * a^3 * \sinh(f*x+e) - 5 * \sinh(f*x+e) * b * a^2 * (-1/a*b)^{(1/2)} / (-1/a*b)^{(1/2)} / (\\ & a+b*\sinh(f*x+e)^2)^{(3/2)} / (a-b)^3 / a / \cosh(f*x+e) / f \end{aligned}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx+e)^2}{(b \sinh(fx+e)^2 + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")`

[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \tanh^2(fx + e)}{b^3 \sinh^6(fx + e) + 3ab^2 \sinh^4(fx + e) + 3a^2b \sinh^2(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(fx + e)}{\left(b \sinh^2(fx + e) + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)

$$3.509 \quad \int \frac{1}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=251

$$\frac{i\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1 \operatorname{EllipticF}\left(ie + ifx, \frac{b}{a}\right)}{3af(a-b)\sqrt{a+b \sinh^2(e+fx)}} - \frac{2b(2a-b) \sinh(e+fx) \cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b \sinh^2(e+fx)}} - \frac{2i(2a-b)\sqrt{a+b \sinh^2(e+fx)}E\left(ie + ifx, \frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1}$$

[Out] $-(b*\cosh[e + f*x]*\sinh[e + f*x])/(3*a*(a - b)*f*(a + b*\sinh[e + f*x]^2)^{(3/2)} - (2*(2*a - b)*b*\cosh[e + f*x]*\sinh[e + f*x])/(3*a^2*(a - b)^2*f*\sqrt{a + b*\sinh[e + f*x]^2}) - (((2*I)/3)*(2*a - b)*\operatorname{EllipticE}[I*e + I*f*x, b/a]*\operatorname{Sqrt}[a + b*\sinh[e + f*x]^2])/(a^2*(a - b)^2*f*\sqrt{1 + (b*\sinh[e + f*x]^2)/a}) + ((I/3)*\operatorname{EllipticF}[I*e + I*f*x, b/a]*\sqrt{1 + (b*\sinh[e + f*x]^2)/a})/(a*(a - b)*f*\sqrt{a + b*\sinh[e + f*x]^2})$

Rubi [A] time = 0.288262, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{2b(2a-b) \sinh(e+fx) \cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b \sinh^2(e+fx)}} - \frac{2i(2a-b)\sqrt{a+b \sinh^2(e+fx)}E\left(ie + ifx \middle| \frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b \sinh^2(e+fx)}{a}} + 1} - \frac{b \sinh(e+fx) \cosh(e+fx)}{3af(a-b)(a+b \sinh^2(e+fx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\sinh[e + f*x]^2)^{-5/2}, x]$

[Out] $-(b*\cosh[e + f*x]*\sinh[e + f*x])/(3*a*(a - b)*f*(a + b*\sinh[e + f*x]^2)^{(3/2)} - (2*(2*a - b)*b*\cosh[e + f*x]*\sinh[e + f*x])/(3*a^2*(a - b)^2*f*\sqrt{a + b*\sinh[e + f*x]^2}) - (((2*I)/3)*(2*a - b)*\operatorname{EllipticE}[I*e + I*f*x, b/a]*\operatorname{Sqrt}[a + b*\sinh[e + f*x]^2])/(a^2*(a - b)^2*f*\sqrt{1 + (b*\sinh[e + f*x]^2)/a}) + ((I/3)*\operatorname{EllipticF}[I*e + I*f*x, b/a]*\sqrt{1 + (b*\sinh[e + f*x]^2)/a})/(a*(a - b)*f*\sqrt{a + b*\sinh[e + f*x]^2})$

Rule 3184

$\operatorname{Int}[(a + b*\sin[(e + f*x)*(x)]^2)^p, x_Symbol] \rightarrow -\operatorname{Simp}[(b*\cos[e + f*x]*\sin[e + f*x]*(a + b*\sin[e + f*x]^2)^{p+1})/(2*a*f*(p+1)*(a +$

b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1) *Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]

Rule 3173

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^2)^(p_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x] * (a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p + 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]

Rule 3172

Int[((A_) + (B_)*sin[(e_) + (f_)*(x_)])^2)/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x], x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ[{a, b, e, f, A, B}, x]

Rule 3178

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2], x_Symbol] := Dist[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3177

Int[Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2], x_Symbol] := Simp[(Sqrt[a] * EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rule 3183

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2], x_Symbol] := Dist[Sqrt[1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]

Rule 3182

Int[1/Sqrt[(a_) + (b_)*sin[(e_) + (f_)*(x_)])^2], x_Symbol] := Simp[(1*EllipticF[e + f*x, -(b/a)]/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \sinh^2(e + fx))^{5/2}} dx &= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{\int \frac{-3a+2b+b \sinh^2(e+fx)}{(a+b \sinh^2(e+fx))^{3/2}} dx}{3a(a - b)} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{\int \frac{-a(3a-b)}{\sqrt{a+b \sinh^2(e+fx)}} dx}{3a} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{\int \frac{-a(3a-b)}{\sqrt{a+b \sinh^2(e+fx)}} dx}{3a} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} + \frac{(2(2a - b)b \cosh(e + fx) \sinh(e + fx))}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} \\
&= -\frac{b \cosh(e + fx) \sinh(e + fx)}{3a(a - b)f (a + b \sinh^2(e + fx))^{3/2}} - \frac{2(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}} - \frac{2i(2a - b)b \cosh(e + fx) \sinh(e + fx)}{3a^2(a - b)^2 f \sqrt{a + b \sinh^2(e + fx)}}
\end{aligned}$$

Mathematica [A] time = 1.25598, size = 190, normalized size = 0.76

$$\frac{ia^2(a - b) \left(\frac{2a + b \cosh(2(e + fx)) - b}{a} \right)^{3/2} \text{EllipticF} \left(i(e + fx), \frac{b}{a} \right) + \sqrt{2}b \sinh(2(e + fx)) (-5a^2 + b(b - 2a) \cosh(2(e + fx)) + 5ab)}{3a^2 f (a - b)^2 (2a + b \cosh(2(e + fx)) - b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2), x]

[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b)*Cosh[2*(e + f*x)]*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cosh[2*(e + f*x)]^(3/2))

Maple [A] time = 0., size = 406, normalized size = 1.6

$$\frac{1}{\cosh(fx+e)f} \sqrt{\left(a+b(\sinh(fx+e))^2\right) \left(\cosh(fx+e)\right)^2} \left(-\frac{\sinh(fx+e)}{3ab(a-b)} \sqrt{\left(a+b(\sinh(fx+e))^2\right) \left(\cosh(fx+e)\right)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] $((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*(-1/3/a/b/(a-b)*\sinh(f*x+e)*((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}/(\sinh(f*x+e)^2+a/b)^{2-2/3*b*\cosh(f*x+e)^2/a^2/(a-b)^2*\sinh(f*x+e)*(2*a-b)/((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^{(1/2)}*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}/((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-2/3*b*(2*a-b)/a^2/(a-b)^2/(-1/a*b)^{(1/2)}*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}/((a+b*\sinh(f*x+e)^2)*\cosh(f*x+e)^2)^{(1/2)}*(\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})-\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})))/\cosh(f*x+e)/(a+b*\sinh(f*x+e)^2)^{(1/2)}/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh(fx+e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh(fx+e)^2 + a}}{b^3 \sinh(fx+e)^6 + 3ab^2 \sinh(fx+e)^4 + 3a^2b \sinh(fx+e)^2 + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*
x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

$$3.510 \quad \int \frac{\coth^2(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=351

$$\frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3a^3 f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{(7a-8b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^3 f(a-b)}$$

```
[Out] Coth[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((3*a - 4*b)*Coth[e +
f*x])/(3*a^2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((7*a - 8*b)*Coth[e
+ f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f) - ((7*a - 8*b)*Ellipt
icE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3
*a^3*(a - b)*f)
```

Rubi [A] time = 0.4119, antiderivative size = 351, normalized size of antiderivative = 1.,
number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$,
Rules used = {3196, 469, 579, 583, 531, 418, 492, 411}

$$\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^3 f(a-b)} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^3 f(a-b)} + \frac{(3a-4b)\coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^2 f(a-b)\sqrt{a+b \sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Coth[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((3*a - 4*b)*Coth[e +
f*x])/(3*a^2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((7*a - 8*b)*Coth[e
+ f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f) - ((7*a - 8*b)*Ellipt
icE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3
```

$a^3(a - b)f$

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 469

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 579

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 583

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,

d, e, f, n, p, q}, x]

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= \frac{\coth(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{-4-3x^2}{x^2\sqrt{1+x^2}(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{3af} \\
&= \frac{\coth(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{4+3x^2}{x^2\sqrt{1+x^2}(a+bx^2)^{3/2}} dx, x, \sinh(e+fx)\right)}{3af} \\
&= \frac{\coth(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)}{3a^3} \\
&= \frac{\coth(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)}{3a^3} \\
&= \frac{\coth(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)}{3a^3} \\
&= \frac{\coth(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)}{3a^3} \\
&= \frac{\coth(e+fx)}{3af(a+b\sinh^2(e+fx))^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)}{3a^3}
\end{aligned}$$

Mathematica [C] time = 2.6774, size = 226, normalized size = 0.64

$$\frac{8ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2} \operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\coth(e+fx)(4b(11a^2-19ab+8b^2)\cosh(2(e+fx))-68a^2b+24a^3+b^2(7a-8b)\cosh(4(e+fx)))}{\sqrt{2}}}{6a^3f(a-b)(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2), x]

[Out] (-(((24*a^3 - 68*a^2*b + 69*a*b^2 - 24*b^3 + 4*b*(11*a^2 - 19*a*b + 8*b^2)*Cosh[2*(e + f*x)] + (7*a - 8*b)*b^2*Cosh[4*(e + f*x)])*Coth[e + f*x])/Sqrt[

2]) - (2*I)*a^2*(7*a - 8*b)*((2*a - b + b*Cosh[2*(e + f*x)]/a)^(3/2)*EllipticE[I*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)]/a)^(3/2)*EllipticF[I*(e + f*x), b/a])/(6*a^3*(a - b)*f*(2*a - b + b*Cosh[2*(e + f*x)]^(3/2))

Maple [A] time = 0.24, size = 640, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2), x)

[Out] $\frac{1}{3} \left(\cosh(fx+e)^2 \right)^{1/2} \left(\frac{b}{a} \cosh(fx+e)^2 + \frac{a-b}{a} \right)^{1/2} b \left(3 \operatorname{EllipticF} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) a^2 - 11 \operatorname{EllipticF} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) a^2 b + 8 \operatorname{EllipticF} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) b^2 \right) + 7 \operatorname{EllipticE} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) a^2 b - 8 \operatorname{EllipticE} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) b^2 \right) \sinh(fx+e) \cosh(fx+e)^2 + \left(\cosh(fx+e)^2 \right)^{1/2} \left(\frac{b}{a} \cosh(fx+e)^2 + \frac{a-b}{a} \right)^{1/2} \left(7 \operatorname{EllipticE} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) a^2 b - 15 \operatorname{EllipticE} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) a^2 b + 8 \operatorname{EllipticE} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) b^3 \right) + 3 \operatorname{EllipticF} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) a^3 - 14 \operatorname{EllipticF} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) a^2 b + 19 \operatorname{EllipticF} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) a^2 b - 8 \operatorname{EllipticF} \left(\sinh(fx+e) \left(-\frac{1}{a} b \right)^{1/2}, \left(\frac{a}{b} \right)^{1/2} \right) b^3 \right) \sinh(fx+e) + \left(-7 \left(-\frac{1}{a} b \right)^{1/2} a^2 b + 8 \left(-\frac{1}{a} b \right)^{1/2} b^3 \right) \cosh(fx+e)^6 + \left(-11 \left(-\frac{1}{a} b \right)^{1/2} a^2 b + 26 \left(-\frac{1}{a} b \right)^{1/2} a^2 b - 16 \left(-\frac{1}{a} b \right)^{1/2} b^3 \right) \cosh(fx+e)^4 + \left(-3 \left(-\frac{1}{a} b \right)^{1/2} a^3 + 14 \left(-\frac{1}{a} b \right)^{1/2} a^2 b - 19 \left(-\frac{1}{a} b \right)^{1/2} a^2 b + 8 \left(-\frac{1}{a} b \right)^{1/2} b^3 \right) \cosh(fx+e)^2 \left(-\frac{1}{a} b \right)^{1/2} / \sinh(fx+e) / a^3 / (a-b) / (a+b \sinh(fx+e)^2)^{3/2} / \cosh(fx+e) / f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx+e)^2}{\left(b \sinh(fx+e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2), x, algorithm="maxima")

[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh^2(fx + e) + a} \coth^2(fx + e)}{b^3 \sinh^6(fx + e) + 3ab^2 \sinh^4(fx + e) + 3a^2b \sinh^2(fx + e) + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(fx + e)}{\left(b \sinh^2(fx + e) + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)

$$3.511 \quad \int \frac{\coth^4(e+fx)}{(a+b \sinh^2(e+fx))^{5/2}} dx$$

Optimal. Leaf size=385

$$\frac{(3a-8b)\operatorname{sech}(e+fx)\sqrt{a+b \sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)), 1-\frac{b}{a}\right)}{3a^4 f \sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b \sinh^2(e+fx))}{a}}} + \frac{8(a-2b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^4 f}$$

```
[Out] -((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(a - 3*b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (8*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f) + ((3*a - 8*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*b*f) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^4*f)
```

Rubi [A] time = 0.561806, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$, Rules used = {3196, 468, 579, 583, 531, 418, 492, 411}

$$\frac{8(a-2b)\tanh(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^4 f} - \frac{8(a-2b)\coth(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^4 f} + \frac{(3a-8b)\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b \sinh^2(e+fx)}}{3a^4 f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2), x]
```

```
[Out] -((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(a - 3*b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (8*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f) + ((3*a - 8*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*b*f) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^4*f)
```

]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]]/(3*a^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^4*f)

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 468

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e*x)^(m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b - a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 579

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m + n*(p + q + 2) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]

Rule 583

Int[((g_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := Simp[(e*(g*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) - e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && LtQ[m, -1]

Rule 531

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (

```
f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_)*(x_)^2]*Sqrt[(c_) + (d_)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_)*(x_)^2]/((c_) + (d_)*(x_)^2)^(3/2), x_Symbol] := Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)]/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^4(e+fx)}{(a+b\sinh^2(e+fx))^{5/2}} dx &= \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4(a+bx^2)^{5/2}} dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{3(a-2b)}{x^4\sqrt{1+x^2}}\right)}{3abf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)\operatorname{sech}(e+fx)}\right) \operatorname{Subst}\left(\int \frac{3(a-2b)}{x^4\sqrt{1+x^2}}\right)}{3abf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a-8b)}{3abf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{8(a-2b)}{3abf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{8(a-2b)}{3abf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{8(a-2b)}{3abf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{8(a-2b)}{3abf} \\
&= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf(a+b\sinh^2(e+fx))^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{8(a-2b)}{3abf}
\end{aligned}$$

Mathematica [C] time = 2.78841, size = 247, normalized size = 0.64

$$\frac{i\left(2a^2b\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\left((8b-5a)\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + 8(a-2b)E\left(i(e+fx)\left|\frac{b}{a}\right.\right)\right) + \frac{ib\coth(e+fx)\operatorname{csch}^2(e+fx)(-2(a+b\sinh^2(e+fx)))}{6a^4bf(2a+b\cosh(2(e+fx)))}\right)}{6a^4bf(2a+b\cosh(2(e+fx)))}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]

[Out] $((-I/6)*((I*b*(8*a^3 - 63*a^2*b + 92*a*b^2 - 40*b^3 - 2*(8*a^3 - 38*a^2*b + 63*a*b^2 - 30*b^3)*\text{Cosh}[2*(e + f*x)] - b*(13*a^2 - 36*a*b + 24*b^2)*\text{Cosh}[4*(e + f*x)] - 2*a*b^2*\text{Cosh}[6*(e + f*x)] + 4*b^3*\text{Cosh}[6*(e + f*x)])*\text{Coth}[e + f*x]*\text{Csch}[e + f*x]^2)/\text{Sqrt}[2] + 2*a^2*b*((2*a - b + b*\text{Cosh}[2*(e + f*x)]) / a)^{(3/2)}*(8*(a - 2*b)*\text{EllipticE}[I*(e + f*x), b/a] + (-5*a + 8*b)*\text{EllipticF}[I*(e + f*x), b/a])) / (a^4*b*f*(2*a - b + b*\text{Cosh}[2*(e + f*x)])^{(3/2)})$

Maple [B] time = 0.226, size = 924, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)

[Out] $1/3*(-8*(-1/a*b)^{(1/2)}*a*b^2*\sinh(f*x+e)^8+16*(-1/a*b)^{(1/2)}*b^3*\sinh(f*x+e)^8+3*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a^2*b*\sinh(f*x+e)^5-16*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a*b^2*\sinh(f*x+e)^5+16*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^3*\sinh(f*x+e)^5+8*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a*b^2*\sinh(f*x+e)^5-16*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*b^3*\sinh(f*x+e)^5-13*(-1/a*b)^{(1/2)}*a^2*b*\sinh(f*x+e)^6+16*(-1/a*b)^{(1/2)}*a*b^2*\sinh(f*x+e)^6+16*(-1/a*b)^{(1/2)}*b^3*\sinh(f*x+e)^6+3*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a^3*\sinh(f*x+e)^3-16*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a^2*b*\sinh(f*x+e)^3+16*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticF}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a*b^2*\sinh(f*x+e)^3+8*((a+b*\sinh(f*x+e)^2)/a)^{(1/2)}*(\cosh(f*x+e)^2)^{(1/2)}*\text{EllipticE}(\sinh(f*x+e)*(-1/a*b)^{(1/2)},(a/b)^{(1/2)})*a^2*b*\sinh(f*x+e)^3-4*(-1/a*b)^{(1/2)}*a^3*\sinh(f*x+e)^4-7*(-1/a*b)^{(1/2)}*a^2*b*\sinh(f*x+e)^4+24*(-1/a*b)^{(1/2)}*a*b^2*\sinh(f*x+e)^4-5*(-1/a*b)^{(1/2)}*a^3*\sinh(f*x+e)^2+6*(-1/a*b)^{(1/2)}*a^2*b*\sinh(f*x+e)^2-(-1/a*b)^{(1/2)}*a^3/(-1/a*b)^{(1/2)}/\sinh(f*x+e)^3/a^4/(a+b*\sinh(f*x+e)^2)^{(3/2)}/\cosh(f*x+e)/f$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^4}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")

[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{\sqrt{b \sinh(fx + e)^2 + a} \coth(fx + e)^4}{b^3 \sinh(fx + e)^6 + 3ab^2 \sinh(fx + e)^4 + 3a^2b \sinh(fx + e)^2 + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx + e)^4}{\left(b \sinh(fx + e)^2 + a\right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.512 $\int (a + b \sinh^2(e + fx))^p (d \tanh(e + fx))^m dx$

Optimal. Leaf size=122

$$\frac{\cosh^2(e + fx)^{\frac{m+1}{2}} (d \tanh(e + fx))^{m+1} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{m+1}{2}; \frac{m+1}{2}, -p; \frac{m+3}{2}; -\sinh^2(e + fx) \right)}{df(m+1)}$$

[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*(Cosh[e + f*x]^2)^((1 + m)/2)*(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rubi [A] time = 0.129349, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3197, 511, 510}

$$\frac{\cosh^2(e + fx)^{\frac{m+1}{2}} (d \tanh(e + fx))^{m+1} (a + b \sinh^2(e + fx))^p \left(\frac{b \sinh^2(e + fx)}{a} + 1 \right)^{-p} F_1 \left(\frac{m+1}{2}; \frac{m+1}{2}, -p; \frac{m+3}{2}; -\sinh^2(e + fx) \right)}{df(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]

[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*(Cosh[e + f*x]^2)^((1 + m)/2)*(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sinh[e + f*x]^2)/a)^p)

Rule 3197

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*((d_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*(d*Tan[e + f*x])^(m + 1)*(Cos[e + f*x]^2)^((m + 1)/2))/(d*f*Sinh[e + f*x]^(m + 1)), Subst[Int[((ff*x)^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] && !InegerQ[m]

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p]]/(1 + (b*x

```

n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

```

Rule 510

```

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

```

Rubi steps

$$\begin{aligned}
 \int (a + b \sinh^2(e + fx))^p (d \tanh(e + fx))^m dx &= \frac{\left(i \cosh^2(e + fx)^{\frac{1+m}{2}} (i \sinh(e + fx))^{-1-m} (d \tanh(e + fx))^{1+m} \right) \text{Subst} \left(\right)}{df} \\
 &= \frac{\left(i \cosh^2(e + fx)^{\frac{1+m}{2}} (i \sinh(e + fx))^{-1-m} (a + b \sinh^2(e + fx))^p \left(1 + \frac{b}{a} \sinh^2(e + fx) \right) \right)}{df} \\
 &= \frac{F_1 \left(\frac{1+m}{2}; \frac{1+m}{2}, -p; \frac{3+m}{2}; -\sinh^2(e + fx), -\frac{b \sinh^2(e + fx)}{a} \right) \cosh^2(e + fx)^{\frac{1+m}{2}}}{df(1 + \frac{b}{a} \sinh^2(e + fx))}
 \end{aligned}$$

Mathematica [F] time = 9.49784, size = 0, normalized size = 0.

$$\int (a + b \sinh^2(e + fx))^p (d \tanh(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]

[Out] Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m, x]

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \left(a + b (\sinh(fx + e))^2 \right)^p (d \tanh(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)`

[Out] `int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p \left(d \tanh(fx + e) \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="maxima")`

[Out] `integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(fx + e)^2 + a\right)^p \left(d \tanh(fx + e)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="fricas")`

[Out] `integral((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(f*x+e)**2)**p*(d*tanh(f*x+e))**m,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(fx + e)^2 + a \right)^p (d \tanh(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="giac")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)

3.513 $\int (a + b \sinh^2(c + dx))^p \tanh^3(c + dx) dx$

Optimal. Leaf size=110

$$\frac{\operatorname{sech}^2(c + dx) (a + b \sinh^2(c + dx))^{p+1}}{2d(a - b)} - \frac{(a - b(p + 1)) (a + b \sinh^2(c + dx))^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{b \sinh^2(c + dx) + a}{a - b}\right)}{2d(p + 1)(a - b)^2}$$

[Out] -((a - b*(1 + p))*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)^2*d*(1 + p)) + (Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d)

Rubi [A] time = 0.120445, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3194, 78, 68}

$$\frac{\operatorname{sech}^2(c + dx) (a + b \sinh^2(c + dx))^{p+1}}{2d(a - b)} - \frac{(a - b(p + 1)) (a + b \sinh^2(c + dx))^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{b \sinh^2(c + dx) + a}{a - b}\right)}{2d(p + 1)(a - b)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]

[Out] -((a - b*(1 + p))*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)^2*d*(1 + p)) + (Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d)

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_)*tan[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 78

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> -Simp[(b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],

x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

Rule 68

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rubi steps

$$\begin{aligned} \int (a + b \sinh^2(c + dx))^p \tanh^3(c + dx) dx &= \frac{\text{Subst}\left(\int \frac{x^{(a+bx)^p}}{(1+x)^2} dx, x, \sinh^2(c + dx)\right)}{2d} \\ &= \frac{\text{sech}^2(c + dx) (a + b \sinh^2(c + dx))^{1+p}}{2(a - b)d} - \frac{(a - b(1 + p)) \text{Subst}\left(\int \frac{(a+bx)^p}{1+x} dx\right)}{2(-a + b)d} \\ &= -\frac{(a - b(1 + p)) {}_2F_1\left(1, 1 + p; 2 + p; \frac{a + b \sinh^2(c + dx)}{a - b}\right) (a + b \sinh^2(c + dx))^{1+p}}{2(a - b)^2 d(1 + p)} \end{aligned}$$

Mathematica [A] time = 0.238538, size = 90, normalized size = 0.82

$$\frac{(a + b \sinh^2(c + dx))^{p+1} \left((-a + bp + b) {}_2F_1\left(1, p + 1; p + 2; \frac{b \sinh^2(c + dx) + a}{a - b}\right) + (p + 1)(a - b) \text{sech}^2(c + dx) \right)}{2d(p + 1)(a - b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]

[Out] (((-a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)] + (a - b)*(1 + p)*Sech[c + d*x]^2)*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)^2*d*(1 + p))

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int (a + b(\sinh(dx + c))^2)^p (\tanh(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)`

[Out] `int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \tanh(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="maxima")`

[Out] `integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(dx + c)^2 + a\right)^p \tanh(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="fricas")`

[Out] `integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**3,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \tanh(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="giac")

[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)

3.514 $\int (a + b \sinh^2(c + dx))^p \tanh(c + dx) dx$

Optimal. Leaf size=63

$$\frac{(a + b \sinh^2(c + dx))^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{b \sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)}$$

[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d*(1 + p))

Rubi [A] time = 0.0547016, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3194, 68}

$$\frac{(a + b \sinh^2(c + dx))^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{b \sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x], x]

[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d*(1 + p))

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 68

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Simp[((b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -(d*(a + b*x))/(b*c - a*d)]]/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]

Rubi steps

$$\int (a + b \sinh^2(c + dx))^p \tanh(c + dx) dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x} dx, x, \sinh^2(c + dx)\right)}{2d}$$

$$= -\frac{{}_2F_1\left(1, 1 + p; 2 + p; \frac{a+b \sinh^2(c+dx)}{a-b}\right) (a + b \sinh^2(c + dx))^{1+p}}{2(a-b)d(1+p)}$$

Mathematica [A] time = 0.070719, size = 65, normalized size = 1.03

$$-\frac{(a + b \cosh^2(c + dx) - b)^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{b \cosh^2(c+dx)}{a-b} + 1\right)}{2d(p+1)(a-b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x], x]

[Out] -((a - b + b*Cosh[c + d*x]^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Cosh[c + d*x]^2)/(a - b)]/(2*(a - b)*d*(1 + p))

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int (a + b (\sinh(dx + c))^2)^p \tanh(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c), x)

[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \tanh(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="maxima")

[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(dx + c)^2 + a\right)^p \tanh(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="fricas")

[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(dx + c)^2 + a\right)^p \tanh(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="giac")

[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)

$$3.515 \quad \int \coth(c + dx) \left(a + b \sinh^2(c + dx) \right)^p dx$$

Optimal. Leaf size=54

$$\frac{\left(a + b \sinh^2(c + dx) \right)^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{b \sinh^2(c + dx)}{a} + 1 \right)}{2ad(p + 1)}$$

[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a*d*(1 + p))

Rubi [A] time = 0.0632654, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3194, 65}

$$\frac{\left(a + b \sinh^2(c + dx) \right)^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{b \sinh^2(c + dx)}{a} + 1 \right)}{2ad(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]

[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a*d*(1 + p))

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 65

Int[((b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c]/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rubi steps

$$\int \coth(c + dx) (a + b \sinh^2(c + dx))^p dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x} dx, x, \sinh^2(c + dx)\right)}{2d}$$

$$= -\frac{{}_2F_1\left(1, 1 + p; 2 + p; 1 + \frac{b \sinh^2(c+dx)}{a}\right) (a + b \sinh^2(c + dx))^{1+p}}{2ad(1 + p)}$$

Mathematica [A] time = 0.0602517, size = 54, normalized size = 1.

$$-\frac{(a + b \sinh^2(c + dx))^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{b \sinh^2(c+dx)}{a} + 1\right)}{2ad(p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]

[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a*d*(1 + p))

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int \coth(dx + c) (a + b (\sinh(dx + c))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)

[Out] int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \coth(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")

[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(dx + c)^2 + a\right)^p \coth(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")

[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)**2)**p,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh(dx + c)^2 + a\right)^p \coth(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)

$$3.516 \quad \int \coth^3(c + dx) \left(a + b \sinh^2(c + dx) \right)^p dx$$

Optimal. Leaf size=94

$$\frac{(a + bp) \left(a + b \sinh^2(c + dx) \right)^{p+1} {}_2F_1 \left(1, p + 1; p + 2; \frac{b \sinh^2(c + dx)}{a} + 1 \right)}{2a^2 d (p + 1)} - \frac{\operatorname{csch}^2(c + dx) \left(a + b \sinh^2(c + dx) \right)^{p+1}}{2ad}$$

[Out] -(Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a*d) - ((a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a^2*d*(1 + p))

Rubi [A] time = 0.0907626, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3194, 78, 65}

$$\frac{(a + bp) \left(a + b \sinh^2(c + dx) \right)^{p+1} {}_2F_1 \left(1, p + 1; p + 2; \frac{b \sinh^2(c + dx)}{a} + 1 \right)}{2a^2 d (p + 1)} - \frac{\operatorname{csch}^2(c + dx) \left(a + b \sinh^2(c + dx) \right)^{p+1}}{2ad}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]

[Out] -(Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a*d) - ((a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a^2*d*(1 + p))

Rule 3194

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)]^2)^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p]/(1 - ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]

Rule 78

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],

x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 65

Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])

Rubi steps

$$\begin{aligned} \int \coth^3(c + dx) (a + b \sinh^2(c + dx))^p dx &= \frac{\text{Subst}\left(\int \frac{(1+x)(a+bx)^p}{x^2} dx, x, \sinh^2(c + dx)\right)}{2d} \\ &= -\frac{\text{csch}^2(c + dx) (a + b \sinh^2(c + dx))^{1+p}}{2ad} + \frac{(a + bp) \text{Subst}\left(\int \frac{(a+bx)^p}{x} dx, x, \sinh^2(c + dx)\right)}{2ad} \\ &= -\frac{\text{csch}^2(c + dx) (a + b \sinh^2(c + dx))^{1+p}}{2ad} - \frac{(a + bp) {}_2F_1\left(1, 1 + p; 2 + p; 1 + \frac{b \sinh^2(c + dx)}{a}\right)}{2ad} \end{aligned}$$

Mathematica [A] time = 0.379479, size = 71, normalized size = 0.76

$$-\frac{(a + b \sinh^2(c + dx))^{p+1} \left(\frac{(a+bp) {}_2F_1\left(1, p+1; p+2, \frac{b \sinh^2(c+dx)}{a} + 1\right)}{p+1} + \text{acsch}^2(c + dx) \right)}{2a^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]

[Out] -((a*Csch[c + d*x]^2 + ((a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a])/(1 + p))*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a^2*d)

Maple [F] time = 0.265, size = 0, normalized size = 0.

$$\int (\coth(dx + c))^3 (a + b(\sinh(dx + c))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)`

[Out] `int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \coth(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(dx + c)^2 + a\right)^p \coth(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**3*(a+b*sinh(d*x+c)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \coth(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")

[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)

3.517 $\int (a + b \sinh^2(c + dx))^p \tanh^4(c + dx) dx$

Optimal. Leaf size=103

$$\frac{\sinh^4(c + dx) \sqrt{\cosh^2(c + dx)} \tanh(c + dx) (a + b \sinh^2(c + dx))^p \left(\frac{b \sinh^2(c + dx)}{a} + 1 \right)^{-p} F_1\left(\frac{5}{2}; \frac{5}{2}, -p; \frac{7}{2}; -\sinh^2(c + dx), -\frac{b}{a}\right)}{5d}$$

[Out] (AppellF1[5/2, 5/2, -p, 7/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sqrt[Cosh[c + d*x]^2]*Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x])/(5*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)

Rubi [A] time = 0.12086, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3196, 511, 510}

$$\frac{\sinh^4(c + dx) \sqrt{\cosh^2(c + dx)} \tanh(c + dx) (a + b \sinh^2(c + dx))^p \left(\frac{b \sinh^2(c + dx)}{a} + 1 \right)^{-p} F_1\left(\frac{5}{2}; \frac{5}{2}, -p; \frac{7}{2}; -\sinh^2(c + dx), -\frac{b}{a}\right)}{5d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4,x]

[Out] (AppellF1[5/2, 5/2, -p, 7/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sqrt[Cosh[c + d*x]^2]*Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x])/(5*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&

NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int (a + b \sinh^2(c + dx))^p \tanh^4(c + dx) dx &= \frac{\left(\sqrt{\cosh^2(c + dx) \operatorname{sech}(c + dx)}\right) \operatorname{Subst}\left(\int \frac{x^4(a + bx^2)^p}{(1 + x^2)^{5/2}} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\left(\sqrt{\cosh^2(c + dx) \operatorname{sech}(c + dx)} (a + b \sinh^2(c + dx))^p \left(1 + \frac{b \sinh^2(c + dx)}{a}\right)^{-p}\right)}{d} \\ &= \frac{F_1\left(\frac{5}{2}; \frac{5}{2}, -p; \frac{7}{2}; -\sinh^2(c + dx), -\frac{b \sinh^2(c + dx)}{a}\right) \sqrt{\cosh^2(c + dx) \sinh^4(c + dx)}}{5d} \end{aligned}$$

Mathematica [F] time = 7.10546, size = 0, normalized size = 0.

$$\int (a + b \sinh^2(c + dx))^p \tanh^4(c + dx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4, x]

[Out] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4, x]

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int (a + b (\sinh(dx + c))^2)^p (\tanh(dx + c))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

```
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \tanh(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(dx + c)^2 + a\right)^p \tanh(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \tanh(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

3.518 $\int (a + b \sinh^2(c + dx))^p \tanh^2(c + dx) dx$

Optimal. Leaf size=103

$$\frac{\sinh^2(c + dx) \sqrt{\cosh^2(c + dx)} \tanh(c + dx) (a + b \sinh^2(c + dx))^p \left(\frac{b \sinh^2(c + dx)}{a} + 1 \right)^{-p} F_1 \left(\frac{3}{2}; \frac{3}{2}, -p; \frac{5}{2}; -\sinh^2(c + dx), -\frac{b}{a} \right)}{3d}$$

[Out] (AppellF1[3/2, 3/2, -p, 5/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sqrt[Cosh[c + d*x]^2]*Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x])/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)

Rubi [A] time = 0.107292, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3196, 511, 510}

$$\frac{\sinh^2(c + dx) \sqrt{\cosh^2(c + dx)} \tanh(c + dx) (a + b \sinh^2(c + dx))^p \left(\frac{b \sinh^2(c + dx)}{a} + 1 \right)^{-p} F_1 \left(\frac{3}{2}; \frac{3}{2}, -p; \frac{5}{2}; -\sinh^2(c + dx), -\frac{b}{a} \right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]

[Out] (AppellF1[3/2, 3/2, -p, 5/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sqrt[Cosh[c + d*x]^2]*Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x])/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_)*tan[(e_) + (f_)*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&

NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int (a + b \sinh^2(c + dx))^p \tanh^2(c + dx) dx &= \frac{\left(\sqrt{\cosh^2(c + dx)\operatorname{sech}(c + dx)}\right) \operatorname{Subst}\left(\int \frac{x^2(a+bx^2)^p}{(1+x^2)^{3/2}} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\left(\sqrt{\cosh^2(c + dx)\operatorname{sech}(c + dx)} (a + b \sinh^2(c + dx))^p \left(1 + \frac{b \sinh^2(c + dx)}{a}\right)^{-p}\right)}{d} \\ &= \frac{F_1\left(\frac{3}{2}; \frac{3}{2}, -p; \frac{5}{2}; -\sinh^2(c + dx), -\frac{b \sinh^2(c + dx)}{a}\right) \sqrt{\cosh^2(c + dx) \sinh^2(c + dx)}}{3d} \end{aligned}$$

Mathematica [F] time = 5.42295, size = 0, normalized size = 0.

$$\int (a + b \sinh^2(c + dx))^p \tanh^2(c + dx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]

[Out] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2, x]

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int (a + b (\sinh(dx + c))^2)^p (\tanh(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)`

[Out] `int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \tanh(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="maxima")`

[Out] `integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(dx + c)^2 + a\right)^p \tanh(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="fricas")`

[Out] `integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**2,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \tanh(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

3.519 $\int \coth^2(c + dx) (a + b \sinh^2(c + dx))^p dx$

Optimal. Leaf size=99

$$\frac{\sqrt{\cosh^2(c + dx) \operatorname{csch}(c + dx) \operatorname{sech}(c + dx)} (a + b \sinh^2(c + dx))^p \left(\frac{b \sinh^2(c + dx)}{a} + 1 \right)^{-p} F_1 \left(-\frac{1}{2}; -\frac{1}{2}, -p; \frac{1}{2}; -\sinh^2(c + dx) \right)}{d}$$

[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^p)/(d*(1 + (b*Sinh[c + d*x]^2)/a)^p))

Rubi [A] time = 0.101677, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3196, 511, 510}

$$\frac{\sqrt{\cosh^2(c + dx) \operatorname{csch}(c + dx) \operatorname{sech}(c + dx)} (a + b \sinh^2(c + dx))^p \left(\frac{b \sinh^2(c + dx)}{a} + 1 \right)^{-p} F_1 \left(-\frac{1}{2}; -\frac{1}{2}, -p; \frac{1}{2}; -\sinh^2(c + dx) \right)}{d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]

[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^p)/(d*(1 + (b*Sinh[c + d*x]^2)/a)^p))

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&

NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 510

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \coth^2(c + dx) (a + b \sinh^2(c + dx))^p dx &= \frac{\left(\sqrt{\cosh^2(c + dx) \operatorname{sech}(c + dx)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}(a+bx^2)^p dx, x, \sinh(c + dx)}{d}\right)}{d} \\ &= \frac{\left(\sqrt{\cosh^2(c + dx) \operatorname{sech}(c + dx)} (a + b \sinh^2(c + dx))^p \left(1 + \frac{b \sinh^2(c + dx)}{a}\right)^{-p}\right)}{d} \\ &= \frac{F_1\left(-\frac{1}{2}; -\frac{1}{2}, -p; \frac{1}{2}; -\sinh^2(c + dx), -\frac{b \sinh^2(c + dx)}{a}\right) \sqrt{\cosh^2(c + dx) \operatorname{csch}(c + dx)}}{d} \end{aligned}$$

Mathematica [F] time = 6.36908, size = 0, normalized size = 0.

$$\int \coth^2(c + dx) (a + b \sinh^2(c + dx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]

[Out] Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p, x]

Maple [F] time = 0.218, size = 0, normalized size = 0.

$$\int (\coth(dx + c))^2 (a + b(\sinh(dx + c))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)`

[Out] `int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \coth(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(dx + c)^2 + a\right)^p \coth(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**2*(a+b*sinh(d*x+c)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \coth(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

3.520 $\int \coth^4(c + dx) (a + b \sinh^2(c + dx))^p dx$

Optimal. Leaf size=103

$$\frac{\sqrt{\cosh^2(c + dx)} \operatorname{csch}^3(c + dx) \operatorname{sech}(c + dx) (a + b \sinh^2(c + dx))^p \left(\frac{b \sinh^2(c + dx)}{a} + 1 \right)^{-p} F_1 \left(-\frac{3}{2}; -\frac{3}{2}, -p; -\frac{1}{2}; -\sinh^2(c + dx) \right)}{3d}$$

[Out] -(AppellF1[-3/2, -3/2, -p, -1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]^3*Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^p)/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)

Rubi [A] time = 0.101907, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$, Rules used = {3196, 511, 510}

$$\frac{\sqrt{\cosh^2(c + dx)} \operatorname{csch}^3(c + dx) \operatorname{sech}(c + dx) (a + b \sinh^2(c + dx))^p \left(\frac{b \sinh^2(c + dx)}{a} + 1 \right)^{-p} F_1 \left(-\frac{3}{2}; -\frac{3}{2}, -p; -\frac{1}{2}; -\sinh^2(c + dx) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]

[Out] -(AppellF1[-3/2, -3/2, -p, -1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]^3*Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^p)/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)

Rule 3196

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m + 1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]

Rule 511

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&

NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

Rule 510

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)]/(e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \coth^4(c + dx) (a + b \sinh^2(c + dx))^p dx &= \frac{\left(\sqrt{\cosh^2(c + dx)} \operatorname{sech}(c + dx)\right) \operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2} (a+bx^2)^p}{x^4} dx, x, \sinh(c + dx)\right)}{d} \\ &= \frac{\left(\sqrt{\cosh^2(c + dx)} \operatorname{sech}(c + dx) (a + b \sinh^2(c + dx))^p \left(1 + \frac{b \sinh^2(c + dx)}{a}\right)^{-p}\right)}{d} \\ &= \frac{F_1\left(-\frac{3}{2}; -\frac{3}{2}, -p; -\frac{1}{2}; -\sinh^2(c + dx), -\frac{b \sinh^2(c + dx)}{a}\right) \sqrt{\cosh^2(c + dx)} \operatorname{csch}^3(c + dx)}{3d} \end{aligned}$$

Mathematica [F] time = 6.63468, size = 0, normalized size = 0.

$$\int \coth^4(c + dx) (a + b \sinh^2(c + dx))^p dx$$

Verification is Not applicable to the result.

[In] Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]

[Out] Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p, x]

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int (\coth(dx + c))^4 (a + b(\sinh(dx + c))^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)`

[Out] `int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \coth(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")`

[Out] `integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh(dx + c)^2 + a\right)^p \coth(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")`

[Out] `integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(d*x+c)**4*(a+b*sinh(d*x+c)**2)**p,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sinh(dx + c)^2 + a)^p \coth(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

$$3.521 \quad \int \frac{\coth^3(x)}{a+b \sinh^3(x)} dx$$

Optimal. Leaf size=152

$$\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \sinh(x) + b^{2/3} \sinh^2(x))}{6a^{5/3}} - \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} \sinh(x))}{3a^{5/3}} + \frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{b}\sinh(x)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} - \frac{\log(a + b \sinh^3(x))}{3a}$$

[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sinh[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) - Csch[x]^2/(2*a) + Log[Sinh[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3)*Sinh[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sinh[x] + b^(2/3)*Sinh[x]^2])/(6*a^(5/3)) - Log[a + b*Sinh[x]^3]/(3*a)

Rubi [A] time = 0.251657, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3230, 1834, 1871, 12, 200, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \sinh(x) + b^{2/3} \sinh^2(x))}{6a^{5/3}} - \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} \sinh(x))}{3a^{5/3}} + \frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a-2\sqrt[3]{b}\sinh(x)}}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}} - \frac{\log(a + b \sinh^3(x))}{3a}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]^3/(a + b*Sinh[x]^3),x]

[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sinh[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(5/3)) - Csch[x]^2/(2*a) + Log[Sinh[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3)*Sinh[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sinh[x] + b^(2/3)*Sinh[x]^2])/(6*a^(5/3)) - Log[a + b*Sinh[x]^3]/(3*a)

Rule 3230

Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))^(p_.)*tan[(e_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p]/(1 - ff^2*x^2)^(m + 1)/2), x], x, Sin[e + f*x]/ff], x] /; FreeQ[{a, b, c, e, f, n, p}, x] && LtQ[(m - 1)/2, 0]

Rule 1834

```
Int[((Pq_)*((c_)*(x_)^(m_)))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Dist[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] || !RationalQ[a/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_)*(x_)^3)^(-1), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_)*(x_)^(-1)), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(x)}{a + b \sinh^3(x)} dx &= \text{Subst} \left(\int \frac{1 + x^2}{x^3 (a + bx^3)} dx, x, \sinh(x) \right) \\
&= \text{Subst} \left(\int \left(\frac{1}{ax^3} + \frac{1}{ax} + \frac{-b - bx^2}{a(a + bx^3)} \right) dx, x, \sinh(x) \right) \\
&= -\frac{\text{csch}^2(x)}{2a} + \frac{\log(\sinh(x))}{a} + \frac{\text{Subst} \left(\int \frac{-b - bx^2}{a + bx^3} dx, x, \sinh(x) \right)}{a} \\
&= -\frac{\text{csch}^2(x)}{2a} + \frac{\log(\sinh(x))}{a} - \frac{\text{Subst} \left(\int \frac{b}{a + bx^3} dx, x, \sinh(x) \right)}{a} - \frac{b \text{Subst} \left(\int \frac{x^2}{a + bx^3} dx, x, \sinh(x) \right)}{a} \\
&= -\frac{\text{csch}^2(x)}{2a} + \frac{\log(\sinh(x))}{a} - \frac{\log(a + b \sinh^3(x))}{3a} - \frac{b \text{Subst} \left(\int \frac{1}{a + bx^3} dx, x, \sinh(x) \right)}{a} \\
&= -\frac{\text{csch}^2(x)}{2a} + \frac{\log(\sinh(x))}{a} - \frac{\log(a + b \sinh^3(x))}{3a} - \frac{b \text{Subst} \left(\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx, x, \sinh(x) \right)}{3a^{5/3}} - \frac{b \text{Subst} \left(\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx, x, \sinh(x) \right)}{3a^{5/3}} \\
&= -\frac{\text{csch}^2(x)}{2a} + \frac{\log(\sinh(x))}{a} - \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} \sinh(x))}{3a^{5/3}} - \frac{\log(a + b \sinh^3(x))}{3a} + \frac{b^{2/3} \text{Subst} \left(\int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx, x, \sinh(x) \right)}{3a^{5/3}} \\
&= -\frac{\text{csch}^2(x)}{2a} + \frac{\log(\sinh(x))}{a} - \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} \sinh(x))}{3a^{5/3}} + \frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \sinh(x) + b^{2/3})}{6a^{5/3}} \\
&= \frac{b^{2/3} \tan^{-1} \left(\frac{1 - \frac{2 \sqrt[3]{b} \sinh(x)}{\sqrt[3]{a}}}{\sqrt{3}} \right)}{\sqrt{3} a^{5/3}} - \frac{\text{csch}^2(x)}{2a} + \frac{\log(\sinh(x))}{a} - \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} \sinh(x))}{3a^{5/3}} + \frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} \sinh(x) + b^{2/3})}{6a^{5/3}}
\end{aligned}$$

Mathematica [A] time = 0.338182, size = 136, normalized size = 0.89

$$\frac{(a^{2/3} + (-1)^{2/3} b^{2/3}) \log(-(-1)^{2/3} \sqrt[3]{a} - \sqrt[3]{b} \sinh(x)) + (a^{2/3} + b^{2/3}) \log(\sqrt[3]{a} + \sqrt[3]{b} \sinh(x)) + (a^{2/3} - \sqrt[3]{-1} b^{2/3}) \log(\sqrt[3]{a} + \sqrt[3]{-1} b \sinh(x))}{3a^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]^3/(a + b*Sinh[x]^3), x]

[Out] -Csch[x]^2/(2*a) + Log[Sinh[x]]/a - ((a^(2/3) + (-1)^(2/3)*b^(2/3))*Log[-((-1)^(2/3)*a^(1/3) - b^(1/3)*Sinh[x]] + (a^(2/3) + b^(2/3))*Log[a^(1/3) + b^(1/3)*Sinh[x]] + (a^(2/3) - (-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b^(1/3)*Sinh[x]])/(3*a^(5/3))

$$\frac{\sqrt[3]{1/3} \operatorname{Sinh}[x]}{(3a^{5/3})}$$

Maple [C] time = 0.06, size = 132, normalized size = 0.9

$$-\frac{1}{8a} \left(\tanh\left(\frac{x}{2}\right) \right)^2 - \frac{1}{8a} \left(\tanh\left(\frac{x}{2}\right) \right)^{-2} + \frac{1}{a} \ln\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{3a} \sum_{\substack{_R=\text{RootOf}(a_Z^6-3a_Z^4-8b_Z^3+3a_Z^2-a)}} \frac{-_R^5 a - _R^4 b + 2 _R^3 a}{-_R^5 a - 2 _R^3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)^3/(a+b*sinh(x)^3),x)`

[Out] `-1/8/a*tanh(1/2*x)^2-1/8/a/tanh(1/2*x)^2+1/a*ln(tanh(1/2*x))+1/3/a*sum((-_R^5*a-_R^4*b+2*_R^3*a+4*_R^2*b-_R*a+b)/(-_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))`

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="maxima")`

[Out] Timed out

Fricas [C] time = 11.7566, size = 2762, normalized size = 18.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="fricas")`

[Out] `-1/12*(12*sqrt(1/3)*(a*e^(4*x) - 2*a*e^(2*x) + a)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a +`

$$\begin{aligned}
& 4/a^2) * \arctan(-1/8 * (2 * \sqrt{1/3}) * \sqrt{((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a})^2 * a^4 * e^{(2*x)} + b^2 * e^{(4*x)} + 2 * a * b * e^{(3*x)} - 2 * a * b * e^x - (a^2 * b * e^{(3*x)} + 4 * a^3 * e^{(2*x)} - a^2 * b * e^x) * ((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a) + b^2 + 2 * (2 * a^2 - b^2) * e^{(2*x)} * (((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a) * a^3 - 2 * a^2 * \sqrt{(((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a)^2 * a^2 - 4 * ((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a) * a + 4) / a^2 + \sqrt{1/3} * (((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a)^2 * a^5 * e^x + 4 * a^2 * b * e^{(2*x)} + 4 * a^3 * e^x - 4 * a^2 * b - 2 * (a^3 * b * e^{(2*x)} + 2 * a^4 * e^x - a^3 * b) * ((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a) * \sqrt{(((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a)^2 * a^2 - 4 * ((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a) * a + 4) / a^2) * e^{(-x) / b^2} + 2 * (a * e^{(4*x)} - 2 * a * e^{(2*x)} + a) * ((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a) * \log(((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a) * a^2 * e^x + b * e^{(2*x)} - 2 * a * e^x - b) - ((a * e^{(4*x)} - 2 * a * e^{(2*x)} + a) * ((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a) - 6 * e^{(4*x)} + 12 * e^{(2*x)} - 6) * \log(((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a)^2 * a^4 * e^{(2*x)} + b^2 * e^{(4*x)} + 2 * a * b * e^{(3*x)} - 2 * a * b * e^x - (a^2 * b * e^{(3*x)} + 4 * a^3 * e^{(2*x)} - a^2 * b * e^x) * ((1/2)^{1/3} * (I * \sqrt{3}) + 1) * (1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^{1/3} + 2/a) + b^2 + 2 * (2 * a^2 - b^2) * e^{(2*x)} - 12 * (e^{(4*x)} - 2 * e^{(2*x)} + 1) * \log(e^{(2*x)} - 1) + 24 * e^{(2*x)} / (a * e^{(4*x)} - 2 * a * e^{(2*x)} + a)
\end{aligned}$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)**3/(a+b*sinh(x)**3),x)

[Out] Timed out

Giac [A] time = 1.23755, size = 282, normalized size = 1.86

$$\frac{b \left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|-2 \left(-\frac{a}{b}\right)^{\frac{1}{3}} - e^{(-x)} + e^x\right|\right)}{3 a^2} - \frac{\log\left(\left|-b(e^{(-x)} - e^x)^3 + 8 a\right|\right)}{3 a} + \frac{\log\left(\left|-e^{(-x)} + e^x\right|\right)}{a} - \frac{\sqrt{3}(-ab^2)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}\left(-\frac{a}{b}\right)^{\frac{1}{3}}}{3(-\frac{a}{b})}\right)}{3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="giac")

[Out] 1/3*b*(-a/b)^(1/3)*log(abs(-2*(-a/b)^(1/3) - e^(-x) + e^x))/a^2 - 1/3*log(abs(-b*(e^(-x) - e^x)^3 + 8*a))/a + log(abs(-e^(-x) + e^x))/a - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) - e^(-x) + e^x)/(-a/b)^(1/3))/a^2 - 1/6*(-a*b^2)^(1/3)*log((e^(-x) - e^x)^2 - 2*(-a/b)^(1/3)*(e^(-x) - e^x) + 4*(-a/b)^(2/3))/a^2 - 1/2*(3*(e^(-x) - e^x)^2 + 4)/(a*(e^(-x) - e^x)^2)

$$3.522 \quad \int \frac{\coth(x)}{\sqrt{a+b \sinh^3(x)}} dx$$

Optimal. Leaf size=28

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b \sinh^3(x)}}{\sqrt{a}} \right)}{3\sqrt{a}}$$

[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])

Rubi [A] time = 0.0763304, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3230, 266, 63, 208}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b \sinh^3(x)}}{\sqrt{a}} \right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]^3], x]

[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])

Rule 3230

Int[((a_) + (b_)*((c_)*sin[(e_) + (f_)*(x_)])^(n_))^(p_)*tan[(e_) + (f_)*(x_)]^(m_), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p]/(1 - ff^2*x^2)^(m + 1)/2], x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2, 0]

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{a + b \sinh^3(x)}} dx &= \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx^3}} dx, x, \sinh(x) \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx}} dx, x, \sinh^3(x) \right) \\
&= \frac{2 \text{Subst} \left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^3(x)} \right)}{3b} \\
&= -\frac{2 \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^3(x)}}{\sqrt{a}} \right)}{3\sqrt{a}}
\end{aligned}$$

Mathematica [A] time = 0.0235918, size = 28, normalized size = 1.

$$-\frac{2 \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^3(x)}}{\sqrt{a}} \right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^3], x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Maple [A] time = 0.214, size = 21, normalized size = 0.8

$$-\frac{2}{3} \operatorname{Arctanh} \left(\sqrt{a + b (\sinh(x))^3} \frac{1}{\sqrt{a}} \right) \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)/(a+b*sinh(x)^3)^(1/2),x)

[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))/a^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b \sinh(x)^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")

[Out] integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a + b \sinh^3(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)**3)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b \sinh(x)^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)
```

$$3.523 \quad \int \coth(x) \sqrt{a + b \sinh^3(x)} dx$$

Optimal. Leaf size=45

$$\frac{2}{3} \sqrt{a + b \sinh^3(x)} - \frac{2}{3} \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^3(x)}}{\sqrt{a}} \right)$$

[Out] $(-2*\text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[x]^3]/\text{Sqrt}[a]])/3 + (2*\text{Sqrt}[a + b*\text{Sinh}[x]^3])/3$

Rubi [A] time = 0.0822686, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2}{3} \sqrt{a + b \sinh^3(x)} - \frac{2}{3} \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^3(x)}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[x]*\text{Sqrt}[a + b*\text{Sinh}[x]^3], x]$

[Out] $(-2*\text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[x]^3]/\text{Sqrt}[a]])/3 + (2*\text{Sqrt}[a + b*\text{Sinh}[x]^3])/3$

Rule 3230

$\text{Int}[(a_ + (b_)*((c_)*\sin[e_ + (f_)*(x_)]))^{(n_)}]^{(p_)}*\tan[e_ + (f_)*(x_)]^{(m_)}, x_Symbol] \rightarrow \text{With}[\{ff = \text{FreeFactors}[\text{Sin}[e + f*x], x]\}, \text{Dist}[ff^{(m + 1)}/f, \text{Subst}[\text{Int}[(x^m*(a + b*(c*ff*x)^n)^p]/(1 - ff^2*x^2)^{(m + 1)/2}, x], x, \text{Sin}[e + f*x]/ff], x]] \text{ /; FreeQ}\{a, b, c, e, f, n, p\}, x] \ \&\& \ \text{I} \ \text{LtQ}[(m - 1)/2, 0]$

Rule 266

$\text{Int}[(x_)^{(m_)}*(a_ + (b_)*(x_)^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p}, x], x, x^n], x] \text{ /; FreeQ}\{a, b, m, n, p\}, x] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \coth(x) \sqrt{a + b \sinh^3(x)} dx &= \text{Subst} \left(\int \frac{\sqrt{a + bx^3}}{x} dx, x, \sinh(x) \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \frac{\sqrt{a + bx}}{x} dx, x, \sinh^3(x) \right) \\
&= \frac{2}{3} \sqrt{a + b \sinh^3(x)} + \frac{1}{3} a \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx}} dx, x, \sinh^3(x) \right) \\
&= \frac{2}{3} \sqrt{a + b \sinh^3(x)} + \frac{(2a) \text{Subst} \left(\int \frac{1}{\frac{a}{-b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^3(x)} \right)}{3b} \\
&= -\frac{2}{3} \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a + b \sinh^3(x)}}{\sqrt{a}} \right) + \frac{2}{3} \sqrt{a + b \sinh^3(x)}
\end{aligned}$$

Mathematica [A] time = 0.0225346, size = 45, normalized size = 1.

$$\frac{2}{3}\sqrt{a + b \sinh^3(x)} - \frac{2}{3}\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^3],x]

[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sinh[x]^3])/3

Maple [A] time = 0.042, size = 34, normalized size = 0.8

$$-\frac{2}{3}\operatorname{Artanh}\left(\sqrt{a + b(\sinh(x))^3}\frac{1}{\sqrt{a}}\right)\sqrt{a} + \frac{2}{3}\sqrt{a + b(\sinh(x))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(a+b*sinh(x)^3)^(1/2),x)

[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sinh(x)^3)^(1/2)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(x)^3 + a} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)

Fricas [B] time = 15.5511, size = 5184, normalized size = 115.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")

[Out]
$$\frac{1}{6} \sqrt{a} (\cosh(x) + \sinh(x)) \log(-b^2 \cosh(x)^{12} + 12b^2 \cosh(x) \sinh(x)^{11} + b^2 \sinh(x)^{12} - 6b^2 \cosh(x)^{10} + 64ab \cosh(x)^9 + 6(11b^2 \cosh(x)^2 - b^2) \sinh(x)^{10} + 15b^2 \cosh(x)^8 + 4(55b^2 \cosh(x)^3 - 15b^2 \cosh(x) + 16ab) \sinh(x)^9 - 192ab \cosh(x)^7 + 3(165b^2 \cosh(x)^4 - 90b^2 \cosh(x)^2 + 192ab \cosh(x) + 5b^2) \sinh(x)^8 + 24(33b^2 \cosh(x)^5 - 30b^2 \cosh(x)^3 + 96ab \cosh(x)^2 + 5b^2 \cosh(x) - 8ab) \sinh(x)^7 + 192ab \cosh(x)^5 + 4(128a^2 - 5b^2) \cosh(x)^6 + 4(231b^2 \cosh(x)^6 - 315b^2 \cosh(x)^4 + 1344ab \cosh(x)^3 + 105b^2 \cosh(x)^2 - 336ab \cosh(x) + 128a^2 - 5b^2) \sinh(x)^6 + 15b^2 \cosh(x)^4 + 24(33b^2 \cosh(x)^7 - 63b^2 \cosh(x)^5 + 336ab \cosh(x)^4 + 35b^2 \cosh(x)^3 - 168ab \cosh(x)^2 + 8ab + (128a^2 - 5b^2) \cosh(x)) \sinh(x)^5 - 64ab \cosh(x)^3 + 3(165b^2 \cosh(x)^8 - 420b^2 \cosh(x)^6 + 2688ab \cosh(x)^5 + 350b^2 \cosh(x)^4 - 2240ab \cosh(x)^3 + 320ab \cosh(x) + 20(128a^2 - 5b^2) \cosh(x)^2 + 5b^2) \sinh(x)^4 - 6b^2 \cosh(x)^2 + 4(55b^2 \cosh(x)^9 - 180b^2 \cosh(x)^7 + 1344ab \cosh(x)^6 + 210b^2 \cosh(x)^5 - 1680ab \cosh(x)^4 + 480ab \cosh(x)^2 + 20(128a^2 - 5b^2) \cosh(x)^3 + 15b^2 \cosh(x) - 16ab) \sinh(x)^3 + 6(11b^2 \cosh(x)^{10} - 45b^2 \cosh(x)^8 + 384ab \cosh(x)^7 + 70b^2 \cosh(x)^6 - 672ab \cosh(x)^5 + 320ab \cosh(x)^3 + 10(128a^2 - 5b^2) \cosh(x)^4 + 15b^2 \cosh(x)^2 - 32ab \cosh(x) - b^2) \sinh(x)^2 + b^2 - 16(b \cosh(x)^8 + 8b \cosh(x) \sinh(x)^7 + b \sinh(x)^8 - 3b \cosh(x)^6 + (28b \cosh(x)^2 - 3b) \sinh(x)^6 + 16a \cosh(x)^5 + 2(28b \cosh(x)^3 - 9b \cosh(x) + 8a) \sinh(x)^5 + 3b \cosh(x)^4 + (70b \cosh(x)^4 - 45b \cosh(x)^2 + 80a \cosh(x) + 3b) \sinh(x)^4 + 4(14b \cosh(x)^5 - 15b \cosh(x)^3 + 40a \cosh(x)^2 + 3b \cosh(x)) \sinh(x)^3 - b \cosh(x)^2 + (28b \cosh(x)^6 - 45b \cosh(x)^4 + 160a \cosh(x)^3 + 18b \cosh(x)^2 - b) \sinh(x)^2 + 2(4b \cosh(x)^7 - 9b \cosh(x)^5 + 40a \cosh(x)^4 + 6b \cosh(x)^3 - b \cosh(x)) \sinh(x)) \sqrt{a} \sqrt{(b \sinh(x)^3 + 3(b \cosh(x)^2 - b) \sinh(x) + 4a) / (\cosh(x)^2 - 2 \cosh(x) \sinh(x) + \sinh(x)^2)} + 12(b^2 \cosh(x)^{11} - 5b^2 \cosh(x)^9 + 48ab \cosh(x)^8 + 10b^2 \cosh(x)^7 - 112ab \cosh(x)^6 + 80ab \cosh(x)^4 + 2(128a^2 - 5b^2) \cosh(x)^5 + 5b^2 \cosh(x)^3 - 16ab \cosh(x)^2 - b^2 \cosh(x)) \sinh(x) / (\cosh(x)^{12} + 12 \cosh(x) \sinh(x)^{11} + \sinh(x)^{12} + 6(11 \cosh(x)^2 - 1) \sinh(x)^{10} - 6 \cosh(x)^{10} + 20(11 \cosh(x)^3 - 3 \cosh(x)) \sinh(x)^9 + 15(33 \cosh(x)^4 - 18 \cosh(x)^2 + 1) \sinh(x)^8 + 15 \cosh(x)^8 + 24(33 \cosh(x)^5 - 30 \cosh(x)^3 + 5 \cosh(x)) \sinh(x)^7 + 4(231 \cosh(x)^6 - 315 \cosh(x)^4 + 105 \cosh(x)^2 - 5) \sinh(x)^6 - 20 \cosh(x)^6 + 24(33 \cosh(x)^7 - 63 \cosh(x)^5 + 35 \cosh(x)^3 - 5 \cosh(x)) \sinh(x)^5 + 15(33 \cosh(x)^8 - 84$$

```

*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20
*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*si
nh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 1
5*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 1
0*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(
(b*sinh(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(cosh(x) + sinh(x)), 1/3*(sqrt(-a)*(cosh(x) + sinh(x))
*arctan(8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*sqrt((b*sinh
(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
sinh(x)^2)))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 - 3*b*cos
h(x)^4 + 3*(5*b*cosh(x)^2 - b)*sinh(x)^4 + 16*a*cosh(x)^3 + 4*(5*b*cosh(x)^3
- 3*b*cosh(x) + 4*a)*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 - 6*b*cos
h(x)^2 + 16*a*cosh(x) + b)*sinh(x)^2 + 6*(b*cosh(x)^5 - 2*b*cosh(x)^3 + 8*a
*cosh(x)^2 + b*cosh(x))*sinh(x) - b)) + sqrt((b*sinh(x)^3 + 3*(b*cosh(x)^2
- b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)
+ sinh(x))]

```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^3(x)} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sinh(x)**3)**(1/2),x)

[Out] Integral(sqrt(a + b*sinh(x)**3)*coth(x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(x)^3 + a} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)

$$3.524 \quad \int \frac{\coth(x)}{\sqrt{a+b \sinh^n(x)}} dx$$

Optimal. Leaf size=29

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{an}}$$

[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)

Rubi [A] time = 0.0865435, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3230, 266, 63, 208}

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b \sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{an}}$$

Antiderivative was successfully verified.

[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]^n], x]

[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)^(m_.), x_Symbol] := With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{\sqrt{a + b \sinh^n(x)}} dx &= \text{Subst} \left(\int \frac{1}{x \sqrt{a + bx^n}} dx, x, \sinh(x) \right) \\
&= \frac{\text{Subst} \left(\int \frac{1}{x \sqrt{a+bx}} dx, x, \sinh^n(x) \right)}{n} \\
&= \frac{2 \text{Subst} \left(\int \frac{1}{\frac{a}{-b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^n(x)} \right)}{bn} \\
&= -\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b \sinh^n(x)}}{\sqrt{a}} \right)}{\sqrt{an}}
\end{aligned}$$

Mathematica [A] time = 0.0237434, size = 29, normalized size = 1.

$$-\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b \sinh^n(x)}}{\sqrt{a}} \right)}{\sqrt{an}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^n], x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A] time = 0.033, size = 24, normalized size = 0.8

$$-2 \frac{1}{n\sqrt{a}} \operatorname{Arctanh} \left(\frac{\sqrt{a + b (\sinh(x))^n}}{\sqrt{a}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(coth(x)/(a+b*sinh(x)^n)^(1/2),x)`

[Out] `-2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b \sinh(x)^n + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")`

[Out] `integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)`

Fricas [A] time = 1.83506, size = 390, normalized size = 13.45

$$\left[\frac{\log \left(\frac{b \cosh(n \log(\sinh(x))) + b \sinh(n \log(\sinh(x))) - 2\sqrt{b \cosh(n \log(\sinh(x))) + b \sinh(n \log(\sinh(x))) + a} \sqrt{a+2a}}{\cosh(n \log(\sinh(x))) + \sinh(n \log(\sinh(x)))} \right)}{\sqrt{an}}, 2\sqrt{-a} \arctan \left(\frac{\sqrt{b \cosh(n \log(\sinh(x))) + b \sinh(n \log(\sinh(x))) + a}}{\sqrt{a}} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")`

[Out] `[log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x)))) - 2*sqrt(b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*log(sinh(x))) + sinh(n*log(sinh(x)))))/(sqrt(a)*n), 2*sqrt(-a)*arctan(sqrt(b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(-a)/a)/(a*n)]`

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a + b \sinh^n(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sinh(x)**n)**(1/2), x)

[Out] Integral(coth(x)/sqrt(a + b*sinh(x)**n), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b \sinh^n(x) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2), x, algorithm="giac")

[Out] integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)

3.525 $\int \coth(x) \sqrt{a + b \sinh^n(x)} dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{a + b \sinh^n(x)}}{n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

[Out] $(-2*\text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[x]^n]/\text{Sqrt}[a]])/n + (2*\text{Sqrt}[a + b*\text{Sinh}[x]^n])/n$

Rubi [A] time = 0.0919908, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2\sqrt{a + b \sinh^n(x)}}{n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Coth}[x]*\text{Sqrt}[a + b*\text{Sinh}[x]^n], x]$

[Out] $(-2*\text{Sqrt}[a]*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sinh}[x]^n]/\text{Sqrt}[a]])/n + (2*\text{Sqrt}[a + b*\text{Sinh}[x]^n])/n$

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p]/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff, x] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50


```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \coth(x) \sqrt{a + b \sinh^n(x)} dx &= \text{Subst} \left(\int \frac{\sqrt{a + bx^n}}{x} dx, x, \sinh(x) \right) \\
&= \frac{\text{Subst} \left(\int \frac{\sqrt{a+bx}}{x} dx, x, \sinh^n(x) \right)}{n} \\
&= \frac{2\sqrt{a + b \sinh^n(x)}}{n} + \frac{a \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \sinh^n(x) \right)}{n} \\
&= \frac{2\sqrt{a + b \sinh^n(x)}}{n} + \frac{(2a) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b \sinh^n(x)} \right)}{bn} \\
&= -\frac{2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+b \sinh^n(x)}}{\sqrt{a}} \right)}{n} + \frac{2\sqrt{a + b \sinh^n(x)}}{n}
\end{aligned}$$

Mathematica [A] time = 0.0208877, size = 45, normalized size = 0.96

$$\frac{2\sqrt{a + b \sinh^n(x)} - 2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+b \sinh^n(x)}}{\sqrt{a}} \right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^n],x]

[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]^n])/n

Maple [A] time = 0.016, size = 38, normalized size = 0.8

$$\frac{1}{n} \left(2 \sqrt{a + b (\sinh(x))^n} - 2 \sqrt{a} \operatorname{Arctanh} \left(\frac{\sqrt{a + b (\sinh(x))^n}}{\sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(coth(x)*(a+b*sinh(x)^n)^(1/2),x)

[Out] 1/n*(2*(a+b*sinh(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2)))

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(x)^n + a} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)

Fricas [A] time = 1.81846, size = 552, normalized size = 11.74

$$\left[\frac{\sqrt{a} \log \left(\frac{b \cosh(n \log(\sinh(x))) + b \sinh(n \log(\sinh(x))) - 2 \sqrt{b \cosh(n \log(\sinh(x))) + b \sinh(n \log(\sinh(x))) + a \sqrt{a + 2a}}}{\cosh(n \log(\sinh(x))) + \sinh(n \log(\sinh(x)))} \right) + 2 \sqrt{b \cosh(n \log(\sinh(x)))} \right]}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x)))) - 2*sqrt(b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*log(sinh(x))) + sinh(n*log(sinh(x)))))) + 2*sqrt(b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(-a)/a) + sqrt(b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a))/n]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh(x)^n + a} \coth(x) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)
```


Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*      is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*      antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```

22     If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25     "C"],
26 If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46     If[AtomQ[expn],
47         1,
48     If[ListQ[expn],
49         Max[Map[ExpnType,expn]],
50     If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52             ExpnType[expn[[1]]],
53         If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55                 1,
56                 Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58     If[Head[expn]===Plus || Head[expn]===Times,
59         Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60     If[ElementaryFunctionQ[Head[expn]],
61         Max[3,ExpnType[expn[[1]]],
62     If[SpecialFunctionQ[Head[expn]],
63         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64     If[HypergeometricFunctionQ[Head[expn]],
65         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66     If[AppellFunctionQ[Head[expn]],
67         Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68     If[Head[expn]===RootSum,

```

```

69   Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
70   If[Head[expn]===Integrate || Head[expn]===Int,
71     Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
72   9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp, Log,
78     Sin, Cos, Tan, Cot, Sec, Csc,
79     ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
80     Sinh, Cosh, Tanh, Coth, Sech, Csch,
81     ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
82   }, func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   }, func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]
99
100
101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1}, func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 #
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 #
11 see problem 156, file Apostol_Problems

```

```

11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;
14
15     leaf_count_result:=leafcount(result);
16     #do NOT call ExpnType() if leaf size is too large. Recursion problem
17     if leaf_count_result > 500000 then
18         return "B";
19     fi;
20
21     leaf_count_optimal:=leafcount(optimal);
22
23     ExpnType_result:=ExpnType(result);
24     ExpnType_optimal:=ExpnType(optimal);
25
26     if debug then
27         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
28     fi;
29
30 # If result and optimal are mathematical expressions,
31 # GradeAntiderivative[result,optimal] returns
32 #   "F" if the result fails to integrate an expression that
33 #     is integrable
34 #   "C" if result involves higher level functions than necessary
35 #   "B" if result is more than twice the size of the optimal
36 #     antiderivative
37 #   "A" if result can be considered optimal
38
39 #This check below actually is not needed, since I only
40 #call this grading only for passed integrals. i.e. I check
41 #for "F" before calling this. But no harm of keeping it here.
42 #just in case.
43
44
45 if not type(result,freeof('int')) then
46     return "F";
47 end if;
48
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then

```



```

56     if debug then
57         print("both result and optimal complex");
58     fi;
59     #both result and optimal complex
60     if leaf_count_result<=2*leaf_count_optimal then
61         return "A";
62     else
63         return "B";
64     end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71     else # result do not contain complex
72         # this assumes optimal do not as well
73         if debug then
74             print("result do not contain complex, this assumes optimal do not
as well");
75         fi;
76         if leaf_count_result<=2*leaf_count_optimal then
77             if debug then
78                 print("leaf_count_result<=2*leaf_count_optimal");
79             fi;
80             return "A";
81         else
82             if debug then
83                 print("leaf_count_result>2*leaf_count_optimal");
84             fi;
85             return "B";
86         end if
87     end if
88     else #ExpnType(result) > ExpnType(optimal)
89         if debug then
90             print("ExpnType(result) > ExpnType(optimal)");
91         fi;
92         return "C";
93     end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417

```

```

102 is_contains_complex:= proc(expression)
103   return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)
119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'^^') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'+'') or type(expn,'*') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))

```

```

149   elif AppellFunctionQ(op(0,expn)) then
150       max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152       max(8,apply(max,map(ExpnType,[op(expn)]))) else
153       9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159     member(func,[
160         exp,log,ln,
161         sin,cos,tan,cot,sec,csc,
162         arcsin,arccos,arctan,arccot,arcsec,arccsc,
163         sinh,cosh,tanh,coth,sech,csch,
164         arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168     member(func,[
169         erf,erfc,erfi,
170         FresnelS,FresnelC,
171         Ei,Ei,Li,Si,Ci,Shi,Chi,
172         GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173         EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177     member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181     member(func,[AppellF1])
182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple

```

```

196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #           Port of original Maple grading function by
3 #           Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #           added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:

```

```

42     if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43         return True
44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,'^^')
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
72 ))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+' or type
77 (expn,'*')
78         m1 = expnType(expn.args[0])
79         m2 = expnType(list(expn.args[1:]))
80         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82         return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84         m1 = max(map(expnType, list(expn.args)))
85         return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88         m1 = max(map(expnType, list(expn.args)))

```

```

85     return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
86 elif is_appell_function(expn.func):
87     m1 = max(map(expnType, list(expn.args)))
88     return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
89 elif isinstance(expn,RootSum):
90     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
91     return max(7,m1)
92 elif str(expn).find("Integral") != -1:
93     m1 = max(map(expnType, list(expn.args)))
94     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
95 else:
96     return 9
97
98 #main function
99 def grade_antiderivative(result,optimal):
100
101     leaf_count_result  = leaf_count(result)
102     leaf_count_optimal = leaf_count(optimal)
103
104     expnType_result  = expnType(result)
105     expnType_optimal = expnType(optimal)
106
107     if str(result).find("Integral") != -1:
108         return "F"
109
110     if expnType_result <= expnType_optimal:
111         if result.has(I):
112             if optimal.has(I): #both result and optimal complex
113                 if leaf_count_result <= 2*leaf_count_optimal:
114                     return "A"
115                 else:
116                     return "B"
117             else: #result contains complex but optimal is not
118                 return "C"
119         else: # result do not contain complex, this assumes optimal do not as
well
120             if leaf_count_result <= 2*leaf_count_optimal:
121                 return "A"
122             else:
123                 return "B"
124     else:
125         return "C"

```

4.0.4 SageMath grading function

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by

```

2 #           Albert Rich to use with Sagemath. This is used to
3 #           grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #           'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
33         flatten(tree(anti))))
34         return round(1.35*len(flatten(tree(anti)))) #fudge factor
35         #since this estimate of leaf count is bit lower than
36         #what it should be compared to Mathematica's
37
38 def is_sqrt(expr):
39     debug=False;
40     if expr.operator() == operator.pow: #isinstance(expr,Pow):
41         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
42             if debug: print ("expr is sqrt")
43             return True
44         else:
45             return False
46     else:
47         return False

```

```

48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func ," is special_function")
83         else:
84             print ("func ", func ," is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
91     ']
92
93 def is_appell_function(func):

```



```

93     return func.name() in ['hypergeometric']    #[appellf1] can't find this in
          sagemath
94
95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
104             return expn in expn.parent().base_ring() or expn in expn.parent().
          gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list:    #isinstance(expn,list):
121         return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
          Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0]))    #max(2,expnType(expn.
          args[0]))
127     elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer:    #isinstance(expn.args[1],Integer)
129             return expnType(expn.operands()[0])    #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational:    #isinstance(expn.args[1],
          Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
          Rational)
132                 return 1

```

```

133         else:
134             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137         elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138             m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139             m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140             return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
141         elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
142             return max(3,expnType(expn.operands()[0]))
143         elif is_special_function(expn.operator()): #is_special_function(expn.func)
144             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
145             return max(4,m1) #max(4,m1)
146         elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
148             return max(5,m1) #max(5,m1)
149         elif is_appell_function(expn.operator()):
150             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
151             return max(6,m1) #max(6,m1)
152         elif str(expn).find("Integral") != -1: #this will never happen, since it
153             #is checked before calling the grading function that is passed.
154             #but kept it here.
155             m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
156             return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
157         else:
158             return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
176                     expnType_optimal)
177
178     if expnType_result <= expnType_optimal:
179         if result.has(I):
180             if optimal.has(I): #both result and optimal complex
181                 if leaf_count_result <= 2*leaf_count_optimal:
182                     return "A"
183             else:
184                 return "B"
185         else: #result contains complex but optimal is not
186             return "C"
187     else: # result do not contain complex, this assumes optimal do not as
188         well
189         if leaf_count_result <= 2*leaf_count_optimal:
190             return "A"
191         else:
192             return "B"
193     else:
194         return "C"
```